A STUDY OF THE FORCE REACTIONS UPON FLOW RESTRICTING ELEMENTS USED IN CONJUNCTION WITH NOZZLES AND ORIFICES

by

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PLATES

NOMENCLATURE

a	Acceleration - ft/sec^2
A	Area - L^2
C	Capacity - lb_m/psi
D	Diameter - L^2
F	Force - lb_f
k	Spring constant - lb_f/in
m	Mass - lb
p	Pressure - psia
Q	Flow - lb_m/sec
s	Complex variable
V	Velocity - ft/sec
α	Plate-to-nozzle distance (lift)
β	Arbitrary gas demand constant
ϕ	Arbitrary damping constant
ρ	Density - lb_m/ft^3

SUBSCRIPTS

i	Inlet conditions
n	Nozzle
o	Outlet conditions
p	Plate
pp	Spring seat
s	Spring

INTRODUCTION

Because of the increasing use of automatic controls in the process industries, it has become necessary for manufacturers to know more about the design characteristics of the components of these controls. One of the components about which little is known is the flow restricting element.

> The use of flow restricting elements in conjunction with orifices and nozzles is and has been the basis for controlling flow, pressure, and liquid level in chemical, refining, power, and gas distribution loops. It is used either in the final control element which may be a large valve, or in the measuring means which may be a relay-operated pilot using a very small orifice and flapper. (King, 4).

At present, devices that can be used for the automatic control of the above processes are designed empirically, that is to say that when the specifications for a new control device are known, an attempt to design the device will be made by an experienced engineer. He will make a first design based on his experience, then have a model constructed and tested to determine if it meets the specifications. If it does not, he then makes a change in the model which, based on his past experience, will correct the error. The model is again tested and the procedure repeated until the specifications are met and a new controller is available, or until it is determined that the specifications cannot be met and the project is discontinued.

The above procedure is inherently a long and costly one. However, because of the present complexity of automatic controllers, particularly self-operated ones, and because of the lack of knowledge about the various physical components of the controllers, the procedure is necessary.

If mathematical relations could be developed for the various components, then the problem could be reduced to equations which could be solved by computers. In addition, the computer would permit a much greater selection of design characteristics.

A typical example of such a controller is the house service regulator generally used where natural gas is available. It is employed to reduce intermediate distribution pressures of up to 100 psig to a pressure of a few inches of water required in the outlet piping. The gas is required for such domestic uses as stoves, water heaters, and furnaces.

A cross section of a house service regulator is pictured in Plate I. It is interesting to note the operating characteristics of this regulator. If the regulator were operating with a constant rate of flow, then the plate would be at some distance from the nozzle and there would be a force balance at the spring seat. If the demand for gas is suddenly increased, the pressure will drop in the lower casing, reducing the force on the diaphragm and allowing the spring to push the diaphragm downward, thus increasing the distance between the nozzle and the plate. This allows more flow through the nozzle, thus increasing the pressure in the lower casing. Due to the increase in pressure the spring seat moves upward, narrowing the gap between the nozzle and the plate and moving the spring seat to a new balance point. It is easy to see that this chain of events could become oscillatory, thus emphasizing the fact that the stability of the regulator may be a problem. The stability of a house

EXPLANATION OF PLATE I

Cross-section view of regulator.

Reference letters:

A - plate

B - nozzle

C - spring seat

D - diaphragm

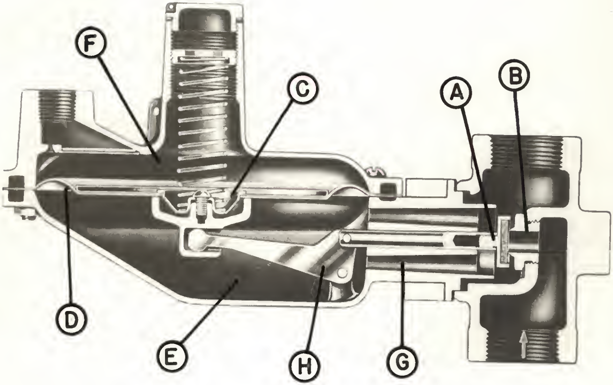
E - lower casing

F - spring casing

G - flow restriction

H - valve lever assembly

PLATE I



service regulator is of prime importance. It is installed when gas service is started and is intended to last for a period of ten to fifteen years. Instability will soon result in the regulator wearing out, and furthermore may cause dissatisfaction to the user due to the fluctuating outlet pressure or the noise that will be transmitted through the house piping. It has been found that some regulators exhibit instability at low flows.

The gas regulator is essentially a proportional control device and exhibits the characteristics of such control. That is to say, there is a deviation of the outlet pressure from the original set point as the flow requirement changes. It is a natural tendency for the pressure to drop as the flow increases. This is a very undesirable characteristic and an attempt is made to offset this by changing the shape of the plate and the inlet to the lower chamber. By suitable changes the pressure may be made to increase as the flow increases.

Several features of this regulator are hard to determine mathematically for design purposes. Of these, four stand out as being of primary importance:

1. The force on the plate.
2. The damping provided by the spring casing.
3. The friction in the valve lever assembly.
4. The transmission of the pressure from the outlet to the lower casing.

It is the purpose of this paper to investigate the first and to make assumptions about the latter such that some preliminary equations about the regulator can be written and an attempt made to solve these equations on the analogue computer.

DESIGN AND OPERATION OF TEST EQUIPMENT

Because the purpose of this thesis was to investigate the forces produced in the flow restricting region, it was necessary to locate this region. A device to do this, pictured in Plate II, was constructed. It consists of a standard one and one-half-inch regulator body, on top of which a simple screw mechanism has been placed to raise or lower a flat plate with regard to the nozzle in the regulator body. Upstream pressure was controlled by an operator and the downstream flow was measured by a rotometer. The distance between the plate and the nozzle was measured by the indicator dial. The dial divisions were five ten thousandths. To take a reading, the plate was positioned a specified distance from the nozzle and then the pressure difference across the regulator body was adjusted to the desired value. After each adjustment the flow was read from one of a bank of rotometers selected so that their ranges overlap. The readings were taken at specified intervals until the flow ceased to increase. When the flow remains constant regardless of the plate position, there is no longer any flow restriction due to the plate. These data are included in the Appendix.

The design of a test fixture to measure the total force on the plate was more involved. Most early researchers used a device such as the one in Plate III (Bouasse, 2 and Morley, 7). The air nozzle would be placed horizontally and directed against

EXPLANATION OF PLATE II

Flow restriction test fixture.

Reference letters:

A - $1\frac{1}{2}$-inch regulator body

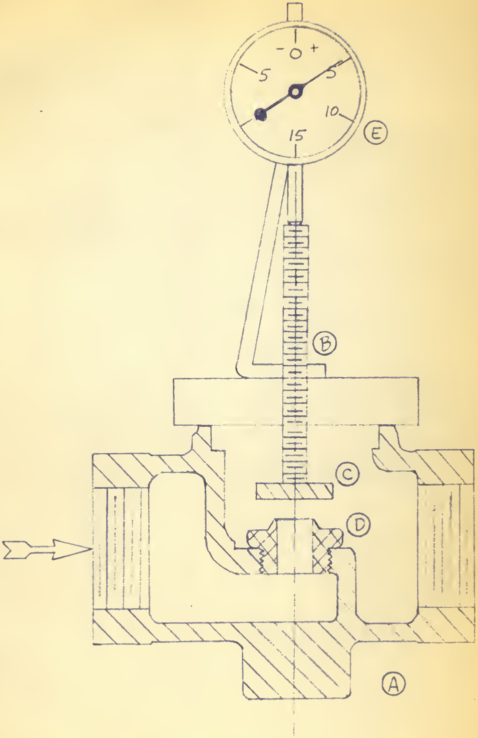
B - 1/4-inch screw to raise and lower plate

C - plate

D - nozzle

E - dial indicator to measure nozzle-to-plate distance

PLATE II



EXPLANATION OF PLATE III

Test fixture used by other researchers to determine the force on a plate outside the flow restricting range.

Reference letters:

A - brass plate

B - spindle

CC - spring steel strips

D - bracket

E - pillar

F - wooden stand

G - grooved pulley

HH - cord

PLATE III

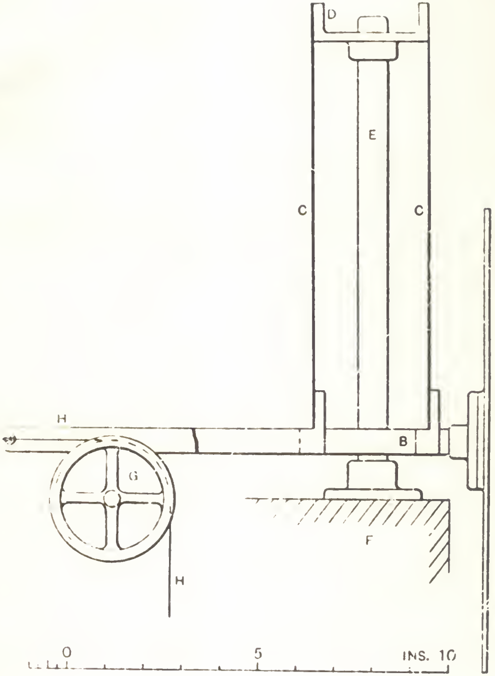


plate (A) toward the left. The spring steel strips (CC) would allow the plate to move to the left. Weights would be placed on cord (H) to restore the plate to its original position. Generally some form of damping would be provided below the point at which the weights are added. Despite this, the fixture has a tendency to be unstable and oscillate. This makes it difficult to obtain accurate measurements of either plate position with regard to the nozzle or the total force on the plate.

Two of the authors listed in the Bibliography (Welanetz, 9 and Willis, 10) investigated a problem associated with the present work, that of the attraction between an orifice and a flat plate. One of these (Welanetz, 9) used a calibrated spring to measure forces and experienced no problems of stability. However, his forces were suction forces tending to pull the plate to the orifice. It was felt that this system would not be satisfactory for the present investigation where the nozzle repels the plate.

A feature which is very desirable in a test fixture but which was not present in the devices mentioned, is the continuous recording of test readings; in other words, a plot of the test measurements. If this feature can be incorporated, then the rate of data taking can be increased tremendously. In the flow test fixture, it was necessary to set the plate at specified distances and take flow readings. This takes time but was a satisfactory method because no discontinuities were expected. However, in measuring the force reactions on the plate, the experimenter has no assurance that the force will not suddenly change as the plate-to-nozzle distance is varied. In fact, early researchers (Bouasse, 2 and Morley, 7) mention such problems and the difficulties they experienced as they attempted to take data near the flow restricting region. Because of this, the test fixture was designed as in Plate IV, with a screw adjustment to vary the plate-to-nozzle distance. A cantilever beam was used to measure the force on the plate which was transmitted to it through an anodized aluminum rod. The anodizing reduced the friction between the rod and its retaining block. The position of the plate was measured by the differential transformer. Four SR-4 strain gages were mounted on the cantilever beam to serve as a bridge circuit to magnify the strain and provide temperature compensation. Plate V exhibits one of the strain-measuring elements used. Two cantilever beams of this type were used. One, designed for large forces, had a thickness of 0.100 inch and the other, for greater accuracy in measuring small forces, had a thickness of 0.032 inch. Both beams are six inches long and one and one-fourth inches wide. The strain gages on all test fixtures were covered with wax to prevent damage. (A) in Plate VI is a force ring with four SR-4 strain gages. The ring, 0.060 inch thick and one and seven-eighths inches in diameter, was used as shown in Plate VII. Since a cantilever beam undergoes considerable movement to produce the strain required to measure a force, the force ring was used as a check. It requires very little deformation to produce the strain required for force measurement, but was slightly unstable. Plate VII shows the pressure gage used by the operator to adjust the pressure toward the desired value. The pressure was recorded by the use of a transducer. The outputs of the

EXPLANATION OF PLATE IV

Cantilever beam test fixture.

Reference letters:

A - screw adjustment

B - plate

C - nozzle

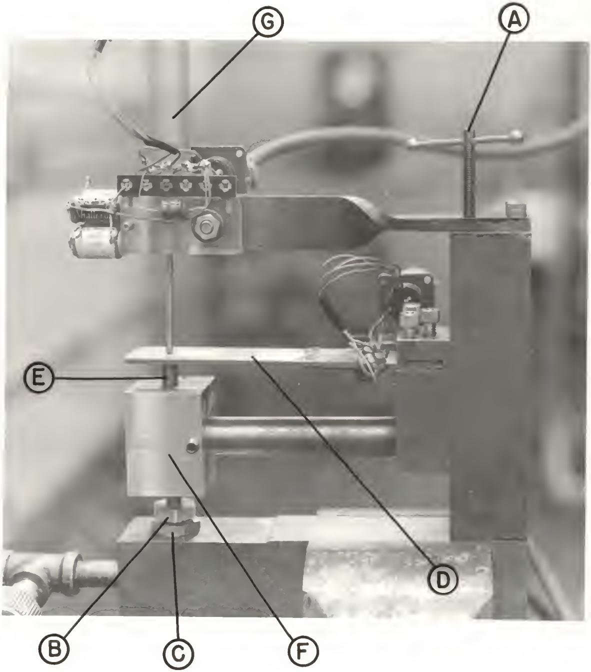
D - cantilever beam

E - rod

F - retaining block

G - differential transformer

PLATE IV



EXPLANATION OF PLATE V

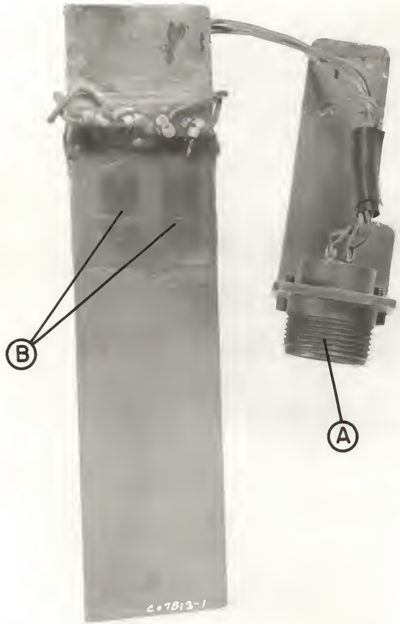
Cantilever beam.

Reference letters:

A - connection for recording instrument

B - strain gages

PLATE V



EXPLANATION OF PLATE VI

Force ring

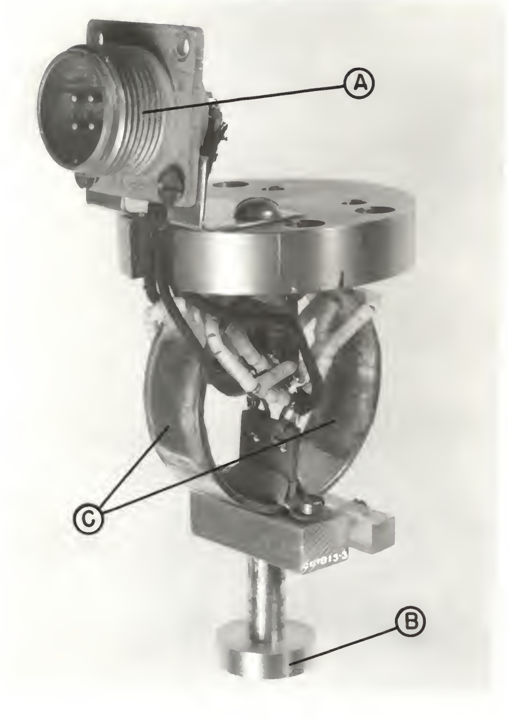
Reference letters:

A - connection for recorder

B - plate

C - strain gages

PLATE VI



EXPLANATION OF PLATE VII

Force ring test fixture.

Reference letters:

A - screw adjustment

B - plate

C - nozzle

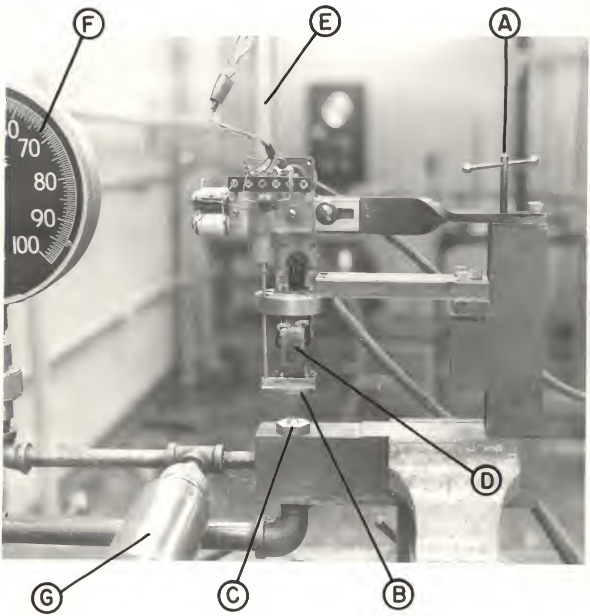
D - force ring

E - differential transformer

F - pressure gage

G - pressure transducer

PLATE VII



differential transformer, the pressure transducer, and the strain gage bridge on the cantilever beam or force ring were then plotted simultaneously by use of a Sanborn recorder, so that the influence of a change in one parameter could be seen on the others.

The complete test arrangement is pictured in Plate VIII. The test fixture bridge circuits were connected to the four-channel Sanborn amplifier and recorder. A pneumatic null balance pressure controller without amplification was used to control the inlet pressure. A pressure controller was connected to the input side of the diaphragm to keep the inlet pressure constant at low pressures. At higher pressures it was more satisfactory to control the inlet pressure by hand. A large air tank supplied an essentially constant-pressure source of air. A rack of water and mercury manometers was used instead of a pressure gage to measure low pressures.

This equipment was made available by the Fisher Governor Company. All tests were conducted in their research laboratories, at Marshalltown, Iowa.

In an attempt to examine the physical picture of the flow, pictures were taken with a "Schlieren" apparatus. The "Schlieren" is pictured in Plate IX. The optical components are: the light unit, the condenser lens unit with the first grating, the main objective lens, the second grating, and the camera. For focusing and visual observation, the camera would be replaced with a ground glass viewer. A complete description of the theory and operating principles of the instrument can be found in the references (Mortensen, 8).

EXPLANATION OF PLATE VIII

A view of the test area showing the complete instrumentation.

Reference letters:

A - test fixture

B - four-channel Sanborn recorder

C - null balance pressure controller

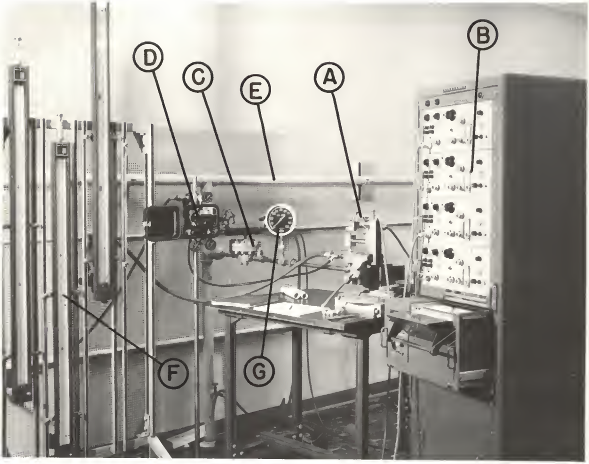
D - pressure controller for diaphragm pressure

E - supply air

F - mercury and water manometers

G - pressure gage

PLATE VIII



EXPLANATION OF PLATE IX

Schlieren

Reference letters:

A - light unit

B - condenser lens

C - first grating

D - main objective lens

E - second grating

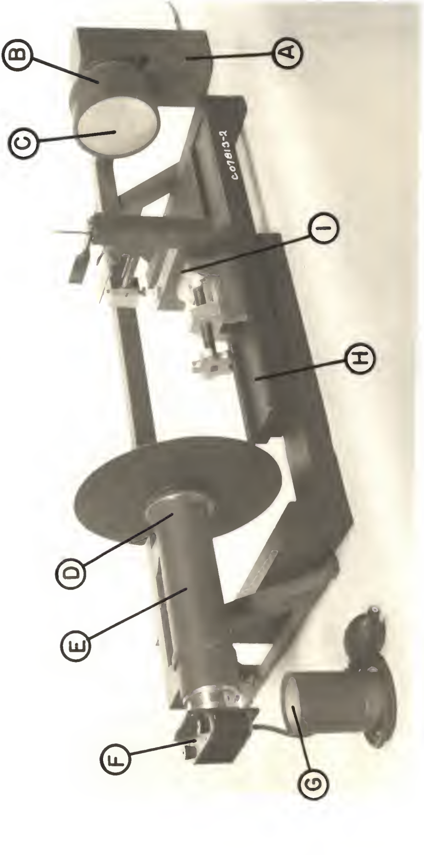
F - camera

G - ground glass for observation

H - track for focusing

I - test fixture

PLATE IX



The test equipment was placed approximately half way between lenses (B) and (D), and then moved in track (H) until the proper focus was obtained. By proper manipulation of the light and the position of the two gratings, changes in density of the escaping air jet became visible and were photographed on microfile film.

DISCUSSION AND RESULTS

The results of the investigation of the flow restriction characteristics of a flat plate can be seen in Plate X. The curve is plotted dimensionlessly and a new term, lift, is introduced. Lift is defined as the distance between the nozzle and plate and will be used to describe this distance henceforth in this article.

The curve shows that flow restriction ceases at 0.38 D, where D is the diameter of the nozzle, rather than 0.25 D where the cross-section area of the nozzle is equal to the curved surface area of a cylinder whose height is equal to the lift and whose diameter is equal to the nozzle diameter.

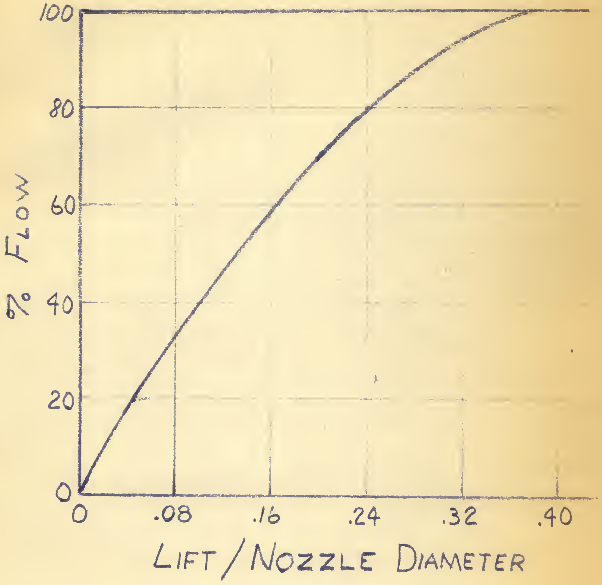
One author (Boehnlein. 1) used water as his working fluid and found that for this case flow restriction ceases when the lift is equal to the nozzle diameter.

If the plate diameter was made slightly larger than the diameter of the nozzle, any further increase in plate diameter does not affect the flow restriction curve. The amount larger depends on the pressure drop across the nozzle. At subsonic

EXPLANATION OF PLATE X

Dimensionless plot of flow restriction characteristics.

PLATE X



velocities a very slight increase in plate diameter is satisfactory, while at supersonic velocities a somewhat greater diameter is required because the jet is underexpanded and expands upon leaving the nozzle. The minimum diameter of plate required is also influenced by the slope of the walls of the nozzle. If they are diverging, a still larger plate is required. The nozzles used in this report had parallel walls. A plate diameter of one and one-fourth the nozzle diameter was sufficient to remove any variation in flow restriction due to plate diameter.

A slight variation in the curve was noticed for different pressure drops. However, it was felt this variation was within experimental error.

Plate XI is a series of Schlieren pictures showing the jet as the plate is moved into the flow restricting range. In Fig. 1, a shock line may be seen just in front of a formation on the plate. This formation is called a wall jet (Glauert, 3). As the lift is decreased, the shock wave is pushed into the outlet of the jet. When the wave is just at the outlet, flow restriction starts.

Although the purpose of this investigation was to obtain information on a particular combination of nozzle and plate, it was felt that more basic information should also be obtained. It was desirable to determine the force on the plate for variations of lift, pressure drop, plate configuration, plate diameter, nozzle configuration, nozzle diameter, and plate material. Some basis of comparison is necessary if all of these variables are to be investigated. It was decided to adopt as a basis for

EXPLANATION OF PLATE XI

Schlieren photographs of the flat plate moving into a flow restricting position.

Fig. 1. Wall jet with shock wave.

Fig. 2. Wall jet with shock wave approaching nozzle.

Fig. 3. Flow restriction.

PLATE XI

Fig. 1.

Fig. 2.

Fig. 3.

comparison the flat plate (Figs. 5 and 6) and the nozzle (Figs. 1 and 3) shown in Plates XII and XIII. The plate was made of flat aluminum with a diameter of 13/16 inch, the diameter of standard house regulator plates. The nozzle was the standard one-quarter-inch diameter nozzle of a type used in house regulators. A plot of force on the plate versus its position for four pressure differences is shown in Plate XIV. This is a photo copy of the original data sheet. Several points are of special interest. It can be seen that the total force on the plate is not a constant but has a dip of -15 per cent in the flow restricting region. Two discontinuities occur in this plot. A discontinuity occurs when the force changes with no change in pressure or lift. It was for this reason that the pressure was plotted even though it was to be held constant. When the flow is changing, it is very difficult to maintain a constant pressure and any change in pressure might produce what would appear to the observer to be a discontinuity. In order to insure that the rate of change of lift would not influence the data, several rates were tried. A lift rate of 0.5 inch per minute used in these tests permitted rapid data taking, yet did not affect the results. The first discontinuity occurs just after the lift has been increased to 0.01 inch, the second at 0.31 inch. The flow restricting range for a 1/4-inch nozzle would be approximately 0.1 inch. Therefore the first one occurs in the flow restricting range and the second one out of it. No discontinuities were found when water was used as the working fluid (Boehnlein, 1).

It should be further pointed out from Plate XIV that the

EXPLANATION OF PLATE XII

Plate details.

Fig. 1. Front view of plates with rubber inserts.

Fig. 2. Standard house service regulator plate.

Fig. 3. Flat plate with rubber insert.

Fig. 4. Standard house service regulator plate with inside walls sloped to 45 degrees.

Figs. 5 and 6. Flat plate of variable diameter used for standard of comparison.

PLATE XII

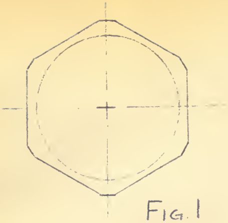


Fig. 1

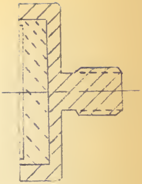


Fig 2

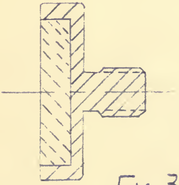


Fig 3

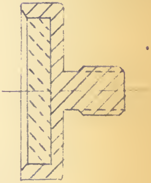


Fig 4

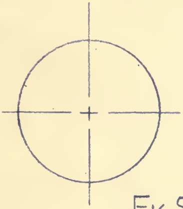


Fig. 5

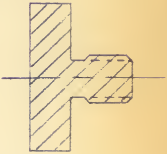


Fig 6

EXPLANATION OF PLATE XIII

Nozzle details.

Fig. 1. Top view of all nozzles.

Fig. 2. Flat orifice.

Fig. 3. Standard house service regulator nozzle.

Fig. 4. Elongated nozzle to reduce base effects.

Fig. 5. Nozzle machined to A.S.M.E. entrance specifications.

Fig. 6. Sloped side nozzle.

Fig. 7. Details of all nozzle outlets showing rounding to prevent cutting the rubber insert in the plates.

PLATE XIII

FIG. 1

FIG 2

FIG 3

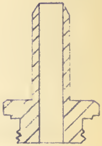


FIG 4

FIG 5

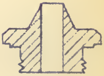


FIG 6

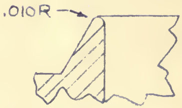


FIG 7 DETAIL OF NOZZLE OUTLET

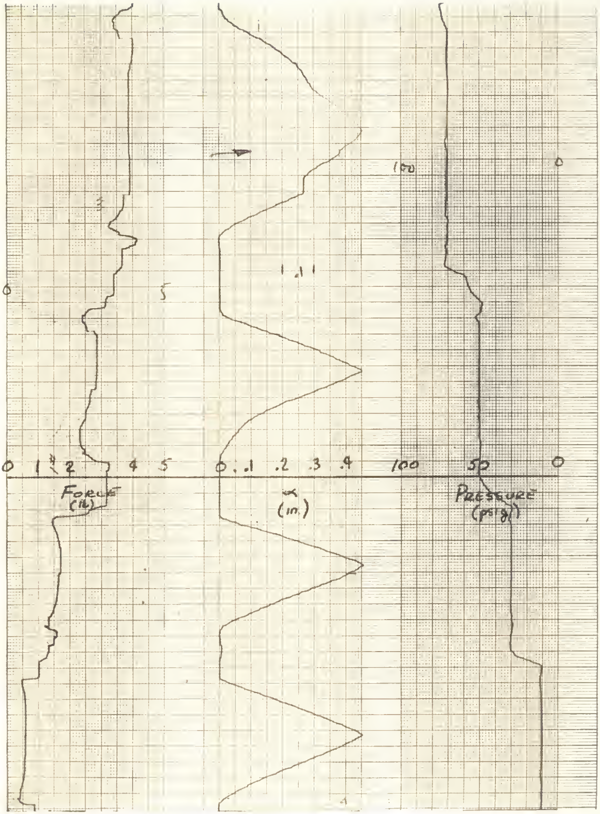
EXPLANATION OF PLATE XIV

Plot of force on the plate and lift versus time at several constant pressures for a standard flat plate and regulator nozzle.

Nozzle diameter - 1/4 inch.

Plate diameter - 13/16 inch.

PLATE XIV



second discontinuity occurs at a different value of lift when the lift is increasing than when it is decreasing. The first discontinuity has this same property. When the lift is decreasing, the first one occurs just as the flow is shut off.

No physical significance can be attached to the first discontinuity at present but the second one occurs when there is a change in the direction of the radial flow from the plate. As the lift is increased from zero, the flow out of the nozzle hits the plate and is diverted downward, forming an umbrella. A strong region of low pressure is established between the base of the nozzle and the plate. This region is maintained as the lift is increased and causes the flow to leave the plate at an angle rather than tangentially. After the lift has increased to a certain point, the low pressure region suddenly disappears and the flow leaves the plate in a tangential direction. Plate XV illustrates the Schlieren photographs of this phenomenon. Figure 1 is a free jet, with the plate out of the flow restricting range, showing the characteristic expansions and compressions of a three-dimensional underexpanded jet. Figure 2 is a picture of the jet when the plate is in the flow restricting range and has just changed from a radial tangential flow to one which is curved downward. The air contains a slight amount of oil and it can be seen to have coated the thin glass plate on the left with a coat of oil curving downward and slightly in toward the nozzle.

Plate XVI shows a dimensionless plot of the data taken. Curve (A) represents the experimental results. Curve (B) is the force predicted from momentum flow characteristics. No information is known at present about the pressure between the nozzle

EXPLANATION OF PLATE XV

Schlieren photographs

Fig. 1. Schlieren photograph of compressions and expansions in a free jet.

Fig. 2. Schlieren photograph showing curvature of flow leaving the plate.

PLATE XV

Fig. 1.

Fig. 2.

EXPLANATION OF PLATE XVI

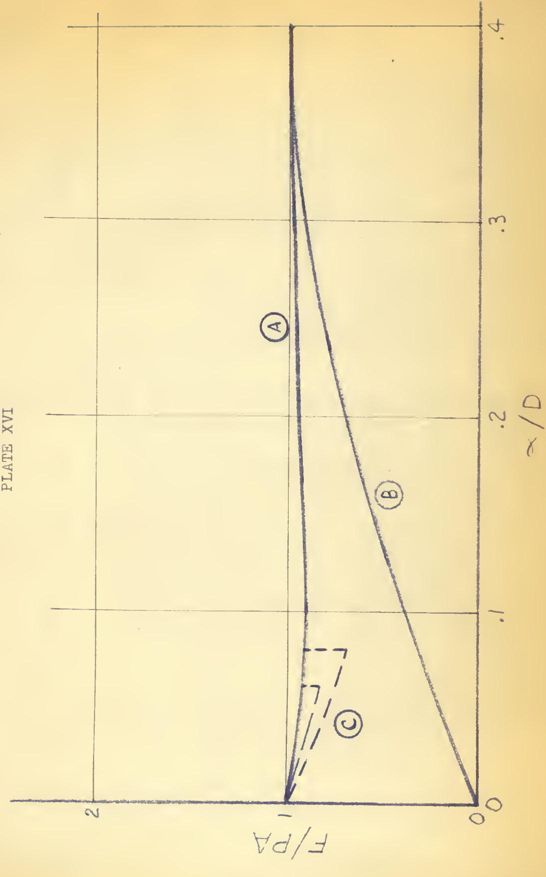
Dimensionless comparison of recorded data.

Curve A. Test results.

Curve B. Force on the plate from momentum flow considerations.

Curve C. First discontinuity as the plate leaves the nozzle.

PLATE XVI



and plate. The curves (C) represent the first discontinuity. As the pressure drop increases the discontinuity becomes smaller, disappearing altogether when the pressure drop approaches 100 psi. It is of interest to note that even though the flow at α/D equals 0.4 is radially downward, the calculated force on the plate, assuming tangential flow, and the test results agree. Curve (A) is a composite curve of data taken at several nozzle diameters and pressure drops. Nozzle diameter was varied from 1/8 to 1/2 inch and pressures were varied from five to 90 psig. The plate diameter was held constant at 13/16 inch.

The second discontinuity appeared to be a function of the distance between the nozzle base and the plate. Therefore this distance was increased from 1/8 inch to 1/4 inch and the nozzle was again tested with the same plate. The results are exhibited in Plate XVII. At low pressures both discontinuities were removed; however, the force relation still showed the same dip as in Plate XVI. In an attempt to remove this dip, the nozzle base-to-tip distance was increased to one inch, as shown in Fig. 4, Plate XIII. This removed all discontinuities, but not the dip. It may therefore be concluded that the dip is a characteristic of a nozzle and flat plate.

In a further variation of nozzle design, the nozzles shown in Figs. 5 and 6, Plate XIII, were tested. The results obtained with the first can be seen in Plate XVIII. The major change was an increase in flow, thus an increase in total force outside the flow restricting region and a more pronounced dip in total force through the flow restricting region.

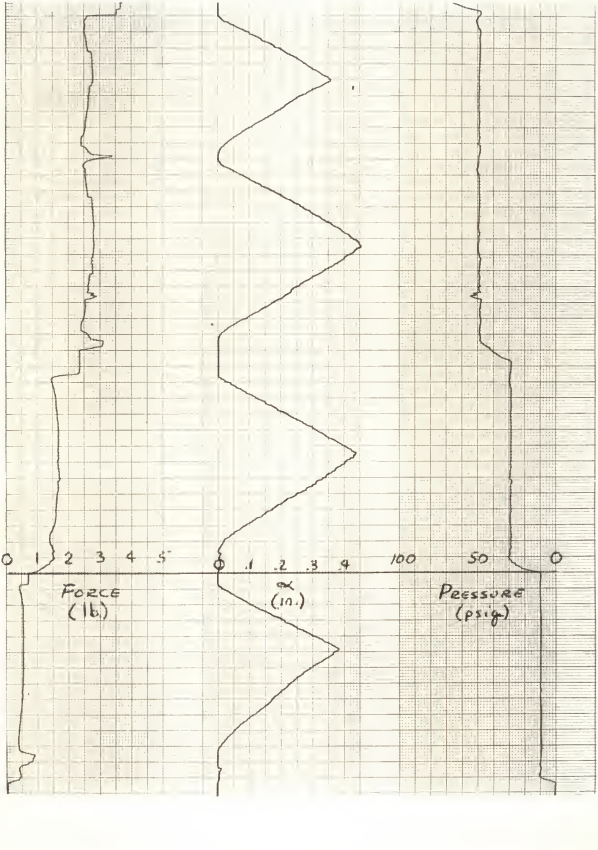
EXPLANATION OF PLATE XVII

Plot of force on the plate and lift versus time at several constant pressures for an increased nozzle length.

Nozzle diameter - 1/4 inch.

Plate diameter - 13/16 inch.

PLATE XVII



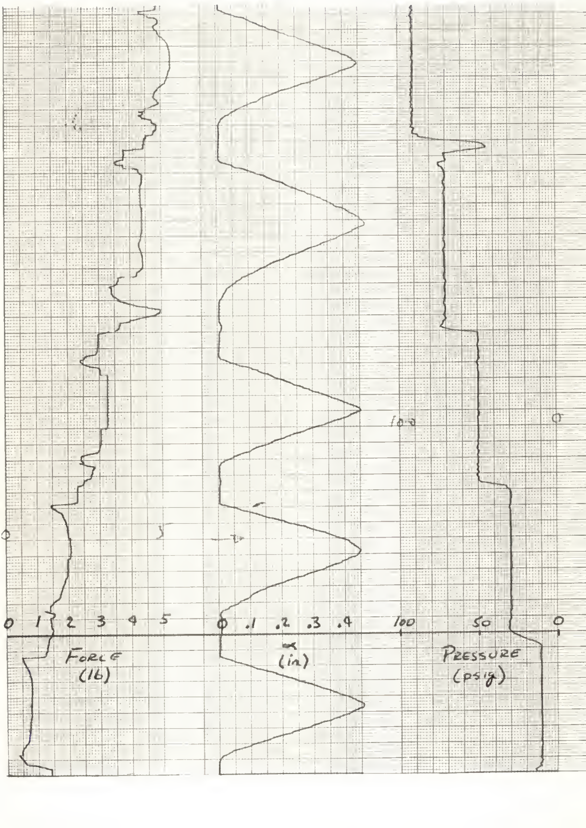
EXPLANATION OF PLATE XVIII

Plot of force on the plate and lift versus time at several constant pressures for a flat plate and a nozzle with entrance machined to A.S.M.E. specifications.

Nozzle diameter - 1/4 inch.

Plate diameter - 13/16 inch.

PLATE XVIII



Because the surfaces of the flow restricting plates used in regulators must seal tight, they are fitted with a rubber surface. Plate XIX shows the test results obtained using a plate with a rubber insert pictured in Fig. 3, Plate XII. The rubber removed the first discontinuity, but not the second. Again the dip characteristic was unaffected.

Plates XX and XXI are the results obtained with the nozzles shown as Figs. 4 and 2 in Plate XII. These plates are the two types actually used in the regulator. The plate with the sloping inner walls had no first discontinuity but the characteristic dip was changed so that when the second discontinuity occurred, the force decreased instead of increasing as it had in the other tests. As the pressure drop is increased, this plate develops much greater changes in force, tending to become unstable. The second plate with square inner walls had a completely new force characteristic. The force started increasing as the lift was increased and continued increasing until the flow reached a maximum. It then remained constant as the lift was further increased. At a pressure drop of 50 psi, the plate became unstable and no further data could be taken.

Two references were located which dealt with the forces on a plate when the plate was restricting flow (Welanetz, 9 and Willis, 10). Both of these papers dealt with a jet of air issuing from a flat nozzle or orifice. The authors were primarily interested in the force of attraction between the orifice and plate. Data were taken for this case in order to have a comparison with previous work. The data taken are presented in Plate XXII.

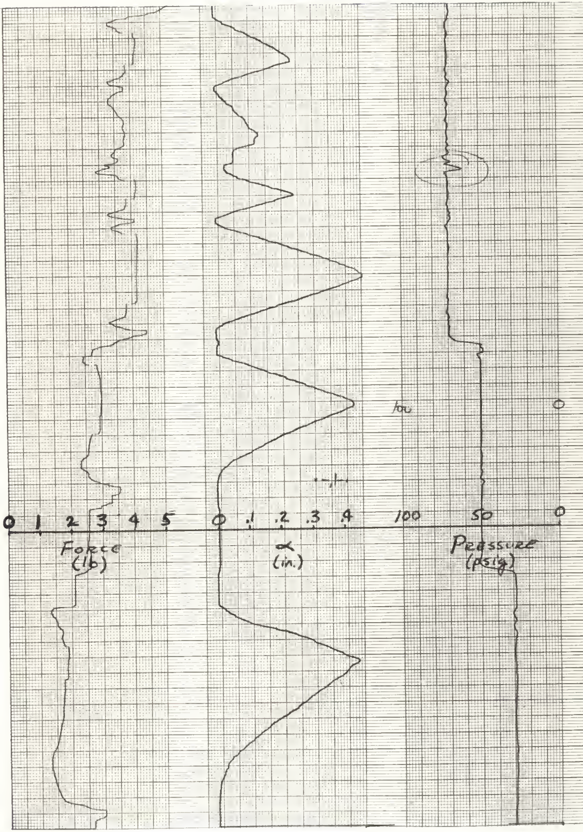
EXPLANATION OF PLATE XIX

Plot of force on the plate and lift versus time for several constant pressures for a regulator nozzle and a flat plate with a rubber insert.

Nozzle diameter - 1/4 inch.

Rubber insert diameter - 3/4 inch.

PLATE XIX



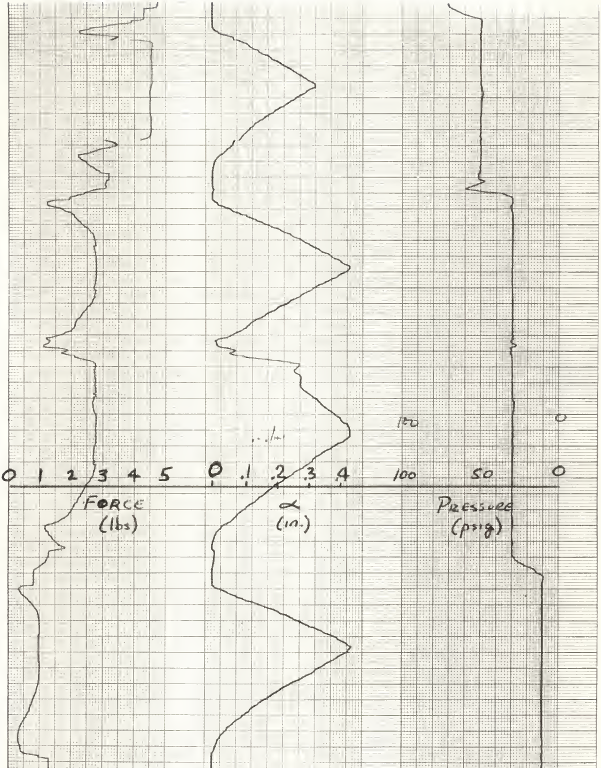
EXPLANATION OF PLATE XX

Plot of force on the plate and lift versus time for several constant pressures with standard regulator nozzle and plate with sloping walls.

Nozzle diameter - 1/4 inch.

Standard regulator plate - 45 degrees.

PLATE XX



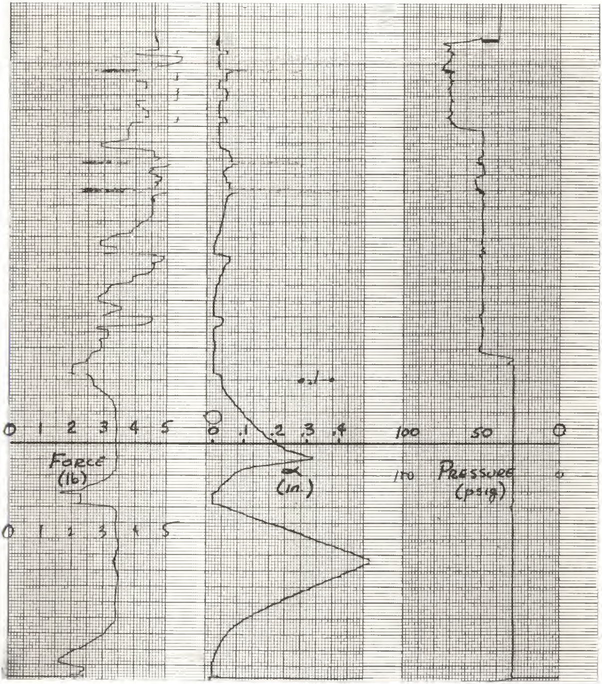
EXPLANATION OF PLATE XXI

Plot of force on the plate and lift versus time for several constant pressures with standard regulator plate and nozzle.

Nozzle diameter - 1/4 inch.

Standard regulator plate.

PLATE XXI



0 1 2 3 4 5
Force (lb)
0 1 2 3 4 5
0 .1 .2 .3 .4
α (in.)
.1
100 50 0
150 Pressure 0
(psig)

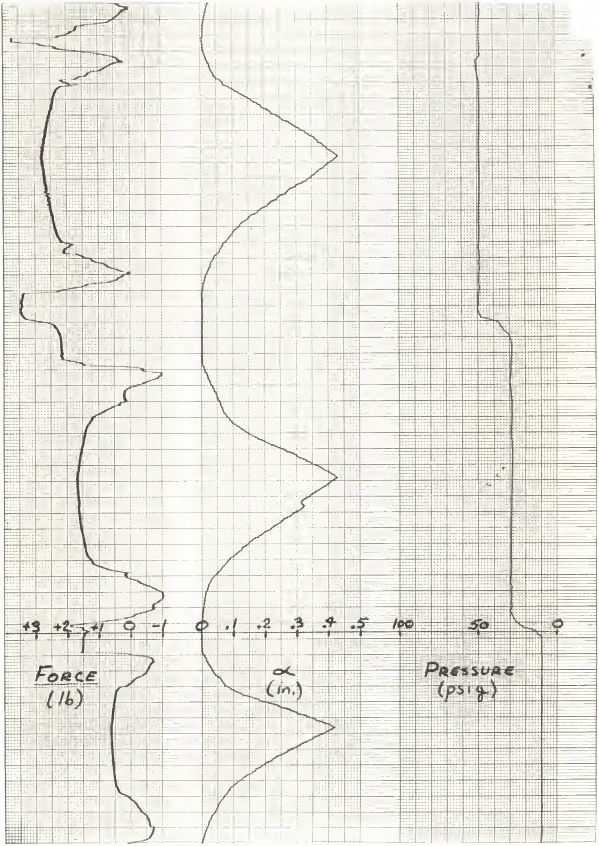
EXPLANATION OF PLATE XXII

Plot of force on the plate and lift versus time for a flat nozzle and plate.

Nozzle diameter - 1/4 inch.

Plate diameter - 13/16 inch.

PLATE XXII



The greatest force of attraction occurs during the flow restricting range at a lift of 0.04 D_n for a 13/16-inch diameter plate and a nozzle diameter of one-fourth inch. The diameter of the plate is very critical. By reducing the diameter to 5/8 inch, the force of attraction will be reduced from 1.10 to 0.20 pound for a pressure drop of 30 psi. These results agree with those of Willis (10).

The data for the portion of the curve from zero force to maximum force agrees with previous work (Welanetz, 9). However, at this point the theory developed by this author appears to break down and could not be checked by his tests or the present ones.

It should be noted also that the present tests were conducted with much greater pressure drops and gas flows than previous tests.

Despite the greater pressure drops, no data-taking problems such as were experienced by earlier authors were encountered. Retests of a combination of plate and orifice resulted in the same plot of readings. The reproducibility of the data justifies the original test fixture design.

APPLICATION OF RESULTS

Having determined the characteristics of the force on the flow restricting plate, a mathematical model of the regulator was derived.

The first step was to make some assumptions about the characteristics of the regulator that had not been determined. With reference to Plate XXIII, the spring case was assumed to provide no damping, friction in the valve lever assembly was neglected, and it was assumed that there was no resistance to flow through the lower casing outlet.

Two equations can be written for the regulator with two variables--lift (α) and regulator exhaust pressure (p). This should make a solution possible.

The first equation is a force balance around the spring seat. The lift which is measured at the nozzle is magnified four times at the spring seat and the force due to the jet is reduced.

A force balance shows the following results:

$$F_{pp} = F_s - \alpha K - Ap + F_p$$

where F_{pp} = force on the spring seat (lb_f)

F_s = force due to the original deflection of the spring (lb_f)

α = lift (in)

K = spring constant (lb_f/in)

A = effective area of the diaphragm (in^2)

p = pressure in lower casing (psig)

F_p = force from the plate

F_p will be assumed a constant for the regulator plate with the sloping sides and a constant $+M\alpha$ for the other regulator plate. The present equations will be developed for the regulator plate with sloping sides and mention will be made later of the results using the other plate.

F_s will be determined by the initial pressure setting

EXPLANATION OF PLATE XXIII

Cross-section view of regulator.

Reference letters:

A - plate

B - nozzle

C - spring seat

D - diaphragm

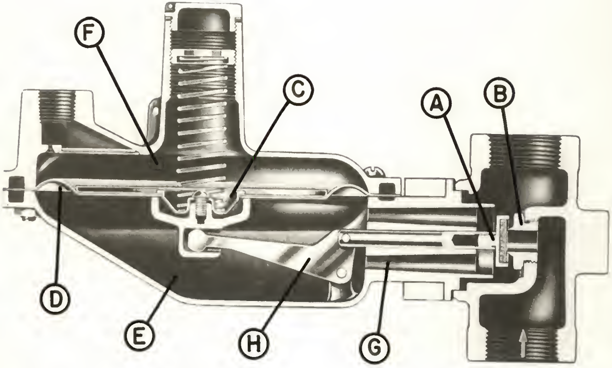
E - lower casing

F - spring casing

G - flow restriction

H - valve lever assembly

PLATE XXIII



desired. If the set point is six inches of water, F_s was found to be 4.64 lb_f. Rewriting the equation with the constants results in the following equation:

$$F_{pp} = 5.13 - 20\alpha - 23.8\,p$$

Setting F_{pp} = ma, where

m = mass of the spring seat and diaphragm head (0.385 lb_m)

and a = acceleration

$$.385\,\frac{d^2\alpha}{dt^2} = 5.13 - 20\alpha - 23.8\,p$$

or

$$\alpha'' = 13.32 - 52\alpha - 61.8\,p \qquad (1)$$

The second equation expresses the rate of change of pressure in the lower casing.

$$\frac{dp}{dt} = \frac{Q_i - Q_o}{C}$$

where Q_i = flow into the chamber (lb_m/sec)

Q_o = flow out of the chamber (lb_m/sec)

C = capacity of the chamber (lb_m/psi)

The flow restriction of the plate will be assumed linear for the derivation. If the flow is assumed incompressible,

$$Q_i = A_n V_i \rho_i f(\alpha) \quad \text{and} \quad Q_o = \beta V_o p_o$$

where A_n = area of nozzle

V_i = velocity at nozzle outlet

V_o = velocity at outlet valve

ρ_i = density at nozzle outlet

ρ_a = atmospheric density

$f(\alpha)$ = flow restriction of plate = 10α

β = an arbitrary constant depending upon gas demand

For a 1/4-inch nozzle and a 10-psi pressure drop,

$$Q_i = .0234\,\alpha\,\sqrt{147 - 4.7\,p - p^2}$$

and $$Q_o = .1835\,\beta\,\sqrt{p}$$

The volume of the lower casing is 19.9 cu in. Thus the capacity (C) = 5.87×10^{-5} lb_m/psi.

Substituting these relations into the expression for the rate of change of pressure gives the following results:

$$\frac{dp}{dt} = \frac{.0234\,\alpha\,\sqrt{147 - 4.7\,p - p^2}}{5.87 \times 10^{-5}} - \frac{.1835\,\beta\,\sqrt{p}}{5.87 \times 10^{-5}}$$

$$p' = 398\,\alpha\,\sqrt{147 - 4.7\,p - p^2} - \beta\,3130\,\sqrt{p} \qquad (2)$$

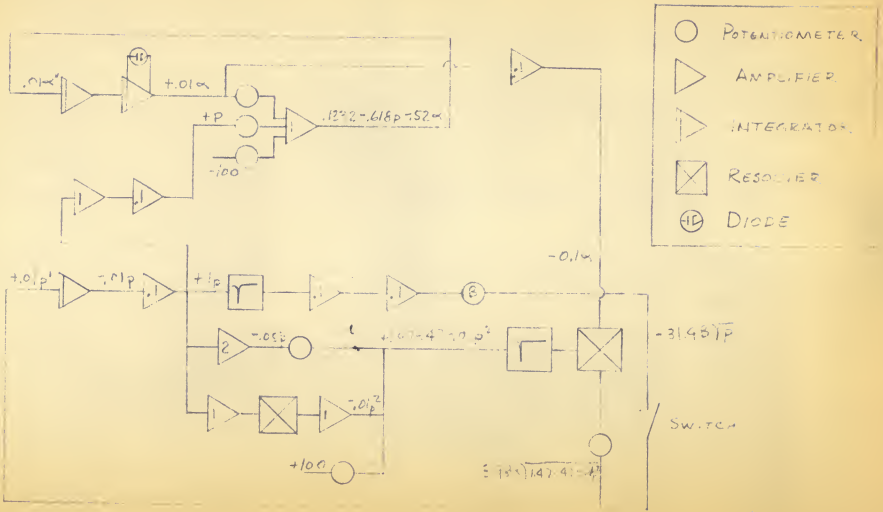
These two equations are very difficult if not impossible to solve by the methods of ordinary differential equations. However, the equations can be solved by use of an analogue computer. A simplified block diagram is shown in Plate XXIV. The wiring diagram is shown in Plate XXV. The switch represents the step function change in demand that occurs when a demand is increased. The diode in the second integrator is a limiter to insure that α cannot be negative. This is a boundary value on the original problem. An amplitude change has been made on both α and p. This would enable this problem to be set up on the analogue computer located at Kansas State College.

As a check on the validity of the equations, some more assumptions were made so that the equations could be solved by Laplace transform methods. First, it was assumed that the flow into the regulator is a function of α and the line pressure only. This assumption neglects terms of the order of the square root of 1.30 while the major term is the square root of 147.0.

EXPLANATION OF PLATE XXIV

Analogue computer solution in block diagram form.

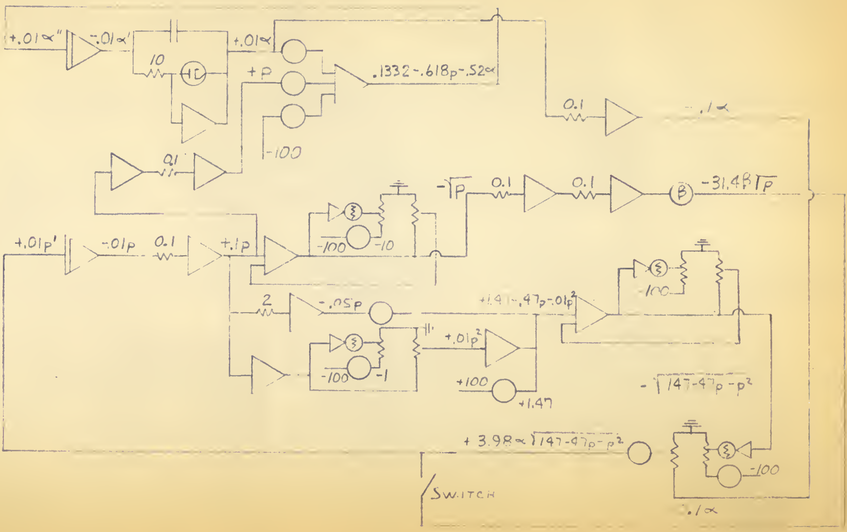
PLATE XXIV


POTENTIOMETER
AMPLIFIER
INTEGRATOR
RESOLVER
DIODE
SWITCH
+.01α
+P
-100
+100
-0.1α

EXPLANATION OF PLATE XXV

Wiring diagram for analogue computer solution.

PLATE XXV


+.01α''
-.01α'
10
+.01α
+p
-100
.1332-.618p-.52α
0.1
-.1α
0.1
-√p
0.1
0.1
β
-31.4β√p
+.01p'
-.01p
0.1
+.1p
-100
-10
2
-.05p
+1.41-.47p-.01p²
+.01p²
-100
-1
-100
+100
+1.47
-√147-47p-p²
+3.98α√147-47p-p²
-100
SWITCH
.1α

Next the flow out of the regulator is assumed to be a function of p rather than the square root of p. This is a good assumption for small pressure drops and cases where p varies only slightly.

Applying these assumptions, the equations reduce to the following:

$$\alpha'' + 52\alpha = 13.32 - 61.8\, p \qquad (1)$$

$$p' + \beta 3130\, p = 4825\alpha \qquad (3)$$

The Laplace transforms of these equations result in the following if initial conditions equal zero.

$$(s^2 + 52)\bar{\alpha} = \frac{13.32}{s} - (61.8)\,\bar{p} \qquad (4)$$

$$(s + \beta 3130)\,\bar{p} = (4825)\bar{\alpha} \qquad (5)$$

If the initial conditions of p are applied, equation (5) becomes

$$(s + \beta 3130)\,\bar{p} = (4825)\bar{\alpha} + .217 \qquad (6)$$

or

$$\bar{p} = \left(\frac{4825}{s + \beta 3130}\right)\bar{\alpha} + \frac{.217}{s + \beta 3130}$$

Substituting (6) into (4),

$$(s^2 + 52)\bar{\alpha} = \frac{13.32}{s} - \left(\frac{61.8 \times 4825}{s + 3130\beta}\right)\bar{\alpha} + \frac{61.8 \times .217}{s + 3130\beta}$$

$$\bar{\alpha} = \frac{13.32\,(s + 3130\beta) + 61.8(.217)s}{s(s^3 + 3130\beta\, s^2 + 52\, s + 162{,}760\beta + 298{,}185)}$$

Since the problem is one of stability, only the denominator is of interest. It is:

$$F(s) = s(s^3 + 3130\beta\, s^2 + 52\, s + 162{,}760\beta + 298{,}185) \qquad (7)$$

The use of Routh's criterion results in the following equation for stability.

$$-\frac{298,185}{3130\,\beta} > 0 \qquad (8)$$

Since this criteria cannot be met, the equations are unstable for any value of β.

An analogue computer solution of equations (1) and (2) would probably result in the same instability. This would be the expected result from consideration of the physical system because the first assumptions result in the loss of all damping from the equations. If these unknowns were evaluated and placed in equation (1), the equations would show that they are stable for a range of values of β.

Let ϕ be the evaluation of this damping. Equation (1) now becomes:

$$\alpha'' + \phi\alpha' + 52\alpha = 13.32 - 61.8\ p \qquad (9)$$

or transformed

$$(s^2 + \phi\ s + 52)\bar{\alpha} = \frac{13.32}{s} - (61.8)\ \bar{p}$$

$$(s^2 + \phi\ s + 52)\bar{\alpha} = 13.32 - \left(\frac{61.8 \times 4825}{s + 3130\,\beta}\right)\bar{\alpha} + \frac{61.8 \times .217}{s + 3130\,\beta}$$

$$\bar{\alpha} = \frac{13.32\ (s + 3130\,\beta) + 61.8(.217)s}{s\left[s^3 + (\phi + 3130\beta)s^2 + (3130\beta\phi + 52)s + 162,760\beta + 298,185\right]}$$

The Routh relationship for stability is:

$$3130\,\beta\phi + 52 - \frac{162,760\,\beta + 298,185}{\phi + 3130\,\beta} > 0$$

or $\quad (3130\,\beta + \phi)(3130\,\beta\phi + 52) - 162,760\,\beta - 298,185 > 0$

For a small value of ϕ, say one, this equation reduces to:

$$(3130\,\beta + 1)(3130\,\beta + 52) - 162,760\,\beta - 298,185) > 0$$

At $\beta > .174$, the above equation will be stable so it now possesses the properties of a regulator. It has been observed that instability occurs at low flows.

If the equations were derived using the second regulator plate, it can be seen that the coefficient of s in equation (7) is reduced. This increases the possibility of instability. Therefore the rubber plate with the sloping edges will be more stable. It will be recalled that it was observed during the force test that the sloping sided plate was the more stable of the two regulator plates.

CONCLUSIONS

After careful consideration of the test results, the following conclusions can be made.

When a flat plate is placed directly in front of the flow from a nozzle and moved toward the nozzle, it starts restricting the flow at a nozzle-to-plate distance of 0.38 D, where D is the diameter of the nozzle.

Because of the ease of data taking, reproducibility of data, and the dimensionless reduction of the data, it can be concluded that the test fixture designed for this thesis is a satisfactory one. Results obtained from this test fixture indicates that the total force on the plate is a function of the lift and not a constant through the flow restricting range.

The variation of the force in the flow restricting region, -15 per cent, would indicate that for design considerations it may be considered a constant. On this basis, a mathematical model can be derived which may be solved on the analogue computer.

By making some simplifying assumptions, the equations of this mathematical model may be solved by Laplace transform methods. If an arbitrary small damping constant is included in the equations, the solutions will be unstable only at low flows. The actual regulator is also unstable at low flows.

SUGGESTIONS FOR FUTURE RESEARCH

Any further work in this area should be directed into one of two channels.

The development of a theory for the force on the plate would be of basic interest. As a basis for this development, investigations should be made into the physical occurrences in the hodograph plane. This could well be the subject of a doctor's dissertation.

Of secondary interest is the complete analogue computer solution to the equations presented in this article and the development of a more exact mathematical model of the house regulator. Any work in this area should include complete instrumentation of the regulator and tests of it during operating conditions.

ACKNOWLEDGMENTS

The author would like to express his appreciation to the many persons without whose efforts this paper would not have been possible.

Special thanks should be given to Dr. R. G. Nevins, major instructor, and to the Fisher Governor Company, who sponsored this research.

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APPENDIX

Table 1. Flow restriction data for .125-inch diameter nozzle.

	ΔP[1] = 25		ΔP = 50		ΔP = 90	
Lift[2]	Flow[3]	Per cent	Flow	Per cent	Flow	Per cent
.005			182	20.2	300	21.2
.010	174	31.6	295	32.8	542	38.2
.015	233	42.4	435	48.4	730	51.4
.020	285	51.8	530	58.9	855	60.2
.025	320	58.2	615	68.3	980	69.0
.030	420	76.4	710	78.9	1150	81.0
.035	470	85.5	798	88.7	1290	90.8
.040	528	96.0	880	97.8	1400	98.7
.045	550	100.0	900	100.0	1420	100.0
.050	550		900		1420	

Table 2. Flow restriction data for .250-inch diameter nozzle.

	ΔP[1] = 10		ΔP = 50		ΔP = 90	
Lift[2]	Flow[3]	Per cent	Flow	Per cent	Flow	Per cent
.010			580	17.6	990	18.3
.020	325	28.2	980	29.7	1660	30.8
.030	465	40.3	1430	43.4	2400	44.5
.040	660	57.1	2008	60.8	3460	64.1
.050	747	64.7	2348	71.2	3980	73.7
.060	815	70.5	2592	78.5	4420	81.9
.070	925	80.1	2980	90.2	4900	90.7
.080	1030	89.3	3220	97.6	5320	98.5
.090	1035	89.7	3280	99.4	5380	99.6
.100	1115	96.6	3300	100.0	5380	99.6
.110	1155	100.0	3300		5400	100.0

Table 3. Flow restriction data for .500-inch diameter nozzle.

	ΔP[1] = 10			ΔP = 10	
Lift[2]	Flow[3]	Per cent	Lift	Flow	Per cent
.010	272	6.3	.140	3900	90.4
.020	620	14.4	.150	3980	92.2
.030	1800	41.7	.160	4030	93.4
.050	1700	39.4	.170	4130	95.6
.070	2380	55.2	.180	4260	98.7
.090	2790	64.6	.190	4260	98.7
.110	3250	75.3	.200	4320	100.0
.120	3340	77.3	.210	4320	
.130	3650	84.5			

[1] ΔP = pressure difference (psi).
[2] Lift = distance from plate to nozzle (inches).
[3] Flow = flow through nozzle (SCFH).

A STUDY OF THE FORCE REACTIONS UPON FLOW RESTRICTING ELEMENTS USED IN CONJUNCTION WITH NOZZLES AND ORIFICES

by

PHILIP COURTNEY GREGORY

B. S., Kansas State College
of Agriculture and Applied Science, 1957

AN ABSTRACT OF
A MASTER'S THESIS

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Mechanical Engineering

KANSAS STATE COLLEGE
OF AGRICULTURE AND APPLIED SCIENCE

1958

Because of the increasing use of automatic controls in the process industry, it has become necessary for manufacturers to know more about the design characteristics of the components of these controls. A typical example is the house service gas regulator. An analysis of a gas regulator shows two areas in which design characteristics are not known. There is no knowledge available on the damping characteristics or of the forces on the plate in the regulator used to restrict or shut off the flow of gas through the regulator. It is this second problem, the force on a plate used as a flow restricting element, that is investigated in this thesis.

The author describes the design of suitable test fixtures to locate the region of flow restriction and measure the forces. Data presented shows that flow restriction ceases at a plate-to-nozzle distance of 0.38 D, where D is the diameter of the nozzle. The force on the plate is shown to drop off and rise again through the flow restricting range. A dip of -15 per cent of total force occurs.

By making certain assumptions about the damping characteristics of the regulator, a mathematical model is constructed and the analogue computer wiring to accomplish the solution to these equations is presented. As a check on the mathematical model, assumptions are made which permit solutions by the Laplace transform methods. These solutions show that if an arbitrary small value of damping is included, the regulator is unstable only at low flows. Instability of the actual regulator also occurs at low flows.