A COMPUTER IMPLEMENTATION OF A MATHEMATICAL MODEL OF AN O-TYPE TRAVELING WAVE TUBE AMPLIFIER

by

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1.0 Introduction

There is a need for a large signal theory of traveling wave tube amplifiers (TWTA) because they are usually operated close to saturation and the linear theory developed by Pierce [1] is not adequate to describe this type of operation. There have been attempts by Rowe [2] and others to model the non-linear large signal operation of a TWTA but they suffer from the drawback that the electron stream is divided into a finite number of discrete charge groups and the motion of each charge group is considered independent of the other groups [3]. There is a new non-linear theory developed by N. Kalyanasundaram [3] which overcomes this drawback.

The intent of this work was to independently verify N. Kalyanasundaram's large signal theory of an O-type traveling wave tube amplifier by writing a FORTRAN computer program based on Kalyanasundaram's equations and comparing the results of the computer simulations to Kalyanasundaram's results. The results were then to be extended and improved by using more terms in Kalyanasundaram's infinite series solution and by reducing the step size in the numerical integrations.

A FORTRAN computer program was written which implemented Kalyanasundaram's equations. The results produced by this program were compared to Kalyanasundaram's results and were not found to be in agreement. The agreement of the results was improved by changing the sign of the phase factor used by Kalyanasundaram; however, there were still some differences in the results. There was also a possible problem with the convergence test used by Kalyanasundaram and the FORTRAN program in that the test did not guarantee that the solution converged, but only that the solution did not change by more than a specified amount from one iteration to the next.

Included in this report is J. R. Pierce's linear small signal theory of TWTA's. Pierce's theory is included so that the results of the small signal theory, Kalyanasundaram's large signal theory, and the FORTRAN program can be compared for the small signal case. This is done so that Kalyanasundaram's theory can be verified for the small signal case. For small signals both theories should give similar results. Pierce's small signal theory, and Kalyanasundaram's small signal results are similar for the gain of a TWTA.

This report first develops Pierce's linear theory of TWTA's. Next, Kalyanasundaram's TWTA equations are given and the results are compared to Pierce's results for a specific small signal case. Last, this report describes the development of a FORTRAN computer program which implements Kalyanasundaram's equations and compares Kalyanasundaram's results and the FORTRAN program's results. The FORTRAN program results and Kalyanasundaram's results are similar qualitatively; however, they are not identical.

Future work towards reconciling the differences between the FORTRAN program's results and N. Kalyanasundaram's results should include a complete rederivation of Kalyanasundaram's large signal theory to verify the equations in his paper. Then with any discrepancies uncovered the FORTRAN program should be modified accordingly. Only then should extensions to Kalyanasundaram's examples be attempted.

2.0 Pierce's Linear Theory of a Traveling Wave Tube Amplifier.

This section will describe Pierce's linear small signal theory of a traveling wave tube amplifier (TWTA). Fig. 2.1 below shows a schematic of a typical traveling wave tube. The parts of this which will be discussed are the electron beam and the slow wave structure. A slow wave structure is used to slow the speed of the traveling wave to be slightly slower than the speed of the electron stream. The electron stream has to travel slightly faster than the wave so that energy can be transferred from the electron stream to the RF wave which causes the power amplification of the RF signal which is desired. A helix is used as the slow wave structure as shown in Fig. 2.2. A helix is basically a single wire wound like a corkscrew which slows the forward travel of the voltage wave by effectively increasing the distance the voltage wave must travel per unit of travel along the axis of the tube.

To derive equations which describe the portion of the tube shown in Fig. 2.2, the helix is simulated by a transmission line, which extends infinitely in the $\bar{z}$ direction and has distributed parameters L and C per unit length, as shown in Fig. 2.3. The helix is modeled by the transmission line because the mathematics is well known for transmission lines and this results in a problem which is

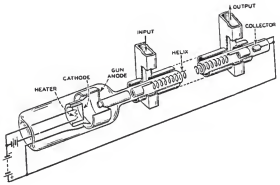

Figure 2.1. Schematic of a traveling wave tube amplifier.
(from Pierce page 7 [1])

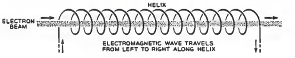

Figure 2.2. Portion of the traveling wave tube amplifier needed for the analysis.
(from Pierce page 7 [1])

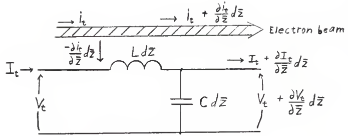

Figure 2.3. The transmission line equivalent circuit, which extends infinitely in the $\bar{z}$ direction, of the helix used by Pierce, which has parameters L and C per unit length and carries a voltage V_t and current I_t. The coupling of the electron beam is by a distributed source $J_t = -\partial i_t/\partial \bar{z}$.

easier to solve rather than using field theory to find the solution.

Pierce first considers the disturbance produced in the circuit by a bunched electron stream. Refer to Fig. 2.3 for this development. The main simplifying assumption which Pierce makes is that all electrons in the electron flow are acted on by the same a-c field [4]. This is a good assumption when the diameter of the electron beam is small. It is also assumed that the electrons are displaced by the a-c field in the axial direction only because TWT's use strong magnetic focusing fields to limit radial movement of the electrons. Pierce also assumes all a-c and circuit quantities, in complex form, vary with time and distance as

$\exp(jwt - \Gamma\bar{z})$ to get a self-consistent solution. Also, non-relativistic equations of motion are used throughout the development.

Applying Kirchhoff's current and voltage laws and transmission line theory [5] to the circuit in Fig. 2.3 results in

$$\partial I_t/\partial \bar{z} = -C\partial V_t/\partial t - \partial i_t/\partial \bar{z} \tag{2.1}$$

and

$$\partial V_t/\partial \bar{z} = -L\,\partial I_t/\partial t \tag{2.2}$$

where

$$V_t = \mathrm{Re}\{V\cdot\exp(jwt - \Gamma\bar{z})\} \tag{2.3}$$

is the transmission line voltage, V is a complex number which has the magnitude and phase of V_t,

$$i_t = -I_0 + \mathrm{Re}\{i\cdot\exp(jwt - \Gamma\bar{z})\} \tag{2.4}$$

is the electron beam convection current, i is a complex number which has the magnitude and phase of the a-c part of i_t, and I_0 is the magnitude of average electron convection current. Note that in Eq.(2.4) $-I_0$ is used because the electrons are traveling in the positive $\bar{z}$ direction.

$$I_t = \mathrm{Re}\{I\cdot\exp(jwt - \Gamma\bar{z})\} \tag{2.5}$$

is the transmission line current, and I is a complex number which has the magnitude and phase of I_t.

Pierce is interested in cases in which all a-c quantities, in complex form, vary with distance as $\exp(-\Gamma\bar{z})$

because he is looking for a wave type solution for the traveling wave tube. This allows replacement of differentiation with respect to $\bar{z}$ by multiplication by $-\Gamma$. Differentiation with respect to t is replaced with multiplication by jw. The impressed current per unit length is given by Pierce's equation (2.3) [6] as

$$J_t = -\partial i_t/\partial \bar{z} \tag{2.6}$$

so, Eq.(2.1) and Eq.(2.2) become

$$-\Gamma I = -jBV + \Gamma i \tag{2.7}$$

$$-\Gamma V = -jXI \tag{2.8}$$

where

$$B = Cw \tag{2.9}$$

is the shunt suseptance per unit length, and

$$X = Lw \tag{2.10}$$

is the series reactance per unit length.

BX is chosen so that the phase velocity of the circuit in Fig. 2.3 is the same as that for a particular traveling wave tube helix and X/B is chosen so that $-\partial V_t/\partial \bar{z}$ is equal to the axial electric field component for that helix. This establishes the definition of the transmission line model for the helix.

Solving for I in Eq.(2.7) and Eq.(2.8) and setting the results equal and rearranging results in

$$V(\Gamma^2 + BX) = -j\Gamma Xi \qquad (2.11)$$

If there were no impressed current, the right side of Eq.(2.11) would be zero and Eq.(2.11) would be the normal transmission line equation. B and X can be replaced by the propagation constant and characteristic impedance of the line with beam absent as follows.

$$\Gamma_1 = j(BX)^{1/2} \qquad (2.12)$$

where Γ_1 is the propagation constant for the line in the absence of the electron beam. Thus, the forward wave on the line, with the electron beam absent, varies with distance as $Re\{\exp(-\Gamma_1\bar{z})\}$ and the backward wave as $Re\{\exp(+\Gamma_1\bar{z})\}$. These are sometimes called the cold waves.

The characteristic impedance, K, of the line itself is from elementary transmission line theory [7]

$$K = (X/B)^{1/2} \qquad (2.13)$$

Eq.(2.12) and Eq.(2.13) can be used to replace X and B by Γ_1 and K. From Eq.(2.13) and Eq.(2.12)

$$X = -jK\Gamma_1 \qquad (2.14)$$

Substituting Eq.(2.13) and Eq.(2.14) into Eq.(2.11) results in

$$V = \frac{-\Gamma\Gamma_1 Ki}{(\Gamma^2 - \Gamma_1{}^2)} \qquad (2.15)$$

which is Pierce's equation (2.10) [6]. Thus the a-c part of the convection current i_t is the source of the line voltage V_t.

Now that the transmission line voltage V_t has been found in terms of the electron convection current i_t the next part of the problem is to find the disturbance produced on the electron beam by the fields of the transmission line.

The force exerted on an electron by the electric field is

$$F = -eE \tag{2.16}$$

where $e = 1.602 \times 10^{-19}$ coulomb, is the fundamental charge. From Newton's second law of motion the force exerted on the electron is

$$F = m_e \frac{d(v_o + v_t)}{dt} \tag{2.17}$$

where $m_e = 9.1095 \times 10^{-31}$kg is the mass of an electron, $v_t = \mathrm{Re}\{v \cdot \exp(jwt - \Gamma \bar{z})\}$ is the a-c component of the electron velocity, and v_o is the average velocity of the electrons. From the transmission line assumption above the electric field component parallel to the beam is

$$E = - \frac{\partial V_t}{\partial z} \tag{2.18}$$

Equating Eq.(2.16) and Eq.(2.17) and substituting in Eq.(2.18) gives

$$\frac{d(v_o + v_t)}{dt} = n \frac{\partial V_t}{\partial \bar{z}} \tag{2.19}$$

where $n = 1.759 \times 10^{11}$ coulomb/kg is the charge to mass ratio of electrons.

The derivative in Eq.(2.19) represents the change of velocity following a single electron and obviously there is no change in the average velocity v_o. The change in the a-c component of the velocity is expressed by taking the total derivative of v_t since velocity is a function of time and distance.

$$\frac{dv_t}{dt} = \frac{\partial v_t}{\partial t} + \frac{\partial v_t}{\partial \bar{z}} \frac{d\bar{z}}{dt} = n \frac{\partial V_t}{\partial \bar{z}} \tag{2.20}$$

Eq.(2.20) can be rewritten as shown below using $dz/dt = v_o + v_t$

$$\frac{\partial v_t}{\partial t} + \frac{\partial v_t}{\partial \bar{z}} (v_o + v_t) = n \frac{\partial V_t}{\partial \bar{z}} \tag{2.21}$$

Pierce assumes that the a-c velocity v_t is small compared to v_o so v_t is neglected in the parentheses in Eq.(2.21) to produce a linear differential equation. Since Pierce assumes that the a-c parts of all quantities, in complex form, vary as $\exp(jwt - \Gamma\bar{z})$ he replaces differentiation with respect to time with multiplication by jw and differentiation with respect to distance with multiplication by $-\Gamma$. Thus Eq.(2.21) becomes

$$(jw - v_o\Gamma)v = -n\Gamma V \tag{2.22}$$

Solving Eq.(2.22) for velocity so that velocity can be eliminated yields

$$v = \frac{-n\Gamma V}{v_o(j\beta_e - \Gamma)} \tag{2.23}$$

where $$\beta_e = w/v_o \tag{2.24}$$

The next equation to work with is the equation of conservation of charge, which is Pierce's equation (2.17) [6].

$$\frac{\partial i_t}{\partial \bar{z}} = -\frac{\partial p_t}{\partial t} \tag{2.25}$$

where $p_t = \mathrm{Re}\{p\cdot\exp(jwt - \Gamma\bar{z})\}$ is the a-c component of the linear charge density and p is a complex number which has the magnitude and phase of p_t. Replacing differentiation with respect to time with multiplication by jw and differentiation with respect to distance with multiplication by $-\Gamma$ and solving for the a-c charge density, p, results in

$$p = \frac{-j\Gamma i}{w} \tag{2.26}$$

The total convection current is the total velocity times the total linear charge density :

$$-I_0 + i_t = (v_o + v_t)(p_o + p_t) \tag{2.27}$$

By neglecting products of a-c quantities in comparison with products of an a-c quantity and a d-c quantity and recognizing that $-I_0 = v_o p_o$ results in

$$i = p_o v + v_o p \tag{2.28}$$

Substituting p from Eq.(2.26) into Eq.(2.28) along with w from Eq.(2.24) and solving for i gives

$$i = \frac{j\beta_e p_o v}{(j\beta_e - \Gamma)} \tag{2.29}$$

Substituting Eq.(2.23) which gives the velocity in terms of the voltage into Eq.(2.29) and using $p_o = -I_0/v_o$ and using $v_o = (2nV_0)^{1/2}$ the convection current given in terms of the voltage is seen to be

$$i = \frac{jI_0\beta_e\Gamma V}{2V_0(j\beta_e - \Gamma)^2} \tag{2.30}$$

which is Pierce's equation (2.22) [6].

In Eq.(2.30) the convection current is given in terms of the voltage and in Eq.(2.15) the voltage is given in terms of the convection current. Any value of Γ which satisfies both equations provides a self-consistent solution which is called a natural mode of propagation along the circuit and the electron beam. By combining Eq.(2.30) and Eq.(2.15) and eliminating the convection current and the voltage results in Pierce's equation (2.23) [6]

$$1 = \frac{jKI_0\beta_e\Gamma^2\Gamma_1}{2V_0(\Gamma_1^2 - \Gamma^2)(j\beta_e - \Gamma)^2} \tag{2.31}$$

which is valid for any electron velocity given by β_e and any wave velocity and attenuation given by the circuit propagation constant Γ_1 [8].

Now Pierce considers a special case where he assumes that the electron speed is made equal to the speed of the wave in the absence of electrons. This case is considered because it is of practical interest since the speed of the wave and the electron stream need to be approximately equal to achieve maximum power gain. This case is also considered because it has an exact solution. So, Pierce has

$$-\Gamma_1 = -j\beta_e \tag{2.32}$$

which is Pierce's equation (2.24) [6].

Since Pierce is looking for a wave with about the same speed as the electrons, he assumes that the propagation constant differs from β_e by a small amount ε, giving his equation (2.25) [6].

$$-\Gamma = -j\beta_e + \varepsilon \tag{2.33}$$

Substituting Eq.(2.32) and Eq.(2.33) into Eq.(2.31) gives

$$1 = \frac{-KI_0\beta_e{}^2(-\beta_e{}^2 - 2j\beta_e\varepsilon + \varepsilon^2)}{2V_0(2j\beta_e\varepsilon - \varepsilon^2)(\varepsilon^2)} \tag{2.34}$$

which is Pierce's equation (2.26) [6].

For typical traveling wave tubes, ε is much smaller than β_e so in the numerator Pierce neglects terms involving $\beta_e\varepsilon$ and ε^2 compared with $\beta_e{}^2$ and in the denominator Pierce neglects the term ε^2 compared with the term $\beta_e\varepsilon$. This results in

$$\varepsilon^3 = -j\beta_e{}^3\frac{KI_0}{4V_0} \tag{2.35}$$

For simplification Pierce defines the terms C and δ with his equation (2.28) and equation (2.29) [6].

$$KI_0/(4V_0) = C^3 \tag{2.36}$$

$$\varepsilon = \beta_e C\delta \tag{2.37}$$

Substituting Eq.(2.36) and Eq.(2.37) into Eq.(2.35) results in

$$\delta = (-j)^{1/3} \tag{2.38}$$

The roots of Eq.(2.38) are

$$\delta_1 = (3/4)^{1/2} - j/2 \tag{2.39}$$

$$\delta_2 = - (3/4)^{1/2} - j/2 \tag{2.40}$$

$$\delta_3 = j \tag{2.41}$$

The three roots represent the three forward waves. This is because the waves propagate down the line as

$$\mathrm{Re}[\exp(-\Gamma\bar{z})] = \mathrm{Re}[\exp(\mathrm{Re}\{\delta\}C\beta_e\bar{z})\cdot\exp(-j\beta_e(1 - \mathrm{Im}\{\delta\}C)\bar{z})] \tag{2.42}$$

and $$|\mathrm{Im}\{\delta\}C| < 1 \tag{2.43}$$

Eq.(2.43) is true because for typical traveling wave tubes [9] C will be approximately 0.02.

From Euler's theorem and the definition of Eq.(2.3) it is known that if the exponential has a negative imaginary argument the wave is forward traveling and, if the argument is positive the the wave is backward traveling. For a forward traveling wave if the real argument of the exponential is positive it will be an increasing wave, if the argument is negative it will be a decreasing wave. For a backward traveling wave if the real argument of the exponential is positive it will be a decreasing wave, if the argument is negative it will be an increasing wave. The wave corresponding to δ_1 is an increasing wave which travels a little more slowly than the electrons, the wave corresponding to δ_2 is a decreasing wave which travels a little more slowly than the electrons, and the wave corresponding to δ_3 is unattenuated and travels faster than the electrons. Eq.(2.31) was of fourth order so it is seen that a wave is missing. The missing root was eliminated by the approximations which are only valid for forward waves. Pierce shows that the other wave is a backward wave which means that it propagates in a direction opposite to electron velocity, and its propagation constant is given by Pierce's equation (2.32) [6] as

$$-\Gamma = j\beta_e(1 - C^3/4) \tag{2.44}$$

Since the transmission line voltage is

$$V_t = \mathrm{Re}\{V \cdot \exp(jwt - \Gamma \bar{z})\} \tag{2.45}$$

the rate at which a voltage wave will increase or decrease is

$$G = |\exp(-\Gamma \bar{z})| \tag{2.46}$$

which in dB is

$$G_{dB} = 20 \cdot \mathrm{Re}\{-\Gamma\} \cdot \bar{z} \cdot \log_{10} e \tag{2.47}$$

using

$$R_e\{-\Gamma\} = R_e\{\delta\}\beta_e C \tag{2.48}$$

the gain in dB becomes

$$G_{dB} = 20\beta_e C \cdot R_e\{\delta\} \cdot \bar{z} \cdot \log_{10} e \qquad (2.49)$$

Converting the gain formula from distance units $\bar{z}$ to number of wavelengths N requires the use of

$$\beta_e = w/v_o \qquad (2.50)$$

$$\text{wavelength} = \bar{z}/N \qquad (2.51)$$

and

$$v_o = \text{wavelength} \cdot w/(2\pi) \qquad (2.52)$$

which yields

$$\beta_e = 2\pi N/\bar{z} \qquad (2.53)$$

Substituting Eq.(2.53) into Eq.(2.49) yields gain in dB in terms of number of wavelengths, N, as

$$G_{dB} = 40\pi NC \cdot \mathrm{Re}\{\delta\} \cdot \log_{10} e \qquad (2.54)$$

For the forward increasing wave

$$\mathrm{Re}\{\delta\} = (3/4)^{1/2} \qquad (2.55)$$

which yields

$$G_{dB} = 40\pi NC \cdot (3/4)^{1/2} \cdot \log_{10} e = B \cdot C \cdot N \qquad (2.56)$$

where $B = 40\pi(3/4)^{1/2}\log_{10} e = 47.3$, N is the number of wavelengths and C is as defined in Eq.(2.36). The above value for G_{dB} is the approximate gain for the tube because the contribution of the other three waves is negligible.

Now the power flow in the transmission line will be related to the electric field of the helix. From transmission line theory [10] the power flow in the circuit without the electron beam is given by

$$P = |V|^2/(2K) \qquad (2.57)$$

where K is the characteristic impedance of the line. Eq.(2.57) relates the power flow in the transmission line to the electric field E of the helix because $E = |\Gamma V|$. A quantity which Pierce uses as a circuit parameter to connect the characteristic impedance, K, of the transmission line to calculations for the electric field of the helix is

$$E^2/(\beta^2 P) = 2K \qquad (2.58)$$

which is Pierce's equation (2.42) [6]. Using Eq.(2.36) and Eq.(2.58) the unitless gain parameter, C, is related to the electric field and the power in the transmission line by

$$C^3 = (2K)(I_0/(8V_0)) = (E^2/\beta^2 P)(I_0/(8V_0)) \qquad (2.59)$$

where E is the magnitude of the electric field, and P is the RF power in the circuit.

From the analysis of the fields of a sheath helix given in Pierce's chapter 3 [11] and appendix 2 [12] Pierce obtains for the field at the electron beam radius

$$(E^2/\beta^2 P)^{1/3} = (\beta/\beta_o)^{1/3}(\tau/\beta)^{4/3}F(\tau\bar{b})[I_o{}^2(\tau\bar{a}) - I_1{}^2(\tau\bar{a})]^{1/3} \tag{2.60}$$

where $\bar{a}$ is the electron beam radius,

$$\beta = w/v_p \tag{2.61}$$

$$\beta_o = w/c \tag{2.62}$$

and $$F(\tau\bar{b}) = \left(\frac{\tau\bar{b}}{240}\frac{I_o}{K_o}\left[\frac{I_1}{I_o} - \frac{I_o}{I_1} + \frac{K_o}{K_1} - \frac{K_1}{K_o} + \frac{4}{\tau\bar{b}}\right]\right)^{-1/3} \tag{2.63}$$

with the I_n's and K_n's being modified Bessel functions of argument $\tau\bar{b}$, and order n, $\bar{b}$ the radius of the helix,

$$\tau = w(v_p{}^{-2} - c^{-2})^{1/2} \tag{2.64}$$

c the speed of light in vacuum, and v_p the phase velocity of the increasing wave.

For a helix the phase velocity [13] of the increasing wave is given by

$$v_p = c \cdot \sin(\theta) \tag{2.65}$$

with θ being the pitch angle of the helix. Substituting Eq.(2.65) into Eq.(2.60) and using the fact that $\beta = w/v_p$ and $\beta_o = w/v_o$ results in

$$E^2/(\beta^2 P) = (1 - \sin^2\theta)^2 F^3(\tau\bar{b})[I_o{}^2(\tau\bar{a}) - I_1{}^2(\tau\bar{a})]/\sin(\theta) \tag{2.66}$$

So that comparisons can be made between Pierce's theory and Kalyanasundaram's theory, conversion from wavelengths in Pierce's gain formula to Kalyanasundaram's normalized unit of length is required. Kalyanasundaram defines normalized length [3] as

$$z = \frac{w\bar{z}}{v_o} \tag{2.67}$$

where $\bar{z}$ is the axial distance coordinate in meters,w is the frequency of the RF signal in rad/sec, and v_o is the initial electron velocity in meters/sec.

Eq.(2.68) relates $\bar{z}$ to N and Eq.(2.69) relates wavelength to the phase velocity of the increasing wave and the RF frequency of the signal.

$$N = \bar{z}/(\text{wavelength of RF signal}) \tag{2.68}$$

$$\text{wavelength} = 2\pi v_p/w \tag{2.69}$$

Substituting Eq.(2.68) and Eq.(2.69) into Eq.(2.67) and assuming that v_p is approximately equal to v_o gives

$$N = z/(2\pi) \tag{2.70}$$

Substituting Eq.(2.70) into Eq.(2.56) gives the gain of the traveling wave tube in terms of the normalized distance coordinate z.

$$G_{dB} = 7.528 \cdot C \cdot z \tag{2.71}$$

where C is defined in Eq.(2.59) through Eq.(2.64).

Eq.(2.71) can be used to compare Kalyanasundaram's results to Pierce's.

Table 2.1 shows the values which were used by Kalyanasundaram [14] in his numerical solution of the traveling wave tube.

Table 2.1. Values used in Kalyanasundaram's numerical solution of a TWTA.

Accelerating voltage, V_0	= 5.6 kV
Beam current, I_0	= 0.06 A
Helix pitch , Θ	= 0.1349 radians
Normalized beam radius, $a = \bar{a}/\bar{b}$	= 0.44

For the above case and assuming that the phase velocity of the wave and the electron velocity are equal, the theoretical small signal gain of a traveling wave tube is

$$G_{dB} = 0.258z \text{ dB} \qquad (2.72)$$

Removing the assumption that the cold wave phase velocity and the electron velocity are equal and also removing the narrow beam assumption the gain G_{dB} decreases because the value of B will be less than 47.3. The quantity B decreases because not as much energy will be transferred from the electron stream to the increasing voltage wave. Space charge effects which are due to the capacitive impedance and the diameter of the beam will also reduce the value of B. The new value of B determined from Pierce's Fig. A6.4 [15] and Fig. 8.11 [16] for the case of a solid electron beam for a TWT with the parameters listed in Table 2.1 is

$$B = 32 \tag{2.73}$$

This results in

$$G_{dB} = 0.18z \text{ dB} \tag{2.74}$$

From the above development a number for the small signal gain of a traveling wave tube which has parameters as listed in Table 2.1 was obtained. This value of G_{dB} = 0.18z dB will be compared to N. Kalyanasundaram's results for the gain of a TWTA for small signals given in Section 3.0.

3.0 Kalyanasundaram's Large Signal Analysis of A Traveling Wave Tube Amplifier.

This section contains a description of Kalyanasundaram's large signal theory of a traveling wave tube amplifier. Included are his equations which he programmed. Some of the equations may appear to be ambiguous; however, there will be no attempt to interpret his equations in this section. Section 4.0 will interpret the equations and describe how they were programmed on a VAX 750 at Kansas State University.

The purpose of Kalyanasundaram's work was to mathematically model a traveling wave tube amplifier without resorting to using the transmission line analogy as Pierce did. Kalyanasundaram did this by using the Eulerian formulation for Maxwell's field equations and the Lagrangian formulation for the electron ballistic equation. Kalyanasundaram then substituted the expression for the field into the electron ballistic equation. This resulted in a double Fourier series expansion over time and space to obtain the axial electric field of the tube.

With this approach Kalyanasundaram found a steady state solution for a single frequency RF input signal. The assumptions listed below were used by Kalyanasundaram to achieve the solution.

1) A sheath-helix model is used for the slow wave circuit [3].

2) Operation of the amplifier is axially symmetric [3].

3) The electron beam is axially confined and partially fills the tube [3].

4) Nonrelativistic operation is assumed, so that the RF magnetic force terms can be dropped from the ballistic equation [3].

5) The effect of the transverse electric field components on the electron motion is negligible [3].

6) There is no initial transverse motion of the electrons [3].

7) The velocity, v_o, and the charge density, ρ_o, of the entering electron stream are constant. The DC electron velocity is assumed to be close to the cold wave phase velocity, v_p, of the slow wave circuit at the input signal frequency, to meet the condition of approximate synchronism between the electron beam and the traveling electromagnetic wave [3].

Table 3.1 below lists the dimensional and nondimensionalised variables used by Kalyanasundaram in his development.

Table 3.1 Defining relations for the TWT variables used by Kalyanasundaram.

Dimensional variables	Nondimensionalised variables
v_o: the initial electron velocity	
w : angular frequency of the RF signal	
$\bar{z}$: axial coordinate	$z = w\bar{z}/v_o$
$\bar{r}$: radial coordinate	$r = \bar{r}/\bar{b}$
$\bar{t}$: time	$t = w\bar{t}$
$\bar{t}_o$: electron entrance time	$t_o = w\bar{t}_o$
$\bar{t}(\bar{z},\bar{r},\bar{t}_o)$: electron arrival time at the position specified by $\bar{z}$ and $\bar{r}$	$t(z,r,t_o) = w\bar{t}(\bar{z},\bar{r},\bar{t}_o)$
$\bar{a}$: radius of electron beam	$a = \bar{a}/\bar{b}$
$\bar{b}$: radius of sheath helix	
$\bar{d}$: interaction length of tube	$d = w\bar{d}/v_o$
$\bar{p}(\bar{z},\bar{r},\bar{t})$: electron charge density	$p(z,r,t) = v_o^2 Z_o \bar{p}(\bar{z},\bar{r},\bar{t})/wA_o$

A_o is the amplitude of the axial electric field component at $\bar{z} = 0$ and $\bar{r} = \bar{b}$. Z_o is the intrinsic impedance of vacuum.

Table 3.2 below lists the constants used in the development of the TWT equations.

Table 3.2 Constants used in the equations

Z_o	: intrinsic impedance of vacuum	376.7 ohms
M_e	: mass of an electron	9.1095×10^{-31} kg
e	: charge on an electron	1.602×10^{-19} C
c	: speed of light	2.9979×10^{8} m/s

Kalyanasundaram uses the ballistic equation to obtain an integral equation for the arrival time at point z,r of an electron entering the tube at time t_o. This equation is

$$t(z,r,t_o) = t_o + \int_0^z dx/\{1 - 2\varepsilon\int_0^x f_1(s,r,t(s,r,t_o))ds\}^{1/2} \tag{3.1}$$

where $\varepsilon = A_o e/M_e w v_o$, and t_o is the electron entrance time, for $0 \le z \le d$ and $0 \le r \le a$. He solves this integral equation by iteration based on an initial assumption for the arrival time.

Kalyanasundaram uses the following equation [14] to calculate the axial electric field component by forming a temporal Fourier series.

$$f_1(z,r,t) = \sum_{m=1}^{M}(f_{1m}(z,r)\exp(jmt) + c.c.) \tag{3.2}$$

where c.c. denotes the complex conjugate of the expression.

Recursively, the temporal Fourier coefficients [14] are given by

$$f_{1m}(z,r) = \delta_{1m} A I_o(p_1 r)\exp(-jk_1 z)/2I_o(p_1) + [F_{1m}(z,r) + jF_{2m}(z,r)]/2m \qquad m = 1,2,\ldots,M \tag{3.3}$$

with δ_{1m} being the Kronecker delta and the phase factor of the RF input signal given by

$$A = \exp(j\Phi) \tag{3.4}$$

with Φ being the phase angle of the RF input signal. The values of p_m, which are eigenvalues for the sheath helix when there is no electron beam present, and k_m are calculated from the dispersion relation [3], which is derived from the sheath helix model, given below.

$$\frac{(a_2 p_m)^2 I_o(mp_m) K_o(mp_m)}{a^2{}_1 I'_o(mp_m) K'_o(mp_m)} + \cot^2\Theta = 0 \tag{3.4}$$

$$k^2{}_m = a^2{}_1 + a^2{}_2 p^2{}_m \tag{3.5}$$

$$a_1 = v_o/c \tag{3.6}$$

$$a_2 = v_o/w_o b \tag{3.7}$$

with I_o, K_o, I_o', and K_o' being Bessel functions of argument mp_m and order 0.

The spatial Fourier series [14] of the axial electric field is formed below.

$$F_{lm}(z,r) = \sum_{n=0}^{N}(2 - \delta_{0n})[F_{lmn}(r)\cos nk_d z - I_o(mp_m r)F_{lmn}(1)\cdot\cos mk_m z/I_o(mp_m)], \quad l = 1,2 \tag{3.4}$$

with
$$k_d = \pi/d \tag{3.4a}$$

The Fourier coefficients for the spatial Fourier series are determined from the equations [14] below.

$$F_{1mn}(r) = \int_0^a [H_{1mn}(r,y)f_{smn}(y) - H_{2mn}(r,y)f_{cmn}(y)]y\,dy \tag{3.5}$$

$$F_{2mn}(r) = \int_0^a [H_{1mn}(r,y)f_{cmn}(y) + H_{2mn}(r,y)f_{smn}(y)]y\,dy \tag{3.6}$$

where

$$H_{1mn}(r,y) = a_o p^2{}_{mn} C_o(p_{mn}r)C_o(p_{mn}y)[b_{mn} + I_{mn}(r,y)] \tag{3.7}$$

$$H_{2mn}(r,y) = a_o p^2{}_{mn} J_o(p_{mn}y)J_o(p_{mn}r)a_{mn} \tag{3.8}$$

$$a_o = p_o/\pi a_1 d \tag{3.9}$$

with p_o being the normalized electron beam current density at the entrance of the tube. J_o, a_{mn}, b_{mn}, and C_o are Bessel functions which are defined below.

$$f_{smn}(y) = \int_0^d \cos nk_d x \, dx \int_{-\pi}^{\pi} \sin mt(x,y,\tau) \, d\tau \quad (3.10)$$

and

$$f_{cmn}(y) = \int_0^d \cos nk_d x \, dx \int_{-\pi}^{\pi} \cos mt(x,y,\tau) \, d\tau \quad (3.11)$$

The following Bessel function equations are needed to interpret the above equations.

$$H_l(p_{mn}X) = J_l(p_{mn}X) - jY_l(p_{mn}X), \quad l=0,1 \quad (3.12)$$

which is the Hankel function of the second kind of order l. Also, C_o is defined by

$$C_o(p_{mn}X) = \begin{cases} J_o(p_{mn}X) & \text{for } 0 \le n < ma_1/k_d \\ I_o(p_{mn}X) & \text{for } n > ma_1/k_d \end{cases} \quad (3.13)$$

with J_o and I_o being Bessel functions of argument $p_{mn}X$ and order 0. D_o is defined by

$$D_o(p_{mn}X) = \begin{cases} \pi Y_o(p_{mn}X)/2 & \text{for } 0 \le n < ma_1/k_d \\ K_o(p_{mn}X) & \text{for } n > ma_1/k_d \end{cases} \quad (3.14)$$

with Y_o and K_o being Bessel functions of argument $p_{mn}X$ and order 0. The Bessel function a_{mn} is defined by

$$a_{mn} = \begin{cases} \mathrm{Re}[\pi H_o(p_{mn})/2J_o(p_{mn})\Omega_{mn}] & \text{for } 0 \le n < ma_1/k_d \\ 0 & \text{for } n > ma_1/k_d \end{cases} \quad (3.15)$$

with H_o being the Hankel function of the second kind with argument p_{mn} and order 0. The Bessel function b_{mn} is defined by

$$b_{mn} = \begin{cases} \mathrm{Im}[\pi H_o(p_{mn})(1 - Q^{-1}_{mn})/2J_o(p_{mn})] \\ \qquad\qquad \text{for } 0 \le n < ma_1/k_d \\ K_o(p_{mn})(Q^{-1}_{mn} - 1)/I_o(p_{mn}) \\ \qquad\qquad \text{for } n > ma_1/k_d \end{cases} \tag{3.16}$$

The Bessel function I_{mn} is defined by

$$I_{mn}(r,y) = D_o(p_{mn}\max(r,y))/C_o(p_{mn}\max(r,y)) \tag{3.17}$$

The Bessel function Q_{mn} is defined by

$$Q_{mn} = \begin{cases} 1 + a^2{}_2p^2{}_{mn}I_o(p_{mn})K_o(p_{mn})\tan^2\theta/m^2a^2{}_1I'_o(p_{mn})K'_o(p_{mn}) \\ \qquad\qquad \text{for } n > ma_1/k_d \\ 1 + a^2{}_2p^2{}_{mn}J_o(p_{mn})H_o(p_{mn})\tan^2\theta/m^2a^2{}_1J'_o(p_{mn})H'_o(p_{mn}) \\ \qquad\qquad \text{for } 0 \le n < ma_1/k_d \end{cases} \tag{3.18}$$

An argument which appears in many of the above Bessel functions is defined below.

$$p_{mn} = |((m^2a^2{}_1 - n^2k_d{}^2)/a^2{}_2)^{1/2}| \tag{3.19}$$

Next, Kalyanasundaram defines a parameter, α, which relates the power in the input RF signal power, P_{in}, to the dc power of the electron beam. The dc beam power is given in Eq.(3.21).

$$\alpha = 10\log_{10}(P_{in}/P_{dc}) \tag{3.20}$$

$$P_{dc} = \text{beam current} \cdot \text{beam voltage} \tag{3.21}$$

Kalyanasundaram solved the above equations iteratively on an ICL 2955 mainframe for some typical values of α. Table 3.3 below shows the parameters which remained constant for all runs.

Table 3.3 Parameters for the solution of the TWT equations.

beam voltage,	$V_0 = 5600$ V
beam current,	$I_0 = 0.06$ A
helix pitch,	$\tan(\theta) = 0.1357$
normalized beam radius,	$a = 0.44$
$a_2 = v_o/w\bar{b}$	$a_2 = 0.453152$
RF phase factor,	$A = 1.0$
axial step size,	$dz = 0.20$
radial step size,	$dr = 0.11$
time step size,	$dt_o = \pi/12$
number of temporal harmonics;	$M = 3$
number of spatial harmonics;	$N = 48$

Note that integral step sizes are given in Table 3.3; however, there was no indication as to which method of numerical integration he used.

Fig. 3.1a and Fig. 3.1b below shows some plots of electron exit times for d = 120 and α = -30 dB, -50 dB, and $-\infty$ dB. An α of $-\infty$ dB corresponds to an RF input power of 0.

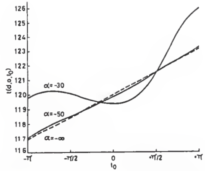

Figure 3.1a. Electron exit time versus entrance time for electrons at the center of the tube. (from Kalyanasundaram [14] p. 164)

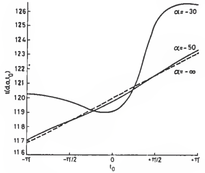

Figure 3.1b. Electron exit time versus entrance time for electrons at the edge of the beam. (from Kalyanasundaram [14] p. 164)

The plots in Fig. 3.1a and Fig. 3.1b for $\alpha = -30$ show electron overtaking which is known to occur when the RF input power is large enough. Also it should be noted that there is little variation in electron exit times between the center of the beam and the edge of the beam. Pierce's theory assumes there is no variation.

Fig. 3.2a and Fig. 3.2b below shows plots of gain over input power versus normalized distance for an α of -40 dB. Gain is defined by

$$\bar{f}_{1m}(z,r) = |f_{1m}(z,r)/f_{11}(0,r)| \tag{3.22}$$

The slope of the increasing part of the graph in Fig. 3.2a, measured from z = 30 to z = 90, is 0.217 dB/z. The slope of the increasing part of the graph in Fig. 3.2b, measured from z = 60 to z = 90, is 0.2 dB/z. The value of the slope predicted by Pierce's formulas for small signals was between 0.26 dB/z and 0.18 dB/z. This seems to indicate that the two theories' gain formulas agree for small signals; however, it would be better to compare the slopes for a smaller α for a better comparison.

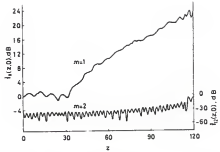

Figure 3.2a. Plot of gain at the center of the tube versus normalized distance for the TWT defined by the parameters in Table 3.3.
(from Kalyanasundaram [14] p. 165)

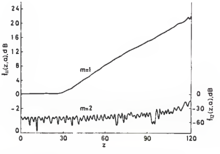

Figure 3.2b. Plot of gain at the edge of the beam versus normalized distance for the TWT defined by the parameters listed in Table 3.3.
(from Kalyanasundaram [14] p. 165)

In Fig. 3.2 note that the gain does not start to increase until $z = 30$. This is because the electrons need time to bunch before they can transfer energy to the electromagnetic field.

The gain comparisons given above seem to verify Kalyanasundaram's large signal theory in the limiting case of small input signals.

4.0 Fortran Computer Program Development of a Traveling Wave Tube Amplifier Mathematical Model.

The purpose of this work was to develop a Fortran computer program named TUBE which would solve the equations developed by Kalyanasundaram. The complete program listing is given in the appendix. First, the program was used to solve the same case as Kalyanasundaram solved and the results were compared. Next, the results were going to be extend by taking more terms of the Fourier series expansion and taking smaller steps in the numerical integrations. This second step was to be implemented on a CRAY supercomputer because the program took approximately 12 hours on a VAX 750 to perform 8 iterations. The second step was never taken because there appears to be some sort of numerical instability in the program. The results produced by TUBE agree qualitatively with Kalyanasundaram's results but they are not exactly the same as his results.

All equations in Section 3.0 which were ambiguous will be given in this section in the form that the program used them. Refer to the appendix for all program listings.

The program begins by accepting the TWT parameters and the numeric integration step sizes from the user. All integrals are performed with trapezoidal integration. Table 4.1 below contains the parameters and step sizes used in the solution of the equations. These parameters and step sizes are the same as those used by Kalyanasundaram.

Table 4.1. TWT parameters and integral step sizes, which remain constant, used in the solution of the TWT equations.

radial step size;	$dr = 0.11$
axial step size;	$dz = 0.2$
time step size;	$dt_o = \pi/12$
beam voltage;	$V_0 = 5600$ volts
beam current;	$I_0 = 0.06$ amps
normalized beam radius;	$a = 0.44$
normalized tube length;	$d = 120$
$a_2 = v_o/(w_o\bar{b})$;	$a_2 = 0.453152$
helix pitch;	$\tan(\theta) = 0.1357$
phase factor of input signal;	$A = -1.0$
number of temporal harmonics;	$M = 3$
number of spatial harmonics;	$N = 48$

Note that there are two important differences between Table 3.3 which lists the parameters Kalyanasundaram used and the parameters listed above. The first is that the phase factor, A, listed above is -1.0 and in Table 3.3 it is 1.0. Changing the phase factor, A, produced better agreement between Kalyanasundaram's results and the FORTRAN program's results. The next difference is that the quantity a_2 has been strictly interpreted by adding parenthesis. The reason for interpreting a_2 as shown in Table 4.1 is that it is known that a_2 has to be a unitless quantity.

Next, the program calculates the initial velocity of the electrons via the relativistic kinetic energy equation.

$$K.E. = eV_0 = m_e c^2[(1 - v^2{}_o/c^2)^{-1/2} - 1] \quad (4.1)$$

Solving Eq.(4.1) for initial electron velocity, v_o, the equation below is obtained which is implemented by the program named TUBE.

$$v_o = c(1 - [eV_0/(M_e c^2 + 1)]^{-2})^{1/2} \quad (4.3)$$

The program calculates the phase velocity of the increasing wave using the equation below which was derived from Pierce's analysis of a sheath helix [11].

$$v_p = c \cdot \sin(\Theta) \quad (4.4)$$

Next, the program calculates the normalizing factor a_1

$$a_1 = v_o/c \quad (4.5)$$

From the analysis of the electric field of a sheath helix derived by Pierce [11], the amplitude of the axial electric field component, A_o, at the entrance of the tube at the helix radius is found.

$$A^2{}_o = P_{in} c v_p w^2 (v^{-2}{}_p - c^{-2}) F^3(\tau\bar{b})[I^2{}_o(\tau\bar{b}) - I^2{}_1(\tau\bar{b})] \quad (4.6)$$

where $\tau = w(v_p{}^{-2} - c^{-2})^{1/2}$. (4.6a)

The program next calculates ε as shown below.

$$\varepsilon = A_o e/(m_e w v_o) \quad (4.7)$$

The non-normalized electron beam current density, $\bar{p}_o$, at the tube entrance is given by

$$\bar{p}_o = \frac{I_0}{\pi(\bar{a})^2 v_o} \qquad (4.8)$$

Using Kalyanasundaram's normalizing factors [3] $\bar{a}$ can be represented in terms of the normalizing factors. This is shown in Eq.(4.9) through Eq.(4.11)

$$\bar{a} = a\bar{b} \qquad (4.9)$$

$$\bar{b} = \frac{v_o}{wa_2} \qquad (4.10)$$

$$\bar{a} = \frac{av_o}{wa_2} \qquad (4.11)$$

Substituting Eq.(4.11) into Eq.(4.8) gives the non-normalized electron beam current density at the entrance of the tube in terms of the normalizing factors.

$$\bar{p}_o = \frac{I_0 w^2 a^2{}_2}{\pi v^3{}_o a^2} \qquad (4.12)$$

Kalyanasundaram's normalization of $\bar{p}_o$ [3], is shown below.

$$p_o = \frac{v^2{}_o \bar{p}_o Z_o}{wA_o} \qquad (4.13)$$

Substituting Eq.(4.12) into Eq.(4.13) yields the normalized beam current density equation, which is valid at the entrance of the tube, and which is implemented by the program named TUBE.

$$p_o = \frac{Z_o I_0 w a^2{}_2}{A_o \pi v_o a^2} \tag{4.14}$$

TUBE calculates the quantity a_o using

$$a_o = \frac{p_o}{\pi a_1 d} \tag{4.15}$$

which is one of Kalyanasundaram's normalizing factors.

The program calls a subroutine, ZEROIN, to solve for p_m, which is the eigenvalue of the cold wave problem for the sheath helix at the angular frequency mw, in the dispersion relation [3] given below.

$$\frac{(a_2 p_m)^2 I_o(mp_m) K_o(mp_m)}{a^2{}_1 I_1(mp_m) K_1(mp_m)} - \cot^2\theta = 0 \tag{4.16}$$

for $m = 1, 2, 3$

TUBE relates the quantity k_m [3] to p_m using

$$k_m = a^2{}_1 + (a_2 p_m)^2 \tag{4.17}$$

for $m = 1, 2, 3$

The quantity p_{mn} [3] is calculated by TUBE using

$$p_{mn} = \frac{|[(ma_1)^2 - (nk_d)^2]|^{1/2}}{a_2} \qquad (4.18)$$

for m = 1, 2, 3

n = 0, 1, 2, ... 48

The quantities a_{mn}, and b_{mn} [14] are calculated by subroutines AMN and BMN which implement Eq.(4.19), Eq.(4.20) and Eq.(4.21).

$$a_{mn} = \begin{cases} \mathrm{Re}\left\{\dfrac{\pi H_o(p_{mn})}{2J_o(p_{mn})Q_{mn}}\right\} & \text{for } 0 \le n \le ma_1/k_d \\ 0 & \text{for } n > ma_1/k_d \end{cases}$$

for m = 1, 2, 3

n = 0, 1, 2, ... 48

(4.19)

$$b_{mn} = \begin{cases} \mathrm{Im}\left\{\dfrac{\pi H_o(p_{mn})(1 - Q^{-1}{}_{mn})}{2J_o(p_{mn})}\right\} & \text{for } 0 \le n \le ma_1/k_d \\ \dfrac{K_o(p_{mn})(Q^{-1}{}_{mn} - 1)}{I_o(p_{mn})} & \text{for } n > ma_1/k_d \end{cases}$$

for m = 1 , 2, 3

n = 0, 1, 2, ...48

(4.20)

where

$$
Q_{mn} = \begin{cases} 1 - \dfrac{(a_2 p_{mn} \tan(\theta))^2 I_o(p_{mn}) K_o(p_{mn})}{(ma_1)^2 I_1(p_{mn}) K_1(p_{mn})} & \text{for } n > ma_1/k_d \\[2ex] 1 - \dfrac{(a_2 p_{mn} \tan(\theta))^2 J_o(p_{mn}) H_o(p_{mn})}{(ma_1)^2 J_1(p_{mn}) H_1(p_{mn})} & \text{for } 0 \le n \le ma_1/k_d \end{cases}
$$

$$
\text{for } m = 1, 2, 3 \quad n = 0, 1, 2, \ldots 48 \tag{4.21}
$$

The program next calculates H_{1mn} and H_{2mn}, which are Bessel functions, using Eq.(4.22) through Eq.(4.26). The variables r and y in these equations are radial variables which run from the center of the electron beam to the edge of the electron beam. With a radial step size of 0.11 and normalized beam radius of 0.44 the variables r and y each take on the values 0, 0.11, 0.22, 0.33, 0.44. The integer m runs from 1 to 3 and the integer n runs from 0 to 48.

$$
H_{1mn}(r,y) = a_o p^2{}_{mn} C_o(p_{mn} r) C_o(p_{mn} y)[b_{mn} + I_{mn}(r,y)] \tag{4.22}
$$

$$
H_{2mn}(r,y) = a_o p^2{}_{mn} J_o(p_{mn} y) J_o(p_{mn} r) a_{mn} \tag{4.23}
$$

where

$$
C_o(p_{mn} X) = \begin{cases} J_o(p_{mn} X) & \text{for } 0 \le n \le ma_1/k_d \\ I_o(p_{mn} X) & \text{for } n > ma_1/k_d \end{cases} \tag{4.24}
$$

$$
D_o(p_{mn} X) = \begin{cases} \pi Y_o(p_{mn} X)/2 & \text{for } 0 \le n \le ma_1/k_d \\ K_o(p_{mn} X) & \text{for } n > ma_1/k_d \end{cases} \tag{4.25}
$$

and

$$I_{mn}(r,y) = D_o(p_{mn}\max(r,y))/C_o(p_{mn}\max(r,y)) \qquad (4.26)$$

The program next creates an initial guess array, called TIME(Z,R,T), for normalized electron arrival times at all normalized radial and axial electron positions. Eq.(4.27) defines the array. The initial guess array uses Kalyanasundaram's time normalization, $T = w\bar{T}$, and assumes that no electron overtaking occurs so that the electrons pass through the tube in a linear manner. The integer variable Z runs from 0 to 600 which represents a tube of normalized length of d = 120. The integer variable R runs from 0 to 4 which represents an electron beam of normalized radius a = 0.44. The integer variable T runs from 0 to 24 which represents 24 electrons per period.

$$TIME(Z,R,T) = T \cdot D\_T + Z \cdot D\_Z - \pi$$

where $$D\_T = \pi/12$$

and $$D\_Z = 0.20 \qquad (4.27)$$

Now that an initial guess has been made for the solution of the electron position times, the program begins iterating to find the solution for the actual electron position times. The program begins the iteration by integrating Eq.(4.28) and Eq.(4.29) using the trapezoidal rule.

$$f_{smn}(y) = \int_0^d \cos(nk_d x) \int_{-\pi}^{\pi} \sin(m\cdot TIME(Z,R,T))\ dt\ dx \tag{4.28}$$

$$f_{cmn}(y) = \int_0^d \cos(nk_d x) \int_{-\pi}^{\pi} \cos(m\cdot TIME(Z,R,T))\ dt\ dx \tag{4.29}$$

In the sums which replace the above integrals in trapezoidal integration dt = $\pi/12$ and dx = 0.20 . The program calls a subroutine called F1_X which calculates the inner integrals in Eq.(4.28) and Eq.(4.29) using trapezoidal integration.

The program next calculates the quantities $F_{1mn}(r)$ and $F_{2mn}(r)$, which are the Fourier coefficients of the spatial Fourier expansion of the axial electric field, using Eq.(4.30) and Eq.(4.31). The integrals are implemented with trapezoidal integration.

$$F_{1mn}(r) = \int_0^a [H_{1mn}(r,y)f_{smn}(y) - H_{2mn}(r,y)f_{cmn}(y)]y\ dy \tag{4.30}$$

$$F_{2mn}(r) = \int_0^a [H_{1mn}(r,y)f_{cmn}(y) + H_{2mn}(r,y)f_{smn}(y)]y\ dy \tag{4.31}$$

The program next performs the Fourier series sum of the spatial harmonics of the axial electric field component for n = 0 to 48. This is implemented by TUBE as shown below.

$$F_{lm}(z,r) = \sum_{n=0}^{N=48} (2 - \delta_{0n})[F_{lmn}(r)\cos(nk_d z) - I_o(mp_m r)F_{lmn}(1)\cos(mk_m z)/I_o(mp_m)] \qquad l = 1,\ 2 \tag{4.32}$$

The program next calculates the Fourier coefficients needed for the temporal Fourier series expansion using

$$f_{lm}(z,r) = \delta_{lm} A I_o(p_1 r) EXP(-jk_1 z)/(2I_o(p_1)) + [F_{1m}(z,r) + jF_{2m}(z,r)]/(2m) \tag{4.33}$$

TUBE performs the temporal Fourier series expansion of the axial electric field using

$$f_1(z,r,t) = \sum_{m=1}^{M=3} 2\mathrm{Re}\{f_{1m}(z,r)\exp(jm\cdot TIME(Z,R,T)\} \tag{4.34}$$

Now TUBE performs the double integral in Eq.(4.35) using trapezoidal integration. The result is the new estimate of electron position times.

$$t(z,r,t_o) = t_o + \int_0^z \frac{1}{1 - 2\varepsilon \int_0^x f_1(s,r,t(s,r,t_o))\, ds^{\ 1/2}}\, dx \tag{4.35}$$

After the double integral in Eq.(4.35) is performed, convergence is checked. The convergence test used compares the old electron exit times to the new electron exit times. The convergence test is shown below.

If $|t_i(d,r,t_o) - t_{i+1}(d,r,t_o)| < 0.2$ for all r and t_o

then the solution has converged. The convergence test shown above was used so that the results of the computer simulation could be compared to Kalyanasundaram's results which were also based on this convergence test. It should be noted that this is not a mathematically strict method for testing convergence but was used for comparison purposes.

Fig. 4.1 and Fig. 4.2 below show plots of electron exit times for an α of -30 dB. Comparing these plots to Kalyanasundaram's plots one can see they are qualitatively similar but they are not exactly the same. The electron exit time plots show electron overtaking which is known to occur when the input signal level is large enough. Fig. 4.3 and Fig. 4.4 show gain curves for an α of -30. These curves indicate the tube is beginning to saturate because the curves are beginning to flatten out at the end. The gain of the tube is calculated from the normalized Fourier coefficient magnitudes of the axial electric field. The plots in Fig. 4.3 and Fig. 4.4 which represent power gain are generated by

$$10\cdot\log_{10}[\bar{f}_{1m}(z,r)] = 10\cdot\log_{10}[|f_{1m}(z,r)/f_{11}(0,r)|] \quad (4.36)$$

Fig. 4.5 and Fig. 4.6 show plots of electron exit times for an α of -40 dB. Note that the plots show very little radial variation. Pierce's theory assumed that there was no radial variation. Gain plots for α = -40 dB are shown in Fig. 4.7 and Fig. 4.8. The slope of these plots, measured from z = 60 to z = 90, is 0.2 dB/z. These are similar to Kalyanasundaram's plots but not identical. The plots in Fig. 4.7 and Fig. 4.8 show some type of periodic numerical noise for which the explanation is not known.

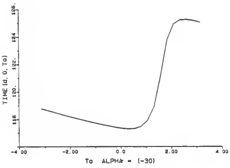

Figure 4.1. Electron exit time versus entrance time for electrons at the center of the tube. $\alpha = -30$, $d = 120$, 9 iterations performed in arriving at the solution.

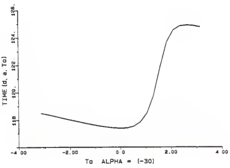

Figure 4.2. Electron exit time versus entrance time for electrons at the edge of the beam. $\alpha = -30$, $d = 120$, 9 iterations performed in arriving at the solution.

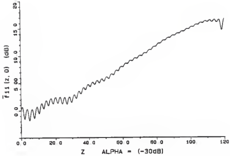

Figure 4.3. TWT power gain versus normalized distance along the tube at the center of the tube. α = -30, d = 120, 9 iterations performed in arriving at the solution.

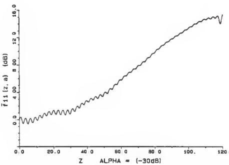

Figure 4.4. TWT power gain versus normalized distance along the tube at the edge of the electron beam. α = -30, d = 120, 9 iterations performed in arriving at the solution.

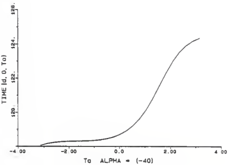

Figure 4.5. Electron exit time versus entrance time for electrons at the center of the tube. α = -40, d = 120, 9 iterations performed in arriving at the solution.

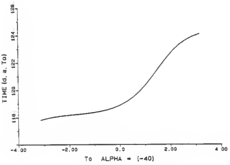

Figure 4.6. Electron exit time versus entrance time for electrons at the edge of the beam. α = -40, d = 120, 9 iterations performed in arriving at the solution.

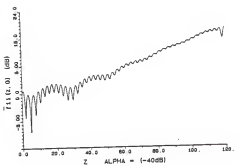

Figure 4.7. TWT power gain versus normalized distance along the tube at the center of the tube. α = -40, d = 120, 9 iterations performed in arriving at the solution.

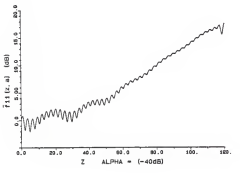

Figure 4.8. TWT power gain versus normalized distance along the tube at the edge of the electron beam. α = -40, d = 120, 9 iterations performed in arriving at the solution.

From the plots on the previous pages it is seen that the FORTRAN program, TUBE, gives results for gain which are similar to Kalyanasundaram's and Pierce's. The results for electron exit times are also similar to Kalyanasundaram's. The electron exit time plots show electron overtaking which is known to occur for large signal input. The electron exit time plots are linear for small signal input. There is some numerical noise in the gain plots which indicates some difference in the solution method used by Kalyanasundaram compared to the method used in TUBE. The cause of the noise needs to be determined before further investigation can be done.

5.0 Conclusion

This report described the development of Pierce's linear theory for the small signal power gain of a traveling wave tube. Using Pierce's theory the gain for a specific TWT was determined to be 0.18 dB/z. The purpose of the development of the Pierce theory was to verify Kalyanasundaram's non-linear theory in the limiting case of a small signal input.

This independent verification of Kalyanasundaram's theory has shown that his new theory produces gain results which are similar to Pierce's for small signals. For a signal input of $\alpha = -40$ Kalyanasundaram's theory predicted a gain of 0.21 dB/z. It was also seen that Kalyanasundaram's theory predicted electron overtaking which is known to occur for large signal inputs. The FORTRAN program, which was developed at Kansas State University, was seen to produce results which were similar qualitatively to Kalyanasundaram's but not identical. The gain predicted by TUBE for $\alpha = -40$ was 0.21 dB/z. TUBE also displayed electron overtaking for an input of $\alpha = -30$.

Further work which needs to be done includes a rederivation of Kalyanasundaram's equations. After finding and correcting any discrepancies, the FORTRAN program should be modified accordingly. When this is done the theory should be extended by including more terms in the Fourier series expansion. Also modeling a more practical traveling wave tube which includes losses would be beneficial.

References

[1] J. R. Pierce, Traveling Wave Tubes. New York: D. Van Nostrand, 1950.

[2] J. E. Rowe, Nonlinear Electron Wave Interaction Phenomena. New York: Academic Press, 1965

[3] N. Kalyanasundaram, "Large-signal field analysis of an O-type travelling wave amplifier. Part 1: Theory," IEE Proceedings, Vol. 131, Pt. 1, No. 5, pp 145-152, October 1985.

[4] J. R. Pierce, Traveling Wave Tubes. p 8, New York: D. Van Nostrand, 1950.

[5] S. Y. Liao, Microwave Devices and Circuits. p 213, Englewood Cliffs, New Jersey: Prentice Hall, 1980.

[6] J. R. Pierce, Traveling Wave Tubes. pp 5-18, New York: D. Van Nostrand, 1950.

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[10] M. A. Plonus, Applied Electromagnetics. p 577, New York: McGraw-Hill, 1978.

[11] J. R. Pierce, Traveling Wave Tubes. pp 19-48, New York: D. Van Nostrand, 1950.

[12] J. R. Pierce, Traveling Wave Tubes. pp 229-232, New York: D. Van Nostrand, 1950.

[13] S. Y. Liao, Microwave Devices and Circuits. p 139, Englewood Cliffs, New Jersey: Prentice Hall, 1980.

[14] N. Kalyanasundaram and R. Chinnadurai, "Large-signal field analysis of an O-type travelling wave amplifier. Part 2: Numerical results," IEE Proceedings, Vol. 133, Pt. 1, No. 4, pp 163-168, August 1986.

[15] J. R. Pierce, Traveling Wave Tubes. p 249, New York: D. Van Nostrand, 1950.

[16] J. R. Pierce, Traveling Wave Tubes. p 127, New York: D. Van Nostrand, 1950.

Appendix

FORTRAN Program Listings

```
***************************************************************
*       Department of Electrical and Computer Engineering     *
*                   Kansas State University                   *
*                                                             *
*       VAX FORTRAN source filename: TUBE.FOR                 *
***************************************************************
*
*   REFERENCES:       N. Kalyanasundaram, "Large-signal field
*                     analysis of an O-type travelling-wave
*                     amplifier. Part 1: Theory," IEE PROCEEDINGS,
*                     Vol. 131, Pt. 1, No. 5, pp 154-152,
*                     October 1984.
*
*                     N. Kalyanasundaram, and R. Chinnadurai,
*                     "Large-signal field analysis of
*                     an O-type travelling-wave amplifier. Part 2:
*                     Numerical results," IEE PROCEEDINGS,
*                     Vol. 133, Pt. 1, No. 4, pp 163-168,
*                     August 1986.
*
*
*   All equation numbers in this program refer to the above
*   references.
*
*
*   ROUTINE:          MAIN PROGRAM
*                     TUBE
*
*   DESCRIPTION       This program solves equations 29 and 42.
*                     The program first accepts constants related
*                     to the travelling wavetube. The program then
*                     normalizes the constants to be used in the
*                     calculations. Next Equation 22b is solved
*                     for the pm's. Next an initial guess is made
*                     of the solution. Next equations 42 and 29
*                     are itterated until the solution converges.
*
*
*   DOCUMENTATION
*   FILES:            None.
*
*   ARGUMENTS:
*
*
*   RETURN:           Not used.
*
*   ROUTINES          AMN, BMN, BESC, BESD, BESK01, BESJ01, BESY01,
*   CALLED:           KRON_DEL, ZEROIN, F1_X
*
*   AUTHOR:           Bradley P. Badke
*
```

```
*   DATE CREATED:   28SEP87    Version 1.0
*
*   REVISIONS:      None.
*
*****************************************************************

        PROGRAM         TUBE

        IMPLICIT         NONE

      REAL    a_o, aa, A_0, AMN, A(3,0:48), A1, A2, AP, ALPHA,
     +         BMN, B(3,0:48), BESJ01, BESK01, BESI01,
     +         BESC, BESD, B_RAD, BI1GB, BI0GB, BK1GB, BK0GB,
     +         C,
     +         D_R, D_Z, D_T, d,
     +        e, EPS,
     +         f1(0:600,0:5,0:24), F1MN(0:5), F2MN(0:5),
     +         fcmn(0:5), fsmn(0:5), F1_x, F2_x,
     +         F1M(0:600,0:5), F2M(0:600,0:5), F_GAMMA_B,
     +         FARR(0:600)

      REAL    GAMMA_B,
     +         H1MN(4,0:48,0:5,0:5),H2MN(4,0:48,0:5,0:5),
     +         H_PIT,
     +        I_0,
     +         K_d, K_m(3), KRON_DEL, K_1,
     +         Me,
     +         NZ,
     +         PI, P(3,0:48), P_m(3), Pin,
     +        Qo, QUIT,
     +        TIME(0:600,0:5,0:24), TEMP,
     +         VELo, VELp, VOLT,
     +         Zo, ZEROIN

      INTEGER          CAP_M, CAP_N, M, N, NUM_R, NUM_T,
     +        NUM_Z, R, T, X, Y, Z, I, L, MAX_IT,
     +         LL

      CHARACTER*10 NAME,TIMEA,TIMEO,FOURIER

      COMPLEX          Lo_f1m(3,0:600,0:5)

      COMMON  TIME

      EXTERNAL BESK01,BESJ01,BESI01,BESY01,BESC,BESD,ZEROIN,
     +          F1_X,AMN,BMN,KRON_DEL

      DATA PI/3.141592654/,e/1.602E-19/,Zo/376.7/,
     +     Me/9.1095E-31/,C/2.9979E8/

**    Accept TWT constants from the user.
```

```
PRINT*,'ENTER THE RADIAL STEP SIZE (0.11)'

READ(*,*) D_R

PRINT*,'ENTER THE AXIAL STEP SIZE (0.20)'

READ(*,*) D_Z

PRINT*,'ENTER THE TIME STEP SIZE. (PI/12 = 0.2617994)'

READ(*,*) D_T

PRINT*,'ENTER THE ACCELERATING VOLTAGE IN VOLTS. (5600)'

READ(*,*) VOLT

PRINT*,'ENTER THE BEAM CURRENT IN AMPS. (0.06)'

READ(*,*) I_0

PRINT*,'ENTER THE NORMALIZED INTERACTION
+        LENGTH OF THE TUBE. d = 120'

READ(*,*) d

PRINT*,'ENTER THE NORMALIZED BEAM RADIUS. a = 0.44'

READ(*,*) aa

PRINT*,'ENTER a2. a2 = 0.453152'

READ(*,*) A2

PRINT*,'ENTER THE TANGENT OF THE HELIX PITCH.
+        (TAN(PSI) = 0.1357)'

READ(*,*) H_PIT

PRINT*,'ENTER THE PHASE FACTOR OF THE
+        INPUT SIGNAL. (1)'

READ(*,*) AP

PRINT*,'ENTER THE NUMBER OF dB THE RF INPUT
+        POWER IS BELOW THE DC BEAM POWER. (-30 etc)'

READ(*,*) ALPHA

Pin = I_0*VOLT*(10**(ALPHA/10))

PRINT*,'Enter the number of temporal harmonics,
+        "CAP_M = 3"'
```

```
      READ(*,*) CAP_M

      PRINT*,'Enter the number of spatial harmonics,
     +          "CAP_N = 48"'

      READ(*,*) CAP_N

      PRINT*,'ENTER THE MAXIMUM NUMBER OF ITTERATIONS'

      READ*,MAX_IT

**  This is the relativistic K.E. equation which gives
**  initial electron velocity.

      VELo = C*SQRT(1 - (e*VOLT/(Me*C**2) + 1) ** -2)

**  This is the phase velocity equation for a helix
**  slow wave structure.

      VELp = C * SIN(ATAN(H_PIT))
      PRINT*,'VELp',VELp

**  This is a normalizing factor.  (Eqn. 4)

      A1 = SQRT(1 - (e*VOLT/(Me*C**2) + 1) ** -2)

**  The following equations were derived from Pierce's
**  Traveling Wave Tubes (Appendix 2). This is done to
**  determine the strength of the electric field at the
**  entrance of the tube.

      GAMMA_B = VELo * SQRT( (VELp ** -2) - (C ** -2) ) / A2

      BI1GB = BESI01(GAMMA_B,1,1)
      BI0GB = BESI01(GAMMA_B,0,1)
      BK1GB = BESK01(GAMMA_B,1,1,NZ)
      BK0GB = BESK01(GAMMA_B,0,1,NZ)
      PRINT*,BI1GB,BI0GB,BK1GB,BK0GB

**   Note when comparing F_GAMMA_B in the program to
**   Pierce's formula that my F_GAMMA_B is cubed.

       F_GAMMA_B = 240*BK0GB/GAMMA_B/BI0GB/(
     +            BI1GB/BI0GB - BI0GB/BI1GB
     +            + BK0GB/BK1GB - BK1GB/BK0GB
     +            + 4/GAMMA_B)

**  Multiply in Pierce's correction factor for off axis
**  fields.

**    F_GAMMA_B = F_GAMMA_B*(BI0GB**2.0 - BI1GB**2.0)
```

```
**  Calculate a_o for use in equation 42e.

      a_o = Zo*I_0*(A2**2)/A1/d/VELo/(PI**2)/(aa**2)/
     +       SQRT(Pin*C*VELp*F_GAMMA_B)/(VELp**-2 - C**-2)

**  Calculate EPS for use in equation 29.

      EPS = SQRT(Pin*C*VELp*F_GAMMA_B)*((VELp ** -2) - (C** -2))
     +       *e/Me/VELo

**  Total number of radial terms is NUM_R + 1.

      NUM_R = NINT(aa/D_R)

**  Total number of electrons is NUM_T + 1.

      NUM_T = NINT(2*PI/D_T)

**  Total number of axial terms is NUM_Z + 1.

      NUM_Z = NINT(d/D_Z)

      K_1 = VELo/VELp

      K_d = PI/d

** Evaluate all AMN, BMN,PMN, K_m, and P_m so it is not done
** more than necessary. AMN, and BMN are equation 41. PMN is
** equation 19. K_m, and P_m are found from the roots of
** equation 22b and 22c.

      DO 10 M = 1, CAP_M

         DO 5 N = 0, CAP_N

            P(M,N) = SQRT(ABS((((M*A1)**2) - ((N*K_d)**2))/
     +                (A2**2)))

            A(M,N) = AMN(P(M,N),A1,A2,N,M,K_d,H_PIT)

            B(M,N) = BMN(P(M,N),A1,A2,N,M,K_d,H_PIT)

**  The following code evaluates equation (42e).

      DO 4 Y = 1 , NUM_R

         DO 3 R = 0, NUM_R
```

```
            H1MN(M,N,R,Y) = a_o * (P(M,N)**2) *
     +                  BESC(P(M,N)*R*D_R,N,M,A1,K_d)
     +                  * BESC(P(M,N)*Y*D_R,N,M,A1,K_d)
     +                  * (B(M,N)
     +                + (BESD(P(M,N)*D_R*AMAX0(R,Y),N,M,A1,K_d)
     +                / BESC(P(M,N)*D_R*AMAX0(R,Y),N,M,A1,K_d)))

            H2MN(M,N,R,Y) = a_o * (P(M,N)**2) *
     +                  BESJ01(P(M,N)*Y*D_R,0) *
     +                  BESJ01(P(M,N)*R*D_R,0) *
     +                  A(M,N)

3           CONTINUE

**  Evaluate H1MN(1,Y) and H2MN(1,Y) because they are needed
**  to evaluate F1MN(1) and F2MN(1). Note that H1MN(R,Y) and
**  H2MN(R,Y) are not needed for R between aa and 1.

          H1MN(M,N,NUM_R + 1,Y) = a_o * (P(M,N)**2) *
     +                        BESC(P(M,N),N,M,A1,K_d)
     +                        * BESC(P(M,N)*Y*D_R,N,M,A1,K_d)
     +                        * (B(M,N)
     +                        + (BESD(P(M,N),N,M,A1,K_d)
     +                        / BESC(P(M,N),N,M,A1,K_d)))

          H2MN(M,N,NUM_R + 1,Y) = a_o * (P(M,N)**2) *
     +                        BESJ01(P(M,N)*Y*D_R,0) *
     +                        BESJ01(P(M,N),0) *
     +                        A(M,N)


4       CONTINUE

**  Finished with equation (42e).

5           CONTINUE

**      Solve for the P_m's and K_m's from equations
** 22b and 22c.

            P_m(M) = ZERDIN(1.0,3.0,0.00000000001,A2,A1,H_PIT,M)

            K_m(M) = SQRT(A1**2 + (A2*P_m(M))**2)

10      CONTINUE

**      Create the initial guess array.

        DO 40 Z = 0, NUM_Z

             DO 30 R = 0, NUM_R
```

```
            DO 20 T = 0, NUM_T

                TIME(Z,R,T) = T * D_T + Z * D_Z - PI

20          CONTINUE

30        CONTINUE

40      CONTINUE

**  Do not allow more than MAX_IT itterations for convergence!!!

        DO 310 L = 1, MAX_IT

        DO 43 Z = 0, NUM_Z
        DO 42 R = 0, NUM_R
        DO 41 T = 0, NUM_T
           f1(Z,R,T) = 0.0
41      CONTINUE
42      CONTINUE
43      CONTINUE

**      Itterate 42 once.

        DO 265 M = 1, CAP_M
        DO 52 Z = 0, NUM_Z
        DO 51 R = 0, NUM_R
             F1M(Z,R) = 0.0
             F2M(Z,R) = 0.0
51      CONTINUE
52      CONTINUE
        DO 190 N = 0, CAP_N

** The following code evaluates fsmn(y) and fcmn(y)
** from equation (42f). Trapezoidal integration is
** performed. The variable y runs from the center
** of the tube to the outer radius of the electron
** beam.

        DO 90 Y = 1, NUM_R

** The following DO loop evaluates the outer integrals of
** equation (42f) from the entrance of the tube to the
** normalized end of the tube. Trapezoidal integration
** is used.

** F1_X and F2_X are the inner integrals of equation 42f.

** Note, COS(0) = 1

            fsmn(Y) = F1_X(0,Y,M,NUM_T,F2_X,D_T)/2
            fcmn(Y) = F2_X/2
```

```
          DO 80 X = 1, NUM_Z - 1

                fsmn(Y) = fsmn(Y) + F1_X(X,Y,M,NUM_T,F2_X,D_T)*
     +                    COS(N*K_d*X*D_Z)
                fcmn(Y) = fcmn(Y) + F2_X*COS(N*K_d*X*D_Z)

80        CONTINUE

** Note that N*PI = N*K_d*X*D_Z

             fsmn(Y) = (fsmn(Y) + (F1_X(NUM_Z,Y,M,NUM_T,F2_X,D_T)*
     +                 COS(N*PI)/2))*D_Z
             fcmn(Y) = (fcmn(Y) + (F2_X*COS(N*PI)/2))*D_Z

90       CONTINUE

**  Finished with equation (42f).

**  Evaluate equation (42d). Trapezoidal integration will
**  be performed.

        DO 140 R = 0, NUM_R

**  Initialize the integrals.

           F1MN(R) = 0

           F2MN(R) = 0

**  The first term is zero so dont evaluate it.

              DO 130 Y = 1, NUM_R - 1

                  F1MN(R) = (( H1MN(M,N,R,Y) * fsmn(Y) -
     +                      H2MN(M,N,R,Y) * fcmn(Y) ) * Y * D_R )
     +                      + F1MN(R)

                  F2MN(R) = (( H1MN(M,N,R,Y) * fcmn(Y) +
     +                      H2MN(M,N,R,Y) * fsmn(Y) ) * Y * D_R )
     +                      + F2MN(R)

130           CONTINUE

              F1MN(R) = ( F1MN(R) + ((H1MN(M,N,R,NUM_R) *
     +                  fsmn(NUM_R) - H2MN(M,N,R,NUM_R) *
     +                  fcmn(NUM_R)) * aa / 2)) * D_R

              F2MN(R) = ( F2MN(R) + ((H1MN(M,N,R,NUM_R) *
     +                  fcmn(NUM_R) + H2MN(M,N,R,NUM_R) *
     +                  fsmn(NUM_R)) * aa / 2)) * D_R
```

```
140     CONTINUE

**  Now integrate F1MN(1) AND F2MN(1). Note the values
**  between R = aa and R = 1 are not needed.

        F1MN(NUM_R + 1) = 0

        F2MN(NUM_R + 1) = 0

**  The first term is zero so don't evaluate it.

        DO 150 Y = 1, NUM_R - 1

           F1MN(NUM_R + 1) = (( H1MN(M,N,NUM_R + 1,Y) * fsmn(Y) -
     +              H2MN(M,N,NUM_R + 1,Y) * fcmn(Y) ) * Y * D_R )
     +                     + F1MN(NUM_R + 1)

            F2MN(NUM_R + 1) = (( H1MN(M,N,NUM_R + 1,Y) * fcmn(Y) +
     +              H2MN(M,N,NUM_R + 1,Y) * fsmn(Y) ) * Y * D_R )
     +                     + F2MN(NUM_R + 1)

150     CONTINUE

           F1MN(NUM_R + 1) = ( F1MN(NUM_R + 1) +
     +                    ((H1MN(M,N,NUM_R + 1,NUM_R) *
     +                fsmn(NUM_R) - H2MN(M,N,NUM_R + 1,NUM_R) *
     +                 fcmn(NUM_R)) * aa / 2)) * D_R

           F2MN(NUM_R + 1) = ( F2MN(NUM_R + 1) +
     +                    ((H1MN(M,N,NUM_R + 1,NUM_R) *
     +                 fcmn(NUM_R) + H2MN(M,N,NUM_R + 1,NUM_R) *
     +                 fsmn(NUM_R)) * aa / 2)) * D_R

**  Finished with equation (42d).

**  Evaluate equation (42c)

       DO 180 Z = 0, NUM_Z

           DO 170 R = 0, NUM_R

              F1M(Z,R) = F1M(Z,R) + (2 - KRON_DEL(0,N)) *
     +               (F1MN(R)*COS(N*K_d*Z*D_Z) -
     +              (BESI01(M*P_m(M)*R*D_R,0,1)*F1MN(NUM_R +1) *
     +               COS(M*K_m(M)*Z*D_Z)/BESI01(M*P_m(M),0,1)))

              F2M(Z,R) = F2M(Z,R) + (2 - KRON_DEL(0,N)) *
     +               (F2MN(R)*COS(N*K_d*Z*D_Z) -
     +              (BESI01(M*P_m(M)*R*D_R,0,1)*F2MN(NUM_R +1) *
     +              COS(M*K_m(M)*Z*D_Z)/BESI01(M*P_m(M),0,1)))

170         CONTINUE
```

```
180     CONTINUE

**      This next CONTINUE is from the N to CAP_N loop.

190     CONTINUE

**  Evaluate equation (42a).

        DO 260 Z = 0, NUM_Z

        DO 250 R = 0, NUM_R

**  Now equation (42b) will be evaluated.

           Lo_f1m(M,Z,R)=KRON_DEL(1,M)*AP*BESI01(P_m(1)*R*D_R,0,1)
     +                  *CEXP(CMPLX(0.0,-1.0*K_m(1)*Z*D_Z))/
     +                  (2*BESI01(P_m(1),0,1)) +
     +                  (CMPLX(F1M(Z,R),F2M(Z,R))/(2*M))


**  Evaluate equation 42a.

        DO 240 T = 0, NUM_T

           f1(Z,R,T)=2*REAL(Lo_f1m(M,Z,R)*CEXP(CMPLX(0.0,M*TIME(Z,R,T))))
     +                   + f1(Z,R,T)

240     CONTINUE

250     CONTINUE

260     CONTINUE

**   This next CONTINUE is from the M to CAP_M loop.

265     CONTINUE

**  Evaluate equation (29). Trapezoidal integration is used.

**  Note, TIME(0,R,T) = Entrance time

        DO 300 T = 0, NUM_T

        DO 290 R = 0, NUM_R

CC      TEMP = TIME(NUM_Z,R,T)

        FARR(0) = f1(0,R,T)/2

        DO 280 Z = 1, NUM_Z
```

```
          FARR(Z) = FARR(Z - 1) + f1(Z,R,T)

** Note, F(0,Z,R,T) = 1

          TIME(Z,R,T) = 0.5

          DO 270 X = 1, Z - 1

            TIME(Z,R,T) = TIME(Z,R,T) +
     +      SQRT(1/(1 - 2*EPS*D_Z*(FARR(X) - 0.5*f1(X,R,T))))

270      CONTINUE
          TIME(Z,R,T) = (TIME(Z,R,T) +

     +    SQRT(1/(1 - 2*EPS*D_Z*(FARR(Z) - 0.5*f1(Z,R,T))))/2)
     +      * D_Z + T*D_T - PI

280    CONTINUE

***   Keep a check on converngence here. Compare TIME(NUM_Z,R,T)
***   to TEMP.

        QUIT = MAX(ABS(TEMP - TIME(NUM_Z,R,T)), 0.2)

290    CONTINUE

300    CONTINUE

       IF (QUIT .EQ. 0.2) THEN

          GOTO 320

       ENDIF

310    CONTINUE


**     Save the needed results.

       TIME0='TIME0'
       TIMEA='TIMEA'
       FOURIER='FOURIER'

320    open (unit=10,file=TIMEA,status='NEW')
         WRITE(10,*),1
       WRITE(10,*),L
       WRITE(10,*),ALPHA
       DO 330 T=0,NUM_T

          WRITE (10,*), TIME(NUM_Z,NUM_R,T)
```

```
330     CONTINUE

        close (unit=10)

        open (unit=10,file=TIME0,status='NEW')
        WRITE(10,*),0
        WRITE(10,*),L
        WRITE(10,*),ALPHA
        DO 340 T=0,NUM_T

             WRITE (10,*), TIME(NUM_Z,0,T)

340     CONTINUE

        close (unit=10)

        open (unit=10,file=FOURIER,status='new')
        WRITE(10,*),L
        WRITE(10,*),ALPHA
        DO 360 Z = 0, NUM_Z
        DO 350 R = 0, NUM_R
        WRITE(10,*), LO_fim(1,Z,R)
350     CONTINUE
360     CONTINUE
        close(unit=10)

        STOP
        END
```

```
*****************************************************************
*       Department of Electrical and Computer Engineering       *
*                  Kansas State University                      *
*                                                               *
*       VAX FORTRAN source filename: AMN.FOR                    *
*****************************************************************
*
*   REFERENCES:     N. Kalyanasundaram, "Large-signal field
*                   analysis of an O-type travelling wave
*                   amplifier. Part 1: Theory," IEE PROCEEDINGS,
*                   Vol. 131, Pt. 1, No. 5, pp 145-152,
*                   October 1984.
*
*                   N. Kalyanasundaram and R. Chinnadurai,
*                   "Large-signal field analysis of an
*                   O-type travelling wave amplifier. Part 2:
*                   Numerical results,"        IEE PROCEEDINGS,
*                   Vol. 133, Pt. 1, No. 4, pp 163-168,
*                   August 1986.
*
*
*
*                   All equation numbers in this program refer to
*                   the above two references.
*
*   ROUTINE:        real function subprogram
*                   AMN(ARG, A1, A2, N, M, K_d, H_PIT)
*
*   DESCRIPTION:    Returns AMN (Eqn. 41).
*
*
*   DOCUMENTATION
*   FILES:          None.
*
*   ARGUMENTS:
*       ARG         (input) real
*                   The value at which AMN is evaluated.
*
*       A1          (input) real
*                   The initial velocity of the electron
*                   divided by the speed of light. (Eqn. 4)
*
*       A2          (input) real
*                   A normalization factor. (Eqn. 4)

*
*       M           (input) integer
*                   The value of the outer loop.
*
*       N           (input) integer
*                   The value of the inner loop
*
```

```
*       K_d           (input) real
*                     PI divided by the normalized tube length.
*                     (Eqn. 14b)
*
*       H_PIT         (input) real
*                     The tangent of the helix pitch.
*
*
*   RETURN:           Not used.
*
*   ROUTINES
*   CALLED:           BESJ01 (Evaluates the J Bessel function)
*                     BESY01 (Evaluates the Y Bessel function)
*
*   AUTHOR:           Bradley P. Badke
*
*   DATE CREATED:     21NOV87    Version 1.0
*
*   REVISIONS:        None.
*
*******************************************************************

      REAL FUNCTION AMN(ARG, A1, A2, N, M, K_d, H_PIT)

      IMPLICIT        NONE

      REAL    ARG, A1, A2, K_d, BESJ01, H_PIT, PI, ANSJ,
     +        BESY01

      INTEGER         M, N

      COMPLEX          BESQMN, BESH_P, BESHO

      EXTERNAL BESJ01,BESY01

      PI = 3.141592654

** Calculate the Hankel function of the second kind
** of order zero.

      BESHO = CMPLX(ANSJ,-1*BESY01(ARG,0,ANSJ))

** Calculate the derivative of the Hankel function
** of the second kind of order zero.

      BESH_P = CMPLX(-1*ANSJ,BESY01(ARG,1,ANSJ))

      IF (N .LT. M*A1/K_d) THEN

** Calculate QMN (equation 20b)

      BESQMN = 1 - ((A2 * ARG * H_PIT)**2) * BESJ01(ARG,0) *
```

```
+           BESH0/( ((M * A1)**2) * BESJ01(ARG,1)
+          * BESH_P )

  AMN = REAL( PI*BESH0/( 2*BESJ01(ARG,0) *
+       BESQMN) )

  ELSE

  AMN = 0.0

  ENDIF

  RETURN

  END
```

```
*****************************************************************
*       Department of Electrical and Computer Engineering       *
*                   Kansas State University                     *
*                                                               *
*       VAX FORTRAN source filename: BMN.FOR                    *
*****************************************************************
*
*   REFERENCES:       N. Kalyanasundaram, "Large-signal field
*                     analysis of an O-type travelling-wave
*                     amplifier. Part 1: Theory," IEE PROCEEDINGS,
*                     Vol. 131, Pt. 1, No. 5, pp 145-152,
*                     October 1984.
*
*                     N. Kalyanasundaram, R. Chinnadurai,
*                     "Large-signal field analysis of an
*                     O-type travelling-wave amplifier. Part 2:
*                     Numerical results," IEE PROCEEDINGS,
*                     Vol. 133, Pt. 1, No. 4, pp 163-168,
*                     August 1986.
*
*
*                     All equation numbers in this program refer
*                     to the above references.
*
*   ROUTINE:          real function subprogram
*                     BMN(ARG, A1, A2, N, M, K_d, H_PIT)
*
*   DESCRIPTION:      Returns BMN evaluated at ARG. (Eqn. 41)
*
*   DOCUMENTATION
*   FILES:            None.
*
*   ARGUMENTS:
*       ARG           (input) real
*                     The value at which BMN is evaluated.
*
*       A1            (input) real
*                     The initial velocity of the electron
*                     divided by the speed of light. (Eqn. 4)
*
*       A2            (input) real
*                     The initial velocity of the electron
*                     divided by the frequency of the input RF
*                     signal in rad/sec and the helix radius.
*                     A2 = (Vo/(Wo * b))    (Eqn. 4)
*
*       M             (input) integer
*                     The value of the outer loop.
*
*       N             (input) integer
*                     The value of the inner loop.
```

```
*
*      K_d          (input) real
*                   PI divided by the normalized tube length.
*                   (Eqn. 14b)
*
*
*      H_PIT        (input) real
*                   The tangent of of the pitch angle of the
*                   helix.
*
*   RETURN:         Not used.
*
*   ROUTINES
*   CALLED:         BESJ01, BESI01, BESY01, BESK01
*                   (All of which evaluate Bessel functions)
*
*   AUTHOR:         Bradley P. Badke
*
*   DATE CREATED:   21NOV87    Version 1.0
*
*   REVISIONS:      None.
*
*************************************************************

        REAL FUNCTION BMN(ARG, A1, A2, N, M, K_d, H_PIT)

       IMPLICIT        NONE

       INTEGER         M, N

       REAL    ARG, A1, A2, K_d, H_PIT, BESI01,BESJ01,
     +         BESK01, PI, BESY01, ANSJ

       COMPLEX         BESQMN, BESH0, BESH_P

       EXTERNAL BESJ01,BESK01,BESI01,BESY01

       PI = 3.141592654

       IF (N .LT. M*A1/K_d) THEN

** Calculate the Hankel function of the second kind
** of order zero.

       BESH0 = CMPLX(ANSJ,-1*BESY01(ARG,0,ANSJ))

** Calculate the derivative of the Hankel function of the
** second kind of order zero.

       BESH_P = CMPLX(-1*ANSJ,BESY01(ARG,1,ANSJ))

** Calculate QMN (equation 20b)
```

```
      BESQMN = 1 - ((A2 * ARG * H_PIT)**2) * BESJ01(ARG,0) *
     +          BESH0/( ((M * A1)**2) * BESJ01(ARG,1)
     +         * BESH_P )

      BMN = AIMAG(PI*BESH0*(1 - (1/BESQMN))/(2*BESJ01(ARG,0)))

      ELSE

** Calculate QMN (equation 20b)

      BESQMN = 1 - ((A2 * ARG * H_PIT)**2) * BESI01(ARG,0,1)
     +         * BESK01(ARG,0,1,0)/( ((M * A1)**2) *
     +          BESI01(ARG,1,1) * BESK01(ARG,1,1,0) )

      BMN = BESK01(ARG,0,1,0) *
     +       ((1/BESQMN) - 1)/BESI01(ARG,0,1)

      ENDIF

      RETURN

      END
```

```
*****************************************************************
*       Department of Electrical and Computer Engineering      *
*                  Kansas State University                      *
*                                                               *
*       VAX FORTRAN source filename: KRON_DEL.FOR               *
*****************************************************************
*
*   ROUTINE:          function subprogram
*                     KRON_DEL(L,K)
*
*   DESCRIPTION:      Calculates the Kronecker delta.
*                     Returns 1 if L = K.
*                     Returns 0 if L not equal to K
*
*   DOCUMENTATION
*   FILES:            None.
*
*   ARGUMENTS:
*         L           (input) integer
*
*         K           (input) integer
*
*   RETURN:           Not used.
*
*   ROUTINES
*   CALLED:           None.
*
*   AUTHOR:           Bradley P. Badke
*
*   DATE CREATED:     27NOV87     Version 1.0
*
*   REVISIONS:        None.
*
*****************************************************************
        REAL FUNCTION KRON_DEL(L,K)
        IMPLICIT    NONE
        INTEGER L, K
        IF ( L .EQ. K ) THEN
            KRON_DEL = 1.0
        ELSE
            KRON_DEL = 0.0
        ENDIF
        RETURN
        END
```

```
***************************************************************
*       Department of Electrical and Computer Engineering     *
*                   Kansas State University                   *
*                                                             *
*       VAX FORTRAN source filename: F1_X.FOR                 *
***************************************************************
*
*   REFERENCE:      N. Kalanasundaram, and R. Chinnadurai,
*                   "Large-signal field analysis of
*                   an O-type travelling wave amplifier Part 2:
*                   Numerical results," IEE PROCEEDINGS,
*                   Vol. 133, Pt. 1, No. 4, pp 145-152,
*                   August 1986,
*
*
*   All equation numbers in this program refer to the above
*   reference.
*
*
*   ROUTINE:        function subprogram
*                   F1_X(X,Y,M,NUM_T,F2_X,O_T)
*
*   DESCRIPTION:    Calculates the inner integrals of
*                   Eqn. 42f using trapezoidal integration.
*
*
*   DOCUMENTATION
*   FILES:          None.
*
*   ARGUMENTS:
*       X          (input)  integer
*
*   RETURN:         Not used.
*
*   ROUTINES
*   CALLED:         None.
*
*   AUTHOR:         Bradley P. Badke
*
*   DATE CREATED:   30DEC87    Version 1.0
*
*   REVISIONS:      None.
*
****************************************************************
      REAL FUNCTION F1_X(X,Y,M,NUM_T,F2_X,O_T)
        IMPLICIT NONE
      REAL F2_X, O_T, TIME(0:600,0:5,0:24)
        INTEGER X, Y, NUM_T, M, I
        COMMON TIME
        F1_X = (SIN(M*TIME(X,Y,0)) + SIN(M*TIME(X,Y,NUM_T)))/2
        F2_X = (COS(M*TIME(X,Y,0)) + COS(M*TIME(X,Y,NUM_T)))/2
```

```
      DO 10 I = 1, NUM_T - 1
         F1_X = F1_X + SIN(M*TIME(X,Y,I))
         F2_X = F2_X + COS(M*TIME(X,Y,I))
10    CONTINUE
      F1_X = F1_X*D_T
      F2_X = F2_X*D_T
      RETURN
      END
```

```
C       This function subprogram is a sligthly modified
C       translation of the ALGOL 60 procedure, ZERO, given
C       in Richard Brent, Algorithms for minimization
C       without derivatives, Prentice-Hall, Inc. (1973).
C
C
C       This program was modified on September 31, 1987
C       by Bradley P. Badke MSEE Kansas State University
C       to find the zeros of Eqn. 22b from "N.
C       Kalyanasundaram, "Large-signal field
C       analysis of an O-type travelling wave
C       amplifier Part 1: Theory," IEE PROCEEDINGS,
C       Vol. 131, Pt. 1, No. 5, pp 145-152,
C       October 1984.
C
C
C
        REAL FUNCTION ZEROIN(AX,BX,TOL,A2,A1,H_PIT,m)

        IMPLICIT        NONE

          REAL AX,BX,TOL,A2,A1,H_PIT,BESK01,BESI01

        INTEGER m, NZ

        EXTERNAL  BESK01,BESI01
C
C  A zero of equation (22b) is computed in the interval AX,BX.
C  One of the values has to be negative and the other positive.
C
C  INPUT..
C
C  AX     LEFT ENDPOINT OF INITIAL INTERVAL
C
C  BX     RIGHT ENDPOINT OF INITIAL INTERVAL
C
C  A1     The initial velocity of the electrons divided by
C         the speed of light. (Eqn. 4)
C
C  A2     The initial velocity of the electrons divided by
C         the RF frequency (rad/sec) and the non-normalized
C         radius of the sheath helix. (Eqn. 4)
C
C  m      The integer value which describes the root to be found.
C
C  H_PIT  The tangent of the helix pitch.
C
C  TOL    DESIRED LENGTH OF THE INTERVAL OF UNCERTAINTY OF THE
C         FINAL RESULT ( .GE. 0.0)
C
C
```

```
C  OUTPUT..
C
C  ZERDIN ABCISSA APPRDXIMATING A ZERO OF EQUATION 22b  IN THE
C  INTERVAL AX,BX
C
C
C      IT IS ASSUMED THAT EQN. 22b EVALUATED AT (AX) AND
C  EQN. 22b EVALUATED AT (BX) HAVE DPPDSITE SIGNS WITHDUT
C  A CHECK. ZEROIN  RETURNS A ZERD, X, IN THE GIVEN INTERVAL
C  (AX,BX) TD WITHIN A TDLERANCE  4*MACHEPS*ABS(X) + TDL,
C  WHERE MACHEPS  IS THE RELATIVE  MACHINE PRECISIDN.
C      THIS FUNCTIDN SUBPROGRAM IS A SLIGHTLY  MODIFIED
C  TRANSLATIDN  DF THE ALGOL 60 PRDCEDURE  ZERO  GIVEN IN
C  RICHARD BRENT, ALGORITHMS FOR MINIMIZATIDN WITHOUT
C  DERIVATIVES, PRENTICE - HALL, INC. (1973).
C
C
      REAL  A,B,C,D,E,EPS,FA,FB,FC,TOL1,XM,P,Q,R,S
C
C  CDMPUTE EPS, THE RELATIVE MACHINE PRECISIDN
C
      EPS = 1.0
   10 EPS = EPS/2.0
      TDL1 = 1.0 + EPS
      IF (TOL1 .GT. 1.0) GO TD 10
C
C INITIALIZATION
C
      A = AX
      B = BX

CC FA and FB are equation (22b).

      FA = ((A2*A)**2)*BESI01(m*A,0,1)*BESK01(m*A,0,1,NZ) -
     +     (((1/H_PIT)*A1)**2)*BESI01(m*A,1,1)*BESK01(m*A,1,1,NZ)

      FB = ((A2*B)**2)*BESI01(m*B,0,1)*BESK01(m*B,0,1,NZ) -
     +     (((1/H_PIT)*A1)**2)*BESI01(m*B,1,1)*BESK01(m*B,1,1,NZ)
C
C BEGIN STEP
C
   20 C = A
      FC = FA
      D = B - A
      E = D
   30 IF (ABS(FC) .GE. ABS(FB)) GD TO 40
      A = B
      B = C
      C = A
      FA = FB
      FB = FC
      FC = FA
```

```
C
C CONVERGENCE TEST
C
   40 TOL1 = 2.0*EPS*ABS(B) + 0.5*TOL
      XM = .5*(C - B)
      IF (ABS(XM) .LE. TOL1) GO TO 90
      IF (FB .EQ. 0.0) GO TO 90
C
C IS BISECTION NECESSARY
C
      IF (ABS(E) .LT. TOL1) GO TO 70
      IF (ABS(FA) .LE. ABS(FB)) GO TO 70
C
C IS QUADRATIC INTERPOLATION POSSIBLE
C
      IF (A .NE. C) GO TO 50
C
C LINEAR INTERPOLATION
C
      S = FB/FA
      P = 2.0*XM*S
      Q = 1.0 - S
      GO TO 60
C
C INVERSE QUADRATIC INTERPOLATION
C
   50 Q = FA/FC
      R = FB/FC
      S = FB/FA
      P = S*(2.0*XM*Q*(Q - R) - (B - A)*(R - 1.0))
      Q = (Q - 1.0)*(R - 1.0)*(S - 1.0)
C
C ADJUST SIGNS
C
   60 IF (P .GT. 0.0) Q = -Q
      P = ABS(P)
C
C IS INTERPOLATION ACCEPTABLE
C
      IF ((2.0*P) .GE. (3.0*XM*Q - ABS(TOL1*Q))) GO TO 70
      IF (P .GE. ABS(0.5*E*Q)) GO TO 70
      E = D
      D = P/Q
      GO TO 80
C
C BISECTION
C
   70 D = XM
      E = D
C
C COMPLETE STEP
C
```

```
   80 A = B
      FA = FB
      IF (ABS(D) .GT. TOL1) B = B + D
      IF (ABS(D) .LE. TOL1) B = B + SIGN(TOL1, XM)

CC FB is equation (22b).

      FB = ((A2*B)**2)*BESI01(m*B,0,1)*BESK01(m*B,0,1,NZ) -
     +     (((1/H_PIT)*A1)**2)*BESI01(m*B,1,1)*BESK01(m*B,1,1,NZ)

      IF ((FB*(FC/ABS(FC))) .GT. 0.0) GO TO 20
      GO TO 30
C
C DONE
C
   90 ZEROIN = B
      RETURN
      END
```

```
****************************************************************
*       Department of Electrical and Computer Engineering      *
*                    Kansas State University                   *
*                                                              *
*       VAX FORTRAN source filename: BESC.FOR                  *
****************************************************************
*
*   REFERENCES:     N. Kalyanasundaram, "Large-signal field
*                   analysis of an O-type travelling-wave
*                   amplifier. Part 1: Theory," IEE PROCEEDINGS,
*                   Vol. 131, Pt. 1, No. 5, pp 145-152,
*                   October 1984.
*
*
*                   N. Kalyanasundaram, and R. Chinnadurai,
*                   "Large-signal field analysis of an O-type
*                   travelling-wave amplifier. Part 2:
*                   Numerical results," IEE PROCEEDINGS,
*                   Vol. 133, Pt. 1, No. 4, pp 163-168,
*                   August 1986.
*
*                   All equation numbers in this program refer
*                   to the above two references.
*
*   ROUTINE:        function subprogram
*                   BESC(ARG, N, M, A1, K_d)
*
*   DESCRIPTION:    Returns BESJ0(ARG) if N < M*A1/K_d else
*                   return BESI0(ARG). BESC is called while
*                   calculating equation 42e.
*
*   DOCUMENTATION
*   FILES:          None.
*
*   ARGUMENTS:
*       N           (input) integer
*                   The number of the inner loop.
*
*       M           (input) integer
*                   The number of the outer loop.
*
*       A1          (input) real
*                   The initial velocity divided by the speed
*                   of light. (Eqn. 4)
*
*       K_d         (input) real
*                   The normalized length of the tube divided
*                   by PI. (Eqn. 14b)
*
*       ARG         (input) real
*                   The value at which BESC0 is evaluated.
```

```
*
*
*   RETURN:         Not used.
*
*   ROUTINES
*   CALLED:         BESJ01, BESI01 (These routines evaluate
*                   Bessel functions)
*
*   AUTHOR:         Bradley P. Badke
*
*   DATE CREATED:   18NOV87  Version 1.0
*
*   REVISIONS:      None.
*
*******************************************************************

      REAL FUNCTION BESC(ARG, N, M, A1, K_d)

      IMPLICIT        NONE

      REAL    A1, ARG, K_d, BESJ01, BESI01

      INTEGER         N, M

      EXTERNAL BESJ01,BESI01

      IF (N .LT. M*A1/K_d ) THEN

              BESC = BESJ01(ARG,0)

      ELSE

              BESC = BESI01(ARG,0,1)

      ENDIF

      RETURN

      END
```

```
*****************************************************************
*      Department of Electrical and Computer Engineering        *
*                  Kansas State University                      *
*                                                               *
*      VAX FORTRAN source filename: BESO.FOR                    *
*****************************************************************
*
*   REFERENCES:     N. Kalyanasundaram, "Large-signal field
*                   analysis of an O-type travelling wave
*                   amplifier Part 1: Theory," IEE PROCEEDINGS,
*                   Vol. 131, Pt. 1, No. 5, pp 145-152,
*                   October 1985.
*
*                   N. Kalyanasundaram and R. Chinnadurai,
*                   "Large-signal field analysis of an O-type
*                   travelling wave amplifier Part 2:
*                   Numerical results," IEE PROCEEDINGS,
*                   Vol. 133, Pt. 1, No. 4, pp 163-168,
*                   August 1986.
*
*
*   ROUTINE:        function subprogram
*                   BESO(ARG, N, M, A1, K_d)
*
*   DESCRIPTION:    Returns PI*Y0(ARG)/2 for N < M*A1/K_d else
*                   returns K0(ARG). Called when calculating
*                   equation 42e.
*
*   DOCUMENTATION
*   FILES:          None.
*
*   ARGUMENTS:
*        ARG        (input) real
*                   The value at which the Bessel function
*                   is to be evaluated.
*
*        N          (input) integer
*                   The value of the inner loop.
*
*        M          (input) integer
*                   The value of the outer loop.
*
*        K_d        (input) real
*                   The normalized tube length divided by PI.
*                   (Eqn. 14b)
*
*        A1         (input) real
*                   The initial velocity divided by the
*                   speed of light.  (Eqn. 4)
*
*   RETURN:         Not used.
```

```
*
*   ROUTINES
*   CALLED:          BESK01, BESY01 (These routines evaluate
*                    Bessel functions)
*
*   AUTHOR:          Bradley P. Badke
*
*   DATE CREATED:    09NOV87    Version 1.0
*
*   REVISIONS:       None.
*
*******************************************************************

      REAL FUNCTION BESD(ARG, N, M, A1, K_d)

      IMPLICIT        NONE

      INTEGER         N, M, NZ

      REAL    ARG, A1, K_d, ANSJ, PI, BESK01, BESY01

      EXTERNAL BESK01,BESY01

      PI = 3.141592654

      IF (N .LT. M*A1/K_d) THEN

              BESD = (PI/2)*BESY01(ARG,0,ANSJ)

      ELSE

              BESD = BESK01(ARG,0,1,NZ)

      ENDIF

      RETURN

      END
```

A COMPUTER IMPLEMENTATION OF A MATHEMATICAL MODEL OF AN O-TYPE TRAVELING WAVE TUBE AMPLIFIER

by

BRADLEY PAUL BADKE

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Abstract

This report discusses the development of a large signal theory for a traveling wave tube amplifier (TWTA). There is a need for a large signal theory since TWTA's are often operated near saturation and there has yet to be developed a satisfactory model for large signal operation of a TWTA. A good large signal model of a TWTA would be useful when designing a TWTA for a particular application.

This report describes the computer implementation of a mathematical model of a traveling wave tube amplifier (TWTA). The first topic considered is the small signal theory of TWTA's developed by J. R. Pierce. From Pierce's small signal theory the gain of the amplifier considered was 0.18 dB/z.

Next, the large signal theory of TWTA's, developed by N. Kalyanasundaram, is discussed. For the amplifier considered, the small signal gain of the amplifier is 0.2 dB/z, which is close to the gain predicted by Pierce's theory. Kalyanasundaram's theory also shows electron overtaking which is known to occur when a large signal is input to the TWTA.

Next, independent verification of Kalyanasundaram's theory is discussed. A computer program named TUBE was developed based on Kalyanasundaram's equations. For the amplifier considered, the gain of the TWTA was 0.2 dB/z which is close to the small signal gain predicted by

Pierce's theory. TUBE also shows electron overtaking for large input signals. The results produced by TUBE are qualitatively similar to Kalyanasundaram's results but they are not identical. The reason for the discrepancy is not known.