A POSITION SERVOMECHANISM WITH THE GAIN
MODULATED BY THE OUTPUT VELOCITY

by

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SYMBOLS

a, b, c	– Parameters defining a system
e	– D-c voltage
E	– Error signal
J	– Inertia of motor rotor, gears, pots, and reflected load
K_1, K_2, K_3	– Gain constants
p	– Operator d/dt
R	– Coefficient of resistance of motor and reflected load
t	– Time in seconds
W	– Undamped natural angular frequency in radians per second
x	– Input to servo in the time domain
y	– Output of servo in the time domain
X	– Input to servo in the Laplace domain
Y	– Output of servo in the Laplace domain
$\dot{y}$	– First derivative with respect to time
$\ddot{y}$	– Second derivative with respect to time
ζ	– Damping factor

INTRODUCTION

The design of a linear servomechanism (hereinafter fore-
shortened to servo) for a given performance is constrained by
the physical characteristics of its components. Often this dif-
ficulty is overcome by selecting higher performance components.
Most of the time, however, the desired specifications are beyond
the linear systems capability.

If nonlinearities are introduced, the system's performance
can sometimes be improved without an improvement in the charac-
teristics of the components. The use of nonlinear elements or
operations results in nonlinear differential equations which are
extremely difficult to solve. It is worth while to investigate
the nonlinear servos because of their better performance as con-
trasted to linear servos. It is a major task to choose a non-
linear element or operation suitable to achieve the performance
desired. One possible solution is to simply have a catalog of
nonlinear systems available and investigate the various ones for
application to the problem at hand.

The usual procedure in designing a nonlinear servo is to
begin with a linear servo and improve its performance by the
introduction of nonlinearities.

A method of introducing nonlinearities of a multiplicative
type into a given linear servo is presented in this paper. The
gain of the system is multiplied by a function of the magnitude,
not the sense, of the output velocity. The resulting response
is compared to the response of an equivalent linear second order

servo for evaluation. The resulting nonlinear servo has superior performance as compared to an equivalent linear servo.

OTHER NONLINEAR SERVOS

It is worth while to investigate other nonlinear servos so that some aspects of the proposed servo can be compared to other nonlinear servos.

An outstanding contribution to nonlinear control systems was made by Flügge-Lotz, Taylor, and Lindberg in 1958 (1). The technique employs precise variation of proportional and derivative feedback parameters. It is achieved by a logic circuit which switches in combinations of proportional and velocity feedback to give the desired optimal performance.

A simple nonlinear second-order servo presented by Zaborszky (4), in 1958, emphasized simplicity. This is accomplished with a minimum of equipment and circuitry.

This servo utilizes the fact that the peak time and overshoot associated with a step function response of a second-order position servo are independent of the input height, and depends only on the system itself. A timer-actuated potentiometer ahead of a linear second-order servomechanism reduces the input to a level which allows the height of the first overshoot to be equal to the height of the original step input. At that time, the timer switches in the complete step input. Because the system is now at zero error, this switching does not affect the servo's output.

C. L. Smith and C. T. Leondes (3) investigated the practical problems encountered in the design of relay servos with attendant nonlinearities. The performance and suitability of various types were compared. The goal of the paper was to provide the designer with a means of confidently selecting servo parameters to account for the physical limitation of this equipment.

One system discussed was the "optimum" relay servo. The fundamental idea in relay servos is to utilize the full torque output of the motor at all times in order to provide the maximum possible acceleration of the load. This is in contrast with the linear servo which operates at a fraction of its torque potential. The relay servo will give the fastest response obtainable using a given servo motor.

A servo described by Lewis (2) in 1952, uses nonlinear damping to improve the step response of a linear position servo. The feedback is so arranged that the servo has small damping during the early portion of the transient response, resulting in high initial velocity. As the output approaches the input, the damping becomes large, and thus the overshoot is minimized. The resulting step response is improved over that of the linear servo.

EVALUATION OF THESE NONLINEAR SERVOS

The above examples illustrate some of the existing non-linear servos. Each has its advantages along with its limitations and disadvantages.

Zaborszky (4) was concerned with only a step response to a servo with fixed parameters. With these limitations he obtained a very fast response with the addition of only a timer-actuated potentiometer to the original linear servo.

The discontinuous feedback technique by Flügge-Lotz, Taylor, and Lindberg (1) results in a system that can follow random inputs with great accuracy. This requires the use of logic circuit and switching circuits and the servo is characterized by the expected equipment complexity. A common drawback of most relay servos is the rather complex switching and monitoring equipment required to switch at the right time to obtain the desired response. A relay servo never achieves zero steady-state conditions. If these drawbacks are tolerable, the relay control system will give the fastest response obtainable using a given servo motor.

The purpose of the ensuing investigation is to study non-linear methods of a special sort which are applicable to low-power continuous servos and which improve their initial state performance without excessive equipment complexity.

THE SECOND-ORDER LINEAR SERVO

The characteristics of a linear servo can be described by a linear differential equation with constant coefficients. Linear control system analysis simply sets up a criterion to yield optimum values for the coefficients. Consider a linear second-order position servo with unity feedback. The closed-loop

differential equation is:

$$J\ddot{y} + R\dot{y} = K_1E \qquad (1)$$

$$E = x - y$$

Therefore $\qquad J/K_1\,\ddot{y} + R/K_1\dot{y} + y = x \qquad (2)$

where J, R, K_1 are constants, x is the input, and y is the
output.

Equation (2) is a linear second-order differential equation with constant coefficients of the form:

$$a\ddot{y} + b\dot{y} + cy = x \qquad (3)$$

The three constants (a, b, c) completely characterize the linear system.

For a given linear system, the inertia, J, is usually fixed. The parameters left to be varied are the gain, K, and coefficient of resistance, R.

Subtractive velocity feedback is commonly used to increase the servo's damping. With this velocity feedback, the error signal is

$$E = x - y - K_2\dot{y}$$

Therefore the differential equation is:

$$J/K_1\ddot{y} + (R/K_1 + K_2)\dot{y} + y = x \qquad (4)$$

The coefficient of resistance is now $(R/K_1 + K_2)$ rather than R/K_1.

Applying the Laplace transformation to equation (4), with initial conditions equal to zero, yields

$$\left[Jp^2 + (R + K_1K_2)p + K_1\right] y = K_1x \qquad (5)$$

The transfer function y/x is:

$$\frac{y}{x} = K_1/j \cdot \frac{1}{p^2 + (\frac{R + K_1 K_1}{J})p + K_1/j} \tag{6}$$

$$\frac{y}{x} = K_1/J \cdot \frac{1}{p^2 + 2 \int W_p + W^2} \tag{7}$$

where $\quad W = (K_1/J)^{1/2}, \quad \int = \frac{R + K_1 K_2}{2(K_1/J)^{1/2}}$

The three possibilities for damping factor , $\int$, are:

    1. Critically damped, $\int = 1$.

    2. Overdamped, $\int > 1$.

    3. Underdamped, $\int < 1$.

The limited performance due to the physical characteristics of the components of a linear second-order servo is now illustrated. Assume it is desired to achieve the fastest step response possible without overshoot. The solution is that of critical damping, where $\int = 1$. An increase in gain gives a faster response but results in overshoot. If the rise time required was less than that of critical damping, the given servo could not meet the specifications.

The next question is how could the given linear servo be used to obtain a step response faster than that of critical damping without overshoot? Nonlinearization of the given servo is a solution to the above problem. The servo being investigated has a step-function response which is superior to the response of the equivalent linear servo.

NONLINEARITY BY MULTIPLICATIVE OPERATION

The feature that enables the proposed nonlinear servo to achieve a faster time response is that its gain is variable. The gain is a function of the output velocity. To make it insensitive to the sign of the velocity, the absolute value of the output velocity is used. If the linear servo is overdamped or critically damped, the gain needs to increase if the servo is to reach zero error faster.

A block diagram of the proposed servo is shown in Fig. 1. Note the only additional components required are a multiplier and an absolute value device.

The closed-loop differential equation of the servo is:

$$J\ddot{y} + R\dot{y} = K_1(e + |K_3\dot{y}|)E \qquad (8)$$
$$E = x - y - K_2\dot{y}$$

Reassembling equation (8) yields

$$J\ddot{y} + \left[R + K_1K_2(e + |K_3\dot{y}|)\right]\dot{y}$$
$$+ K_1(e + |K_3\dot{y}|)y = K_1(e + |K_3\dot{y}|)x \qquad (9)$$

The nonlinearity of this servo is the result of a multiplicative operation. From the block diagram in Fig. 1, it can be seen that the error $(x - y - K_2py)$ is multiplied by $(e + |K_3py|)$. e is a constant equal to one for simplicity. The inherent possibilities are next investigated by analog computer simulation.
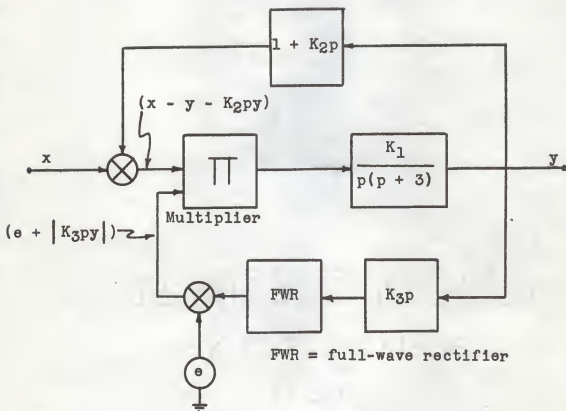
Fig. 1. Block diagram of a nonlinear servo
with the gain modulated by the output
velocity. Referred to as Servo (3).

ANALOG COMPUTER SIMULATION

Because of difficulty in obtaining an analytical solution
of nonlinear servos, this investigation leaned heavily on an
analog computer simulation. The evaluation of the nonlinear
servo is made by comparing its responses to the responses of
two equivalent linear servos. By equivalent is meant that they
are identical except for the nonlinearities and a velocity
feedback loop in one case.

The three servos simulated are referred to hereinafter as
servos (1), (2), and (3), and are identified as:

Servo (1) - A linear second-order position servo with pro-
portional feedback, as in Fig. 2.

Servo (2) - A linear second-order position servo with pro-
portional and velocity feedback, as in Fig. 3.

Servo (3) - A nonlinear second-order position servo with
velocity feedback and gain modulated by a function of the out-
put velocity magnitude. Figure 1 is a block diagram of this
nonlinear servo.

The analog computer simulation is shown in Fig. 5. By ad-
justing values of K_1, K_2, and K_3, all three servos were simu-
lated using the same diagram. By doing this, the comparison of
the three servos was more accurate because the characteristics
of the multiplier were included in each simulation.

The portion of the analog computer diagram shown in Fig. 4
gives the transfer function, $\dfrac{1}{p(p + 3)}$, representing the motor,
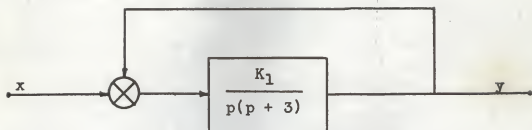gears, load, and pot. To obtain a variation in the R/J term,

Fig. 2. Block diagram of a second-order position servo with proportional feedback. Referred to as Servo (1).
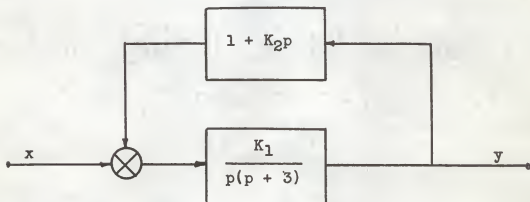

Fig. 3. Block diagram of a second-order position servo with proportional and velocity feedback. Referred to as Servo (2).
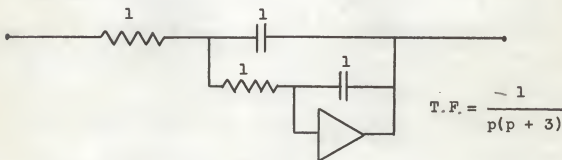

$$T.F. = \frac{-1}{p(p + 3)}$$

Fig. 4. Portion of analog computer diagram that represents motor, gears, load, and pots.
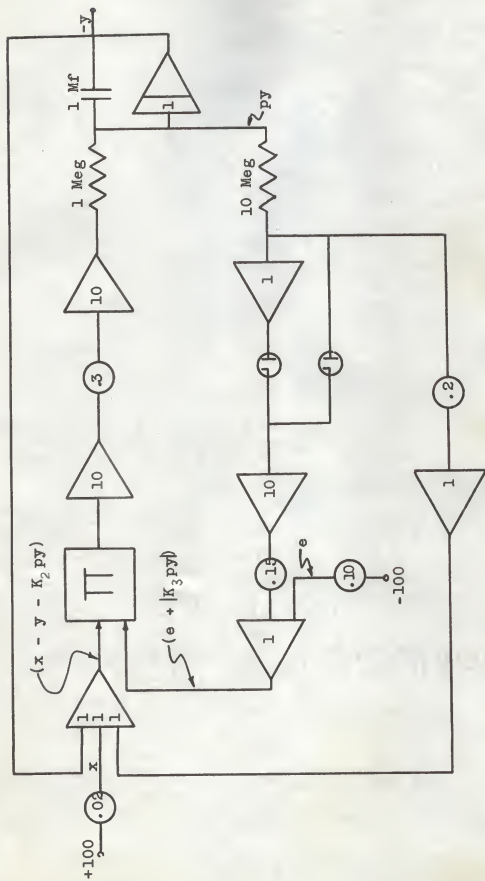
Fig. 5. Analog computer diagram for the simulation of all three servos.

it is only a matter of varying the sizes of the resistors and/
or capacitors.

COMPARING THE LINEAR AND NONLINEAR SERVOS

The first comparison of the three servos was made by com-
paring step-responses of the three servos. The step-responses
were compared at four different values of K_1. K_1 was adjusted
so that servo (1) had a damping factor of critical damping, over-
damping, and underdamping. For the critically damped and over-
damped cases, the step-responses for servos (1) and (3) are
compared. For underdamping the three servos are compared.

With K_1 adjusted so that servo (1) was overdamped, K_2 and
K_3 of servo (3) were adjusted to yield the fastest rise time
with little or no overshoot. At this low value of K_1, equal to
one, servo (3) has a much faster rise time as compared to servo
(1). Measuring rise time from 0.1 to 0.9 of the step-function,
the rise time of servo (3) is 84.4 per cent faster than servo
(1). This is achieved without the usual overshoot associated
with increasing the rise time of a linear servo. The step-
responses obtained from the simulation with K_1 equal to one are
shown in Fig. 6.

The fastest rise time without overshoot is obtained for
servo (1) when K_1 is adjusted to yield critical damping. With
K_1 of servo (3) adjusted to the same value and K_2 and K_3 ad-
justed to yield the fastest rise time without overshoot, servo
(3) has a 68.6 per cent faster rise time than servo (1). This

is possible because of the nonlinearities present in servo (3). The step-response comparisons are shown in Fig. 7.

K_1 was adjusted to two different values that make servo (1) underdamped. For each value of K_1 the step-responses of the three servos were compared. Servo (2) was included for comparison with the nonlinear servo, which also uses velocity feedback as a means of increasing damping. For each, values of K_1, K_2 of servo (2), and K_2 and K_3 of servo (3) were adjusted to yield the fastest rise time with little or no overshoot.

For both cases of underdamping, servo (1) has a considerable amount of overshoot and transient oscillations. By using velocity feedback to increase damping, servo (2) eliminates the overshoot but with a 36 per cent slower rise time. Servo (3) achieves a 28 per cent faster rise time than servo (1) and a 54 per cent faster rise time than servo (2). With K_1 equal to 30 on all three servos, servo (3) has a rise time 16.6 per cent faster than servo (2). Again this increase in step-function performance is due to the modulation of the gain by the output velocity which introduces nonlinearities into the servo.

Figures 8 and 9 show the comparison of the step-responses obtained from the analog simulation of the three servos. K_1 was equal to 10 and 30 which yields underdamping for servo (1).

Another very desirable feature of servo (3) is its relatively consistent step-response for a given range of variation of the R/J term of the transfer function. To illustrate this, a step-function response was obtained on servo (2) and servo (3) for three different values of the R/J term. The three values
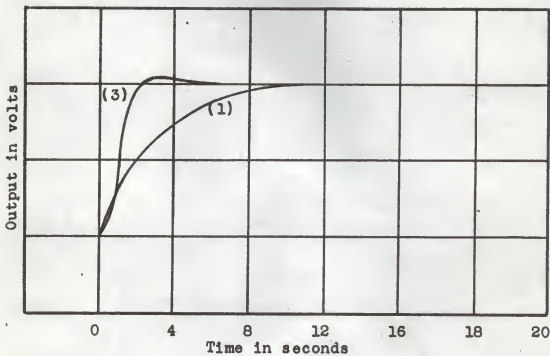
Fig. 6. Step-function responses of
servo (1) and servo (3).

	K_1	K_2	K_3	Rise time in seconds
Servo (1)	1	0	0	.6.4
Servo (3)	1	.1	5	1.0
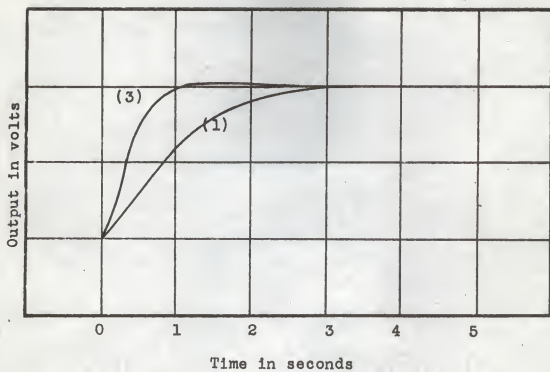
Fig. 7. Step-function responses of
servo (1) and servo (3).

	K_1	K_2	K_3	Rise time in seconds
Servo (1)	3	0	0	1.75
Servo (3)	3	.25	4	0.55
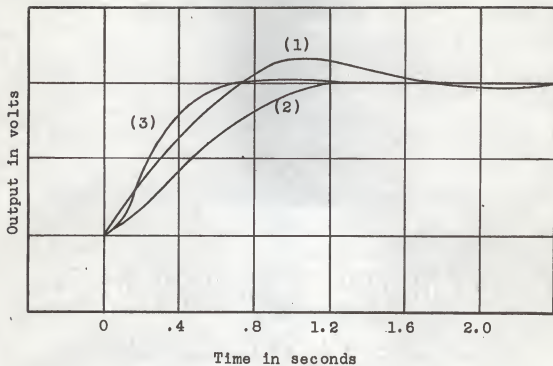
Fig. 8. Step-function responses of
servos (1), (2), and (3).

	K_1	K_2	K_3	Rise time in seconds
Servo (1)	10	0	0	0.5
Servo (2)	10	.2	0	0.78
Servo (3)	10	.2	1.5	0.36
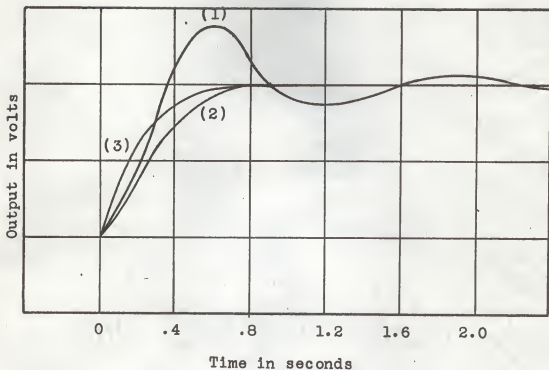
Fig. 9. Step-function responses of servos (1), (2), and (3).

	K_1	K_2	K_3	Rise time in seconds
Servo (1)	30	0	0	0.3
Servo (2)	30	.2	0	0.49
Servo (3)	30	.2	1.5	0.42

chosen were R/J equal to 3, 7, and 12.

The step-responses of servo (2) for the three values of the R/J term have considerable more variation than does the step-responses of servo (3). In contrast to servo (2), the step-responses of servo (3) are virtually independent of the R/J term within a given range. This additional feature is important in many applications and could be called an adaptive feature. The adaptive feature allows the servo to operate under varying parameters but with consistent responses. The allowable variations have limitations and would require further investigation if this feature were to be utilized fully.

Insertion of an automatic gain control circuit could further enhance the adaptive behavior of this servo with respect to changes in R.

The step-responses of servo (2) with the three values of the R/J term are shown in Fig. 10. The corresponding responses of servo (3) are shown in Fig. 11.

To compare the three servos to other types of inputs, they were subjected to sinusoidal and triangular inputs of .2 and .8 cycle per second. The output and error signals were recorded for comparison and are shown in Figs. 12 through 15. In all cases the error signal of servo (3) is smaller than that of servos (1) and (2).

The frequency responses of the three servos were taken for three different values of K_1. In all three cases servo (3) has a wider band width than does servos (1) and (2). The frequency responses are shown in Figs. 16, 17, and 18.
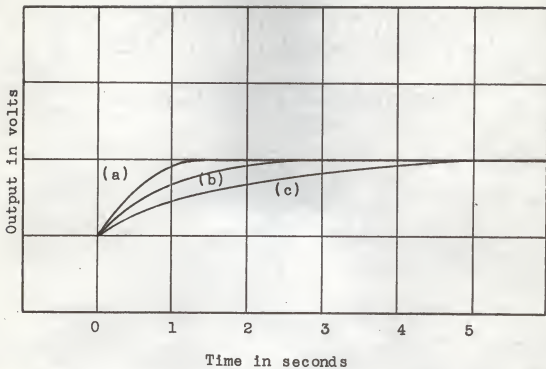
Fig. 10. Step-function responses of servo (2)
with the R/J term, representing the motor,
gears, pots, and load in the transfer
function, being varied.

$$(a). \ TF = \frac{1}{p(p + 3)}$$

$$(b). \ TF = \frac{1}{p(p + 7)}$$

$$(c). \ TF = \frac{1}{p(p + 12)}$$
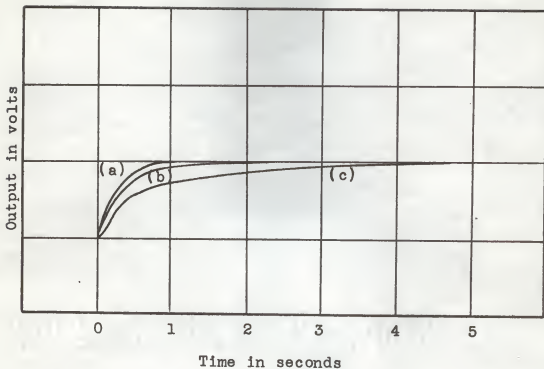
Servo (2) - $K_1 = 10$, $K_2 = .2$, $K_3 = 0$

Fig. 11. Step-function responses of servo (3)
with the R/J term, representing the motor,
gears, pots, and load in the transfer
function, being varied.

$$(a). \quad TF = \frac{1}{p(p + 3)}$$

$$(b). \quad TF = \frac{1}{p(p + 7)}$$

$$(c). \quad TF = \frac{1}{p(p + 12)}$$

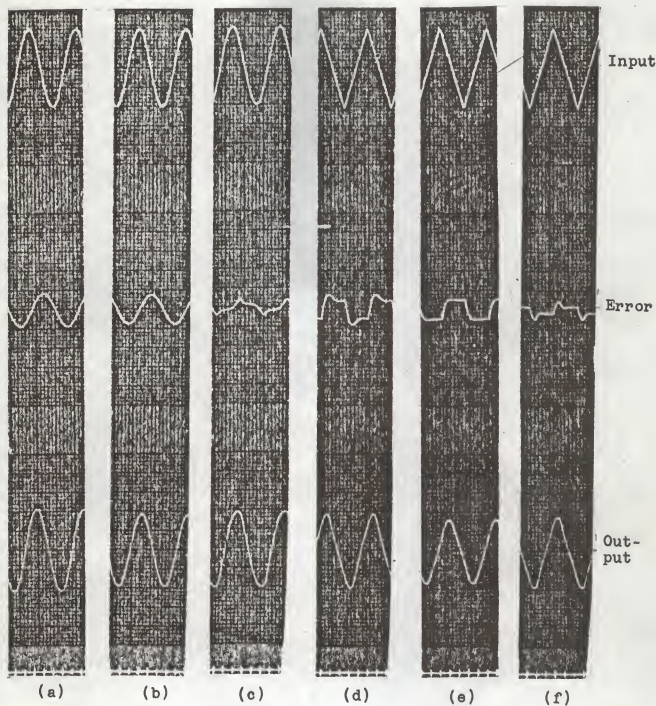Servo (3) - $K_1 = 10$, $K_2 = .2$, $K_3 = 1.5$

Fig. 12. Sinusoidal and triangular responses
of .2 cycle per second.

	K_1	K_2	K_3	Curves
Servo (1)	10	0	0	(a), (d)
Servo (2)	10	.2	0	(b), (e)
Servo (3)	10	.2	1.5	(c), (f)

Input

Error

Out-
put

(a)   (b)   (c)   (d)   (e)   (f)
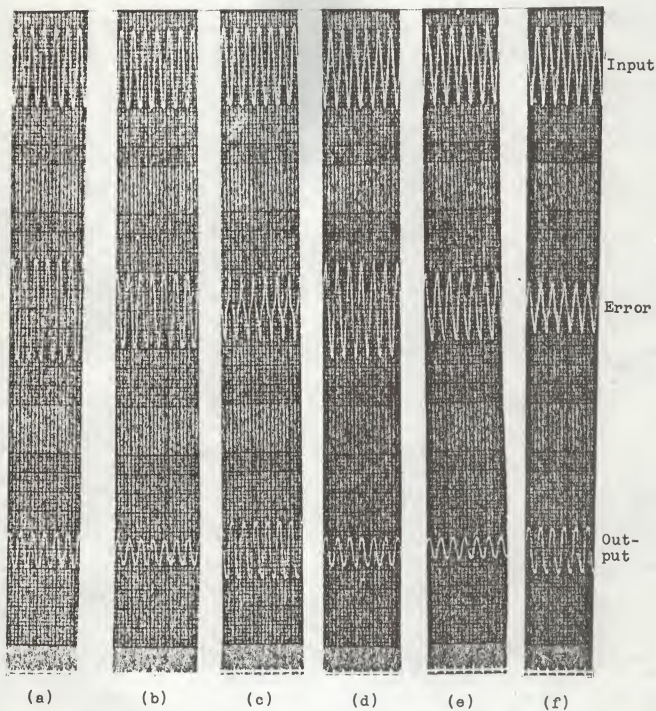
Fig. 13. Sinusoidal and triangular responses
of .8 cycle per second.

	K_1	K_2	K_3	Curves
Servo (1)	10	0	0	(a), (d)
Servo (2)	10	.2	0	(b), (e)
Servo (3)	10	.2	1.5	(c), (f)

Fig. 14.   Sinusoidal and triangular responses
            of .2 cycle per second.

	K_1	K_2	K_3	Curves
Servo (1)	30	0	0	(a), (d)
Servo (2)	30	.2	0	(b), (e)
Servo (3)	30	.2	1.5	(c), (f)

Input

Error

Output
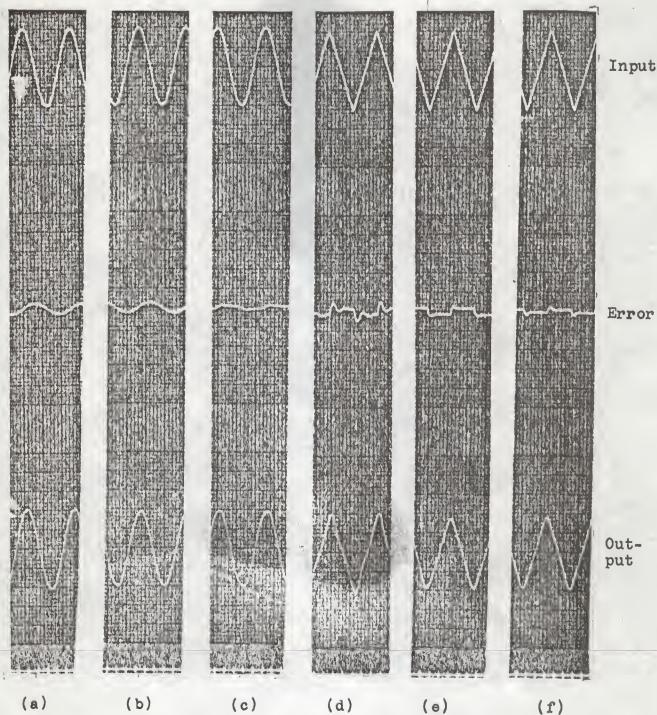
(a)   (b)   (c)   (d)   (e)   (f)
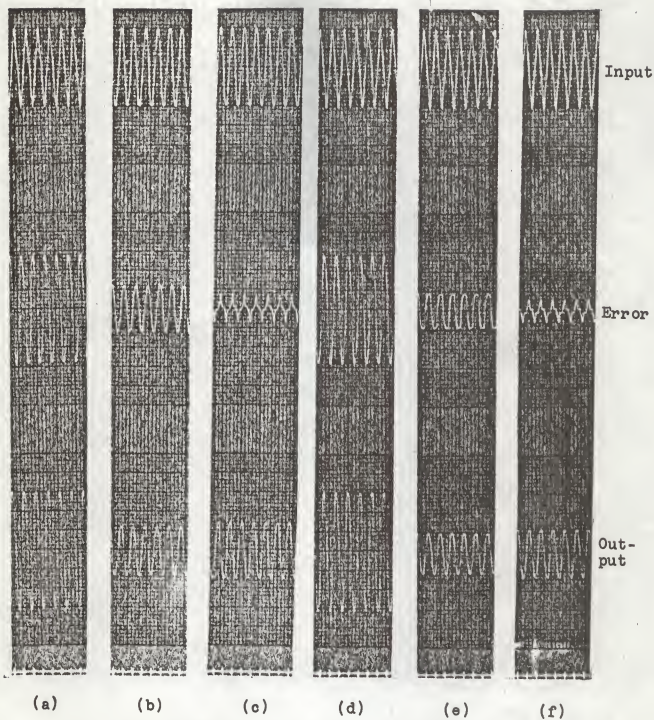
Fig. 15. Sinusoidal and triangular responses of .8 cycle per second.

	K_1	K_2	K_3	Curves
Servo (1)	30	0	0	(a), (d)
Servo (2)	30	.2	0	(b), (e)
Servo (3)	30	.2	1.5	(c), (f)
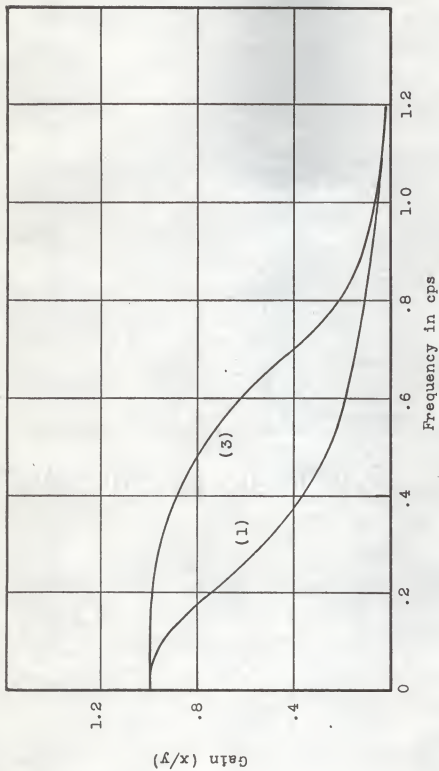
Fig. 16. Frequency responses of servos (1) and (3).

	K_1	K_2	K_3
Servo (1)	3	0	0
Servo (3)	3	.25	4
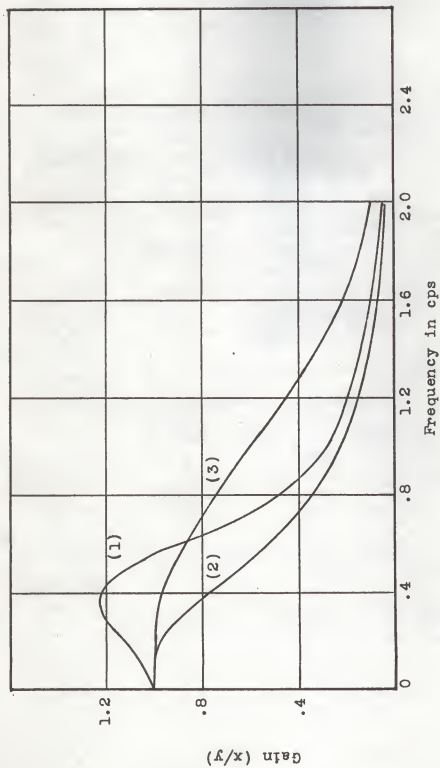
Fig. 17. Frequency responses of servos (1), (2), and (3).

	K_1	K_2	K_3
Servo (1)	10	0	0
Servo (2)	10	.2	0
Servo (3)	10	.2	1.5

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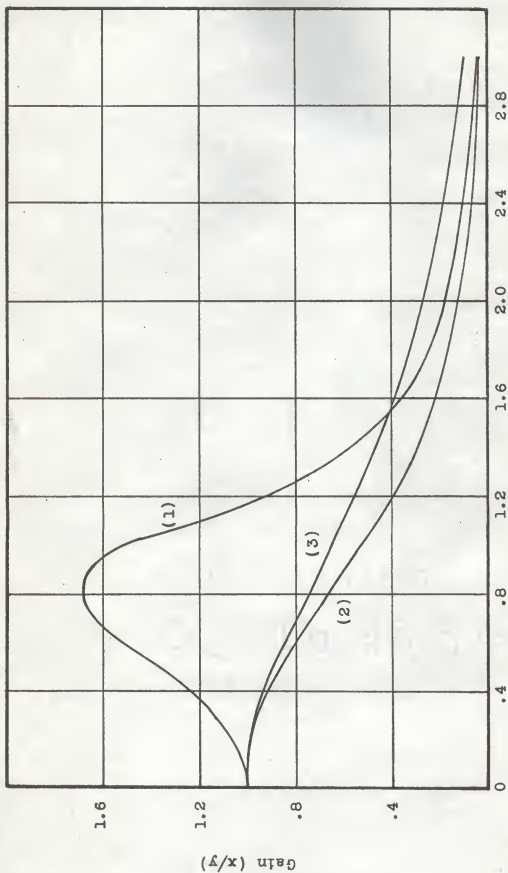
Fig. 18. Frequency responses of servos (1), (2), and (3).

	K_1	K_2	K_3
Servo (1)	30	0	0
Servo (2)	30	.2	0
Servo (3)	30	.2	1.5

ANALOG COMPUTER SIMULATION OF THE
PHYSICAL PROTOTYPE

The foregoing analog computer investigation is exploratory in nature with the intention of investigating introduction of nonlinearities of a special sort. Utilization of this proposal in a prototype servo can result in circuitry simplification if appropriate changes in the control equation are made.

In the analog computer simulation of servo (3), the velocity component was subtracted from the error, $(x - y)$, and then multiplied by $(e + |py|)$. The resulting control equation is:

$$y = A_{(p)} \ (e + |py|)(x - y - py) \tag{10}$$

If the error, $x - y$, is multiplied by $(e + |py|)$ and then the velocity component subtracted, the prototype servo is physically simplified. With this alteration the control equation becomes:

$$y = A_{(p)} \left[(e + |py|)(x - y) - py\right] \tag{11}$$

Behavior of the prototype servo is discussed in the following section.

Because the control equation of the prototype servo has the same form as equation (11), additional analog computer results were obtained to compare with the results of the prototype. In this simulation, equation (11) was used for the control equation. Comparison with the initial simulation is also made to observe the effects of subtracting the velocity feedback from the error after the multiplication rather than before.

The simulation of equation (11) will be referred to as

servo (4). Servo (4) is identical to servo (3) except the velocity feedback is subtracted from the error, $(x - y)$, after the multiplication operation. The same analog computer diagram in Fig. 5 was used with the appropriate change in the velocity feedback loop.

Servo (4) has a faster rise time than does servo (3) but requires more damping to give a response without overshoot. This is probably due to the velocity feedback component not being increased by the multiplication operation as much as the error is increased.

With K_1 equal to 3, servo (4) has a time rise 22 per cent faster than servo (3) but requires K_2 equal to 2.3 compared to 0.25 for servo (3). With K_1 equal to 10, servo (4) has a 38.5 per cent faster rise time than servo (3). In this case, K_2 was equal to 13 compared to .2 for servo (3). The step-responses of servos (1), (3), and (4) with K_1 adjusted to 3, are shown in Fig. 19. The responses with K_1 equal to 10, are shown in Fig. 20.

If K_2 is reduced sufficiently to allow servo (3) or (4) to overshoot, the resulting overshoot is not the ordinary form. It is unlike the damped oscillatory overshoot associated with linear servos, but it is a single overshoot and then decaying to the steady-state value. This particular shape of the overshoot might be a clue in applying the proposed method of introducing nonlinearities to achieve a zero velocity-error servo. Figure 21 shows a step-response of servo (4) with this type of overshoot being illustrated.
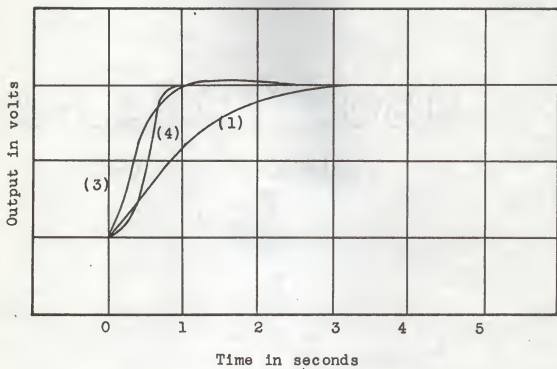
Fig. 19. Step-responses of servos
(1), (3), and (4).

	K_1	K_2	K_3	Rise time in seconds
Servo (1)	3	0	0	1.75
Servo (3)	3	.25	4	0.55
Servo (4)	3	2.3	4	0.45
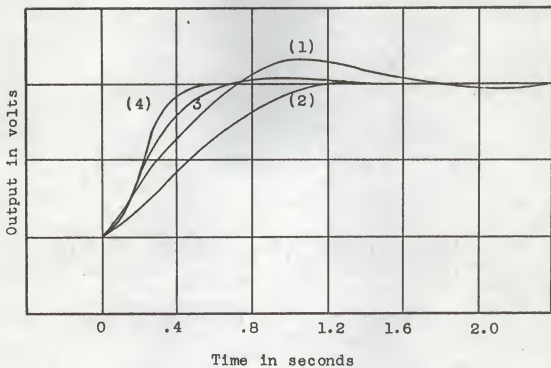
Fig. 20.   Step-function responses of
servos (1), (2), (3), and (4).

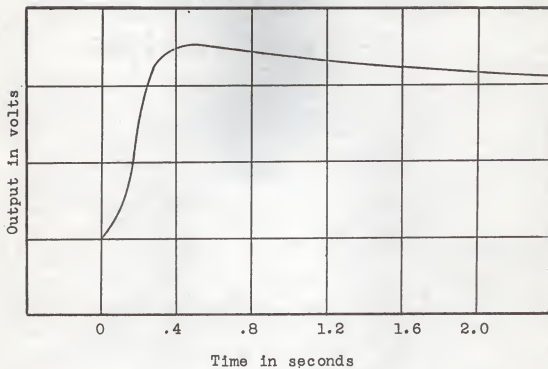|	K_1	K_2	K_3	Rise time in seconds
Servo (1)	10	0	0	.5
Servo (2)	10	.2	0	.78
Servo (3)	10	.2	1.5	.36
Servo (4)	10	13	1.5	.26

Fig. 21. Step-response of servo
(4) with overshoot.

$K_1 = 10$, $K_2 = 1.5$, $K_3 = 8.0$

THE PROTOTYPE NONLINEAR SERVO

Because of favorable results obtained on the analog computer, it was desirable to construct a prototype of the nonlinear servo. By doing this, the analog computer results are verified and the simplicity of the servo is demonstrated.

One of the outstanding features of the proposed servo is its adaptability to existing position servos. In constructing the prototype nonlinear servo, the only additional component required is a full-wave rectifier which is used as an absolute value device. The original servo is a linear position servo with proportional and velocity feedback.

The prototype diagram is shown in Fig. 22. The components consist of the following: a two-phase, 400-cycle, low-torque servo motor; three potentiometers; a differentiating circuit; a full-wave rectifier; a modulator; and an amplifier. The list does not include a multiplier and the proposed servo is nonlinear due to the multiplicative operation. A novel feature of the proposed servo is that it utilizes multiplication already present in the servo.

The purpose of the multiplier is to increase the gain of the servo during the transient time. This is accomplished by multiplying the error signal by a function of the output velocity magnitude. The voltage applied to the input and output potentiometer is $(e + |py|)$, where e is a constant. The input voltage is $x(e + |py|)$. The output voltage is $y(e + |py|)$. The modulator input is the difference between the input and the output
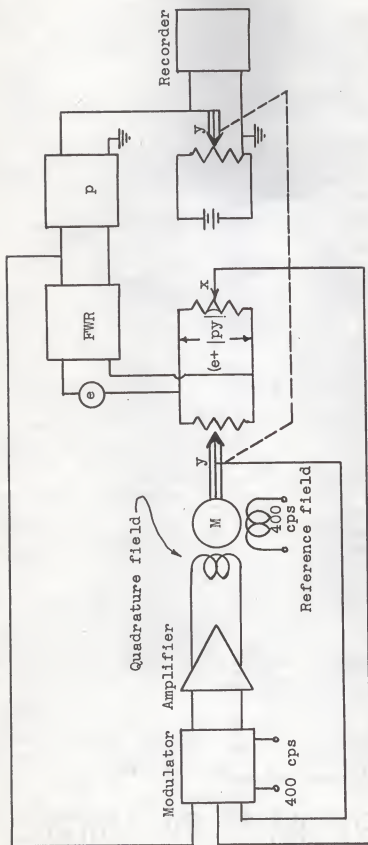
Fig. 22. A position servo with the gain modulated by the output velocity.

voltage: $(e + |py|)(x - y)$.

The third input to the modulator is the velocity feedback component used to increase the damping factor. Because of the low-torque servo motor, a differentiator circuit was used with a second output potentiometer to obtain the output velocity. A constant voltage was applied to this output potentiometer and the differentiator measures the rate which the output changes. This auxiliary potentiometer was also used with a visicorder to record the output position.

It is desirable to vary the gain by a function of the magnitude of the output velocity and not the sense. A full-wave rectifier yields an output which is always of the same sign and it functions as an absolute value device. The output of the differentiator is applied to a full-wave rectifier whose output, added to a constant voltage, e, is the voltage applied to the input and output potentiometers.

The resulting input to the modulator is:

$$(e + |py|)(x - y) - py \tag{12}$$

This is operated on by the amplifier, motor, gears, and potentiometers which results in the following control equation:

$$y = A_{(p)}\left[(e + |py|)(x - y) - p\overline{y}\right] \tag{13}$$

Again, this is unlike the control equation of servo (3) but identical to servo (4). The prototype servo, as used, cannot achieve the same control equation as servo (3) because of the manner in which the prototype servo multiplies.

Step-responses were taken for the prototype servo for three values of gain. The gain is adjusted to yield overdamping,

critical damping, and underdamping for the prototype servo when
connected as a linear position servo with proportional feedback.
Step-responses of the nonlinear servo are compared to the linear
servo with proportional feedback for the overdamped and the
critically damped cases. For the underdamped case the step-
responses of the above linear and nonlinear servo plus a linear
servo with proportional and velocity feedback, similar to
servo (2), are compared.

When the gain is adjusted to yield overdamping and critical
damping, the nonlinear servo has a faster rise time than does
the linear servo with proportional feedback. This faster rise
time is achieved without overshoot. Step-responses of the two
servos are shown in Figs. 23 and 24 for the underdamped and
critically damped cases.

For the underdamped case, the nonlinear servo obtains a
rise time equal to the linear servo with proportional feedback
and with little overshoot. It has a faster rise time than does
the linear servo with proportional and velocity feedback com-
parison of the step-response is shown in Fig. 25.

To illustrate the feature of one overshoot and decaying
to steady-state value, the velocity feedback was reduced to
allow the servo to overshoot. The overshoot response is shown
in Fig. 26. Note its similarity with the overshoot response
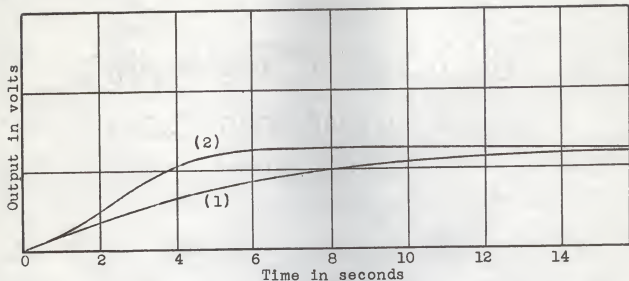obtained from servo (4), shown in Fig. 21.

Fig. 23. Step-responses of the linear and nonlinear prototype
servos with the gain adjusted to yield overdamping.

(1). Linear position servo with proportional feedback.
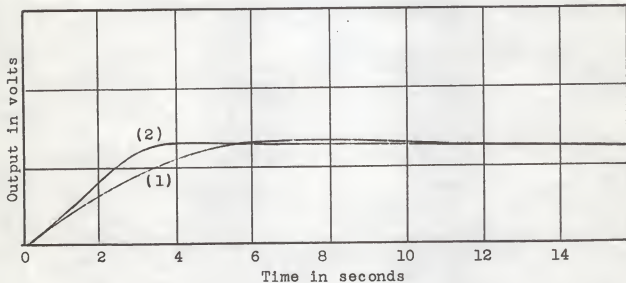(2). Position servo with the gain modulated by the out-
put velocity.

Fig. 24. Step-responses of the linear and nonlinear prototype
servos with the gain adjusted to yield critical damping.

(1). Linear position servo with proportional feedback.
(2). Position servo with the gain modulated by the out-
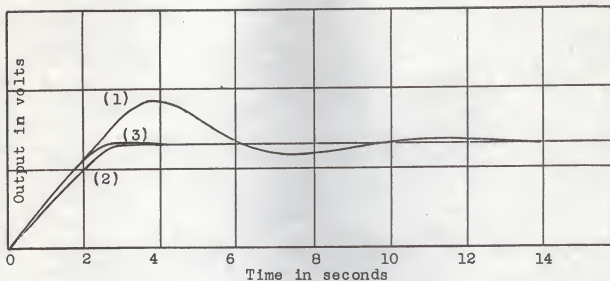put velocity.

Fig. 25. Step-responses of the two linear prototype servos and the nonlinear prototype servo with the gain adjusted to yield underdamping on servo (1).

(1). Linear position servo with proportional feedback.
(2). Linear position servo with proportional and velocity feedback.
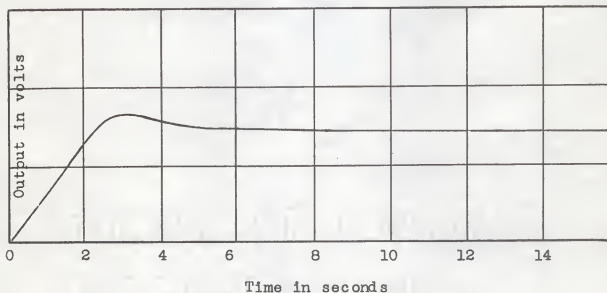(3). Position servo with the gain modulated by the output velocity.

Time in seconds

Fig. 26. Step-response of the prototype nonlinear servo with overshoot.

CONCLUSION

The effects of nonlinearization of a second-order position servo by a continuous multiplicative operation has been demonstrated. Its output is characterized by a small time delay and with no transient overshoot. Because the servo utilizes the existing multiplication feature of a position servo, it can be readily applied to existing servos to improve their performance.

The investigation also indicates other inherent features of importance. The adaptive properties are illustrated by its independence of the R/J term in a limited range. Also mentioned is the zero velocity-error response capability.

The experimental results indicate that nonlinearities introduced by multiplying the error by a function of the output velocity magnitude yields a nonlinear servo with superior performance compared to an equivalent linear servo. Performance was investigated by comparing step-function, sinusoidal, and triangular responses. Because superposition does not hold for nonlinear systems, the servo's response to an arbitrary input is not known. Hence no attempt has been made to extend the investigation beyond the range conducted.

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A POSITION SERVOMECHANISM WITH THE GAIN
MODULATED BY THE OUTPUT VELOCITY

by

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Intentional introduction of nonlinearities is a necessity to meet more demanding servo performance specifications than can be realized with linear servos. This investigation was made to study the effects of introducing nonlinearities into a position servo by a continuous multiplicative operation. The gain of a second-order position servo is increased during transient time by multiplying the error by a function of the output velocity magnitude. Velocity feedback is used to increase the damping so the transient overshoot is eliminated or minimized.

By contrast, other investigators have employed discontinuous switching and nonlinear time constant changes to effect changes in servo performance. There are a few investigators who have manipulated the gain factor of a servo to achieve the same ends.

To avoid solving the nonlinear differential equation which represents the proposed servo, an analog computer simulation was made. The simulation provided a simple means of exploration and evaluation of the servo. The control equation used in the initial simulation is:

$$y = A(p)\left[(e + |p\overline{y}|)\right]\left[x - y - p\overline{y}\right]$$

Note that the error, $(x - y - py)$, is multiplied by e, a constant, plus the absolute value of the output velocity. The absolute value is used to enable the servo to function without limitation to the sign of the input.

The performance is evaluated by comparison of step-function, sinusoidal, and triangular responses of the nonlinear servo to two linear servos. The two linear servos consist of a linear

second-order position servo with proportional feedback and one
with proportional and velocity feedback. The responses were
obtained for values of gain that yield overdamping, critical
damping, and underdamping for the linear servo with proportional
feedback.

Experimental results from the simulation show that the non-
linear servo has a step-response of less time delay with no
transient overshoot than either of the linear servos. Also the
bandwidth of the nonlinear servo is considerably wider than the
linear servos. Further, the nonlinear performance is virtually
independent of a given variation in the damping factor.

To verify the analog computer results and to illustrate the
simplicity of the nonlinear servo, a prototype servo was con-
structed. The multiplicative operation is achieved by utiliz-
ing the multiplication feature already present in a position
servo. This results in a slight change in the control equation
representing the prototype servos. The control equation with
the appropriate change is:

$$y = A(p)\left[(e + |py|)(x - y) - p\overline{y}\right]$$

Additional analog computer simulation was made using the pro-
totype's control equation. Using this control equation, similar
results were found but a considerable increase in the damping
factor was required to obtain a response with no overshoot.
This was probably due to the velocity feedback component not
being increased with the gain during transient time.

Experimental results obtained from the simulation and the prototype indicate that nonlinearities introduced by multiplying the error by a function of the output velocity magnitude renders a nonlinear servo with superior performance compared to an equivalent linear servo.