THE MEASUREMENT OF LIQUID DIELECTRIC CONSTANTS AT MICROWAVE FREQUENCIES

by

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INTRODUCTION

The measurement of dielectric constants at microwave frequencies has come to be of considerable interest in recent years. From a practical standpoint this information is needed because of the increased use of microwaves in communications. It is of interest from a theoretical standpoint because the production of microwaves made possible the securing of data in a part of the electromagnetic spectrum that could previously not be studied.

There are in general two methods by which the dielectric constant at microwave frequencies can be measured. The first method involves the use of a resonant chamber. Here a change in resonant frequency is measured due to the introduction of the dielectric material. This method is limited by the fact that it cannot give the loss tangent, but it has the advantage of using a small amount of material.

The second method involves the use of standing waves. It has the advantage of giving the value for the loss tangent. It is this second method that is described in this research paper.

THEORY

The dielectric constant ε is defined by $D = \varepsilon E$ where D is the electric flux density and E is the electric field intensity. If E is periodic then D must be periodic. In general, however, D is not in phase with E so that ε may be conveniently represented by $\varepsilon = \varepsilon_1 - i\,\varepsilon_2$ where $i = \sqrt{-1}$. Tanδ is defined by $\tan\delta = \frac{\varepsilon_2}{\varepsilon_1}$. It may be shown that ε_2 is proportional to the energy loss in the dielectric (1). Therefore $\tan\delta$ is called the loss tangent.

To find relations to calculate the dielectric constant from obtainable data the following electromagnetic field equations are used:

$$\nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t} \qquad \text{Faraday's Law}$$

$$\nabla \times \underline{H} = \underline{J} + \frac{\partial \underline{D}}{\partial t} \qquad \text{Ampere's Law}$$

$$\nabla \cdot \underline{D} = q$$

Gauss' Law

$$\nabla \cdot \underline{B} = 0$$

$$\underline{D} = \varepsilon \underline{E}$$

$$\underline{B} = \mu \underline{H}$$

These symbols have the following meaning in MKS units:

$\underline{E}$ electric field intensity

$\underline{H}$ magnetic field intensity

$\underline{D}$ electric flux density

$\underline{J}$ convection current density

q charge density

ε dielectric constant

μ permeability

$\underline{B}$ magnetic flux density

Assuming the oscillator a sine wave generator, solutions of the above equations can be found that satisfy the boundary condition inside the waveguide showing that electromagnetic waves can be propagated there. The condition is that the tangential component of $\underline{E}$ to the wall of the waveguide (considered a perfect conductor) must everywhere be zero.

These solutions are of the form:

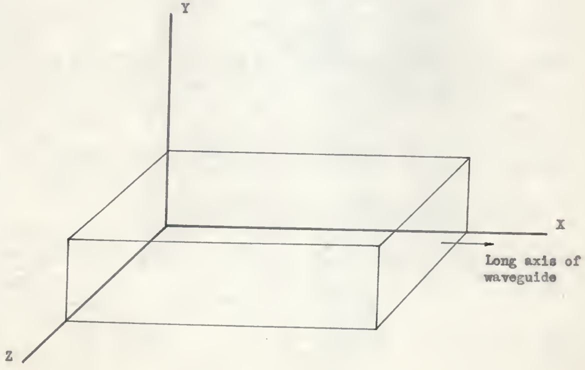
$$E_x = H_y = E_z = 0$$

$$E_y = Ae^{\gamma x}$$

$$H_z = Ce^{\gamma x}$$

$$H_x = Be^{\gamma x}$$

Where $\gamma = \alpha + i\beta$ is the propagation constant. α is the attenuation and $\beta = \frac{2\pi}{\lambda_g}$ is the phase constant with λ_g the wavelength in the guide. The mode of the propagated wave is considered to be TE_{01}. The waveguide is oriented with respect to the coordinate system as shown:



Using these solutions it may be shown that the following relation must hold:

<u>Equation</u> (<u>1</u>). $$\mathcal{E} = \frac{\dfrac{1}{\lambda_c^2} - \left(\dfrac{\gamma}{2\pi}\right)^2}{\dfrac{1}{\lambda_c^2} + \dfrac{1}{\lambda_g^2}}$$

where λ_c, called the cutoff wavelength, is the longest wave that will propagate down the waveguide. For rectangular waveguide λ_c is twice the width of the waveguide (2).

The waveguide is terminated by a conducting sheet so that reflection occurs. The following relations then hold for the standing wave inside the guide:

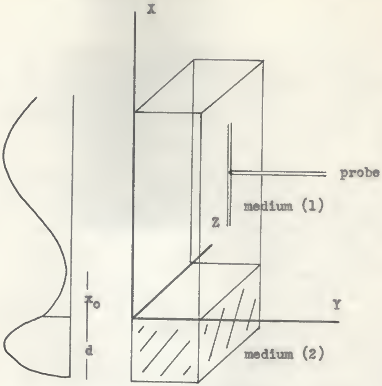
$$E_{y1} = A_{i1}\, e^{\gamma_1 x} + A_{r1}\, e^{-\gamma_1 x}$$

$$H_{z1} = C_{i1}\, e^{\gamma_1 x} - C_{r1}\, e^{-\gamma_1 x}$$

$$E_{y2} = A_{i2}\, e^{\gamma_2 x} + A_{r2}\, e^{-\gamma_2 x}$$

$$H_{z2} = C_{i2}\, e^{\gamma_2 x} - C_{r2}\, e^{-\gamma_2 x}$$

where A and C are the amplitudes, i refers to incident, r refers to reflected, 1 refers to medium 1, 2 refers to medium 2 and the waveguide containing two dielectric materials is oriented as shown on page 5.



Let x_o be the distance from the surface of the dielectric to the first minimum of the standing wave. At this point the incident and reflected waves must have a phase difference of π radians. Also at the reflector, i.e. at $x = -d$, there must be a voltage node. Making use of these conditions and neglecting the attenuation in the air filled part of the guide, the following relation results, assuming that μ for dielectrics = 1 (3):

$$\frac{\tanh \gamma_2 d}{\gamma_2 d} = \frac{\dfrac{E\ min}{E\ max} - i \tan \dfrac{360\ x_o}{\lambda g}}{1 - i \dfrac{E\ min}{E\ max} \tan \dfrac{360\ x_o}{\lambda g}} \times \frac{\lambda g}{2\pi d i}$$

Equating the real parts of this expression gives:

Equation (2).
$$\frac{-\lambda_g \tan \dfrac{360\, x_o}{\lambda_g}\left(1 - \dfrac{E^2 \min}{E^2 \max}\right)}{2\pi d\left(1 + \dfrac{E^2 \min}{E^2 \max}\tan^2 \dfrac{360\, x_o}{\lambda_g}\right)} =$$

$$\frac{\beta_2 d \tan\beta_2 d\,(1 - \tanh^2\alpha_2 d) + \alpha_2 d\,(1 + \tan\beta_2 d)\tanh\alpha_2 d}{(\alpha_2^2 d^2 + \beta_2^2 d^2)\,(1 + \tanh^2\alpha_2 d \tan\beta_2 d)}$$

and for the imaginary parts,

Equation (3).
$$\frac{-\lambda_g \dfrac{E \min}{E \max}\left(1 + \tan^2 \dfrac{360\, x_o}{\lambda_g}\right)}{2\pi d\left(1 + \dfrac{E^2 \min}{E^2 \max}\tan^2 \dfrac{360\, x_o}{\lambda_g}\right)} =$$

$$\frac{\alpha_2 d \tan\beta_2 d - \alpha_2 d \tanh^2\alpha_2 d \tan\beta_2 d - \beta_2 d \tanh\alpha_2 d - \beta_2 d \tanh\alpha_2 d \tan^2\beta_2 d}{(1 + \tanh^2\alpha_2 d \tan\beta_2 d)\,(\alpha_2^2 d^2 + \beta_2^2 d^2)}$$

If $\dfrac{E \min}{E \max}$ and $\alpha_2 d$ are small, equation (2) gives:

Equation (4).
$$\frac{\tan\beta_2 d}{\beta_2 d} = -\frac{\lambda_g}{2\pi d}\tan\frac{360\, x_o}{\lambda_g}$$

and equation (1) gives:

Equation (5).
$$\mathcal{E}_1 = \frac{\dfrac{1}{\lambda_c^2} + \left(\dfrac{\beta_2 d}{2\pi d}\right)^2}{\dfrac{1}{\lambda_c^2} + \dfrac{1}{\lambda_g^2}}$$

Also for small α_2 and $\frac{E\ min}{E\ max}$, equation (3) gives:

$$\alpha_2 d \doteq \frac{\beta_2^2\, d^2\, \lambda_g}{2\pi d} \times \frac{E\ min}{E\ max} \times \frac{1 + \tan^2 \dfrac{360\, x_o}{\lambda_g}}{\beta_2 d\,(1 + \tan^2 \beta_2 d) - \tan\beta_2 d}$$

It can be shown that:

$$\frac{E\ min}{E\ max} = \frac{\pi \Delta x}{\lambda_g}$$

where Δx is the width of the minimum of the standing wave to the double power points. An equivalent value for α_2 is obtained from equation (1) which is:

$$\alpha_2 d = - \tan\delta \left[\frac{1}{\lambda_c^2} + \left(\frac{\beta_2}{2\pi}\right)^2\right] \left(\frac{2\pi}{\beta_2}\right)^2$$

Substituting this into the above expression the value for the loss tangent is given by (4):

Equation (6).
$$\tan\delta = \frac{\Delta x}{d} \times \frac{\dfrac{1}{\lambda_c^2} + \dfrac{1}{\lambda_g^2} - \dfrac{1}{\lambda_c^2 \mathcal{E}_1}}{\dfrac{1}{\lambda_c^2} + \dfrac{1}{\lambda_g^2}} \times \frac{\beta_2 d\left(1 + \tan^2 \dfrac{360\, x_o}{\lambda_g}\right)}{\beta_2 d\,(1 + \tan^2 \beta_2 d) - \tan\beta_2 d}$$

APPARATUS

Plate I is a complete diagram of the apparatus used. Microwave energy is obtained from a reflex klystron tube type 2K25, which has a frequency range of approximately 8700 to 9700 megacycles per second. The tube is mounted on a section of waveguide and the energy is introduced into the guide by means of a coaxial line directly from the tube. The accelerating voltage is obtained from a regulated 300 volt power supply. Bias is supplied by a 90 volt battery and the modulating voltage is obtained from the negative sawtooth of an oscilloscope. The function of the diode is to keep the base level of the sawtooth voltage at the value of the bias voltage. The base level height and the amplitude of the sawtooth are controlled by the two potentiometers, R_2 and R_3 respectively.

The attenuator can be adjusted to remove any fractional amount of energy being propagated in the waveguide. Sometimes attenuation is necessary to isolate the oscillator from the rest of the circuit.

The wavemeter is of the transmission type. It is calibrated so the frequency being transmitted is known from the setting of the micrometer screw.

The standing wave detector is shown in Plate II.

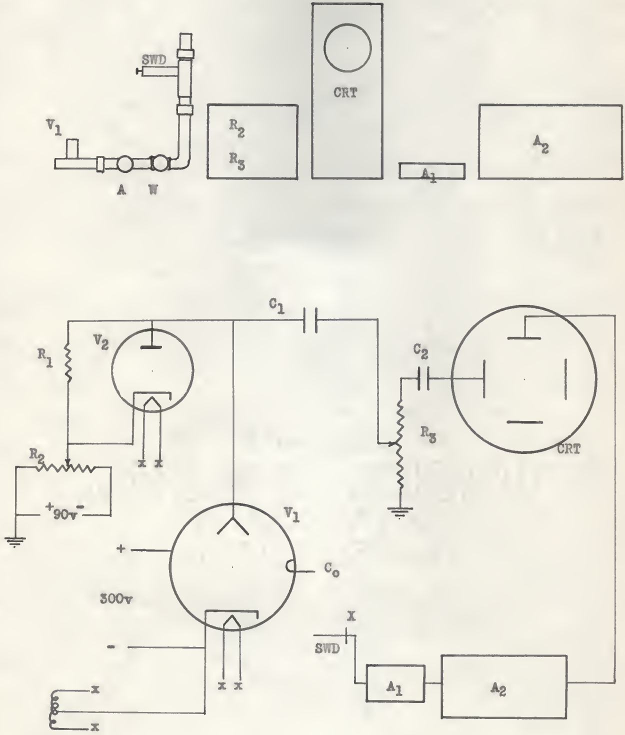
Crystals of the type used have been tested to see that the "square law" is obeyed over the region in which the crystal is used (5).

The amplifier has been tested to see that amplification is linear over the region in which it is used. Under typical operation it is set to give a gain of about 10,000.

EXPLANATION OF PLATE I

C_1, C_2	1 Mfd., 600 v. condenser
R_1	120,000 ohm resistor
R_2	20,000 ohm potentiometer
R_3	100,000 ohm potentiometer
V_1	2 K 25 reflex klystron
V_2	6 H 6 diode
CRT	Oscilloscope
X	1 N 23 crystal
A_1	Preamplifier
A_2	Amplifier
SWD	Standing wave detector
C_o	Coaxial line from klystron
A	Attenuator
W	Wavemeter

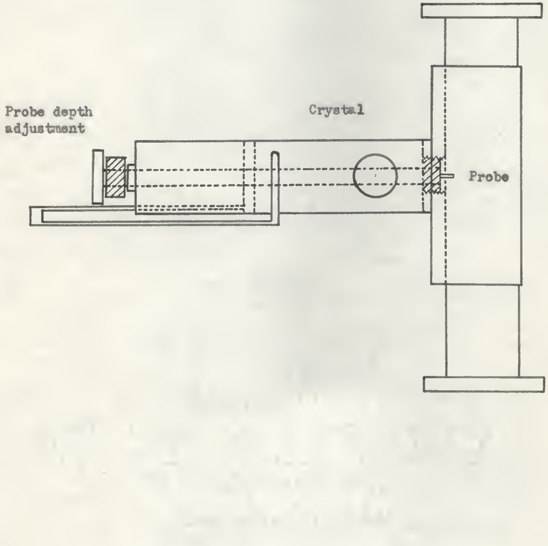
PLATE I


SWD
V1
R2
R3
CRT
A1
A2
A W
C1
V2
R1
C2
R3
CRT
R2
+90v-
x x
V1
+
Co
300v
-
X
SWD
x
x
x x
A1
A2

EXPLANATION OF PLATE II

Standing Wave Detector

PLATE II



PROCEDURE

The following equations have been developed in the theory:

Equation (4).
$$\frac{\tan \beta_2 d}{\beta_2 d} = -\frac{\lambda_g}{2\pi d} \tan \frac{360\, x_o}{\lambda_g}$$

Equation (5).
$$\mathcal{E}_1 = \frac{\frac{1}{\lambda_c^2} + \left(\frac{\beta_2 d}{2\pi d}\right)^2}{\frac{1}{\lambda_c^2} + \frac{1}{\lambda_g^2}}$$

Equation (6).
$$\tan\delta = \frac{\Delta x}{d} \times \frac{\frac{1}{\lambda_c^2} + \frac{1}{\lambda_g^2} - \frac{1}{\lambda_c^2 \mathcal{E}_1}}{\frac{1}{\lambda_c^2} + \frac{1}{\lambda_g^2}} \times \frac{\beta_2 d \left(1 + \tan^2 \frac{360\, x_o}{\lambda_g}\right)}{\beta_2 d\,(1 + \tan^2 \beta_2 d) - \tan \beta_2 d}$$

The apparatus was originally set up with the long axis of the waveguide in a vertical position and the shorted end at the bottom as shown on page 5. The liquid of unknown dielectric constant was poured into the waveguide to a depth d. The traveling probe was then adjusted to find the position of minimum of the standing wave. This position was indicated by the minimum height of the pip on the oscilloscope screen. The distance from the probe to the surface of the liquid was measured with a vernier caliper to give x_o. Then the probe was displaced to either side of the minimum until the pip on the screen was twice minimum height. This gave the double power points. The distance between the double power points was measured with a traveling microscope to give Δx.

For the waveguide used λ_c = 5.7920 centimeters and the free space wavelength was obtained from the calibrated frequency meter. λ_g was calculated from equation:

$$\frac{1}{\lambda^2} = \frac{1}{\lambda_g^2} + \frac{1}{\lambda_c^2}$$

With these data the following procedure was used to calculate $\mathcal{E}_1$ and $\tan\delta$. λ_g, d and x_o were substituted into equation (4) which was solved for $\beta_2 d$ by trial. Putting $\beta_2 d$ into equation (5) gives the value of $\mathcal{E}_1$. Which value of $\beta_2 d$ to use could be determined only by taking two sets of data using different values of d. Only one $\beta_2 d$ from each set gave a consistent value for $\mathcal{E}_1$.

After obtaining the value of Δx everything was known on the right hand side of equation (6) so that $\tan\delta$ could be calculated.

In attempting to use this procedure it was found that the values for $\tan\delta$ could not be repeated satisfactorily. Also the fact that the experiment needed to be repeated several times to find $\mathcal{E}_1$ resulted in a large amount of calculating. Subsequently equations (4), (5), and (6) were examined to find values for the arbitrary constants so as to make the errors minimum. The uncertainty in $\mathcal{E}_1$ may be calculated as follows:

Equation (4) is written as:

<u>Equation</u> (<u>7</u>). $$\frac{\tan\ (\beta_2 d + \Delta\beta_2 d)}{\beta_2 d + \Delta\beta_2 d} = -\frac{\lambda_g}{2\pi d}\tan\frac{360(x_o + \Delta x_o)}{\lambda_g}$$

where $\Delta\beta_2 d$ is the error in $\beta_2 d$ due to the error Δx_o in x_o. The contribution to $\Delta\beta_2 d$ due to an error in d is neglected at this time. The error in λ_g is neglected compared to the error in x_o or in d.

Divide equation (7) by equation (4) to get:

$$\frac{\beta_2 d}{\beta_2 d + \Delta\beta_2 d} \times \frac{\tan(\beta_2 d + \Delta\beta_2 d)}{\tan \beta_2 d} = \frac{\tan \dfrac{360\,(x_o + \Delta x_o)}{\lambda_g}}{\tan \dfrac{360\, x_o}{\lambda_g}}$$

or

$$\left[1 - \frac{\Delta\beta_2 d}{\beta_2 d}\right] \times \frac{\tan(\beta_2 d + \Delta\beta_2 d)}{\tan \beta_2 d} = \frac{\tan \dfrac{360\,(x_o + \Delta x_o)}{\lambda_g}}{\tan \dfrac{360\, x_o}{\lambda_g}}$$

Since the dielectric constant is of the order of 2.3 and when d is not less than 3 centimeters, $\dfrac{\Delta\beta_2 d}{\beta_2 d}$ can be neglected compared to 1 if the error in $\mathcal{E}_1$ is to be small. This gives:

$$\frac{\tan(\beta_2 d + \Delta\beta_2 d)}{\tan \beta_2 d} = \frac{\tan \dfrac{360\,(x_o + \Delta x_o)}{\lambda_g}}{\tan \dfrac{360\, x_o}{\lambda_g}}$$

Using equation (4) this may be rearranged without approximation to give:

$$\tan\Delta\beta_2 d = \frac{\beta_2 d \dfrac{\lambda_g}{2\pi d} \tan \dfrac{360\,\Delta x_o}{\lambda_g}}{\left[1 - \left(\dfrac{\beta_2 d\, \lambda_g}{2\pi d}\right)^2\right]\left[\sin \dfrac{360\, x_o}{\lambda_g} \;\cos \dfrac{360\, x_o}{\lambda_g} \tan \dfrac{360\,\Delta x_o}{\lambda_g} + \sin^2 \dfrac{360\, x_o}{\lambda_g}\right] - 1}$$

which is approximately $\Delta\beta_2 d$. Setting $\tan \frac{360\,\Delta x_o}{\lambda_g} = \frac{2\pi\,\Delta x_o}{\lambda_g}$ the fractional error in $\beta_2 d$ is given by:

Equation (8).

$$\frac{\Delta\beta_2 d}{\beta_2 d} = \left\{\left[1 - \left(\frac{\beta_2 d\,\lambda_g}{2\pi d}\right)^2\right]\left[\frac{2\pi\,\Delta x_o}{\lambda_g}\sin\frac{360\,x_o}{\lambda_g}\cos\frac{360\,x_o}{\lambda_g} + \sin^2\frac{360\,x_o}{\lambda_g}\right] - 1\right\}^{-1}\frac{\Delta x_o}{d}$$

Since $\lambda_g \doteq 4$ centimeters, and for $\varepsilon_1 > 2.3$ and $d > 3$ centimeters, equation (5) shows that $\frac{\beta_2 d\,\lambda_g}{2\pi d} > 1$. Therefore the maximum value of the term containing sines and cosines gives the least error. The least value of the same term gives the largest error. The following is the range of probable fractional error in $\beta_2 d$:

$$\frac{\Delta x_o}{d} > \frac{\Delta\beta_2 d}{\beta_2 d} > \left(\frac{2\pi d}{\beta_2 d\,\lambda_g}\right)^2 \frac{\Delta x_o}{d}$$

Now consider the contribution to $\Delta\beta_2 d$ due to an error in d. Starting as before write:

$$\frac{\tan(\beta_2 d + \Delta\beta_2 d)}{\tan\beta_2 d} = \frac{\frac{\lambda_g}{2\pi d} + \frac{\lambda_g}{2\pi}\Delta\frac{1}{d}}{\frac{\lambda_g}{2\pi d}} = 1 + d\,\Delta\frac{1}{d} = 1 - \frac{\Delta d}{d}$$

or

$$\frac{\tan\beta_2 d + \tan\Delta\beta_2 d}{\tan\beta_2 d\,(1 - \tan\beta_2 d\,\tan\Delta\beta_2 d)} = 1 - \frac{\Delta d}{d}$$

$$\tan\Delta\beta_2 d = -\frac{\tan\beta_2 d}{\sec^2\beta_2 d - \frac{\Delta d}{d}\tan^2\beta_2 d} \times \frac{\Delta d}{d}$$

$$\tan\Delta\beta_2 d = -\frac{1}{\frac{1}{\tan\beta_2 d} + \tan\beta_2 d} \times \frac{\Delta d}{d}$$

since $\sec\beta_2 d \geq \tan\beta_2 d$ and $\frac{\Delta d}{d}$ is small. This shows that the contribution to $\Delta\beta_2 d$ due to Δd may be neglected for all x_o compared to the contribution due to Δx_o since Δx_o and Δd are of the same magnitude.

From equation (5):

$$\Delta\mathcal{E}_1 = \frac{\frac{2\beta_2 d\Delta\beta_2 d}{(2\pi d)^2}}{\frac{1}{\lambda_c^2} + \frac{1}{\lambda_g^2}}$$

Dividing the above expression by equation (5) there results for the fractional error in $\mathcal{E}_1$,

$$\frac{\Delta\mathcal{E}_1}{\mathcal{E}_1} = \frac{\Delta x_o}{\lambda_g^2 d\beta_2 d\left(\frac{1}{\lambda_c^2} + \frac{1}{\lambda_g^2}\right)}$$

The error in $\tan\delta$ is calculated as follows, starting with the equation:

$$\tan\delta = \frac{\Delta x}{d} \times \frac{\frac{1}{\lambda_c^2} + \frac{1}{\lambda_g^2} - \frac{1}{\lambda_c^2 \ \varepsilon_1}}{\frac{1}{\lambda_c^2} + \frac{1}{\lambda_g^2}} \times \frac{\beta_2 d\left(1 + \tan^2 \frac{360\ x_o}{\lambda_g}\right)}{\beta_2 d\ (1 + \tan^2 \beta_2 d) - \tan \beta_2 d}$$

consider first the error in the third factor. If the value of the factor is changed by, say, less than one tenth, if $\tan \beta_2 d$ is neglected, then the error in $\beta_2 d$ may be neglected if the other numbers give errors of the order of 1 per cent. This is saying that:

$$\beta_2 d\ (1 + \tan^2 \beta_2 d) > 10 \tan \beta_2 d$$

This inequality is satisfied if $\beta_2 d > 5$. Since this is nearly always true the fractional error in the third factor is given approximately by:

$$\frac{\Delta\left(1 + \tan^2 \frac{360\ x_o}{\lambda_g}\right)}{1 + \tan^2 \frac{360\ x_o}{\lambda_g}} - \frac{\Delta\,(1 + \tan^2 \beta_2 d)}{1 + \tan^2 \beta_2 d}$$

or

$$2 \tan \frac{360\ x_o}{\lambda_g} \times \frac{2\pi\,\Delta x_o}{\lambda_g} - 2 \tan \beta_2 d\,\Delta\beta_2 d$$

Using equations (4) and (8) this reduces to:

$$2\left[\frac{2\pi}{\lambda_g} - \frac{(\beta_2 d)^2 \lambda_g}{2\pi d^2}\ \frac{1}{\left[\left(\frac{\beta_2 d\,\lambda_g}{2\pi d}\right)^2 - 1\right]\left[\frac{2\pi\,\Delta x_o}{\lambda_g} \sin\frac{360\ x_o}{\lambda_g} \cos\frac{360\ x_o}{\lambda_g} + \sin\frac{360\ x_o}{\lambda_g}\right] + 1}\right] \tan\frac{360 x_o}{\lambda_g}\,\Delta x_o$$

A graph of $\frac{\text{fractional error in } \tan\delta}{x_0}$ vs x_0 is shown in Figure 1. Although the graph shows that the error should be low when x_0 is near $\frac{1}{2}\lambda_g$ data should probably not be taken there. This is because the factors inside the brackets in the derived expression will likely not cancel out in a practical case. (For example the contribution to $\beta_2 d$ due to Δd has been neglected). Since the tangent becomes large the error again becomes large. For data taken in this region care should at least be taken that when equation (4) is solved for $\beta_2 d$, the value should be substituted back into the equation to solve for $\tan \beta_2 d$. This will usually cause the error in $\tan \frac{360\, x_0}{\lambda_g}$ and $\tan \beta_2 d$ to be in the same direction.

$\frac{\Delta x}{d}$ contributes considerably to the error in $\tan\delta$. This may be lessened by increasing the depth to increase the absorption so the minimum is broadened. The rest of the factors do not contribute significantly to the error in $\tan\delta$.

For the apparatus used according to the foregoing procedure it is seen that the major contributing factor to the errors is the uncertainty in x_0. This uncertainty is about 1.5 millimeters which is mostly due to surface tension and the evaporation of the liquid. The inability to find the exact position of the minimum also contributes.

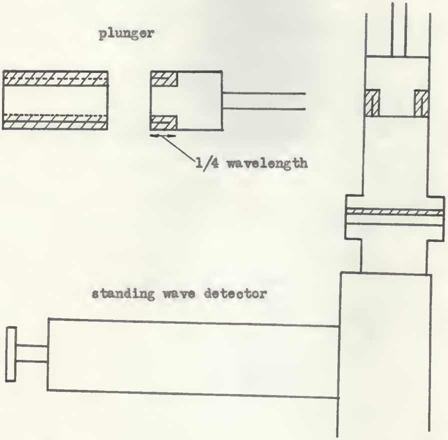
For such an error in x_0 the value of ε_1 may be obtained with considerable accuracy. The calculated values of $\tan\delta$, however, may contain large errors.

To overcome the surface tension effect and evaporation a thin plastic sheet is used between two sections of waveguide. A shorting plunger has been designed so that the depth of the liquid can be varied to cause the minimum detected by the probe to fall at any desired position. A diagram of the plunger is shown in Plate III.

EXPLANATION OF PLATE III

Diagram of Plunger and Standing Wave Detector

PLATE III



By use of a plunger a slightly different procedure is used to find $\mathcal{E}_1$. According to equation $\frac{\mathcal{E}_1}{\lambda^2} = \frac{1}{\lambda_g^2} + \frac{1}{\lambda_c^2}$ if the wavelength in the liquid is known $\mathcal{E}_1$ can be calculated. The wavelength in the liquid is obtained by leaving the probe fixed in position and measuring the displacement of the plunger for a number of half wavelengths. The number of half wavelengths traversed by the plunger are counted by noting the number of times the pip on the screen passes through a minimum value. This method gives the value of $\mathcal{E}_1$ to within .1 per cent. The answer is also obtained with much less calculation.

After the value for $\mathcal{E}_1$ was obtained it was planned that the probe could be set at a desired point at which the error in measuring $\tan\delta$ would be the least, i.e. so x_0 equals $\frac{1}{2}\ \lambda_g$, and the plunger could be moved until a minimum fell at that point. The plunger was then to be left in position and the data obtained in the usual way to calculate $\tan\delta$, using the previously calculated value for $\mathcal{E}_1$. On attempting to do this, however, it was found that the plunger and waveguide were not built accurately enough to determine d by direct measurement. In effect reflection did not take place at the front face of the plunger and also occurred with some loss. Thus a fixed short has been used to obtain the loss tangent. If the value of x_0 is too far from a $\frac{1}{2}\ \lambda_g$ position with a fixed length of sample a piece of waveguide of different length can be used to get the reflector in the desired position.

It is supposed that the slot and the probe in the waveguide also cause error by absorption and distortion of the field. The effect of the slot has not been studied. Figure 2 shows that the effect of the probe is quite low. Losses in the walls of the guide are too low to be detected.

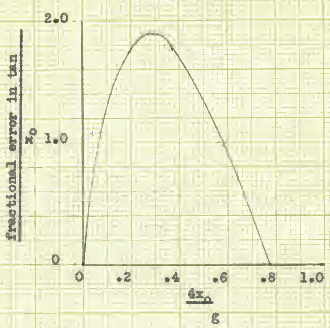


Fig. 1. Fractional error in tan as function of $\frac{4x_o}{\lambda g}$

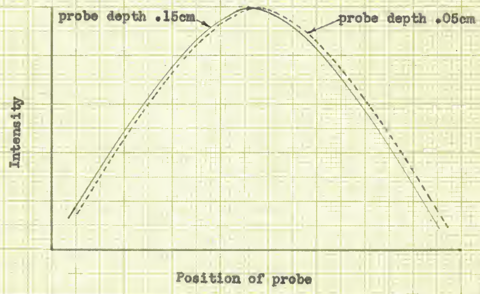


Fig. 2. Intensity as a function of the position of the probe for different probe depths.

RESULTS

Following is a table showing the measured wavelengths in a solution of n butyl alcohol in benzene (not thiophene free) for different concentrations. A curve is plotted in Figure 3 showing the relation.

Table 1. Results of dielectric constant measurements.

% Alcohol	Measured wavelength	Wavelength from graph λ_1	Error of measured wavelength
0	2.390 cm	2.391 cm	.001 cm
2.04	2.350	2.352	.002
4.12	2.327	2.324	.003
6.25	2.305	2.299	.006
8.42	2.274	2.276	.002
10.62	2.255	2.254	.001
12.89*	2.192	2.229	
15.18	2.205	2.205	.000
0	2.392	2.391	.001

Average error = .002 cm

*Later measurements show this reading to be in error.

Wavemeter reading = .3220 inches

Frequency (from calibration curve) = $.8983 \times 10^{10}$ cps

Free space wavelength = $\lambda = \dfrac{2.9978 \times 10^{10}}{.8983 \times 10^{10}} = 3.337$ cm

Cutoff wavelength = λ_c = twice waveguide width = 5.7920 cm

$$\varepsilon_1 = \lambda^2\left(\frac{1}{\lambda_1^2} + \frac{1}{\lambda_c^2}\right) = 2.280 \text{ for benzene}$$

Average error in ε_1 = .15 per cent

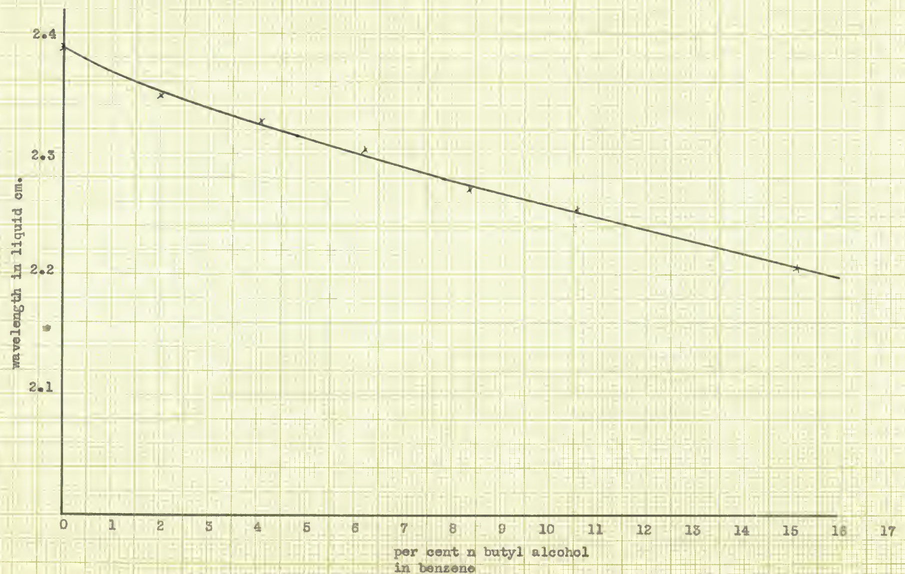


Fig. 3. Wavelength as function of per cent n butyl alcohol in benzene at frequency 8.983 x 10^9 cps

Roberts and Von Hipple obtained the value 2.25 (3) and Abadie obtained 2.29 (6) as the dielectric constant of benzene (thiophene free) at wavelengths ranging from 3-17 cm wavelengths. ε_1 vs per cent n butyl alcohol in benzene is plotted in Figure 4.

Table 2 gives pertinent data to calculate $\tan\delta$.

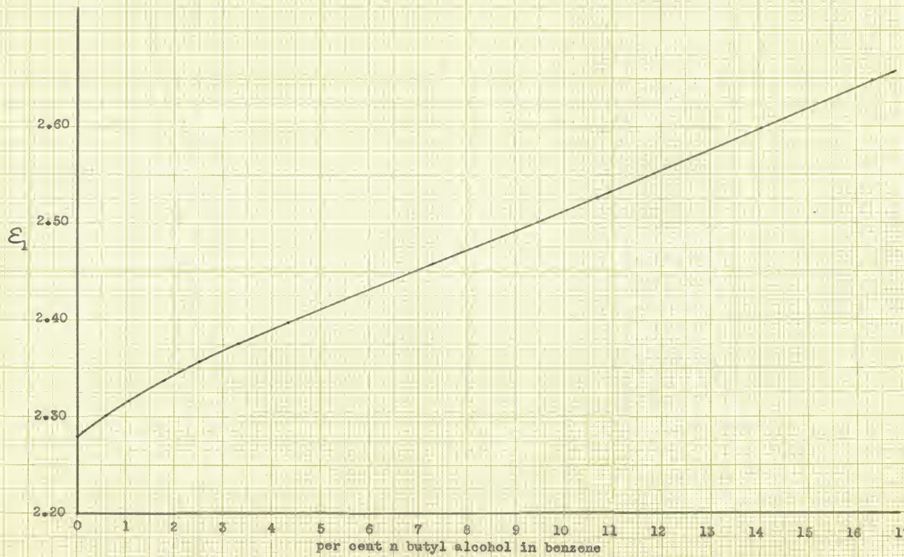


Fig. 4. Real part of dielectric constant as function of per cent n butyl alcohol in benzene at frequency 8.983 x 10^9 cps.

Table 2. Results of loss tangent measurements.

% Alcohol	From graph ε_1	Measured Δx	Measured x_0	$\tan \frac{360\, x_0}{\lambda_g}$	$\tan \beta_2 d$	$\tan \delta$
0	2.28	.192	1.76	- .487	.833	.0059
2.00	2.34	.585	1.48	-1.21	2.09	.0112
4.02	2.38	1.069	1.82	- .378	.661	.0411
5.98	2.43	.883	2.03	- .038	.068	.0371
7.80	2.47	.800	1.86	- .306	.546	.0282
9.77	2.51	.760	1.91	- .227	.411	.0287
11.7	2.55	.825	1.87	- .257	.468	.0305
13.8	2.58	.868	1.91	- .227	.418	.0330
16.1	2.64	.883	1.91	- .227	.422	.0333
			Using new sample of benzene			
3.00	2.37	.933	.28	.476	-.833	.0284
4.00	2.39	.734	.05	.078	-.137	.0304
5.12	2.42	1.060	0	0	0	.0444
6.26	2.44	.777	1.96	-.148	.263	.0311
7.52	2.46	.639	1.89	-.261	.467	.0236
8.58	2.48	.667	1.85	-.325	.582	.0232
16.6	2.65	.793	1.86	-.308	.573	.0276
19.6	2.71	.782	1.83	-.358	.677	.0258
21.6	2.77	.801	1.82	-.376	.718	.0257
24.6	2.83	.808	1.85	-.325	.630	.0275
			Using new sample of benzene			
4.45	2.40	1.310	2.02	-.054	.095	.0541
4.85	2.41	1.236	2.04	-.021	.037	.0516
5.36	2.42	.600	2.04	-.021	.037	.0250

Wavemeter reading = .3412 inches

Frequency (from calibration curve) = $.8949 \times 10^{10}$ cps

λ_c = cutoff wavelength = 5.7920 cm

$$\lambda_g = \text{wavelength in waveguide} = \frac{1}{\frac{1}{\lambda^2} - \frac{1}{\lambda_c^2}} = 4.105 \text{ cm}$$

Depth = d = 20.6 cm

The curve is shown in Figure 5.

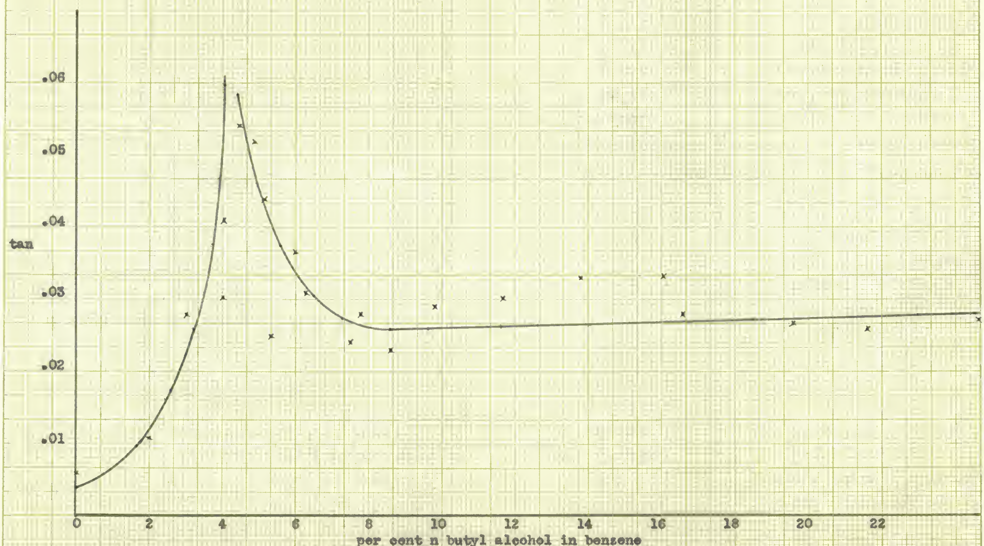
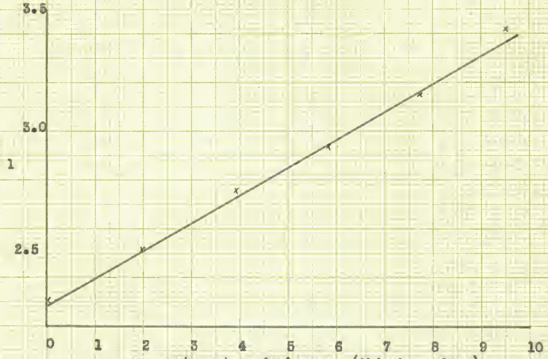


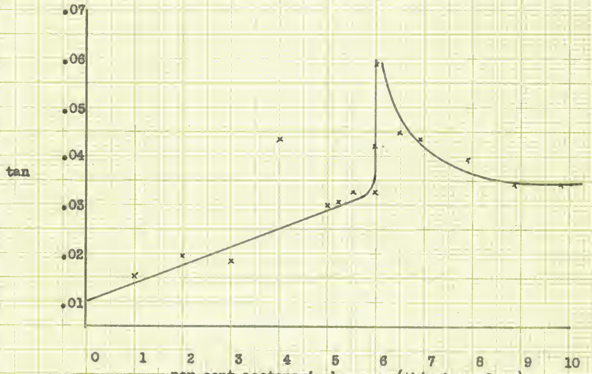
Fig. 5. Loss tangent as function of per cent n butyl alcohol in benzene at frequency 8.983 x 10^9 cps.

Figures 6 and 7 show the results for measurements of acetone in thiophene free benzene.



per cent acetone in benzene (thiophene free)

Fig. 6. Real part of dielectric constant for solutions of acetone in benzene at frequency 8.956 x 10^9 cps.



per cent acetone in benzene (thiophene free)

Fig. 7. Loss tangent for solutions of acetone in benzene at frequency 8.956 x 10^9 cps.

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THE MEASUREMENT OF LIQUID DIELECTRICS AT MICROWAVE FREQUENCIES

by

HENRY UNRUH, JR.

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AN ABSTRACT OF A THESIS

Department of Physics

KANSAS STATE COLLEGE
OF AGRICULTURE AND APPLIED SCIENCE

1952

The complex dielectric constant is represented by $\varepsilon = \varepsilon_1 - i\ \varepsilon_2$. The loss tangent is represented by $\tan\delta = \frac{\varepsilon_2}{\varepsilon_1}$.

Microwave energy is introduced into the waveguide by means of a klystron tube. A piece of waveguide contains the liquid of unknown dielectric constant and is terminated by a short circuit. The reflected wave together with the incident wave gives a standing wave in the waveguide. A traveling probe is used to detect the standing wave.

The dielectric constant and loss tangent are calculated by the following approximate equations:

<u>Equation (1).</u>

$$\frac{\varepsilon_1}{\lambda^2} = \frac{1}{\lambda_c^2} + \frac{1}{\lambda_1^2}$$

<u>Equation (2).</u>

$$\tan\delta = \frac{\Delta x}{d} \times \frac{\frac{1}{\lambda_c^2} + \frac{1}{\lambda_g^2} - \frac{1}{\lambda_c^2 \varepsilon_1}}{\frac{1}{\lambda_c^2} + \frac{1}{\lambda_g^2}} \times \frac{\beta_2 d \ (1 + \tan^2 \frac{360\ x_o}{\lambda_g})}{\beta_2 d\ (1 + \tan^2 \beta_2 d) - \tan\beta_2 d}$$

where

λ = free space wavelength

λ_c = cutoff wavelength = twice waveguide width

λ_1 = wavelength in liquid

λ_g = wavelength in waveguide in air

d = depth of liquid

Δx = distance between twice minimum heights of standing wave

x_o = distance from liquid surface to first minimum

$\beta_2 = \frac{2\pi}{\lambda_1}$

λ_l is determined by using a traveling short. λ_g is calculated from equation (1) by setting $\mathcal{E} = 1$ and $\lambda_l = \lambda_g$.

From the above equations it was shown that the error in $\tan\delta$ is least when x_0 is near $\frac{1}{2}\lambda_g$. For such values of x_0 it should be possible to find $\tan\delta$ with an accuracy of .001. Values of $\mathcal{E}_1$ are obtained with an accuracy of .5 per cent.

For benzene (not thiophene free) $\mathcal{E}_1 = 2.280$ and $\tan\delta = .004$.

Measurements of solutions of n butyl alcohol in benzene give nearly a straight line relationship for $\mathcal{E}_1$ vs. concentration. A plot of $\tan\delta$ vs. concentration gives a sharp peak in the curve at a concentration of approximately 4 per cent.