MODELING AND OPTIMIZATION OF THE HYDRAULIC REGIME OF ACTIVATED SLUDGE SYSTEMS

by

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B.S., National Taiwan University, China 1965

A MASTER'S THESIS

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Chemical Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1968

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CHAPTER I

INTRODUCTION

There has been much interest recently in the hydraulic regime of biological waste treatment processes (1, 2, 3, 4, 5)*. The pattern of flow into the system, the recycle flow, and the mixing and distribution of liquid and of the materials dissolved or suspended in the liquid within the compartments of the process are all important parts of the hydraulic regime. Since the hydraulic regime may greatly influence the rate of biological growth, it is a fundamentally important consideration in the improvement of a biological waste treatment process.

The step aeration waste treatment process shown in Figure 1, in which the influent is introduced at several locations, was first described by Gould (6) in 1942. Although this process is fairly old and widely used, little work has been done to develop an optimal step aeration design. The paper by Polonesik, Grieves, and Milbury (3) is probably the first reported effort to optimize the design of a step aeration activated sludge process. These investigators examined the behavior of three completely mixed tanks connected in series using two different models to describe the growth kinetics. In a later investigation (2, 5), a discrete version of the maximum principle was used to optimize several different step aeration systems. These investigators considered systems represented by several completely mixed tanks connected

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* Numbers in parentheses refer to references given on page 67

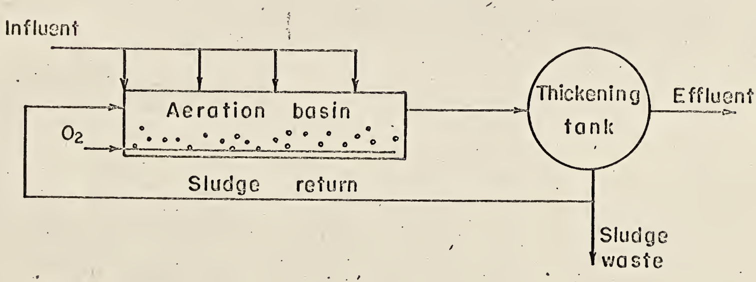


Fig. 1. Flow diagram of step aeration process.

in series, several tanks with plug flow connected in series, a composite system composed of a tank with complete mixing followed by a tank with plug flow, and a plug flow system with continuous allocation of influent along the length of the system.

The present study is a continuation of the work reported previously (1, 2, 5). In this study the step aeration process and the conventional process shown in Figure 2, in which all of the influent is fed to the first tank, are compared under optimal conditions. The effects of recycle of organisms and endogenous respiration which were not considered in the previous study of the step aeration process (2, 5) are included in this investigation. In the present study only systems composed of tanks with complete mixing are considered. The analysis is limited to the secondary portion of the waste treatment system where aerobic biological oxidation is taking place and it is primarily concerned with the optimization of the hydraulic regime of this portion of the system. The difference between the oxygen demand pattern of the step aeration and the conventional process is illustrated in Figure 3.

In this work, the process is analyzed by employing mathematical modeling and optimization procedures to determine the optimum values of several of the design variables. In employing this approach, it is necessary to have a mathematical model which describes the growth kinetics of the biological waste treatment process and a mathematical model which represents the hydrodynamic behavior of the flow system. An economic model which relates the design variables to the various treatment costs, such as capital

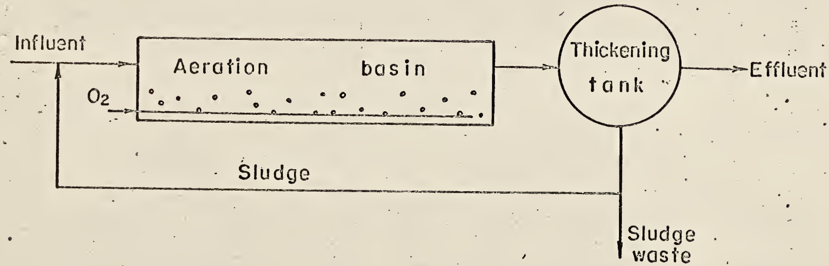


Fig. 2. Flow diagram of conventional activated sludge process.

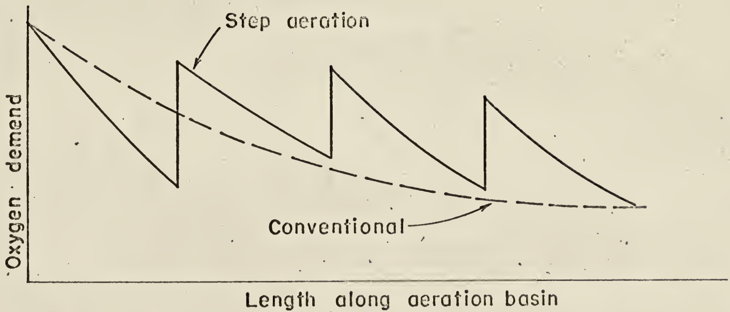


Fig. 3. Comparison of oxygen demands exerted in the conventional and step aeration processes.

and operating costs, is also often used in this type of investigation. In addition to the kinetic, hydrodynamic, and economic models, one must have an objective function to be optimized, that is, one must have the objective of the design stated in mathematical terms. The process and economic models and the objective function together provide a mathematical statement of the problem.

When simulation and optimization procedures are employed to solve mathematically stated optimum design problems, engineering judgement and experience must be used in evaluating the results. Since the process and economic models and the objective function are only approximations, the optimum of the mathematical problem will probably deviate from the true optimum. Inspite of this, studies of this type can be useful in obtaining a better understanding of the biological waste treatment process and in predicting the effect of specific parameters and variables on the performance of the system.

CHAPTER II

MATHEMATICAL ANALYSIS OF THE PROCESS

1. PRINCIPLE OF AERATION AND KINETIC MODEL

(a) Principle of Biological Oxidation (7)

Biological oxidation is simply a conversion process therein dissolved organic compounds are converted into bacterial cells, which can then be removed from the waste water. The generalized reaction for the removal of soluble organics may be considered as follows:

$$\text{Organics} + O_2 + \text{Nutrients} \xrightarrow{\text{Organisms}} CO_2 + H_2O + \text{Organisms.}$$

(b) Growth Pattern (8)

The curve in Figure 4 illustrates the classic growth pattern exhibited by microorganisms in a batch culture. Examination of the curve reveals that growth passes through three different phases. Initially, all nutrients are present in excess of the requirements of the microorganism, and growth is unrestricted. During this period, called the constant growth phase, the concentration of microorganisms increases at an exponential rate. At some concentration, one of the nutrients becomes growth limiting and the culture proceeds into the declining growth phase. In response to the increasing competition of the microorganisms for the remaining limiting nutrient, the rate of growth decreases until growth finally halts. The remaining portion of the curve represents the decrease of the microorganism resulting from autooxidation which

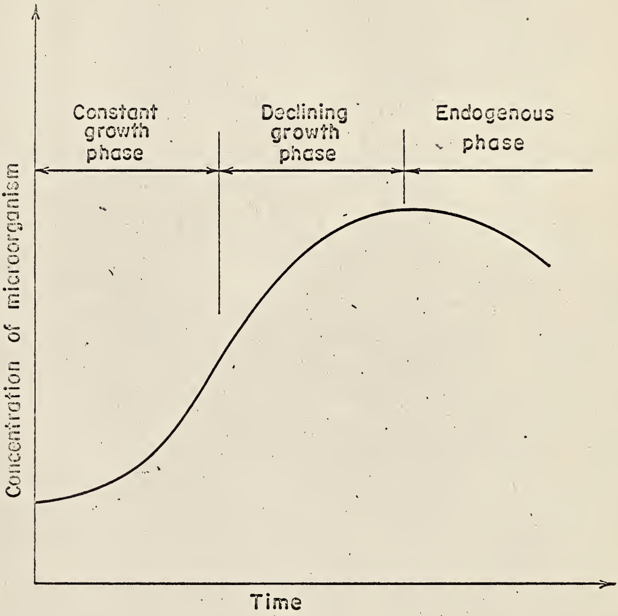


Fig. 4. Classic growth pattern.

occurs after the depletion of the available organics. This is often called the endogeneous respiration phase of activated sludge.

(c) Kinetic Model

The Michaelis-Menten form of rate equation is often used to describe the growth kinetics of biological systems. It is known that this is a gross over-simplification of the very complex phenomena that are occurring and that Tsuchiya, Fredrickson, and Aris (9) have recently presented the results of their initial attempt for a more complete treatment. Since much more work will be necessary before such a treatment will be usable for engineering design purposes, the Michaelis-Menten form of rate equation will be used in this study. Although it is widely recognized that there are a number of different types of micro-organisms present in biological waste treatment systems and that different types of micro-organisms predominate under different conditions (10), no attempt will be made to include the effects of interactions between different types of micro-organisms in the mathematical model of the growth process.

The growth of activated sludge micro-organisms will be expressed in terms of a single growth rate equation which is at all times a function of the concentrations of organic nutrients and active sludge organisms. If oxygen and other trace nutrients are available in sufficient quantities, the kinetic model for micro-organism growth is assumed to be

$$\frac{dx_2}{dt} = \frac{k\ x_1\ x_2}{K + x_1} - k_D\ x_2 \tag{1}$$

where

$\frac{dx_2}{dt}$ = growth rate, mg/liter hr,

x_1 = concentration of organic nutrients, mg/liter,

x_2 = concentration of active micro-organisms, mg/liter,

k = maximum specific growth rate when the organic concentration is not limiting the rate of growth, hr^{-1},

K = the concentration of organics at which the specific growth rate observed is one half the maximum value,

k_D = specific endogenous microbial attrition rate, hr^{-1}.

When growth occurs according to equation (1), the organic nutrients are being consumed at a rate

$$-\frac{dx_1}{dt} = \frac{k\ x_1\ x_2}{Y(K + x_1)} \tag{2}$$

where

$-\frac{dx_1}{dt}$ = rate with which organic nutrients are consumed, mg/liter hr,

Y = nutrient conversion yield factor.

Simplified forms of equations (1) and (2) result if K is much larger than x_1 or if K is much smaller than x_1. When K >> x_1, equation (1) reduces to

$$\frac{dx_2}{dt} = \frac{k\ x_1\ x_2}{K} - k_D\ x_2 \tag{3}$$

while when K << x_1, equation (1) reduces to

$$\frac{dx_2}{dt} = kx_2 - k_D\ x_2$$
$$= (k - k_D)x_2 \tag{4}$$

Similar simplifications can be written for equation (2).

2. THE MATHEMATICAL REPRESENTATION OF THE PROCESS

(a) Flow Models

Since the conventional actived sludge process can be considered as a special case of the step aeration process in which all the feed are allocated to the first tank, only the step aeration process is described here.

In step aeration, the influent is fed to the system at several different locations. At each location the influent is mixed with the fluid at that location. Material balance equations can be used to describe the resulting concentrations if complete mixing of the influent and the fluid is assumed to occur at each location. In this investigation we shall assume that the influent at each location is instantaneously mixed with the fluid at that location. Mathematically, we shall assume that this mixing of influent takes place at an individual mixing point of negligible volume and that a completely mixed stream leaves each mixing point and passes to an aeration or reaction tank where growth occurs. In this way, the material balance equations for the aeration tanks will reflect only the effect of the growth process. This mathematical modeling approach is illustrated in Figure 5 where each circle and each box denotes a set of material balance equations. Complete mixing will always be assumed for both the mixing points and the aeration tanks where growth is assumed to occur.

The secondary portion of the biological waste treatment system is assumed to be composed of a sequence of N completely mixed tanks connected in series followed by a secondary clarifier. Each square box in Figure 5 corresponds to a completely mixed aeration tank. The circle preceeding each square box corresponds to the mixing point where untreated influent can be added to the fluid flowing from one tank to the next. The final circle corresponds to the secondary clarifier where the sludge micro-organisms are allowed to settle. A portion of the sludge from the bottom of the clarifier is removed and sent to the sludge disposal system and the remainder is recycled.

(b) Simplifying Assumptions (5)

The following assumptions and simplifications are made in specifying the process and developing the mathematical representation for the process.

(1) The system is isothermal (and is under the steady state condition).

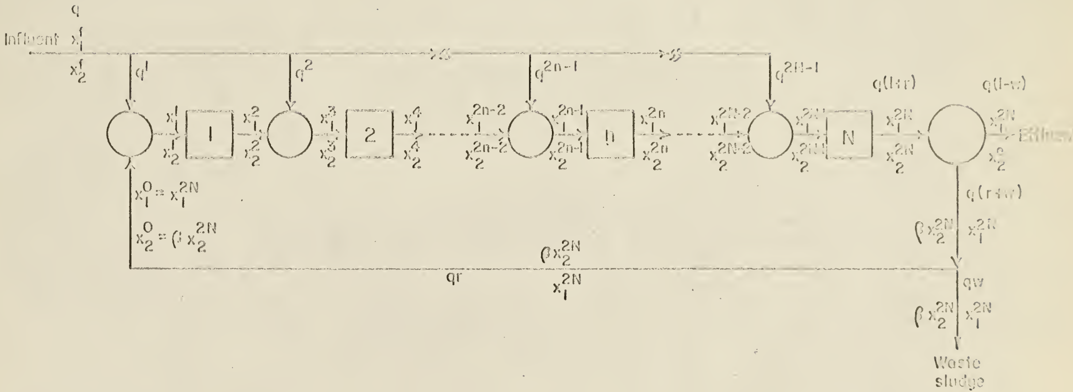


Fig. 5. Schematic diagram of the step aeration biological waste treatment system.

(2) Physical properties such as density, diffusivity, and viscosity are constant.

(3) Y is dependent only upon the property of the waste itself, and independent of the age of the organisms and the effect of other physical conditions such as the concentrations of organics and organisms.

(4) Oeganics and organisms are distinctly separate entities in solution.

(5) Endogeneous respiration does not influence the ststem performance.

(6) The sludge and waste streams are completely mixed at each point where the waste is introduced.

(7) Sufficient oxygen is supplied for the oxidation.

(8) The fluid is a continuum and there is no segregation.

Some of these assumptions depart from reality. They are justified on the grounds that they simplify the relationship of the process without appreciably changing its basic characteristics.

(c) The Mathematical Representation

In Figure 5, q is the volumetric flow rate of the feed or influent to the overall system, qr is the recycle flow rate, qw is the volume flow rate to the sludge digester, x_1^{2n-1} is the concentration of organic nutrients in the stream entering the nth aeration tank from the mixing point preceding this tank while x_1^{2n} is the organic nutrient concentration of the stream leaving this aeration tank, x_2^{2n-1} and x_2^{2n} are the concentrations of active organisms in the entering and exit streams of the nth aeration tank respectively, and x_3^{2n-1} and x_3^{2n} are the volumetric flow rates of the entering and exit streams of the nth aeration tank, respectively. In Figure 5, q^{2n-1} is the volumetric flow rate of the raw waste introduced to the mixing point that precedes the nth aeration tank and V^{2n} denotes the volume of the nth aeration tank.

(1) Analysis of a mixing point.

In order to establish a mathematical model for each mixing point, we consider the mixing point preceding the nth aeration tank as shown in Fig. 6.a. The essential governing equations are written based on the assumed complete mixing flow model using the notation established above. The organic nutrient balance around this mixing point gives

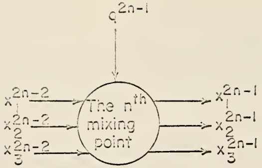
$$q^{2n-1}\, x_1^f + x_3^{2n-2}\, x_1^{2n-2} = x_3^{2n-1}\, x_1^{2n-1} \tag{5}$$

where x_1^f is the concentration of organic nutrients in the untreated waste fed to the system. A balance of the active sludge organisms around this mixing point gives

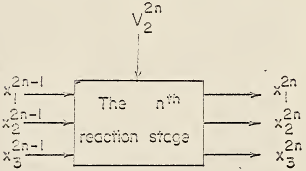
$$q^{2n-1}\, x_2^f + x_3^{2n-2}\, x_2^{2n-2} = x_3^{2n-1}\, x_2^{2n-1} \tag{6}$$

where x_2^f is the organism concentration in the raw waste. A balance on the volumetric flow, x_3, gives

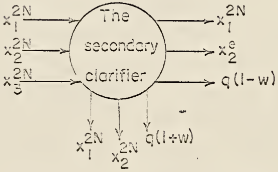
$$q^{2n-1} + x_3^{2n-2} = x_3^{2n-1} \tag{7}$$



(a). Mixing point.



(b). Reaction stage.



(c). The secondary clarifier.

Fig. 6. A schematic representation of a mixing point, a reaction stage and the secondary clarifier.

(2) Analysis of an aeration tank.

An organic nutrient balance around the nth aeration tank as shown in Fig. 6b gives

$$x_3^{2n-1}\, x_1^{2n-1} - x_3^{2n}\, x_1^{2n} - \frac{k\, x_1^{2n}\, x_2^{2n}}{Y(K + x_1^{2n})}\, V^{2n} = 0 \tag{8}$$

when the growth kinetics are described by equations (1) and (2). Similarly, an organism balance around the nth aeration tank gives

$$x_3^{2n-1}\, x_2^{2n-1} - x_3^{2n}\, x_2^{2n} + \left(\frac{k\, x_1^{2n}\, x_2^{2n}}{K + x_1^{2n}} - k_D\, x_2^{2n}\right)V^{2n} = 0 \tag{9}$$

Since the volumetric flow rate does not change at the aeration tank, we have

$$x_3^{2n-1} = x_3^{2n} \tag{10}$$

for the nth aeration tank.

(3) Analysis of the secondary clarifier.

For a system with N aeration tanks, the flow entering the secondary clarifier is denoted by x_3^{2N} which is equal to $q(1+r)$. The effluent flow from the clarifier is $q(1-w)$ and the bottoms flow $q(r+w)$. This is schematically represented in Fig. 6c. Since the organic waste is assumed to pass through the clarifier unchanged, the concentration in the effluent and bottoms is given by x_1^{2N}. If the sludge is concentrated in the clarifier to a bottoms concentration of βx_2^{2N}, the organism balance around the clarifier is

$$q(1+r)x_2^{2N} = q(1-w)x_2^e + q(r+w)\beta x_2^{2N} \qquad (11)$$

where x_2^e is the organism concentration in the effluent stream and β is the separator concentration efficiency. For any given set of influent flow rate and concentration, recycle flow rate, and waste sludge flow rate, the value of β is assumed constant in this investigation.

Since the recycle stream is connected to the first mixing point, the organic and organism concentrations and the flow of this stream must satisfy the relations

$$x_1^0 = x_1^{2N} \qquad (12)$$

$$x_2^0 = \beta x_2^{2N} \qquad (13)$$

$$x_3^0 = qr \qquad (14)$$

Equations (5) through (14) provide a mathematical model of the step aeration waste treatment system composed of N completely mixed aeration tanks connected in series. Since equations (5) through (10) can be written for each stage consisting of a mixing point and an aeration tank, there are $6N + 4$ equations in the model. The model for the conventional process in which all of the influent is fed to the first mixing point can be obtained by letting

$$q^{1} = q \tag{15}$$

and

$$q^{2n-1} = 0 \tag{16}$$

for $n = 2, 3, \ldots, N$.

(d) Economic Model

There are a number of costs associated with the biological waste treatment process. Both capital and operating costs must be considered in developing a complete economic model. Some of the capital costs which should be considered include the costs due to the biological chamber, the secondary clarifier, the sludge digester, and the operations and maintenance building. Some of the operating costs which should be considered include plant maintenance costs, power costs, the cost of nutrients and chemical additives, operating labor costs, and administrative costs. McBeath and Eliassen (11) have presented an economic model in their sensitivity analysis of activated sludge economics. Their model, or models similar to theirs, can be used in design optimization studies.

In the simple economic model which will be used here, the volume of the biological chamber will be minimized for a fixed effluent quality. In McBeath and Eliassen's economic model, the capital cost of the biological chamber is a function of the total volume of the aeration tank (11). Thus, minimizing the total volume of the biological chamber minimizes the capital cost of the biological unit according to their cost equation.

The mathematical objective function for this problem is thus to minimize the total volume of the biological growth chamber

$$V_T = \sum_{n=1}^{N} V^{2n} \tag{17}$$

where V^{2n} is the volume of the nth aeration tank. Although this mathematical objective function is rather simple, it can be used to obtain useful information about the effect of the hydraulic flow pattern on the performance of the system. Information about the optimal allocation of influent to the system and the optimal distribution of volume among the aeration tanks can be gained using this objective function.

CHAPTER III

OPTIMIZATION STUDY OF THE PROCESS

1. OPTIMIZATION STUDIES OF STEP AERATION AND CONVENTIONAL SYSTEMS

As described in the previous chapter, equations (5) through (14) provide a mathematical model of the step aeration activated sludge system shown in Figure 5. If equations (15) and (16) are substituted into these equations, a mathematical model of the conventional activated sludge system can be obtained. In the optimization analysis, the recycle ratio r, the separator concentration efficiency β, and the concentration of the organic waste in the effluent from the secondary clarifier will be treated as fixed parameters for each optimization calculation. The minimization of equation (17) subject to the equality constraints presented in equations (5) through (14) can be readily accomplished when the values of q, x_1^f, x_2^f, x_1^{2N}, r, and β are fixed and N is fairly small.

An analysis of the degrees of freedom of these optimization problems reveals that there are N-1 degrees of freedom for the conventional system in which all of the influent is fed to the first tank while there are 2N-2 degrees of freedom for the step aeration system. The details of the system analysis are listed in Appendix I. Thus, in searching for the optimal design of the conventional system, the volumes of all except one of the tanks may be treated as independent variables. For the step aeration system, one may assume that the 2N-2 independent design variables consist of the allocation of feed and volume to all except one of the N tanks.

In order to make the results as general as possible, the concentrations and flow rates are put into a dimensionless form. The organic waste concentrations are made dimensionless by dividing them by the concentration of the organic waste in the influent, x_1^f, the organism concentrations by the product, Yx_1^f, and the flow rates to individual mixing points by the flow rate of the influent to the system, q. This gives rise to the following dimensionless variables.

$$y_1^{2n} = \frac{x_1^{2n}}{x_1^f}, \quad y_2^{2n} = \frac{x_2^{2n}}{Yx_1^f}, \quad y_3^{2n} = \frac{x_3^{2n}}{q}, \quad \theta_1^{2n-1} = \frac{q^{2n-1}}{q},$$

where y_1^{2n} and y_2^{2n} are the dimensionless concentrations of the organic waste and organisms respectively in the nth tank, y_3^{2n} the dimensionless flow from the nth aeration tank, and θ_1^{2n-1} the dimensionless flow of influent to the nth mixing point. θ_2^{2n} defined below is the holding time in hours for the nth aeration tank.

$$\theta_2^{2n} = \frac{V^{2n}}{x_3^{2n}}$$

The constant K may also be made dimensionless by dividing it by x_1^f; that is

$$K_1 = \frac{K}{x_1^f}$$

Thus, K_1 is the dimensionless organic concentration at which the specific growth rate observed is one half the maximum value.

In addition to this, the objective function, equation(17), may be divided by q(1+r) to give

$$S = \frac{V_T}{q(1+r)} = \sum_{n=1}^{N} \frac{V^{2n}}{q(1+r)} \tag{18}$$

S is indeed the total mean holding time for the conventional system, but it may not be the total mean holding time for the step aeration system, since the flow rate through some of the tanks may be less than q(1+r). Nevertheless, S is the total required volume per unit of the total flow rate to the aecondary clarifier, which is identical for both conventional and step aeration processes under the equivalent operating conditions. Thus the use of S still enables one to compare both processes on a consistent basis. In the presentation and discussion of the results, equation (18) will be referred to as the total holding time, even though this may not be the ture mean holding time for the step aeration system.

In this investigation a modified direct pattern search technique originally developed and the simplex method were used to obtain the optimal results (12). The details of the mathematical optimization procedure are contained in Appendix II and the modified search technique is described in Appendix III.

2. RESULTS AND DISCUSSION

The minimization of equation (18) for the model shown in Figure 5, which is described mathematically by equations (5) through (14), was carried out for conventional and step aeration activated sludge systems with one, two, and three aeration tanks. Optimal results were obtained for 90, 95, 98, and 99% treatment for several values of the parameters β and K_1. The following values for the constants and parameters were used in this investigation.

$y_2^f = 0$, dimensionless

$k = 0.1\ hr^{-1}$

$k_D = 0.002\ hr^{-1}$

$r = 0.25$, dimensionless

$\beta = 4.0$, dimensionless

$K_1 = 0.01, 0.02, 0.05, 0.1, 0.2, 0.5$, dimensionless

$y_1^{2N} = 0.01, 0.02, 0.05, 0.1$, dimensionless.

The results of this investigation are presented in Figures 7 through 25. Tabular values of the results are presented in Tables 1 through 5. Figures 7, 8, and 9 show the variation in the objective function (total holding time) with per cent of treatment for $x_1 = 0.01, 0.05$, and 0.5, respectively. In these Figures, the solid lines give the optimal results for the conventional system in which all of the influent is fed to the first tank while the dashed lines present the optimal results for the step aeration system. For all systems considered, the minimum required holding time increases as the per cent of treatment increases; however, this increase is much more rapid for the one tank system. As the number of aeration tanks in the conventional activated sludge system increases, the total volume required for treatment de-

Table 1. Optimal results for β = 4.0 for systems of the conventional process with 90 and 95% treatment.

	Tank One			Tank Two				
K_1	θ^1	y_1^1	y_2^1	θ^2	y_1^2	y_2^2	θ_T	V_R
[illegible]	2.250	0.10	3.521				2.250	
[illegible]	2.459	0.05	3.709				2.459	
[illegible]	1.975	0.176	3.450	0.236	0.10	3.524	2.210	0.982
[illegible]	1.940	0.152	3.619	3.276	0.05	3.718	2.268	0.922
[illegible]	2.459	0.10	3.514				2.459	
[illegible]	2.881	0.05	3.694				2.881	
[illegible]	1.930	0.218	3.405	0.402	0.10	3.520	2.333	0.949
0.01	1.980	0.173	3.593	0.463	0.05	3.712	2.442	0.848
[illegible]	3.093	0.10	3.492				3.093	
[illegible]	4.167	0.05	3.468				4.167	
[illegible]	2.010	0.256	3.358	0.665	0.10	3.509	2.675	0.865
0.05	2.208	0.189	3.561	0.751	0.05	3.694	2.958	0.710
0.10	4.167	0.10	3.456				4.167	
0.10	6.383	0.05	3.572				6.383	
0.10	2.263	0.271	3.327	0.976	0.10	3.491	3.240	0.778
0.10	2.638	0.195	3.527	1.187	0.05	3.664	3.825	0.599
[illegible]	6.383	0.10	3.384				6.383	
[illegible]	11.111	0.05	3.420				11.111	
[illegible]	2.834	0.278	3.286	1.549	0.10	3.453	4.382	0.687
0.20	3.548	0.198	3.470	2.052	0.05	3.603	5.599	0.504
[illegible]	13.636	0.10	3.168				13.636	
[illegible]	28.205	0.05	2.964				28.205	
[illegible]	4.665	0.283	3.180	3.292	0.10	3.342	7.957	0.584
[illegible]	6.455	0.200	3.304	4.829	0.05	3.422	11.283	0.400

Table 2. Optimal results for β = 4.0 for systems of the conventional process with 98 and 99% treatment.

	Tank One			Tank Two				
K	θ^1	y_1^1	y_2^1	θ^2	y_1^2	y_2^2	θ_T	V_R
0.01	3.093	0.02	3.802				3.093	
0.01	4.167	0.01	3.802				4.167	
0.01	2.021	0.110	3.743	0.354	0.02	3.831	2.375	0.768
0.01	2.121	0.083	3.794	0.380	0.01	3.865	2.501	0.600
0.02	4.167	0.02	3.763				4.167	
0.02	6.383	0.01	3.722				6.383	
0.02	2.152	0.118	3.726	0.514	0.02	3.820	2.666	0.640
0.02	2.333	0.086	3.777	0.595	0.01	3.849	2.929	0.459
0.05	7.527	0.02	3.646				7.527	
0.05	13.636	0.01	3.485				13.636	
0.05	2.594	0.123	3.692	0.952	0.02	3.788	3.546	0.471
0.05	3.000	0.088	3.733	1.231	0.01	3.801	4.231	0.310
0.10	13.636	0.02	3.450				13.636	
0.10	28.205	0.01	3.089				28.205	
0.10	3.357	0.125	3.642	1.686	0.02	3.735	5.042	0.370
0.10	4.139	0.089	3.660	0.333	0.01	3.722	6.472	0.229
0.20	28.205	0.02	3.058				28.205	
0.20	72.414	0.01	2.297				72.414	
0.20	4.946	0.126	3.545	3.216	0.05	3.628	8.162	0.289
0.20	6.587	0.089	3.517	4.653	0.01	3.563	11.240	0.155
0.50	108.330	0.02	1.882				108.330	
0.50	—	—	—				—	
0.50	10.311	0.127	3.257	8.364	0.05	3.308	18.676	0.172
0.50	15.160	0.090	3.087	13.185	0.01	3.085	28.345	0

Table 3. Optimal results for $\beta = 4.0$ for system of the conventional process with 90, 95, 98 and 99% treatment.

	Tank One			Tank Two			Tank Three				
K_1	θ^1	y_1^1	y_2^1	θ^2	y_1^2	y_2^2	θ^3	y_1^3	y_2^3	θ_T	V_R
0.01	1.960	0.180	3.445	0.163	0.127	3.497	0.085	0.10	3.524	2.208	0.981
0.01	1.864	0.176	3.596	0.259	0.090	3.679	0.130	0.05	3.719	2.252	0.916
0.01	1.862	0.156	3.701	0.312	0.056	3.799	0.140	0.02	3.834	2.314	0.748
0.01	1.914	0.134	3.750	0.327	0.036	3.846	0.133	0.01	3.870	2.374	0.570
0.02	1.845	0.244	3.381	0.297	0.153	3.470	0.179	0.10	3.521	2.321	0.944
0.02	1.814	0.220	3.549	0.385	0.102	3.664	0.196	0.05	3.714	2.395	0.831
0.02	1.861	0.188	3.664	0.452	0.060	3.789	0.207	0.02	3.827	2.520	0.575
0.02	1.947	0.159	3.717	0.469	0.040	3.833	0.230	0.01	3.861	2.646	0.415
0.05	1.742	0.323	3.296	0.557	0.174	3.441	0.316	0.10	3.512	2.615	0.845
0.05	1.833	0.270	3.487	0.613	0.115	3.638	0.350	0.05	3.701	2.797	0.671
0.05	2.025	0.211	3.622	0.688	0.065	3.763	0.412	0.02	3.805	3.125	0.415
0.05	2.214	0.171	3.679	0.755	0.041	3.803	0.492	0.01	3.831	3.460	0.254
0.10	1.812	0.360	3.246	0.780	0.186	3.414	0.494	0.10	3.497	3.086	0.741
0.10	2.000	0.295	3.443	0.886	0.121	3.612	0.576	0.05	3.678	3.462	0.542
0.10	2.350	0.223	3.579	1.046	0.067	3.728	0.746	0.02	3.769	4.143	0.304
0.10	2.667	0.180	3.628	1.230	0.042	3.756	0.942	0.01	3.781	4.840	0.172
0.20	2.076	0.384	3.196	1.133	0.195	3.378	0.821	0.10	3.467	4.030	0.631
0.20	2.438	0.309	3.391	1.349	0.124	3.565	1.022	0.05	3.632	4.809	0.433
0.20	3.081	0.228	3.511	1.739	0.067	3.659	1.411	0.02	3.696	6.232	0.221
0.20	3.668	0.183	3.539	2.164	0.043	3.663	1.871	0.01	3.682	7.703	0.106
0.50	3.071	0.398	3.104	2.122	0.199	3.288	1.760	0.10	3.375	6.954	0.510
0.50	3.943	0.314	3.266	2.742	0.125	3.436	2.364	0.05	3.494	9.048	0.321
0.50	5.434	0.232	3.318	3.944	0.068	3.455	3.615	0.02	3.478	12.993	0.120
0.50	7.013	0.184	3.279	5.263	0.043	3.385	4.965	0.01	3.384	17.240	0

Table 4. Optimal results for β = 4.0 for systems of step aeration with 90, 95, 98, and 99% treatment (2 tank reactors in series).

	Tank One						Tank Two							
K_1	θ_2^2	θ_1^1	y_1^1	y_1^2	y_2^1	y_2^2	θ_2^4	θ_1^3	y_1^3	y_1^4	y_2^3	y_2^4	θ_T	V_R
0.01	0.700	0.473	0.689	0.094	4.867	5.449	1.173	0.524	0.476	0.10	3.153	3.521	1.873	0.832
0.01	0.800	0.536	0.699	0.0925	4.710	5.302	1.220	0.463	0.428	0.05	3.342	3.711	2.020	0.821
0.01	1.210	0.703	0.743	0.0839	4.010	4.651	1.100	0.297	0.301	0.02	3.552	3.821	2.310	0.747
*0.01	2.121	1.000	0.802	0.083		3.794	0.380	0	0.083	0.01	3.450	3.865	2.501	0.600
0.02	0.784	0.502	.701	0.117	4.672	5.242	1.260	0.498	0.469	0.10	3.155	3.515	2.043	0.831
0.02	0.992	0.603	0.722	0.113	4.339	4.933	1.301	0.397	0.394	0.05	3.368	3.703	2.294	0.796
*0.02	2.152	1.000		0.118		3.726	0.514	0	0.118	0.02	3.726	3.820	2.666	0.640
*0.02	2.333	1.000		0.086		3.777	0.595	0	0.086	0.01	3.777	3.849	2.929	0.459
0.05	1.115	0.630	0.744	0.182	3.978	4.526	1.388	0.370	0.424	0.10	3.187	3.501	2.504	0.810
0.05	1.904	0.900	0.793	0.178	3.208	3.807	1.052	0.100	0.244	0.05	3.502	3.689	2.956	0.709
*0.05	2.656	1.000		0.117		3.700	0.893	0	0.117	0.02	3.700	3.788	3.548	0.471
*0.05	3.000	1.000		0.088		3.733	1.231	0	0.088	0.01	3.733	3.801	4.231	0.310
0.10	1.689	0.795	0.785	0.236	3.333	3.866	1.501	0.205	0.361	0.10	3.232	3.482	3.190	0.766
*0.10	2.638	1.000		0.195		3.527	1.187	0	0.195	0.05	3.527	3.664	3.825	0.599
*0.10	3.357	1.000		0.125		3.642	1.686	0	0.125	0.02	3.642	3.735	5.042	0.370
*0.10	4.139	1.000		0.089		3.660	2.333	0	0.089	0.01	3.660	3.722	6.472	0.229
*0.20	2.834	1.000		0.278		3.286	1.549	0	0.278	0.10	3.286	3.453	4.382	0.687
*0.20	3.548	1.000		0.198		3.470	2.052	0	0.198	0.05	3.470	3.603	5.600	0.504
*0.20	4.946	1.000		0.126		3.527	3.216	0	0.126	0.02	3.527	3.628	8.162	0.289
*0.20	6.587	1.000		0.089		3.517	4.653	0	0.089	0.01	3.517	3.563	11.240	0.155
*0.50	4.665	1.000		0.283		3.180	3.292	0	0.283	0.10	3.180	3.342	7.957	0.594
*0.50	6.455	1.000		0.200		3.304	4.829	0	0.200	0.05	3.304	3.422	11.283	0.400
*0.50	10.311	1.000		0.127		3.257	8.364	0	0.127	0.02	3.257	3.308	18.676	0.172
*0.50	15.160	1.000		0.090		3.087	13.185	0	0.090	0.01	3.087	3.085	28.345	0

*The result of step aeration concides with that of the conventional process.

Table 5. Optimal results for $\beta = 4.0$ for systems of step aeration with 90, 95, 98 and 99% treatment. (3 tank reactors in series)

	Tank One						Tank Two						Tank Three							
K_1	θ_2^2	θ_1^1	y_1^1	y_1^2	y_2^1	y_2^2	θ_2^4	θ_1^3	y_1^3	y_1^4	y_2^3	y_2^4	θ_2^6	θ_1^5	y_1^5	y_1^6	y_2^5	y_2^6	θ_T	V_R
0.01	0.364	0.308	0.597	0.095	6.309	6.799	0.565	0.326	.429	.095	4.291	4.617	.812	.366	.360	0.10	3.266	3.521	1.741	.774
0.01	0.414	0.356	0.608	0.096	6.123	6.624	0.663	.367	.437	.094	4.128	4.464	.789	.277	.294	0.05	3.475	3.714	1.866	.759
0.01	0.657	0.489	0.668	0.092	5.184	5.747	1.104	.511	.463	.089	3.398	3.764	.273	.154	.090	0.02	3.763	3.830	2.034	.658
0.01	0.700	0.513	0.676	0.093	5.064	5.634	1.159	.487	.446	.064	3.441	3.814	.279	0	.064	0.01	3.814	3.866	2.138	.513
0.02	.396	.329	.611	.128	6.071	6.543	.630	.342	.452	.125	4.113	4.433	.871	.329	.355	0.10	3.266	3.516	1.897	0.771
0.02	.517	.415	.643	.131	5.572	6.073	.898	.432	.473	.120	3.681	4.026	.670	.152	.228	0.05	3.535	3.708	2.086	0.724
0.02	.670	.514	.679	.146	5.003	5.524	1.257	.486	.478	.091	3.376	3.753	.373	0	.091	0.02	3.753	3.822	2.300	0.552
0.02	1.392	.778	.759	.123	3.748	4.371	.803	.222	.279	.054	3.595	3.813	.346	0	.054	0.01	3.813	3.855	2.542	0.398
0.05	.549	.405	.657	.180	5.345	5.809	.843	.389	.486	.176	3.645	3.947	.905	.206	.311	0.10	3.298	3.503	2.296	.742
0.05	.742	.518	.691	.186	4.309	5.301	1.301	.482	.500	.149	3.257	3.599	.537	0	.149	0.05	3.599	3.694	2.580	.619
0.05	1.680	.881	.783	.200	3.362	3.931	.916	.119	.276	.073	3.556	3.752	.488	0	.073	0.02	3.752	3.801	3.079	.409
*0.05	2.214	1.000	.802	.171	3.065	3.679	.755	0	.171	.041	3.679	3.803	.492	0	.041	0.01	3.803	3.831	3.460	.254
0.10	.796	.509	.703	.237	4.599	5.052	1.340	.491	.537	.225	3.067	3.370	.714	0	.225	0.10	3.370	3.490	2.850	.684
0.10	1.033	.616	.726	.234	4.235	4.713	1.477	.384	.469	.151	3.266	3.573	.828	0	.151	0.05	3.573	3.668	3.339	.523
*0.10	2.350	1.000	.804	.223	3.015	3.579	1.046	0	.223	.067	3.579	3.728	.746	0	.067	0.02	3.228	3.769	4.143	.304
*0.10	2.667	1.000	.802	.180	3.025	3.628	1.230	0	.180	.042	3.628	3.756	.942	0	.042	0.01	3.756	3.781	4.840	.172
0.20	1.111	0.598	.735	.295	4.075	4.503	1.676	0.402	.522	.225	3.053	3.339	1.088	0	.225	0.10	3.339	3.456	3.876	0.607
0.20	1.980	0.872	.788	.299	3.231	3.231	1.642	0.128	.371	.135	3.325	3.549	1.179	0	.135	0,05	3.549	3.626	4.799	0.432
*0.20	3.081	1.000	.804	.228	2.957	3.511	1.739	0	.288	.067	3.511	3.659	1.411	0	.067	0.02	3.659	3.696	6.232	0.221
*0.20	3.668	1.000	.802	.183	2.946	3.539	2.164	0	.183	.043	3.539	3.663	1.871	0	.043	0.01	3.663	3.682	7.703	0.106
0.50	2.644	.895	.803	.384	2.944	3.344	2.355	.105	.436	.209	3.062	3.273	1.938	0	0.209	0.10	3.273	3.369	6.937	.509
*0.50	3.943	1.000	.810	.314	2.796	3.266	2.742	0	.316	.125	3.266	3.436	2.364	0	0.125	0.05	3.436	3.494	9.048	.321
*0.50	5.434	1.000	.804	.232	2.782	3.318	3.944	0	.232	.068	3.318	3.455	3.615	0	0.068	0.02	3.455	3.478	12.993	.120
*0.50	7.013	1.000	.802	.184	2.707	3.279	5.263	0	.184	.043	3.279	3.385	4.965	0	0.043	0.01	3.385	3.384	17.240	0

*The result of step aeration concides with that of the conventional process.

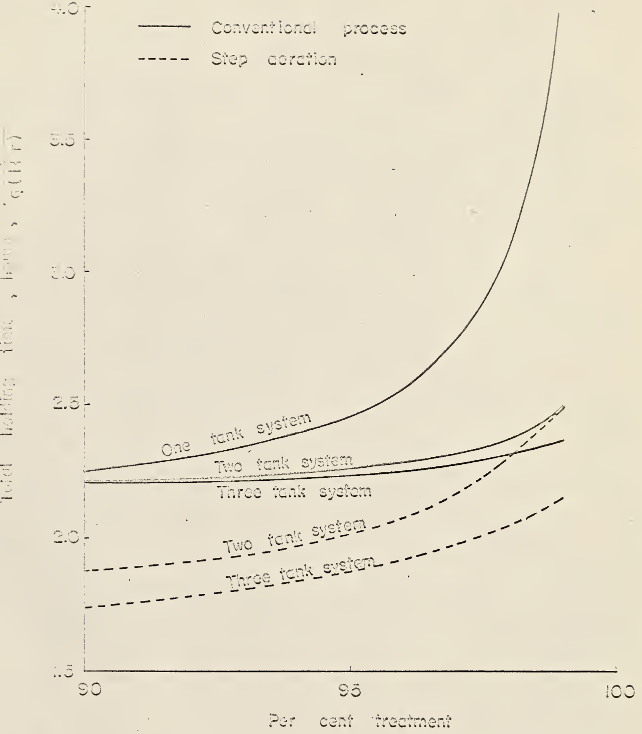


Fig. 7 . Variation of total holding time with per cent treatment for $K_1 = 0.01$ and $\beta = 4.0$.

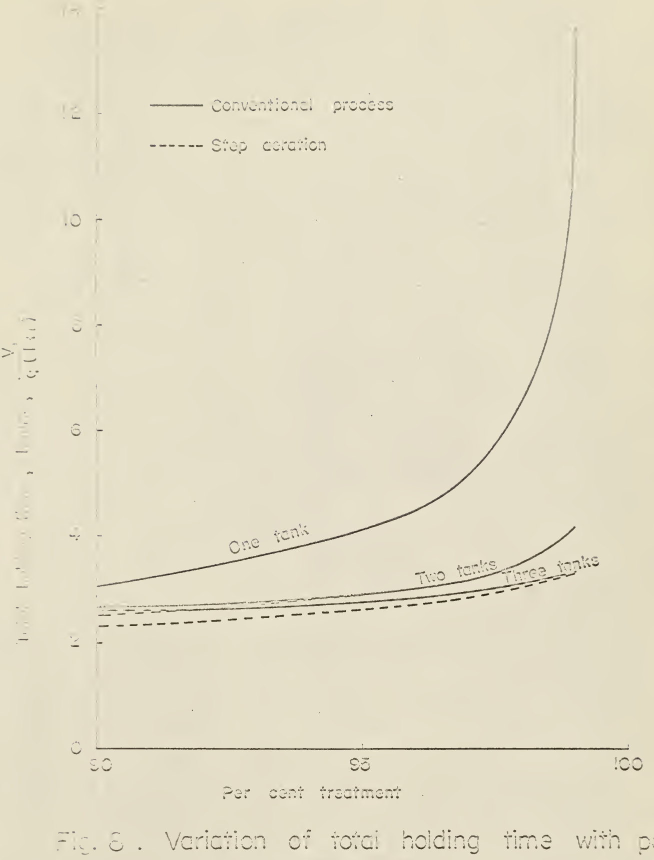


Fig. 8. Variation of total holding time with per cent treatment for $K_1 = 0.05$ and $\beta = 4.0$.

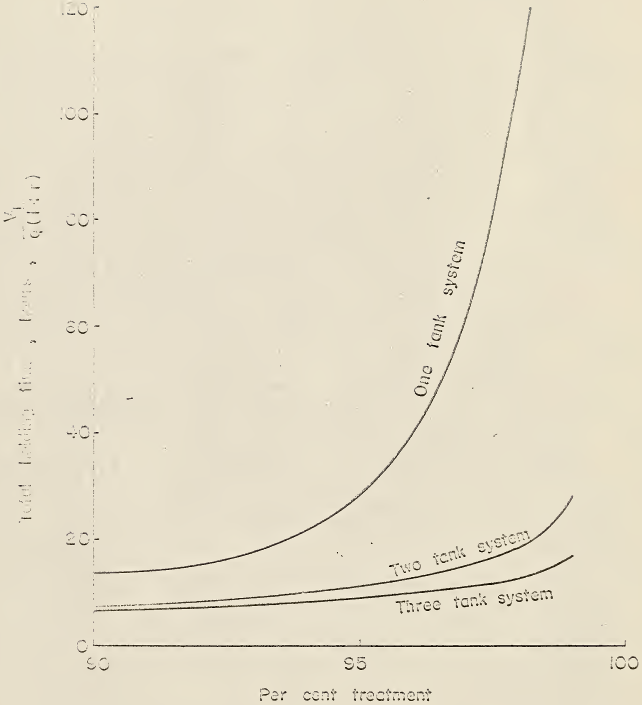


Fig. 9. Variation of total holding time with per cent treatment for $K_1 = 0.5$ and $\beta = 4.0$.

creases. This decrease in required volume is much larger for 99% treatment than for 90% treatment.

The effect of step aeration on the required volume is shown in Figures 7 and 8; however, for $K_1 = 0.5$, the effect of step aeration is very small and thus, separate curves are not shown in Figure 9 for the step aeration system. In Figure 2, where the value of K_1 is small ($K_1 = 0.01$), the effect of step aeration is significant in that the required volume is reduced considerably by using a three tank step aeration system. Figure 8 shows that an optimal step aeration system can be used to reduce the required volume for 90% treatment, but that very little volume reduction is obtained for 99% treatment. While the savings in volume due to using additional aeration tanks is greatest for 99% treatment, the savings in volume due to using the step aeration system rather than the conventional system is greatest for 90% treatment.

Figure 10 shows the variation of the objective function with the parameter K_1 for 90% treatment. When K_1 is small ($K_1 = 0.01$), there is only a very small volume reduction that can be obtained by using an optimal multi-tank conventional aeration system instead of a single tank system with complete mixing; however, when K_1 is large ($K_1 = 0.5$) the results predict that a substantial savings in volume can be obtained by using an optimal multi-tank system. On the other hand the results indicate that the savings in volume that can be obtained by using step aeration is greatest when K_1 is small.

The variation of the objective function with per cent treatment with K_1 as a parameter is shown in Figures 11 and 12 for two

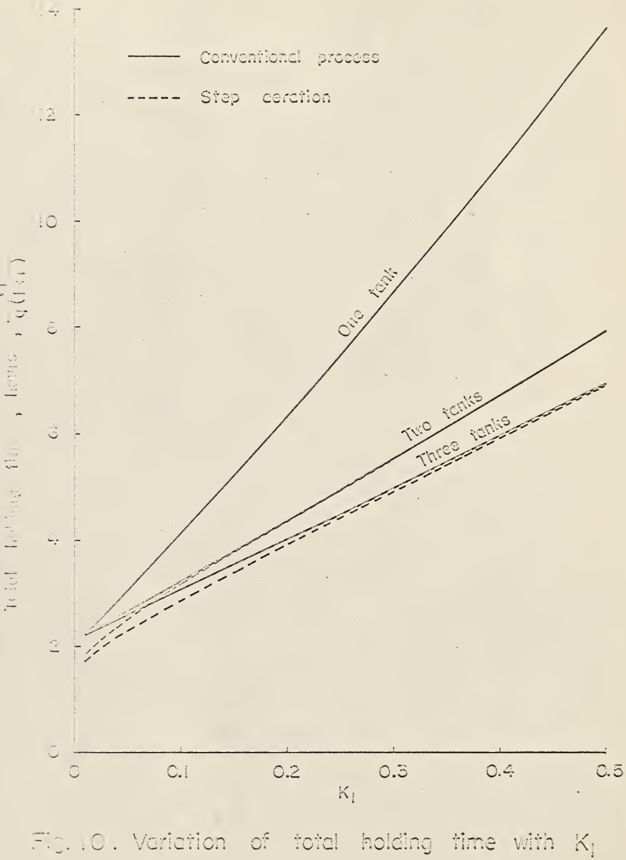


Fig. 10. Variation of total holding time with K_1 for 90 % treatment and β = 4.0 .

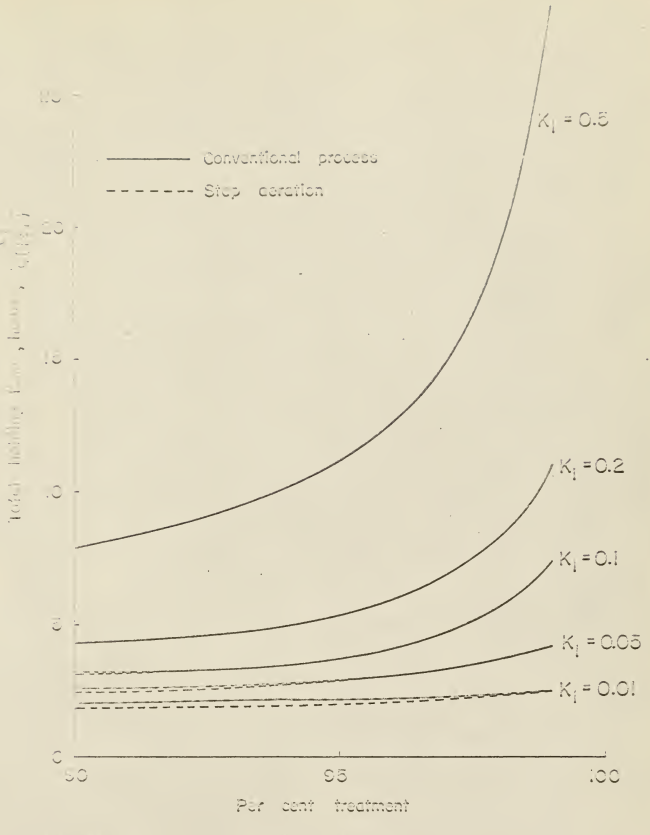


Fig. [illegible]. Variation of total holding time with per cent treatment for the two tank system with K_1 as a parameter and $\beta = 4.0$.

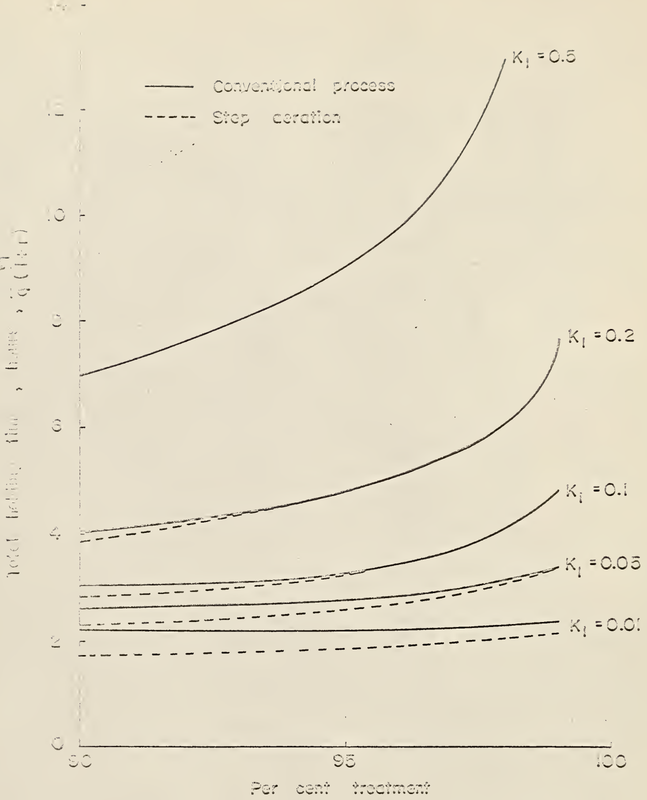


Fig. 12. Variation of total holding time with per cent treatment for the three tank system with K_1 as a parameter and $\beta = 4.0$.

and three tank systems, respectively. These results show that the total required holding time increases much more rapidly with per cent treatment when K_1 is large. These results also show how the effect of step aeration decreases as K_1 increases and per cent treatment increases.

In Figures 13 through 16 the volume ratio, which is defined as the minimum volume requirement for a particular multi-tank system divided by the volume requirement for a single tank system, is considered. Figures 13 and 14 show the variation of the volume ratio with per cent treatment for various values of K_1. Figure 13 shows that for 90% treatment and $K_1 = 0.01$ the predicted reduction in volume obtained by using an optimal three tank conventional system rather than a one tank system is only about 2% while the corresponding reduction predicted for the optimal three tank step aeration system is about 23%. The results presented in Figure 14 predict that when K_1 is small there are conditions where it is better to use an optimal two tank step aeration system than a three tank conventional system.

The results in Figures 13 and 14 also predict that when K_1 is large, the volume requirement is greatly reduced by using multi-tank systems rather than a single tank system; however, the additional volume reduction due to step aeration appears to be quite small when K_1 is large. As per cent treatment increases, the predicted reduction in volume due to using multi-tank systems increases. This is because the volume requirement for the single tank system increases very rapidly as per cent treatment increases.

Figures 15 and 16 show how the volume ratio with the para-

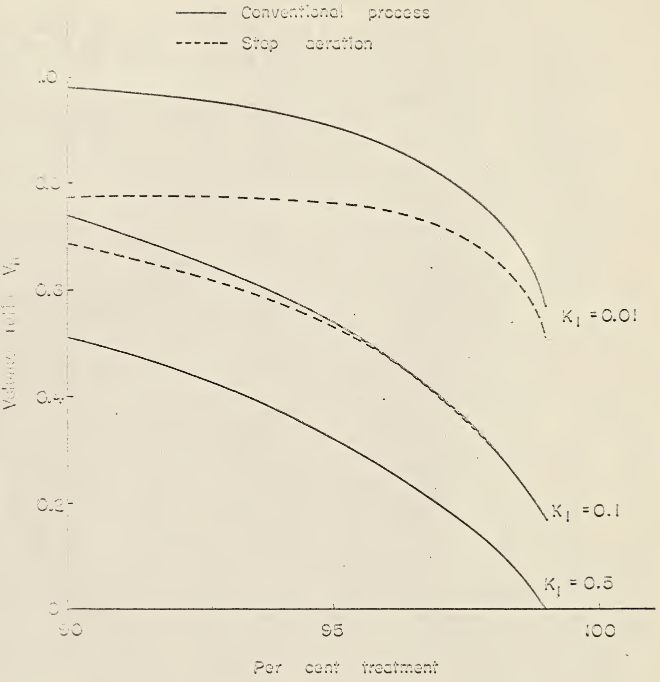


Fig. 13. Ratio of optimal volume requirement for the three tank systems to volume requirement for the one tank system versus per cent treatment for β = 4.0

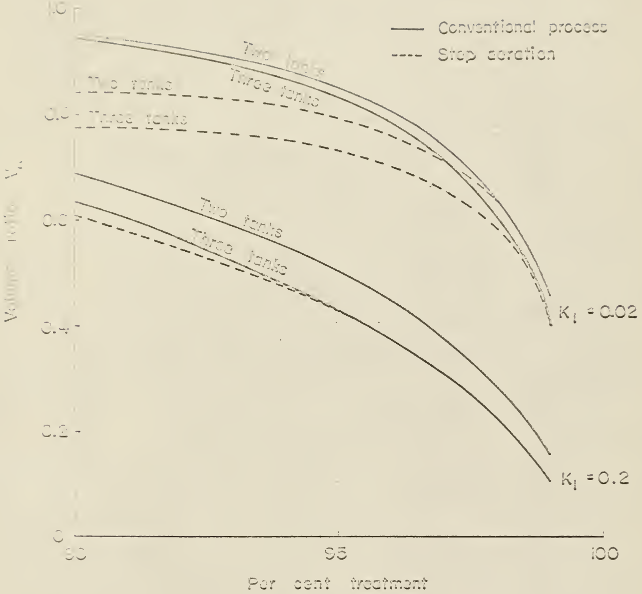


Fig. 14. Ratio of optimal volume requirement for multitank systems to volume requirement for a one tank system versus per cent treatment for β = 4.0 .

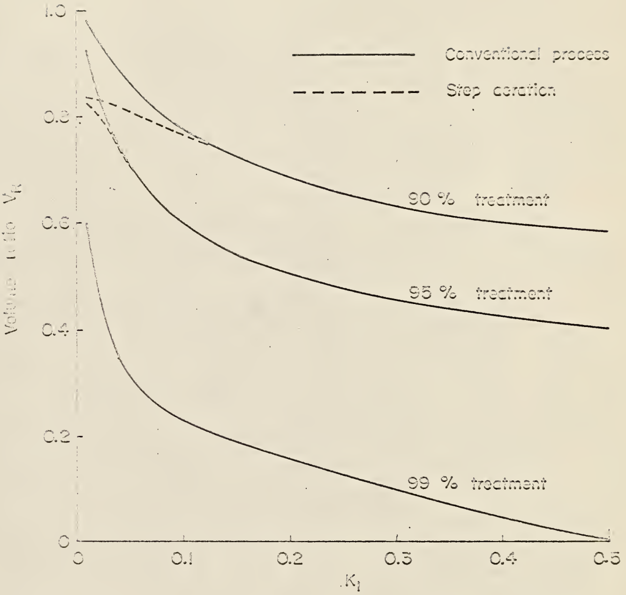


Fig. 15. Ratio of optimal volume requirement for the two tank system to the volume requirement for the one tank system versus K_1 for 90 %, 95% and 99 % treatment for $\beta = 4.0$.

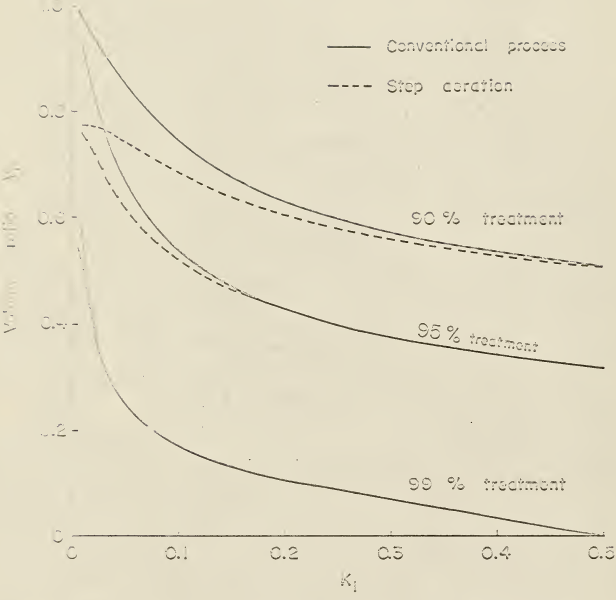


Fig. 16. Ratio of optimal volume requirement for the three tank system to volume requirement for the one tank system versus K_1 for 90, 95 and 99% treatment for $\beta = 4.0$.

meter K_1 in the kinetic model used in this study. As K_1 increases, the volume ratio decreases for both the conventional and the step aeration systems. Figure 15, which is for the two tank system, shows that there is little to be gained by using step aeration in a two tank system unless K_1 is less than 0.1. The results for the three tank systems in Figure 16 also predict that the greatest advantages are gained using step aeration when K_1 is small.

The variation of the per cent of feed allocated to the first tank with per cent treatment for the two tank step aeration system is shown in Figure 17. For $K_1 = 0.01$ and 90 to 95% treatment the results show that about one half of the influent should be allocated to each tank of the two tank system; however, for values of K_1 greater than 0.2 the results predict that all of the influent should be fed to the first tank as in the conventional process. For 99% treatment, there is no difference between the optimal two tank step aeration system and the optimal conventional process because all of the feed is allocated to the first tank even for the case where $K_1 = 0.01$.

The variation of the per cent of volume allocated to the first tank with per cent treatment for the two tank systems is shown in Figure 18. As the per cent treatment increases, the optimal allocation of the total volume to the first tank of the two tank conventional system gradually decreases; however, the optimal allocation of per cent volume to the first tank of the two tank step aeration system increases with per cent treatment. This increase of per cent volume allocated to the first tank with per cent treatment closely follows the increase in feed allocation to

Fig. 17. Variation of per cent of feed allocation to the first tank of the two tank system with per cent treatment for $\beta = 4.0$ with K_1 as parameter.

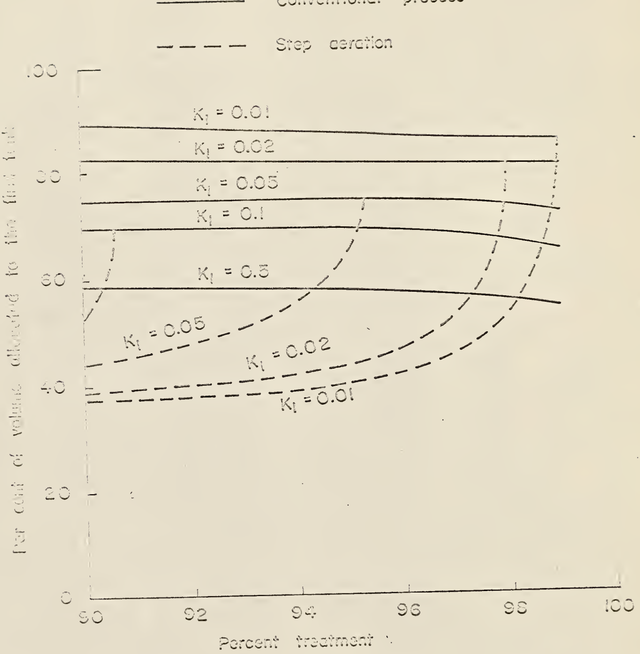


Fig. 18. Variation of per cent of volume allocation to the first tank of the two tank system with per cent treatment for $\beta = 4.0$ with K_1 as parameter.

the first tank shown in Figure 17. As K_1 increases, the per cent volume allocated to the first tank decreases for the conventional process. However, for the step aeration process, the optimal allocation of per cent volume to the first tank increases as K_1 is increased.

The variation of the optimal feed allocation to the three tank system with K_1 is shown for 90% treatment in Figure 19. The variation of the optimal allocation of volume with K_1 is shown for this same system and the conventional process in Figure 20. For the step aeration system abrupt changes in the slope of the curves occur when the allocation to a particular tank reaches zero. In Figure 19 the optiaml allocation of influent to the first tank increases with K_1 while the allocation to the second tank first increases and then decreases. In Figure 20 the optimal allocation of volume to these two tanks follows a similar pattern. The volume allocation to the third tank decreases until the feed allocation to this tank reaches zero; however, the allocation of volume to the third tank then increases as K_1 increases.

For the conventional process the optimal allocation of total volume to the first tank decreases as K_1 increases while the allocation to the other two increases with K_1. For the conventional systems considered here, the allocation of per cent volume is always largest for the first tank and smallest for the last tank, as shown in Figure 20.

In Figure 21 the variation of the organic concentration in the first tank of the two system is plotted as a function of K_1 with per cent treatment as a parameter. For the conventional

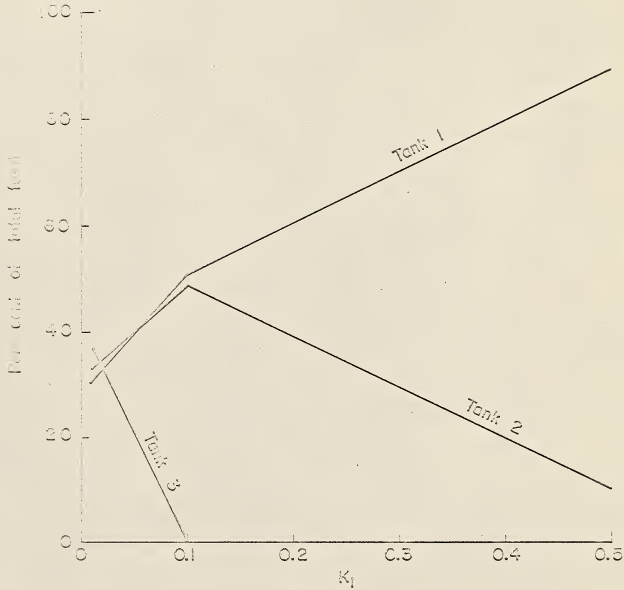


Fig. 19. Variation of feed allocation to the three tank system with K_1 for 90 % treatment and $\beta = 4.0$.

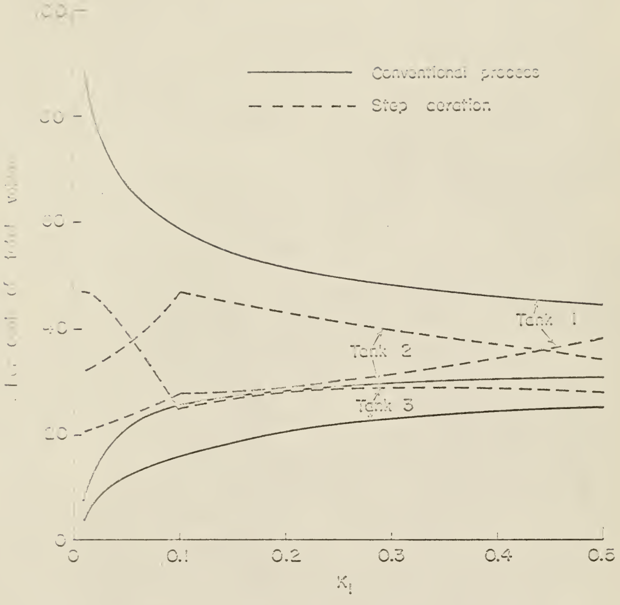


Fig. 20. Variation of volume allocation to the three tank system with K_1 for 90 % treatment and $\beta = 4.0$.

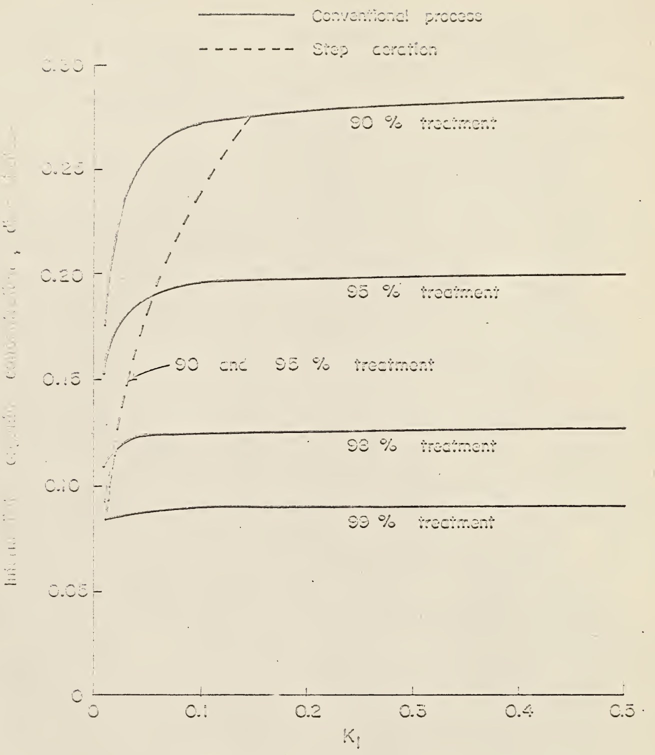


Fig. 21. Variation of the intermediate organic concentration with K_1 for the two tank system.

tem, this concentration does not change significantly with K_1 when K_1 is large; however, it does decrease significantly as per cent of treatment increases. For step aeration on the other hand, as long as the optimal step aeration system differs from the conventional system the organic concentration in the first tank does not depend on the per cent of treatment, but instead, it increases rapidly as K_1 increases. However, when K_1 increases to the point where all of the feed is allocated to the first tank, then from that point on the results for the two systems are indentical. This point depends on the per cent of treatment desired and it occurs for each case where the dashed line meets the solid line for that particular per cent of treatment; that is, at an organic concentration of about 0.12 for 98% treatment, 0.19 for 95% treatment, and 0.28 for 90% treatment.

Figure 22 presents similar results for the organic concentration in the first tank of the three tank system. As in Figure 21, as long as there is allocation to more than one tank, the optimal results for the step aeration system can be represented by a single dashed line, which does not depend on the per cent of treatment desired.

The variation of the organic concentration in the second tank with K_1 is shown in Figure 23. The curves for the conventional process are similar to those in Figures 21 and 22; however, the predicted optimal results for step aeration are different. In Figures 21 and 22 the results for step aeration were almost independent of per cent of treatment as long as there was allocation of feed to more than one tank; however, this is not true for the

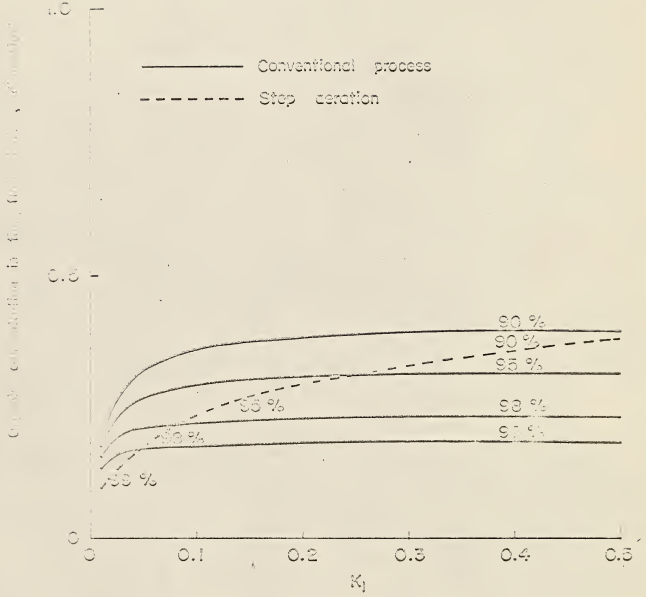


Fig. 22. Variation of the intermediate organic concentration from the first tank with K_1 for the three tank system.

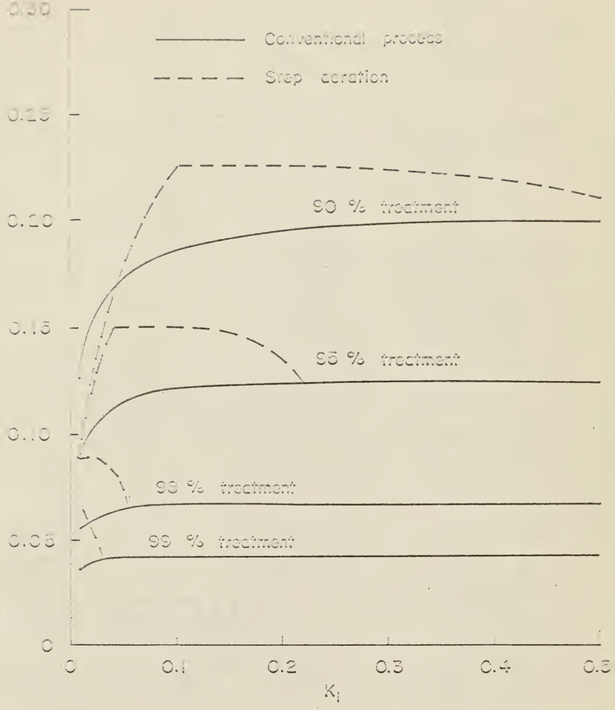


Fig. 25. Variation of organic concentration in the second tank with K_1 for the three tank system , β = 4.0 .

organic concentration in the second tank of the three tank step aeration process. As long as there is allocation of feed to the third tank, this organic concentration is almost independent of per cent treatment; however, when there is allocation of feed to only the first two tanks, it is greatly dependent on per cent of treatment. Figure 23 shows that the organic concentration in the second tank of the optimal step aeration process may exceed the organic concentration in that tank for the optimal conventional process. The abrupt changes in slope in the curves for the step aeration process occur where the allocation of feed to a particular tank reaches zero. For example, for 95% treatment there is a change in slope at about $K_1 = 0.04$ and $K_1 = 0.22$. The first occurs because the allocation of feed to the third tank reaches zero when K_1 is increased to about the value of 0.04 while the second change in slope occurs because the allocation of feed to the second tank stops when K_1 is increased approximately to 0.22.

Figures 24, 25 and 26 provide some information about the sensitivity and shape of the contour surface for the two tank step aeration system. Figure 24 shows how the optimal total holding time varies with the allocation of feed to the first tank. This curve was obtained by fixing the allocation of feed and then optimizing the objective function by proper choice of the one remaining independent variable.

Figure 25 shows a plot of total holding time versus the dimensionless organic concentration in the inlet stream of the second reaction stage with the fraction of allocation of sewage to the first tank of a two tank system as parameter. The curve

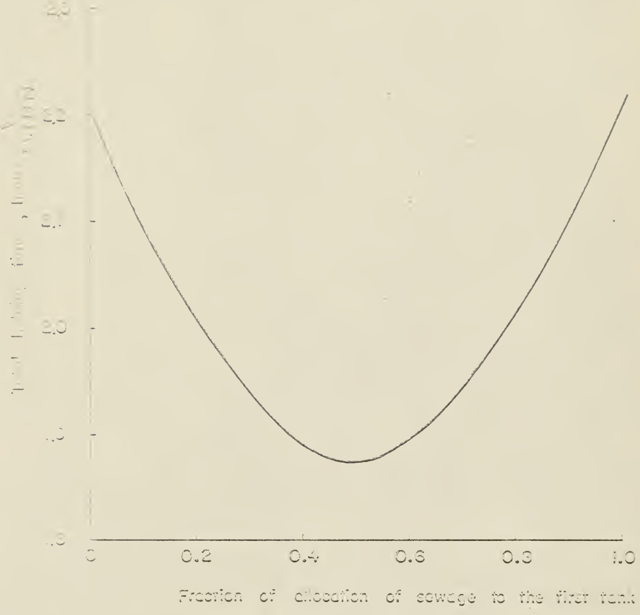


Fig. 24. Variation of total holding time with fraction of allocation of sewage to the first tank of a two tank system for 90 % treatment, $K_1 = 0.01$ and $\beta = 4$.

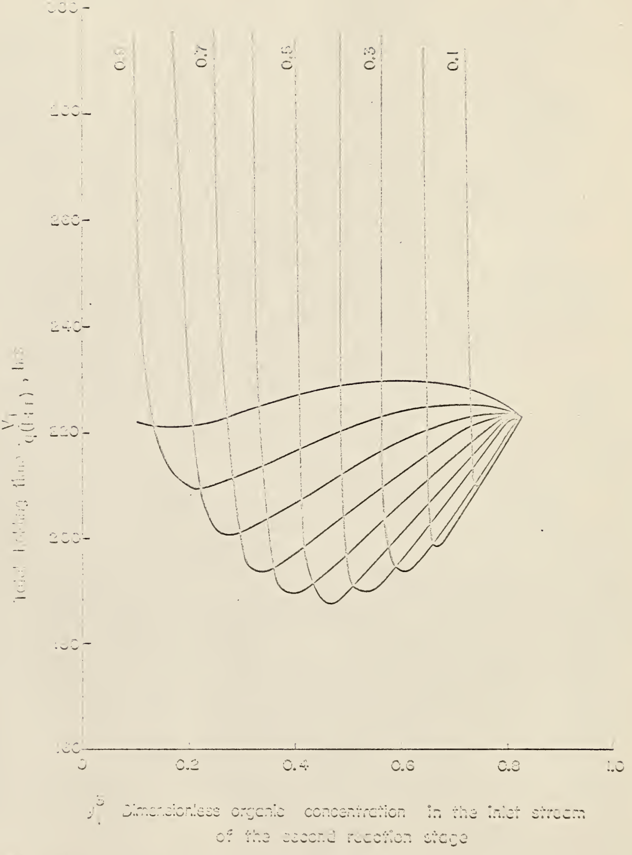


Fig. 23. Variation of total holding time with dimensionless organic concentration in the inlet stream of the second reaction stage with fraction of allocation of sewage to the first tank of a two tank system as parameter for 90 % treatment, K = 0.01 and β = 4.0 .

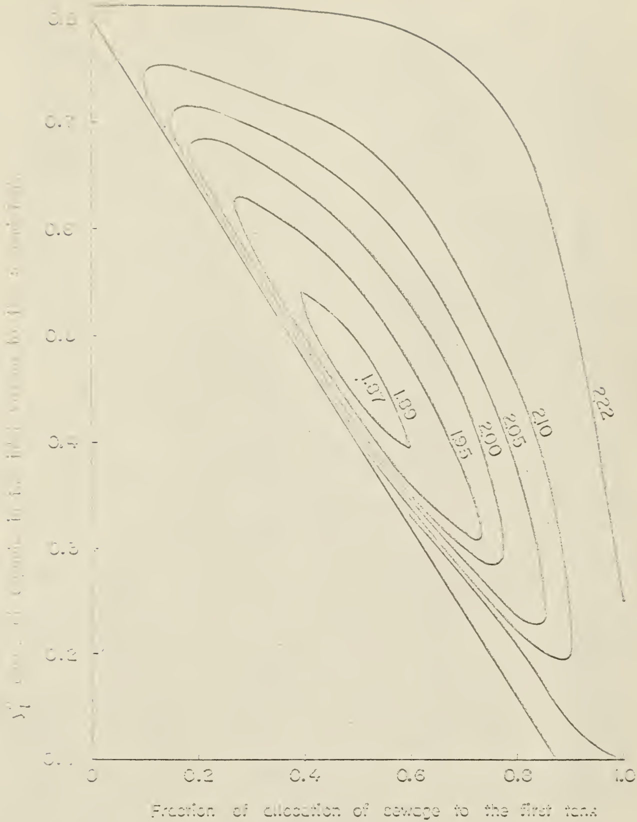


Fig. 25. Surface contours of constant total holding time for the two tank step aeration system.

was obtained by numerical simulation.

Figure 26 shows how two particular variables effect the value of the objective function. Each surface contour is for a particular value of the objective function; the point 1.87 denotes the minimum point. Since the value of the organic concentration from the second mixing point to the second tank is influenced by the feed allocated to the second mixing point, the results appear in the region to the right of the diagonal line.

Some results were also obtained for $\beta = 4.5$ and 4.75, but the effect of β on the optimal results is not as significant as that of K_1 and per cent treatment. Some of the effects of these values of β are reported elsewhere (1).

Since it may be possible to achieve the optimal pattern of flow and mixing within an activated sludge system without greatly affecting the costs of separation and sludge disposal, it is desirable to increase our knowledge about the effects of the hydraulic regime on the performance of the system. Although the results presented here indicate that step aeration can be used to reduce the volume requirements under some conditions, one should remember that the mathematical model used here to describe the biological waste treatment process is only an approximation of the actual process.

The results of this investigation show that the values of K_1 and per cent treatment greatly influence the optimal design. When K_1 is very small step aeration can be used to significantly reduce the volume requirement; however, when K_1 is large, there is almost no incentive to try to design an optimal step aeration system.

The per cent treatment desired also influences the desirability of designing a step aeration system because the percentage reduction in volume is much greater for 90% treatment than for 99% treatment. The relationship between K_1 and the dimensionless effluent organic concentration, y_1^{2N}, appears to be an important indicator in deciding whether or not step aeration should be employed. When K_1 is less than y_1^{2N} the possible advantages of using step aeration should be considered; however, when K_1 is greater than y_1^{2N} the advantages of step aeration are almost always small.

The results of this investigation predict that an optimal conventional activated sludge system composed of several completely mixed tanks connected in series will require less volume than a system with one completely mixed aeration tank when either K_1 or the percent treatment or both are large. When K_1 is small, the optimal design for 90% treatment approaches that of one completely mixed aeration tank; however, for all other conditions investigated an optimal multi-tank system requires significantly less volume than a single tank system.

The parameters K_1 and per cent treatment affect the type of system that is to be designed; however, they also greatly affect the optimal values of the design variables. For the conventional system, when K_1 is small most of the volume should be allocated to the first tank; however, when K_1 is large an optimal design would require a much greater allocation of volume to the other tanks. For the step aeration system, there is also considerable variation in the optimal allocation of influent and volume with K_1 and per cent treatment. Thus, it is desirable to know the values of these parameters for each waste treatment problem.

Additional experimental research is needed in order to determine the value of K for different wastes. Values of K reported by Washington, Hetling, and Rao (13) for different substrates range from 2 to 20 mg/liter. Milbury, Pipes and Grieves (10) reported a K value of 110 mg/liter COD for dried skim milk. Since the concentration of the influent entering a waste treatment system is usually between about 100 and 1000 mg/liter, the value of K_1 should usually be between about 0.002 and 1.0.

The concentrations of the organic nutrients and the microorganisms in each tank are important variables and they can be used to provide some explanation for the outcome of this optimization study. For the conventional system, the microorganism concentration increases gradually from tank to tank in going from the first to the last tank; however, the increase from the first to the last tank is usually less than 10% for the results reported here. On the other hand the organic concentration rapidly decreases from tank to tank in going from the first to the last tank. When K_1 is large this latter effect predominates and the results tend towards those for a first order chemical reaction where the optimal result is equal allocation of volume to each tank (14). When K_1 is small, the rate of growth of microorganisms is almost independent of the organic concentration. Thus, as K_1 approaches zero, the optimal conventional system approaches the complete mixing activated sludge system which is composed of a single well mixed aeration tank and a secondary clarifier.

When the step aeration system is studied one notes that by adding only a portion of the influent to the first tank, one obtains a higher microorganism concentration in that tank. This increased micro-organism concentration provides the incentive for using step aeration; however, since the organic concentration in the first tank is less than for the conventional system, step aeration is most advantageous, when K_1 is small and the organic concentration does not greatly affect the rate of growth.

A series of tanks with complete mixing in each tank is assumed for this study because this type of flow behavior can be realized in practice. It is known that a system where plug flow is assumed would theoretically give a smaller volume requirement for the conventional systems considered here (4). A step aeration system in which the influent is optimally allocated along the length of a tank in which plug flow is assumed to occur (2) will also give a smaller volume requirement than that obtained for the step aeration systems considered in this investigation. However, it is difficult to design an activated sludge waste treatment system in which plug flow can be assumed, because aeration and mixing must be continuously provided and the residence time of the waste is quire large. Since the tanks in series system can also be used as a model for a flow system with some longitudinal mixing, it seemed desirable to investigate this system.

Albury, Pipes and Grieves (15) have reported that they have obtained improved performance of a laboratory size activated sludge system by using a compartmentalized aeration tank. Their experimental investigation is one indication that optimal allocation of volume among the compartments of a tank may lead to improved performance; however, additional laboratory studies are needed in order to determine all of the effects of optimizing the pattern of flow in an activated sludge system.

CHAPTER IV

CONCLUSION AND RECOMMENDATION

A systems engineering approach to the design of a waste treatment system often involves the use of mathematics. An analysis which employs mathematical optimization procedures may often be used to find the values of the design variables in a systems design study. When this approach is used, one must have a mathematical model of the biological waste treatment process that relates the design variables to the behavior of the system and an economic model that relates the design variables to the various treatment costs that one must consider, such as capital and operating costs. In addition to the process and economic models, one must have an objective function to be optimized, that is, one must have the objective of the design stated in mathematical terms. The process and economic models and the objective function together provide a mathematical statement of the design problem.

In the systems approach, simulation and mathematical optimization procedures are used to find the optimum of the mathematical design problem. However, since the process and economic models and the objective function are only approximations, the optimum of the mathematical problem will probably deviate from the true optimum. Although the development of mathematical models that accurately describe the biological waste treatment process is difficult because of the many factors affecting biological growth, studies of this type can be useful in obtaining a better understanding of the process and in predicting the effect of specific parameters and variables on the performance of the system. This investigation shows that the constant K_1, which is the dimensionless organic nutrient concentration at which the specific growth rate is one half the maximum value, and the per cent treatment are two important variables which affect the optimal design of activated sludge systems. The optimum type of system and the optimum values of the design variables change significantly with changes in K_1 and per cent treatment.

The results of this analysis predict that an optimal step aeration system requires less volume than a conventional system when K_1 and per cent treatment are small, but that there is no advantage to using step aeration when K_1 and per cent treatment are large. The results also predict that optimal multi-tank conventional systems require less volume than single tank system and that the greatest savings in volume requirement occur when K_1 and per cent treatment are large.

Since, as mentioned previously, the mathematical model of a process is just an approximation, experimental research with step aeration systems is needed to verify the results presented here. Although it would be difficult to experimentally optimize a laboratory size activated sludge system where the per cent treatment is specified, one could experimentally optimize a laboratory size system of fixed total volume by adjusting the allocation of volume and influent among the tanks until the maximum per cent treatment is obtained. A direct search optimization procedure could be used to guide the adjustment of feed and volume allocation from experiment to experiment so that the optimum may be found as rapidly as possible.

NOMENCLATURE

k	Maximum specific growth rate when the organic concentration is not limiting the rate of growth, hr^{-1}.
K	The concentration of organic at which the specific growth rate observed is one half the maximum value, mg/liter.
k_D	Specific endogeneous microbial attrition rate, hr^{-1}.
K_1	The dimensionless organic concentration at which the specific growth rate observed is one half the maximum value.
q	Volumetric flow rate of feed to the overall system, liters/hr.
q^{2n-1}	The volumetric flow rate of feed introduced to the mixing point at that precedes the nth aeration tank, liters/hr.
r	Recycle ratio.
S	Object function
V_T	Total volume of the biological growth chamber, liters.
V^{2n}	The volume of the nth aeration tank, liters.
w	Withdrawal ratio.
x_1	Concentration of organic nutrients, mg/liter.
x_2	Concentration of active micro-organisms, mg/liter.
x_1^f	The concentration of organics in the feed, mg/liter
x_2^f	The concentration of organisms in the feed, mg/liter.
x_1^{2n-1}	The concentration of organic nutrients entering the nth aeration tank, mg/liter.
x_2^{2n-1}	The concentration of organisms entering the nth aeration tank, mg/liter.

x_3^{2n-1} The volumetric flow rate of the entering stream of the nth aeration tank, liters/hr.

x_3^{2n} The volumetric flow rate of the exit stream of the nth aeration tank, liters/hr.

y_1^{2n} The dimensionless concentration of the organic waste in the nth tank.

y_2^{2n} The dimensionless concentration of organisms in the nth tank.

y_3^{2n} The dimensionless flow rate from the nth tank.

t_1^{2n-1} The dimensionless flow rate of influent to the nth mixing point.

θ_2^{2n} The holding time for the nth tank, hrs.

ACKNOWLEDGMENTS

The author wishes to express his sincere appreciation to Dr. Liang-tseng Fan and Dr. Larry E. Erickson for their constant enthusiasm and advice in this work; Dr. William H. Honstead and Dr. R. G. Akins for their help in reading the manuscript; the Kansas State University Computing Center for the use of their facilities; and the K.S.U. Engineering Experiment Station, the Kansas Water Resources Research Institute, and Office of Water Resources Research, and the Federal Water Pollution Control Administration, U. S. Department of the Interior for supporting this work (Proj. A-019-KAN and Proj. WP-01141-01).

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APPENDIX I

DETERMINATION OF THE NUMBER OF DEGREES OF FREEDOM OF THE BIOLOGICAL WASTE TREATMENT SYSTEMS

An analysis of the system to determine the number of independent variables or degrees of freedom is often required for designing and optimizing complex systems. In this analysis, an N stage system is considered; however, each stage is assumed to consist of a mixing point and reaction stage; thus, the actual model we consider here is composed of 2N stages (see Figure 27).

(1) Type and Number of System Variables

a) Dimensionless volumetric flow rate of feed,

θ_1^{2n-1}, $n = 1, 2, \ldots, N$; N

b) Dimensionless organic concentration,

y_1^i, $i = 0, 1, \ldots, 2N$; 2N + 1

c) Dimensionless organism concentration,

y_2^i, $i = 0, 1, \ldots, 2N$; 2N + 1

d) Dimensionless volumetric flow rate through each stage,

y_3^i, $i = 0, 1, \ldots, 2N$; 2N + 1

e) Reactor volume at each stage,

θ_2^{2n}, $n = 1, 2, \ldots, N$; N

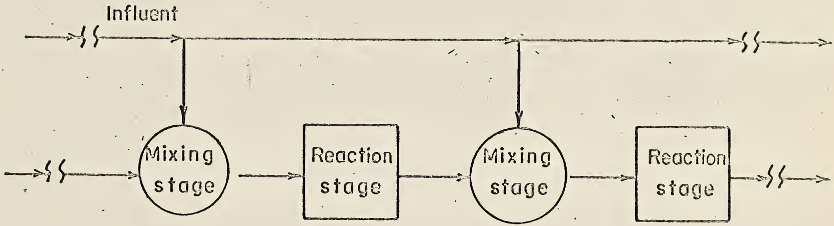

Fig. 27. Model of step aeration activated sludge process.

f) Dimensionless concentrations of organic and organism in feed,

y_1^f and y_2^f 2

Total number of system variables $= 3(2N + 1) + 2N + 2$
$= 8N + 3$

(2) Type and Number of Relations Among System Variables

a) Material balance of organic component,

(i) Mixing points,

$$y_1^{2n-2}\, y_3^{2n-2} + \theta_1^{2n-1} = y_1^{2n-1}\, y_3^{2n-1}, \quad n = 1, 2, \ldots, N; \ N$$

(ii) Reaction stages (Tank reactor is under consideration),

$$y_1^{2n-1}\, y_3^{2n-1} + V_s\, \theta_2^{2n} = y_1^{2n}\, y_3^{2n}, \quad n = 1, 2, \ldots, N; \ N$$

b) Material balance of organism component,

(i) Mixing stages,

$$y_2^{2n-2}\, y_3^{2n-2} + \theta_1^{\theta n-1}\, y_2^{f} = y_2^{2n-1}\, y_3^{2n-1}, \quad n = 1,2, \ldots, N; \ N$$

(ii) Reactor stages,

$$y_2^{2n-1}\, y_3^{2n-1} + r\theta_2^{2n} = y_2^{2n}\, y_3^{2n}, \quad n = 1, 2, \ldots, N; \ N$$

c) Overall material balance at each stage,

(i) Mixing stages,

$$y_3^{2n-2} + \theta_1^{2n-1} = y_3^{2n-1}, \qquad n = 1, 2, \ldots, N; \quad N$$

(ii) Reaction stages,

$$y_3^{2n-1} = y_3^{2n} \qquad n = 1, 2, \ldots, N; \quad N$$

d) Total number of relations = 6N

(3) Degree of Freedom of System

From the total number of variables and relations obtained in previous sections, the number of degree of freedom for the entire system is

$$F = 8N + 5 - 6N = 2N + 5$$

If the system is specified by the variables, y_1^0, y_2^0, y_3^0, y_1^{2N}, y_1^f, y_2^f with the equality constraint

$$\sum_{n=1}^{N} \theta_1^{2n-1} = 1,$$

then the number of degrees of freedom becomes

$$F = 2N + 5 - 7 = 2N - 2$$

for the step aeration process. Moreover, among the 2N-2 variables, N-1 variables of influent allocation should be specified. In other words, N-1 variables have to be specified as the influent allocation. However, for the conventional process all the influent is allocated to the first tank, there are only N-1 decision variables to be considered.

APPENDIX II

MATHEMATICAL OPTIMIZATION PROCEDURE

The mathematical problem of minimizing equation (18) subject to equations (5) - (14) for fixed values of y_1^{2N}, β, and r can be accomplished using direct search optimization procedures. If the allocation of influent, θ_1^{2n-1}, to N-1 of the N mixing points and the organic waste concentration, y_1^{2n}, in all of the aeration tanks except the last one are taken as the independent decision variables of the optimization problem, the objective function, equation (18), can be evaluated. If the dimensionless variables defined earlier are used, the problem is that of minimizing

$$S = \frac{V_T}{q(1+r)} = \frac{1}{1+r} \sum_{n=1}^{N} \frac{V^{2n}}{q} = \frac{1}{1+r} \sum_{n=1}^{N} y_3^{2n} \, \theta_2^{2n} \qquad \text{(A-II-1)}$$

subject to the equality constraints

$$y_1^{2n-2} \, y_3^{2n-2} + \theta_1^{2n-1} = y_1^{2n-1} \, y_3^{2n-1} \qquad \text{(A-II-2)}$$

$$y_2^{2n-2} \, y_3^{2n-2} + \theta_1^{2n-1} \, y_2^{f} = y_2^{2n-1} \, y_3^{2n-1} \qquad \text{(A-II-3)}$$

$$y_3^{2n-1} = y_3^{2n-2} + \theta_1^{2n-1} \qquad \text{(A-II-4)}$$

$$y_1^{2n-1} - y_1^{2n} - \theta_2^{2n} \left[\frac{k \, y_1^{2n} \, y_2^{2n}}{K_1 + y_1^{2n}}\right] = 0 \qquad \text{(A-II-5)}$$

$$y_2^{2n-1} - y_2^{2n} + \theta_2^{2n} \left[\frac{k\, y_1^{2n}\, y_2^{2n}}{K_1 + y_1^{2n}} - k_D\, y_2^{2n}\right] = 0 \qquad \text{(A-II-6)}$$

$$y_3^{2n} = y_3^{2n-1} \qquad \text{(A-II-7)}$$

$$\sum_{n=1}^{N} \theta_1^{2n-1} = 1 \qquad \text{(A-II-8)}$$

$$y_1^{0} = y_1^{2N} \qquad \text{(A-II-9)}$$

$$y_2^{0} = \beta y_2^{2N} \qquad \text{(A-II-10)}$$

and

$$y_3^{0} = r \qquad \text{(A-II-11)}$$

This optimization problem can be put in a form in which the objective function, S, can be easily computed for various values of the desogn variables. We first solve equation (A-II-5) for θ_2^{2n} to obtain

$$\theta_2^{2n} = \frac{(y_1^{2n-1} - y_1^{2n})(K_1 + y_1^{2n})}{k\, y_1^{2n}\, y_2^{2n}} \qquad \text{(A-II-12)}$$

Substituting this result into equation (A-II-6) gives

$$y_2^{2n} = y_2^{2n-1} + \frac{(y_1^{2n-1} - y_1^{2n})[k\, y_1^{2n} - k_D(K_1 + y_1^{2n})]}{k\, y_1^{2n}} \qquad \text{(A-II-13)}$$

By rearranging equation (A-II-3), we obtain

$$y_2^{2n-1} = \frac{y_2^{2n-2}\, y_3^{2n-2} + \theta_1^{2n-1}\, y_2^{f}}{y_3^{2n-1}} \tag{A-II-14}$$

Combining equations (A-II-13) and (A-II-14) gives

$$y_2^{2n} = \frac{y_2^{2n-2}\, y_3^{2n-2} + \theta_1^{2n-1}\, y_2^{f}}{y_3^{2n-1}} + \frac{(y_1^{2n-1} - y_1^{2n})[k\, y_1^{2n} - k_D(K_1 + y_1^{2n})]}{k\, y_1^{2n}}$$

or

$$y_2^{2n} = A^{2n}\, y_2^{2n-2} + B^{2n} \tag{A-II-15}$$

where

$$A^{2n} = \frac{y_3^{2n-2}}{y_3^{2n-1}} \tag{A-II-16}$$

and

$$B^{2n} = \frac{(y_1^{2n-1} - y_1^{2n})[k\, y_1^{2n} - k_D(K_1 + y_1^{2n})]}{k\, y_1^{2n}} + \frac{\theta_1^{2n-1}\, y_2^{f}}{y_3^{2n-1}}$$

.... (A-II-17)

Equation (A-II-10) may be written in the form

$$y_2^{0} = A^{0}\, y_2^{2N} \tag{A-II-18}$$

by letting $A^0 = B$. Substitution of y_2^0 into the equation for y_2^2, we obtain

$$y_2^2 = A^2 A^0 y_2^{2N} + B^2$$

Substitution of this expression into equation (A-II-15) for n = 12 gives

$$y_2^4 = A^4 A^2 A^0 y_2^{2N} + A^4 B^2 + B^4$$

by induction, the expression for the nth tank can be obtained as

$$y_2^{2n} = A^{2n} A^{2n-2} \ldots A^2 A^0 y_2^{2N} + A^{2n} A^{2n-2} \ldots A^4 B^2$$
$$+ \ldots + A^{2n} B^{2n-2} + B^{2n} \qquad \text{(A-II-19)}$$

When n = N, equation (A-II-19) may be written in the form

$$y_2^{2N} = \frac{A^{2N} A^{2N-2} \ldots A^4 B^2 + \ldots + A^{2N} B^{2N-2} + B^{2N}}{[1 - (A^{2N} A^{2N-2} \ldots A^2 A^0)]} \qquad \text{(A-II-20)}$$

For a desired degree of treatment, y_1^{2N} is fixed. Selecting values for the variables θ_1^{2n-1} and y_1^{2n}, $n = 1, 2, \ldots, N-1$ is sufficient to specify the values of the dependent variables and the objective function, S. To obtain the minimum value of S, the values of θ_1^{2n-1} and y_1^{2n}, $n = 1, 2, \ldots, N-1$ must be the optimum values; that is, the values which allows S to take on its minimum value must be selected.

The suggested computational procedure to compute S is as follows:

1. Assume values for θ_1^{2n-1} and y_1^{2n}, $n = 1, 2, \ldots, N-1$.

2. Compute θ_1^{2N-1} using equation (A-II-8).

3. Compute y_3^{2n} and y_3^{2n-1} for n = 1, 2, ..., N using equation (A-II-4) and (A-II-7).

4. Compute y_1^{2n-1} for n = 1, 2, ..., N using equation (A-II-2).

5. Compute A^{2n} and B^{2n} for n = 1, 2, ..., N using equations (A-II-16) and (A-II-17).

6. Compute y_2^{2N} using equation (A-II-20).

7. Compute y_2^0 using equation (A-II-18) and y_2^{2n} for n = 1, 2, ..., N-1 using equation (A-II-15).

8. Compute y_2^{2n-1} for n = 1, 2, ..., N using equation (A-II-14).

9. Compute θ_2^{2n} for n = 1, 2, ..., N using equation (A-II-12).

10. Compute S using equation (A-II-1).

A direct search optimization procedure may be used to systematically assume sets of values of θ_1^{2n-1} and y_1^{2n} for n = 1, 2, ..., N-1 until the optimum values of these design variables have been found. For the problem considered here a modified direct pattern search technique and the simplex method has been written as an optimization subroutine to determine the optimum values of these variables. In assuming sets of values of θ_1^{2n-1} and y_1^{2n} only values between zero and one were allowed.

For the problem in which step aeration is considered the above procedure can be used directly. When the conventional system is considered, the decision variables related to the allocation of feed are fixed such that $\theta_1^1 = 1$ and all other values of θ_1 are zero; however, the same method can still be used to obtain the optimum design for the conventional system.

APPENDIX III

A MODIFIED PATTERN SEARCH TECHNIQUE (1)

The general concept of this method is to set up a pattern of $K \geq n+1$ vertices, that is, to select K points in the space of n independent variables and evaluate the objective function values at these selected points. Then, by comparing the objective function values at these points, the vertex with the highest function value (i.e. the worst point in minimization) is replaced by another point with a lower value of the objective function, which is determined according to certain operations. This method forces the objective function to approach the minimum by, at each stage of the operation, discarding the worst point of the pattern and adapting a better point to form a new pattern. This procedure is repeated until the minimum point is reached.

In this method, $K = n+1$ points are used, of which one is the given or starting point. The additional (K-1) points required to set up the initial pattern are obtained one at a time by the use of a step size increment for each of independent variables, i.e., $\bar{x}_i = x_i^0 + \Delta x_i$ for $i = 1, 2, \ldots, n$. In other words, if $P^1(x_1^0, x_2^0, \ldots, x_n^0)$ is the starting point, then the further (K-1) points are set up in this way:

$$P^2 : (x_1^0 + \Delta x_1, x_2^0, \ldots, x_n^0)$$

$$P^3 : (x_1^0, x_2^0 + \Delta x_2, \ldots, x_n^0)$$

$$\vdots$$

$$P^n : (x_1^0, x_2^0, \ldots, x_{n-1}^0 + \Delta x_{n-1}, x_n^0)$$

$$P^{n+1} : (x_1^0, x_2^0, \ldots, x_n^0 + \Delta x_n)$$

The objective function is then evaluated at each vertex, and thesearch is carried on by the following operating procedures: At first we write S_j for the objective function value at P_j and define

$S_h = \max_j (S_j)$ where subscript h stands for "the highest",

$S_m = \operatorname{med}_j (S_j)$ where subscript m stands for "the second highest",

and

$S_\ell = \min (S_j)$ where subscript ℓ stands for "the lowest".

We further define $\overline{P}$ with the coordinates of $\overline{x}_i = \sum_{j=1}^{K-1} x_i^j/K-1$, $i = 1, 2, \ldots, N$, as the centroid of the points with $j \neq h$ and write $\overline{P_iP_j}$ for the distance from P_i to P_j. At each stage in the process P_h is replaced by a new point obtained by these three operations - reflection, contraction, and expansion.

i) Reflection

By using a positive reflection coefficient α, the reflection of P_h is denoted by P*, and its coordinates are defined by the relation,

$$P^* = \overline{P} + \alpha(\overline{P} - P_h)$$

Note that P* is co-linear with $\overline{P}$ and P_h, on the far side of $\overline{P}$ from P_h with

$$\overline{\bar{P}P^*} = \alpha\ \overline{P_h\ \bar{P}}$$

If S* lies between S_h and S , then P_h is replaced by P* and the search is started again with the new pattern.

ii) Expansion

If $S^* < S_\ell$, i.e. reflection has produced a new minimum, then we wxpand P* to P** by the relation

$$P^{**} = \bar{P} + \gamma(P^* - \bar{P})$$

In other words, the expansion coefficient γ, which is greater than unity, is the ratio of the distance $\overline{P^{**}\bar{P}}$ to $\overline{P^*\bar{P}}$. If $S^{**} < S^*$, we replace P_h by P** and before starting, define a new centroid $\bar{P}$ with

$$\bar{X}_i = \frac{(2n-1)x_i^1 + \sum_{j=2}^{K-1} (2n-2)x_i^j}{(2n-1) + (K-2)\ (2n-2)}$$

iii) Contraction:

If on reflecting P to P* we find that $S^* > S_m$, then we define a new P_h to be either the old P_h or P*, whichever has the lower S value, and form

$$P^* = \bar{P} + \beta(P_h - \bar{P})$$

The contraction coefficient β is the ratio of the distance $\overline{P^*\ \bar{P}}$ to $\overline{P_h\bar{P}}$ and has a value between 0 and 1. Unless $S^* > S_h$, we accept P* for P_h and restart the search. However, for such a failed contraction we replace all the P_j's by $(P_j + P\ )/2$ before restarting the process.

A good expansion may be thought of as resulting from a right direction toward the valley, so it is reasonable to select a centroid much near the best point of the pattern instead of using the conventional way for defining the centroid together with a slightly larger reflection coefficient. A failed contraction seldom happens, but can occur when a valley is curved. Therefore, the action of contracting the pattern towards the lowest point will eventually bring all points into the valley. A flow chart to describe the complete method is given in Fig. AIII-1.

The criterion for stopping the computation is to compare the "standard error" of the S's in the form $\sqrt{\Sigma(S_i - \overline{S})^2/n}$ as in the simplex method with a pre-set value, and to stop when it falls below this value. The success of this criterion depends on the pattern not becoming too small in relation to the curvature of the surface until the final minimum is reached.

NUMERICAL EXAMPLES

Two functions, all of which have a minimum of zero and have been used before for testing minimization search techniques, were used to test the method. These were:

(1) Rosenbrock's parabolic valley (Rosenbrock (1960))

$$S(x_1, x_2) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2,$$

starting point (-1.2, 1).

(2) Powell's quadratic function (Powell (1962))

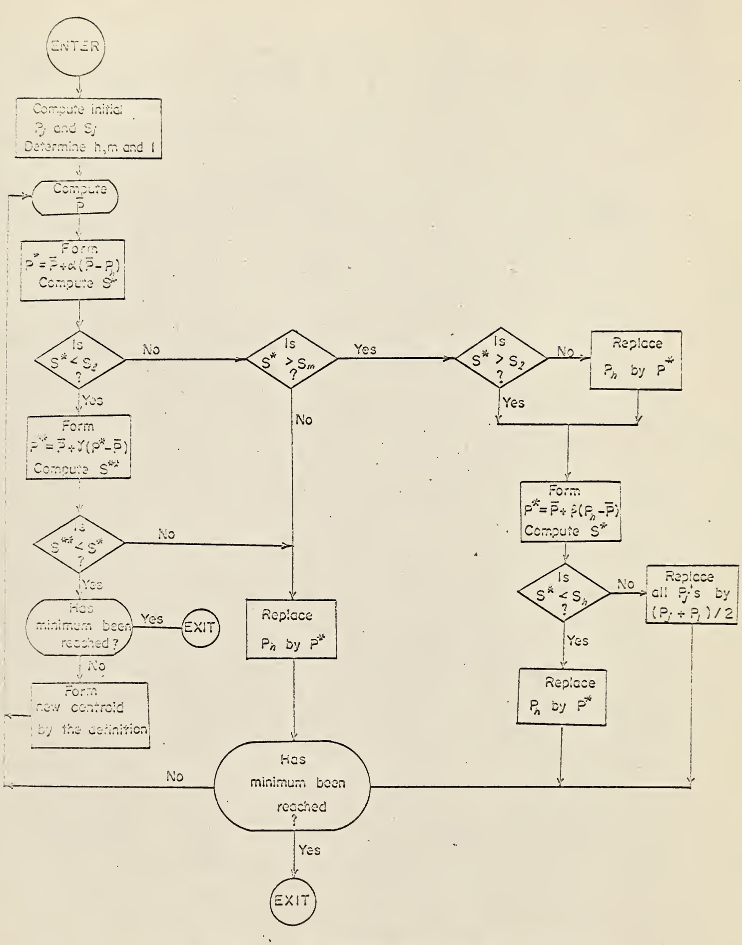

Fig. A.III-1 . The general flow chart for the new method .

$$S(x_1, x_2, x_3, x_4) = (x_1 + 10x_2)^2 + 5(x_3 - x_4)^2$$
$$+ (x_2 - 2x_3)^4 + 10(x_1 - x_4)^4,$$

starting point (3, -1, 0, 1).

The stopping criterion used was $\sqrt{\Sigma(S_j - \bar{S})^2/n} < 3 \times 10^{-9}$. However, the results shown in the tables are picked as the function values are down below 1.0×10^{-8}. It is certain that the size and orientation of the initial pattern had a significant effect on the speed of convergence. Also, the definition of the new centroid, due to a good expansion, showed a big effect on the speed of reaching the minimum. The first set of results investigated the different strategies, which include different values of α, β, and γ and different definition of the new centroid due to a good expansion. The second compared the results for the best strategy with those of the simplex (1965).

The first trial with function (1) used all combinations of $\alpha = 1.0, 1.2, 1.0 \rightarrow 1.2$; $\beta = 1/2$; $\gamma = 2$; initial step-lengths 0.2, 0.8, 1.2, 2.4; and the weight for the points of the pattern is (2.1), (3.2), (4.3), and (5.4). Part of the results are presented in Table A III-1.

On using function (2), the same strategies were used and part of the results are presented in Table AIII-2.

The analyzing of the results shows the following strategy gives the best result, i.e.,

α : $1.0 \rightarrow 1.2$ (1.2 follows the good expansion)

β : 0.5

TABLE AIII-1

Number of Evaluations for Function 1

$\epsilon = 3.0 \times 10^{-9}$

Step-Length	Strategy(α,β,γ) $(1, \frac{1}{2}, 2)$	$(1. \rightarrow 1.2, \frac{1}{2}, 2)$	$(1.2, \frac{1}{2}, 2)$
0.2	144	149	155
0.8	112	98	107
1.2	134	121	160
2.4	150	158	75

Definition of new centroid $\overline{P}$ with

$$\overline{x}_i = \frac{(2n-1)x_i^j \sum_{j=2}^{K-1} (2n-2)x_i^j}{(2n-1) + (K-2)(2n-2)}$$

TABLE AIII-2

Number of Evaluations for Function 2

$\epsilon = 3.0 \times 10^{-9}$

Step-Length	$(1, \frac{1}{2}, 2)$	$(1 \rightarrow 1.2, \frac{1}{2}, 2)$	$(1.2, \frac{1}{2}, 2)$
0.5	250	215	250
1.0	240	226	250
2.0	250	207	245
4.0	245	213	250
The same definition of the new centroid as in Table 1.			

γ : 2.0

and the definition of the new centroid $\overline{P}$ with

$$\bar{x}_i = \frac{(2n-1)x_i^1 + \sum_{j=2}^{K-1} (2n-2)x_i^j}{(2n-1) + (K-2)\ (2n-2)}$$

Results obtained with the above strategy are shown in Table AIII-3.

DISCUSSION

For comparison, the simplex method has been built in as a part of the present computer program. The best strategy stated in the simplex original paper was used to treat with the four step-lengths and the results are presented in Table AIII-4. The mean numbers of evaluations for functions given by equations (1) and (2) by using the modified method are 131 and 215 respectively. By using the simplex, the results are 143 and 225, respectively. Of course, the modified method may not always have the advantage over the simplex. Sometimes, however, in using the two equations as the testing sample, it appears to hold a slight advantage over the simplex method. The problem which was originally used by Box to test a constrained maximization procedure, has been changed to a minimization problem. The modified method and the simplex method have then been employed to solve the problem. The original problem has a maximum value of 1 at the point $(3, \sqrt{3})$; however, the transformed problem has a minimum value of zero at $(3, \sqrt{3})$. In other words, the transformed minimization function has the minimum value at the same vertex. The problem is stated as follows:

TABLE AIII-3

Minimum Number of Evaluations Required For Different Step-Lengths for Functions (1) and (2) from Tables (1) and (2).

Step-Length	Function (1)	(2)
0.2	149	
0.5		215
0.8	98	
1.0		226
1.2	121	
2.0		207
2.4	158	
4.0		213
Mean	131	215

TABLE AIII-4

Comparison of the results obtained with the best strategies by the modified method and simplex method

Step-Length	The Modified Method		The Simplex Method	
	Function (1)	Function (2)	Function (1)	Function (2)
0.2	149		146	
0.5		215		225
0.8	98		129	
1.0		226		233
1.2	121		158	
2.0		207		223
2.4	158		140	
4.0		213		220
Mean	131	215	143	225

Minimize the function f, of 2 variables, subject to 3 constraints given below;

$$f = 1 - [9 - (x_1 - 3)^2]\frac{x_2^3}{27\sqrt{3}} .$$

subject to

$$0 \leq x_1$$

$$0 \leq x_2 \leq \frac{x_1}{\sqrt{3}}$$

$$0 \leq x_3 = x_1 + \sqrt{3}\,(x_2) \leq 6$$

The initial point used in this problem was

$$x_1 = 1$$

$$x_2 = 0.5$$

Corresponding to f = 0.98664.

The optimum value is 0 at $x_1 = 3$, $x_2 = \sqrt{3}$.
The simplex method has given rise to a function value of 2×10^{-7} at (3, 1.732) after 179 evaluations; however, the modified method yield a function value of 2×10^{-7} at (3, 1.732) after 161 evaluations and incidently it reached a function value of 10^{-99} after 196 evaluations! Comparison of the results obtained with the best strategies by the modified method and the simplex method is also presented in Table AIII-4.

The method has been written as a subroutine in FORTRAN II language. The details of the computer program are described in the comment statements. Although the search deck has been built for minimization problems, the same deck can also be used for maximization problems. Specifically, when S_j is to be maximized, $(- S_j)$ is used as the objective function to be minimized in the search deck. This eventually gives rise to a desired result since the maximum of an objective function is equivalent to the negative of the minimization of its negative, i.e. max (S_j) = - min $(-S_j)$. The flow charts of the modified method and the built-up computer program are presented in Figures AIII-1 through AIII-6.

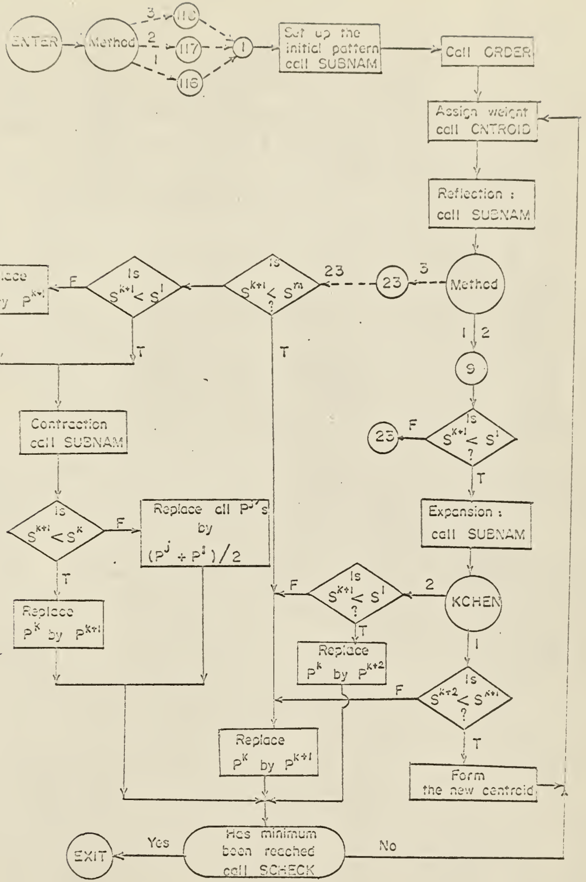

Fig. AIII-2. The flow chart for the build-up computer program.

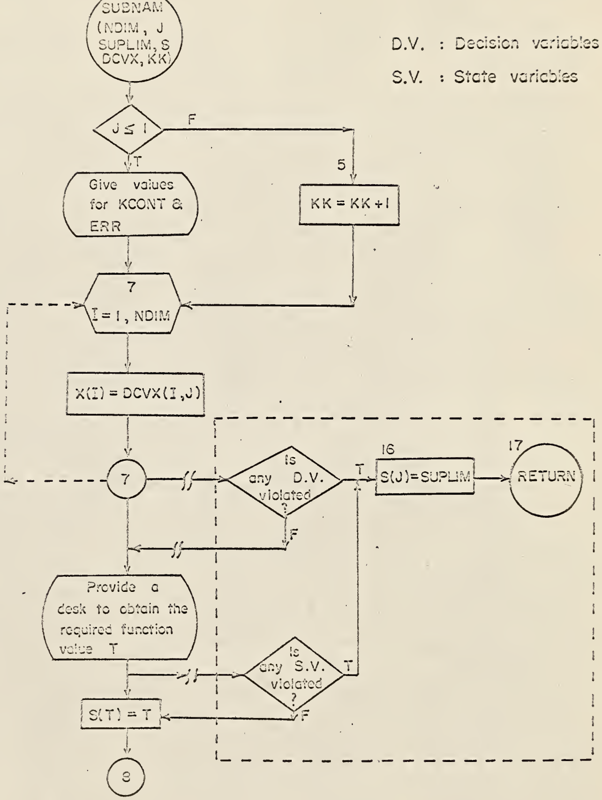

Fig. AIII – 3(a). The first part of the flow chart of subroutine SUBNAM.

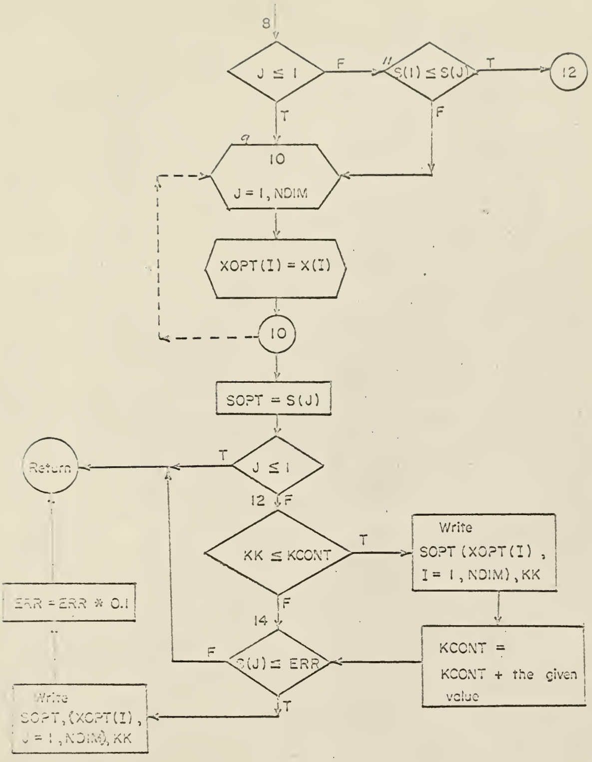

Fig. AIII – 3(b). The second part of the flow chart of subroutine SUBNAM.

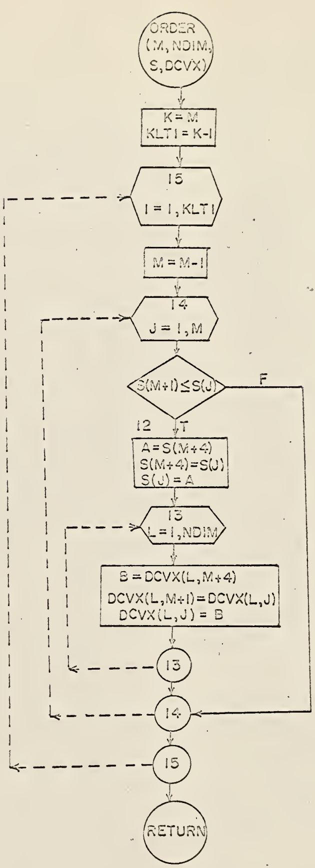

Fig. A III-4. Flow chart of subroutine ORDER.

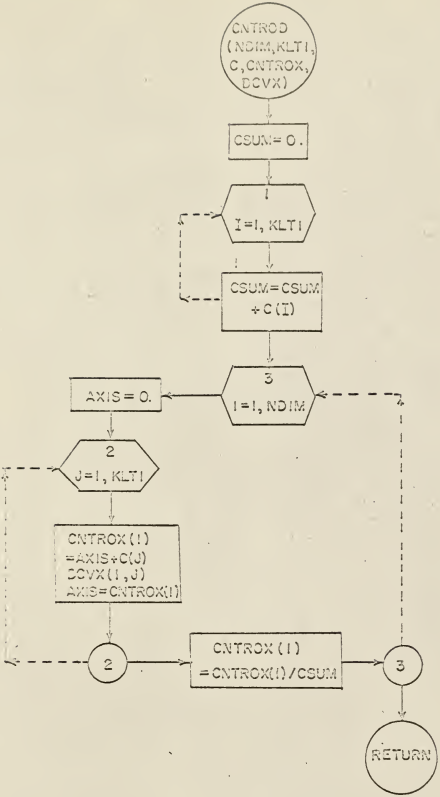

Fig. A III – 5. The flow chart for subroutine CNTROD .

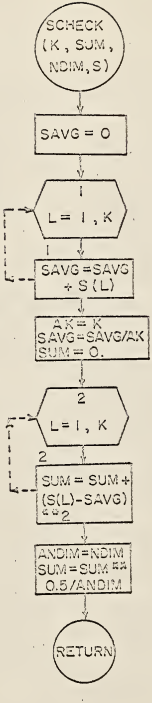

Fig. AIII-6. Flow chart for subroutine SCHECK.

PROGRAM SYMBOLS	EXPLANATION	MATHEMATICAL NOTATION
ALPHA	The reflection coefficient of the worst point with respect to the centroid.	α
BETA	The contraction coefficient of the worse reflected point to the centre.	β
C(L)	The weight assigned to the Lth vertex of the pattern.	C_{ℓ}
CNTROD(I)	The Ith decision variable of the centroid.	$\bar{x}_i$
DCVX(I,J)	The Ith decision variable x_i at the Jth vertex in an N dimensional space	x_i^j
DLTVX(I,L)	The increment of the Ith decision variable from the starting vertex to the Lth remaining vertex of the initial pattern.	Δx_i^{ℓ}
ERROR	The prescribed accuracy of the function value for stopping the computation.	
GAMMA	Expansion coefficient.	γ
J	The Jth vertex of the pattern.	j
K	The maximum points used for setting up the initial pattern.	k
KK	Number of acrual function evaluation.	
MAXNO	Maximum number of function evaluation set for terminating the computation.	
MDIM	Maximum dimensionality used in the search deck.	k+2
METHOD	1 indicates the modified method; 2, the simplex method and 3, the Box method.	
NDIM	Number of decision variables.	n

NDIMP1	The number of vertices other than the starting point for forming the initial pattern.	$k-1$.
NOPT	Number of vertices of the pattern to which the desired information will be written out.	
S(J)	The function value of the Jth vertex.	S_j
SUPLIM	Superlimit in constrained optimization problem, which is positively inifinite in minimization (negatively inifinite in maxization) when the constraints are violated.	

REFERENCE

(1) Chen, Gilbert K. C., Fan, L. T. and Wen, C. Y., "A Modified Direct Pattern Search Technique," Unpublished Report (1968).

APPENDIX IV
COMPUTER PROGRAMS

```
//MASTER    JOB   (15 47600 2,GILBERTCHEN,MSGLEVEL=1
//GREX YZG EXEC PROC=GOMATFOR
//GO.SYSIN DD  *
 JOB            GILBERT,RUN=CHECK,TIME=15,PAGES=400,LINES=60,KP=26
C
C
C      A NEW PATTERN SEARCH TECHNIQUE DEVELOPED AND WRITTEN BY
C      GILBERT K. C. CHEN  DEPT. OF CHEM. ENGG.  KSU  DEC. 1967
C
C
C        PURPOSE
C           TO FIND THE BEST FUNCTION VALUE OF A FUNCTION WITH N
C           INDEPENDENT VARIABLES AND THE SET OF INDEPENDENT VARI-
C           ABLES WHICH PRODUCES THIS OUTCOME.
C
C        USAGE
C
C           A PART OF THE SUBROUTINE CALLED SUBNAM SHOULD BE
C           WRITTEN AND PLUGGED IN THE PROVIDED SUBROUTINE DECK
C           TOGETHER WITH SOME ARRANGEMENTS BY THE USER, IF NECE-
C           SSARY.
C
C        DESCRIPTION OF PARAMETERS
C
C           ALPHA.. REFLECTION FACTOR WITH A VALUE BETWEEN 1.0
C                   AND 1.5.
C
C           BETA.. CONTRACTION FACTOR BETA= 0.5 HAS BEEN SET IN
C                  THE SEARCH DECK. ITS RANGE LIES BETWEEN 0 AND
C                  1.
C
C           C(J).. THE WEIGHT OF THE JTH VERTEX OF THE PATTERN.
C                  ..DIMENSION..(K-1).
C
C           CNTROX(I)..THE ITH DECISION VARIABLE AT THE CENTROID
C                      OF THE PATTERN. ..DIMENSION..(K).
C
C           DCVX(I,J)..THE ITH INDEPENDENT VARIABLE AT THE JTH
C                      VERTEX OF THE N DIMENSIONAL SPACE. ..DI-
C                      MENSION..(NDIM,K),(K=NDIM+JMCHEN),...WHERE
C                      JMCHEN=1 IN THE NEW METHOD AND THE SIMPLEX
C                      , =NDIM IN THE BOX METHOD.
C
C           DLTVX(I,J)..THE INCREMENT OF THE ITH INDEPENDENT VA-
C                       RIABLE FROM THE INITIAL VERTEX TO THE LTH
C                       REMAINING VERTEX OF THE INITIAL PATTERN.
C                       ..DIMENSION..(NDIM,K).
C
C           ERROR.. THE PRESCRIBED ACCURACY OF THE FUNCTION VALUE
```

```
          FOR STOPPING THE COMPUTATION.

   GAMMA.. EXPANSION FACTOR GAMMA=2 HAS BEEN SET .

    K.. MAXIMUM NO. VERTICES USED FOR SETTING UP THE INITIAL
        PATTERN.

    KK.. NO OF ACTUAL FUNCTION EVALUATION.

   MAXNO..  MAX. NO. OF FUNCTION EVALUATION SET BY THE
            USER FOR TERMINATING THE COMPUTATION WHEN THE
            NO. OF FUNCTION EVALUATION EXCEEDS THIS GIVEN
            VALUE. MAX. VALUE=99998.

   METHOD.. =1 ..THE NEW DEVELOPED SEARCH TECHNIQUE.
            =2 ..THE SIMPLEX METHOD.
            =3 ..THE BOX METHOD

    NDIM.. NO. OF DECISION VARIABLES, N.

    NDIMP1.. THE NO. OF VERTICES OTHER THAN THE STARTING
             POINT IN FORMING THE INITIAL PATTERN. NDIMP=
             K-1.

    NOPT.. NO. OF VERTICES OF THE PATTERN TO WHICH THE DE-
           SIRED INFORMATION WILL BE WRITTEN OUT.  MAX. NO.
           =K.

    S(J).. THE FUNCTION VALUE OF THE JTH VERTEX. ..DIMEN-
           SION..(K).

    SUPLIM.. THIS IS A SUPERLIMIT SET BY THE USER IN CON-
             STRAINED OPTIMIZATION PROBLEMS, WHICH IS PO-
             SITIVELY INFINITE IN MINIMIZATION (NEGATIVELY
             INFINITE IN MAXIMIZATION) WHEN THE CONSTRAINTS
             ARE VIOLATED.


 REMARKS

    THE DIMENSION STATEMENT IN THE DECK HAS BEEN BUILT FOR
    A FUNCTION WITH 27 DECESION VARIABLES WHEN METHOD 1
    AND 2 ARE USED. IF METHOD 3 IS USED. IT CAN ONLY USED
    FOR A FUNCTION WITH 14 DECESION VARIABLES.

    THE DATA OF THE PARAMETERS SHOULD BE PROVIDED BY THE
    USER ARE
    1) NDIM
    2) DCVX(I,1)
    3) DLTVX(I,J),I=1,NDIM AND J=1,NDIMP1
    4) NOPT
    5) MAXNO
    6) METHOD
```

```
C        7) ERROR (E.G. 1.0E- 8)
C        8) SUPLIM = 0 IN UNCONSTRATED PROBLEM, IN CONSTRAINED
C           PROBLEM SEE DESCRIPTION OF SUPLIM.
C
C     ** THE USERS ARE ENCOURAGED TO READ THROUGH CAREFULLY THE
C        COMMENT STATEMENTS IN THE PROVIDED DECK.
C
C     ** THE SEARCH DECK HAS BEEN BUILT FOR MINIMIZATION PRO-
C        BLEM. HOWEVER THE SAME DECK CAN ALSO BE USED FOR MAX.
C        PROBLEM IF -S(J) INSTEAD OF S(J)(S(J) IS THE REAL
C        FUNCTION VALUE THE MAX. PROBLEM) IS USED, I.E. S(J)=
C        -T.
C
C    ILLUSTRATION...UNCONSTRAINED PROBLEM
C
C        TO MINIMIZE S(X,Y)=X*X+Y*Y+1. WITH AN ARBITRARY START-
C        ING POINT, S(10,5).
C        THE INPUT DATA ARE
C        1) NDIM=2
C        2) NDIMP1=2 FOR METHOD 1 AND 2, FOR METHOD 3 NDIMP1=3
C        3) DCVX(1,1)=10. ,DCVX(2,1)=5.
C        4) DLTVX(1,1)=0.5   ,DLTVX(2,1)=0.
C           DLTVX(1,2)=0.    ,DLTVX(2,1)=0.25 .
C        5) NOPT=2
C        6) MAXNO=1000
C        7) ERROR=1.0E-08
C        8) METHOD=1
C        9) SUPLIM=0.
C
C
C        THE PART OF SUBROUTINE TO BE WRITTEN AND PLUGGED IN IS
C           T=X(1)*X(1)+X(2)*X(2)+1
C        IN ORTHER WORDS, THE CONTINUOUS THREE CARDS ARE
C          7 CONTINUE
C            T=X(1)*X(1)+X(2)*X(2)+1
C            S(J)=T
C
C
```

```
C
C     THIS IS THE MAIN PROGRAM FOR PROVIDING THE NECESSARY DATA OF
C     THE PARAMETERS.
      DIMENSION DLTVX(27,28),S(30),DCVX(27,30)
  101 FORMAT(10I5)
  102 FORMAT(7E10.4)
  103 FORMAT(/16H EVALUATION NO =I5/)
  104 FORMAT(6E13.6)
      READ(1,101)NDIM,NOPT,NDIMP1,MAXNO,METHOD
      READ(1,102)ERROR,SUPLIM
      READ(1,102)(DCVX(I,1),I=1,NDIM)
      READ(1,102)((DLTVX(I,J),I=1,NDIM),J=1,NDIMP1)
      WRITE(3,101)NDIM,NOPT,NDIMP1,MAXNO,METHOD
      WRITE(3,104)ERROR,SUPLIM
      WRITE(3,104)(DCVX(I,1),I=1,NDIM)
      WRITE(3,104)((DLTVX(I,J),I=1,NDIM),J=1,NDIMP1)
      CALL GKCHEM(NDIM,METHOD,MAXNO,ERROR,SUPLIM,DLTVX,DCVX,S,KK)
      WRITE(3,104)S(NDIM+2),(DCVX(I,NDIM+2),I=1,NDIM)
      WRITE(3,104)((DCVX(I,J),I=1,NDIM),J=1,NOPT)
      WRITE(3,104)(S(I),I=1,NOPT)
      WRITE(3,103)KK
      END
```

```
C
C
C     THE NEW PATTERN SEARCH TECHNIQUE DEVELOPED AND WRITTEN BY
C     GILBERT CHEN CHEM. ENGG. KSU DEC. 1967
C
C     THE FOLLOWING PROGRAM HAS BEEN WRITTEN IN FORTRAN II AND
C     PUNCHED IN KEYPUNCHER 26.
C
C
      SUBROUTINE GKCHEN(NDIM,METHOD,MAXNO,ERROR,SUPLIM,DLTVX,DCVX
     1,S,KK)
      DIMENSION DLTVX(27,28),C(28),DCVX(27,30),S(30),CNTROX(27)
  110 FORMAT(/19H THIS IS NEW METHOD/)
  111 FORMAT(/16H THIS IS SIMPLEX/)
  112 FORMAT(/12H THIS IS BOX/)
  113 FORMAT(/16H ****WARNING****/)
  114 FORMAT(49H INADEQUATE GIVEN MAX NO FOR FUNCTION EVALUATION,)
  115 FORMAT(47H INCREASING THE MAXNO OR CHANGING THE STEP SIZE/)
      GO TO (116,117,118),METHOD
C
C     THE SEARCH BEGINS WITH THE CHOSEN METHOD.
C
C     THIS IS THE NEW METHOD.
C
  116 JMCHEN=1
      KCHEN=1
      ALPHO=1.0
      BETA=0.5
      COEFF=1.2
      GAMMA=2.0
      WRITE(3,110)
      GO TO 1
C
C     THIS IS THE SIMPLEX.
C
  117 JMCHEN=1
      KCHEN=2
      ALPHO=1.0
      BETA=0.5
      GAMMA=2.0
      WRITE(3,111)
      GO TO 1
C
C     THIS IS BOX.
C
  118 JMCHEN=NDIM
      ALPHO=1.3
      BETA=0.5
      WRITE(3,112)
C
C     NO STATEMENTS FROM NOW ON CAN BE REMOVED EXCEPT YOU ARE SURE
C     WHAT TO DO.
C
```

```
C     SET UP THE INITIAL PATTERN
C
C     1) EVALUATION OF THE GIVEN INITIAL POINT
C
    1 J=1
      KK=1
      CALL SUBNAM(NDIM,J,SUPLIM,S,DCVX,KK)
      K=NDIM+JMCHEN
      KLT1=K-1
C
C     2) EVALUATION OF THE REMAINING POINTS OF THE INITIAL PATTERN
C
      DO 3  J=2,K
      DO 2 I=1,NDIM
    2 DCVX(I,J)=DCVX(I,1)+DLTVX(I,J-1)
      CALL SUBNAM(NDIM,J,SUPLIM,S,DCVX,KK)
    3 CONTINUE
    4 I=K
      ALPHA=ALPHO
C
C     ORDERING THE FUNCTION VALUES OF THE PATTERN
C
      CALL ORDER(M,NDIM,S,DCVX)
C
C     DEFINING THE CENTROID TO OBTAIN THE FURTHER SEARCH
C
      DO 5  I=1,KLT1
    5 C(I)=1.
      CALL CNTROD(NDIM,KLT1,C,CNTROX,DCVX)
C
C     REFLECTING OPERATION
C
    6 DO 7  I=1,NDIM
    7 DCVX(I,K+1)=CNTROX(I)+ALPHA*(CNTROX(I)-DCVX(I,K))
      J=K+1
      CALL SUBNAM(NDIM,J,SUPLIM,S,DCVX,KK)
      IF(KK-MAXNO)8,8,36
    8 GO TO (9,9,23),METHOD
C
C     NO EXPANSION IN BOX METHOD,THAT IS THE SIGNIFICANT DIFFER-
C     ENCE.
C
    9 IF(S(K+1)-S(1))10,10,23
C
C     EXPANDING OPERATION
C
   10 DO 11 I=1,NDIM
   11 DCVX(I,K+2)=CNTROX(I)+GAMMA*(DCVX(I,K+1)-CNTROX(I))
      J=K+2
      CALL SUBNAM(NDIM,J,SUPLIM,S,DCVX,KK)
      IF(KK-MAXNO)12,12,36
C
C     THE DIFFERENCE OF THE NEW METHOD FROM THE SIMPLEX
```

```
C
   12 GO TO (16,13),KCHEN
   13 IF(S(K+2)-S(1))14,14,21
   14 S(K)=S(K+2)
      DO 15 L=1,NDIM
   15 DCVX(L,K)=DCVX(L,K+2)
      GO TO 35
   16 IF(S(K+2)-S(K+1))17,17,21
   17 S(K)=S(K+2)
      DO 18 L=1,NDIM
   18 DCVX(L,K)=DCVX(L,K+2)
      M=K
      CALL ORDER(M,NDIM,S,DCVX)
      CALL SCHECK(K,SUM,NDIM,S)
      IF(SUM-ERROR)37,37,19
C
C     DEFINING THE NEW CNTROD ACCORDING TO THE IDEA OF THE NEW
C     METHOD
C
   19 CVALUE=2*NDIM-1
      DO 20 I=1,KLT1
      C(I)=CVALUE
   20 CVALUE=2*NDIM-2
      CALL CNTROD(NDIM,KLT1,C,CNTROX,DCVX)
      ALPHA=ALPHO*COEFF
      GO TO 6
   21 S(K)=S(K+1)
      DO 22 L=1,NDIM
   22 DCVX(L,K)=DCVX(L,K+1)
      GO TO 35
   23 IF(S(K+1)-S(K-1))21,21,24
   24 IF(S(K+1)-S(K))25,25,27
   25 S(K)=S(K+1)
      DO 26 I=1,NDIM
   26 DCVX(I,K)=DCVX(I,K+1)
C
C     CONTRACTING OPERATION
C
   27 DO 28 I=1,NDIM
   28 DCVX(I,K+1)=CNTROX(I)+BETA*(DCVX(I,K)-CNTROX(I))
      J=K+1
      CALL SUBNAM(NDIM,J,SUPLIM,S,DCVX,KK)
      IF(KK-MAXNO)29,29,36
   29 IF(S(K+1)-S(K))30,30,32
   30 S(K)=S(K+1)
      DO 31 I=1,NDIM
   31 DCVX(I,K)=DCVX(I,K+1)
      GO TO 35
C
C     SHRINKING THE PATTERN DUE TO A BAD CONTRACTION
C
   32 DO 34 J=2,K
      DO 33 I=1,NDIM
```

```
33 DCVX(I,J)=(DCVX(I,1)+DCVX(I,J))/2.
   CALL SUBNAM(NDIM,J,SUPLIM,S,DCVX,KK)
34 CONTINUE
   IF(KK-MAXNO)35,35,36
35 CALL SCHECK(K,SUM,NDIM,S)
   IF(SUM-ERROR)37,37,4
C
C     THE SEARCH IS INCOMPLETE ACCORDING TO THE+GIVEN INODEQUATE
C     MAXNO.
C
36 WRITE(3,113)
   WRITE(3,114)
   WRITE(3,115)
   GO TO 40
C
C     THE SEARCH IS COMPLETED, RETURN TO THE MAIN PROGRAM AFTER
C     EVALUATING THE CNTROD OF THE PATTERN.
C
37 DO 38 I=1,KLT1
38 C(I)=1.
   CALL CNTROD(NDIM,KLT1,C,CNTROX,DCVX)
   DO 39 I=1,NDIM
39 DCVX(I,K+1)=CNTROX(I)
   J=K+1
   CALL SUBNAM(NDIM,J,SUPLIM,S,DCVX,KK)
40 RETURN
   END
C
C
C     THIS SUBROUTINE SUBNAM SHOULD PROVIDED BY USER FOR OBTAIN-
C     ING THE REQUIRED OBJECTIVE FUNCTION VALUE.
C
C     KCONT...A CONTROL NUMBER SET FOR OUTPUT. FOR EVERY KCONT
C             NO. OF FUNCTION EVALUATIONS
C
C     ERR.....A FUNCTION VALUE SET FOR THE DATA TO BE WEITTEN OUT
C             AS THE COMPUTED FUNCTION VALUE DROPPED A TENTH ORDER
C             EACH TIME.
C     SOPT....THE BEST FUNCTION VALUE HAS BEEN FOUND AT EACH STAGE
C             THE COMPUTATION.
C
C     XOPT(I).THE CORRESPONDING ITH DECISION VARIABLE OF SOPT.
C
      SUBROUTINE SUBNAM(NDIM,J,SUPLIM,S,DCVX,KK)
      DIMENSION S(30),DCVX(27,30),X(27),XOPT(27)
    1 FORMAT(31H THE OPTIMUM FUNCTION VALUE IS E13.6)
    2 FORMAT(6E13.6)
    3 FORMAT(10I4)
      IF(J-1)4,4,5
    4 KCONT=10
      ERR=10.
      GO TO 6
    5 KK=KK+1
```

```
C
C     TRANSLOCATION OF THE VALUES OF THE INDEPENDENT VARIABLES
C     FROM THE SEARCH DECK TO THOSE USED IN THIS SPECIAL PROBLEM,
C     WHERE X(I) IS THE ITH INDEPENDENT VARIABLE IN THE USER PRO-
C     BLEM.
C
    6 DO 7  I=1,NDIM
      X(I)=DCVX(I,J)
    7 CONTINUE
C
C     THE USER SHOULD PROVIDE A PART OF THIS SUBROUTINE FOR OB-
C     TAINING THE REQUIRED FUNCTION VALUE AT EACH VERTEX BETWEEN
C     THIS COMMENT STATEMENT AND THE FOLLOWING STATEMENT IN WHICH
C     T MEANS THE REQUIRED FUNCTION VALUE.
C
      CALL        WASTE (J,NDIM,XX,AK1,BETA,R,AK)
C
      S(J)=T
C     STORAGE OF BETTER FUNCTION VALUE WITH THE CORRESPONDING IN-
C     DEPENDENT AND DEPEDENT VARIABLES, IF NECESSARY.
      IF(J-1)9,9,11
    9 DO 10 I=1,NDIM
      XOPT(I)=X(I)
   10 CONTINUE
      SOPT=T
      IF(J-1)17,17,12
   11 IF(S(1)-S(J))12,9,9
   12 IF(KK-KCONT)14,13,13
   13 WRITE(3,1)SOPT
      WRITE(3,2)(XOPT(I),I=1,NDIM)
      WRITE(3,3)KK
      KCONT=KCONT+10
   14 IF(S(J)-ERR)15,15,17
   15 WRITE(3,1)SOPT
      WRITE(3,2)(XOPT(I),I=1,NDIM)
      WRITE(3,3)KK
      ERR=ERR*0.1
      GO TO 17
   16 S(J)=SUPLIM
   17 RETURN
      END
```

```
      SUBROUTINE ORDER(N,NDIM)
      DIMENSION S(30),DCVX(27,30)
      K=N
      KLT1=K-1
      DO  5 I=1,KLT1
      M=K-1
      DO  4 J=1,M
      IF(S(M+1)-S(J))2,2,4
    2 A=S(M+1)
      S(M+1)=S(J)
      S(J)=A
      DO  3 L=1,NDIM
      B=DCVX(L,M+1)
      DCVX(L,M+1)=DCVX(L,J)
      DCVX(L,J)=B
    3 CONTINUE
    4 CONTINUE
    5 CONTINUE
      END
```

```
C
C
C     THE SUBROUTINE FOR ORDERING THE FUNCTION VALUES OF THE PATTERN
C
C
C
C     A NECESSARY PART OF THE WHOLF SEARCH DECK BUILT FOR OBTAINING
C     THE CENTROID OF THE PATTERN EXCLUSIVE OF THE WORST POINT.
C
      SUBROUTINE CNTROD(NDIM,KLT1,C,CNTROX,DCVX)
      DIMENSION C(28),CNTROX(27),DCVX(27,30)
C     SEARCHING FOR THE BETTER POINT OF THE SPACE
      CSUM=0.
      DO 1  I=1,KLT1
    1 CSUM=CSUM+C(I)
      DO 3  I=1,NDIM
      AXIS=0.
      DO 2  J=1,KLT1
      CNTROX(I)=AXIS+C(J)*DCVX(I,J)
      AXIS=CNTROX(I)
    2 CONTINUE
      CNTROX(I)=CNTROX(I)/CSUM
    3 CONTINUE
      RETURN
      END
```

```
C     THE BUILT-IN SUBROUTINE FOR CHECKING WHETHER THE OPTIMUM
C     POINT HAS BEEN ACHIEVED
C     THE CRITERION USED IS SQRT(((AVG.(S)-S(J))**2/NDIM),J=1,K)
C     .L.E. ERROR.
C     ERROR.
C
      SUBROUTINE SCHECK(K,SUM,NDIM,S)
      DIMENSION S(30)
      SAVG=0.
      DO 1  L=1,K
    1 SAVG=S(L)+SAVG
      AK=K
      SAVG=SAVG/AK
      SUM=0.
      DO 2  L=1,K
    2 SUM=SUM+(S(L)-SAVG)**2
      ANDIM=NDIM
      SUM=SUM **0.5/ANDIM
      RETURN
      END
```

```
      SUBROUTINE WASTE (J,NDIM,XX,AK1,BETA,R,AK)
      DIMENSION S(7),DCVX(4,7),G(4),TH3(6),TH10(5),TH30(7),X10(7)
      DIMENSION X1(6),X2(6),X3(6),TH1(5),TH2(6),A(6),B(6),AX2(6)
      DIMENSION BX2(6),X20(6)
      COMMON S,DCVX,G,ERROR,DCVZ,NJ
    1 FORMAT(3H J=I2,6H S(J)=E13.6)
    3 FORMAT(6E13.6)
   66 FORMAT(4H BT=F5.3,4H K1=F5.3,7H X1(6)=F5.3)
    6 FORMAT(4H K1=F5.2)
    7 FORMAT(6(13H ------------))
   33 FORMAT(6(13H ************)/)
  400 FORMAT(12H 1ST TANK     F7.3,4X,F6.3,4X,F6.3,2X,F6.3,4X,
     1F7.3,2X,F7.3,4X,F6.3)
  401 FORMAT(12H 2ND TANK     F7.3,4X,F6.3,4X,F6.3,2X,F6.3,4X,
     1F7.3,2X,F7.3,4X,F6.3)
  402 FORMAT(12H 3RD TANK     F7.3,4X,F6.3,4X,F6.3,2X,F6.3,4X,
     1F7.3,2X,F7.3,4X,F6.3)
  404 FORMAT(84H            VOL.       OF FEED     INLET(1)  OUTLET(2)
     1INLET(1)   OUTLET(2)   OF VOL.)
  405 FORMAT(67H                                          ----------- --
     1-----------)
      X(3)=DCVZ(3,NJ)
      X(4)=DCVZ(4,NJ)
      X2F=0.
      AKD=0.002
      NSTG=(NDIM+2)/2
      NSTG2=NSTG*2
      NDIMH=NDIM/2
      X1(2*NSTG)=XX
      P=1.+R
      DO 5 I=1,NDIM
      IF(I-NDIMH)44,44,4
   44 II=NSTG-I+1
      TH1(2*II-1)=DCVX(1,J)
      GO TO 5
    4 X1(2*I-4)=DCVX(I,J)
    5 CONTINUE
      DO 503 I=2,NSTG
      IF(TH1(2*I-1))56,500,500
  500 IF(TH1(2*I-1)-1.)501,501,56
  501 IF(X1(2*I-2))56,56,502
  502 IF(X1(2*I-2)-((1.+X1(2*NSTG)*R)/(1.+R)))503,503,56
  503 CONTINUE
    9 AA=0.
      DO 10 N=2,NSTG
      AA=AA+TH1(2*N-1)
   10 CONTINUE
      IF(AA-1.)101,101,56
  101 TH1(1)=1.-AA
      DO 13 N=1,NSTG
      IF(N-1)100,100,11
  100 X2(2*N-1)=R+TH1(2*N-1)
      GO TO 12
```

```
11 X3(2*N-1)=X3(2*N-2)+TH1(2*N-1)
12 X3(2*N)=X3(2*N-1)
13 CONTINUE
   X1(1)=(X1(2*NSTG)*R+TH1(1)),X3(1)
   DO 40 N=2,NSTG
  X1(2*N-1)=(X1(2*N-2)*X3(2*N-2)+TH1(2*N-1))/X3(2*N-1)
40 CONTINUE
   DO 41 N=1,NSTG
   IF(X1(2*N-1)-X1(2*N))56,41,41
41 CONTINUE
   DO 70 N=1,NSTG
   B(2*N)=(X1(2*N-1)-X1(2*N))*(AK*X1(2*N)-AKD*(AK1+X1(2*N)))
   B(2*N)=B(2*N)/(AK*X1(2*N))+TH1(2*N-1)*X2F/X3(2*N-1)
   IF(N-1)65,65,60
65 A(2*N)=R/X3(2*N)
   GO TO 70
60 A(2*N)=X3(2*N-2)/X3(2*N)
70 CONTINUE
   DO 19 N=1,NSTG
   AO=BETA
   DO 15 I=1,N
   AX2(N)=AO*A(2*I)
   AO=AX2(N)
   IF(N-1)15,16,15
15 CONTINUE
   GO TO 17
16 BX2(N)=B(2*N)
   GO TO 19
17 BX2(N)=B(2*N)+A(2*N)*BX2(N-1)
19 CONTINUE
   X2(2*NSTG)=BX2(NSTG)/(1.-AX2(NSTG)).
   M=NSTG-1
   DO 30 N=1,M
   IF(N-1)31,31,32
31 X2(2*N)=A(2*N)*BETA*X2(2*NSTG)+B(2*N)
   GO TO 30
32 X2(2*N)=A(2*N)*X2(2*N-2)+B(2*N)
30 CONTINUE
   DO 21 N=1,NSTG
   IF(N-1)190,190,20
190 X2(2*N-1)=X2(2*NSTG)*BETA*R/X3(2*N-1)
   GO TO 21
20 X2(2*N-1)=(X2(2*N-2)*X3(2*N-2)+TH1(2*N-1)*X2F)/X3(2*N-1)
21 CONTINUE
   SO=0.
   DO 22 N=1,NSTG
   TH2(2*N)=(X1(2*N-1)-X1(2*N))*(AK1+X1(2*N))/(AK*X1(2*N)*X2(2*N)
   TH3(2*N)=TH2(2*N)*X3(2*N)/(1.+R)
   T=SO+TH3(2*N)
   SO=T
22 CONTINUE
   S(J)=T
C     EXCHANGE OF MINIMUM DECISION AND STATE VARIABLES
```

```
      IF(J-1)222,222,223
  222 DO 224 N=1,NSTG2
      X10(N)=X1(N)
      X20(N)=X2(N)
  224 CONTINUE
      DO 225 N=1,NSTG
      TH30(2*N)=TH3(2*N)
      TH10(2*N)=TH1(2*N)
  225 CONTINUE
      ST=S(J)
      R1=TH3(2)/S(J)
      R2=TH3(4)/S(J)
      R3=TH3(6)/S(J)
      IF(J-1)220,220,226
  223 IF(S(1)-S(J))226,222,222
  226 IF(J-(NDIM+2))220,221,221
      WRITE(3,1)J,ST
      WRITE(3,6)AK1
      WRITE(3,7)
      WRITE(3,406)
      WRITE(3,405)
      WRITE(3,404)
      WRITE(3,400)TH30(2),TH10(1),X10(1),X10(2),X20(1),X20(2),R1
      WRITE(3,401)TH30(4),TH10(3),X10(3),X10(4),X20(3),X20(4),R2
      WRITE(3,402)TH30(6),TH10(5),X10(5),X10(6),X20(5),X20(6),R3
      WRITE(3,33)
  220 RETURN
   56 T   =10.**6
      RETURN
      END
```

MODELING AND OPTIMIZATION OF THE HYDRAULIC REGIME OF ACTIVATED SLUDGE SYSTEMS

by

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B.S., National Taiwan University, China 1965

AN ABSTRACT OF A MASTER'S THESIS

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Chemical Engineering

KANSAS STATE UNIVERSITY
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1968

ABSTRACT

The pattern of flow into the system, the recycle flow, and the mixing within the system are important variables which need to be considered in the design biological waste treatment systems. In this investigation, optimization procedures are used to determine the optimum flow regime for several types of activated sludge systems. Step aeration and conventional activated sludge systems composed of several completely mixed aeration tanks connected in series are optimized and the results are compared. The analysis indicates that the degree of treatment and the Michaelis-Menten constant, which is the dimensionless organic nutrient concentration at which the observed specific growth rate is one half the maximum value, are important parameters in selecting the optimal flow regime.