COMBINED FANNO-LINE, RAYLEIGH-LINE HEAT TRANSFER AND FRICTION EFFECTS ON STEADY ONE-DIMENSIONAL GAS FLOW IN CONSTANT-AREA PASSAGES

by

WILBUR PO-SAM HSIEH 265

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Approved by:

Wilson Tripp

Major Professor

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NOMENCLATURE

A : cross-sectional flow area, sq.ft.

A_w : wetted area, sq.ft.

c_p : specific heat at constant pressure, Btu/slug ^oR

c_v : specific heat at constant volume, Btu/slug ^oR

D : equivlaent hydraulic diameter, $4Ax/A_w$, ft.

e : base of natural logarithms

e_e : expansion efficiency, defined by Equation (A-2)

f : local friction coefficient

F : a funcion of the Mach number, defined by Equation (18)

$\overline{f}$: mean value of friction coefficient, $\frac{1}{L}\int_o^L f dx$

h coefficient of convective heat transfer, Btu/sec sq.ft. ^oR

H : a function of the Mach number, defined by Equation (19)

J : mechanical equivalent of heat (778 foot-pounds per Btu)

k : ratio of specific heats

L_{max} : maximum length of flow passage in subsonic region, ft.

L : a function of the Mach number, defined by Equation (51)

M : Mach number

P : static pressure, lb/sq.ft. abs.

P_o : total pressure, lb/sq.ft. abs.

Q : heat flow per unit mass, Btu/slug

R : gas constant, ft-lb/slug ^oR

r : a ratio defined by Equation (13)

T : static temperature, ^oR

T_o : total temperature, ^oR

T_w : flow passage wall temperature, ^oR

T_{aw} : adiabatic wall temperature, ^oR

U : a function of the Mach number, defined by Equation (36)

V : axial velocity in flow passage, ft/sec

v : specific volume, cu.ft./slug

W : work per unit mass, ft-lb/slug

w : rate of mass flow, slug/sec

X : friction-distance parameter, $\int_0^x \frac{4f}{D}\,dx$

x : axial distance through flow passage, ft.

Greek Letters

ρ : density, slug/cu.ft.

ϕ : defined by Equation (A-3)

τ_w : wall shearing stress, lb/sq.ft.

η : Mach number squared

θ : defined by Equation (69)

μ : defined by Equation (71)

Subscripts

1 : signifies properties at initial section of flow passage, i.e., at x = 0

2 : signifies properties at final section of flow passage, i.e., at x = 1

F : properties evaluated at Fanno line

R : properties evaluated at Rayleigh line

Superscript

$()^*$: signifies properties at Mach number unity

$()'$: signifies the first derivative

INTRODUCTION

The well-known Fanno-line process deals with a perfect gas flowing in a duct of constant cross-sectional area with friction in which there is no heat transfer to or from the gas. Making use of the principles of conservation of mass, momentum, and energy, as well as the perfect gas relation, it is possible to separate the variables and express them in terms of the Mach number. On the other hand, Rayleigh worked on the same type of flow except that he considered the case of heat transfer with no friction. In connection with these two processes, there exist two very convenient tables (2) for the processes which enable us to determine the ratios of the fluid properties at any section downstream for a given initial Mach number.

In practical cases, however, the theoretical adiabatic or ideal flow is seldom encountered. Therefore, it is proposed to investigate the case when both friction and heat transfer are present. A number of studies of the steady flow of a compressible fluid have been published. Steady flow in ducts with friction and heat addition was investigated by Hicks, Montgomery, and Vasserman (1) and by Shapiro (2). Each of them presents the problem by a different approach, but none of them has been able to solve for the fluid properties and present them in general forms.

Research of steady, one-dimensional compressible fluid flow in a constant-area passage with friction and heat transfer is being conducted in the Department of Mechanical Engineering, Kansas State University under the direction of Professor Wilson Tripp. Since the Fanno-line process is affected only by the wall friction of the passage and the Rayleigh-line process is affected only by heat transfer from the wall, it is advisable to combine the two effects on the fluid properties in such a way as to make use of the information already available from Fanno's and Rayleigh's investigations. It is Professor Tripp's suggestion to introduce a new function r(M) which denotes the proportionality of the two effects. The function r is so defined that the flow process which is to be represented identical with the Fanno-line process when r is equal to zero. With the new function r, we follow a procedure similar to that employed in Shapiro's analysis, and we are able to express the fluid properties in terms of the Mach number and r in differential form. The differential equations can not be integrated unless r is known. The next step which must then be taken is to determine the function, r.

It is possible to ignore the nature of the local frictional effect by introducing a parameter, namely, the friction-distance parameter, which is defined as the average value of the wall friction over the entire length of the flow passage. When a constant average value of wall friction is assumed, the function, r, depends on the nature of the heat transfer only.

Three special cases of heat transfer are considered, namely, constant heat flux, constant wall temperature, and constant ratio of wall temperature to total temperature. For the three cases, it is assumed that the Reynolds' analogy remains valid and that the recovery factor is unity. According to Shapiro (2), experiments show that Reynolds' analogy has an accuracy of a few per cent for fully developed turbulent gas flows. With Reynolds' analogy, it is possible to relate the total temperature to the length variable, x.

In the special cases of constant heat flux and constant wall temperature, the "L" and "U" functions of the Mach number assume special physical significance. In either case, after some manipulation, it is possible to derive an ordinary differential equation for the specialized function which applies to the specialized case. Exact solution of the differential equation seems to be quite difficult. However, Simpson's rule may be used for direct numerical integration. After values of the function at each Mach number have been obtained, r is determined through an equation which will be presented in the text. Since the fluid properties are expressed in terms of the Mach number and r in differential forms, these properties can be obtained by a method of numerical integration once r is found.

METHOD OF ANALYSIS

The analysis closely follows that of Shapiro (2), and Hicks, Montgomery, and Wasserman (1). Before carrying out the analysis, the following assumptions are made:

1. The flow is steady and one-dimensional; i.e., all properties are uniform over each cross section.
2. Changes in stream properties are continuous.
3. The fluid is a perfect gas.
4. The ratio of specific heats and the molecular weight are constant.
5. Body forces are neglected.
6. Heat is transferred continuously and completely but only transversely throughout the passage.

The conventional variables (pressure, temperature, density, and velocity) in one-dimensional flow are connected by four relations derivable from the first law of thermodynamics, the conservation of mass, the conservation of momentum, and the equation of state for a perfect gas. Consider two consecutive sections which are separated by an infinitesimal distance. Between the two sections, an infinitesimal amount of heat is added, wall friction is present, and the fluid undergoes a change in momentum. The four governing differential equations describing the process are :

Conservation of energy $\quad c_p dT + \frac{1}{J}\frac{d(V^2)}{2} = dQ \qquad (1)$

Conservation of mass $\quad d(\rho V) = 0 \qquad (2)$

Conservation of momentum $\quad \rho AVdV + AdP + \tau_w dA_w = 0 \qquad (3)$

Equation of state $\quad dP = Rd(\rho T) \qquad (4)$

The first equation is divided by $c_p T$, and by introducing the definition of the Mach number

$$M^2 = \frac{V^2}{kRT} \tag{4a}$$

It can be written as

$$\frac{dT}{T} + \frac{k-1}{2} M^2 \frac{dV^2}{V^2} = \frac{dQ}{c_pT} \tag{5}$$

Dividing Equation (3) by the product, PA, with $P = \frac{\rho V^2}{kM^2}$,

$$\frac{dP}{P} + kM^2 \frac{dV}{V} + \tau_w \frac{dA_w}{PA} = 0 \tag{6}$$

The drag coefficient, or the coefficient of friction, is defined as the ratio of the wall shearing stress to the dynamic head of the stream. Thus,

$$f = \frac{\tau_w}{\rho V^2/2}$$

The hydraulic diameter is defined as four times the ratio of cross-sectional area to wetted perimeter,

$$D = 4 \frac{A}{dA_w} dx$$

We now introduce the latter two expressions into Equation (6), which gives

$$\frac{dP}{P} + \frac{kM^2}{2} 4f \frac{dx}{D} + \frac{kM^2}{2} \frac{dV^2}{V^2} = 0 \tag{7}$$

It may be seen that the terms, $\frac{dQ}{c_pT}$ and $\frac{4f}{D}$ dx, in Equations (5) and (7) are not in convenient forms; therefore, we introduce two new functions H and F, defined by the equations,

$$H\frac{dV^2}{2V^2} = \frac{dQ}{c_pT} \tag{8}$$

and $$F \frac{dV^2}{V^2} = \frac{4f}{D} dx \tag{9}$$

Finally, from Equations (5) and (8)

$$\frac{dT}{T} = \left(H - (k - 1) M^2\right) \frac{dV}{V} \tag{10}$$

and from Equations (7) and (9),

$$\frac{dP}{P} = - kM^2 (1 + F)\frac{dV}{V} \tag{11}$$

In order to solve for the fluid properties in terms of the Mach number, we introduce a new function, r, which is defined by the equation,

$$n = r\, n_F + (1 - r)\, n_R,$$

where n is a function of the Mach number and depends upon the problem concerned. It is found in Appendix A that

$$n_F = 1 + (k - 1)\, M^2$$

and $n_R \doteq k\, M^2.$

where n_F and n_R denote the function, n, for the Fanno process and the Rayleigh process, respectively. Therefore, n can be written as

$$n = r + (k - r)\, M^2. \tag{12}$$

Solving for r, Equation (12) becomes

$$r = \frac{n - kM^2}{1 - M^2} \tag{13}$$

It is known from thermodynamics that

$$\frac{dP}{P} = n \frac{d\rho}{\rho} \tag{14}$$

Only if n is a constant can Equation (13) be integrated to the form $P = c\rho^n$.

By logarithmic differentiation of the equation of state, the continuity equation, and the definition of the Mach number,

$$\frac{dP}{P} = \frac{d\rho}{\rho} + \frac{dT}{T}\,, \tag{15}$$

$$\frac{d\rho}{\rho} = - \frac{dV}{V}\,, \tag{16}$$

and $$\frac{dM^2}{M^2} = 2\,\frac{dV}{V} - \frac{dT}{T} \tag{17}$$

Equating the right-hand sides of Equations (11) and (14), we have

$$kM^2\,(1 + F)\,\frac{dV}{V} = - n\,\frac{d\rho}{\rho} \tag{17a}$$

From Equations (12), (16) and (17a), the function of F is given by

$$F = \frac{r\,(1 - M^2)}{kM^2}\,. \tag{18}$$

Eliminating $\frac{d\rho}{\rho}$ and $\frac{dP}{P}$ in Equation (15) by substitution from Equations (16) and (11), we have

$$\frac{dT}{T} = \left[1 - kM^2\,(1 + F)\right]\frac{dV}{V}\,.$$

Substituting Equation (18) for F and equating the right-hand side of the resulting equation to the right-hand side of Equation (10),

$$\left(H \div (k - 1) M^2\right) \frac{dV}{V} = \left((1 - r) - (k - r) M^2\right) \frac{dV}{V}$$

Thus, the function, H, is given by

$$H = (1 - r)(1 - M^2) \tag{19}$$

It is seen that if $\frac{dV}{V}$ can be related M and r, the rest of the fluid properties can be expressed in terms of the Mach number and r.

From Equations (10), (17) and (19), we have

$$\frac{dM^2}{M^2} = \left((1 + r) + (k - r) M^2\right) \frac{dV}{V} \tag{19a}$$

or

$$\frac{dV}{V} = \frac{dM^2}{(1 + r)M^2} - \frac{(k - r)dM^2}{(1 + r)\left[(1 + r) + (k - r)M^2\right]} \tag{20}$$

With Equation (19a), Equation (17) becomes

$$\frac{dT}{T} = \frac{1 - r}{1 + r}\frac{dM^2}{M^2} - \frac{2}{1 + r}\frac{(k - r)dM^2}{\left((1 + r) + (k - r)M^2\right)} \tag{21}$$

Combination of Equations (15), (16), (20) and (21) gives

$$\frac{dP}{P} = -\left\{\frac{r}{1 + r}\frac{dM^2}{M^2} + \frac{1}{1 + r}\frac{(k - r)dM^2}{(1 + r) + (k - r)M^2}\right\} \tag{22}$$

The differential equations for the total temperature and the total pressure can be found by adding the terms $\frac{dK}{K}$ and $\frac{k}{k-1}\frac{dK}{K}$ to Equations (21) and (22), respectively, where $K = 1 + \frac{1}{2}(k - 1) M^2$. Therefore,

$$\frac{dT_o}{T_o} = \frac{1-r}{1+r}\frac{dM^2}{M^2} - \frac{2}{1+r}\frac{(k-r)dM^2}{(1+r) + (k-r)M^2} + \frac{(k-1)dM^2}{2 + (k-1)M^2} \tag{23}$$

and

$$\frac{dP_o}{P_o} = \frac{k}{k-1}\frac{(k-1)dM^2}{2+(k-1)M^2} - \left\{\frac{r}{1+r}\frac{dM^2}{M^2} + \frac{1}{1+r}\frac{(k-r)dM^2}{\left[(1+r) + (k-r)M^2\right]}\right\} \tag{24}$$

Next we want to determine the friction-distance parameter which is defined as

$$\frac{4\bar{f}}{D} L_{max} = \int_o^{L_{max}} \frac{4f}{D}\, dx \tag{25}$$

where

$$\overline{f} = \frac{1}{L_{max}} \int_0^{L_{max}} \frac{4f}{D}\, dx$$

From Equations (9), (18), and (20), we have

$$\frac{4f}{D}\, dx = \frac{2r(1 - M^2)}{kM^2} \frac{dM^2}{M^2[(1 + r) + (k - r)M^2]} \tag{26}$$

After rearrangement, Equation (26) can be written as

$$\frac{4f}{D}\, dx = \frac{2r}{k(1+r)^2} \left\{ \frac{1+r}{M^4} - \frac{k+1}{M^2} + \frac{(k+1)\ (k-r)}{(1+r) + (k-r)M^2} \right\} dM^2 \tag{27}$$

The entropy change is defined by

$$dS = c_p \frac{dT}{T} - \frac{R}{J} \frac{dP}{P} \tag{28}$$

If r is equal to some constant — such as r = 1 for the Fanno-line process — Equations (20) to (28) can be integrated to give

$$\frac{V}{V^*} = \left\{ \frac{(k + 1)\ M^2}{(1 + r) + (k - r)\ M^2} \right\}^{\frac{1}{1+r}}$$

$$\frac{T}{T^*} = \left\{ \frac{(k + 1)M^{(1-r)}}{(1 + r) + (k - r)\ M^2} \right\}^{\frac{2}{1+r}}$$

$$\frac{P}{P^*} = \left\{ \frac{(k + 1)}{[(1 + r) + (k - r)]M^{2r}} \right\}^{\frac{1}{1+r}}$$

$$\frac{T_o}{T_o^*} = \left\{ \frac{2 + (k - 1)M^2}{k + 1} \right\} \left\{ \frac{(k + 1)M^{1-r}}{(1 + r) + (k - r)\ M^2} \right\}^{\frac{2}{1+r}}$$

$$\frac{P_o}{P_o^*} = \left\{ \frac{2 + (k - 1)M^2}{k + 1} \right\}^{\frac{k}{k-1}} \left\{ \frac{k + 1}{[(1 + r) + (k - r)M^2]M^{2r}} \right\}^{\frac{1}{1+r}}$$

$$\frac{4\overline{f}}{D} L_{max} = \frac{2r}{k(1 + r)^2} \left\{ (1 + r)\frac{1 - M^2}{M^2} + (k + 1) Ln \frac{(k + 1)M^2}{(1 + r) + (k - r)M^2} \right\}$$

$$S - S^* = c_p Ln \left\{ \frac{(k+1)M^{1-r}}{(1+r) + (k-r)M^2} \right\}^{\frac{2}{1+r}} - \frac{R}{(1+r)J}\, Ln \frac{k + 1}{[(1+r) + (k-r)M^2]M^{2r}}$$

where the limits of integration are taken from the initial state where the Mach number is M to the final state where the flow is choked.

If r is not a constant, Equations (20) to (28) cannot be integrated unless r is a known function of the Mach number. On the other hand, numerical integration of the equations cannot be carried out unless values of r for corresponding Mach numbers have been found. At this point, the remaining problem is to determine r.

As we have mentioned in the introduction, by defining a friction-distance parameter, we may ignore the nature of the local friction effect on the fluid properties. Under these assumptions, the nature of heat addition dominates the whole problem. For different patterns of heat addition, the function of r can be determined through the mathematical interpretation of the problem. The following cases serve as illustrations of the applicability of this method for the determination of r, either by exact solution or by numerical method.

Case 1. Constant Heat Flux

This is the case where the hsat flux per unit of wall area is the sams for all values of x. Such a situation results, for example, when a tube is heated electrically by passing current either through the wall of the tube itself, or through resistance wires wrapped uniformly around the tube.

Consider an infinitesimal length of a duct. The rate of heat transferred from wall to fluid is equal to the rate at which heat is absorbed by the fluid. This can be written as

$$W\, dQ = \rho A V c_p\, dT_o = h\,(T_w - T_{aw})\, dA_w \tag{29}$$

The recovsry factor is taken as unity, i.e., $T_{aw} = T_o$. Thus, Equation (29) becomes

$$\frac{dT_o}{T_w - T_o} = \frac{h}{\rho V c_p}\,\frac{4}{D}\, dx \tag{30}$$

Furthermore, Reynolds' analogy, which relates the friction factor and the coefficient of heat transfer, is assumed to be valid. This gives

$$\frac{h}{\rho V c_p} = \frac{f}{2} \tag{31}$$

Introducing Equation (31) into (30) results in

$$\frac{dT_o}{T_w - T_o} = \frac{2f}{D}\, dx \tag{32}$$

Equation (32) shows the relation between the change in total temperature and the length of the flow passage.

Assuming that the coefficient of heat transfer, h, is constant, it follows from Equation (29) that $T_w - T_o$ is independent of x for constant heat flux. Therefore, from Equation (32), we have a very important relation

$$\frac{dT_o}{dx} = \text{constant} \tag{33}$$

From Equation (32)

$$dT_o = (T_w - T_o)\,\frac{2f}{D}\,dx \tag{34}$$

$\frac{dT_o}{T_o}$ and $\frac{4f}{D}dx$ have been expressed in terms of the Mach number and r. Dividing both sides of Equation (34) by T_o, and substituting Equations (23) and (26) for $\frac{dT_o}{T_o}$ and $\frac{4f}{D}dx$, we have

$$\frac{T_w - T_o}{T_o} = \frac{2kM^2(1 - r)}{r[2 + (k - 1)M^2]} \tag{35}$$

Let

$$U = \frac{T_w - T_o}{T_o}\,, \quad K = 2 + (k - 1)M^2\,, \quad \eta = M^2 \tag{36}$$

where U is a function of the Mach number. With Equation (36), Equation (35) becomes

$$U = \frac{2k\eta(1 - r)}{rK} \tag{37}$$

Solving for r, Equation (37) gives

$$r = \frac{2k\eta}{KU + 2k\eta} \tag{38}$$

Keeping in mind that $T_w - T_o$ is a constant, logarithmic differentiation of Equation (36) with respect to M gives

$$\frac{U'}{U} = -\left(\frac{dT_o}{T_o}\right)/dM \tag{39}$$

where $U' = \frac{dU}{dM}$. Substitution of Equation (23) for $\frac{dT_o}{T_o}$ results in

$$\frac{U'}{U} = \frac{-4}{M[r(1 - \eta) + k\eta + 1]} + \frac{4}{MK}$$

Solving this last equation for r, we have

$$r = \frac{4(1 - \eta)U + (k\eta + 1)MKU'}{(1 - \eta)(4U - MKU')} \tag{40}$$

Since the functions of r expressed in Equations (38) and (40) have the same physical meaning, i.e. they represent the same form of heat addition, they must be equal to each other. Equating Equations (38) and (40), we have

$$\frac{4(1-\eta)U + (k\eta + 1)MKU'}{(1-\eta)(4U - MKU')} = \frac{2k\eta}{UK + 2k\eta}$$

or

$$(k\eta + 1)MKUU' + 2k\eta MKU' + 4(1-\eta)U^2 = 0 \tag{41}$$

Again, the general solution of Equation (41) cannot be given in a closed form; therefore, it is necessary to obtain the solution by a numerical method. Solving Equation (41) for U', gives

$$U' = \frac{4(\eta - 1)U^2}{2k\eta MK + (k\eta + 1)MKU} \tag{42}$$

If the initial value of U is known, Equation (42) can be integrated immediately by numerical methods. From Equation (36), it is possible to determine the initial value for U, since the initial conditions of T_w and T_0 can be determined by direct measurement. It is also practically possible to assign an initial value for U as a parameter. $U_1 = 4, 3, 2, 1$ and 0.5 have been assigned; families of curves of the fluid properties are presented in Appendix B.

The method employed to solve Equation (42) uses Simpson's rule (3). The interval of the Mach number chosen is 0.01. With the initial value of U, the slope of U at the first point is found by substituting U_1 into Equation (42). Since each subdivision of the Mach number is small, it is possible to assume that the slope of U between U_1 and U_2 is constant, thus U_2 is given by

$$U_2^1 = U_1 + \Delta M U_1' \tag{43}$$

as a first approximation. Here, the superscript denotes the number of the iteration. Substitution of U_2^1 into Equation (42) determines the slope of U at the second point. Using the mean value theory (4), the second approximation of U at the second point is given by

$$U_2^2 = U_1 + \frac{\Delta M}{2}(U_2^{1'} + U_1') \tag{44}$$

where ΔM is the subdivision of the Mach number.

Substituting Equation (44) into (42) gives an improved value for the slope at the second point. Repeating this process gives more and more accurate values for U and U' at the second point. Following the same procedure, it is possible to obtain values of U and U' at each succeeding point within the interval under consideration.

Since the value of U' at each point has been determined, Simpson's rule is used for numerical integration to find highly accurate values of U. Simpson's rule is given by:

For even points $\quad U_i^n = U_i^{n-1} + \frac{\Delta M}{12}(5U_{i-1}^{n-1'} + 8U_i^{n-1'} - U_{i+1}^{n-1'})$

For odd points $\quad U_i^n = U_i^{n-1} + \frac{\Delta M}{12}(5U_i^{n-1'} + 8U_{i-1}^{n-1'} - U_{i-2}^{n-1'})$

If we substitute the values of U obtained by Simpson's rule into Equation (42) and integrate again, we may get even better results. Such an operation can be done many times, depending on how accruately we would like to determine the value of U. As a matter of fact, computer results show that no further improvement in U can be made with more than three iterations of integration by Simpson's rule and three iterations of the previous approximation method.

Once U has been found, r is determined through the relation given by Equation (38). Simpson's rule is used to integrate Equation (26) for the friction-distance parameter, $\frac{4\bar{f}}{D}L$. The total temperature ratio is determined by Equation (36), since $T_w - T_o$ is equal to a constant for the case of constant heat flux. Therefore, from Equation (36)

$$\frac{T_o}{T_{o1}} = \frac{U_1}{U} \tag{45}$$

After the total temperature ratio is determined, the rest of the fluid properties can be obtained by the following relations:

$$\frac{T}{T_1} = \frac{T_o}{T_{o1}} \frac{K_1}{K} \tag{46}$$

$$\frac{V}{V_1} = \frac{M}{M_1}\sqrt{\frac{T}{T_1}} \tag{47}$$

$$\frac{P}{P_1} = \frac{V_1}{V} \frac{T}{T_1} \tag{48}$$

$$\frac{P_o}{P_{o1}} = \frac{P}{P_1} \left(\frac{K}{K_1}\right)^{\frac{k}{k-1}} \tag{49}$$

$$S - S_1 = c_p \ Ln\frac{T}{T_1} - \frac{R}{J} \ Ln\frac{P}{P_1} \tag{50}$$

Case 2. Constant Wall Temperature

An example of this type of heating can be found in condensers where tubes with good conductivity are surrounded by condensing steam.

As in the case of constant heat flux, Reynolds' analogy is assumed to be valid and the recovery factor is taken as unity. In order to find an expression for r in terms of the Mach number only, we introduce a new function of the Mach number such that

$$L(M) = \frac{T_w - T_o}{T_o} \tag{51}$$

Therefore, Equation (32) can be written as

$$\frac{dT_o}{L(M)\ T_o} = \frac{2f}{D}\, dx \tag{52}$$

With Equations (23) and (26), Equation (52) becomes

$$\frac{1}{L(M)} = \frac{\frac{r(1-\eta)}{k\eta} \quad \frac{1}{\eta[(1+r)+(k-r)\eta]}}{\frac{2}{\eta[(1+r)+(k-r)\eta]} - \frac{k-1}{K} - \frac{1}{\eta}}$$

or

$$\frac{1}{L} = \frac{rK}{2k\eta(1-r)}$$

Solving for r, we have

$$r = \frac{2k\eta}{KL + 2k\eta} \tag{53}$$

Equation (51) can be written as

$$T_o(L + 1) = T_w \tag{54}$$

Taking logarithmic differentiation with respect to M on both sides of Equation (54) results in

$$\left(\frac{dT_o}{T_o}\right)/dM = -\frac{L'}{L+1} \tag{55}$$

With Equation (23), Equation (55) becomes

$$\frac{4}{M[(1 + r) + (k - r)\eta]} - \frac{4}{MK} = -\frac{L'}{L + 1}$$

where $L' = \frac{dL}{dM}$. Solving the above equation for r, we have

$$r = \frac{4(1 - \eta)(L + 1) + MK(k\eta + 1)L'}{(1 - \eta)[4(L + 1) - MKL']} \tag{56}$$

Equations (53) and (56) represent the same r for the case of constant wall temperature, and they are equal to each other. Thus, equating Equations (53) and (56), there results the differential equation for L

$$\frac{4(1 - \eta)(L + 1) + MK(k\eta + 1)L'}{(1 - \eta)[4(L + 1) - MKL']} = \frac{2k\eta}{KL + 2k\eta} \ .$$

After simplication and rearrangement, it becomes

$$MK(k\eta + 1)LL' + 2k\eta MKL' + 4(1 - \eta)(L + 1)L = 0 \tag{57}$$

As in the previous case, the general solution of Equation (57) can not be given in closed form. Therefore, a numerical method is necessary for obtaining values of L. If we take a closer look at Equation (57), it is found that Equation (57) has a form similar to that for the case of constant heat flux. In Equation (51), initial pipe section for parametric values of L_1 were chosen as 4, 3, 2, 1, and 0.5 for numerical integration process.

Solving for L', Equation (57) gives

$$L' = \frac{4(\eta - 1)(L + 1)L}{MK(k\eta + 1)L + 2k\eta MK} \tag{58}$$

The method of numerical integration mentioned previously is used.

With zero as a starting value for the friction-distance parameter, the numerical integration formular is given by Simpson's rule

For odd points $$F_i^n = F_i^{n-1} + \frac{\Delta M}{12}(5F_{i-1}^{n-1'} + 8F_i^{n-1'} - F_{i+1}^{n-1'}) \qquad (59)$$

For even points $$F_i^n = F_i^{n-1} + \frac{\Delta M}{12}(5F_i^{n-1'} + 8F_{i-1}^{n-1'} - F_{i-2}^{n-1'}) \qquad (60)$$

Since the wall temperature is a constant, the total temperature ratio is given by

$$\frac{T_o}{T_{o1}} = \frac{L_1 + 1}{L + 1} \qquad (61)$$

After the total temperature ratio is determined, the rest of the fluid properties can be found by substituting Equation (61) into (46) and by using the relations from Equations (47) to (50).

Case 3. Constant Ratio of Wall Temperature to Total Temperature

In the present case, it is assumed that the wall temperature is given by the relation

$$T_w = C_7 T_o \qquad (62)$$

where C_7 is a constant.

Again, Reynolds' analogy is assumed to be valid and the recovery factor is taken as unity. Thus Equation (32) can be written as

$$\frac{dT_o}{(C_7 - 1)} = \frac{2f}{D}\,dx \qquad (63)$$

From Equations (34) and (35)

$$\left(\frac{dT_o}{T_o}\right)/\left(\frac{2f}{D}\,dx\right) = \frac{2k\eta(1 - r)}{rK}$$

or

$$\frac{2k\eta(1 - r)}{rK} = (C_7 - 1) \qquad (64)$$

Solving for r, we have

$$r = \frac{2k\eta}{(C_7 - 1)K + 2k\eta} \tag{65}$$

and, if we let $C_8 = C_7 - 1$,

$$r = \frac{2k\eta}{C_8 K + 2k\eta} \tag{66}$$

It may be seen from Equation (63), that the plotting of T_o/T_{o1} versus the friction-distance parameter, $4\bar{f}x/D$ on a semi-logarithmic paper gives a straight line. From Equation (13) r has the value of unity for the Fanno-line process. Thus from Equation (66), we have $C_8 = 0$ or $C_7 = 1$, we find that the wall temperature is equal to the total temperature, as seen from Equation (62). The physical meaning of $T_w = T_o$ implies that the wall of the passage is insulated or implies an adiabatic process, which is the case of the Fanno-line process.

Next we want to examine what the value of C_7 would be for the Rayleigh-line process. It is found from Equation (66) that $C_8 = \infty$ or $C_7 = \infty$ with $r = 0$. From Equation (62), we have $T_w = \infty$, which is imposaible. On the other hand, we know that the recovery factor is defined as the ratio of the frictional temperature increase of the wall to that due to adiabatic compression, or

$$\text{R.F.} = \frac{2(T_w - T)}{T(k - 1)\eta} \tag{67}$$

As the Rayleigh-line process deals with no friction, Equation (67) implies the recovery factor equals zero. As we recall, the analysis of the present case is under the assumption that the recovery factor is taken as unity. Thus we may conclude when the wall temperature is proportional to the total temperature, friction must present.

DISCUSSION

It is known from the Fanno-line process that the flow is "choked" at sonic velocity. The behavior of choking for the Fanno-line process is characteristic of a reduction in mass flow rate if an additional length of the flow passage is added to the section where sonic velocity has been reached. The phenomenon of choking also exists in Rayleigh-line process when sonic velocity has been reached; an increase in the heat flow will cause the mass flow rate to decrease. As for the present case when the flow is under the simultaneous effects of friction and heat transfer, it is desirable to study whether such a phenomenon exists. In order to do so, we start with the basic equations which are given below.

$$M^2 = \eta = \frac{V^2}{kRT} \tag{4a}$$

$$\frac{dV}{V} + \frac{d\rho}{\rho} = 0 \tag{2}$$

$$kM^2 \frac{dV}{V} + \frac{dP}{P} = - \frac{kM^2}{2} \frac{4f}{D} dx \equiv - \frac{\rho \, dZ}{P} \tag{7}$$

$$(k - 1)M^2 \frac{dV}{V} + \frac{dT}{T} = \frac{dQ}{c_pT} \tag{5}$$

$$- \frac{dP}{P} + \frac{dT}{T} + \frac{d\rho}{\rho} = 0 \tag{15}$$

From Equation (4a)

$$\frac{dV}{V} = \tfrac{1}{2}\left(\frac{d\eta}{\eta} + \frac{dT}{T}\right) \tag{68}$$

From Equations (5) and (68)

$$\frac{(k-1)\eta}{2} \frac{d\eta}{\eta} + \left[1 + \frac{(k-1)}{2}\right] \frac{dT}{T} = \frac{dQ}{c_pT} \equiv d\theta \tag{69}$$

From Equations (15) and (68)

$$\tfrac{1}{2} \frac{d\eta}{\eta} + \frac{dP}{P} - \frac{dT}{2T} = 0 \tag{70}$$

From Equations (7) and (68)

$$-\frac{k\eta}{2}\frac{d\eta}{\eta} - \frac{dP}{P} - \frac{k\eta}{2}\frac{dT}{T} - \frac{dZ}{RT} = d\mu \qquad (71)$$

where the dimensionless quantities $d\theta$, and $d\mu$ have been introduced to simpligy the following analysis.

If the determinant formed by the coefficients of $d\eta/\eta$, dP/P and dT/T in Equations (69), (70) and (71) is not identically zero, the equations may be solved uniquely for these three differentials. The solution is obtained as follows.

$$d\eta/\eta = (1 - \eta)^{-1}\left[(1 + k\eta)d\theta + Kd\mu\right] \qquad (72)$$

$$dP/P = (1 - \eta)^{-1}\left\{-k\eta d\theta - \left[1 + (k - 1)\eta\right] d\mu\right\} \qquad (73)$$

$$dT/T = (1 - \eta)^{-1}\left[(1 - k\eta)d\theta - (k - 1)\eta d\mu\right] \qquad (74)$$

Equations (72) to (74) can be rewritten as

$$\eta' = \frac{\eta}{1 - \eta}\frac{1 + k\eta}{c_pT}Q' + \frac{\eta}{1 - \eta}\frac{2 + (k - 1)\eta}{RT}Z' \qquad (75)$$

$$P' = -\frac{P}{1 - \eta}\frac{k\eta}{c_pT}Q' - \frac{P}{1 - \eta}\frac{1 + (k - 1)\eta}{RT}Z' \qquad (76)$$

$$T' = \frac{T}{1 - \eta}\frac{1 - k\eta}{c_pT}Q' - \frac{T}{1 - \eta}\frac{(k - 1)\eta}{RT}Z' \qquad (77)$$

where the primes indicate differentiation with respect to x. Solutions of this system of equations exist at all values of η, except $\eta = 1$.

The Phenomenon of Choking. The general Equations (69) to (71) impose restrictions on the relations among the flow variables, and Q and Z. When these restrictions take the form of upper or lower limits on the value of the Mach number, the associated phenomena are termed "choking" processes. As an exaple, it is well known that the ideal nozzle has for given subsonic entry conditions a maximum mass flow beyond which the discharge can not be increased no matter how much the exit pressure is lowered.

The nature of choking may be studied with the help of Equation (75), whioh was derived simply from the basic equations. It will be shown that unless heat and friction variations are such that $(1-\eta)$ times the right-hand side of Equation (75) changes from positive to negative as x increases, the Mach number in the tube cannot become greater than unity if the entrance velocity is subsonic and cannot become less than unity if the entrance velocity is supersonic, provided that the flow variables remain continuous.

For convenience, designate by Y the factor $(1-\eta)$ times the right-hand side of Equation (75). The quantity Y is seen to consist of a sum of terms in Q' and Z' multiplied by functions that are always positive.

Suppose now that Y is always negative. Then, if the entering flow at x_1 is subsonic, $d\eta/dx = Y/(1-\eta) < 0$, and the Mach number decreases; if the entering flow is supersonic, $d\eta/dx = Y/(1-\eta) > 0$, and the Mach number increases.

Suppose now that Y is always positive. Then, if the entering flow at x_1 is subsonic, $d\eta/dx = Y/(1-\eta) > 0$, and the Mach number increases. But η cannot increase past unity as x increases. For suppose $\eta=1$ at $x=x_o$ and is greater than unity in the right-hand neighborhood of x_o (exclusive of x_o); then $d\eta/dx$ is negative in this neighborhood, because $(1-\eta)$ is less than zero and Y is greater than zero. Now η is equal to unity at $x = x_o$, is continuous, and has a negative derivative in the neighborhood mentioned. Hence η is less than unity in this neighborhood, which contradicts the assumption. Therefore, η cannot be greater than unity if Y is always positive, η is continuous, and $\eta(x_1)$ is less than unity. In general, a non-continuous solution exists for values of $x = x_o$ if Y is always positive. This statement, and the foregoing proof, are valid even if η' at $x = x_o$ does not exist. An analogous development may be made for $\eta(x_1)$

greater than unity with the conclusion that, with Y positive and η continuous, η cannot be less than unity.

If Y changes from positive to negative at $x = x_0$, however, the value of η will cross unity at that point, but if Y is initially negative, η goes away from unity as previously shown and can only turn toward unity if Y changes from negative to positive; this change must be made at some value of η other than unity. After Y has changed to positive, the situation reduces to the case that Y is always positive, (described by the paragraph beginning on line 10, page 20) in which the η where Y changes sign is now taken as the entrance η.

It has been shown that up to some fixed point in the tube, which is either the exit or the point at which Y changes from positive to negative, η and the Mach number do not become greater than unity if the entering velocity is subsonic, nor less than unity if the entering velocity is supersonic. Furthermore, if Y is positive up to the point at which η is limited, the derivative of η before this point is always positive if the entering flow is subsonic, and is always negative if the entering flow is eupersonic. Thus, for positive Y and subsonic entrance velocity η cannot exceed the limiting value of unity, for η is alwaye increasing from ite initial value and cannot exceed unity, by analogous considerations for positive Y and supersonic entrance velocity η cannot be less than eome limiting value greater than unity. This limitation is essentially the choking phenomenon.

CONCLUSION

The flow of steady, one-dimensional compressible fluid in constant-area passages under the simultaneous effects of friction and heat addition was investigated. A function r denoting the proportionality of the two effects was introduced. Making use of the principles of conservation of mass, energy and momentum, as well as the equation of state and the definition of the Mach number, it was possible to express the fluid properties in terms of the Mach number and the function r in logarithmic differential forms.

In order to simplify the investigation, a friction-distance parameter was defined as the mean wall friction over the length of the flow passage under consideration. With this parameter, the nature of the local wall friction could be ignored; therefore, the whole problem was dominated by the nature of heating process. Thus, the specification of the heating process is essential to the determination of the function r as well as to the integration of the fluid properties.

Three particular cases of subsonic heating, i.e., constant heat flux, constant wall temperature, and constant ratio of wall temperature to total temperature were surveyed under the assumptions of the validity of Reynolds' analogy and the recovery factor equal to unity. Through the physical and mathematical meaning of the individual problem, it was possible to determine the function r. Thus, the logarithmic differentiation of the fluid properties could be integrated, since r was determined. Owing to the fact that the function r could not be represented in an exact form, Simpson's rule was used for numerical integration. Arbitrary Mach numbers of 0.1 and 0.4 were taken as the initial conditions, and the initial ratio of wall temperature to the total fluid temperature was chosen as a parameter and a starting value for numerical integration.

The changes of r with Mach number are shown in Figures 1-a through 1-c. It is found that r increases with the Mach number in the subsonic region, and that the rate of increase of r decreases with increase in

Mach number as M approaches unity. Physically, this means that the frictional effect is less important in comparison with that of heat addition at low Mach numbers and becomes more important at higher Mach numbers.

It was found that all the fluid properties change very rapidly as choking is being reached, i.e., as M approaches unity. Pressure loss is very noticeable at high Mach numbers and also for high ratios of T_{w1}/T_{o1}. The frictional effect becomes less significant as the amount of heat addition is increased.

Finally, an attempt to solve for the function r in an exact form was made by the writer, but without success. He hopes that it will be investigated further by those who are interested in this subject.

ACKNOWLEDGMENT

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APPENDICES

APPENDIX A

Derivation of n_F and n_R

Consider an expansion process of a perfect gas. If the process is reversible, we know from thermodynamics that the work done by a unit pound of the fluid is given by

$$d\,W = P\,dv/J \tag{A-1}$$

But in practice no process is reversible, therefore the actual work output must be less than that of the ideal. For this reason we define a fraction factor as the ratio of the actual work over the ideal work.

$$e_e = dW_{act}/dW \tag{A-2}$$

where

$$0 \leq e_e \leq 1$$

Next we introduce a thermal efficiency factor which is given by

$$\phi = e_e\left(\frac{Pdv}{J}\right)/dQ \tag{A-3}$$

or

$$dQ = \frac{e_e Pdv/J}{\phi} \tag{A-4}$$

Consider an irreversible diabatic expansion process of a perfect gas. From the first law of thermodynamics

$$dQ = du + dW$$

With (A-2) and (A-4), it can be written as

$$\frac{e_e Pdv/J}{\phi} = c_v dT + \frac{e_e Pdv}{J} \tag{A-5}$$

Tho differential of the equation of state gives

$$RdT = Pdv + vdP \tag{A-6}$$

Combining equations (A-5) and (A-6)

$$\frac{e_e Pdv}{\phi} = \frac{Pdv + vdP}{k - 1} + e_e Pdv \tag{A-7}$$

After rearrangement, we have

$$vdP + e_e(k - 1)(1 - 1/\phi)Pdv + 1 = 0 \tag{A-8}$$

Let

$$n = 1 + e_e(k - 1)(1 - 1/\phi) \tag{A-9}$$

Therefore, Equation (A-8) becomes

$$vdP + nPdv = 0 \tag{A-10}$$

If n is a constant, integration of Equation (A-10) gives

$$Pv^n = \text{constant}$$

Rayleigh Line. This is a reversible diabatic process for which $e_e = 1$. Thus, Equation (A-9) gives

$$n_R = 1 + (k - 1)(1 - 1/\phi) \tag{A-11}$$

Solving (A-7) for ϕ, with $e_e = 1$, we have

$$\phi = \frac{k - 1}{vdP/Pdv + k} \tag{A-12}$$

Since $\frac{dV}{V} = \frac{dv}{v}$, with r = o, Equations (20) and (22) become

$$\frac{dv}{v} = \frac{dM^2}{M^2(1 + kM^2)} \tag{A-13}$$

$$\frac{dP}{P} = \frac{-kdM^2}{1 + kM^2} \tag{A-14}$$

Dividing Equation (A-14) by (A-13) yields

$$vdP/Pdv = -kM^2 \tag{A-15}$$

Substituting (A-15) into (A-12), we have

$$\phi = \frac{k - 1}{k(1 - M^2)} \tag{A-16}$$

Equations (A-11) and (A-16) give

$$n_R = kM^2 \tag{A-17}$$

Fanno Line . This is known as an irreversible adiabatic process for which $\phi = \infty$, or $1/\phi = 0$. Thus, Equation (A-9) gives

$$n_F = 1 + e_e(k - 1) \tag{A-18}$$

Solving Equation (A-7) for e_e, with $1/\phi = 0$, we have

$$e_e = -\frac{1 + vdP/Pdv}{k - 1} \tag{A-19}$$

With $r \doteq 1$, Equations (20) and (22) become

$$\frac{dv}{v} = \frac{dM^2}{2M^2\left(1 + \frac{k-1}{2} M^2\right)} \tag{A-20}$$

$$\frac{dP}{P} = -\frac{1 + (k-1)M^2}{2M^2\left(1 + \frac{k-1}{2} M^2\right)} dM^2 \tag{A-21}$$

Dividing (A-21) by (A-20), we have

$$vdP/Pdv = -\left[1 + (k-1)M^2\right] \tag{A-22}$$

Substituting (A-22) into (A-19) yields

$$e_e = M^2 \tag{A-23}$$

Equations (A-18) and (A-23) give

$$n_F = 1 + (k-1)M^2 \tag{A-24}$$

APPENDIX B

(GRAPHS)

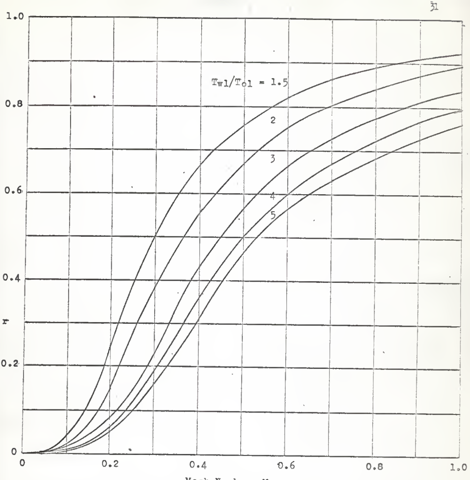

Fig. 1-a. r versus Mach number --- Combined friction and heat transfer in a constant-area passage for the case of constant heat flux, with k = 1.4, R.F. =1, and M_1 = 0.1.

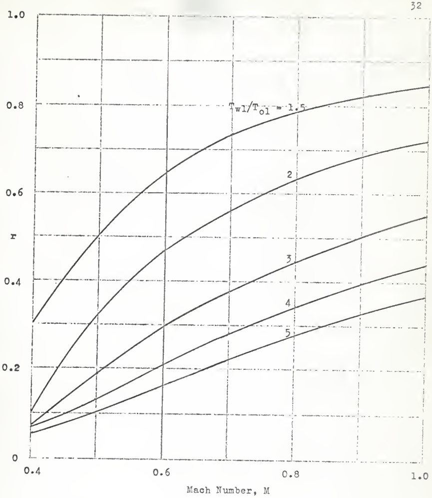

Fig. 1-b. r versus Mach number --- Combined friction and heat transfer in a constant-area passage, with k = 1.4, R.F. = 1, and $M_1 = 0.4$, for the case of constant wall temperature.

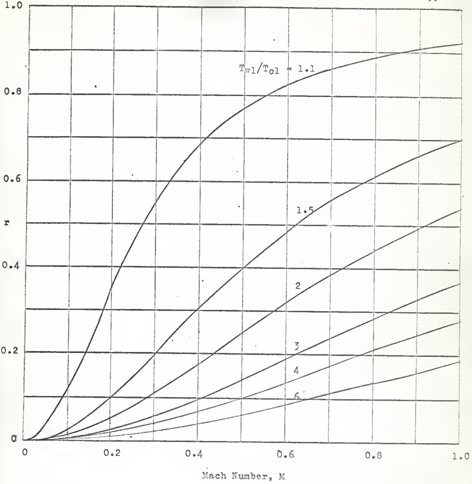

Fig. 1-c. r versus Mach number.--- Combined friction and heat transfer in a constant-area passage for the case of constant ratio of the wall temperature to the total temperature, with $k = 1.4$, R.F. = 1, and $M_1 = 0.1$.

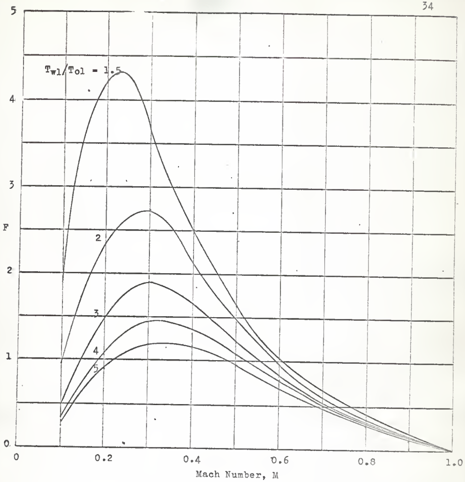

Fig. 2-a. F versus Mach number --- Combined friction and heat transfer in a constant-area passage for the case of constant heat flux, with k = 1.4, R.F. = 1, and M_1 = 0.1

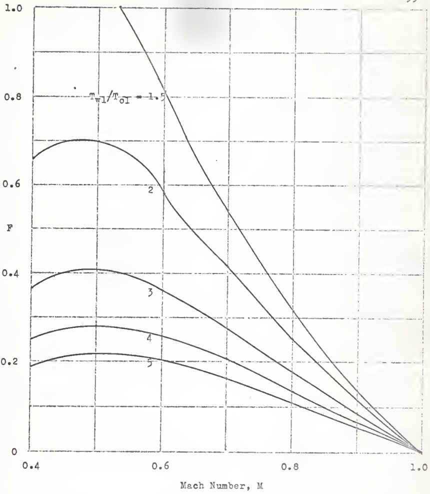

Fig. 2-b. F versus Mach number --- Combined friction and heat transfer in a constant-area passage for the case of constant wall temperature, with k = 1.4, R.F. = 1, and $M_1 = 0.4$

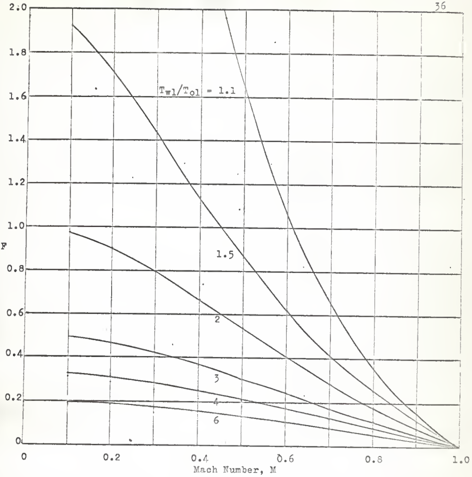

Fig. 2-c. F versus Mach number --- Combined friction and heat transfer in a constant-area passage for the case of constant ratio of the wall temperature to the total temperature, with $k = 1.4$, R.F. = 1, and $M_1 = 0.1$.

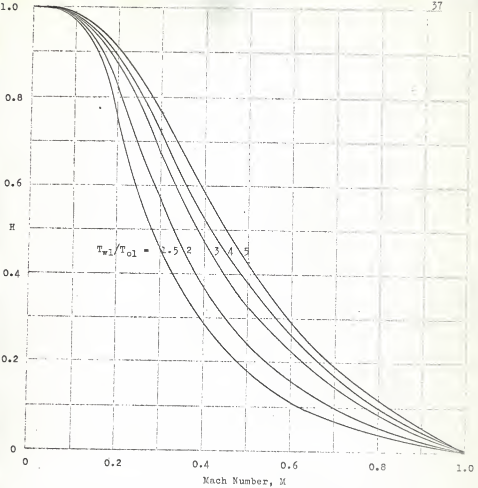

Fig. 3-a. H versus Mach number --- Combined friction and heat transfer in a constant-area passage for the case of constant heat flux, with $k = 1.4$, R.F. = 1, and $M_1 = 0.1$.

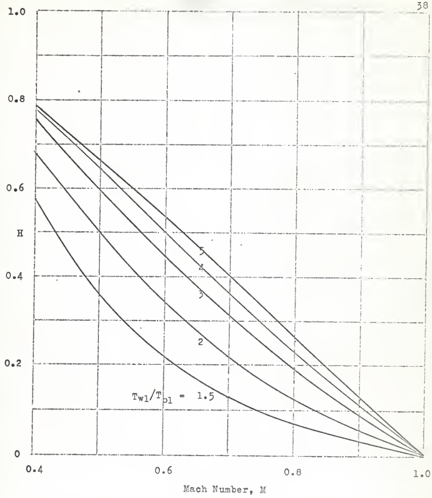

Fig. 3-b. H versus Mach number --- Combined friction and heat transfer in a constant-area passage for the case of constant wall temperature, with $k = 1.4$, R.F. = 1, and $M_1 = 0.4$.

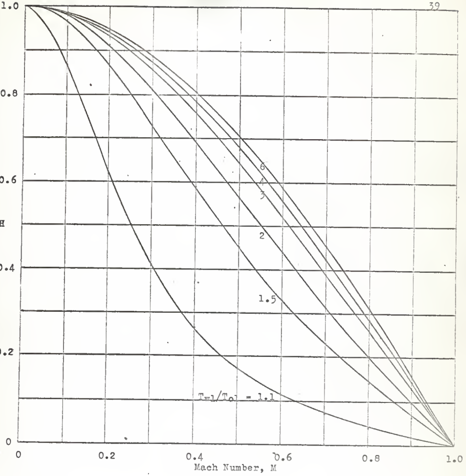

Fig. 3-c. H versus Mach number --- Combined friction and heat transfer in a constant-area passage for the case of constant ratio of the wall temperature to the total temperature, with k = 1.4, R.F. = 1, and M_1 = 0.1.

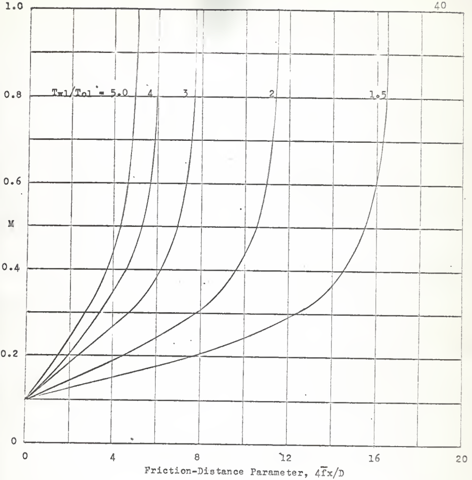

Fig. 4-a. Mach number versus distance --- Combined friction and heat transfer in a constant-area paesage for the case of constant heat flux, with k = 1.4, R.F. = 1, and M_1 =.1.

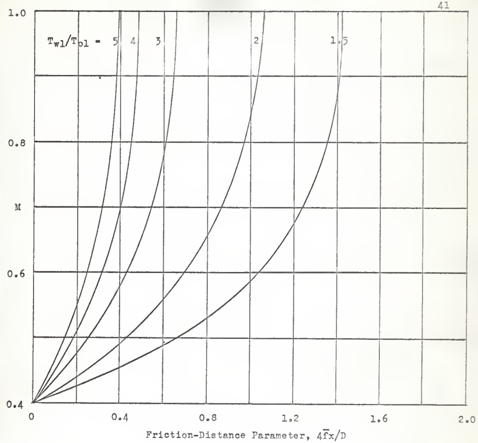

Fig. 4-b. Mach number versus distance --- Combined friction and heat transfer in a constant-area passage for the case of constant wall temperature, with $k = 1.4$, R.F. = 1, and $M_1 = 0.4$.

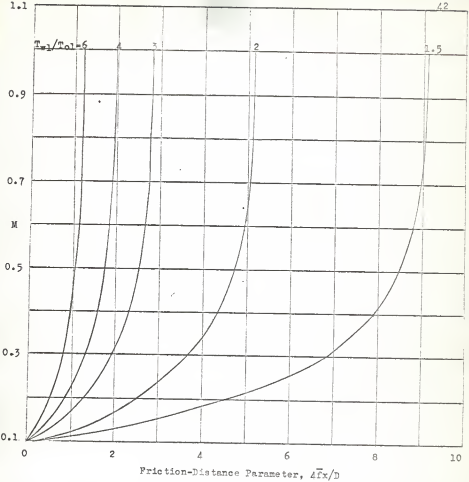

Fig. 4-c. Mach number versus distance --- Combined friction and heat transfer in a constant-area passage for the case of constant ratio of the wall temperature to the total temperature, with k = 1.4, R.F. = 1, and M_1 = 0.1.

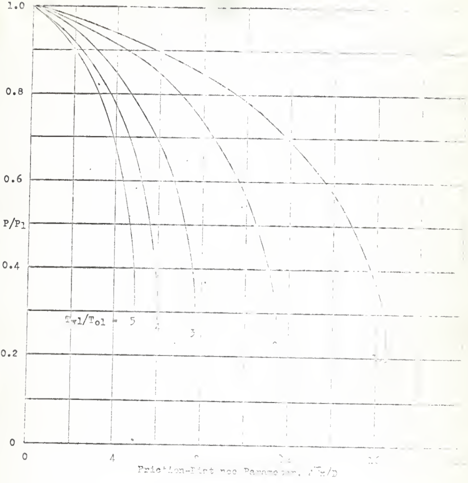

Fig. 5-a. Static pressure ratio versus distance --- Combined friction and heat transfer in a constant-area passage [illegible] of constant heat flux, with k = 1.4, [illegible] = 1, [illegible]

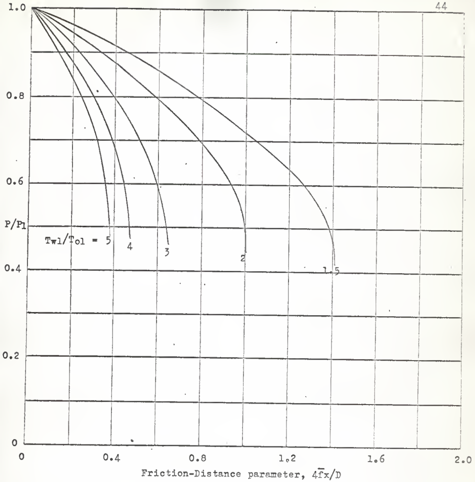

Fig. 5-b. Static pressure ratio versus distance --- Combined friction and heat transfer in a constant-area passage for the case of constant wall temperature, with $k = 1.4$, R.F. = 1, and $M_1 = 0.4$.

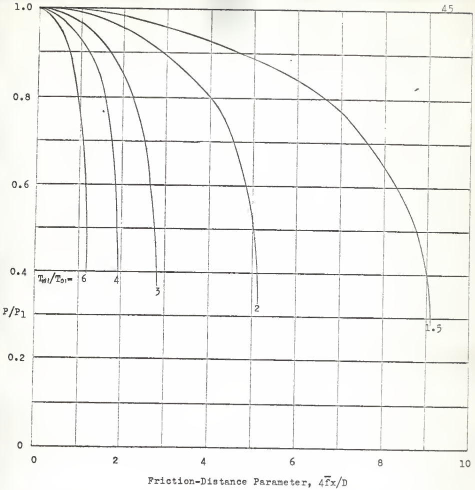

Fig. 6-o. Static pressure ratio versus distance --- Combined friction and heat transfer in a constant-area passage for the case of constant ratio of the wall temperature to the total temperature, with $k = 1.4$, R.F. = 1, and $M_1 = 0.1$.

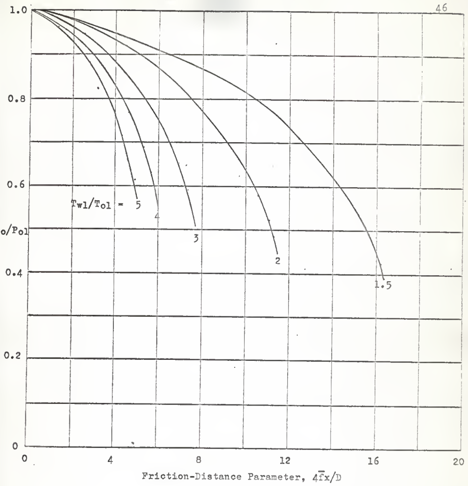

Fig. 6-a. Total pressure ratio versus distance --- Combined friction and heat transfer in a constant-area passage for the case of constant heat flux, with $k = 1.4$, R.F. = 1, and $M_1 = 0.1$.

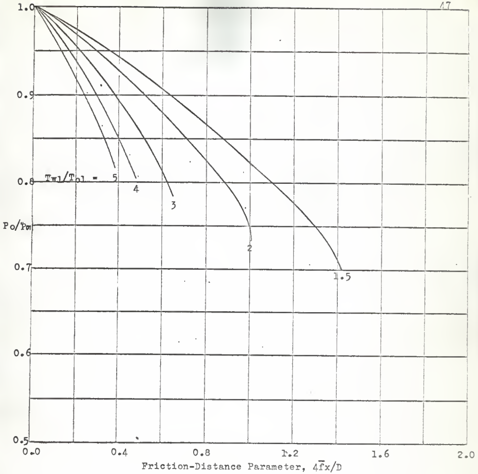

Fig. 6-b. Total pressure ratio versus distance --- Combined friction and heat transfer in a constant-area passage for the case of constant wall temperature, with k = 1.4, R.F. = 1, and M_1 = 0.4.

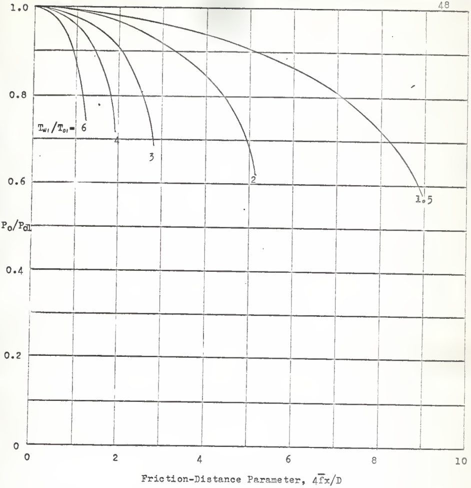

Fig. 6-c. Total pressure ratio versus distance --- Combined friction and heat transfer in a constant-area passage for the case of constant ratio of the wall temperature to the total temperature, with k = 1.4, R.F. =1, and M_1 = 0.1.

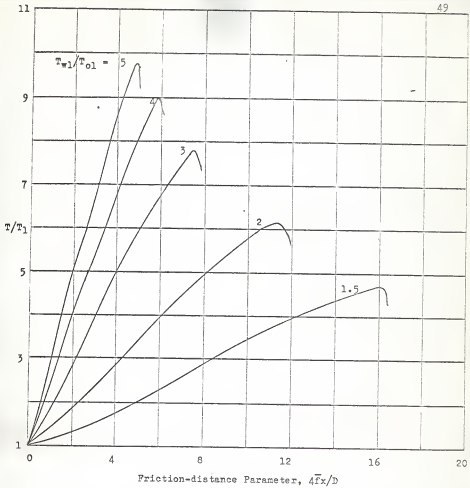

Fig. 7-a. Static temperature ratio versus distance --- Combined friction and heat transfer in a constant-area passage for the case of constant heat flux, with $k = 1.4$, R.F. = 1, and $M_1 = 0.1$.

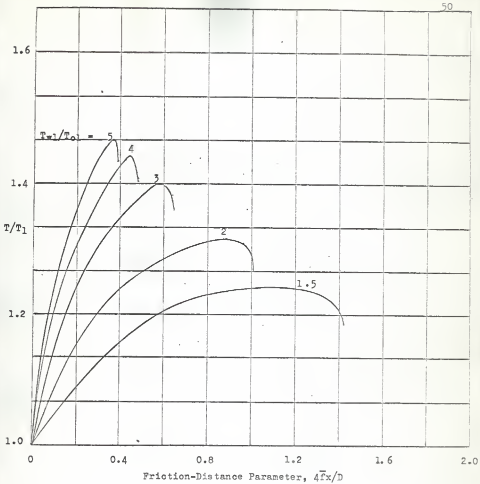

Fig. 7-b. Static temperature ratio versus distance --- Combined friction and heat transfer in a constant-area passage for the case of constant wall temperature, with $k = 1.4$, R.F. = 1, and $M_1 = 0.4$.

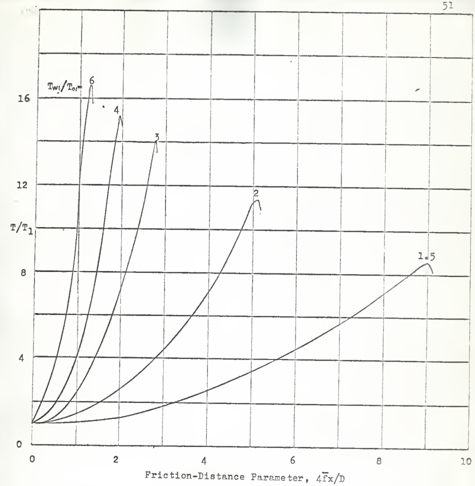

Fig. 7-c. Static temperature ratio versus distance --- Combined friction and heat transfer in a constant-area passage for the case of constant ratio of the wall temperature to the total temperature, with k = 1.4, R.F. = 1, and M_1 = 0.1.

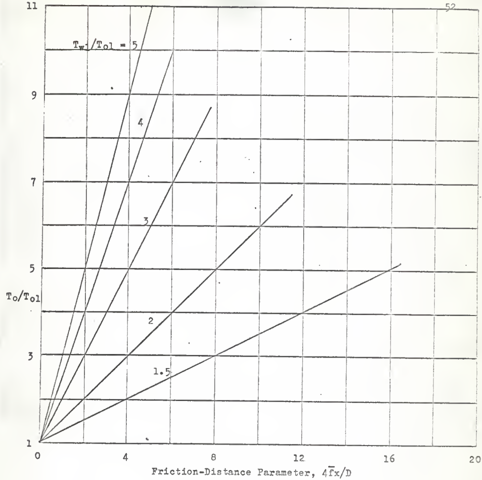

Fig. 8-a. Total temperature ratio versus distance --- Combined friction and heat transfer in a constant-area passage for the case of constant heat flux, with k = 1.4, R.F. = 1, and M_1 = 0.1.

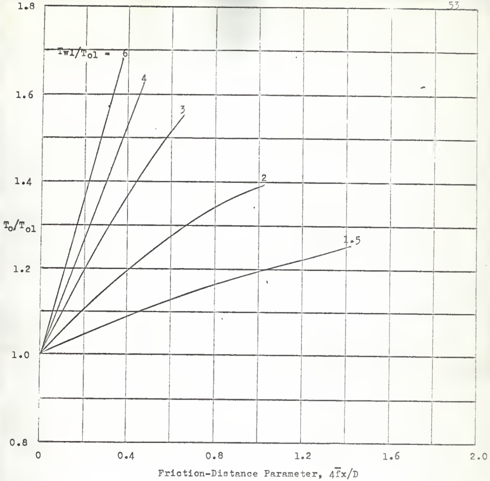

Fig. 8-b. Total temperature ratio versus distance --- Combined friction and heat transfer in a constant-area passage for the case of constant wall temperature, with $k = 1.4$, R.F. = 1, and $M_1 = 0.4$.

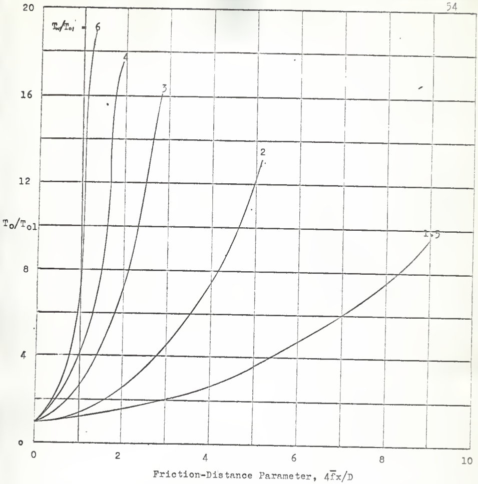

Fig. 8-c. Total temperature ratio versus distance --- Combined friction and heat transfer in a constant-area passage for the case of constant ratio of the wall temperature to the total temperature, with k = 1.4, R.F. = 1, and M_1 = 0.1.

COMBINED FANNO-LINE, RAYLEIGH-LINE HEAT TRANSFER AND FRICTION EFFECTS ON STEADY ONE-DIMENSIONAL GAS FLOW IN CONSTANT-AREA PASSAGES

by

WILBUR PO-SAM HSIEH

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The purpose of this report is to investigate the combined effects of friction and heat transfer on the fluid properties of steady, one-dimensional compressible flow in constant-area passages. A new function of the Mach number $r(M)$ is introduced which denotes the proportionality of the two effects. When the flow process is effected only by friction, r takes the value unity all through the process and thus becomes the Fanno-line process. When the flow process is effected by heat transfer only, r takes the value zero all through the process and thus becomes the Rayleigh-line process.

In order to simplify the investigation, a friction-distance parameter is introduced which is defined as the mean value of the wall friction over the length of the flow passage under consideration. With the friction-distance parameter, the nature of the local wall friction can be ignored, and, the nature or the form of heat transfer dominates the problem.

Making use of the principles of conservation of mass, energy and momentum, as well as the thermal equation of state and the definition of the Mach number, it is possible to express the fluid properties in terms of the Mach number and the function r in logarithmic differential forms. The function r must be determined before we can carry out the integration for the fluid properties. The determination of r is impossible unless the heat addition process has been specified, because for different forms of heat addition will result in different funcions of r. As an illustration of the applicability of this new method to combine the two effects, three special forms of heat transfer were investigated, namely, constant heat flux, constant wall temperature and constant ratio of the wall temperature to the total temperature.

For the three cases, it is assumed that the recovery factor takes the value unity and the Reynolds' analogy remains valid. For the first and the second cases, another function of the Mach number is introduced which is defined as the ratio of the difference between the wall and total temperature to the total temperature. Through the physical meaning of the individual problem, it is possible to obtain a differential equation for this function. The author was not able to obtain a general solution of this function in a closed form. However, numerical methods are effective after a starting value is specified. The numerical method involved is divided into two steps. In the first step, "successive improvement" is used. The approximate value of the function is used for the next step in which Simpson's rule is used for numerical integration. With this procedure of numerical approximation, it is possible to obtain values of the function as accurate as desired. After the new function has been solved, r can be determined. With the values of r, the fluid properties can be solved by using Simpson's rule for numerical integration. For the third case, a very simple relation between r and the Mach number is obtained, and the fluid properties are determined by the same way mentioned above.

Families of curves of the fluid properties are presented in Appendix B. Arbitrary initial Mach numbers of 0.1 and 0.4 are used. It may be seen from the plotting of r versus the Mach number that the value of r increased with the Mach number in the subsonic region, and finally tends to certain value. Physically, this means that the frictional effect is less important compared with that of heat addition near the initial section. As the Mach number increases, the frictional effect becomes more important and finally tends to a certain ratio with the heating effect up to a Mach number of unity.