A THERMISTOR ANEMOMETER FOR VERY LOW AIR VELOCITIES

by

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DEFINITIONS OF THERMISTOR TERMS AND SYMBOLS

The following definitions and symbols are essentially the same as those specified in Electronic Industries Association (EIA) standard RS-275 (Feb. 1963), and are adapted from ref. 30.

1. THERMISTOR, a thermally sensitive resistor whose primary function is to exhibit a change in electrical resistance with a change in its body temperature. Most thermistors are metallic oxide semiconductors which exhibit high negative temperature coefficient of resistance.

2. ZERO-POWER RESISTANCE, (R, R_T, $R_o(T)$, R_o @T), the resistance value of a thermistor at a specified temperature with zero electrical power dissipation. For purposes of measurement the power dissipation in the thermistor is limited, so that any further decrease in power will result in not more than ± 0.01 % change in resistance.

3. STANDARD REFERENCE TEMPERATURE, the temperature of the thermistor body at which nominal zero-power resistance is

specified (25° C ± 0.05° C, unless otherwise specified).

4. RESISTANCE RATIO, (R_{T_1} / R_{T_2}, $R_o(T_1) / R_o(T_2)$, $R_o@T_1 / R_o@T_2$), the ratio of the zero-power resistances of a thermistor measured at two specified reference temperatures. The most common forms of specifying resistance ratio are either

$$\left(\frac{R_{0^\circ\ C}}{R_{50^\circ\ C}}\right)$$

or

$$\left(\frac{R_{25^\circ\ C}}{R_{125^\circ\ C}}\right)$$

5. ZERO-POWER RESISTANCE-TEMPERATURE CHARACTERISTIC, the relationship between the zero-power resistance of a thermistor and its body temperature. This characteristic may be approximated by the classical thermistor formula:

$$R_T = R_{T_o}\ e^{\beta\left(\frac{1}{T} - \frac{1}{T_o}\right)} \qquad (1)$$

with:

$$T_1 \leq T \leq T_2$$

$$T_1 \leq T_o \leq T_2$$

where β is the "material constant" of the thermistor, assumed to be constant over the temperature range $T_1 \leq T \leq T_2$, and T and T_o are absolute temperatures in degrees Kelvin. The zero-power resistance-temperature characteristic is often also found in terms of the logarithm of zero-power resistance and inverse absolute temperature. Expressed mathematically,

$$\log_e R_T = a + \beta / T;\ T_1 \leq T \leq T_2 \qquad (2)$$

where $a = \log_e R_{T_o} - \beta/T_o$, subject to the constraint $T_1 \leq T_o \leq T_2$.

6. BETA, (β), the material constant of a thermistor. Although β increases slightly with increasing temperature it may be considered constant over limited temperature spans of approximately 30° C to 70° C in width, depending upon the thermistor material and the absolute temperature at which the center of the span is located. The beta (β) of a thermistor may be determined from equations (1) or (2). Over the temperature span ($T_1 \leq T \leq T_2$) for which β may be considered constant, equation (2) is linear and may be plotted as a straight line between the points $1/T_1$ and $1/T_2$. Unless otherwise specified, β is derived from measurements at 0° C and 50° C and 125° C as follows:

$$\beta_{0^\circ C - 50^\circ C} = 1765.37 \log_e \frac{R_{0^\circ C}}{R_{50^\circ C}}; \qquad (3\text{-a})$$

$$\beta_{25^\circ C - 125^\circ C} = 1187.08 \log_e \frac{R_{25^\circ C}}{R_{125^\circ C}} \qquad (3\text{-b})$$

7. RESISTANCE RATIO-TEMPERATURE CHARACTERISTIC, (r_T, $r(T)$, r @ T), is defined as:

$$r_T = R_T / R_{T_o} = e^{\beta(1/T - 1/T_o)}$$

8. TEMPERATURE COEFFICIENT OF RESISTANCE. (α_T, $\alpha(T)$, α @ T), the ratio at a specified temperature, T, of the rate of change of zero-power resistance with temperature to the zero-power resistance of a thermistor. It is commonly expressed in percent per degree or in Ohms/Ohm/degree and is expressed

Mathematically as,

$$\propto_T = \frac{1}{R_T} \cdot \frac{dR_T}{dT} \simeq - \frac{\beta}{T^2} \qquad (4)$$

Unless otherwise specified, $\propto_T$ is given at 25° C and is obtained from equation (4) as:

$$\propto_{25^\circ C} = 0.019859 \log_e \frac{R_{0^\circ C}}{R_{50^\circ C}} \qquad (5)$$

9. <u>DISSIPATION CONSTANT</u>, (δ), the ratio, at a specified ambient temperature, of the power dissipated in a thermistor to the resultant change in its body temperature. Since the temperature rise in the thermistor due to dissipated power depends on the rate at which heat is transferred from it, the dissipation constant depends on the method of mounting the unit as well as the medium or environment in which the unit is located. Unless otherwise specified, the dissipation constant is given for the thermistor when supported by its leads in still air, at an ambient temperature of 25° C, within a test chamber having a volume greater than 1,000 times the volume of the thermistor under test. Usually, the power dissipated is taken as the power required to raise the body temperature of the thermistor by 50° C (from 25° ± 0.2° C to 75° ± 0.2° C). Since in still air a significant portion of the heat is transferred through the thermistor leads, the length of unsupported leads is critical. In the case of the thermistor chosen to be used in the anemometer (glass-coated bead) the minimum specified lead length is used. The dissipation constant is generally specified as a minimum value.

When a nominal value is given, the tolerance is ± 25%.

10. SELF-HEATED THERMISTOR, a thermistor whose body temperature is significantly higher than the temperature of its ambient medium as a result of the power being dissipated in it.

11. THERMAL TIME CONSTANT, (τ), the time required for a thermistor to change 63.2% of the difference between its initial and final body temperatures, when subjected to a step function change in temperature under zero-power conditions. Since the thermal time constant depends upon the rate of heat transfer between the thermistor and its surroundings, the method of mounting the unit as well as the surrounding medium must be specified. The test conditions are usually the same as those used for obtaining the dissipation constant, and the tests can be performed sequentially. In practice, with the thermistor stabilized at 75° C (after the dissipation constant has been determined), the power is switched to its "zero-power" level and the time required for the thermistor to cool to 43.4° C is its time constant.

For thermistor structures for which the heat capacity of the structure is significantly different from that of the enclosed thermistor element (i.e. bead-in-glass probes, bead-in-glass rods, etc.), the thermal time constant cannot be measured by allowing the thermistor to cool from a self-heated condition. In such cases, it becomes necessary to pre-heat the entire thermistor structure to the reference temperature (i.e. 75° C) by placing the unit in a controlled constant temperature bath or chamber. After the temperature of the unit has stabilized, the unit is then removed to still air at the reference ambient temperature (i.e. 25° C). The time required for the thermistor to cool to 43.4° C is its time constant.

The thermal time constant is generally specified as a maximum value. When a nominal value is given, the tolerance is $\pm$ 25%, unless otherwise specified.

12. <u>CURRENT TIME CHARACTERISTIC</u>, the relationship, at a specified ambient temperature, between the current through a thermistor and the time elapsed from the application of a step function of voltage. Unless otherwise specified, the test temperature shall be $25^\circ \pm 0.2^\circ$ C and the mounting and test conditions shall be as specified under dissipation constant.

13. <u>STATIC VOLTAGE-CURRENT CHARACTERISTIC</u>, the relationship, at a specified ambient temperature, between the voltage across a thermistor and the current through it under conditions of thermal equilibrium. For very small currents, for which the power dissipation is low, this characteristic approximates the linear relationship given by Ohm's Law. As self-heating of the thermistor is progressively increased, the slope of the characteristic, dV/di, continues to decrease until it becomes equal to zero. If the current is increased beyond this point ($dV/di = 0$), the slope continues to decrease and becomes negative. In this region, the thermistor is said to exhibit a negative resistance characteristic.

14. PEAK VOLTAGE, (V_{max}, V_m, V_p), the voltage point, on the static voltage-current characteristic, for which the slope of the characteristic is equal to zero ($dV/di=0$).

15. NEGATIVE RESISTANCE CHARACTERISTIC, that portion of the voltage-current characteristic beyond the point at which the peak voltage occurs.

16. STABILITY, the ability of a thermistor to retain specified characteristics after being subjected to designated environmental and/or electrical test conditions. Unless otherwise specified, the stability is given in terms of an allowable percentage change per year in R_o @ 25 °C. The stability for a small bead thermistor is 0.1 - 0.2 %, at a maximum operating temperature of 150 - 200 °C.

17. MAXIMUM OPERATING TEMPERATURE, the maximum ambient temperature at which a thermistor will operate for an extended period of time with acceptable stability of its characteristics.

CHAPTER I

INTRODUCTION

The idea of designing an anemometer to measure very low air velocities was motivated by one of the research projects presently being done in the Mechanical Engineering Department of Kansas State University. This project, called "Air Distributing Ceiling" (Dept. project No. 2432), is sponsored by the American Society of Heating, Refrigerating and Air Conditioning Engineers (ASHRAE Research Project - 55) and by the Acoustical Manufacturers Association (AMA). It requires accurate measurement of low air velocities at several points inside a test room, since the comfort criteria adopted in the project is a function of these velocities. The room is maintained at a constant temperature of about 75° F.

The desirable characteristics in an anemometer for such an application would be:

1. The instrument should be remote reading, so that air velocity measurements can be made without the operator entering the test room.

2. It should have a permanent calibration curve, with little or no drift after extensive use.

3. The anemometer, once calibrated, should have small sensitivity to minor ambient temperature fluctuations.

4. The anemometer should permit its connection to standard auxiliary equipment, in order to provide a continuous and automatic recording of air velocities.

5. An anemometer, in general, should be sensitive and accurate over a wide range of velocities. In this application, it should be sensitive from 10 to 200 fpm.

6. Its response should be completely independent of the particular direction of the moving air for "comfort" applications.

7. The anemometer should have a very fast response to velocity changes. Although in this application measurements are done in steady state, some fluctuation usually occurs in the velocity of the moving air and it is important to know the magnitude.

8. Production and calibration of the anemometer should be relatively easy.

9. Its construction should be inexpensive.

10. The sensor element should be as small as possible in order to avoid any perturbation of the flow pattern.

In order to achieve the objectives stated above, a compromise solution was sought whenever some of those requisites were contradictory. As an example, the compromise solution employed in order to fulfill the requirements of sensitivity and stability reduced the sensitivity to provide stability.

A thermistor was chosen to be the sensor element of the anemometer, due to the characteristics that will be discussed later.

In the following discussion, the circuit of the anemometer is synthesized and analysed, and a theoretical prediction of the performance of the instrument is calculated as the instrument output versus air velocity. These data are then compared with the experimental performance curve of the anemometer.

1.1. A Brief Literature Review

The present literature dealing with anemometers is very extense, for this instrument is known since the seventeenth century, when Robert Hook (1635-1703) built a mechanical device that indicated the magnitude of the wind velocity. Of course today's status-of-art of the anemometer is very well developed, with all sorts of instruments, based on different principles of operation, being available.

In this section, only the main types of thermal anemometers used in ventilating and air-conditioning practice, their particular applications, the range of air velocities in which they can be used, their precision and limitations, will be discussed. Part of the information presented here was obtained from the ASHRAE Guide and Data Book (Fundamentals and Equipment, 1963, pp. 256-258).

1. Hot wire anemometer. This instrument is the most re-

liable and precise for measurements of velocities in the range from 1,000 to 60,000 fpm. For these velocities its precision varies from 1 to 0.01 % for the lower and higher limits of the range, respectively. It is also the most satisfactory instrument for measuring turbulence due to the small size of the probe and to its very small thermal time constant. The sensing element is an electrically heated platinum or tungsten wire mounted between two small steel rods. The usual dimensions of the wire are from 0.0001 to 0.0003 inches in diameter, and from 0.025 to 0.06 inches in length. A constant current flowing through the wire produces its heating due to the Joule effect. In steady-state operation, the heat generated in the wire is equal to the heat loss to the ambient. This heat loss is a function of the wire temperature, ambient temperature, and the air velocity. L.V. King, in a famous work done in 1914 (ref.6), derived the following equation relating the heat loss and the above parameters:

$$Q = (C_1 + C_2.v^{1/2}) \cdot (T - T_o)$$

where Q is the rate of heat loss from the wire, C_1 and C_2 are constants, and T and T_o are the wire and ambient temperatures, respectively. The velocity of the moving air is v.

As the velocity of the air fluctuates, the temperature of the wire also fluctuates, and so does the wire resistance. Since the wire current is maintained constant, the voltage applied to the wire also fluctuates. Therefore, the fluctuations of the voltage are proportional to the fluctuations in air velocity. The main limitation of this instrument is the high cost and complexity of the electronic equipment necessary to its operation. In measurements of turbulence an additional complexity is required to compensate for wire lag and for nonlinear characteristics. Various arrangements and methods of using hot wires are available to allow the instrument to measure the direction of the moving air.

The precison of the hot wire anemometer in the range of velocities from 5 to 1,000 fpm is 20 and 1 %, respectively. For further information on this instrument, reference 7, pp. 121-128, should be consulted.

2. Kata thermometer, is an instrument useful for studying air currents in free spaces. It consists simply of an alcohol thermometer with a large bulb, and with only two divisions etched on its stem. In operation, this instrument is heated in a water bath until the alcohol has risen sufficiently above the upper division of the stem. It is then thoroughly dried and placed at the point where the velocity is to be measured. The measure of the air velocity is given by the time elapsed since the thermometer cools from the temperature indicated by its upper division to its lower division temperature, once the instrument was previously calibrated in this fashion. Usually the divisions are done at 95 and 100 ^{o}F, and at 125 and 130 ^{o}F.

The velocity range in which this anemometer is used is from 5 to 300 fpm, with a precision of 15 and 5 %, respectively. Its limitations are: it is awkward to use, is affected by radiation, and does not measure velocity fluctuations. However, it is an inexpensive instrument.

3. Heated-bulb thermometer, in which a heating wire is wound around the bulb of a mercury-in-glass thermometer, and the temperature difference between this thermometer and a similar unheated one serves as an index of the air velocity. Again, the accuracy of this instrument is not good at low air velocities, and the thermal time constant is relatively high, which limits its use to a stable velocity field.

4. Heated-thermocouple anemometer, consists of a hollow sphere (usually copper) with a thermocouple embedded in its surface, a heating resistor inside the sphere, and another thermocouple, placed close to the sphere, measuring the ambient temperature. This anemometer is calibrated to give air velocity in terms of the differential emf between the heated and unheated thermo-junctions, when it is exposed to the air stream.

The main advantage of this anemometer is that it provides combined measurements of air temperature and velocity at the same point of the space, which is very useful for air distribution studies. Since the size of the sphere should be limited to a minimum of about 5 mm in diameter (to provide space for the inner resistor), the temperature differential must be small to minimize natural convection effects. This fact limits considerably the sensitivity of the instrument when measuring low air velocities (less than 100 fpm). Ref.3 discusses this type of anemometer. Ref.11 describes a similar anemometer, in which the sphere was replaced by small heaters.

Another similar anemometer, designed to measure very low air velocities is described in ref. 17. This instrument differs from the hot-sphere type only in the mode of operation, which is based on the transient cooling of the sphere when exposed to moving air, instead of the steady-state operation of the former. The time required for the sphere to cool from an initial temperature to a lower value is a measure of the air velocity. Again, the thermal time constant of the sphere limits the aplication of this anemometer to measurements of stable velocities and the principle of operation does not permit an automatic recording of the measurements.

Several other anemometers, of non-thermal type, such as the pitot tube, swinging-vane type, venturi-type multiplying pitot tube, revolving-vane type, impact tube and side-wall or static tap anemometer, may be used in applications other than the present one, and their description and characteristics are given in detail in the ASHRAE Guide (op.cit., pp. 257-262).

The uses and applications of thermistors are not new. In 1946, a little before the semiconductors "revolution", Becker, et al., published the classical paper "Properties and Uses of Thermistors" (ref. 1), where a detailed discussion of the physical properties and applications of thermistors is given. The reason for using thermistors as velocity sensors is obviously due to their characteristics. Presently, thermistors are available in several different shapes and in very small sizes, with thermal time constants of the order of milliseconds, with very stable characteristics, and with close manufacture tolerances either in size or in their properties, which allows a good degree of interchange among units. Many other important reasons for choosing thermistors and the particular electric circuit adopted in the design are given in Chapter II. This circuit is classical and the principle of operation of the instrument is very simple. It was even mentioned in ref. 1 (1946). However, the selection of the particular thermistors, the determination of the values for the circuit elements that would provide a desired instrument response to a given air velocity range, the method of relating the effect of air velocity to the corresponding change in the thermistor dissipation constant, and the theoretical calculation of the anemometer sensitivity to changes in ambient temperature, were developed in this work. The circuit analysis used here is similar to the one discussed in ref. 14, where a circuit is designed for a thermal conductivity gas analyzer.

Extensive work has been and is still being done, in order to measure turbulence using thermistors (refs. 9 and 10).

Ref. 13 discusses the application of thermistors in measuring air and water velocities using a single thermistor operated at constant current and then at constant temperature.

In ref. 18 a simple anemometer is designed, using two thermistors. One thermistor is heated by an electric resistance while the other, the sensor, is exposed to the moving air. The advantage of this instrument is that it allows the measurement of very low air velocities, but has the inconvenient of requiring frequent calibrations due to its mode of operation.

In ref. 23 an ingenious technique is explained in order to build an accurate flowmeter, for very low flow rates, using a single thermistor. This instrument requires the rigorous control of the fluid temperature at a constant reference temperature, and cannot be used to measure velocities in a free space.

Other references on the use of thermistors as velocity sensors are given in refs. 12, 24, 27 and 29.

1.2. Applications of Self-Heated Thermistors

A self-heated thermistor in which power is dissipated has the following heat transfer equation:

$$P = hA\,(T - T_0) + C\,\frac{dT}{dt} \tag{6}$$

where $P = Vi$, P being the power dissipated in the thermistor, V and i the thermistor voltage and current at the same instant t, respectively. C is the heat capacity of the thermistor, h is the film coefficient of heat transfer between the thermistor of area A and the surrounding medium, and T and T_0 are the instantaneous temperatures of the thermistor and medium, respectively.

It is usual, in thermistor practice, to represent the product hA by the symbol δ, called the "dissipation constant" of the thermistor. As is obvious from the physical meaning of δ, it is neither a constant nor a property of the thermistor as the name may suggest, but it is a function of the thermodynamic properties of the fluid, the thermistor body temperature, the rate of heat conduction through the leads of the thermistor to its mount, of the heat loss due to radiation (that is only significant when the medium is a gas at low pressure, ref. 14, page 1), and of the velocity of the medium.

In the following analysis, only the influence of the motion of the fluid, the free convection to the still medium and the heat conduction through the leads will be considered. These last two factors are accounted for experimentally by the measurement of the dissipation constant of the thermistor, in still air at 25° C (77° F) done by the manufacturers. The influence of small variations in the thermistor body temperature upon the value of δ will be neglected since it is of minor order when compared to the other factors. A design body temperature of 125° C will be used in this work. (The reasons for choosing this temperature will be explained later).

The transient solution of equation (6) originates the Current-Time Characteristic of self-heated thermistors.

The steady-state solution of equation (6), that occurs when $\frac{dT}{dt} = 0$, gives rise to the Voltage-Current Characteristic or Static, Voltage-Current Transfer Characteristic of a thermistor.

The so-called Zero-Power Resistance-Temperature Characteristic is obtained directly from equation (1), in which case the thermistor body temperature is equal to the surrounding medium temperature, neglecting radiation.

The applications of thermistors can be divided into three major categories, according to whether its mode of operation follows one of the above three characteristics. Applications based on Current-Time Characteristic are: time delay, overload protection, sequential switching and surge suppression.

Applications based on the Zero-Power Resistance-Temperature Characteristic are mainly precision temperature measurement, control and compensation. Precision control of baths temperature, with controller differentials of $\pm$ 0.0005° C or less are possible, using thermistors.

Thermistor applications based on the Voltage-Current Characteristic will next be examined in more detail since the anemometer is included in this group.

The application depending upon the Voltage-Current Characteristic may be subdivided into four groups, each group being characterized by a particular type of excitation applied to the self-heated thermistor.

Table I below summarizes the main applications within each group and the type of thermistor excitation (ref. 27, pages 13 and 14).

TABLE I

THERMISTOR APPLICATIONS DEPENDING UPON THE VOLTAGE CURRENT CHARACTERISTIC

GROUP I: VARIATION IN DISSIPATION CONSTANT

a. Vacuum Manometer
b. Anemometers, Flow Meters, Fluid Velocity
c. Thermal Conductivity Analysis, Gas Detectors, Gas Chromatography
d. Liquid Level Measurement, Control and Alarm

GROUP 2: VARIATION IN CIRCUIT PARAMETERS

a. Oscillator Amplitude and/or Frequency Regulation
b. Gain or Level Stabilization and Equilization
c. Voltage Regulators
d. Volume Limiters
e. Expanders and Compressors
f. Switching Devices

GROUP 3: VARIATION IN AMBIENT TEMPERATURE

a. Temperature control or alarm

GROUP 4: MICROWAVE POWER MEASUREMENT

a. Bolometers

In the first group the excitation is a change in the thermistor dissipation constant. If the thermal conductivity of the medium changes, the instrument may be either a manometer, or a conductivity cell or a liquid-level indicator. If the dissipation constant changes due to a variation in the film coefficient of heat transfer between the thermistor and the medium, produced by a variation in the fluid velocity, the instrument is an anemometer or a flow meter. By analogy with a vacuum tube circuit, the bias point is determined by the intersection of a fixed load line (of slope equal to the Thevenin equivalent impedance as seen by the thermistor, but of opposite sign, and intersecting the voltage axis at the Thevenin voltage) with a family of thermistor voltage-current transfer curves, generated by different values of the dissipation constant.

In the second group, the excitation is of electrical origin and the bias point is found at the intersection of a variable Thevenin load line (a rotation, a translation or both) with a single thermistor voltage-current curve.

The operating point in the third group, where the excitation is thermal, is obtained at the intersection of the Thevenin load line with different voltage-current curves generated by different ambient temperatures.

The fourth group consists of applications in which the thermistor excitation is a change in the applied microwave or r - f power.

In order to design an instrument based on a change in the thermistor dissipation constant, it is necessary to know the quantitative relationship between this change and its physical agent which, in the present application, is a variation in air velocity. Usually this relationship is obtained directly from experiment. In this work the relation between the thermistor dissipation constant and air velocity will be derived from heat transfer considerations, in section 2.2. Based on this derivation, a theoretical prediction of the anemometer performance will be made and later compared with the corresponding experimental data.

CHAPTER II

DESIGN OF THE ANEMOMETER CIRCUIT

2.1. General Remarks

The circuit chosen is a two-thermistor modified Wheatstone bridge of both deflection and voltage-sensitive type. This bridge was chosen because it provides both high output for a low thermistor excitation, and an excellent balanced condition when the still air temperature fluctuates slightly. Its schematic representation is shown in figure 1-a. The elements composing the circuit are: E, the source voltage; R_1, the source resistance, whose main function is to minimize the non-linearity of the bridge output E_o; two identical resistors, R_2, one in each upper bridge arm; two identical resistors, R_3, in the left arms of the bridge; a balance, which is a potentiometer of value $2R_3$, that permits the initial zero adjustment, necessary due to small differences in the actual value of the bridge components; two identical thermistors, one in each lower bridge arm, of nominal resistance of 2,000 Ohm at 25° C; R_G is the resistance of the read-out galvanometer, a voltmeter in this application.

The two thermistors were chosen based on their very small size, that will minimize the natural convection effects at very low air velocity, their stability, small thermal time

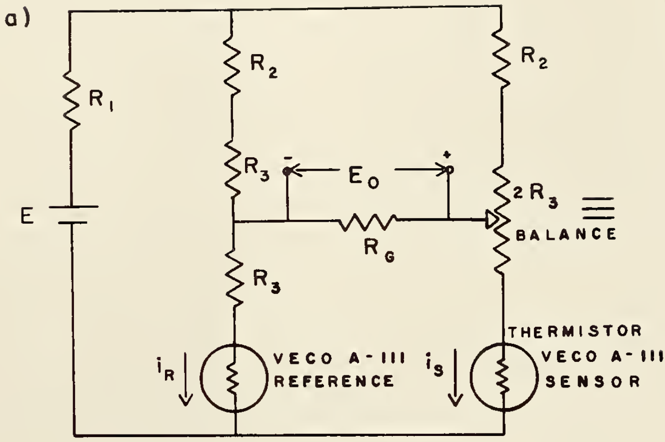
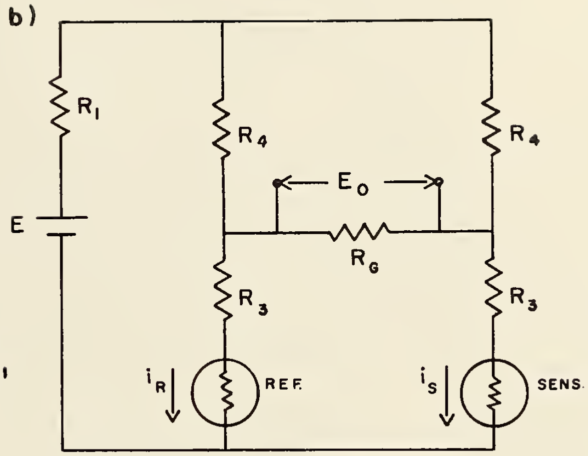


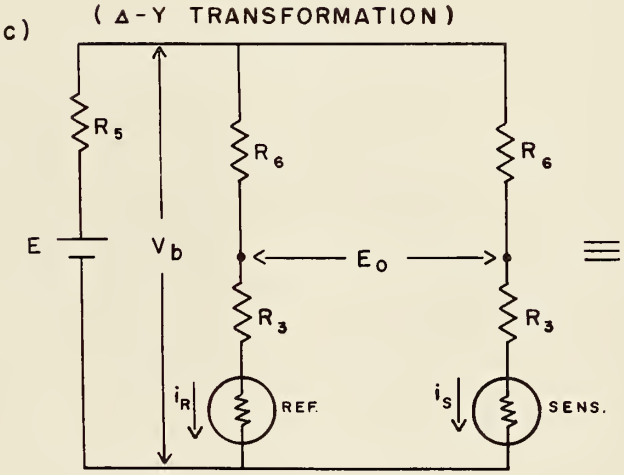
Figure 1-a — Proposed Circuit



where: $R_4 = R_2 + R_3$

Figure 1-b — Intermediate Transformed Circuit

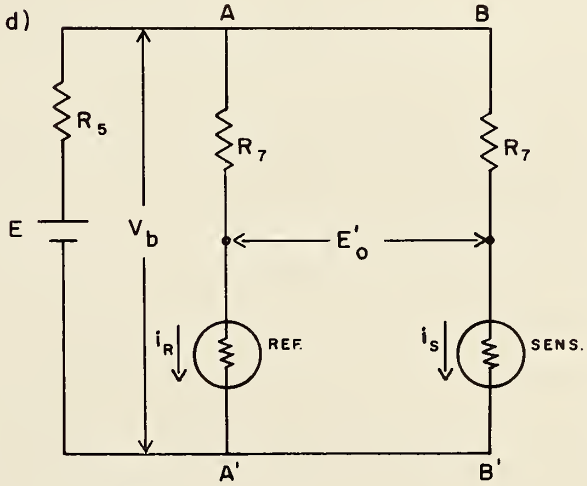
(Δ-Y TRANSFORMATION)



where: $R_5 = R_1 + \frac{R_4^2}{2R_4 + R_G}$

and: $R_6 = \frac{R_4 \cdot R_G}{2R_4 + R_G}$

Figure 1-c — Intermediate Transformed Circuit



where: $R_7 = R_3 + R_6$

and: $E'_0 = E_0 + R_3(i_R - i_S)$

Figure 1-d — Final Transformed Circuit

constant, and sensitivity. The thermistors are manufactured by Victory Engineering Corporation, Model A-111. They are matched thermistors of bead shape. Their nominal characteristics are given in Table II.

TABLE II

THERMISTOR VECO A-111 CHARACTERISTICS

Diameter:	0.013 inches
Resistance at 25° C (77° F):	2,000 ± 25% Ohm
Resistance at 0° C (32° F):	5,000 Ohm
Resistance at 50° C (122° F):	900 Ohm
Resistance at 300° C (572° F):	12 Ohm
Beta (β) at 25° C:	3050 ± 200° k
Ratio $\frac{R_o (0^\circ C)}{R_o (50^\circ C)}$:	5.03 to 6.31
Temperature coefficient of Resistance ($\propto$) at 25° C:	-3.4 %/°C
Dissipation Constant (δ) at 25° C, in still air:	0.10 MW/°C
Thermal Time Constant:	1 second (in still air)
Maximum Continuous Power:	45 MW (at 25° C in still air)
Maximum Operating Temperature:	300° C
Matching:	Voltage-Current Type

According to the manufacturers, thermistors of the Voltage-Current Matching group "are matched in their self-heated state, that is, where sufficient current is passed through the thermistors to heat them electrically above their surrounding medium to an appreciable degree. In general three or four voltage-current points are selected from the static characteristic curve of the thermistor."

Only one of the thermistors will be directly exposed to the moving medium, and this will be the velocity sensor. The other thermistor will be exposed to the same medium, to the same temperature and pressure, but shielded from any movement of the medium. Care must be taken in order to assure that this reference thermistor is placed close enough to the sensor thermistor, so that the temperature of the medium will be essentially the same in the neighborhood of both thermistors. Also, the thermal time constant of this shield should be as small as possible to avoid the introduction of errors due to medium temperatures which change as a function of time.

The thermistors are mounted on a header of shape and dimensions (in inches) shown in figure 2. This figure also shows the Zero-Power Resistance-Temperature Characteristic plotted for several types of thermistors. Curve "B" is for the VECO A-111 thermistors used in the anemometer.

2.2. Heat Transfer Analysis of the Self-Heated Thermistor

As mentioned at the end of section 1.2., it is necessary to obtain a relationship between the thermistor dissipation constant and the air velocity, in order to design the optimum circuit for a desired range of air velocities to be measured.

The assumptions needed for such derivation are:

1. The thermistor will be assumed spherical in shape, although actually it is slightly elipsoidal.

2. To calculate the fluid properties at the film temperature, it is necessary to assume a design body temperature for the thermistor. This temperature will be assumed to be 125° C, which is a reasonable value for the operation of the

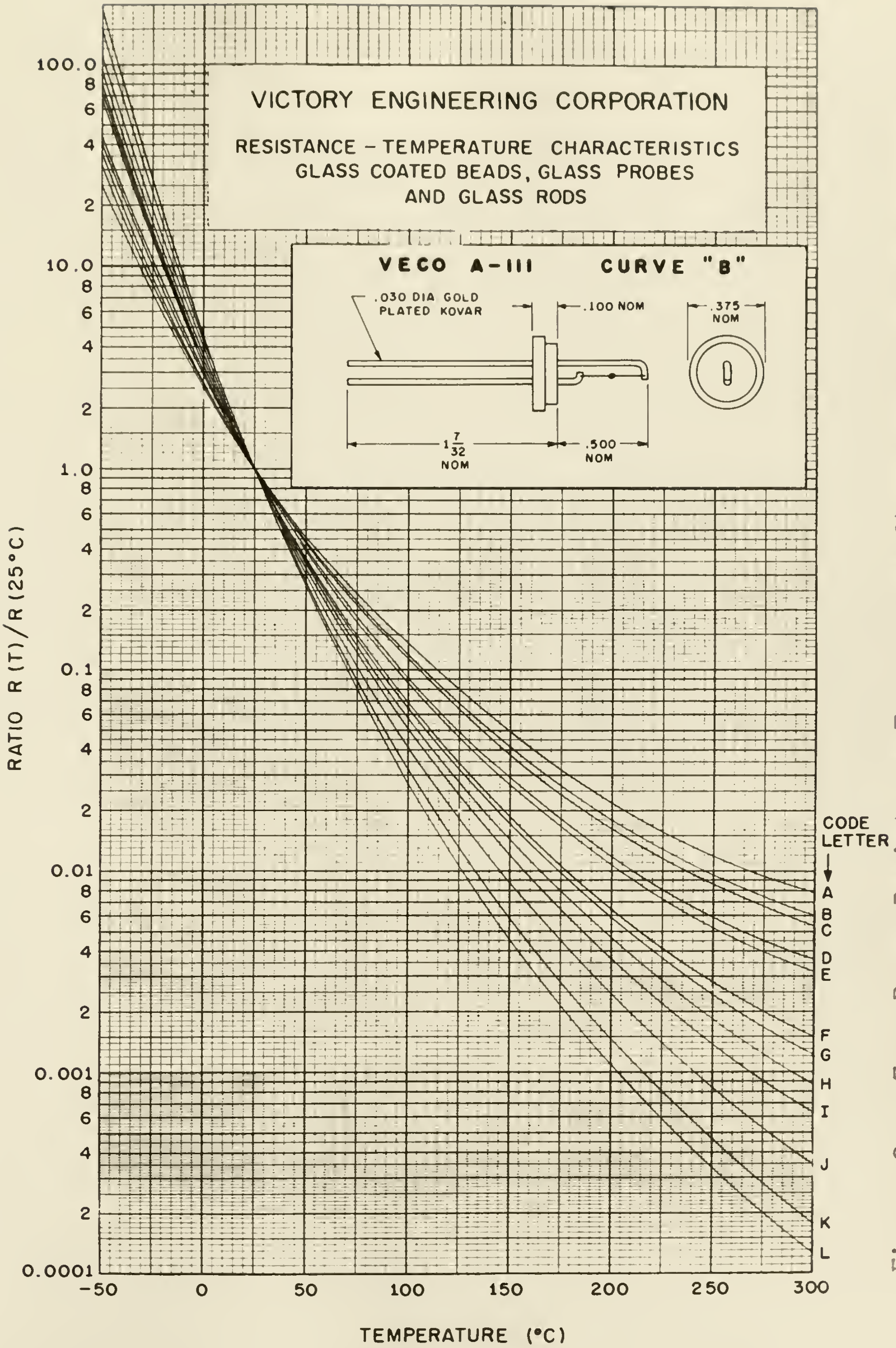


Figure 2 — Zero-Power Resistance-Temperature Characteristics for Several Thermistor Shapes

thermistor, since it will be 100° C above the standard ambient temperature. This large difference in temperature will render the instrument very sensitive to low air velocities and, as will be explained later, will minimize the undesired sensitivity of the thermistor to small ambient temperature fluctuations. Natural convection due to this difference in temperature will not constitute a serious problem, since both thermistors will operate at the same temperature in still air and the bridge output will be zero in this condition. This is another advantage of the use of the reference thermistor. Nevertheless, it is necessary to have an estimate of the influence of natural convection effects upon the velocity field surrounding the thermistor. This may be established as follows:

It is desired to know under what circumstances the effects of natural convection can be neglected and which are the conditions where both effects are of the same order of magnitude. From an analysis based on the Navier-Stokes and energy equations, and on the theory of similarity, applied to the thermal and velocity boundary layers, it is possible to conclude that the body force which is due to the buoyancy of the hotter fluid particles (that is, due to their thermal expansion) is of the same order of magnitude as the inertia and viscous forces, if the ratio of Grashof number to the square of the Reynolds number is of the order of unit. This occurs at very small fluid velocities and high temperature differences, which is precisely the present case. In other words, the ratio $\frac{Gr}{Re^2}$ gives a qualitative indication of the influence of buoyancy on forced convection, and when Gr is equal or larger than Re^2, natural convection effects cannot be ignored compared to forced convection.

Therefore, it is necessary to calculate this ratio for the lower velocity values of the range of interest. By definition:

$$Gr = \frac{\rho^2 g \beta (T - T\infty) R^3}{\mu^2},$$

where R is the sphere radius, since it is the most appropriate characteristic length for natural convection on spheres;

$\beta = \frac{\rho\infty - \rho}{\rho(T - T\infty)}$, which, for an ideal gas, reduces to $\frac{1}{T\infty}$.

The characteristic length to be used in the Reynolds number must also be the sphere radius, since the analysis done requires uniformity in the characteristic lengths used. Therefore:

$$Re = \frac{\rho v R}{\mu}$$

The numerical values for the numbers are:

$$Gr = 0.15,$$

based on a temperature difference of 100° C (180° F) and on $t\infty = 25°$ C (77° F).

$$Re = 0.069v,$$

v in fpm.

For the lowest velocity in the range of interest, namely 10 fpm, $\frac{Gr}{Re^2}$ is equal to:

$$\frac{Gr}{Re^2} = 0.315$$

For 20 fpm and 100 fpm the ratio is 0.0787 and 0.00315, respectively. Therefore, it seems from this qualitative calculation that the influence of natural convection upon forced convection is not very intense, although the velocities are very low and the temperature difference high.

3. The effect of film temperature changes, caused by variations in the thermistor body temperature upon the heat transfer coefficient will be neglected, since this influence is of minor order, when compared to the influence of air velocity changes.

4. The heat conduction through the thermistor leads to its mount is appreciable, and must, therefore, be considered. This effect will be accounted for by using an equivalent heat transfer area for the system sphere and leads, calculated based upon the experimental value of the dissipation constant measured, in still air at 25° C supplied by the manufacturer of the thermistor.

For extremely low values of the Reynolds number, the heat transferred from a heated sphere to its surrounding medium may be considered as a conduction through a stagnant medium of infinite radius. The heat conducted through this medium can be calculated as follows. For a hollow sphere, of inside diameter D_i, at temperature T_i, and outside diameter D_o, at a temperature T_o, the radial heat-flow through it is given by:

$$Q = \frac{\pi (T_i - T_o)}{(\frac{1}{2k}) \cdot (\frac{1}{D_i} - \frac{1}{D_o})} \tag{7}$$

where k is the thermal conductivity of the medium.

Now, if D_o approaches infinity, equation (7) is reduced to:

$$Q = 2k\pi D_i \ (T_T - T_\infty) \tag{8-a}$$

where T_T is the thermistor body temperature and T_∞ is the ambient temperature far from the thermistor.

From the definition of film coefficient, the same amount of heat, Q, may be expressed as follows:

$$Q = h\pi D_i^2 \ (T_T - T_\infty) \tag{8-b}$$

From equations (8-a) and (8-b) it follows:

$$\frac{hD}{k} = 2 \tag{9}$$

where h is the film coefficient of heat transfer between the thermistor of diameter D and the medium of thermal conductivity k. Since the dimensionless group in equation (9) is, by definition, the Nusselt number, it may be writen:

$$Nu = 2 \tag{10}$$

Considering a boundary layer of thickness γ around the thermistor, instead of an infinite fluid, so that in this layer the temperature changes from T_T to T_∞, then D_o becomes equal to $(D_i + 2\gamma)$ and equation (7) is reduced to:

$$Q = 2k\pi D_i \ (\frac{D_i + 2\gamma}{2\gamma})(T_T - T_\infty)$$

Again, the heat conducted is equal to the heat convected from the sphere and therefore:

$$Q = 2k\pi D_i \ (\frac{D_i + 2\gamma}{2\gamma})(T_T - T_\infty) = h\pi D_i^2 \ (T_T - T_\infty) \tag{11}$$

From equation (11):

$$Nu = \frac{hD}{k} = 2 + \frac{D}{\gamma} \qquad (12)$$

where h, k and D are the same as in equation (9).

The term $\frac{D}{\gamma}$ is a function of both Prandtl and Reynolds numbers, and its value for flow past spheres, is given by the following experimental equation (cf. ref. 4) obtained by Kudrjashev:

$$\frac{D}{\gamma} = 0.388\ (Pr)^{1/2}\ (Re)^{1/2} \qquad (13)$$

Finally, the desired relation between the film coefficient (and thus the dissipation constant) and the air velocity is obtained from the substitution of equation (13) into equation (12):

$$Nu = 2 + 0.388\ (Pr)^{1/2}\ (Re)^{1/2} \qquad (14)$$

It should be observed from the derivation of equation (14) that this equation is valid only for very small Reynolds numbers. Several other correlations between Nusselt and Reynolds numbers are available from literature for the heat transfer from a sphere to its surrounding medium. However, those equations are better for Reynolds numbers in a higher range than in the present application. The following correlations and their respective Reynolds number ranges, are given for comparison (ref. 7):

$$Nu = 2.0 + 0.60\ (Pr)^{1/3}\ (Re)^{1/2},\ 1 < Re < 70{,}000,$$

from Froessling;

$$Nu = 2.0 + 1.3\ (Pr)^{0.15} + 0.66\ (Pr)^{0.31}\ (Re)^{1/2},\ 10 < Re < 10^5,$$

from Kramers.

The air properties necessary to obtain a numerical relation between h and the air velocity v, should be taken at the "film temperature", namely the arithmetic mean between the thermistor design body temperature (125° C) and the standard ambient temperature (25° C). This temperature is, therefore, 75° C (167° F). Air properties at 160° F are given in table III.

TABLE III

AIR PROPERTIES AT 160° F AND 1 ATM

Density (ρ):	0.06399 lbm/ft^3
Specific heat at constant pressure (C_p)	0.241 BTU/lbm°F
Thermal conductivity (k):	0.0169 BTU/hr.ft.°F
Viscosity (μ):	0.0496 lbm/hr.ft.
Kinematic viscosity ($\nu = \frac{\mu}{\rho}$):	0.775 ft^2/hr
Prandtl Number (Pr):	0.707

The next step is to calculate the equivalent area for heat transfer, as explained in assumption number 4 above.

From equation (6), section 1.2, the thermistor dissipation constant, δ, is equal to:

$$\delta = hA \qquad (15)$$

From equation (14) when v = 0(Re = 0):

$$Nu = \frac{hD}{k} = 2$$

or

$$h = \frac{2k}{D} \qquad (16)$$

Substituting h from equation (16) into equation (15) and rearranging:

$$A = \frac{\delta D}{2k}$$

or:

$$\pi D^2 = \frac{\delta D}{2k}$$

or:

$$D = \frac{\delta}{2k\pi} \qquad (17)$$

If the experimental value of $\delta = 0.10$ MW/^{o}C, in still air at 25^{o} C, is substituted in equation (17) the resulting value is the equivalent diameter De:

$$De = \frac{0.10 \times 3.4128 \times 12}{1,800 \times 2 \times 0.0169 \times 3.1416} = 0.0214 \text{ inches,}$$

that is, almost twice the nominal value of the actual thermistor diameter (0.013 inches).

From equation (14):

$$\frac{h\left(\frac{0.0214}{12}\right)}{0.0169} = 2 + 0.388\ (0.707)^{1/2} \left(\frac{v \times \frac{0.0214}{12}}{0.775}\right)^{1/2}$$

then:

$$h = 18.92 + 1.148\ (v)^{1/2} \qquad (18)$$

where v is in feet per minute and h is in BTU/hr.ft.^{2} ^{o}F.

Therefore δ is:

$$\delta = hA = 0.10 + 0.605 \times 10^{-2} \sqrt{v} \qquad (19)$$

with v in fpm and δ in MW/°C.

Figure 3 shows equation (19) plotted over the range of air velocities from 0 to 200 fpm. For this range, the Reynolds number varies from 0 to 27.6, which indeed represents very low values.

2.3. Synthesis and Analysis of the Anemometer Circuit

Once the relationship between the thermistor dissipation constant and the air velocity was obtained, the next step in the design was to generate a family of thermistor voltage-current transfer curves for several values of dissipation constants within the range of velocities in which the anemometer will operate.

A graphical solution will be used in the design, since it is the best approach to the large-signal analysis of self-heated thermistors. An analytical solution would require the simultaneous solution of non-linear equations.

Equation (6), applied to a steady-state operation of the thermistor is reduced to

$$P = Vi = \delta\,(T - T_0) \qquad (20)$$

With the help of equations (19) and (20), and the Zero-Power Resistance-Temperature Characteristic from figure 2, and of the equation:

$$P = \frac{V^2}{R}\,,$$

the family of curves "1", shown in figure 4, was obtained. A digital computer solution, in tabular form, is given in

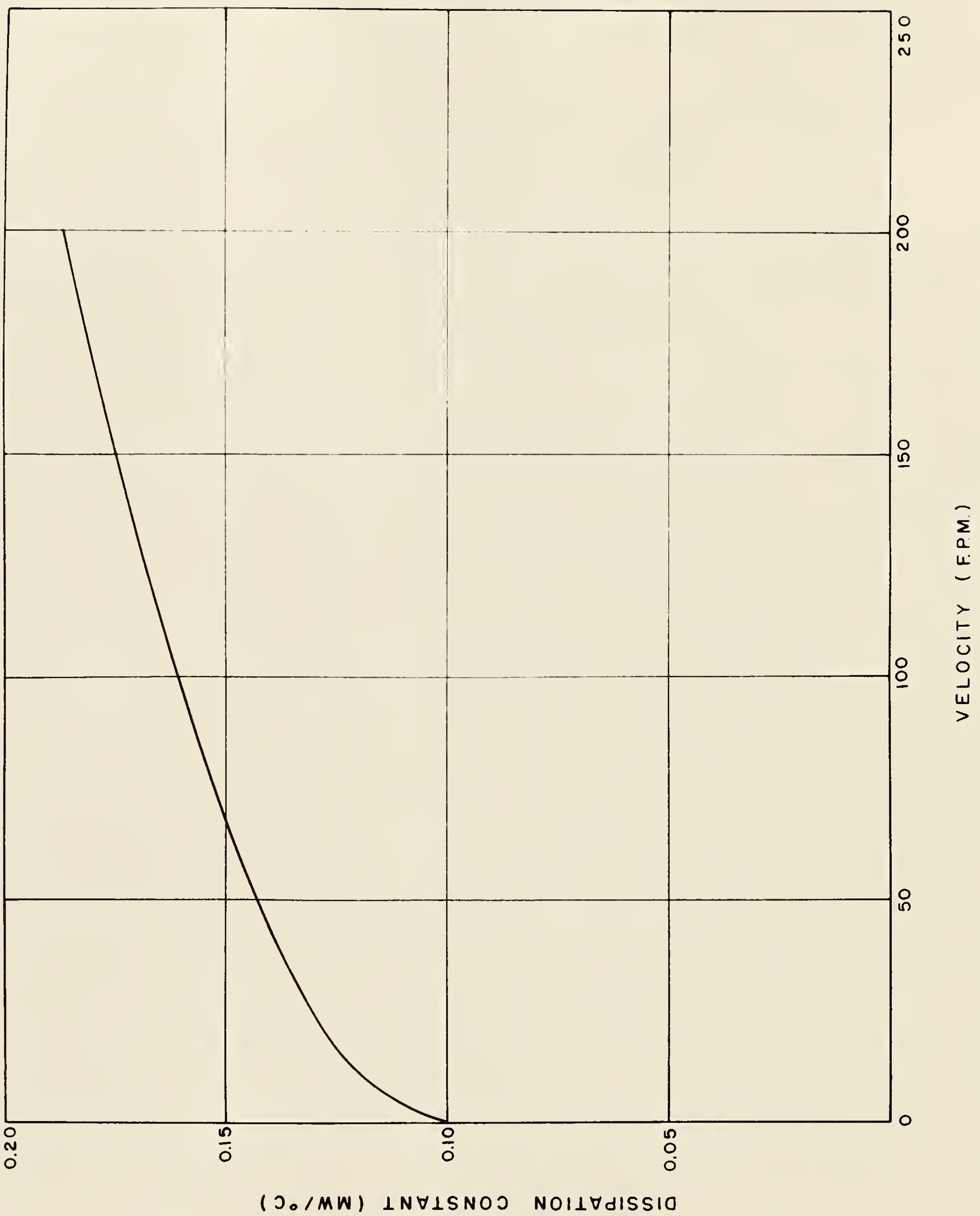


Figure 3 — Thermistor Dissipation Constant as a Function of Air Velocity, from Equation (19)

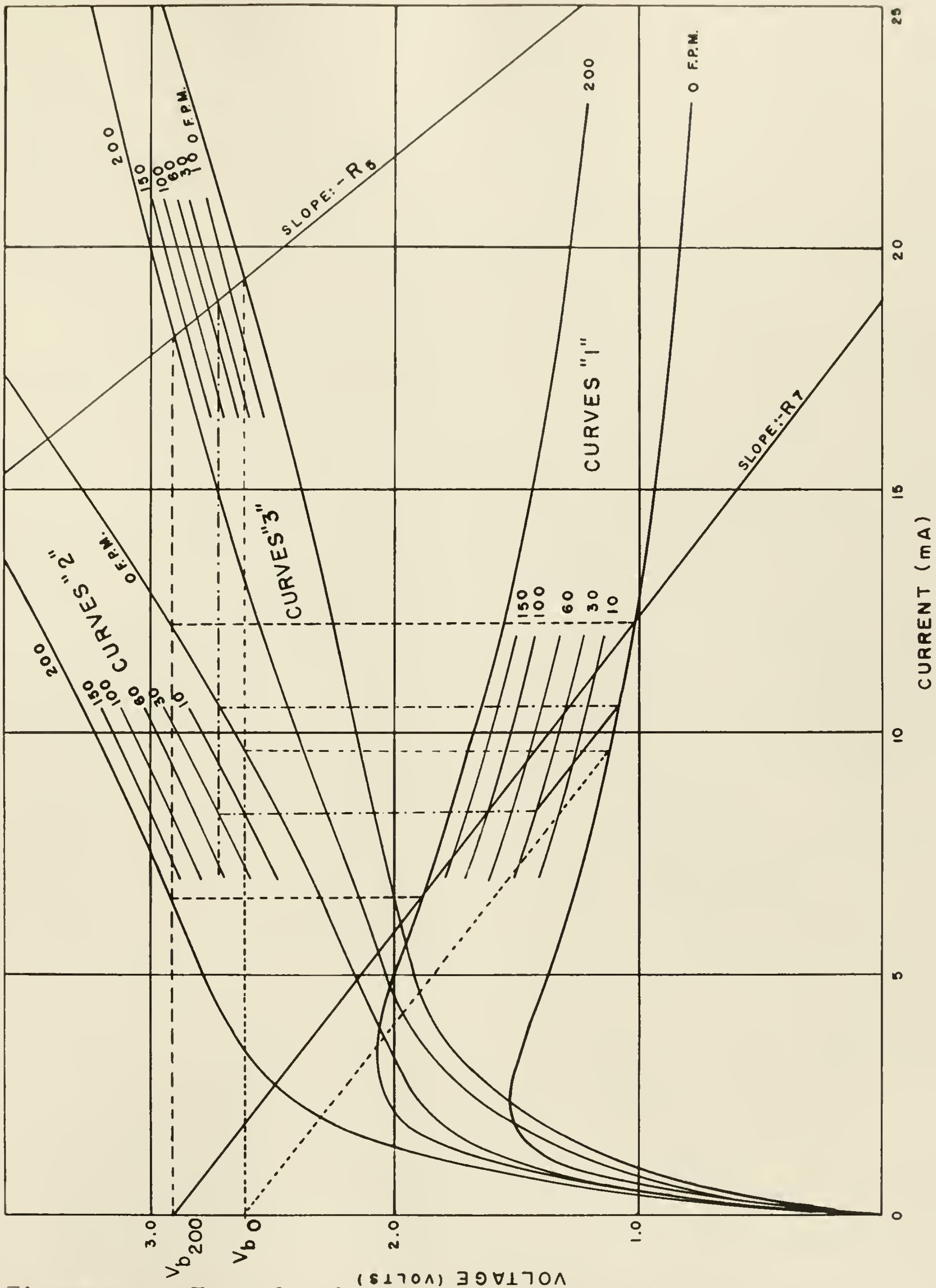


Figure 4 — Transfer Curves for the Thermistors and Circuit Branches, and Graphical Analysis of the Proposed Circuit

Appendix A, for values of air velocities starting at 0 fpm and increasing up to 200 fpm, in steps of 10 fpm. Also, the corresponding value of δ is given, at the right side of the velocity, in each table. The first column of each table shows the thermistor temperatures, starting at 25° C and increasing to 300° C. The second column, indicated as RATIO, is the ratio $R(T)/R(25^\circ C)$, obtained, for each of the temperatures, from figure 2. The third column is the thermistor resistance, obtained from the multiplication of column 2 by $R\ (25^\circ C) =$ 2,000 Ohms. The fourth column (T - TA) is the temperature difference between the thermistor body and the ambient temperatures. The fifth column is the power (in MW) dissipated in the thermistor, and was obtained from equation 20, multiplying column 4 by δ. The sixth column is the thermistor voltage, obtained from $V = (P \times R)^{1/2}$. The last column is the current, i, across the thermistor, obtained from $i = \frac{P}{V}$.

Although δ varies slightly with temperature, it was assumed constant in the above solution. The error introduced by this assumption is very small, as may be seen from the comparison between the first table in Appendix A and the experimental data supplied by the manufacturer, shown in table VII, although the resistances of the actual thermistors are 2,120 Ohms and 2,108 Ohms instead of the nominal value of 2,000 Ohms used. β also varies with temperature but its variation is small enough to be considered constant between ambient temperature and the temperature at the peak voltage.

In order to check the curves obtained in the previously described manner, the temperature at maximum voltage of the curve for 0 fpm will be obtained by another procedure. From equations (1) and (6), after setting $\frac{dV}{di}$ equal to zero, it results:

$$T_p = \frac{\beta}{2} - \sqrt{\left(\frac{\beta}{2}\right)^2 - \beta T_o}$$

$$T_p = \frac{3050}{2} - \sqrt{\left(\frac{3050}{2}\right)^2 - 3050\ (273 + 25)} = 335^o k = 62^o\ C$$

From the first table in Appendix A, it may be seen that at a thermistor temperature near the value of 65° C the voltage passes through a maximum, thus in good agreement with the adopted solution.

Returning to the design, the next step was to transform the proposed circuit until it assumed a convenient form to be analyzed. Referring to figure 1, the change in the circuit when passing from the initial circuit, represented in figure 1-a, to circuit b is simply the grouping of resistors R_2 and R_3 into a single resistor R_4 (the balance of value $2R_3$ was split into two resistors of equal value, R_3). Next, two transformations were done simultaneously to pass from circuit b to c: first, resistors R_4 (left bridge arm), R_G and R_4 (right bridge arm) are operated on with a Δ - Y transformation resulting in the resistors R_6 and $\frac{R_4^2}{2R_4 + R_G}$. This resistor, then, is grouped with R_1, resulting in resistor R_5. The value of R_6 is $\frac{R_4 R_G}{2R_4 + R_G}$. The last step, shown in circuit d, is to group R_3 and R_6, in both sides of the bridge, yielding resistors

R_7 and a new bridge output, necessary for the subsequent analysis, related to the actual bridge output E_o, by the equation

$$E_o' = E_o + R_3 \; (i_R - i_s) \qquad (21)$$

where i_R and i_s are the currents flowing through the reference and sensor thermistors, respectively.

The circuit represented in figure 1-d is now ready to be analyzed by the large-signal theory, following the method of Sapoff-Oppenheim discussed in reference 14.

As mentioned before, the family of curves "1" in figure 4 are the thermistor transfer curves. Curve 0 fpm represents either the reference or sensor thermistor in still air at 25° C. Curves 10, 30, etc., 200 fpm represent the sensor thermistor in air moving at velocities of 10, 30, etc., 200 fpm, respectively, also at 25° C.

In order to obtain the family of transfer curves "3", corresponding to different air velocities, for the network defined by the terminals AB - A'B' (figure 1-d), it is necessary to generate, as an intermediate step, the family of curves "2" for the circuit branches defined by terminals A-A' and B-B' (figure 1-d). But, to obtain the family of curves "2", the value of the resistors R_7 must be known. Therefore, it is necessary to determine this value, based on a desired circuit response. The condition for maximum sensitivity, and consequently minimum stability, at a given operating point, exists when the load line of slope $- R_7$ is tangent to the thermistor transfer curve at that operating point. This is so because, for this situation, any slight shift in either the thermistor transfer curve or the load line will produce a large change in the location of the operating point. (This fact can also be proved mathematically by the small-signal theory, after obtaining the small-signal equivalent circuit. In this circuit, the bridge output is inversely proportional to the sum of the thermistor dynamic resistance - equal to $\left.\frac{dV}{di}\right|_{\delta,\, T_o = \text{const.}}$ - with R_7. Therefore, if R_7 is equal in magnitude to the slope of the thermistor transfer curves at the same operating point, the sum will be equal to zero, since the dynamic resistance is negative. Thus, the bridge output will increase, mathematically, to infinity.)

As the angle at the intersection point between the load line and the thermistor transfer curve increases, the stability of the circuit increases and the sensitivity decreases. A reasonable compromise seems to be to choose the slope of the load line equal to twice the magnitude of the slope of a line tangent to the thermistor transfer curve at its inflection point, independently of the particular location of the operating point (ref. 14, page 4). With respect to the location of the operating points, it is desirable to place them so that they will be in the portion of maximum separation (and maximum output) between the transfer curves for the limit case when the air velocity is 200 fpm. Also, they should not be located too close to the peaks of the transfer curves, in order to minimize the thermistor sensitivity to ambient temperature fluctuations, and, finally, the operating points should be at a temperature near the design value chosen, namely, 125° C.

Based upon all the above considerations, the load line of slope $- R_7$ shown in fig. 4 was traced, and the corresponding value of R_7 was found to be equal to 154 Ohms. The operating point for the reference thermistor is, from figure 4, at $V_R = 1.02$ V and $i_R = 12.0$ mA, and for the sensor at $V_S = 1.88$V and $i_S = 6.4$ mA. The bridge voltage, V_{b200}, is equal to 2.90 V. From these values and from equation (21), the maximum bridge output is:

$$E_o = E_o' - R_3 (i_R - i_S) = (1.88 - 1.02) - R_3 (12.0 - 6.4) \times 10^{-3}$$

$$E_o = 0.86 - 5.6 \times 10^{-3} R_3$$

From this last expression and from the fact that i_R is always greater than i_S, R_3 should be chosen as small as possible, and yet effective as a balance, in order to avoid an appreciable reduction in the bridge output E_o. Since this calculation is being done for the least favorable case, namely the case with the maximum value of i_R and the minimum value of i_S, in all other cases the reduction in E_o will be even smaller than in this case (v = 200 fpm), although the values of E_o' will also be smaller.

Choosing R_3 equal to 13 Ohms:

$$E_o = 0.787 \text{ V.}$$

Therefore, this value of E_o is still large enough to be easily measurable.

In this application, the bridge output will be measured by a precision digital voltmeter (Beckman, Model 4011 RVP) that has an automatic printer synchronized to it (Beckman Printer, Model 1453). This digital voltmeter has an input impedance of 10 Megaohm (at a null).

Therefore:

$$R_G = 10 \times 10^6 \text{ Ohm}$$

Next, the values of R_6, R_4 and R_2 can be readily determined.

$$R_6 = R_7 - R_3 = 154 - 13 = 141 \text{ Ohm}$$

$$R_4 = \frac{R_6 R_G}{R_G - 2R_6} = 141 \text{ Ohm}$$

$$R_2 = R_4 - R_3 = 141 - 13 = 128 \text{ Ohm}$$

The remaining unknown circuit elements are R_5, R_1 and E. But R_5 and E are related by the fact that the total current flowing through the bridge is $(i_R + i_S)_{200}$. The constraint equation is therefore:

$$E = V_{b_{200}} + R_5\ (i_R + i_S)_{200} \qquad (22)$$

or:

$$E = 2.90 + (12.0 + 6.4) \times 10^{-3}\ R_5$$

Since:

$$R_5 = R_1 + \frac{R_4{}^2}{2R_4 + R_G}\ ,$$

it follows that:

$$R_5 = R_1 + \frac{(141)^2}{282 + 10 \times 10^6} = R_1 + 0.0199$$

Therefore, considering the precision of the nominal values of the resistor within ± 1%:

$$R_5 = R_1 ,$$

and:

$$E = 2.90 + 18.4 \times 10^{-3} R_1 \qquad (23)$$

Any set of values for E and R_1 satisfying equation (23) will yield the limit condition that a maximum output of 0.787 V exist when the air velocity is 200 fpm.

The value of E will be chosen dictated by the available supply and mainly because it was found, according to reference 14, page 5, that by increasing progressively the values of both E and R_1, subject to the constraint of equation (23), the non-linearity of the output voltage versus change in thermistor dissipation constant is "reduced and is minimized when the source approaches a constant current source".

A value of 8.50 V for E was chosen. Therefore, by equation (23), the corresponding value of R_1 is:

$$R_1 = \frac{8.50 - 2.90}{18.4 \times 10^{-3}} = 305 \text{ Ohm}$$

Thus, having determined all circuit elements, it is now possible to obtain the bridge output for different air velocities. As mentioned before, the family of curves "2" is an intermediate step necessary to obtain the family of curves "3", that are the transfer curves for the network defined by terminals AB-A'B' (fig. 1-d), and that will provide the complete analysis of the synthesized circuit. Each curve of family "2", the family of transfer curves for the circuit branches A-A' and B-B', was obtained by addition, along each constant current line, of the voltage drop across the resistors R_7, at that constant current with the respective thermistor transfer curve. Curve 0 fpm, of family "2", is the transfer curve for either branch A-A' and B-B', when the air velocity is zero. Curves 10, 30, etc., 200 fpm, of family "2", are the transfer curves for the circuit branch B-B' when the air velocities are 10, 30, etc., 200 fpm, respectively. The last step is to generate the desired transfer curves "3", by graphical addition, along constant voltage lines, of the transfer curves for the circuit branches A-A' and B-B'.

The bridge voltage corresponding to any air velocity may now be obtained from the intersection of the family of transfer curves "3" with the load line of slope - R_5, given by the equation:

$$V_{b_j} = E - R_5\ (i_R + i_S)_j \tag{24}$$

where j denotes the corresponding air velocity.

The constraint of equation (22) requires that the load line of slope - R_5 pass through the operating point located at the intersection of transfer curve "200 fpm" of family "3", with the voltage coordinate $V_{b_{200}}$. The corresponding operating points on the thermistor transfer curves, and consequently the bridge output for any air velocity, can then be obtained from the intersection of the family of thermistor transfer curves "1" with a load line of slope - R_7, given by the equation:

$$V_k = V_{b_j} - R_7 i_k \tag{25}$$

where k = R or S, for the reference or sensor thermistors, respectively. The same operating points may also be obtained by the traverse, first at constant voltage, between families "3" and "2", and then along constant current lines from family "2" to family "1". This procedure is shown in figure 4, for air velocities of 0, 30 and 200 fpm.

There is an interesting aspect in the dynamics of the circuit, namely a feed-back effect caused by resistor R_5. In order to see this, suppose that $R_5 = 0$. Therefore, the load line of slope $-R_5$ would be horizontal, coinciding with the voltage coordinate $V_{b_{200}}$, and all operating points in family "1" would be located along the same fixed load line of slope $-R_7$, with the reference thermistor operating point fixed for all values of air velocity. In this situation, the bridge would be connected to a constant voltage source. Instead, with $R_5 \neq 0$, the operating points of both thermistors shift with a change in air velocity, and the load line of slope $-R_7$ is translated to the portion of the curves where the separation between them is larger, which causes an increase in the bridge output at lower air velocities. Consequently, the bridge voltage changes, when $R_5 = 305$ Ohm, from $V_{b_{200}} = 2.90$ V to $V_{b_o} = 2.60$ V.

As R_5 increases, the source E approaches a constant current source.

2.4. Theoretical Prediction of the Anemometer Performance

From section 2.3., the prediction of the anemometer performance is simply a straightforward calculation of the bridge output E_o, produced by air velocities varying from 0 to 200 fpm.

Before doing this calculation, it is interesting to check the precision of the graphical solution of section 2.3. by calculating the bridge voltage V_{b_o} by four different means and comparing them.

1. From figure 4, the bridge voltage corresponding to still air may be obtained from the intersection with the voltage axis, of a load line of slope - R_7 passing through the operating points of both thermistors in curve 0 fpm of family "1".

This value is read to be 2.6 V.

2. From equation (25), with $j = 0$ and $k = R$ (in this case $i_R = i_S$):

$$V_{b_o} = V_R + R_7 i_S$$

From figure 4:

$$V_R = V_S = 1.10 \text{ V},$$

and

$$i_R = i_S = 9.6 \text{ mA}$$

Therefore:

$$V_{b_o} = 1.10 + 154 \times 9.6 \times 10^{-3} = 2.6 \text{ V}$$

3. A constant current line drawn from the coinciding thermistors operating points in curve 0 fpm (family "1") will

intersect the corresponding curve "0 fpm" for the circuit branches A-A' or B-B' (in family "2") at a voltage of 2.6, which is the bridge voltage.

4. With respect to the circuit branch A-R_5-E-A' (fig. 1-d), the bridge voltage can be obtained from equation (22) applied to 0 fpm.

$$V_{b_o} = E - R_5\ (i_R + i_S)_o = E - R_1\ (i_R + i_S)_o,$$

since $R_5 = R_1$

$$V_{b_o} = 8.5 - 305\ (9.6 + 9.6) \times 10^{-3} = 2.6 \text{ V}$$

The operating points of both thermistors in still air are completely determined. From the first table in Appendix A, at a thermistor current of 9.6 mA, it can be read:

$V = 1.10$ V

$P = 10.56$ MW

$R = 115$ Ohm

$T = 130^\circ$ C

Therefore, the design value, for the thermistor body temperature of 125° C, adopted at the beginning of this work is a reasonable value.

At v = 200 fpm, entering the last table in Appendix A with the current values read in figure 4:

Reference thermistor ($i_R = 12.0$ mA)

$V_R = 1.02$ V

$P_R = 12.25$ MW

$R_R = 85$ Ohm

$T_R = 150^\circ$ C

Sensor thermistor (i_S = 6.4 mA)

V_S = 1.88 V

P_S = 12.0 MW

R_S = 294 Ohm

T_S = 90° C

Therefore, the characteristics of the operating points of both thermistors can be easily determined for any value of air velocities within the range from 0 to 200 fpm. With the help of equation (21), E_o can be calculated for those values of air velocity. Table IV shows this calculation done for air velocities of 0, 10, 30, 60, 100, 150 and 200 fpm.

TABLE IV

BRIDGE OUTPUT CALCULATION

v (fpm)	E'_o (volt)	i_R (mA)	i_S (mA)	(i_R-i_S) (mA)	$R_3(i_R-i_S)\times10^{-3}$ (volt)	E_o (volt)
0	0.000	9.6	9.6	0.0	0.000	0.000
10	0.190	10.2	8.9	1.3	0.0169	0.173
30	0.325	10.6	8.4	2.2	0.0286	0.296
60	0.500	10.9	7.7	3.2	0.0416	0.458
100	0.660	11.4	7.2	4.2	0.0547	0.605
150	0.790	11.7	6.7	5.0	0.0650	0.725
200	0.860	12.0	6.4	5.6	0.0727	0.787

2.5. Anemometer Sensitivity to Ambient Temperature Fluctuations

It is important to know the effect of changes in ambient temperature upon the thermistor operating points, in order to predict the deviation from the calibration curve obtained at the standard ambient temperature of 25° C (77° F), caused by those changes.

Since the thermistor resistance is highly dependent upon its temperature, it is obvious that a change in the temperature of its surrounding medium will change its resistance, if the thermistor is in thermal equilibrium with the ambient medium. In terms of the Voltage-Current Characteristic Curve, a change in ambient temperature will produce a shift in this curve. The shift is composed of both a translation and a rotation in the Voltage-Current Curve. However, if the thermistor is being operated in a self-heated condition, that is, with its steady-state temperature much higher than the ambient temperature, then its sensitivity to variations in ambient temperature will decrease appreciably. The mathematical proof of this statement follows. This proof will also provide a quantitative relationship between the change in one of the thermistor parameters caused by a given change in ambient temperature.

The change in the thermistor operating voltage caused by a variation in the temperature of its surroundings will be analyzed, since this case is of interest in this application. Considering the thermistor initially operating in steady-state condition at a voltage V and at a current i, equation (6) reduces to:

$$P = Vi = \delta\ (T - T_o) \tag{26}$$

or:

$$i^2 \cdot R = \delta\,(T - T_0) \qquad (27)$$

If V is represented as

$$V = f(i, T_0, \delta),$$

then:

$$dV = \left(\frac{\partial V}{\partial i}\right)_{T_0,\delta} di + \left(\frac{\partial V}{\partial T_0}\right)_{i,\delta} dT_0 + \left(\frac{\partial V}{\partial \delta}\right)_{i,T_0} d\delta \qquad (28)$$

But, since δ is considered, to a first order approximation, a function of the air velocity only, which is maintained constant in the following analysis, equation (28) is reduced to:

$$dV = \left(\frac{\partial V}{\partial i}\right)_{T_0} di + \left(\frac{\partial V}{\partial T_0}\right)_{i} dT_0 \qquad (29)$$

Equation (27) can be written, in differential form, when i is considered constant, as:

$$i^2 dR = \delta\, dT - \delta\, dT_0 \qquad (30)$$

Multiplying and dividing the left-hand side of this equation by R, it follows:

$$i^2 R\, \frac{dR}{R} = \delta\, dT - \delta\, dT_0$$

Rearranging and substituting i^2R by P:

$$P\left(\frac{1}{R}\,\frac{dR}{dT}\right) = \delta - \delta\,\frac{dT_0}{dT} \qquad (31)$$

But, the expression in parenthesis in equation (31) is, by definition (see equation (4)), the temperature coefficient of resistance. Therefore equation (31) can be written as:

$$-\frac{P\beta}{T^2} = \delta\left(1 - \frac{dT_0}{dT}\right) \qquad (32)$$

Rearranging equation (32):

$$\frac{dT}{dT_o} = \frac{1}{1 + \frac{\beta P}{\delta T^2}} \tag{33}$$

Since the current i was considered as a constant parameter, equation (33) can be written:

$$\left(\frac{\partial T}{\partial T_o}\right)_i = \frac{1}{1 + \frac{\beta P}{\delta T^2}} \tag{34}$$

Since:

$$V = iR,$$

$$dV = idR,$$

or, dividing both sides by RdT:

$$\frac{dV}{RdT} = i \frac{dR}{RdT} ,$$

or

$$\frac{dV}{RdT} = i \left(- \frac{\beta}{T^2}\right),$$

or

$$\frac{1}{V} \cdot \frac{dV}{dT} = - \frac{\beta}{T^2} \tag{35}$$

Multiplying equation (35) by equation (33):

$$\frac{1}{V} \cdot \frac{dV}{dT_o} = - \frac{\beta}{T^2} \cdot \frac{1}{1 + \frac{\beta P}{\delta T^2}} \tag{36}$$

Defining the thermistor voltage sensitivity to ambient temperature variations as:

$$\bar{V}_{T_o} \equiv \frac{1}{V} \left(\frac{\partial V}{\partial T_o}\right)_i \qquad (37)$$

it follows from equation (36) that:

$$\bar{V}_{T_o} = - \frac{\beta}{T^2} \cdot \frac{1}{1 + \frac{\beta P}{\delta T^2}} \qquad (38)$$

The term $\left(\frac{\partial V}{\partial i}\right)_{T_o}$ in equation (29) is the thermistor dynamic resistance at a given operating point. Representing this resistance by R_d, and substituting it, together with the value of $\left(\frac{\partial V}{\partial T_o}\right)_i$ from equations (37) and (38), in equation (29), the desired thermistor voltage change caused by a change in T_o, becomes:

$$dV = R_d d_i + \bar{V}_{T_o} \, V dT_o \qquad (39)$$

For example, assume a 3° C (5.4° F) fluctuation in the value of T_o. The corresponding change in the bridge output E_o can be calculated as follows. The most critical change in E_o will occur at the air velocity of 200 fpm, since the thermistors operating points are at their maximum separation ($T_R = 150^\circ$ C and $T_S = 90^\circ$ C).

For the reference and sensor thermistors, equation (38) becomes:

$$(\bar{V}_{T_o})_{Ref} = - \frac{3050}{(273 + 150)^2} \cdot \frac{1}{1 + \frac{3050 \times 12.25}{0.10\ (273 + 150)^2}}$$

$$(\bar{V}_{T_o})_{Ref} = -\ 0.00552\ {}^oC^{-1}$$

$$(\bar{V}_{T_o})_{Sens} = -\frac{3050}{(273 + 90)^2} \cdot \frac{1}{\frac{3050 \times 12.0}{0.1856\ (273 + 90)^2}}$$

$$(\bar{V}_{T_o})_{Sens} = -\ 0.00927\ {}^oC^{-1}$$

Considering first the voltage change at constant current, equation (39) becomes:

$$(dV)_i = \bar{V}_{T_o}\ VdT_o$$

Thus:

$$(dV)_{i,\ Ref} = -\ 0.00552 \times 1.02 \times 3 = -\ 0.0169\ V$$

$$(dV)_{i,\ Sens} = -\ 0.00927 \times 1.88 \times 3 = -\ 0.0522\ V$$

As an approximation, the current will change proportionally to the inverse of the magnitude R_7, since the thermistor operating point changes along a load line of slope $- R_7$ (assuming V_b constant).

Therefore:

$$d_i = \frac{(dV)i}{-R_7}$$

$$(d_i)_{Ref} = \frac{-\ 0.0169}{-\ 154} = 0.11\ mA$$

$$(d_i)_{Sens} = \frac{-\ 0.0522}{-\ 154} = 0.34\ mA$$

This assumption, namely, that V_b is constant, is reasonable since, for this increase in the current across both thermistors, the voltage drop across R_5 increases only 0.13 V above the normal value of 5.6 V, that is, only 2.5%. This assumption represents an overestimation of the effect of the ambient temperature, and makes possible an analysis that otherwise would have been extremely complex.

From figure 4, at the thermistors operating points:

$$(R_d)_{Ref} = -32 \text{ V/A}$$

and

$$(R_d)_{Sens} = -65 \text{ V/A}$$

Therefore, by equation (39), the total voltage change is:

$$(dV)_{Ref} = -32 \times 0.11 \times 10^{-3} - 0.0169 = -0.0204 \text{ V}$$

and:

$$(dV)_{Sens} = -65 \times 0.34 \times 10^{-3} - 0.0522 = -0.0743 \text{ V}$$

As mentioned before, the thermistor sensitivity to fluctuations in ambient temperature indeed decreases with the increase in the thermistor operating temperature, as the last two figures confirm.

Therefore:

$$V_S^* = 1.88 - 0.074 = 1.806 \text{ V}$$

$$V_R^* = 1.02 - 0.0204 = 1.000 \text{ V},$$

where * means values at $T_o^* = T_o + dT_o = 25 + 3 = 28^\circ$ C.

$$E_o'^* = 1.806 - 1.000 = 0.806 \text{ V}$$

$$i_R^* = 12.0 + 0.11 = 12.11 \text{ mA}$$

$$i_S^* = 6.40 + 0.34 = 6.74 \text{ mA}$$

Finally, the bridge output E_o^* at $T_o^* = 28^\circ$ C and $v = 200$ fpm, will be:

$$E_o^* = E_o'^* - R_3 (i_R^* - i_S^*) = 0.806 - 13 (12.11 - 6.74) \times 10^{-3}$$

$$E_o^* = 0.736 \text{ V}$$

Therefore, the decrease in E_o is 6.5%, when the ambient temperature increases 3° C (5.4° F), at 200 fpm. This deviation decreases as the thermistor operating points come closer, which occurs at lower velocities. In the limit case, namely, in still air, the deviation is 0% and the bridge should be balanced at any ambient temperature.

CHAPTER III

CONSTRUCTION AND INSTRUMENTATION

3.1. Description of the Anemometer and Auxiliary Equipment

The anemometer was built with the circuit values obtained in Section 2.3 and its final circuit is shown in figure 5. The source used is a regulated d.c. power-supply, HeathKit model IP-20. The output voltage is recorded automatically by a printer, Beckman model 1453, syncronized to a precision digital voltmeter, Beckman model 4011 RVP. The anemometer control box is provided with a voltmeter (Simpson, 1000 Ohm/V, represented by V in figure 5) that can indicate either the input voltage, 8.5 V, or the bridge voltage V_b variable with the air velocity. This selection is possible by means of a d.p.d.t. switch, that also has an OFF position. This OFF position should be used during an experiment in order to disconnect the voltmeter from the circuit and thus eliminating possible errors. A potentiometer of total resistance of 200 Ohm is also provided to adjust the source resistance, R_1, at its calculated value of 305 Ohm. The balance is a precision potentiometer that allows a total of ten complete turns.

The digital voltmeter has a sensitivity control that can be adjusted to a desired level; also, it is possible to adjust the frequency at which the readings should be printed.

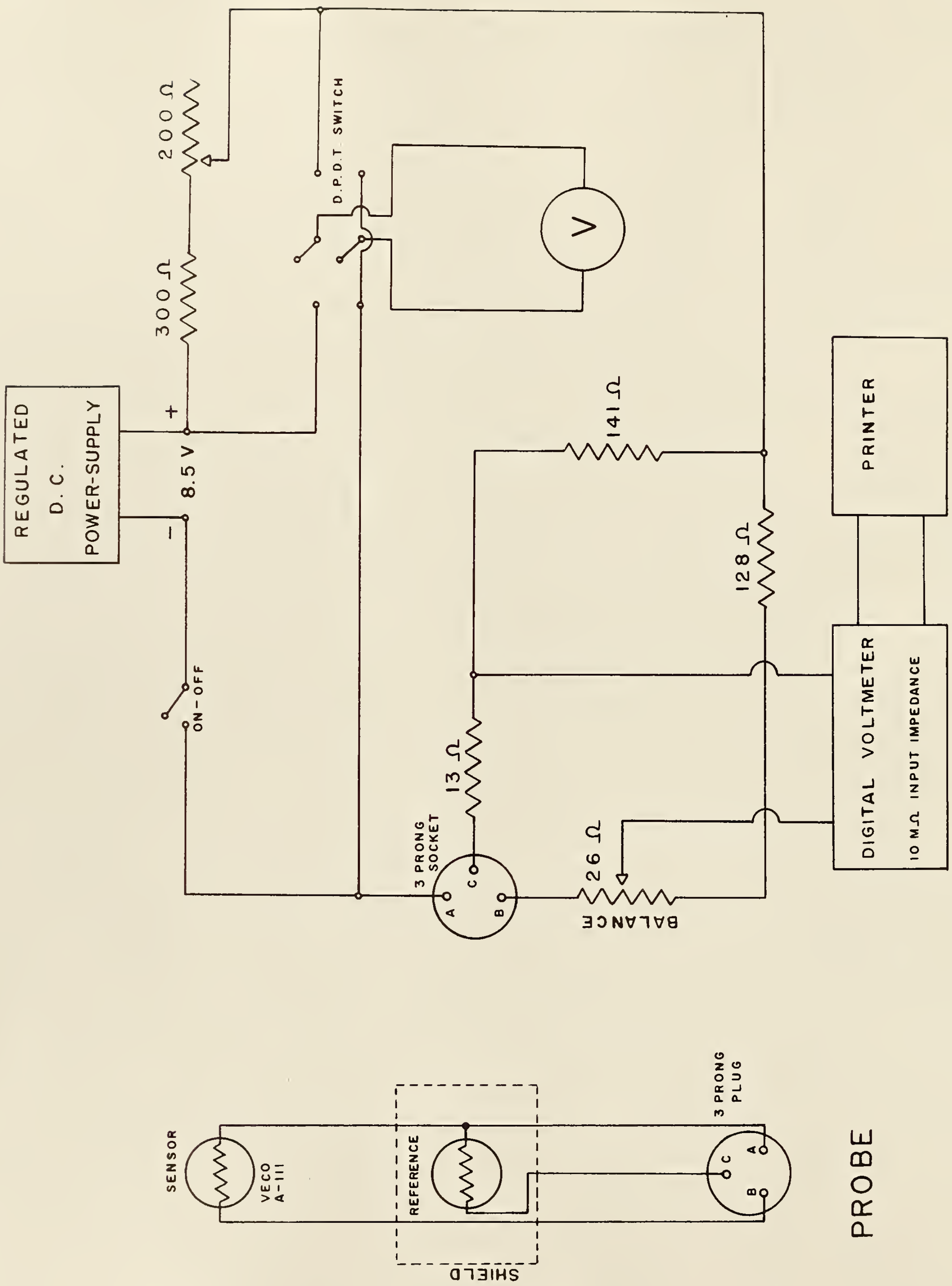


Figure 5 — Actual Circuit Diagram of the Anemometer

The digital voltmeter provides readings of 1/1000 volts in the range from 0.000 V to 9.999 V.

The instruments are shown in figure 6, where 1 is the anemometer control box and the probe, 2, the power-supply, and 3 is the digital voltmeter assembly (the nanovolt amplifier is out of the circuit), with a scanner and the printer.

The probe circuit shown in figure 5 was built inside a 3/8" O.D. brass tube, of thin wall (0.3 mm.) in order to minimize the thermal time constant of the probe. The sensor thermistor is mounted at the end of the probe, as may be seen in figure 7, enlarged eight times actual size. The probe has a sliding sleeve and a cap to protect the sensor when not in use, and to permit the bridge balance in still air. In order to maintain the pressure inside the shield (the brass tube) equal to the external, a small orifice was made in the tubing, underneath the sliding sleeve. The reference thermistor is mounted inside the brass tube, near the handle. The handle is a good thermal insulator (fiberboard).

3.2. Calibration Wind Tunnel

The calibration of the anemometer was done in a wind tunnel available in the Mechanical Engineering Department of Kansas State University.

The tunnel, shown in figure 8, has a 12" nozzle at the test section, 1. The air is drawn through this nozzle by a fan at the end of the duct. In the measuring section, 2, a pitot tube is at the center of a 2" nozzle. The pressure

Figure 6 — Instrumentation: No. 1: Anemometer Control Box and Probe; No. 2: Regulated D.C. Power-Supply; No. 3: Digital Voltmeter and Printer Assembly

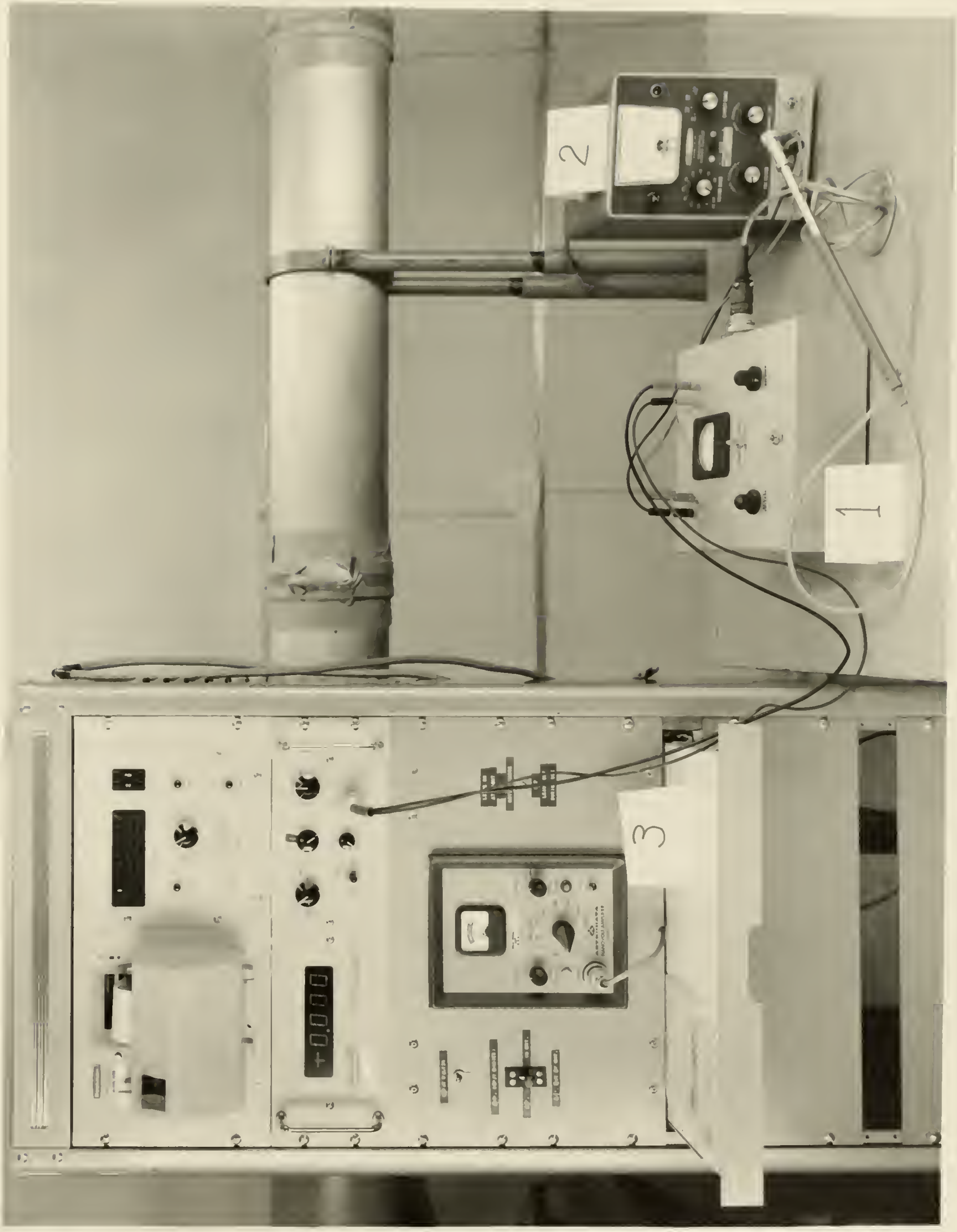


Figure 6

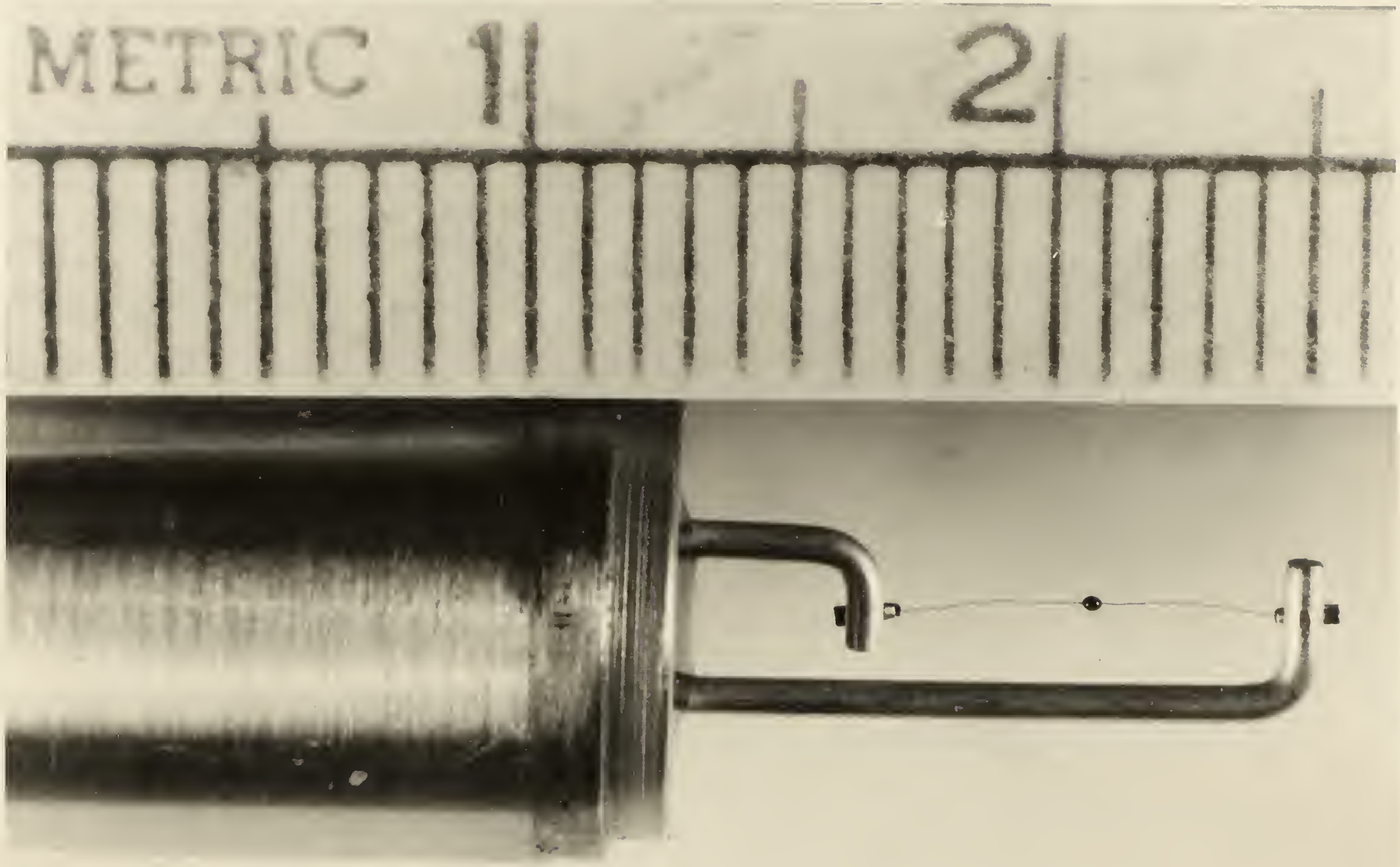


Figure 7 — Closeup of the Probe, Showing Sensor Thermistor (Magnification 8x)

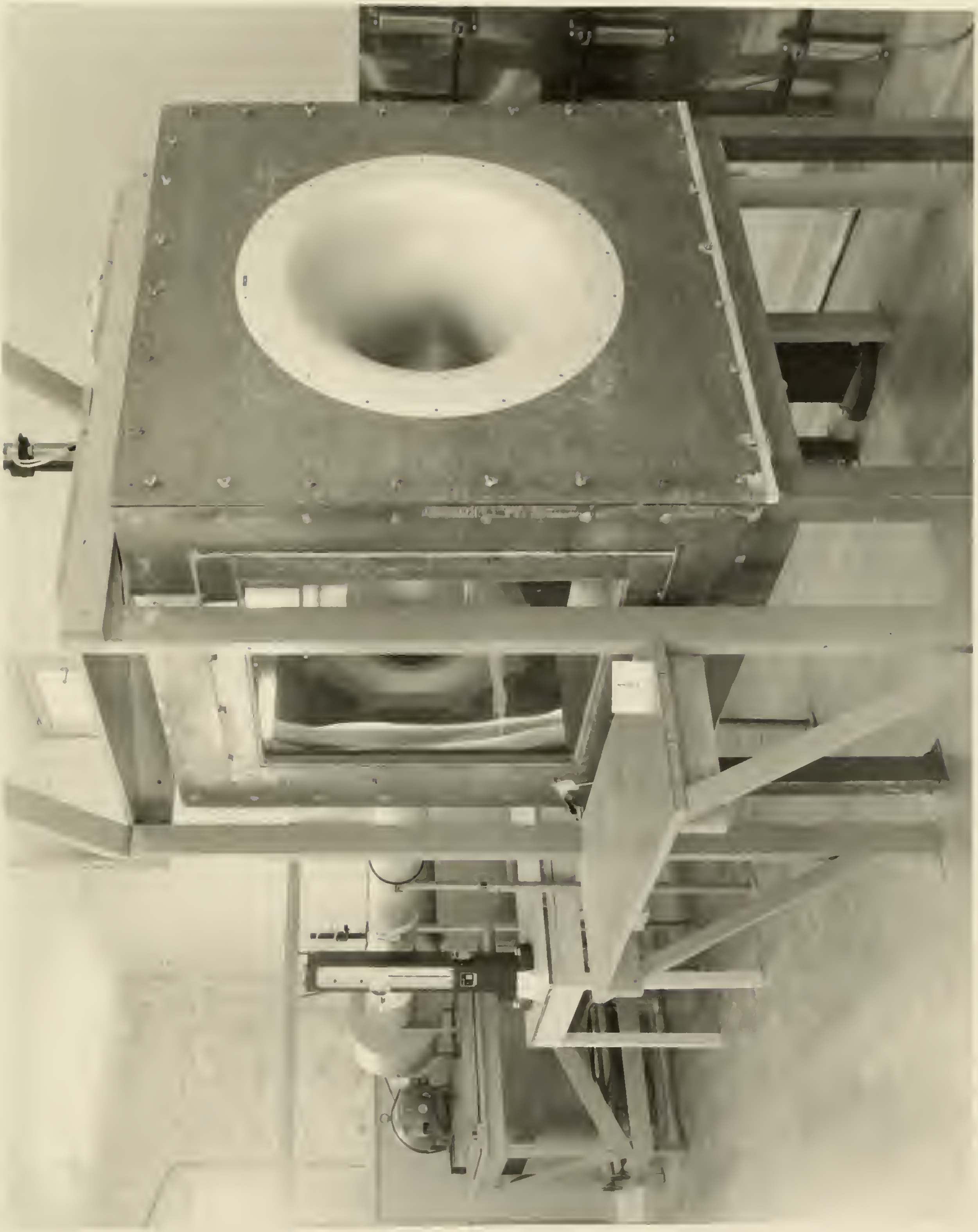


Figure 8 — Wind Tunnel Used to Calibrate the Anemometer

difference is measured by a micromanometer (Meriam, Model 34FB2) connected to the pitot tube. The flow rate may be adjusted by a damper located at the exhaust of the fan. The anemometer probe was carefully placed at the center of the 12" nozzle, inside the test section.

At the measuring section, the velocity at the center of the 2" nozzle can be obtained from Bernouli's equation, as:

$$V_2 = \sqrt{\frac{2g_c\ h_w}{\rho}} = 1096.5\sqrt{\frac{h_w}{\rho}} \tag{40}$$

where V_2 is the velocity (in fpm) at the center of the 2" nozzle, ρ is the air density (pound per cubic foot) and h_w is the velocity pressure directly read in the micromanometer (in inches of water).

In order to relate this velocity to the velocity at the center of the 12" nozzle, V_1, where the probe is located, the continuity equation yields:

$$\rho_1\ A_1\ C_{\text{℄}1}\ V_1 = \rho_2\ A_2\ C_{\text{℄}2}\ V_2 \tag{41}$$

where subscripts 1 and 2 refer to the test section and the measuring section, respectively. A is the nozzle area, and $C_{\text{℄}}$ is the "center-line coefficient" of the nozzles, necessary since V_1 and V_2 are measured only at the center of the nozzles. $C_{\text{℄}1}$ and $C_{\text{℄}2}$ were both previously determined by experiment.

By definition:

$$C_{\text{℄}} \equiv \int_0^1 \sqrt{\frac{h}{h_{\text{℄}}}}\cdot d\left(\frac{r}{R}\right)^2 \tag{42}$$

where h is the head pressure at a radius r, $h_{\text{℄}}$ is the head pressure at the center of the nozzle and R is the nozzle radius. Measuring h as a function of r, $C_{\text{¢}}$ was obtained from the graphical integration of equation (42).

Assuming incompressible flow (for very low Mach numbers):

$$\rho_1 = \rho_2 = \frac{P}{RT}$$

and equations (40) and (41) give:

$$V_1 = 1096.5 \, \frac{A_2 C_{\text{¢} 2}}{A_1 C_{\text{¢} 1}} \sqrt{\frac{(t + 459.67)}{PB} \cdot h_w}$$

Finally it results:

$$V_1 = 26.766 \sqrt{\frac{(t + 459.67)}{P_B} \cdot h_w} \qquad (43)$$

where t is the air temperature (in degrees Fahrenheit), P_B is the barometric pressure (in inches of mercury) and h_w is the velocity pressure (in inches of water).

Equation (43) was used in a computer program to reduce the data taken in the experiments, and the computer output gives h_w, velocity and E_o, in this order, besides the ambient temperature and the barometric pressure existing during the experiment.

CHAPTER IV

CONCLUSIONS AND COMMENTS

4.1. Purpose of the Experiments

The main objectives of the experiments were:

1. To obtain a calibration curve for the instrument at an ambient temperature of 25° C (77° F).

2. To compare the experimental calibration curve with the theoretical prediction of the anemometer performance at 25° C.

3. To investigate the influence of changes in ambient temperature upon the output of the instrument.

4. To determine the time required for the output of the instrument to reach a steady value, when the air temperature suddenly changes to another constant value (as if inserted into a duct carrying warm air).

5. To measure the anemometer output with different directions of the air velocity.

6. To investigate the reproductibility of the initial calibration curve after an extensive period of operation of the instrument to determine drift, if any.

7. To determine the sensitivity of the instrument to changes in air velocity at very low values of velocity.

8. To measure the stability of the instrument readings.

9. To investigate the output of the instrument to air velocities higher than the ones in the designed operation

range, namely, from 10 to 200 fpm.

10. To determine the influence of fluctuations in the input voltage upon the instrument readings.

4.2. Results

The following results were obtained from the experiments: (results are given in the same order as the objectives that motivated the experiments).

1. Several runs were made, at 77° F, and two sets of data points from two runs, chosen at random, are given in Appendix B, on pages 97 and 98.

The first of the two runs has 37 data points with the air velocity varying from 0 to 200 fpm. From the data it can be seen that for an extremely low air velocity such as 7 fpm, the output of the instrument is relatively high, namely, 0.059 V.

The calibration curve for the instrument is given in figure 9, obtained as a mean curve from the data from eight runs taken at the standard temperature of 25° C (77° F).

Table V gives the numerical values of air velocity and their corresponding instrument output. The last digit of the output, that is, millivolts, is doubtful since the instrument is very sensitive and small disturbances in air velocity produced fluctuations in the output of ± 0.005 V. Therefore it is desirable, and more realistic, to consider the output of the instrument limited to 1/100 of a volt.

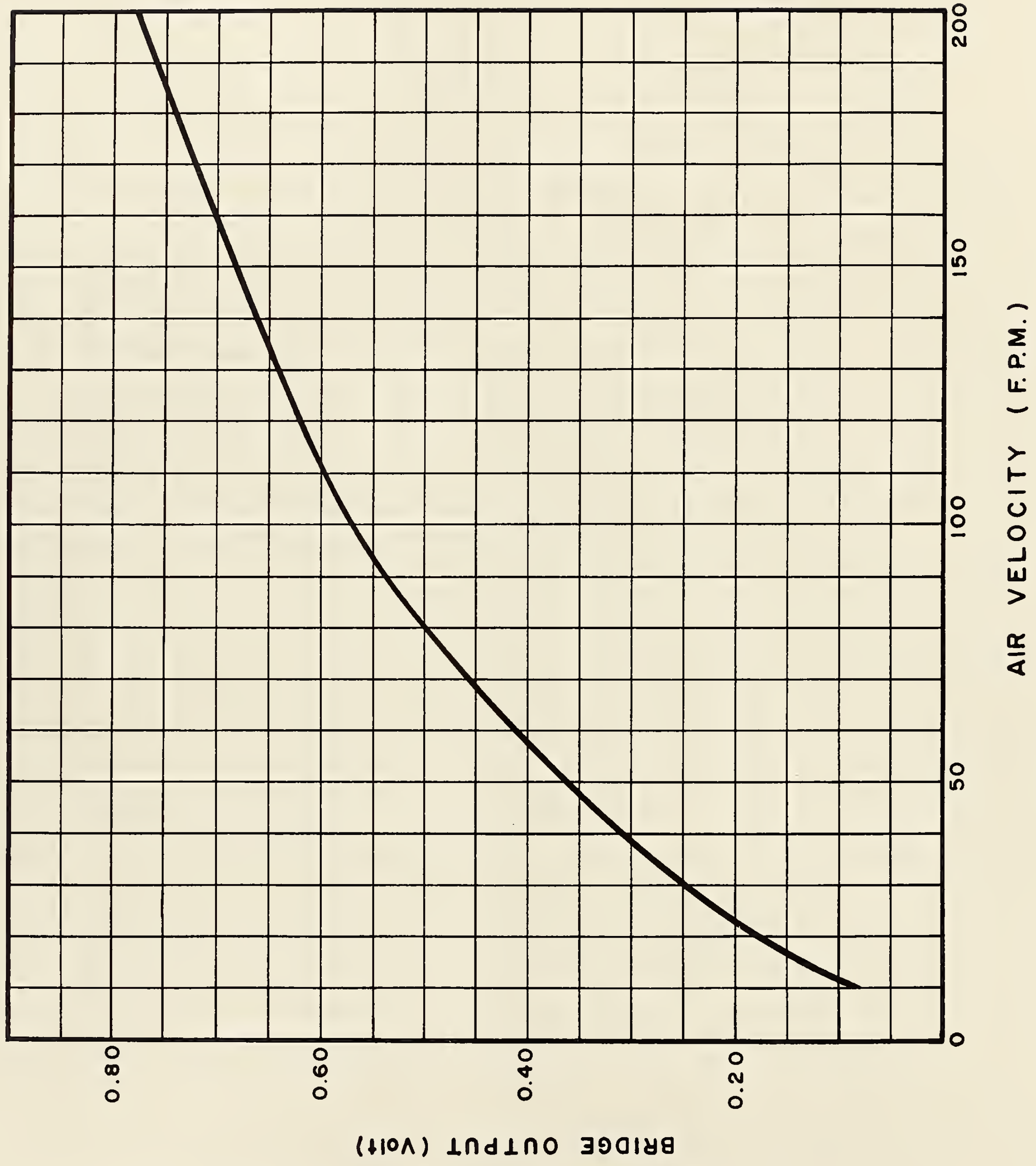


Figure 9 — Anemometer Calibration Curve, at 25° C (77° F)

TABLE V

AIR VELOCITY VS. INSTRUMENT OUTPUT, AT 25° C (77° F)

AIR VELOCITY (fpm)	OUTPUT (VOLT)	AIR VELOCITY (fpm)	OUTPUT (VOLT)
10	0.09	85	0.52
15	0.14	90	0.54
20	0.18	95	0.55
25	0.22	100	0.57
30	0.25	100	0.60
35	0.28	120	0.62
40	0.31	130	0.64
45	0.34	140	0.66
50	0.37	150	0.68
55	0.39	160	0.70
60	0.42	170	0.72
65	0.44	180	0.74
70	0.46	190	0.76
75	0.48	200	0.78
80	0.50		

2. The experiments showed excellent agreement with the theoretically predicted performance.

Figure 10 shows the plots of the theoretical curve and the 37 experimental points from the first run in Appendix B. It is seen that the instrument response follows reasonably the predicted trend and output magnitudes, with only slightly lower values for velocities higher than 30 fpm.

Table VI gives the deviations between both curves for several air velocities.

TABLE VI

DEVIATIONS BETWEEN THEORETICAL AND EXPERIMENTAL CURVES AT 77° F

VELOCITY (fpm)	THEORETICAL OUTPUT (VOLT)	EXPER. OUTPUT (VOLT)	DEVIATION (%)
0	0.00	0.00	0
10	0.17	0.09	47%
30	0.29	0.26	10%
60	0.45	0.42	7%
100	0.60	0.56	7%
150	0.72	0.67	7%
200	0.78	0.77	2%

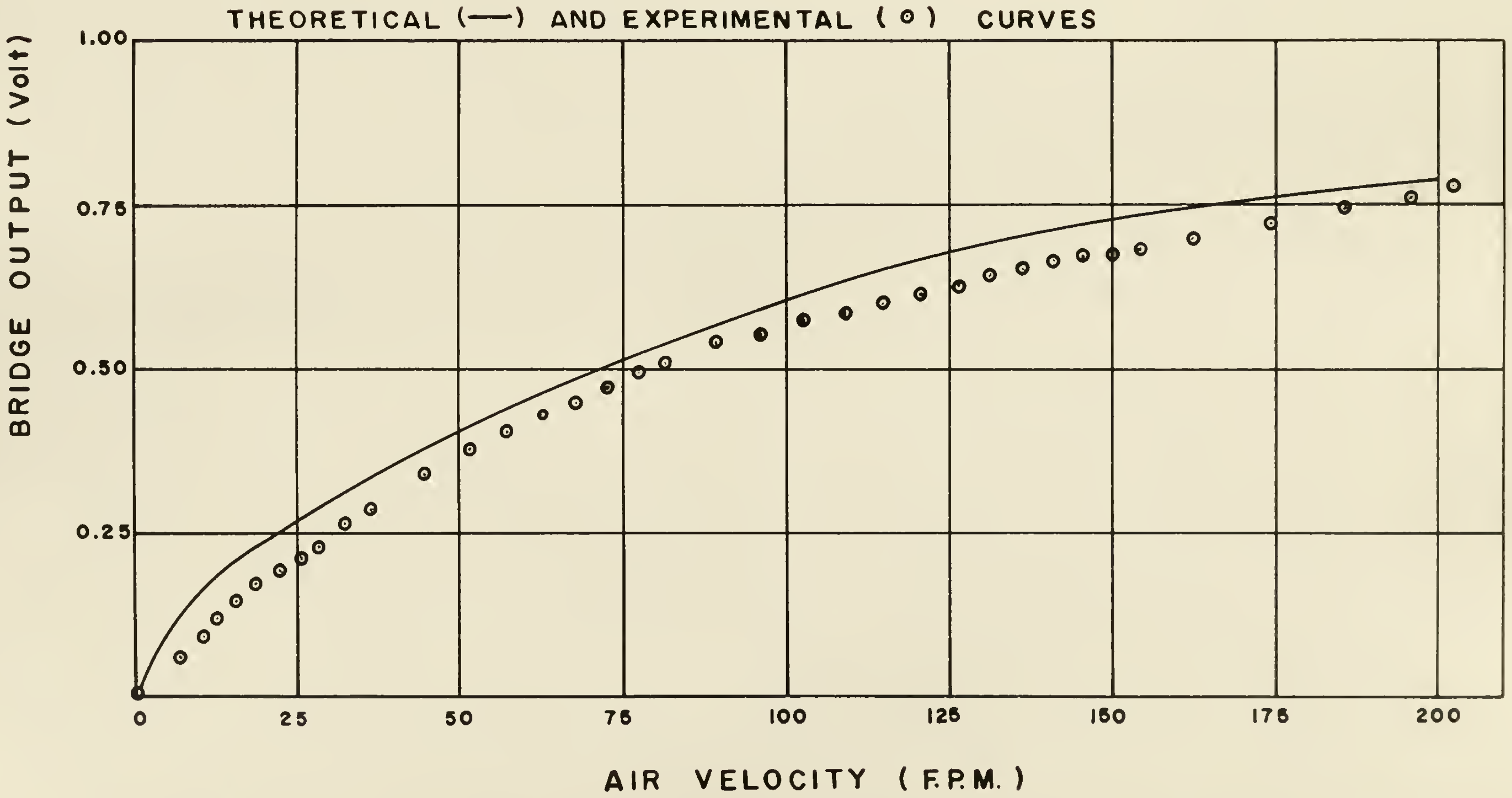


Figure 10 — Comparison Between the Theoretically Predicted and Experimental Anemometer Data, at 25° C (77° F)

With the exception of the deviation at 10 fpm, the other values are reasonable, considering all the assumptions made in the derivation of the relationship between the thermistor dissipation constant and air velocity. The main reasons for the large deviation at 10 fpm are the difficulties either in obtaining a good precision in the graphical solution of figure 4 (for this low velocity), and in obtaining a steady air flow pattern, free from turbulence. Several smoke studies were made inside the 12" nozzle of the wind tunnel in order to visualize the flow pattern for velocities lower than 30 fpm.

Also, all values used in the theoretical prediction were nominal values. For instance, the resistances of the actual thermistors, at 25° C, are 2120 Ohms and 2108 Ohms instead of 2000 Ohms used in the analysis. Table VII shows four points of the Voltage-Current Characteristics of the matched thermistors, measured by the manufacturers.

TABLE VII

A-111 THERMISTOR ASSEMBLY VOLTAGE-CURRENT MATCHING DATA (AT 25° C, IN STILL AIR)

CURRENT (mA)	VOLTAGE (VOLTS) Ro=2120 Ohms	VOLTAGE (VOLTS) Ro=2108 Ohms
2.0	1.488	1.480
5.0	1.380	1.375
10.0	1.160	1.160
15.0	1.040	1.039

3. As predicted in section 2.5, the instrument is very sensitive to large ambient temperature changes. In that section, a deviation of 6.5% in the least favorable air velocity, namely 200 fpm, was predicted, for a 3° C (5.4° F) increase in ambient temperature.

Several experiments were made at different ambient temperatures and for the particular predicted case, the experimental deviation was found to be of the order of 8%, with 5° F increase in T_o.

In Appendix B, the third and fourth runs are for 74° F and 80° F respectively. From those data points it can be seen that the output of the instrument at 80° F is slightly lower than the corresponding velocities at 77° F, and a little higher at 74° F, although not very different from the values at the standard temperature.

For variations of ± 1° F, about the standard temperature, no change was observed in two runs taken at 76° F and one run at 78° F.

For a variation of 23° F, that is, $T_o^* = 100°$ F, the measured output was 0.39 V at 90 fpm. At this velocity, at 77° F, the output is 0.54 V (average value of 8 runs). This deviation represents a decrease in output of 27%.

For a decrease in T_o of 9° F ($T_o^* = 68°$ F), the measured output was 0.815 V, at 154 fpm. At this velocity, at 77° F, the output was 0.680 V. Therefore the increase in E_o is 16.5 %.

Normal variations in the barometric pressure (of the order of 0.5 inch of Hg) had no influence on the anemometer output. Nevertheless it is necessary to investigate the influence of larger variations.

4. The time after a sudden change of 23° F in the air temperature until the instrument output reached a steady and constant value was of the order of 5 minutes, both for the heating and cooling of the probe.

This experiment was performed with the help of a piece of duct with electric heaters inside. With the heaters already warmed up, the duct was placed, as quickly as possible, in front of the 12" nozzle, with the probe in its center. In this fashion, the air temperature inside the test section rapidly increased. In order to obtain a desired temperature rise, it was necessary to change the flow rates and the number of heaters turned on. The disadvantage of this method is the obvious perturbation of the flow pattern caused by the convection currents.

5. In order to investigate the effect of the direction of the moving air upon the output of the anemometer, several tests were made with the probe in different positions. No appreciable change that could indicate sensitivity to a particular air direction was observed.

6. After an initial four-hour period to allow the aging of the thermistors (no readings were made during this period), no appreciable drift in the calibration curve

was observed, neither after extensive continuous operation, nor after a long period of intermittent use.

The run shown on page 98, Appendix B, was taken almost one month after the run on page 97 with the instrument being used intensely during this interval.

7. The sensitivity of the instrument appears to be excellent. Imperceptible changes in air velocity were rapidly indicated. The time necessary for the total system (anemometer and digital voltmeter) to respond was very small, of the order of 2 to 3 seconds. Most of this time was used by the digital voltmeter to adjust itself to the different input.

8. The stability of the instrument is very good, depending obviously upon the stability of the air velocity. The visually observed flow patterns (by means of smoke) agreed with the instrument indication, that is, when the flow was stable, the instrument output was also stable. When a disturbance was noticed in the flow, the instrument output showed some instability. When this disturbance was small, the sensitivity control available in the digital voltmeter permitted to obtain a stable output by adjusting it.

9. Although the velocity range of interest in this application is from 10 to 200 fpm, an investigation of a possible saturation of the anemometer was done. Using another anemometer as a reference (since the wind tunnel did

not permit obtaining velocities higher than 250 fpm without changing nozzles), the instrument proved capable of indicating velocities up to 500 fpm. For example, the output at 450 fpm was 0.94 V. No tests were made at velocities higher than 500 fpm. The anemometer used as reference was the "Anemotherm" (model 60, M.E.Dept.No. 1006) manufactured by Anemostat Corp.

10. Fluctuations of the order of ± 0.100 V in the input did not affect the anemometer output more than 1%. This fact indicates that an inexpensive power-supply, or simply a battery, could be used to provide the 8.5 V input voltage.

4.3. Recommendations and Applications

1. The first and most important recommendation of practical interest is to compensate the influence of changes in ambient temperature upon the output of the anemometer over a given range of temperatures. This problem is very much simplified when the calibration curves for different ambient temperatures are known from experiment. One possible means to achieve the compensation would be the addition of a network to the original circuit, using another thermistor operating at ambient temperature and in parallel with a resistor. This network should be introduced in series with the output voltmeter. In the case of the digital voltmeter, with a negligible current flowing through it, it is necessary to use a shunt to allow a current to flow through the compensating network.

Of course, the circuit should be redesigned and recalibratted, and its analysis would be much more difficult.

2. When a precision digital voltmeter is not available, a simple milliammeter may be used to indicate the anemometer output. The calculation of the new circuit parameters in order to allow this change is easy and can be done as follows. For a full scale (1 mA) deflection of the milliammeter when the air velocity is 200 fpm, the resistance in series with the milliammeter should be:

$$R_G = E_o / i_G = 0.787/(1 \times 10^{-3}) = 787 \text{ Ohms.}$$

With this value of R_G, the other circuit parameters can be readily calculated as shown in section 2.3.

3. An important application for the anemometer is to measure also the temperature at the point where the velocity is measured. This objective can be achieved simply by switching the sensor thermistor to a conventional thermistor bridge for temperature measurements, built in the same control box of the anemometer. References 15, 16, 26, 27 and 29, explain this circuit in detail.

4. Other applications are to introduce range controls in the anemometer circuit to measure high velocities, and to design a new probe to render the instrument sensitive to the direction of velocity. By measuring the maximum output when the probe is rotated, it is possible to determine the direction of the moving air.

BIBLIOGRAPHY

1. Becker, J. A., C. B. Green, and G. L. Pearson. Properties and uses of thermistors. A.I.E.E. Trans. 65:711-725. November, 1946.

2. Clayton, B. R., and E. G. Farmer. An instrument for measuring low fluid velocities. J. Sci. Instr. 40:579-582. 1963.

3. Fechheimer, C. J. Hot sphere anemometer. The Electrical Journal, 27: 536. 1930.

4. Grober, H., S. Erk, and U. Grigull. Fundamentals of Heat Transfer. McGraw-Hill Book Company, Inc., New York, 1961, pp. 282-283.

5. Jones, J. I. P. Continuous computation of the standard deviations of longitudinal and lateral wind velocity components. Brit. J. Appl. Phys. 15(5):467-480. May, 1964.

6. King, Louis Vessot. On the convection of heat from small cylinders in a stream of fluid: Determination of the convection constants of small platinum wires with applications to hot-wire anemometry. Phil. Trans. Royal Society (London) ser. A, Vol. 214:373-432. 1914.

7. Knudsen, J. G., and D. L. Katz. Fluid Dynamics and Heat Transfer. McGraw-Hill Book Co., Inc., New York, 1958, pp. 511-512.

8. Kratz, A. P., A. E. Hershey, and R. B. Engdahl. Development of instruments for study of air distribution in rooms. A.S.H.V.E. Trans. 46:351-366. 1940.

9. Lambert, R. B., H. A. Snyder, and S. K. F. Karlsson. Hot thermistor anemometer for finite amplitude stability measurements. Rev. Sci. Instr. 36(7):924-928. July, 1965.

10. Niiler, Pearn P. Performance of a thermistor anemometer in constant density shear flow. Rev. Sci. Instr. 36(7):921-924. July, 1965.

11. Nottage, H. B.
A simple heated-thermocouple anemometer. A.S.H.V.E. Trans. 56:431-439. 1950.

12. Pollack, H.
An electronic anemometer. Popular Electronics, February, 1957, pp. 51-54.

13. Rasmussen, Robert A.
Application of thermistors to measurements in moving fluids. Rev. Sci. Instr. 33:38-42. January, 1962.

14. Sapoff, M., and R. M. Oppenheim.
Theory and application of self-heated thermistors. Paper presented before the 1963 I.E.E.E.International Convention on March 26, 1963.

15. Saulson, Donald S.
The thermistor bridge. Electro-Technology (New York), pp. 73-75. September, 1961.

16. Schwelb, O., and G. C. Temes.
Thermistor-resistor temperature-sensing networks. Electro-Technology (New York), pp. 71-75. November, 1961.

17. Seban, R. A. W. H. Hillendahl, E. J. Gallagher, and A. L. London.
A thermal anemometer for low velocity flow. A.S.M.E. Trans. 65:843-846. 1943.

18. Seymour, Errol V.
Simple miniature probe anemometer. Engineer (London) 216:899-900. November 29, 1963.

19. Smith, Otto J. M.
Thermistors. Part I. Static characteristics.
Part II.Dynamic characteristics. Rev. Sci. Instr. 21:344-356. 1950.

20. Smith, R. W.
An evaluation of thermistors. Gen. Motors Eng. Journal, p. 14, October-November-December, 1960.

21. Tuve, G. L., and D. K. Wright, Jr.
Air flow measurements at intake and discharge openings and grilles. A.S.H.V.E. Trans. 46:313-328. 1940.

22. Tuve, G. L., D. K. Wright, Jr., and L. J. Seigel.
The use of air-velocity meters. A.S.H.V.E. Trans. 45:645-660. 1939.

23. Vepřek, J. A.
Thermistor flowmeter. J. Sci. Instr. 40:66-68. February, 1963.

24. Wingo, H. E.
Thermistors measure low liquid velocities. Control Engineering 6:131-133. October, 1959.

25. Bead thermistors. Victory Eng. Corp. Springfield, New Jersey. Technical Bulletin MB011.

26. Bridge techniques in temperature measurement. Electronic Products, p. 44. November, 1964.

27. Hand book of thermistor applications. Victory Eng. Corp., Springfield, New Jersey. Form No. V-1133.

28. Matched thermistors. Victory Eng. Corp., Springfield, New Jersey. Form No. SM 111.

29. Thermistors and varistors data book. Victory Eng. Corp., Springfield, New Jersey. Form No. V 42.

30. VECO thermistor terminology. Victory Eng. Corp., Springfield, New Jersey. Technical Bulletin MTD 131.

APPENDIX A

THERMISTOR VOLTAGE-CURRENT CHARACTERISTICS AS A FUNCTION OF AIR VELOCITY

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              VEL=    .OFPM          DELTA=.1000MW/C

 T (C)     RATIO     R (OHM)   T-TA (C)   P (MW)   V (VOLT)     I (MA)
  25.0    1.0000     2000.0        .0       .00       .000        .000
  30.0     .8500     1700.0       5.0       .50       .921        .542
  35.0     .7000     1400.0      10.0      1.00      1.183        .845
  40.0     .6000     1200.0      15.0      1.50      1.341       1.118
  45.0     .5000     1000.0      20.0      2.00      1.414       1.414
  50.0     .4500      900.0      25.0      2.50      1.500       1.666
  55.0     .3800      760.0      30.0      3.00      1.509       1.986
  60.0     .3300      660.0      35.0      3.50      1.519       2.302
  65.0     .2900      580.0      40.0      4.00      1.523       2.626
  70.0     .2500      500.0      45.0      4.50      1.500       3.000
  75.0     .2200      440.0      50.0      5.00      1.483       3.370
  80.0     .1950      390.0      55.0      5.50      1.464       3.755
  85.0     .1700      340.0      60.0      6.00      1.428       4.200
  90.0     .1500      300.0      65.0      6.50      1.396       4.654
  95.0     .1350      270.0      70.0      7.00      1.374       5.091
 100.0     .1200      240.0      75.0      7.50      1.341       5.590
 110.0     .0950      190.0      85.0      8.50      1.270       6.688
 120.0     .0760      152.0      95.0      9.50      1.201       7.905
 130.0     .0600      120.0     105.0     10.50      1.122       9.354
 140.0     .0500      100.0     115.0     11.50      1.072      10.723
 150.0     .0420       84.0     125.0     12.50      1.024      12.198
 160.0     .0350       70.0     135.0     13.50       .972      13.887
 170.0     .0290       58.0     145.0     14.50       .917      15.811
 180.0     .0245       49.0     155.0     15.50       .871      17.785
 190.0     .0210       42.0     165.0     16.50       .832      19.820
 200.0     .0180       36.0     175.0     17.50       .793      22.047
 220.0     .0138       27.6     195.0     19.50       .733      26.580
 240.0     .0108       21.6     215.0     21.50       .681      31.549
 260.0     .0086       17.2     235.0     23.50       .635      36.963
 280.0     .0070       14.0     255.0     25.50       .597      42.678
 300.0     .0060       12.0     275.0     27.50       .574      47.871
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```
           VEL= 10.0FPM            DELTA=.1190MW/C

  T (C)      RATIO      R (OHM)   T-TA (C)   P (MW)   V (VOLT)    I (MA)
   25.0     1.0000      2000.0        .0       .00       .000       .000
   30.0      .8500      1700.0       5.0       .59      1.005       .591
   35.0      .7000      1400.0      10.0      1.19      1.290       .921
   40.0      .6000      1200.0      15.0      1.78      1.463      1.219
   45.0      .5000      1000.0      20.0      2.38      1.542      1.542
   50.0      .4500       900.0      25.0      2.97      1.636      1.818
   55.0      .3800       760.0      30.0      3.57      1.647      2.167
   60.0      .3300       660.0      35.0      4.16      1.657      2.512
   65.0      .2900       580.0      40.0      4.76      1.661      2.864
   70.0      .2500       500.0      45.0      5.35      1.636      3.272
   75.0      .2200       440.0      50.0      5.95      1.618      3.677
   80.0      .1950       390.0      55.0      6.54      1.597      4.096
   85.0      .1700       340.0      60.0      7.14      1.558      4.582
   90.0      .1500       300.0      65.0      7.73      1.523      5.077
   95.0      .1350       270.0      70.0      8.33      1.499      5.554
  100.0      .1200       240.0      75.0      8.92      1.463      6.098
  110.0      .0950       190.0      85.0     10.11      1.386      7.296
  120.0      .0760       152.0      95.0     11.30      1.310      8.624
  130.0      .0600       120.0     105.0     12.49      1.224     10.204
  140.0      .0500       100.0     115.0     13.68      1.169     11.698
  150.0      .0420        84.0     125.0     14.87      1.117     13.307
  160.0      .0350        70.0     135.0     16.06      1.060     15.149
  170.0      .0290        58.0     145.0     17.25      1.000     17.248
  180.0      .0245        49.0     155.0     18.44       .950     19.401
  190.0      .0210        42.0     165.0     19.63       .908     21.621
  200.0      .0180        36.0     175.0     20.82       .865     24.051
  220.0      .0138        27.6     195.0     23.20       .800     28.995
  240.0      .0108        21.6     215.0     25.58       .743     34.416
  260.0      .0086        17.2     235.0     27.96       .693     40.322
  280.0      .0070        14.0     255.0     30.34       .651     46.556
  300.0      .0060        12.0     275.0     32.72       .626     52.221
```

```
            VEL=  20.0FPM             DELTA=.1270MW/C

 T (C)      RATIO      R (OHM)    T-TA (C)   P (MW)   V (VOLT)     I (MA)
  25.0     1.0000      2000.0         .0       .00      .000        .000
  30.0      .8500      1700.0        5.0       .63     1.038        .611
  35.0      .7000      1400.0       10.0      1.27     1.333        .952
  40.0      .6000      1200.0       15.0      1.90     1.511       1.259
  45.0      .5000      1000.0       20.0      2.54     1.593       1.593
  50.0      .4500       900.0       25.0      3.17     1.690       1.878
  55.0      .3800       760.0       30.0      3.81     1.701       2.239
  60.0      .3300       660.0       35.0      4.44     1.712       2.595
  65.0      .2900       580.0       40.0      5.08     1.716       2.959
  70.0      .2500       500.0       45.0      5.71     1.690       3.380
  75.0      .2200       440.0       50.0      6.35     1.671       3.798
  80.0      .1950       390.0       55.0      6.98     1.650       4.232
  85.0      .1700       340.0       60.0      7.62     1.609       4.734
  90.0      .1500       300.0       65.0      8.25     1.573       5.245
  95.0      .1350       270.0       70.0      8.89     1.549       5.738
 100.0      .1200       240.0       75.0      9.52     1.511       6.299
 110.0      .0950       190.0       85.0     10.79     1.432       7.537
 120.0      .0760       152.0       95.0     12.06     1.354       8.909
 130.0      .0600       120.0      105.0     13.33     1.264      10.541
 140.0      .0500       100.0      115.0     14.60     1.208      12.085
 150.0      .0420        84.0      125.0     15.87     1.154      13.747
 160.0      .0350        70.0      135.0     17.14     1.095      15.650
 170.0      .0290        58.0      145.0     18.41     1.033      17.818
 180.0      .0245        49.0      155.0     19.68      .982      20.043
 190.0      .0210        42.0      165.0     20.95      .938      22.336
 200.0      .0180        36.0      175.0     22.22      .894      24.846
 220.0      .0138        27.6      195.0     24.76      .826      29.954
 240.0      .0108        21.6      215.0     27.30      .767      35.554
 260.0      .0086        17.2      235.0     29.84      .716      41.655
 280.0      .0070        14.0      255.0     32.38      .673      48.095
 300.0      .0060        12.0      275.0     34.92      .647      53.948
```

VEL= 30.0FPM DELTA=.1330MW/C

T (C)	RATIO	R (OHM)	T-TA (C)	P (MW)	V (VOLT)	I (MA)
25.0	1.0000	2000.0	.0	.00	.000	.000
30.0	.8500	1700.0	5.0	.66	1.063	.625
35.0	.7000	1400.0	10.0	1.33	1.364	.974
40.0	.6000	1200.0	15.0	1.99	1.547	1.289
45.0	.5000	1000.0	20.0	2.66	1.630	1.630
50.0	.4500	900.0	25.0	3.32	1.729	1.922
55.0	.3800	760.0	30.0	3.99	1.741	2.291
60.0	.3300	660.0	35.0	4.65	1.752	2.655
65.0	.2900	580.0	40.0	5.32	1.756	3.028
70.0	.2500	500.0	45.0	5.98	1.729	3.459
75.0	.2200	440.0	50.0	6.65	1.710	3.887
80.0	.1950	390.0	55.0	7.31	1.689	4.330
85.0	.1700	340.0	60.0	7.98	1.647	4.844
90.0	.1500	300.0	65.0	8.64	1.610	5.368
95.0	.1350	270.0	70.0	9.31	1.585	5.872
100.0	.1200	240.0	75.0	9.97	1.547	6.446
110.0	.0950	190.0	85.0	11.30	1.465	7.713
120.0	.0760	152.0	95.0	12.63	1.385	9.117
130.0	.0600	120.0	105.0	13.96	1.294	10.787
140.0	.0500	100.0	115.0	15.29	1.236	12.367
150.0	.0420	84.0	125.0	16.62	1.181	14.068
160.0	.0350	70.0	135.0	17.95	1.121	16.015
170.0	.0290	58.0	145.0	19.28	1.057	18.234
180.0	.0245	49.0	155.0	20.61	1.005	20.511
190.0	.0210	42.0	165.0	21.94	.960	22.858
200.0	.0180	36.0	175.0	23.27	.915	25.426
220.0	.0138	27.6	195.0	25.93	.846	30.654
240.0	.0108	21.6	215.0	28.59	.785	36.384
260.0	.0086	17.2	235.0	31.25	.733	42.628
280.0	.0070	14.0	255.0	33.91	.689	49.218
300.0	.0060	12.0	275.0	36.57	.662	55.207

```
            VEL= 40.0FPM          DELTA=.1380MW/C

  T (C)      RATIO      R (OHM)   T-TA (C)   P (MW)  V (VOLT)    I (MA)
   25.0     1.0000      2000.0        .0       .00      .000      .000
   30.0      .8500      1700.0       5.0       .69     1.083      .637
   35.0      .7000      1400.0      10.0      1.38     1.389      .992
   40.0      .6000      1200.0      15.0      2.07     1.576     1.313
   45.0      .5000      1000.0      20.0      2.76     1.661     1.661
   50.0      .4500       900.0      25.0      3.45     1.762     1.957
   55.0      .3800       760.0      30.0      4.14     1.773     2.333
   60.0      .3300       660.0      35.0      4.83     1.785     2.705
   65.0      .2900       580.0      40.0      5.52     1.789     3.085
   70.0      .2500       500.0      45.0      6.21     1.762     3.524
   75.0      .2200       440.0      50.0      6.90     1.742     3.960
   80.0      .1950       390.0      55.0      7.59     1.720     4.411
   85.0      .1700       340.0      60.0      8.28     1.677     4.934
   90.0      .1500       300.0      65.0      8.97     1.640     5.468
   95.0      .1350       270.0      70.0      9.66     1.614     5.981
  100.0      .1200       240.0      75.0     10.35     1.576     6.566
  110.0      .0950       190.0      85.0     11.73     1.492     7.857
  120.0      .0760       152.0      95.0     13.11     1.411     9.287
  130.0      .0600       120.0     105.0     14.49     1.318    10.988
  140.0      .0500       100.0     115.0     15.87     1.259    12.597
  150.0      .0420        84.0     125.0     17.25     1.203    14.330
  160.0      .0350        70.0     135.0     18.63     1.141    16.313
  170.0      .0290        58.0     145.0     20.01     1.077    18.574
  180.0      .0245        49.0     155.0     21.39     1.023    20.893
  190.0      .0210        42.0     165.0     22.77      .977    23.283
  200.0      .0180        36.0     175.0     24.15      .932    25.900
  220.0      .0138        27.6     195.0     26.91      .861    31.224
  240.0      .0108        21.6     215.0     29.67      .800    37.062
  260.0      .0086        17.2     235.0     32.43      .746    43.421
  280.0      .0070        14.0     255.0     35.19      .701    50.135
  300.0      .0060        12.0     275.0     37.95      .674    56.236
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```
              VEL=  50.0FPM            DELTA=.1430MW/C

  T (C)      RATIO      R (OHM)   T-TA (C)   P (MW)   V (VOLT)     I (MA)
   25.0     1.0000      2000.0         .0       .00       .000       .000
   30.0      .8500      1700.0        5.0       .71      1.102       .648
   35.0      .7000      1400.0       10.0      1.43      1.414      1.010
   40.0      .6000      1200.0       15.0      2.14      1.604      1.336
   45.0      .5000      1000.0       20.0      2.86      1.691      1.691
   50.0      .4500       900.0       25.0      3.57      1.793      1.993
   55.0      .3800       760.0       30.0      4.29      1.805      2.375
   60.0      .3300       660.0       35.0      5.00      1.817      2.753
   65.0      .2900       580.0       40.0      5.72      1.821      3.140
   70.0      .2500       500.0       45.0      6.43      1.793      3.587
   75.0      .2200       440.0       50.0      7.15      1.773      4.031
   80.0      .1950       390.0       55.0      7.86      1.751      4.490
   85.0      .1700       340.0       60.0      8.58      1.707      5.023
   90.0      .1500       300.0       65.0      9.29      1.669      5.566
   95.0      .1350       270.0       70.0     10.01      1.643      6.088
  100.0      .1200       240.0       75.0     10.72      1.604      6.684
  110.0      .0950       190.0       85.0     12.15      1.519      7.998
  120.0      .0760       152.0       95.0     13.58      1.436      9.453
  130.0      .0600       120.0      105.0     15.01      1.342     11.185
  140.0      .0500       100.0      115.0     16.44      1.282     12.823
  150.0      .0420        84.0      125.0     17.87      1.225     14.587
  160.0      .0350        70.0      135.0     19.30      1.162     16.606
  170.0      .0290        58.0      145.0     20.73      1.096     18.907
  180.0      .0245        49.0      155.0     22.16      1.042     21.268
  190.0      .0210        42.0      165.0     23.59       .995     23.702
  200.0      .0180        36.0      175.0     25.02       .949     26.365
  220.0      .0138        27.6      195.0     27.88       .877     31.785
  240.0      .0108        21.6      215.0     30.74       .814     37.727
  260.0      .0086        17.2      235.0     33.60       .760     44.201
  280.0      .0070        14.0      255.0     36.46       .714     51.035
  300.0      .0060        12.0      275.0     39.32       .686     57.245
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```
                 VEL= 60.0FPM          DELTA=.1470MW/C

   T (C)     RATIO       R (OHM)   T-TA (C)   P (MW)   V (VOLT)     I (MA)
    25.0    1.0000       2000.0         .0      .00       .000       .000
    30.0     .8500       1700.0        5.0      .73      1.117       .657
    35.0     .7000       1400.0       10.0     1.47      1.434      1.024
    40.0     .6000       1200.0       15.0     2.20      1.626      1.355
    45.0     .5000       1000.0       20.0     2.94      1.714      1.714
    50.0     .4500        900.0       25.0     3.67      1.818      2.020
    55.0     .3800        760.0       30.0     4.41      1.830      2.408
    60.0     .3300        660.0       35.0     5.14      1.842      2.792
    65.0     .2900        580.0       40.0     5.88      1.846      3.184
    70.0     .2500        500.0       45.0     6.61      1.818      3.637
    75.0     .2200        440.0       50.0     7.35      1.798      4.087
    80.0     .1950        390.0       55.0     8.08      1.775      4.553
    85.0     .1700        340.0       60.0     8.82      1.731      5.093
    90.0     .1500        300.0       65.0     9.55      1.693      5.643
    95.0     .1350        270.0       70.0    10.29      1.666      6.173
   100.0     .1200        240.0       75.0    11.02      1.626      6.777
   110.0     .0950        190.0       85.0    12.49      1.540      8.109
   120.0     .0760        152.0       95.0    13.96      1.456      9.585
   130.0     .0600        120.0      105.0    15.43      1.360     11.341
   140.0     .0500        100.0      115.0    16.90      1.300     13.001
   150.0     .0420         84.0      125.0    18.37      1.242     14.790
   160.0     .0350         70.0      135.0    19.84      1.178     16.837
   170.0     .0290         58.0      145.0    21.31      1.111     19.170
   180.0     .0245         49.0      155.0    22.78      1.056     21.563
   190.0     .0210         42.0      165.0    24.25      1.009     24.031
   200.0     .0180         36.0      175.0    25.72       .962     26.731
   220.0     .0138         27.6      195.0    28.66       .889     32.227
   240.0     .0108         21.6      215.0    31.60       .826     38.251
   260.0     .0086         17.2      235.0    34.54       .770     44.815
   280.0     .0070         14.0      255.0    37.48       .724     51.744
   300.0     .0060         12.0      275.0    40.42       .696     58.040
```

```
              VEL= 70.0FPM           DELTA=.1506MW/C

  T (C)      RATIO      R (OHM)   T-TA (C)   P (MW)   V (VOLT)     I (MA)
   25.0     1.0000      2000.0         .0      .00       .000       .000
   30.0      .8500      1700.0        5.0      .75      1.131       .665
   35.0      .7000      1400.0       10.0     1.50      1.452      1.037
   40.0      .6000      1200.0       15.0     2.25      1.646      1.372
   45.0      .5000      1000.0       20.0     3.01      1.735      1.735
   50.0      .4500       900.0       25.0     3.76      1.840      2.045
   55.0      .3800       760.0       30.0     4.51      1.853      2.438
   60.0      .3300       660.0       35.0     5.27      1.865      2.826
   65.0      .2900       580.0       40.0     6.02      1.869      3.222
   70.0      .2500       500.0       45.0     6.77      1.840      3.681
   75.0      .2200       440.0       50.0     7.53      1.820      4.136
   80.0      .1950       390.0       55.0     8.28      1.797      4.608
   85.0      .1700       340.0       60.0     9.03      1.752      5.155
   90.0      .1500       300.0       65.0     9.78      1.713      5.712
   95.0      .1350       270.0       70.0    10.54      1.687      6.248
  100.0      .1200       240.0       75.0    11.29      1.646      6.860
  110.0      .0950       190.0       85.0    12.80      1.559      8.208
  120.0      .0760       152.0       95.0    14.30      1.474      9.701
  130.0      .0600       120.0      105.0    15.81      1.377     11.479
  140.0      .0500       100.0      115.0    17.31      1.316     13.160
  150.0      .0420        84.0      125.0    18.82      1.257     14.970
  160.0      .0350        70.0      135.0    20.33      1.192     17.042
  170.0      .0290        58.0      145.0    21.83      1.125     19.403
  180.0      .0245        49.0      155.0    23.34      1.069     21.826
  190.0      .0210        42.0      165.0    24.84      1.021     24.323
  200.0      .0180        36.0      175.0    26.35       .974     27.057
  220.0      .0138        27.6      195.0    29.36       .900     32.619
  240.0      .0108        21.6      215.0    32.37       .836     38.717
  260.0      .0086        17.2      235.0    35.39       .780     45.360
  280.0      .0070        14.0      255.0    38.40       .733     52.374
  300.0      .0060        12.0      275.0    41.41       .704     58.747
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```
            VEL= 80.0FPM          DELTA=.1540MW/C

T (C)      RATIO      R (OHM)  T-TA (C)   P (MW)  V (VOLT)    I (MA)
 25.0     1.0000      2000.0        .0      .00      .000      .000
 30.0      .8500      1700.0       5.0      .77     1.144      .673
 35.0      .7000      1400.0      10.0     1.54     1.468     1.048
 40.0      .6000      1200.0      15.0     2.31     1.664     1.387
 45.0      .5000      1000.0      20.0     3.08     1.754     1.754
 50.0      .4500       900.0      25.0     3.85     1.861     2.068
 55.0      .3800       760.0      30.0     4.62     1.873     2.465
 60.0      .3300       660.0      35.0     5.39     1.886     2.857
 65.0      .2900       580.0      40.0     6.16     1.890     3.258
 70.0      .2500       500.0      45.0     6.93     1.861     3.722
 75.0      .2200       440.0      50.0     7.70     1.840     4.183
 80.0      .1950       390.0      55.0     8.47     1.817     4.660
 85.0      .1700       340.0      60.0     9.24     1.772     5.213
 90.0      .1500       300.0      65.0    10.01     1.732     5.776
 95.0      .1350       270.0      70.0    10.78     1.706     6.318
100.0      .1200       240.0      75.0    11.55     1.664     6.937
110.0      .0950       190.0      85.0    13.09     1.577     8.300
120.0      .0760       152.0      95.0    14.63     1.491     9.810
130.0      .0600       120.0     105.0    16.17     1.392    11.608
140.0      .0500       100.0     115.0    17.71     1.330    13.307
150.0      .0420        84.0     125.0    19.25     1.271    15.138
160.0      .0350        70.0     135.0    20.79     1.206    17.233
170.0      .0290        58.0     145.0    22.33     1.138    19.621
180.0      .0245        49.0     155.0    23.87     1.081    22.071
190.0      .0210        42.0     165.0    25.41     1.033    24.596
200.0      .0180        36.0     175.0    26.95      .984    27.360
220.0      .0138        27.6     195.0    30.03      .910    32.985
240.0      .0108        21.6     215.0    33.11      .845    39.151
260.0      .0086        17.2     235.0    36.19      .788    45.870
280.0      .0070        14.0     255.0    39.27      .741    52.962
300.0      .0060        12.0     275.0    42.35      .712    59.406
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```
              VEL= 90.0FPM            DELTA=.1575MW/C

  T (C)      RATIO      R (OHM)   T-TA (C)   P (MW)   V (VOLT)     I (MA)
   25.0     1.0000      2000.0        .0       .00       .000       .000
   30.0      .8500      1700.0       5.0       .78      1.157       .680
   35.0      .7000      1400.0      10.0      1.57      1.484      1.060
   40.0      .6000      1200.0      15.0      2.36      1.683      1.403
   45.0      .5000      1000.0      20.0      3.15      1.774      1.774
   50.0      .4500       900.0      25.0      3.93      1.882      2.091
   55.0      .3800       760.0      30.0      4.72      1.894      2.493
   60.0      .3300       660.0      35.0      5.51      1.907      2.890
   65.0      .2900       580.0      40.0      6.30      1.911      3.295
   70.0      .2500       500.0      45.0      7.08      1.882      3.764
   75.0      .2200       440.0      50.0      7.87      1.861      4.230
   80.0      .1950       390.0      55.0      8.66      1.838      4.712
   85.0      .1700       340.0      60.0      9.45      1.792      5.272
   90.0      .1500       300.0      65.0     10.23      1.752      5.841
   95.0      .1350       270.0      70.0     11.02      1.725      6.390
  100.0      .1200       240.0      75.0     11.81      1.683      7.015
  110.0      .0950       190.0      85.0     13.38      1.594      8.394
  120.0      .0760       152.0      95.0     14.96      1.508      9.921
  130.0      .0600       120.0     105.0     16.53      1.408     11.739
  140.0      .0500       100.0     115.0     18.11      1.345     13.458
  150.0      .0420        84.0     125.0     19.68      1.285     15.309
  160.0      .0350        70.0     135.0     21.26      1.219     17.428
  170.0      .0290        58.0     145.0     22.83      1.150     19.843
  180.0      .0245        49.0     155.0     24.41      1.093     22.320
  190.0      .0210        42.0     165.0     25.98      1.044     24.874
  200.0      .0180        36.0     175.0     27.56       .996     27.669
  220.0      .0138        27.6     195.0     30.71       .920     33.358
  240.0      .0108        21.6     215.0     33.86       .855     39.594
  260.0      .0086        17.2     235.0     37.01       .797     46.388
  280.0      .0070        14.0     255.0     40.16       .749     53.560
  300.0      .0060        12.0     275.0     43.31       .720     60.078
```

```
              VEL=100.0FPM              DELTA=.1605MW/C

  T (C)      RATIO     R (OHM)   T-TA (C)   P (MW)  V (VOLT)     I (MA)
   25.0     1.0000     2000.0         .0      .00      .000        .000
   30.0      .8500     1700.0        5.0      .80     1.168        .687
   35.0      .7000     1400.0       10.0     1.60     1.498       1.070
   40.0      .6000     1200.0       15.0     2.40     1.699       1.416
   45.0      .5000     1000.0       20.0     3.21     1.791       1.791
   50.0      .4500      900.0       25.0     4.01     1.900       2.111
   55.0      .3800      760.0       30.0     4.81     1.912       2.517
   60.0      .3300      660.0       35.0     5.61     1.925       2.917
   65.0      .2900      580.0       40.0     6.42     1.929       3.327
   70.0      .2500      500.0       45.0     7.22     1.900       3.800
   75.0      .2200      440.0       50.0     8.02     1.879       4.270
   80.0      .1950      390.0       55.0     8.82     1.855       4.757
   85.0      .1700      340.0       60.0     9.63     1.809       5.321
   90.0      .1500      300.0       65.0    10.43     1.769       5.897
   95.0      .1350      270.0       70.0    11.23     1.741       6.450
  100.0      .1200      240.0       75.0    12.03     1.699       7.082
  110.0      .0950      190.0       85.0    13.64     1.609       8.473
  120.0      .0760      152.0       95.0    15.24     1.522      10.015
  130.0      .0600      120.0      105.0    16.85     1.422      11.850
  140.0      .0500      100.0      115.0    18.45     1.358      13.585
  150.0      .0420       84.0      125.0    20.06     1.298      15.454
  160.0      .0350       70.0      135.0    21.66     1.231      17.593
  170.0      .0290       58.0      145.0    23.27     1.161      20.031
  180.0      .0245       49.0      155.0    24.87     1.104      22.532
  190.0      .0210       42.0      165.0    26.48     1.054      25.110
  200.0.     .0180       36.0      175.0    28.08     1.005      27.932
  220.0      .0138       27.6      195.0    31.29      .929      33.674
  240.0      .0108       21.6      215.0    34.50      .863      39.969
  260.0      .0086       17.2      235.0    37.71      .805      46.828
  280.0      .0070       14.0      255.0    40.92      .756      54.068
  300.0      .0060       12.0      275.0    44.13      .727      60.647
```

VEL=110.0FPM DELTA=.1635MW/C

T (C)	RATIO	R (OHM)	T-TA (C)	P (MW)	V (VOLT)	I (MA)
25.0	1.0000	2000.0	.0	.00	.000	.000
30.0	.8500	1700.0	5.0	.81	1.178	.693
35.0	.7000	1400.0	10.0	1.63	1.512	1.080
40.0	.6000	1200.0	15.0	2.45	1.715	1.429
45.0	.5000	1000.0	20.0	3.27	1.808	1.808
50.0	.4500	900.0	25.0	4.08	1.918	2.131
55.0	.3800	760.0	30.0	4.90	1.930	2.540
60.0	.3300	660.0	35.0	5.72	1.943	2.944
65.0	.2900	580.0	40.0	6.54	1.947	3.357
70.0	.2500	500.0	45.0	7.35	1.918	3.836
75.0	.2200	440.0	50.0	8.17	1.896	4.310
80.0	.1950	390.0	55.0	8.99	1.872	4.801
85.0	.1700	340.0	60.0	9.81	1.826	5.371
90.0	.1500	300.0	65.0	10.62	1.785	5.951
95.0	.1350	270.0	70.0	11.44	1.757	6.510
100.0	.1200	240.0	75.0	12.26	1.715	7.147
110.0	.0950	190.0	85.0	13.89	1.624	8.552
120.0	.0760	152.0	95.0	15.53	1.536	10.108
130.0	.0600	120.0	105.0	17.16	1.435	11.960
140.0	.0500	100.0	115.0	18.80	1.371	13.712
150.0	.0420	84.0	125.0	20.43	1.310	15.598
160.0	.0350	70.0	135.0	22.07	1.243	17.757
170.0	.0290	58.0	145.0	23.70	1.172	20.217
180.0	.0245	49.0	155.0	25.34	1.114	22.741
190.0	.0210	42.0	165.0	26.97	1.064	25.344
200.0	.0180	36.0	175.0	28.61	1.014	28.192
220.0	.0138	27.6	195.0	31.88	.938	33.987
240.0	.0108	21.6	215.0	35.15	.871	40.341
260.0	.0086	17.2	235.0	38.42	.812	47.263
280.0	.0070	14.0	255.0	41.69	.763	54.571
300.0	.0060	12.0	275.0	44.96	.734	61.211

```
            VEL=120.0FPM            DELTA=.1663MW/C

  T (C)     RATIO       R (OHM)  T-TA (C)  P (MW)  V (VOLT)     I (MA)
   25.0    1.0000       2000.0        .0     .00      .000       .000
   30.0     .8500       1700.0       5.0     .83     1.188       .699
   35.0     .7000       1400.0      10.0    1.66     1.525      1.089
   40.0     .6000       1200.0      15.0    2.49     1.730      1.441
   45.0     .5000       1000.0      20.0    3.32     1.823      1.823
   50.0     .4500        900.0      25.0    4.15     1.934      2.149
   55.0     .3800        760.0      30.0    4.98     1.947      2.562
   60.0     .3300        660.0      35.0    5.82     1.959      2.969
   65.0     .2900        580.0      40.0    6.65     1.964      3.386
   70.0     .2500        500.0      45.0    7.48     1.934      3.868
   75.0     .2200        440.0      50.0    8.31     1.912      4.347
   80.0     .1950        390.0      55.0    9.14     1.888      4.842
   85.0     .1700        340.0      60.0    9.97     1.841      5.417
   90.0     .1500        300.0      65.0   10.80     1.800      6.002
   95.0     .1350        270.0      70.0   11.64     1.772      6.566
  100.0     .1200        240.0      75.0   12.47     1.730      7.208
  110.0     .0950        190.0      85.0   14.13     1.638      8.625
  120.0     .0760        152.0      95.0   15.79     1.549     10.194
  130.0     .0600        120.0     105.0   17.46     1.447     12.062
  140.0     .0500        100.0     115.0   19.12     1.382     13.829
  150.0     .0420         84.0     125.0   20.78     1.321     15.731
  160.0     .0350         70.0     135.0   22.45     1.253     17.908
  170.0     .0290         58.0     145.0   24.11     1.182     20.389
  180.0     .0245         49.0     155.0   25.77     1.123     22.935
  190.0     .0210         42.0     165.0   27.43     1.073     25.560
  200.0     .0180         36.0     175.0   29.10     1.023     28.432
  220.0     .0138         27.6     195.0   32.42      .946     34.277
  240.0     .0108         21.6     215.0   35.75      .878     40.685
  260.0     .0086         17.2     235.0   39.08      .819     47.666
  280.0     .0070         14.0     255.0   42.40      .770     55.036
  300.0     .0060         12.0     275.0   45.73      .740     61.733
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```
            VEL=130.0FPM            DELTA=.1690MW/C

T (C)      RATIO     R (OHM)   T-TA (C)   P (MW)   V (VOLT)    I (MA)
 25.0     1.0000     2000.0        .0       .00      .000       .000
 30.0      .8500     1700.0       5.0       .84     1.198       .705
 35.0      .7000     1400.0      10.0      1.69     1.538      1.098
 40.0      .6000     1200.0      15.0      2.53     1.744      1.453
 45.0      .5000     1000.0      20.0      3.38     1.838      1.838
 50.0      .4500      900.0      25.0      4.22     1.950      2.166
 55.0      .3800      760.0      30.0      5.07     1.962      2.582
 60.0      .3300      660.0      35.0      5.91     1.975      2.993
 65.0      .2900      580.0      40.0      6.76     1.980      3.413
 70.0      .2500      500.0      45.0      7.60     1.950      3.900
 75.0      .2200      440.0      50.0      8.45     1.928      4.382
 80.0      .1950      390.0      55.0      9.29     1.903      4.881
 85.0      .1700      340.0      60.0     10.14     1.856      5.461
 90.0      .1500      300.0      65.0     10.98     1.815      6.051
 95.0      .1350      270.0      70.0     11.83     1.787      6.619
100.0      .1200      240.0      75.0     12.67     1.744      7.267
110.0      .0950      190.0      85.0     14.36     1.652      8.695
120.0      .0760      152.0      95.0     16.05     1.562     10.277
130.0      .0600      120.0     105.0     17.74     1.459     12.160
140.0      .0500      100.0     115.0     19.43     1.394     13.940
150.0      .0420       84.0     125.0     21.12     1.332     15.858
160.0      .0350       70.0     135.0     22.81     1.263     18.053
170.0      .0290       58.0     145.0     24.50     1.192     20.554
180.0      .0245       49.0     155.0     26.19     1.132     23.121
190.0      .0210       42.0     165.0     27.88     1.082     25.766
200.0      .0180       36.0     175.0     29.57     1.031     28.662
220.0      .0138       27.6     195.0     32.95      .953     34.554
240.0      .0108       21.6     215.0     36.33      .885     41.014
260.0      .0086       17.2     235.0     39.71      .826     48.052
280.0      .0070       14.0     255.0     43.09      .776     55.481
300.0      .0060       12.0     275.0     46.47      .746     62.232
```

VEL=140.0FPM DELTA=.1715MW/C

T (C)	RATIO	R (OHM)	T-TA (C)	P (MW)	V (VOLT)	I (MA)
25.0	1.0000	2000.0	.0	.00	.000	.000
30.0	.8500	1700.0	5.0	.85	1.207	.710
35.0	.7000	1400.0	10.0	1.71	1.549	1.106
40.0	.6000	1200.0	15.0	2.57	1.756	1.464
45.0	.5000	1000.0	20.0	3.43	1.852	1.852
50.0	.4500	900.0	25.0	4.28	1.964	2.182
55.0	.3800	760.0	30.0	5.14	1.977	2.601
60.0	.3300	660.0	35.0	6.00	1.990	3.015
65.0	.2900	580.0	40.0	6.86	1.994	3.439
70.0	.2500	500.0	45.0	7.71	1.964	3.928
75.0	.2200	440.0	50.0	8.57	1.942	4.414
80.0	.1950	390.0	55.0	9.43	1.917	4.917
85.0	.1700	340.0	60.0	10.29	1.870	5.501
90.0	.1500	300.0	65.0	11.14	1.828	6.095
95.0	.1350	270.0	70.0	12.00	1.800	6.668
100.0	.1200	240.0	75.0	12.86	1.756	7.320
110.0	.0950	190.0	85.0	14.57	1.664	8.759
120.0	.0760	152.0	95.0	16.29	1.573	10.353
130.0	.0600	120.0	105.0	18.00	1.470	12.250
140.0	.0500	100.0	115.0	19.72	1.404	14.043
150.0	.0420	84.0	125.0	21.43	1.341	15.975
160.0	.0350	70.0	135.0	23.15	1.273	18.186
170.0	.0290	58.0	145.0	24.86	1.200	20.706
180.0	.0245	49.0	155.0	26.58	1.141	23.291
190.0	.0210	42.0	165.0	28.29	1.090	25.956
200.0	.0180	36.0	175.0	30.01	1.039	28.873
220.0	.0138	27.6	195.0	33.44	.960	34.809
240.0	.0108	21.6	215.0	36.87	.892	41.316
260.0	.0086	17.2	235.0	40.30	.832	48.406
280.0	.0070	14.0	255.0	43.73	.782	55.890
300.0	.0060	12.0	275.0	47.16	.752	62.691

```
              VEL=150.0FPM            DELTA=.1741MW/C

  T (C)      RATIO      R (OHM)   T-TA (C)   P (MW)   V (VOLT)    I (MA)
   25.0     1.0000      2000.0         .0       .00       .000       .000
   30.0      .8500      1700.0        5.0       .87      1.216       .715
   35.0      .7000      1400.0       10.0      1.74      1.561      1.115
   40.0      .6000      1200.0       15.0      2.61      1.770      1.475
   45.0      .5000      1000.0       20.0      3.48      1.866      1.866
   50.0      .4500       900.0       25.0      4.35      1.979      2.199
   55.0      .3800       760.0       30.0      5.22      1.992      2.621
   60.0      .3300       660.0       35.0      6.09      2.005      3.038
   65.0      .2900       580.0       40.0      6.96      2.009      3.465
   70.0      .2500       500.0       45.0      7.83      1.979      3.958
   75.0      .2200       440.0       50.0      8.70      1.957      4.447
   80.0      .1950       390.0       55.0      9.57      1.932      4.955
   85.0      .1700       340.0       60.0     10.44      1.884      5.542
   90.0      .1500       300.0       65.0     11.31      1.842      6.141
   95.0      .1350       270.0       70.0     12.18      1.813      6.718
  100.0      .1200       240.0       75.0     13.05      1.770      7.376
  110.0      .0950       190.0       85.0     14.79      1.676      8.825
  120.0      .0760       152.0       95.0     16.53      1.585     10.431
  130.0      .0600       120.0      105.0     18.28      1.481     12.342
  140.0      .0500       100.0      115.0     20.02      1.414     14.149
  150.0      .0420        84.0      125.0     21.76      1.352     16.095
  160.0      .0350        70.0      135.0     23.50      1.282     18.323
  170.0      .0290        58.0      145.0     25.24      1.210     20.862
  180.0      .0245        49.0      155.0     26.98      1.149     23.467
  190.0      .0210        42.0      165.0     28.72      1.098     26.152
  200.0      .0180        36.0      175.0     30.46      1.047     29.091
  220.0      .0138        27.6      195.0     33.94       .967     35.072
  240.0      .0108        21.6      215.0     37.43       .899     41.628
  260.0      .0086        17.2      235.0     40.91       .838     48.771
  280.0      .0070        14.0      255.0     44.39       .788     56.312
  300.0      .0060        12.0      275.0     47.87       .757     63.164
```

VEL=160.0FPM DELTA=.1765MW/C

T (C)	RATIO	R (OHM)	T-TA (C)	P (MW)	V (VOLT)	I (MA)
25.0	1.0000	2000.0	.0	.00	.000	.000
30.0	.8500	1700.0	5.0	.88	1.224	.720
35.0	.7000	1400.0	10.0	1.76	1.571	1.122
40.0	.6000	1200.0	15.0	2.64	1.782	1.485
45.0	.5000	1000.0	20.0	3.53	1.878	1.878
50.0	.4500	900.0	25.0	4.41	1.992	2.214
55.0	.3800	760.0	30.0	5.29	2.006	2.639
60.0	.3300	660.0	35.0	6.17	2.019	3.059
65.0	.2900	580.0	40.0	7.06	2.023	3.488
70.0	.2500	500.0	45.0	7.94	1.992	3.985
75.0	.2200	440.0	50.0	8.82	1.970	4.478
80.0	.1950	390.0	55.0	9.70	1.945	4.989
85.0	.1700	340.0	60.0	10.59	1.897	5.580
90.0	.1500	300.0	65.0	11.47	1.855	6.183
95.0	.1350	270.0	70.0	12.35	1.826	6.764
100.0	.1200	240.0	75.0	13.23	1.782	7.426
110.0	.0950	190.0	85.0	15.00	1.688	8.885
120.0	.0760	152.0	95.0	16.76	1.596	10.502
130.0	.0600	120.0	105.0	18.53	1.491	12.427
140.0	.0500	100.0	115.0	20.29	1.424	14.246
150.0	.0420	84.0	125.0	22.06	1.361	16.206
160.0	.0350	70.0	135.0	23.82	1.291	18.449
170.0	.0290	58.0	145.0	25.59	1.218	21.005
180.0	.0245	49.0	155.0	27.35	1.157	23.628
190.0	.0210	42.0	165.0	29.12	1.105	26.332
200.0	.0180	36.0	175.0	30.88	1.054	29.291
220.0	.0138	27.6	195.0	34.41	.974	35.313
240.0	.0108	21.6	215.0	37.94	.905	41.914
260.0	.0086	17.2	235.0	41.47	.844	49.106
280.0	.0070	14.0	255.0	45.00	.793	56.699
300.0	.0060	12.0	275.0	48.53	.763	63.598

```
              VEL=170.0FPM              DELTA=.1789MW/C

T (C)      RATIO      R (OHM)   T-TA (C)   P (MW)   V (VOLT)    I (MA)
 25.0     1.0000      2000.0         .0       .00       .000       .000
 30.0      .8500      1700.0        5.0       .89      1.233       .725
 35.0      .7000      1400.0       10.0      1.78      1.582      1.130
 40.0      .6000      1200.0       15.0      2.68      1.794      1.495
 45.0      .5000      1000.0       20.0      3.57      1.891      1.891
 50.0      .4500       900.0       25.0      4.47      2.006      2.229
 55.0      .3800       760.0       30.0      5.36      2.019      2.657
 60.0      .3300       660.0       35.0      6.26      2.032      3.080
 65.0      .2900       580.0       40.0      7.15      2.037      3.512
 70.0      .2500       500.0       45.0      8.05      2.006      4.012
 75.0      .2200       440.0       50.0      8.94      1.983      4.508
 80.0      .1950       390.0       55.0      9.83      1.958      5.022
 85.0      .1700       340.0       60.0     10.73      1.910      5.618
 90.0      .1500       300.0       65.0     11.62      1.867      6.225
 95.0      .1350       270.0       70.0     12.52      1.838      6.810
100.0      .1200       240.0       75.0     13.41      1.794      7.477
110.0      .0950       190.0       85.0     15.20      1.699      8.946
120.0      .0760       152.0       95.0     16.99      1.607     10.574
130.0      .0600       120.0      105.0     18.78      1.501     12.511
140.0      .0500       100.0      115.0     20.57      1.434     14.343
150.0      .0420        84.0      125.0     22.36      1.370     16.316
160.0      .0350        70.0      135.0     24.15      1.300     18.574
170.0      .0290        58.0      145.0     25.94      1.226     21.148
180.0      .0245        49.0      155.0     27.72      1.165     23.788
190.0      .0210        42.0      165.0     29.51      1.113     26.510
200.0      .0180        36.0      175.0     31.30      1.061     29.489
220.0      .0138        27.6      195.0     34.88       .981     35.552
240.0      .0108        21.6      215.0     38.46       .911     42.198
260.0      .0086        17.2      235.0     42.04       .850     49.439
280.0      .0070        14.0      255.0     45.61       .799     57.083
300.0      .0060        12.0      275.0     49.19       .768     64.029
```

```
              VEL=180.0FPM            DELTA=.1812MW/C

T (C)      RATIO      R (OHM)   T-TA (C)   P (MW)   V (VOLT)     I (MA)
 25.0     1.0000      2000.0         .0      .00       .000        .000
 30.0      .8500      1700.0        5.0      .90      1.241        .730
 35.0      .7000      1400.0       10.0     1.81      1.592       1.137
 40.0      .6000      1200.0       15.0     2.71      1.805       1.504
 45.0      .5000      1000.0       20.0     3.62      1.903       1.903
 50.0      .4500       900.0       25.0     4.53      2.019       2.243
 55.0      .3800       760.0       30.0     5.43      2.032       2.674
 60.0      .3300       660.0       35.0     6.34      2.045       3.099
 65.0      .2900       580.0       40.0     7.24      2.050       3.535
 70.0      .2500       500.0       45.0     8.15      2.019       4.038
 75.0      .2200       440.0       50.0     9.06      1.996       4.537
 80.0      .1950       390.0       55.0     9.96      1.971       5.055
 85.0      .1700       340.0       60.0    10.87      1.922       5.654
 90.0      .1500       300.0       65.0    11.77      1.879       6.265
 95.0      .1350       270.0       70.0    12.68      1.850       6.854
100.0      .1200       240.0       75.0    13.59      1.805       7.524
110.0      .0950       190.0       85.0    15.40      1.710       9.003
120.0      .0760       152.0       95.0    17.21      1.617      10.641
130.0      .0600       120.0      105.0    19.02      1.510      12.591
140.0      .0500       100.0      115.0    20.83      1.443      14.435
150.0      .0420        84.0      125.0    22.65      1.379      16.420
160.0      .0350        70.0      135.0    24.46      1.308      18.693
170.0      .0290        58.0      145.0    26.27      1.234      21.283
180.0      .0245        49.0      155.0    28.08      1.173      23.941
190.0      .0210        42.0      165.0    29.89      1.120      26.680
200.0      .0180        36.0      175.0    31.71      1.068      29.678
220.0      .0138        27.6      195.0    35.33       .987      35.780
240.0      .0108        21.6      215.0    38.95       .917      42.468
260.0      .0086        17.2      235.0    42.58       .855      49.756
280.0      .0070        14.0      255.0    46.20       .804      57.449
300.0      .0060        12.0      275.0    49.83       .773      64.439
```

```
             VEL=190.0FPM              DELTA=.1834MW/C

T (C)      RATIO      R (OHM)    T-TA (C)   P (MW)   V (VOLT)     I (MA)
 25.0     1.0000      2000.0          .0      .00       .000        .000
 30.0      .8500      1700.0         5.0      .91      1.248        .734
 35.0      .7000      1400.0        10.0     1.83      1.602       1.144
 40.0      .6000      1200.0        15.0     2.75      1.816       1.514
 45.0      .5000      1000.0        20.0     3.66      1.915       1.915
 50.0      .4500       900.0        25.0     4.58      2.031       2.257
 55.0      .3800       760.0        30.0     5.50      2.044       2.690
 60.0      .3300       660.0        35.0     6.41      2.058       3.118
 65.0      .2900       580.0        40.0     7.33      2.062       3.556
 70.0      .2500       500.0        45.0     8.25      2.031       4.062
 75.0      .2200       440.0        50.0     9.17      2.008       4.565
 80.0      .1950       390.0        55.0    10.08      1.983       5.085
 85.0      .1700       340.0        60.0    11.00      1.934       5.688
 90.0      .1500       300.0        65.0    11.92      1.891       6.303
 95.0      .1350       270.0        70.0    12.83      1.861       6.895
100.0      .1200       240.0        75.0    13.75      1.816       7.570
110.0      .0950       190.0        85.0    15.58      1.721       9.058
120.0      .0760       152.0        95.0    17.42      1.627      10.706
130.0      .0600       120.0       105.0    19.25      1.520      12.667
140.0      .0500       100.0       115.0    21.09      1.452      14.522
150.0      .0420        84.0       125.0    22.92      1.387      16.520
160.0      .0350        70.0       135.0    24.75      1.316      18.806
170.0      .0290        58.0       145.0    26.59      1.241      21.412
180.0      .0245        49.0       155.0    28.42      1.180      24.086
190.0      .0210        42.0       165.0    30.26      1.127      26.842
200.0      .0180        36.0       175.0    32.09      1.074      29.858
220.0      .0138        27.6       195.0    35.76       .993      35.996
240.0      .0108        21.6       215.0    39.43       .922      42.725
260.0      .0086        17.2       235.0    43.09       .860      50.057
280.0      .0070        14.0       255.0    46.76       .809      57.797
300.0      .0060        12.0       275.0    50.43       .777      64.829
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```
                VEL=200.0FPM          DELTA=.1856MW/C

 T (C)      RATIO      R (OHM)   T-TA (C)   P (MW)   V (VOLT)    I (MA)
  25.0     1.0000      2000.0        .0        .00      .000       .000
  30.0      .8500      1700.0       5.0        .92     1.256       .738
  35.0      .7000      1400.0      10.0       1.85     1.611      1.151
  40.0      .6000      1200.0      15.0       2.78     1.827      1.523
  45.0      .5000      1000.0      20.0       3.71     1.926      1.926
  50.0      .4500       900.0      25.0       4.64     2.043      2.270
  55.0      .3800       760.0      30.0       5.56     2.057      2.706
  60.0      .3300       660.0      35.0       6.49     2.070      3.137
  65.0      .2900       580.0      40.0       7.42     2.075      3.577
  70.0      .2500       500.0      45.0       8.35     2.043      4.087
  75.0      .2200       440.0      50.0       9.28     2.020      4.592
  80.0      .1950       390.0      55.0      10.20     1.995      5.116
  85.0      .1700       340.0      60.0      11.13     1.945      5.723
  90.0      .1500       300.0      65.0      12.06     1.902      6.341
  95.0      .1350       270.0      70.0      12.99     1.872      6.936
 100.0      .1200       240.0      75.0      13.92     1.827      7.615
 110.0      .0950       190.0      85.0      15.77     1.731      9.112
 120.0      .0760       152.0      95.0      17.63     1.637     10.770
 130.0      .0600       120.0     105.0      19.48     1.529     12.743
 140.0      .0500       100.0     115.0      21.34     1.460     14.609
 150.0      .0420        84.0     125.0      23.20     1.395     16.618
 160.0      .0350        70.0     135.0      25.05     1.324     18.919
 170.0      .0290        58.0     145.0      26.91     1.249     21.540
 180.0      .0245        49.0     155.0      28.76     1.187     24.230
 190.0      .0210        42.0     165.0      30.62     1.134     27.002
 200.0      .0180        36.0     175.0      32.48     1.081     30.037
 220.0      .0138        27.6     195.0      36.19      .999     36.211
 240.0      .0108        21.6     215.0      39.90      .928     42.981
 260.0      .0086        17.2     235.0      43.61      .866     50.356
 280.0      .0070        14.0     255.0      47.32      .813     58.142
 300.0      .0060        12.0     275.0      51.04      .782     65.217
```

APPENDIX B

EXPERIMENTAL DATA

```
                AMBIENT TEMPERATURE = 77.0 F
                BAROMETRIC PRESSURE = 29.03 INCH

      HW (INCH)       VEL (FPM)      EMF (VOLT)

         .000            .000          0.000
         .004           7.278          0.059
         .008          10.293          0.089
         .012          12.606          0.119
         .019          15.863          0.142
         .025          18.196          0.171
         .038          22.434          0.191
         .050          25.733          0.211
         .060          28.189          0.227
         .079          32.346          0.266
         .099          36.210          0.284
         .150          44.572          0.343
         .200          51.467          0.376
         .247          57.196          0.402
         .300          63.034          0.429
         .350          68.085          0.448
         .400          72.785          0.470
         .450          77.201          0.494
         .500          81.377          0.509
         .600          89.144          0.539
         .700          96.286          0.550
         .796         102.677          0.571
         .900         109.178          0.586
        1.000         115.084          0.599
        1.099         120.647          0.611
        1.206         126.383          0.625
        1.304         131.418          0.643
        1.400         136.170          0.651
        1.500         140.949          0.662
        1.600         145.571          0.670
        1.700         150.052          0.672
        1.800         154.402          0.681
        2.000         162.754          0.700
        2.300         174.534          0.720
        2.600         185.568          0.749
        2.900         195.982          0.758
        3.100         202.627          0.779
```

```
          AMBIENT TEMPERATURE = 77.0 F
          BAROMETRIC PRESSURE = 28.89 INCH

HW (INCH)        VEL (FPM)       EMF (VOLT)

   .000              .000          0.000
   .020            16.314          0.149
   .045            24.472          0.195
   .075            31.593          0.259
   .100            36.481          0.288
   .154            45.271          0.350
   .200            51.592          0.372
   .300            63.187          0.418
   .400            72.962          0.469
   .502            81.737          0.504
   .600            89.360          0.529
   .700            96.519          0.551
   .800           103.184          0.578
   .982           114.320          0.601
  1.200           126.374          0.630
  1.400           136.499          0.659
  1.600           145.924          0.679
  2.000           163.148          0.715
  2.500           182.405          0.741
  3.000           199.815          0.770
```

```
          AMBIENT TEMPERATURE = 74.0 F
          BAROMETRIC PRESSURE = 29.42 INCH

    HW (INCH)        VEL (FPM)        EMF (VOLT)

       .000             .000            0.000
       .018           15.294            0.140
       .023           17.288            0.162
       .045           24.182            0.193
       .063           28.613            0.226
       .106           37.115            0.289
       .141           42.806            0.321
       .185           49.033            0.369
       .246           56.541            0.398
       .295           61.917            0.430
       .338           66.276            0.449
       .392           71.374            0.478
       .439           75.532            0.489
       .500           80.609            0.514
       .588           87.416            0.541
       .716           96.462            0.561
       .798          101.836            0.580
       .900          108.149            0.592
       .996          113.771            0.604
      1.092          119.128            0.621
      1.196          124.671            0.638
      1.300          129.979            0.650
      1.402          134.982            0.661
      1.497          139.480            0.674
      1.581          143.340            0.682
      1.781          152.136            0.690
```

```
                 AMBIENT TEMPERATURE = 80.0 F
                 BAROMETRIC PRESSURE = 28.84 INCH

          HW (INCH)       VEL (FPM)      EMF (VOLT)

             .000             .000          0.000
             .017           15.096          0.130
             .032           20.712          0.168
             .048           25.367          0.200
             .072           31.068          0.239
             .103           37.159          0.271
             .154           45.437          0.329
             .199           51.651          0.365
             .247           57.544          0.403
             .316           65.087          0.420
             .358           69.278          0.441
             .406           73.776          0.462
             .455           78.101          0.471
             .503           82.117          0.495
             .595           89.312          0.521
             .702           97.011          0.540
             .797          103.367          0.564
             .899          109.782          0.581
             .984          114.855          0.596
            1.109          121.932          0.613
            1.215          127.626          0.622
            1.300          132.015          0.639
            1.414          137.682          0.641
            1.499          141.760          0.649
            1.593          146.137          0.657
            1.702          151.054          0.665
            1.724          152.027          0.668
```

A THERMISTOR ANEMOMETER FOR VERY LOW AIR VELOCITIES

by

FRANCISCO S. MARTINO

B. S., Instituto Tecnológico de Aeronáutica, Brazil, 1964

AN ABSTRACT OF A MASTER'S THESIS

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Mechanical Engineering

KANSAS STATE UNIVERSITY

Manhattan, Kansas

1967

A thermistor anemometer is designed to measure very low air velocities (from 10 to 200 fpm). Its circuit is synthesized and analyzed by a graphical technique analogous to that used in the analysis of vacuum-tube circuits. From this analysis a theoretical prediction of the performance of the instrument is obtained as a curve of the anemometer output versus air velocity. This curve is then compared with the experimental performance, and a good agreement is found between them.

Several characteristics of the anemometer are investigated experimentally. Some of those characteristics are: sensitivity and stability, reproducibility of the calibration curve obtained, influence of changes in ambient temperature upon the instrument output, influence of the direction of the moving air, and range of velocities in which the anemometer may be used. It was found that the anemometer has an excellent sensitivity, good stability, its calibration curve is reproducible (at the same ambient temperature) after extensive use, and that the instrument is practically insensitive to the direction of moving air.

As predicted in the theoretical analysis, the instrument is very sensitive to changes in ambient temperature. In order to make the anemometer of practical interest, the addition of a temperature compensation network is recommended. As another modification it is suggested that the instrument should also measure the temperature of the air at the same point in which its velocity is measured, by simply switching the sensor thermistor to a conventional bridge circuit for temperature measurement built in the same control box of the anemometer.