

STUDENT'S GUIDE SHEETS BASED  
UPON THE CONTRACT METHOD OF TEACHING FOR  
THE FIRST SEMESTER OF NINTH GRADE ALGEBRA

by

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## INTRODUCTION

There is little doubt but that the old-fashioned recitation method of teaching is something of the past.

A modern curriculum cannot become final, but the work here presented represents one of the more modern methods, usually called the "Contract Plan" of teaching.

In this method, the individual pupil and not the class is the unit of instruction. Each pupil is given a guide sheet or lesson assignment with study helps covering the unit's work. The units here given can be adopted to any text. The pupil has the privilege of choosing his own contract. If he chooses the "B" Contract, that means that he must be able to work the whole of Contracts "D" and "C" besides Contract "B". In other words, the contracts are accumulated. If in giving the unit test, the pupil usually does better on it than the contract he chooses, he should be encouraged to strive for a higher contract. For this would reveal that he is capable of doing better work.

The first step in using this method of teaching is to make a preliminary appraisal of each pupil's experience as related to the unit in order to determine how much teaching the individual needs.

A pre-test is usually given before beginning the unit's work since it helps to reveal cases where some pupils may have studied this work before, and thus helps in presenting the new unit and determines the point of view from which the unit should be attacked for a particular class.

The second step consists in telling the story of the unit to the whole group. Present the unit as a whole in ten to twenty minutes by bringing out the important facts, their relation to each other, and to the unit. Then encourage the pupil to study further and in detail the various facts brought out by the presentation. This results in the laboratory plan or supervised study period.

During this period, the teacher has the opportunity to observe the study habits of her pupils and to define their difficulties in learning certain phases of the work. It is suggested that those who wish help during the class period write their names on slips of paper and place them in order on the teacher's desk. This will do away with wasting time by raising hands. Each pupil is free to work at his own rate of speed within a set time limit, at work given in the form of a contract which he chooses to complete.

This time limit per unit should be made flexible so

that an extra day or two can be added if necessary.

A ten-minute test given in the middle of the unit's work is advisable as it reveals whether the pupil is making the correct progress. Care should be taken so that there will be plenty of time for reviews.

Be sure to give each pupil a chance to bring his findings on the unit's work to the discussion table, as it will encourage socialization and it will also give the pupils working on the "D" Contract a chance to see and hear what those working on the "B" and "A" Contracts are discovering.

The "Mastery Test" should follow this discussion period. This test should have a totally different purpose from that of determining what marks should be recorded, since those are determined by the contracts one completes, but to see if the pupils have really learned the unit's work. Pupils failing on parts are retaught and another similar test is given. This is repeated until the work is mastered.

This method of teaching should make the class period a productive thought provoking hour for every member of the class. It is a question of working up to capacity while working, and does away with the necessity of having

the brighter pupils wasting time, waiting for the slower ones to catch up.

An added advantage is in the case a pupil is absent, he can work at home. In case the teacher is absent, a substitute teacher can more easily carry out the work for the pupils know just what is expected of them. The pupils also have more social contact in that they can work in small groups. It often helps to prepare them for their life work by developing in them responsibility and initiative. They are responsible only to themselves for the daily work they accomplish. It teaches them how to be good managers so as not to have all the work to do at one time. The work is more enjoyable to them and many strive for the "A" Contract.

The teacher benefits in that she no longer needs to see that each pupil is given enough to do or enough help. It is their duty to go to her if they need it. She no longer is necessary in the class room every minute of the period as the pupils can go on without her. She can help the slower ones during the class period, rather than after school, which is almost impossible in some communities.

This work can be done successfully by pupils of the ninth grade if emphasis is placed on leading the pupils to discover a process of how to work problems rather than of telling them how to do them.

## UNIT I. INTRODUCTION TO ALGEBRA

Time: 2 - 3 days

How do you suppose we ever decided to call this subject algebra? The Arabs introduced into Europe in the ninth century a book called "Al-jabr w'al muqābalah", which was shortened to algebra. From that time forth we have used the word algebra.

## Lesson I

Open your books to the "Table of Contents" and compare it with that of another text or two. Do you find that they are quite similar or widely different? Can you explain your findings? Under which of the following would the author of each of these texts be more easily classified: Student, superintendent, publisher, teacher, or a principal?

Contract D. Read the Introduction and Preface of your text. Choose some topic of historical value dealing with mathematics and write a short outline of the interesting things you find.

Contract C. Name some professions which presuppose a knowledge of algebra.

Contract B. Report on origin of symbols of operation by Slaughter and Lennes, "Elementary Algebra", p. 7, and Hawkes, Luby, and Touton's "First Course in Algebra", pp. 4, 5.

Contract A. Give an example of the type which the person must do for your professions listed under Contract C.

Name all the occupations you can think of where no mathematics is used.

## Lesson II

### Arithmetic Foundation Drill

Everybody write out the answers to the following:

- (1)  $5 + 0$ . (2)  $0 \div 7$ . (3)  $8 - 0$ . (4)  $7 \times 0$ . (5)  $0 \times 6$ .  
 (6)  $37 + 8$ . (7)  $74 - 9$ . (8)  $46 + 7$ . (9)  $36 - 9$ . (10)  
 $25 + 8$ . (11)  $3/4 + 1/2$ . (12)  $5/6 + 1/3$ . (13)  $2/3 + 3/4$ .  
 (14)  $3/5 - 1/2$ . (15)  $3/8 + 5/6$ . (16)  $3/4 \times 8/5$ . (17)  
 $2/3 \times 9/10$ . (18)  $5/3 \times 6/10$ . (19)  $3 \ 3/8 \times 2 \ 1/2$ . (20)  
 $6 \ 1/4 \div 2 \ 1/2$ .

Place the decimal point where it belongs in Ex. 21 - 24. The products and quotients are correct otherwise.

- (21)  $4.7 \times .32 = 1504$ . (22)  $.03 \times .68 = 204$ .  
 (23)  $11.97 \div 2.1 = 57$ . (24)  $.445 \div .05 = 89$ .

Solve:



- (25)  $2\frac{3}{5} + 3\frac{1}{2}$ . (26)  $82.5 + 16.07$ . (27)  $18.75 - 9.8$ .  
(28)  $7\frac{3}{5} \times 85$ . (29)  $16\frac{2}{3} \times 36$ . (30)  $7.5 \times 15.3$ .  
(31)  $84.152 \div 3.14$ . (32)  $146.054 \div 2.06$ . (33)  $37\frac{1}{2} \div 3\frac{1}{8}$ .  
(34)  $8\%$  of 2750. (35)  $62\frac{1}{2}\%$  of 56. (36)  $120\%$  of 75.

If upon working the above problems, you discover that you do not know how to add a number to zero or zero to a number, how to subtract zero from a number, or how to multiply a number by zero, or zero by a number, or to divide zero by a number, review your rules along these lines, since you will make good use of them throughout the course.

Also be sure you know how to handle arithmetic computations dealing with fractions, decimals, and mixed numbers before you go on. Your teacher will give you supplementary work along these lines if you ask for it.

## PRE-TEST ON UNIT II

1. Express  $x$  feet and 2 inches in terms of inches.
2. How many cents are there in  $2x$  quarters?
3. What does  $x + 5$  indicate?  $5x$ ?
4. If  $h = 6$  and  $b = 7$ , what would be the value of  $a$  in the formula,  $a = bh$ ?
5. Find the area of a triangle whose altitude is 8 inches, and base 12 inches.
6. Write the formulas for:
  - a. The area of a square whose side is  $x$ .
  - b. The area of a rectangle.
  - c. The area of a circle.
  - d. The area of a parallelogram.

## UNIT II. THE FORMULA

Time: 7 - 8 days

One of our chief reasons for studying algebra is to solve difficult problems and to solve them quickly. In order to do this, we must be able to write our English words by the use of symbols in shorthand form. Our aim is to appreciate the significance of the formula and to strengthen our ability to write the shorthand of algebra and to translate it into words.

This is all clearly explained in your text. Read it carefully.

Contract D. Before leaving this unit, you should have mastered the following types of problems to a degree of perfection.

1. The use of letters in algebra.
2. How to write certain formulas, as  $A = bh$ , from the verbal form.
3. How to evaluate formulas by substitution.
4. Important formulas and how to apply them, as  $A = bh$ ,  $A = \frac{bh}{2}$ ,  $V = lwh$ ,  $C = 2\pi r$ ,  $P = 2l + 2w$ , which are commonly used in mensuration.
5. The interest formula and its various forms and

uses, as finding the time when  $P$ ,  $I$ , and  $r$  are known.

6. How to express the meaning of these formulas in sentence form.
7. How to change formulas into other forms, as  $d = rt$ ;  $r = d/t$ ;  $t = d/r$ .

Contract C. Be able to explain:

1. How we can find the value of  $\pi$  by constructing a circle.
2. How to find either the length, width, height, or volume of a rectilinear solid when only one unknown is given.
3. How to determine the area of any or all of the faces of a rectilinear solid.
4. How to find the area of the curved surface of a cylinder or its total area.
5. How to find the volume of a cylinder.

Write and solve some practical applications of the above, as a circular flower bed, fencing a garden, or a race track.

Contract B. Learn the following:

1. How to compare the entire surfaces or the volumes of two or more different cubes.
2. How to find the number of gallons in a cylin-

dricul or a rectilinear tank when its dimensions are given in inches or feet.

3. How to construct a table, given the formula and the values of all the letters but one, as  $A = bh$ . Construct a table when  $h = 5$ .

When $b =$	0	2	3	5	9	12	15	20
Then $A = 5b =$	0	10	15	25	45			

Using the formula  $p = 3s$ , see if you can find the perimeter of an equilateral triangle when  $s = 5$ . Name and work out others.

4. How to decide which oranges are the most economical to buy, by comparing the amount of juice in each, to their respective prices.

Be able to explain what is meant by "Dependence and Variables". In the formula  $C = 2\pi r$ , the length of the circumference ( $C$ ) depends on the radius ( $r$ ). The quantities  $C$  and  $r$  may change in value and are therefore called variables.

#### Contract A.

1. Be able to obtain formulas from tables as: Find the formula from the following table and complete.

When $b =$	1	2	4	5	7	9	12	15
then $A =$	4	8	16	20				

The answer would be  $A = 4b$ .

See if you can construct others for the area of a circle, perimeter of a rectangle. etc.

2. Be able to draw a graph from a formula.

3. Be able to obtain the area of an irregular plot of ground by dividing it up into rectangles and triangles.

4. Know how to proportion the usual distribution of your time in a circular graph form.

5. Name other uses for this type of a graph.

6. Make a survey of formulas from the "Popular Science" or "Science and Mechanics".

7. Make a line graph of the lowest, average, and highest temperatures for your city in the last 25 years.

## UNIT II. MASTERY TEST

Directions: In order to complete any contract, you are required to pass the tests on it and all the contracts preceding it, i.e., those completing Contract C are required to pass both tests D and C. Everyone should work as many problems in each contract test beyond his own contract as is possible for him to do.

Contract D. Write a shorthand statement for each of the following:

1. Five times a number (n) plus 3.
2. The length times the width divided by two.
3. The quotient of x divided by y.
4. One-half a number (n) equals six.
5. The cost of y divided by seven.
6. Four plus a number (n).

Evaluate the following:

7.  $A = \frac{1}{2}bh$ ; find A when  $b = 6$  and  $h = 5$ .
8.  $A = bh$ ; find h when  $A = 16$  and  $b = 2$ .
9.  $V = lwh$ ; find V when  $l = 7$ ,  $w = 3$ ,  $h = 4$ .
10.  $V = lwh$ ; find w when  $V = 420$ ,  $h = 7$ ,  $l = 12$ .
11.  $d = rt$ ; find r when  $d = 120$ ,  $t = 3$  hrs.
12. Find the area of a parallelogram with a base of 4 ft., and an altitude of  $2\frac{1}{2}$  ft.

13. If the side of an equilateral triangle is  $4x$ , write a formula for its perimeter ( $p$ ).

14. If  $y$  stands for any number, what is the meaning of  $3y$ ,  $y + 5$ ,  $5 - y$ ,  $y/6$ ?

15. Find the amount invested at  $4\%$  for 3 years if the interest amounted to \$10.44.

16.  $A = lw$ ; solve for  $l$ ; for  $w$ .

17.  $I = prt$ ; solve for  $r$ ; for  $t$ .

18.  $V = lwh$ ; solve for  $h$ ; for  $w$ .

Contract C.

19. A rectangular strip of ground is 20 ft. long and 12 ft. wide. Along the two longer sides is a flower bed one foot wide. The rest of the plot is in grass. How many sq.ft. of grass are there?

20. The dimensions of a rectangular solid are  $l$ ,  $w$ , and  $h$ . Write a formula for (1) its total surface ( $S$ ); (2) its volume ( $V$ ).

21. Write the formula which shows how many cents ( $c$ ) in ( $d$ ) dollars, ( $q$ ) quarters, and ( $n$ ) nickels.

22. A rectangular tin box has a square base, the edge of which is 8 inches long. It is 6 inches high. What is the volume of this box? How much tin is used in making the bottom and sides of this box?



Contract B.

23. A cylindrical boiler was 14 inches in diameter and 6 ft. high. How many gallons of water did it hold? (One gal. = 231 cu.in.). Find the area in sq.in. of its total surface (S).

24. Compare the volumes of two cubes whose edges are 6 ft. and 3 ft. respectively. Compare their areas.

25. Construct a table of values for the formula  $c = 8n$  for the following values of  $n$ : 5, 9, 12, 15, 25.

Contract A.

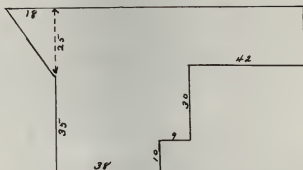
26. This table shows how to find the postage required to send a parcel into the third zone.

Wt. in lbs. (x)	1	2	3	5	10	20	70
Cost (C)	\$.08	.14	.20	.32	.62	1.22	4.22

Write the formula for the above table.

27. If you spend 9 hours daily in sleeping, 2 hours eating, 2 hours in dressing, 6 hours at school, 2 hours in outside activities, 1 hour playing, and 2 hours working, draw a circular graph showing this.

28. Find the area of the plot of land shown here.



29. (a) Draw a graph of the formula  $p = 3s$ . Let  $s$  have the values 0, 3, 6, 9, 12, 15.
- (b) From the graph, determine  $p$  when  $s$  is 8.
- (c) From the graph, determine  $s$  when  $p$  is 30.
- (d) Complete: As  $s$  increases,  $p$  \_\_\_\_\_.

## PRE-TEST FOR UNIT III

1. (a) What is the left side of the equation  $6a = 12$ ?  
(b) What is its root?  
(c) What number satisfies it?  
(d) To solve it, what do you do to both sides?
2. What does  $x + 7$  mean?  $7x$ ?  $7 - x$ ?  $x/7$ ?
3. What does  $A_3$  mean?  $S_8$ ?  $M_x$ ?
4. Solve the following equations and check:
  - (a)  $x/2 = 5$ .
  - (b)  $a + 3 = 8$ .
  - (c)  $y - 7 = 2$ .
  - (d)  $3m = 15$ .
5. Terms such as  $3x^2$  and  $5x^2$  are called \_\_\_\_\_.

## UNIT III. SIMPLE EQUATIONS

Time: 4 weeks

What is an equation? Can you recall working with equations before? What is a formula? Is it an equation? Do all equations need to be formulas? What always separates the left member from the right member?

Our aim in this unit is to use the equation as an instrument in the solution of verbal problems. We also desire to strengthen our ability of problem solution, and to develop self-reliance in the handling of numerical problems through the consistent use of checks.

Contract D. Before attempting the "Mastery Test" on this unit, you should have completely mastered the work on the following types of problems:

1. How to write verbal statements in shorthand form, as:

Three times  $x = 3x$ .

Seventy-five per cent of the cost of an algebra book is 96 cents. What does it cost?

Solution: a. Let  $x =$  cost of the book in cents.

b.  $\therefore .75x = 75\%$  of the cost of the book.

c.  $\therefore .75x = 96$

d.  $x = 128$

$\therefore$  the book cost \$1.28.

Check. Does  $.75 \times \$1.28 = \$.96$ ? Yes.

Always write out all the steps as given above and check your result.

2. How to solve the multiplication type as  $ax = b$ , or  $3x = 15$ .

Note: In this type of problems, all the letters except  $x$  represent numerals. The  $x$  is the unknown.

3. How to solve the division type, as  $x/a = b$ , or  $x/2 = 10$ .

4. How to solve the addition type, as  $x + b = c$ , or  $x + 3 = 15$ .

5. How to solve the subtraction type, as  $x - b = c$ , or  $x - 8 = 10$ .

6. How to use the fundamental axioms.

7. How to check any type of an equation.

8. How to combine like terms.

9. How to solve the following types of equations consisting of more than one step, as:

(a)  $x/a + b = c$ , or  $x/3 + 5 = 14$ .

(b)  $x/a - b = c$ , or  $x/3 - 5 = 1$ .

(c)  $x/a \pm b/c = d$ , or  $x/2 \pm 4/3 = 5$ .

(d)  $ax \pm b = c$ , or  $2x + 5 = 7$ , and  $4x - 3 = 9$ .

(e)  $ax + b = cx + d$ , where  $a$  is greater than  $c$  and  $b$  is less than  $d$ , as  $5x + 4 = x + 7$ .

(f)  $ax + b + cx = d$ , or  $7x + 9 + x = 25$ .

(g)  $ax + bx + cx + dx + e = f$ , or  $8x + x + 5x + 3x + 9 = 43$ .

10. How to write the equation and how to solve and check those based on verbal statements, as:

If 8 years be subtracted from Ann's age, the result is 6. How old is Ann?

Solution: Let  $x$  = Ann's age.

$x - 8 = 8$  years subtracted from Ann's age.

$x - 8 = 6$  correct equation.

$A_8 + 8 \quad + 8$

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$x = 14$  Ann's age.

Check:  $14 - 8 = 6$

$6 = 6$ .

Know that  $A_4$  means to add 4 to both sides of the equation.

Know that  $S_3$  means to subtract 3 from both sides of the equation.

Know that  $D_5$  means to divide each term of the equation by 5.

Know that  $M_2$  means to multiply each term of the equation by 2.

Contract C. More complicated forms of the above verbal type problems will be expected of you. Instead of only one unknown, you should be able to solve problems containing two unknowns, both expressed in terms of one unknown letter. There may also be a necessity of taking two or more steps in solving these equations, such as:

Frank is 5 years older than Fred. The sum of their ages is 25. How old is each?

Solution: Let  $x$  = Fred's age.

Then  $x + 5$  = Frank's age.

$x + x + 5 = 25$  correct equation.

$$2x + 5 = 25$$

$$\begin{array}{r} S_5 \quad -5 \quad -5 \\ \hline \end{array}$$

$$2x = 20$$

$D_2 \quad x = 10$  Fred's age.

$x + 5 = 15$  Frank's age.

Check: Does  $10 + 15 = 25$ ? Yes.

Contract B. Emphasis should be placed on verbal problems containing three unknowns. The way you derive your equations, and your method of solving and checking them, are very important. Care should be taken so that the most economical unknown is represented by ( $x$ ). Then state the others in terms of it. This will lead to clear, straight-forward statements and fewer errors will result.

For example: The sum of the ages of three children of a certain family is 36 years. Charles' age is one year more than twice John's age; Mary's age is one year less than 3 times John's age. Find their ages.

Solution: Let  $x$  = John's age.

Then  $2x + 1$  = Charles' age,

and  $3x - 1$  = Mary's age.

$x + 2x + 1 + 3x - 1 = 36$ , correct equation.

$$6x = 36$$

$D_6$              $x = 6$  John's age

$2x + 1 = 13$  Charles' age

$3x - 1 = 17$  Mary's age.

Check: Does  $6 + 13 + 17 = 36$ ? Yes.

Contract A. Bring to class several verbal problems based on your everyday experiences or on those of your friends. Can you find any involving fractions?



## UNIT III. MASTERY TEST

Directions: In order to complete any contract, you are required to pass the tests on it and all the contracts preceding it, i.e., those completing Contract C are required to pass both tests D and C. Everyone should work as many problems in each contract test beyond his own contract as is possible for him to do.

Contract D. Write the following statements in shorthand form:

1. A man is  $x$  years old. Express his age 8 years from now.
2. A man is  $x$  years old. Express his age 8 years ago.
3. A man is  $x$  years old. Express 8 times his age.
4. A baseball team played  $n$  games and lost 7. The number of games won was?
5. The cost ( $c$ ) of one article when the total cost ( $T$ ) of  $n$  articles is known.
6. Represent the smaller number ( $s$ ) when the larger one is ( $l$ ) and their difference is ( $d$ ).
7. Six diminished by twice a number ( $n$ ).
8. Represent the number of days ( $D$ ) in  $x$  weeks and 5 days.

Form equations only:

9. Nine less than 3 times a number is 27.

10. If 4 is subtracted from 11 times a number, the remainder is 7.

11. Eight more than 5 times a number is 43.

12. Does 4 satisfy the equation  $7x + 3x = 40$ ?

13. Is 5 a root of the equation  $3y - 9 = 12$ ?

14. What does  $A_2$  mean?  $S_4$ ?  $M_3$ ?

Solve for the value of the unknown and check your results.

15.  $15t = 80$ .

18.  $x + 11 = 29$ .

16.  $m - 17 = 13$ .

19.  $3/5x = 18$ .

17.  $x/7 = 2$ .

20.  $2x - 9 = 15$ .

Solve and check the following:

21. The sum of the angles in a triangle is 180 degrees. If two of them are equal to each other, and the third angle is 55 degrees, how many degrees are there in each equal angle?

22. Find two parts of 92, one of which is  $1/3$  of the other.

Contract C. Solve and check the following:

23. John's radio set cost him 4 times as much as Henry's. Both sets cost \$125 together. What was the cost of each?

24. The length of the school playground is twice the width. The perimeter is 1800 ft. What are the length and width?

25. Divide 21 into three parts such that the first is twice the second, and the second is twice the third.

Contract B. Solve for the value of the unknown in the following and check your results.

$$26. \frac{3}{4}x = 12.$$

$$27. 3x + 11x = 280.$$

$$28. \frac{x}{2} + 7 = 12.$$

$$29. 21z - z + 8 = 88.$$

$$30. 3x + 5 - x = x + 9.$$

$$31. 3y + y - 8 + 7y + 2 = y + 28.$$

Solve and check the following:

32. A plumber and two helpers together earned \$15 a day. How much did each man earn in a day if the plumber earned 4 times as much as each helper?

33. Mary has 3 times as far to walk to school as Helen has and their difference is  $\frac{1}{2}$  mile. How far does each walk?

34. Ann has \$8.50 in a savings account and is depositing \$.50 a week. Edna has \$12.25 and is depositing \$.25 a week. In how many weeks will Ann have as much as Edna?

Contract A. Solve and check the following:

35. There are two numbers of which the larger is 6

times the smaller. The larger is also 25 more than the smaller. What are the two numbers?

36. The perimeter of a triangle is 20 inches. The second side is 2 inches longer than the first side, and the third side is 3 inches longer than the first side. Find the length of each side of the triangle.

37. A is ten years older than C, and B is 6 years younger than C. The sum of their ages six years ago was 40 years. How old is each now?

## PRE-TEST ON UNIT IV

1. How would you indicate a rise of 7 degrees in temperature? A fall of 7 degrees?

2. Add:

$$\begin{array}{r} +3 \\ -4 \\ \hline \end{array} \quad \begin{array}{r} +6 \\ +1 \\ \hline \end{array} \quad \begin{array}{r} -9 \\ -2 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ 4 \\ \hline \end{array} \quad \begin{array}{r} -12 \\ +9 \\ \hline \end{array} \quad \begin{array}{r} +3 \\ -3 \\ \hline \end{array} \quad \begin{array}{r} 0 \\ -2 \\ \hline \end{array} \quad \begin{array}{r} 1 \\ +9 \\ \hline \end{array} \quad \begin{array}{r} -4a^2 \\ -8a^2 \\ \hline \end{array} \quad \begin{array}{r} -2x \\ +13x \\ \hline \end{array}$$

3. Subtract the lower number from the upper number:

$$\begin{array}{r} -5 \\ +2 \\ \hline \end{array} \quad \begin{array}{r} -6 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} +8x \\ +10x \\ \hline \end{array} \quad \begin{array}{r} -7 \\ -12 \\ \hline \end{array} \quad \begin{array}{r} +8 \\ -11 \\ \hline \end{array} \quad \begin{array}{r} +13 \\ -4 \\ \hline \end{array} \quad \begin{array}{r} -8 \\ +15 \\ \hline \end{array} \quad \begin{array}{r} +0 \\ -6 \\ \hline \end{array} \quad \begin{array}{r} -6 \\ -6 \\ \hline \end{array} \quad \begin{array}{r} -y \\ +y \\ \hline \end{array}$$

4. Multiply:

$$\begin{array}{r} 3 \\ -2 \\ \hline \end{array} \quad \begin{array}{r} +7 \\ +4 \\ \hline \end{array} \quad \begin{array}{r} -7x \\ -2x \\ \hline \end{array} \quad \begin{array}{r} -9 \\ +3 \\ \hline \end{array} \quad \begin{array}{r} +8x \\ -12y \\ \hline \end{array} \quad \begin{array}{r} -5a^2 \\ +8a^2 \\ \hline \end{array} \quad \begin{array}{r} -8 \\ 0 \\ \hline \end{array} \quad \begin{array}{r} 0 \\ +3 \\ \hline \end{array}$$

5. Divide:

27 by -3; -9 by +3; 5 by 0; 0 by -8; -15 by -5; 12 by +4;

35x by -7;  $-3x^3y^2$  by xy.

## UNIT IV. POSITIVE AND NEGATIVE NUMBERS

Time: 7 weeks

A. ADDITION OF POSITIVE AND NEGATIVE NUMBERS

Positive numbers such as we have in arithmetic may be pictured by lengths of lines. We can choose a line segment and mark off on it units of equal length to represent our numbers, or parts of a unit for fractions. If this line segment is extended on the left, it can be marked off similarly. This will give us a number scale which we can find in most any standard text. Study carefully the characteristics of this scale. Will a thermometer scale make a good number scale? Can you think of others?

Contract D. Before attempting the "Mastery Test" on this part of the unit, you should have complete mastery of the following:

1. The meaning and important uses of positive and negative numbers.
2. How to mentally picture signed numbers.
3. How to add signed numbers and fractions.
4. How to add two or more monomials either vertically or horizontally.

5. How to add two or more polynomials and check your results.

6. How to solve equations involving two or more additions, as:

$$ax - b = c - dx, \text{ or } 9x - 12 = 1 - 4x,$$

where  $a$  is greater than  $d$  and  $b$  is greater than  $c$ .

Know the definitions of: directed numbers, similar terms, coefficient, exponent, monomials, polynomials, absolute values, descending and ascending powers, binomial, trinomial, negative and positive terms.

Know the rules for addition.

Contract C. Be able to determine the following:

1. How to find the average hourly temperature reading for any day.
2. How to solve verbal problems based on addition.

Contract B.

1. Write out a brief summary of the uses of positive and negative numbers, showing their relationship to arithmetic. State if you think they will be of benefit to you, and why.

Contract A.

1. Write out a few practical applications where you can make use of these rules in everyday life, as:

$$\begin{array}{r}
 3 \text{ yds. } 2 \text{ ft. } 6 \text{ in.} \\
 5 \text{ yds. } \quad \quad \quad 4 \text{ in.} \\
 \hline
 8 \text{ yds. } 2 \text{ ft. } 10 \text{ in.}
 \end{array}$$

2. Know how to check the addition of any polynomials.

### B. SUBTRACTION OF POSITIVE AND NEGATIVE NUMBERS

Construct a number scale similar to the one you used in learning how to add directed numbers. See if you can determine the rules for subtraction from it. If you cannot, study this method from some standard text. Do you find any similarity between the addition and subtraction rules?

Contract D. A semi-unit test will be given at the close of this part of the unit. You should have mastered the following before attempting to take it.

1. How to subtract one signed number from another and check your results.
2. How to subtract one monomial from another and check your results.
3. How to subtract one polynomial from another and check your results.
4. How to solve equations involving two or more subtractions, as:



$$ax + b = c + dx, \text{ or } 6x + 2 = 12 + x,$$

where  $b$  is less than  $c$  and  $d$  is less than  $a$ .

5. How to remove one symbol of grouping and collect like terms in solving algebraic expressions containing parentheses preceded by either a plus or a minus sign.
6. How to solve an equation involving one parenthesis.

Know the definitions of the following: subtrahend, minuend, supplementary and complementary angles, and consecutive numbers.

Know the rules for subtractions and for removing parentheses.

Contract C. Learn to do the following:

1. How to perform the combination of addition and subtraction with polynomials.
2. How to solve equations or algebraic expressions involving two parentheses.
3. How to solve verbal problems based on subtraction.

Contract B. Study carefully the solution of equations, both numerical and literal, containing two or more parentheses, some of which are within others.

Contract A. Learn the following:

1. How to check the subtraction of the more difficult polynomials.
2. How to inclose two or more terms in parentheses preceded by either a minus or plus sign.

UNIT IV. MASTERY TEST ON THE ADDITION  
AND SUBTRACTION OF SIGNED NUMBERS

Directions: In order to complete any contract, you are required to pass the tests on it and all the contracts preceding it, i.e., those completing Contract C are required to pass both tests D and G. Everyone should work as many problems in each contract test beyond his own contract as is possible for him to do.

Contract D.

1. Add:

$$\begin{array}{r} +10 \\ -3 \\ \hline \end{array} \quad \begin{array}{r} -7 \\ +2 \\ \hline \end{array} \quad \begin{array}{r} -8 \\ -4 \\ \hline \end{array} \quad \begin{array}{r} +2x \\ +8x \\ \hline \end{array} \quad \begin{array}{r} 0 \\ -2 \\ \hline \end{array} \quad \begin{array}{r} x \\ -7x \\ \hline \end{array} \quad \begin{array}{r} -y \\ +y \\ \hline \end{array} \quad \begin{array}{r} -y \\ -y \\ \hline \end{array} \quad \begin{array}{r} 3r \\ -r \\ \hline \end{array} \quad \begin{array}{r} 5x \\ 0 \\ \hline \end{array}$$

2. Subtract:

$$\begin{array}{r} 3 \\ -9 \\ \hline \end{array} \quad \begin{array}{r} -12 \\ 5 \\ \hline \end{array} \quad \begin{array}{r} +6 \\ +6 \\ \hline \end{array} \quad \begin{array}{r} 0 \\ -9 \\ \hline \end{array} \quad \begin{array}{r} x \\ 0 \\ \hline \end{array} \quad \begin{array}{r} -x \\ -x \\ \hline \end{array} \quad \begin{array}{r} 3y \\ -2y \\ \hline \end{array} \quad \begin{array}{r} 5r \\ -5r \\ \hline \end{array} \quad \begin{array}{r} -7y \\ +9y \\ \hline \end{array} \quad \begin{array}{r} 12y \\ -y \\ \hline \end{array}$$

3. In the expression  $3m^2 - 9mn + n^2$ :

a.  $3m^2$ ,  $-9mn$ ,  $+n^2$  are called \_\_\_\_\_?

b.  $3m^2$  is a \_\_\_\_\_?

c.  $-9mn$  is a \_\_\_\_\_?

d. The numerical coefficient of  $n^2$  is \_\_\_\_\_?

4. Add:

$$\begin{array}{r} 9ab^2 \\ -3ab^2 \\ +7ab^2 \\ ab^2 \\ -2ab^2 \\ \hline \end{array}$$

Add and check the following:

$$\begin{array}{r} 5. \quad 3x + 2y - z \\ \quad -x + y + z \\ \hline \end{array}$$

$$\begin{array}{r} 6. \quad -9x^2 + 7xy + 8 \\ \quad \quad 8x^2 + xy - 7 \\ \hline \end{array}$$

$$\begin{array}{r} 7. \quad 3x^2 \quad + 7y^2 \\ \quad - 3xy + y^2 \\ \hline \end{array}$$

Subtract and check the following:

$$\begin{array}{r} 8. \quad 3x - y + 4z \\ \quad 3x + 2y - z \\ \hline \end{array}$$

$$\begin{array}{r} 9. \quad 7r^2 + 2st - 9t^2 \\ \quad - r^2 + st - 2t^2 \\ \hline \end{array}$$

10. From  $5a + 7b - 3c$  subtract  $a - 2b - 5c$ .

11. Combine like terms:  $5r - 3s + t + 4r - 2t$

$+ 7s - t + 3r - s$ .

Solve the following equations and check your results:

12.  $11x = 144 - 5x$ .

13.  $17y - 38 = 100 - 6y$ .

14.  $18x + 4 = 5x + 147$ .

15.  $5x - (3x - 7) = 13$ .

16.  $5y - (8 - 6y) = 80$ .

17. How many degrees are there in the angle which is 11 times as large as its supplement?

Contract C. Solve the following:

18. How much more than  $x^2 - 8$  is  $-5x^2 + 11$ ?

19. Subtract  $-5r^2 - rs - 5s^2$  from the sum of  $r^2 + 3rs - 5s^2$  and  $2r^2 + 7s^2$ .

20. By how much does  $e^2 - 15e + 2$  exceed  $e^2 + 5e - 2$ ?

21. How much smaller is  $-4a + 2b - 14c$  than  $a - 6b - 7c$ ?

22. From the sum of  $y^2 - z^2$ ,  $y^2 - yz - 2z^2$ , and  $2yz + z^2$  subtract  $z^2 - y^2 + 2yz$ .

Solve the following equations:

23.  $(10x + 6) - (11 - 15x) = 20x$ .

24.  $1 - (6x - 5) = 2x + (15 - 9x)$ .

25. There are three consecutive integers whose sum is 114. What are they?

26. If you subtract 32 from 7 times a certain number, the remainder is 24. What is the number?

Contract B. Solve the following equations:

27.  $b - [2 + (11 - b)] = 1$ .

28.  $3x - [x - (x + a)] = c$ . Solve for  $x$ .

29. How large is the angle which is 6 more than 3 times as large as its complement?

Contract A.

30. Check No. 18 above by letting  $x = 3$ .

31. Check No. 19 above by letting  $r = 2$  and  $s = 1$ .

32. Check No. 22 above by letting  $y = 2$  and  $z = 3$ .

33. One number is 8 times as large as a smaller one. If the smaller one be subtracted from the larger, the remainder is 21. Find the two numbers.

C. MULTIPLICATION OF POSITIVE  
AND NEGATIVE NUMBERS

The rules for multiplying positive and negative numbers are easy to learn, but you should carefully study the explanation of them in your text so as to be able to understand them.

Contract D. Learn the following perfectly:

1. How to multiply signed numbers.
2. How to multiply monomials containing exponents and coefficients.
3. How to multiply a polynomial by a monomial.
4. How to multiply a polynomial by a binomial.
5. How to solve simple fractional equations.

Know the laws of order, grouping, exponents, removing parentheses, and for multiplication.

Be able to define multiplier, product, and multiplicand.

Contract C. Learn the following:

1. How to multiply a polynomial by any polynomial.
2. How to solve verbal problems of the multiplication type, such as: The sum of two numbers is 55; twice the greater equals three times the

smaller, plus 15. Find each number.

Solution: Let  $x$  = the smaller part.

Then  $55 - x$  = the greater part.

$2(55 - x) = 3x + 15$ , correct equation.

$110 - 2x = 3x + 15$ , or rearranging

$3x + 15 = 110 - 2x$

$$\begin{array}{r} A_{2x} \quad +2x \qquad \qquad \qquad + 2x \\ \hline 5x + 15 = 110 \end{array}$$

$$\begin{array}{r} S_{15} \quad \quad - 15 \quad -15 \\ \hline 5x = 95 \end{array}$$

D<sub>5</sub>  $x = 19$  smaller part.

$55 - x = 36$  larger part.

Check: Does  $19 + 36$  give 55? Yes.

Contract B. Know how to solve verbal problems of the more complicated types involving multiplication and addition or subtraction, as: If 5 times the complement of a certain angle be subtracted from 2 times its supplement, the result is 15 degrees. What is the angle?

Solution: Let  $x$  = the angle.

Then  $90 - x$  = the complement

and  $180 - x$  = the supplement.

$2(180 - x) - 5(90 - x) = 15$  correct equation.

$360 - 2x - 450 + 5x = 15$

$3x - 90 = 15$

$$\begin{array}{r} A_{90} \quad \quad + 90 \quad +90 \\ \hline \end{array}$$

$$3x = 105$$

$x = 35$  degrees, the size of the angle.

$$90 - x = 55, \text{ its complement.}$$

$$180 - x = 145, \text{ its supplement.}$$

Check: Does 2 times 145 minus 5 times 55 = 15? Yes.

Contract A. Write out some practical applications of multiplication from everyday life. It will not be necessary that you solve all of them.

#### D. DIVISION OF POSITIVE AND NEGATIVE NUMBERS

Look through the explanation of division of signed numbers and see if you can make up your own rules. How do they differ from those for multiplication?

Contract D. You should have complete mastery of the following before attempting to take the "Mastery Test" over the whole of Unit IV.

1. How to divide one signed number by another.
2. How to divide one monomial containing exponents and coefficients by another.
3. How to divide a polynomial by a monomial.
4. How to divide a polynomial by a trinomial or binomial.
5. How to solve equations with negative roots.
6. How to find the equations for, and how to solve



and check all types of verbal problems based on simple linear equations, such as percentage problems, mixture problems, coin problems, motion problems, age problems, and other applied problems.

Know the laws of order, exponents, and your rules for division.

Be able to define dividend, divisor, quotient, and remainder.

Contract C. Learn how to do the following:

1. How to divide one polynomial by another polynomial.
2. How to solve the more difficult verbal problems.

Contract B.

1. Be able to multiply or divide three or more place figures, which are perfect multiples of smaller figures, by the use of the laws of exponents, such as: Divide 1024 by 128 by using a table of the various powers of 2.

Solution: 
$$\frac{1024}{128} = \frac{2^{10}}{2^7} = 2^3 = 8.$$

2. Make a study of a number of problems that can be solved by signed numbers, as games, ages, financial conditions, present weather condi-

tions, and business problems .

Contract A.

1. Bring to class some outside project based upon what you have already learned in algebra. This may be a group of practical problems and their solutions, a poem, play, poster, etc.
2. Make a study of scales and find how to use them with positive and negative numbers.

## UNIT IV. MASTERY TEST

Directions: In order to complete any contract, you are required to pass the tests on it and all the contracts preceding it, i.e., those completing Contract C are required to pass both tests D and C. Everyone should work as many problems in each contract test beyond his own contract as is possible for him to do.

Contract D. Add, then subtract the following:

$$1. \begin{array}{r} -7x + 3y + 4z \\ 6x - 3y - z \end{array} \qquad 2. \begin{array}{r} r^2 - 3rs + s^3 \\ -2r^2 + rs \end{array}$$

3. From  $5x - 3y - z$  subtract  $4x - 2y - z$ , and check.

4. Remove parentheses and simplify:

$$(3y - 7) - (4y - 9).$$

Solve for  $x$  and check:

$$5. 14x + 7 = 9x + 32.$$

$$6. 15(2x - 1) + 8 = 5x - 57.$$

Multiply:

$$7. \begin{array}{r} 3 \quad -1 \quad 0 \quad 12 \quad 4x \quad -xy \quad -3x^2y \quad -9y^3 \\ -7 \quad 5 \quad -9 \quad 0 \quad 3x^2 \quad -3xy \quad xy^2 \quad -3xy^3 \end{array}$$

Divide:

$$8. (+18) \text{ by } (+2) \qquad 25y^4 \text{ by } 5y^4$$

$$(-15) \text{ by } (-3) \qquad 15t \text{ by } (-3)$$

$(-10) \text{ by } (-1)$

$xyz \text{ by } xy$

$(-4) \text{ by } (+8)$

$(-12xy^2) \text{ by } (-6xy)$

$(+27) \text{ by } (-3)$

$(cd) \text{ by } (-d)$

$y^3 \text{ by } y^2$

$(-8y^2) \text{ by } (2y)$

$a^3b \text{ by } a$

$16m^2n^3 \text{ by } (-2mn)$

$3xy^3 \text{ by } y^3$

Find:

9.  $(6a - 12b) \div 6$ .

10.  $(-2m^2 + 6m - 4) \div (-2)$ . 14.  $12 \cdot -3/4 \cdot 2/3$ .

11.  $(-y^4)(-y^5) \cdot 7y \cdot -2x^3y$ . 15.  $2m(5m^2 - 4m + 3)$ .

12.  $(-1)(-1)(-1)(-1)(-1)$ .

16. 
$$\frac{-x^2 + 3x - 4}{-1}$$

13. 
$$\frac{y^2 + 8y + 15}{y + 3}$$

Multiply:

17.  $y^2 + 8y + 15 \text{ by } y + 3$ .

18.  $12r + 3s \text{ by } r - 4s$ .

19.  $5x^2 - 5x - 2 \text{ by } 2x + 7$ .

Divide:

20.  $-10x^3y + 6x^2y^2 - 2xy^3 \text{ by } -2xy$ .

21.  $10r - 8 + 9r^3 - 15r^2 \text{ by } 3r - 4$ .

Solve:

22.  $1/2x - 1/5x = 9/10$ .

23.  $m - 10 = 3m/4 - m/6$ .

24. What is the area of a rectangle whose base is

$(5x + y)$  in., and whose altitude is  $(2x - y)$  in.?

Contract C.

25. Multiply  $x^3 - 2x^2 + 8 + 4x$  by  $2 - x$ .

26. Divide  $a^3 - 8$  by  $a - 2$ .

27. Divide  $27b^3 + 1$  by  $3b + 1$ .

28. Two-thirds of a certain number is 20 more than one-fourth of the number. What is the number?

29. A is now 4 times as old as B. In 10 years, he will be only twice as old as B is then. How old is each now?

30. Two men start from the same place, one going south and the other going north. One goes twice as fast as the other. In 5 hours they are 450 miles apart. Find the rate at which each man travels. Write the equation only.

31. Mary weighs 80 lbs. and sits 4 ft. from the fulcrum. Where must John sit if he weighs 90 lbs.? Write the equation only.

Contract B. Solve for  $x$  and check your result:

32.  $12x^2 + 3x + 2 [2x - 5x(x - 1) - 10] - (2x^2 + 7x) = 20.$

33. Find 3 consecutive even numbers such that the sum of 3 times the first and twice the second equals 4 times the third.

34. The total income from 2 investments is \$135. Part of the money is invested at 6% and part at 7%. The total amount invested is \$2000. Find the two sums.

35. One car running 40 miles an hour left a certain place 4 hours later than another car running in the same direction at the rate of 25 miles an hour. In how many hours will the faster car overtake the other? Write the equation only.

36. Eighteen coins, all dimes and quarters, amount to \$2.25. Find the number of each kind of coin. Write the equation only.

Contract A.

37. Inclose the last three terms in parentheses preceded by a minus sign:  $2x^3 - y^2 - 2y - 1$ .

38. A collection of nickels, dimes, and quarters amounts to \$4. There are 10 more nickels than dimes, and 2 less quarters than dimes. Find the number of each.

39. I bought 45 stamps for \$1.05. If part of them were 2-cent stamps and part 3-cent stamps, how many of each did I buy?

40. A grocer mixes two kinds of tea. He mixes 20 lbs. more of the 60¢ variety than of the 75¢ variety. How many pounds of each does he mix if the total value is \$45.75? Write the equation only.

## PRE-TEST ON UNIT V

1. Find the product of the following the easiest way you can and show your work.

$$a(3x + 5y - z)$$

$$(2a + 7)(a - 1)$$

$$(x - y)(x - y)$$

$$(a + b)(a - b)$$

$$(2a + 7)(2a + 7)$$

$$x(r + 3)(2r - 9)$$

2. Factor the following and explain how you did it by using the easiest method you know.

$$(7r - 7t)$$

$$(a^2 - 2ab + b^2)$$

$$(2a^2 + a - 21)$$

$$(25x^2 - y^2)$$

$$(9m^2 + 12mn + 4n^2)$$

$$(3x^2y - 11xy - 20y)$$

3. Solve for the unknown letter.

$$a^2 - 5a + 6 = 0$$

$$a^2 - 25 = 0$$

## UNIT V. PRODUCTS AND FACTORING

Time: 5 weeks

You have already learned one method of finding the products of two or more quantities by multiplication and how to find the factors of two quantities by division. Our aim here is to find a simpler method for some special cases in order to develop speed and accuracy in handling them. Also to understand the fact that algebra has broader powers than arithmetic.

Study this through carefully in your text.

Contract D. A complete mastery of the following should be had before attempting the unit test.

1. How to find the product of any two binomials by the cross-product method, as  $(ax + b)(cx + d)$ .
2. How to find the special products listed below:
  - a. The square of any binomial and check, as  $(ax + b)^2$ .
  - b. The product of the sum and difference of any two terms and check, as  $(x + y)(x - y)$ .
3. How to find the product of any two binomials and a monomial and check.



4. How to factor and check the following types:
  - a. Removing a common monomial factor by inspection, as  $ab + ac + ad$ .
  - b. Factoring a trinomial of the type  $ax^2 + bx + c$ , or  $3x^2 - 5x - 2$ .
  - c. Factoring any trinomial square, as  $x^2 + 2bx + b^2$ .
  - d. Factoring the difference of two squares, as  $a^2 - b^2$ .
5. How to solve quadratic equations by the factoring method.

Know the definitions of quadratic equations, and prime and highest common factors.

Contract C. Learn the following:

1. How to find the products or the factors of the above named "special forms or products" by inspection.
2. How to factor with speed the type  $ax^2 + bx + c$ , or  $6x^2 + 24x + 18$ , where three factors are obtained, one a monomial factor.

Contract B.

1. Learn how to factor polynomials that may be grouped to show the difference between two squares, as  $a^2 + 2ab + b^2 - c^2$ .

2. Make charts showing how to illustrate the finding of the products of two binomials.
3. Make charts to illustrate the use of factoring in the solution of formulas.
4. Read in Hamilton-Bliss-Kupfer, "Essentials of Junior High School Mathematics", Book III, pages 160-164-168-173-177.
5. Read in Engelhardt and Haertter, pages 166-186.

Contract A.

1. Learn how to solve literal quadratic equations.
2. Learn how to solve equations containing special products.
3. Make a survey of rectangles, triangles, and circles whose area can be found by the "cross-product" method and whose dimensions can be found by the factoring method.
4. Bring to class some problems based on your everyday experience, such as finding the unused area in a cookie pan by using different sized cookie cutters, etc.

## UNIT V. MASTERY TEST

Directions: In order to complete any contract, you are required to pass the tests on it and all the contracts preceding it, i.e., those completing Contract C are required to pass both tests D and C. Everyone should work as many problems in each contract test beyond his own contract as is possible for him to do.

Contract D. Multiply:

- |                        |                        |
|------------------------|------------------------|
| 1. $(-3x)^2$           | 2. $(+7y^2)^3$         |
| 3. $(-2xy^3)^2$        | 4. $3a(-2y^2)^3$       |
| 5. $3a(r + s - t)$     | 6. $-2y(a - b + c)$    |
| 7. $a(a^2 - 3a - 1)$   | 8. $(3x - b)(3x + b)$  |
| 9. $(x + y)(x + y)$    | 10. $(2y - 7)(y + 8)$  |
| 11. $(ab - c)(ab - c)$ | 12. $(4d - 3)(4d - 3)$ |
| 13. $(3x + 2)(x - 1)$  | 14. $(x - y)(x + y)$   |

## Find the prime factors of:

- |                         |                            |
|-------------------------|----------------------------|
| 15. $3x - 7xy$          | 16. $2ax + 8bx - 4cx$      |
| 17. $5x - 5y$           | 18. $-2r^3x + 4r^2y - 6rz$ |
| 19. $ax^2 - ay^2$       | 20. $4c^2 - 1$             |
| 21. $m^2 - 7mn - 44n^2$ | 22. $25y^2 - 9x^2$         |
| 23. $x^2y^2 + xy - 6$   | 24. $4x^2 - 12xy + 9y^2$   |
| 25. $4x^2 + 4xy + y^2$  | 26. $6a^2 - 13a + 6$       |

Solve the following quadratic equations by factoring:

$$27. 3x^2 + 8x + 5 = 0. \quad 28. 6a^2 - 7a + 1 = 0.$$

Contract C. Multiply:

$$29. b(2a + 7)(2a + 7). \quad 30. (7m^2 - 3n)(5m^2 - 2n).$$

$$31. m(3m + 2)(2m + 3). \quad 32. (8x^2 - 5y)(3x^2 - 2y).$$

$$33. (a + b)(c + d). \quad 34. (2x^2 + y)(3x - 7).$$

Find the prime factors of:

$$35. 1 - 9y^4. \quad 36. 16a^4 - 1$$

$$37. 6m^4 + 9m^2 - 6. \quad 38. a^2/4 - 4a + 16$$

$$39. 169a^2b + 78ab^2 + 9b^3.$$

$$40. 4x^4y^2 + 2x^3y^3 - 7x^3y^2.$$

$$41. 18at^2 + 48at + 32a. \quad 42. 18a^2c^2 - 57ac^2 + 45c^2.$$

Solve the following quadratic equations by factoring:

$$43. b - 16 + 15b^2 = 0. \quad 44. t^2 = 9 - 8t.$$

$$45. x^2 = 2/3x + 5/3.$$

Contract B. Find the prime factors of:

$$46. 196y^2 - \frac{25}{169}a^4. \quad 47. a^2 - b^2 + 2bc - c^2.$$

$$48. r^2 - a^2 - 2ab - b^2.$$

Solve for the unknown:

$$49. (2x - 1)(x + 2) - (x + 1)(x - 5) = 24 + x^2.$$

$$50. 3(2a - 5)(a + 4) - 6(a + 8)(a - 2) = 9.$$

$$51. (2x + 3)^2 - (4x + 1)(x + 2) = 19.$$

52. The square of a certain number equals the sum of 5 times the given number and 6. What is the number?

Contract A.

53. The base of a certain rectangle exceeds its altitude by 8 inches. If the base and altitude are both decreased by 4 inches, the old area exceeds the new area by 192 square inches. Find its dimensions.

54. The square of the larger of two consecutive integers is 29 more than the square of the smaller. What are the integers?

55. Given, a circular metal plate of radius  $R$ . From this plate 16 circular pieces, each of radius  $r$ , are to be cut.

- a. Write a formula for the remaining area  $A$  of the surface of the plate.
- b. Write in factored form the formula obtained.
- c. Evaluate  $A$  if  $R = 12$  in. and  $r = 2$  in.

## TEST OVER ELEMENTARY ALGEBRA, FIRST SEMESTER

## Part I. True - False

Directions: If the statement is true, mark it plus (+); if false, zero (0).

- \_\_\_ 1. A monomial is an expression consisting of one term.
- \_\_\_ 2. A formula is an equation.
- \_\_\_ 3. We cannot multiply a number by zero.
- \_\_\_ 4. We cannot divide a number by zero.
- \_\_\_ 5.  $(-5)^3$  means 3 times  $(-5)$ .
- \_\_\_ 6. The sum of twice a number and 6 may be expressed as  $2x + 6$ .
- \_\_\_ 7. The root of the equation  $15x + 4 = 11 - 6x$  is  $1/3$ .
- \_\_\_ 8.  $(3x - 2)(x + 4) = 3x^2 + 12x - 8$ .
- \_\_\_ 9. If the same number is added to both sides of an equation, the equality is destroyed.
- \_\_\_ 10. When terms are multiplied, we must multiply the exponents of like letters to obtain the product.
- \_\_\_ 11. The quotient of two negative numbers is always positive.

- \_\_\_\_\_ 12. The sum of  $x$  and  $y$  may be written  $xy$ .
- \_\_\_\_\_ 13. Angles are measured by a small unit called a degree.
- \_\_\_\_\_ 14. Subtract exponents in division.
- \_\_\_\_\_ 15. The quotient is the result obtained by multiplication.
- \_\_\_\_\_ 16. A trinomial is an expression of one term.
- \_\_\_\_\_ 17. A small number written to the right and a little above the number is called the coefficient of the number.
- \_\_\_\_\_ 18. An expression which contains two or more terms joined with plus or minus signs is a polynomial.
- \_\_\_\_\_ 19. An algebraic statement that two quantities are equal is called an equation.
- \_\_\_\_\_ 20. In addition of positive and negative numbers, add like signs and subtract unlike signs.

## Part II. Matching Test

Directions: Write the letter of the correct product in the blank space before the problem.

- |                            |                     |
|----------------------------|---------------------|
| ___ 1. $(3x - 2)(2x - 3)$  | a. $6x^2 + 13x + 6$ |
| ___ 2. $(6x - 1)(x - 6)$   | b. $4x^2 + 12x + 9$ |
| ___ 3. $(2x - 3)(3x - 1)$  | c. $6x^2 - 13x + 6$ |
| ___ 4. $(6x + 1)(x - 3)$   | d. $6x^2 + 19x + 3$ |
| ___ 5. $(x + 3)(4x - 3)$   | e. $4x^2 + 9x - 9$  |
| ___ 6. $(2x - 3)(2x - 3)$  | f. $6x^2 - 17x - 3$ |
| ___ 7. $(3x - 1)(2x + 3)$  | g. $6x^2 - 37x + 6$ |
| ___ 8. $(4x + 1)(x + 6)$   | h. $4x^2 - 9$       |
| ___ 9. $(2x + 3)(2x + 3)$  | i. $4x^2 - 12x + 9$ |
| ___ 10. $(6x + 1)(x + 3)$  | j. $6x^2 - 11x + 3$ |
| ___ 11. $(3x + 2)(2x + 3)$ | k. $4x^2 + 25x + 6$ |
| ___ 12. $(2x - 3)(2x + 3)$ | l. $6x^2 + 7x - 3$  |



## Part III. Completion Test

Directions: In each blank write the word which should be supplied where the corresponding number appears in the incomplete statements.

1.  $(a - b)^2$  means the (1) of the (2) of (3) and (4). (1) \_\_\_\_\_  
(2) \_\_\_\_\_  
(3) \_\_\_\_\_  
(4) \_\_\_\_\_
2.  $(xy)^2$  means the (5) of the (6) of (7) and (8). (5) \_\_\_\_\_  
(6) \_\_\_\_\_  
(7) \_\_\_\_\_  
(8) \_\_\_\_\_
3.  $x^2 + y^2$  means the (9) of the (10) of (11) and (12). (9) \_\_\_\_\_  
(10) \_\_\_\_\_  
(11) \_\_\_\_\_  
(12) \_\_\_\_\_
4. The formula  $V = lwh$  shows that the (13) of a (14) is equal to the (15) of the (16) times the (17) times the (18). (13) \_\_\_\_\_  
(14) \_\_\_\_\_  
(15) \_\_\_\_\_  
(16) \_\_\_\_\_  
(17) \_\_\_\_\_  
(18) \_\_\_\_\_
5. The square of the sum of any two numbers equals the (19) of the first number, (20) two times the (21) of the two numbers, (22) the (23) of the (24) number. (19) \_\_\_\_\_  
(20) \_\_\_\_\_  
(21) \_\_\_\_\_  
(22) \_\_\_\_\_  
(23) \_\_\_\_\_  
(24) \_\_\_\_\_  
(25) \_\_\_\_\_  
(26) \_\_\_\_\_
6. In subtraction, change the sign of the (25) and proceed as in (26). (27) \_\_\_\_\_  
(28) \_\_\_\_\_  
(29) \_\_\_\_\_  
(30) \_\_\_\_\_
7. In adding polynomials, each group of (27) terms should be written in a column.
8. If in solving some special problem, it happens that

two expressions are (28), or represent the same thing, we get an (29). It is a tool to be used in solving problems in which (30) numbers are to be found.

## Part IV

Directions: Write the correct answer to each problem on the line to the right of the problem.

1. If  $3x - 7 = 8$ , find  $x$ . \_\_\_\_\_
2. If  $a = 3$  and  $b = 2$ , find the value of  $3a^2 - 2ab$ . \_\_\_\_\_
3. If  $2x - 7y = 10$ , find  $y$  when  $x = 3$ . \_\_\_\_\_
4. Find the product of  $4a^3$  and  $-9ab^4$ . \_\_\_\_\_
5. Find the sum of  $-7a^2$  and  $4a^2$ . \_\_\_\_\_
6. What is the remainder when  $-8r$  is subtracted from  $-3r$ ? \_\_\_\_\_
7. What is the quotient when  $6a^8$  is divided by  $3a^4$ ? \_\_\_\_\_
8. Express  $x^2 - 11x + 24$  as a product of two factors. \_\_\_\_\_
9. What is the product of  $(3a^2 - 7)$  and  $(a^2 + 2)$ ? \_\_\_\_\_
10. Find the values for  $x$  in  $4x^2 - 11x - 3 = 0$ . \_\_\_\_\_
11. If  $y/2 = 20$ , what does  $y$  equal? \_\_\_\_\_
12. If a man travels  $x$  miles in one

hour, how far does he travel in  $y$  hours? \_\_\_\_\_

13. How many truck loads of  $R$  tons each are contained in a car load of  $T$  tons of coal? \_\_\_\_\_

14. If  $bx - c = a$ , then  $x$  equals \_\_\_\_\_

15. If  $i = E/R$ , then  $R$  equals \_\_\_\_\_

16. Subtract  $3x^3 - 7x^2 + 8$  from  $x^3 + 2x^2 - 6x + 3$ . \_\_\_\_\_

17. Add  $2x^2y + 7y^3, -3x^3 + xy^2, 3x^3 + x^2y - y^3$ . \_\_\_\_\_

18. Divide  $a^3 - b^3$  by  $a - b$ . \_\_\_\_\_

19. Solve for  $x$  in  $2x - 7(3 - x) = x + 8$ . \_\_\_\_\_

20. Combine  $3x - 7y - x - 7 - 3y + 7x + 4$ . \_\_\_\_\_

21. Express as a formula the area  $A$  of a right triangle whose altitude is  $a$  and base  $b$ . \_\_\_\_\_

22. If 5 pencils cost  $y$  cents, what will 2 cost? \_\_\_\_\_

23. If the first number is  $x$  and the second is  $y$ , represent the square of the sum of the two numbers. \_\_\_\_\_

24. If  $p$  pounds of copper cost  $t$  \_\_\_\_\_

dollars, how many cents per pound did it cost? \_\_\_\_\_

25. The greater of two numbers is 5 times the smaller. The sum of the two numbers is 144. Write the equation. \_\_\_\_\_

26. The result obtained by subtracting 24 from 7 times a number is 25. Find the number. \_\_\_\_\_

27. There are three consecutive numbers whose sum is 126. Find them.

Equa. (1) \_\_\_\_\_ . Ans. (2) \_\_\_\_\_

28. Two angles A and B are complementary. Angle A is 21 degrees smaller than angle B. How large is each angle?

Equa. (1) \_\_\_\_\_ . Ans. (2) \_\_\_\_\_

29. A grocer has some 40¢ tea and some 90¢ tea. How many lbs. of each must he take to form a mixture of 100 lbs. which he can sell for 60¢ a lb.?

Equa. (1) \_\_\_\_\_ . Ans. (2) \_\_\_\_\_

30. The age of A is 4 times the age of B. Four years from now A's will be one year more than 3 times B's age then. What are their present ages?

Equa. (1) \_\_\_\_\_ . Ans. (2) \_\_\_\_\_

## APPENDIX

## Key for Pre-test for Unit II

- |  |              |
|--|--------------|
| 1. $12x + 2$ .   | 2. $50x$ .   |
| 3. 5 more than $x$ ; 5 times $x$ .                                   |              |
| 4. $A = 42$ .  | 5. 48 sq.in. |
| 6. (a) $x^2$ ; (b) $bh$ or $ab$ ; (c) $\pi r^2$ ; (d) $bh$ or $ab$ . |              |

## Key for Mastery Test on Unit II

Contract D.

- |  |                               |
|--|-------------------------------|
| 1. $5n + 2$ .  | 2. $lw/2$ .                   |
| 3. $x/y$ .   | 4. $n/2 = 6$ .                |
| 5. $y/7$ .   | 6. $4 + n$ .                  |
| 7. $A = 15$ .  | 8. $h = 8$ .                  |
| 9. $V = 84$ .  | 10. $w = 5$ .                 |
| 11. $r = 40$ .   | 12. $A = 10$ sq.ft.           |
| 13. $p = 12x$ .  |                               |
| 14. 3 times $y$ ; 5 more than $y$ ; 5 diminished by $y$ ; $1/6$ of $y$ . |                               |
| 15. 87.  | 16. $l = A/w$ ; $w = A/l$ .   |
| 17. $r = 1/pt$ ; $t = 1/pr$ .  | 18. $h = V/lw$ ; $w = V/ln$ . |

Contract C.

19. 200 sq.ft.
20.  $S = 2lh + 2wk + 2lw$ .  $V = lwh$ .

Contract C.

21.  $c = 100d + 25q + 5n$ .      22. 384 cu.in., 256 sq.in.

Contract B.

23. 48 gal.  $S = 3476$  sq.in.

24. 216 to 27, or one is 8 times the volume of the other.

216 to 54, or the area of one is 4 times that of the other.

25. 40; 72; 96; 120; 200.

Contract A.

26.  $c = 6x + 2$ .

28. 3795 sq.ft.

29. (b)  $p = 24$ ; (c)  $S = 10$ ; (d) increases.

Evaluation of problems: Total score 75 points.

Ex. 1 - 6 ----- 1 point each.

7 - 11, 14, 17, 18, 19, 20, 21, 26 --- 2 points each.

12, 13, 16 ----- 3 points each.

22, 23, 24, 29 ----- 4 points each.

15, 25, 27, 28 ----- 5 points each.

## Key for Pre-test for Unit III

1. (a) 6a; (b) 2; (c) 2; (d) divide both sides by 6.

2. 7 more than  $x$ ; 7 times  $x$ ; 7 diminished by  $x$ ;  $x$  divided by 7.3. Add 3; subtract 8; multiply by  $x$ .4. (a)  $x = 10$ ; (b)  $a = 5$ ; (c)  $y = 9$ ; (d)  $m = 5$ .

## 5. Similar terms.

## Key for Mastery Test for Unit III

Contract D.

1.  $x + 8$ . 2.  $x - 8$ . 3.  $8x$ . 4.  $n - 7$ . 5.  $c = T/n$ .  
 6.  $s = -d$ . 7.  $6 - 2n$ . 8.  $D = 7x + 5$ .  
 9.  $3n - 9 = 27$ . 10.  $11x - 4 = 7$ . 11.  $8 + 5x = 43$ .  
 12. Yes. 13. No. 14. Add 2; subtract 4; multiply by 3.  
 15.  $t = 5 \frac{1}{3}$ . 16.  $m = 30$ . 17.  $x = 14$ . 18.  $x = 18$ .  
 19.  $x = 30$ . 20.  $x = 12$ . 21.  $62 \frac{1}{2}$  degrees.  
 22. 23. 69.

Contract C.

23. Henry's, \$25; John's, \$100. 24. Width, 300; length, 600.  
 25. 3, 6, 12.

Contract B.

26.  $x = 16$ . 27.  $x = 20$ . 28.  $x = 10$ . 29.  $x = 4$ .  
 30.  $x = 4$ . 31.  $y = 3 \frac{2}{5}$ . 32. helper, \$2.50;  
 plumber, \$10. 33.  $\frac{1}{4}$ ,  $\frac{3}{4}$ . 34. 15.

Contract A.

35. 5, 30. 36. 5, 7, 8. 37. 28, 12, 18.

Evaluation of problems: Total score 74 points.

- Ex. 1 - 8, 15 - 20, 26 - 31 ----- 1 point each.  
 9 - 13 ----- 2 points each.  
 14, 21, 25, 36 ----- 3 points each.  
 22, 23, 24, 32 - 36, 37 ----- 4 points each.



## Key for Pre-test on Unit IV

1. +7 degrees; -7 degrees.
2. -1, +7, -11, 9, -3, 0, -2, 10,  $-12a^2$ ,  $11x$ .
3. -7, -5,  $-2x$ , 5, 19, 17, -23, 6, 0,  $-2y$ .
4. -6, 28,  $14x^2$ , -27,  $-96xy$ ,  $-40a^4$ , 0, 0.
5. -9, -3, 0, impossible. 3, 3,  $-5x$ ,  $-3x^2y$ .

## Key for Mastery Test on Unit IV - A and B

Contract D.

1. 7, -5, -12,  $10x$ , -2,  $-6x$ , 0,  $-2y$ ,  $2r$ ,  $5x$ .
2. 12, -17, 0, 8,  $x$ , 0,  $5y$ ,  $10r$ ,  $-16y$ ,  $13y$ .
3. (a) Terms or monomials; (b) positive term; (c) negative term; (d) 1.
4.  $12ab^2$ . 5.  $2x + 3y$ . 6.  $-x^2 + 8xy - 15$ .
7.  $3x^2 - 3xy + 8y^2$ . 8.  $-3y + 5z$ . 9.  $8r^2 + st - 7t^2$ .
10.  $4a + 9b + 2c$ . 11.  $12r + 3s - 2t$ . 12.  $x = 9$ .
13.  $y = 6$ . 14.  $x = 11$ . 15.  $x = 3$ . 16.  $y = 8$ .
17. 165 degrees.

Contract C.

18.  $-6x^2 + 19$ . 19.  $8r^2 + 4rs + 7s^2$ . 20.  $-20e + 4$ .
21.  $5a - 8b + 7c$ . 22.  $3y^2 - yz - 3z^2$ . 23.  $x = 1$ .
24.  $x = 9$ . 25. 37, 38, 39. 26. 8.

Contract B.

27.  $b = 7$ . 28.  $x = \frac{c - a}{3}$ . 29. 69 degrees.

Contract A.

30. -35. 31. 47. 32. -21. 33. 3. 24.

Evaluation of problems: Total score 112 points.

Ex. 1 and 2 ----- 10 points each.

18, 30, 31, 32 ----- 2 points each.

4 - 10, 11, 12, 13, 14, 17, 20, 21.

25, 26, 29, 33 ----- 3 points each.

3, 15, 16, 23, 24, 27, 28 ----- 4 points each.

19 and 22 ----- 6 points each.

## Key for Mastery Test on Unit IV

Contract D.

1.  $-x + 3z$ ;  $-13x + 6y + 5z$ .

2.  $-r^2 - 2rs + s^2$ ;  $3r^2 - 4rs + s^2$ .

3.  $x - y$ . 4.  $2 - y$ . 5.  $x = 5$ . 6.  $x = -2$ .

7. -21; -5; 0; 0;  $12x^3$ ;  $3x^2y^2$ ;  $-3x^3y^3$ ;  $27xy^6$ .

8. 9; 5; 10;  $-1/2$ ; -9;  $y$ ;  $a^2b$ ;  $3x$ ; 5;  $-5t$ ;  $s$ ;  $2y$ ;  $-a$ ;  
 $-4y$ ;  $-8mn^2$ .

9.  $a - 2b$ . 10.  $m^2 - 3m + 2$ . 11.  $-14x^3y^{11}$ . 12. -1.

13. -6. 14.  $10m^3 - 8m^2 + 6m$ . 15.  $x^2 - 3x + 4$ .

16.  $y + 5$ . 17.  $y^3 + 11y^2 + 39y + 45$ .

18.  $12r^2 - 45rs - 12s^2$ . 19.  $6x^3 + 11x^2 - 39x - 14$ .

20.  $5x^2 - 3xy + y^2$ . 21.  $3r^2 - r + 2$ . 22.  $x = 3$ .

23.  $m = 24$ . 24.  $10x^2 - 3xy - y^2$  sq.in.

Contract C.

25.  $16 - 8x^2 + 4x^3 - x^4$ . 26.  $a^2 + 2a + 4$ .

27.  $9b^2 - 3b + 1$ . 28.  $x = 48$ . 29. A, 20 yrs.; B, 5 yrs.

30.  $5x + 10x = 450$ . 31.  $80 \cdot 4 = 90x$ .

Contract B.

32.  $x = 4$ . 33. 12, 14, 16. 34.  $\$1500$  at 7%;  $\$500$  at 6%.

35.  $40x = 25(x + 4)$ . 36.  $25x + 10(18 - x) = 225$ , or

$10x + 25(18 - x) = 225$ .

Contract A.

37.  $2x^3 - (y^2 + 2y + 1)$ . 38. Dimes, 10; quarters, 8;

nickels, 20. 39. 2-cent stamps, 30; 3-cent stamps, 15.

40.  $60(x + 20) + 75x = 4575$ .

Evaluation of problems: Total score 133 points.

Ex. 1 - 3 ----- 3 points each.

4, 5, 11 - 21, 27, 28 ----- 2 points each.

6, 7, 34 - 40 ----- 3 points each.

8, 22 - 26, 29 - 33 ----- 4 points each.

9 ----- 8 points.

10 ----- 15 points.

## Key for Pre-test for Unit V

1.  $3ax + 5ay - az.$

$2a^2 + 5a - 7.$

$x^2 - 2xy + y^2.$

$a^2 - b^2.$

$4a^2 + 28a + 49.$

$2r^2x - 5rx - 27x.$

2.  $7(r - t).$

$(a - b)(a - b).$

$(2a + 7)(a - 3).$

$(5x - y)(5x + y).$

$(3m + 2n)(3m + 2n).$

$y(3x + 4)(x - 5).$

3.  $a = 3$  or  $2.$

$a = 5$  or  $-5.$

## Key for Mastery Test for Unit V

Contract D.

1.  $9x^2.$  2.  $343y^6.$  3.  $4x^2y^6.$  4.  $-24ay^6.$

5.  $3ar + 3as - 3at.$  6.  $-2ay + 2by - 2cy.$

7.  $a^3 - 3a^2 - a.$  8.  $9x^2 - b^2.$  9.  $x^2 + 2xy + y^2.$

10.  $2y^2 + 9y - 56.$  11.  $a^2b^2 - 2abc + c^2.$

12.  $16d^6 - 24d + 9.$  13.  $3x^2 - x - 2.$  14.  $x^2 - y^2.$

15.  $x(3 - 7y)$ . 16.  $2x(a + 4b - 2c)$ . 17.  $5(x - y)$ .  
 18.  $2r(-r^2x + 2ry - 3z)$ . 19.  $\pi(x + y)(x - y)$ .  
 20.  $(2e + 1)(2e - 1)$ . 21.  $(m - 11n)(m + 4n)$ .  
 22.  $(5y + 3x)(5y - 3x)$ . 23.  $(xy - 2)(xy + 3)$ .  
 24.  $(2x - 3y)(2x - 3y)$ . 25.  $(2x + y)(2x + y)$ .  
 26.  $(3a - 2)(2a - 3)$ . 27.  $x = -5/3, -1$ .  
 28.  $a = 1/6, 1$ .

Contract C.

29.  $4a^2b + 28ab + 49b$ . 30.  $35m^4 - 29m^2n + 6n^2$ .  
 31.  $-6m^3 - 13m^2 - 6m$ . 32.  $24x^4 - 31x^2y + 10y^2$ .  
 33.  $ac + bc + ad + bd$ . 34.  $6x^3 + 3xy - 14x^2 - 7y$ .  
 35.  $(1 + 3y^2)(1 - 3y^2)$ . 36.  $(4a^2 + 1)(2a + 1)(2a - 1)$ .  
 37.  $3(2m^2 - 1)(m^2 + 2)$ . 38.  $(a/2 - 4)(a/2 - 4)$ .  
 39.  $b(13a + 3b)(13a + 3b)$ . 40.  $x^3y^2(4x + 2y - 7)$ .  
 41.  $2a(3t + 4)(3t + 4)$ . 42.  $3c^2(3a - 5)(2a - 3)$ .  
 43.  $b = -16/15, 1$ . 44.  $t = -9, 1$ . 45.  $x = 5/3, -1$ .

Contract B.

46.  $(14y + 5/13a^2)(14y - 5/13a^2)$ .  
 47.  $a + (b - c)$   $a - (b - c)$ .  
 48.  $r + (a + b)$   $r - (a + b)$ .  
 49.  $x = 3$ . 50.  $a = 1$ . 51.  $x = 4$ . 52.  $6, -1$ .

Contract A.

53. 22, 30. 54. 14, 15. 55. (a)  $A = \pi R^2 - 16\pi r^2$ .  
 (b)  $A = \pi(R + 4r)(R - 4r)$ . (c)  $A = 251 \frac{3}{7}$  sq.in.

Evaluation of problems: Total score 172 points .

Ex. 1 - 7 -----	1 point each.
8 - 48 -----	3 points each.
49 - 55 -----	6 points each.

### Key for Semester Test

#### Part I. True - False

1. +.	8. 0.	15. 0.
2. +.	9. 0.	16. 0.
3. 0.	10. 0.	17. 0.
4. +.	11. +.	18. +.
5. 0.	12. 0.	19. +.
6. +.	13. +.	20. 0.
7. +.	14. +.	

#### Part II. Matching Test

1. c.	5. e.	9. b.
2. g.	6. i.	10. d.
3. j.	7. <i>l</i> .	11. a.
4. f.	8. k.	12. h.

## Part III

- |                |              |                      |
|----------------|--------------|----------------------|
| 1. square.     | 11. x.       | 21. product.         |
| 2. difference. | 12. y.       | 22. plus.            |
| 3. a.          | 13. volume.  | 23. square.          |
| 4. b.          | 14. solid.   | 24. second.          |
| 5. square.     | 15. product. | 25. subtrahend.      |
| 6. product.    | 16. length.  | 26. addition.        |
| 7. x.          | 17. width.   | 27. like or similar. |
| 8. y.          | 18. height.  | 28. equal.           |
| 9. sum.        | 19. square.  | 29. equation.        |
| 10. squares.   | 20. plus.    | 30. unknown.         |

## Part IV

1.  $x = 5$ . 2. 15. 3.  $y = -4/7$ . 4.  $-36a^4b^4$ . 5.  $-3a^2$ .  
 6.  $5x$ . 7.  $2a^4$ . 8.  $(x - 8)(x - 3)$ . 9.  $3a^4 - a^2 - 14$ .  
 10.  $-1/4$ , 3. 11.  $y = 40$ . 12.  $xy$ . 13. T/R.  
 14.  $\frac{a+c}{b}$ . 15.  $E/1$ . 16.  $-2x^3 + 9x^2 - 6x - 5$ .  
 17.  $3x^2y + xy^2 + 6y^3$ . 18.  $a^2 + ab + b^2$ . 19.  $x = 3 \frac{5}{8}$ .  
 20.  $9x - 10y - 3$ . 21.  $A = ab/2$ . 22.  $2y/5$ .  
 23.  $(x + y)^2$ . 24.  $100t/p$ . 25.  $x + 5x = 144$ . 26. 7.  
 27. 41, 42, 43. 28.  $A = 34 \frac{1}{2}$ ,  $B = 55 \frac{1}{2}$ .  
 29. 40 lbs. at 90 cents; 60 lbs. at 40¢. 30.  $A = 36$ ;  
 $B = 9$ .

Evaluation of problems: Total score 71 points.

Score for Part I = No. right - No. wrong.

Score for Part II = No. right.

Score for Part III = No. right.

Score for Part IV = Ex. 1 - 26 ----- 1 point each.

27 - 30 ----- 1 point each

correct equation

and answer.



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