BOLTED CONNECTIONS

by

SHANKER L. BATHWAL

B.S., Birla Institute of Technology, Ranchi, 1961

A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Mechanical Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1963

Approved by:

J C Lindholm
Major Professor

TABLE OF CONTENTS

INTRODUCTION 1

EQUATION PROPOSED FOR BOLTED CONNECTIONS 2

PREVIOUS INVESTIGATIONS 7

DETERMINING THE EFFECTIVE AREA OF BOLTED PARTS 12

SUMMARY AND RECOMMENDATIONS 15

ACKNOWLEDGEMENT 17

BIBLIOGRAPHY 18

APPENDIX I 19

APPENDIX II 23

INTRODUCTION

The design of a fastening in which two or more members are bolted together by a bolt or group of bolts appears at first to be a simple problem, nevertheless, little fundamental information is available on which to base a rational design procedure. During the past war complexities of the problem were emphasized by the many failures of connecting rod bolts and cylinder head bolts encountered during the development of new aircraft engines.

In the design of an assembly of two or more metal sections fastened together by bolts, some designers assume that the compression members of the assembly are rigid and do not deform when the bolts are tightened. However, no metal is incompressible though it may be very stiff and the interactions which occur should be considered in the design.

The extent to which the load on a bolt is increased by the application of an external force to the joint depends partly upon the magnitude of the external force and in part upon the relative stiffness of the bolt as compared with that of the members joined by the bolt. Relative stiffness depends upon the effective area, modulus of elasticity, and the effective length of bolt and parts.

Of all the factors affecting the bolted connections, the effective area of the part is the only one not well-defined. Thus, it is the object of the investigation in this report.

EQUATION PROPOSED FOR BOLTED CONNECTIONS

A number of approaches have been used in analyzing a bolted connection. The earlier equations were based on certain simplifying assumptions which reflected the general understanding of the problem. As more was understood about the material properties and the elasticity of the materials, other equations were developed to better reflect the true situation.

In the equations to follow the symbols are:

F_t = Total load on bolt (lb.)

F_i = Initial load due to tightening (lb.)

F_e = External or applied load (lb.)

E_j = Modulus of elasticity (psi.)

A_j = Cross-sectional area (sq. in.)

L_j = Length parallel to center line of bolt (in.)

Subscript j refers to the bolt or any one of bolted parts, i.e., j = b, g, 1, 2, or 3.

Figure I shows a bolted joint illustrative of that found frequently and will be used in explaining the equations applied to such connections.

The earliest equation used is that given by Kimbal and Bar (1). The total load acting on the bolt is assumed to be the sum of the initial and external load. In the analysis it is assumed that the bolted parts are rigid.

The initial load is obtained from the equation

$$F_i = 16{,}000\ D$$

where D is the shank diameter of the bolt (in.)

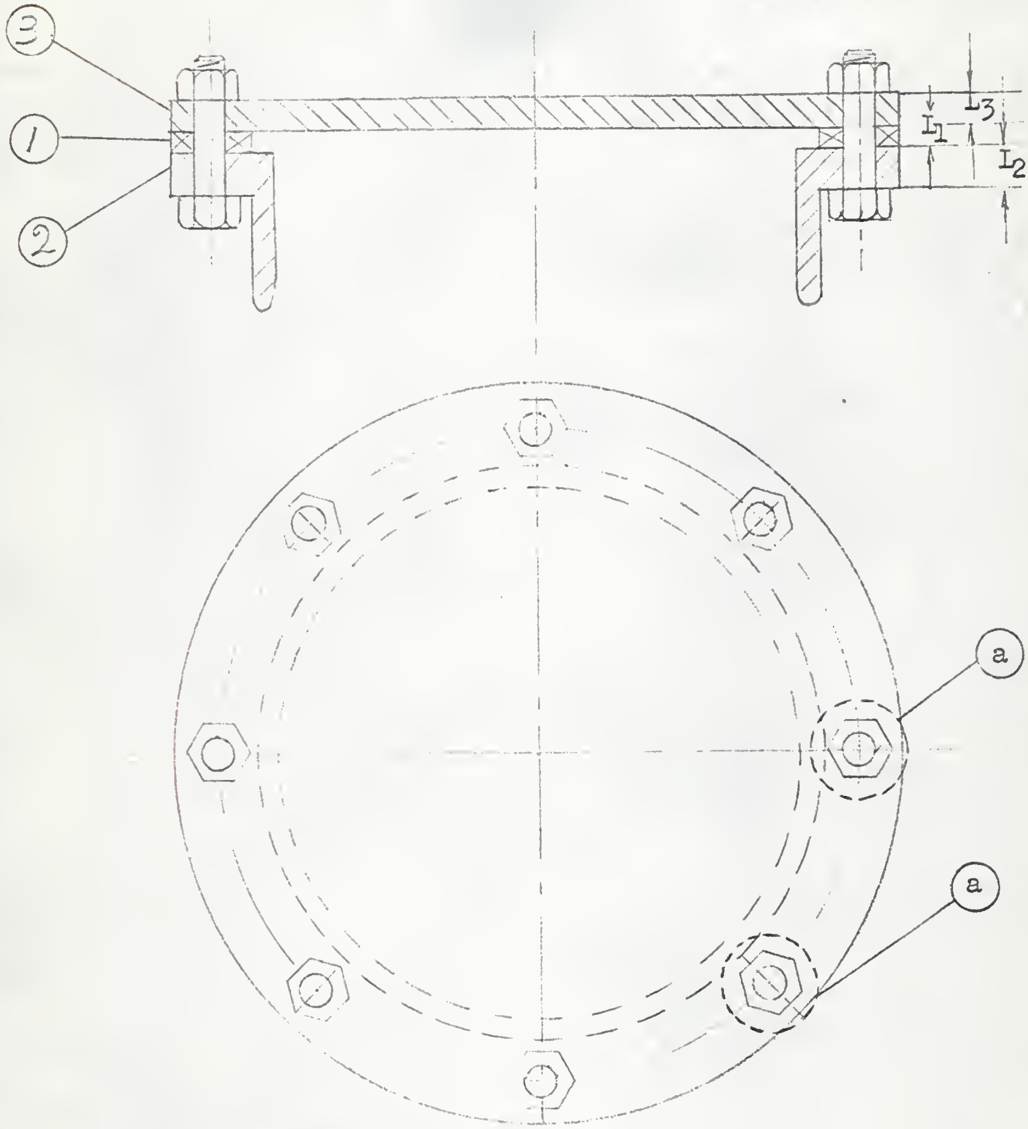

Fig. 1

FLANGE JOINT

Part 1 - Gasket
Part 2 - Flange
Part 3 - Cover Plate

(a) Effective area of part for each bolt by Radzimvosky equation (see page 8).

Initial load due to tightening is a function of the bolt diameter. It depends upon the mechanic, how experienced he is, the length of wrench used, as well as the condition of bolt and nut. The above equation is given for average conditions.

The external load for the flanged connection, Figure I, is

$$F_e = \frac{pA}{n}$$

where p = Internal pressure (psi.)

$A = 0.7854\ (D_i)^2_g$(sq. in.)

$(D_i)g$ = Inside deameter of the gasket (in.)

n = Number of bolts

According to this approach the total load on the bolt is

$$F_t = F_i + F_e \qquad \text{eq. (A)}$$

$$= 16{,}000\ D + \frac{pA}{n}$$

Doughtie and Carter (2) developed an expression assuming the bolt and gasket to have elastic behavior, but the flange and plate members remained rigid. The deformations of the bolt and gasket are directly proportional to the load. The total load on the bolt is

$$F_t = F_i + mFe \qquad \text{eq. (B)}$$

where $m = \dfrac{\left(\frac{EA}{L}\right)b}{\left(\frac{EA}{L}\right)_b + \left(\frac{EA}{L}\right)_g}$

The quantity m will have values ranging from zero to one depending upon the gasket used. If the gasket is hard, thin, and of large area, the term $\left(\frac{EA}{L}\right)_g$ will be large compared to $\left(\frac{EA}{L}\right)_b$ and m will approach zero. For gaskets that are soft the term $\left(\frac{EA}{L}\right)_g$ becomes small compared to $\left(\frac{EA}{L}\right)_b$, then the value of m approaches one. The absence of the gasket between the members is the same as having a gasket of infinite stiffness, that is, $\left(\frac{EA}{L}\right)_g$ becomes infinite, then m becomes zero and the equation reduces to $F_t = F_i$. This implies the bolt load is independent of the external load. Equation (B) is valid only as long as the gasket remains in contact with the other members of the connections.

Vallance and Doughtie (3) give the following values for m

Type of Joint	m
Soft packing with stud....................	1.00
Soft packing with through bolt............	.75
Asbestos..................................	.60
Soft copper with long through bolt........	.50
Hard copper gasket with long through bolt.	.25
Metal to metal with rough bolt............	.00

The most recent development is that of Radzimovsky (4) and presented in present machine design books such as Faires (5). The elasticity of all the parts is considered.

The total load on the bolt is

$$F_t = F_i + kF_e \qquad \text{eq. (C)}$$

where

$$k = \frac{\left[\left(\frac{L}{AE}\right)_1 + \left(\frac{L}{AE}\right)_2 + \left(\frac{L}{AE}\right)_3 + \text{-------}\right]}{\left(\frac{L}{AE}\right)_b + \left[\left(\frac{L}{AE}\right)_1 + \left(\frac{L}{AE}\right)_2 + \left(\frac{L}{AE}\right)_3 + \text{---}\right]}$$

In general form

$$k = \frac{\sum_{1}^{N} \left(\frac{L}{AE}\right)_n \text{ parts}}{\sum_{1}^{N} \left(\frac{L}{AE}\right)_n \text{ parts} + \frac{L}{AE}\,_b}$$

where n = 1, 2, 3, 4...........N

N = Total number of bolted parts.

$$F_i = \frac{T}{CD}$$

T = Tightening tongue (in. lb.)

C = Friction factor

The equation is based on the following assumptions; (a) the load is distributed uniformly over the area, (b) all parts are elastic, and (c) bolted parts are in compression.

If the bolt used is not of uniform cross-section then that term in the equation for k must be modified as follows

$$\left(\frac{L}{AE}\right)_b = \left[\frac{1}{E} \sum_{1}^{M} \left(\frac{L}{A}\right)_m\right]_b$$

Where M is the total number of different bolt cross-sections.

The terms in k are defined satisfactorily except for the effective area of the parts. Some work has been done in an attempt to define it, however, only specific cases have been considered. This means that for most joints the designer must make assumptions in selecting the area of the parts.

PREVIOUS INVESTIGATIONS

Radzimvosky (4) stated, that for two plates of infinite area bolted together, the cross-section can be represented by compression cones as shown in Figure 2a, which cut the bearing surfaces under the nut and the head at an angle of 45 degrees. Elasticity of the double cone can be determined approximately by replacing the cone with a hollow cylinder using the mean cone diameter as the outside diameter and the same inside diameter, d, as the cone (bolt hole). Thus, the effective cross-section area is obtained from the following equation:

$$A_p = \frac{\pi}{4}(D_e^{2}-d^{2}) \qquad \text{eq. (d)}$$

where

$$D_e = D_h + \frac{L_1 + L_2}{2}$$

D_h = Diameter of the washer face of the nut (in.)

If a gasket is placed between the flange and the cover plate as shown in Figure 2b then

$$D_e = D_h + \frac{L_1 + L_2 + L_3}{2}$$

In general form

$$D_e = D_h + \frac{1}{2}\sum_{1}^{N}(L)_n$$

where $n = 1, 2, 3, 4 \ldots\ldots\ldots\ldots N$

N = Total number of bolted parts.

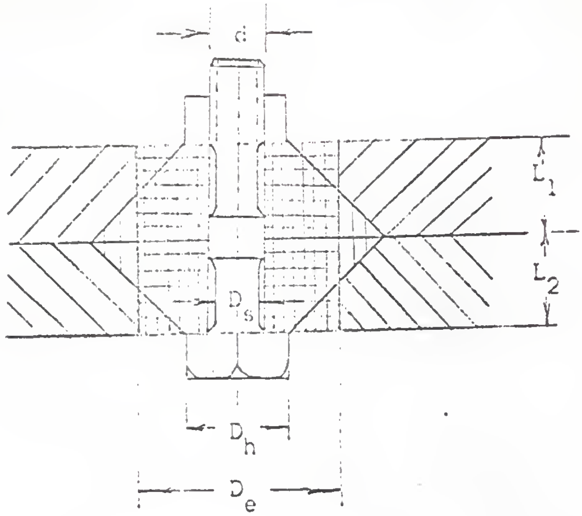

Figure 2(a) Schematic representation of infinite plates undergoing compressive deformation.

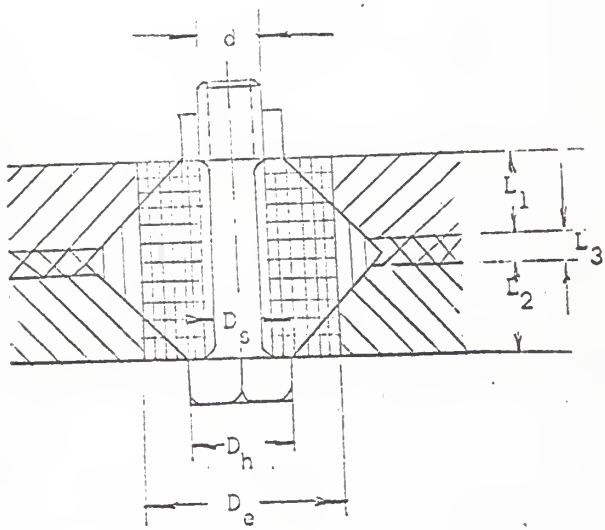

Figure 2(b) Infinite plates with gasket undergoing compressive deformation.

Radzimvosky also suggested that the stiffness and therefore effective area may have to be found experimently if the machine member is complex in form such as the connecting rod. Thum and Debas (6) tested a connecting rod joint and found the ratio between bolt stiffness and stiffness of the corresponding parts of the rod to be 1:1.5. The bolt used was 0.8978 in. shank diameter. For this particular case the effective area of the parts was 0.950 sq. in.

In the previous discussion it has been assumed that the bolt load is distributed uniformaly over the entire gasket flange area. In an actual case the load acts under the bolt head. Considering the load acting at a point, Robert (7) presented an expression for deflection on the basis of the theory of a beam on an elastic foundation. Deflection varies from a maximum value at the bolt to a minimum at a point midway between consective bolts.

Figure 3(a) represents the combination of flange and gasket.

Figure 3(b) represents the combination of flange and gasket on the elastic foundation with a series of loads - P - representing the bolts, having spacing τ between the bolts.

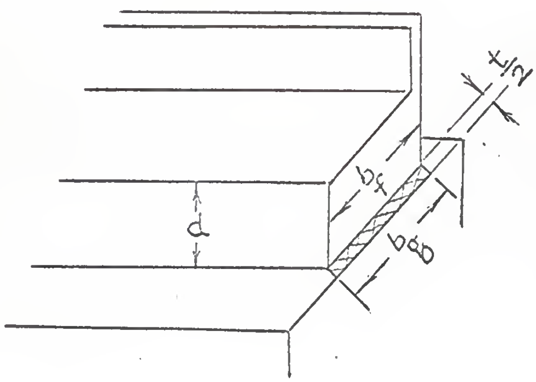

Figure 3(a)

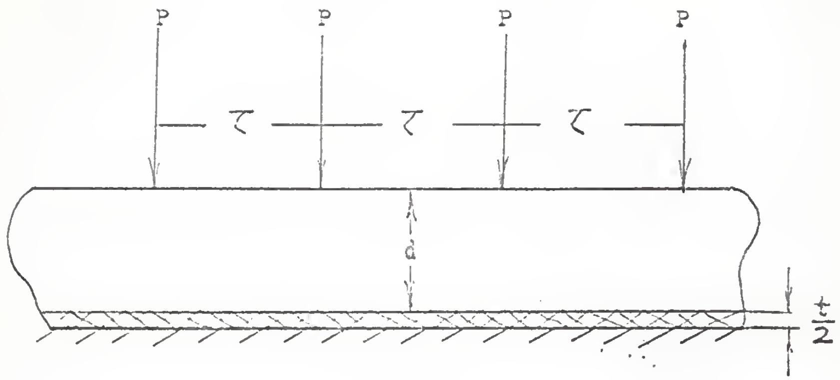

Figure 3(b)

Deflection of a beam on an elastic foundation is

$$y = \frac{P\beta}{2K} e^{-\beta x} (\cos\beta x + \sin\beta x) \qquad \text{eq. (E)}$$

where P = Applied load (lb.)

$$\beta = \sqrt[4]{\frac{K}{4E_f I}}$$

x = Distance from point of concentrated load

K = Modulus of foundation

$$K = \frac{2}{\frac{d}{b_f E_f} + \frac{t}{b_g E_g}}$$

I = Moment of inertia of beam

$$= \frac{b_f d^3}{12}$$

by substitution

$$\beta = \sqrt[4]{\frac{\dfrac{2}{\dfrac{d}{b_f E_f} + \dfrac{t}{b_g E_g}}}{4\, E_f \left(\dfrac{b_f d^3}{12}\right)}} = \frac{1.565}{d} \sqrt[4]{\frac{1}{1 + S}}$$

where $S = \dfrac{t b_f E_f}{d b_g E_g}$

In the case where t = o, i.e., metal to metal joint, $S = 0$ and $\beta = \dfrac{1.565}{d}$ which is the result obtained by Soderberg (8).

For analysis of leakage evidence Robert also tested four flange joints to varify his analytical work. Two joints were designed not to leak according to theory. Tests showed no evidence of leakage on these joints. The other two joints, designed for use in a refinery, should have leaked according to the theory. Leakage occured and in order to make one of the joints tight it was necessary to remove the gasket and solder the flanges together.

DETERMINING THE EFFECTIVE AREA OF BOLTED PARTS

Most bolted connections are not of infinite plates as considered by Radzimvosky. The flange connection shown in Figure 1 is more common. No investigation has been reported for determining an actual effective area of the parts for this type of joint. Using the idea of a beam on an elastic foundation suggested by Robert an attempt was made to determine the effective area of the parts. One of the flange joints designed and tested by Robert was used for this development. It was modified by removing the gasket. The flange area per bolt for the joint was 24 sq. in.. One procedure would be to use this as the effective area.

The first approach was to determine an average deflection and use it in equation (F) to determine an effective area.

$$A = \frac{PL}{yE} \qquad \text{eq. (F)}$$

where L = Effective length of parts (in.)

y = Average deflection of beam (in.)

E = Modulus of elasticity of flange (psi.)

P = Applied load (lb.)

Using a unit load for P the deflection y was calculated for different positions between load application by use of equation (E) (See Appendix I). Figure 4 shows the defection from the point of load application to the midpoint between loads. The average deflection was 0.473 (10^{-24}) in.. Substituting this value in eq. (F), the effective area of the part is 3.52 (10^{16}) which is many orders of magnitude greater than the flange area.

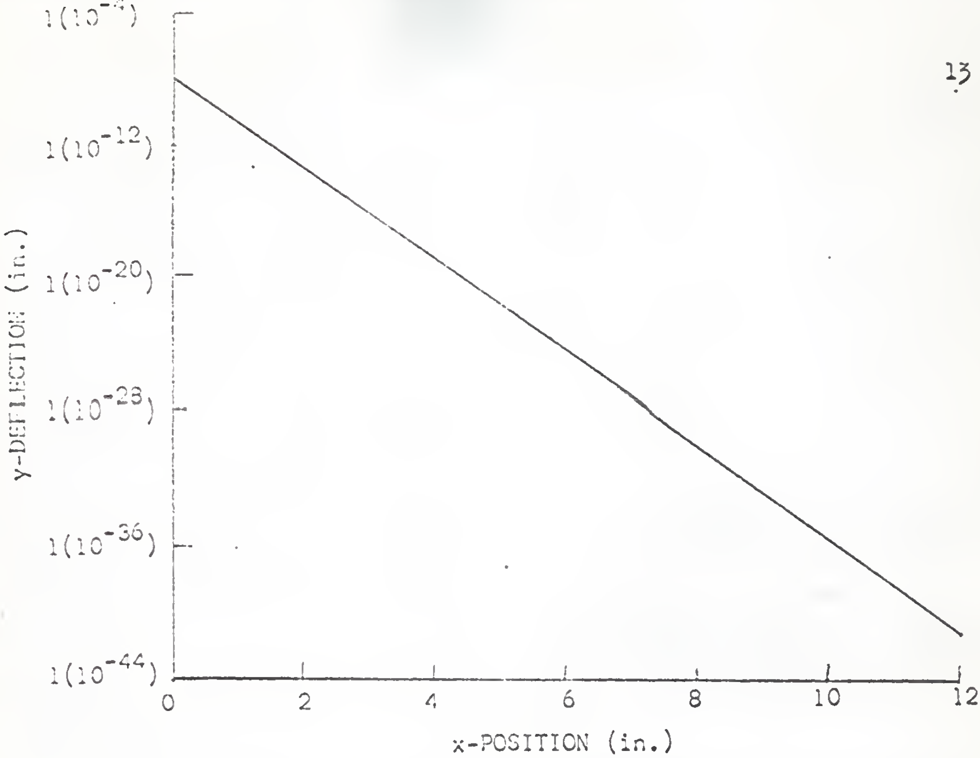

Fig. 4. Deflection - beam on an elastic foundation

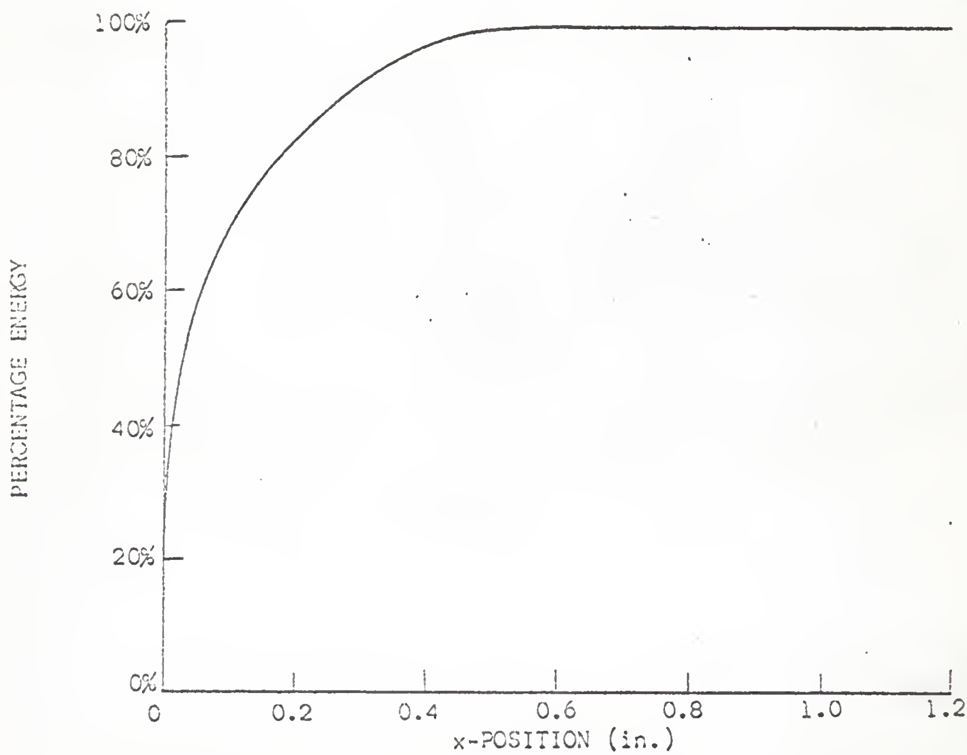

Fig. 5. Energy distribution - beam on an elastic foundation

Since the effective area obtained by using the average deflection was too large the idea of a root-mean-square value of deflection was considered. By using the root-mean-square value of $y = 3.77\ (10^{-9})$ in. in equation (F) the effective area of the part is 4.45 sq. in. This value is more realistic.

The next approach was by use of an energy method. By the strain energy method the percentage of energy absorbed in the beam as a function of position from the point of load application to the midpoint between loads was determined (Appendix II) and is plotted in Figure 5. This figure shows that 99.8% of the energy is absorbed in the beam within 0.56 in. from the point of load application. Using this the effective area is $2\ (x)\ (b_f) = 2\ (0.56)\ (1) = 1.12$ sq. in..

According to Radzimvosky equation, the effective area would be 0.049 sq. in. assuming a point load. However, if it is assumed that 3/8 in. bolts were used the area becomes 0.4075 sq. in..

SUMMARY AND RECOMMENDATIONS

Five defferent effective areas were determined by applying the following different approaches; (1) the area of flange itself, (2) using the average deflection for the beam on an elastic foundation, (3) using a root-mean-square value of deflection of a beam on an elastic foundation, (4) by strain energy method and (5) by using Radzimvosky's equation for infinite plates.

The first approach to find the effective area of parts was on the basis that the whole area of the flange between two consecutive bolts will be effective. This value is a maximum limit that can be taken for consideration.

In the next approach the effective area was found by using an average deflection for the beam on an elastic foundation. Since the value obtained by this approach was many orders of magnitude greater than the area of the flange, this method is not satisfactory.

Using the root-mean-square value of deflection for the beam on an elastic foundation, the area was of smaller magnitude than the area of the flange. This area may be taken into consideration.

Strain energy method seems to be more realistic approach. The effective area was that portion of beam which contains 99.8% of the strain energy. This seems to be a reasonable value.

The Radzimvosky equation gave a very small effective area for a point load. Assuming a 3/8 in. diameter bolt a more realistic value was calculated.

The following are recommendations based on the results of this study.

1. An experimental investigation of this flange joint should be undertaken to determine the effective area as a check of the different methods presented.

2. Apart from this experiment, work may be conducted for bolted joints such as infinite, circular and square plates, long narrow parts and also with gaskets of various stiffness between the parts to develop a general analytical equation for determining the actual effective area of parts.

ACKNOWLEDGEMENT

The author wishes to express his appreciation to Dr. John C. Lindholm, major professor, for advice and counsel during the preparation of this report.

BIBLIOGRAPHY

1. Kimbal, D. S. and J. H. Bar
   Element of Machine Design. John Willey and Sons, Third edition. p. 278

2. Doughtie, V. L. and W. I. Carter
   Bolted assemblies, Machine Design, February, 1950. p. 127

3. Vallance, V. L. and V. Doughtie
   Design of machine member. McGraw Hill Book Company, 1951.

4. Radzimosky, E. I.
   Bolt design for repeated loading. Machine Design, Volume 24. November, 1952. p. 135.

5. Faires, V. M.
   Design of machine element. Third Edition, New York. The Macmillan Company, 1961.

6. Thum, A. and F. Debas
   vorspannug und Dauverhaltbarkeit von Schraubenverbindungen, V. D. I. Verlag, Berlin, 1936.

7. Robert, S. I.
   Gasket and bolted joint. A. S. M. E. Journal Applied Mechanics. Volume 72. p. 169.

8. Soderberg, C. H.
   Discussion on practical aspect of turbine cylinder joint. Journal of Applied Mechanics. Trans. A. S. M. E. Volume 61. 1939. p. 31.

9. Timosinko, D.
   Strength of material. Part 2 Van Nostrand Company, New York, Second Edition, 1941.

A P P E N D I C E S

APPENDIX I

Calculation of effective area of parts by using average deflection of beam on an elastic foundation. Figure 6 represents the flange as a beam on an elastic foundation with a series of point loads P representing the bolts having a spacing τ.

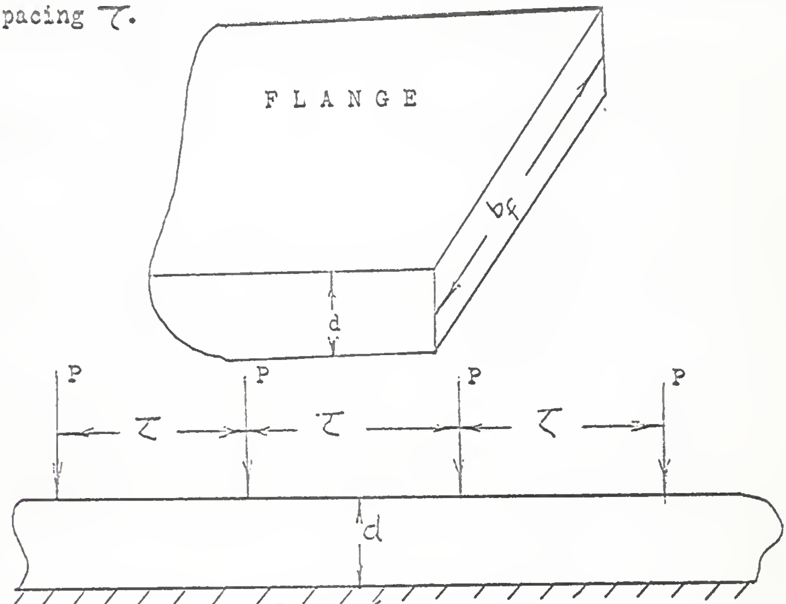

Figure 6.

Data obtained from Robert's experiment are:

τ = Bolt spacing = 24 in.

d = Thickness of flange = $\frac{1}{4}$ in.

b_f = Width of flange = 1 in.

E_f = Modulus of elasticity = 30×10^6 psi.

P = Applied load = 1 lb.

Deflection of a beam on an elastic foundation is

$$y = \frac{P\beta}{2K} e^{-\beta x} (\cos\beta x + \sin\beta x)$$

where K = Modulus of foundation, psi.

$$= \frac{2}{\frac{d}{b_f\ E_f}} = 2.4\ (10^8)\ \text{psi.}$$

$$\beta = \sqrt[4]{\frac{K}{4E_f I}}$$

I = Moment of inertia of beam

$$I = \frac{b_f\ d^3}{12} = 0.013\ \text{in.}^4$$

$$\beta = 6.26$$

by substitution

$$y = 1.305(10^{-8})\ e^{-6.26x}(\cos 6.26x + \sin 6.26x)$$

where x = Distance from point of concentrated load (in.)

Values of y are given in table I for different positions x.

TABLE I

x (in.)	y (in.)
0	1.305 (10^{-8})
1	0.238 (10^{-10})
2	0.434 (10^{-13})
3	0.790 (10^{-16})
4	0.143 (10^{-18})
5	0.261 (10^{-21})
6	0.473 (10^{-24})
7	0.858 (10^{-27})
8	0.155 (10^{-29})
9	0.280 (10^{-32})
10	0.504 (10^{-35})
11	0.907 (10^{-38})
12	0.162 (10^{-40})

Average deflection

$$y = 0.473\ (10^{-24})\ \text{in.}$$

Root-mean-square deflection

$$y = 3.77\ (10^{-9})\ \text{in.}$$

The effective area of part was obtained from the following equation:

$$A = \frac{PL}{yE}$$

where L = Effective length of parts

P = Applied load

y = Deflection of beam

E = Modulus of elasticity of flange

For average deflection

$$A = \frac{1(0.5)}{0.473(10^{-24})\ 30(10^{6})}$$

$$= 3.52(10^{16})\ \text{sq. in.}$$

For root-mean-square deflection

$$A = \frac{1(0.5)}{3.77(10^{-9})\ 30(10^{6})}$$

$$= 4.45\ \text{sq. in.}$$

APPENDIX II

Effective area of parts by using energy method

Strain energy stored in beam on an elastic foundation is

$$U = \frac{EI}{2} \int_0^L \frac{d^2y}{dx^2}\, dx$$

where y is the deflection of a beam on an elastic foundation given by

$$Y = \frac{P\beta}{2K}\, e^{-\beta x} (\cos \beta x + \sin \beta x)$$

$$U = \frac{EI}{2} \int_0^L \left[\frac{P\beta^3}{K}\, e^{-\beta x} (\sin \beta x - \cos \beta x) \right]$$

$$= \frac{PEI\beta^5}{4K^2} \left[\frac{1}{2} (1 + \phi) - e^{-2\beta x} \right]$$

$$\text{where } \phi = e^{-2\beta x} (\sin 2\beta x + \cos 2\beta x)$$

The percentage strain energy absorbed in the beam to position x.

$$\%\ \text{energy} = \frac{\left[U\right]_0^x}{\left[U\right]_0^\infty} (100)$$

$$\frac{\dfrac{PEI\beta^5}{4K^2} \left[0 - \dfrac{1}{2} (1 + \phi)\, e^{-2\beta x} \right]}{\dfrac{PEI\beta^5}{4K^2} \left[0 - \dfrac{1}{2} \right]}$$

$$= 2\left[\frac{1}{2}(1+\phi) - e^{-2\beta x}\right](100)$$

$$\beta = 6.26 \text{ (from appendix I)}$$

The energy distribution was determined and tabulated in table II. The data are plotted in figure 5.

Since 99.8% of the energy was absorbed in the beam within 0.56 in. from the point of load application. The effective area of parts was determined using $2(x)(b_f) = 2(0.56)(1) = 1.12$ sq. in.

TABLE II

2 x	ϕ	$1+\phi$	$\frac{1+\phi}{2}$	$e^{-2\beta x}$	$\frac{1}{2}(1+\phi)-e^{-2\beta x}$	% Energy	x
0	1.0	2.0	1.0	-1.0	0	0	0
0.5	0.8231	1.8231	0.911	-0.606	0.305	60.1	0.04
1.0	0.5083	1.5083	0.704	-0.367	0.337	67.4	0.08
1.5	0.2384	1.2384	0.619	-0.2232	0.395	78.1	0.12
2.0	0.0667	1.0667	0.533	-0.135	0.398	79.6	0.16
2.5	-0.0166	0.9834	0.4917	-0.082	0.4097	81.94	0.20
3.0	-0.0166	0.958	0.4788	-0.0497	0.4291	85.8	0.24
3.5	-0.042	0.9611	0.4805	-0.0302	0.4503	90	0.28
4.0	-0.0389	0.9742	0.4871	-0.01832	0.4688	93.76	0.32
4.5	-0.0258	0.9868	0.4832	-0.01111	0.4721	94.42	0.36
5.0	-0.0132	0.9954	0.4877	-0.00674	0.4810	96.2	0.40
5.5	0	1.0	0.5	-0.0041	0.4959	99.18	0.44
6.0	0.0017	1.0017	0.5000	-0.00248	0.4976	99.52	0.48
6.5	0.0018	1.0018	0.50000	-0.0015	0.4986	99.72	0.52
7.0	0.0013	1.0013	0.5000	-0.0009	0.49915	99.83	0.56

BOLTED CONNECTIONS

by

SHANKER L. BATHWAL

B.S., Birla Institute of Technology, Ranchi, 1961

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Mechanical Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1963

A number of approaches have been used in analyzing bolted connections. The earlier equations were based on certain simplifying assumptions which reflected the general understanding of the problem. As more was understood about the material properties and elasticity of materials different equations were developed to better reflect the true satuation.

The earliest equation which assumed the bolted parts rigid is that by Kimbal and Bar.

$$F_t = F_i + F_e$$

Doughtie and Carter developed an equation on the assumption that the bolt and gasket were the only elastic members.

$$F_t = F_i + mF_e$$

$$m = \frac{\left(\frac{EA}{L}\right)_b}{\left(\frac{EA}{L}\right)_b + \left(\frac{EA}{L}\right)_g}$$

Radzimvosky developed an equation assuming all the parts elastic.

$$F_t = F_i + kF_e$$

$$k = \frac{\sum_{I}^{N} \left(\frac{L}{AE}\right)_n \text{parts}}{\sum_{I}^{N} \left(\frac{L}{AE}\right)_n \text{parts} + \left(\frac{L}{AE}\right)_b}$$

In the above equations, F_t, F_i and F_e are total load initial load and external load per bolt, respectively.

The terms in the expression for k are satisfactorily defined except for the area of the parts. This report discusses five different approaches for determining the effective area; (1) The area of flange itself (2) using the average deflection for a beam on an elastic foundation (3) using a root-mean-square value of deflection of a beam on an elastic foundation (4) by strain energy method (5) by using Radzimvosky equation for an infinite plate.