FREQUENCY CHARACTERISTICS

OF A

STOPPED ORGAN PIPE

by

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B. S., Kansas State Teachers' College, Pittsburg, 1926

A THESIS

submitted in partial fulfillment of the requirements

for the degree of

MASTER OF SCIENCE

KANSAS STATE AGRICULTURAL COLLEGE

1928

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INTRODUCTION

This research began with an attempt to calibrate a variable pitch organ pipe for use as a source of sound vibrations in the study of resonators. It was hoped that by means of such a pipe any vibration frequency between 256 v.p.s. and 520 v.p.s. might be obtained with a certainty of $\pm$ .1 v.p.s. As the work progressed it was found that several factors affecting the frequency of such a source had to be eliminated, or measured and corrected. These included length, temperature, pressure, and mouth opening. The first was the means whereby the variations of frequency were to be obtained; the last could be eliminated by making the mouth of such size and shape that the fundamental tone would speak over the octave range. Temperature and pressure, however, seemed to be more difficult to control, consequently it was decided to determine their effects both for this purpose, and to check up on previous determinations of these factors. As the work developed it was found that the correction to length due to the mouth opening could be computed very easily from the same data.

Among those who have assisted in making this work successful I am especially indebted to Professor E. V. Floyd, who directed my research, and rendered me valuable assistance in various ways. The use of the chromatic octave set of tuning forks was made possible by the generosity of Dr. F. E. Kester of Kansas University. I am also grateful to Professor J. O. Hamilton and to several other members of the physics department for suggestions and assistance in various matters.

APPARATUS: DESCRIPTION AND OPERATION.

The apparatus and equipment used in this work, drawings and diagrams of which appear on subsequent pages, consisted of a fan driven by an A.C. motor, an air pressure regulating tank, an adjustable, stopped organ pipe, a phonelescope, an adjustable resonator, and a chromatic octave set of Koenig tuning forks. The forks gave vibration frequencies corresponding to the Bach equitempered scale, A-435 Ut_3-258.57 at 21.5° C., Ut_4-517.15 at 21.5° C. The frequency of Ut_3 was determined by comparison with a fork of 256 v.p.s. which had recently been calibrated by the Bureau of Standards. A-435 was determined by comparison with an orchestral bell of 440 v.p.s.

The effective length of the pipe could be adjusted by means of a threaded piston rod connected through suitable mechanism to a hand wheel within easy reach of the operator. The distance from base block to piston face was read from a scale and vernier attached to the pipe. A thermometer was kept with its bulb in the air stream below the mouth of the pipe to indicate the temperature. A manometer in communication with the same cavity indicated the pressure, and was read by means of a cathetometer whose accuracy was checked

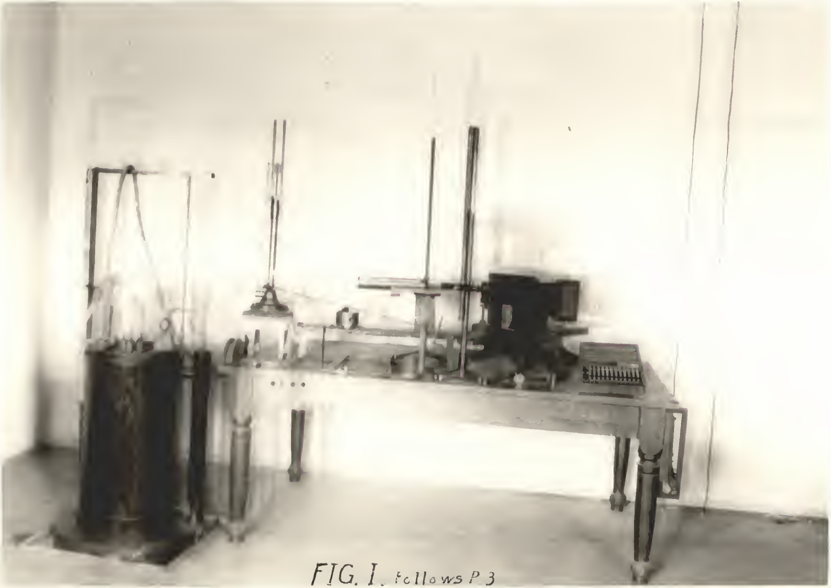


FIG. I. Follows P. 3

FLOOR PLAN SKETCH OF

APPARATUS.

1. Motor.
2. Fan.
3. Pressure regulating tank.
4. Adjustable organ pipe.
5. Tuning fork.
6. Adjustable resonator.
7. Phonelescope.
8. Light tube with ground glass screen at end near 9.
9. Operator's position.
10. Cathetometer.
11. Hand wheel for adjusting ipe.
12. Incandescent lamp.
13. Slit.

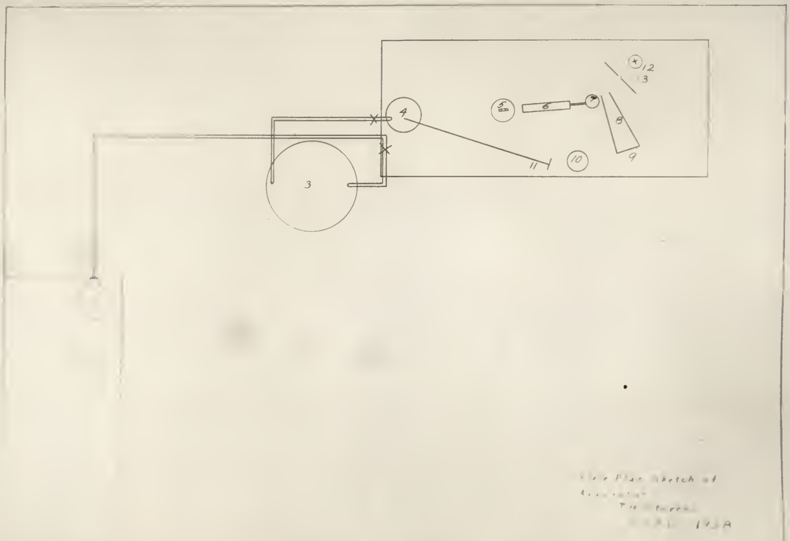
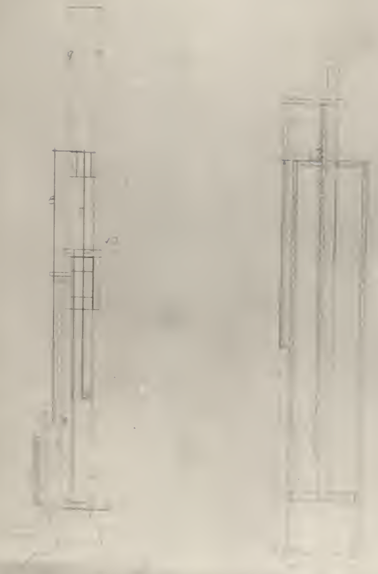


FIG. II _ Follows P. 3A.

DETAILS OF VERNIER ORGAN PIPE.

1. Air inlet.
2. Mouth of pipe.
3. Piston.
4. Adjusting Rod.
5. Metric scale for indicating pipe length.
6. Vernier for reading scale.
7. Piston rod, threaded.
8. Rod for attaching vernier to piston.
9. Guide rods for upper end of piston rod.
10. Gears for moving piston.
11. Leveling screw.
12. Manometer.
13. Thermometer.



Adjustable Resonator

Vernier Pipe

FIG. III. To Follow P 3B

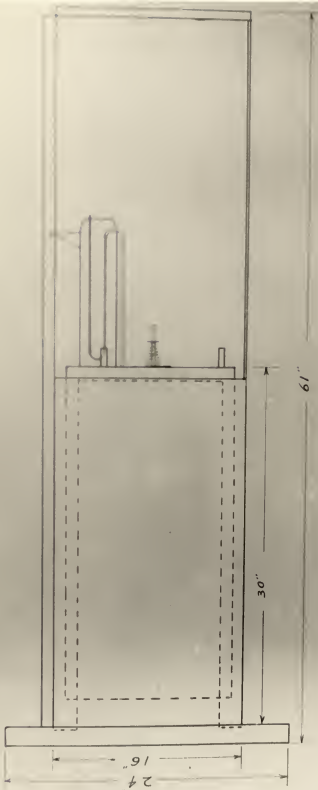


FIG. IV

AIR SUPPLY TANK
Scale

as $\pm$ .05 mm. In the early part of the work a manometer tube of about three mm. bore was used, but it was found to be unreliable due to capillary action and was finally replaced by a tube of about seven mm. diameter, and this in turn by one of ten mm. diameter.

The adjustable resonator was made in the physics department shop by Professor Floyd and myself. It consisted of a piece of brass pipe two inches in diameter and about ten inches in length, partially closed at one end by a brass disc bored five-eights inch diameter. Within the pipe a piston moved on the end of a small pipe that served to transmit the sound energy to the phonelescope and to carry a rack, enabling it to be moved by means of a pinion. When the forks were in use they were held in a lathe chuck of about nine pounds weight.

As stated in the introduction, the first consideration was that of determining the length-frequency characteristic of the pipe. To determine the length of the pipe needed to give a certain frequency, I set the proper fork directly in front of the mouth of the resonator as shown in Figs. 1 and 2. Then I adjusted the resonator so that when the fork was vibrating it gave a maximum displacement of the light on the screen. Then, with the fork vibrating and the pipe speaking I adjusted the pipe length by turning the hand wheel until the phonelescope showed no beats more rapid than one in ten or more seconds. The frequency was found from a table of frequencies for the Bach A-435 scale, and the corresponding pipe length read from the scale and vernier to tenths of a millimeter. This operation was repeated for each tone in the chromatic octave, and the length-frequency characteristic curve was plotted from the data so obtained. Throughout this determination the temperature was kept between 21° and 22°C., and the pressure was nearly 4.52 cm. water as could be read with the cathetometer.

Two of the above described determinations were run at temperatures of 28°C. and 20°C. respectively in an effort to obtain the temperature-frequency coefficient of the pipe. They were run before the adoption of the water manometer and cathetometer for measuring pressures, so that they cannot be taken as wholly reliable. Later, in another part of my work I was able to check up on this factor in another way which I consider more accurate.

During the time that I was trying to make the above described determinations, I noticed that fluctuations of pressure due to faulty operation of the fan caused appreciable variations in the frequency of the pipe. In fact, this led to the use of the cathetometer. I happened one day to hear a closed tube blown as a whistle, and thought I detected a reduction of pitch with increased wind force. At about the same time I read Rayleigh's comment to the effect that increased wind pressure invariably raises the pitch of an organ pipe. As a result, I attacked the problem of finding out if there were any condition under which increased pressure would result in a lowering of the pitch.

For this purpose I set the pipe in approximate unison with each of the forks in succession, then varied the pressure from the lowest that would cause the pipe to speak its fundamental tone (lower critical pressure) to the highest pressure that could be obtained, or that would permit the fundamental to speak (upper critical pressure). At various pressures within this range I determined the frequency of the pipe's tone by counting the beats between it and the fork. In this part of my work I found the visual method of counting beats the better near unison, and the auditory method better for more rapid beats. In so far as possible I used the two methods as a check upon each other.

From the data of the immediately preceding determination it is possible to compute the mouth correction for any condition of pressure and frequency contained therein, by the equation,

$$c = \frac{V}{4N} - L$$

in which c is the correction in centimeters of pipe length;

V is the velocity of sound in free air at the prevailing temperature; N is the frequency of vibration; and L is the measured length of the pipe.

The pressure-frequency characteristic curves plotted from the first list of data were very rough if made to pass through the points as read. I felt sure of the frequencies, for they were carefully checked to a much closer approximation than the distances of the points from the approximation curve indicate. The manometer then came up for criticism, and one of the larger bore was substituted to see what its effect upon the regularity of the curves would be. It was an improvement, but capillarity seemed to have considerable effect in it, so an even larger one was used in checking some of the data.

REFERENCES TO LITERATURE.

Previous to this time very little work seems to have been done on the above mentioned subjects. Most text-books of physics give the simple inverse ratio for the length-frequency characteristic. For example, Weld and Palmer (Text-book of Modern Physics, page 362) say:

"Since the distance from a loop to a node is one-quarter of a wave-length, the length of a closed organ pipe is one-quarter of the wave length of its fundamental tone; hence the fundamental of a short pipe is of higher pitch than the fundamental of a long one."

The mouth correction is discussed as follows by Lord Rayleigh (Theory of Sound, p. 218, Vol. 2, Ed. 2.)

"We have seen how to take account of an upper open end, but according to the rule of Cavaille-Coll the whole addition which must be made to the measured length of an open pipe in order to bring about agreement with the simple formula, t* $= \frac{L}{a} = \frac{2\,l}{a}$, amounts to as much as 3 1/3 R, very much greater than the correction (1.2 R) necessary for a simple tube of circular section open at both ends. This discrepancy is sometimes attributed to the blast. But it must be remembered that the lower end is very much less open than the upper end, and that if a sensible correction on account of deficient openness is required of the latter, a much more important correction will probably be necessary for the former. Observations by the author have shown this to be the case. - - - - The considerable correction to length found by Cavaille-Coll is not attributable to the blast, but to the contracted character of the lower end treated as open in the elementary theory. - - - - The rise of pitch due to the

* t = time of 1 vibration; L = wave length; l = pipe length; a = velocity of sound.

wind increases with pressure. Thus, in the case referred to above, the pipe under a pressure of 2.7 cm. of water gave a note about 2 vibrations per second sharper than that of the fork, but when the wind pressure was raised to 10.7 cm. the excess was as much as 11 vibrations per second. When the pressure was raised much farther, the pipe was overblown and gave the octave of its proper pitch. This, of course, corresponds to another mode of vibration of the aerial column."

The following is in regard to the effects of wind pressure and temperature on organ pipe frequencies as given by Cavaille-Coll (Helmholtz', Sensations of Tone, p. 89, Ed. 4)

"As to strength of wind, as pressure varies from 2 3/4 inches to 3 1/4 inches, the pitch number increases by about 1 in 300, but as pressure varies from 3 1/4 inches to 4 inches, the pitch number increases by about 1 in 440, the whole increase of pressure from 2 3/4 to 4 inches increases the pitch number by 1 in 180.

For temperature, I found by numberous observations at very different temperatures that the following practical rule is sufficient for reducing the pitch number observed at a given temperature, to that due to another. It is not quite accurate, for the air blown from the bellows is often lower than the external temperature. Let P be the pitch number observed at a given temperature, and d the difference of temperature, in degrees F. Then the pitch number is $P(1 \pm .00104\, d)$ according as the temperature is higher or lower. The practical operation is as follows: supposing $P = 528$ and $d = 14$ increase of temperature. To 528 add 4 in 100 or 21.12, giving 549.12. Divide by 1000 to two places of decimals, giving .55. Multiply by $d = 14$, giving 7.70. Adding this to 528, we get 535.7 for the pitch number at the new temperature."

LENGTH-FREQUENCY CHARACTERISTIC.

The accompanying data and the graph thereof show the true relationship between the length of a stopped organ pipe, at constant temperature and pressure, and the frequency of its prime tone. The important points to be observed are, (1) that the relationship is not linear, but approximates in the graph, an arc of a conic, perhaps an hyperbola. (2) The length of the pipe needed for C_4 is less than half that for C3. As will be shown later in this work, the mouth correction for the shorter position is greater than for the longer. Hence, a shorter actual length is required, as the addition for the total effective length brings it to the proper value. For example, the measured length for C3 is 30.87 cm.; the mouth correction is 2.45 cm.

effective length 33.32 cm. For C4 the effective length is 13.9 - 2.94 = 16.84. 2 x 16.84 = 33.68, which gives the length for the lower tone within 2.5 v.p.s. This includes temperature change and experimental error, and is of the order of 1% of the frequency range over which the calculation is made.

Calibration Data.

Tuning fork.		Pipe length, 21.5°C.
Tone	Frequency, 21.5°C.	
C3	258.57	31.40 cm.
C#	273.94	29.35 "
D	290.23	27.49 "
D#	307.49	25.74 "
E	325.75	24.00 "
F	345.13	22.38 "
F#	365.68	20.90 "
G	387.42	19.55 "
G#	410.44	18.20 "
A	434.85	16.74 "
A#	460.70	15.60 "
B	488.11	14.49 "
C	517.15	13.47 "

The graphical representation of these points appears on the next page.

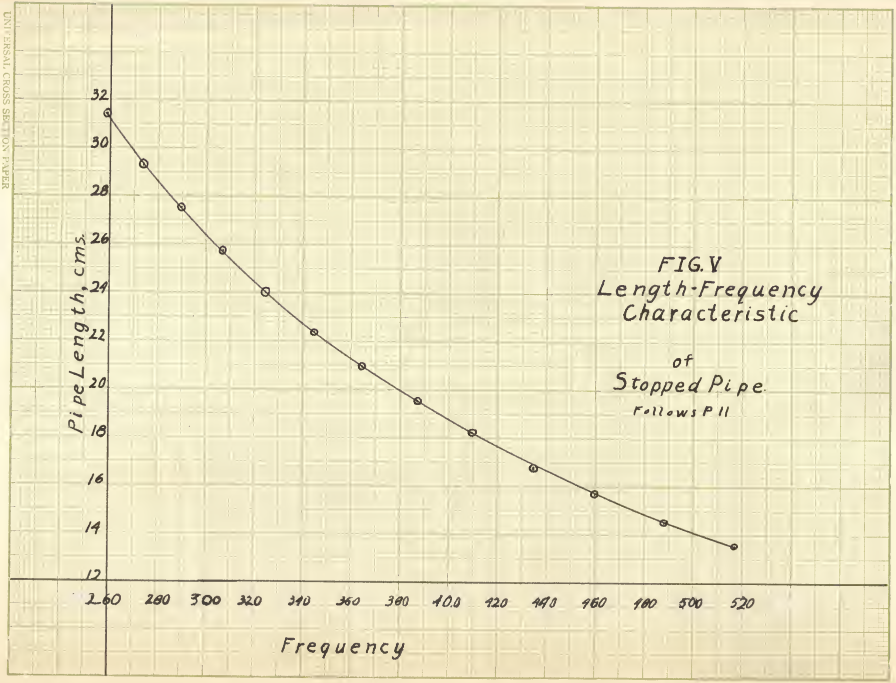


FIG. V
Length-Frequency
Characteristic
of
Stopped Pipe
Follows P 11

TEMPERATURE-FREQUENCY COEFFICIENT.

In order to determine the relation between temperature and frequency of the pipe, I made two length-frequency determinations at temperatures of approximately 28°C. and 20°C. respectively, keeping the pressure practically constant in both cases. This was done before the adoption of the cathetometer for reading pressures, so that its value is only approximate and it serves to give by its average, an indication of the true coefficient. The following data shows the method of arriving at the probable value for the temperature-frequency coefficient.

Data.

$\theta_1^{o} - \theta_2^{o}$	$L_1 - L_2$	V.p.s. per mm. length	Tone	$\frac{1}{N}\frac{\Delta N}{\Delta\theta}$
7.6°C.	.41 cm.	.7	C3	.00146
7.6 "	.26 "	.75	C#	.00936
7.6 "	.26 "	.9	D	.00106
7. "	.17 "	1.0	D#	.00789
5.6 "	.31 "	1.1	E	.00187
5.3 "	.23 "	1.25	F	.00157
6.6 "	.23 "	1.45	F#	.00138
7.7 "	.24 "	1.65	G	.00132
7.2 "	.24 "	1.75	G#	.00141
7.4 "	.21 "	1.95	A	.00127
6.6 "	.25 "	2.25	A#	.00184
6.2 "	.23 "	2.7	B	.00205
---	--	2.8	C	-----
		Mean - - - - - -		.0027
		Cavaille-Coll's value		.00188

C# and D# are of entirely different order from the remainder of the values, hence are probably errors. Leaving them out, the mean of the other ten is .00152.

Typical Calculation.

$$\frac{1}{N}\ \frac{\Delta N}{\Delta \theta} = \frac{10(L_1 - L_2)\ \text{v.p.s. per mm.}}{N(\theta_1 - \theta_2)}$$

For C3 this becomes,

$$\frac{4.1 \times .7}{7.6 \times 258.6} = .00146$$

In this case, ΔN represents a difference of one vibration per second in frequency, and $\Delta\theta$ a temperature change of one degree C. Reference to the pressure-frequency curves taken at various temperatures will serve as a check upon the work, and will be discussed later.

PRESSURE-FREQUENCY COEFFICIENT.

Many things may be considered under this head. The rate of change of frequency with change of pressure is the outstanding feature. Pressure above which the tome is uncertain and not good for musical purposes is perhaps of chief importance to musicians and instrument makers. Upper and lower critical pressures for the fundamental tone of the pipe is also of interest. These will now be discussed in the order given above.

As may be seen by examining the accompanying curves, the frequency rises rapidly with increase of pressure over about the lower one-half or one-third of the total pressure range. It then reaches a point at which the frequency is practically constant over a considerable increase of pressure, then in a few instances decreases slightly, to rise again before changing over to the upper partial. This is in contradistinction to the statements of Rayleigh and Cavaille Coll that frequency rises with pressure.

Due to the lack of facilities for maintaining constant pressure above 10 centimeters of water, all but two or three of the curves are incomplete, and I have indicated by dotted lines the curves as they will probably be found when this work is supplemented and completed. My prediction is that all of the curves will be found to have the general shape of those given for $C^{\#}$ and C3, the difference being that the bend of the curve will come at a higher pressure for each successive semitone in the octave, and the pressure range of slight frequency variation will increase for the higher notes of the octave. I also expect to find that each of the partials follows a similar type of variation in this respect.

On the C_3 graph, (next page) I have plotted three curves as follows: (1) from data taken at 21°C. using the original small bore menometer; (2) from data taken at 24.2°C. with a manometer of 7 mm. bore; (3) from data taken at 20.5°C. with a 1 cm. diameter manometer.

It may be seen that the critical pressures correspond very closely in the three determinations. Due to the method used for finding them, the values given by (3) are perhaps the more reliable. Also, there is little doubt that curve (3) represents more accurately than the others the behavior of the pipe in this range.

Examination of (3) shows that near the lower critical pressure the frequency rises rather slowly, but between 3 and 4 cms. of water, the frequency rises very rapidly. Between 4 and 5 cms. it increases at the rate of about one vibration per second per cm. water pressure.

To compute the pressure-frequency coefficient within any range one may use the expression

$$\text{Coefficient} = \frac{\Delta N}{N \cdot \Delta P}$$

where N is the frequency, ΔN is the frequency change for 1 cm. change in pressure, and ΔP is 1 cm. of water change in pressure. In the above case it becomes

$$\frac{1}{258.6 \times 1} = .00386$$

That is, between 4 and 5 cms. pressure the frequency changes at the average rate of about .00386 times itself per one cm. change in pressure. If the pressure-frequency characteristic were a straight line with the slope $\frac{\Delta N}{\Delta P}$, the frequency at any pressure would be

$$N = 258.6(1 \pm .00386(4.5 \pm P_2)$$

where 4.5 is the pressure giving 258.6 and P_2 is any other pressure. In view of the fact that the curve is not a straight line, the above cannot be used, but instead the coefficient must be calculated, if desired, at the point under consideration on the curve.

The vertical lines at the right and left extremities of the curve are the upper and lower critical pressure lines, respectively. That is, the fundamental tone of the pipe speaks only at pressures indicated between them.

The vertical broken line at 5 cm. pressure is perhaps of as much interest and more value than the critical pressure lines. Near that part of the curve, the tone of the pipe gave strong evidence of the presence of upper partials, and above that point the tone became unsteady, or what musicians term a split tone. The actual frequency varied within a few seconds between .5 or 1 v.p.s. above the fork, to as much as 3 or 4 v.p.s. above. This would indicate that it would be inadvisable for several reasons to try to use this pipe above 5 cm. pressure as a musical instrument, especially in a sustained chord. A serious difficulty that arises in this connection is due to the fact that the loudness of a wind instrument tone is changed by changing the air pressure with which the pipe is blown. Unless all the instruments of the (supposedly) same pitch have the same pressure-frequency coefficient, serious dissonances are sure to arise in sustained chords unless some means is employed to compensate this change in frequency. Every wind instrument except the organ may be thus compensated, so the organ must be equipped with separate pipes voiced for the various pressures.

I wish to call attention to the minimum point on the curve at about 7 cm. pressure. So far as I have been able to discover, this has not been detected previous to this time, and adds a special qualification to the work of Rayleigh and Cavaille-Coll on this subject.

Data.

Pipe length, 30.87 cm. Small manometer. 21°C.

Pressure	Beats	Seconds	Beats/ Second	N
1.32 cm.	80	20	4	254.6
1.95 "	100	38	2.6	256.0
2.70 "	80	31.2	2.5-	256.1

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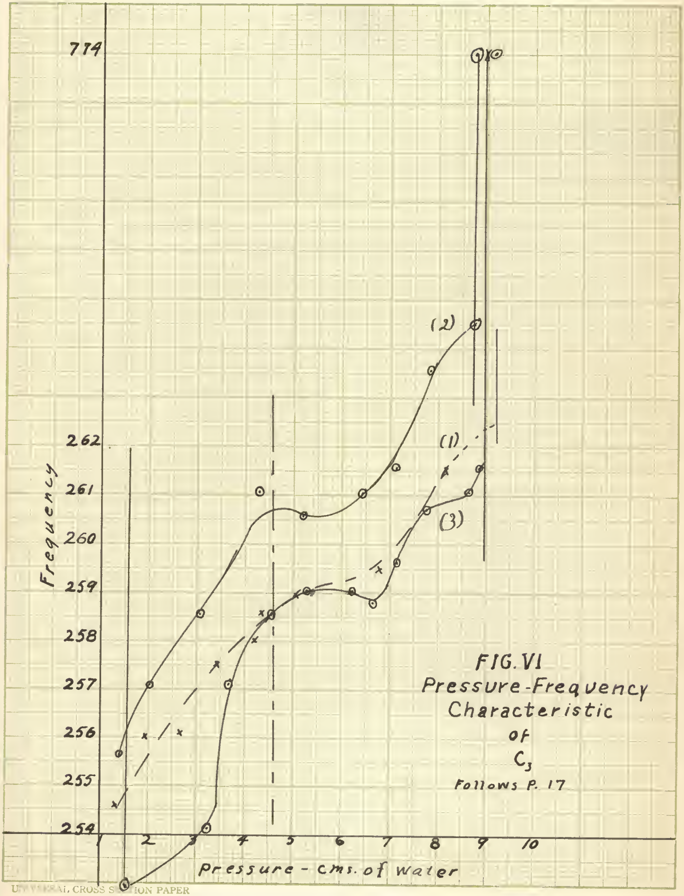

FIG. VI
Pressure-Frequency
Characteristic
of
C_3
Follows P. 17

3.43 cm.	50	45	1.1	257.5
4.20 "	30	50	.6	258.0
4.34 "	Unison			258.6
5.06 "	15	42	.37	258.97
6.78 "	30	35	.85	259.45
8.18 "	80	27.6	2.9	261.50
9.18	Second partial			774.

Data.

Pipe length, 30.87 cm.

Pressure	Beats	Seconds	Beats/sec.	N
1.40 cm.	60	20	3	255.57
2.10 "	20	12.8	1.56	257.01
3.04 "	Unison			258.57
4.29 "	30	12	2.5	261.07
5.20 "	30	15	2	260.570
6.46 "	30	12	2.5	261.07
7.13 "	32	10	3	261.57
7.83 "	30	6	5	263.57
8.74 "	30	5.2	6	264.5

Data.

Pipe length, 30.95 cm.

Pressure	Beats	Seconds	Beats/sec.	N
1.56 cm.	40	7	5.7	252.87
3.23 "	30	9	4.44	254.13
3.56 "	30	20.6	1.45	257.12
4.53	Unison			258.57
5.27	12	24	.5	259.07
6.21	12	25	.48	259.05
6.62	5	21	.24	258.81

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7.15 cm.	20	21	1	259.57
7.76 "	30	14	2.14	260.71
8.66 "	30	12	2.5	261.07
8.83 "	30	10	3	261.57
8.99 "	Second partial			

C#3

The data taken at 21.4°C. with the small manometer made a smooth curve of the general shape of the others in the lower part of the octave, and was considered as sufficiently accurate for the purpose. Because of limited time I did not repeat this data with the larger manometer.

It may be seen that this curve corresponds in general to that for C . The pipe began to speak its fundamental tone at 1.6 cms. water pressure. The frequency rose rapidly to about 5 cms. pressure, then decreased slightly at 7 cms. and increased again before the fundamental gave place to the next partial, the third harmonic, at 9.7 cms. pressure.

Comparing with the preceding curve, it is evident that the "breaking point" or the point where the split tone is noticeable is at a slightly higher pressure than in the former curve. Between 4 and 5 cms. the coefficient is .00329.

Data.

Fork C3#　　　　Pipe length, 29.01

Pressure	Beats	Seconds	Beats/sec.	N
1.53	50	10	5	268.94
2.9	50	16.2	3	270.94
4.03	20	31	.66	273.21
4.73	Unison			273.94
5.50	20	36.4	.55	274.49
6.06	20	31.8	.64	274.58
7.59	20	35	.57	274.51
8.57	30	23	1.3	275.24
9.31	50	20	2.5	276.44
9.69	Second partial			821.82

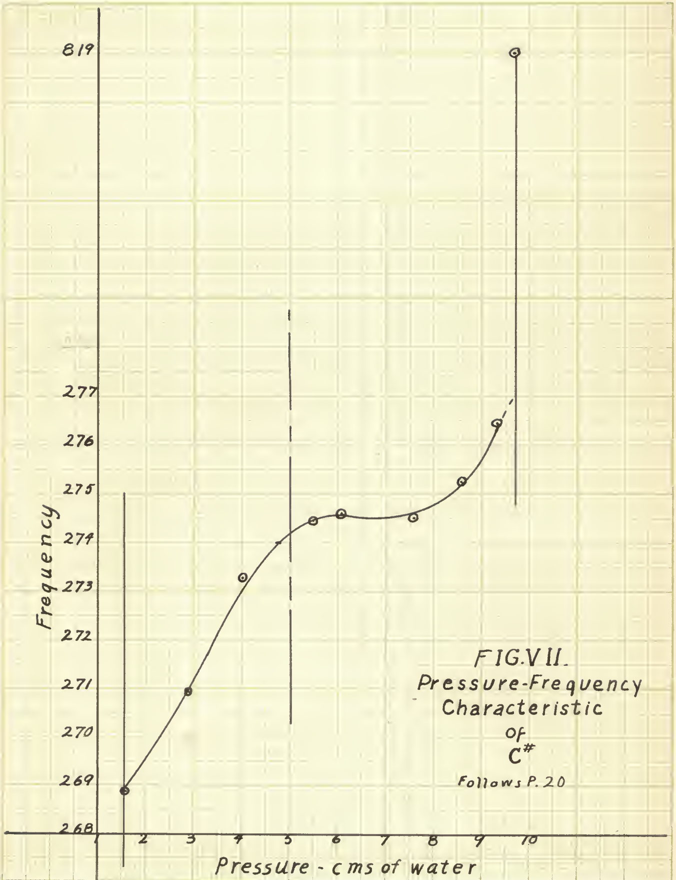

FIG. VII.
Pressure-Frequency
Characteristic
of
C#

Follows P. 20

D: Frequency 290.23

In this curve also the downward trend in the region of the split tone is apparent, but due to the lack of sufficient air pressure the full curve between critical pressures could not be determined and I have indicated by a dotted line the probable course of the curve in the unknown region. This is done on the strength of the evidence of the two preceding curves. The breaking point is at about 5.5 cms. pressure and the subsequent "flat" part of the curve is broader than in either of the preceding curves.

Both the upper and lower critical pressures are somewhat higher than those of the two preceding curves, and these points will be seen to rise on through the octave, the upper one the more rapidly.

The coefficient at 4 to 5 cms. pressure is .00605.

Data.

Temperature 21.5°C. Pipe at 27.35 cm.

Pressure	Beats	Seconds	Beats/sec.	N
1.91 cm.	20	3	6-	284.
3.42 "	60	19	3.4	286.83

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4.65 cms.	8	24	.33	289.90
5.43 "	10	23	.43	290.66
7.19 "	20	30	.66	290.89
7.34 "	20	27	.74	290.97
8.04 "	30	30	1	291.23
10.74 "	Second partial			870.69

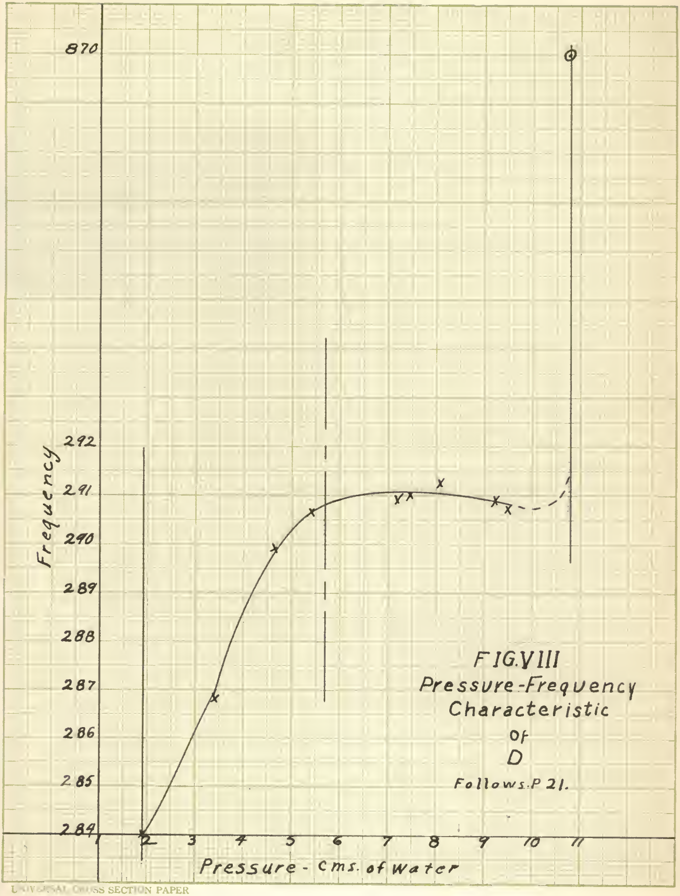

FIG. VIII
Pressure-Frequency
Characteristic
of
D

Follows P 21.

D#

Here again the curve has the same general shape up to 9.5 cm. pressure as the former curves have to the point of negative coefficient. As in the curve for 'D', I have dotted the probable course of the curve on up to the upper critical pressure. Note that the breaking point is at slightly higher pressure, and the flat part of the curve broader than in any of those previously considered. Due to the fact that the 4 to 5 cm. pressure range is on the steeply rising part of the curve, the coefficient is somewhat greater, viz. .00752. This curve as determined by means of the small manometer was considered fairly reliable, and was not repeated.

Data.

Temperature, 21.5°C. Pipe length, 25.55 cm.

Pressure	Beats	Seconds	Beats/Sec.	N
2.18	30	6	5	302.49
3.96	30	15	2	305.49
4.73	Unison			307.49
5.71	10	3.5	.28	307.77
6.57	50	22.5	2.08	309.57

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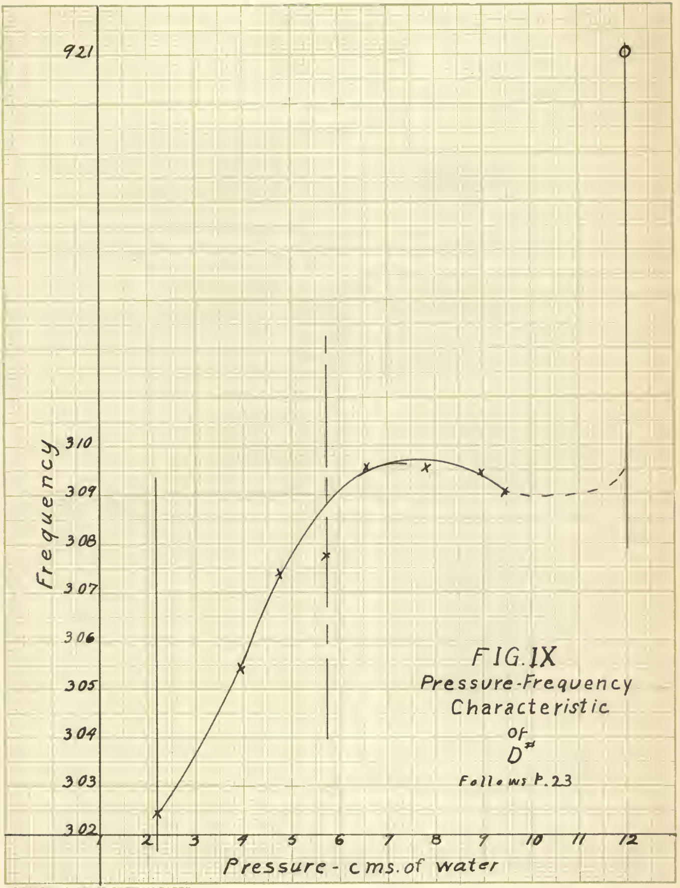

FIG. IX
Pressure-Frequency
Characteristic
of
$D^{\#}$

Follows p. 23

7.86	50	24.2	2.06	309.55
8.99	50	25	2	309.49
9.47	50	32	1.56	309.05
12.00	Second partial			922.47

E: Frequency 325.75

The first determination of points for this curve gave a rather rough line, curve (1), so I repeated the work with the second manometer. (Results shown by curve 2.) Here again I have had to supply the probable continuation of the curve above 9.5 cm. pressure. The critical pressures are seen to be somewhat higher, and the breaking point is more difficult to determine, but is between 6 and 7 cms pressure. The coefficient for (2) between 4 and 5 is .0076.

Data.

Temperature (1) 21.6°C; (2) 23.6°C. Pipe length 24 cm.

Pressure	Beats	Seconds	Beats/Sec.	N
2.10	20	3	6.6	319.15
4.17	30	10	3	322.75
5.00	20	35	.57	325.18
5.11	Unison			325.75
6.23	30	19	1.58	327.33
7.35	40	17	2.35	328.10
8.07	50	17	3	328.75
9.28	50	16	3.1	328.85
9.43	50	16	3.1	328.85
14	Second partial			977.25

E: Temperature 23.6°C.

Pressure	Beats	Seconds	Beats/Sec.	N
1.81	40	7	5.7	320.05
2.46	40	10	4	321.75
3.13	40	11.5	3.48	322.27
3.76	30	18	1.66	324.09
4.55	Unison			325.75
5.26	30	16	1.87	327.62
5.97	40	15.5	2.66	328.41
	Pipe at 24.2; add 2.3 v.p.s.			
7.82	30	14	4.44	330.19
8.82	30	14-	4.5	330.25

The last two readings were made by using the length-frequency characteristic to determine the change in frequency due to the greater pipe length. This added to the actual difference between pipe and fork gives the difference as it would be with the pipe at its original length.

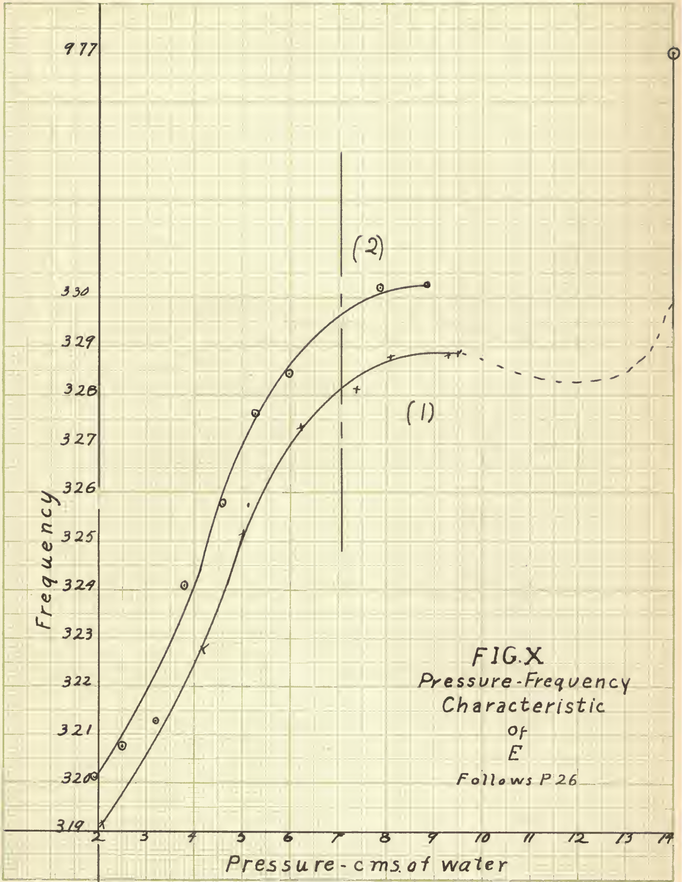

FIG. X
Pressure-Frequency
Characteristic
of
E
Follows P 26

F: Frequency 345.13

In this curve the breaking point is at about 8 cm. pressure, and the final point on the curve is apparently at the maximum point just before the downward turn of the split tone. I have not attempted a continuation of this curve, but would be inclined to make it similar to the preceding ones. The coefficient between 4 and 5 cm. pressure, which is decidedly on the straight part of the curve, is .0065.

Data.

Temperature, 22.3°C. Pipe length, 22.47 cm.

Pressure	Beats	Seconds	Beats/Sec.	N
1.94	30	4.1	7.13	338.
2.72	20	3.5	5.7	339.43
3.95	50	16	3.1	342.03
5.00	20	17	1.17	343.96
5.19	Unison			345.13
5.48	20	20	1	346.13
6.00	30	18	1.66	346.79
6.41	30	16	1.87	347.00
7.70	30	11	2.7	347.83
8.74	30	8	3.75	348.88
9.66	30	8	3.75	348.88
15	Second partial			1035.

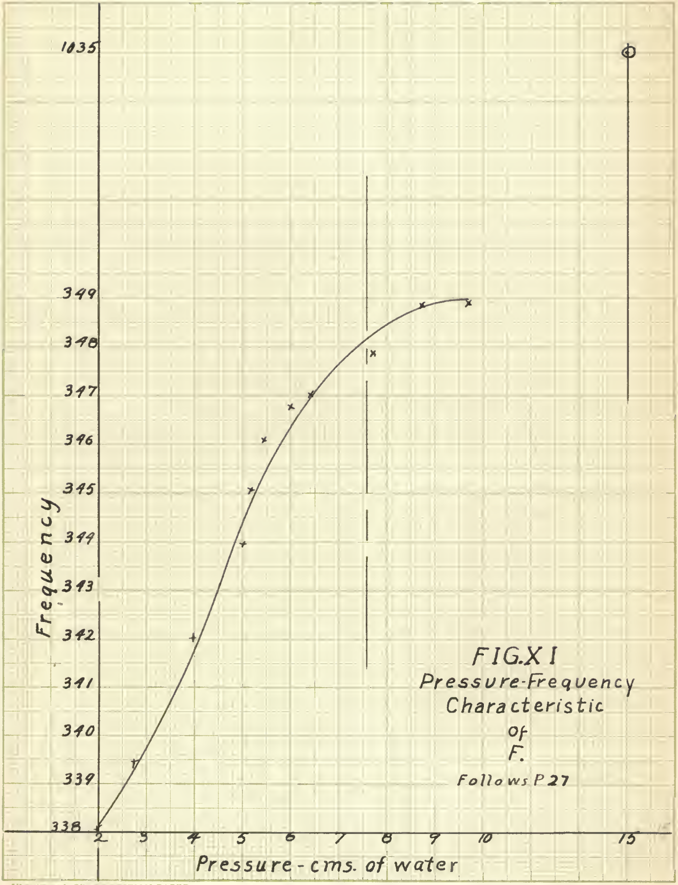

FIG. XI
Pressure-Frequency
Characteristic
of
F.

Follows P 27

F#: Frequency 365.68

Here again I have plotted two curves. (1) is from data taken with the small manometer, (2) using the large bore (1 cm.) manometer. Both are at practically 21°C. In (2) there is evidence of the breaking point at about 9 cm. pressure. When the pipe was sounding at this point the split tone was quite evident. This is the highest point in the octave at which I detected the split tone: hence the subsequent curves will represent only the lower straight parts of the curves found for the lower tones. Each will be respectively a smaller fraction of the entire curve between the critical pressures.

For this tone on curve (2), between 4 and 5 cm. pressure, the coefficient is .0046.

Data.

Curve (1) Temperature 22.3°C. Pipe length 20.84 cm.

Pressure	Beats	Seconds	Beats/Sec.	N
2.17	30	7.5	4	361.68
3.68	20	32	.62	365.06
4.36	Unison			365.68

Cont'd on next page

5.35	30	16	1.87	367.55
5.85	30	10.5	2.83	368.53
6.90	20	3	6.66	372.34
Pipe at 21.31; add 7.98				
10.04	50	17	2.94	
			7.98	
		Total -	10.92	376.60

Data.

F#. Large manometer — Pipe length, 20.81

Pressure	Beats	Seconds	Beats/Sec.	N
2.46	40	10	4	361.68
3.35	30	18	1.66	364.02
4.26	15	27	.55	365.13
4.55	Unison			365.68
4.85	20	32.8	.61	366.29
5.93	30	15	2	367.68
7.12	40	9.5	4.21	369.89
8.42	40	8.5	4.7	370.38
9.26	40	6.5	6.16	371.84

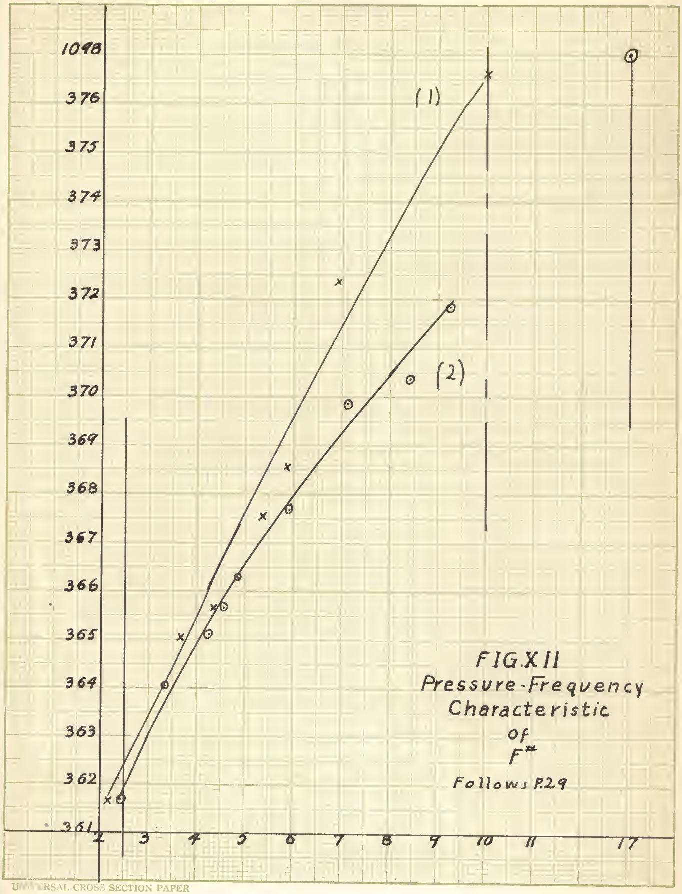

FIG.XII
Pressure-Frequency
Characteristic
of
F*
Follows P.29

G: Frequency 387.42

Although I took data for this tone with each of the three sizes of manometer, I have plotted only that for the largest, because I consider it the more significant of the three. The points are so placed that one might draw a straight line through them with only a small departure from any one point. However, it may be that the tone of the pipe actually follows such a curve as I have drawn, in all cases, in the lower part of its pressure range. Certainly there is no sufficient reason for so far disregarding my readings as to draw a straight line approximating the curve. The coefficient at 4 to 5 cm. is .00387.

Data.

Temperature 21°C. Pipe length 19.43 cm.

Pressure	Beats	Seconds	Beats/Sec.	N
2.74	60	20	3	384.42
3.45	20	15	1.33	386.09
4.52	Unison			387.42
4.88	10	16	.63	388.05
5.31	20	19	1.05	388.47
6.54	40	16	2.5	389.92
8.56	40	8.5	4.7	392.12
9.55	40	5	8	395.42
18.50	Second partial			1162.26

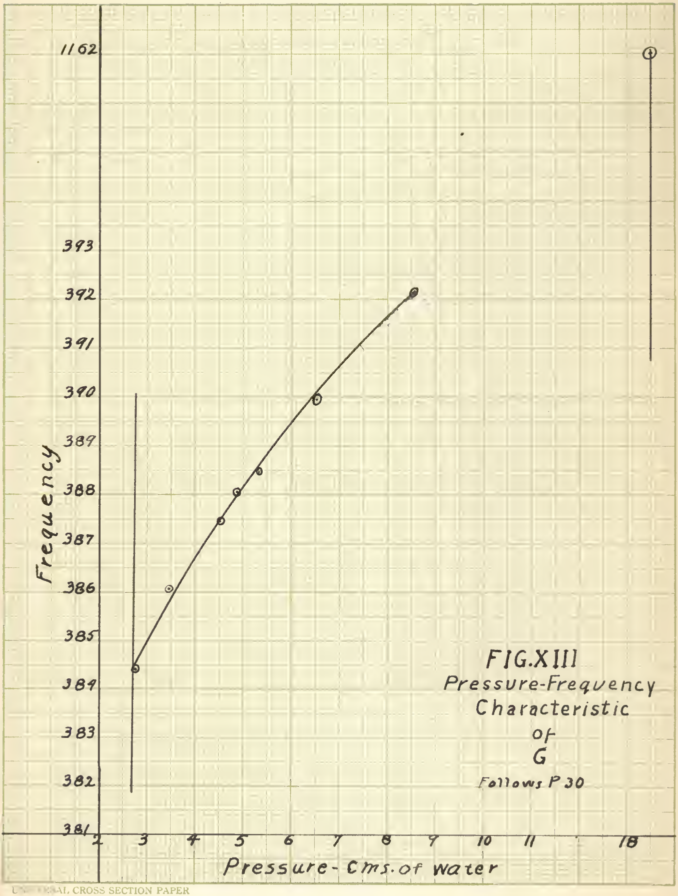

FIG. XIII
Pressure-Frequency
Characteristic
of
G

Follows P 30

G#: Frequency 410.44

Here again I have graphed only the final data from readings of the large manometer. The portion of the curve thus obtained is evidently a straight line, the variation of the points from it being so small that it may be due to addition of various experimental errors, such as the effect of capillarity. The 4 to 5 cm. coefficient is .0046.

Data.

Temperature 21.5°C. Pipe length, 18.23 cm.

Pressure	Beats	Seconds	Beats/Sec.	N
3.09	40	15	2.66	407.78
3.84	30	28.6	1.05	409.39
4.52	Unison			410.44
4.89	15	23.6	.63	411.07
6.63	40	10	4	414.44
7.86	40	7.8	5.12	415.56
8.69	40	5.8	7	417.44
20.00	Second partial			1231.32

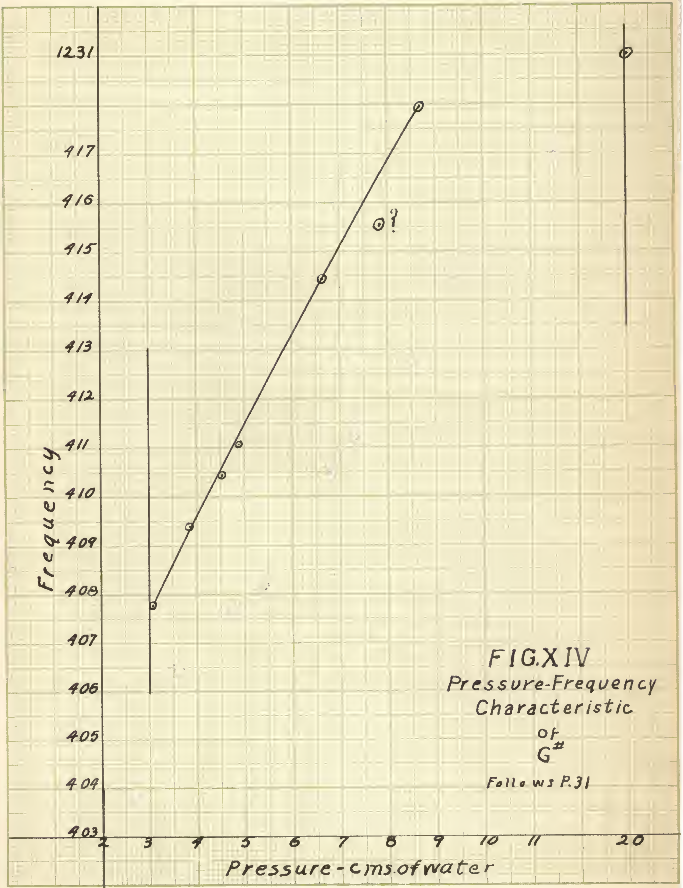

FIG. XIV
Pressure-Frequency Characteristic of $G^\#$

Follows P. 31

A: Frequency 434.85

Only the final determination is graphed here. An idea of the small part of the total characteristic curve for this tone that is shown here may be obtained from a consideration of the following:

Total pressure range between critical pressures - 20 cm

Pressure range represented by curve 5 "

Part of total characteristic represented - 25%

Considering the fact that about 50% of the characteristic was needed in the former curves to determine even the breaking point, it is evident that this and the remaining curves tell very little about the actual operation of the pipe at the important pressures. The 4 to 5 cm. coefficient is .0039.

Data.

Temperature 21.8°C. Pipe length, 17.03 cm.

Pressure	Beats	Seconds	Beats/Sec.	N
3.30	30	8.5	3.53	451.32
3.84	30	16	1.9	432.95
4.50	Unison			434.85
4.90	20	22	.91	435.76
6.92	40	11	3.63	438.48
8.62	40	7.2	5.35	440.40
23.00	Second partial			1304.55

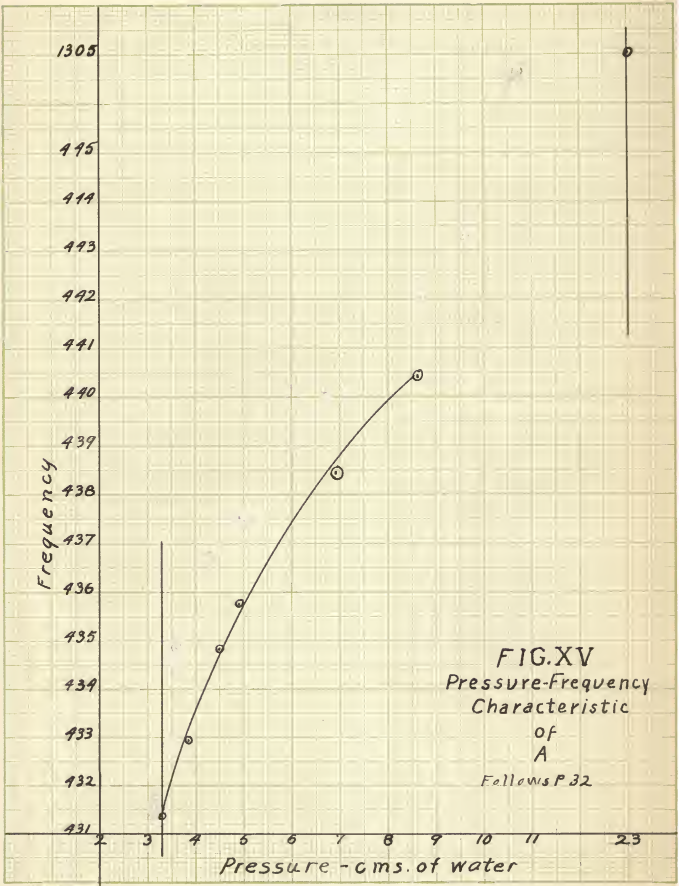

FIG. XV
Pressure-Frequency Characteristic of A

Follows P 32

A#: Frequency 460.70

Here the fraction of the total characteristic represented is about 23%. The 4 to 5 cm. coefficient is .0039.

Data.

Temperature - 21.9°C. Pipe length, 15.89 cm.

Pressure	Beats	Seconds	Beats/Sec.	N
2.68	40	15	2.66	458.04
4.15	30	28.8	1.04	459.66
4.51	Unison			460.70
4.99	30	27.6	1.06	461.76
6.48	40	10	4	464.70
8.76	40	6	6.66	467.36
26.00	Second partial			1382.10

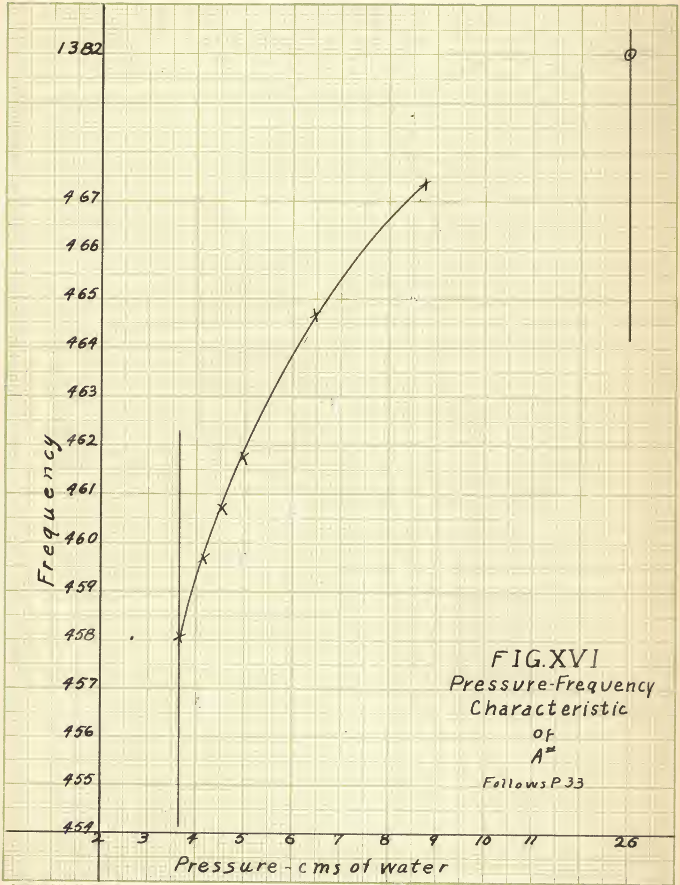

FIG. XVI
Pressure-Frequency
Characteristic
of
A
Follows P 33

B: Frequency 488.11

This curve is from data taken with the small manometer. It was sufficiently near a straight line that I did not feel justified in repeating it due to lack of time. The part of the total characteristic shown is 18.7%. The 4 to 5 cm. pressure coefficient is .0055.

Data.

Temperature 21.2°C. Pipe length 14.87 cm.

Pressure	Beats	Seconds	Beats/Sec.	N
3.75	20	3	6.66	481.45
4.53	30	13	2.3	485.81
5.23	20	31	.64	487.47
5.52	Unison			488.11
6.76	30	15	2	490.11
7.88	30	7	4.28	492.39
8.94	20	4	5	493.11
9.64	20	3	6.66	494.77
32	Second partial			1464.33

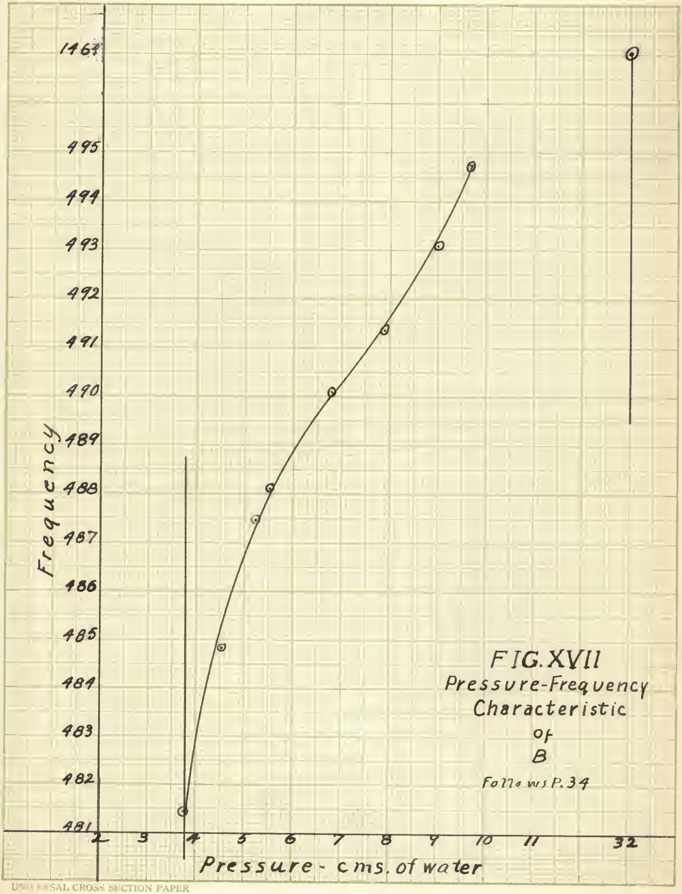

FIG. XVII
Pressure-Frequency
Characteristic
of
B

Follows P. 34

C: Frequency 517.15

This curve is from data taken with the second manometer. The part of the total curve represented is 8.3%. The 4 to 5 cm. coefficient can not be obtained due to the fact that 4.5 is the lower critical pressure. The 4.5 to 5.5 value is .0067.

Data.

Temperature 21.6°C. Pipe length 13.9 cm.

Pressure	Beats	Seconds	Beats/Sec.	N.
4.29	20	3.2	6.25	510.90
5.11	20	7.4	2.7	514.41
5.32	20	11.5	1.74	515.41
5.77	10	28	.36	516.79
6.15	Unison			517.15
6.36	10	15	.66	517.81
7.27	20	8	2.5	519.65
7.83	20	3.8	5.26	522.41
9.74	30	5	6	523.15
10.28	30	4.2	7.14	524.29
42	Second partial			1551.45

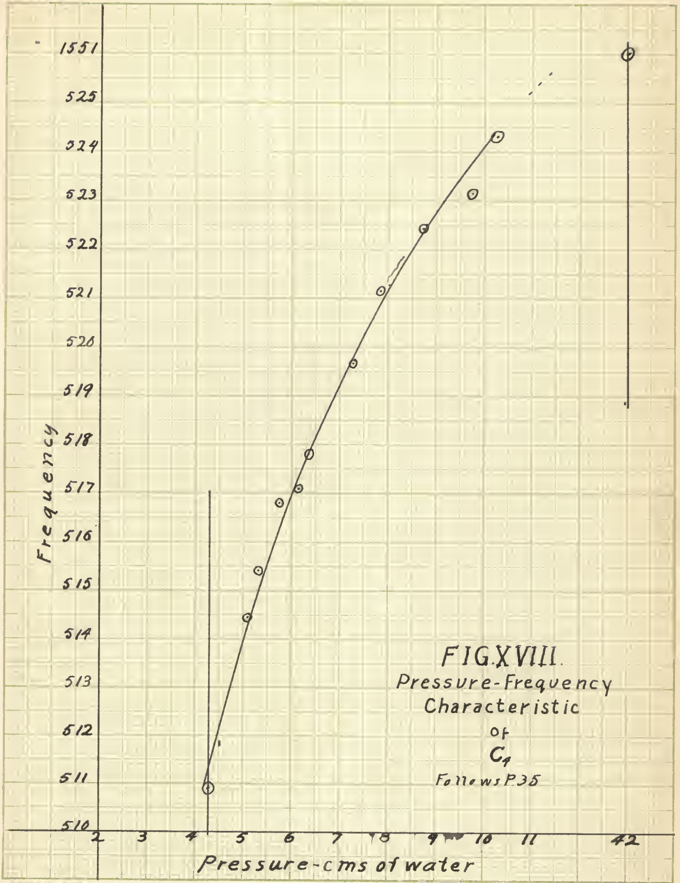

FIG. XVIII
Pressure-Frequency Characteristic of C_4

Follows P.35

CRITICAL PRESSURES.

The relation of the lower critical pressure to frequency, or to position of the tone in the octave, is not definite but in general it increases as the frequency rises, and the length of the pipe decreases. The pressure necessary to bring in the fundamental at any point varies considerably with different trials. In general, the second partial is the first sound besides the wind-rush, and is suddenly replaced at about the pressures given, by the fundamental.

The upper critical pressure, on the other hand, follows very closely the arc of a parabola. As the frequency of vibration rises, with decreasing length, a point is approached at which a great, or practically infinite pressure would be necessary to elicit the upper partial. No doubt further work on mouth corrections to length will be necessary before this matter can be developed further.

PRESSURE FREQUENCY COEFFICIENT

The relation of pressure-frequency coefficient to frequency, or to pipe length is not easily deducible from the

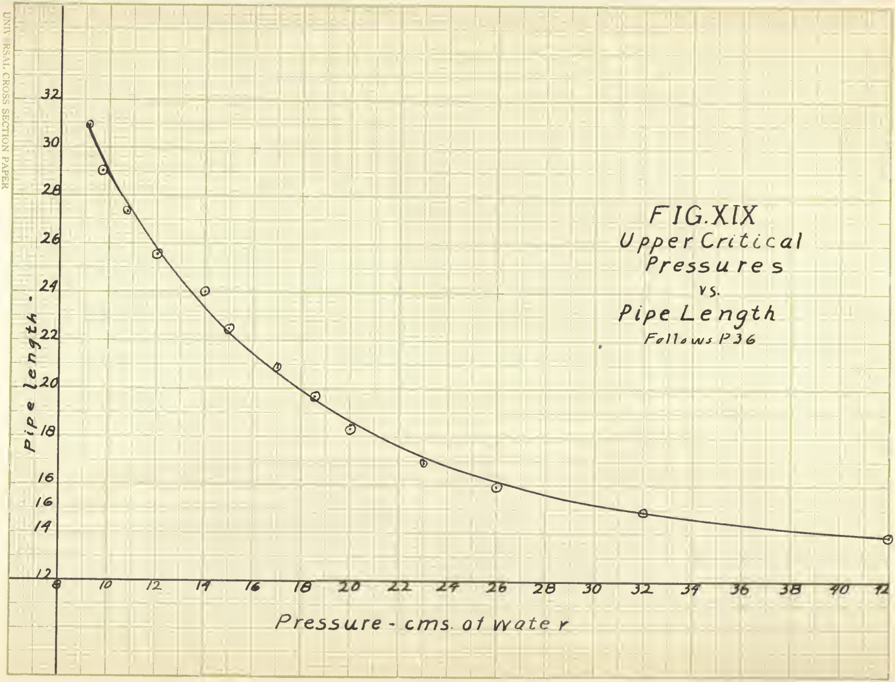
FIG. XIX
Upper Critical
Pressures
vs.
Pipe Length
Follows P36
Pipe length -
32
30
28
26
24
22
20
18
16
16
14
12
8
10
12
14
16
18
20
22
24
26
28
30
32
34
36
38
40
42
Pressure - cms. of water

data contained herein. In view of the fact that the coefficient at 4.5 cm. pressure is for a different RELATIVE part of successive curves, it hardly seems to me to have any particular significance except as it affects the calibration of such a pipe as I used in this experiment. If it were possible to do so, it seems to me that a determination of the pressure-pipe-length characteristic at constant coefficient, perhaps at the breaking point of each curve in the above list, would be of considerable interest and value. As stated previously, the breaking points of only a few of the lower tones of the octave could be found, due to lack of constant air pressure above that which will sustain a column of water ten centimeters in height.

MOUTH CORRECTION.

The work done to date on the question of mouth corrections is, so far as I can find, summed up in the quotations from Rayleigh that are given in the first part of this work. Rayleigh states that for an open organ pipe, the total addition to the length necessary to give the acoustic length of the pipe is 3.3 R . Subtracting .6 R for the open end, which of course is not present in a closed pipe, there is left 2.7 times the radius of the circular part of the pipe for the mouth alone. Such is his treatment of the question. It is given without any consideration whatever for length of pipe, temperature, or wind pressure.

From the data given above it is a simple matter to calculate the mouth correction at any point on any of the pressure-frequency curves by the formula

$$c = \frac{V\theta}{4N} - L$$

where c is the correction in centimeters length, V- is the velocity of sound in free air at the prevailing temperature, N is the frequency of vibration, and L is the measured length of the pipe. The following table shows the correction for each tone of the octave at 4.5 cm. pressure and 8 cm. pressure respectively.

Data.

Tone.	Pipe-Length.	θ^{o}C.	Correction at pressure of 4.5 cm.	Correction at pressure of 8 cm.
C3	30.87 cm.	21	2.45 cm.	2.17 cm.
"	30.87 "	24.2	2.32 "	
C#	29.01 "	21.5	2.47 "	2.36 "
D	27.35 "	21.8	2.42 "	2.28 "
D#	25.55 "	21.8	2.50 "	2.31 "
E	24.00 "	21.6	2.61 "	2.23 "
"	"	23.8	2.57	---

Cont'd on next page

F	22.47 cm.	22.2°	2.68 cm.	1.587 cm.
F#	20.84 "	22.3	2.74 "	2.24 "
G	19.64 "	22.5	2.76 "	2.44 "
G#	18.33 "	19.9	2.78 "	2.53 "
A	16.88 "	20.5	2.84 "	2.59 "
A#	15.94 "	21.	2.865 "	2.60 "
B	14.87 "	21.3	2.87 "	2.63 "
C	13.9 "	21.3	2.94 "	2.63 "
"	"	24.8	2.93 "	2.63 "

Without plotting curves, it may be seen that the mouth correction is not constant, as it would be if it were a definite function of the diameter. Instead, it is seen to increase as the pipe length decreases. It decreases as the pressure is increased and also as the temperature increases. If it were possible to vary the area or shape of the mouth opening, some relation between correction and size or shape of opening might appear. In fact, when I was getting the pipe in shape for this experiment, I found that the size of the mouth opening had a great deal to do with the pressure at which the fundamental tone of the pipe would speak. Since the pressure appears to influence the correction, it seems reasonable to conclude that the size of mouth would also have an effect.

In so far as my work can be used as a guide, it seems that the mouth correction varies with temperature, length, and pressure so that the frequency of the pipe follows the simple rule,

$$N = \frac{V}{4L}$$

wherein N is the frequency, L the acoustic length, and V is the velocity of sound in free air.

As a check upon the accuracy of my measurements, I calculated the acoustic lengths for 258 v.p.s. and 516 v.p.s. and found the inverse octave relationship within .1%.

Referring again to the table of data for the mouth correction, it may be seen that readings were taken in a few instances at various temperatures with identical conditions of air pressure and pipe length. See also curves for C_3, page 15. Evidently the temperature frequency coefficient may be calculated from these readings to check the previous method. (Page 13). For C_3 I find, by this means, that the coefficient is .0025; for E it is .0024; and for C_4, .002. Evidently, further work is necessary before anything like an accurate value for the temperature-frequency coefficient can be found.

SUMMARY

In actual practice, the frequency of a stopped pipe is not inversely proportional to the length of the pipe itself. The variation from such a relationship is caused by variation in the mouth correction at various lengths of the pipe.

The correction to length, necessitated by lack of openness of the mouth, is not a function of the pipe diameter as Rayleigh, Helmholtz, and others have assumed. It is inversely proportional to the length of pipe, pressure of wind with which the pipe is blown, and the temperature at which the pipe is operated. It is probable that the shape or area of the mouth opening also has an influence upon the correction.

Increasing the pressure of the wind with which a stopped pipe is blown does not always increase the vibration frequency, although in general it does. This is another feature which is not considered in any treatise on the subject of sound.

Increase of temperature results in an increase of frequency. The exact value of the temperature-frequency coefficient is not known.

Date Due
