

EFFECT OF CHANGE IN DIAMETER OF THE  
BOLTED MEMBERS ON ITS EFFECTIVE AREA

by

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## INTRODUCTION

In a bolted joint the increase of load in the bolt due to the external force is determined by the magnitude of the external force and the relative stiffness of the bolt as compared with that of the members joined by the bolt (see appendix II). For the proper design of the bolted assembly it is necessary to determine the relative stiffness of the assembly. This depends upon the Modulus of Elasticity, effective length and effective area of the bolt and connected members. The stiffness of the bolt is influenced to some extent by localized deformations due to bending of the threads and warping or flexing of the nut and bolted sections. These effects may be aggravated in some instances by poor thread cutting or by a lack of squareness producing improper fit at bearing faces of nut and bolted head which results in additional flexing of the parts. The stiffness of the members joined is influenced to an appreciable extent by the surface roughness, lack of squareness, quality of fit, foreign particles and burrs on the surfaces joined. Much work has been done on the stiffness of the bolt but very little work is done on the stiffness of the connected members.

The stress distribution in two steel plates under compression was studied at various diameters to determine the effective area of bolted members and the findings are presented in this paper.

PREVIOUS WORK ON THE EFFECTIVE AREA OF A BOLTED CONNECTION

Shankar L. Bathwal (1)\* discussed five different approaches to determine the effective area of a bolted joint, no two gave results in reasonable agreement.

G. C. Srivastava (2) made an experimental investigation to determine the effective area of a bolted joint. The joint consisted of two aluminum plates 5 inches in diameter and 1 inch in thickness with a bolt hole of 1 inch diameter. The bolt was simulated by two steel studs. The stress distribution was found by measuring strains at different locations by means of strain gages. Using the experimentally determined strains effective areas were calculated based on (1) Maximum strain (2) Mean effective strain (3) Energy stored in the plates due to compression assuming linear relationship between the load and deformation and (4) Actual energy stored in the plates as obtained by the load deformation curves. The results were as shown below

Table 1

Method	Without gasket		With gasket	
	Effective area in sq. in.	Equivalent Dia. in.	Effective area in sq. in.	Equivalent Dia. in in.
1	0.91	1.47	1.18	1.58
2	11.80	4.00	8.12	3.37
3	0.91	1.47	1.116	1.556
4	1.03	1.52	1.07	1.54

\*Refer to Bibliography

A gasket between the plates increased the effective area. As the effective area of the connected members increased the relative stiffness constant,  $m$ , was decreased. Hence the total load in the bolt with a gasket is less than that without a gasket under the same initial and external loads.

The effective area calculated experimentally by Srivastava was smaller than the area based on Radzimvosky's equation. This means that the design is less safe when based on the effective area calculated from Radzimvosky's equation.

## EXPERIMENTAL INVESTIGATION

It was proposed to investigate the variation in the effective area with the change in diameter of the connected members of a bolted assembly without a gasket. In the test two steel plates were used as the portion of connected members around a single bolt. The plates with a bolt hole were compressed by two steel studs to give the effect similar to a bolt and nut. A compression testing machine, as shown in Fig. 1, was used to load the assembly.

The dimensions of the test pieces were as shown below:

Diameter of steel plates in inches	4.5
Diameter of bolt hole in inches	1.0
Stem diameter of studs in inches	1.0
Head diameter of the stud in inches	1.5
Height of the stud head in inches	1.0

Holes of 1/4 inch diameter, five in each plate were drilled and then reamed. They were located such that no two were on the same radial line and at different distances from the center of the plate. Strain gages were installed in each hole and one in the bolt hole near the contact surface as shown in Fig. 2. The distance of the gages from the center of the hole were as shown in the table below:

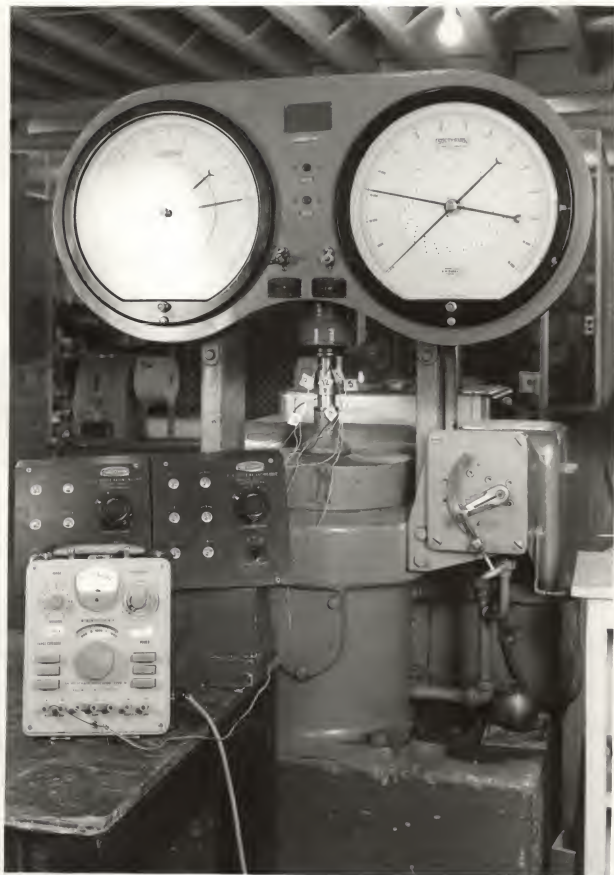


Fig. 1. The Test Set Up

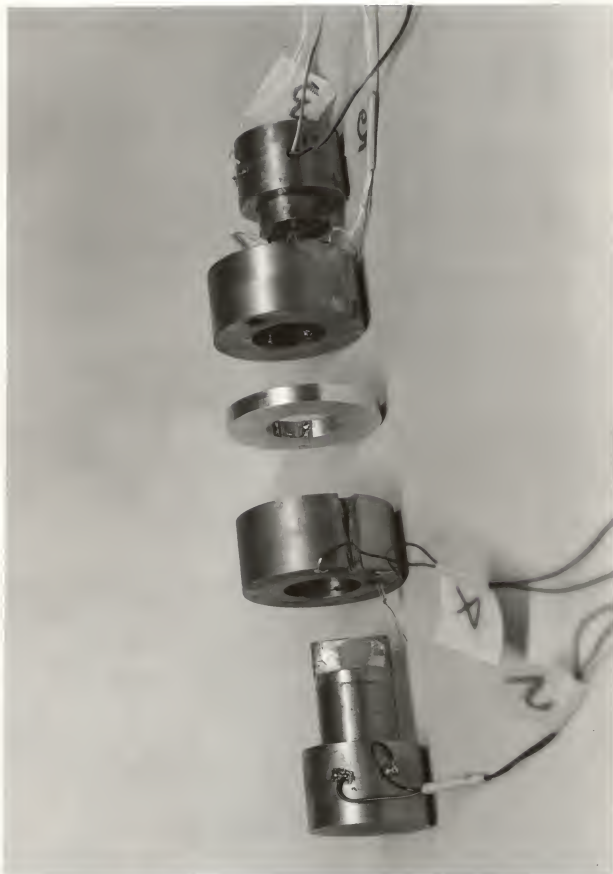


Fig. 2. The two steel test-pieces with studs and aluminum gasket.



Table 2

Gage No. in Plate 1	Gage No. in Plate 2	Distance in inches from the Center of the Hole.
1		0.50
	2	0.56
3		0.65
	4	0.81
5		0.97
	6	1.10
7		1.27
	8	1.42
9		1.56
	10	1.72
11		1.88

Loads from 4,000 to 16,000 lbs. in increments of 2,000 lbs. were applied. At each load the strains in all the gages were recorded.

The magnitude of strains and pattern of stress distribution were found to vary significantly due to any small relative rotation of one part of the assembly with respect to the other. A typical variation of strains for two different positions is as shown in Tables 8 and 9, Appendix I. It was concluded that this variation was due to the lack of perfect flatness between the contact surfaces of the plates and bearing surfaces of the studs and plates. The contact surfaces of the plates were ground. Still the surfaces were not flat enough to give the same strain readings at any position when one part of the assembly was rotated relative to the other parts.

All the parts of the assembly were marked. The variation of stress distribution, hence the effective area of the bolted members was studied due to change in the diameter of the connected members. The testing was restricted to the marked position of the assembly.

The diameter of the steel plates was reduced in steps from 4.5 inches to 2.5, 2-1/16 inches and 1.5 inches. The experiment was repeated at each diameter.

## DISCUSSION

Under the ideal conditions of perfect flatness of the contact and bearing surfaces the stress would be maximum under the load and decrease with the distance from the center. Also the strain in each gage would vary linearly with the load.

The strain vs. load was plotted for each gage and for each setup. The strain vs. distance from the center was plotted for each size of the plates at 16,000 lbs. load only. (see Figs. 3 to 7) The 16,000 lb. load corresponds to the initial tension in a steel bolt of 1 inch root diameter as given by Kimball and Barr (4). See also appendix II.

For the 4.5 inch diameter plates the strains in gages 6 and 7 were linear with the load. The strains in gages 8 and above were insignificant. The strains in gages 1, 2, 3, 4 and 5 were linear except at lower loads. The magnitude of the strain in gage 2 was larger than in other gages. Evidently it could be concluded that as the load is applied, some regions were stressed before the other regions came into contact. At higher loads uniform contact was obtained. Hence the strain vs. load was linear.

At 2.5 inches diameter the maximum strain was decreased. The strains in all gages were tending linear with the load. Still the strain in gage 2 was larger than in other gages.

Strain Vs Load at various Locations  
 Bolted Connection  
 Two steel plates diameter in inches 4.5

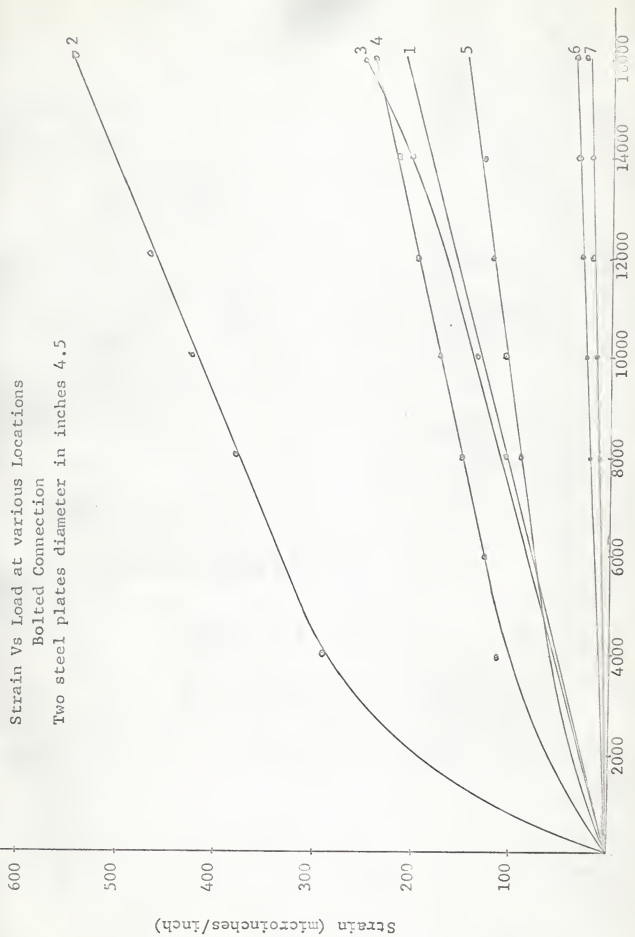


Fig. 3. Strain Vs Load

Strain Vs Load at various locations  
 Bolted Connection  
 Two steel plates, diameter in inches 2.5

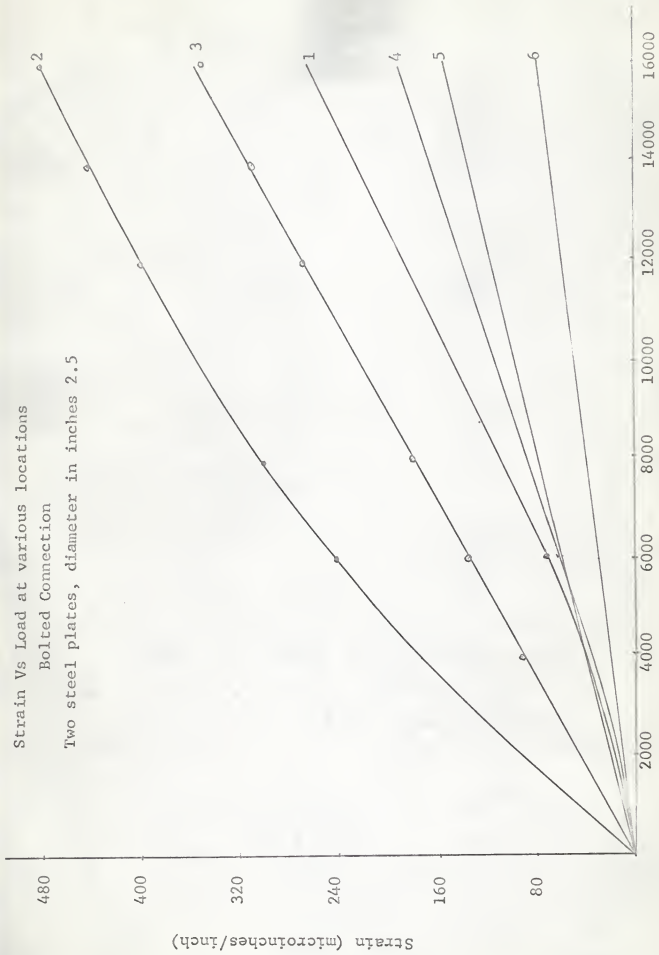


Fig. 4. Strain Vs Load

Strain Vs Load at various Locations

Bolted Connection

Two steel plates, diameter in inches 2-1/16

420

350

200

210

140

70

Strain (microinches/inch)

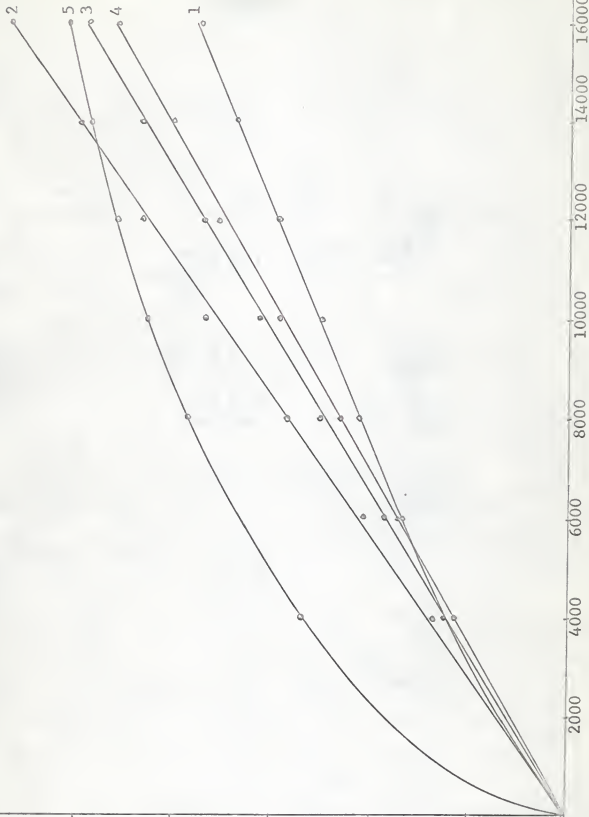


Fig. 5. Strain Vs Load

Strain Vs Load at various locations  
Bolted Connection

Two steel plates, diameter in inches 1.5

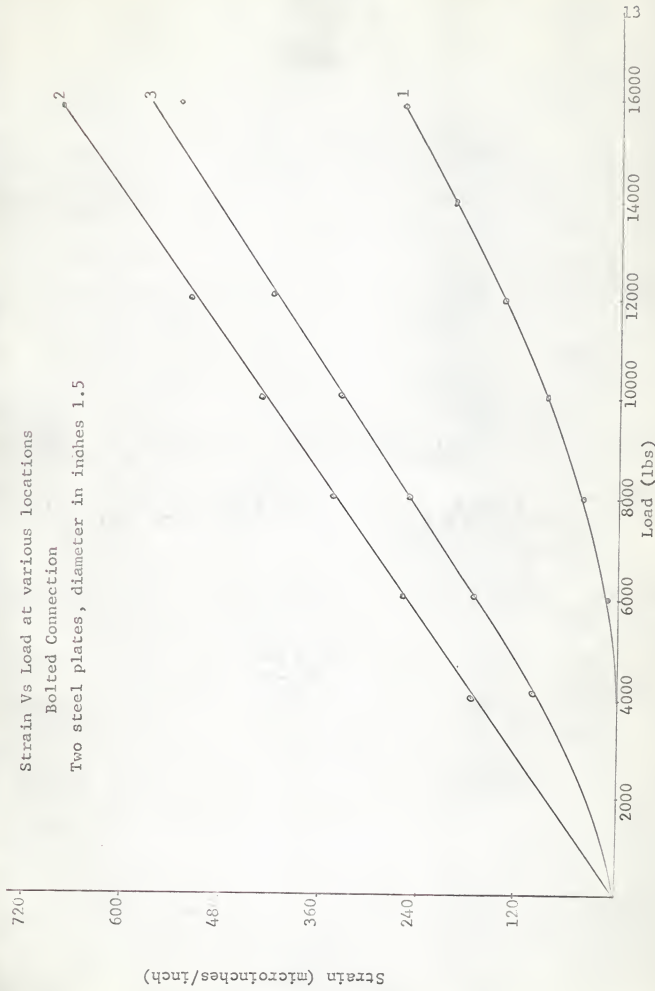


Fig. 6. Strain Vs Load

Strain Vs distance for various diameters  
at 16,000 load

Bolted Connection

Two Steel Plates:

- curve A - 4.5 inches diameter
- curve B - 2.5 inches diameter
- curve C - 2-1/16 inches diameter
- curve D - 1.5 inches diameter

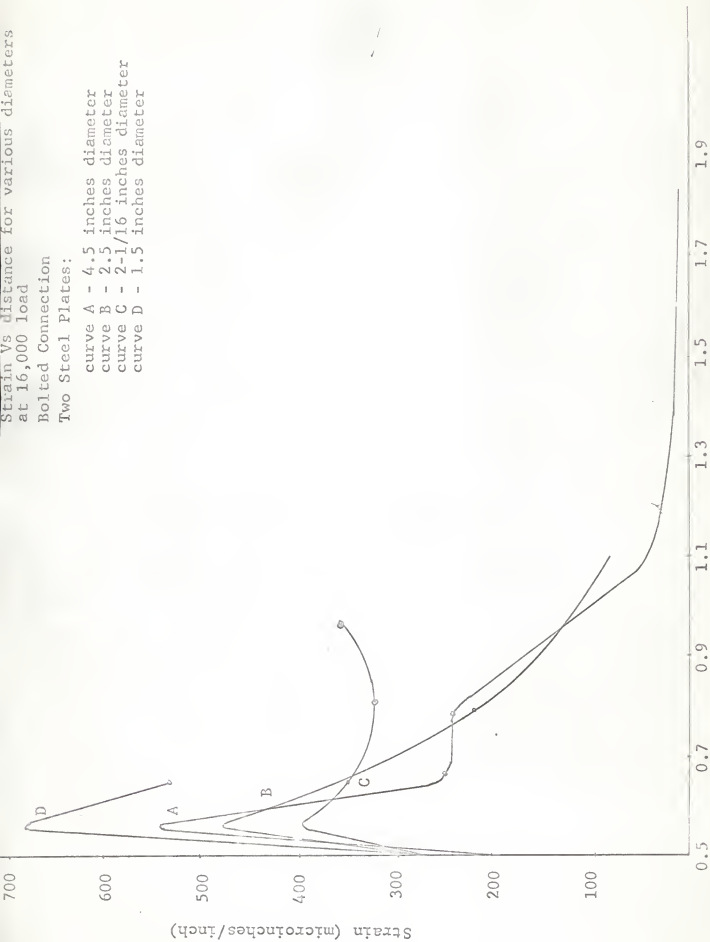


Fig. 7. Strain Vs Distance



At 2-1/16 inch diameter the maximum was decreased further. Strains in gages 3 and 4 were linear with the load. Strain in gage 5 was greater than in other gages till 12,000 lbs. For 4.5 and 2.5 inches diameters the strain in gage 5 was less than the strain in gages 1, 2, 3 and 4.

The maximum strain at 1.5 inches diameter was greater than the maximum strain at 4.5, 2.5 and 2-1/16 inches diameters. The strain in gage 2 was linear with the load. The strain in gage 3 was linear above 4,000 lbs. There was no strain in the gage 1 at 4,000 lbs.

The strain gages 1, 2 and 3 were located under the head of the stud. The strain in gage 1 was always less than the strains in gage 2 and 3 for all sizes. This indicates that the contact at the region near gage 1, (either at the contact surfaces of the plates or at the bearing surfaces of the stud and plates) came after the regions near gages 2 and 3 were stressed to an extent.

## EFFECTIVE AREA

The determination of the relative stiffness of a bolted connection is governed by "how the effective area is defined". The criterion of its definition should be such as to give a safe design of the bolted connection. The effective area calculated on the basis of the maximum strain would give a value less than that calculated by any other method (see the results of Srivastava). Hence the effective area of the bolted members is defined as the area of the bolted members around a bolt over which the strain is uniform and equal to the maximum strain in the members due to external load.

The effective areas were calculated based on the measured maximum strains. These may or may not be the maximum strains in the plates as discussed on page 7.

The effective area and equivalent diameter for each of the four sizes tested were calculated. (see appendix III) The relative stiffness factor,  $m$ , was also calculated for each size.

The results are tabulated below:

Table 3

No.	Diameter of plates in inches	Effective area in sq. inches	Equivalent Diameter in inches	$m$
1	4.50	0.979	1.500	0.406
2	2.50	1.110	1.550	0.415
3	2.06	1.350	1.650	0.368
4	1.5	0.705	1.414	0.500

From the calculated equivalent or effective diameter of the bolted members the ratio of the equivalent diameter to the actual diameter of the plates ( $D_e/D_o$ ) was calculated. This ratio was plotted against the ratio of the inside diameter to the outside diameter of the plates ( $D_i/D_o$ ) in Fig. 8. This ratio approaches 1.0 as the outside diameter is decreased.

Table 4

	outside diameter in inches	inside diameter in inches	$D_i/D_o$	$D_e/D_o$
1	4.5	1	0.222	0.333
2	2.5	1	0.400	0.620
3	2-1/16	1	0.484	0.800
4	1.5	1	0.666	0.944

$$\frac{D_e}{D_o} \quad \text{Vs} \quad \frac{D_i}{D_o}$$

Bolted Connection

Two steel plates

$D_e$  = calculated equivalent diameter in inches

$D_o$  = outside diameter in inches

$D_i$  = inside diameter in inches

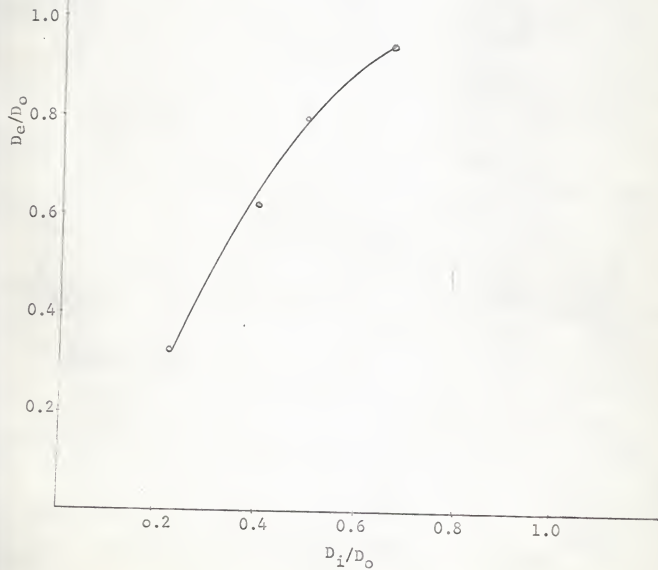


Fig. 8.  $D_e/D_o$ , Vs  $D_i/D_o$

## SUMMARY AND RECOMMENDATIONS

The effect of change in diameter of the bolted members on its effective area in a bolted assembly was studied. Two steel plates with a bolt hole and two steel studs were used as test pieces to simulate the bolted assembly. The strain in the plates was measured at different locations by strain gages installed in 1/4 inch holes. Due to the lack of flatness of the contact and bearing surfaces the testing was restricted to a single position. The diameter of the bolted members (steel plates) was reduced in steps from 4.5 inches to 2.5, 2-1/16, and 1.5 inches. The experiment was repeated for each size of the plates. The effective area of the bolted members was calculated on the basis of measured maximum strain in the members due to 16,000 lbs. load which corresponds to the initial tension in the steel bolt of 1 in. basic diameter.

The equivalent diameter of the bolted members and the relative stiffness factor of the bolted assembly,  $m$ , were also calculated.

It was found that the magnitude and pattern of stress distribution was influenced by the lack of flatness of the contact surfaces of the plates and the bearing surfaces of the studs and plates.

The ratio  $D_e/D_0$  approached unity as the diameter of the bolted member was reduced. The effective diameter was always less than the actual diameter of the bolted members.

The relative stiffness factor  $m$  was found varying from 0.37 to 0.50. It shows that the load in the bolt due to external force varies accordingly and depends largely upon the nature of the contact surfaces.

The lack of flatness of the contact surfaces has less influence for ductile materials than for nonductile materials because the ductile materials will yield under load.

In practice the contact surfaces in a bolted assembly would not even be as flat as the test pieces were. In the design of a bolted assembly the maximum strain due to localized stresses should be predicted and given consideration.

It was not possible to measure the actual maximum strain in the plates because of the limitation on the number of gages installed in the plates to measure strains. Moreover, the location of maximum strain is quite arbitrary and depends upon the nature of the contact surfaces.

For further work in this area, it is recommended that

(1) Photoelastic techniques may be used to study the stress distribution in bolted member and the effect of reducing the diameter of the bolted members. It would give a more continuous indication of the variation of stress distribution than strain gage techniques.

(2) Experimental investigation may be made on a number of plates to make a statistical study of the variation of effective area.

#### ACKNOWLEDGMENT

The author wishes to express his sincere appreciation to Dr. John C. Lindholm, major professor, for his advice and counsel during the investigation and subsequent interpretation of the results. Also, the author expresses his appreciation to Professor Frank J. McCormick for help in developing the experimental technique of mounting the strain gages and in running the experiment.

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APPENDICES

## APPENDIX 1

Table 8. Strain (microinches/in) vs Load for various locations. Bolted connection. Two steel plates, Diameter 4.5 inches, Plate Position 1.

gage no.	Loads in lbs.						
	4000	6000	8000	10000	12000	14000	16000
1	145	190	240	265	290	325	350
2	300	370	430	465	510	550	590
3	42	70	110	130	170	200	240
4	90	120	160	180	210	235	265
5	75	100	125	140	160	180	205
6	100	125	140	150	160	170	180
7	10	15	20	20	20	25	30
8	20	25	30	35	40	45	50
9	10	10	15	20	20	20	25
10	10	10	10	10	10	15	20
11	15	20	20	25	25	30	30

Table 9. Strain (microinches/inch) vs Load for various locations. Bolted connection. Two steel plates, Diameter 4.5 inch. Plate Position 2

gage no.	Loads in lbs.						
	4000	6000	8000	10000	12000	14000	16000
1	70	110	140	180	210	240	260
2	380	440	500	560	610	660	700
3	40	50	80	120	150	180	220
4	120	130	170	180	210	220	245
5	35	45	70	90	110	120	145
6	130	150	165	180	200	205	220
7	10	15	20	25	30	30	30
8	10	10	25	25	30	35	40
9	10	10	15	20	20	20	30
10	0	5	5	5	10	10	15
11	10	20	20	25	25	20	20

Table 10. Strain (microinches/inch) vs Load at various locations. Bolted connection. Two steel plates. Diameter 4.5 inches.

gage no.	Loads in lbs.						
	4000	6000	8000	10000	12000	14000	16000
1	60	75	100	130	160	185	220
2	290	335	380	425	470	505	545
3	50	78	105	135	170	200	248
4	115	125	150	175	195	215	240
5	60	75	90	105	120	132	150
6	10	15	20	25	30	35	40
7	10	10	15	15	15	18	20
8	10	10	15	15	15	18	20
9	10	10	10	10	15	15	15
10	5	5	5	10	10	10	10
11	10	15	15	15	20	20	25

Table 11. Strain (microinches/inch) vs load at various locations. Bolted connection. Two steel plates. Diameter 2.5 inches.

gage no.	Loads in lbs.						
	4000	6000	8000	10000	12000	14000	16000
1	40	70	110	150	190	220	260
2	170	240	300	350	400	440	480
3	90	135	180	225	270	310	350
4	40	60	85	110	140	165	190
5	35	55	75	90	115	130	150
6	20	30	40	50	58	65	75

Table 12. Strain (microinches/inch) vs load at various locations. Bolted connection. Two steel plates. Diameter 2-1/16 inches.

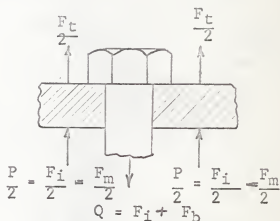
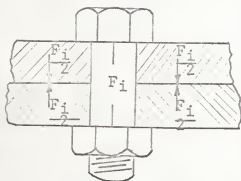
gage no.	Loads in lbs.						
	4000	6000	8000	10000	12000	14000	16000
1	88	115	150	175	205	235	260
2	90	140	200	260	305	350	395
3	85	130	175	220	260	305	340
4	80	120	160	205	250	280	320
5	190	230	270	300	320	340	355

Table 13. Strain (microinches/inch) vs load at various locations. Bolted connection. Two steel plates. Diameter 1.5 inches.

gage no.	Loads in lbs.						
	4000	6000	8000	10000	12000	14000	16000
1	0	10	40	85	140	200	260
2	180	260	350	440	525	590	680
3	90	170	250	340	425	490	580

## APPENDIX II

When a bolt is tightened the forces in the bolted members and bolt are as shown in Fig. 9. When an external load is applied to the assembly the forces in the members and bolt are as shown in Fig. 10.



$F_i$  = Initial bolt tension, lb.

$F_t$  = External tensile load added to members joined, lb.

$F_b$  = Added bolt tension caused by load  $F_t$  on joint, lb.

$F_m$  = Decreased compression between members joined due to added load  $F_t$ , lb.

$P$  = Total load in the members.

$Q$  = Total load in the bolt.

$k$  = Stiffness

$E$  = Elasticity of Modulus

$L$  = Length

$m$  = Relative stiffness constant

subscripts b and m denote bolt and members.

External load applied to the pretightened assembly increases the deformation in the bolt and decreases the deformation in the bolted members. The deformation depends upon the stiffness and load acting. Stiffness constant is the ratio of the load applied to the deflection produced by that load.

The deformation due to a tension or compressive load is given as

$$\text{Deformation} = \frac{(\text{Load}) (\text{Length})}{(\text{Area}) (\text{Modulus of Elasticity})}$$

$$\begin{aligned} \text{Stiffness constant } k &= \frac{\text{load}}{\text{deformation}} \\ &= \frac{(\text{Area}) (\text{Modulus of Elasticity})}{\text{Length}} \end{aligned}$$

The increase in the deformation of the bolt due to the external load,

$$\Delta S_b = \frac{F_b}{k_b}$$

The decrease in the deformation of the members due to the external load;

$$\Delta S_m = \frac{F_m}{k_m}$$

On the assumption that the members have not separated,

$$\Delta S_b = \Delta S_m$$

hence

$$\frac{F_b}{k_b} = \frac{F_m}{k_m} \quad (a)$$

Also 
$$F_t = F_m + F_b \quad (b)$$

From equations (a) and (b)

$$F_b = \frac{k_b}{k_b + k_m} F_t$$

The total load in the bolt:

$$\begin{aligned} Q &= F_i + F_b = F_i + \frac{k_b}{k_b + k_m} F_t \\ &= F_i + m F_t \end{aligned} \quad (1)$$

The distribution of loads and strains in a bolted joint is explained by the diagram in Fig. 11.

Line OA has slope equal to the stiffness constant of bolt,  $k_b$ , Line CD has slope equal to the stiffness constant of bolted members,  $k_m$ . The lines OA and CD intersect at E. The ordinate of E gives the initial tension in the bolt and the abscissa of E gives initial elongation in the bolt. CE' is the compression of bolted members due to initial tension. When the external load  $F_t$  is applied to the joint, the total tension in the bolt is given by  $Q = F_i + F_b$  and total compression load in the members is given by  $P = (F_i - F_m)$ .





### APPENDIX III

Effective area was calculated using the maximum strain:

For 4.5 inches diameter plates:

$$F = 16,000, E = 30 \times 10^6$$

$$\epsilon_{\max} = 545 \times 10^{-6} \text{ inches/inch}$$

$$A = \frac{16,000}{545 \times 10^{-6} \times 30 \times 10^6} = 0.979 \text{ sq. in.}$$

Equivalent Diameter:

$$A = \frac{\pi}{4} (D_e^2 - d^2)$$

$$0.979 = \frac{\pi}{4} (D_e^2 - 1)$$

$$D_e^2 = \frac{4}{\pi} (0.979) + 1$$

$$D_e = 1.5 \text{ inches.}$$

$$m = \frac{\left(\frac{AE}{L}\right)_b}{\left(\frac{AE}{L}\right)_b + \left(\frac{AE}{L}\right)_m} = \frac{(0.785)}{(0.785) + (0.979)} = 0.446$$

For 2.5 inches diameter:

$$\epsilon_{\max} = 480 \times 10^{-6} \text{ inch/inch.}$$

$$A = \frac{16,000}{480 \times 10^{-6} \times 30 \times 10^6} = 1.11 \text{ sq. in.}$$

$$D_e^2 = \frac{4}{\pi} (1.11) + 1$$

$$D_e = 1.55 \text{ inches}$$

$$m = \frac{0.785}{0.785 + 1.11} = 0.415$$

For 1/16 inches diameter:

$$\epsilon_{\max} = 395 \times 10^{-6} \text{ inch/inch.}$$

$$A = \frac{16,000}{395 \times 10^{-6} \times 30(10)^6} = 1.35 \text{ sq. in.}$$

$$D_e^2 = \frac{4}{\pi} (1.35) + 1$$

$$D_e = 1.65 \text{ inches.}$$

$$m = \frac{0.785}{0.785 + 1.35} = 0.368$$

For 1.5 inches diameter:

$$\epsilon_{\max} = 680 \times 10^{-6} \text{ inch/inch.}$$

$$A = \frac{16,000}{680 \times 10^{-6} \times 30 \times 10^6} = 0.785 \text{ sq. in.}$$

$$D_e^2 = \frac{4}{\pi} (0.785) + 1 = 2$$

$$D_e = 1.414 \text{ inches.}$$

$$m = \frac{0.785}{0.785 + 0.785} = 0.50$$

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AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Mechanical Engineering

KANSAS STATE UNIVERSITY  
Manhattan, Kansas

1967

An equation for the total load in bolt of the bolted assembly without a gasket was developed assuming all the parts elastic.

$$Q = F_i + mF_t$$

$$m = \frac{k_b}{k_b + k_m}$$

where

$Q$  = total load in lb in a bolt

$F_i$  = Initial tension due to tightening in lb in the bolt

$F_t$  = External load in lbs.

$m$  = relative stiffness constant of the bolted assembly

$k_b$  = stiffness constant of bolt in lb/in.

$k_m$  = stiffness constant of members in lb/in.

Total load in the bolt depends upon the value of  $m$ . The value of  $m$  depends upon the effective area, effective length and modulus of elasticity of the bolt and bolted members. This report presents the effect of change in diameter of the connected members on its effective area. It was found that the lack of flatness of contact and bearing surfaces of the bolted assembly had a great influence on the stress distribution in the bolted members.

Effective areas for four sizes of connected members were calculated based on the measured maximum strains in the members. The results obtained are tabulated below.

Table 14

No.	Diameter in inches	Effective area in sq. in.	Equivalent diameter in in.
1	4.5	0.979	1.50
2	2.5	1.11	1.55
3	2.0625	1.35	1.65
4	1.5	0.785	1.414

The value of  $m$  was calculated in each case and found varying from 0.368 to 0.5 as the diameter of the bolted members was decreased the ratio of calculated equivalent diameter to the actual diameter of the bolted members approached unity.

The effective area calculated in each case was found to be less than the area calculated by standard design procedures. This indicates that the standard design procedure gives an optimistic design.