EDDY CURRENT LOSSES IN SOLID AND LAMINATED IRON

by

ASHOK KUMAR DALAL

B. S. (Mechanical) & B. S. (Electrical)
Vallabh University, Anand, India, 1961

A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Electrical Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1963

M. C. Cottom

TABLE OF CONTENTS

INTRODUCTION

When iron is subjected to an alternating magnetomotive force, energy is lost in the iron owing to the phenomena of eddy current and hysteresis. In most applications the designer is interested in minimizing these losses to improve the efficiency. However, many parts of machines cannot be adequately laminated and solid surfaces are exposed to alternating fields. On the other hand, there are applications like magnetic clutches and eddy current brakes, where one tries to maximize these losses.

Therefore the problem of estimating losses in solid and laminated iron has been of prime concern in this report. The greatest problem arises owing to the extremely nonlinear relation, which the hysteresis loop represents, between B and H, for a ferromagnetic material. Therefore an exact mathematical calculation is very difficult.

However, an approximate solution may be obtained by assuming the permeability to be constant, that is, the hysteresis loop is condensed to a single line through origin. Solution of Maxwell's field equations in a half plane shows that the flux density and current density are maximum at the surface, and decrease exponentially with penetration into the material and are also phase shifted.

This theory may yield good results so long as the peak magnetization is low, or essentially the material is being worked on the linear portion of the magnetization curve.

The analytical derivation of the formulas in this report is by the rigorous solution of Maxwell's field equations, assuming the B-H relation for iron to be an ideal rectangular magnetization curve. Actually the B-H curve of steel departs materially from the assumed rectangular shape.

Also the wave shapes of applied magnetomotive force and flux occasionally depart materially from pure sine waves. These and other factors can be taken into account empirically. The value of B_0, the saturation flux density, is so determined to be 3/4 B_m, where B_m is the flux density from the static magnetization curve of the material, corresponding to the peak of the imposed mmf at the surface.

EDDY CURRENT LOSSES FOR SINUSOIDAL MMF AND IDEAL MAGNETIZATION CURVE

Assumed Magnetization Curve

The assumed ideal magnetization curve under which the eddy current phenomena are being analyzed is shown in Figure 1.

The material is magnetized to saturation for any value of magnetic intensity, h, except for h=0, when it is only possible for flux density to change. This does not imply that when h=0 the flux density must change. Thus it is possible to have regions within the material where h is zero but the flux density may have any constant value less than or equal to the saturation flux density B_0. The particular constant value depends on the state in which the material was left during some previous cycle of operation.

Physical Picture of Phenomena

Consider an iron plate of infinite dimensions in the xy plane, with its thickness, 2d, in the z direction as shown in Figure 2.

Let a magnetizing winding, not shown in the figure, produce and maintain on each face of the plate a uniform sinusoidal magnetizing force $H=H_m \text{Sin } wt$, in the y direction, with no variation in the x direction. Thus x-polarized plane waves impinge on the outside faces of the plate and propagate inwards

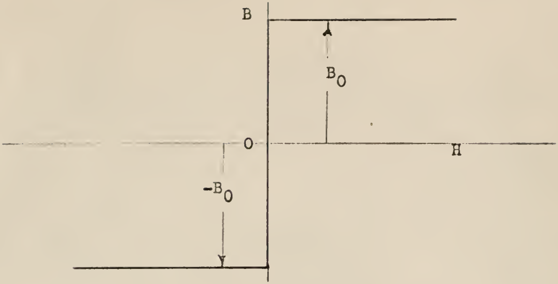


Fig..1.. Assumed magnetization curve.

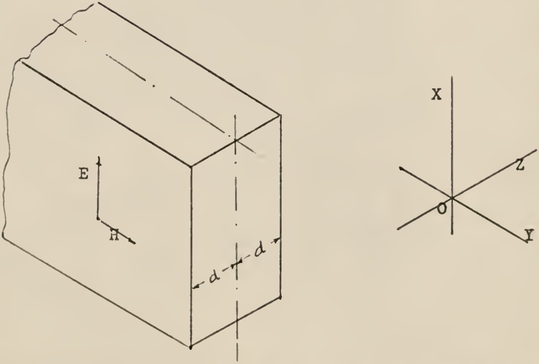


Fig..2.. Plane waves penetrating inwards from both faces of the plate..

towards the center of the plate. Since this wave penetration is symmetrical with respect to the middle of the plate, we need consider the phenomena in the half thickness of the plate only. As shown in Appendix III the h wave enters the plate from the surface and penetrates inward with resulting attenuation. The movement of the front of the wave, h=0, shown in Figure 30 is of particular interest because, due to the assumed B-H curve, change in magnetization takes place only at this point. The wave front, h=0, with h increasing, is accompanied by a flux density wave of constant amplitude B_0. Thus we see that flux wave density B_0 penetrates from the faces into the plate. The flux waves reach their maximum depth at the end of a half cycle. This extreme depth to which fields penetrate and beyond which there are no fields at all is called "depth of penetration", δ.

Two cases now arise, one in which depth of penetration, δ, is less than the half thickness, d, of the plate, and the other in which δ is greater than d. In the first case, the fields penetrate to depth on each side of the plate and the inner part of the plate remains un-magnetized. In the second case, the two flux waves will meet at the center of the plate before the end of the half period and for the remaining part of the half cycle there will be no change of flux in the plate.

At the end of a half period, h at the surface becomes zero and is at the beginning of its negative half wave. Now again a wave front, h=0, but with h decreasing, penetrates into the plate. This wave front is accompanied by a flux density wave of constant amplitude, $-B_0$, and reverses the direction of magnetization in the plate. Thus we see that the plate is alternately magnetized to density $\pm B_0$ up to the depth δ, or d, whichever is smaller. The transition of magnetic induction takes place at a point $\mathfrak{f}(t)$, which defines an xy plane, referred to here as the "separating surface".

The separating surface moves parallel to itself with time. This concept is actually taken from MacLean[12] because of its hypothetical physical meaning, but is extended to the general case of arbitrary thickness of the plate.

Maxwell's Equations

There are seven equations, commonly known as Maxwell's Equations, which summarize the fundamental principles expressing the nature and behavior of electromagnetic fields, whether static or dynamic.

Two Maxwell's field equations in media at rest and 1 in integral form, neglecting displacement current are as follows:

I. The line integral of magnetic intensity over any closed line path is equal to the current enclosed by that path;

$$\text{i. e.,} \quad \oint h\, dl = \int_S J\, ds$$

J = current density.

Now, if γ is the conductivity $= \dfrac{\text{Current}}{\text{Volt}}$

$$\text{or} \quad \int_S J\, ds = \gamma \int_S e\, ds,$$

e = electric field intensity.

Therefore,

$$\oint h\, dl = \gamma \int_S e\, ds. \qquad \text{1a}$$

II. The line integral of electric field intensity in a closed line path is the negative time derivative of the magnetic flux linking the path.

$$\oint e\, dl = -\frac{d\phi}{dt}$$

$$\text{but } \phi = \int_S b\, ds.$$

b = magnetic flux intensity.

Therefore, $\oint e\, dl = -\frac{d}{dt}\int_S b\, ds.$ 1b

In the above equations all variables are in the MKS system.

Now $$\bar{h} = \bar{i}h_x + \bar{j}h_y + \bar{k}h_z$$

$$\bar{e} = \bar{i}e_x + \bar{j}e_y + \bar{k}e_z$$

In the problem under consideration, the electric field has only an x component $\bar{i}e_x$, and the magnetic field has only a y component $\bar{j}h_y$;

i.e., $$\bar{e} = \bar{i}e_x$$

$$\bar{h} = \bar{j}h_y$$

This reduces Maxwell's equations to,

$$\oint \bar{j}h_y \cdot dl = \gamma\int_S e\, ds \quad 2a$$

$$\oint \bar{i}e_x \cdot dl = -\frac{d}{dt}\int_S b\, ds = -\frac{d\phi}{dt} \quad 2b$$

where ϕ is the flux passing through the surface around which the line integral is taken.

Boundary Conditions

Let E, B and H be the values of the variables at the surface; i.e., z = 0. We seek the solution of equations (2a, b) compatible with Figure 1, the assumed magnetization curve, and with an impressed sinusoidal magnetizing force at the surface $H = H_m \mathrm{Sin}\ t$ 3

The rectangular magnetization curve can be expressed as,

$$B = B_0 \mathrm{Sig.}H \quad 4$$

"sigH" means "Sign of H" and is the unit square wave associated with sinusoidal H.

Solution of Maxwell's Equations

Consider a yz section of half the plate as shown in Figure 3. At any instant, let the separating surface be at depth ξ and moving towards the center of the plate with a velocity V. Then in a time dt, the flux (B_0Vdt) is built up and at the same time an equal amount of residual flux is destroyed.

Therefore, the rate of change of flux per unit length in the x direction is given by

$$\frac{d\phi}{dt} = 2B_0V \quad (5)$$

The line integral in equation (2b) is easily evaluated in this case by taking the surface S in the xz plane, Figure 4. Consider the closed loop abmc, in Figure 4. There is no change of flux taking place in this area;

$$\text{therefore,} \quad e_{ab} = e_{cm} = 0 \quad (6)$$

which shows that there is no electric field beyond the separating surface.

Now consider the closed loop cmgf, which encloses the separating surface. From Equation 2b and 5

$$e_{cm} + e_{gf} = 2B_0V = 2B_0\left(\frac{d\xi}{dt}\right)$$

Since $e_{cm} = 0$, by Equation 6,

$$e_{gf} = 2B_0\frac{d\xi}{dt} \quad (7)$$

There is no change of flux to the left of the separating surface and hence, considering loop fgkh,

$$e_{fg} = e_{hk} = E \quad (8)$$

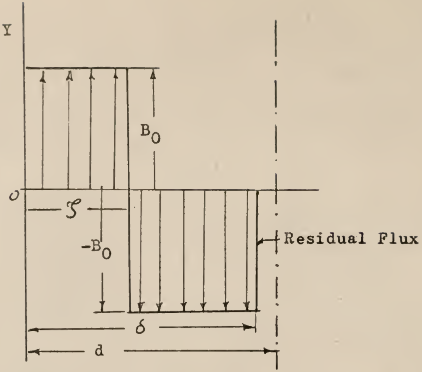


Fig.3.. Separating surface penetrating the plate..

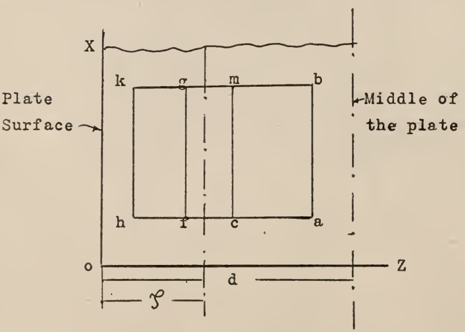


Fig..4.. XY Section of the plate.

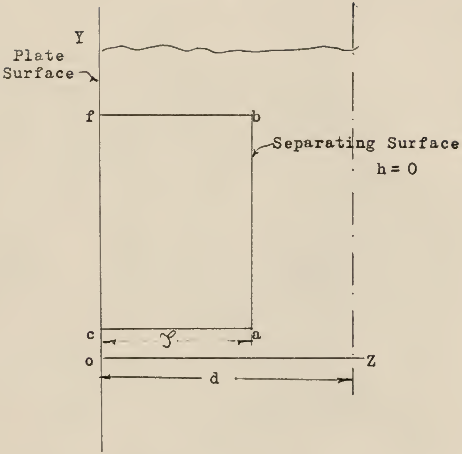


Fig.. 5.. YZ Section of the plate.

This shows that the electric field in the region to the left of the separating surface, that is $0 < z < \delta$, is the same everywhere and its magnitude is equal to E, the value of the electric field at the outside surface, and the discontinuity in the electric field at the separating surface.

To evaluate the line integral in Equation 2a, consider the loop abcf in Figure 5, with surface in the yz plane.

The magnetic field intensity, h, is zero at the separating surface and has no component in the z direction.

$$\int_{abcf} h \, dl = H$$

$$= \text{Current enclosed in the loop abcf.}$$

$$= \gamma \delta(t) E$$

$$\text{Now } \frac{dh}{dz} = \frac{H}{\delta(t)} = \gamma E. \tag{9}$$

From the above equation we draw the following conclusions:

I. The induced eddy current flows with uniform density in the region between the outside surface of the plate and the separating surface, and zero beyond.

II. The magnetic field intensity, h, decreases linearly from its value H at the outside surface, ($z=0$), to zero at the separating surface ($z=\delta$).

III. The electric field intensity, e, and magnetic field intensity, h, are zero beyond the separating surface.

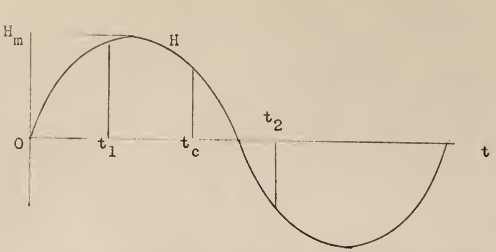
Field Configuration at Different Times of the Cycle

The field configuration at four different instants is given in Figure 6. The four instants are indicated on the sinusoidal H curve in Figure 6a. In Figure 6b, $\xi < d$, the electric field has the value e=E or zero as shown, and h is linear to ξ and then zero as shown. When the separating surface moving inwards reaches the center of the plate, that is at $\xi = d$ (say at $t = t_c$) the whole material is magnetized to saturation and no change of flux takes place for the remaining part of the cycle, that is during the interval between t_c and $\frac{\pi}{\omega}$. After half a cycle, that is at $t = \frac{\pi}{\omega}$, the separating surface again starts from each face and reverses the direction of magnetization. During the interval t_c to $\frac{\pi}{\omega}$, there is no change of flux in the plate and hence, e=0. Substituting from Figure 6b in Equation 9 we find

$$\frac{dh}{dz} = \frac{H}{\xi} = \gamma E \qquad \xi < d \text{ or } 0 < t < t_c \qquad (10)$$

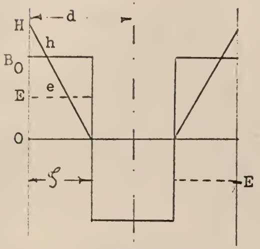
$$= 0 \qquad t_c < t < \frac{\pi}{\omega} \qquad (11)$$

meaning that after t_c, h everywhere jumps to value H at the surface, thus becoming uniform. This is shown in Figure 6d. After a half cycle, that is at $t = \frac{\pi}{\omega}$, H, the magnetic field intensity at the surface, is equal to zero and therefore, h is zero everywhere in the material, and a new separating surface of opposite magnetic induction starts from the two ends. This condition is shown in Figure 6e.



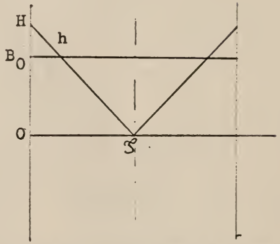
t_c = The instant at which separating surface reaches the middle of the plate.

(a)



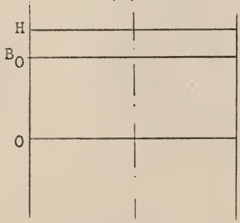
Flux has penetrated to depth $\delta < d$, $t = t_1$.

(b)



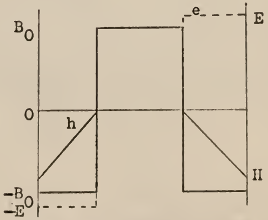
Flux wave has reached the center of the plate $\delta = d$. $t = t_c$

(c)



After the flux wave has reached the center of the plate, $h = H$, uniform.

(d)



A new magnetic flux wave has penetrated for a distance $t = t_2$

(e)

Fig. 6. Field configuration at four different instants.

Thus we have replaced Maxwell's equations by the following pairs:

$$\frac{H}{\xi} = \gamma E \qquad \left\{ \begin{array}{l} 0 < t < t_c \\ \xi < d \end{array} \right. \qquad \begin{array}{r} 12a \\ 12b \end{array}$$

$$E = 2B_0 \frac{d\xi}{dt}$$

or $h = H$

$$e = 0 \qquad t_c < t < \frac{\pi}{\omega}$$

Eliminating E between equations 12a and 12b.

$$2\xi\frac{d\xi}{dt} = \frac{H}{\gamma B_0} = \frac{H_m \operatorname{Sin} Wt}{\gamma B_0 \operatorname{Sig} H}$$

$$\frac{d(\xi^2)}{dt} = \frac{H_m}{\gamma B_0} \left| \operatorname{Sin} Wt \right| \qquad 13$$

Equation 13 shows that the transition point ξ always moves from the outside surface towards the center of the plate where it ceases to exist, and repeats every half cycle when H becomes zero at the surface and subsequent half cycles of $\xi(t)$ are identical.

Depth of Penetration

If the plate is thick enough, then the transition point, ξ, will penetrate to its maximum depth at the end of the half cycle. This maximum depth to which fields penetrate is called the depth of penetration, δ. The separating surface as a function of time, in the first half period can be evaluated as follows:

$$\frac{d(\xi)^2}{dt} = \frac{Hm}{\gamma B_o} \text{ Sin}\omega t \ dt \qquad t < \frac{\pi}{\omega}$$

$$\xi^2 = \int_0^t \frac{Hm}{\gamma B_o} \text{ Sin}\omega t \ dt = \frac{Hm}{\gamma B_o} \frac{\text{I-Cos}\omega t}{W}$$

giving
$$\xi = \left[\frac{2Hm}{\omega \gamma B_o}\right]^{\frac{1}{2}} \text{Sin} \frac{\omega t}{2} \qquad 0 < t < \frac{\pi}{\omega} \qquad 14$$

The depth of penetration is simply the maximum value of ξ which is attained at the end of each half period

$$\delta = \left[\frac{2Hm}{\gamma \omega B_o}\right]^{\frac{1}{2}} \qquad 15$$

$$\delta = \left[\frac{2}{\gamma \omega B_o/H_m}\right]^{\frac{1}{2}}$$

The field will penetrate to this depth when d, the half thickness of the plate, is greater than or equal of δ .

It should be noted, that in the linear case where permeability is constant and equal to B_0/H_m at the surface, the depth of penetration or skin depth is also given by equation 15, but it has significance as the depth at which the field has decreased to $1/\epsilon$ (about 36.9% of the value at the surface)

In a thin plate, that is where the half thickness of the plate is less than the depth of penetration, the two waves moving inward from the outside faces meet at the center before the end of half period. The solution of the problem for this portion of the half period is identical with that for the thick plate, because the electromagnetic wave cannot distinguish whether it is penetrating a thick plate or thin plate, until it reaches the middle and confronts the wave approaching from the opposite side.

Therefore

$$\mathcal{S} = \left[\frac{2H_m}{\gamma \omega \beta_o}\right]^{1/2} \sin \frac{wt}{2} \qquad 0 < t < t_c$$

$$= \delta \quad \sin \frac{\omega t}{2} \qquad 16$$

where t_c is the time when the separating surface reaches the middle of the plate.

$$d = \delta \sin \frac{wt_c}{2} \qquad 17$$

$$t_c = \frac{2}{w} \sin^{-1}\left(\frac{d}{\delta}\right) \qquad 18$$

We can calculate the value of E from Equation 12a, which can be written

$$E = \frac{1}{\gamma} \frac{H_m \sin wt}{\delta \sin \frac{\omega t}{2}} \qquad 0 < t < t_c \qquad 19$$

$$= 0 \qquad t_c < t < \frac{\pi}{\omega}$$

in the first half period. Since in successive half periods, H reverses and does not, E reverses each half cycle.

Equation 19 can be written,

$$E = \frac{2H_m}{\gamma\delta} \cos \frac{wt}{2} \qquad 0 < t < t_c \qquad 20$$

$$= 0 \qquad t_c < t < \frac{\pi}{\omega}$$

A plot of $\mathcal{S}(t)$, $E(t)$ and H is given in Figure 7.

Power Loss From Poynting Vector

To determine the power loss per unit area, designate by H and E the peak complex amplitude of H and the fundamental component of E. The Poynting Vector, N into the metal would be:

$$\bar{N} = N + jM = \frac{1}{2}\bar{E}\,\bar{H}^*$$

where $\bar{H}^*$ means the conjugate of $\bar{H}$. The real part of $\bar{N}$ is N, which gives the power flow. Since H is sinusoidal, the harmonics of E do not contribute to the power flow.

A complex surface impedance for plane wave (wave impedance at the surface), $\bar{\eta}$ is usually designated by

$$\bar{\eta} = R + jX = \frac{\bar{E}}{\bar{H}} \qquad 22$$

which gives

$$\begin{aligned} N = \text{Re}\ \bar{N} &= \frac{1}{2}\ \text{Re}\ \bar{E}\,\bar{H}^* \\ &= \frac{1}{2}\ \text{Re}\ \bar{\eta}\ \bar{H}\,\bar{H}^* \\ &= \frac{1}{2}\ \text{Re}\ \bar{\eta}\ H_m^2 \\ &= \frac{1}{2}\ H_m^2\ \text{Re}\,\bar{\eta} \\ &= \frac{1}{2}\ H_m^2\ R \end{aligned} \qquad 23$$

Fourier Coefficent for The Fundamental of E

The fundamental of E can be calculated by Fourier Series:

$$E = 2\sum_{1}^{\infty}(A_n \cos n\omega t + B_n \sin n\omega t) \qquad 24$$

$$\text{where } A_n = \frac{1}{T}\int_0^T E(t) \cos n\omega t \ dt. \qquad 25$$

$$B_n = \frac{1}{T}\int_0^T E(t) \sin n\omega t \ dt.$$

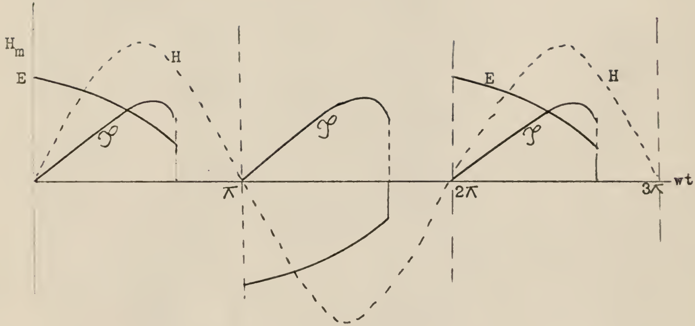


Fig. 7. Wave shape of H, E and $\mathcal{S}$.

For the fundament l, that is for n = 1, using Figure 7 and equation 20

$$A_1 = \frac{2Hm}{\gamma\delta}\,\frac{\omega}{2\pi}\int_0^{t_c} 2\ \mathrm{Cos}\,\frac{\omega t}{2}\,\mathrm{Cos}\,\omega t\ \ dt.$$

$$= \frac{Hm\,\omega}{\gamma\delta\pi}\int_0^{t_c}\left(\mathrm{Cos}\left(\frac{3\omega t}{2}\right) + \mathrm{Cos}\left(\frac{\omega t}{2}\right)\right) dt.$$

$$= \frac{2Hm}{\gamma\delta\pi}\left[\frac{1}{3}\,\mathrm{Sin}\left(\frac{3\omega t_c}{2}\right) + \mathrm{Sin}\left(\frac{\omega t_c}{2}\right)\right] \qquad 26$$

$$B_1 = \frac{2Hm\,\omega}{\gamma\delta\pi}\int_0^{t_c} \mathrm{Cos}\,\frac{\omega t}{2}\,\mathrm{Sin}\,\omega t\quad dt.$$

$$= \frac{Hm\,\omega}{\gamma\delta\pi}\int_0^{t_c} \mathrm{Sin}\left(\frac{3\omega t}{2}\right) + \mathrm{Sin}\left(\frac{\omega t}{2}\right)\quad dt.$$

$$= \frac{2Hm}{\gamma\delta\pi}\left[\frac{4}{3} - \frac{1}{3}\,\mathrm{Cos}\left(\frac{3\omega t_c}{2}\right) - \mathrm{Cos}\left(\frac{\omega t_c}{2}\right)\right] \qquad 27$$

From Equ tion 27

$$\mathrm{Sin}\left(\frac{\omega t_c}{2}\right) = \frac{d}{\delta}$$

$$\mathrm{Cos}\left(\frac{\omega t_c}{2}\right) = \left[1 - (d/\delta)^2\right]^{\frac{1}{2}}$$

$$\mathrm{Sin}\left(\frac{3\omega t_c}{2}\right) = 3\ \mathrm{Sin}\left(\frac{\omega t_c}{2}\right) \quad 4\ \mathrm{Sin}^3\left(\frac{\omega t_c}{2}\right)$$

$$= \left[3\ d/\delta - 4(d/\delta)^3\right] \qquad 29$$

$$\mathrm{Cos}\left(\frac{3\omega t_c}{2}\right) = 4\ \mathrm{Cos}^3\,\frac{\omega t}{2} - 3\ \mathrm{Cos}\left(\frac{\omega t_c}{2}\right)$$

$$= 4\left[1-(d/\delta)^2\right]^{3/2} - 3\left[1-(d/\delta)^2\right]^{\frac{1}{2}} \qquad 30$$

Substituting Equations 17, 28, and 29 and 30 in Equation 26 and 27:

$$A_1 = \frac{4H_m}{\gamma \delta \pi} \left[d/\delta - 2/3 \ (d/\delta)^3 \right] \qquad 31$$

$$B_1 = \frac{8H_m}{3\gamma\delta\pi} \left[1 - (1 - d^2/\delta^2)^{3/2} \right] \qquad 32$$

Therefore the fundamental of E

$$\bar{E}_1 = \frac{8H_m}{3\gamma\pi\delta} \left[\left(3\frac{d}{\delta} - 2\frac{d^3}{\delta^3}\right) \cos\omega t + 2\left\{1-\left(1-\frac{d^2}{\delta^2}\right)^{3/2} \sin\omega t\right\} \right] \qquad 33$$

and the peak complex amplitude of E and H is given by

$$\bar{E} = \frac{8H_m}{3\gamma\delta\pi} \left[\left(3\frac{d}{\delta} - 2\frac{d^3}{\delta^3}\right) - j2\left\{1 - \left(1-\frac{d^2}{\delta^2}\right)^{3/2}\right\} \right] \qquad 34$$

$$\bar{H} = -jH_m \qquad 35$$

Substituting these in Equation 22, the surface impedance

$$\bar{\eta} = R + jX = \frac{\bar{E}}{\bar{H}}$$

$$= \frac{16}{3\pi\gamma\delta} \left[\left\{1 - \left(1-\frac{d^2}{\delta^2}\right)^{3/2}\right\} + \frac{j}{2}\left(3\frac{d}{\delta} - 2\frac{d^3}{\delta^3}\right) \right] \qquad 36$$

The angle θ in the complex surface impedance $\bar{\eta}$ is equal to:

$$\theta = \tan^{-1}\frac{X}{R}$$

$$\tan\theta = \frac{X}{R} = \frac{3\frac{d}{\delta} - 2\left(\frac{d}{\delta}\right)^3}{2\left[1 - \left(1-\frac{d^2}{\delta^2}\right)^{3/2}\right]} \qquad 37$$

Eddy Current Loss and Power Factor Formula

The power factor that the eddy current losses reflect into the exciting winding is

$$P.F = \cos\theta = \cos\left[\tan^{-1} \frac{\frac{3d}{\delta} - 2\left(\frac{d}{\delta}\right)^3}{2\left[1-\left(1-\frac{d^2}{\delta^2}\right)^{3/2}\right]}\right] \qquad 38$$

The surface resistance is the real part of the surface impedance and is given by

$$R = \frac{16}{3\pi\gamma\delta}\left[1 - \left(1-\frac{d^2}{\delta^2}\right)^{3/2}\right] \qquad 39$$

The loss per unit area N, given by Equation 23 is thus:

$$N = \tfrac{1}{2}\, R\, H_m^2$$

$$= \frac{8}{3\pi}\,\frac{Hm^2}{\gamma\delta}\left[1-\left(1-\frac{d^2}{\delta^2}\right)^{3/2}\right]$$

$$= \frac{8}{3\pi}\,\frac{Hm^2}{\gamma\delta}\, L_F \qquad 40$$

where L_F is the lamination factor

$$L_F = \left[1-\left(1-\frac{d^2}{\delta^2}\right)^{3/2}\right]$$

Solid Iron Case

By solid iron we imply here the case when the half thickness of the plate is equal to or greater than the depth of penetration. The moving surface as a function of time is given by Equation 14.

$$\delta = \left[\frac{2H_m}{\omega \gamma B_0}\right]^{\frac{1}{2}} \sin \frac{\omega t}{2} \qquad 0 < t < \frac{\pi}{\omega}$$

$$= \delta \sin \frac{\omega t}{2} \qquad 41$$

The electric field at the surface can be obtained from Equation 20, realizing that now the field penetrates up to the end of the half period.

$$E = \frac{2H_m}{\gamma \delta} \cos \frac{\omega t}{2} \qquad 0 < t < \frac{\pi}{\omega}. \qquad 42$$

A plot of $\delta(t)$, $E(t)$ and $H(t)$ is given in Figure 8. Here again, since the applied magnetic field is sinusoidal, only the fundamental of the electric field will contribute to power flow into the material.

The fundamental of the electric field is determined by Fourier Series. In determining the Fourier coefficients A_1 and B_1 the upper limit of integration has now to be changed to $T = \frac{\pi}{\omega}$ instead of $T = t_c$.

This yields
$$A_1 = \frac{4 H_m}{3\pi \gamma \delta} \qquad 43$$

$$B_1 = \frac{8 H_m}{3\pi \gamma \delta} \qquad 44$$

whereby
$$E = \frac{8H_m}{3\pi \gamma \delta} \left(\cos \omega t + 2 \sin t \right) \qquad 45$$

and the peak complex amplitude of E and H is given by

$$E = \frac{8 H_m}{3\pi \gamma \delta} \quad (1 - 2j) \qquad 46$$

$$H = -jH_m \qquad 47$$

This gives
$$N = \frac{8}{3\pi} \quad \frac{H_m^2}{\gamma \delta} \qquad 48$$

$$P.F. = \cos \left(\tan^{-1} \tfrac{1}{2}\right) = 0.894 \qquad 49$$

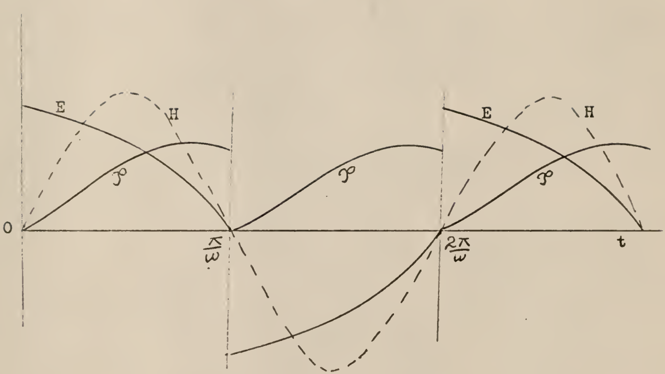


Fig.. 8. Wave shape of H, E and $\mathcal{S}$ for the solid iron case..

It can be seen from Equation 48 and 49 that the power loss per square meter, and p.f. for the solid case can be readily obtained from Equation 40 and 38 by substituting $d/\delta = 1$, the loss per square meter is independent of the value d, as long as d is greater than δ. Therefore, the maximum value of d/δ to be used in formula 38 and 40 is unity. MacLean was the first to solve the solid iron case and obtained the loss formula, Equation 48. His analysis has been extended to thin plates, thus yielding the general formula, Equation 38 is applicable irrespective of the thickness of the material.

SEARCH COIL VOLTAGE

Computation of Search Coil Voltage

The variation of flux in the iron ring specimens on the basis of the theory presented here can be calculated as follows:

Let n = the number of laminations in the ring.

d = the half thickness of laminations

l = the width of laminations as shown in the Figure 9.

Consider the general case when $d < \mathcal{S}$, let the separating surface be at a depth $\mathcal{S}$ from the outside surface of the laminations.

Φ is the instantaneous value of the flux in the iron ring.

$$\Phi = 2n\ell B_0 (d-2\mathcal{S}). \qquad 50$$

Substituting the value of from Equation 16

$$\Phi = 2nlB_0(d-2\delta \sin \frac{\omega t}{2}) \qquad 0 < t < t_c \qquad 51$$

where t_c is the time the separating surface takes to travel from the outside surface of the plate to its middle.

$$t_c = \frac{2}{\omega} \operatorname{Sin}^{-1} \frac{d}{\delta} \qquad \text{from Equation 18}$$

and $$\phi = 2nlB_Cd \qquad t_c \angle t \angle \frac{\pi}{\omega} \qquad 52$$

The voltage induced in a search coil of N_t turns wound around the iron ring is given by

$$e = -N_t \frac{d\phi}{dt}$$

$$= -2N_t nlB_C w\delta \operatorname{Cos}\frac{\omega t}{2} \qquad o \angle t \angle t_c$$

$$e = o \qquad t_c \angle t \angle \frac{\pi}{\omega} \qquad 53$$

$$e = o \qquad \text{for } t_c \angle t \angle \frac{\pi}{\omega}$$

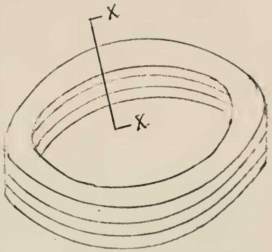
The flux wave of density B_C penetrates into the material during the time $t_c = \frac{2}{\omega} \operatorname{Sin}^{-1} \frac{d}{\delta}$, and induces a voltage in the search coil which is given by Equation 53. During the remainder of the half cycle the flux remains unchanged and the induced voltage is zero.

Wave Shape of Search Coil Voltage

The wave shape of the search coil voltage and induced flux depends on the value of t_c and hence, on the value of d/δ only. These wave shapes derived theoretically are shown in Figure 10 for various values of d/δ.

Duration of Eddy Current as a Function of d/δ

According to this theory, as evident from Figure 10, the duration of the induced voltage and hence, eddy currents, depends on the value of the fraction d/δ. For the solid iron case; ie., when $d/\delta \geq 1$, the eddy currents flow continuously, but as d/δ decreases, the eddy currents flow



Search coil specimen.

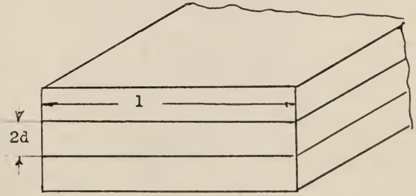


Fig..9. Section of search coil specimen at X-X..

for a smaller and smaller fraction of the half period as derived below.

Let F_e = fraction of the half period during which eddy current flows

$$F_e = \frac{t_c}{\pi/2} = \frac{\frac{2}{\omega}\ \sin^{-1} d/\delta}{\pi/\omega} = \frac{2}{\pi}\ \sin^{-1}\left(\frac{d}{\delta}\right) \qquad 54$$

A plot of F_e versus d/δ is given in Figure 11. This theory shows that in thin plates eddy currents do not flow continuously, but only for a fraction of the time during each half cycle.

MAJOR INFLUENCING FACTORS

Saturation Flux Density B_0

The value of B_0, the saturation flux density, to be used in any problem is open to question. If it is assumed that under dynamic conditions, induced density at any instant corresponds to the value of the magnetic field intensity, H, given by the static magnetization curve of the material, then the value of B_0 will correspond to H_m at the outside surface only. Each succeeding inner layer is magnetized by a progressively smaller number of exciting ampere turns, because of the shielding effect of the eddy current in the region between the outside surface and the layer under consideration.

The approximate distribution of the flux density in the succeeding layer of the material, from the surface to the depth of penetration, can be derived as follows. Figure 12 gives the eddy current distribution, neglecting phase shift from layer to layer, and assuming the arithmetical sum of eddy currents in the various layers to be equal to the exciting ampere turns at the surface. Under this assumption the area under the eddy current density curve, Figure 12, is the total exciting ampere turns at

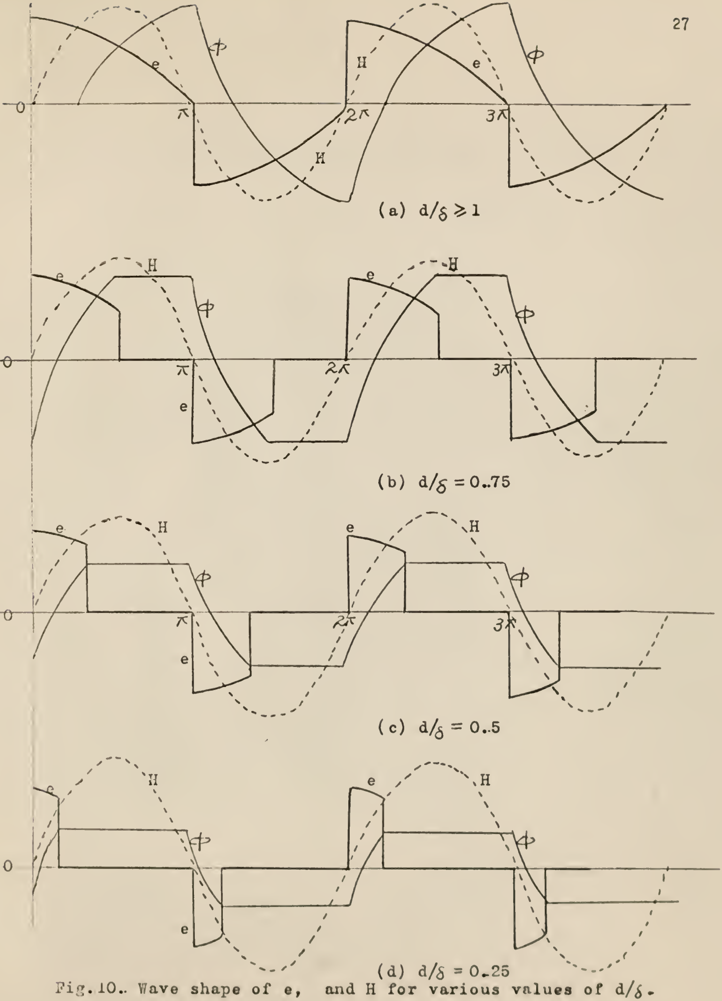
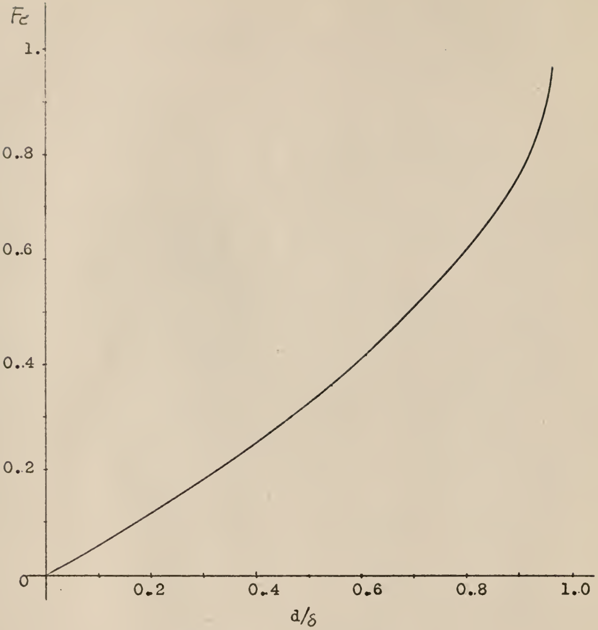


Fig. 10. Wave shape of e, and H for various values of d/δ.



d = Half thickness of lamination.

δ = Depth of penetration.

F_e = Fraction of the half period during which eddy current flows.

Fig. 11. Duration of eddy current as a fraction of d/δ.

the surface. The resulting exciting mmf at any inner layer is therefore given in this figure by the area under the eddy current density beyond the layer to the right.

Figure 13 gives the resultant exciting ampere turns at various depths from the surface. Figure 14 gives a typical B-H curve. The flux density, in kilolines per square inch, as a function of depth from the surface is derived from Figure 13 and 14 and is plotted in Figure 15 for various peak exciting ampere turns per inch. In Figure 16 this peak flux density has been expressed as a percentage of B_m, the peak surface flux density.

An examination of Figure 16 shows that even for the case of strong magnetization (curve 1, for 600 ampere turns per inch), at depth $\frac{1}{2}$ and (3/4) from the surface, the peak flux density reduces to about 80% and 60% respectively, of the peak flux density at the surface.

In order to find a calibration factor applicable over the entire range, the best value to be used for B_0 is equal to 3/4 B_m, where B_m is the flux density corresponding to H_m, the peak value of mmf impressed at the surface, obtained from the d-c magnetization curve of the material.

For a rectangular magnetization curve the power loss, using $B_0 = B_m$ as saturation flux density, is given by Equation 48.

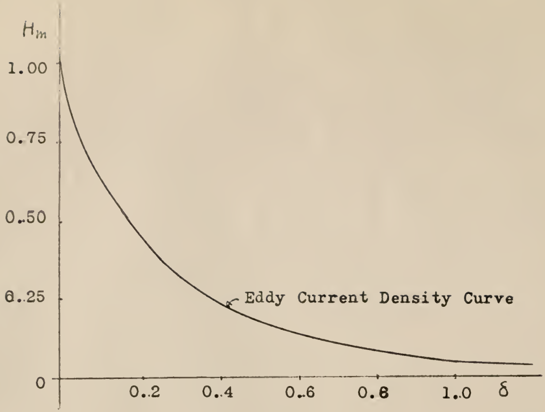


Fig. 12. Eddy current density curve.

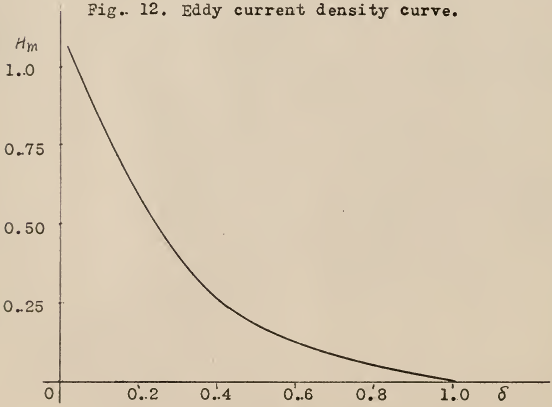


Fig. 13. Resultant exciting AT as a Fraction of surface mmf.

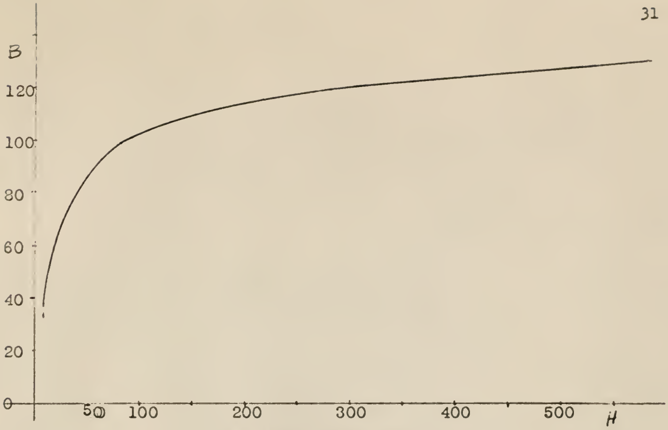


Fig.14. B-H curve of typical steel plate.

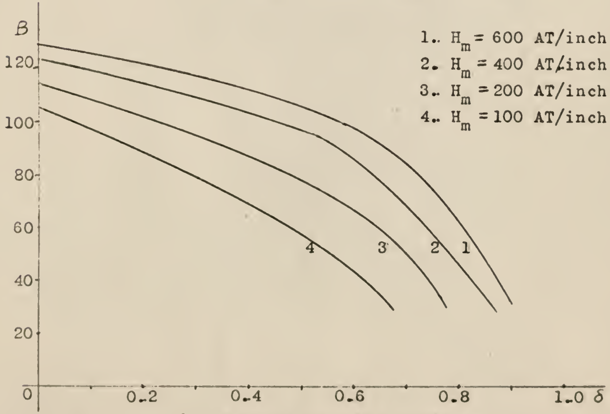


Fig.15. Peak flux density at layer inside.

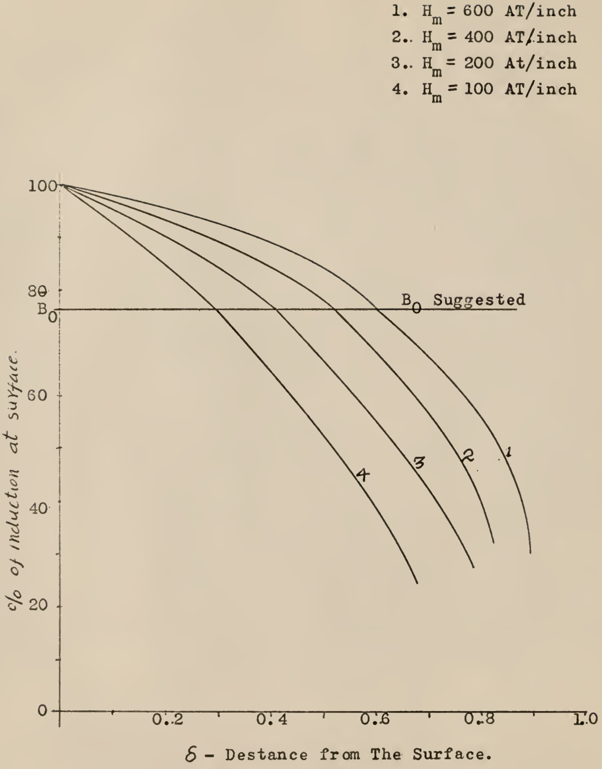


Fig..16.. Peak flux density at layers inside the material as percentage of B_0 at the surface.

$$N = \frac{8}{3\pi}\,\frac{Hm^2}{\gamma\delta} \qquad \text{but} \qquad \delta = \left[\frac{2\,Hm}{\gamma\,\omega\,B_o}\right]^{\frac{1}{2}} \qquad 15$$

$$= \left[\frac{8}{3\pi}\,\frac{Hm^4}{\gamma^2} \times \frac{\gamma\,\omega\,B_o}{2\,Hm}\right]^{\frac{1}{2}} = \left[\frac{8}{3\pi}\,\frac{Hm^3\,\omega\,Bm}{2\gamma}\right]^{\frac{1}{2}} \qquad 55$$

$$N = 0.85\ N_1 \qquad 56$$

For a rectangular magnetization curve using $B_0 = (3/4)\ B_m$

$$N = \frac{8}{3\pi}\sqrt{\frac{3}{4}}\left[\frac{Hm^3\ \omega\ B_m}{2\gamma}\right]^{\frac{1}{2}}$$

$$= 0.734\ N_1 \qquad 57$$

Since the true magnetization curve of iron lies between the linear and rectangular curve, as shown in the Figure 17 the rigorous solution for the linear case as well as the rectangular case establishes the limits between which the actual iron performs.

Effect of Temperature

The temperature coefficient of resistance of iron is higher than for copper. For steel plate, this coefficient per degree centigrade at 25° C is approximately 0.0045. Therefore, a temperature rise of 50° C would increase the resistivity of iron by about 22 percent. The eddy current loss formula, Equation 40, needs to be examined to determine the effect of temperature of these losses.

$$N = \left[\frac{8}{3\pi}\ \frac{H_m^2}{\gamma\delta}\ 1 - \left(1 - \frac{d^2}{\delta^2}\right)^{3/2}\right]$$

$$= \frac{8}{3\pi}\ \frac{Hm^2}{\gamma\delta}\left[\frac{3}{2}\left(\frac{d^2}{\delta^2}\right) - \frac{3}{8}\ \frac{d}{\delta} - \frac{1}{16}\left(\frac{d}{\delta}\right)^6\right] \qquad 58$$

and for solid iron; i.e., when $d/\delta = 1$, this reduces to

$$N = \frac{8}{3\pi}\ \frac{Hm^2}{\gamma\delta}$$

$$= \frac{8}{3\pi}\ H_m^{3/2}\left[\frac{\omega B_0}{2\gamma}\right]^{\frac{1}{2}}$$

substituting the value of $\delta = \left[\frac{2\ Hm}{\gamma\omega B_0}\right]^{\frac{1}{2}}$

$$N = \frac{8}{3\pi}\ H_m^{3/2}\left[\frac{B_0\omega}{2}\right]^{\frac{1}{2}}\sqrt{\rho} \qquad 59$$

where ρ is resistivity in the MKS units

This shows that in solid iron, the eddy current losses are proportional to the square root of resistivity.

For very thin plate; i.e., when $d/\delta < 1$, higher powers of d/δ can be neglected in Equation 4.4 giving

$$N = \frac{8}{3\pi}\ \frac{Hm^2}{\gamma\delta}\left[\frac{3}{2}\left(\frac{d^2}{\delta^2}\right)\right]$$

$$= \frac{4}{\pi}\ Hm^2\rho\ \frac{d^2}{\delta^3}$$

$$= \frac{4}{\pi}\ Hm^2\rho d^2\left[\frac{\omega B_0}{2\ Hm}\right]^{3/2}$$ obtained by substituting the value of δ

$$= \frac{\sqrt{2}}{\pi}\ Hm^{\frac{1}{2}}\left[B_0\omega\right]^{3/2} d^2 \times \frac{1}{\sqrt{\rho}} \qquad 60$$

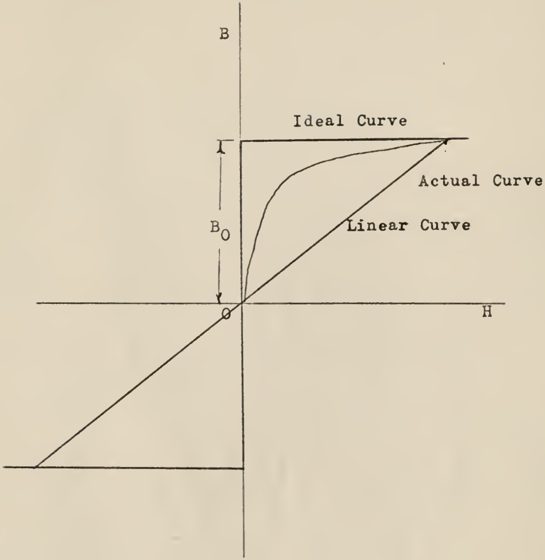


Fig. 17. Ideal, actual and linear magnetization Curves.

This shows that for thin laminations, the eddy current losses are inversely proportion l to the square root of resistivity. Therefore, a temperature rise of 50° C would increase the eddy current losses in solid iron by about 10 to 11% and decrease the losses in thin laminations by approximately the same amount.

Figure 18 gives the effect of temperature on iron losses for different thickness of specimens. For example, Curve A shows that the eddy current loss in a (1/8)" ring, for an increased mmf of 350 rms ampere turns per inch, decreased by about 12% for temperature rise of 50° C. At lower values of H, (1/16)" laminations behave like solid iron and, hence the losses increase as $\sqrt{\rho}$, with increase in temperature.

Hysteresis

Hysteresis loss in solid iron has been a subject of great controversy. Some authors believe that in solid ferromagnetic materials the hystersis losses are negligible compared to eddy current loss.

When considering hysteresis, the ideal magnetization curve is that of a single crystal, shown in Figure 19.

The coercive force is shown as h_k. When hysteresis is neglected, the flux wave of saturation density, B_0, penetrate the material from the surface when the field mmf, H, passes through zero. With hysteresis, the field mmf has to reach the value h_k, the coercive force, before the flux waves penetrate the material. The magnetic field, h, inside the material does not decrease from the value H at the surface to zero at the separating surface, Figure 19 but decreases to h_k. This results in an electric field and hence, residual eddy currents of constant density in the region

Numbers On Curves Indicate R.M.S. AT/inch.
Ech Vertical Divison Represents 5 %
Change of Value At Room Temperature.

% Change In Iron Losses With Tempreature.

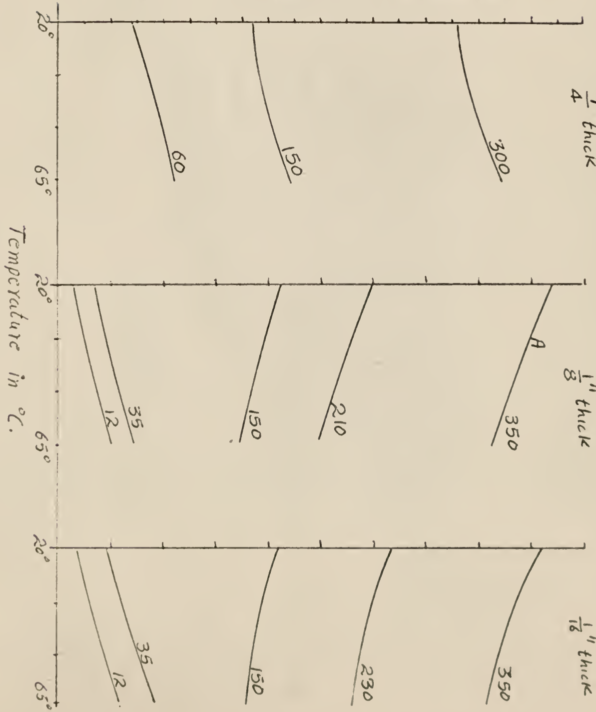


Fig. 18. Effect of temperature on losses.

between the separating surface and d or δ , whichever is smaller. Under these conditions the depth of penetration of flux is smaller because of the smaller effective field force, than in the case when hysteresis is neglected. Also, the peak value of the induced emf is smaller. The power loss due to eddy current now becomes

N_h = Eddy current loss per square meter taking hysteresis into account

$$N_h = \left[1 - \frac{h_E}{4\ Hm}\right] N \qquad 61$$

where N is power loss neglecting hysteresis.

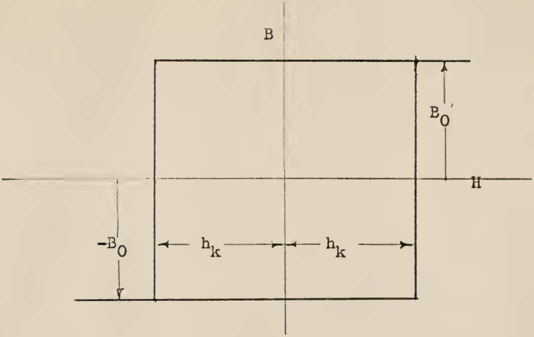
The formula shows that, except for very low values of induction, the reduction in eddy current losses caused by hysteresis is small. This reduction is offset in some degree by the hysteresis loss.

The view that in massive iron hysteresis loss is negligible compared to eddy current loss seems reasonable, because, except for low values of induction, change of current amplitude is larger for a small change of flux density amplitude, and eddy current loss is a function of the former, and hysteresis loss of the latter.

Example:

Calculate the eddy current and hysteresis losses in 0.025" motor grade steel, at different frequencies.

The hysteresis loss per cubic inch at 60 cps. for a maximum induction of 115 Kilolines per square inch is 0.544 watts. If we assume that the hysteresis loop has the same area even at higher frequencies, the hysteresis loss at any frequency is



Ideal magnetization curve with hysteresis.

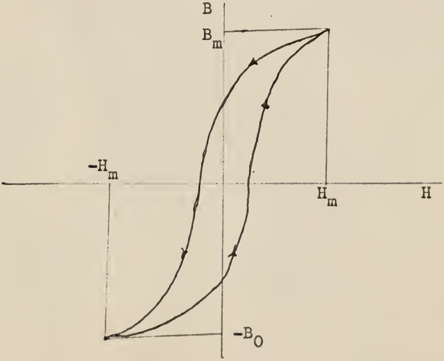


Fig. 19. Hysteresis loop.

$$P = 0.544 \; \frac{\delta_f}{d} \; \frac{f}{60} \qquad \text{watts per cubic inch}$$

where δ_f is the depth of penetration at frequency f. The maximum value of the fraction $\frac{\delta_f}{d}$ to be used is unity. H_m the peak ampere turns per inch, for maximum induction of 115 Kilolines per square inch equals 425.

$$H_{rms} = 425/\sqrt{2} = 300$$

The eddy current losses have been calculated by Equation 2.40

f = 100000 cycles/ second.

$$P = 0.544 \; \frac{100000}{60} = 90.5 \text{ watts}$$

$$P_e = \text{Eddy current loss per cubic inch} = \frac{110\ 2}{0.025}$$

$$= 8880 \text{ watts}$$

Therefore, the ratio of hysteresis loss to eddy current loss

$$P_h/P_e = 0.0102$$

f = 2000 c/s

$$P_h = 0.544 \; \frac{2000}{60} = 18.1 \text{ watts}$$

$$P_e = 900 \text{ watts}$$

$$P_h/P_e = 0.02$$

f = 500 c/s

$$P_h = 4.5 \text{ watts}$$

$$P_e = 115 \text{ watts}$$

$$P_h/P_e = 0.032$$

As the frequency was decreased from 100000 c/s, the hysteresis loss increased from one percent to four percent of eddy current loss. At 60 c/s, the hysteresis loss becomes more predominant and may be as much as 60% to 70% of the total losses. Thus, we see that with the decrease of d/δ; i.e., as we go from solid material to thinner laminations, hysteresis losses as a percentage of the total losses, increase.

POWER FACTOR OF CIRCUIT

Most problems are conveniently solved by circuit theory and, therefore, we require the proper values of resistance and reactance parameters to be used in the equivalent circuit, to take into account the effect of alternating fields in iron. Due to nonlinearity of the magnetization curve of iron, both the voltage and the magnetizing current cannot be simultaneously sinusoidal, and therefore, it should be realized that in this case power factor does not have its usual meaning.

Equivalent Circuit

Figure 20 gives the approximate equivalent circuit of the ring specimens. The R_{h+e} branch is a resistive branch which is theoretically connected in parallel with the pure inductance of the coil, in order to provide a path wherein a number of watts equal to the actual core loss can be hypothetically spent. The subscript h + e refers to the hysteresis and eddy current components of the core losses. The fact that core loss in question is a function of the flux established by the inductance coil, together with the fact that the flux established by the inductance of voltage drop across the ohmic resistance, dictates the circuit position of the hypothetical R_{h+e} branch.

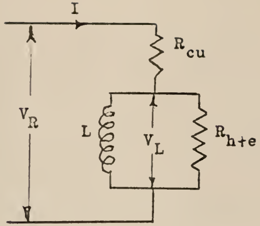


Fig.. 20.. Equivalent circuit..

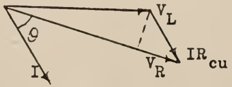


Fig.. 21.. Phasor diagram.

Method of Calculations

Since the analysis is to be based entirely on experimental readings of V, I, R_{cu} and power loss, W, neither R_{h+e}, nor L can be determined accurately. An approximate procedure is employed, which is consistent with the accuracy of the experimental readings and the accuracy with which the equivalent circuit of Figure 20 simulates the phenomena. The phasor diagram is given by Figure 21.

From the equivalent circuit and phasor diagram, the applied voltage is balanced by V_L and the voltage drop across the copper resistance of the coil,

$$\bar{V}_R = \bar{V}_L + \bar{I}R_{cu} \qquad 62$$

$$\cos \theta = W/V_R I$$

$$V_L = (V_R - IR_{cu} \cos \theta)^2 + (IR_{cu} \sin \theta)^2 \qquad 63$$

$$P_{iron} = W - P_{cu}$$

Power factor of the iron circuit $= P_{iron}/V_L I$

The measured values of the parameters were determined by the above equation. The analytical value of the power factor is given by

$$\text{P.F.} = \cos \left[\tan^{-1} \frac{\frac{3d}{\delta} - 2\frac{d^3}{\delta^3}}{2\left\{1 - \left(1 - \frac{d^2}{\delta^2}\right)^{3/2}\right\}} \right] \qquad 64$$

where $\delta = \left[\frac{2 H_m \rho}{\omega B_0}\right]^{\frac{1}{2}}$

The value of the power factor depends on the value of d/δ only. When d/δ is equal to or greater than one, the power factor is 0.894. As d/δ decreases, the power factor also decreases according to the above equation.

EDDY CURRENT LOSSES FOR SINUSOIDAL FLUX

Introduction

Solution of Maxwell's field equation with applied sinusoidal magnetomotive force and the idealized saturation curve is visualized on the eddy current phenomenon in iron. This concept of flux waves of constant flux density penetrating into the material, resulting in layers of material being alternately magnetized in one direction or the other, is not only simple to understand, but actually yields mathematical results that are simpler than those obtained by the classical theory.

The close correlation between the wave form of search coil voltages, derived analytically, seems to indicate that the magnetization of iron does actually take place as predicted by the rectangular theory, even though the actual magnetization curve is not rectangular.

Solution of the Problem

Consider a packet of n laminations of thickness 2d each, on which is imposed a sinusoidal flux with amplitude Φ_M. Flux waves of saturation density B_C, enter the laminations from both sides, until the end of each half cycle, to the depth δ and there is no flux beyond.

$$\Phi_m = \frac{\Phi_m}{2n}$$ is the amplitude of the imposed flux per laminated surface (65)

$$= B_0 \delta$$

Let ϕ = instantaneous value of flux through half the thickness of the lamination

$$= \phi_m \sin t \qquad (66)$$

Again, due to symmetry, let us consider the solution for only half the thickness of one lamination, and place the coordinate axis, $Z = 0$, at the left face of the lamination with increasing towards its center, as shown in Figure 22 when $\phi = 0$, the separating surface must be at $z = d/2$ from the surface, giving alternate magnetization in each $\delta/2$ region. When flux is maximum, the separating surface must be at depth δ, H at the surface is equal to zero and a new separating surface starts inward, reversing the direction of magnetization.

The separating surface, $z = \mathcal{S}(t)$, as a function of time, can be easily derived from the above consideration to be

$$\mathcal{S}(t) = \frac{\delta}{2}(1 + \sin \omega t) \qquad (67)$$

The induced voltage at each layer is given by Equation 7

$$E = e = 2B_0 \frac{d\mathcal{S}}{dt}$$

$$= 2B_0 \frac{\delta\omega}{2} \cos \omega t$$

$$= B_0 \delta \omega \cos \omega t. \qquad (68)$$

As the separating surface moves from the surface, ($z = 0$), to the depth of penetration, δ , the portion of the material from the outside to the separating surface carries eddy current with uniform current density. Therefore, magnetic field intensity at the surface

$$
\begin{aligned}
H &= \text{total eddy current in the material} \\
&= \gamma E \delta \\
&= \frac{\omega \gamma B_0 \delta^2}{2} \cos\omega t \ (1 + \sin\omega t)
\end{aligned}
\qquad 69
$$

Poynting Vector and Eddy Current Loss

Since E is sinusoidal, only the fundamental component of H contributes to power flow into the material. The fundamental component of H is given by Fourier Series.

$$H_1 = \gamma \omega B_0 \delta^2 \left[\tfrac{1}{2} \cos\omega t + \tfrac{2}{3} \sin\omega t \right] \qquad 70$$

If E and H_1 are the peak complex amplitudes of E and H_1, at the surface, then

$$
\begin{aligned}
N &= \text{real part of } \bar{N} = 1/2\ \bar{E}\ \bar{H}^* \\
&= \frac{\gamma \omega^2 B_0 \Phi_m}{4} \\
&= \frac{\gamma \omega^2 \Phi_m^3}{4 B_0} = \frac{\gamma \omega^3 B_0^2 \delta^3}{4}
\end{aligned}
\qquad 71
$$

A plot of $\mathcal{S}$, H and E as a function of time is given in Figure 23

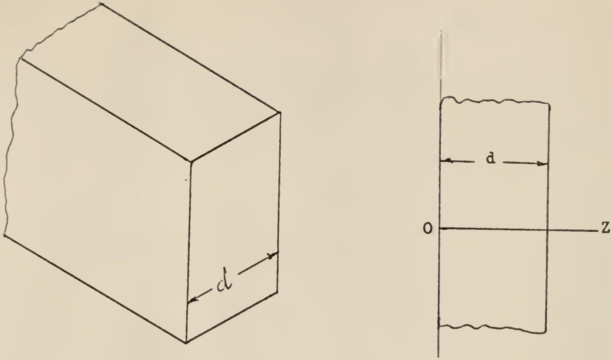


Fig. 22. Plane wave on the surface of the plate.

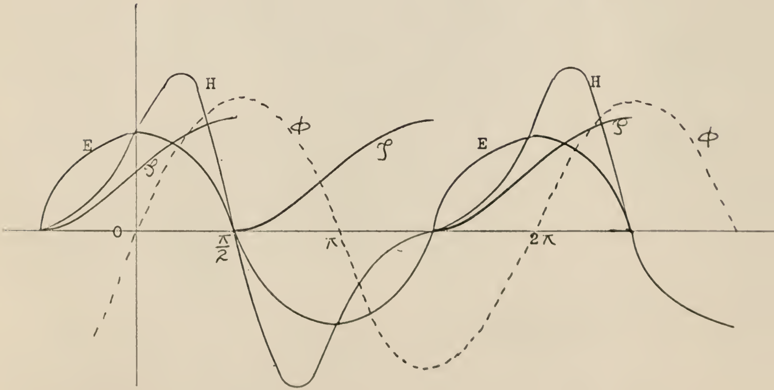


Fig. 23. Wave shape of H, E, $\mathfrak{I}$, and Φ for sinusoidal flux.

CONCLUSION

Solution of Maxwell's Equations, when a sinusoidal mmf is impressed on a core composed of steel plates, and under the conditions of the ideal B-H curve, shows that a wave of flux density B_0 enters each plate from the outside surface at the start of each half cycle and penetrates to a depth δ, the "depth of penetration."

If δ is less than d, the half-thickness of the plate, the inner part of the plate remains unmagnetized and the magnetic flux as well as the eddy currents are confined to a layer of depth δ, on each surface of the plate. If δ is greater than d, the flux density waves from the two sides meet in the center of the plate before the end of the half cycle, and for the remainder of the half cycle the plate remains magnetized to the value B_0. In this case, therefore, the eddy current flows during the first portion of each half cycle and is zero during the period in which the plate is fully magnetized.

Equations 18 and 19 give the value of $\mathcal{S}$ and e_x respectively in terms of H_m and t, and are plotted in Figure 7 as t. We can conclude from this figure that the value of e_x reverses at every half period but (t) does not reverse.

Equation 40 gives the power loss per unit area. From that formula we can conclude that the power loss decreases as d/δ decreases. For the solid iron case; i.e., $d/\delta \geq 1$, the value of the lamination factor is unity. The power factor, that the eddy current losses reflects into the exciting winding is given by Equation 38 for solid iron. The p.f. is Cos $(\tan^{-1} \frac{1}{2})$ 0.899. As d/δ decreases the power factor increases.

The effect of different values of d/δ on induced Search Coil Voltage and flux for sinusoidal mmf is given in Figure 30. For the solid iron case, the eddy current flows continuously, but as d/δ decreases, the eddy current flows for a smaller fraction of the half period, thus decreasing the total losses.

Equation 59 and Equation 60 give the power losses for solid and laminated iron respectively in terms of resistivity.

From Figure 18 we can conclude that for the solid iron case, as temperature increases, losses increase and for the laminated iron case, as temperature increases losses decrease.

Figures 25, 26, 27 and 28 give losses in watts per cubic inch for different values of frequency.

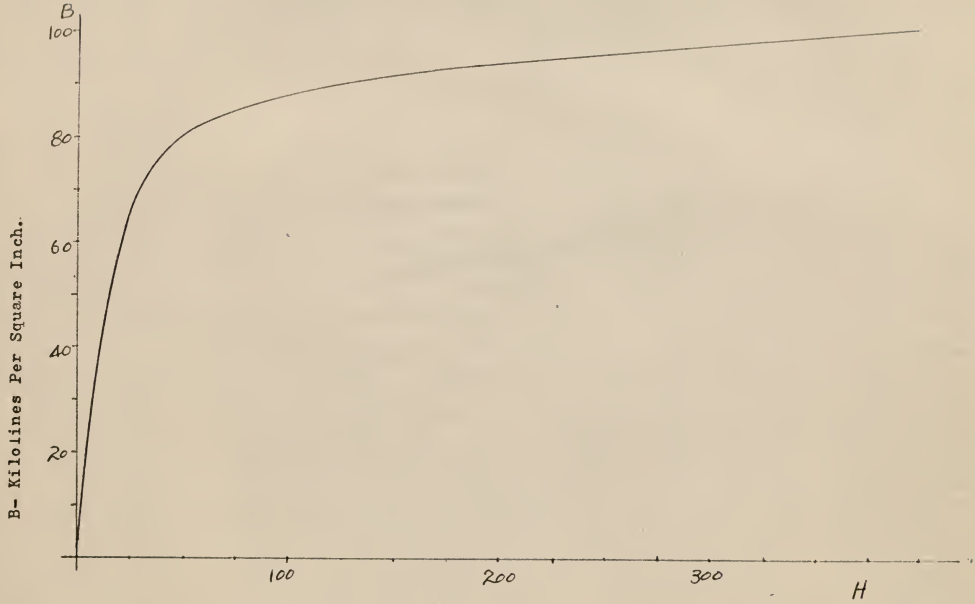


Fig. 24. Magnetization curve of $\frac{1}{4}$" ring specimen.

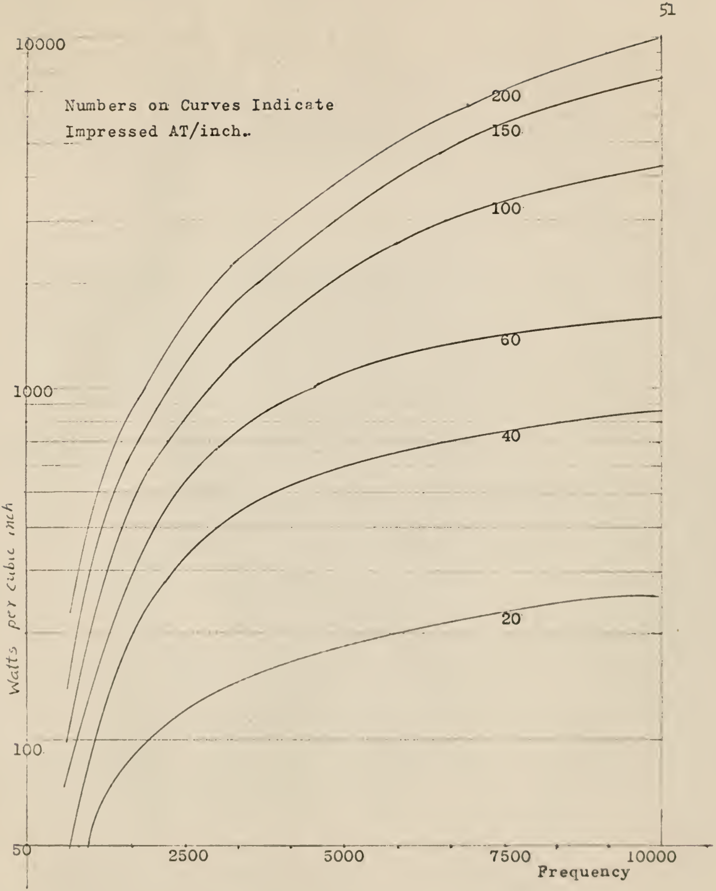


Fig.25.Iron loss curves for I-5 steel,0.025" thick lamination.

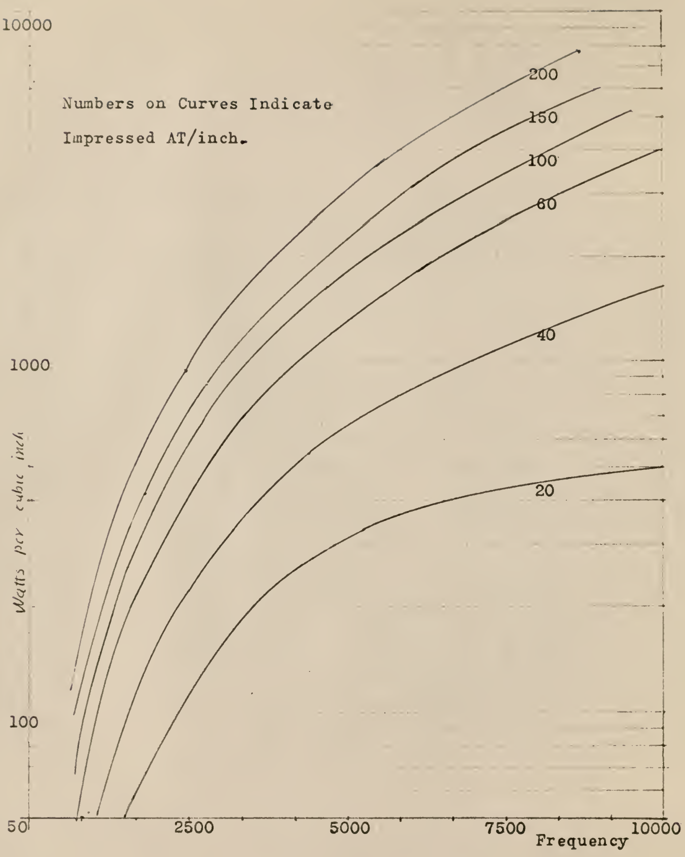


Fig.26.. Iron loss curves for I-5 steel 0.014" thick lamination.

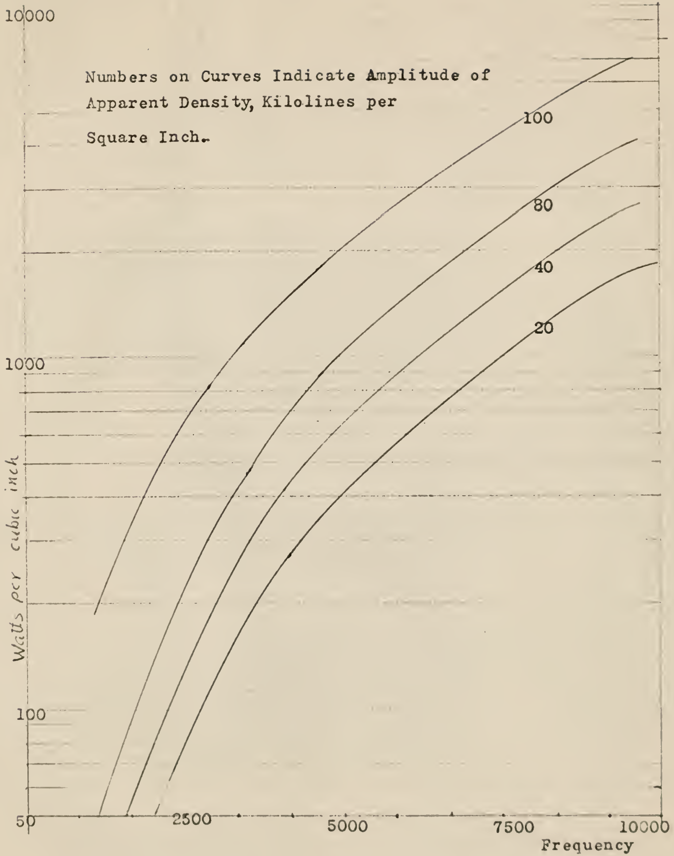


Fig. 27. Iron loss curves for I-5 steel, 0.025" thick lamination.

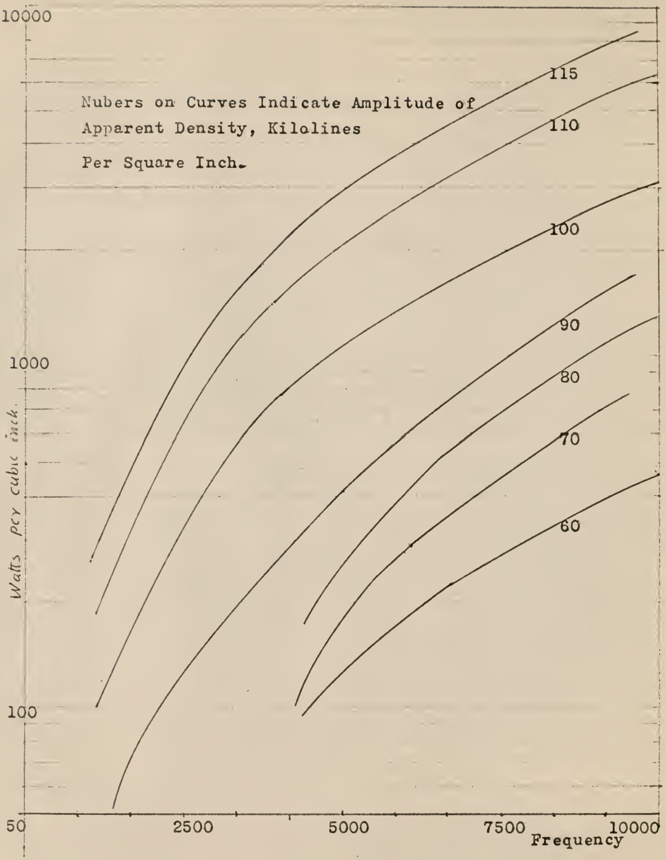


Fig. 28. Iron loss curves for I-5 steel, 0.014" thick lamination.

ACKNOWLEDGEMENT

The author wishes to express his heartfelt and sincere gratitude to Mr. M. C. Cottom, Professor of Electrical Engineering, Department of Electrical Engineering, for his able guidance, helpful direction, and valuable suggestions in the writing of this report.

BIBLIOGRAPHY

Aspden, H.
Eddy-Currents in solid cylindrical cores having non-uniform permeability. Jour. of Applied Physics 23:523-528. May, 1952.

Attwood, S. S.
Electric and magnetic fields. 3rd ed. New York: John Wiley & Sons, 1949.

Baird, J.
Eddy-Current losses in induction motor end turn clamping rings. Trans. of AIEE 73 (part 3):660-664.

Blake, L. R.
Eddy-Current anomaly in ferromagnetic lamina at high frequencies. Proc. of IEE 96 (part c):705.

Brailsford, F.
Investigation of the eddy-current anomaly in electrical sheet steels. Jour. of IEE 95 (part b):38-48. February, 1948.

Brown, E. H., Jr.
Magnetic losses at temperature. Jour. of Applied Physics 30:112-114. January, 1959.

Deuring, W. G.
Induced losses in steel plates in presence of alternating currents. Trans. of AIEE 76:166-171. June, 1957.

Greig, J., and K. Sathirakul.
Pole face losses in alternators. Proc. of IEE 108 (part c):130-138.

Hale, J. W., and F. R. Richardson.
Mathematical description of core losses. Trans. of AIEE 72 (part 1):495-501. September, 1953.

Jones, D. A.
Theoretical and analogue approach to stray eddy-current loss in laminated magnetic cores. Proc. of IEE 108 (part c):509-515. September, 1961.

Lee, E. W.
Eddy-Current losses in thin ferromagnetic sheets. Proc. of IEE 105 (part c):337-342. September, 1958.

MacLean, W.
Theory of strong electromagnetic waves in massive iron. Jour. of Applied Physics 25:1267-1270. October, 1954.

McConnell, H. M.
Eddy-Current phenomena in ferromagnetic materials. Trans. of AIEE 73 (part 1):226-235. July, 1954.

Moore, A. D.
Eddy-Currents in discs: driving and damping forces and torques. Trans. of AIEE 66:1-11. 1947.

Pasculle, M. J.
Armature tooth pulsation on iron losses. Trans. of AIEE 78 (part 1) :1069-1074. January, 1960.

Ponwell, E. P., and S. W. Gough.
Eddy-Current brakes. Jour. of Scientific Instruments 12:161-165. May, 1953.

Poritsky, H., and R. P. Jerrard.
Eddy-Current losses in a semi-infinite solid due to nearby alternating currents. Trans. of AIEE 73 (part 1):97-106. May, 1954.

Rosenberg, E.
Eddy-Currents in iron mass. Electrician 91:188. August, 1923.

Russell, R. L., and K. H. Norvsworthy.
Eddy-Currents problems involving two-dimensional fields. Proc. of IEE 107 (part c):11-18. March, 1960.

Schinoler, M. J.
Effect of flux distribution on iron losses. Trans. of AIEE 78 (part 1):1069-1074. January, 1960.

Say, M. G.
The performance and design of alternating current machines. 2nd ed. London: Sir Isaac Pitman & Sons, 1960.

APPENDIX I

List of Symbols

Symbols	Description
A	Cross sectional area.
B	Magnetic flux density at the surface.
B_m	Amplitude of flux density B.
B_0	Saturation flux density.
d	Half depth of material.
e	Electric field intensity.
E	Electric field intensity at the surface.
E_1	Fundamental component of E.
$\bar{E}$	Peak complex Amplitude of E.
h	Magnetic field intensity.
H	Magnetic field intensity at the surface.
H_m	Amplitude of H.
$\bar{H}$	Peak complex Amplitude of H.
n	Number of laminations.
N_t	Number of turns in the coil.
N	Power loss in Solid iron per square meter of effective surface.
xyz	Coordinate system.
γ	Conductivity
δ	Depth of penetration
ρ	Coordinate of the Separating surface.
μ	Permeability

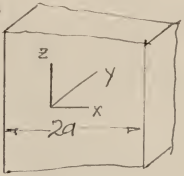
Symbols	Description
ρ	Resistivity
ω	Electrical angular velocity.

APPENDIX II

Eddy Current Losses with Constant Permeability

Let the magnetizing force, H, have a y component only, and its value at the surface of the plate, that is at x= a, be a sinusoidal function

$$H = H_m \sin\omega t$$



The plate is of infinite dimension in the y and z direction and, therefore, e and eddy current will have only z direction. All quantities are expressed in the MKS system, and γ is the conductivity. Let E, B, and H be the values of the variables e, b, and h at the surface.

Under these conditions Maxwell's equation yields

$$\text{Curl } h = k \frac{\partial h}{\partial x} = k(\gamma + j\omega\epsilon)\, e$$

or

$$\frac{\partial h}{\partial x} = (\gamma + j\omega\epsilon)\, e \qquad \text{72a}$$

$$\text{Curl } e = -i \frac{\partial e}{\partial x} = -i\, j\omega h\mu$$

$$\frac{\partial e}{\partial x} = j\omega h\mu \qquad \text{72b}$$

Differentiating Equation 72b with respect to x and substituting in Equation 72a

$$\frac{\partial^2 e}{\partial x^2} = j\omega\mu \frac{\partial h}{\partial x}$$

$$= j\omega\mu(\gamma + j\omega\epsilon)\, e$$

$$\frac{\partial^2 e}{\partial x^2} = j\omega\mu\gamma e + j^2\omega^2\mu\epsilon t$$

The second term on the right hand side can be dropped as displacement current in metal will not be appreciable even at highest radio frequencies. Therefore, the skin effect equation reduces to

$$\frac{\partial^2 e}{\partial x^2} = j\omega\mu e \gamma \qquad 73$$

The solution of this equation yields

$$e = A_1 e^{mx} + A_2 e^{-mx} \qquad 74$$

where $m = \sqrt{j\omega\mu\gamma}$

The boundary conditions are

at $x = 0$ $\qquad J = 0$

$\qquad e = 0$

at $x = \pm a$ $\qquad H = H_m \mathrm{Sin}\,\omega t \qquad 75$

From Equation 75

$$0 = A_1 + A_2$$

$$e = A_1(e^{mx} - e^{-mx})$$

$$= 2A_1 \mathrm{Sinh}\, mx \qquad 76$$

From Equation 76 and 72b

$$\frac{\partial e}{\partial x} = 2A_1 m \,\mathrm{Cosh}\, mx = j\omega H_m \mu$$

and from boundary conditions, Equation 75

$$2A_1 m \;\mathrm{Cosh}\, mx = j\omega H_m \mu$$

or $$A_1 = \frac{j\omega\mu H_m}{2 \;\mathrm{Cosh}\, ma}$$

$$e = \frac{j\omega\mu H_m}{\text{Cosh ma}} \text{ Sinh mx}$$

at x = a
$$e = E = \frac{j\omega\mu H_m}{m} \text{ Tanh ma}$$

If we designate by $\bar{H}$ and $\bar{E}$, the peak complex amplitude of H and E, then the Poynting Vector, $\bar{N}$, into the metal would be

$$\bar{N} = \tfrac{1}{2}\bar{E}\ \bar{H} = \frac{j\omega\mu H_m^2}{2} \text{ Tanh ma}$$

The Poynting Vector at both faces of the plate (x = a) is in the direction of the x axis and pointing towards the center of the plate (x = 0).

Total loss due to eddy current = real part $\iint N \cdot ds$.

The integral is taken over closed surface, in the yz direction.

Total loss per unit surface in the yz direction

$$Q = \text{Re}\left[\frac{j\omega\mu}{2m}\ \bar{H}_m^2\ \frac{\text{Sinh ma}}{\text{Cosh ma}}\right] \qquad 77$$

$$m = j$$
$$= (1+j)\beta \qquad \text{where } \beta = \sqrt{\frac{\mu\gamma\omega}{2}}$$

$$Q = \text{Re}\ \frac{H_m^2}{2}\sqrt{\frac{\omega\mu}{\gamma}}\ \sqrt{j}\ \times\ \frac{\text{Sinh }(1+j)\beta a}{\text{Cosh }(1+j)\beta a}$$

$$= \frac{H_m^2}{2}\sqrt{\frac{\omega\mu}{\gamma}}\ \text{Re}\left[\frac{1+j}{\sqrt{2}}\ \frac{\text{Sinh }(1+j)\beta a}{\text{Cosh }(1+j)\beta a}\right] \qquad 78$$

Rationalizing the quantity in the bracket.

$$Q = \frac{H_m^2}{2}\sqrt{\frac{\omega\mu}{\gamma}}\quad \text{Re}\left[\frac{(1+j)\text{ Sinh }(1+j)\beta a\text{ Cosh }(1-j)\beta a}{\text{Cosh }(1+j)\beta a\ \text{ Cosh }(1-j)\beta a}\right]$$

$$= \frac{H_m^2}{2} \sqrt{\frac{\omega\mu}{\gamma}} \quad \frac{\text{Sinh } 2\beta a - \text{Sinh } 2\beta a}{\text{Cosh } 2\beta a + \text{Cosh}\beta a} \qquad 79$$

APPENDIX III

Penetration of H into Material of Constant Permeability

Assume an infinite half-space of conductivity γ and permeability μ, with surface in the xy plane, is excited by a magnetizing winding, not shown, such that the flux has only an x component. The z direction extends into the material normal to the xy plane. Neglecting displacement current, the field equation is

$$\frac{\partial^2 H_x}{\partial z^2} = \mu\gamma \frac{\partial H_x}{\partial t}$$

The solution is

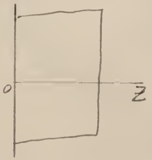
$$H_x = \mathrm{Re}\ H_m\, e^{\left(j\omega t - \sqrt{j\omega\gamma\mu}\, z\right)}$$

where H_m is the amplitude of the field intensity impressed at the surface ($z = 0$).

Since $\sqrt{j} = \frac{1 + j}{\sqrt{2}}$ (taking the root with positive sign) the above equation can be written as

$$H_m = \mathrm{Re}\ H_m e^{-\sqrt{\frac{\omega\gamma\mu}{2}}z}\, e^{j\left(\omega t - \sqrt{\frac{\omega\gamma\mu}{2}} z\right)}$$

The behavior of H_x as a function of time and of depth is illustrated in Figure 30. Examination of this figure shows that the wave front h = 0, marked by A on the various curves penetrates into the material.



Semi-infinite solid configuration Figure 29

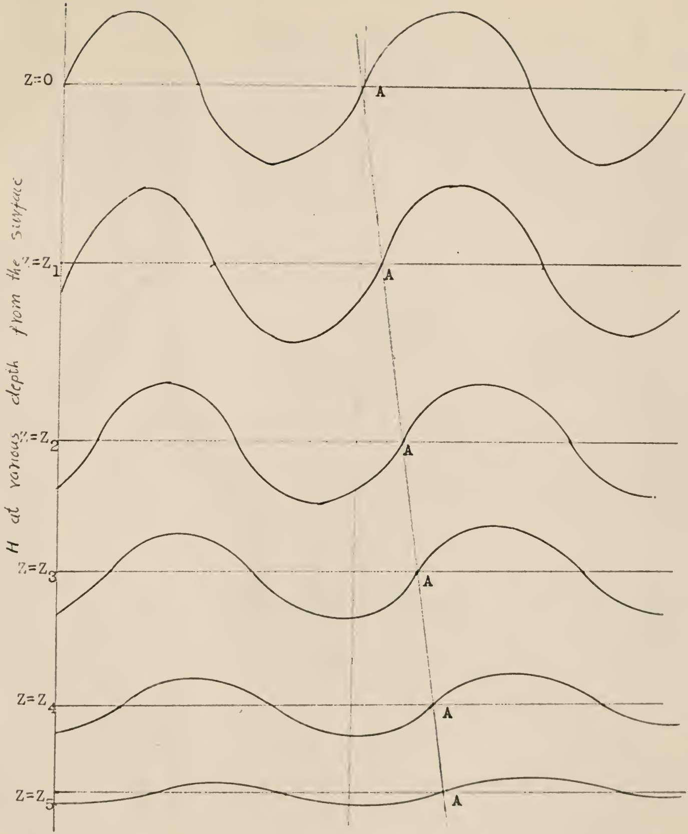


Fig.30.. Penetration of H as a function of time.
(Constant Permeability)

EDDY CURRENT LOSSES IN SOLID AND LAMINATED IRON

by

ASHOK KUMAR DALAL

B.S. (Mechanical) & B.S. (Electrical)

Vallabh University, Anand, India, 1961

Abstract of

A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Electrical Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1963

In this report there are developed, on the basis of simple physical conceptions, formulas for the calculation of eddy currents in solid and laminated iron. A simple formula for the calculation of the power factor that the magnetic circuit reflects into the magnetizing winding is also presented.

The theory employed in developing the formulas, assumes that the permeability of steel is constant for all flux densities below a limiting saturation value, B_0, giving a rectangular magnetization curve.

The eddy current loss and power factor of magnetic circuit are calculated by the Poynting Vector Theorem, assuming the applied mmf is sinusoidal. Eddy current loss formulas, when the impressed flux is sinusoidal are also presented.

Actually, the B-H curve of steel departs materially from the assumed rectangular shape. Also, both the applied mmf and flux wave may depart materially from pure sine waves. The value of B_0 to be used in the formulas has been determined empirically to be $(3/4)B_m$, where B_m is the flux density obtained from the static magnetization curve of the material corresponding to H_m, the peak of the impressed mmf.

Date Due
