STATISTICAL ANALYSIS OF METAL CUTTING DATA

by

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INTRODUCTION

The application of statistical principles and experimentations has been widely practiced in industrial research for the past two or three decades. The basic advantages of such endeavors are that, by proper statistical analysis of experimental data it is possible to obtain a clearer perspective of the relationship between the factors affecting the experiment and the variations in the magnitude of the test results created by the fractional effects. Tests of analysis of variance, regression, correlation, etc. all lead to justifiable conclusions pronounced with a satisfactory degree of confidence.

The principal aim of this thesis was to associate certain important machinability factors with the variations in the cutting forces generated at the tool tip, while machining stainless steel (SAE 304) with cemented carbide tools, on a Reed-Prentice engine lathe.

The analysis of cutting force data was performed according to statistical principles. The statistical analysis employed to investigate the effects of tool angle, cutting speed, and rate of feed on the magnitude of cutting forces, was as follows:

1) Three-factorial-design and preliminary analysis of variance based on this design.
2) Breakdown analysis of the factorial effects and determination of the linear, quadratic and cubic effects of the factors by means of regression analysis and orthogonal comparisons of the treatment effects.
3) Application of correlation technique to compare the mutual effects between the cutting forces.

The main experiment performed was one of three factor factorial in a randomized complete block design, which was comprised of three replicates, three tools of different side rake angles, three levels of feed and four different cutting speeds. Experiments were performed in compliance with the requirements of a three factor factorial under randomized complete block design. Other than altering the tools, feed and speed, all relevant factors, which may have an influence on the cutting forces, were either held constant or minimized as far as technically feasible.

Since the stainless steel work-piece procured was in the form of an 8 5/8" O. D., welded pipe of 5/16" wall thickness, it was found necessary to design and construct a special fixture to hold the stainless steel pipe, so as to turn the pipe between the lathe centers.

The measurements of the cutting forces were accomplished by a lathe tool dynamometer, and a strain gage amplifying unit.

All machining operations were performed dry, without the presence of any cutting fluid.

All calculations pertaining to the statistical analysis of the data were carried out on the IBM 1620 digital computer, using the programing statements classified as Fortran II.

CUTTING FORCES AND THEIR IMPORTANCE

The components of the total force on the tool during machining include the tangential, feed and radial forces as characterized in Fig. 1. Assuming that the cutting tool is perfectly sharp and that the total resultant force acts at a point on the tool tip, the resultant total force may be broken down into components and established in a three dimensional array (1) and

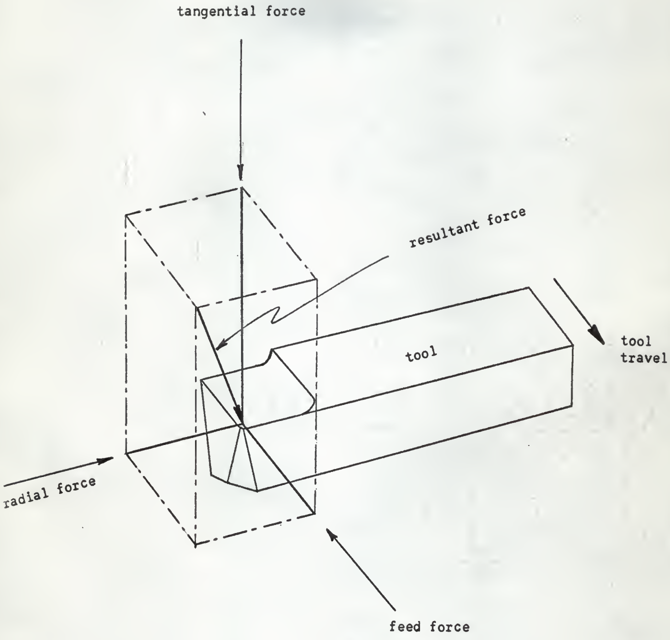


Fig. 1. Cutting forces acting on a tool tip during machining.

(2). The importance of studying the nature of these forces arises from the fact that a knowledge of these forces contributes significantly to understanding metal cutting phenomena associated with power requirements, tool wear, temperature distribution, etc. (2). The tangential force characterizes the power consumption during metal cutting and is generally termed the major cutting force. It may be applied to obtain power requirements to effect metal cutting. The feed force is the force exerted at the tool tip, in opposition to the tool travel. Experiments (2) and (3) have suggested the possibility of estimating tool wear and tool life by means of feed force variations. The radial force is the thrust imparted on the tool by the work-piece during machining and as such does not constitute a highly relevant source of information. Though by considerable research and experimental investigations many substantial evidences and inferences have been established as to the behavior and general characteristics of the cutting forces (4), (5), (6), there appears to be extensive scope for probing more deeply into the analysis of cutting force generation. In the past, conventional methods of machinability research have always centered around single variable testings. By applying a multivariable testing technique, it is possible to note not only the effects of the main factors but also to review the combinatorial effects of these main factors on the variations of cutting forces. Such an effort assumes greater prominence when, as with the magnitude of the cutting forces, the factors influencing them are numerous. One of the aims of this paper has been to investigate such effects. Then again by regression analysis it is possible to define mathematically the nature of the graphs depicting the variations in cutting forces caused by alterations in the levels of machining factors. Such a presentation has valuable possibilities in that empirical formulae may be derived to predict the magnitude of the cutting forces, knowing the influences of the machining factors.

It may also be noted that this paper aims at investigating whether mutual relationships between the different cutting forces are significant or otherwise. This is attempted by performing a correlation test on the feed and tangential forces.

Experiments utilizing the techniques of analysis of variance to estimate the effects of machining conditions on dependent variables have been attempted by very few persons. Mennel and Jeffery (7) conducted statistical experiments to examine several accelerated tool-life testing methods so as to determine their validity under various machining conditions. Kibbey and Morris (8) experimented with the effects of different variables associated with metal cutting with ceramic tools. Apart from these two references, no article pertaining to statistical analysis of metal cutting data, had come to notice at the time of writing this thesis.

DESIGN OF THE EXPERIMENT

Analysis of Variance

The statistical technique of analysis of variance was chosen as the statistical method best suited to fulfill the aims of this paper. One of the features of the analysis of variance is that the variability of the components in an experiment is measured in terms of sums of squares of deviations about the general mean or average. These components of total variability may be classified under two main categories:

1) Those ascribable to cause variations. These include the effects of different factors or combinations of factors.

2) That due to random variations, or variations between observations due to non-assignable or uncontrollable factors. This residual variance may then be used to test the significance of the effects of several factors on the experimental data.

Thus, analysis of variance is a powerful technique to use when variations in data may come from several variables. The results of the analysis will inform the investigator which variables have pronounced effects on the data. The greater the residual variance the greater the other variances must prove to be significant.

Snedecor's Variance Ratio or F-test, will reveal if non-equal variances are significantly different or are different only because of random or uncontrollable variations in the data, and that due to sampling error.

To test for a significant difference in variances, the hypothesis is made that there is no significant differences in the variances. This null hypothesis is tested by means of tables of percentage points of the F distribution (9). These tables show (for a given ratio of variances and given degrees of freedom) the probability of error if the hypothesis is rejected.

Selection of Factors

Though the factors that affect the machining conditions are numerous it was considered appropriate to limit the controlling factors to cutting speed, rate of feed, and the side rake angle of the cutting tool. It should be emphasized that apart from these three factors, there are many others which have a direct bearing on the values of the cutting forces. To name a few, depth of cut, alterations of other tool angles, applications of different forms of cutting fluids, machining at different temperatures, etc., could all be expected to create dissimilar machining conditions and thereby alter the magnitudes of the cutting forces. However, building a statistical model to accommodate all these different factors would amount to much complexity in the experimental design. Further, controlling all these factors at different levels according to the design requirements is often impracticable in many instances. Therefore it was decided to consider for this experiment, only three of the more important factors which have decisive influence on the cutting forces. The three factors considered are:

1) The side rake angle of the tool.

2) The rate of feed in inches/rev.

3) The cutting speed in ft./min.

The experimental data measured were the tangential and feed forces. Here again it may be suggested that many other forms of experimental data could be gathered to supplement the cutting forces, as the test criteria. For example, the chip thickness, surface finish, tool wear, tool temperature and others may be investigated as dependent variables. In this project, however, the variables of interest were the cutting forces and hence effort was made to measure and analyze only these forces.

It should also be noted that, though there would be three components of cutting forces, it was decided to neglect the radial cutting force, and measure only the tangential and feed forces. The strain gage amplifying recorder, used in this experiment, provided facilities to measure only two forces simultaneously. Further, the radial force is associated more with depth of cut, and since depth of cut was one of the factors held constant, it was apparent that a study of radial force variations under the prescribed experimental design would not provide significant information.

Selection of the Levels of Factors

Tool Angle. Single point tools are generally classified according to the nature of the side rake angle. The inclination of the tool face from the horizontal plane (whether it be negative, zero or positive) is the decisive factor characterizing the tool. It was considered of importance, therefore, to include these three different forms of cutting tools in the basic model so as to judge the performance of each of them separately and in conjunction, on the evolution of the cutting forces. It was thereupon decided to provide three tools, one with the side rake angle of -5° and the other two possessing side rake angles of 0° and $+5^{\circ}$, respectively. These three tools were classified as A_1, A_2 and A_3, with

A_1 = tool possessing side rake angle of -5°.

A_2 = tool possessing side rake angle of 0°.

A_3 = tool possessing side rake angle of $+5^{\circ}$.

Table 10 gives the other nomenclatures of the tools.

Rate of Feed. The magnitude and span of the levels of feed were influenced by running trial tests on the test material prior to the actual experiment. Before choosing the rates of feed, it was decided to observe a particular value for the depth of cut. It was observed in these trial tests that a depth of cut of less than 1/8 of an inch did not produce cutting force data of significant variability and satisfactory magnitude. It was therefore proposed to fix depth of cut at 1/8 inch and to maintain this constant throughout the experiment, along with other constant factors.

The necessity of accepting a depth of cut of 1/8 inch brought along with it limitations on the rate of feed and cutting speed. Continuing the trial tests at a depth of cut of 1/8 inch, it became apparent that a feed rate of over .006 ipr resulted in excessive tool wear. Further, the tool wear was found to be extremely rapid and gave rise to chipping and cracking of the brazed carbide tip tools. The tool wear observed at high feed rates was so high that total destruction of the tools became a problem. Further, the excessive wear of the tools, in effect, generated non-uniform, fluctuating forces, difficult to measure and gage accurately. Hence it was concluded that the maximum level of feed should be .006 ipr. According to the availability of the feeds on the lathe it was decided to fix the levels of feed at .002 ipr, .004 ipr, and .006 ipr, respectively, and these are designated as F_1, F_2 and F_3, respectively.

Cutting Speed. Persisting with the trial tests, it became clear that at speeds exceeding 240.5 sfpm excessive heat generation becomes a cause of concern. Hence the upper limit of speed of cutting was fixed at 110 rpm or 240.5 sfpm and then the four levels of speed selected as follows:

S_1 = 28 rpm	61.5 sfpm
S_2 = 59 rpm	129.5 sfpm
S_3 = 91 rpm	199.0 sfpm
S_4 = 110 rpm	240.5 sfpm

It should be pointed out that in all three cases of angle, feed and speed, the levels have been approximately equally spaced. This is not a restriction as far as the factorial experimental design is concerned, but forms a helpful technique when considering orthogonal polynomials later in the analysis (10).

Advantages Pertaining to Factorial Models

For the study of the variations brought about by deliberate changes in the experimental conditions, a generally useful technique is provided by the factorial experiment. The general intention of an experiment similar to the one used is to appraise the effects of one or more factors on the experimental outcome of the proposed study. A considerable advantage is gained if the experiment is so designed that the effect of changing any one variable can be assessed independently of the others. One way of achieving this objective is to decide on a set of values or levels, for each of the factors to be studied, and to carry out one or more trials of the process with each of the possible combinations of the levels of the factors. A further interesting property of a factorial design is that it is possible to study the interacting nature of the variables. By interaction is meant the failure of the levels of one factor to retain the same order and magnitude of performance (within random sampling errors) throughout all levels of another factor. Thus, if the effect of one factor is dependent on the level of another factor, then the two factors are said to interact. It is imperative to note that in experiments where numerous factors appear, a study of interaction between factors aids in illuminating the combinatorial effects of the factors on the experimental data. Furthermore, when factors are not independent the simple effects of a factor vary according to the particular combination of the other factors with which these are produced.

The main advantages of a factorial arrangement may be summarized as follows (11):

1) Greater efficiency in the use of available experimental resources is achieved.
2) Information is obtained about various interactions.
3) The experimental results are applicable over a wide range of conditions; that is, due to the combining of the various factors in one experiment, the results are of a more comprehensive nature.

Replications

Having decided upon three levels of tool angle, three levels of feed and four of speed as factor levels, there arise in all 3 x 3 x 4 = 36 different tool angle-feed-speed combinations, or 36 different treatment possibilities within the scope of this experiment. The question arose as to whether the factorial design should be one of a completely randomized model or a completely randomized block design with replicates. There are two main advantages associated with the completely randomized block design.

1) A great precision is attained by replication and by increasing the degrees of freedom associated with the error variance which would be used to test the significance of the main effects to those of the interactions.
2) If it is not known whether the testing material is of homogenous nature, replication of completely randomized blocks help to signify such a lack of homogeneity. Prior to the experiment under study it was not known whether or not the different lengths of stainless

steel pipe would exhibit the same machinable properties. It was therefore decided to replicate the completely randomized blocks three times, considering three sections of a 12-inch stainless steel pipe as constituting the test material specimen for replication. If the variability as a result of replication did not prove significant, then further replication need not be considered. If it had been found significant, then a greater number of replicates could be taken to substantiate the decisions.

Randomization within Blocks

The randomized block technique requires complete randomization of the different combinations of levels of factors within each block. In order to effect such a proposal, 36 small cardboard discs were cut, each marked with a number, ranging from one to 36.

No. 1 designated a machining combination A_1, F_1, S_1 where A_1 represents side rake angle -5^{o}, F_1 a feed of .002 ipr, and S_1 a speed of 61.5 sfpm.

Similarly, No. 2 designated a combination of A_1, F_1, S_2 and so forth. Tables 1, 2 and 3 show the designated machining combinations and the results of drawing out a chip at a time to give the sequence of treatment combinations. Machining was then performed keeping strictly to the randomized pattern within each block.

EXPERIMENTAL EQUIPMENT

The experiments were conducted on a Reed-Prentice engine lathe, 5 HP capacity, 30" between centers.

Table 1. The sequence of machining condition for replicate No. 1 under complete randomization.

Angle	Feed	Speed	Angle	Feed	Speed
A_1	F_1	S_3	A_1	F_3	S_3
A_1	F_3	S_4	A_3	F_2	S_4
A_2	F_3	S_2	A_3	F_3	S_1
A_3	F_1	S_2	A_3	F_1	S_1
A_1	F_2	S_1	A_2	F_1	S_3
A_2	F_2	S_3	A_2	F_1	S_2
A_2	F_3	S_3	A_1	F_1	S_1
A_1	F_3	S_1	A_1	F_2	S_2
A_2	F_1	S_1	A_1	F_1	S_1
A_2	F_2	S_2	A_1	F_2	S_4
A_1	F_1	S_4	A_2	F_1	S_4
A_2	F_2	S_4	A_3	F_2	S_1
A_2	F_2	S_1	A_2	F_3	S_1
A_1	F_3	S_2	A_3	F_3	S_2
A_1	F_2	S_3	A_3	F_2	S_3
A_3	F_3	S_4	A_2	F_3	S_4
A_3	F_2	S_2	A_3	F_1	S_3
A_3	F_1	S_4	A_3	F_3	S_3

A_1 = -5° side rake
A_2 = 0° side rake
A_3 = $+5^{\circ}$ side rake

F_1 = .002 ipr
F_2 = .004 ipr
F_3 = .006 ipr

S_1 = 61.5 sfpm
S_2 = 129.5 sfpm
S_3 = 199.0 sfpm
S_4 = 240.5 sfpm

Table 2. The sequence of machining conditions for replicate No. 2 under complete randomization.

Angle	Feed	Speed	Angle	Feed	Speed
A_1	F_2	S_3	A_3	F_2	S_3
A_2	F_3	S_3	A_2	F_3	S_4
A_1	F_1	S_1	A_3	F_1	S_2
A_3	F_1	S_3	A_1	F_1	S_4
A_3	F_3	S_1	A_3	F_2	S_2
A_2	F_3	S_1	A_3	F_1	S_1
A_2	F_1	S_1	A_1	F_1	S_2
A_2	F_3	S_2	A_2	F_2	S_2
A_3	F_3	S_3	A_1	F_2	S_4
A_3	F_3	S_4	A_2	F_2	S_4
A_3	F_3	S_2	A_2	F_1	S_2
A_2	F_2	S_3	A_3	F_2	S_4
A_2	F_1	S_4	A_1	F_3	S_4
A_1	F_1	S_3	A_1	F_3	S_2
A_1	F_2	S_1	A_2	F_2	S_1
A_1	F_3	S_3	A_3	F_1	S_4
A_1	F_3	S_1	A_1	F_2	S_2
A_3	F_2	S_1	A_2	F_1	S_3

$A_1 = -5^\circ$ side rake
$A_2 = 0^\circ$ side rake
$A_3 = +5^\circ$ side rake

F_1 = .002 ipr
F_2 = .004 ipr
F_3 = .006 ipr

S_1 = 61.5 sfpm
S_2 = 129.5 sfpm
S_3 = 199.0 sfpm
S_4 = 240.5 sfpm

Table 3. The sequence of machining conditions for replicate No. 3 under complete randomization.

Angle	Feed	Speed	Angle	Feed	Speed
A_3	F_3	S_3	A_3	F_1	S_4
A_3	F_1	S_3	A_3	F_2	S_2
A_2	F_3	S_4	A_3	F_3	S_4
A_3	F_2	S_3	A_1	F_2	S_3
A_3	F_3	S_2	A_1	F_3	S_2
A_2	F_3	S_1	A_2	F_2	S_1
A_3	F_2	S_1	A_2	F_2	S_4
A_2	F_1	S_4	A_1	F_1	S_4
A_1	F_2	S_4	A_2	F_2	S_2
A_1	F_1	S_2	A_2	F_1	S_1
A_1	F_2	S_2	A_1	F_3	S_1
A_1	F_1	S_1	A_2	F_3	S_3
A_2	F_1	S_2	A_2	F_2	S_3
A_2	F_1	S_3	A_1	F_2	S_1
A_3	F_1	S_1	A_3	F_1	S_2
A_3	F_3	S_1	A_2	F_3	S_2
A_3	F_2	S_4	A_1	F_3	S_4
A_1	F_3	S_3	A_1	F_1	S_3

$A_1 = -5^\circ$ side rake
$A_2 = 0^\circ$ side rake
$A_3 = +5^\circ$ side rake

$F_1 = .002$ ipr
$F_2 = .004$ ipr
$F_3 = .006$ ipr

$S_1 = 61.5$ sfpm
$S_2 = 129.5$ sfpm
$S_3 = 199.0$ sfpm
$S_4 = 240.5$ sfpm

The main equipment employed consisted of a lathe tool dynamometer and a Sanborn strain gage amplifier, Model #60-1300B. They were used in collaboration, to measure the cutting forces generated at the tool tip, while machining. The arrangement was such as to measure only the feed and tangential cutting forces. The thrust force was not measured as the Sanborn strain gage amplifier did not have the facility to measure more than two forces at a time. Prior to the experiment, the dynamometer and the Sanborn recorder were tested and calibrated for accuracy.

Lathe Dynamometer

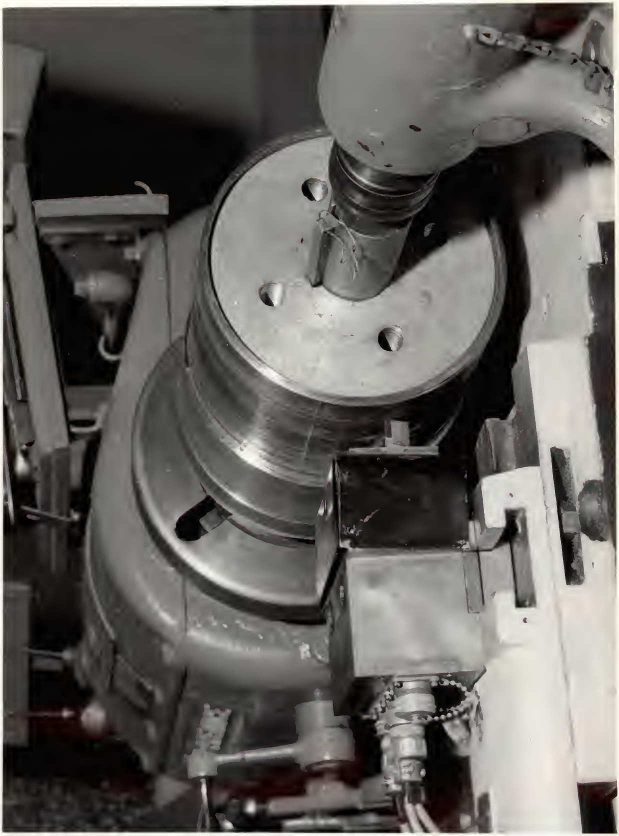
A three-dimensional lathe dynamometer was used to measure the force components acting on the lathe tool. The dynamometer consisted of a unit machined from a single steel block in order to provide the maximum possible stiffness for the required sensitivity and to achieve the high degree of linearity and freedom from hysteresis effects. All the deflection that takes place under load was purely elastic and free of friction and it was then measured with suitably placed resistance strain gages to provide the means for converting forces into a conveniently measured electrical quantity. Figures 2, 3 and 4 show pictures of the lathe dynamometer with the tool in position and the unit in relative position on the tool post. Figure 5 gives a diagramatic picture of the dynamometer showing the locations of the strain gages.

The strain gages were made up of thin tantalum strain gage wires, bonded, and cemented on to the dynamometer block with epoxy resins, at positions indicated. Strain gages C_5, C_6, C_7 and C_8 were intended to measure the

EXPLANATION OF FIGURE 2

Photograph showing the lathe dynamometer and the stainless steel work-piece mounted on the special fixture.

Fig. 2



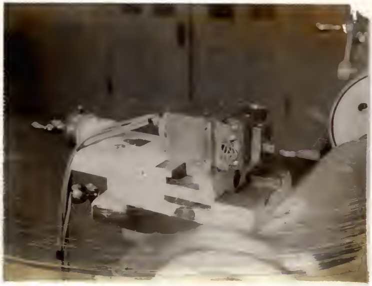


Fig. 3. View of the dynamometer with the cover off, showing the location of the strain gages.

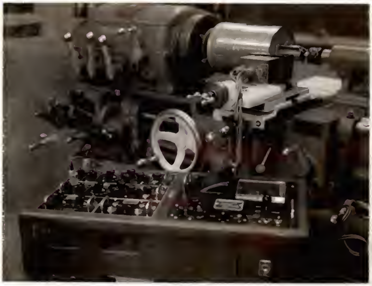


Fig. 4. Photograph of the experimental set up, showing the work-piece and dynamometer with the Sanborn strain gage amplifier in the foreground.

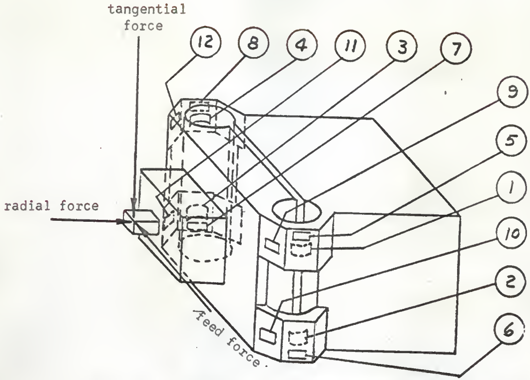


Fig. 5. Diagrammatic sketch of lathe dynamometer used during the experiment.

Gages 1 to 4 are used to measure radial force.
Gages 5 to 8 are used to measure tangential force.
Gages 9 to 12 are used to measure feed force.

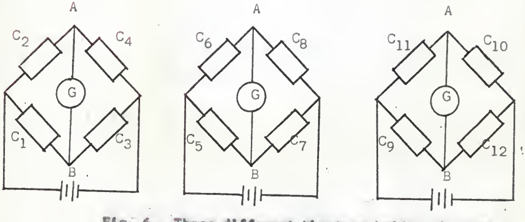


Fig. 6. Three different Wheatson bridge circuits formed by the appropriate strain gages to measure the respective cutting forces.

tangential force, C_9, C_{10}, C_{11} and C_{12}, measured the feed force and C_1, C_2, C_3, C_4 the radial force component. All three measuring gage units contain four 120 ohm strain gages and connections are brought to three separate Wheatson bridge connections representing the three directional forces as indicated in Fig. 6.

The dynamometer works under the principle that when the tool is under the effect of a particular force the corresponding strain gages become longer and thinner or shorter and thicker, as a result of the force acting along it. This results in an increase or decrease of the resistance of the strain gages which in effect unbalances the Wheatson bridge circuit causing a voltage difference at the bridge terminals which if measured would provide an estimate of the force responsible for the change in resistance of the strain gages.

The different strain gages affixed on the dynamometer are connected in such a way as to form three independent Wheatson bridge circuits as in Fig. 6. The four arms of each of the Wheatson bridges are formed by those appropriate strain gages intended to measure the three cutting forces. Thus the Wheatson bridge circuit comprising strain gages C_1, C_2, C_3 and C_4 would measure the radial force, that containing C_5, C_6, C_7 and C_8 measures the tangential force and the third Wheatson bridge with strain gages C_9, C_{10}, C_{11} and C_{12} measures the feed force. Voltage input is between C and D and the voltage output between A and B.

It is evident from Fig. 5, that if a vertical force similar to the tangential force were to act on the tool tip, then due to the bending of the octogonal ring of the dynamometer, strain gages C_6 and C_7 would be compressed and strain gages C_5 and C_8 would be stretched as they would be in tension.

When the strain gages C_6 and C_7 undergo compression their resistance to the electric current would be decreased and since C_5 and C_8 are under tension their resistance would be increased.

Initially when there were no forces acting on the tool, all four strain gages C_5, C_6, C_7, and C_8 possessing equal resistances would balance the Wheatson bridge ACBD and hence would show no voltage difference between A and B and therefore no current would pass through G. When, however, a vertical force becomes active at the tool tip, the changes in strain gage resistances would unbalance the bridge circuit. If C_5 is stretched so that its resistance increases by a small incremental unit (+R) then the voltage at B would be increased from zero to a small amount (+V), which would result in a small current flowing through G. Simultaneously C_6, the strain gage under compression, would suffer a lessening of its resistance by (-R) resulting in the voltage at B to change by (+V) - (-V) i.e. + 2V. At the same time the resistance of C_7 which is in compression and C_8 which is in tension, would change by (-R) and (+R), respectively. The net result of all these changes in resistances would be to increase the voltage at B by (+4V), and thereby causing a proportional unit of current to flow through G. The magnitude of the force causing the resistance changes in the strain gages may then be measured by measuring the unbalanced voltage of (+4V).

Connections from A and B are taken to the Sanborn strain gage amplifier wherein the unbalance of the Wheatson bridge caused by the increase in voltage at B is measured and is indicated by the deflection of a stylus needle beneath which passes a running graph paper. Similarly either the radial or the feed force may be measured simultaneously on the Sanborn strain gage amplifying recorder.

Balancing of the Sanborn Amplifier

Before setting up the Sanborn amplifier for calibration, the instrument was balanced. The balancing procedures may be summarized as follows (15):

1) After warming the Sanborn amplifier for half an hour, throw the coarse-fine switch to the coarse position and with the attentuator at the OFF position, observe the position of the stylus (it will normally be near the center of the recording chart).

2) Advance the attentuator to the X100 position. Unless the bridge circuit is accidentally in balance, the stylus will be deflected upscale. Using the resistance balance control, the stylus is brought back toward the position it occupied when the attentuator was at the OFF position. When the minimum position is found with the resistance balance control, then the final adjustment may be done by using the capacity balance.

3) The attentuator is next advanced to X20 position and the process of bringing back the stylus needle to its original starting position repeated. Continue these operations until the attentuator has been brought to X1 position.

4) Return the attentuator to the OFF position and throw the coarse-fine switch to the fine position.

5) Using the zero control, set the stylus at the line desired for the base line, according to the expected amplitude and direction of the deflections to be recorded.

6) The attentuator knob is next advanced to the desired sensitivity level and if necessary reset the resistance balance slightly to bring the stylus back to the base line position which had been set with the zero

control. If the adjustment is properly made, the attentuator knob may be turned from one position to another without disturbing the stylus position. It is understood, of course, that these adjustments are made with no load on the strain-sensitive elements, or in the case of temperature measurements, with the temperature at what will be considered the reference level.

7) The electrical sensitivity of the system can now be checked by pushing the calibrating button and the sensitivity required for the dynamometer may be adjusted to the desired value by the use of the gain control.

8) Since the position of the gain control may affect the base line position slightly, the recorder may be tested to see if it is in a balanced position by turning the attentuator to different settings and noting that there are no deflections of the stylus needle. The strain gage amplifier and recorder are now ready for use with the dynamometer.

Calibration of the Dynamometer

Having balanced the Sanborn recorder, the complete unit of dynamometer and the Sanborn recorder are calibrated. The method employed to calibrate the unit was to provide a known, varying force at the tool tip and to measure the deflection of the stylus at desired force intervals, so that graphs may be drawn with the stylus deflection on one axis and the force responsible for these corresponding deflections on the other. Having obtained such graphs, it may then be used to determine forces during experiments, by noting the stylus deflections created by the forces.

The following steps were observed when performing calibration of the equipment:

1) The dynamometer was firmly affixed to the base plate of a Southwark Emery Universal Testing machine, of 120,000 lbs. capacity (hydraulic).

2) A tool of 5/8 inch square section and 4½ inch long was inserted into the dynamometer and securely clamped by means of Allen screws.

3) A point was carefully marked on the tool tip such as to be in close approximation with the tool point wherein the cutting forces may be expected to act. A small hole of 1/16 inch diameter was drilled at this location (Fig. 7).

4) A tool overhang of 3/4 inch was adjusted. For all further experiments thereafter the overhang of the tool was fixed at 3/4 inch. A greater overhang of the tool, it was felt, would limit the maximum rigidity of the tool and thereby produce undesirable deflection of the tool instead of the desired absolute dynamometer deflection.

5) In order to apply the vertical load at the concentrated location on the tool tip, a small steel ball of 1/8 inch diameter was placed on the hole drilled on the tool face. The load from the overhead ram was transmitted to the ball point by means of a slender cylindrical steel rod of 1/2-inch diameter and 3-inches in length, placed vertically above the steel ball and perpendicular to the overhead ram.

6) The overhead ram was slowly lowered so as to hold the steel rod in place.

7) The motor controlling the flow of graph paper on the Sanborn strain gage amplifier was turned on and the zero deflection under no load noted to coincide with the datum line.

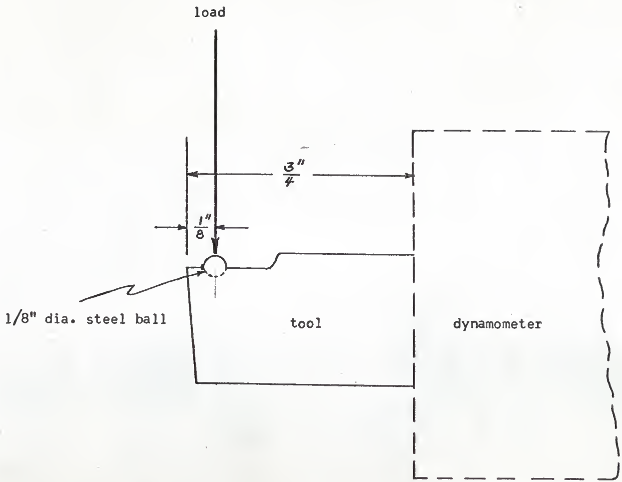


Fig. 7. Tool set-up for dynamometer calibration.

8) The attentuator switch was turned to X10 position and the ram slowly depressed to obtain a total sweep of the stylus representing the tangential force.

9) The maximum load required to effect a total sweep of 50 mm's on the graph sheet was noted.

10) Next the attentuator was set at X5 and a similar maximum safe load to effect a total stylus deflection was observed.

11) These loads of 600 lbs. and 1,100 lbs. for attentuator settings of X10 and X5 were considered satisfactory upper limits beyond which the experimental force would not exceed.

12) The attentuator was once more set at X10 and the stylus needle corresponding to no load position adjusted. Now by lowering the ram the dynamometer was loaded at a uniformly continuous vertical load. As the testing machine indicated loads of 0, 50, 100, 150 ...... pounds of force, the corresponding stylus deflections were also noted by depressing the marker button which etches a short mark at the edge of the graph paper. Thus, the stylus deflections pertaining to loads at every 50 pound increment were noted.

13) The loading was thus continued till the maximum load was reached; thereafter the loading was reversed and similar recordings made as the load decreased from the maximum to the zero load.

14) The same procedure was repeated by turning the attentuator to X5.

15) Tables 4 and 5 show the recordings of the stylus deflection in mm at every increasing and decreasing increment of 50 pound forces.

Table 4. The deflection in millimeters of the Sanborn amplifier stylus needle during calibration. Type of force - vertical. Attentuator setting - X10.

Force lbs.	Up	Down	Average
50	.75	.25	0.5
100	1.0	1.0	1.0
150	2.0	1.25	1.6
200	2.25	2.0	2.12
250	3.0	2.5	2.7
300	3.75	3.1	3.40
350	4.25	4.0	4.10
400	5.0	4.8	4.9
450	5.75	5.5	5.6
500	6.5	6.0	6.25
550	7.1	7.0	7.05
600	8.0	7.75	7.82

Table 5. The deflection in millimeters of the Sanborn amplifier stylus needle during calibration. Type of force - vertical. Attentuator setting - X5.

Force lbs.	Up	Down	Average
50	1.0	1.0	1.0
100	2.8	2.0	2.4
150	5.0	5.2	5.1
200	7.0	6.0	6.5
250	9.5	9.3	9.2
300	11.5	11.0	11.25
350	14.0	14.2	14.1
400	16.5	16.0	16.25
450	19.0	19.2	19.1
500	21.5	21.0	21.25
550	24.0	24.2	24.1
600	26.5	26.5	26.5
650	29.0	29.2	29.1
700	31.5	31.5	31.5
750	34.0	34.2	34.1
800	36.5	36.5	36.5
850	39.0	39.2	39.1
900	41.1	40.5	41.1
950	43.2	43.4	43.3
1000	45.6	45.5	45.6
1050	48.0	48.6	48.3
1100	50.0	49.5	49.75

16) The average of the stylus deflections corresponding to increasing forces of 0, 50, 100, 150 ...... etc. and the decreasing forces of 150, 100, 50, 0, etc. were calculated and graphs drawn to represent them as shown in Figs. 8 and 9.

17) A line of best fit was drawn to establish a linear relationship between load and the corresponding stylus deflections. This graph would then provide a reference for further experiments.

18) In order to measure the feed or horizontal force, the dynamometer was affixed on one of its sides and the whole procedure repeated. Tables 6 and 7 show the recordings of the stylus deflection as the horizontal force is varied. Figures 8 and 9 represent these force-deflection relationship.

Work Piece. The work piece employed was a stainless steel pipe of 8 5/8 inch O. D. and 5/16 inch wall thickness. The type of stainless steel employed was Austenitic Chromium-Nickel steel classified as stainless steel SAE 304 (18/8 variety). Tables 8 and 9 give the composition and physical properties of this type of stainless steel.

Fixture. A special fixture had to be manufactured in order to hold the work piece between lathe centers. Figure 10 shows the form of the fixture utilized. It consisted of two cast aluminum discs of shapes shown in the figure. These cast aluminum discs were assembled onto a 2-inch diameter and 16-inch long, high carbon steel shaft as in Fig. 10. One of the discs was fixed onto an end of the shaft by means of threads and the other was a movable type held in any specific location by means of a 4-inch steel key, for which a key way was sunk on the shaft. The stainless steel pipe was slipped over the aluminum castings and one end bolted to one of the aluminum castings. The

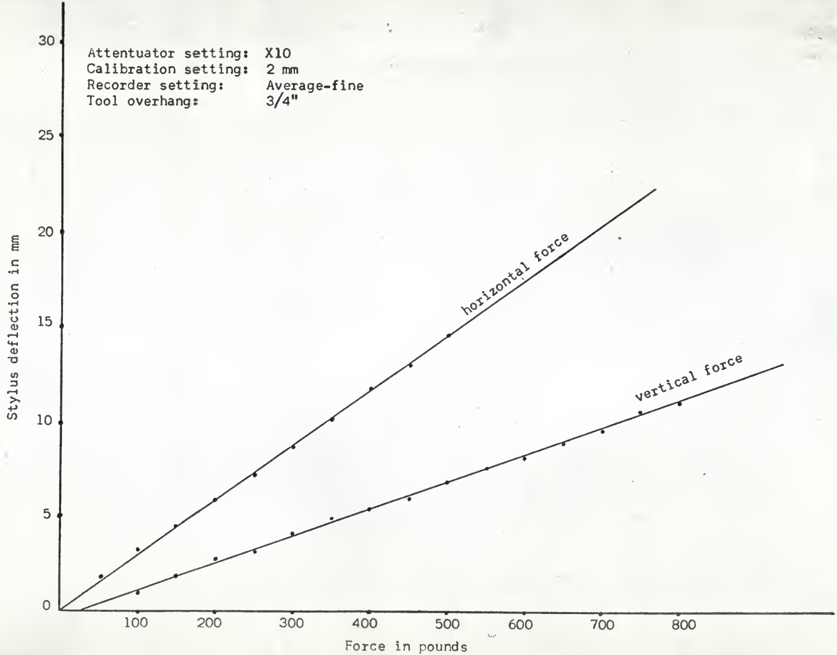


Fig. 8. Calibration graph showing the Sandborne recorder stylus deflection vs. the vertical and horizontal forces at attentuator setting X10.

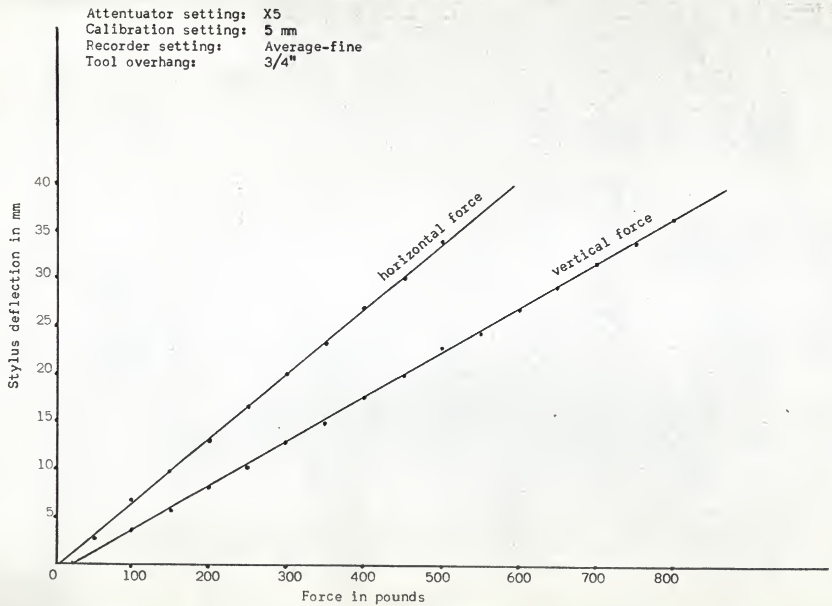


Fig. 9. Calibration graph showing the Sandborne recorder stylus deflection vs. the vertical and horizontal forces at attentuator setting X5.

Table 6. The deflection in millimeters of the Sanborn amplifier stylus needle during calibration. Type of force - horizontal. Attentuator setting - X10.

Force lbs.	Up	Down	Average
50	2.0	1.8	1.9
100	3.1	3.0	3.1
150	4.6	4.1	4.35
200	6.0	5.8	5.9
250	7.1	7.0	7.1
300	8.9	8.5	8.7
350	10.1	10.1	10.1
400	11.6	11.6	11.6
450	13.1	13.0	13.1
500	15.0	14.5	14.7

Table 7. The deflection in millimeters of the Sanborn amplifier stylus needle during calibration. Type of force - horizontal. Attentuator setting - X5.

Force lbs.	Up	Down	Average
50	3	2.8	2.9
100	6.9	6.0	6.45
150	9.0	9.2	9.1
200	13.2	12.9	13.0
250	16.4	16.0	16.2
300	20.1	19.9	20.0
350	23.0	23.2	23.1
400	27.0	27.0	27.0
450	30.6	30.6	30.6
500	34.0	34.2	34.1

Table 8. Chemical composition of non-hardenable austentic chromium nickel stainless steel. SAE-304.

Alloying elements	Percentage
Chromium	18.0 - 20.0
Nickel	8.0 - 11.0
Other elements	Non significant
Carbon	0.08 max.
Manganese	2.00 max.
Silicon	0.75 max.

Table 9. Physical properties.

Yield strength $lbs/in.^2$	Ultimate strength $lbs/in.^2$	% Elongation in 2"	Coefficient of thermal expansion
30,000	75,000	50.0	11.2 in./in./ $^{o}F \times 10^{-6}$

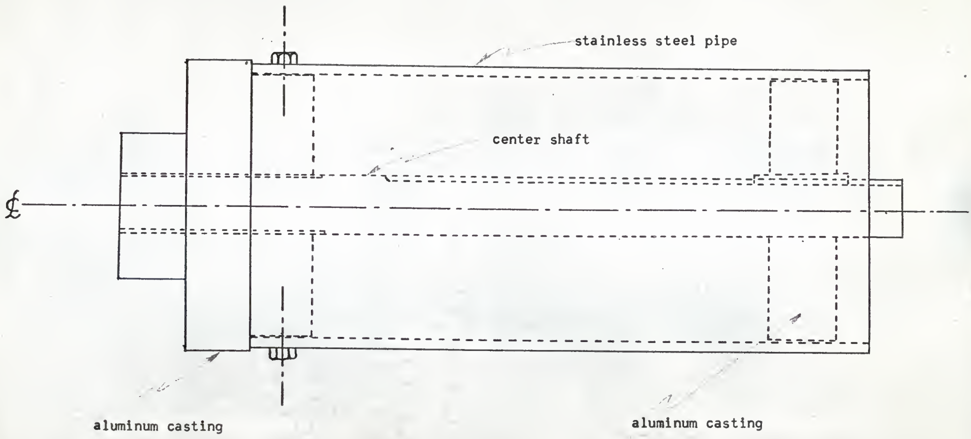


Fig. 10. Fixture employed to hold the stainless steel pipe during the experiment.

other end was supported by the movable casting. The whole assembly was then mounted on the lathe, by holding the inside shaft between the lathe centers.

Tools. The tools employed were brazed carbide tip tools (Grade E in Catalog No. VR 10861), supplied by Vascaloy-Ramet Co., Chicago, Illinois. The tool shanks were of 5/8 inch square section and 4½ inches in length. The carbide tip was a half-inch square piece of 3/16 inch thickness, brazed onto the tool shank. No chip breakers were provided.

EXPERIMENTAL PROCEDURE

Preliminary Operations

Work-Piece. Having placed the work-piece and fixture between the centers of the lathe, the work-piece was turned on the outside to obtain a completely cylindrical surface. After preliminary machining the pipe diameter was found to be exactly 8½ inches throughout its one-foot length. This was taken as the outside diameter of the work-piece. Three equidistant marks were made on the stainless steel pipe to represent the three replications.

Tools. Three tools were ground accurately to have -5°, 0°, and $+5^{\circ}$ side rake angles, respectively. All other angles and nose radius were kept constant. Table 10 gives the nomenclature of the three different forms of tools employed. The ground tools were checked for accuracy using a tool angle measuring protractor. A diamond coated abrasive stone was used to hone the surface of the tools to remove any irregularities in grinding.

Table 10. Nomenclature of the three different forms of tools employed.

	Tool A_1	Tool A_2	Tool A_3
Type	Brazed Carbide Tool		
Grade	E(V.R.)	E(V.R.)	E(V.R.)
Side rake angle	-5°	0°	$+5^{\circ}$
Back rake angle	0°	0°	0°
Side cutting edge angle	0°	0°	0°
End cutting edge angle	5°	5°	5°
End relief angle	5°	5°	5°
Side relief angle	5°	5°	5°
Nose radius	1/16"	1/16"	1/16"

Running the Experiment

The dynamometer was fixed onto the tool post of the lathe and the Sanborn amplifier (recorder) connected to the dynamometer. The Sanborn amplifier was balanced and the stylus positions adjusted.

The experiment was run in a randomized manner following the pattern shown in Tables 1, 2 and 3. The alterations in feed and speed were effected by changing the appropriate levers on the lathe. When tools had to be changed, each tool when inserted into the dynamometer was checked for overhang.

The cutting forces were obtained on the graph sheet within the Sanborn recorder.

After each cut the tools were closely inspected for wear and built-up-edge formations. Any built-up-edge present on the tool was carefully removed by using an abrasive stone and then honing with the diamond coated abrasive stone. If the wear on the tool was prominent when viewed under a microscope, the tool was removed and reground to the initial specifications. Considerable wear on the cutting edge and tool flank was observed periodically.

One of the factors most difficult to be kept under constant condition was the tool and work-piece temperatures. Therefore, the time between each cut was prolonged sufficiently to allow the work-piece and the tool to attain normal temperatures.

After the experiment, the readings from the graph sheets of the Sanborn amplifier were analyzed and converted to readings in force units by referring to the calibration graphs in Figs. 8 and 9.

The results are listed in Tables 11 and 12.

EXPECTATIONS OF MEAN SQUARES IN FACTORIAL DESIGN

Mathematical Model

The mathematical model postulated for data obtained under three-factor factorial is as follows:

$$Y_{ijkm} = M + R^*_i + A^*_j + B^*_k + C^*_m$$

$$+ (AB)^*_{jk} + (AC)^*_{jm} + (BC)^*_{km}$$

$$+ (ABC)^*_{jkm} + E^*_{ijkm}$$

Table 11. Feed forces obtained during the machining of stainless steel (SAE 304) in a three-factor factorial randomized complete block design.

		Replicate 1				Replicate 2				Replicate 3			
		S_1	S_2	S_3	S_4	S_1	S_2	S_3	S_4	S_1	S_2	S_3	S_4
	F_1	217	212	145	140	215	180	126	126	212	212	155	135
A_1	F_2	280	255	225	217	282	240	227	215	285	260	225	217
	F_3	285	270	250	212	280	280	230	210	300	300	272	212
	F_1	140	212	217	245	200	210	220	250	165	200	212	245
A_2	F_2	217	230	190	217	215	230	185	220	212	245	180	225
	F_3	285	225	225	285	279	230	210	295	265	240	220	280
	F_1	125	110	105	125	178	132	132	130	180	132	105	128
A_3	F_2	185	165	130	145	180	155	125	145	165	150	155	148
	F_3	212	245	215	155	215	255	200	160	212	245	225	155

Table 12. Tangential forces obtained during the machining of stainless steel (SAE 304) in a three-factor factorial randomized complete block design.

		Replicate 1				Replicate 2				Replicate 3			
		S_1	S_2	S_3	S_4	S_1	S_2	S_3	S_4	S_1	S_2	S_3	S_4
	F_1	250	230	190	180	230	225	200	205	250	235	215	215
A_1	F_2	320	300	295	275	310	320	280	275	330	310	300	270
	F_3	370	360	320	330	380	380	320	340	362	360	340	340
	F_1	245	195	215	215	250	200	220	195	250	215	235	215
A_2	F_2	295	285	260	285	300	295	275	295	295	285	270	295
	F_3	430	350	320	360	400	360	335	340	380	368	330	345
	F_1	230	195	215	170	235	200	235	170	250	200	215	160
A_3	F_2	320	250	280	255	320	240	270	250	315	235	275	245
	F_3	360	380	360	295	362	372	350	300	360	370	360	295

where i = Replicates = r = 3 Nos.

j = Side rake angle = a = 3 levels

k = Rate of feed = b = 3 levels

m = Cutting speed = c = 4 levels

and

M = Mean effect

R^*_i = Effect of the i^{th} replicate

A^*_j = Effect of the j^{th} level of side rake angle

B^*_k = Effect of the k^{th} level of rate of feed

C^*_m = Effect of the m^{th} level of speed

$(AB)^*_{jk}$ = Effect of the interaction of the j^{th} level of side rake angle with the k^{th} level of rate of feed

$(AC)^*_{jm}$ = Effect of the interaction of the j^{th} level of side rake angle with the m^{th} level of cutting speed

$(BC)^*_{km}$ = Effect of the interaction of the k^{th} level of feed rate with the m^{th} level of cutting speed

$(ABC)^*_{jkm}$ = Second order interaction effect of the j^{th} level of side rake angle, with the k^{th} level of rate of feed and the m^{th} level of cutting speed

E^*_{ijkm} = Effect of the experimental unit in the i^{th} replicate to which the $(jkm)^{th}$ treatment has been randomly assigned.

Assumptions and Test of Hypotheses

Certain statistical assumptions pertaining to factorial designs of the nature under contemplation are listed below (12):

1) E^*_{ijkm}, the deviations due to natural or unassignable causes are normally independently distributed with a mean of zero and variance σ^2.

2) And that the cumulative effects of the replicates, angles, feed, speed and their first order and second order interactions are all zero, i.e.:

$$\sum_{i=1}^{r} R^*_i = \sum_{j=1}^{a} A^*_j = \sum_{k=1}^{b} B^*_k = \sum_{m=1}^{c} C^*_m$$

$$= \sum_{j=1}^{a} (AB)^*_{jk} = \sum_{k=1}^{b} (AB)^*_{jk} = ------ \sum_{m=1}^{c} (ABC)^*_{jkm} = 0$$

In conjunction with these assumptions there would be the following null hypotheses for testing.

The cutting forces are not significantly affected by the different levels of factors such as side rake angle, feed, and speed and that there exists no interaction effects, between side rake angle, feed, and speed. Each of them may be tested for significance by the F-test by dividing their respective mean squares by the experimental error mean square, and comparing them with the ratios in the F-table at their appropriate degrees of freedom. The level of significance is maintained at .05, for all the tests.

Calculations

The computation of sum of squares and the mean squares were performed on the IBM 1620 digital computer using Fortran II automatic coding system. The computer program is shown in Appendix A.

The first step was to read in the data consisting of the values of the feed or tangential force components obtained while machining under different machining conditions. This was accomplished by feeding in the data in array form. Next, interaction tables of speed x feed, angle x feed, angle x speed, angle x feed x speed were formed by adding the relevant values of the feed or tangential forces common to angle, feed or speed appropriately, as given in Tables 13, 14, 15, and 16.

The calculations involved in obtaining the sums of squares and mean squares for the analysis of variance of a three-factor factorial in a randomized complete block design, are summarized below (13):

$$y^2 = \sum_{i=1}^{r} \sum_{j=1}^{a} \sum_{k=1}^{b} \sum_{m=1}^{c} Y^2_{ijkm} - \frac{T^2}{rabc}$$

$$R_{yy} = \frac{\sum_{i=1}^{r} R_i^2}{abc} - \frac{T^2}{rabc}$$

$$S_{abc} = \frac{\sum_{j=1}^{a} \sum_{k=1}^{b} \sum_{m=1}^{c} T^2_{jkm}}{r} - \frac{T^2}{rabc} = \text{among sub classes (cells)}$$

Sum of squares for the angle x feed x speed table.

$$S_{ab} = \frac{\sum_{j=1}^{a} \sum_{k=1}^{b} T^2_{jk}}{rc} - \frac{T^2}{rabc} = \text{Among sub class (cells) sum}$$

of squares for the angle x feed table.

$$S_{ac} = \frac{\sum_{j=1}^{a} \sum_{m=1}^{c} T^2_{jm}}{rb} - \frac{T^2}{rabc} = \text{Among sub classes (cells) sum}$$

of squares for the angle x speed table.

$$S_{bc} = \frac{\sum_{k=1}^{b} \sum_{m=1}^{c} T^2_{km}}{ra} - \frac{T^2}{rabc} = \text{Among sub classes (cells) sum}$$

of squares for the feed x speed table.

Table 13. Tables formed from data in Table 11.

Rep. x Angle Table

	R_1	R_2	R_3	
A_1	2708	2611	2785	8104
A_2	2688	2744	2689	8121
A_3	1917	2007	2000	5924
	7313	7362	7474	22149

Speed x Feed Table

	S_1	S_2	S_3	S_4	
F_1	1632	1600	1417	1524	6173
F_2	2021	1930	1642	1749	7342
F_3	2333	2290	2047	1964	8634
	5986	5820	5106	5237	22149

Angle x Feed Table

	A_1	A_2	A_3	
F_1	2075	2516	1582	6173
F_2	2928	2566	1848	7342
F_3	3101	3039	2494	8634
	8104	8121	5924	22149

Table 14. Tables formed from data in Table 11.

Angle x Speed Table					
	S_1	S_2	S_3	S_4	
A_1	2356	2209	1855	1684	8104
A_2	1978	2022	1859	2262	8121
A_3	1652	1589	1392	1291	5924
	5986	5820	5106	5237	22149

Angle x Feed x Speed Table					
	S_1	S_2	S_3	S_4	
A_1F_1	644	604	426	401	2075
A_1F_2	847	755	677	649	2928
A_1F_3	865	850	752	634	3101
A_2F_1	505	622	649	740	2516
A_2F_2	644	705	555	662	2566
A_2F_3	829	695	655	860	3039
A_3F_1	483	374	342	383	1582
A_3F_2	530	470	410	438	1848
A_3F_3	639	745	640	470	2494
	5986	5820	5106	5237	22149

Table 15. Tables formed from data in Table 12.

Rep. x Angle Table				
	R_1	R_2	R_3	
A_1	3420	3465	3527	10412
A_2	3455	3465	3483	10403
A_3	3310	3304	3280	9894
	10185	10234	10290	30709

Speed x Feed Table					
	S_1	S_2	S_3	S_4	
F_1	2190	1895	1940	1725	7750
F_2	2805	2520	2505	2445	10275
F_3	3404	3300	3035	2945	12684
	8399	7715	7480	7115	30709

Angle x Feed Table				
	A_1	A_2	A_3	
F_1	2625	2650	2475	7750
F_2	3585	3435	3255	10275
F_3	4202	4318	4164	12684
	10412	10403	9894	30709

Table 16. Tables formed from data in Table 12.

Angle x Speed Table					
	S_1 :	S_2 :	S_3 :	S_4 :	
A_1	2802	2720	2460	2430	10412
A_2	2845	2553	2460	2545	10403
A_3	2752	2442	2560	2140	9894
	8399	7715	7480	7115	30709

Angle x Feed x Speed Table					
	S_1 :	S_2 :	S_3 :	S_4 :	
A_1F_1	730	690	605	600	2625
A_1F_2	960	930	875	820	3585
A_1F_3	1112	1100	980	1010	4202
A_2F_1	745	610	670	625	2650
A_2F_2	890	865	805	875	3435
A_2F_3	1210	1078	985	1045	4318
A_3F_1	715	595	665	500	2475
A_3F_2	955	725	825	750	3255
A_3F_4	1082	1122	1070	890	4164
	8399	7715	7480	7115	30709

$$A_{yy} = \frac{\sum_{j=1}^{a} A_j^2}{rbc} - \frac{T^2}{rabc}$$

$$B_{yy} = \frac{\sum_{k=1}^{b} B_k^2}{rac} - \frac{T^2}{rabc}$$

$$C_{yy} = \frac{\sum_{m=1}^{c} C_m^2}{rab} - \frac{T^2}{rabc}$$

$$(AB)_{yy} = S_{ab} - A_{yy} - B_{yy}$$

$$(AC)_{yy} = S_{ac} - A_{yy} - C_{yy}$$

$$(BC)_{yy} = S_{bc} - B_{yy} - C_{yy}$$

$$(ABC)_{yy} = S_{abc} - A_{yy} - B_{yy} - C_{yy} - (AB)_{yy} - (AC)_{yy} - (BC)_{yy}$$

$$E_{yy} = y^2 - R_{yy} - S_{abc}$$

where,

T = total of all observations

R_i = total of all observations in the i^{th} replicate

T_{jkm} = entry in the $(jkm)^{th}$ cell of the angle x feed x speed table, this entry being the "total" of all observations associated with the $(j)^{th}$ level of angle, the $(k)^{th}$ level of feed and the $(m)^{th}$ level of speed.

T_{jk} = entry in the $(jk)^{th}$ cell of the angle x feed table, this entry being the "total" of all observations associated with the $(j)^{th}$ level of angle and the $(k)^{th}$ level of feed.

T_{jm} = entry in the $(jm)^{th}$ cell of the angle x speed table, this entry being the "total" of all observations associated with the $(j)^{th}$ level of angle and $(m)^{th}$ level of speed.

T_{km} = entry in the $(km)^{th}$ cell of the feed x speed table, this entry being the "total" of all observations associated with the $(k)^{th}$ level of feed and the $(m)^{th}$ level of speed.

A_j = total of all observations associated with the $(j)^{th}$ level of angle.

B_k = total of all observations associated with the $(k)^{th}$ level of feed.

C_m = total of all observations associated with the $(m)^{th}$ level of speed.

A_{yy} = corrected angle sum of squares.

B_{yy} = corrected feed sum of squares.

C_{yy} = corrected speed sum of squares.

$(AB)_{yy}$ = corrected (angle x feed) sum of squares.

$(AC)_{yy}$ = corrected (angle x speed) sum of squares.

$(BC)_{yy}$ = corrected (feed x speed) sum of squares.

$(ABC)_{yy}$ = corrected (angle x feed x speed) sum of squares.

E_{yy} = corrected error sum of squares.

Table 17 gives the general analysis of variance for the three-factor factorial in the randomized complete block design.

Fortran Variable Names. The Floating Point variable names employed in the Fortran computer program are as follows. The program is shown in Appendix A.

Table 17. Estimations of mean square for the 3x3x4 factorial randomized complete block design.

Sources of variation	Degrees of freedom	Mean square	Expected mean square
Replicates	(r-1)	$R_{yy}/(r-1)$	$\sigma^2 + abc \sum_{i=1}^{r} (R^*_i)^2/(r-1)$
Factors			
Angle	(a-1)	$A_{yy}/(a-1)$	$\sigma^2 + rbc \sum_{j=1}^{a} (A^*_j)^2/(a-1)$
Feed	(b-1)	$B_{yy}/(b-1)$	$\sigma^2 + rac \sum_{k=1}^{b} (B^*_k)^2/(b-1)$
Speed	(c-1)	$C_{yy}/(c-1)$	$\sigma^2 + rab \sum_{m=1}^{c} (C^*_1)^2/(c-1)$
Interactions			
Angle feed	(a-1)(b-1)	$AB_{yy}/(a-1)(b-1)$	$\sigma^2 + rc \sum_{j=1}^{a} \sum_{k=1}^{b} (AB^*_{jk})^2/(a-1)(b-1)$
Angle speed	(a-1)(c-1)	$AC_{yy}/(a-1)(c-1)$	$\sigma^2 + rb \sum_{j=1}^{a} \sum_{m=1}^{c} (AC^*_{jm})^2/(a-1)(c-1)$
Angle feed speed	(a-1)(b-1)(c-1)	$(ABC)_{yy}/(a-1)(b-1)(c-1)$	$\sigma^2 + r \sum_{j=1}^{a} \sum_{k=1}^{b} \sum_{m=1}^{c} (ABC^*_{jkm})^2/(a-1)(b-1)(c-1)$

Table 17. (concl.)

Sources of variation	Degrees of freedom	Mean square	Expected mean square
<u>Residual</u>			
Experimental error	$(r-1)(abc-1)$	$E_{yy}/(r-1)(abc-1)$	σ^2
TOTAL	$(rabc-1)$		

A(I, J) = Cell values of input data table.

B(I, J) = Cell values of replicate x angle table.

C(I, J) = Cell values of speed x feed table.

D(I, J) = Cell values of angle x feed table.

E(I, J) = Cell values of angle x speed table.

F(I, J) = Cell values of angle x feed x speed table.

$$\text{SUM} = T$$

$$\text{RAFS} = \sum_{i=1}^{r} \sum_{j=1}^{a} \sum_{k=1}^{b} \sum_{m=1}^{c} Y^2_{ijkm}$$

$$\text{CF} = \frac{T^2}{rabc}$$

$$\text{REPU} = R_{yy}$$

$$\text{AFSQ} = \sum_{j=1}^{a} \sum_{k=1}^{b} \sum_{m=1}^{c} T^2_{jkm}$$

$$\text{AFSU} = S_{abc}$$

$$\text{AXFQ} = \sum_{j=1}^{a} \sum_{k=1}^{b} T^2_{jk}$$

$$\text{AXFU} = S_{ab}$$

$$\text{AXSQ} = \sum_{j=1}^{a} \sum_{m=1}^{c} T^2_{jm}$$

$$\text{AXSU} = S_{ac}$$

$$\text{FXSQ} = \sum_{k=1}^{b} \sum_{m=1}^{c} T^2_{km}$$

FXSU = S_{bc}

ASS = A_{yy}

FSS = B_{yy}

SSS = C_{yy}

RMS = $(R_{yy})/(r-1)$ = Replicate mean square

AMS = $(A_{yy})/(a-1)$ = Angle mean square

FMS = $(B_{yy})/ (b-1)$ = Feed mean square

SMS = $(C_{yy})/(c-1)$ = Speed mean square

AFMS = $(AB)_{yy}/(a-1)(b-1)$ = Angle x feed mean square

ASMS = $(AC)_{yy}/(a-1)(c-1)$ - Angle x speed mean square

FSMS = $(BC)_{yy}/(b-1)(c-1)$ = Feed x speed mean square

AFS = $[S_{abc} - A_{yy} - B_{yy} - C_{yy} - (AB)_{yy} - (AC)_{yy} - (BC)_{yy}]/$
$(a-1)(b-1)(c-1)$ = Angle x feed x speed mean square

ERR = $(E_{yy})/(r-1)(abc-1)$ = Experimental error

A check has been provided in the program to observe if the calculations are correct. This is done by calculating the total sum of squares, independently and also by adding the different source sums of squares. These are typed as RAFT and TSS, respectively.

Table 18 gives the computed calculations of analysis of variance for the feed force analysis. Table 19 shows the analysis of variance for the tangential force analysis.

The test of significance on the variables is performed by dividing each of the variable mean squares by the experimental error mean square, and comparing the values for significance with the theoretical variance ratios tabulated in Snedecor's table of variance ratio. The level of significance was chosen as 5 percent (9).

Table 18. Analysis of variance for effect of factors (angle, feed and speed) on the feed force in the machining of stainless steel, for 3x3x4 factorial complete randomized block design.

Sources of variation	D.F.	M.S.	Critical F values at .05	Calculated F values
Replicates	2	190.55	3.13	1.42
Factors				
Angle	2	44351.8	3.13	331.6*
Feed	2	42096.25	3.13	319.2*
Speed	3	6883.63	2.74	51.2*
Interactions				
Angle x feed	4	4158.33	2.50	31.1*
Angle x speed	6	5064.4	2.23	37.8*
Feed x speed	6	514.95	2.23	3.86*
Angle x feed x speed	12	1913.15	1.89	14.16*
Residual				
Experimental error	70	134.76		

* Significant at .05 level.

Table 19. Analysis of variance for effect of factors (angle, feed and speed) on the tangential force in the machining of stainless steel, for 3x3x4 factorial complete randomized block design.

Sources of variation	D.F.	M.S.	Critical F values at .05	Calculated F values
Replicates	2	74.2	3.13	.844
Factors				
Angle	2	2440.6	3.13	27.7*
Feed	2	169088.5	3.13	1911.2*
Speed	3	10830.8	2.74	122.1*
Interactions				
Angle x feed	4	557.76	2.50	6.38*
Angle x speed	6	1728.68	2.23	19.54*
Feed x speed	6	672.92	2.23	7.57*
Angle x feed x speed	12	785.53	1.89	8.85*
Residual				
Experimental error	70	88.83		

* Significant at .05 level.

DISCUSSION

Results of Analysis of Variance on Feed Force

Tests of analysis of variance provide information to the effect that all three main factors, along with their first and second order interactions, are significantly responsible for affecting the variations in the magnitude of the feed force. The variations between replicates were, however, insignificant. This suggests that the work-piece was of a homogeneous nature. Table 18 shows the results of the analysis of variance tests.

Since the highest order interaction term, namely, the angle x feed x speed interaction appears to be significant, it would be pertinent to review the angle-feed-speed effects more elaborately and then constitute a breakdown analysis by tool angles and analyze each of the three as a two-factor analysis.

Angle-Feed-Speed Interaction. This interaction being significant, it may be concluded that angle, feed, and speed effects are not independent in their effect on the feed forces. Some combinations of angle, feed or speed are conceivably better or worse when considering the minimization of feed force. It also shows that the variations of feed force are not similar to each other as the alterations in the levels of the machining factors are altered. Hence, there exists a failure for the two factor interaction of angle x feed to remain relatively constant at all levels of the third factor speed, or it may also be stated as the failure for the angle x speed interaction to remain relatively constant at all levels of feed.

From Fig. 11, it is apparent that there is a general decrease in feed force when speed is increased and whatever be the angle-feed combination, excepting at two instances, (A_2F_1) 0^0 side rake angle and .002 ipr, and (A_2F_3) 0^0 side rake angle and .006 ipr.

It is also evident from the graph that at the low speed of (S_1) 61.5 sfpm, there appears to be very little difference among the feed force generating under the following combination of angle and feed, (A_3F_1) $+5^0$ side rake angle and .002 ipr, (A_3F_2) $+5^0$ side rake angle and .004 ipr, and (A_2F_1) 0^0 side rake angle and .002 ipr. Two more similar sets of isolated feed force characteristics are shown by angle-feed combinations of (A_1F_1) -5^0 side rake angle and .002 ipr, (A_3F_3) $+5^0$ side rake angle and .006 ipr, (A_2F_2) 0^0 side rake angle and .004 ipr, and those by (A_2F_3) 0^0 side rake angle and .006 ipr, (A_1F_3) -5^0 side rake angle and .006 ipr, and (A_1F_2) -5^0 side rake angle and .004 ipr.

The graphs in Fig. 11 also point out the fact that low rates of feed and a tool angle (A_3) $+5^0$ side rake angle at all levels of speed contribute to low feed forces. From this it may be estimated that an angle-feed-speed combination of (A_3-F_1-S_4) $+5^0$ side rake angle, .002 ipr, and 240.5 sfpm provides the best combination of machining conditions when aiming to minimize cutting forces.

As the second order interaction is highly significant, it was felt that a further break-down analysis by angles may prove more informative. Therefore, three separate two-way analysis of variance were attempted between feed and speed under the three different angles, independently. Tables 20, 21 and 22 give the analysis of variance for feed and speed under (A_1) -5^0

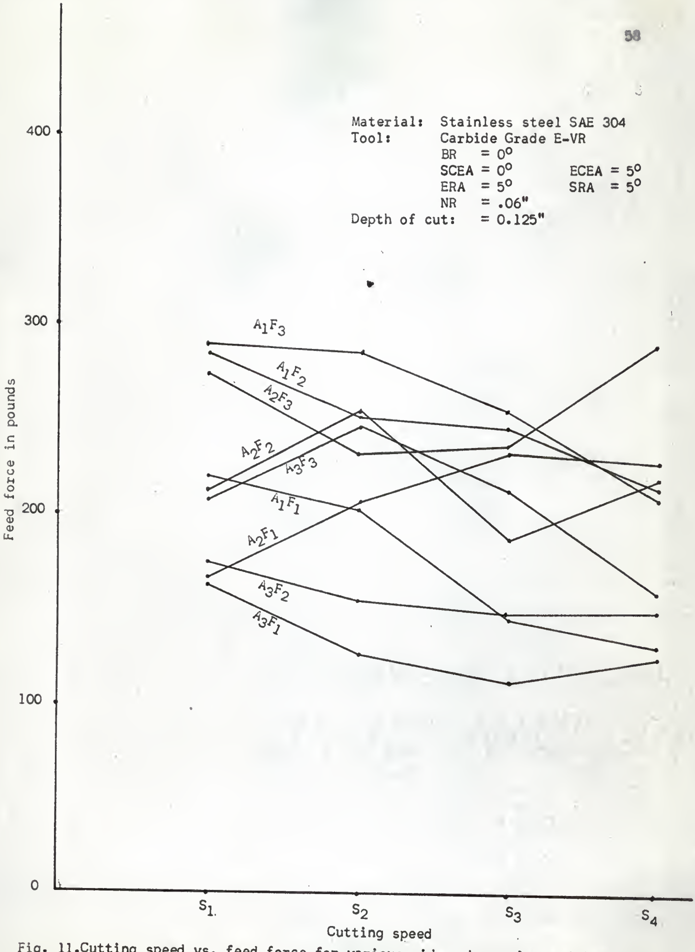


Fig. 11.Cutting speed vs. feed force for various side rake angles and feeds.

Table 20. Two factorial analysis of variance under -5° side rake angle considering feed and speed on the feed force.

Sources of variation	D.F.	M.S.	Critical F value at .05	Calculated F value
Replicates	2	67.1	3.44	0.794
Feed	2	25141.85	3.44	313.3*
Speed	3	10688.6	3.05	132.4*
Feed x speed	6	475.4	2.55	5.96*
Experimental error	22	80.42		

* Significant at .05 lev

Table 21. Two factorial analysis of variance under 0° side rake angle, considering feed and speed, on the feed force.

Sources of variation	D.F.	M.S.	Critical F value at .05	Calculated F value
Replicates	2	172.1	3.44	1.48
Feed	2	6941.07	3.44	59.8*
Speed	3	3178.99	3.05	27.2*
Feed x speed	6	2397.32	2.55	20.5*
Experimental error	22	116.7		

* Significant at .05 level.

Table 22. Two factorial analysis of variance under +5° side rake angle, considering feed and speed, on the feed force.

Sources of variation	D.F.	M.S.	Critical F value at .05	Calculated F value
Replicates	2	419.1	3.44	2.54
Feed	2	18330.76	3.44	112.8*
Speed	3	3145.41	3.05	19.2*
Feed x speed	6	1646.61	2.55	10.34*
Experimental error	22	164.2		

* Significant at .05 level.

side rake angle, (A_2) 0° side rake angle, and (A_3) +5° side rake angle, respectively. Figures 12, 13, and 14 represent the interaction effects.

Feed-Speed Factorials Under (A_1) -5° Side Rake Angle. Figure 12 shows that the feed speed interaction is not so prominent in this case. A marked uniformity is emphasized, as speed increases under different levels of feed, for they all show a reduction in the size of the feed force. However, increase in the levels of feed at different speeds suggests an increase in feed force. Therefore, a feed of .002 ipr and a speed of 240.5 sfpm appear to be the machining conditions most desired under (A_1) -5° side rake angle.

Feed-Speed Factorials Under (A_2) 0° Side Rake Angle. It seems far more difficult to generalize between feed and speed under (A_2) +0° side rake angle, as in Fig. 13. At low feed of (F_1) .002 ipr, there is an increase in feed force at higher speeds. This, however, becomes inapplicable at (F_2) .004 ipr, and (F_3) .006 ipr where medium speed promotes low feed forces. A feed speed

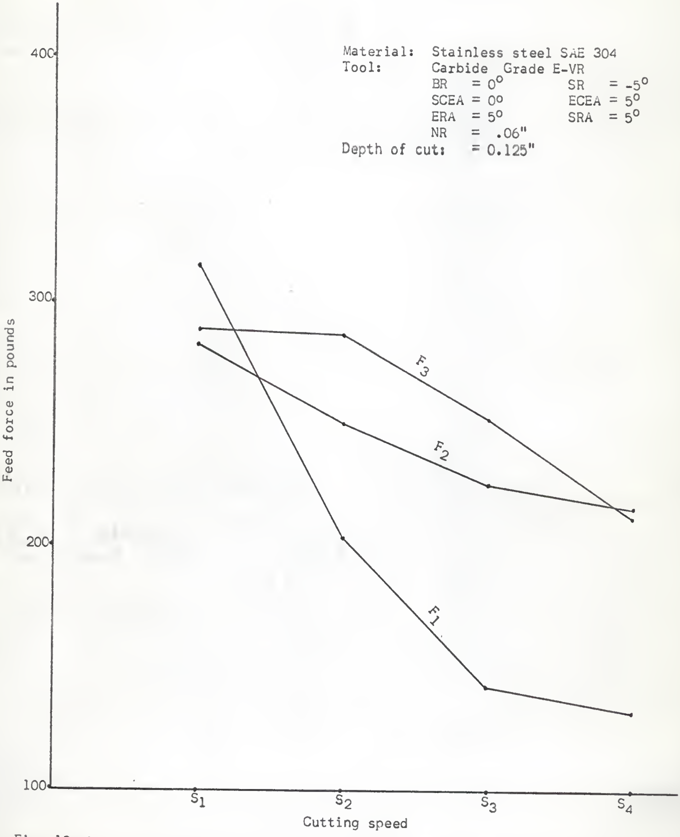


Fig. 12. Cutting speed vs. feed force for various feeds under -5° side rake angle.

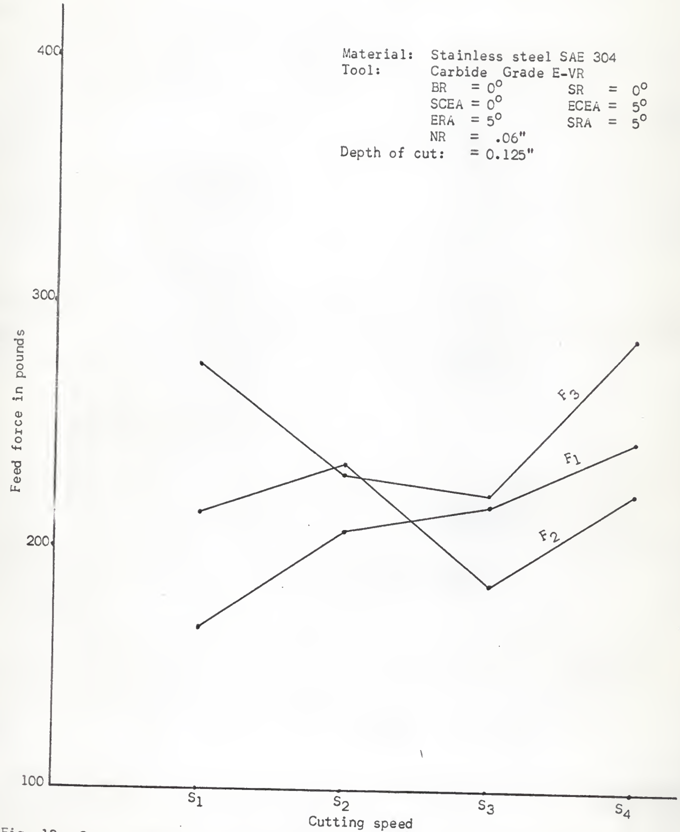


Fig. 13. Cutting speed vs. feed force for various feeds under 0° side rake angle.

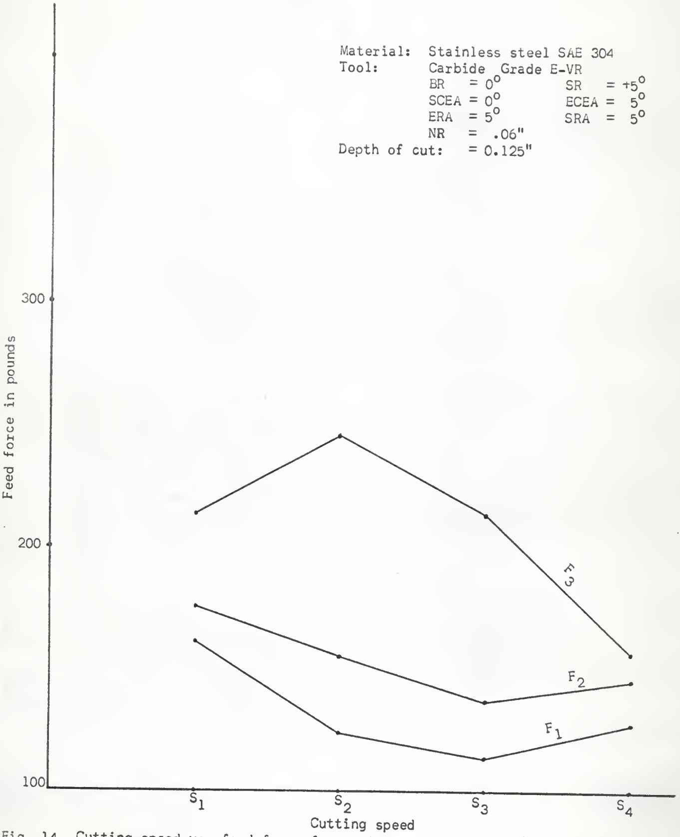


Fig. 14. Cutting speed vs. feed force for various feeds under +5° side rake angle.

combination recommendable at this angle appears to be F_1S_1, i.e., a feed of .002 ipr and a speed of 61.5 sfpm.

Feed-Speed Factorials Under (A_3) +5° Side Rake Angle. The feed speed interaction again appears significant as in Table 22. Here, as under (A_1) -5° side rake angle, the tendency is for the feed forces to lessen at low feeds and high speeds, as shown in Fig. 14.

Analysis of Response Curves in Feed-Speed Factorials Under Angles. It seems reasonable at this juncture to investigate thoroughly how the feed force characteristics vary with changes in the levels of feed and speed under different angles. This is made possible by a breakdown analysis, wherein the main effects, and the interactions are further subdivided into linear, quadratic and cubic effects, and tested for significance (14).

The method adopted for partitioning the main effect and interaction sums of squares into linear and higher orders was by adopting the techniques of orthogonal polynomials (14). The computation was carried out on the IBM 1620 computer, according to the proper computational procedure to arrive at the results. Appendix B gives the Fortran specifications used in arriving at the results.

Tables 23, 24 and 25 give the results of analysis, after partitioning the sums of squares into linear, quadratic and cubic components of speed; linear and quadratic components of feed and combinatorial components of the interaction.

The following mean squares are found significant under (A_1) -5° side rake angle:

1) The linear and cubic effects of speed.

2) The linear and quadratic effects of feed.

Table 23. Analysis of variance showing the linear, quadratic and cubic effects of the feed-speed two-factorial breakdown analysis under -5° side rake angle for the feed force.

Sources of variation	D.F.	Sum of square	M.S.	Critical F value at .05	Calculated F value
Replicates	2	67.1	33.5	3.44	0.41
Treatments					
S_1	1	31204.99	31204.9	4.30	389.90*
S_q	1	15.99	15.9	4.30	0.197
S_c	1	844.99	844.9	4.30	11.50*
F_1	1	43861.49	43861.5	4.30	592.20*
F_q	1	6422.22	6422.2	4.30	88.10*
S_1F_1	1	112.13	112.1	4.30	1.54
S_1F_q	1	348.09	348.1	4.30	4.06
S_qF_1	1	580.16	580.2	4.30	7.84*
S_qF_q	1	647.98	648.1	4.30	8.85*
S_cF_1	1	433.19	433.2	4.30	5.94*
S_cF_q	1	331.34	331.3	4.30	4.04
Experimental error	22	1765.01	80.42		

$$\text{Feed force} = b_0 + b_1S + b_2S^3 + b_3F + b_4F^2 + b_5S^2F + b_6S^2F^2 + b_7S^3F$$

* Significant at .05 level.

Table 24. Analysis of variance showing the linear, quadratic, cubic effects of the feed-speed two-factorial breakdown analysis under side rake angle 0°, for the feed force.

Sources of variation	D.F.	Sum of square	M.S.	Critical F value at .05	Calculated F value
Replicates	2	172.1	86.5	3.44	0.73
Treatments					
S_l	1	2637.34	2637.3	4.30	22.41*
S_q	1	3580.02	3580.0	4.30	30.81*
S_c	1	3319.61	3319.6	4.30	28.22*
F_l	1	11397.03	11397.0	4.30	97.43*
F_q	1	2485.12	2485.1	4.30	21.63*
S_lF_l	1	3842.00	3842.0	4.30	33.16*
S_lF_q	1	2651.46	2651.5	4.30	22.82*
S_qF_l	1	5551.03	5551.0	4.30	47.98*
S_qF_q	1	678.34	678.3	4.30	5.84*
S_cF_l	1	2.07	2.07	4.30	0.02
S_cF_q	1	1659.01	1659.01	4.30	14.30*
Experimental error	22	2561.4	116.7		

$$\text{Feed force} = b_0 + b_1S + b_2S^2 + b_3S^3 + b_4F + b_5F^2 + b_6SF + b_7SF^2 + b_8S^2F + b_9S^2F^2 + b_{10}S^3F^2$$

* Significant at .05 level.

Table 25. Analysis of variance showing the linear, quadratic, cubic effects of the speed-feed two-factorial breakdown analysis under +5° side rake angle for the feed force.

Sources of variation	D.F.	Sum of square	M.S.	Critical F value at .05	Calculated F value
Replicates	2	419.2	209.6	3.44	1.28
Treatments					
S_l	1	9102.22	9102.2	4.30	55.61*
S_q	1	40.11	40.11	4.30	0.25
S_c	1	293.89	293.89	4.30	1.78
F_l	1	34655.97	34655.9	4.30	215.64*
F_q	1	2005.55	2005.6	4.30	12.53*
S_lF_l	1	653.34	653.3	4.30	3.98
S_lF_q	1	205.51	205.5	4.30	1.24
S_qF_l	1	7561.49	7561.5	4.30	4.61*
S_qF_q	1	1266.72	1266.7	4.30	7.32*
S_cF_l	1	187.49	187.5	4.30	1.14
Experimental error	22	3610.2	164.2		

Feed force = $b_0 + b_1S + b_2F + b_3F^2 + b_4S^2F + b_5S^2F^2$

* Significant at .05 level.

3) The quadratic effect of speed on both the linear and quadratic of feed and finally the cubic effect of speed on the linear of feed.

This prompts the possibility of suggesting the formation of an empirical formula to mathematically designate the variability of feed forces under different levels of feed and speed. Thus an equation of the form shown below may be formulated:

$$\text{Feed force} = b_0 + b_1S + b_2S^3 + b_3F + b_4F^2 + b_5S^2F + b_6S^2F^2 + b_7S^3F$$

where b_0 is a constant, and b_1 to b_7 are non-linear regression coefficients associated with the given data, and S to F denote speed and feed levels, respectively. A formula of this nature would be found applicable within those levels of feed and speed enveloped by the experimental design under consideration. Applications of such formulae assume great importance when considering the fact that as of now, the cutting forces are measured primarily from observational data and not from any mathematical formulae.

The analysis of the feed-speed factorial under (A_2) $+0^{\circ}$ side rake angle shows all effects other than the cubic of speed on the linear of feed to be significant as shown in Table 24, suggesting an equation for the feed force of this nature,

$$\text{Feed force} = b_0 + b_1S + b_2S^2 + b_3S^3 + b_4F + b_5F^2 + b_6SF + b_7SF^2 + b_8S^2F + b_9S^2F^2 + b_{10}S^3F^2.$$

Table 25 constitutes the break-down analysis of feed-speed factorial under (A_3) +5° side rake angle, giving rise to an equation of this form:

$$\text{Feed force} = b_0 + b_1S + b_2F + b_3F^2 + b_4S^2F + b_5S^2F^2$$

Results of Analysis of Variance
on Tangential Force

The analysis of variance on the tangential force is shown in Table 19 and represented graphically in Fig. 15, as the angle-feed-speed interaction.

The tests of significance show that the main effects of angle, feed and speed and their higher order interactions, proves them to be significant. However, the interactions are significant to a lesser degree than the main effects.

Though interaction effects occur it is obvious from Fig. 15 that very definite grouping of the tangential forces are observed under different levels of feed. There appears to be lesser variations within levels of feed, than between feeds. A gradual declination of tangential force with increase in speed is once more conspicuous. Apart from these two marked tendencies, there is very little to suggest a pronounced angular superiority to lessen tangential forces.

Nevertheless, a break-down analysis was performed at different angles as shown in Tables 26, 27, and 28 and Figs. 16, 17, and 18. The analysis under (A_1) -5° side rake angle, brings out an interesting fact in that interaction between feed and speed becomes non-significant. At angle (A_2) 0° side rake angle, and (A_3) +5° side rake angle there are suggestions of interactions.

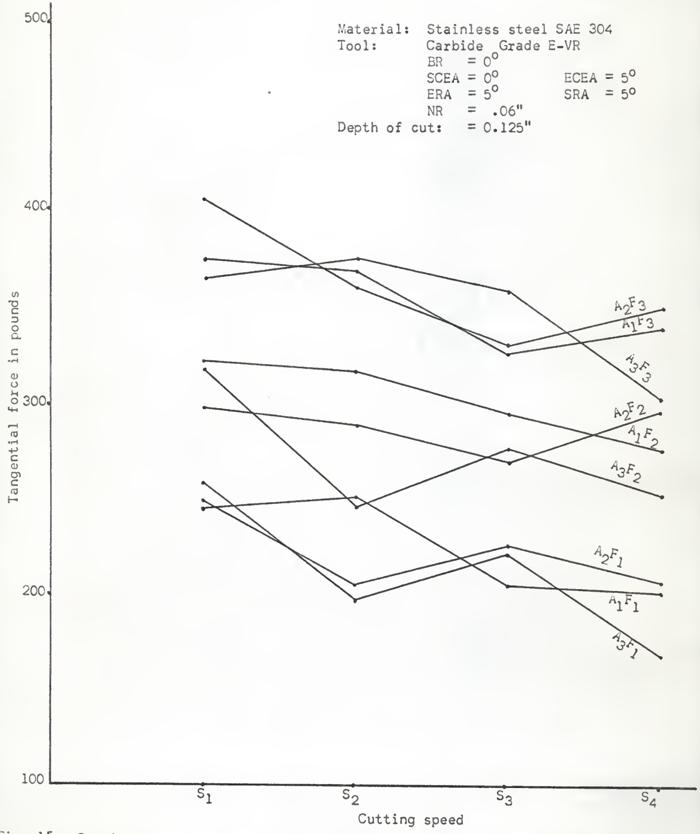


Fig. 15. Cutting speed vs. tangential force for various side rake angles and feeds.

Table 26. Two-factorial analysis of variance under -5^{0} side rake angle considering feed and speed, on the tangential force.

Sources of variation	D.F.	M.S.	Critical F value at .05	Calculated F value
Replicates	2	48.31	3.44	0.48
Feed	2	52628.01	3.44	531.12*
Speed	3	3839.55	3.05	38.64*
Feed x speed	6	215.36	2.55	2.16
Experimental error	22	99.81		

* Significant at .05 level.

Table 27. Two-factorial analysis of variance under 0^{0} side rake angle considering feed and speed, on the tangential force.

Sources of variation	D.F.	M.S.	Critical F value at .05	Calculated F value
Replicates	2	33.92	3.44	0.27
Feed	2	58029.61	3.44	473.14*
Speed	3	3142.84	3.05	25.56*
Feed x speed	6	842.83	2.55	6.84*
Experimental error	22	123.13		

* Significant at .05 level.

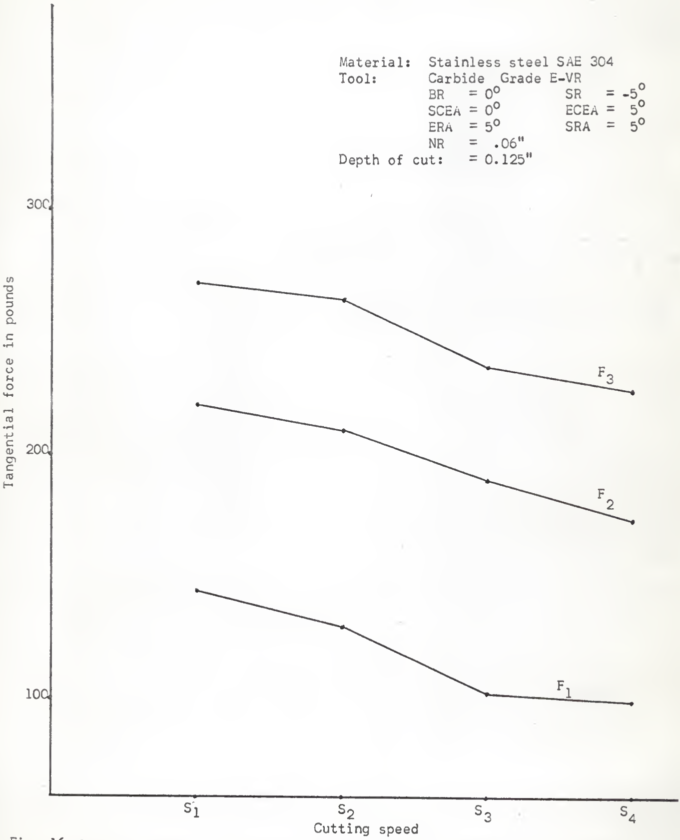


Fig. 16. Cutting speed vs. tangential force for various feeds under -5° side rake angle.

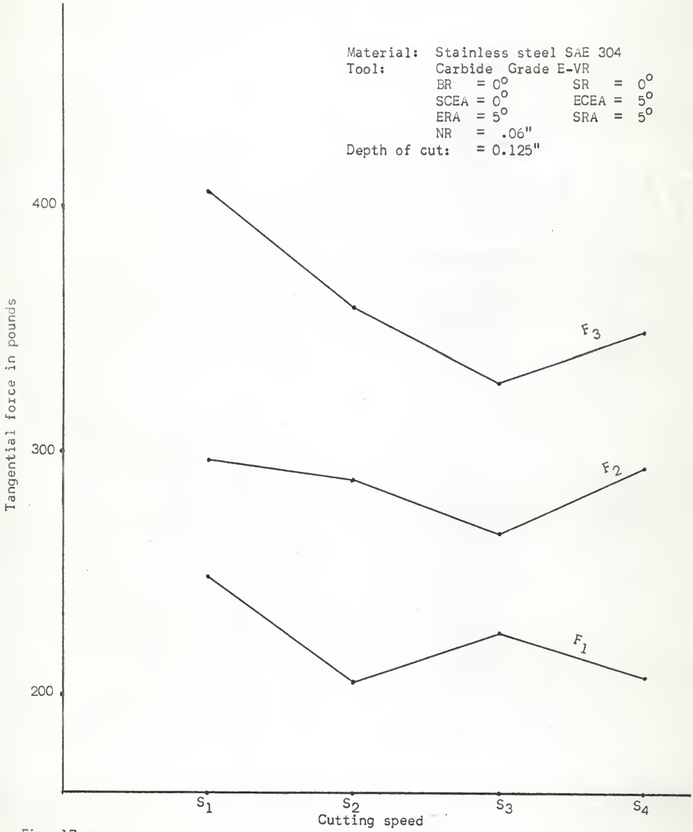


Fig. 17. Cutting speed vs. tangential force for various feeds under 0° side rake angle.

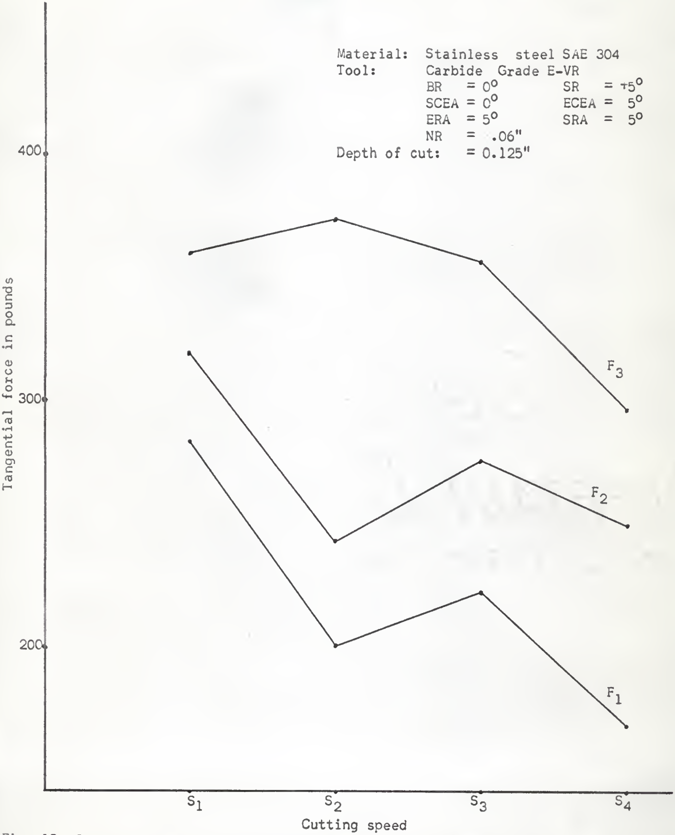


Fig. 18. Cutting speed vs. tangential force for various feeds under +5° side rake angle.

Table 28. Two-factorial analysis of variance under +5° side rake angle, considering feed and speed, on the tangential force.

Sources of variation	D.F.	M.S.	Critical F value at .05	Calculated F value
Replicates	2	44.12	3.44	1.08
Feed	2	59547.19	3.44	1451.21*
Speed	3	7305.88	3.05	178.10*
Feed-speed	6	1415.24	2.55	34.38*
Experimental error	22	41.1		

* Significant at .05 level.

Analysis of Response Curves in Feed Speed Factorials Under Different Side Rake Angles. Tables 29, 30, and 31 give the analysis of variance when subjected to subdivisions of linear, quadratic and cubic effects. From the results of tests of significance, the following equations representative of the tangential force may be modulated:

Under A_1: $$\text{Tangential force} = b_0 + b_1S + b_2S^3 + b_3F + b_4F^2$$

Under A_2: $$\text{Tangential force} = b_0 + b_1S + b_2S^2 + b_3F + b_4SF + b_5SF^2 + b_6S^3F + b_7S^3F^2;$$ and

Under A_3: $$\text{Tangential force} = b_0 + b_1S + b_2S^2 + b_3S^3 + b_4F + b_5F^2 + b_6S^2F + b_7S^2F^2 + b_8S^3F + b_9S^3F^2$$

where b_0, b_1 ....b_9 are all constants associated with coefficients of regression and S, S^2, S^3, F, F^2, etc. depict the linear and higher orders of speed and feed, respectively.

Table 29. Analysis of variance showing the linear, quadratic, cubic effects of speed-feed, two-factorial breakdown analysis under -5° side rake angle, for the tangential force.

Sources of variation	D.F.	Sum of square	M.S.	Critical F value at .05	Calculated F value
Replicates	2	96.60	48.30	3.44	0.48
Treatments					
S_l	1	10518.75	10518.75	4.30	105.82*
S_q	1	75.11	75.11	4.30	0.76
S_c	1	924.79	924.79	4.30	92.61*
F_l	1	103622.02	103622.02	4.30	1035.01*
F_q	1	1634.01	1634.01	4.30	16.44*
S_lF_l	1	20.02	20.02	4.30	0.22
S_lF_q	1	6.67	6.67	4.30	0.06
S_qF_l	1	2.04	2.04	4.30	0.02
S_qF_q	1	224.01	224.01	4.30	2.24
S_cF_l	1	147.40	147.40	4.30	1.47
S_cF_q	1	462.03	462.03	4.30	4.62*
Experimental error	22	2195.0	99.81		

Tangential force = $b_0 + b_1S + b_2S^3 + b_3F + b_4F^2 + b_5S^3F^2$

* Significant at .05 level.

Table 30. Analysis of variance showing the linear, quadratic, cubic effects of speed-feed, two-factorial breakdown analysis under 0^{o} side rake angle for the tangential force.

Sources of variation	D.F.	Sum of square	M.S.	Critical F value at .05	Calculated F value
Replicates	2	67.82	33.91	3.44	0.27
Treatments					
S_1	1	5478.05	5478.05	4.30	44.52*
S_q	1	3948.03	3948.03	4.30	32.10*
S_c	1	2.45	2.45	4.30	0.02
F_1	1	115925.97	115925.97	4.30	938.86*
F_q	1	133.38	133.38	4.30	1.08
S_1F_1	1	691.19	691.19	4.30	5.62*
S_cF_q	1	1276.89	1276.89	4.30	10.35*
S_qF_1	1	433.49	433.49	4.30	3.54
S_qF_q	1	117.56	117.56	4.30	0.95
S_cF_1	1	1428.29	1428.29	4.30	11.61*
S_cF_q	1	1109.39	1109.39	4.30	8.96*
Experimental error	22	2710.7	123.10		

$$\text{Tangential force} = b_0 + b_1S + b_2S^2 + b_3F + b_4SF + b_5SF^2 + b_6S^3F + b_7S^3F^2$$

* Significant at .05 level.

Table 31. Analysis of variance showing the linear, quadratic, cubic effects of speed-feed, two-factorial breakdown analysis under +5° side rake angle for tangential force.

Sources of variation	D.F.	Sum of square	M.S.	Critical F value at .05	Calculated F value
Replicates	2	88.24	44.12	3.44	1.07
Treatments					
S_1	1	16397.35	16397.35	4.30	398.01*
S_q	1	336.11	336.11	4.30	8.17*
S_c	1	5184.19	5184.19	4.30	125.98*
F_1	1	118863.27	118863.27	4.30	2881.32*
F_q	1	231.12	231.12	4.30	5.62*
S_1F_1	1	23.40	23.40	4.30	0.57
S_1F_q	1	83.13	83.13	4.30	2.23
S_qF_1	1	1276.04	1276.04	4.30	31.08*
S_qF_q	1	4592.01	4592.01	4.30	111.82*
S_cF_1	1	1261.02	1261.02	4.30	30.81*
S_cF_q	1	1255.83	1255.83	4.30	30.59*
Experimental error	22	897.0	41.1		

$$\text{Tangential force} = b_0 + b_1S + b_2S^2 + b_3S^3 + b_4F + b_5F^2 + b_6S^2F + b_7S^2F^2 + b_8S^3F + b_9S^3F^2$$

* Significant at .05 level.

Correlation Between Feed Forces and Tangential Forces. Since the feed and tangential forces were measured at the same time, a correlation test of significance to judge whether any interdependency exists between the feed forces and the tangential forces were attempted. The correlation tests were performed under the three different angles, each combination of feed and speed being considered to give two dependent variables in feed force and tangential force. The correlation coefficient was calculated by applying the formula,

$$\text{Correlation Coefficient} = \frac{\Sigma(X_i - \overline{X})(Y_i - \overline{Y})}{\sqrt{\Sigma(X_i - \overline{X})^2 \Sigma(Y_i - \overline{Y})^2}}$$

The tests gave the following correlation coefficients:

For -5° side rake angle C.C. = 0.8709

For 0° side rake angle C.C. = 0.5432

For $+5^{\circ}$ side rake angle C.C. = 0.8798

All three correlation coefficients appear significant at their appropriate degrees of freedom. The relationship between feed force and tangential force at Angle (A_1) -5° side rake angle, and (A_3) $+5^{\circ}$ side rake angle, appear to be almost the same, increase in one resulting in an increase in the latter. At Angle (A_2) 0° side rake angle, though there is evidence of a positive correlationship between feed and tangential forces, the increase of one in relation to the other is less than that for the other two angles. Figures 19, 20, and 21 depict this attitude.

CONCLUSION

The statistical analysis helped to formulate the following inferences. Positive tool angles, low feeds and high speeds are generally advantageous

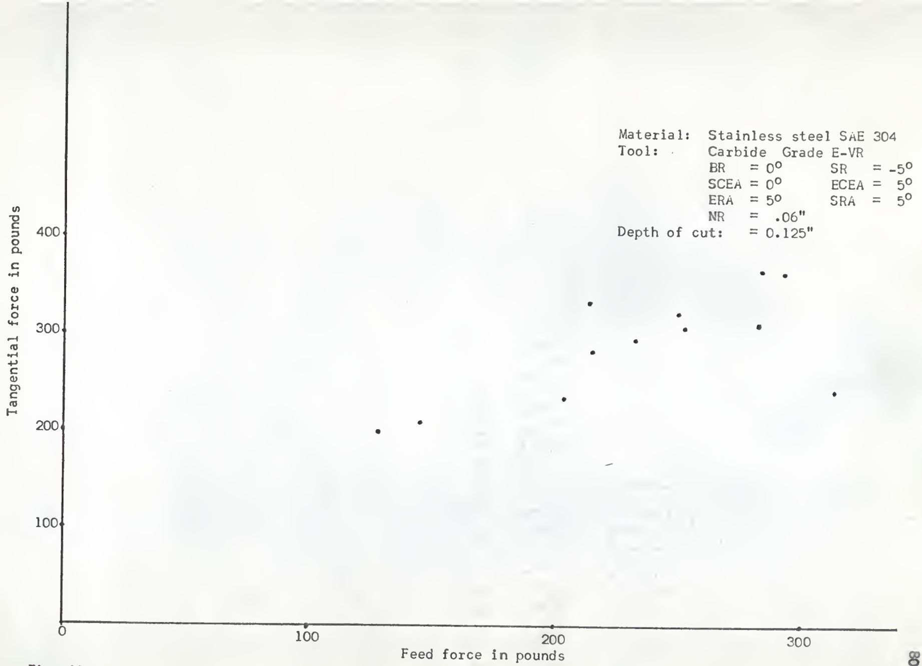


Fig. 19. The correlation between feed force and tangential force under -5° side rake angle.

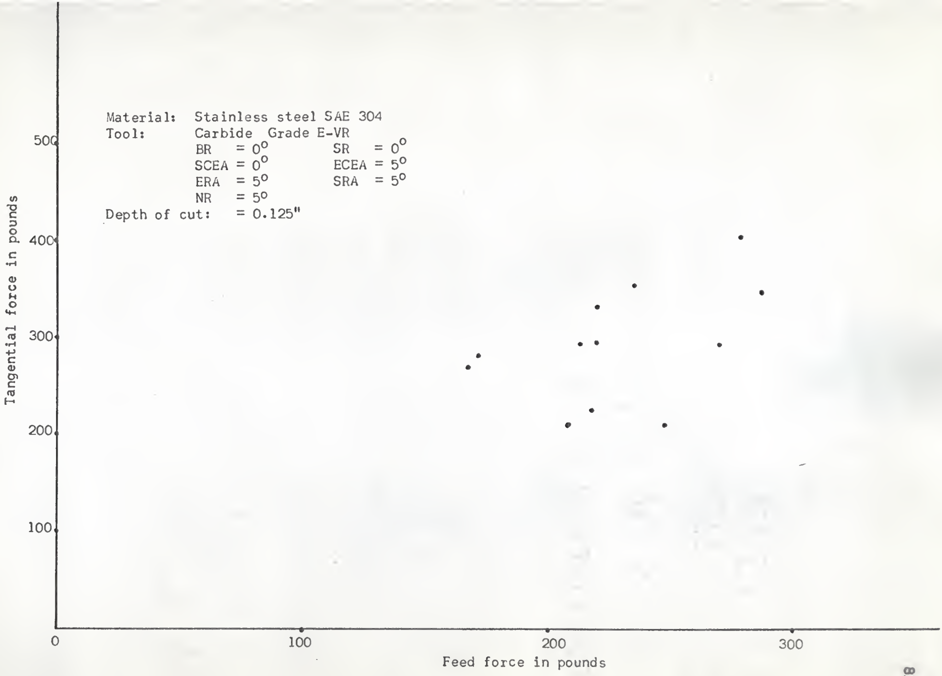


Fig. 20. The correlation between feed force and tangential force under 0° side rake angle.

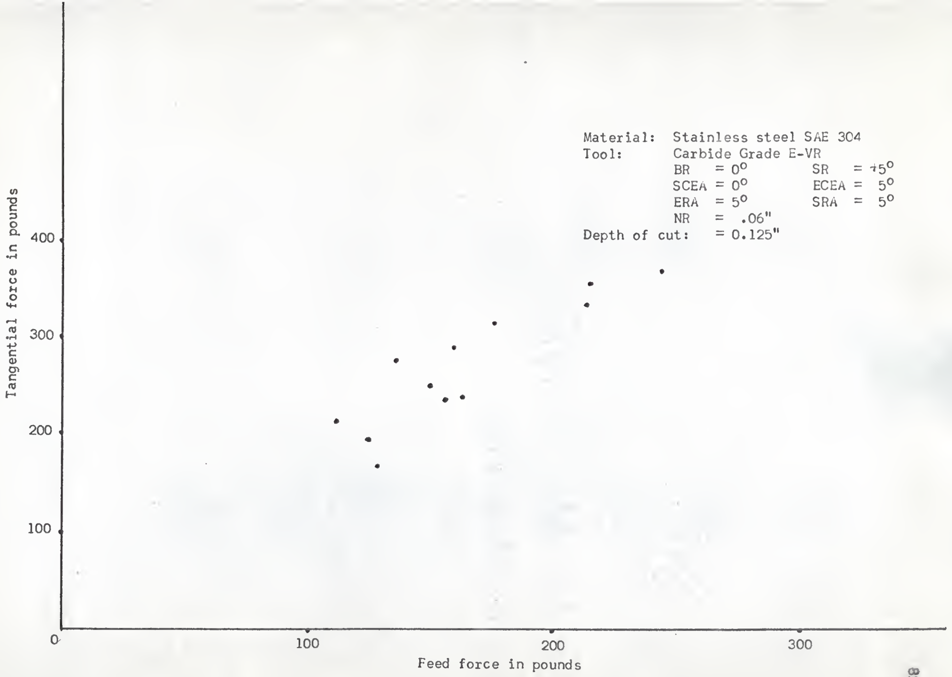


Fig. 21. The correlation between feed force and tangential force under +5° side rake angle.

to employ if reduction of cutting forces are desired, when machining stainless steel with carbide tools.

It has been shown by many observers (16, 17) that tools with positive side rake angles generate lesser cutting forces than those with zero or negative side rake angles. The reason for this is that when employing tools with positive side rake angles, the shear plane angle is larger than when employing tools with zero or negative side rake angles (16, 17). Figure 22 shows the relative shear plane angles when machining with the three different forms of tools. A decrease in shear plane angle suggests that the extent of the shear plane will be increased and consequently, this results in higher cutting forces. This is so because the area of shear is also increased with a lengthening of the shear plane.

The experiments also brought out the fact that at low levels of feed such as at .002 ipr, the feed force is considerably less than at high levels of feed. When the feed is comparatively less, the horizontal thrust made by the tool on the work-piece would also be less and hence the low feed force.

The tendency for the cutting forces to decrease as speed increases is thought to be due to a decrease in coefficient of friction, which is evidently so decreased that the cutting forces are also decreased correspondingly. This was found to be the general trend from the machining data obtained. Any deviation from this may be considered as characteristic of these particular machining conditions.

Interactions Between Side Rake Angle, Feed and Speed

Both on feed forces and tangential forces, the interaction effects of side rake angle, feed and speed were found to be highly significant. The

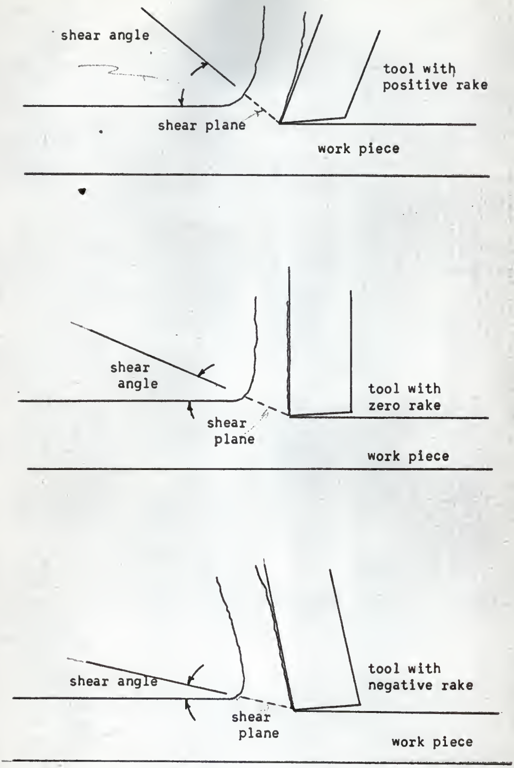


Fig. 22. Schematic diagram showing the effects of tools with different rake angles on shear plane inclination during machining.

magnitude of these forces are therefore dependent on different levels of side rake angle, levels of feed and speed.

Break-down Analysis of Feed Forces and Tangential Forces

At all levels of side rake angle, the feed speed interactions were found to be significant on the feed force.

However, at side rake angle of -5°, the feed-speed interaction did not prove significant when considering the tangential forces. At levels of 0° and $+5^{\circ}$ side rake angles, however, the speed-feed interaction appears to be having significant effect on the tangential forces.

Feed Speed Relationships on Forces

It was found possible to designate mathematical equations to express the magnitude of cutting forces in relation to the magnitude of feed and speed, under different side rake angles.

Correlation Between Forces

Significant correlation was found to exist between the feed and tangential forces under different side rake angle levels. It was of interest to note that at -5° and $+5^{\circ}$ side rake angles the correlation between the feed force and the tangential forces were about the same. This would therefore suggest that the interdependency between the forces at these two side rake angles appears to be the same, whereas the correlation coefficient between the two forces at 0° side rake angle was found to be much less than that for the -5° and $+5^{\circ}$ side rake angles.

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APPENDICES

Appendix A - Computer Program for Calculating Mean Squares

```
      DIMENSION A(12,9),B(4,4),C(5,4),D(4,4),E(5,4),F(5,10)
      PRINT 500
  500 FORMAT(12H INPUT DATA )
    1 FORMAT(12F5.0)
      READ 1,((A(I,J),I=1,12),J=1,9)
    2 FORMAT(12F5.0)
      PRINT2,((A(I,J),I=1,12),J=1,9)
      PRINT100
  100 FORMAT(18H REP X ANGLE INTER)
      DO 5 K=1,3
      DO 5 L=1,3
    5 B(K,L)=0.
      DO 10 I=1,4
      DO 10 J=1,3
   10 B(1,1)=B(1,1)+A(I,J)
      DO 15 I=5,8
      DO 15 J=1,3
   15 B(2,1)=B(2,1)+A(I,J)
      DO 20 I=9,12
      DO20 J=1,3
   20 B(3,1)=B(3,1)+A(I,J)
      DO 30 I=1,4
      DO 30 J=4,6
   30 B(1,2)=B(1,2)+A(I,J)
      DO40 I=5,8
      DO40 J=4,6
   40 B(2,2)=B(2,2)+A(I,J)
      DO 50 I=9,12
      DO 50 J=4,6
   50 B(3,2)=B(3,2)+A(I,J)
      DO 60 I=1,4
      DO 60 J=7,9
   60 B(1,3)=B(1,3)+A(I,J)
      DO 70 I=5,8
      DO 70 J=7,9
   70 B(2,3)=B(2,3)+A(I,J)
      DO 80 I=9,12
      DO 80J=7,9
   80 B(3,3)=B(3,3)+A(I,J)
      B(4,1)=B(1,1)+B(2,1)+B(3,1)
      B(4,2)=B(1,2)+B(2,2)+B(3,2)
      B(4,3)=B(1,3)+B(2,3)+B(3,3)
      B(4,4)=B(4,1)+B(4,2)+B(4,3)
      B(1,4)=B(1,1)+B(1,2)+B(1,3)
      B(2,4)=B(2,1)+B(2,2)+B(2,3)
      B(3,4)=B(3,1)+B(3,2)+B(3,3)
   90 FORMAT(4F10.3)
      PRINT 90,((B(K,L),K=1,4),L=1,4)
      PRINT 106
  106 FORMAT(19H SPEED X FEED INTER)
```

```
      DO 6 I=1,5
      DO 6 J=1,4
    6 C(I,J)=0.
      M=0
      DO 16 K=1,4
      IP1=M+1
      IP2=M+9
      DO 26 I=IP1,IP2,4
      DO 26 J=1,7,3
   26 C(K,1)=C(K,1)+A(I,J)
      M=M+1
   16 CONTINUE
      M=0
      DO 36 K=1,4
      IP1=M+1
      IP2=M+9
      DO 46 I=IP1,IP2,4
      DO 46 J=2,8,3
   46 C(K,2)=C(K,2)+A(I,J)
      M=M+1
   36 CONTINUE
      M=0
      DO 56 K=1,4
      IP1=M+1
      IP2=M+9
      DO 66 I=IP1,IP2,4
      DO 66 J=3,9,3
   66 C(K,3)=C(K,3)+A(I,J)
      M=M+1
   56 CONTINUE
      DO 76 N=1,4
      C(5,1)=C(5,1)+C(N,1)
      C(5,2)=C(5,2)+C(N,2)
   76 C(5,3)=C(5,3)+C(N,3)
      DO 96 L=1,3
      C(1,4)=C(1,4)+C(1,L)
      C(2,4)=C(2,4)+C(2,L)
      C(3,4)=C(3,4)+C(3,L)
      C(4,4)=C(4,4)+C(4,L)
   96 C(5,4)=C(5,4)+C(5,L)
      PRINT 86,((C(I,J),I=1,5),J=1,4)
   86 FORMAT(5F10.3)
      PRINT124
  124 FORMAT(19H ANGLE X FEED INTER)
      DO4 K=1,3
      DO4 L=1,3
    4 D(K,L)=0.
      DO14 I=1,12
   24 D(1,1)=D(1,1)+A(I,1)
   34 D(2,1)=D(2,1)+A(I,4)
   44 D(3,1)=D(3,1)+A(I,7)
```

```
54 D(1,2)=D(1,2)+A(I,2)
64 D(2,2)=D(2,2)+A(I,5)
74 D(3,2)=D(3,2)+A(I,8)
84 D(1,3)=D(1,3)+A(I,3)
94 D(2,3)=D(2,3)+A(I,6)
14 D(3,3)=D(3,3)+A(I,9)
   D(4,1)=D(1,1)+D(2,1)+D(3,1)
   D(4,2)=D(1,2)+D(2,2)+D(3,2)
   D(4,3)=D(1,3)+D(2,3)+D(3,3)
   D(4,4)=D(4,1)+D(4,2)+D(4,3)
   D(1,4)=D(1,1)+D(1,2)+D(1,3)
   D(2,4)=D(2,1)+D(2,2)+D(2,3)
   D(3,4)=D(3,1)+D(3,2)+D(3,3)
104 FORMAT(4F10.3)
114 PRINT104,((D(I,J),I=1,4),J=1,4)
   PRINT 103
103 FORMAT(20H ANGLE X SPEED INTER)
   DO 203 K=1,4
   DO203 L=1,3
203 E(K,L)=0.
   M=0
   DO 33 N=1,4
   IP1=M+1
   IP2=M+9
   DO 13 I=IP1,IP2,4
   DO13 J=1,3
 13 E(N,1)=E(N,1)+A(I,J)
   M=M+1
 33 CONTINUE
   M=0
   DO 43 N=1,4
   IP1=M+1
   IP2=M+9
   DO53 I=IP1,IP2,4
   DO53 J=4,6
 53 E(N,2)=E(N,2)+A(I,J)
   M=M+1
 43 CONTINUE
   M=0
   DO 63 N=1,4
   IP1=M+1
   IP2=M+9
   DO 73 I=IP1,IP2,4
   DO 73 J=7,9
 73 E(N,3)=E(N,3)+A(I,J)
   M=M+1
 63 CONTINUE
   E(5,1)=E(1,1)+E(2,1)+E(3,1)+E(4,1)
   E(5,2)=E(1,2)+E(2,2)+E(3,2)+E(4,2)
   E(5,3)=E(1,3)+E(2,3)+E(3,3)+E(4,3)
   E(1,4)=E(1,1)+E(1,2)+E(1,3)
```

```
      E(2,4)=E(2,1)+E(2,2)+E(2,3)
      E(3,4)=E(3,1)+E(3,2)+E(3,3)
      E(4,4)=E(4,1)+E(4,2)+E(4,3)
      E(5,4)=E(5,1)+E(5,2)+E(5,3)
   83 FORMAT(5F10.3)
      PRINT83,((E(M,N),M=1,5),N=1,4)
      PRINT 107
  107 FORMAT(27H ANGLE X FEED X SPEED INTER)
      DO 7 K=1,5
      DO 7 L=1,10
    7 F(K,L)=0.
      M=0.
      DO 17 J=1,4
      IP1=M+1
      IP2=M+9
      DO 27 I=IP1,IP2,4
      F(J,1)=F(J,1)+A(I,1)
      F(J,2)=F(J,2)+A(I,2)
      F(J,3)=F(J,3)+A(I,3)
      F(J,4)=F(J,4)+A(I,4)
      F(J,5)=F(J,5)+A(I,5)
      F(J,6)=F(J,6)+A(I,6)
      F(J,7)=F(J,7)+A(I,7)
      F(J,8)=F(J,8)+A(I,8)
   27 F(J,9)=F(J,9)+A(I,9)
      M=M+1
   17 CONTINUE
      DO 37 M=1,4
      F(5,1)=F(5,1)+F(M,1)
      F(5,2)=F(5,2)+F(M,2)
      F(5,3)=F(5,3)+F(M,3)
      F(5,4)=F(5,4)+F(M,4)
      F(5,5)=F(5,5)+F(M,5)
      F(5,6)=F(5,6)+F(M,6)
      F(5,7)=F(5,7)+F(M,7)
      F(5,8)=F(5,8)+F(M,8)
   37 F(5,9)=F(5,9)+F(M,9)
      DO 47 N=1,9
      F(1,10)=F(1,10)+F(1,N)
      F(2,10)=F(2,10)+F(2,N)
      F(3,10)=F(3,10)+F(3,N)
      F(4,10)=F(4,10)+F(4,N)
   47 F(5,10)=F(5,10)+F(5,N)
   57 FORMAT(5F10.3)
      PRINT 57,((F(I,J),I=1,5),J=1,10)
      SUM=0.
      RAFS=0.
      DO 108 I=1,12
      DO 108 J=1,9
      SUM=SUM+A(I,J)
  108 RAFS=RAFS+A(I,J)**2.
```

```
      CF=(SUM**2.)/108.
      REPU=(((B(1,4)**2.)+(B(2,4)**2.)+(B(3,4)**2.))/36.)-CF
      AFSQ=0.
      DO 118 I=1,4
      DO 118 J=1,9
  118 AFSQ=AFSQ+F(I,J)**2.
      AFSU=(AFSQ/3.)-CF
      AXFQ=0.
      DO 128 I=1,3
      DO 128 J=1,3
  128 AXFQ=AXFQ+D(I,J)**2.
      AXFU=(AXFQ/12.)-CF
      AXSQ=0.
      DO 138 I=1,4
      DO 138 J=1,3
  138 AXSQ=AXSQ+E(I,J)**2.
      AXSU=(AXSQ/9.)-CF
      FXSQ=0.
      DO 148 I=1,4
      DO 148 J=1,3
  148 FXSQ=FXSQ+C(I,J)**2.
      FXSU=(FXSQ/9.)-CF
      ASS=(((B(4,1)**2.)+(B(4,2)**2.)+(B(4,3)**2.))/36.)-CF
      FSS=(((C(5,1)**2.)+(C(5,2)**2.)+(C(5,3)**2.))/36.)-CF
      SSS=(((C(1,4)**2.)+(C(2,4)**2.)+(C(3,4)**2.)+(C(4,4)**2.))/27.)-CF
      RMS=REPU/2.
      AMS=ASS/2.
      FMS=FSS/2.
      SMS=SSS/3.
      AFMS=(AXFU-ASS-FSS)/4.
      ASMS=(AXSU-ASS-SSS)/6.
      FSMS=(FXSU-FSS-SSS)/6.
      AFS=(AFSU-(ASS+FSS+SSS+4.*AFMS+6.*ASMS+6.*FSMS))/12.
      ERR=((RAFS-CF)-(REPU+AFSU))/70.
      PRINT 158
  158 FORMAT(40H    RMS        AMS        FMS        SMS    )
      PRINT 168,RMS,AMS,FMS,SMS
  168 FORMAT(4F11.3)
      PRINT 178
  178 FORMAT(40H   AFMS       ASMS       FSMS       AFS    )
      PRINT 188,AFMS,ASMS,FSMS,AFS
  188 FORMAT(4F10.3)
      PRINT 198
  198 FORMAT(10H    ERR   )
      RAFT=RAFS-CF
      TSS=AFSU+REPU+ERR*70.
      PRINT 200,ERR
  200 FORMAT(F10.3)
      PRINT 300
  300 FORMAT(51H        TSS            RAFT          CHECK ON TOTAL SS )
      PRINT 333,TSS,RAFT
  333 FORMAT(F14.6,F14.6)
      END
```

Appendix B - Computer Program for Calculating the Linear and Higher Order Effects of Feed and Speed on the Cutting Forces, under Different Side Rake Angles

```
      DIMENSION A(12,9),E(5,4),F(5,10),Y(12,9)
    1 FORMAT(12F5.0)
      READ 1,((A(I,J),I=1,12),J=1,9)
      READ 83,((E(M,N),M=1,5),N=1,4)
   83 FORMAT(5F10.3)
      READ 57,((F(I,J),I=1,5),J=1,10)
   57 FORMAT(5F10.3)
    9 FORMAT(12F5.0)
      READ 9,((Y(I,J),I=1,12),J=1,9)
      I=1
      J=1
      K=2
      L=3
  404 SL=(((-3.*E(1,I))+(-1.*E(2,I))+(1.*E(3,I))+(3.*E(4,I)))**2.)/180.
      SQ=(((1.*E(1,I))+(-1.*E(2,I))+(-1.*E(3,I))+(1.*E(4,I)))**2.)/36.
      SC=(((-1.*E(1,I))+(3.*E(2,I))+(-3.*E(3,I))+(1.*E(4,I)))**2.)/180.
      FL=(((-1.*F(5,J))+(0.*F(5,K))+(1.*F(5,L)))**2.)/24.
      FQ=(((1.*F(5,J))+(-2.*F(5,K))+(1.*F(5,L)))**2.)/72.
      L=L+3
      K=K+3
      J=J+3
      PRINT 411,SL,SQ,SC,FL,FQ
  411 FORMAT(5F10.3)
      I=I+1
      IF(I-4)404,436,436
  436 I=1
      M=1
      N=2
      K=3
  437 F1=(-3.*F(1,M))+(-1.*F(2,M))+(1.*F(3,M))+(3.*F(4,M))
      F2=(-3.*F(1,N))+(-1.*F(2,N))+(1.*F(3,N))+(3.*F(4,N))
      F3=(-3.*F(1,K))+(-1.*F(2,K))+(1.*F(3,K))+(3.*F(4,K))
      SLFL=(((-1.*F1)+(0.*F2)+(1.*F3))**2.)/120.
      SLFQ=(((1.*F1)+(-2.*F2)+(1.*F3))**2.)/360.
      FF1=(1.*F(1,M))+(-1.*F(2,M))+(-1.*F(3,M))+(1.*F(4,M))
      FF2=(1.*F(1,N))+(-1.*F(2,N))+(-1.*F(3,N))+(1.*F(4,N))
      FF3=(1.*F(1,K))+(-1.*F(2,K))+(-1.*F(3,K))+(1.*F(4,K))
      SQFL=(((-1.*FF1)+(0.*FF2)+(1.*FF3))**2.)/24.
      SQFQ=(((1.*FF1)+(-2.*FF2)+(1.*FF3))**2.)/72.
      FFF1=(-1.*F(1,M))+(3.*F(2,M))+(-3.*F(3,M))+(1.*F(4,M))
      FFF2=(-1.*F(1,N))+(3.*F(2,N))+(-3.*F(3,N))+(1.*F(4,N))
      FFF3=(-1.*F(1,K))+(3.*F(2,K))+(-3.*F(3,K))+(1.*F(4,K))
      SCFL=(((-1.*FFF1)+(0.*FFF2)+(1.*FFF3))**2.)/120.
      SCFQ=(((1.*FFF1)+(-2.*FFF2)+(1.*FFF3))**2.)/240.
      M=M+3
      N=N+3
      K=K+3
      PRINT 424,SLFL,SLFQ,SQFL,SQFQ,SCFL,SCFQ
```

```
424 FORMAT(6F11.2)
    I=I+1
    IF(I-4)437,438,436
438 L=1
    M=3
    K=1
    N=3
403 CR=0.
    SIGY=0.
    SUMT=0.
    TSUM=0.
    DO 400 I=1,12
    DO400 J=L,M
    CR=CR+A(I,J)
400 SIGY=SIGY+A(I,J)**2.
    CF=(CR**2.)/36.
    YSQ=SIGY-CF
    DO 401 I=1,4
    DO 401 J=K,N
401 SUMT=SUMT+F(I,J)**2.
    TSUM=(SUMT/3.)-CF
    PRINT 402,CF,YSQ,TSUM
402 FORMAT(3F14.2)
    K=K+3
    N=N+3
    L=L+3
    M=M+3
    IF(M-9)403,403,666
666 M=1
    N=3
 24 XY=0.
    XZ=0.
    YZ=0.
    XSUM=0.
    YSUM=0.
    DO 20 I=1,12
    DO 20 J=M,N
    XSUM=XSUM+A(I,J)
 20 YSUM=YSUM+Y(I,J)
    XAV=XSUM/36.
    YAV=YSUM/36.
    DO 21 I=1,12
    DO 21 J=M,N
    XY=XY+(A(I,J)-XAV)*(Y(I,J)-YAV)
    XZ=XZ+(A(I,J)-XAV)**2.
 21 YZ=YZ+(Y(I,J)-YAV)**2.
    CORR=XY/(XZ*YZ)**.5
    PRINT 23,CORR
 23 FORMAT(F14.6)
    M=M+3
    N=N+3
    IF(N-9)24,24,25
 25 ND=1
    END
```

STATISTICAL ANALYSIS OF METAL CUTTING DATA

by

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The object of this thesis was to analyze statistically data comprising of metal cutting forces so as to observe the manner in which the prominent variables in metal cutting process, and their possible interactions affect the magnitude of the tangential and feed forces. The statistical model conceived was one of 3 factor factorial completely randomized block design. The variable factors under control were side rake angles (-5°, 0°, $+5^{\circ}$), three levels of feed (.002 ipr, .004 ipr, and .006 ipr) and four levels of speed (61.5 sfpm, 129.5 sfpm, 199.0 sfpm, and 240.5 sfpm).

The experiment was conducted on stainless steel SAE 304, using brazed carbide tools of grade E-VR. Conventional form of machining was performed and all metal cutting performed dry.

A three dimensional lathe tool dynamometer was used to measure the feed and tangential forces. The dynamometer, along with a Sanborn strain gage amplifying unit was calibrated and checked for accuracy prior to the experiment.

All calculations pertaining to the experiment were done on the IBM 1620 digital computer using the Fortran II machine language.

The investigations showed that all factors and their interactions were prominently significant, on both the feed force as well as the tangential forces. A further breakdown analysis was then attempted to observe whether the feed and speed have any significant influence on the cutting forces under the tools possessing the three different side rake angles. While analyzing the feed forces it was noticed that the effects of factors, feed, speed and their interaction feed-speed assume significant proportion under all three different side rake angles. However, analysis of tangential forces showed that though feed and speed effects are significant under all side rake angles, the feed-speed interaction was non-significant under -5^{0} side rake angle.

An attempt was also made to study the response curves under different side rake angles and to establish empirical formulae to measure the feed and tangential forces by means of orthogonal polynomials, with feed and speed as the variables.

Finally, the correlation between the feed and tangential forces under different side rake angles were investigated by computing the coefficient of correlation between the two forces. It became apparent that at side rake angles of -5^{0} and $+5^{0}$, the correlation coefficient appeared to be very much the same in 0.8709 and 0.8793, respectively, and at 0^{0} side rake angle it was considerably lower, being 0.5432.