# A METHOD FOR MEASURING THE DIELECTRIC CONSTANT AT MICROWAVE FREQUENCIES USING A STANDING WAVE DETECTOR

by

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# INTRODUCTION

There have been several methods devised for measuring the dielectric constant and loss tangent for both solids and liquids at microwave frequencies. R. M. Redheffer (1) lists these methods under two categories: transmission in a guide and free space transmission. In his rather complete summary, Redheffer describes methods for determining the dielectric constant and loss tangent by transmission through the dielectric material or by reflection from the surface of the material, either one of which is applicable to free space transmission. Transmission in a wave guide offers possibilities for transmission through the dielectric or reflection from the dielectric without short circuiting the guide. If this modification is used, both phase and amplitude measurements are made in the process. Two other methods which have been developed more extensively, perhaps, than those previously mentioned are the resonant cavity method and the short circuited transmission line method.

The resonant cavity method has performed quite satisfactorily for low loss and medium loss solids (2, 3). Modifications of this method have proved useful in determining the dielectric constant of gases (4, 5). However, the somewhat laborious operatione involved in calculating the dielectric constant once the measurements have been made become even more cumbersome when high loss dielectric materials are involved. M. E. Olmstead (6), in his work with low loss dielectrics, found that the machining and surfacing of the solid materials were rather critical and rather an inconvenience in the operation. Olmstead also discovered some difficulties in using the resonant cavity method for liquids.

For liquids of low, medium, or high loss and for high loss solids, the short circuited transmission line method has proved more efficient (7, 8), and several abbreviated procedures have been developed with special apparatus and for special materials (9, 10). All techniques, however, are developed from the general method of Roberts and von Hippel (7) and its approximation for use with low loss dielectrics by Dakin and Works (8). This method involves measurement of the standing wave ratio (abbreviated SWR). A standing wave is produced by the combined effect of the incident radiation and the radiation reflected from the short circuiting plate in the wave guide. With suitable apparatus, the core of which is a standing wave detector, the relative intensities at various points along the standing wave may be measured; thence the ratio of the maximum intensity to the minimum intensity of the standing wave may be determined. From the information thus obtained, the dielectric constant and the loss tangent may be calculated.

# THEORY

## General Concepts

The current density in a dielectric medium is

$$(1) \qquad \underline{J} = \sigma_1 \underline{E} - \varepsilon_1 (\partial \underline{E}/\partial t)$$

where $\sigma_1$ is the conductivity of the medium, $\varepsilon_1$ its electric inductive capacity, $\underline{E}$ the electric field intensity in the medium, and the $\partial \underline{E}/\partial t$ the rate of change of the electric field intensity with respect to time.

By Ohm's law, $\sigma_1 \underline{E}$ is the conduction current including both true conduction current and current supplying hysteresis losses. $\varepsilon_1 (\partial \underline{E}/\partial t)$ is the displacement current according to Kirchoff's laws.

For a harmonic voltage, $\underline{E} = \underline{E}_0 e^{j\omega t}$, and $(\partial \underline{E}/\partial t) = j\omega \underline{E}$. Equation (1) may now be written

$$(2) \qquad \underline{J} = (\sigma_1 - j\omega \varepsilon_1) \underline{E}.$$

The conduction current is in phase with the electric field and therefore represents a power loss. The displacement is out of phase with the electric field and thus does not represent a power loss. Since the conduction current is usually small, equation (2) may be written

$$(3) \qquad \underline{J} = j\omega \varepsilon_1 \left[ 1 - j(\sigma_1/\omega \varepsilon_1) \right] \underline{E}.$$

For convenience, a complex dielectric constant is defined as

(4) $$\varepsilon = \varepsilon_1\left[1 - j(\sigma_1/\omega\varepsilon_1)\right];$$

thus expression (3) is simplified to

(5) $$\underline{J} = j\omega\varepsilon\underline{E}.$$

The complex dielectric constant is usually expressed as

(6) $$\varepsilon = \varepsilon' - j\varepsilon''.$$

Comparison with equation (4) shows that $\varepsilon' = \varepsilon_1$ is the real part and $\varepsilon'' = (\sigma_1/\omega)$ is the imaginary part of the complex dielectric constant.

The ratio $(\varepsilon''/\varepsilon') = \tan\delta$ is called the loss tangent. Thus from equation (6) the complex dielectric constant may be expressed as

(7) $$\varepsilon = \varepsilon'(1 - j\tan\delta).$$

The ratio $(\varepsilon'/\varepsilon_0) = k_e$ is defined as the specific electric inductive capacity, $\varepsilon_0$ being the dielectric constant of free space. More commonly it is called the relative dielectric constant or simply the dielectric constant.

The wavelength $\lambda_1$ in the dielectric medium is related to the wavelength $\lambda_0$ in free space by the expression

(8) $$\lambda_1(\varepsilon'\mu')^{\frac{1}{2}} = \lambda_0(\varepsilon_0\mu_0)^{\frac{1}{2}}$$

so that

$$(9) \qquad \lambda_1 = \lambda_0/k_e^{\frac{1}{2}}$$

since $\mu'/\mu_0 = k_m$ is in general unity for dielectric materials.

Equation (9) suggests a means whereby the dielectric constant may be determined if the wavelength of the electromagnetic radiation within the dielectric medium may be compared directly or indirectly with the wavelength of the same radiation in free space. This comparison is implicit in the development of the working equations for calculating the complex dielectric constant based upon the concept of wave impedance.

## Derivation of General Expression

A standing wave in a hollow wave guide may be represented as the sum of two traveling waves progressing in opposite directions parallel to the axis of symmetry. The transverse field components $E(z)_1$ and $H(z)_1$ in air at a variable distance z from the surface of the dielectric are given by the expressions

$$(10a) \quad E(z)_1 = A_1 e^{\gamma_1 z} + B_1 e^{-\gamma_1 z} = A_1 (e^{\gamma_1 z} + r_0 e^{-\gamma_1 z}),$$

$$(10b) \quad H(z)_1 = (A_1/\bar{Z}_1) e^{\gamma_1 z} - (B_1/\bar{Z}_1) e^{-\gamma_1 z} = (A_1/\bar{Z}_1)(e^{\gamma_1 z} - r_0 e^{-\gamma_1 z}).$$

$A_1$ and $B_1$ are the amplitudes of the incident and reflected waves, respectively, at the dielectric surface. $\bar{Z}$ represents the intrinsic impedance of the wave in air. The ratio

$$(11) \qquad A_1/B_1 = r_0 = e^{-2\varphi}$$

where $\varphi = \rho - j\psi$ defines a reflection coefficient which characterizes the terminating section of the wave guide containing the dielectric medium.

The wave impedance of the terminating section is related to $r_0$ and $\varphi$ and is given by the ratio of the electric to the magnetic field intensities at the dielectric surface

$$(12a) \qquad \overline{Z}(0) = E(0)/H(0) = \overline{Z}_1(1 - e^{-2\varphi})/(1 - e^{-2\varphi})$$

since $z = 0$ at the surface. Further reduction of (12a) gives

$$(12b) \qquad \overline{Z}(0) = \overline{Z}_1 \coth \varphi .$$

Normally the attenuation in the air-filled section of wave guide can be neglected; thus $\gamma_1 = \alpha_1 - j\beta_1$ reduces to

$$(13) \qquad \gamma_1 = j\beta_1 = j\,2\pi/\lambda_1$$

where $\lambda_1$ is the wavelength in air inside the wave guide. Since $\gamma_1$ is imaginary, $|e^{\pm\gamma_1 z}| = 1$; and from (10a) we arrive at the following expressions for the maximum and minimum field amplitudes of the standing wave:

(14a) $$E_{max} = |A_1| \ (1 + |r_o|) = |A_1| \ (1 + e^{-2\rho}),$$

(14b) $$E_{min} = |A_1| \ (1 - |r_o|) = |A_1| \ (1 - e^{-2\rho}).$$

The ratio of the maximum field amplitude to the minimum field amplitude is

(15) $$(E_{min}/E_{max}) = \tanh \rho.$$

this ratio being the reflection coefficient.

The first minimum outside the dielectric medium occurs at a distance $z_o$ from the dielectric boundary where the incident and reflected electric waves have a phase difference of $\pi$ radians; *i.e.* (since $\psi$ is the phase difference of the two waves at the boundary of the dielectric)

(16a) $$\left[\psi + (2\pi z_o/\lambda_1)\right] - \left[-\psi - (2\pi z_o/\lambda_1)\right] = \pi$$

or

(16b) $$(2\pi z_o/\lambda_1) = (\pi/2) - \psi.$$

The wave impedance $\bar{Z}(0)$ can now be expressed in terms of directly measurable parameters, $z_o$, $\lambda_1$, and $(E_{min}/E_{max})$.

Expanding coth $\varphi$ in (12b) we obtain for the wave impedance

(17) $$\bar{Z}(0) = \bar{Z}_1 \coth \varphi = \bar{Z}_1 \cosh(\rho - j\psi)/\sinh(\rho - j\psi)$$

$$\bar{Z}_1 \coth \varphi = \bar{Z}_1(\tanh \rho - j \cot \psi)/(1 - j \tanh \rho \cdot \cot \psi),$$

and

$$\bar{Z}(0) = \bar{Z}_1 \frac{(E_{min}/E_{max}) - j \tan(2\pi z_0/\lambda_1)}{1 - j(E_{min}/E_{max}) \tan(2\pi z_0/\lambda_1)} . \tag{18}$$

The impedance $\bar{Z}(0)$ is determined by the length d of the dielectric column and the propagation constant $\gamma_2$ of the dielectric-filled section of wave guide.

The standing wave in the dielectric is described by

$$E(z)_2 = A_2 e^{\gamma_2 z} + B_2 e^{-\gamma_2 z} \tag{19a}$$

$$H(z)_2 = (A_2/\bar{Z}_2) e^{\gamma_2 z} - (B_2/\bar{Z}_2) e^{-\gamma_2 z} \tag{19b}$$

At $z = -d$ the wave guide is terminated by a metal plate which introduces at this boundary a voltage node; thus we can write

$$E(z)_2 = 0 = A_2 e^{-\gamma_2 d} - B_2 e^{\gamma_2 d} . \tag{20}$$

Thence $(B_2/A_2) = -(e^{-\gamma_2 d}/e^{\gamma_2 d})$ so that equations (16) become

$$E(z)_2 = A_2 (e^{\gamma_2 z} - e^{-2\gamma_2 d} e^{-\gamma_2 z}) \tag{21a}$$

$$H(z)_2 = (A_2/\bar{Z}_2) (e^{\gamma_2 z} + e^{-2\gamma_2 d} e^{-\gamma_2 z}) \tag{21b}$$

At the boundary between the dielectric-filled section and the air-filled section of wave guide $z = 0$; thus, solving for $\bar{Z}(0)$, we obtain

$$(22) \qquad \bar{Z}(0) = E(0)_2/H(0)_2 = \bar{Z}_2 \tanh \gamma_2 d.$$

The characteristic impedance $\bar{Z}_2$ and the propagation constant $\gamma_2$ of any TEM or TE wave in a dielectric medium are related in the following manner.

$$(23a) \qquad \gamma_2 \bar{Z}_2 = j\omega\mu_2 \ ,$$

where $\mu_2$ is the permeability of the dielectric medium. The permeability of dielectric materials is generally $\mu_o$ of free space so that

$$(23b) \qquad \gamma_2 \bar{Z}_2 = j\omega\mu_o = \gamma_1 \bar{Z}_1.$$

Thus

$$(24) \qquad \bar{Z}(0) = \bar{Z}_1 \ (\gamma_1/\gamma_2) \tanh \gamma_2 d.$$

Equating (24) and (18) and remember that $\gamma_1 = j(2\pi/\lambda_1)$, we obtain

$$(25a) \qquad \frac{(\tanh \gamma_2 d)}{\gamma_2 d} = -j(\lambda_1/2\pi d) \ \frac{(E_{min}/E_{max}) - j\tan(2\pi z_o/\lambda_1)}{1 - j(E_{min}/E_{max})\tan(2\pi z_o/\lambda_1)}$$

or for convenience,

$$\text{(25b)} \qquad \frac{\tanh \gamma_2 d}{\gamma_2 d} = Ce^{j\zeta}.$$

The function $Ce^{j\zeta}$ is found by measuring the thickness d of the sample, the wavelength $\lambda_1$ in the air-filled section of wave guide, the ratio of the minimum electric field strength to the maximum electric field strength $E_{min}/E_{max}$, and the distance $z_0$ of the first minimum from the surface of the dielectric. The magnitude of this complex expression is given by

$$\text{(26)} \qquad C = (\lambda_1/2\pi d) \left[ \frac{E_{min}/E_{max})^2 + \tan^2(2\pi z_0/\lambda_1)}{1 + (E_{min}/E_{max})^2 \tan^2(2\pi z_0/\lambda_1)} \right]^{\frac{1}{2}},$$

and the tangent of the phase angle of the quantity is

$$\text{(27)} \qquad \tan \zeta = \frac{(E_{min}/E_{max}) \left[ 1 + \tan^2(2\pi z_0/\lambda_1) \right]}{\tan(2\pi z_0/\lambda_1) \left[ 1 - (E_{min}/E_{max})^2 \right]}.$$

The next step is the determination of $Te^{j\tau} = \gamma_2 d$ from graphs or from a series approximation of the function

$$\text{(28)} \qquad \tanh(Te^{j\tau})/Te^{j\tau} = Ce^{j\zeta}.$$

The hyperbolic functions are multivalued so that the measurements with a single thickness d of the dielectric may leave the value of $\gamma_2$ in doubt. But if two different thicknesses are used, only one set of values of T/d and $\tau$ will satisfy both experimental results. When the correct set of values for T/d and $\tau$ have been established, the loss factor $\tan \delta_2$ may be found immediately from the relation

$$\cot \tau = \frac{\alpha}{\beta} = \frac{\tan \delta_2}{2} \tag{29}$$

The complex dielectric constant $\varepsilon_2$ can be found from $\gamma_2$ by the general relation

$$\gamma_2^{\,2} = (2\pi/\lambda_c)^2 - \omega_c^{\,2}\mu_2\varepsilon_2 . \tag{30}$$

The cutoff wavelength $\lambda_c$ is determined by the geometry of the wave guide and the mode of propagation. $\omega_c$ is the cutoff frequency. The value of $\gamma_1$ is given by

$$\gamma_1^{\,2} = (2\pi/\lambda_c)^2 - \omega_c^{\,2}\mu_1\varepsilon_1 = -(2\pi/\lambda_1)^2 . \tag{31}$$

Recalling that $\mu_2 = \mu_1 = \mu_o$ for dielectrics

$$\varepsilon_2 = \varepsilon_1 \; \frac{(1/\lambda_c)^2 - (\gamma_2 d/2\pi d)^2}{(1/\lambda_c)^2 + (1/\lambda_1)^2} . \tag{32}$$

$\varepsilon_1$ for air is very nearly equal to $\varepsilon_1' = k_1\varepsilon_o$ which in turn is very nearly $\varepsilon_o$ so that with negligible error for liquid and solid dielectrics the ratio $\varepsilon_2/\varepsilon_o$ may be readily be found from (32). $k_2 = \varepsilon_2'/\varepsilon_o$ is related to the above ratio by means of equation (7); whence the expression

$$\varepsilon_2'/\varepsilon_o = \left| \frac{\varepsilon_2/\varepsilon_o}{1 - j\tan\delta_2} \right| = \frac{|\varepsilon_2/\varepsilon_o|}{(1 + \tan^2\delta_2)^{\frac{1}{2}}} \tag{33}$$

The determination of $\varepsilon_2'$ may be summarized as follows:

The ratio $E_{min}/E_{max}$, the distances $z_o$ and d, and the wavelength $\lambda_1$

are measured. From these measured values, C and $\zeta$ are calculated by means of equations (26) and (27). From C and $\zeta$ values of T and $\tau$ are found from graphs or by a series approximation. $\gamma_2$ is found by taking the quotient of $Te^{j\tau}/d$. Finally $\epsilon_2'$ is determined from $\gamma_2$ by (32) and (33).

Expression for $E_{min}/E_{max}$ in the Neighborhood of a Minimum

With a low loss dielectric, the measurements are carried through only in the neighborhood of the minimum; because the maximum is flatter and the detector is likely to be overloaded. A simple relationship is derived to calculate the ratio of minimum to maximum field strength.

From (10a)

(34a) $$\begin{aligned} \left|E(z)_1\right| &= \left|A_1\right| \cdot \left|e^{\gamma_1 z} - e^{-2\varphi}\, e^{-\gamma_1 z}\right| \\ &= \left|A_1\right| \cdot \left|e^{\gamma_1 z}\right| \cdot \left|1 - e^{-2(\gamma_1 z + \varphi)}\right| \end{aligned}$$

and since $\gamma_1$ is imaginary in the approximation, $\left|e^{\pm \gamma_1 z}\right| = 1$ so that

(34b) $$\left|E(z)_1\right| = \left|A_1\right| \cdot \left|1 - e^{-2\left[j(2\pi z_1/\lambda_1) + \rho + j\psi\right]}\right|,$$

the phase angle being $\theta = (2\pi z_1/\lambda_1) - \psi$, giving

(35) $$\left|E(z)_1\right| = \left|A_1\right| \cdot \left|1 - e^{-2(\rho - j\theta)}\right|.$$

The ratio of this field strength to the minimum given by (14b) is

$$(36) \qquad (|E(z)_1| / E_{min}) = (e^{\rho - j\theta} - e^{-(\rho - j\theta)})/(e^{\rho} - e^{-\rho}).$$

Further reduction of (36) gives

$$(37) \qquad (|E(z)_1| / E_{min}) = \cos\theta - j\ (\sin\theta / \tanh\rho).$$

Squaring this expression and solving for $\tanh\rho$ and remembering that $\tanh\rho = (E_{min}/E_{max})$, we obtain

$$(38) \qquad (E_{min}/E_{max}) = \sin\theta / \left[ (|E(z)_1|^2 / E_{min}^2) - \cos^2\theta \right]^{\frac{1}{2}}$$

It is convenient to measure $E(z)_1$ at points on either side of the minimum at which the power obtained is twice the power obtained at the minimum. Thus $E(z)_1$ will be measured at a point $z_a < z_o$, at the minimum $z_o$, and at a point $z_b > z_o$. The phase angle between either $z_a$ or $z_b$ and $z_o$ is

$$(39) \qquad \theta = 2\pi(z_b - z_o)/\lambda_1 = 2\pi(z_o - z_a)/\lambda_1$$
$$= \pi(z_b - z_a)/\lambda_1$$

The ratio of the power $P(z)_1$ and the minimum power $P_{min}$ is related to the ratio of the corresponding field strengths as

$$(40) \qquad P(z)_1/P_{min} = E(z)_1^2/E_{min}^2$$

In this case $P(z)_1/P_{min} = 2$. Setting $(z_b - z_a) = \Delta z$ we find

$$(41) \qquad E_{min}/E_{max} = \sin(\pi \Delta z/\lambda_1)/\ 2 - \cos^2(\pi \Delta z/\lambda_1)\ ^{\frac{1}{2}},$$

which reduces to

$$(42) \qquad E_{min}/E_{max} = \sin(\pi \Delta z/\lambda_1)/\left[1 - \sin^2(\pi \Delta z/\lambda_1)\right]^{\frac{1}{2}}.$$

If the ratio $E_{min}/E_{max}$ is very small, the approximation $\sin\theta \sim \theta$ can be used. In this case, $\sin^2\theta << 1$ so that (42) becomes

$$(43) \qquad E_{min}/E_{max} = \pi \Delta z/\lambda_1.$$

## Approximation for Low Loss Dielectrics

When the loss factor $\tan\delta < 0.1$, a simpler solution, usually of sufficient accuracy, may be obtained by separating equation (25a) into its real and imaginary components.

Rationalizing the right member of (25a):

$$(44) \qquad \frac{-\lambda_1\left[\tan(2\pi z_o/\lambda_1)\ (1 - E_{min}^2/E_{max}^2) - j\,(E_{min}/E_{max})\ (1 + \tan^2(2\pi z_o/\lambda_1)\right.}{2\pi d\left[1 + (E_{min}/E_{max})^2 \tan^2(2\pi z_o/\lambda_1)\right]}.$$

Recalling that $\gamma_2 = \alpha_2 + j\beta_2$, and rationalizing the left member of (25a):

$$(45)\quad \frac{\alpha_2 d \sinh(\alpha_2 d)\cosh(\alpha_2 d) + \beta_2 d \sin(\beta_2 d)\cos(\beta_2 d) - j\left[\alpha_2 d \sin(\beta_2 d)\cos(\beta_2 d) - \beta_2 d \sinh(\alpha_2 d)\cosh(\alpha_2 d)\right]}{(\alpha_2^2 + \beta_2^2)\, d^2 \left[\cosh^2(\alpha_2 d)\cos^2(\beta_2 d) + \sinh^2(\alpha_2 d)\sin^2(\beta_2 d)\right]}.$$

For a low loss dielectric, $(\alpha_2 d)^2$ and $(E_{min}/E_{max})^2$ are small and may be neglected without appreciable error; thus equating the real parts of (44) and (45) gives the following expression (where $\alpha_2 d = \tanh \alpha_2 d$):

$$(46)\qquad -\lambda_1 \frac{\tan(2\pi z_0/\lambda_1)}{2\pi d} = \frac{\tan \beta_2 d}{\beta_2 d}$$

This equation may readily be solved for $\beta_2 d$ by reference to a table of $(\tan\theta)/\theta$ or by graphing $(\tan\theta)/\theta$ vs. $\theta$ around the value of $(\tan\theta)/\theta$ as determined from the left hand member of equation (46).

Since $\gamma_2^2 \sim -\beta_2^2$, referring to relation (32) the real part of the complex dielectric constant may be obtained:

$$(47)\qquad \varepsilon_2'/\varepsilon_0 = \frac{(1/\lambda_c)^2 + (\beta_2 d/2\ d)^2}{(1/\lambda_c)^2 + (1/\lambda_1)^2}$$

An expression for the attenuation constant $\alpha_2$ can be obtained by equating the imaginary parts of (44) and (45) utilizing the approximations mentioned above:

$$(48)\qquad \frac{\lambda_1 (E_{min}/E_{max})\left[1 + \tan^2(2\pi z_0/\lambda_1)\right]}{2\pi d} = \frac{\beta_2 d\ (1 + \tan^2\beta_2 d)\,\alpha_2 d - \alpha_2 d \tan\beta_2 d}{(\beta_2 d)^2}.$$

Recalling that $E_{min}/E_{max} = \pi \Delta z/\lambda_1$ and solving for $\alpha_2$:

$$(49) \quad \alpha_2 = 2 \ (\pi^2/\beta_2) \ (\beta_2 d/2\pi d)^2 \ (\Delta z/d) \ \frac{\beta_2 d \left[1 + \tan^2(2\pi z_0/\lambda_1)\right.}{\beta_2 d \ (1 + \tan^2\beta_2 d) - \tan\beta_2 d} .$$

Now solving (47) for $(\beta_2 d/2\pi d)^2$ we obtain (writing $k_2$ for $\varepsilon_2'/\varepsilon_0$)

$$(50) \qquad (\beta_2 d/2\pi d)^2 = k_2 \left[(1/\lambda_c)^2 + (1/\lambda_1)^2 - (1/\lambda_c^2 k_2)\right] .$$

An equivalent relation for $\alpha_2$ is $(\pi/\lambda_2)\tan\delta_2$. Recalling that $2\pi/\beta_2 = \lambda_2$ we can now write (49) as

$$(51) \qquad (\pi/\lambda_2)\tan\delta_2 = \pi k_2 \lambda_2 \ (\Delta z/d) \left[(1/\lambda_c)^2 + (1/\lambda_1)^2 - (1/\lambda_c^2 k_2)\right] \cdot \frac{\beta_2 d \left[1 + \tan^2(2\pi z_0/\lambda_1)\right]}{\beta_2 d \left[1 + \tan^2(\beta_2 d)\right] - \tan(\beta_2 d)}$$

$\lambda_2$ is related to $\lambda_1$ by the expression

$$(52) \qquad k_2 \lambda_2^{\ 2} = \frac{1}{(1/\lambda_c)^2 + (1/\lambda_1^{\ 2} k_1)}$$

Since $k_1$ is very nearly unity, expression (47) can now be written

$$(53) \qquad \tan\delta_2 = (\Delta z/d) \ \frac{(1/\lambda_c)^2 + (1/\lambda_1)^2 - (1/\lambda_c^{\ 2} k_2}{(1/\lambda_c)^2 + (1/\lambda_1)^2} \cdot \frac{1 + \tan^2(2\pi z_0/\lambda_1)}{\left[1 + \tan^2(\beta_2 d)\right] - \tan(\beta_2 d)/\beta_2 d} .$$

## Correction of Tan $\delta_2$ for Wave Guide Losses

Since there is always some dissipation in the walls of the wave guide, a measurement of $E_{min}/E_{max}$ includes those losses together with the losses due to the dielectric. The wave guide losses must be subtracted, the proper procedure being first to correct the measured value of $E_{min}/E_{max}$ for the loss due to the wave guide in front of the sample between the boundary of the dielectric and the position of the measured minimum. The total loss factor of the wave guide and the dielectric material are then calculated by means of equation (53) and the loss factor of the wave guide subtracted from the result.

The loss tangent for the wave guide is obtained from (53) by measuring $\Delta$ z with the wave guide empty. In this case $k_2$ would be very nearly unity and d would be the distance from the measured minimum to the short circuited end of the wave guide. Both $\tan(2\pi s_0/\lambda_1)$ and $\tan(\beta_2 d)$ would be very small in this case and thus equation (53) reduces to

$$(54) \qquad \tan\delta_1 = (\Delta s/d)\left\{1/\left[(\lambda_1/\lambda_c)^2 + 1\right]\right\}.$$

## Summary of Working Equations

A summary of the working equations necessary for the calculation of the complex dielectric constant and loss tangent gives the following;

$E_{min}/E_{max}$ is determined from expression (42) if the ratio is large. For values of $(\pi\Delta s/\lambda_1) < 0.30$, $\sin(\pi\Delta z/\lambda_1) \simeq (\pi\Delta z/\lambda_1)$. If $E_{min}/E_{max}$

is small (i.e. $(\pi \Delta z/\lambda_1) \ll 1$), expression (43) may be used.

For medium and high loss dielectrics $k_2$ is found using equations (32) and (33). $\gamma_2 d$ is found by means of equations (26), (27), and a graphical solution of (28). Equation (29) gives the loss tangent.

For low loss dielectrics which have $\tan \delta_2 < 0.1$, $k_2$ is found by means of equation (47), and expression (46) will yield the value of $\beta_2 d$ either graphically or from tables.

The loss factor of the wave guide is given by equation (54).

The cutoff wavelength in a rectangular wave guide is given generally for any type mode ($TE_{mn}$, $TM_{mn}$, $TEM_{mn}$) by the relation

$$(55) \qquad \lambda_c = 2 \left[ (m/a)^2 + (n/b)^2 \right]^{-\frac{1}{2}} ,$$

where a is the greatest inner dimension of the wave guide, and b is the least inner dimension of the wave guide. For the $TE_{10}$ mode, which is the dominant mode of transmission in a rectangular wave guide, this relation reduces to

$$(56) \qquad \lambda_c = 2a.$$

# APPARATUS

## General Description

A reflex klystron oscillator, an attenuating pad, and a transmission type cavity resonator were mounted as a unit on a length of 1" x 4" board. This unit was mounted horizontally on a frame of wood so that by means of a 90° E-plane bend the standing wave detector could be mounted vertically. This allowed a liquid dielectric to present a plane boundary perpendicular to the direction of propagation of the electromagnetic radiation without the use of a window of mica, teflon, or other material. The wave guide circuit was terminated by a cell which contained the dielectric material. This cell was short-circuited with a metal plate to provide a reflecting surface to the incident radiation.

The wave guide junctions were aligned with special care and clamped tightly together by means of machine screws; any slight misalignment would cause spurious reflections which would reduce the power transmitted through the wave guide. The flange-flange type of coupling was chosen instead of either the choke-flange or the choke-choke type because there was less loss due to the former.

## Microwave Generator

A reflex klystron oscillator was used as the source of microwave energy. The oscillator circuit was essentially that described by Olmstead (11). This circuit employed frequency modulation of the klystron by applying a saw-tooth modulating voltage to the reflector of the klystron. To stabilize the base level of the saw-tooth voltage, a diode restorer circuit was used in which the base level of the modulating voltage could be adjusted by varying the negative cathode bias of the diode restorer. The saw-tooth was obtained from the sweep voltage applied to one of the horizontal deflecting plates of the oscilloscope used as the indicator.

Overcoupling of the klystron oscillator with the remainder of the transmission line may cause spurious modes to be generated, which in turn cause variations in the frequency and the power output of the klystron. To prevent overcoupling, an adjustable attenuating pad was placed in the transmission circuit immediately following the oscillator. During measurements this attenuator was adjusted to give the maximum attenuation which allowed a readable trace to appear on the screen of the oscilloscope.

The saw-tooth modulating voltage produced a wide band of frequencies. Since a single frequency, or at most an extremely narrow band of frequencies, was desired, a transmission type cavity resonator was used as a narrow band-pass filter and also as a frequency monitor. The narrow band of frequencies produced an envelope which appeared on the oscilloscope screen as a narrow pulse of energy, the height of which was proportional to the power generated in the wave guide.

The frequency at which it was desired to make the measurements was selected by setting the micrometer screw of the cavity resonator to the setting corresponding to the frequency; then by mechanically adjusting the r-f gap of the reflex klystron and electronically adjusting the modulating voltage applied to the reflector, maximum transmission of power through the cavity resonator was obtained.

## Standing Wave Detector

Properties of the Slotted Line and Probe. The technique involved in the use of the standing wave detector is based on two assumptions. The first assumption is that the slotted line itself can be treated as a uniform lossless transmission line; the second is that the presence of the probe does not appreciably affect the electromagnetic field in the line. However, unless certain considerations are given to the design and construction of the standing wave detector, serious disturbances can affect the validity of these assumptions. These disturbances are of two kinds: those which cannot be avoided so long as the probe and slot are of finite size and a finite amount of power is required to actuate the indicator, and those which can be attributed to accidental mechanical irregularities such as departure from perfect symmetry. In general, if the second class of disturbances are adequately dealt with, the first will be negligible.

The presence of the slot affects the propagation constant of the line as well as its characteristic impedance with the result that the wavelength in the slotted section differs slightly from that in an unslotted section of the same cross section, being somewhat greater in the slotted section.

The electric field penetrates to some extent the slot; and, since the wall is of finite thickness, this field extends into the exterior region with a consequent loss of power by radiation. However, this affect is very small when the wall thickness is comparable to the slot width because the slot then acts as a wave guide far beyond the cutoff frequency for the penetrating field. Thus the attenuation of the slot wave is so rapid that the coupling to the exterior region is negligible.

Another property of the slotted wave guide is that a new mode of propagation is made possible by the presence of the slot. This slot wave is generally caused by departure from symmetry. The possibility of departures from symmetry must be given due attention in the design and use of the standing wave detector. Even weak excitation of the slot wave may have a disturbing effect upon standing wave measurements through a resonance effect. The probe divides the slot into two sections, and as the probe travels the length of the slot the lengths of these sections will vary allowing opportunity for a slot wave resonance if the slot wave is excited.

Perfect geometrical symmetry of the probe and slotted section will prevent excitation of the slot wave and will insure that the slotted section will act as a uniform and nearly lossless transmission line supporting only one mode of propagation.

The influence of the probe susceptance upon the standing wave pattern is such that the maxima are shifted away from the reflecting termination. The minima also are shifted similarly; however the effect is small compared with the shift of the maxima. Because of this, the neighborhood of the minima rather than the maxima is preferred to establish the phase of the reflection coefficient of the load in a standing wave measurement.

Electrically the probe may be represented in an equivalent circuit as a simple shunt admittance between the generator and the load, this admittance being the sum of the conductance and the susceptance of the probe. This is not the only possibility, but it is convenient since the probe is considered as an electric antenna and its reading refers to the transverse electric field.

Design and Construction. The standing wave detector used in the experiment consisted of a slotted section of wave guide into which a probe was extended to detect the electric field intensity of a standing wave produced by reflecting the transmitted wave in a short circuited wave guide. The probe extracted a small portion of the power transmitted in the wave guide and was connected to a circuit comprised of a crystal rectifier, a linear amplifier, and an oscilloscope.

The slotted section was an eight inch section of 5/8" x $1\frac{1}{4}$" brass wave guide fitted at each end with a butt flange. The length of the slot was about four inches which was more than twice the wavelength of three centimeters used in the measurements. This allowed the probe to travel through at least four maxima and four minima if necessary to determine the wavelength in the wave guide. The slot was 2 mm in width and situated in the center of a broad wall of the guide. Since the slot ran parallel to the lines of surface current flow of the $TE_{10}$ (dominant) mode of transmission, its presence produced a negligible effect on the original field configuration.

The probe carriage was made of stock brass in the form of a frame which fit around the perimeter of the wave guide. This frame was held in intimate contact with the slotted section by means of pressure exerted by a phosphor-bronze spring between the under side of the frame and the wave guide. The carriage could be moved to and fro along the length of the slotted section to explore all but a small segment of the slot with the probe. Movement of the carriage was by means of a screw adjustment; the distance moved being indicated by means of a vernier scale to to 0.01 cm.

The support of the probe was accomplished by a tuning stub of one inch brass tubing standing seven inches above the probe carriage. The penetration of the probe into the slot was controlled by means of a threaded polystyrene insulator. The probe shaft, which ran up through the center of the tuning stub, passed through the center of a movable tuning plunger and was supported at the top of the stub by means of a brass yoke. A fiber sleeve provided insulation of the probe shaft from the shell of the tuning stub.

Tuning was accomplished by moving the plunger up and down until a point was located at which the pick-up was a maximum. An external scale and indicator provided an arbitrary reference for the position of the plunger. The tuning stub acted as a resonating column; the maximum power pick-up was at locations of the stub which were multiples of half-wave lengths from the probe tip.

The power picked up by the probe was conducted to a 1N23A crystal detector, one side of which was in contact with the probe shaft and the other side grounded to the probe carriage through the tuning stub. The contact of the crystal with the probe shaft was maintained at constant pressure by means of a small coil spring in the crystal housing. It was found that less noise in the output signal was introduced by the crystal if the base of the crystal was the grounded part. In order to transmit the rectified probe voltage to the amplifier, connections were made to the probe shaft and the probe carriage.

It was found that the probe could penetrate the slot as much as 1.0 mm without causing appreciable shift in the minima due to the susceptance of the probe. Thus the depth could be chosen up to 1.0 mm in order to obtain a readable signal. Calibration of the probe depth adjustment gave for one complete roation of the probe shaft a change of 0.105 cm in the penetration of the probe. The probe, however, was mounted eccentrically in polystyrene insulator screw. This caused a null to occur when the probe became off center with respect to the slot; this null was due to the anti-symmetrical effect thus produced. The eccentricity was measured with a micrometer comparator microscope and found to be $\pm$ 0.012 cm. The error due to this was easily corrected in the calculations.

The terminating section of the standing wave detector was a section of 5/8" x $1\frac{1}{4}$" wave guide, one and one-half inches in length, fitted with a butt flange on one end and short circuited at the other end with a piece of sheet brass bolted to the guide and soldered to insure good electrical contact and to provide a liquid-tight seal. The total length of the cell plus the thin lead gasket used in order to secure a liquid-tight joint between the cell and the standing wave detector was 2.56 cm so that the total distance from the short circuit to the zero position of the probe was 6.17 cm.

In taking the measurements for the determination of the dielectric constant it was found convenient to use the terminal end of the slot as a reference point for the length of the liquid column so that d was taken to be 6.17 cm.

The zero on the scale for the probe carriage position was - 0.02 cm. With the aid of a micrometer comparator microscope, this scale was measured and the probable error in reading the scale was calculated to be ± 0.016 cm.

## Linear Amplifier

The rapid response of an a.c. amplifier made the use of it together with an oscilloscope preferred over a meter as an indicator for the pulses of crystal current. Since the crystal was a square-law detector, a linear amplifier was necessary in order that the output of the amplifier would be proportional to the power propagated in the wave guide.

The amplifier consisted of a video amplifier and power supply on one chassis and a preamplifier on a separate chassis. The circuits were designed by the Clinton Laboratories of the Monsanto Chemical Company according to the designations A-1 amplifier and power supply and A-1-A preamplifier.

The video amplifier consisted of two negative feedback sections plus a cathode follower stage which served to isolate the amplifier from its load. Each section had two amplifying stages and a cathode follower. The bandwidth response of the amplifier could be varied by a position switch giving a selection of bandwidths corresponding to 2 Mc/sec., 0.5 Mc/sec., or 0.1 Mc/sec.

The preamplifier used four 6AK5 tubes and also employed a negative feedback circuit. Its power was supplied from the main chassis. The signal to noise ratio was affected appreciably by the first stage of the preamplifier requiring a high input impedance; special care had to be exercised in the selection of a tube for the first stage. In the author's case, over two dozen tubes were tried before a suitable one was found. Increasing the input capacitance to 0.01$\mu$f materially reduced overshoot.

A well regulated power supply was needed to produce sufficient stability. Employing a 6AS7 triode as the regulator tube, the power supply exhibited, a very nearly linear increase from 269 to 271 v.d.c. supply voltage at 200 ma load current (normal) was obtained with a variation of line voltage from 90 to 125 volts a.c. This two volt change caused no measurable departure from linearity in the out-put of the amplifier. The lower voltages which were required to operate some of the stages were obtained by the use of voltage dropping resistors.

The overall gain of the amplifier and preamplifier, using a 1000 cps sweep frequency, to modulate the klystron was $10^5$ for the 2 Mc band, 2 x $10^5$ for the 0.5 Mc band, and 3 x $10^5$ for the 0.1 Mc band. When a 500 cps sweep frequency was used, the gain was slightly less; but a much clearer envelope was traced on the oscilloscope screen, so this frequency was used to modulate the klystron for the measurements.

The amplifier was designed to deliver only positive pulses; however either a positive or a negative pulse could be applied to the amplifier depending upon the coupling between the two feedback sections of the video amplifier. Since the circuit of the crystal detector and standing wave detector was such as to deliver a positive pulse, the appropriate coupling was made in the amplifier.

## EXPERIMENTAL TECHNIQUES

### Method of Measurement

The method of measurement consisted of short-circuiting the wave guide with a nearly perfectly reflecting surface and placing the dielectric against this reflecting surface. The process of reflection sets up standing waves in the space region in front of the sample as a result of the superposition of the incident and reflected waves. A TE mode of oscillation was used for the measurements; i.e. the direction of the electric field was maintained perpendicular to the direction of propagation of the radiation.

The separation of the first minimum from the face of the sample depended upon the wavelength of the wave in the dielectric sample and the thickness of the sample, since the first minimum was an integral number of half-wavelengths from the reflecting surface behind the sample. Insertion of the dielectric shifted the minima toward the termination because the wavelength in the dielectric material was less than the wavelength in the air. The separation of the first minimum from the sample was $z_0$ in the expressions developed in the theory.

If there were any attenuation in the dielectric, there would be a reduction in the standing wave ratio since the energy of the radiation would be partially dissipated in the dielectric. The quantity of this dissipation is indicated by the loss tangent. There was, of course, a finite standing wave ratio with no dielectric present because of the slight attenuation by the walls of the transmission line and losses at the junction of the standing wave detector and the dielectric cell. The reciprocal of the standing wave ratio $E_{min}/E_{max}$ was used in the working equations for calculating the dielectric constant and loss tangent.

To Plot the Function $Ce^{j\zeta} = (\tanh Te^{j\tau})/(Te^{j\tau})$

Rectangular coordinates were established, the argument $\tau$ being the ordinate, and the absolute value of T being the abscissa. C and $\zeta$ were the parameters of intersecting curves plotted on these coordinates. To facilitate the plotting of the function, the quantity $Te^{j\tau}$ was set equal to $\alpha + j\beta$. Using this substitution and manipulating the resulting expression, C and $\zeta$ were found in terms of $\alpha$ and $\beta$.

$$C = \left[\frac{\tanh^2\alpha - \tan^2\beta}{(\alpha^2 + \beta^2)\ (1 + \tanh^2\alpha\tan^2\beta\ )}\right]^{\frac{1}{2}}$$

$$\tan\zeta = \frac{\alpha\tan\beta(1 - \tanh^2\ ) - \beta\tanh\alpha(1 + \tan^2\beta)}{\beta\tanh\ (1 + \tan^2\ ) - \beta\tan\beta(1 - \tanh^2\ )}$$

Values were assigned to T such that $|T| \geq 0$; then for each value of T, there were assigned to $\tau$ values in the range $0 \leq \tau \leq \pi/2$. For each of the assigned values of T and $\tau$, $\alpha$ and $\beta$ were determined from the relations $T^2 = \alpha^2 + \beta^2$ and $\tan\tau = \beta/\alpha$. C and $\tan\zeta$ were calculated using the values of $\alpha$ and $\beta$ thus determined. It was found convenient to finally express $\tau$ and $\zeta$ in degrees.

When $\alpha$ becomes large, $\tanh\alpha \rightarrow 1$. Under this condition, $C = 1/T$, and $|\zeta| = \tau$. To four figure accuracy, $\tanh\alpha = 1$ for $\alpha > 5.298$.

Sample Calculation for Glycerine

**Data.**

$$s_o' = 2.12\bar{9} \text{ cm}$$
$$s_o'' = 4.1\bar{9}0$$
$$\lambda_1 = 4.1\bar{2}2$$
$$d = 6.1\bar{7}$$
$$\lambda_c = 6.02\bar{4}$$

Note: $s_o' = 2.129$ cm, $\lambda_1/2 = 2.061$ cm; thus $s_o'$ is the second minimum from the air dielectric boundary rather than the first minimum. The first minimum at $s_o = 0.068$ cm lay too close to the dielectric to determine its width.

As the amplitude of the reflected wave decreases, the ratio $E_{max}/E_{min}$ decreases exponentially. In a short-circuited wave guide in air this decrease is very nearly linear within several wavelengths of the termination. Since there was a difference of only 0.02 cm between the widths of the second and third minima, it was assumed that no appreciable error would be introduced by extrapolating linearly through $(E_{min}/E_{max})''$ and $(E_{min}/E_{max})'$ to $(E_{min}/E_{max})$.

Calculation of $E_{min}/E_{max}$.

$$E_{min}/E_{max} = \frac{\sin(\pi \Delta z/\lambda_1)}{\left[1 + \sin^2(\pi \Delta z/\lambda_1)\right]^{\frac{1}{2}}}$$

For $z_o' = 2.129$ cm, $\Delta z' = 0.447$ cm;

$$(E_{min}/E_{max})' = \frac{\sin(0.447\,\pi/4.122)}{\left[1 + \sin^2(0.447\,\pi/4.122)\right]^{\frac{1}{2}}} = 0.3171$$

For $z_0'' = 4.\overline{1}90$ cm, $\Delta z'' = 0.470$ cm;

$$(E_{min}/E_{max})'' = \frac{\sin(0.470\ \pi/4.122)}{\left[1 + \sin^2(0.470\ \pi/4.122)\right]^{\frac{1}{2}}} = 0.3301.$$

$$\begin{aligned} E_{min}/E_{max} &= (E_{min}/E_{max})' - \left[(E_{min}/E_{max})'' - (E_{min}/E_{max})'\right] \\ &= 2\ (E_{min}/E_{max})' - (E_{min}/E_{max})'' \\ &= 0.6342 - 0.3301. \end{aligned}$$

$$E_{min}/E_{max} = 0.3041.$$

Calculation of $\zeta$.

$$\tan \zeta = \frac{(E_{min}/E_{max})\left[1 + \tan^2(2\pi z_0/\lambda_1)\right]}{\tan(2\pi z_0/\lambda_1)\left[1 - (E_{min}/E_{max})^2\right]}$$

$$= \frac{(0.3041)\left[1 + \tan^2(2\pi 2.129/4.122)\right)}{\tan(2\pi 2.129/4.122)\left[1 - (0.341)^2\right]}$$

$$= 3.260.$$

$$\zeta = 72^{\circ}\ 57'$$

Calculation of C.

$$C = (\lambda_1/2\pi d)\left[\frac{(E_{min}/E_{max})^2 + \tan^2(2\pi z_0/\lambda_1)}{1 + (E_{min}/E_{max})^2\ \tan^2(2\pi z_0/\lambda_1)}\right]^{\frac{1}{2}}$$

$$= (4.122/2\ \ 6.17)\quad \frac{(0.3041)^2 - (0.1039)^2}{1 - (0.3041)^2\ (0.1039)^2}^{\frac{1}{2}}.$$

$$C = 0.03412\ .$$

In the above calculations for $\zeta$ and C, $(2\pi z_0/\lambda_1) = 5^{\circ}\ 56'$

Determination of T and $\tau$. T and $\tau$ are generally more easily determined by graphical methods from the plot of the function $(\tanh Te^{j\tau})/Te^{j\tau} = Ce^{j\zeta}$. However, in the range of $C < 0.03636$ and $\zeta < 78.0°$, it can be shown that $T = 1/C$ and $\tau = |\zeta|$ very nearly. Using this fact, the values for T and $\tau$ were found to be:

$$T = 29.31;$$

$$\tau = 72° \; 57'.$$

Calculation of $\varepsilon_2/\varepsilon_o$.

$$\varepsilon_2/\varepsilon_o = \frac{(1/\lambda_c)^2 - (\gamma_2 d/2\pi d)^2}{(1/\lambda_c)^2 + (1/\lambda_1)^2};$$

$$|\gamma_2 d| = T = 29.31;$$

$$|\varepsilon_2/\varepsilon_o| = \frac{(1/6.024)^2 - (29.31/2\pi 6.17)^2}{(1/6.024)^2 - (1/4.122)^2}.$$

$$|\varepsilon_2/\varepsilon_o| = 6.296$$

Calculation of $\tan\delta_2$.

$$\tan\delta_2 = 2\cot\tau = 2(0.3067)$$

$$\tan\delta_2 = 0.6134$$

Since $\tan \delta_1 < 0.0068$, $\tan \delta_1$ is assumed to be negligible for these calculations

Calculation of $\varepsilon_2'/\varepsilon_0$.

$$\varepsilon_2'/\varepsilon_0 = \frac{|\varepsilon_2/\varepsilon_0|}{(1 + \tan^2 \delta_2)^{\frac{1}{2}}}$$

$$= \frac{6.296}{\left[1 + (0.6134)^2\right]^{\frac{1}{2}}}$$

$$\varepsilon_2'/\varepsilon_0 \doteq 5.367$$

## DISCUSSION OF RESULTS

Table 1. Results of dielectric constant and loss tangent measurements.

| Substance | Dielectric Constant ($\varepsilon_2/\varepsilon_0$) | Loss Tangent ($\tan \delta_2$) |
|---|---|---|
| Benzene, thiophrene free | 2.29 | $< 0.007$ |
| | 2.24 | $< 0.007$ |
| | 2.49 | $< 0.007$ |
| Transformer oil | 3.25 | $< 0.007$ |
| | 2.20 | $< 0.007$ |
| | 2.22 | $< 0.007$ |
| Acetone, C. P. | 1.76 | 0.019 |
| | 1.55 | 0.028 |
| Glycerine, C. P. | 4.13 | 0.72 |
| | 5.15 | 0.79 |
| | 5.49 | 0.62 |
| | 5.37 | 0.61 |

Note: All measurements were made at a frequency of 8900 Mc.

The sets of values of $\varepsilon_2'/\varepsilon_0$ and $\tan\delta_2$ for benzene and transformer oil fall within the range of values obtained by other investigators using similar or different methods.

Because of the high vapor pressure and consequent rapid evaporation of the acetone, it was difficult to obtain reliable readings for it. Since these factors would decrease the length of the sample and cause the region above the dielectric to be rich in vaporous acetone, the effect would be to decrease the value of $\varepsilon_2'/\varepsilon_0$ and increase $\tan\delta_2$. No data was found in the literature for the dielectric constant or loss tangent of acetone at microwave frequencies.

The transformer oil, due to its moderate viscosity, would adhere to the sides of the wave guide above the dielectric surface if care were not taken to prevent sloshing the oil. This would cause a decrease in the length of the sample and give an effective increase in the loss of the air-filled section of wave guide. This would cause a decrease in $\varepsilon_2'/\varepsilon_0$.

The high viscosity and surface tension of the gylcerine caused no little trouble in filling the dielectric cell. Extreme care had to be exercised to prevent sloshing on the sides of the air-filled section of the wave guide. Also care had to be taken not to overflow the glycerine through the slot (of which the terminal end was used as the liquid level reference). The dominant effect of these items would be to increase markedly the apparent loss tangent of the glycerine. The absolute value of the complex dielectric constant would be increased also; but the net effect, due to the marked increase in $\tan\delta_2$, would be to decrease $\varepsilon_2'/\varepsilon_0$. This is evidenced in the first two sets of values recorded for glycerine.

The use of an electrically thin window to separate the liquid from the air-filled section was considered, but the idea was abandoned. The only position in which the window could be mounted in order to render it easily accessible for cleaning would be between the flanges at the joint of the standing wave detector and the dielectric cell. The construction of the probe carriage did not permit the probe to approach close enough to this point to obtain readings of the first minimum and, in most cases, the second minimum. Hence, the technique described earlier was used. However, in view of the problems which have been mentioned above, it is indicated that redesign of the instrument to utilize the window would mitigate the resulting errors and aid in obtaining more reliable readings. Teflon or mica is suggested for use as the window, the former being preferred

Another source of error lay in the lead seal between the standing wave detector and the dielectric cell. This apparently caused an increase in $\tan \delta_2$. The effect was most noticeable for high loss materials. Some later trials using copper as the seal showed some improvement. The latter two sets of values for glycerine were determined using a copper seal. These values fall within the range of values recorded in the literature.

A source of large error lay in the effect of the probe and slot. There were some irregularities in the width and depth of the slot despite attempts to machine it to close tolerance. Any departure from symmetry in the travel of the probe along the slot would cause erroneous determinations of both $\Delta z$ and $z_0$. To mitigate this effect it is advantageous to shield the probe by means of a tongue one-half wavelength in length, just narrower than the width of the slot, and even with the inner wall of the wave guide in depth. In this way the probe is shielded from the effects of the electric field in the slot.

Variation of the effective capacitance between the probe and the wave guide through the carriage is also a cause of erroneous measurements. It would be easier to hold an air gap within a tolerance of a few thousandths of an inch than to insure continual clean contact between the probe carriage and the wave guide as the carriage travels along the guide.

Reasonable care had to be taken to insure that the surface of the liquid was perpendicular to the direction of propagation of the radiation. If the surface is within $2^{0}$ of the perpendicular, the error will be very small for most determinations (except for extremely high loss materials).

The method used is not new in principle, but it differs in the use of a frequency modulated oscillator rather than a fixed stabilized oscillator driven by a pulse or square wave as described by Roberts and von Hippel (12), Dakin and Works (13), and other investigators. A definite disadvantage of the frequency-modulated oscillator seems to be an appreciable width of the voltage minima due to the narrow band of frequencies transmitted through the resonant cavity monitor instead of a single frequency which would be transmitted by the stabilized oscillator. However, it has a definite advantage in eliminating concern as to the stability of the oscillator power supply as the frequency at which the measurement is made, in the case of the frequency modulated oscillator, is dependent upon the setting of the resonant cavity. The frequency drift which is attendant to an unstable fixed oscillator or to a square wave with a slow rise time is thus mitigated. Also the generation of spurious modes due to a fluctuation of the power output of the oscillator is minimised.

## ACKNOWLEDGMENTS

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APPENDIX

EXPLANATION OF PLATE I

Figure 1. Diagram of a slotted section of wave guide.

Figure 2. Diagram of the standing wave pattern in the wave guide.

PLATE I

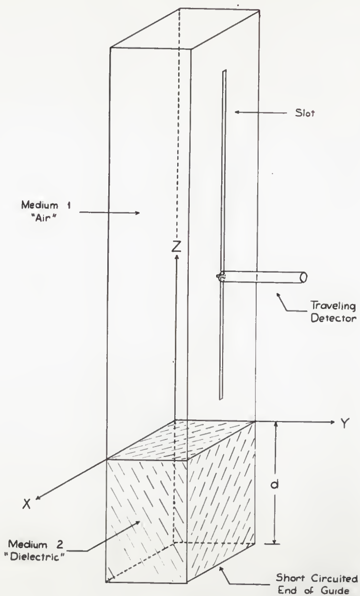



Figure 1

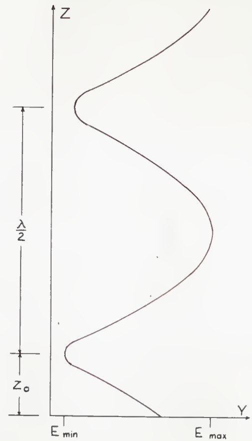



Figure 2

EXPLANATION OF PLATE II

Figure 1. Block diagram of the microwave circuit used to measure the complex dielectric constant.

Figure 2. Cross section sketch of the standing wave detector used in the experiment.

PLATE II

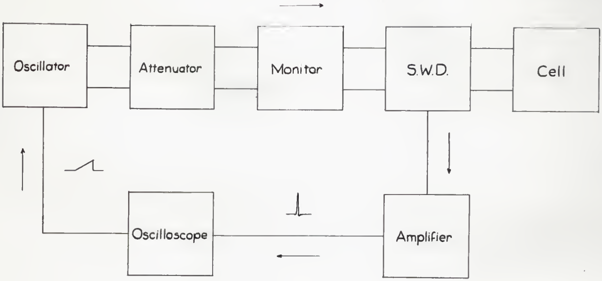



Figure 1

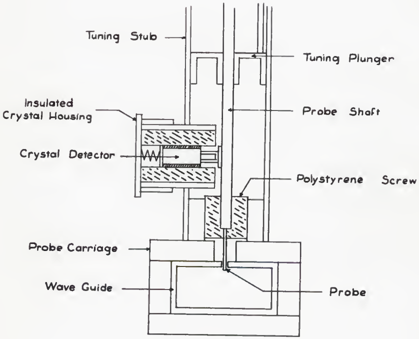



Figure 2

## EXPLANATION OF PLATE III

Figure 1. An equivalent circuit for the probe in which the probe is represented as a shunt admittance $Y_p = G_p + B_p$. $G_p$ is the probe conductance, $B_p$ is the probe susceptance, and $G_g$ and $G_l$ are the conductance of the generator and the load respectively.

Figure 2. Illustrating the distortion of the standing wave pattern due to the susceptance of the probe.

PLATE III

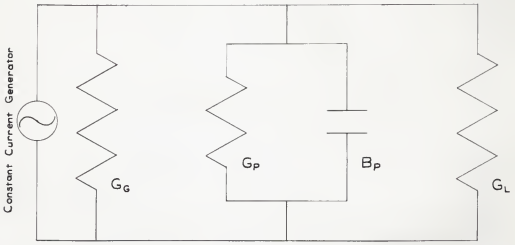



Figure 1

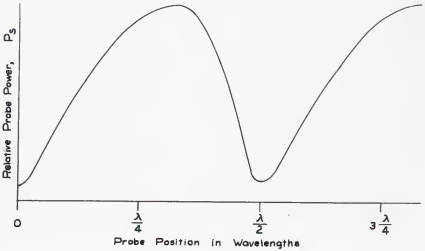



Figure 2

## EXPLANATION OF PLATE IV

Figure 1. Patterns of standing waves in a short circuited wave guide:
(a) with no dielectric;
(b) with one-half wavelength of dielectric;
(c) with one-third wavelength of dielectric.

Figure 2. Chart of the function $Ce^{j\gamma} = (\tanh Te^{j\tau})/(Te^{j\tau})$ in the range $27.5 \leq T \leq 30.5$, $60.0^{\circ} \leq \tau \leq 90.0^{\circ}$.

PLATE IV

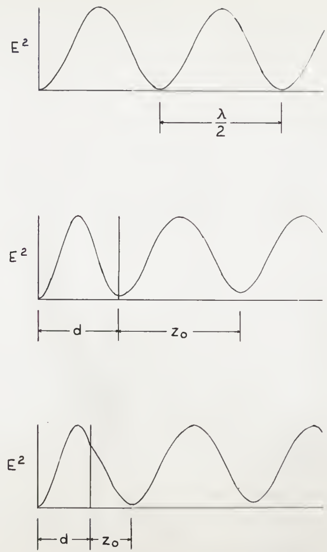
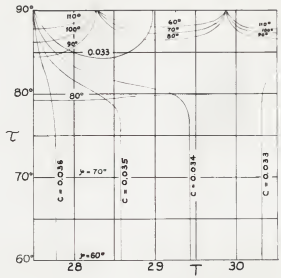



Figure 1




$$Ce^{j\phi} = \frac{\tanh Te^{j\tau}}{Te^{j\tau}}$$

Figure 2

# A METHOD FOR MEASURING THE DIELECTRIC CONSTANT AT MICROWAVE FREQUENCIES USING A STANDING WAVE DETECTOR

by

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## AN ABSTRACT

A method for measuring the complex dielectric constant and loss tangent is described in which the voltage standing wave ratio is utilized by measurement with a standing wave detector. The general method consists of reflecting a three centimeter wave from a column of the dielectric material which is placed against a perfectly reflecting surface in order to generate standing waves. The insertion of the dielectric causes a reduction of the standing wave ratio and a shift in the minimum of the standing wave toward the dielectric medium. This reduction and shift are mathematically related to the dielectric constant and loss factor.

The general concept of the complex dielectric constant is discussed briefly, and a general expression for calculating the dielectric constant and loss tangent is developed using the concept of intrinsic impedance. From this expression is derived an approximation suitable for reasonably accurate calculation of the dielectric constant and loss factor of low loss materials. A description is given of the apparatus and techniques used.

Measurements were conducted on benzene, transformer oil, acetone, and glycerine. A sample calculation is given for glycerine.