MEASUREMENT OF THE
ZERO POWER TRANSFER FUNCTION OF THE KANSAS STATE
UNIVERSITY TRIGA MARK II NUCLEAR REACTOR

by

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B. S., Kansas State University, 1964

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A MASTER'S THESIS

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Nuclear Engineering

KANSAS STATE UNIVERSITY

Manhattan, Kansas

1966

Approved by:

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TABLE OF CONTENTS

## LIST OF TABLES

# LIST OF FIGURES

# NOMENCLATURE

| | |
|---|---|
| $A$ | Relative scale factor |
| $A_o$ | Amplitude of reactor output signal |
| $A_1$ | Amplitude of reactor input signal |
| $A_i$ | Experimentally measured gain at frequency $\omega_i$ |
| $A_r$ | Amplitude attentuation caused by displacement transducer |
| $A_s$ | Exposed surface area of movable cadmium strip |
| $A(\omega_i)$ | Theoretical gain at frequency $\omega_i$ |
| $C_1$-$C_5$ | Constants from least squares analysis |
| $C_{11}$-$C_{22}$ | Elements of inverse coefficient matrix |
| $C_i$ | Concentration of ith group of delayed neutron precursors |
| $C_{io}$ | Steady state concentration of ith group of delayed neutron precursors |
| $D_\theta$ | Distance corresponding to complete cycle from strip chart recorder |
| $d$ | Displacement of oscillator movable cadmium strip |
| $d_o$ | Amplitude of movable cadmium strip displacement |
| $d_\theta$ | Distance corresponding to phase shift from strip chart recorder |
| $F_r$ | Amplitude response of strip chart recorder |
| $f_o$ | Frequency of measurement |
| $f_{-3db}$ | Frequency at which displacement transducer output is attenuated by three decibels |
| $G$ | Gain portion of reactor transfer function |
| $h$ | Width of oscillator cadmium strip |
| $j$ | Square root of minus one |
| $K$ | Proportionality constant |

| | |
|---|---|
| $k$ | Effective multiplication constant |
| $k_o$ | Steady state effective multiplication constant |
| $\ell$ | Prompt neutron lifetime |
| $\ell_o$ | First guess for prompt neutron lifetime |
| $N$ | Total number of data points |
| $n$ | Neutron density |
| $n_o$ | Steady state neutron density |
| $r_1$ | Radius of stationary cadmium strip |
| $r_2$ | Radius of movable cadmium strip |
| $S$ | Weighted sum of squares of deviations |
| $s$ | Laplace transform variable |
| $T$ | Integration time |
| $V_1$ | Integral of absolute value of reactor input signal |
| $V_2$ | Integral of absolute value of reactor output signal |
| $V_3$ | Integral of product of reactor input and output signals |
| $W_i$ | Weighting factor for data point i |

<u>Greek Symbols</u>

| | |
|---|---|
| $\alpha$ | Ratio of delayed neutron fraction to the prompt neutron lifetime |
| $\beta$ | Total delayed neutron fraction |
| $\beta_i$ | Delayed neutron fraction for delayed group i |
| $\Delta$ | Coefficient matrix from least squares analysis |
| $\Delta^{-1}$ | Inverse of coefficient matrix |
| $\Delta\ell$ | Calculated correction for prompt neutron lifetime |
| $\delta C_i$ | Time variation of concentration of ith group of delayed neutron precursors |

| | |
|---|---|
| $\delta k$ | Time variation of effective multiplication constant |
| $\delta n$ | Time variation of neutron density |
| $\theta(\omega_i)$ | Theoretical phase shift at frequency $\omega_i$ |
| $\theta_i$ | Experimentally measured phase shift at frequency $\omega_i$ |
| $\lambda_i$ | Decay constant of the ith delayed neutron group |
| $\rho$ | Reactivity |
| $\sigma(\text{ext})$ | Estimated standard deviation of a function of unit weight |
| $\sigma_A$ | Estimated standard deviation of A |
| $\sigma_\ell$ | Estimated standard deviation of $\ell$ |
| $\phi$ | Phase shift introduced by displacement transducer |
| $\omega$ | Angular frequency |

# INTRODUCTION

For safe operation of a nuclear reactor, it is essential that the kinetic and dynamic response of the reactor be known. One method of determining this behavior is the application of conventional frequency response techniques to find the transfer function. Once the transfer function is known, it may be used in the usual manner for the design of control systems (21) and for stability analysis (26).

Transfer functions may be defined relating any parameters which affect reactor behavior, but the expression of general interest relates changes in reactivity to changes in the neutron population. Thus, if the reactivity is forced to vary in a sinusoidal manner, the neutron population will also vary sinusoidally at the same frequency, but with a different amplitude and phase. By forcing sinusoidal variation of the reactivity over the frequency range of interest, information is obtained which allows calculation of the reactor gain and phase shift.

If a reactor is operated at a sufficiently low power level, it is found that the transfer function is determined by the delayed neutron parameters and the prompt neutron lifetime. Thus, from measurements at "zero" power, it is possible to determine the ratio of the effective fraction of delayed neutrons to the prompt neutron lifetime.

The device used to produce a sinusoidal reactivity change in a reactor is termed a pile oscillator. A variety of such devices have been built (3, 10, 15, 16, 18, 19, 20) which make use of the principle that, for a "black" absorber, the reactivity worth is proportional to the exposed surface area of the absorber. Thus, to produce the desired reactivity signal, it is necessary only to build a mechanism which will cause the exposed surface area of a neutron absorber to vary in a sinusoidal manner.

The purpose of the work presented here was the design and construction of a suitable pile oscillator, and the measurement of the zero power transfer function of the Kansas State University TRIGA Mark II reactor. Computer programs were developed to perform least squares analyses on the measured gain and phase shift to determine the ratio of the effective delayed neutron fraction to the prompt neutron lifetime.

## 2.0 PILE OSCILLATORS

Due to the varied types and designs of reactors, it is virtually impossible to specify one general pile oscillator design suitable for all reactors. For purposes of this work, a pile oscillator will be defined as any device which causes the reactivity of a reactor to vary in a sinusoidal manner.

Oscillators may be separated into two general categories: 1) those using a linear harmonic motion to produce the desired sinusoidal reactivity variation, and 2) those using a rotary motion. Regardless of what type oscillator is considered, a set of basic requirements must be fulfilled. One requirement is that the frequency range extend at least from 0.1 cps to approximately one decade above the break frequency of the reactor. The break frequency is determined by the ratio of the effective fraction of delayed neutrons, $\beta$, to the prompt neutron lifetime, $\ell$. Next, the oscillator must produce a sufficiently large variation in the reactivity, that the resulting variations in the reactor power yield a usable signal to noise ratio at high frequencies. At the same time, it is necessary that the variations be sufficiently small that excessively large power variations do not result at the low frequencies. The reactivity waveform produced by the oscillator should be an accurate approximation of a sine wave. Finally, in order that perturbation effects be minimized, the oscillator should be as small as possible. These requirements are contradictory and require that compromises be made.

A variety of oscillator designs have been successfully applied. The first pile oscillator was used by Harrer, Boyar and Krucoff in their now classic experimental determination of the zero power transfer function of the CP-2 reactor (15). The oscillator used was of the linear motion type in which a movable cadmium rod was oscillated in and out of a stationary cadmium cylinder. Pawlicki (20) has used a rotary oscillator in which a sinusoidally shaped piece of cadmium is alternately exposed or masked by a rotating cadmium half-cylinder to measure the frequency response of the Argonaut. Measurement of the transfer functions of the KEWB (7) and SRE (16) reactors has been accomplished using quite similar rotary oscillators in which cadmium or boral "spots" are alternately masked or exposed by rotating cadmium or boral "shades". An interesting feature of these two oscillators is that the spots and shades are arranged such that four perturbation cycles occur per revolution of the oscillator rotor.

Three types of oscillators were investigated in the frequency response measurements on the ZPR-III mockup assembly of EBR-I, Mark III (5). A steel half rod and an eccentric rod were tested, but the design used was a steel rod which contained an eccentric hole filled with boron. An oscillator consisting of a cadmium half-cylinder rotating inside a stationary cadmium half-cylinder was used for transfer function measurements on SPERT I (4). The frequency response of Zeus, a zero power mockup of the Dounreay Fast Reactor, was measured by using a rotating rod which was filled half on one side with fuel and the other with natural uranium (2). An eccentric cadmium vane, rotating past a stationary semicircular cadmium vane was used for zero power transfer function determination of Zephyr (2). A linear motion oscillator, which utilized an electromagnetic drive unit, has been used to measure the transfer function of the UWNR (1). A unique feature of this oscillator is that reactivity inputs such as square waves, triangular waves, or random noise may be used in addition to the more standard sine wave.

Perhaps the most direct approach to producing the desired reactivity variation in a reactor is the oscillation of an existing control rod. This method falls in the category of linear motion type oscillators. The zero power and high power transfer function of the EBWR have been measured by precisely this method (9). Control rod oscillation has also been used for relative stability analysis and determination of core dynamic parameters on the SRE (14). One major problem associated with this method is that the mass which must be oscillated is usually relatively large and the frequency range which can be covered is thus quite limited.

At least two transfer function measurements have been completed on TRIGA reactors. One, by Beg (3) on the University of Illinois reactor, made use of a cadmium semicircle which was rotated past a specially shaped stationary piece of cadmium to measure the at-power internal feedback mechanism of the transfer function. The other measurements were made by Park (18) on the Atomic Energy Research Institute of Korea TRIGA Mark II. The oscillator used was of the rotary type with sinusoidally shaped cadmium "spots" which were rotated past stationary rectangular cadmium "shades". Both the zero power and at-power transfer functions were measured.

The oscillator used in this work was of the linear motion type. The absorber used was cadmium, in the form of two coaxial cylinders. The outer cylinder was stationary, while the inner cylinder was attached to a tube which was oscillated by converting a rotary motion to a linear harmonic motion. The displacement of the movable cylinder is given by

$$d(t) = d_o \sin\omega t, \tag{1}$$

where $d(t)$ = displacement of the movable cylinder at time t

$d_o$ = amplitude of displacement.

It is assumed that the cadmium used represents a "black" absorber, and thus, the reactivity worth is proportional to the exposed cadmium surface area, i.e.

$$\rho(t) = - KA_s(t) \tag{2}$$

where $\rho(t)$ = reactivity at time t

K = proportionality constant

$A_s(t)$ = exposed surface area at time t.

The exposed surface area is made up of two components, the stationary cylinder which is always exposed and the moving cylinder which is alternately exposed and masked. Thus, $A_s$ is given by

$$A_s(t) = 2\pi \, [r_1 h + r_2 d_o \sin\omega t] \tag{3}$$

where $r_1$ = radius of the stationary cylinder

$r_2$ = radius of the movable cylinder

h = height of stationary cylinder

$d_o \sin\omega t$ = exposed height of movable cylinder.

Thus, it is found that the reactivity as a function of time is

$$\rho(t) = - 2K\pi \, [r_1 h + r_2 d_o \sin\omega t]. \tag{4}$$

Equation (4) indicates that the variation in reactivity is some constant value upon which a sinusoidal variation is superimposed.

## 3.0 ZERO POWER REACTOR TRANSFER FUNCTION

The zero power, or open loop, reactor transfer function may be derived from the point reactor kinetic equations. These equations have been developed in Appendix A using a slowing-down diffusion model based on Fermi Age theory. For a system to be approximately represented by this mathematical model, it should satisfy the following set of conditions (17).

1. The medium is isotropic and homogeneous.
2. The macroscopic absorption cross-section of the medium is much less than the macroscopic scattering cross-section.
3. The general dimensions of the system are much larger than characteristic neutron lengths, such as the diffusion length.
4. The regions within the reactor which are to be described by the combination slowing-down diffusion model are free of localized neutron sources and sinks.
5. These regions do not contain large voids.
6. Neutron scattering is isotropic in the center-of-mass system.
7. The mass number of the moderator is sensibly larger than unity.

The above conditions appear somewhat severe, but lead directly to equations which have been successfully used in studies which arise from the characteristic "in-hour" equation (12), reactor noise measurements (24), stability analysis (21), and transfer function measurements (10).

The resulting time dependent, one velocity, source free, bare reactor, spatially independent, or point reactor kinetic equations are,

$$\frac{dn(t)}{dt} = \frac{n(t)}{\ell}\left[(1 - \beta)k(t) - 1\right] + \sum_{i=1}^{6} \lambda_i C_i(t) \tag{5}$$

and

$$\frac{dC_i(t)}{dt} = \frac{k(t)\beta_i n(t)}{\ell} - \lambda_i C_i(t), \tag{6}$$

where $n(t)$ = neutron density at time t ($n/cm^3$)

$C_i(t)$ = concentration of the ith group of delayed neutron precursors at time t ($atoms/cm^3$)

$\beta$ = fraction of fission neutrons which are delayed

$\ell$ = prompt neutron lifetime (sec)

$k(t)$ = effective multiplication constant at time t

$\lambda_i$ = decay constant of the ith delayed neutron group ($sec^{-1}$)

$\beta_i$ = fraction of fission neutrons which are delayed in the ith delayed neutron group.

The effective multiplication constant may be forced to vary with time by some device, such as a pile oscillator. It will be assumed that the forced variations in k, and the resulting variations in n and $C_i$, are small compared with the steady state values. The functions may thus be represented as small time dependent variations (denoted by a delta) superimposed on some constant steady state value (denoted by a zero subscript).

$$\begin{aligned} k(t) &= k_o + \delta k(t) \\ n(t) &= n_o + \delta n(t) \\ C_i(t) &= C_{io} + \delta C_i(t) \end{aligned} \tag{7}$$

If equations (7) are substituted into (5) and (6), and terms involving the product of $\delta k(t)$ and $\delta n(t)$ are neglected, the result is

$$\frac{d\delta n(t)}{dt} = \frac{1}{\ell}\,[k_o n_o + n_o \delta k(t) + k_o \delta n(t) - n_o - \delta n(t)] - \sum_{i=1}^{6} \frac{d\delta C_i(t)}{dt} \tag{8}$$

and

$$\frac{d\delta C_i(t)}{dt} = \frac{\beta_i}{\ell}\,[k_o n_o + n_o \delta k(t) + k_o \delta n(t)] - \lambda_i C_{io} - \lambda_i \delta C_i(t). \tag{9}$$

Equation (6) may be solved for $C_i(t)$ and the result substituted into (5). The resulting equation, along with equation (6), for the steady state condition yield

$$\frac{dn(t)}{dt} = 0 = \frac{1}{\ell}\,[k_o n_o - n_o] \tag{10}$$

and

$$\frac{dC_i(t)}{dt} = 0 = \frac{k_o \beta_i n_o}{\ell} - \lambda_i C_{io}. \tag{11}$$

Equations (10) and (11), when substituted into (8) and (9) yield

$$\frac{d\delta n(t)}{dt} = \frac{1}{\ell}\,[n_o \delta k(t) + k_o \delta n(t) - \delta n(t)] - \sum_{i=1}^{6} \frac{d\delta C_i(t)}{dt} \tag{12}$$

and

$$\frac{d\delta C_i(t)}{dt} = \frac{\beta_i}{\ell}\,[n_o \delta k(t) + k_o \delta n(t)] - \lambda_i \delta C_i(t). \tag{13}$$

The open loop transfer function is defined as the ratio of the Laplace transform of the neutron density variation to the Laplace transform of the input reactivity variation times the reciprocal of the steady state neutron density $n_o$. The Laplace transform of equation (13) may be solved for $C_i(s)$ and the result substituted into the Laplace transform of equation (12) to yield an expression which may be solved directly for the transfer function (see Appendix B for a detailed derivation). The result of these operations is

$$G(s) \equiv \frac{\delta n(s)}{n_o \delta k(s)} = \frac{1 - S \sum_{i=1}^{6} \frac{\beta_i}{S + \lambda_i}}{S\ell \left[1 + \frac{k_o}{\ell} \sum_{i=1}^{6} \frac{\beta_i}{S + \lambda_i}\right] - (k_o - 1)}, \tag{14}$$

where the initial conditions $\delta n(0^+)$ and $\delta C_i(0^+)$ have been chosen to be zero.

One method for measuring transfer functions is to utilize a pure sinusoidal input signal. Mathematically this imples that $s = j\omega$, where $j$ is the square root of minus one and $\omega$ is the frequency of the input signal in radians per second (28). For such an input signal, the output will be sinusoidal of the same frequency, but will be of a different phase and amplitude. Thus if the effective multiplication constant is forced to vary in a sinusoidal manner, and the mean power level of the reactor is stable, so that $k_o = 1.0$, equation (14) becomes

$$G(j\omega) = \frac{1 - j\omega \sum_{i=1}^{6} \frac{\beta_i}{j\omega + \lambda_i}}{j\omega \left[\ell + \sum_{i=1}^{6} \frac{\beta_i}{j\omega + \lambda_i}\right]}. \tag{15}$$

Equation (15) is commonly called the zero power reactor transfer function. This is because only neutronic effects have been considered. As the power level of the reactor is increased, more heat is generated and this heat causes temperature rises in the reactor core components. These temperature rises result in changes of the physical properties of the components and are reflected as changes in reactivity (26). These reactivity changes appear as a feedback loop which has not been considered in the above development. Thus, for equation (15) to describe the system, the power level of the reactor must be such that temperature effects are negligible.

Another limitation imposed on equation (15) is the small signal assumption used in the derivation. If the amplitude of the input reactivity signal is too large, non-linearity of the system is observed. It has been stated that the small signal theory holds for peak-to-peak power variations of less than about fifteen per cent (6).

It is interesting to note that the zero power transfer function depends only on the delayed neutron parameters $\beta_i$ and $\lambda_i$, the prompt neutron lifetime $\ell$, and the frequency of oscillation.

# 4.0 EXPERIMENTAL FACILITIES

## 4.1 Description of the Reactor

The experimental measurements presented in this thesis were made on the Kansas State University TRIGA Mark II reactor.

The TRIGA Mark II is a swimming pool type reactor developed by General Atomic. The core is composed of cylindrical fuel-moderator and graphite dummy elements arranged in concentric arrays to form a right circular cylinder. Approximately sixteen feet of water above the core provide shielding in the vertical direction. Shielding in the radial direction is provided by one and one-half feet of water and approximately eight feet of concrete. The fuel elements are composed of a homogeneous mixture of 91% Zr, 1% H and 8% U by weight, the uranium being 20% enriched.

The core is surrounded by a one foot thick radial graphite reflector. Four, six-inch inside diameter beam ports, penetrate the concrete shielding and reactor tank to terminate at the outer surface of the reflector. A void is provided at one point in the graphite reflector, so that the "fast" beam port penetrates essentially to the outer surface of the core.

A central thimble irradiation facility is provided in the form of a 1.33 inch inside diameter tube. This tube extends from the top of the reactor tank to the bottom of the core, passing through the radial center of the core.

## 4.2 Description and Location of Equipment

The pile oscillator used was custom built in the machine shop of the

Department of Nuclear Engineering. It was of the linear motion type.

The oscillator drive consisted of a Zero-Max type 60P400M CCW unit, the output shaft speed of which could be varied effectively over the range 4 rpm to 400 rpm. This speed range necessitated the addition of a gear train, the ratio of which could be manually set at 12.25:1.0, 1.0:1.0, or 1.0:12.25. With this drive system, the frequency range 0.01 to 40.0 cps was covered. A machine drawing of the oscillator is included in Appendix J.

The rotary motion from the drive system was converted to a harmonic motion by means of an off-set cam and connecting rod assembly. The amplitude of the harmonic motion was controlled by adjusting the off-center distance of the connecting rod pin (see Fig. 1). The connecting rod was mechanically coupled to the 0.5 inch outside diameter aluminum oscillator drive tube. The drive tube then extended from the top of the reactor to the center of the core, some nineteen feet below, inside the stationary oscillator outer housing.

A 0.25 inch wide, 1.29 inch diameter, 20 mil thick cadmium strip was located at the bottom of the drive tube. Another cadmium strip, 0.25 inch wide, 1.43 inches diameter and 20 mil thick, was located on the oscillator outer housing in such a position that the sinusoidally moving strip was alternately masked by the stationary strip, or exposed to the neutron flux.

The motion of the oscillator, which was proportional to the reactivity input signal to the reactor, was measured by mechanically connecting a Sanborn Model 7DCDT-500 displacement transducer to the drive tube. The

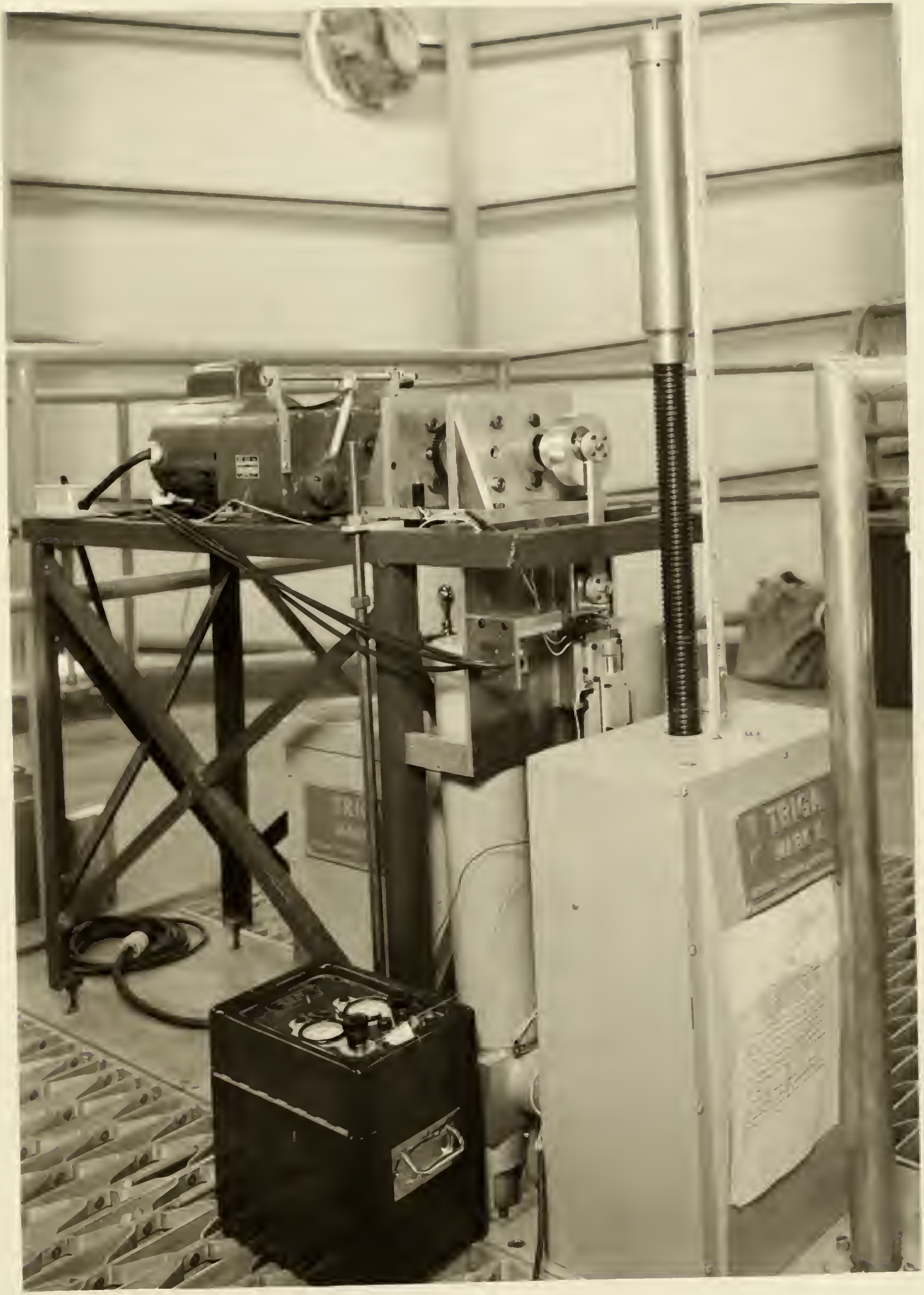

Figure 1. Pile oscillator system

excitation voltage required by this transducer was six volts dc and was obtained from four 1.5 volt dry cells connected in series.

The response of the reactor to the reactivity input signal was observed by means of a compensated neutron sensitive ionization chamber manufactured by Daystrom. The chamber was located in the fast beam port (see Figure 2). A B. J. Electronics Model DV-1 high voltage supply was used to supply the required 600 volts to the chamber. The output of the chamber was fed to a Keithley 410 Micro-microammeter and then to the top of the reactor for amplification and recording (see Fig. 3). Both the reactor input and output signals were recorded on a KRS Electronics Data-Stact portable instrumentation recorder for later analysis.

The equipment used for data analysis included a Pace TR-10 portable analog computer, an Offner Type RS Dynograph, a Beckman Universal EPUT and Timer, and a Standard Electric precision timer (see Fig. 4). A complete listing of equipment used, including manufacturers, serial numbers and Kansas State University inventory numbers, may be found in Table I.

## 4.3 Procedure

Since the pile oscillator was designed for location in the position normally occupied by the central thimble, and demand for use of the central thimble was high, it was necessary to remove the oscillator following each data collection run.

The first step for a data run was the removal of the central thimble and insertion of the oscillator (details of the installation are included

| Item | Manufacturer | Serial Number | K. S. U. Inventory No. |
|---|---|---|---|
| Pace TR-10 portable analog computer | Electronics Associates, Incorporated | | N.E. 816 |
| Type RS Dynograph | Offner | | N.E. 733 |
| Type 545 oscilloscope | Tektronix, Incorporated | 12084 | N.E. 69 |
| Battery box, model P.A.B. | J. Beeber Co., Inc. | | |
| Data-Stact portable instrumentation recorder, model MD-2 | KRS Electronics | 140 | |
| Low frequency function generator | Hewlett Packard | 037 08739 | Ch.E. FG1 |
| Universal EPUT and timer, model 5230 | Beckman | 652 | M.E. 1745 |
| Oscillator drive unit | Zero-Max | 60P400M CCW | |
| 410 Micro-microammeter | Keithley Instruments | 23495 | N.E. 818 |
| High voltage supply, model DV-1 | B. J. Electronics | 8251 | N.E. 275 |
| Compensated neutron sensitive ionization chamber, model 3 | Daystrom | R209 | |
| DC LVDT displacement transducer, model 7DCDT-500 | Sanborn | | |
| Precision timer, model S-60 | Standard Electric Time Co. | | N.E. 91 |

Table I: Equipment used for transfer function measurement

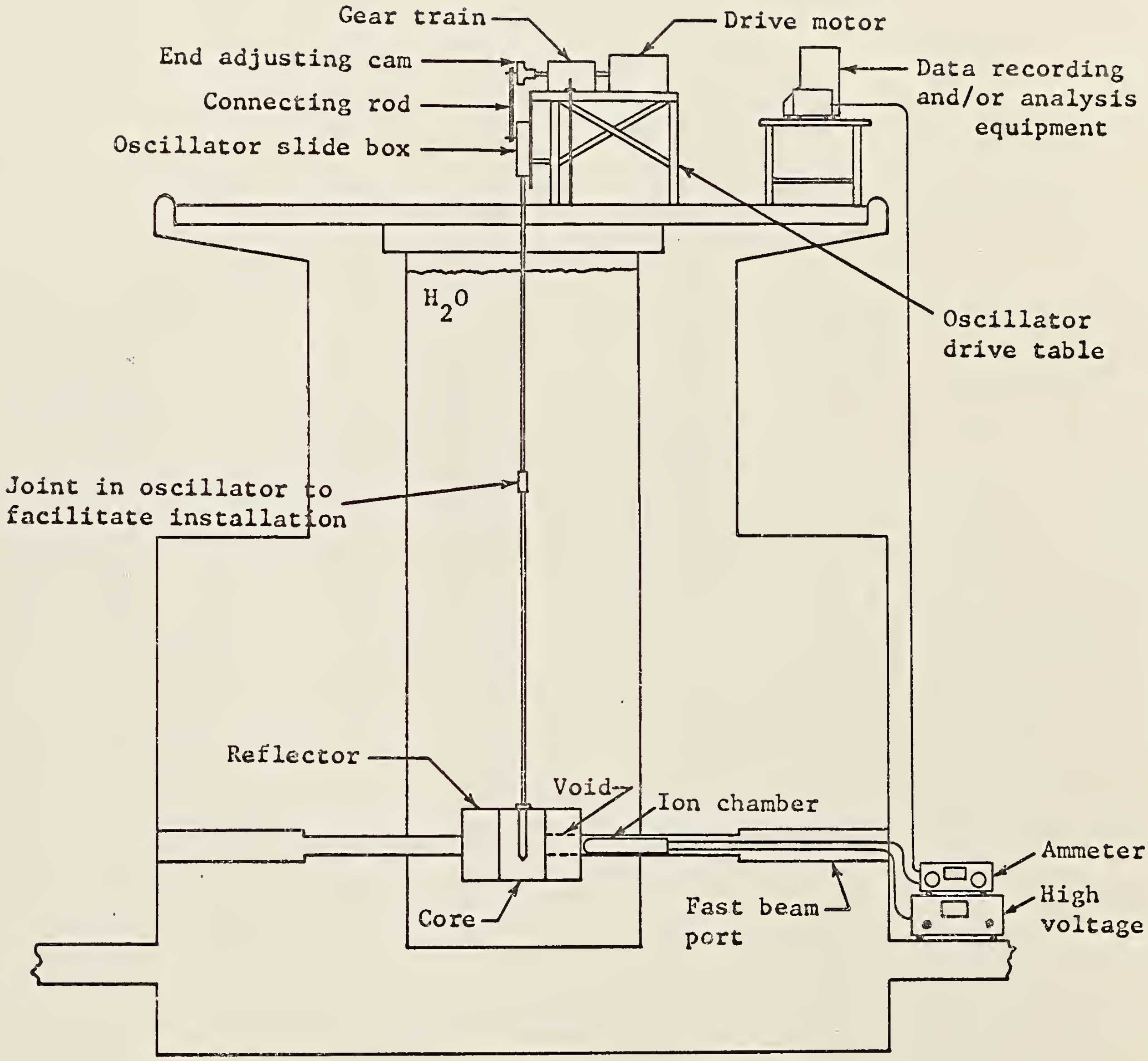


Figure 2. Elevation view of pile oscillator installation

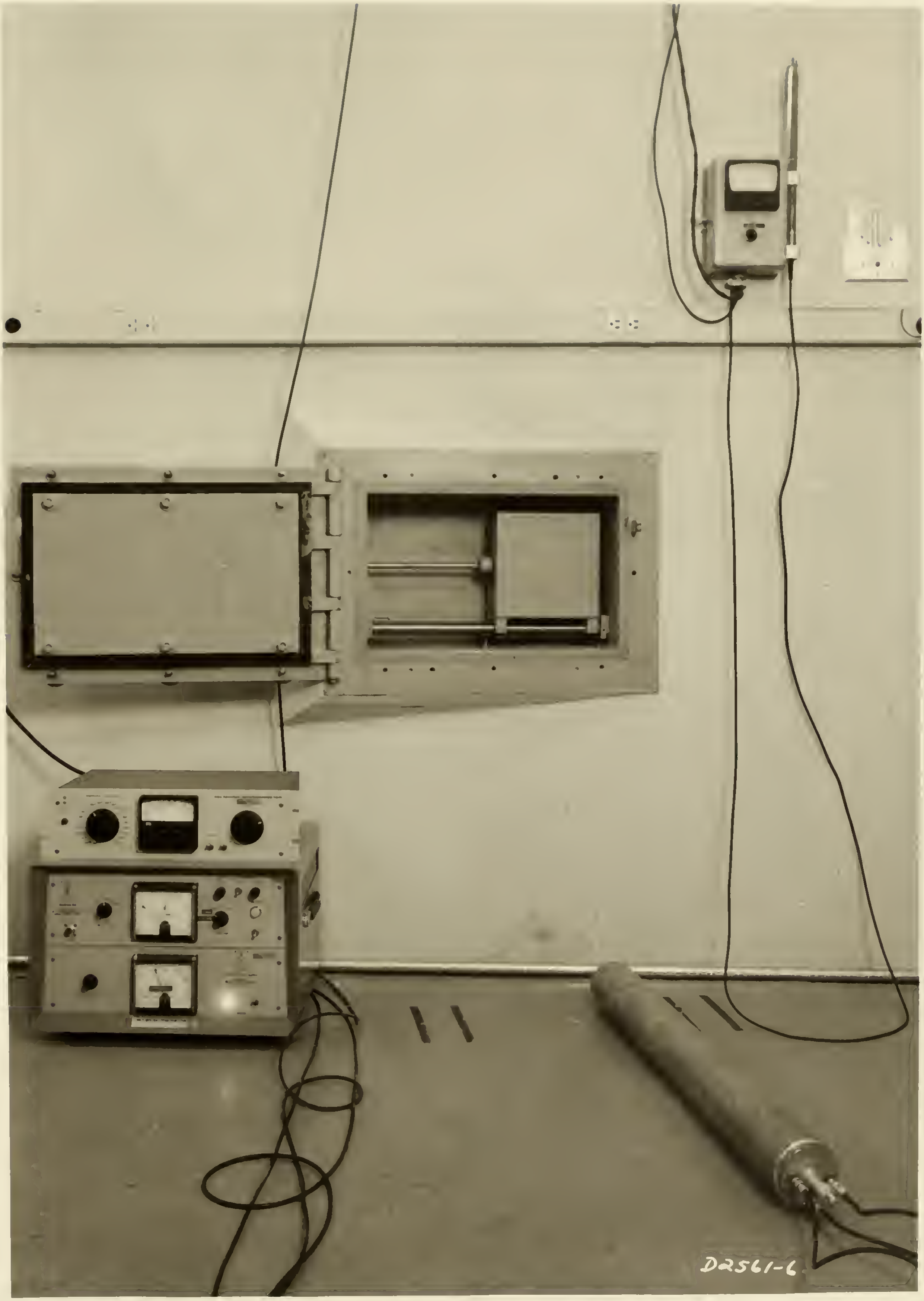


Figure 3. Detection portion of pile oscillator system

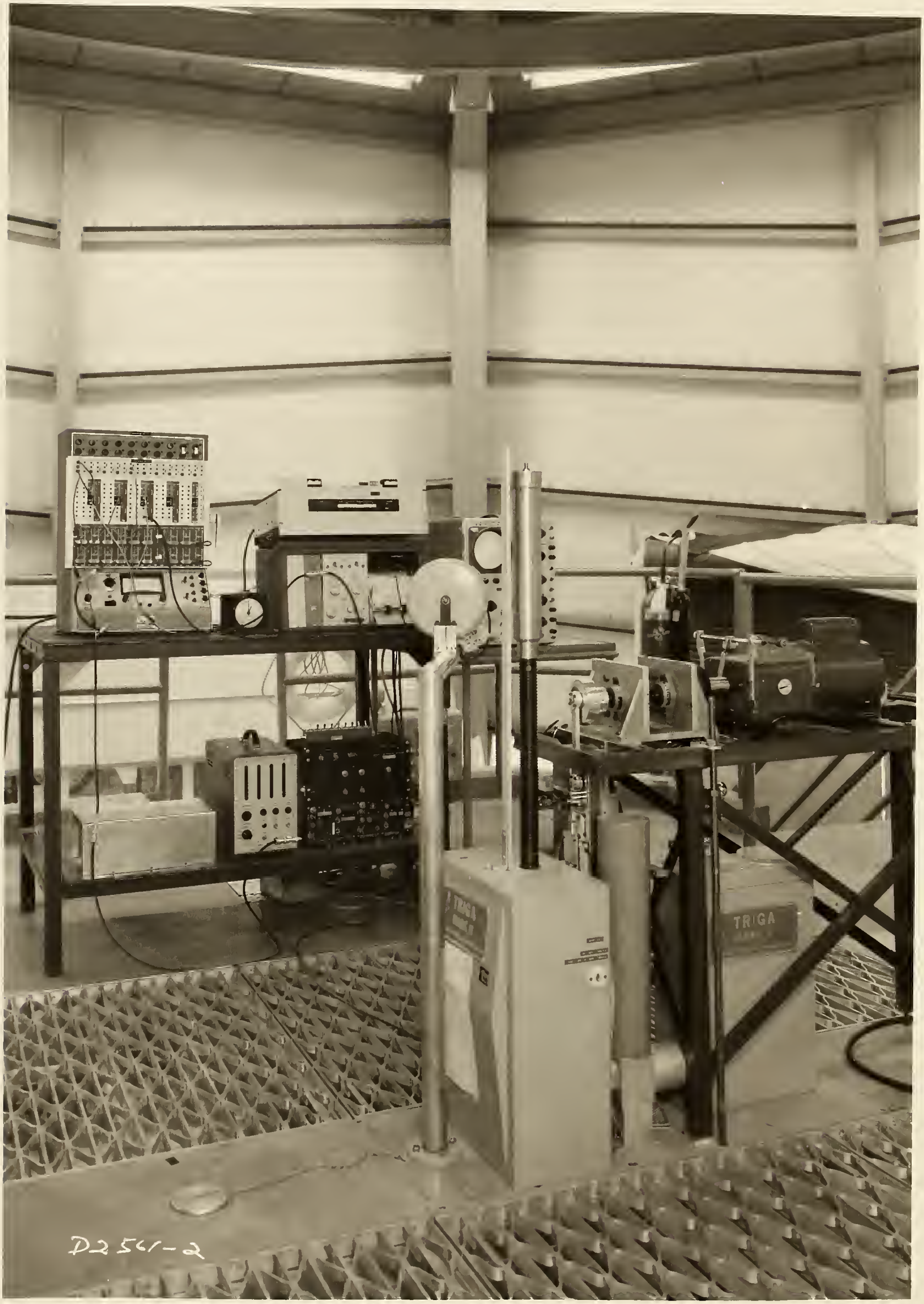


Figure 4. Pile oscillator system, including data collection and analysis equipment

in Appendix J). Next the oscillator drive table was positioned at the top of the reactor tank and the necessary connections were completed. These connections included bolting the oscillator to the table, bolting the table to the center channel assembly and inserting the connecting rod between the gear train and the oscillator drive tube. Details of the connections may be seen in Fig. 1.

The displacement transducer was connected to the table and oscillator drive tube and the six volt dc excitation voltage was applied. The transducer output was electrically connected to the Beckman Universal EPUT and Timer, which was used in the Count mode of operation for oscillator frequency determination above 0.5 cps. This unit registered one count for each reactivity perturbation cycle of the oscillator and thus served as a frequency measuring device when used in conjunction with an elapsed timer. At low frequencies (0.01 cps to 0.5 cps), the frequency of oscillation was measured by using the timer to record the time required for a given number of flashes of a neon lamp. This neon lamp was controlled by a micro-switch which was closed momentarily once each perturbation cycle.

The electrical signal was passed from the Beckman unit to the Pace TR-10 analog computer where it was amplified to a value of approximately 1.0 volt rms. Following amplification, the signal was recorded on the KRS Data-Stact tape recorder.

The shielding plugs were removed from the fast beam port and the Daystrom ion chamber was inserted. The high voltage was connected to the chamber and the output run to the Keithley 410 Micro-microammeter. The output of the Keithley was connected to the Offner Dynograph by means of a fifty foot coaxial cable. This trace was used as a monitor of reactor power during the run. Next the signal was passed to the analog computer for amplification, and was then recorded on the second channel of the KRS tape recorder.

With the equipment all in position and operating, the oscillator was stopped in its mean reactivity worth position. The reactor was brought to a power level of 100 watts and allowed to stabilize at that power in the manual control mode of operation. The needle of the Offner strip chart recorder was adjusted so that the signal from the ion chamber caused a deflection corresponding to the mid-point of the strip chart paper. Next the oscillator was set in motion by switching on the Zero-Max drive unit. Several minutes were then allowed to insure that the power was stabilized at the desired 100 watts. The oscillator was adjusted to the desired frequency by manually varying the speed control on the drive unit. The frequency was measured by either the Beckman unit or the neon lamp and timer, depending on the frequency. When the desired frequency was obtained and the power was stabilized, the "RECORD" button on the KRS tape recorder was depressed and approximately four and one-half minutes of data were recorded. This procedure of setting the frequency and recording the reactor input and output signals was repeated for all the frequencies of interest. A block diagram of the transfer function measurement system is presented in Fig. 5.

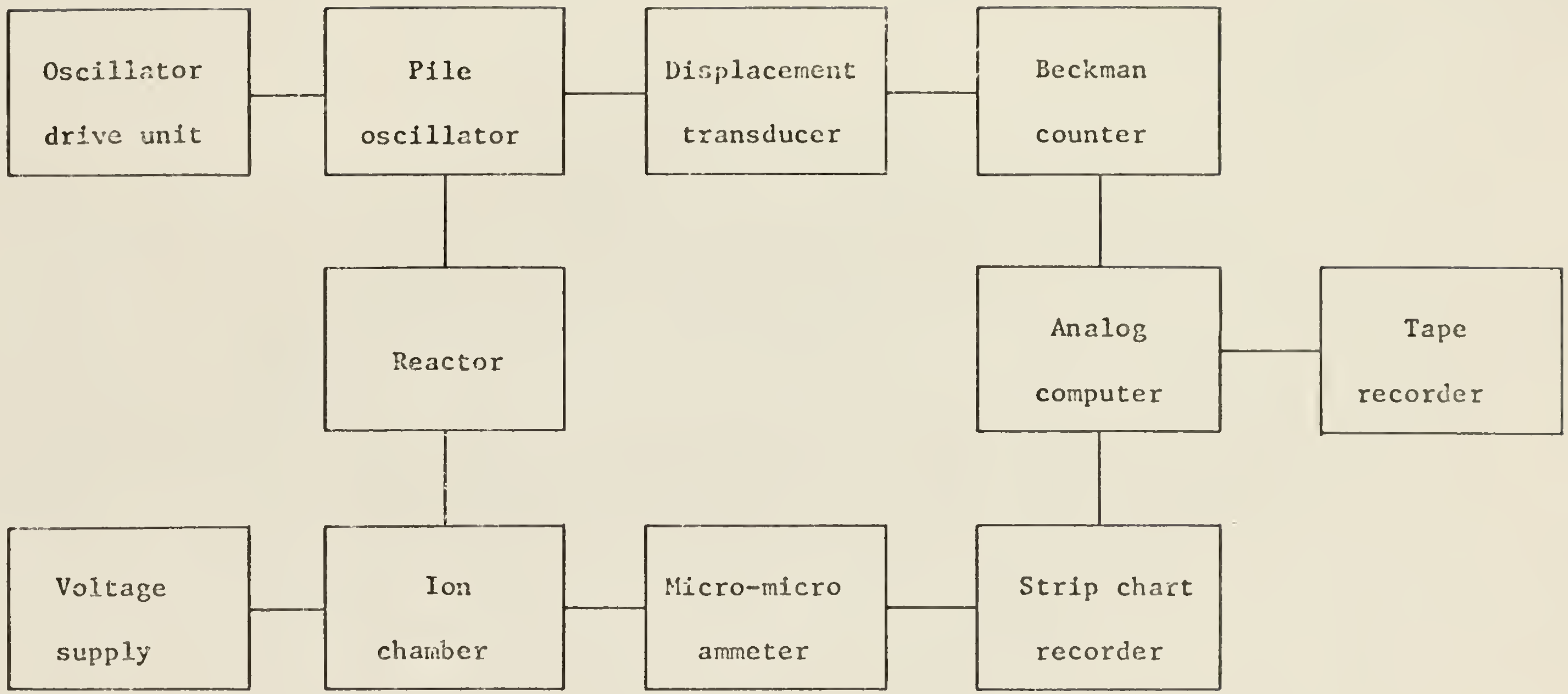


Figure 5. Block diagram of transfer function measurement system

# 5.0 DATA ANALYSIS

## 5.1 Methods Used by Other Investigators

The ultimate goal of any method of data analysis is the extraction of the gain and phase shift information which define the transfer function. The methods of analysis which have been used are as numerous and varied as the types of oscillators in existence. Some of the methods used by other investigators are summarized below.

Dual-channel recording oscillographs have been used frequently (13, 16, 18). Another common method is sine-cosine potentiometer correlation of quadrature components (3, 5, 16, 23, 25, 29). Other methods include a special low frequency wave analyzer (9), a servoscope (20), dual beam oscilloscope (1), digital computer analysis of digital data obtained from photographs of dual beam oscilloscope trace (7), digital system using analog to digital converters (1), and an analog computer method (1).

Two methods of data analysis have been used for the work presented in this thesis. One method involved the use of a dual-channel strip chart recorder and the other involved the use of an analog computer. These are discussed in detail in sections 5.2 and 5.3. The selection of these two methods was based primarily on the availability of equipment.

## 5.2 Dual-channel Strip Chart Recorder Method

A very straight forward approach to the measurement of the gain and phase shift, which defines the reactor transfer function, is to record the reactor input and output signals on a dual-channel strip chart recorder.

The gain, at the frequency of interest, may be determined by measuring the amplitude of the reactor input signal trace and the output trace. The amplification and recorder sensitivities must of course be accounted for in the amplitude measurements. The gain is then given by

$$G = 20 \log_{10} \left\{\frac{A_o}{A_1}\right\} \tag{16}$$

where $A_o$ = amplitude of reactor output signal, from the ion chamber

$A_1$ = amplitude of reactor input signal, from the displacement transducer.

The phase shift may be obtained by comparing the zero crossing of one signal with that of the other. Let this distance be $d_\theta$. The distance representing one complete cycle may be measured as $D_\theta$. The phase shift, $\theta$(in degrees), is now given by

$$\theta = -360 \left\{\frac{d_\theta}{D_\theta}\right\} . \tag{17}$$

At higher frequencies, the displacement transducer attenuation of the amplitude, and the phase shift introduced by the transducer, must be taken into account. The amplitude response, $A_r$, is given by

$$A_r = \left[1 + \left\{\frac{f_o}{f_{-3db}}\right\}^2\right]^{-1/2} . \tag{18}$$

The phase shift is given by

$$\phi = \tan^{-1} \left\{\frac{f_o}{f_{-3db}}\right\} \tag{19}$$

where $\phi$ = phase shift introduced by displacement transducer

$A_r$ = amplitude attentuation caused by displacement transducer

$f_o$ = frequency of measurement

$f_{-3db}$ = 135 cps = frequency at which the displacement transducer output is down by a factor of 3 db.

Curves showing the amplitude and phase response of the displacement transducer may be found in Fig. 6.

Typical strip chart recorder traces and the required measurements, as recorded during data analysis, are shown in Fig. 7.

### 5.3 Analog Computer Method

Another relatively simple method of data analysis involves the use of an analog computer to perform certain integrations (1).

Consider the integral of the absolute value of the reactor input signal, $V_1(T)$,

$$V_1(T) = \int_0^T |A_1 \sin \omega t| dt = \frac{2TA_1}{\pi} \tag{20}$$

where T is the integration time in seconds and corresponds to an integral number of cycles, i.e. $T = 2\pi N/\omega$.

The integral of the absolute value of the reactor output signal may be formed in the same manner as

$$V_2(T) = \int_0^T |A_o \sin(\omega t - \theta)| dt = \frac{2TA_o}{\pi} . \tag{21}$$

With these two voltages, the gain may be calculated from

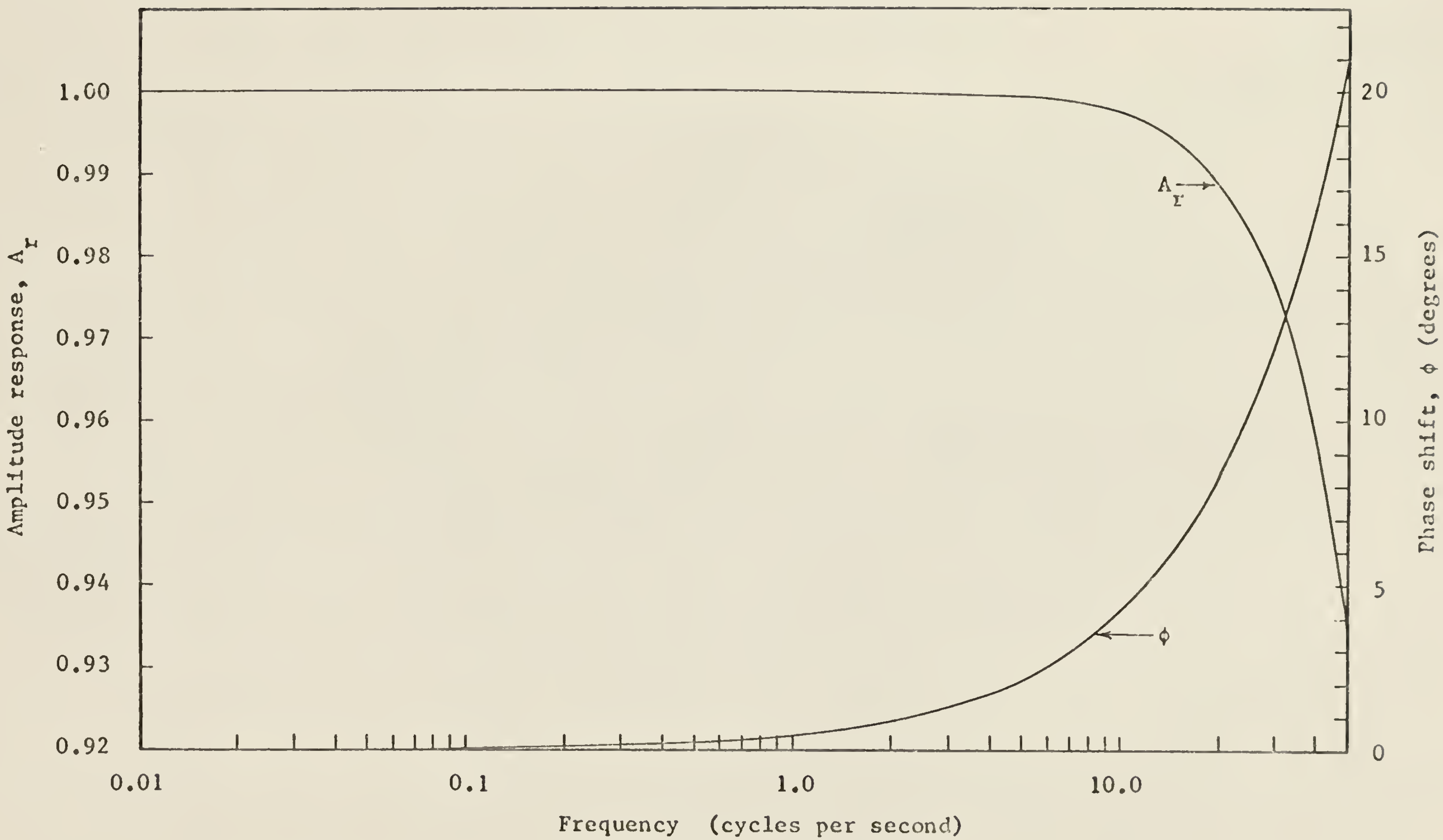


Figure 6. Amplitude and phase response of Sanborn 7DCDT-500 displacement transducer

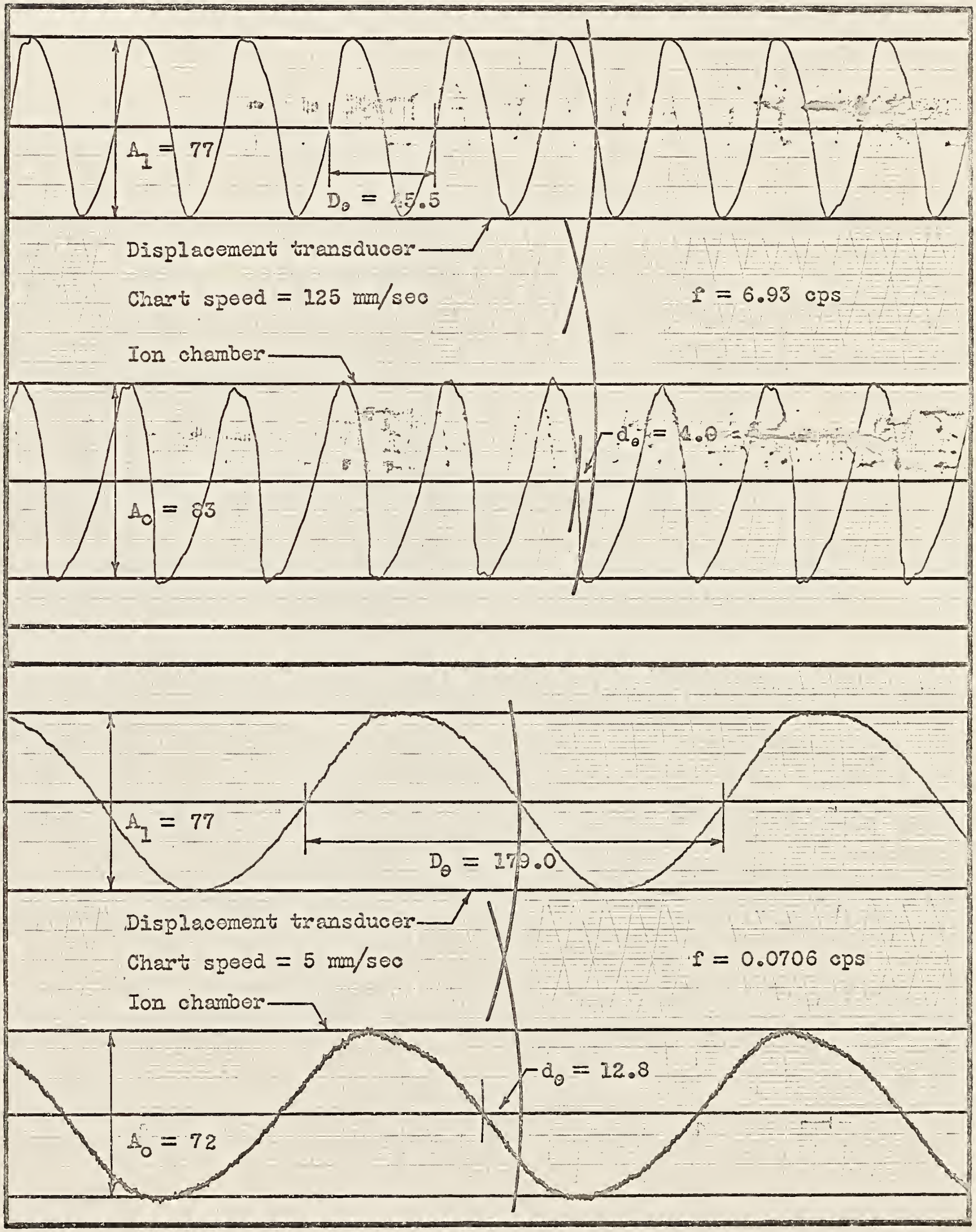


Figure 7. Typical strip chart recorder traces

$$G = 20 \log_{10} \left\{\frac{V_2(T)}{V_1(T)}\right\} . \tag{22}$$

Consider now the integral of the product of the reactor input and output signals, $V_3(T)$,

$$V_3(T) = \int_0^T A_1 A_o \sin\omega t \sin(\omega t - \theta) dt = \frac{T}{2} A_1 A_o \cos\theta. \tag{23}$$

With voltages $V_1(T)$, $V_2(T)$, and $V_3(T)$, the phase shift may be evaluated from

$$\theta = \cos^{-1} \left[ \frac{8T}{\pi^2} \frac{V_3(T)}{V_1(T)\ V_2(T)} \right]. \tag{24}$$

The analog computer circuit used to form the above integrals is shown in Fig. 8. Only one pair of matched diodes was available for use in the absolute value circuit. This necessitated making one pass from the tape recorder to the analog computer to evaluate voltages $V_2(T)$ and $V_3(T)$ and a separate pass to find $V_1(T)$. In actuality, each voltage was measured three times and the average values used in equations (22) and (24).

The time of integration was adjusted so that an integral number of cycles were covered at the lower frequencies, i.e. the integration time for 0.0101 cps was 198 seconds, for two complete cycles. A constant integration time of two hundred seconds was used above 0.3 cps for all but the two highest frequencies, where the time used was thirty seconds. Because of the large number of cycles included in this length of time, the error caused by not considering an integral number of cycles is negligible.

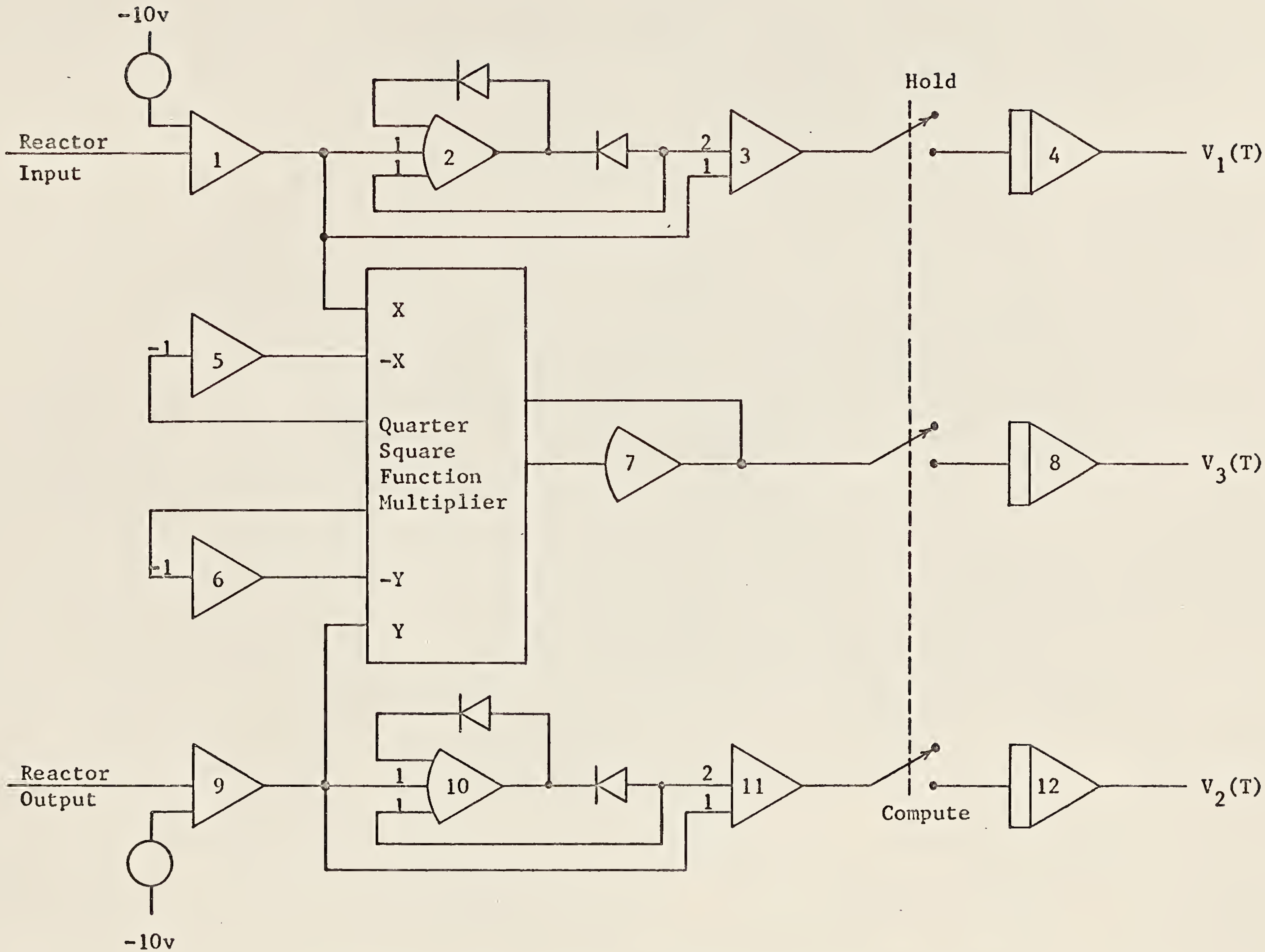


Figure 8. Circuit for analog computer method of data analysis

All voltages were read directly from the null balance circuit included on the analog computer. This circuit made it possible to measure the desired voltages to an accuracy of 0.01 volts.

## 6.0 DISCUSSION AND RESULTS

It was noted in section 5.1 that the dual-channel strip chart recorder and analog computer methods were used for data reduction. No particular problems were encountered with the analog computer method, but the strip chart recorder used did present some difficulties.

The Zero-Max unit used for the oscillator drive was chosen because of its availability. As noted in section 4.2, the use of this unit necessitated the addition of a gear train. The only means of varying the ratio of this gear train was to manually disassemble it and rearrange the gears to obtain the desired ratio. This meant that the frequency range of 0.01 cps to 40.0 cps had to be divided into three discrete regions and the data taken over a period of three different days. This procedure resulted in step discontinuities in the measured gain, as seen in Fig. 9 and Fig. 10. Since only the shape of the gain curve was of interest, it was acceptable to "smooth" these data to obtain the results shown in Fig. 11 and Fig. 12. The smoothed data are also presented in tabular form in Tables III and IV.

The data were smoothed by using a least squares technique to fit a straight line to the theoretical curve in the region of overlap at the points of discontinuity. A straight line of this slope was then least squares fitted to both sets of experimental data for the same region, and the difference between the resulting relative height constants used as an additive factor to shift the data of the lower set upward.

It was desired to obtain the value of $\alpha$, the break frequency, from the resulting smoothed gain curves. This was accomplished by least squares

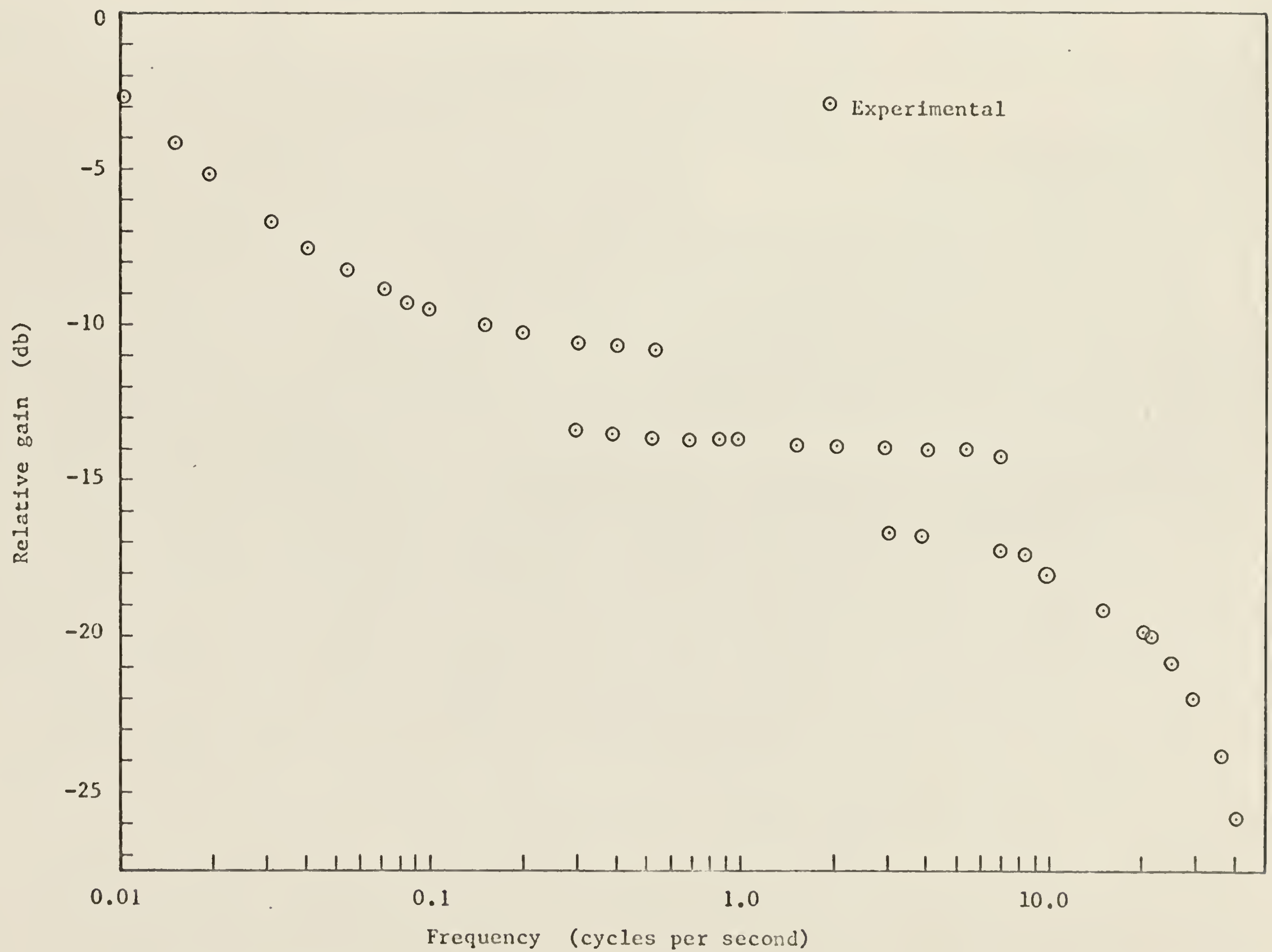


Figure 9. Transfer function gain, analog computer method (not smoothed)

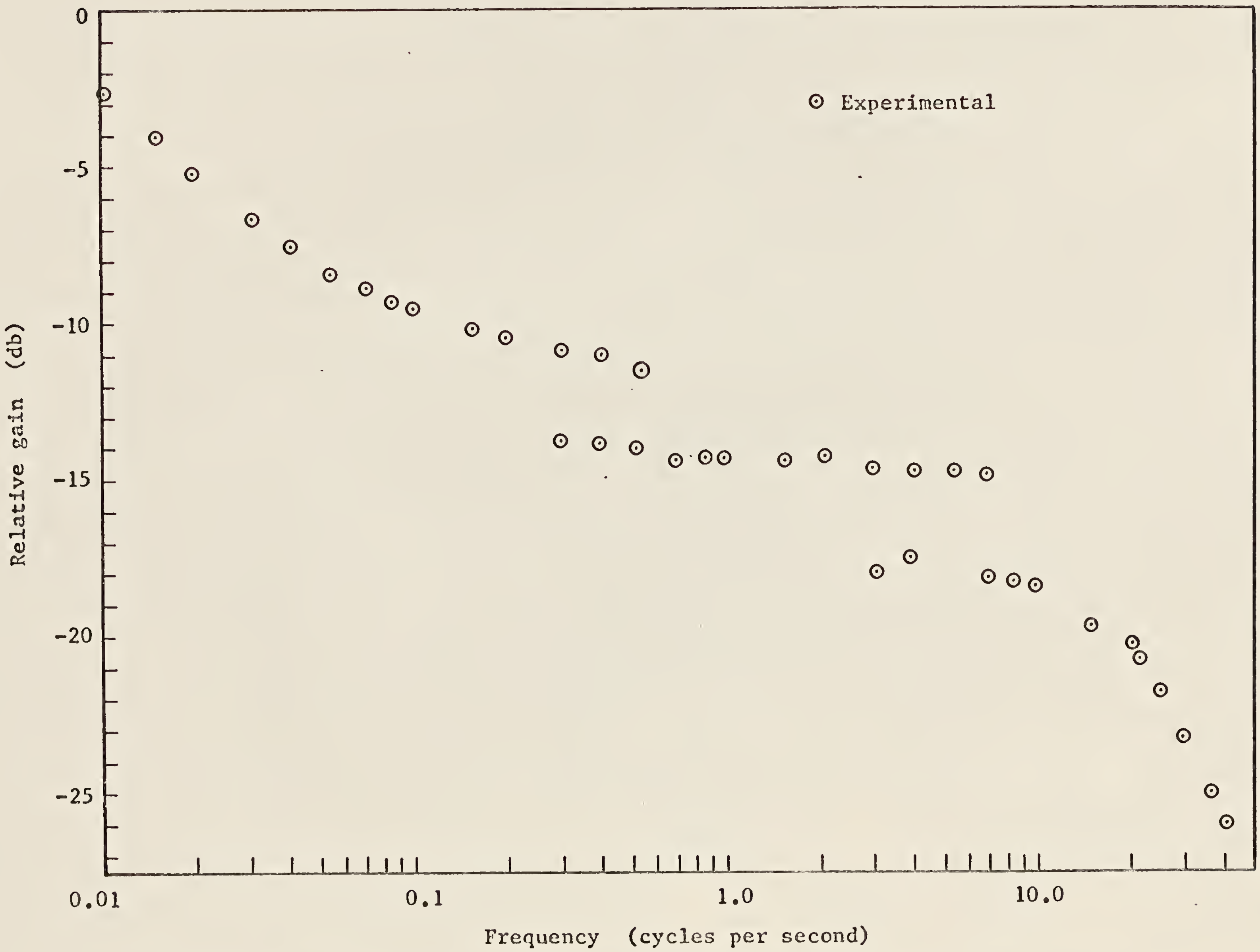


Figure 10. Transfer function gain, strip chart recorder method (not smoothed)

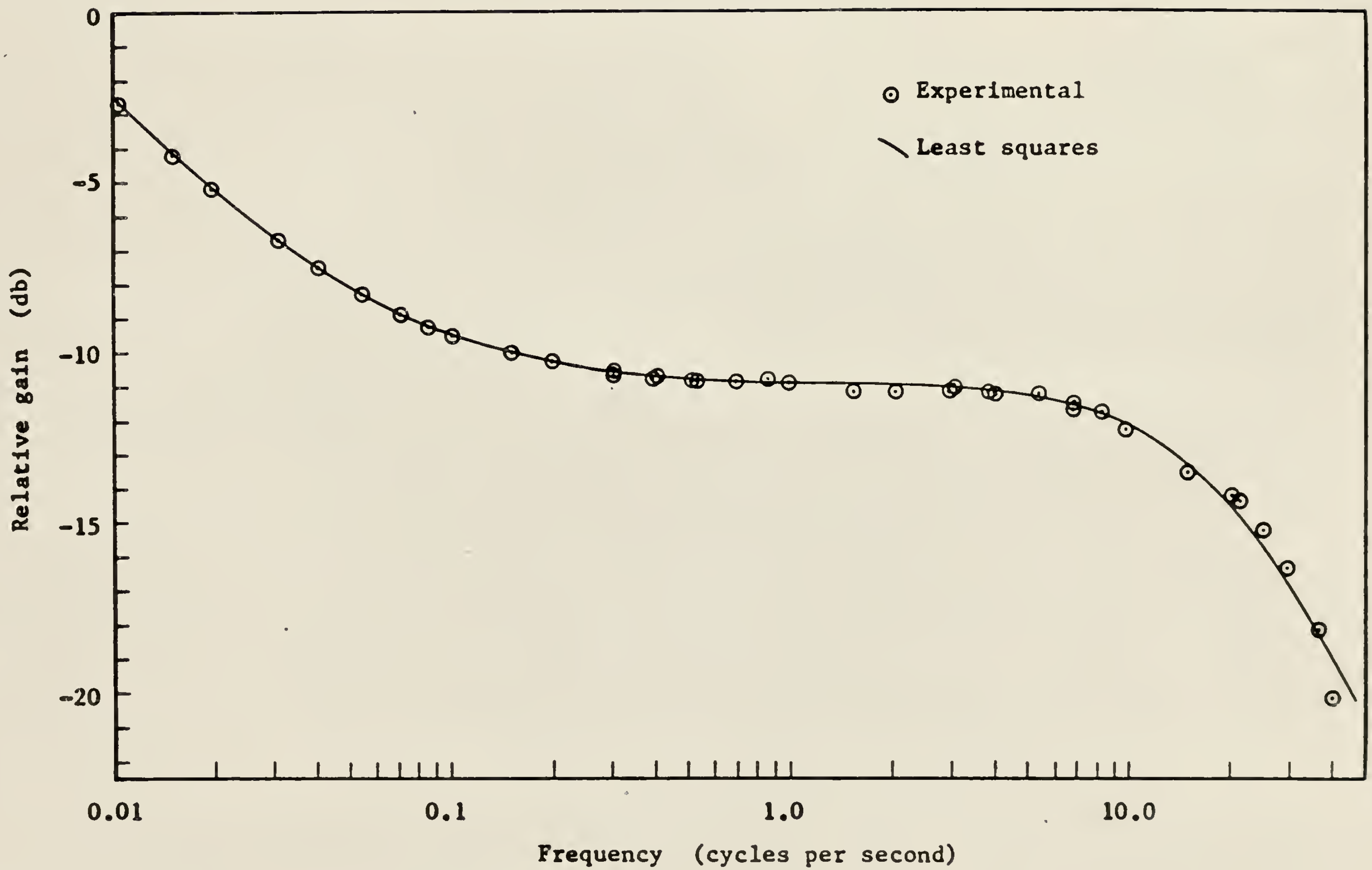


Figure 11. Transfer function gain, analog computer method (smoothed)

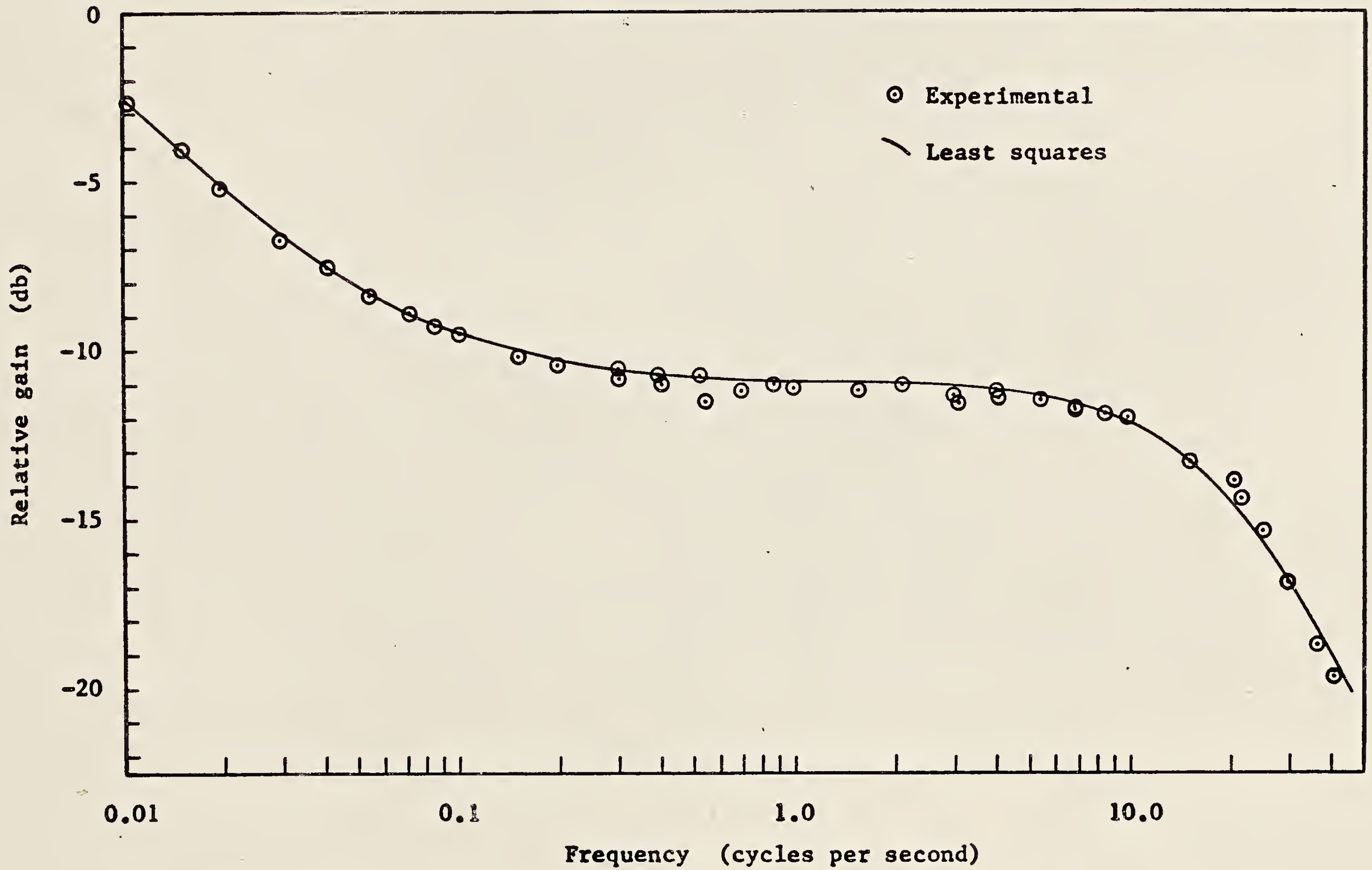


Figure 12. Transfer function gain, strip chart recorder method (smoothed)

fitting the theoretical gain curve to the experimental data. The theoretical zero power reactor transfer function is defined by Eq. (15). This expression may be separated into an amplitude, or gain, and a phase shift (details of the manipulation for this separation are presented in Appendix C). Equation (C-29) defines the theoretical gain.

$$G(j\omega) = A(\omega_i, \ell) \;\underline{/\theta(\omega_i, \ell)} \tag{25}$$

where $A(\omega_i,\ell)$ = magnitude of the transfer function at frequency $\omega_i$, corresponding to prompt neutron lifetime $\ell$,

$\theta(\omega_i,\ell)$ = phase shift at frequency $\omega_i$, and prompt neutron lifetime $\ell$.

A digital computer code, TRANSFUNC1, was written to evaluate the gain and phase shift using six groups of delayed neutrons. This program is described in detail in Appendix G.

The experimental gain data are measured on a relative scale, and thus a relative scale factor must be included in the least squares analysis to place the theoretical gain on the same scale. The weighted sum of the squares of the deviations of the observed gain from the theoretical gain is thus

$$S = \sum W_i \; [A_i - A(\omega_i, \ell) - A]^2 \tag{26}$$

where $\sum$ denotes summation over the index i, where i = 1,2,...,N

N = total number of data points

$A_i$ = experimentally measured gain at frequency $\omega_i$

A = relative scale factor

$W_i$ = weighting factor

The principle of least squares is to minimize the value of S in Eq. (26). It was assumed that the delayed neutron parameters, $\beta_i$ and $\lambda_i$, were known. The parameters to be adjusted in minimizing S are the prompt neutron lifetime and the relative scale factor. The theoretical gain, $A(\omega_i,\ell)$, is non-linear in $\ell$, but may be linearized by expansion in a Taylor series as follows,

$$A(\omega_i,\ell) = A(\omega_i) + \Delta\ell \frac{\partial A(\omega_i)}{\partial \ell_o} \tag{27}$$

where $A(\omega_i) = A(\omega_i,\ell_o)$

$\Delta\ell = \ell - \ell_o$

$$\frac{\partial A(\omega_i)}{\partial \ell_o} = \left.\frac{\partial A(\omega_i,\ell)}{\partial \ell}\right|_{\ell = \ell_o}$$

$\ell_o$ = approximate value of the prompt neutron lifetime.

After substitution of Eq. (27) for $A(\omega_i,\ell)$ in Eq. (26), S becomes

$$S = \sum W_i \left[A_i - A(\omega_i) - \Delta\ell \frac{\partial A(\omega_i)}{\partial \ell_o} - A\right]^2. \tag{28}$$

Now S may be minimized in the usual manner

$$\frac{\partial S}{\partial \ell} = 0 = \sum W_i\left[A_i - A(\omega_i) - \Delta\ell \frac{\partial A(\omega_i)}{\partial \ell_o} - A\right] \frac{\partial A(\omega_i)}{\partial \ell_o} \tag{29}$$

$$\frac{\partial S}{\partial A} = 0 = \sum W_i \left[A_i - A(\omega_i) - \Delta\ell \frac{\partial A(\omega_i)}{\partial \ell_o} - A\right]. \tag{30}$$

Equations (29) and (30) may be expressed in matrix form as

$$\Delta \begin{pmatrix} \Delta \ell \\ A \end{pmatrix} = \begin{pmatrix} C_4 \\ C_5 \end{pmatrix} \tag{31}$$

where $\Delta = \begin{pmatrix} C_1 & C_2 \\ C_2 & C_3 \end{pmatrix}$.

The expressions for the constants $C_1$ through $C_5$ are included in Appendix E. Equation (31) was programmed for digital computer solution in the code TRANSFUNC2. An iterative scheme of solution was chosen whereby a value of $\ell$ was chosen, $\Delta\ell$ calculated, a new value of $\ell$ calculated from that $\Delta\ell$ and in turn a new value of $\Delta\ell$ obtained. This procedure was repeated until the calculated value of $\Delta\ell$ was made less than some arbitrary value $\varepsilon$. The computer code and the method are described in greater detail in Appendices D and E.

An estimate of the variance of the least squares regression parameters $\ell$ and A is available from the above analysis. From Deming (8), it is found that

$$\sigma_\ell^{\,2} = C_{11}\sigma^2(\text{ext}) \tag{32}$$

$$\sigma_A^{\,2} = C_{22}\sigma^2(\text{ext}). \tag{33}$$

The constants $C_{11}$ and $C_{22}$ are elements from the main diagonal of the inverse of the coefficient matrix $\Delta$, i.e.

$$\Delta^{-1} = \begin{pmatrix} C_{11} & C_{12} \\ C_{12} & C_{22} \end{pmatrix} \tag{34}$$

and may be found in Appendix E. The factor noted as $\sigma^2(\text{ext})$ is an estimate of the variance of a function of unit weight, and is given by

$$\sigma^2(\text{ext}) = \frac{S}{N - 2}\,. \tag{35}$$

This statistical analysis was incorporated into the computer code TRANSFUNC2. The delayed neutron parameters used were those determined by Keepin, and shown in Table II (21).

It was assumed in the beginning that the amplitude and phase response, as a function of frequency, of the two channels of the dual-channel strip chart recorder were the same. If this were true, it would not matter what these responses were on an absolute scale, for all measurements were made on a relative basis between the two channels. It was found, however, that the raw gain data determined from the strip chart lagged behind that measured by the analog computer method above 0.2 cps, with the lag increasing with increasing frequency. This led to the speculation of a difference in the amplitude response of the two channels. Consequently, a calibration test was run in which signals of equal magnitude and phase were recorded on both channels. The frequency of the test signal was varied from 0.1 cps to 40.0 cps. As a result of this test, it was found that the ratio of the recorded amplitude from channel one to that from channel two was 0.65, instead of the 1.0 originally assumed. Figure 13 shows the amplitude response determined for the Offner strip chart recorder used. After this factor was included in the analysis of the strip chart determined gain, good agreement with the analog measured gain was obtained.

The phase shift, as measured by the analog computer method, is shown in Fig. 14. The results are somewhat erratic, and appear to become more dispersed with increasing frequency. It was decided to check the value of this data by performing a least squares analysis, similar to that used for the gain data, to find the value of $\alpha$.

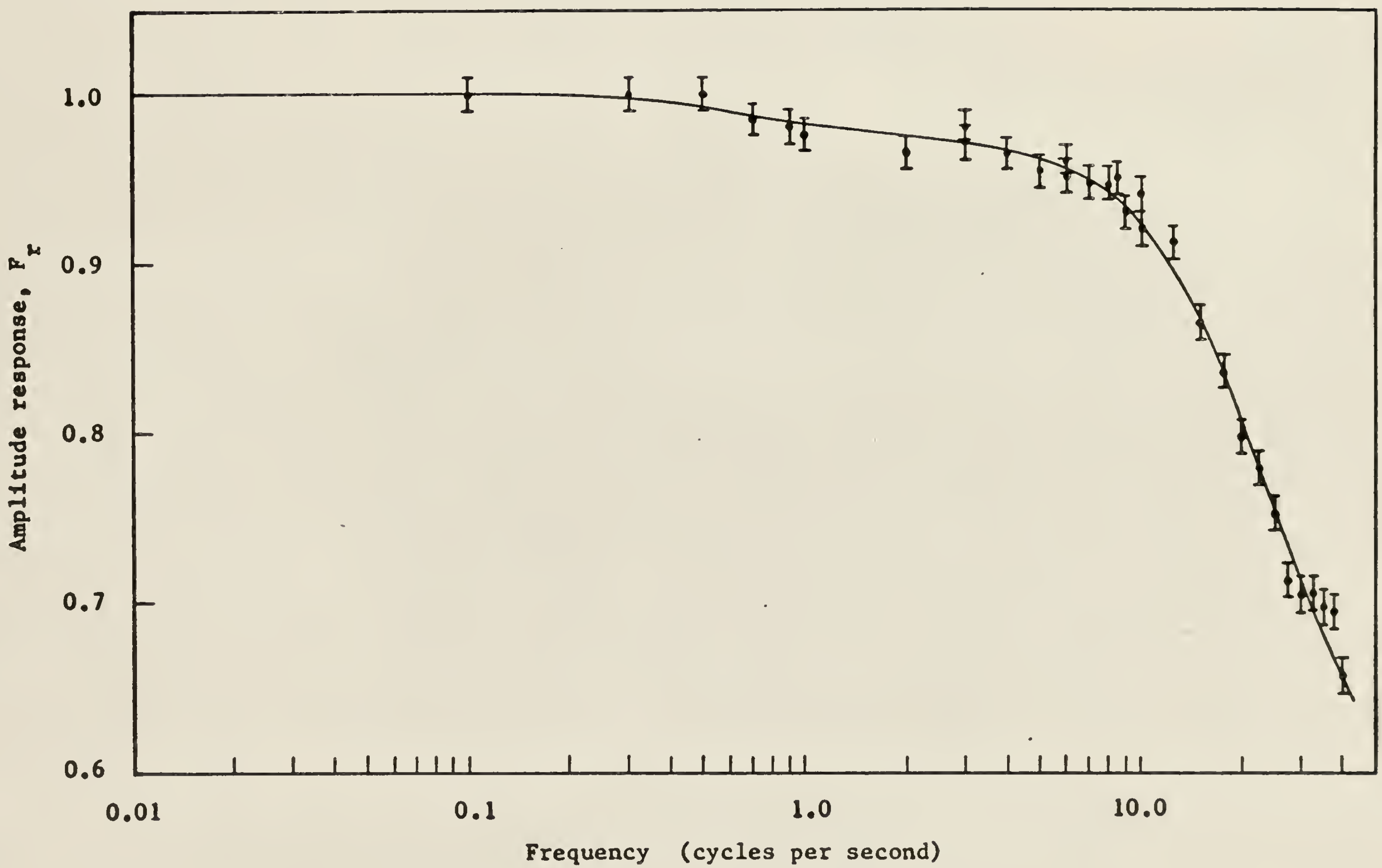


Figure 13. Amplitude response of Offner strip chart recorder

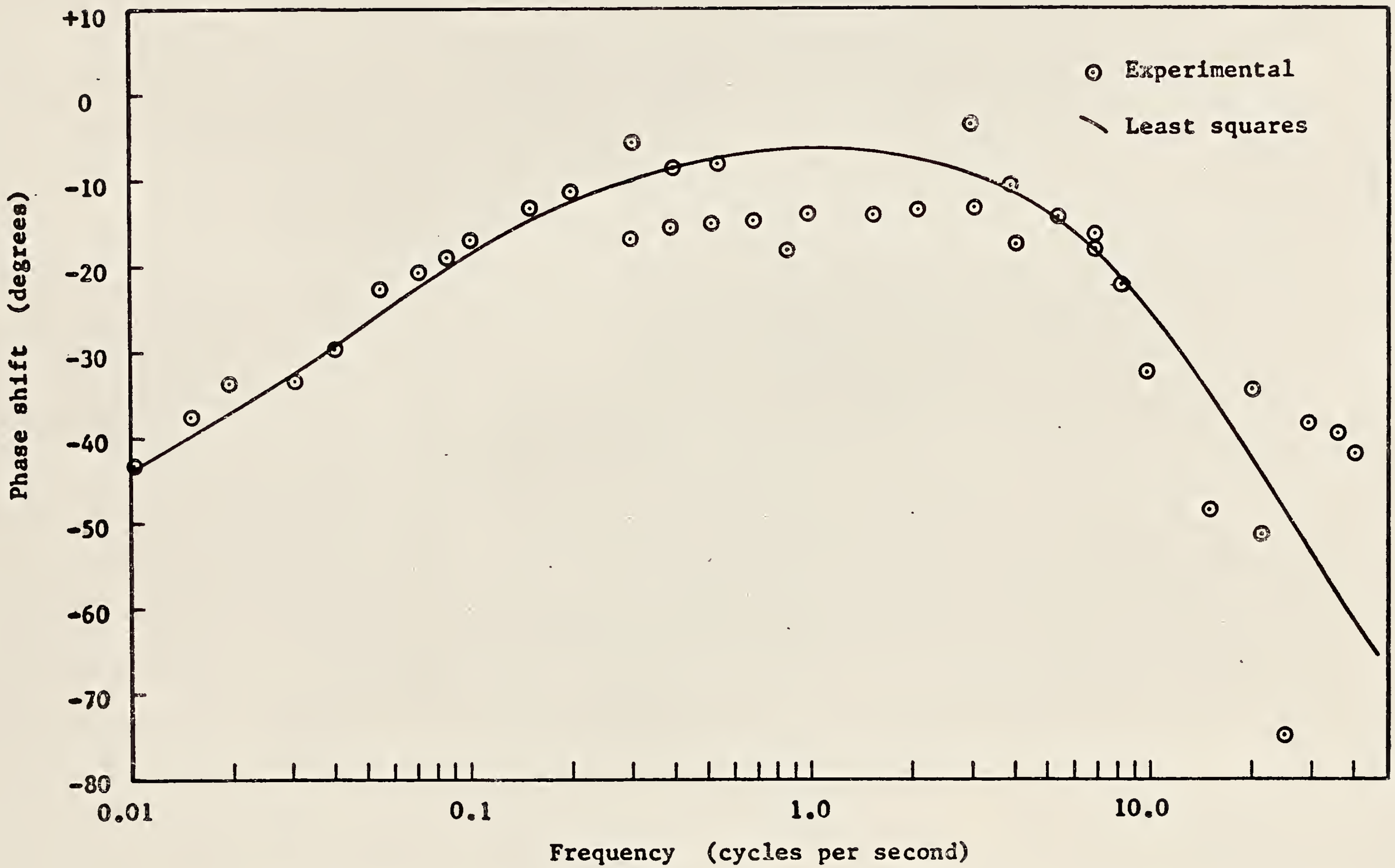


Figure 14. Transfer function phase shift, analog computer method

The theoretical phase shift, $\theta(\omega_i,\ell)$, is given by Eq. (C-30). In this case, there are no relative measurements and the only regression parameter is the prompt neutron lifetime. The expression $\theta(\omega_i,\ell)$ is non-linear in $\ell$ and must be linearized by expansion in a Taylor series.

$$\theta(\omega_i,\ell) \simeq \theta(\omega_i) + \Delta\ell \frac{\partial\theta(\omega_i)}{\partial\ell_o} \tag{36}$$

where $\theta(\omega_i) = \theta(\omega_i,\ell_o)$

$\Delta\ell = \ell - \ell_o$

$$\frac{\partial\theta(\omega_i)}{\partial\ell_o} = \left.\frac{\partial\theta(\omega_i,\ell)}{\partial\ell}\right|_{\ell = \ell_o} .$$

The expression for the weighted sum of the squares of the deviations is

$$S = \sum W_i \left[\theta(\omega_i) + \Delta\ell \frac{\partial\theta(\omega_i)}{\partial\ell_o} - \theta_i\right]^2 \tag{37}$$

where $\theta_i$ = experimentally measured phase shift at frequency $\omega_i$.

S may be minimized by the following procedure,

$$\frac{\partial S}{\partial\ell} = 0 = \sum W_i \left[\theta(\omega_i) + \Delta\ell \frac{\partial\theta(\omega_i)}{\partial\ell_o} - \theta_i\right] \frac{\partial\theta(\omega_i)}{\partial\ell_o} . \tag{38}$$

The digital computer code TRANSFUNC3 was written to perform the calculations indicated in Eq. (38). The procedure of solution was exactly that used before, where a value is guessed for $\ell$ and $\Delta\ell$ calculated. The calculated $\Delta\ell$ is used to find a new $\ell$ and consequently a new $\Delta\ell$.

The variance of the prompt neutron lifetime may be estimated from

$$\sigma_{\ell}^2 = C_{11}\sigma^2(\text{ext}) \tag{39}$$

where $C_{11}$ is defined in Appendix F and an estimate of $\sigma^2(\text{ext})$ is

$$\sigma^2(\text{ext}) = \frac{S}{N - 1} \ . \tag{40}$$

Again the delayed neutron parameters of Keepin were used. The procedure followed was an analysis for the value of the break frequency including all the data points. Next, the highest frequency point was omitted and the remaining data were analyzed for $\alpha$. The data points corresponding to the two highest frequencies were then omitted and the remaining data were analyzed for a third value of $\alpha$. This procedure was repeated until the seven highest frequency data points had been omitted. The results of these calculations are presented in graphical form in Fig. 15.

From Fig. 15, it is seen that the standard deviation of the calculated break frequency from the phase shift data overlaps the value of $\alpha$ determined from the gain data, for all but the two highest frequency data points. This would seem to indicate some systematic error in the last two points. This error could possibly be accounted for by a deviation in the experimental procedure. For all frequencies from thirty cycles per second down, the oscillator was run continuously during data collection. However, at frequencies above this, some heating of the bearings in the connecting rod used to convert the rotary motion to the oscillating motion was observed. In order that this heating be minimized, the oscillator drive was set for the desired frequency and then set in motion, and approximately two and one-half minutes of data recorded on magnetic tape. The power level of

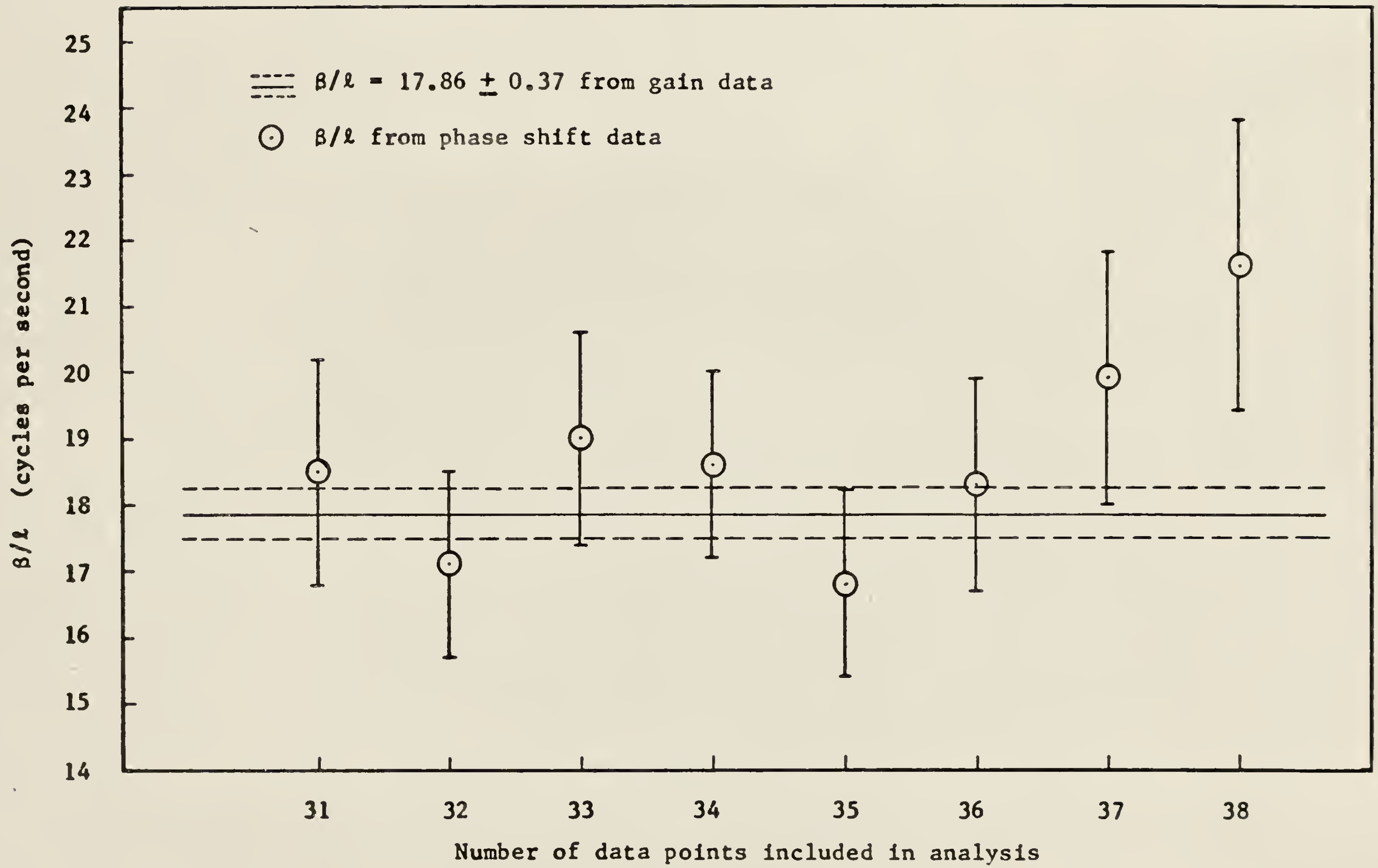


Figure 15. Results of least squares analysis of analog computer method determined transfer function phase shift

the reactor was stabilized before the oscillator was set in motion, but decreased in an exponential manner after oscillation was begun. This decrease in power was caused by the oscillator not being located exactly in the mean reactivity position before the start of oscillation. The result was essentially a negative step reactivity insertion upon which the desired oscillations were superimposed. This step insertion was undesirable, in that unwanted perturbations were caused in the delayed neutron precursors, and in the power level. To minimize the effects of these perturbations, only the last thirty seconds of data recorded were used for analysis. The half lives of the delayed neutron precursors range from 55.7 seconds to 0.23 seconds. Waiting for approximately two minutes after the negative step insertion meant that the precursors were given from 2.5 to over 500 half lives to reach new equilibrium values.

The phase shift, as measured by the strip chart recorder method, is presented in Fig. 16. It may be observed that the measured values lie, in general, above the theoretical curve, up to about 3.0 cps, at which point the data begins to become more dispersed. The observed spread in the data at the higher frequencies was caused by limitations of the strip chart recorder used. Chart speeds of 1, 5, 25 and 125 mm/sec were available and were quite suitable for phase shift measurements up to about 3.0 cps. However, above this frequency, the traces of the reactor input and output signals were compressed to such an extent that accurate readings became virtually impossible. Consider a measurement at 30.0 cps. The measured distance per cycle was 10.8 and the measured phase shift was 2.1.

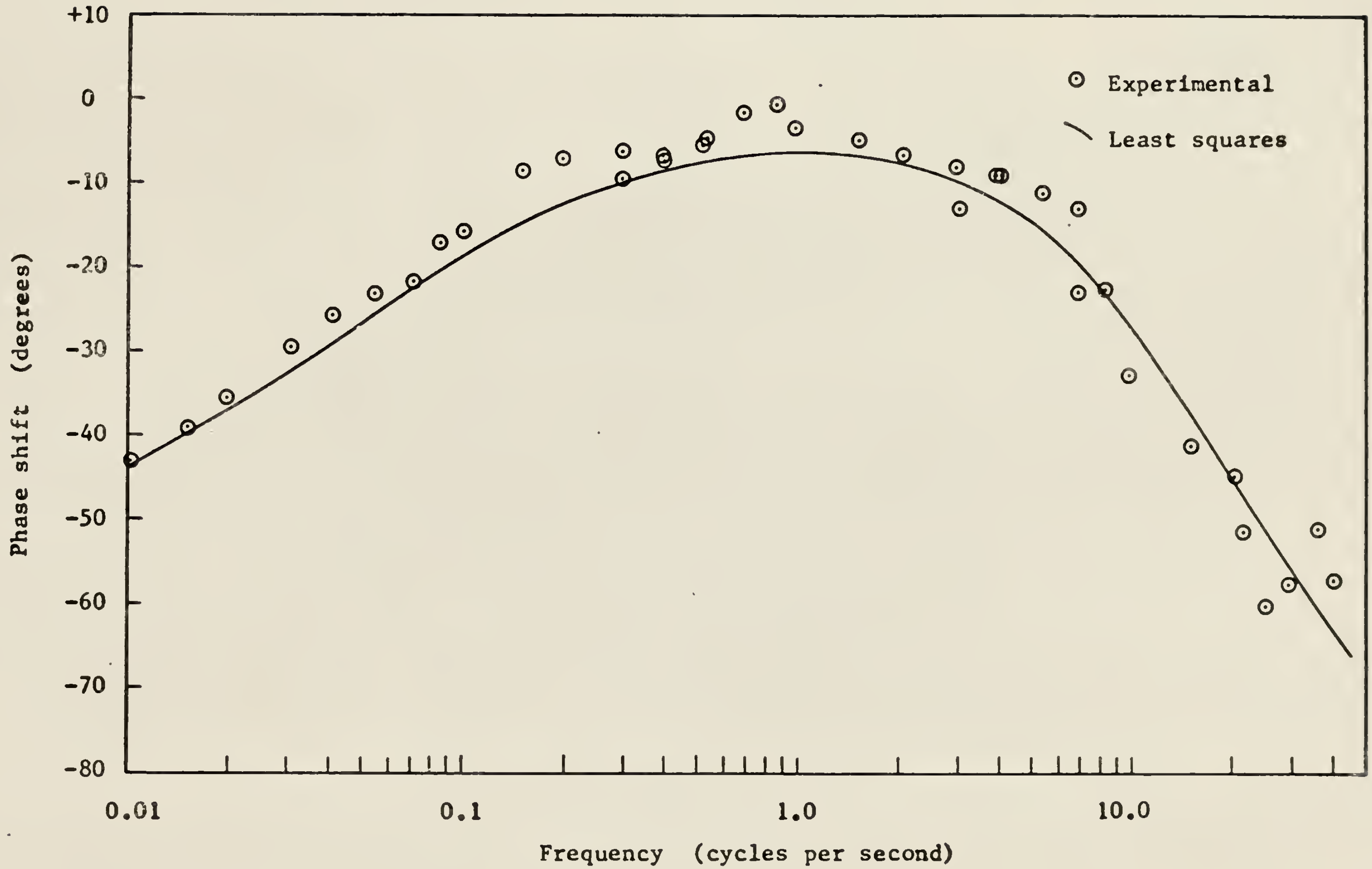


Figure 16. Transfer function phase shift, strip chart recorder method

The units used for measurement were sixty-fourths of an inch and were accurate only to the nearest unit. This yielded a phase shift of −70.1 degrees.

Besides the compression of the time scale at high frequencies, which made phase shift measurement difficult, another factor was discovered which affected the measurements over the entire frequency range. Because of the mechanical construction of the strip chart recorder, it was found that a difference in the needle zero crossing poisition existed between the two channels. As nearly as could be measured, this difference was 2.0 sixty-fourths inch.

The strip chart phase measurements were analyzed by the same technique as the analog phase data. The results are shown in Fig. 17. These calculations seem to indicate a value of $\alpha$ somewhat higher than that obtained from least squares analysis of the corresponding gain data. This indicates the possible presence of some systematic error. It is believed that the source of such an error could have been in the measurement of the difference in the needle zero crossing position between channels one and two, as noted above. This difference was measured to be 2.0 sixty-fourths, but could have been in error by as much as 0.5 sixty-fourths inch. If the value was actually 1.5, the result would be a downward adjustment of all data. This adjustment would range from about 1.5% at the lowest frequency to about 15% at the highest frequency. Such a shifting of the data would result in a larger value of the prompt neutron lifetime from the least squares analysis, and thus a smaller value of $\alpha$.

The value of the ratio of the effective fraction of delayed neutrons

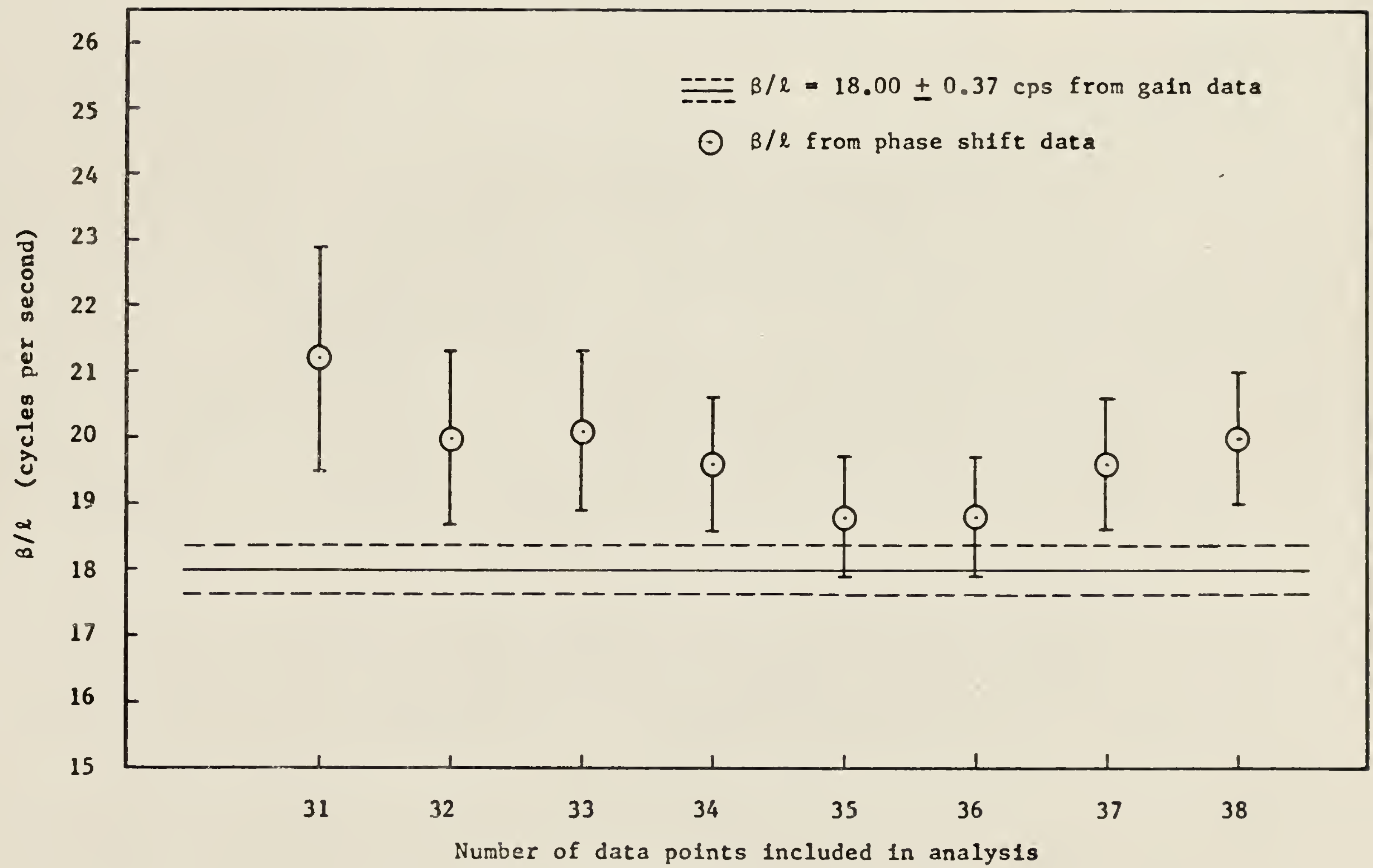


Figure 17. Results of least squares analysis of strip chart recorder method determined transfer function phase shift

to the prompt neutron lifetime, as found from the least squares analysis of the analog computer method measured gain data, was

$$\alpha = 17.86 \pm 0.37 \text{ cps.}$$

The value determined from analysis of the strip chart recorder method measured gain data was

$$\alpha = 18.00 \pm 0.37 \text{ cps.}$$

A value of the break frequency for the Kansas State University TRIGA Mark II reactor may be obtained from values of 0.0073 for the effective fraction of delayed neutrons (25), and 65 microseconds for the prompt neutron lifetime (11), as supplied by General Atomic. These values yield

$$\alpha = 17.87 \text{ cps.}$$

Thus it is seen that the experimentally measured values of the break frequency are in good agreement both with one another and with the value supplied by General Atomic.

It was desired to determine the sensitivity of the calculated break frequency to the delayed neutron parameters used in the least squares analysis. This was done by repeating the analysis of the two sets of gain data using the values of $\beta_i$ and $\lambda_i$ determined by Hughes (21), and tabulated in Table II. The result obtained from the analog computer method gain was

$$\alpha = 18.82 \pm 0.52 \text{ cps.}$$

The value from the strip chart recorder method gain was

$$\alpha = 18.96 \pm 0.48 \text{ cps.}$$

The results of this analysis show that the break frequency calculated using the delayed neutron data of Hughes is approximately 5.4% larger than that obtained using the data of Keepin. It should also be noted that the error indication on the break frequency is about 40% larger for the Hughes data than for the Keepin data.

Table II: Thermal fission delayed neutron constants for $U^{235}$ (21)

| Group | Keepin's values | | Hughes' values | |
|---|---|---|---|---|
| $i$ | $\beta_i$ | $\lambda_i\ (\frac{1}{sec})$ | $\beta_i$ | $\lambda_i\ (\frac{1}{sec})$ |
| 1 | 0.00027 | 3.01 | 0.00025 | 14.0 |
| 2 | 0.00074 | 1.14 | 0.00084 | 1.61 |
| 3 | 0.00253 | 0.301 | 0.0024 | 0.456 |
| 4 | 0.00125 | 0.111 | 0.0021 | 0.151 |
| 5 | 0.00140 | 0.0305 | 0.0017 | 0.0315 |
| 6 | 0.00021 | 0.0124 | 0.00026 | 0.0124 |

Table III: Zero power transfer function gain and phase shift, as measured by the analog computer method

| Frequency (cps) | Gain (db) | Phase shift (deg) |
|---|---|---|
| 0.0101 | -2.67 | -43.2 |
| 0.0150 | -4.19 | -37.6 |
| 0.0196 | -5.09 | -33.5 |
| 0.0305 | -6.71 | -33.1 |
| 0.0403 | -7.52 | -29.4 |
| 0.0544 | -8.26 | -22.4 |
| 0.0706 | -8.89 | -20.3 |
| 0.0850 | -9.27 | -19.1 |
| 0.100 | -9.49 | -17.0 |
| 0.150 | -10.03 | -13.3 |
| 0.198 | -10.26 | -11.3 |
| 0.300 | -10.57 | -5.7 |
| 0.400 | -10.65 | -8.3 |
| 0.537 | -10.83 | -7.9 |
| 0.298 | -10.54 | -16.8 |
| 0.390 | -10.71 | -15.7 |
| 0.517 | -10.80 | -15.2 |
| 0.686 | -10.91 | -14.7 |
| 0.860 | -10.81 | -18.1 |
| 0.986 | -10.92 | -13.9 |
| 1.52 | -11.07 | -14.0 |
| 2.08 | -11.10 | -13.6 |
| 2.98 | -11.07 | -3.5 |
| 4.03 | -11.21 | -17.3 |
| 5.40 | -11.22 | -14.3 |
| 6.93 | -11.45 | -16.2 |
| 3.06 | -10.99 | -13.2 |
| 3.88 | -11.14 | -10.4 |
| 6.92 | -11.59 | -18.0 |
| 8.30 | -11.75 | -22.2 |
| 9.79 | -12.30 | -32.4 |
| 14.95 | -13.50 | -48.7 |
| 20.11 | -14.20 | -34.6 |
| 21.30 | -14.31 | -51.7 |
| 24.95 | -15.21 | -75.1 |
| 29.54 | -16.31 | -38.6 |
| 36.11 | -18.11 | -39.7 |
| 40.30 | -20.09 | -42.0 |

Table IV: Zero power transfer function gain and phase shift, as measured by the strip chart recorder method

| Frequency (cps) | Gain (db) | Phase shift (deg) |
|---|---|---|
| 0.0101 | -2.61 | -43.0 |
| 0.0150 | -4.02 | -38.8 |
| 0.0196 | -5.18 | -35.7 |
| 0.0305 | -6.61 | -29.6 |
| 0.0403 | -7.53 | -25.8 |
| 0.0544 | -8.40 | -23.0 |
| 0.0706 | -8.86 | -21.8 |
| 0.0850 | -9.24 | -17.1 |
| 0.100 | -9.53 | -15.8 |
| 0.150 | -10.20 | -8.6 |
| 0.198 | -10.45 | -7.1 |
| 0.300 | -10.86 | -6.2 |
| 0.400 | -11.04 | -7.3 |
| 0.537 | -11.54 | -4.6 |
| 0.298 | -10.51 | -9.6 |
| 0.390 | -10.61 | -6.6 |
| 0.517 | -10.73 | -5.8 |
| 0.686 | -11.20 | -1.9 |
| 0.860 | -11.03 | -0.8 |
| 0.986 | -11.11 | -3.7 |
| 1.52 | -11.20 | -5.5 |
| 2.08 | -10.99 | -7.1 |
| 2.98 | -11.38 | -9.2 |
| 4.03 | -11.47 | -10.7 |
| 5.40 | -11.44 | -13.4 |
| 6.93 | -11.71 | -15.8 |
| 3.06 | -11.59 | -14.3 |
| 3.88 | -11.16 | -10.5 |
| 6.92 | -11.75 | -25.8 |
| 8.30 | -11.90 | -25.8 |
| 9.79 | -12.04 | -37.0 |
| 14.95 | -13.34 | -47.6 |
| 20.11 | -13.87 | -53.3 |
| 21.30 | -14.39 | -60.2 |
| 24.95 | -15.40 | -70.8 |
| 29.54 | -16.86 | -70.1 |
| 36.11 | -18.65 | -66.0 |
| 40.30 | -19.64 | -73.7 |

## 7.0 CONCLUSIONS

The results of these investigations have led to several conclusions, as stated below.

1. The pile oscillator which was designed and built, is adequate for frequency response measurements on the Kansas State University TRIGA Mark II reactor in the frequency range 0.01 cps to 30.0 cps.
2. The analog computer method of gain measurement is simple and reasonably accurate and rapid to perform.
3. The strip chart recorder method of gain measurement is reasonably accurate, but somewhat tedious and complicated by the necessity of calibrating the amplitude response of the recorder.
4. Neither the analog computer method nor the strip chart recorder method are well suited to accurate measurement of the phase shift portion of the reactor transfer function.
5. Least squares analysis of the experimentally measured gain is an accurate and sensitive method of determining the break frequency.
6. The value determined for the break frequency of the Kansas State University TRIGA Mark II reactor is

$$\alpha = 17.9 \pm 0.4 \text{ cps.}$$

## 8.0 SUGGESTIONS FOR FURTHER STUDY

In this investigation, only the zero power frequency response of the reactor was determined. An obvious extension of this work would be the measurement of the high power transfer function. Knowledge of these two factors allows calculation of the temperature feedback transfer function. Further attention could also be devoted to improvement of the present data collection and analysis systems.

The ion chamber used for detection of the reactor response could be replaced by a detector, such as a fission chamber, which could be positioned at several locations closer to the core. A rotary motion oscillator capable of higher frequencies, i.e., ∿100 cps, could be built to supplement the present linear motion oscillator. Such a system would allow investigation of spatial effects and reflector effects which cannot be observed with the present system.

The data analysis could be improved by cross-correlation of the reactor input and output signals, to determine the desired gain and phase shift information. Cross-correlation and autocorrelation electronic equipment are commerically available which would make such an analysis seem quite attractive.

## 9.0 ACKNOWLEDGMENT

The author wishes to express his gratitude to Dr. W. R. Kimel, Head of the Kansas State University Department of Nuclear Engineering, under whose direction this study was accomplished. Special thanks go to Dr. M. Copic, for his assistance and encouragement, and to Professor R. W. Clack, for the many hours of reactor operation. The author gratefully acknowledges support by the Atomic Energy Commission through the award of a special fellowship in nuclear science and engineering, administered by the Oak Ridge Institute of Nuclear Studies. Sincere appreciation is also extended to the Kansas State University Experiment Station for financial support.

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# 11.0 APPENDICES

# APPENDIX A

## Derivation of the Point Reactor Kinetic Equations

The one velocity, source free, time-dependent point reactor kinetic equations may be derived using a slowing-down diffusion model based on Fermi Age theory. The following development is limited by the assumptions stated in section 3.0, which will not be repeated here.

Consider a neutron balance for the element of volume dV,


In equation form this becomes,

$$\frac{1}{v}\frac{\partial\phi(\underline{r},t)}{\partial t} = D\nabla^2\phi(\underline{r},t) - \Sigma_a\phi(\underline{r},t) + p_{th}g_{th}(1 - \beta)\nu\Sigma_f\phi(\underline{r},t)$$

$$+ p_{th}g_{th}\sum_{i=1}^{6}\lambda_i C_i(\underline{r},t) \qquad \text{(A-1)}$$

where $p_{th}$ = resonance escape probability for neutrons while slowing down to thermal energy,

$g_{th}$ = nonleakage probability for neutrons while slowing down to thermal energy

$\beta$ = fraction of fission neutrons which are delayed,

$\phi(\underline{r},t)$ = neutron flux at position $\underline{r}$ and time t (n/cm$^2$-sec),

$C_i(\underline{r},t)$ = concentration of the ith group of delayed neutron precursors at time t and position $\underline{r}$ (atoms/cm$^3$),

$D$ = diffusion coefficient (cm),

$\Sigma_a$ = macroscopic absorption cross-section (cm$^{-1}$),

$\Sigma_f$ = macroscopic fission cross-section (cm$^{-1}$),

$\nu$ = average number of neutrons produced per thermal fission,

$\lambda_i$ = decay constant of the ith delayed neutron group (sec$^{-1}$).

The differential equation which describes the delayed neutron precursors is,

$$\frac{\partial C_i(\underline{r},t)}{\partial t} = \beta_i \nu \Sigma_f \phi(\underline{r},t) - \lambda_i C_i(\underline{r},t) \tag{A-2}$$

where $\beta_i$ = fraction of fission neutrons which are delayed in the ith delayed neutron group.

These equations may be solved by assuming the separability of time and space.

$$\phi(\underline{r},t) = F(\underline{r})\phi(t) \tag{A-3}$$

and

$$C_i(\underline{r},t) = F(\underline{r})C_i(t) \tag{A-4}$$

$F(\underline{r})$ is assumed to satisfy the expression

$$\nabla^2 F(\underline{r}) + B^2 F(\underline{r}) = 0 \tag{A-5}$$

where $B^2$ = geometric buckling.

Equations (A-3) and (A-4) may be substituted into equations (A-1) and (A-2) to yield,

$$\frac{1}{v} F(\underline{r}) \frac{d\phi(t)}{dt} = \phi(t) D\nabla^2 F(\underline{r}) - \Sigma_a F(\underline{r})\phi(t) + p_{th} g_{th}$$

$$(1 - \beta)\nu\Sigma_f F(\underline{r})\phi(t) + p_{th} g_{th} F(\underline{r}) \sum_{i=1}^{6} \lambda_i C_i(t) \tag{A-6}$$

and

$$F(\underline{r}) \frac{dC_i(t)}{dt} = \beta_i \nu \Sigma_f F(\underline{r}) \phi(t) - \lambda_i F(\underline{r}) C_i(t). \quad \text{(A-7)}$$

But from equation (A-5)

$$\nabla^2 F(\underline{r}) = - B^2 F(\underline{r}). \quad \text{(A-8)}$$

Equation (A-8) may be substituted into equation (A-6) to yield

$$\frac{1}{v} F(\underline{r}) \frac{d\phi(t)}{dt} = -\phi(t) D B^2 F(\underline{r}) - \Sigma_a F(\underline{r}) \phi(t) + p_{th} g_{th} (1 - \beta)$$

$$\nu \Sigma_f F(\underline{r}) \phi(t) + p_{th} g_{th} F(\underline{r}) \sum_{i=1}^{6} \lambda_i C_i(t). \quad \text{(A-9)}$$

It is now noted that the spatial dependence may be cancelled from equations (A-7) and (A-9), and the terms regrouped to yield

$$\frac{1}{v} \frac{d\phi(t)}{dt} = \phi(t) \; [- DB^2 - \Sigma_a + p_{th} g_{th} (1 - \beta) \nu \Sigma_f]$$

$$+ p_{th} g_{th} \sum_{i=1}^{6} \lambda_i C_i(t). \quad \text{(A-10)}$$

and

$$\frac{dC_i(t)}{dt} = \beta_i \nu \Sigma_f \phi(t) - \lambda_i C_i(t). \quad \text{(A-11)}$$

For the one velocity model assumed,

$$\phi(t) = v n(t). \quad \text{(A-12)}$$

Equation (A-12) may be substituted into equation (A-10) to give

$$\frac{dn(t)}{dt} = n(t) \; [- v \Sigma_a (1 + \frac{DB^2}{\Sigma_a}) + v p_{th} g_{th} (1 - \beta) \nu \Sigma_f]$$

$$+ p_{th} g_{th} \sum_{i=1}^{6} \lambda_i C_i(t). \quad \text{(A-13)}$$

The definition of the square of the diffusion length, $L^2 = D/\Sigma_a$, may be applied in equation (A-13). The result is

$$\frac{dn(t)}{dt} = n(t)\ [v\Sigma_a(1 + L^2B^2)][\frac{p_{th}g_{th}(1 - \beta)\nu\Sigma_f}{\Sigma_a(1 + L^2B^2)} - 1]$$

$$+ p_{th}g_{th} \sum_{i=1}^{6} \lambda_i C_i(t). \tag{A-14}$$

The definition of the effective multiplication constant (k) and the prompt neutron lifetime ($\ell$) are,

$$k = \frac{\nu\Sigma_f p_{th}g_{th}}{\Sigma_a(1 + L^2B^2)} \tag{A-15}$$

and

$$\ell = \frac{1}{v\Sigma_a(1 + L^2B^2)} . \tag{A-16}$$

Equations (A-15) and (A-16) may be substituted into equations (A-11) and (A-14) to give

$$\frac{dn(t)}{dt} = \frac{n(t)}{\ell}\ [(1 - \beta)k - 1] + p_{th}g_{th} \sum_{i=1}^{6} \lambda_i C_i(t) \tag{A-17}$$

and

$$\frac{dC_i(t)}{dt} = \frac{k\beta_i n(t)}{\ell p_{th}g_{th}} - \lambda_i C_i(t). \tag{A-18}$$

If it is now assumed that $p_{th}g_{th} = 1.0$, the spatially independent, or point reactor kinetic equations result.

$$\frac{dn(t)}{dt} = \frac{n(t)}{\ell}\ [(1 - \beta)k - 1] + \sum_{i=1}^{6} \lambda_i C_i(t) \tag{A-19}$$

and

$$\frac{dC_i(t)}{dt} = \frac{k\beta_i n(t)}{\ell} - \lambda_i C_i(t). \tag{A-20}$$

## APPENDIX B

### Derivation of the Open Loop Reactor Transfer Function

The effective multiplication constant (k) in the point reactor kinetic equations of APPENDIX A may be considered as a function of time without loss of generality, so long as any variation in k is spatially independent, i.e., uniform throughout the reactor.

It is now assumed that the time dependent quantities may be expressed as small time dependent variations (denoted by a delta) superimposed on some constant steady state value (denoted by a zero subscript).

$$\begin{aligned} k(t) &= k_o + \delta k(t) \\ n(t) &= n_o + \delta n(t) \\ C_i(t) &= C_{io} + \delta C_i(t) \end{aligned} \tag{B-1}$$

Equation (A-20) may be solved for $C_i(t)$ and the result substituted into (A-19) to yield

$$\frac{dn(t)}{dt} = \frac{n(t)}{\ell} [(1 - \beta)k(t) - 1] + \sum_{i=1}^{6} \lambda_i \{\frac{1}{\lambda_i} [\frac{k(t)\beta_i n(t)}{\ell} - \frac{dC_i(t)}{dt}]\} \tag{B-2}$$

or

$$\frac{dn(t)}{dt} = \frac{k(t)n(t)}{\ell} - \frac{n(t)}{\ell} - \sum_{i=1}^{6} \frac{dC_i(t)}{dt} . \tag{B-3}$$

Equations (B-1) may now be substituted into equations (B-3) and (A-20).

$$\frac{d\delta n(t)}{dt} = \frac{1}{\ell} [k_o n_o + n_o \delta k(t) + k_o \delta n(t) - n_o - \delta n(t)] - \sum_{i=1}^{6} \frac{d\delta C_i(t)}{dt} \tag{B-4}$$

and

$$\frac{d\delta C_i(t)}{dt} = \frac{\beta_i}{\ell} [k_o n_o + n_o \delta k(t) + k_o \delta n(t)] - \lambda_i C_{io} - \lambda_i \delta C_i(t), \quad \text{(B-5)}$$

where terms involving the product $\delta n(t)\delta k(t)$ have been neglected. For the steady state condition, equations (A-20) and (B-3) become

$$\frac{dn(t)}{dt} = 0 = \frac{1}{\ell} [k_o n_o - n_o]$$

and (B-6)

$$\frac{dC_i(t)}{dt} = 0 = \frac{k_o \beta_i n_o}{\ell} - \lambda_i C_{io}.$$

Equations (B-6) may be substituted into equations (B-4) and (B-5) to give

$$\frac{d\delta n(t)}{dt} = \frac{1}{\ell} [n_o \delta k(t) + k_o \delta n(t) - \delta n(t)] - \sum_{i=1}^{6} \frac{d\delta C_i(t)}{dt}$$

and (B-7)

$$\frac{d\delta C_i(t)}{dt} = \frac{\beta_i}{\ell} [n_o \delta k(t) + k_o \delta n(t)] - \lambda_i \delta C_i(t).$$

The open loop reactor transfer function, G(s), may now be defined as the ratio of the Laplace transform of the neutron density variation to the Laplace transform of the input reactivity variation times the reciprocal of the steady state neutron density $n_o$.

$$G(s) \equiv \frac{1}{n_o} \frac{L[\delta n(t)]}{L[\delta k(t)]} = \frac{1}{n_o} \frac{\delta n(s)}{\delta k(s)} \quad \text{(B-8)}$$

Taking the Laplace transform of equations (B-7) gives

$$s\delta n(s) = \frac{1}{\ell} [n_o \delta k(s) + k_o \delta n(s) - \delta n(s)] - \sum_{i=1}^{6} s\delta C_i(s) \quad \text{(B-9)}$$

and

$$s\delta C_i(s) = \frac{\beta_i}{\ell}\,[n_o\delta k(s) + k_o\delta n(s)] - \lambda_i\delta C_i(s), \tag{B-10}$$

where the initial conditions, $\delta n(0^+)$ and $\delta C_i(0^+)$, have been chosen to be zero. Equation (B-10) may be solved for $\delta C_i(s)$ and the result substituted into equation (B-9) to yield

$$s\delta n(s) = \frac{1}{\ell}\,[n_o\delta k(s) + k_o\delta n(s) - \delta n(s) - n_o\delta k(s)\sum_{i=1}^{6}\frac{s\beta_i}{s+\lambda_i}$$

$$- k_o\delta n(s)\sum_{i=1}^{6}\frac{s\beta_i}{s+\lambda_i}\ . \tag{B-11}$$

Equation (B-11) may be solved directly for the transfer function

$$\delta n(s)[s\ell - (k_o - 1) + k_o s\sum_{i=1}^{6}\frac{\beta_i}{s+\lambda_i}\,] = n_o\delta k(s)[1 - s\sum_{i=1}^{6}\frac{\beta_i}{s+\lambda_i}\,] \tag{B-12}$$

or

$$G(s) = \frac{\delta n(s)}{n_o\delta k(s)} = \frac{1 - s\displaystyle\sum_{i=1}^{6}\frac{\beta_i}{s+\lambda_i}}{s\ell[1 + \dfrac{k_o}{\ell}\displaystyle\sum_{i=1}^{6}\frac{\beta_i}{s+\lambda_i}\,] - (k_o - 1)}\ . \tag{B-13}$$

If the reactivity input signal is sinusoidal, s may be replaced by $j\omega$. When the reactor is critical, and the power level stable, $k_o \approx 1.0$ and the zero power transfer function becomes

$$G(j\omega) = \frac{1 - j\omega\displaystyle\sum_{i=1}^{6}\frac{\beta_i}{j\omega+\lambda_i}}{j\omega[\ell + \displaystyle\sum_{i=1}^{6}\frac{\beta_i}{j\omega+\lambda_i}\,]}\ . \tag{B-14}$$

# APPENDIX C

## Derivation of Expressions for the Reactor Transfer Function Gain and Phase Shift for Six Groups of Delayed Neu[illegible]ns

The expression for the transfer function was given as equation (B-14) and is repeated here for convenience.

$$G(\omega) = \frac{1 - j\omega \sum_{i=1}^{6} \frac{\beta_i}{j\omega + \lambda_i}}{j\omega[\ell + \sum_{i=1}^{6} \frac{\beta_i}{j\omega + \lambda_i}} \;. \tag{B-14}$$

In order to obtain expressions for the gain and phase shift, it is necessary to separate equation (B-14) into its real and imaginary parts. The notation used for this development is identical to that used in the computer program, TRANSFUNC1, for easy reference. The delayed neutron fractions, $\beta_i$, and the decay constants, $\lambda_i$, are replaced by

$$\begin{aligned} \lambda_1 &= A_1 \\ &\vdots \\ \lambda_6 &= A_6 \\ \beta_1 &= B_1 \\ &\vdots \\ \beta_6 &= B_6 . \end{aligned} \tag{C-1}$$

Consider the expression

$$\sum_{i=1}^{6} \frac{B_i}{j\omega + A_i} = \frac{B_1(j\omega + A_2)\ldots(j\omega + A_6)+\ldots+B_6(j\omega + A_1)\ldots(j\omega + A_5)}{(j\omega + A_1)(j\omega + A_2)\ldots(j\omega + A_5)(j\omega + A_6)}$$

$$= \frac{\sum_{i=1}^{6} B_i \prod_{n \neq i}^{6} (j\omega + A_n)}{\prod_{i=1}^{6} (j\omega + A_i)} = \frac{P(j\omega)}{Q(j\omega)} \;. \tag{C-2}$$

Consider now the expression $Q(j\omega)$.

$$Q(j\omega) = (j\omega + A_1)(j\omega + A_2)(j\omega + A_3)(j\omega + A_4)(j\omega + A_5)(j\omega + A_6) \qquad \text{(C-3)}$$

Use will be made of the notation

$$A_{123\ldots} = A_1A_2A_3$$

$Q(j\omega)$ may be expanded to obtain

$$Q(j\omega) = [-\omega^6 + B\omega^4 + C\omega^2 + D] + j[E\omega^5 + F\omega^3 + G\omega]; \qquad \text{(C-4)}$$

where
$$B = A_1(A_2 + A_3 + A_4 + A_5 + A_6) + A_2(A_3 + A_4 + A_5 + A_6)$$
$$+ A_3(A_4 + A_5 + A_6) + A_4(A_5 + A_6) + A_5A_6,$$
$$C = -[A_{12}(A_{34} + A_{35} + A_{45} + A_{36} + A_{46} + A_{56}) + A_3(A_1 + A_2)$$
$$(A_{45} + A_{46} + A_{56}) + A_{456}(A_1 + A_2 + A_3)], \qquad \text{(C-5)}$$
$$D = A_{123456},$$
$$E = A_1 + A_2 + A_3 + A_4 + A_5 + A_6,$$
$$F = -[A_{12}(A_3 + A_4 + A_5 + A_6) + A_3(A_1 + A_2)(A_4 + A_5 + A_6)$$
$$- A_{56}(A_1 + A_2 + A_3 + A_4) + A_4(A_5 + A_6)(A_1 + A_2 + A_3)]$$

and
$$G = A_{23456} + A_{13456} + A_{12456} + A_{12356} + A_{12346} + A_{12345}.$$

The expression for $P(j\omega)$ is

$$P(j\omega) = B_1(j\omega + A_2)\ldots(j\omega + A_6) + \ldots + B_6(j\omega + A_1)\ldots(j\omega + A_5)$$
$$= [S_1\omega^4 + S_2\omega^2 + S_3] + j[S_6\omega^5 + S_4\omega^3 + S_5\omega], \qquad \text{(C-6)}$$

where
$$S_n = \sum_{i=1}^{6} B_iD_{in} \qquad (n=1,\ldots,5)$$
$$\text{(C-7)}$$
$$S_6 = \sum_{i=1}^{6} B_i$$

and

$$D_{n1} = \sum_{i \neq n}^{6} A_i$$

$$D_{n2} = - \Psi_{i,j,k \neq n}^{6} A_i A_j A_k$$

$$D_{n3} = \prod_{i \neq n}^{6} A_i \qquad \text{(C-8)}$$

$$D_{n4} = - \Psi_{i,j \neq n}^{6} A_i A_j$$

$$D_{n5} = \Psi_{i,j,k,\ell \neq n}^{6} A_i A_j A_k A_\ell .$$

The notation used for $D_{n2}$, $D_{n4}$ and $D_{n5}$ represents the combination of the set of $A_i$'s taken three, two and four at a time, respectively. The number of combinations for n things taken r at a time is (22)

$$_nC_r = \frac{n!}{r!(n - r)!} . \qquad \text{(C-9)}$$

Thus it is seen that $D_{n2}$ is the sum of ten products involving three $A_i$'s, $D_{n4}$ is the sum of ten products involving two $A_i$'s, and $D_{n5}$ is the sum of five products involving four $A_i$'s. For illustration, the following examples are given:

$$D_{11} = \sum_{i \neq 1}^{6} A_i = A_2 + A_3 + A_4 + A_5 + A_6$$

$$D_{12} = - \Psi_{i,j,k \neq 1}^{6} A_i A_j A_k = - (A_{234} + A_{235} + A_{236} + A_{245} + A_{456} + A_{246}$$

$$+ A_{345} + A_{346} + A_{356} + A_{256})$$

$$D_{43} = \prod_{i \neq 4}^{6} A_i = A_{12356} \tag{C-10}$$

$$D_{34} = - \Psi_{i,j \neq 3}^{6} A_i A_j = - [A_1(A_2 + A_4 + A_5 + A_6) + A_2(A_4 + A_5 + A_6)$$

$$+ A_4(A_5 + A_6) + A_5 A_6]$$

$$D_{65} = \Psi_{i,j,k,m}^{5} A_i A_j A_k A_m = A_{1234} + A_{1534} + A_{1254} + A_{1235} + A_{5234}.$$

Equation (B-14) may now be expressed as

$$G(j\omega) = \frac{1 - j\omega \dfrac{P(j\omega)}{Q(j\omega)}}{j\omega[\ell + \dfrac{P(j\omega)}{Q(j\omega)}} = \frac{Q(j\omega) - j\omega P(j\omega)}{j\omega[\ell Q(j\omega) + P(j\omega)]} . \tag{C-11}$$

Equation (C-11) may be expanded to the form

$$G(j\omega) = \frac{[T_1\omega^6 + T_2\omega^4 + T_3\omega^2 + D] + j[T_4\omega^5 + T_5\omega^3 + T_6\omega]}{- [T_7\omega^6 + T_8\omega^4 + T_9\omega^2] + j[-\ell\omega^7 + T_{10}\omega^5 + T_{11}\omega^3 + T_{12}\omega]}, \tag{C-12}$$

where $T_1 = S_6 - 1$

$$T_2 = S_5 + B$$

$$T_3 = S_4 + C$$

$$T_4 = E - S_1$$

$$T_5 = F - S_2$$

$$T_6 = G - S_3 \tag{C-13}$$

$$T_7 = S_6 + \ell E$$

$$T_8 = S_5 + \ell F$$

$$T_9 = S_4 + \ell G$$

$$T_{10} = S_1 + \ell B$$

$$T_{11} = S_2 + \ell C$$

$$T_{12} = S_3 + \ell D.$$

Equation (C-12) may be separated into its real and imaginary parts by multiplying the numerator and denominator by its complex conjugate and re-arranging

$$G(j\omega) = \frac{[BA_1 + BA_2] + j[- BA_3 - BA_4]}{BA_5 + BA_6} \tag{C-14}$$

where:

$$BA_1 = - \omega^2[R_1\omega^{10} + R_2\omega^8 + R_3\omega^6 + R_4\omega^4 + R_5\omega^2 + R_6] \tag{C-15}$$

$$R_1 = T_1T_7$$

$$R_2 = T_1T_8 + T_2T_7$$

$$R_3 = T_1T_9 + T_2T_8 + T_3T_7 \tag{C-16}$$

$$R_4 = T_2T_9 + T_3T_8 + DT_7$$

$$R_5 = T_3T_9 + DT_8$$

$$R_6 = DT_9$$

$$BA_2 = \omega^2[BL_1\omega^{10} + BL_2\omega^8 + BL_3\omega^6 + BL_4\omega^4 + BL_5\omega^2 + BL_6] \tag{C-17}$$

$$BL_1 = - \ell T_4$$

$$BL_2 = T_4T_{10} - \ell T_5$$

$$BL_3 = T_4T_{11} + T_5T_{10} - \ell T_6 \tag{C-18}$$

$$BL_4 = T_4T_{12} + T_5T_{11} + T_6T_{10}$$

$$BL_5 = T_5T_{12} + T_6T_{11}$$

$$BL_6 = T_6T_{12}$$

$$BA_3 = \omega[V_1\omega^{12} + V_2\omega^{10} + V_3\omega^8 + V_4\omega^6 + V_5\omega^4 + V_6\omega^2 + V_7] \tag{C-19}$$

$$V_1 = - \ell T_1$$

$$V_2 = T_1T_{10} - \ell T_2$$

$$V_3 = T_1T_{11} + T_2T_{10} - \ell T_3 \tag{C-20}$$

$$V_4 = T_1T_{12} + T_2T_{11} + T_3T_{10} - \ell D$$

$$V_5 = T_2T_{12} + T_3T_{11} + DT_{10}$$

$$V_6 = T_3T_{12} + DT_{11}$$

$$V_7 = DT_{12}$$

$$BA_4 = \omega^3[X_1\omega^8 + X_2\omega^6 + X_3\omega^4 + X_4\omega^2 + X_5] \tag{C-21}$$

$$X_1 = T_4T_7$$

$$X_2 = T_4T_8 + T_5T_7$$

$$X_3 = T_4T_9 + T_5T_8 + T_6T_7 \tag{C-22}$$

$$X_4 = T_5T_9 + T_6T_8$$

$$X_5 = T_6T_9$$

$$BA_5 = \omega^4[Y_1\omega^8 + Y_2\omega^6 + Y_3\omega^4 + Y_4\omega^2 + Y_5] \tag{C-23}$$

$$Y_1 = T_7^{\,2}$$

$$Y_2 = 2T_7T_8$$

$$Y_3 = T_8^{\ 2} + 2T_7T_9 \tag{C-24}$$

$$Y_4 = 2T_8T_9$$

$$Y_5 = T_9^{\ 2}$$

$$BA_6 = \omega^2[Z_1\omega^{12} + Z_2\omega^{10} + Z_3\omega^8 + Z_4\omega^6 + Z_5\omega^4 + Z_6\omega^2 + Z_7] \tag{C-25}$$

$$Z_1 = \ell^2$$

$$Z_2 = -\ 2\ell T_{10}$$

$$Z_3 = T_{10}^{\ 2} - 2\ell T_{11}$$

$$Z_4 = 2[T_{10}T_{11} - \ell T_{12}] \tag{C-26}$$

$$Z_5 = T_{11}^{\ 2} + 2T_{10}T_{12}$$

$$Z_6 = 2T_{11}T_{12}$$

$$Z_7 = T_{12}^{\ 2}$$

Thus it is seen that the real, R, and the imaginary, I, parts of equation (B-14) may be expressed as

$$R = \frac{BA_1 + BA_2}{BA_5 + BA_6} \tag{C-27}$$

and

$$I = \frac{BA_3 + BA_4}{BA_5 + BA_6}. \tag{C-28}$$

The magnitude of $G(j\omega)$ is the gain, A, and expressed as

$$|G| = A = [R^2 + I^2]^{1/2} = \left[\frac{(BA_1 + BA_2)^2 + (BA_3 + BA_4)^2}{(BA_5 + BA_6)^2}\right]^{1/2} . \tag{C-29}$$

The phase shift, θ, is expressed as

$$\theta = \tan^{-1} \left(\frac{I}{R}\right) = \tan^{-1} \left[- \frac{BA_3 + BA_4}{BA_1 + BA_2}\right]. \qquad \text{(C-30)}$$

Equations (C-29) and (C-30) were solved by the computer program, TRANSFUNC1, described in Appendix G.

# APPENDIX D

## Least Squares and Statistical Analyses

The least squares and statistical analyses used in the treatment of all data presented in this thesis followed the method set forth by Deming (8).

Consider that y is a function of x, $\alpha$ and $\gamma$. A series of measurements are made which yield values $Y_1$, $Y_2$,..., $Y_N$, corresponding to $x_1$, $x_2$,..., $x_N$. It will be assumed that $x_i$ is known without error, while the values $Y_i$ are subject to measurement errors. The true relationship

$$y = f(x,\alpha,\gamma) \tag{D-1}$$

cannot be determined because of the measurements errors, but it is desired to estimate the functional relationship

$$y_i = f(x_i,a,b). \tag{D-2}$$

Here a and b represent estimates of the true parameters $\alpha$ and $\gamma$. If $Y_i$ were known without error, $\alpha$ and $\gamma$ could be obtained directly from two values of $Y_i$ and $x_i$. However, since error is present in $Y_i$, equation (D-2) represents a set of N inconsistent equations.

Define the residual, $V_i$, of point i as

$$\begin{aligned} V_i &= \text{observed value - calculated value} \\ &= Y_i - y_i = Y_i - f(x_i,a,b). \end{aligned} \tag{D-3}$$

The principle of least squares requires that the sum, S, of the weighted squares of the residuals

$$S = \sum_{i=1}^{N} W_i V_i^2 \tag{D-4}$$

shall be made a minimum. The weighting factor, $W_i$, is proportional to the reciprocal of the variance, or the square of the standard deviation, of the observed value $Y_i$. Physically, the weighting factor means that the more accurately an observed $Y_i$ is known, the more the square of its residual will contribute to the value of S, relative to a less accurately known $Y_i$.

The function f(x,a,b) may be linearized by expansion in a Taylor series, where only first order terms are retained, i.e.,

$$f(x_i,a,b) = f_i + \Delta a\, f_{ai} + \Delta b\, f_{bi} \tag{D-5}$$

where $a_o$ = approximate value of a

$b_o$ = approximate value of b

$\Delta a = a - a_o$

$\Delta b = b - b_o$

$$f_{ai} = \left.\frac{\partial f(x,a,b_o)}{\partial a}\right|_{a = a_o}$$

and

$$f_{bi} = \left.\frac{\partial f(x,a_o,b)}{\partial b}\right|_{b = b_o} .$$

Equation (D-5) may be substituted into (D-4) to yield

$$S = \sum W_i \, [Y_i - f_i - \Delta a\, f_{ai} - \Delta b\, f_{bi}]^2 . \tag{D-6}$$

The parameters which may be adjusted to minimize S are a and b. Thus in the usual manner,

$$\frac{\partial S}{\partial a} = 0 = \sum W_i \, [Y_i - f_i - \Delta a\, f_{ai} - \Delta b\, f_{bi}]\, f_{ai} \tag{D-7}$$

$$\frac{\partial S}{\partial b} = 0 = \sum W_i \, [Y_i - f_i - \Delta a\, f_{ai} - \Delta b\, f_{bi}]\, f_{bi} . \tag{D-8}$$

Equations (D-7) and (D-8) may be written in matrix form as

$$\Delta \begin{pmatrix} \Delta a \\ \Delta b \end{pmatrix} = \begin{pmatrix} C_4 \\ C_5 \end{pmatrix} \tag{D-9}$$

where $\Delta = \begin{pmatrix} C_1 & C_2 \\ C_2 & C_3 \end{pmatrix}$

$$C_1 = \sum W_i f_{ai}^2$$

$$C_2 = \sum W_i f_{ai} f_{bi}$$

$$C_3 = \sum W_i f_{bi}^{\ 2}$$

$$C_4 = \sum W_i [Y_i - f_i] f_{ai}$$

$$C_5 = \sum W_i [Y_i - f_i] f_{bi}.$$

Equation (D-9) may be solved for $\Delta a$ and $\Delta b$, and thus better estimates of a and b obtained from

$$a = \Delta a + a_o \tag{D-10}$$

$$b = \Delta b + b_o. \tag{D-11}$$

The values of a and b so obtained may be used to recalculate the constants in equation (D-9), and new values of $\Delta a$ and $\Delta b$ are obtained. This procedure may be repeated until some specified degree of accuracy is reached.

The variance of a function of unit weight may be estimated from

$$\sigma^2(\text{ext}) = \frac{S}{p} \tag{D-12}$$

where p = degrees of freedom and is the number of data points minus the number of parameters used in the analysis.

The variance of the parameters a and b may be estimated from

$$\sigma_a^{\,2} = C_{11}\sigma^2(\text{ext}) \tag{D-13}$$

$$\sigma_b^{\,2} = C_{22}\sigma^2(\text{ext}) \tag{D-14}$$

where $C_{11}$ and $C_{22}$ are the elements from the main diagonal of the inverse of the coefficient matrix, $\Delta$.*

$$\Delta^{-1} = \begin{pmatrix} C_{11} & C_{12} \\ C_{12} & C_{22} \end{pmatrix} \tag{D-15}$$

where $C_{11} = \dfrac{C_3}{C_1\,C_3 - C_2^{\,2}}$

$$C_{12} = \frac{-\,C_2}{C_1\,C_3 - C_2^{\,2}}$$

$$C_{22} = \frac{C_1}{C_1\,C_3 - C_2^{\,2}}\ .$$

The variance of the function y = f(x,a,b,) may be estimated from

$$\sigma_y^{\,2} = \sigma^2(\text{ext})\ [C_{11}f_a^{\,2} + 2C_{12}f_a f_b + C_{22}f_b^{\,2}]. \tag{D-16}$$

*See Appendix K for proof.

# APPENDIX E

## Development of Expressions for Least Squares and Statistical Analyses of Transfer Function Gain Data

Equation (C-29) defines the zero power reactor transfer function gain. It was assumed that the frequency, $\omega_i$, was known without error, but that the experimentally measured gain was subject to statistically distributed measurement errors. The expressions developed in Appendix D may be applied directly to the present case. Equations (D-2) and (D-3) become

$$f(x_i,a,b) = A(\omega_i,\ell) + A \tag{E-1}$$

$$V_i^2 = [A_i - A(\omega_i,\ell) - A]^2 \tag{E-2}$$

where $A(\omega_i,\ell)$ = theoretical gain at frequency $\omega_i$ and prompt neutron lifetime $\ell$, as defined in equation (C-29),

$A_i$ = experimentally measured gain at frequency $\omega_i$,

$A$ = relative scale factor to place theoretical gain on the same relative scale as the experimentally measured gain.

The linearization of $A(\omega_i,\ell) + A$ yields

$$A(\omega_i,\ell) + A \simeq A(\omega_i) + \Delta\ell\frac{\partial A(\omega_i)}{\partial \ell_o} + A \tag{E-3}$$

where $A(\omega_i) = A(\omega_i,\ell_o)$

$\Delta\ell = \ell - \ell_o$

$$\frac{\partial A(\omega_i)}{\partial \ell_o} = \left.\frac{\partial A(\omega_i,\ell)}{\partial \ell}\right|_{\ell = \ell_o}$$

$\ell_o$ = approximate value of the prompt neutro[illegible] fetime.

The weighted sum of the squares of the errors becomes

$$S = \sum W_i [A_i - A(\omega_i) - \Delta\ell \frac{\partial A(\omega_i)}{\partial \ell_o} - A]^2. \tag{E-4}$$

The weighting factors used in this case were assumed to be proportional to the reciprocal of the gain, i.e.,

$$W_i = \frac{1}{A(\omega_i)} . \tag{E-5}$$

The parameters to be adjusted in minimizing S are $\ell$ and A. Thus equations (D-7) and (D-8) become

$$\frac{\partial S}{\partial \ell} = 0 = \sum W_i [A_i - A(\omega_i) - \Delta\ell \frac{\partial A(\omega_i)}{\partial \ell_o} - A] \frac{\partial A(\omega_i)}{\partial \ell_o} \tag{E-6}$$

$$\frac{\partial S}{\partial A} = 0 = \sum W_i [A_i - A(\omega_i) - \Delta\ell \frac{\partial A(\omega_i)}{\partial \ell_o} - A] \tag{E-7}$$

These equations expressed in matrix form are

$$\Delta \begin{pmatrix} \Delta\ell \\ A \end{pmatrix} = \begin{pmatrix} C_4 \\ C_5 \end{pmatrix} \tag{E-8}$$

where $\Delta = \begin{pmatrix} C_1 & C_2 \\ C_2 & C_3 \end{pmatrix}$

$$C_1 = \sum W_i \left[\frac{\partial A(\omega_i)}{\partial \ell_o}\right]^2$$

$$C_2 = \sum W_i \frac{\partial A(\omega_i)}{\partial \ell_o}$$

$$C_3 = \sum W_i$$

$$C_4 = \sum W_i \frac{\partial A(\omega_i)}{\partial \ell_o} [A_i - A(\omega_i)]$$

$$C_5 = \sum W_i [A_i - A(\omega_i)].$$

A first guess for the prompt neutron lifetime, $\ell_o$, was used in equation (E-8) to calculate $\Delta\ell$ and A. A better estimate of $\ell$ was then obtained from

$$\ell = \Delta\ell + \ell_o. \quad \text{(E-9)}$$

This new value of $\ell$ was again used in equation (E-8) to find new values of $\Delta\ell$ and A. This procedure was repeated until the value of $\ell$ changed by less than some arbitrary amount, $\varepsilon$.

The inverse of the coefficient matrix, $\Delta^{-1}$, corresponding to equation (D-15), was

$$\Delta^{-1} = \begin{pmatrix} C_{11} & C_{12} \\ C_{12} & C_{22} \end{pmatrix} \quad \text{(E-10)}$$

where the $C_{ij}$'s are those defined in Appendix D.

The estimation of the variance of $\ell$ and A follows directly from equations (D-13) and (D-14).

$$\sigma_\ell^2 = C_{11}\sigma^2(\text{ext}) \quad \text{(E-11)}$$

$$\sigma_A^2 = C_{22}\sigma^2(\text{ext}) \quad \text{(E-12)}$$

where $\sigma^2(\text{ext}) = \frac{S}{N - 2}$

and N is equal to the total number of data points. Equation (D-16) yields, as an estimate of the variance of the least squares function $A(\omega_i, \ell) + A$,

$$\sigma_f^{\,2} = \sigma^2(\text{ext})\left\{C_{11}\left[\frac{\partial A(\omega_i)}{\partial \ell_o}\right]^2 + 2C_{12}\,\frac{\partial A(\omega_i)}{\partial \ell_o} + C_{22}\right\} . \qquad \text{(E-13)}$$

The iterative scheme set forth above was used in the digital computer program TRANSFUNC2, which performed the indicated least squares and statistical analyses. The program is described in detail in Appendix H.

# APPENDIX F

## Development of Expressions for Least Squares and Statistical Analyses of Transfer Function Phase Shift Data

Equation (C-30) defines the zero power reactor transfer function phase shift. It was assumed that the frequency, $\omega_i$, was known without error, but that the experimentally measured phase shift was subject to statistically distributed measurement errors. The analysis presented in Appendix D may be applied to the present case. Equations (D-2) and (D-3) become

$$f(x_i,a,b) = \theta(\omega_i,\ell) \tag{F-1}$$

$$V_i^{\,2} = [\theta(\omega_i,\ell) - \theta_i]^2 \tag{F-2}$$

where $\theta(\omega_i,\ell)$ = theoretical phase shift at frequency $\omega_i$ and prompt neutron lifetime $\ell$, as defined by equation (C-30),

$\theta_i$ = experimentally measured phase shift at frequency $\omega_i$.

Equation (D-5) yields, for the linearization of $\theta(\omega_i,\ell)$,

$$\theta(\omega_i,\ell) \simeq \theta(\omega_i) + \Delta\ell \frac{\partial\theta(\omega_i)}{\partial\ell_o} \tag{F-3}$$

where $\theta(\omega_i) = \theta(\omega_i,\ell_o)$,

$\Delta\ell = \ell - \ell_o$,

$$\frac{\partial\theta(\omega_i)}{\partial\ell_o} = \left.\frac{\partial\theta(\omega_i,\ell)}{\partial\ell}\right|_{\ell = \ell_o}.$$

The weighted sum of the squares of the errors becomes

$$S = \sum W_i[\theta(\omega_i) + \Delta\ell \frac{\partial\theta(\omega_i)}{\partial\ell_o} - \theta_i]^2. \tag{F-4}$$

In this case, the only adjustable parameter is the prompt neutron lifetime. Thus, equations (D-7) and (D-8) become

$$\frac{\partial S}{\partial \ell} = 0 = \sum W_i[\theta(\omega_i) + \Delta\ell \frac{\partial\theta(\omega_i)}{\partial\ell_o} - \theta_i] \frac{\partial\theta(\omega_i)}{\partial\ell_o} . \tag{F-5}$$

The solution of this equation for $\Delta\ell$ may be expressed in the form

$$\Delta\ell = \frac{C_2}{C_1} \tag{F-6}$$

where $C_1 = \sum W_i[ \frac{\partial\theta(\omega_i)}{\partial\ell_o} ]^2$

$$C_2 = \sum W_i \frac{\partial\theta(\omega_i)}{\partial\ell_o} [\theta_i - \theta(\omega_i)].$$

As suggested in Appendix D, a first guess for the prompt neutron lifetime, $\ell_o$, was used in equation (F-6) to solve for $\Delta\ell$. This value of $\Delta\ell$ was then used to correct $\ell_o$, as

$$\ell = \Delta\ell + \ell_o. \tag{F-7}$$

This new value of $\ell$ was used in equation (F-6) to calculate a new $\Delta\ell$. This procedure was repeated until $\Delta\ell$ attained a value less than some arbitrarily specified $\varepsilon$.

The estimate of the variance of a function of unit weight is, from equation (D-12),

$$\sigma^2(\text{ext}) = \frac{S}{N - 1} \tag{F-8}$$

and from this, the variance of $\ell$ may be estimated.

$$\sigma_\ell^{\,2} = C_{11}\sigma^2(\text{ext}) \tag{F-9}$$

where $C_{11} = \dfrac{1}{C_1}$ .

The variance of the least squares function, $\theta(\omega_i,\ell)$ may be estimated as

$$\sigma_\theta^{\,2} = \sigma^2(\text{ext})C_{11}\left[\frac{\partial\theta(\omega_i)}{\partial\ell}\right]^2. \tag{F-10}$$

The digital computer program TRANSFUNC3, was written to perform the iterative type least squares analysis and the statistical analyses indicated above. The program is described in detail in Appendix I.

# APPENDIX G

## Description of Digital Computer Program used for Calculation of Zero Power Reactor Transfer Function Gain and Phase Shift

This computer code, TRANSFUNC1, was written to calculate the zero power reactor transfer function gain and phase shift from equations (C-29) and (C-30) using six groups of delayed neutrons. The program was written in FORTRAN IV language. The values of the constants $\beta_i$ and $\lambda_i$ are incorporated into the program, eliminating the need to read them in with each set of data. The source program is listed, and the logic diagram is shown, in this appendix. The notation used in the program is identical to that used in the development in Appendix C.

The only required input to the program is one card for each frequency, in cycles per second, for which the gain and phase shift are to be calculated. The input is punched in the format F10.5.

The output of the program is set up for both card and print-out form. The first card of the output contains the statement, "Zero power reactor transfer function". The second card of the output contains the value of AL, the prompt neutron lifetime, used in the calculation. The third card contains headings for the three columns of output, Frequency (cps), Gain (db), and Phase Shift (deg), which follow in the remaining cards of the output. The on-line printer output is identical to the card output.

The theoretical gain and phase shift were calculated for the values of the prompt neutron lifetime determined from least squares analysis of the analog computer and strip chart recorder determined gain data. A print out of the results obtained are included at the end of this appendix.

Table V: Input data and variables required for the theoretical gain and phase shift computer program

| Symbol | Explanation |
|---|---|
| A1 - A6 | Delayed neutron decay constants |
| AL | Prompt neutron lifetime |
| B1 - B6 | Delayed neutron fractions |
| GAIN | Reactor transfer function gain |
| N | Number of data cards to be read |
| RUNNO | Run number, for data identification |
| THETA | Reactor transfer function phase shift |
| W | Frequencies for which the gain and phase shift are to be calculated |

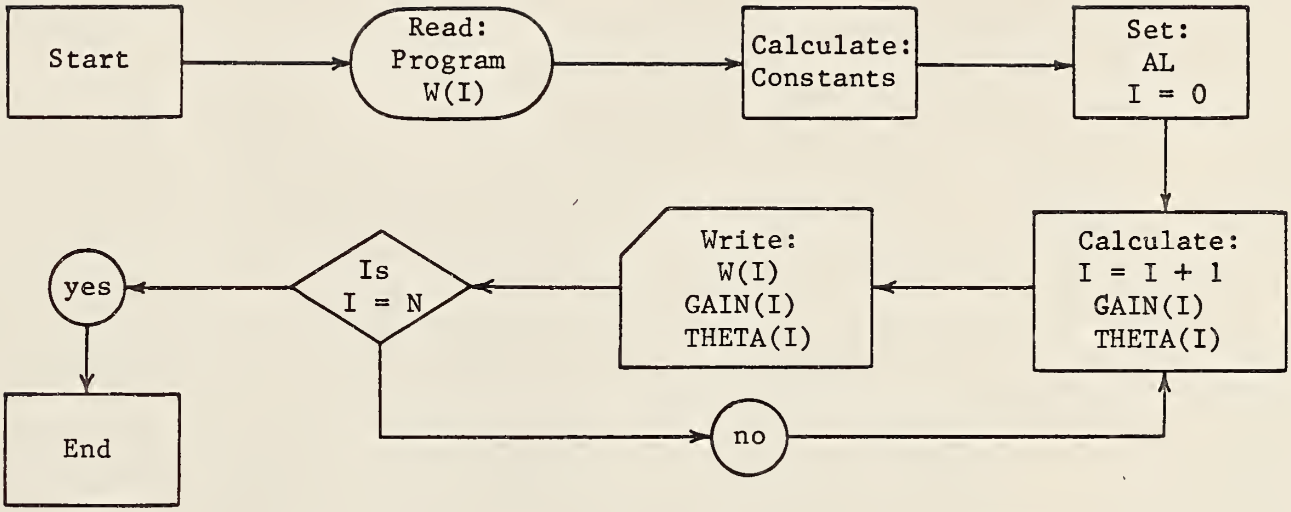


Figure 18. Logic diagram for theoretical transfer function gain and phase shift computer program

```
  THIS PROGRAM EVALUATES THE ZERO POWER REACTOR TRANSFER FUNCTION
  FOR VARIOUS INPUT REACTIVITY OSCILLATION FREQUENCIES
  DIMENSION W(50), GAIN(50), THETA(50)
1 FORMAT(F10.5)
2 FORMAT(1HK,15X,16H FREQUENCY (CPS),10X,9HGAIN (DB),10X,17HPHASE SH
 1IFT (DEG))
3 FORMAT(1H ,18X,F10.5,11X,F11.7,12X,F11.7)
4 FORMAT(1HL,11X,37H ZERO POWER REACTOR TRANSFER FUNCTION)
5 FORMAT(1HK,11X,26H PROMPT NEUTRON LIFETIME =,F10.8,8H SECONDS)
6 FORMAT(1H )
7 FORMAT(F5.1,I3)
8 FORMAT(36HZERO POWER REACTOR TRANSFER FUNCTION)
9 FORMAT(1HK,12X,10HRUN NUMBER,F5.1)
10 FORMAT(10HRUN NUMBER,F5.1)
11 FORMAT(25HPROMPT NEUTRON LIFETIME =,F14.12,8H SECONDS)
12 FORMAT(4X,15HFREQUENCY (CPS),10X,9HGAIN (DB),10X,17HPHASE SHIFT (D
 1EG))
13 FORMAT(7X,F10.5,11X,F11.7,12X,F11.7)
  READ(1,7)RUNNO,N
  READ(1,1)(W(I),I=1,N)
  A1=.0124
  A2=.0305
  A3=.111
  A4=.301
  A5=1.14
  A6=3.01
  B1=.00021
  B2=.00140
  B3=.00125
  B4=.00253
  B5=.00074
  B6=.00027
  B   =A1*(A2+A3+A4+A5+A6)+A2*(A3+A4+A5+A6)+A3*(A4+A5+A6)+A4*(A5+A6)+
 1A5*A6
  C   =-A1*A2*(A3*A4+A3*A5+A4*A5+A3*A6+A4*A6+A5*A6)-A3*(A1+A2)*(A4*A5
 1+A4*A6+A5*A6)-A4*A5*A6*(A1+A2+A3)
  D   =A1*A2*A3*A4*A5*A6
  E   =A1+A2+A3+A4+A5+A6
  F   =-A1*A2*(A3+A4+A5+A6)-A3*(A1+A2)*(A4+A5+A6)-A5*A6*(A1+A2+13+A4)
 1-A4*(A5+A6)*(A1+A2+A3)
  G   =A2*A3*A4*A5*A6+A1*A3*A4*A5*A6+A1*A2*A4*A5*A6+A1*A2*A3*A5*A6+A1
 1*A2*A3*A4*A6+A1*A2*A3*A4*A5
  D11=A6+A2+A3+A4+A5
  D12=-A6*A2*(A3+A4+A5)-A3*A4*(A6+A2+A5)-A5*(A3+A4)*(A6+A2)
  D13=A6*A2*A3*A4*A5
  D14=-A6*(A2+A3+A4+A5)-A2*(A3+A4+A5)-A3*(A4+A5)-A4*A5
  D15=A2*A3*A4*A5+A6*A3*A4*A5+A6*A2*A4*A5+A6*A2*A3*A5+A6*A2*A3*A4
  D21=A1+A6+A3+A4+A5
  D22=-A1*A6*(A3+A4+A5)-A3*A4*(A1+A6+A5)-A5*(A3+A4)*(A1+A6)
  D23=A1*A6*A3*A4*A5
```

```
D24=-A1*(A6+A3+A4+A5)-A6*(A3+A4+A5)-A3*(A4+A5)-A4*A5
D25=A6*A3*A4*A5+A1*A3*A4*A5+A1*A6*A4*A5+A1*A6*A3*A5+A1*A6*A3*A4
D31=A1+A2+A6+A4+A5
D32=-A1*A2*(A6+A4+A5)-A6*A4*(A1+A2+A5)-A5*(A6+A4)*(A1+A2)
D33=A1*A2*A6*A4*A5
D34=-A1*(A2+A6+A4+A5)-A2*(A6+A4+A5)-A6*(A4+A5)-A4*A5
D35=A2*A6*A4*A5+A1*A6*A4*A5+A1*A2*A4*A5+A1*A2*A6*A5+A1*A2*A6*A4
D41=A1+A2+A3+A6+A5
D42=-A1*A2*(A3+A6+A5)-A3*A6*(A1+A2+A5)-A5*(A3+A6)*(A1+A2)
D43=A1*A2*A3*A6*A5
D44=-A1*(A2+A3+A6+A5)-A2*(A3+A6+A5)-A3*(A6+A5)-A6*A5
D45=A2*A3*A6*A5+A1*A3*A6*A5+A1*A2*A6*A5+A1*A2*A3*A5+A1*A2*A3*A6
D51=A1+A2+A3+A4+A6
D52=-A1*A2*(A3+A4+A6)-A3*A4*(A1+A2+A6)-A6*(A3+A4)*(A1+A2)
D53=A1*A2*A3*A4*A6
D54=-A1*(A2+A3+A4+A6)-A2*(A3+A4+A6)-A3*(A4+A6)-A4*A6
D55=A2*A3*A4*A6+A1*A3*A4*A6+A1*A2*A3*A6+A1*A2*A4*A6+A1*A2*A3*A4
D61=A1+A2+A3+A4+A5
D62=-A1*A2*(A3+A4+A5)-A3*A4*(A1+A2+A5)-A5*(A3+A4)*(A1+A2)
D63=A1*A2*A3*A4*A5
D64=-A1*(A2+A3+A4+A5)-A2*(A3+A4+A5)-A3*(A4+A5)-A4*A5
D65=A2*A3*A4*A5+A1*A3*A4*A5+A1*A2*A4*A5+A1*A2*A3*A5+A1*A2*A3*A4
S1=B1*D11+B2*D21+B3*D31+B4*D41+B5*D51+B6*D61
S2=B1*D12+B2*D22+B3*D32+B4*D42+B5*D52+B6*D62
S3=B1*D13+B2*D23+B3*D33+B4*D43+B5*D53+B6*D63
S4=B1*D14+B2*D24+B3*D34+B4*D44+B5*D54+B6*D64
S5=B1*D15+B2*D25+B3*D35+B4*D45+B5*D55+B6*D65
S6=B1+B2+B3+B4+B5+B6
T1=S6-1.
T2=S4+B
T3=S5+C
T4=E-S1
T5=F-S2
T6=G-S3
AL=.00005703
T7=S6+AL*E
T8=S4+AL*F
T9=S5+AL*G
T10=S1+AL*B
T11=S2+AL*C
T12=S3+AL*D
R1=T1*T7
R2=T2*T7+T1*T8
R3=T3*T7+T2*T8+T1*T9
R4=  D*T7+T3*T8+T2*T9
R5=  D*T8+T3*T9
R6=  D*T9
BL1=-AL*T4
BL2=T4*T10-AL*T5
BL3=T4*T11+T5*T10-AL*T6
```

```
BL4=T4*T12+T5*T11+T6*T10
BL5=T5*T12+T6*T11
BL6=T6*T12
V1=-AL*T1
V2=T1*T10-AL*T2
V3=T1*T11+T2*T10-AL*T3
V4=T1*T12+T2*T11+T3*T10-AL*D
V5=T2*T12+T3*T11+T10*D
V6=T3*T12+  D*T11
V7=  D*T12
X1=T4*T7
X2=T4*T8+T5*T7
X3=T4*T9+T5*T8+T6*T7
X4=T5*T9+T6*T8
X5=T6*T9
Y1=T7*T7
Y2=2.*T7*T8
Y3=2.*T7*T9+T8*T8
Y4=2.*T8*T9
Y5=T9*T9
Z1=AL*AL
Z2=-2.*AL*T10
Z3=T10*T10-2.*AL*T11
Z4=2.*(T10*T11-AL*T12)
Z5=T11*T11+2.*T10*T12
Z6=2.*T11*T12
Z7=T12*T12
WRITE(3,4)
WRITE(2,8)
WRITE(3,9)RUNNO
WRITE(2,10)RUNNO
WRITE(3,5)AL
WRITE(2,11)AL
WRITE(3,2)
WRITE(2,12)
WRITE(3,6)
DO 100 I=1,N
FREQ=6.28318531*W(I)
E7=FREQ
E6=E7*E7
E5=E6*E6
E4=E5*E6
E3=E5*E5
E2=E5*E4
E1=E4*E4
BA1=-E6*(E2*R1+E3*R2+E4*R3+E5*R4+E6*R5+R6)
BA2=E6*(E2*BL1+E3*BL2+E4*BL3+E5*BL4+E6*BL5+BL6)
BA3=E7*(E1*V1+E2*V2+E3*V3+E4*V4+E5*V5+E6*V6+V7)
BA4=E7*E6*(E3*X1+E4*X2+E5*X3+E6*X4+X5)
BA5=E5*(E3*Y1+E4*Y2+E5*Y3+E6*Y4+Y5)
```

```
      BA6=E6*(E1*Z1+E2*Z2+E3*Z3+E4*Z4+E5*Z5+E6*Z6+Z7)
      GAIN1=SQRT((((BA1+BA2)**2)+((BA3+BA4)**2))/((BA5+BA6)**2))
      GAIN(I)=((20.)*(ALOG(GAIN1)))/(2.3025850930)
      THETA(I)=(ATAN(-((BA3+BA4)/(BA1+BA2))))*(57.2957795131)
      WRITE(3,3)W(I),GAIN(I),THETA(I)
  100 WRITE(2,13)W(I),GAIN(I),THETA(I)
      END
```

ZERO POWER REACTOR TRANSFER FUNCTION

RUN NUMBER 50.0

PROMPT NEUTRON LIFETIME = .00005703000 SECONDS

| FREQUENCY (CPS) | GAIN (DB) | PHASE SHIFT (DEG) |
|---|---|---|
| .01000 | 52.2923233 | -44.0138468 |
| .01500 | 50.7435177 | -39.8037878 |
| .02000 | 49.6869326 | -36.9439301 |
| .03000 | 48.2890982 | -32.6800969 |
| .04000 | 47.3954026 | -29.3532110 |
| .05000 | 46.7830702 | -26.6100626 |
| .06000 | 46.3458665 | -24.3310519 |
| .08000 | 45.7750210 | -20.8566969 |
| .10000 | 45.4212175 | -18.4047771 |
| .15000 | 44.9214742 | -14.6581695 |
| .20000 | 44.6446553 | -12.5065025 |
| .30000 | 44.3390186 | -10.0593480 |
| .40000 | 44.1745995 | -8.7137796 |
| .50000 | 44.0728455 | -7.8956470 |
| .60000 | 44.0045001 | -7.3785585 |
| .80000 | 43.9202313 | -6.8598790 |
| 1.00000 | 43.8711314 | -6.7340244 |
| 1.50000 | 43.8044217 | -7.2202878 |
| 2.00000 | 43.7617036 | -8.2167242 |
| 3.00000 | 43.6809988 | -10.7459329 |
| 4.00000 | 43.5847223 | -13.5159873 |
| 5.00000 | 43.4679358 | -16.3359688 |
| 6.00000 | 43.3308139 | -19.1321694 |
| 8.00000 | 43.0018590 | -24.5203709 |
| 10.00000 | 42.6133547 | -29.5312280 |
| 15.00000 | 41.4844377 | -40.1727246 |
| 20.00000 | 40.2774508 | -48.3184580 |
| 30.00000 | 37.9903286 | -59.2658704 |
| 40.00000 | 36.0225968 | -65.9546739 |
| 50.00000 | 34.3540589 | -70.3516028 |
| 60.00000 | 32.9242541 | -73.4284234 |
| 80.00000 | 30.5840263 | -77.4171854 |
| 100.00000 | 28.7212176 | -79.8747920 |

ZERO POWER REACTOR TRANSFER FUNCTION

RUN NUMBER 60.0

PROMPT NEUTRON LIFETIME = .000056590000 SECONDS

| FREQUENCY (CPS) | GAIN (DB) | PHASE SHIFT (DEG) |
|---|---|---|
| .01000 | 52.2923920 | -44.0133762 |
| .01500 | 50.7435972 | -39.8031566 |
| .02000 | 49.6870207 | -36.9431546 |
| .03000 | 48.2891993 | -32.6790535 |
| .04000 | 47.3955129 | -29.3519108 |
| .05000 | 46.7831876 | -26.6085085 |
| .06000 | 46.3459898 | -24.3292443 |
| .08000 | 45.7751540 | -20.8543819 |
| .10000 | 45.4213590 | -18.4019557 |
| .15000 | 44.9216349 | -14.6540949 |
| .20000 | 44.6448328 | -12.5011913 |
| .30000 | 44.3392260 | -10.0515896 |
| .40000 | 44.1748349 | -8.7035886 |
| .50000 | 44.0731093 | -7.8830297 |
| .60000 | 44.0047937 | -7.3635175 |
| .80000 | 43.9205920 | -6.8399944 |
| 1.00000 | 43.8715716 | -6.7093014 |
| 1.50000 | 43.8051245 | -7.1835231 |
| 2.00000 | 43.7627642 | -8.1680573 |
| 3.00000 | 43.6830560 | -10.6741302 |
| 4.00000 | 43.5881235 | -13.4222698 |
| 5.00000 | 43.4729852 | -16.2218714 |
| 6.00000 | 43.3377657 | -18.9994555 |
| 8.00000 | 43.0131660 | -24.3562157 |
| 10.00000 | 42.6294156 | -29.3434746 |
| 15.00000 | 41.5121628 | -39.9552513 |
| 20.00000 | 40.3147375 | -48.0986029 |
| 30.00000 | 38.0398665 | -59.0708245 |
| 40.00000 | 36.0785879 | -65.7892387 |
| 50.00000 | 34.4136459 | -70.2107177 |
| 60.00000 | 32.9859959 | -73.3067560 |
| 80.00000 | 30.6480717 | -77.3225168 |
| 100.00000 | 28.7863887 | -79.7977209 |

# APPENDIX H

## Description of Digital Computer Program used for Least Squares and Statistical Analysis of Transfer Function Gain Data

This computer code, TRANSFUNC2, was written to perform the iterative least squares and statistical analyses indicated in Appendix E. The program was written in FORTRAN IV language. A listing of the source deck, and a logic diagram are included in this appendix.

The input to this program was arranged as follows:

FIRST CARD: FORMAT(F5.1,I5,F5.1)

1) RUNNO: Run number, 2) K, and 3) BK: Number of data points.

NEXT K CARDS: FORMAT(F10.5)

W: Frequencies at which the gain was experimentally measured, punched one to a card.

NEXT K CARDS: FORMAT(F9.5)

A: Experimentally measured gain data (db), punched one to a card.

The delayed neutron parameters $\beta_i$ and $\lambda_i$ were incorporated into the program. The iteration accuracy criterion was also included in the program and was not entered as a parameter.

Two types of output were obtained; on-line printer and punched cards. The first punched card contains the statement, "Least squares analysis of gain data". "Data from run number ___", is punched on the second, while the third contains headings for the three columns of output, Prompt Neutron Lifetime, $\Delta\ell$, and ADJUST, which follow on the next few cards. The next three cards contain the final value for the prompt neutron lifetime, an estimate of its standard deviation, and the standard deviation of ADJUST. Next comes a heading card for the three columns of output, Frequency (cps), Gain (db), and an estimate of the standard deviation of the function, which follow in the next K cards. The last two cards contain the ratio of $\beta/\ell$ (cps) and its estimated standard deviation. The on-line printer output is identical to the punched output. A print-out of the results obtained from analysis of the analog computer and strip chart recorder determined gain data is included at the end of this appendix.

Table VI: Input data and variables required for the least squares analysis of gain data computer program

| Symbol | Explanation |
|---|---|
| A1 - A6 | Delayed neutron decay constants |
| ADJUST | Relative scale factor relating the theoretical gain to the experimental gain |
| AL | Prompt neutron lifetime, $\ell$ |
| ALPHA | Ratio of $\beta/\ell$ |
| AT | Theoretical gain |
| B1 - B6 | Delayed neutron fractions |
| DELTAL | Calculated correction for $\ell$, $\Delta\ell$ |
| K | Number of data cards to be read |
| PG | Partial derivative of the theoretical gain with respect to the prompt neutron lifetime |
| RUNNO | Run number, for data identification |
| SIGALP | Estimated standard deviation of $\beta/\ell$ ratio |
| SIGMAA | Estimated standard deviation of ADJUST |
| SIGMAL | Estimated standard deviation of $\ell$ |
| SIGMAY | Estimated standard deviation of least squares function |

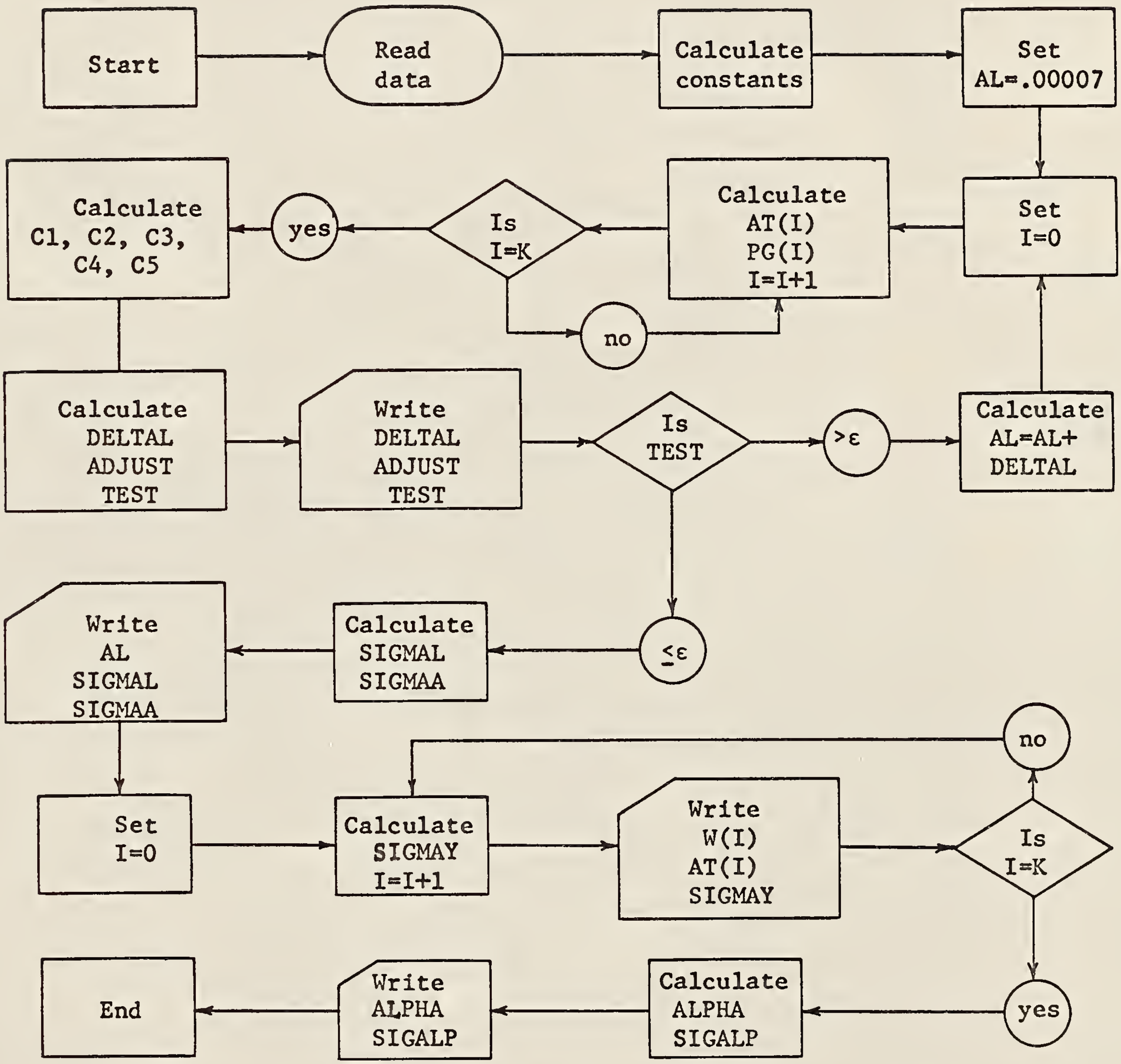


Figure 19. Logic diagram for least squares analysis of transfer function gain and phase shift computer programs

```
C     THIS PROGRAM PERFORMS A LEAST SQUARES ANALYSIS ON EXPERIMENTILLY
C     MEASURED GAIN DATA TO DETERMINE THE BEST VALUE OF THE PROMPT
C     NEUTRON LIFETIME
      DIMENSION W(38),A(38),AT(38),PG(38)
    1 FORMAT(F10.5)
    2 FORMAT(F9.5)
    3 FORMAT(1HL,10X,23HPROMPT NEUTRON LIFETIME,13X,5HDELTA,22X,6HADJUST
     1)
    4 FORMAT(1HK,15X,F14.12,10X,E18.12,10X,E18.12)
    5 FORMAT(1HL,10X,53HLEAST SQUARES BEST VALUE OF PROMPT NEUTRON LIFET
     1IME =,F14.12)
    6 FORMAT(11X,35HLEAST SQUARES ANALYSIS OF GAIN DATA)
    7 FORMAT(F5.1,I5,F5.1)
    8 FORMAT(35HLEAST SQUARES ANALYSIS OF GAIN DATA)
    9 FORMAT(1HK,10X,20HDATA FROM RUN NUMBER,F5.1)
   10 FORMAT(20HDATA FROM RUN NUMBER,F5.1)
   11 FORMAT(23HPROMPT NEUTRON LIFETIME,13X,5HDELTA,22X,6HADJUST)
   12 FORMAT(5X,F14.12,10X,E18.12,10X,E18.12)
   13 FORMAT(53HLEAST SQUARES BEST VALUE OF PROMPT NEUTRON LIFETIME =,F1
     14.12)
   14 FORMAT(1HL,10X,34HSIGMA OF PROMPT NEUTRON LIFETIME =,E18.12)
   15 FORMAT(1HL,10X,17HSIGMA OF ADJUST =,E18.12)
   16 FORMAT(34HSIGMA OF PROMPT NEUTRON LIFETIME =,E18.12)
   17 FORMAT(17HSIGMA OF ADJUST =,E18.12)
   18 FORMAT(1HL,10X,15HFREQUENCY (CPS),10X,9HGAIN (DB),13X,10HSIGMA (DB
     1))
   19 FORMAT(2X,15HFREQUENCY (CPS),10X,9HGAIN (DB),13X,10HSIGMA (DB))
   20 FORMAT(13X,F10.5,11X,F11.7,11X,F11.7)
   21 FORMAT(F10.5,11X,F11.7,11X,F11.7)
   22 FORMAT(1H1)
   23 FORMAT(1HL,10X,20HRATIO OF BETA TO L =,F10.6)
   24 FORMAT(20HRATIO OF BETA TO L =,F10.6)
   25 FORMAT(1HK,10X,26HSIGMA OF BETA TO L RATIO =,F10.6)
   26 FORMAT(26HSIGMA OF BETA TO L RATIO =,F10.6)
  100 READ(1,7)RUNNO,K,BK
      READ(1,1)(W(I),I=1,K)
      READ(1,2)(A(I),I=1,K)
      A1=.0124
      A2=.0305
      A3=.111
      A4=.301
      A5=1.14
      A6=3.01
      B1=.00021
      B2=.00140
      B3=.00125
      B4=.00253
      B5=.00074
      B6=.00027
      B  =A1*(A2+A3+A4+A5+A6)+A2*(A3+A4+A5+A6)+A3*(A4+A5+A6)+A4*(A5+A6)+
```

```
1A5*A6
 C=-A1*A2*(A3*A4+A3*A5+A4*A5+A3*A6+A4*A6+A5*A6)-A3*(A1+A2)*(A4*A5
1A4*A6+A5*A6)-A4*A5*A6*(A1+A2+A3)
 D=A1*A2*A3*A4*A5*A6
 E=A1+A2+A3+A4+A5+A6
 F=-A1*A2*(A3+A4+A5+A6)-A3*(A1+A2)*(A4+A5+A6)-A5*A6*(A1+A2+A3 A4)
1-A4*(A5+A6)*(A1+A2+A3)
 G   =A2*A3*A4*A5*A6+A1*A3*A4*A5*A6+A1*A2*A4*A5*A6+A1*A2*A3*A5*A6+A1
1*A2*A3*A4*A6+A1*A2*A3*A4*A5
 D11=A6+A2+A3+A4+A5
 D12=-(((A6)*(A2))*(A3+A4+A5))-(((A3)*(A4))*(A6+A2+A5))-(((A5)*(A3+
1A4))*(A6+A2))
 D13=A6*A2*A3*A4*A5
 D14=-((A6)*(A2+A3+A4+A5))-((A2)*(A3+A4+A5))-((A3)*(A4+A5))-((A4)*(
1A5))
 D15=(A2*A3*A4*A5)+(A6*A3*A4*A5)+(A6*A2*A4*A5)+(A6*A2*A3*A5)+(A6*A2
1*A3*A4)
 D21=A1+A6+A3+A4+A5
 D22=-(((A1)*(A6))*(A3+A4+A5))-(((A3)*(A4))*(A1+A6+A5))-(((A5)*(A3+
1A4))*(A1+A6))
 D23=A1*A6*A3*A4*A5
 D24=-((A1)*(A6+A3+A4+A5))-((A6)*(A3+A4+A5))-((A3)*(A4+A5))-((A4)*(
1A5))
 D25=(A6*A3*A4*A5)+(A1*A3*A4*A5)+(A1*A6*A4*A5)+(A1*A6*A3*A5)+(A1*A6
1*A3*A4)
 D31=A1+A2+A6+A4+A5
 D32=-(((A1)*(A2))*(A6+A4+A5))-(((A6)*(A4))*(A1+A2+A5))-(((A5)*(A6+
1A4))*(A1+A2))
 D33=A1*A2*A6*A4*A5
 D34=-((A1)*(A2+A6+A4+A5))-((A2)*(A6+A4+A5))-((A6)*(A4+A5))-((A4)*(
1A5))
 D35=(A2*A6*A4*A5)+(A1*A6*A4*A5)+(A1*A2*A4*A5)+(A1*A2*A6*A5)+(A1*A2
1*A6*A4)
 D41=A1+A2+A3+A6+A5
 D42=-(((A1)*(A2))*(A3+A6+A5))-(((A3)*(A6))*(A1+A2+A5))-(((A5)*(A3+
1A6))*(A1+A2))
 D43=A1*A2*A3*A6*A5
 D44=-((A1)*(A2+A3+A6+A5))-((A2)*(A3+A6+A5))-((A3)*(A6+A5))-((A6)*(
1A5))
 D45=(A2*A3*A6*A5)+(A1*A3*A6*A5)+(A1*A2*A6*A5)+(A1*A2*A3*A5)+(A1*A2
1*A3*A6)
 D51=A1+A2+A3+A4+A6
 D52=-(((A1)*(A2))*(A3+A4+A6))-(((A3)*(A4))*(A1+A2+A6))-(((A6)*(A3+
1A4))*(A1+A2))
 D53=A1*A2*A3*A4*A6
 D54=-((A1)*(A2+A3+A4+A6))-((A2)*(A3+A4+A6))-((A3)*(A4+A6))-((A4)*(
1A6))
 D55=(A2*A3*A4*A6)+(A1*A3*A4*A6)+(A1*A2*A4*A6)+(A1*A2*A3*A6)+(A1*A2
1*A3*A4)
 D61=A1+A2+A3+A4+A5
```

```
 D62=-(((A1)*(A2))*(A3+A4+A5))-(((A3)*(A4))*(A1+A2+A5))-(((A5)*(A3+
1A4))*(A1+A2))
 D63=A1*A2*A3*A4*A5
 D64=-((A1)*(A2+A3+A4+A5))-((A2)*(A3+A4+A5))-((A3)*(A4+A5))-((A4)*(
1A5))
 D65=(A2*A3*A4*A5)+(A1*A3*A4*A5)+(A1*A2*A4*A5)+(A1*A2*A3*A5)+(A1*A2
1*A3*A4)
 S1=B1*D11+B2*D21+B3*D31+B4*D41+B5*D51+B6*D61
 S2=B1*D12+B2*D22+B3*D32+B4*D42+B5*D52+B6*D62
 S3=B1*D13+B2*D23+B3*D33+B4*D43+B5*D53+B6*D63
 S4=B1*D14+B2*D24+B3*D34+B4*D44+B5*D54+B6*D64
 S5=B1*D15+B2*D25+B3*D35+B4*D45+B5*D55+B6*D65
 S6=B1+B2+B3+B4+B5+B6
 T1=S6-1.
 T2=S4+B
 T3=S5+C
 T4=E-S1
 T5=F-S2
 T6=G-S3
 PR1=T1*E
 PR2=T1*F+T2*E
 PR3=T1*G+T2*F+T3*E
 PR4=T2*G+T3*F+D*E
 PR5=T3*G+D*F
 PR6=D*G
 PL1=-T4
 PL2=T4*B-T5
 PL3=T4*C+T5*B-T6
 PL4=T4*D+T5*C+T6*B
 PL5=T5*D+T6*C
 PL6=T6*D
 PV1=-T1
 PV2=T1*B-T2
 PV3=T1*C+T2*B-T3
 PV4=T1*D+T2*C+T3*B-D
 PV5=T2*D+T3*C+D*B
 PV6=T3*D+D*C
 PV7=D*D
 PX1=T4*E
 PX2=T4*F+T5*E
 PX3=T4*G+T5*F+T6*E
 PX4=T5*G+T6*F
 PX5=T6*G
 WRITE(3,6)
 WRITE(3,9)RUNNO
 WRITE(3,3)
 WRITE(2,8)
 WRITE(2,10)RUNNO
 WRITE(2,11)
 AL=.00007
```

```
200 T7=S6+AL*E
    T8=S4+AL*F
    T9=S5+AL*G
    T10=S1+AL*B
    T11=S2+AL*C
    T12=S3+AL*D
    R1=T1*T7
    R2=(T2*T7)+(T1*T8)
    R3=(T3*T7)+(T2*T8)+(T1*T9)
    R4=T2*T9+T3*T8+D*T7
    R5=T3*T9+D*T8
    R6=D*T9
    BL1=-AL*T4
    BL2=(T4*T10)-(AL*T5)
    BL3=(T4*T11)+(T5*T10)+(-(AL*T6))
    BL4=(T4*T12)+(T5*T11)+(T6*T10)
    BL5=(T5*T12)+(T6*T11)
    BL6=T6*T12
    V1=-(AL*T1)
    V2=(T1*T10)-(AL*T2)
    V3=(T1*T11)+(T2*T10)-(AL*T3)
    V4=T1*T12+T2*T11+T3*T10-AL*D
    V5=T2*T12+T3*T11+D*T10
    V6=T3*T12+D*T11
    V7=D*T12
    X1=T4*T7
    X2=(T4*T8)+(T5*T7)
    X3=(T4*T9)+(T5*T8)+(T6*T7)
    X4=(T5*T9)+(T6*T8)
    X5=T6*T9
    Y1=T7*T7
    Y2=2.*T7*T8
    Y3=2.*T7*T9+T8*T8
    Y4=2.*T8*T9
    Y5=T9*T9
    Z1=AL*AL
    Z2=-2.*AL*T10
    Z3=T10*T10-2.*AL*T11
    Z4=2.*(T10*T11-AL*T12)
    Z5=T11*T11+2.*T10*T12
    Z6=2.*T11*T12
    Z7=T12*T12
    PY1=E*T7
    PY2=F*T7+E*T8
    PY3=G*T7+E*T9+F*T8
    PY4=G*T8+F*T9
    PY5=G*T9
    PZ1=AL
    PZ2=-AL*B-T10
    PZ3=B*T10-AL*C-T11
```

```
      PZ4=C*T10+B*T11-AL*D-T12
      PZ5=C*T11+D*T10+B*T12
      PZ6=D*T11+C*T12
      PZ7=D*T12
      DO 210 I=1,K
      E1=6.28318531*W(I)
      E2=E1*E1
      E3=E2*E1
      E4=E2*E2
      E6=E3*E3
      E8=E4*E4
      E10=E6*E4
      E12=E6*E6
      BA1=-E2*(E10*R1+E8*R2+E6*R3+E4*R4+E2*R5+R6)
      BA2=E2*(E10*BL1+E8*BL2+E6*BL3+E4*BL4+E2*BL5+BL6)
      BA3=E1*(E12*V1+E10*V2+E8*V3+E6*V4+E4*V5+E2*V6+V7)
      BA4=E3*(E8*X1+E6*X2+E4*X3+E2*X4+X5)
      BA5=E4*(E8*Y1+E6*Y2+E4*Y3+E2*Y4+Y5)
      BA6=E2*(E12*Z1+E10*Z2+E8*Z3+E6*Z4+E4*Z5+E2*Z6+Z7)
      PB1 =-E2*(E10*PR1+E8*PR2+E6*PR3+E4*PR4+E2*PR5+PR6)
      PB2 =E2*(E10*PL1+E8*PL2+E6*PL3+E4*PL4+E2*PL5+PL6)
      PB3 =E1*(E12*PV1+E10*PV2+E8*PV3+E6*PV4+E4*PV5+E2*PV6+PV7)
      PB4 =E3*(E8*PX1+E6*PX2+E4*PX3+E2*PX4+PX5)
      PB5 =2.*E4*(E8*PY1+E6*PY2+E4*PY3+E2*PY4+PY5)
      PB6 =2.*E2*(E12*PZ1+E10*PZ2+E8*PZ3+E6*PZ4+E4*PZ5+E2*PZ6+PZ7)
      AWI=SQRT((((BA1+BA2)*(BA1+BA2))+((BA3+BA4)*(BA3+BA4)))/((BA5+BA6)*
     1(BA5+BA6)))
      AT(I)=((20.)*(ALOG(AWI)))/(2.302585093)
      TE1=(BA5+BA6)*((BA1+BA2)*(PB1 +PB2 )+(BA3+BA4)*(PB3 +PB4 ))
      TE2=(PB5 +PB6 )*((BA1+BA2)*(BA1+BA2)+(BA3+BA4)*(BA3+BA4))
      TE3=(BA5+BA6)*(BA5+BA6)*(BA5+BA6)
  210 PG(I)=(8.6858896*(TE1-TE2))/(TE3*AWI*AWI)
      C1=0.
      C2=0.
      C3=0.
      C4=0.
      C5=0.
      DO 220 I=1,K
      C1=C1+(PG(I)*PG(I))/(AT(I))
      C2=C2+(PG(I))/(AT(I))
      C3=C3+(1.)/(AT(I))
      C4=C4+(PG(I)*(A(I)-AT(I)))/(AT(I))
  220 C5=C5+(A(I)-AT(I))/(AT(I))
      DELTAL=(C3*C4-C2*C5)/(C1*C3-C2*C2)
      ADJUST=(C4-DELTAL*C1)/(C2)
      WRITE(3,4)AL,DELTAL,ADJUST
      WRITE(2,12)AL,DELTAL,ADJUST
      TEST=(DELTAL)/(AL)
      IF(TEST.LT.0.) GO TO 222
  221 IF(TEST.LE..0001) GO TO 230
```

```
      AL=AL+DELTAL
      GO TO 200
  222 TEST=-TEST
      GO TO 221
  230 C11=(C3)/(C3*C1-C2*C2)
      C22=(C1)/(C3*C1-C2*C2)
      C12=-(C2)/(C3*C1-C2*C2)
      SUM=0.
      DO 240 I=1,K
  240 SUM=SUM+((A(I)-AT(I)-ADJUST)*(A(I)-AT(I)-ADJUST))/(AT(I))
      VARI=(SUM)/(BK-2.)
      WRITE(3,5)AL
      WRITE(2,13)AL
      SIGMAL=SQRT(C11*VARI)
      SIGMAA=SQRT(C22*VARI)
      WRITE(3,14)SIGMAL
      WRITE(3,15)SIGMAA
      WRITE(2,16)SIGMAL
      WRITE(2,17)SIGMAA
      WRITE(3,18)
      WRITE(2,19)
      DO 250 I=1,K
      VARIY=VARI*(C11*PG(I)*PG(I)+2.*C12*PG(I)+C22)
      SIGMAY=2.028*SQRT(VARIY)
      AT(I)=AT(I)+ADJUST
      WRITE(2,21)W(I),AT(I),SIGMAY
  250 WRITE(3,20)W(I),AT(I),SIGMAY
      BE=B1+B2+B3+B4+B5+B6
      ALPHA=BE/(AL*6.2831853)
      SIGALP=(BE*SIGMAL)/(AL*AL*6.2831853)
      WRITE(3,23)ALPHA
      WRITE(2,24)ALPHA
      WRITE(3,25)SIGALP
      WRITE(2,26)SIGALP
      WRITE(3,22)
      GO TO 100
      END
```

LEAST SQUARES ANALYSIS OF GAIN DATA
DATA FROM RUN NUMBER 10.0

| PROMPT NEUTRON LIFETIME | DELTA | ADJUST |
|---|---|---|
| .000070000000 | -.139018640000E-04 | -.548389660000E 02 |
| .000056098136 | .946019140000E-06 | -.548249430000E 02 |
| .000057044155 | -.143831800000E-07 | -.548253550000E 02 |
| .000057029772 | .238350120000E-09 | -.548253610000E 02 |

LEAST SQUARES BEST VALUE OF PROMPT NEUTRON LIFETIME = .000057029772
SIGMA OF PROMPT NEUTRON LIFETIME = .117156460000E-05
SIGMA OF ADJUST = .560141180000E-01

| FREQUENCY (CPS) | GAIN (DB) | SIGMA (DB) |
|---|---|---|
| .01010 | -2.5718990 | .1134022 |
| .01500 | -4.0818450 | .1133726 |
| .01960 | -5.0656780 | .1133502 |
| .03050 | -6.5902260 | .1133105 |
| .04030 | -7.4517270 | .1132852 |
| .05440 | -8.2519060 | .1132584 |
| .07060 | -8.8196200 | .1132345 |
| .08500 | -9.1533880 | .1132161 |
| .10000 | -9.4041460 | .1131984 |
| .15000 | -9.9038880 | .1131449 |
| .19800 | -10.1719260 | .1130994 |
| .30000 | -10.4863430 | .1130141 |
| .40000 | -10.6507620 | .1129360 |
| .53700 | -10.7807810 | .1128269 |
| .29800 | -10.4820340 | .1130157 |
| .39000 | -10.6378590 | .1129438 |
| .51700 | -10.7660020 | .1128433 |
| .68600 | -10.8634230 | .1126976 |
| .86000 | -10.9223920 | .1125258 |
| .98600 | -10.9514680 | .1123849 |
| 1.52400 | -11.0232560 | .1116118 |
| 2.07700 | -11.0697090 | .1105314 |
| 2.98300 | -11.1428920 | .1082291 |
| 4.03300 | -11.2441600 | .1050257 |
| 5.39700 | -11.4094890 | .1008068 |
| 6.93300 | -11.6395330 | .0974389 |
| 3.05700 | -11.1493370 | .1080176 |
| 3.87700 | -11.2277140 | .1055209 |
| 6.92300 | -11.6378970 | .0974526 |
| 8.30000 | -11.8783820 | .0969475 |
| 9.78800 | -12.1684920 | .0998851 |
| 14.95000 | -13.3289850 | .1320075 |
| 20.10700 | -14.5736530 | .1724638 |
| 21.27500 | -14.8537370 | .1808029 |
| 24.95400 | -15.7164990 | .2040922 |
| 29.54200 | -16.7369330 | .2270428 |
| 36.10800 | -18.0755520 | .2505628 |
| 40.30000 | -18.8568490 | .2614284 |

RATIO OF BETA TO L = 17.860700
SIGMA OF BETA TO L RATIO = .366912

LEAST SQUARES ANALYSIS OF GAIN DATA
DATA FROM RUN NUMBER 20.0

| PROMPT NEUTRON LIFETIME | DELTA | ADJUST |
|---|---|---|
| .00007000000 | -.144542710000E-04 | -.549672240000E 02 |
| .000055545729 | .106882550000E-05 | -.549508300000E 02 |
| .000056614554 | -.207022870000E-07 | -.549514130000E 02 |
| .000056593852 | .473969730000E-09 | -.549514060000E 02 |

LEAST SQUARES BEST VALUE OF PROMPT NEUTRON LIFETIME = .000056593852
SIGMA OF PROMPT NEUTRON LIFETIME = .117167450000E-05
SIGMA OF ADJUST = .561612670000E-01

| FREQUENCY (CPS) | GAIN (DB) | SIGMA (DB) |
|---|---|---|
| .01010 | -2.6978740 | .1137009 |
| .01500 | -4.2078120 | .1136714 |
| .01960 | -5.1916370 | .1136490 |
| .03050 | -6.7161710 | .1136094 |
| .04030 | -7.5776640 | .1135841 |
| .05440 | -8.3778340 | .1135574 |
| .07060 | -8.9455350 | .1135336 |
| .08500 | -9.2792990 | .1135152 |
| .10000 | -9.5300520 | .1134975 |
| .15000 | -10.0297730 | .1134441 |
| .19800 | -10.2977980 | .1133988 |
| .30000 | -10.6121830 | .1133139 |
| .40000 | -10.7765770 | .1132362 |
| .53700 | -10.9065530 | .1131278 |
| .29800 | -10.6078750 | .1133155 |
| .39000 | -10.7636740 | .1132440 |
| .51700 | -10.8917780 | .1131441 |
| .68600 | -10.9891470 | .1129995 |
| .86000 | -11.0480590 | .1128291 |
| .98600 | -11.0770830 | .1126894 |
| 1.52400 | -11.1485890 | .1119230 |
| 2.07700 | -11.1946420 | .1108517 |
| 2.98300 | -11.2669170 | .1085671 |
| 4.03300 | -11.3667870 | .1053836 |
| 5.39700 | -11.5298110 | .1011772 |
| 6.93300 | -11.7567500 | .0977882 |
| 3.05700 | -11.2732750 | .1083571 |
| 3.87700 | -11.3505670 | .1058761 |
| 6.92300 | -11.7551360 | .0978021 |
| 8.30000 | -11.9925350 | .0972417 |
| 9.78800 | -12.2791320 | .1000949 |
| 14.95000 | -13.4276690 | .1320736 |
| 20.11000 | -14.6633000 | .1727803 |
| 21.28000 | -14.9420160 | .1812105 |
| 24.95000 | -15.7977010 | .2047049 |
| 29.54000 | -16.8138720 | .2279892 |
| 36.11000 | -18.1485450 | .2519081 |
| 40.30000 | -18.9272850 | .2629640 |

RATIO OF BETA TO L = 17.998274
SIGMA OF BETA TO L RATIO = .372622

# APPENDIX I

## Description of Digital Computer Program used for Least Squares and Statistical Analysis of Transfer Function Phase Shift Data

The computer code, TRANSFUNC3, was developed to perform the least squares and statistical analyses presented in Appendix E. The program was written in FORTRAN IV language. A listing of the source deck is included in this appendix. The logic of this program is identical to that of TRANSFUNC2, so reference is made to the logic diagram which was included in Appendix H for that program.

The input for this program was arranged as follows:

FIRST CARD: FORMAT(F5.1,I5,F5.1)

1) RUNNO: Run number, 2) K, and 3) BK: Number of data points.

NEXT K CARDS: FORMAT(F10.5)

W: Frequencies at which the phase shift was measured, punched one to a card.

NEXT K CARDS: FORMAT(F11.6)

P: Experimentally measured phase shift, punched one to a card.

NEXT K CARDS: FORMAT(F8.2)

WT: Weighting factors, punched one to a card.

The delayed neutron constants, $\beta_i$ and $\lambda_i$, and the iteration accuracy criterion have been incorporated into the program.

On-line printer and punched card output are provided. The first two punched cards contain the statements, "Least squares analysis of phase shift" and "Data from run number ___". The next card contains the headings

"Prompt neutron lifetime" and "Deltal" for the two columns of output which follow on the next few cards. The next two cards contain the least squares best value of the prompt neutron lifetime and an estimate of its standard deviation. Next comes a card which contains the headings "Frequency (cps)" and "Phase shift (deg)". The next K cards contain the frequencies at which the phase shift was measured and the corresponding least squares determined theoretical phase shift. The last two cards contain the value of the ratio of the delayed neutron fraction to the prompt neutron lifetime and an estimate of its standard deviation. The on-line printer output is identical to the punched card output.

Table VII: Input data and variables required for the least squares analysis of phase shift data computer program

| Symbol | Explanation |
|---|---|
| A1 - A6 | Decay constants for delayed neutron precursors |
| AL | Prompt neutron lifetime |
| ALPHA | Ratio of $\beta/\ell$ |
| B1 - B6 | Delayed neutron fractions |
| DELTAL | Calculated correction for prompt neutron lifetime estimate |
| K | Number of data cards to be read |
| P | Measured phase shift |
| RUNNO | Run number, for data identification |
| SIGALP | Estimated standard deviation of ALPHA |
| SIGMAL | Estimated standard deviation of prompt neutron lifetime |
| SUM | Weighted sum of squares of deviations |
| THETA | Theoretical phase shift |
| W | Frequency at which phase shift was measured |

```
C     THIS PROGRAM PERFORMS A LEAST SQUARES ANALYSIS ON EXPERIMENTALLY
C     MEASURED PHASE SHIFT DATA TO DETERMINE THE BEST VALUE OF THE
C     PROMPT NEUTRON LIFETIME
      DIMENSION W(50),P(50),THETA(50),PTT(50),WT(50)
    1 FORMAT(F10.5)
    2 FORMAT(F11.6)
    3 FORMAT(1HL,10X,53HLEAST SQUARES BEST VALUE OF PROMPT NEUTRON LIFET
     1IME =,F14.12)
    4 FORMAT(1HK,10X,42HLEAST SQUARES ANALYSIS OF PHASE SHIFT DATA)
    5 FORMAT(F5.1,I5,F5.1)
    6 FORMAT(42HLEAST SQUARES ANALYSIS OF PHASE SHIFT DATA)
    7 FORMAT(1HK,10X,20HDATA FROM RUN NUMBER,F5.1)
    8 FORMAT(20HDATA FROM RUN NUMBER,F5.1)
    9 FORMAT(2X,F14.12,12X,E18.12)
   10 FORMAT(23HPROMPT NEUTRON LIFETIME,10X,6HDELTAL)
   11 FORMAT(53HLEAST SQUARES BEST VALUE OF PROMPT NEUTRON LIFETIME =,F1
     14.12)
   12 FORMAT(1HK,10X,34HSIGMA OF PROMPT NEUTRON LIFETIME =,F14.12)
   13 FORMAT(1HK,10X,23HPROMPT NEUTRON LIFETIME,10X,6HDELTAL)
   14 FORMAT(1HK,12X,F14.12,12X,E18.12)
   15 FORMAT(34HSIGMA OF PROMPT NEUTRON LIFETIME =,F14.12)
   16 FORMAT(1HL,10X,15HFREQUENCY (CPS),10X,17HPHASE SHIFT (DEG))
   17 FORMAT(15HFREQUENCY (CPS),10X,17HPHASE SHIFT (DEG))
   18 FORMAT(1H ,12X,F10.5,15X,F11.7)
   19 FORMAT(2X,F10.5,15X,F11.7)
   20 FORMAT(1H1)
   21 FORMAT(1HL,10X,20HRATIO OF BETA TO L =,F10.6)
   22 FORMAT(1HK,10X,26HSIGMA OF BETA TO L RATIO =,F10.6)
   23 FORMAT(F8.2)
   24 FORMAT(20HRATIO OF BETA TO L =,F10.6)
   25 FORMAT(26HSIGMA OF BETA TO L RATIO =,F10.6)
  180 READ(1,5)RUNNO,K,BK
      READ(1,1)(W(I),I=1,K)
      READ(1,2)(P(I),I=1,K)
      READ(1,23)(WT(I),I=1,K)
      A1=.0124
      A2=.0305
      A3=.111
      A4=.301
      A5=1.14
      A6=3.01
      B1=.00021
      B2=.00140
      B3=.00126
      B4=.00253
      B5=.00074
      B6=.00027
      B  =A1*(A2+A3+A4+A5+A6)+A2*(A3+A4+A5+A6)+A3*(A4+A5+A6)+A4*(A5+A6)+
     1A5*A6
      C=-A1*A2*(A3*A4+A3*A5+A4*A5+A3*A6+A4*A6+A5*A6)-A3*(A1+A2)*(A4*A5
```

```
1A4*A6+A5*A6)-A4*A5*A6*(A1+A2+A3)
 D=A1*A2*A3*A4*A5*A6
 E=A1+A2+A3+A4+A5+A6
 F=-A1*A2*(A3+A4+A5+A6)-A3*(A1+A2)*(A4+A5+A6)-A5*A6*(A1+A2+A3+A4)
1-A4*(A5+A6)*(A1+A2+A3)
 G   =A2*A3*A4*A5*A6+A1*A3*A4*A5*A6+A1*A2*A4*A5*A6+A1*A2*A3*A5*A6+A1
1*A2*A3*A4*A6+A1*A2*A3*A4*A5
 D11=A6+A2+A3+A4+A5
 D12=-(((A6)*(A2))*(A3+A4+A5))-(((A3)*(A4))*(A6+A2+A5))-(((A5)*(A3+
1A4))*(A6+A2))
 D13=A6*A2*A3*A4*A5
 D14=-((A6)*(A2+A3+A4+A5))-((A2)*(A3+A4+A5))-((A3)*(A4+A5))-((A4)*(
1A5))
 D15=(A2*A3*A4*A5)+(A6*A3*A4*A5)+(A6*A2*A4*A5)+(A6*A2*A3*A5)+(A6*A2
1*A3*A4)
 D21=A1+A6+A3+A4+A5
 D22=-(((A1)*(A6))*(A3+A4+A5))-(((A3)*(A4))*(A1+A6+A5))-(((A5-*(A3+
1A4))*(A1+A6))
 D23=A1*A6*A3*A4*A5
 D24=-((A1)*(A6+A3+A4+A5))-((A6)*(A3+A4+A5))-((A3)*(A4+A5))-((A4)*(
1A5))
 D25=(A6*A3*A4*A5)+(A1*A3*A4*A5)+(A1*A6*A4*A5)+(A1*A6*A3*A5)+(A1*A6
1*A3*A4)
 D31=A1+A2+A6+A4+A5
 D32=-(((A1)*(A2))*(A6+A4+A5))-(((A6)*(A4))*(A1+A2+A5))-(((A5-*(A6+
1A4))*(A1+A2))
 D33=A1*A2*A6*A4*A5
 D34=-((A1)*(A2+A6+A4+A5))-((A2)*(A6+A4+A5))-((A6)*(A4+A5))-((A4)*(
1A5))
 D35=(A2*A6*A4*A5)+(A1*A6*A4*A5)+(A1*A2*A4*A5)+(A1*A2*A6*A5)+(A1*A2
1*A6*A4)
 D41=A1+A2+A3+A6+A5
 D42=-(((A1)*(A2))*(A3+A6+A5))-(((A3)*(A6))*(A1+A2+A5))-(((A5)*(A3+
1A6))*(A1+A2))
 D43=A1*A2*A3*A6*A5
 D44=-((A1)*(A2+A3+A6+A5))-((A2)*(A3+A6+A5))-((A3)*(A6+A5))-((A6)*(
1A5))
 D45=(A2*A3*A6*A5)+(A1*A3*A6*A5)+(A1*A2*A6*A5)+(A1*A2*A3*A5)+(A1*A2
1*A3*A6)
 D51=A1+A2+A3+A4+A6
 D52=-(((A1)*(A2))*(A3+A4+A6))-(((A3)*(A4))*(A1+A2+A6))-(((A6)*(A3+
1A4))*(A1+A2))
 D53=A1*A2*A3*A4*A6
 D54=-((A1)*(A2+A3+A4+A6))-((A2)*(A3+A4+A6))-((A3)*(A4+A6))-((A4)*(
1A6))
 D55=(A2*A3*A4*A6)+(A1*A3*A4*A6)+(A1*A2*A4*A6)+(A1*A2*A3*A6)+(A1*A2
1*A3*A4)
 D61=A1+A2+A3+A4+A5
 D62=-(((A1)*(A2))*(A3+A4+A5))-(((A3)*(A4))*(A1+A2+A5))-(((A5)*(A3+
1A4))*(A1+A2))
```

```
      D63=A1*A2*A3*A4*A5
      D64=-((A1)*(A2+A3+A4+A5))-((A2)*(A3+A4+A5))-((A3)*(A4+A5))-((A4)*(
     1A5))
      D65=(A2*A3*A4*A5)+(A1*A3*A4*A5)+(A1*A2*A4*A5)+(A1*A2*A3*A5)+(A1*A2
     1*A3*A4)
      S1=B1*D11+B2*D21+B3*D31+B4*D41+B5*D51+B6*D61
      S2=B1*D12+B2*D22+B3*D32+B4*D42+B5*D52+B6*D62
      S3=B1*D13+B2*D23+B3*D33+B4*D43+B5*D53+B6*D63
      S4=B1*D14+B2*D24+B3*D34+B4*D44+B5*D54+B6*D64
      S5=B1*D15+B2*D25+B3*D35+B4*D45+B5*D55+B6*D65
      S6=B1+B2+B3+B4+B5+B6
      T1=S6-1.
      T2=S4+B
      T3=S5+C
      T4=E-S1
      T5=F-S2
      T6=G-S3
      PR1=T1*E
      PR2=T1*F+T2*E
      PR3=T1*G+T2*F+T3*E
      PR4=T2*G+T3*F+D*E
      PR5=T3*G+D*F
      PR6=D*G
      PL1=-T4
      PL2=T4*B-T5
      PL3=T4*C+T5*B-T6
      PL4=T4*D+T5*C+T6*B
      PL5=T5*D+T6*C
      PL6=T6*D
      PV1=-T1
      PV2=T1*B-T2
      PV3=T1*C+T2*B-T3
      PV4=T1*D+T2*C+T3*B-D
      PV5=T2*D+T3*C+D*B
      PV6=T3*D+D*C
      PV7=D*D
      PX1=T4*E
      PX2=T4*F+T5*E
      PX3=T4*G+T5*F+T6*E
      PX4=T5*G+T6*F
      PX5=T6*G
      AL=.000055
      WRITE(3,4)
      WRITE(2,6)
      DO 185 I=1,K
  185 WT(I)=1./WT(I)
      REP=0.
      K=K+1
      BK=BK+1.
  190 K=K-1
```

```
      BK=BK-1.
      REP=REP+1.
      WRITE(3,7)RUNNO
      WRITE(2,8)RUNNO
      WRITE(3,13)
      WRITE(2,10)
  200 T7=S6+AL*E
      T8=S4+AL*F
      T9=S5+AL*G
      T10=S1+AL*B
      T11=S2+AL*C
      T12=S3+AL*D
      R1=T1*T7
      R2=(T2*T7)+(T1*T8)
      R3=(T3*T7)+(T2*T8)+(T1*T9)
      R4=T2*T9+T3*T8+D*T7
      R5=T3*T9+D*T8
      R6=D*T9
      BL1=-AL*T4
      BL2=(T4*T10)-(AL*T5)
      BL3=(T4*T11)+(T5*T10)+(-(AL*T6))
      BL4=(T4*T12)+(T5*T11)+(T6*T10)
      BL5=(T5*T12)+(T6*T11)
      BL6=T6*T12
      V1=-(AL*T1)
      V2=(T1*T10)-(AL*T2)
      V3=(T1*T11)+(T2*T10)-(AL*T3)
      V4=T1*T12+T2*T11+T3*T10-AL*D
      V5=T2*T12+T3*T11+D*T10
      V6=T3*T12+D*T11
      V7=D*T12
      X1=T4*T7
      X2=(T4*T8)+(T5*T7)
      X3=(T4*T9)+(T5*T8)+(T6*T7)
      X4=(T5*T9)+(T6*T8)
      X5=T6*T9
      C1=0.
      C2=0.
      DO 250 I=1,K
      E1=6.28318531*W(I)
      E2=E1*E1
      E3=E2*E1
      E4=E2*E2
      E6=E3*E3
      E8=E4*E4
      E10=E6*E4
      E12=E6*E6
      BA1=-E2*(E10*R1+E8*R2+E6*R3+E4*R4+E2*R5+R6)
      BA2=E2*(E10*BL1+E8*BL2+E6*BL3+E4*BL4+E2*BL5+BL6)
      BA3=E1*(E12*V1+E10*V2+E8*V3+E6*V4+E4*V5+E2*V6+V7)
```

```
      BA4=E3*(E8*X1+E6*X2+E4*X3+E2*X4+X5)
      PB1 =-E2*(E10*PR1+E8*PR2+E6*PR3+E4*PR4+E2*PR5+PR6)
      PB2 =E2*(E10*PL1+E8*PL2+E6*PL3+E4*PL4+E2*PL5+PL6)
      PB3 =E1*(E12*PV1+E10*PV2+E8*PV3+E6*PV4+E4*PV5+E2*PV6+PV7)
      PB4 =E3*(E8*PX1+E6*PX2+E4*PX3+E2*PX4+PX5)
      PPT    =-((BA1+BA2)*(PB3+PB4)-(BA3+BA4)*(PB1+PB2))/((BA1+BA2)*(BA1+
     1BA2))
      THETA(I)=57.2957795131*ATAN(-(BA3+BA4)/(BA1+BA2))
      PT    =-(BA3+BA4)/(BA1+BA2)
      PTT(I)=(57.2957795131*PPT)/(1.+PT*PT)
      C1=C1+(PTT(I)*PTT(I))*WT(I)
  250 C2=C2+(PTT(I)*(P(I)-THETA(I)))*WT(I)
      DELTAL=C2/C1
      WRITE(3,14)AL,DELTAL
      WRITE(2,9)AL,DELTAL
      TEST=ABS(DELTAL/AL)
      IF(TEST.LE..0001) GO TO 300
      AL=AL+DELTAL
      GO TO 200
  300 C11=1./C1
      SUM=0.
      DO 310 I=1,K
  310 SUM=SUM+((THETA(I)-P(I))*(THETA(I)-P(I)))*WT(I)
      SIGMA=(SUM)/(BK-1.)
      WRITE(3,3)AL
      WRITE(2,11)AL
      SIGMAL=SQRT(C11*SIGMA)
      WRITE(3,12)SIGMAL
      WRITE(2,15)SIGMAL
      WRITE(3,16)
      WRITE(2,17)
      DO 320 I=1,K
      WRITE(2,19)W(I),THETA(I)
  320 WRITE(3,18)W(I),THETA(I)
      ALPHA=S6/(AL*6.2831853)
      SIGALP=(S6*SIGMAL)/(AL*AL*6.2831853)
      WRITE(3,21)ALPHA
      WRITE(2,24)ALPHA
      WRITE(3,22)SIGALP
      WRITE(2,25)SIGALP
      WRITE(3,20)
      IF(REP.GT.7.) GO TO 180
      GO TO 190
      END
```

# APPENDIX J

## Details of Pile Oscillator Construction and Installation

The procedure for installation of the pile oscillator and detector is set forth below.

1. Remove the central thimble by loosening the two nuts on the retaining clamp located beneath the center channel assembly and withdraw from the reactor tank.
2. Insert the nineteen foot long oscillator bottom section and slide box as a single unit in the position vacated by the central thimble.
3. Position the oscillator drive table on the center channel assembly as shown in Fig. 2. Place the three-sixteenth inch shims under the drive table front legs and fasten the slide box to the table using the four bolts provided.
4. Remove the two one-half inch cap screws from the center channel assembly and bolt the drive table down using the special pull down rods and clamps visible in Fig. 4.
5. Insert the connecting rod between the end adjusting cam on the transmission and the slide box. Connect the Sanborn displacement transducer as shown in Fig. 20.
6. Connect the micro-switch to the slide box and bolt the cantilever beam support to the drive table as shown in Fig. 1. Position the cantilever beam so that the micro-switch is in the "on" position when the oscillator reaches the bottom of its travel.

7. Adjust the amplitude of the oscillator to the desired value by loosening the cap screw on the end adjusting cam and rotating the connecting rod shoulder screw mount.
8. Open the "fast" beam port by removing the shielding plugs. Insert the Daystrom ion chamber after connecting the high voltage and output cables (see Fig. 3).
9. Connect the high voltage cable to the high voltage supply and the chamber output to the micro-microammeter. Connect the output from the ammeter to the strip chart recorder using the fifty foot coaxial cable.
10. Connect the ion chamber signal from the strip chart recorder to the analog computer for amplification and then to the tape recorder for recording.
11. Apply the required six volts dc excitation to the displacement transducer and connect the transducer output to the Beckman counter.
12. Connect the displacement transducer signal from the Beckman counter to the analog computer for amplification, if necessary, and then to the tape recorder for recording.

The details of construction of the oscillator bottom section and slide box are shown in Fig. 20.

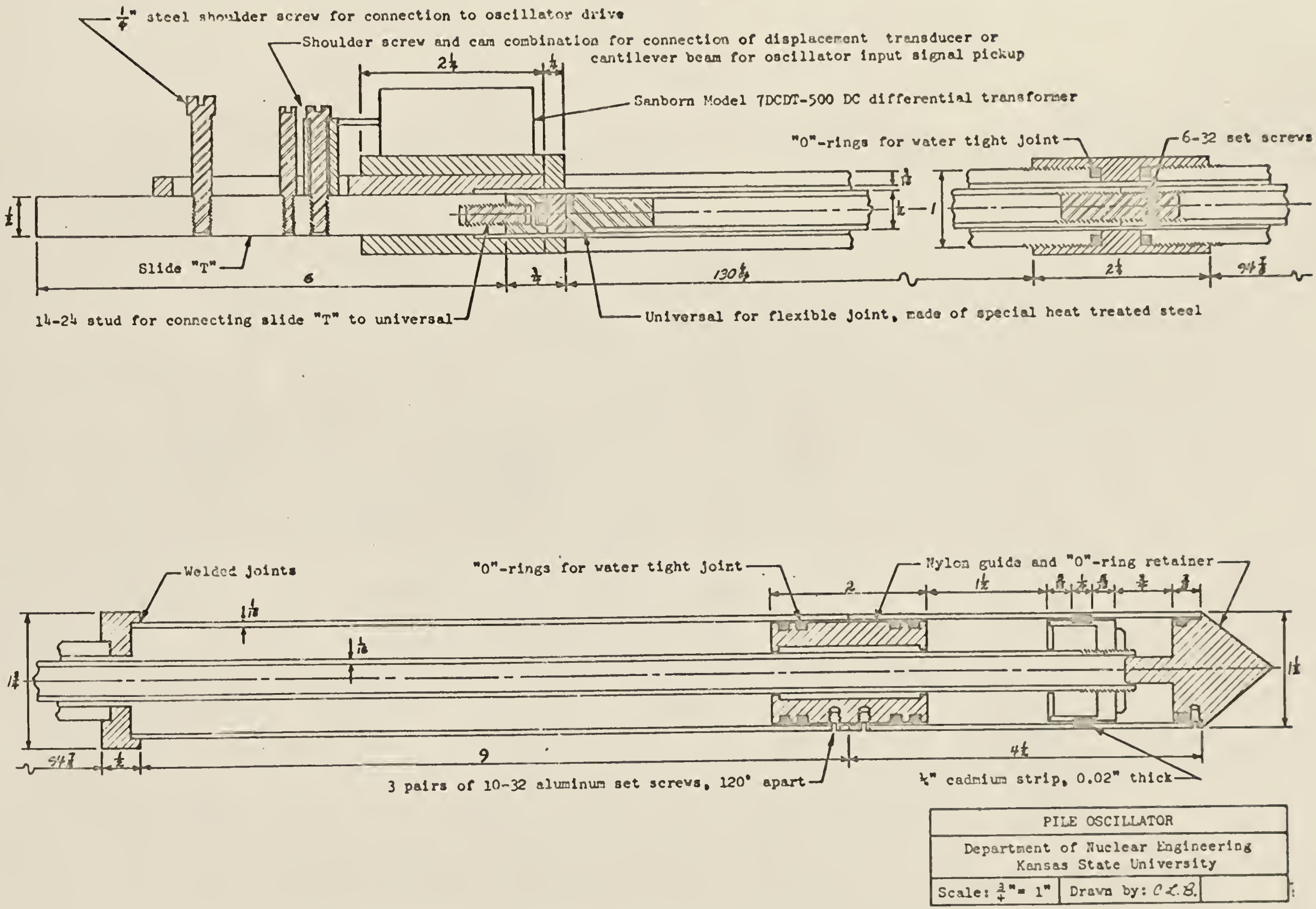


Figure 20. Machine drawing of pile oscillator

# APPENDIX K

## Determination of Variance of Least Squares Parameters

Consider the overdetermined set of linear equations

$$AX = Y, \tag{K-1}$$

where A is an (N,M) matrix, X is an (M,1), and Y is an (N,1). Multiply by the weight matrix $W^{-2}$, which is a diagonal matrix containing weighting factors of $w_{ii} = 1/\sigma_{yi}^2$

$$W^{-2}AX = W^{-2}Y. \tag{K-2}$$

The least squares solution is

$$A^TW^{-2}AX = A^TW^{-2}Y. \tag{K-3}$$

Define

$$\Delta = A^TW^{-2}A, \tag{K-4}$$

which is identical to the coefficient matrix, Δ, of Appendix E, and is a symmetric matrix. The solution for X is

$$X = \Delta^{-1}A^TW^{-2}Y. \tag{K-5}$$

Consider now a variation on X

$$\delta X = \Delta^{-1}A^TW^{-2}\delta Y \tag{K-6}$$

$$\delta X^T = \delta Y^TW^{-2}A\Delta^{-1} \tag{K-7}$$

$$\langle\delta X\delta X^T\rangle = \Delta^{-1}A^TW^{-2}\langle\delta Y\delta Y^T\rangle W^{-2}A\Delta^{-1}, \tag{K-8}$$

where < > indicates the average value and

$$\langle\delta Y\delta Y^T\rangle = W^2. \tag{K-9}$$

Thus

$$\langle \delta X \delta X^T \rangle = \Delta^{-1} A^T W^{-2} A \Delta^{-1} = \Delta^{-1}. \tag{K-10}$$

Thus the elements on the main diagonal of $\Delta^{-1}$ are proportional to the variance of the parameters $x_i$. The constant of proportionality is $\sigma^2$(ext) from Appendix E.

MEASUREMENT OF THE
ZERO POWER TRANSFER FUNCTION OF THE KANSAS STATE
UNIVERSITY TRIGA MARK II NUCLEAR REACTOR

by

CHARLES LEE BEESON

B. S., Kansas State University, 1964

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AN ABSTRACT OF
A MASTER'S THESIS

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Nuclear Engineering

KANSAS STATE UNIVERSITY

Manhattan, Kansas

1966

ABSTRACT

A linear motion pile oscillator was designed and built. The oscillator was then used for zero power frequency response measurements on the Kansas State University TRIGA Mark II reactor. The frequency range investigated was 0.01 cps to 40.0 cps.

Two methods of data reduction were used to determine the reactor gain and phase shift. One method made use of a dual-channel strip chart recorder. The reactor input and output signals were recorded on separate channels, and the desired gain and phase shift information obtained by direct visual measurement. The second method involved the use of an analog computer to integrate the absolute value of the reactor input and output signals, and their product. Using the three voltages thus determined, and the appropriate expressions, the gain and phase shift were calculated. Both data reduction methods were found satisfactory for transfer function gain measurements, the analog computer method being the simpler and more rapid of the two. Neither method was found to be well suited to accurate measurement of the phase shift.

Digital computer codes were developed to perform a least squares fitting of the theoretical gain and phase shift expressions to the measured data. This fitting procedure was used for determining the ratio, $\alpha$, of the effective fraction of delayed neutrons, $\beta$, to the prompt neutron lifetime $\ell$. The value of $\alpha$ determined for the Kansas State University TRIGA Mark II was

$$\alpha = 17.9 \pm 0.4 \text{ cps.}$$

The measured value of $\alpha$ was found to be in good agreement with the value of $\alpha$ = 17.87 cps obtained from General Atomic.