STRESS ANALYSIS OF CABLE STIFFENED STRUCTURES

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SYNOPSIS

This report presents two possible methods of structural analysis for long span cable-stiffened structures.

The first method is to analyze the cable stiffened structure by the principle of superposition, in which the first step is to reduce the structure to be a statical determinate or statical indeterminate in minimum degrees base structure and put all the redundants as external forces to be solved. In this analysis, redundants are the cable stresses. The second step is to find the deflections of the base structure at each cable attachment due to actual loads and unit loads by the method of moment area integration, based on Castigliano's theorem that the deflection at the cable attachment is the summation of deflection due to actual loads and the unit deflections caused by unit stress on cables times the cable stresses. Therefore, a number of simultaneous equations can be found by the equilibrium of cable stress and deflection at each cable attachment. In order to solve those linear equations, the author has used matrix method analysis and set up computer programs for those moment area integration, deflections and matrix inversions to find cable stresses in a quick way.

The second method of analysis is to assume that the cables work as part of the frame reinforcement and, using the principle of transformed areas, the frame can be interporated as a haunched member with variable moment of inertias. That is, the section modules at each end of the frame is increased due to cable stiffening without increasing concrete quantities. Therefore, the cable stiffened structure can be analyzed by using general haunched member theory. The author used a computer and solved this time consuming calculation problem within five minutes.

Numerical examples are given to illustrate those methods and several computer FORGO programs are written to solve those examples with high accuracy. Another new idea developed by the author is using prestress for a further reduction of end mement of the frame. That is, the cable can be pretensioned in a definite amount for a definite amount of mement to be reduced from the frame. This method has been shown in numerical example 1, which is helpful in actual design.

INTRODUCTION

In any kind of structural design, there are two essential conditions for economy to be considered: the efficient utilization of material, and the maximum reduction of the dead weight of the structure.

For a long spanned rigid frame, the very large corner stresses at the fixed-end require that this section be extremely large. This means not only that the concrete and reinforcing steel shall be increased but also the dead loads of the frame. If the span of the rigid frame is very long, it is almost impossible to construct without providing additional support between the span to reduce the fixed end sectional areas. For an economical design, it is possible to use cable stiffening, because a cable-stiffened structure not only can make a very large space without intermediate support but also can reduce the dead loads as well as amounts of construction materials.

The first developed cable stiffened structures were for the girder bridge stiffening. In the time of World War II, there was an extreme shortage of construction materials to rebuild the German bridges which had been destroyed in the war. So German engineers designed a new type of construction which would be safe, durable, economical in cost and in quantity of construction materials, and pleasing architecturally all at the same time. The application of cable stiffening to architectural structures has been developed only in the last ten or twelve years. The typical example of cable stiffened building in the United States is the Pan American Terminal Building at Kennedy International Airport. The purpose underlying the design is to permit aircraft to come underneath, for maintenance in the case of the hangar, and for embarkation of passengers in the case of this terminal building. In both schemes, cables perform an essential role in enabling the tremendous cantilivers to be built. The silhouette is like a vast umbrella covering the activities of travel. Many other buildings have been constructed in recent years in cable stiffening forms or cable suspended forms to make the structures in good architectural shape and economical in material utilization, which is almost impossible in ordinary construction methods.

In this report, the author has taken a typical long span rigid frame with cable stiffening as an example to show how to analyze the cable stiffened structures. Many possible methods of analysis can be employed in analyzing this type of structure. Here the author introduces two kinds of methods of structural analysis which have not been used previously in the cable stiffened structural analysis.

The first one is to solve the cable stiffened structure by means of the superposition principle, which is generally known as the linear structural analysis and is used for solving high order statically indeterminate structures. In this study, the cable forces are taken as the indeterminate redundants to be solved with the frame treated as a base structure without cable stiffening. The moment on the base structure can be found by moment distribution. The deflections at each cable attachment of the frame can be found by superposing all the deflections at that point, that is, the deflection at any cable attachment of the structure is the summation of deflections on that position due to external loads plus the unit deflection on that position due to unit load on that cable times the cable stress plus the unit deflection on that position due to unit load on the cable at other position times the cable stress. Therefore, for each redundant, there is one linear equation of superposed deflections to be written. For many redundants, there are many simultaneous linear equations to be solved. The results of those simultaneous equations are the cable stresses.

The second method is to solve the cable stiffened structure by means of haunch member theory, because in this long span cable stiffened rigid frame, those cables can be regarded as part of the frame reinforcement. By the method of transformed area, those cables can be transfered to be fictitious concrete sectional area. Therefore, the uniform sectional rigid frame can be treated as a frame with variable sections without increasing concrete quantities. In this study, the horizontal girder of the frame is first divided into many equal interval sections and then the center of gravity is located for each section of the concrete with transformed cable sections. The mement of inertias can be found for each section in respect to its central axis. So a variable moment of inertia can be found due to the variable position and quantities of cables. The next step is to solve the fixed end moments for the fictitious variable sectional rigid frame. The theory and numerical example has been shown in this report. In solving these laborious calculations for each section and for the fixed end moment influence lines, a FORGO program has been provided for solving this type of problem.

The purpose of this report is to show how a structure can be stiffened by cables and what is their advantage. Larger space and lighter weight of construction is needed in modern architectural design. Cable stiffening will enable this kind of structure to be built. The cable stiffened rigid frame can be designed for many purposes, such as the construction of a large spaced field house, auditorium, factory and hangar. For the limitation of time and pages, the design of cable stiffened structure will not be included in this report. The author may continue further study after finishing this report.

ELASTIC PROPERTIES OF CABLES

Cable is used for structural purposes in a variety of ways, such as diagonal bracing in certain types of frames, suspended structural system, and as a stiffening for some structures for the purpose of enlarging the structural space without too much increasing in sectional area and reinforcement.

Since a cable is very flexible and cannot resist bending or compression, the characteristic which governs its effect upon the elastic behavior of a structure into which it is incorporated is the product of its cross section area and the value of Young's modulus in tension, AE, which will be referred to as the extensional rigidity.

For a straight bar, the extensional rigidity AE is readily determinable, but if a cable sagging under its own weight is subject to a pull tending to straighten it, the increase in distance between the two ends depends not only upon the elastic stretch of the cable but also upon the alteration in its configuration due to the change in the dip-span ratio. The effective strain will thus be greater than that calculated from the elastic extension of the cable. In Fig. 1, a cable AB of length L is hanged at point A and B, the vertical distance is h and the horizontal distance is d, and T denotes the tension at each end. If point A can be displaced horizontally by a small amount δd, the tension at any point is increased by δT. Let AE be the extensional rigidity for the material of the cable determined from a straight length, and η AE, the effective extensional rigidity of the equivalent straight cable required to replace the actual cable AB. Then

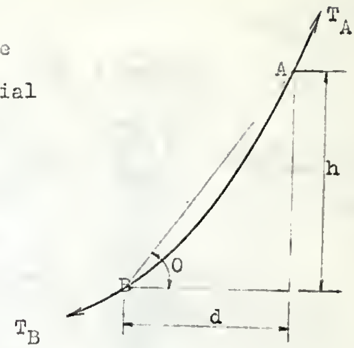


Fig. 1

$$\delta L = \delta d \cos\theta = \frac{L}{\eta AE}\,\delta T_M$$

$$\eta = \frac{L^2}{AEd}\,\frac{\delta T_M}{\delta d} \qquad (1)$$

where T_M denote the tension in the equivalent cable. Let the cable be a constant cross section and total weight W. It will then hang in a column, but the loss of accuracy in assuming the curve to be parabolic is negligible. Take the vertex of the parabola as origin and let the x-coordinates of A and B be $X \pm \frac{1}{2}d$ respectively, The equation for the parabola is then,

$$X^2 = 2ay$$

The constant horizontal tension is

$$H = aw \qquad \text{where } w = W/d \qquad (2)$$

also $T^2 = w^2(a^2 + x^2)$, (3)

and $X = a\tan\theta$ (4)

Where a is given by

$$\frac{a}{d} = \frac{-\sin\theta + \sqrt{4(T_a/W)^2 - \cos^2\theta}}{2\sec\theta}$$

$$= \frac{\sin\theta + \sqrt{4(T_B/W)^2 - \cos^2\theta}}{2\sec\theta} \qquad (5)$$

Since cos θ is small in comparison with any reasonable value of T/W, to a close approximation from (5)

$$T_A - T_B = W\sin\theta$$

$$\tfrac{1}{2}(T_A + T_B) = H\sec\theta = T_M \qquad (6)$$

From (3) and (4) T_M is the actual tension at the point x=X, the length of the cable is

$$S = \tfrac{1}{2}\left\{\frac{(X+\frac{1}{2}d)}{a} \times \frac{T_A}{w} - \frac{(X-\frac{1}{2}d)}{a}\, x.\frac{T_B}{w} + a\log\left(\frac{X+\frac{1}{2}d+\frac{T_A}{w}}{X-\frac{1}{2}d+\frac{T_B}{w}}\right)\right\} \qquad (7)$$

A small change δd in d if h remains contant, gives,

From (3) $\frac{T_M}{w} \cdot \frac{\delta T_M}{w} = X\,\delta X + a\,\delta a$ (8)

From (4) $h\,\delta a = d\,\delta X + X\,\delta d$ (9)

From (7) $\delta S = A\,\delta X + B\,\delta d + C\,\delta a$ (10)

Where $A = \frac{T_A - T_B}{H}$, $B = \frac{T_A + T_B}{2H}$

$$C = \frac{1}{a}(S-AX-Bd) \tag{11}$$

Also $$\delta S = \frac{S}{AE}\delta T_M \tag{12}$$

Eliminating δX and δa from (8) (9) (10) and substituting for δS from (12)

$$\frac{\delta T_M}{w}\left[\frac{T_M}{w}(Ah+Cd) - \frac{wS}{AE}(hX+ad)\right] = \delta d\left[AaX-B(hX+ad) - CX^2\right] \tag{13}$$

The two terms on the left hand side are comparable in size, Ah+Cd and 1/AE both small.

By using the approximation (6) putting S=L

$$\frac{wS}{AE}(hX + ad) = L^2\frac{T_M}{AE}$$

$$AaX - B(hX + ad) - CX^2 = -d\frac{T_M}{w} \tag{14}$$

From (11) $$\frac{Ah+Cd}{d} = \frac{S}{a} - \frac{(T_A+T_B)}{2H} - \frac{d}{a} = \frac{1}{a}\left[\int_{X-\frac{d}{2}}^{X+\frac{d}{2}}\sqrt{(a^2+x^2)}\,dx - \right.$$

$$\left.\frac{d}{2}\sqrt{a^2+(X+\tfrac{1}{2}d)^2} + \sqrt{a^2+(X-\tfrac{1}{2}d)^2}\right]$$

Put $x' = x-$ $a\tan\theta$, than by (4)

$$\frac{Ah+Cd}{d} = \frac{\sec\theta}{a}\left[\int_{-\frac{1}{2}d}^{\frac{1}{2}d}\sqrt{1+\frac{x'(2a\tan\theta + x')}{a^2\sec^2\theta}}\,dx'\right.$$

$$\left.- \tfrac{1}{2}\left(\sqrt{1+\frac{d(d+4a\tan e)}{4a^2\sec^2\theta}} + \sqrt{1+\frac{d(d-4a\tan e}{4a^2\sec^2\theta}}\right)\right]$$

On expanding the right hand side and integrating the expression reduces to the converging series:

$$\frac{Ah+Cd}{a} = -\frac{1}{12}\left(\frac{d\cos\theta}{a}\right)^3 + \frac{(1-5\sin^2\theta)}{160}\left(\frac{d\cos\theta}{a}\right)^5 \qquad (15)$$

Retaining the first term of this series and substituting from this and (14) in (13) that,

$$\frac{\delta T_M}{\delta d} = \frac{W}{\frac{1}{12}\cdot\frac{d^4}{a^3}\cos^3\theta+\frac{wL^2}{AE}} \qquad (16)$$

Hence from (1)

$$\eta = \frac{1}{1+\frac{W^2AE}{12T_M^3}\cos^2\theta} \qquad (17)$$

Also from (16),

$$\delta d = \left(\frac{L\sec\theta}{AE} + \frac{W^2d}{12T_M^3}\right)\delta T_M \qquad (18)$$

and the total displacement of A as the mean tension in the cable increases from T_o to T_M is

$$\Delta = \frac{L\sec\theta}{AE}\,(T_M-T_o) + \frac{W^2d}{24}\left(\frac{1}{T_o^2} - \frac{1}{T_M^2}\right) \qquad (19)$$

Since η depende upon the force in the member, it has a different value for every loading condition of the structure. This variation ought theoretically to be taken into account in calculation, but this would be impracticable. If the load is applied in increments, η being assumed constant during each stage, any required degree of accuracy can be obtained.

To illustrate the importance of accurate control of the initial tension of cable stiffened structures, the case of a beam hanging at four equal points by flexible cables will be considered in some detail. The beam shown in Fig. 2 is a beam of flexural rigidity EI pinned at O, and hanged at A, B, C, D by cables of equal equivalent extensional rigidity AE, with a load w of uniform intensity along the beam.

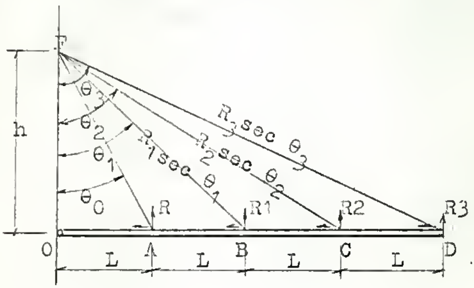


Fig. 2

There are three redundancies which will be taken to be the cables FD, FC, and FB. Let the loads in these cables be R3Secθ_3, R2Secθ_2, R1Secθ_1 respectively, where R1, R2, R3 are the vertical component loads acting perpendicular to the beam. Then by taking moment about O, the corresponding component of FA is 8wL - (2R1 + 3R2 + 4R3). The total strain energy in the structure consists of that due to the bending of the beam and that due to the extension of the cables and will be,

$$\frac{\partial U}{\partial R1} = \frac{\partial U}{\partial R2} = \frac{\partial U}{\partial R3} = 0$$

The contributions to these terms from the bending of the beam are deduced from the bending moment equations,

$$\frac{\partial U}{\partial R1} = \frac{L^3}{EI}\left(\frac{2}{3}R1 + \frac{3}{2}R2 + \frac{7}{3}R3 - \frac{13}{4}wL\right)$$

$$\frac{\partial U}{\partial R2} = \frac{L^3}{EI}\left(\frac{3}{2}R1 + 4R2 + \frac{20}{3}R3 - \frac{103}{12}wL \right)$$

$$\frac{\partial U}{\partial R3} = \frac{L^3}{EI}\left(\frac{7}{3}R1 + \frac{20}{3}R2 + 12R3 - \frac{29}{2}wL \right)$$

The components from the cables are found as follows, taking $\partial U/\partial R1$ as example, it is seen that the unknown R1 occurs in the cable FA and FB. The load in FA is $P = (8wL - 2R1 - 3R2 - 4R3)\mathrm{Sec}\theta_o$, and

$$\left[\frac{\partial U}{\partial R1}\right]_{FA} = \frac{Pl}{\eta AE}\,\frac{\partial P}{\partial R1}$$, where l denotes the length of FA,

that is $h\mathrm{Sec}\theta_o$, also

$$\frac{\partial P}{\partial R1} = -2\mathrm{Sec}\,\theta_o$$

hence $\left[\frac{\partial U}{\partial R1} \right]_{FA} = \frac{2h(2R1+3R2+4R3-8wL)\mathrm{Sec}\,\theta_o}{AE}$

also for FB

$$\left(\frac{\partial U}{\partial R1}\right)_{FB} = \frac{hR1Sec\ \theta_1}{\eta AE}$$

Hence $\left(\frac{\partial U}{\partial R1}\right)_{cables} = \frac{h}{\eta AE}\ 2(2R1+3R2+4R3-8wL)Sec^3\theta+R1Sec^3\theta$

and similarly for the values of $\frac{\partial U}{\partial R2}$ and $\frac{\partial U}{\partial R3}$,

Hence the total value of $\frac{\partial U}{\partial R1}$ is

$$\frac{\partial U}{\partial R1} = \frac{L^3}{EI}\left(\frac{2}{3}R1+\frac{3}{2}R2+\frac{7}{3}R3-\frac{13}{4}wL\right) +$$

$$\frac{h}{\eta AE}\left[(4R1+6R2+8R3-16wL)Sec\theta+R1Sec^3\theta\ \right] = 0$$

The other equations are obtained similarly and finally,

$$8R1+18R2+28R3-39wL+12k\left[(4R1+6R2+8R3-16wL)Sec^3\theta_o+R1Sec^3\theta_1\right] = 0$$

$$18R1+48R2+80R3-103wL+12k\left[(6R1+9R2+12R3-24wL)Sec^3\theta_o+R2Sec^3\theta_2\right] = 0$$

$$28R1+80R2+144R3-174wL+12k\left[(8R1+12R2+16R3-32wL)Sec^3\theta_o+R3Sec^3\theta_3\right] = 0$$

where $k = \frac{h}{\eta AE}\,\frac{EI}{L^3}$.

Two limiting cases provide a check on these equations. In the first case if ηAE is infinitely large, the beam cannot deflect at the points of attachment of the cables and becomes a beam on a series of fixed supports, and can then be analyzed by the theorem of three moments.

$k = 0$, and the equation reduce to

$$8R1 + 18R2 + 28R3 - 39wL = 0$$
$$18R1 + 48R2 + 80R3 - 103wL = 0$$
$$28R1 + 80R2 + 144R3 - 174wL = 0$$

The solution of these equations is

$$P, \quad R3 = \frac{11}{28}wL$$

$$P_A = R2 = 8/7\ wL$$

$$R3 = 13/14\ wL,$$

where P_O and P_A are the normal forces at O and A. These forces are the same as those obtained by applying the theorem of three moments.

The second check is offorded by the case when EI is infinite, that is the beam is rigid. k is then ∞ and the equations become

$$(4R1 + 6R2 + 8R3 - 16wL)\mathrm{Sec}^3\theta_o + R1\mathrm{Sec}^3\theta_1 = 0$$
$$(6R1 + 9R2 + 12R3 - 24wL)\mathrm{Sec}^3\theta_o + R2\mathrm{Sec}^3\theta_2 = 0$$
$$(8R1 + 12R2 + 16R3 - 32wL)\mathrm{Sec}^3\theta_o + R3\mathrm{Sec}^3\theta_3 = 0$$

The solution of these equations is

$$P_A = \frac{8wL\mathrm{Cos}^3\theta_o}{\mathrm{Cos}^3\theta_0 + 4\mathrm{Cos}^3\theta_1 + 9\mathrm{Cos}^3\theta_2 + 16\mathrm{Cos}^3\theta_3}$$

$$R1 = \frac{16wL\mathrm{Cos}^3\theta_1}{\mathrm{Cos}^3\theta_0 + 4\mathrm{Cos}^3\theta_1 + 9\mathrm{Cos}^3\theta_2 + 16\mathrm{Cos}^3\theta_3}$$

$$R2 = \frac{24wLCos^3\theta_2}{Cos^3\theta_0+4Cos^3\theta_1+9Cos^3\theta_2+16Cos^3\theta_3}$$

$$R3 = \frac{32wLCos^3\theta_3}{Cos^3\theta_0+4Cos^3\theta_1+9Cos^3\theta_2+16Cos^3\theta_3}$$

These results can also be obtainable from a direct consideration of the displacements of the rigid beam.

The bending moment diagrams resulting from these two assumptions are shown in Fig. 3. It is evident that the differences are considerable but neither is correct, because neither ηAE nor EI can actually be infinite, and k will have a finite value depending upon the initial tensions in the cables. The bending moment curves are shown for values of 0.01, 0.05, 0.1 and 1.0. The critical point for the design occurs at A, the bending moments there are given in the following table for different assumptions:

k	0	0.01	0.05	0.1	1.0	
$\frac{M_A}{wL^2}$	0.107	0.113	0.171	0.215	0.382	0.499

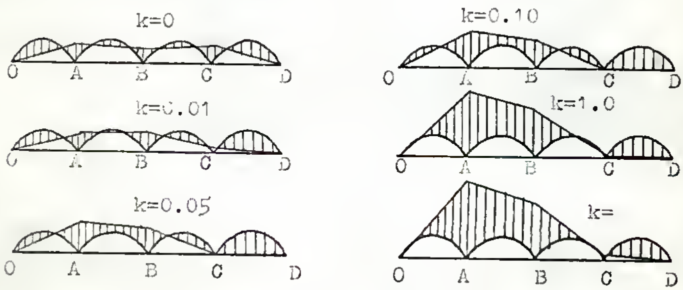


Fig. 3

From Fig. 3 the very big differences in M_A emphasize the importance not only of proper control of initial tensions but also of accurate methods of calculation.

The foregoing analysis has been simplified by the restriction that all cables shall have the same extensional rigidity, it has provided no means whereby the beam can be maintained in a straight line. It serves to show that in a condition of loading in which the cable points on the beam do not remain collinear, the relative elasticity of the beam and cables becomes an important factor. It is assumed that the far cable FD from the end O is displaced by a specified amount and that the cable points remain collinear.

From this assumption, a further discussion on the geometry of the vertical deflection of the beam and displacement of cable attachment is required. In Fig. 4 OB is a column, and a cable BA hangs the beam OA at point A, For the sake of

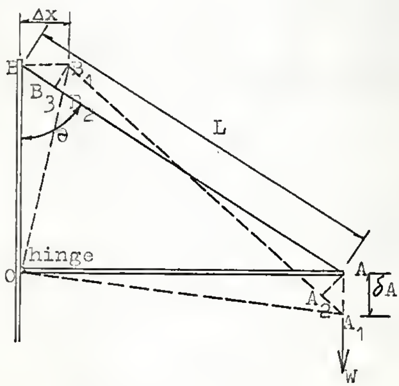


Fig. 4

simplicity, assume the beam is hinged to the column at point O, that is, there is no fixed end moment at the connection. A small deflection is assumed on this structure, that is, the angle between the cable and column does not change as the deformation take place, and the horizontal deformation of point A is negligible. Now the same geometrical relations can be deduced as follows: Let δA be the vertical displacement at point A, assume downward displacement is positive, and let Δx be the horizontal displacement of point B of the column, θ = the angle between the cable and the column, L = the original length of the cable, ΔL = elongation of the cable, As = the cross section area of the cable, Es = the elasticity modulus of the cable, T = the tension in the cable,

$$\Delta L = B_1A_1 - BA \qquad \text{since } B_1A_1 = B_1A_2 + A_2A_1$$

$$BA = BB_3 + B_3A$$

$$\text{and } B_3A = B_1A_1$$

The elongation of the cable BA becomes,

$$\Delta L = B_1A_2 + A_2A_1 - BB_3 - B_1A_2 = A_2A_1 - BB_3$$

$$= \delta A\cos\theta - \ x\sin\theta \qquad (20)$$

$$\Delta L = TL/EAs$$

$$\therefore \quad A\cos\theta = \frac{TL}{EA_s} \qquad \delta A = \frac{TL}{EA_s\cos\theta} \qquad (21)$$

The vertical cable tension

$$T' = \frac{\delta A E A_s}{L} \cos^2\theta$$

Letting δA = unity, and the vertical cable tension shall be

$$T' = \frac{E A_s}{L} \cos^2\theta \qquad (22)$$

STRUCTURAL ANALYSIS BY THE PRINCIPLE OF SUPERPOSITION

The principle of superposition applied for solving high degree indeterminate structures has been considered the most efficient method of structural analysis by using a Computer. In applying this method for stress analysis, it is necessary to reduce the structure to be a base structure form, which is a statically determinate or the minimum degree statically indeterminate structure, and put all the redundants to be solved by linear simultaneous equations. Those equations can be solved by matrix operation.

The principle of superposition is also suitable for analysis of cable stiffened structures. Consider the cable stiffened structure as shown in Fig. 5. It is a single bent rigid frame with six cables hanging on the top of the girder. It is a statically indeterminate structure in nine degrees. By the principle of superposition, the total moment distribution can be considered as composed of two parts,

(1) The moment distribution m_0 due to the applied loads acting only on the base structure without cable stiffening, that is the cable stresses $X_1=X_2 \ldots\ldots =X_n = 0$. It is called the particular solution of the problem because it satisfies the condition of equilibrium but does not satisfy the boundary condition of the problem, since there are six X unknowns to be solved.

(2) The moment distribution m_nX_n due to the action of the arbitrary constant X_n alone on the structure with all other

loads equals to zero. The number of arbitrary constants it has should be the same as the number of moment distributions due to these constants.

Thus the total moment distributions should be

$$M = m_0 + m_1X_1 + m_2X_2 + m_3X_3 \; . \; . \; . + m_nX_n \tag{23}$$

The total strain energy due to flexural bending as discussed in general strength of materials will be

$$U = \int_s \frac{M^2}{2EI} ds \tag{24}$$

It is also necessary to consider the effect of stress resultants other than moment in contributing to the strain energy, and the total energy will be the summation of these energies,

$$U_{total} = U_{bending} + U_{shear} + U_{direct\ force}$$

In equation (24) s is in term of the coordinate and is assumed along the axes of the members of a skeletal frame.

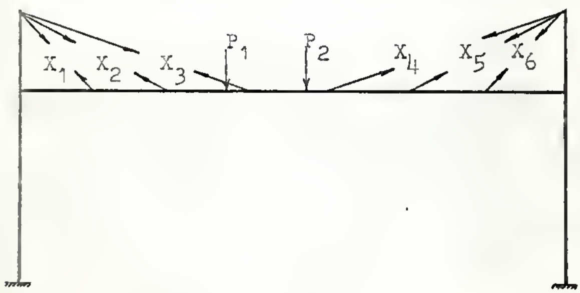


Fig. 5

Applying the theorem of least work, n equations can be obtained for the determination of nX values.

$$\frac{\partial U}{\partial X_1} = \int_s \frac{\partial}{X_1}\left(\frac{M^2}{2EI}\right)ds = 0$$

$$\frac{\partial U}{\partial X_2} = \int_s \frac{\partial}{X_2}\left(\frac{M^2}{2EI}\right)ds = 0$$

$$\frac{\partial U}{\partial X_3} = \int_s \frac{\partial}{X_3}\left(\frac{M^2}{2EI}\right)ds = 0 \qquad (25)$$

$$\vdots \qquad \qquad \vdots$$

$$\frac{\partial U}{\partial X_n} = \int_s \frac{\partial}{X_n}\left(\frac{M^2}{2EI}\right)ds = 0$$

It is known that

$$U = \int_s \frac{M^2}{2EI}ds = \int_s \frac{1}{2EI}(m_0+m_1X_1+m_2X_2+m_3X_3.\ .\ .m_nX_n)ds$$

So that equation (25) becomes:

$$\frac{\partial U}{\partial X_1} = \int_s \frac{m_1}{EI}(m_0+m_1X_1+m_2X_2+m_3X_3.\ .\ .m_nX_n)ds = 0$$

$$\frac{\partial U}{\partial X_2} = \int_s \frac{m_2}{EI}(m_0+m_1X_1+m_2X_2+m_3X_3.\ .\ .m_nX_n)ds = 0$$

$$\frac{\partial U}{\partial X_3} = \int_s \frac{m_3}{EI}(m_0 + m_1X_1 + m_2X_2 + m_3X_3 \ldots + m_nX_n)ds = 0$$

$$\vdots \qquad\qquad\qquad \vdots$$

$$\frac{\partial U}{\partial X_n} = \int_s \frac{m_n}{EI}(m_0 + m_1X_1 + m_2X_2 + m_3X_3 \ldots + m_nX_n)ds = 0$$

It is convenient to expand these equations as follows:

$$\left.\begin{aligned}
& X_1 \int_s \frac{m_1^2}{EI}ds + X_2 \int_s \frac{m_1m_2}{EI}ds + X_3 \int_s \frac{m_1m_3}{EI}ds \ldots + X_n \int_s \frac{m_1m_n}{EI}ds \\
& \qquad + \int_s \frac{m_1m_0}{EI}ds = 0 \\
& X_1 \int_s \frac{m_2m_1}{EI}ds + X_2 \int_s \frac{m_2^2}{EI}ds + X_3 \int_s \frac{m_2m_3}{EI}ds \ldots \\
& \qquad X_n \int_s \frac{m_2m_n}{EI}ds + \int_s \frac{m_2m_0}{EI}ds = 0 \\
& X_1 \int_s \frac{m_3m_1}{EI}ds + X_2 \int_s \frac{m_3m_2}{EI}ds + X_3 \int_s \frac{m_3^2}{EI}ds \ldots \\
& \qquad X_n \int_s \frac{m_3m_n}{EI}ds + \int_s \frac{m_3m_0}{EI}ds = 0 \\
& X_1 \int_s \frac{m_nm_1}{EI}ds + X_2 \int_s \frac{m_nm_2}{EI}ds + X_3 \int_s \frac{m_nm_3}{EI}ds \ldots \\
& \qquad X_n \int_s \frac{m_n^2}{EI}ds + \int_s \frac{m_nm_0}{EI}ds = 0
\end{aligned}\right\} \quad (26)$$

From Castigliano's theorem, it is known that

$\frac{\partial U}{\partial X} = \delta$. So the deflection δ_1 at the position and the direction of X_1 should be:

$$\delta_1 = \frac{\partial U}{\partial X_1} = X_1 \int_s \frac{m_1^2}{EI} ds + X_2 \int_s \frac{m_1 m_2}{EI} ds + X_3 \int_s \frac{m_1 m_3}{EI} ds \ldots$$

$$X_n \int_s \frac{m_1 m_n}{EI} ds + \int_s \frac{m_1 m_0}{EI} ds \tag{27}$$

For simplicity equation (27) can be defined as:

$$\delta_1 = X_1 \int \frac{m_1^2}{EI} ds = X_1 f_{11} \qquad \text{due to } X_1 = 1,$$

where f_{11} = the deflection of the base structure at the position and in the direction of X_1 due to unit X_1 acting alone.

$$\delta_1 = X_2 \int \frac{m_1 m_2}{EI} ds = X_2 f_{12} \qquad \text{due to } X_2 = 1,$$

where f_{12} = the deflection of the base structure at the position and in the direction of X_1 due to a unit X_2 acting alone. And so do the deflections due to other unit X acting alone as,

$$\delta_1 = X_3 \int \frac{m_1 m_3}{EI} ds = X_3 f_{13} \qquad \text{due to } X_3 = 1,$$

$$\delta_1 = X_n \int \frac{m_1 m_n}{EI} ds = X_n f_{1n} \qquad \text{due to } X_n = 1,$$

$$\delta_1 = \int \frac{m_1 m_0}{EI}\, ds = u_1$$

due to applied loads, where u_1 = the deflection of the base structure at the position and in the direction of X_1 due to the applied loads.

Now with the above definition, equation (26) can be re-written in the following form, for the first one:

$$X_1 f_{11} + X_2 f_{12} + X_3 f_{13} \cdot\ \cdot\ \cdot + X_n f_{1n} = -u_1 \tag{28a}$$

Similarly, the second equation, as well as the rest of the equations in relation to each cable attachment can be written as:

$$X_1 f_{21} + X_2 f_{22} + X_3 f_{23} + \cdot\ \cdot\ \cdot + X_n f_{2n} = -u_2$$

$$X_1 f_{31} + X_2 f_{32} + X_3 f_{33} + \cdot\ \cdot\ \cdot + X_n f_{3n} = -u_3$$

$$\vdots$$

$$X_1 f_{n1} + X_2 f_{n2} + X_3 f_{n3} \cdot\ \cdot\ \cdot + X_n f_{nn} = -u_n \tag{28b}$$

Where f_{n1}, f_{n2}, $f_{n3} \cdot\ \cdot\ \cdot f_{nn}$ have meaning similar to f_{11}, f_{12}, $f_{13} \cdot\ \cdot\ \cdot f_{1n}$ and u_n also have the same meaning as u_1 but those are related to X_n. It is to be noted that $f_{12} = f_{21}$, $f_{31} = f_{13} \cdot\ \cdot\ \cdot f_{n1} = f_{1n}$ because,

$$f_{12} = f_{21} = \int_s \frac{m_1 m_2}{EI} ds,$$ which is from the Maxwell's reciprocal theorem due to the physical meaning of f_{12} and f_{21}.

In summation of the above statements, a general solution of a structure with n arbitary constants will lead to a set of n simultaneous equations of the form of equation (28). The equation to be solved will then be:

$$\left.\begin{array}{l} X_1f_{11}+X_2f_{12}+X_3f_{13}\cdot\;\cdot\;\cdot X_jf_{1j}\cdot\;\cdot\;\cdot X_nf_{1n} = -u_1 \\ X_1f_{21}+X_2f_{22}+X_3f_{23}\cdot\;\cdot\;\cdot X_jf_{2j}\cdot\;\cdot\;\cdot X_nf_{2n} = -u_2 \\ X_1f_{31}+X_2f_{32}+X_3f_{33}\cdot\;\cdot\;\cdot X_jf_{3j}\cdot\;\cdot\;\cdot X_nf_{3n} = -u_3 \\ \qquad\vdots \\ X_1f_{i1}+X_2f_{i2}+X_3f_{i3}\cdot\;\cdot\;\cdot X_jf_{ij}\cdot\;\cdot\;\cdot X_nf_{in} = -u_i \\ \qquad\vdots \\ X_1f_{n1}+X_2f_{n2}+X_3f_{n3}\cdot\;\cdot\;\cdot X_jf_{nj}\cdot\;\cdot\;\cdot X_nf_{nn} = -u_n \end{array}\right\} \qquad (29)$$

It should be noted that f_{11}, f_{22}, f_{33}. . .f_{nn} are the deflections of the base structure at the position of cable attachment due to a unit load acting at this point, and the actual deflection at these points should include the elongation of the cable at that point, that is

$$f_{11} = \int\frac{m_1^2}{EI}ds + \frac{L_1}{A_1E}, \quad f_{22} = \int\frac{m_2^2}{EI}ds + \frac{L_2}{A_2E}, \;\ldots\; f_{nn} = \int\frac{m_n^2}{EI}ds + \frac{L_n}{A_nE}$$

Where f_{ij} is called the influence coefficients, which is a function of X. Equation (29) can be written in matrix form as:

$$\begin{bmatrix} f_{11} & f_{12} & f_{13} & \cdots & f_{1j} & \cdots & f_{1n} \\ f_{21} & f_{22} & f_{23} & \cdots & f_{2j} & \cdots & f_{2n} \\ f_{31} & f_{32} & f_{33} & \cdots & f_{3j} & \cdots & f_{3n} \\ f_{i1} & f_{i2} & f_{i3} & \cdots & f_{ij} & \cdots & f_{in} \\ f_{n1} & f_{n2} & f_{n3} & \cdots & f_{nj} & \cdots & f_{nn} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_j \\ X_n \end{bmatrix} = - \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_i \\ u_n \end{bmatrix}$$

Usually, it can be written in a shorthand notation as

$$[F][X] = -[u] \tag{30}$$

Here, $[F]$ is called Flexibility matrix.

The solution of the equations can be obtained by matrix operation. Usually, this matrix operation can be transferred into mechanical language and solved by digital computer. If it is solved by hand, the following operation is suggested.

First, premultiply each side of equation (30) by F^{-1}, that is

$$[F]^{-1}[F][X] = -[F]^{-1}[u]$$

Since $[F]^{-1}[F] = [I]$ is an Idendity matrix, therefore, the solution can be written in the form as

$$[X] = -[F]^{-1}[u] \tag{31}$$

where $[F]^{-1}$ is the inversion of matrix $[F]$. It is obvious that any kind of high degree statically indeterminate structures can be analyzed by matrix method, and it is time saving and convenient to solve this problem by computer.

In addition, the author wants to introduce some matrix operating concerning this kind of problem. From equation (30) since it is known that $[F]$ is a symmetrical matrix, then there can be found a lower triangle matrix $[L]$, such that

$$[F] = [L][L]'$$

where $[L]'$ is the transpose of matrix $[L]$, which is an upper triangular matrix. Now the operation of expression can be carried out in fact as

$$\begin{bmatrix} L11 & 0 & 0 & \cdots & 0 \\ L21 & L22 & 0 & \cdots & 0 \\ L31 & L32 & L33 & \cdots & 0 \\ \\ Ln1 & Ln2 & Ln3 & \cdots & Lnn \end{bmatrix} \begin{bmatrix} L11 & L21 & L31 & \cdots & Ln1 \\ 0 & L22 & L23 & \cdots & Ln2 \\ 0 & 0 & L33 & \cdots & Ln3 \\ \\ 0 & 0 & 0 & \cdots & Lnn \end{bmatrix}$$

$$= \begin{bmatrix} L11 & L12 & L13 & \cdots & L1n \\ L21 & L22 & L23 & \cdots & L2n \\ L31 & L32 & L33 & \cdots & L3n \\ \\ Ln1 & Ln2 & Ln3 & \cdots & Lnn \end{bmatrix}$$

So from the above expression, equation (30) can be written:

$$[L][L]'[X] = -[u] \qquad (32)$$

Similarly, $[F]^{-1} = ([L][L]')^{-1} = [L]^{-1}[L]'^{-1}$

Postmultply each side by $[L]$, and the above equation becomes

$$[F]^{-1} [L] = [L]'^{-1}$$

Therefore, in matrix inversion, it will be easier to find $[L]$ and $[L']^{-1}$ than to find $[F]^{-1}$ directly.

Two numerical examples will be given as follows to demonstrate how to apply the principle of superposition to solve the cable stiffened structural problems.

NUMERICAL EXAMPLE I

Analyze and design the stresses of a cable stiffened rigid frame as shown in Fig. 6. The frame has a span of 140 ft. center to center of columns and the lower column is 50 ft. in height and the upper column is 20 ft. The section of the beam has 5 ft. depth with 2 ft. width and the section of columns is 12 ft. depth by 2 ft width which is fixed supported at each end. The frame is stiffened by six #18s high tension steel bars as cables hanging on the top of the beam and with equal distance of 20 ft. apart. Space between each frame is 10 ft. with 4" roof slab on the top and 12" wall on each side of the frame. Assume

f_s = 35,000 psi for tension cable bars

f_y = 60,000 psi, f_s = 24,000 psi for frame reinforcement

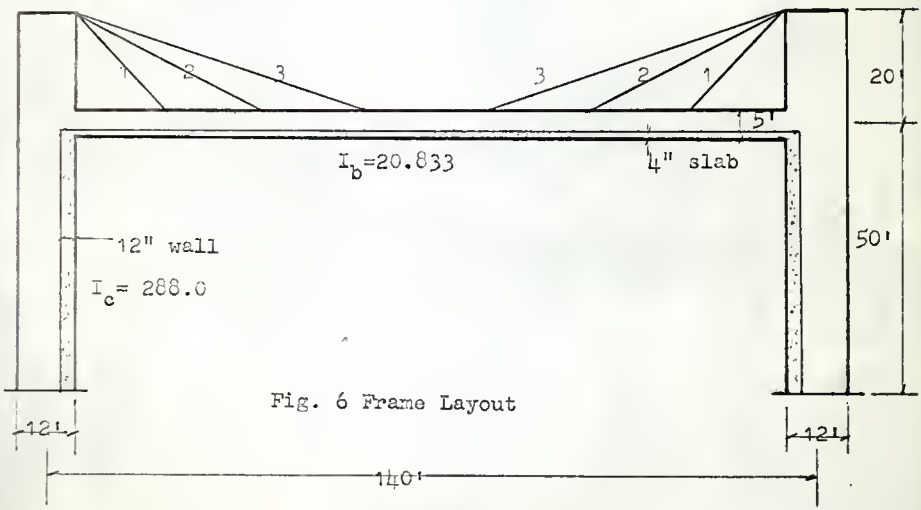


Fig. 6 Frame Layout

fc = 3,000 psi

E_s = 29,000,000 psi E_c = 2,500,000 psi

Cable length L1 = 28.285' L2 = 44.745' L3 = 63.253'

Loading condition

Dead load	beam 5x2x0.15	= 1.5 k/ft
	slab (4/12)8x0.15	= 0.4 k/ft
Live load	snow 35#/sq.ft x 10'	= 0.35/ft

The wind stresses will be shown in the next example

total load = 2.25 k/ft

Check of cable size

Use of #18s as tension cable (2.257") As=4.0 sq. in = 0.02778 sq. ft.

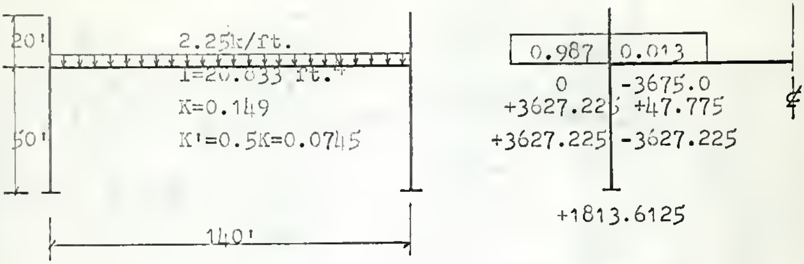
$$As = \frac{\frac{1}{h}\frac{wL^2}{8} x \sec\theta}{Nxf_s} = \frac{\frac{1}{20} x \frac{1}{8}x2.25x140^2x1.4142}{3x35000} =$$

3.62 sq. in < 4.0 o K

Where θ is the angle between the cable and the beam, θ_{max} = 45° for cable 1, h is the length of upper column, N is the number of cables. This formula is derived on the assumption that the bending moment of the beam is taken by those cables. So the assumed As is ok.

Moment for the base structure:

The base structure is shown in Fig. 7, which is the original frame uniformly loaded of 2.25 k/ft on the top of the



Moment distribution

(a) (b)

Fig. 7

beam without cable stiffening. The fixed end moment and the moment diagram can be calculated by the method of moment distribution. (NOTE: All moments are in kips-ft. in this report.)

Moment for the base structure due to unit tension on cable:

The base structure is loaded by cable 1 with a tension force X = 1 on the cable and the fixed end moment is found as shown in Fig. 8a.

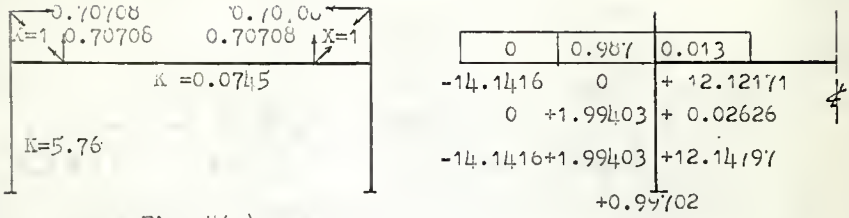


Fig. 8(a)

Moment of the base structure due to unit tension on cable 2:
(see Fig. 8b)

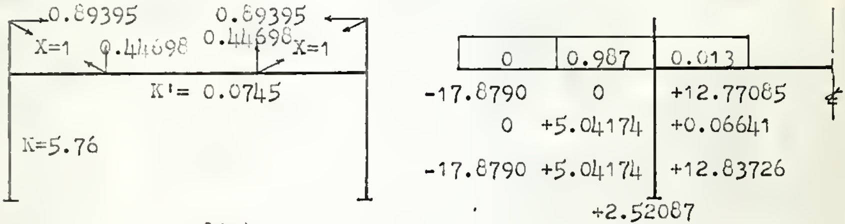


Fig. 8(b)

Moment of the base structure due to unit load on cable 3:
(see Fig. 8c)

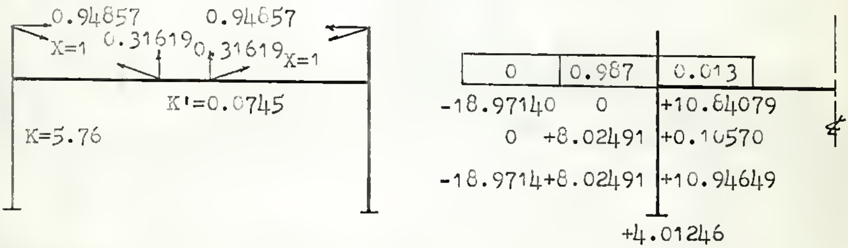
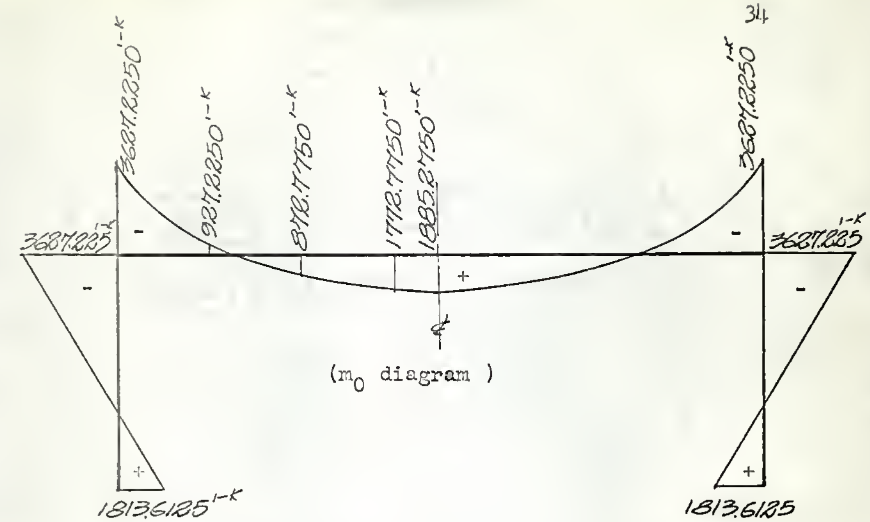
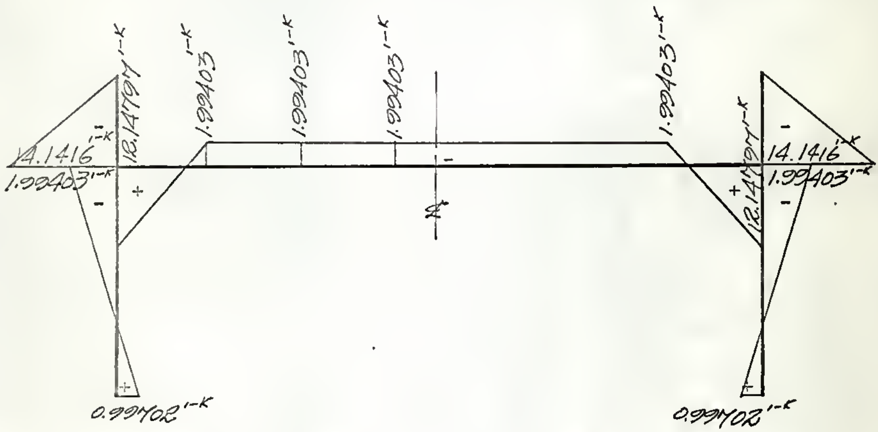


Fig. 8(c)

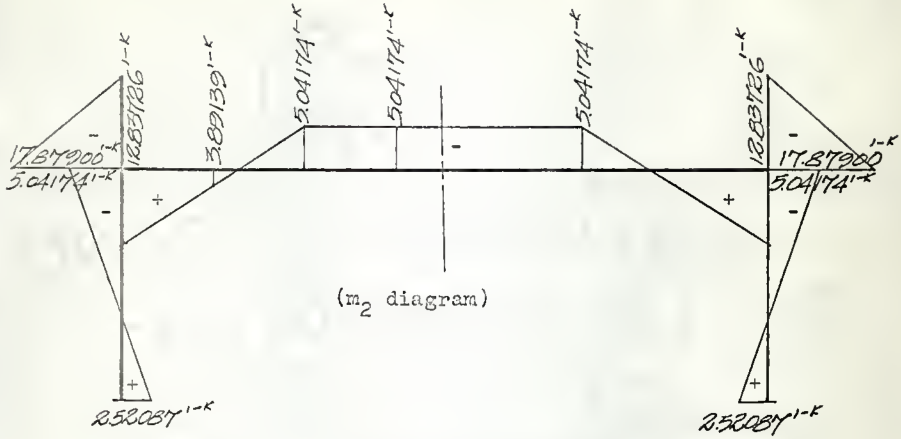


(a) Moment diagram of base structure due to external loads

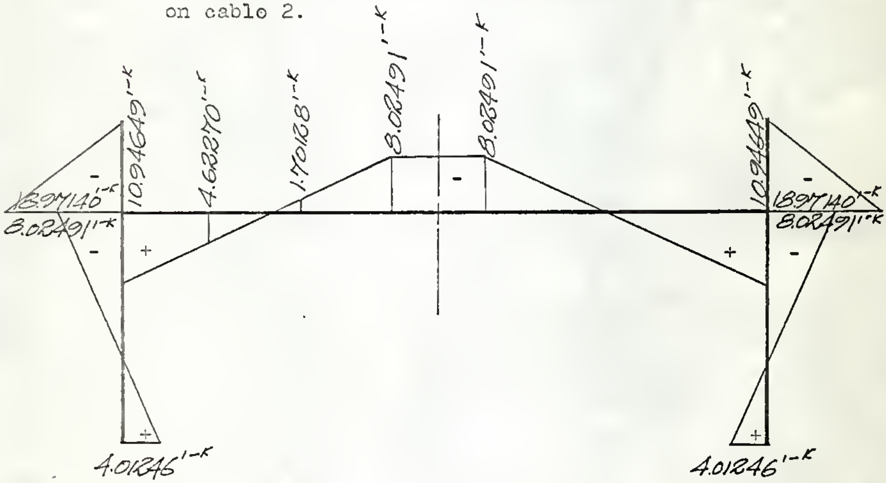


(b) Moment diagram of base structure due to unit load on cable 1.

Fig. 9, a, b



(a) Moment diagram of base structure due to unit load on cable 2.



(b) Moment diagram of base structure due to unit load on cable 3.

Fig. 9, c,d,

Deflections of the base structure:

The producing and integrating of the moment areas have been done by Computer, Programs and data output for this problem are shown in Appendix B. So the deflections due to external loads and unit loads from the Computer output shall be:

$$u_1 = \int_s \frac{m_0 m_1}{EI}\, ds = -\ 18070.64300/E$$

$$u_2 = \int_s \frac{m_0 m_2}{EI}\, ds = -31524.613/E$$

$$u_3 = \int_s \frac{m_0 m_3}{EI}\, ds = -31746.20100/E$$

$$f_{11} = \int_s \frac{m_1^2}{EI}\, ds + \frac{L_1}{A_1 E} = +55.089719/E + 1018.1785 = 1073.268219/E$$

$$f_{12} = \int_s \frac{m_1 m_2}{EI}\, ds = 72.755847/E$$

$$f_{13} = \int_s \frac{m_1 m_3}{EI}\, ds = +66.19547/E$$

$$f_{22} = \int_s \frac{m_2^2}{EI}\, ds + \frac{L_2}{A_2 E} = 125.375590/E + 1610.6911 = 1736.066690/E$$

$$f_{23} = \int_s \frac{m_2 m_3}{EI}\, ds = 123.314750/E$$

$$f_{21} = \int_s \frac{m_2 m_1}{EI}\, ds = 72.755847/E$$

$$f_{33} = \int_s \frac{m_3^2}{EI}\, ds + \frac{L_3}{A_3 E} = 134.565680/E + 2276.9258/E = 2411.49180/E$$

$$f_{32} = \int_s \frac{m_3 m_2}{EI}\, ds = 123.314750/E$$

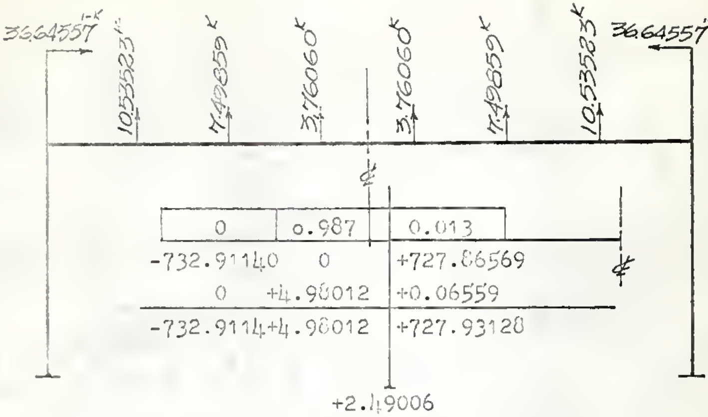
$$f_{31} = \int_s \frac{m_3 m_1}{EI}\, ds = 66.1955470/E$$

Those influence coefficients can be put into a matrix equation as:

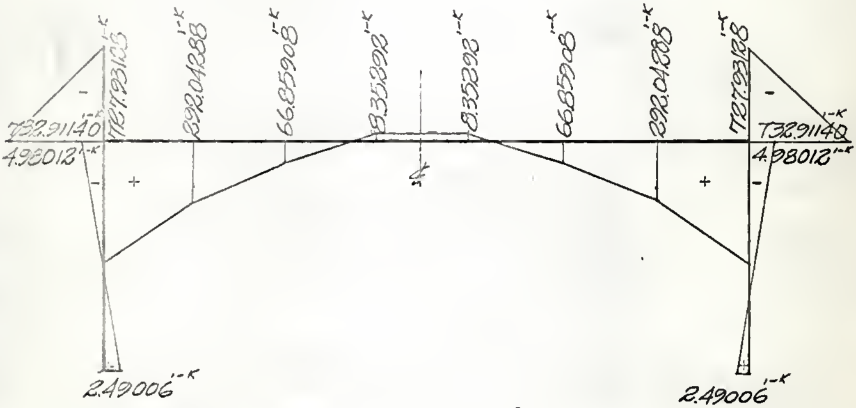
$$\begin{bmatrix} 1073.268219 & 72.755847 & 66.195547 \\ 72.755847 & 1736.066690 & 123.314750 \\ 66.195547 & 123.314750 & 2411.491480 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = - \begin{bmatrix} 18070.643 \\ 31524.613 \\ 31746.201 \end{bmatrix}$$

The inversion of the $[F]$ matrix and the value for $[X]$ matrix have been done by the Computer program 3. So the cable stresses are found:

$$\begin{array}{ccccccc} [X] & & [F]^{-1} & & -[u] & & \\ \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} & = & \begin{bmatrix} 0.00093574 & -0.000037527 & -0.000023767 \\ -0.000037527 & 0.00057962 & -0.000023767 \\ -0.000023767 & -0.000028609 & 0.00041680 \end{bmatrix} & & \begin{bmatrix} -u_1 \\ -u_2 \\ -u_3 \end{bmatrix} & = & \begin{bmatrix} 14.971933 \\ 16.839623 \\ 11.900315 \end{bmatrix} \end{array}$$



(a) Loads on frame due to cable stresses and moment distribution.



(b) Moment diagram of the frame due to cable stresses.

Fig. 10

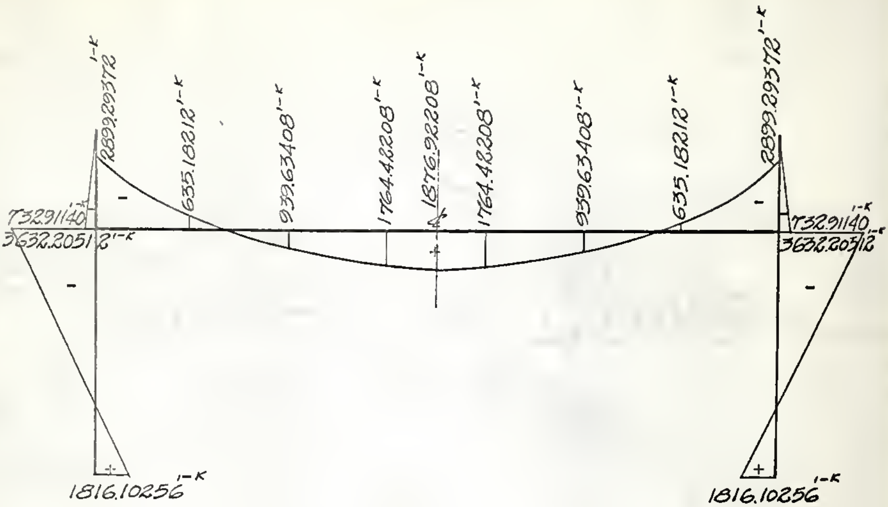


Fig. 11 Moment Diagram of the Frame due to External Loads and Cable Stresses

Fig. 11 shows the actual moment diagram of the cable stiffened rigid frame due to external loads. The moment at the fixed end of the beam is 2899.29372 kips-ft. in comparative to the fixed end moment 3627.2250 kips-ft. of the beam without cable stiffening. So, the moment of the beam has been reduced in an amount of 727.93128 kips-ft. due to the counter-balance of the cable stresses with the external loads on the beam.

If the moment of 2899.29372 kips-ft. is still larger for the beam to take, a further reduction of the moment can be made by prestressing the cables. The amount of moment reduction desired can be interpreted as the amount of prestressing of each cable needed. Assume that the fixed end moment of the beam need to be further reduced in a amount of M kips-ft., which can be done by prestressing those cables. The cable prestressing can be done by the following ways: Figure 12 shows a construction detail of the cable post-tension method. It is done by putting a small jack between the cables and the top of the columns on each side of the frame. After the construction of the frame is finished and has reached 28 days concrete strength, then the cable should be jacked up in a distance Δh. The amount of Δh can be determined by means of interpreting which moments should be reduced by cable prestressing.

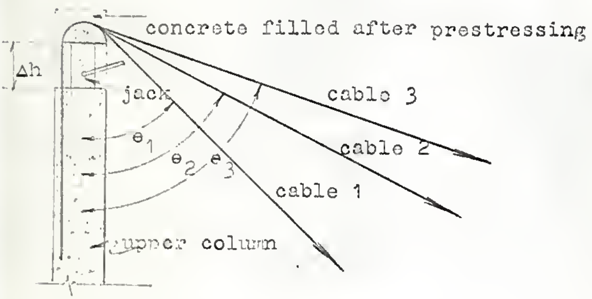


Fig. 12 Cable post-tensioning Layout

It is known from equation (21) that the vertical elongation of the cable shall be

$$\Delta h = \frac{TL}{EA_s \cos\theta}$$ where T is the tension in the cable.

and the vertical component of the cable tension shall be

$$T' = \frac{\Delta h E A_s}{L} \cos^2\theta$$

where θ is the angle between cable and column. So the opposite fixed moment due to cable prestressing can be found without difficulty. Let T'_1, T'_2, T'_3 be the vertical components of the cable tension, and the corresponding angles between column and cable 1, 2, 3 be θ_1, θ_2, θ_3 respectively. Then the fixed end moment due to cable prestressing will be:

$$M = \frac{T'_1(20\text{x}120^2+120\text{x}20^2)}{140^2} + \frac{T'_2(40\text{x}100^2+100\text{x}40^2)}{140^2}$$

$$+ \frac{T'_3(60\text{x}80^2+80\text{x}60^2)}{140^2}$$

$$\therefore M = 17.14290T'_1 + 28.57142T'_2 + 34.28571T'_3$$

The vertical component of cable prestressing in terms of vertical displacement for each cable will be:

$$T'_1 = \frac{\Delta h E A s1}{L_1}\cos^2\theta, \quad T'_2 = \frac{\Delta h E A s2}{L_2}\cos^2\theta, \quad T' = \frac{\Delta h E A s3}{L_3}\cos^2\theta$$

Where $L_1 = 28.285'$ $L_2 = 44.745'$ $L_3 = 63.253'$

and

$\cos\theta_1 = 20/28.285 = 0.70708$ $\cos^2\theta_1 = 0.49996$

$\cos\theta_2 = 20/44.745 = 0.44698$ $\cos^2\theta_2 = 0.19979$

$\cos\theta_3 = 20/63.253 = 0.31620$ $\cos^2\theta_3 = 0.09998$

Thus

$T'_1 = 0.000491\ E\ \ h$, $T'_2 = 0.000125\ E\ \ h$,

$T'_3 = 0.0000439\ E\ \ h$. Substitute T' and found the moment in terms of E h.

Thus

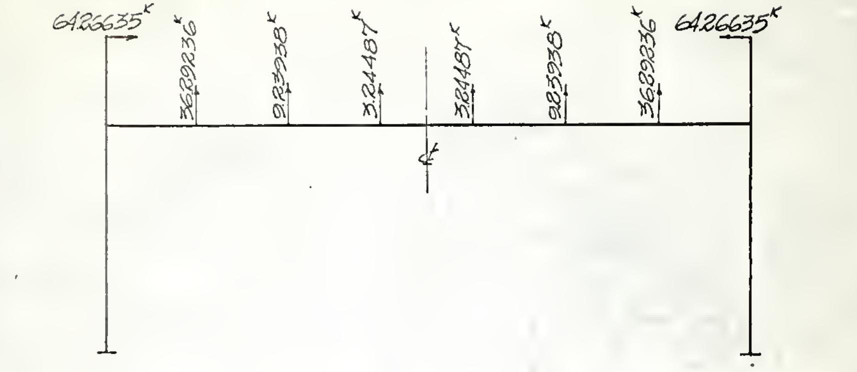
$$M = 0.008417\ E\Delta h + 0.003571\ E\Delta h + 0.0015219\ E\Delta h = 0.013519\ E\Delta h.$$

Now assume that the fixed end moment of the beam will be reduced in an amount of 1000 kips-ft. so the prestressing should be done by jacking up the cables in a vertical distance Δh, which will be equal to

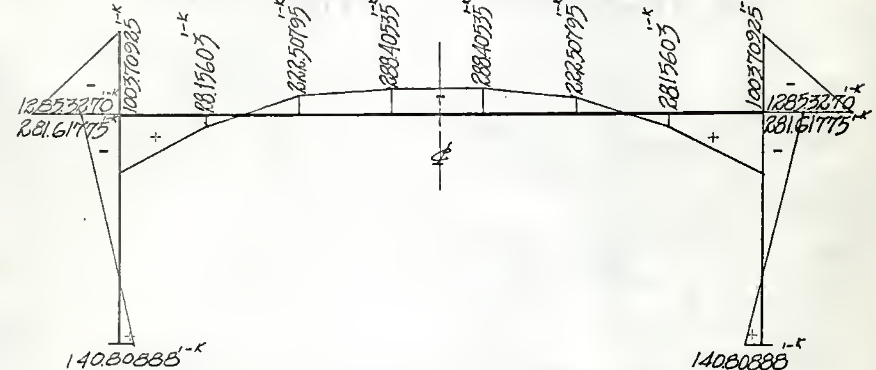
$$\Delta h = \frac{1000}{0.013510E} = \frac{1000 \times 1000}{0.013510 \times 29{,}000{,}000 \times 144} = 1/56.41776$$

$$= 0.01776 \text{ ft.}$$

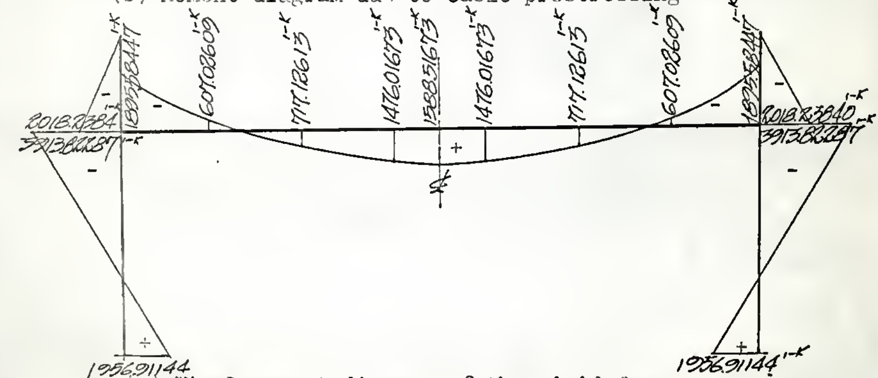
The moment diagram due to cable prestressing and the final moment and shear diagrams are shown in Fig. 13 and Fig. 14.



(a) Loads on frame due to cable prestressing

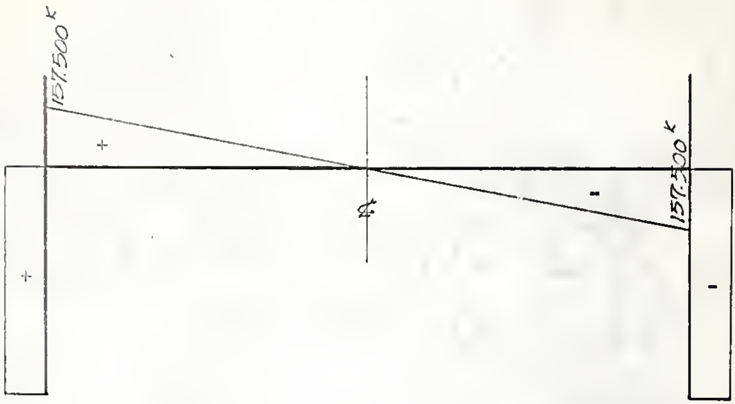


(b) Moment diagram due to cable prestressing

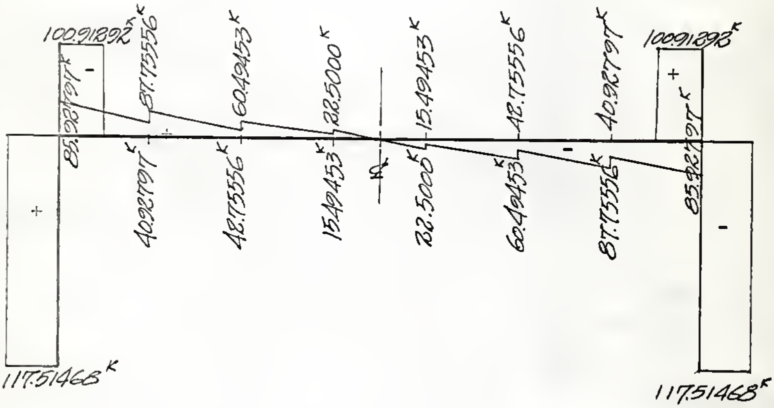


(c) Final moment diagram of the rigid frame

Fig. 13



(a) Shear diagram of the base structure due to external loads without cable stiffening



(b) Final shear diagram of the frame due to external load and cable prestressing.

NUMERICAL EXAMPLE 2

Analyze the cable stiffened rigid frame as shown in numerical example 1, loaded by lateral wind loads of 20 lbs. per sq. ft. on the left side. No live load is considered on the top of the frame, but about 2 k/ft. of beam and slab weight will be acting on the top of the frame.

This problem can be analyzed by two parts; one is symmetrical loading condition and the other is anti-symmetrical loading condition. The summation of the analyzed results of these two loading conditions will give the actual result of

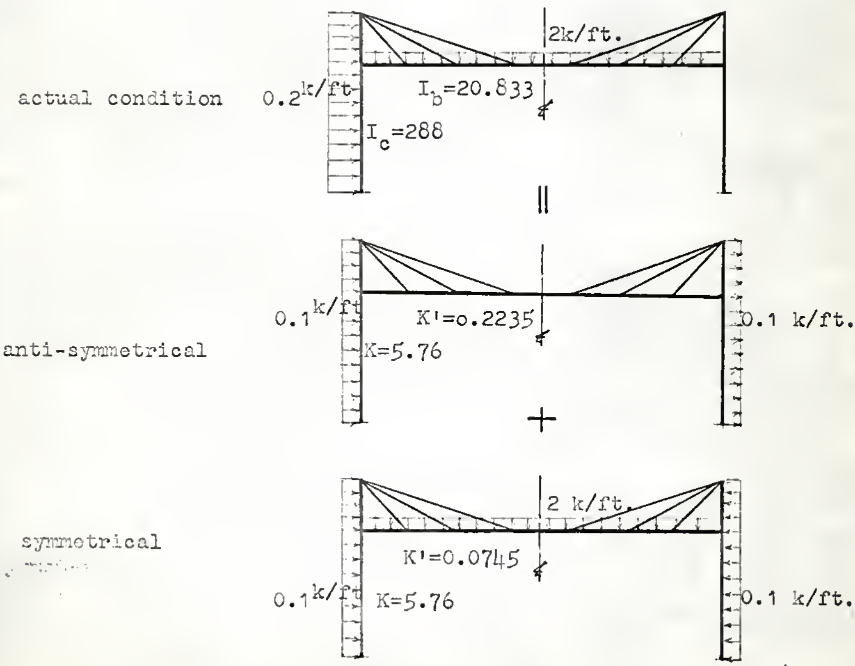
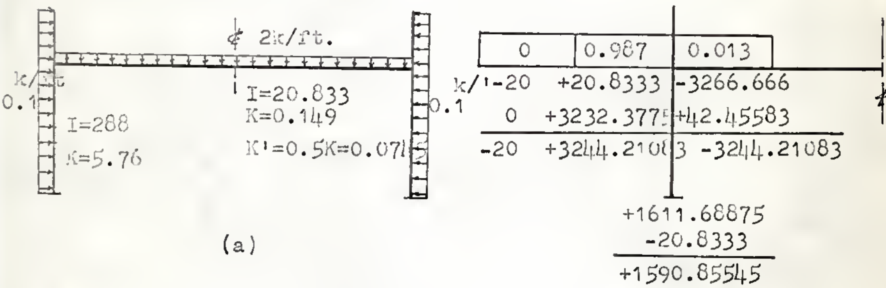


Fig. 15

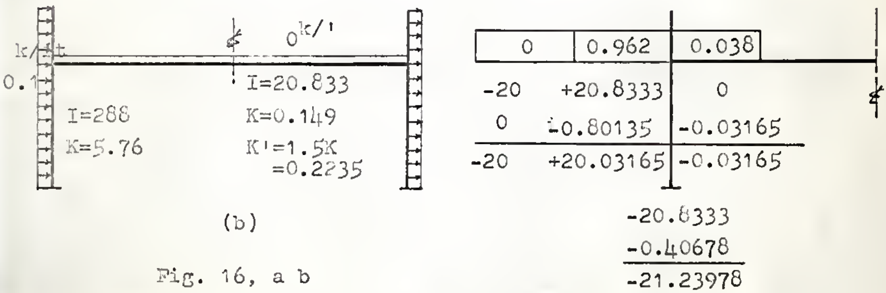
the frame stress analysis as shown in Fig. 15. The lateral wind stresses on the frame will be 0.02 x 10 = 0.2 k/ft.

Moment for the base structure due to external load (symmetrical):



(a)

Moment for the base structure due to external loads (anti-symmetrical):



(b)

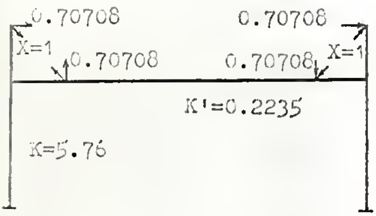
Fig. 16, a b

Moment for the base structure due to unit loads (symmetrical):

The symmetrical condition of unit load on cables had been shown in example 1, and the calculations and moment diagrams can be used in this example. Moment diagrams refer to Fig. 9 (b), (c), (d).

Moment for the base structure due to unit loads (anti-symmetrical):

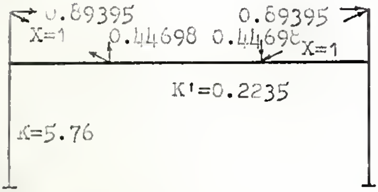
Anti-symmetrical unit load on cable 1:



0	0.962	0.038
-14.1416	0	+8.65837
0	+5.27525	+0.20838
-14.1416	+5.27525	+8.86675
	+2.63763	

(a)

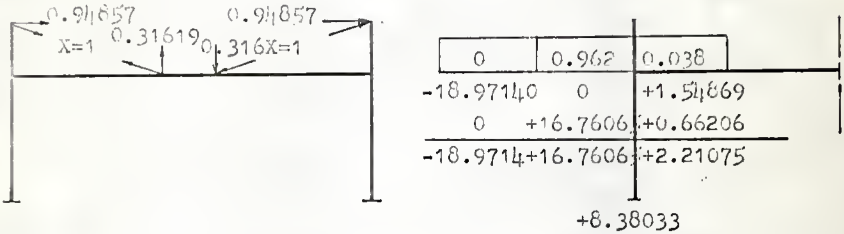
Anti-symmetrical unit load on cable 2:



0	0.962	0.038
-17.8790	0	+5.47323
0	+11.93435	+0.47142
-17.8790	+11.93435	+5.94465
	+5.96718	

(b)

Anti-symmetrical unit load on cable 3:



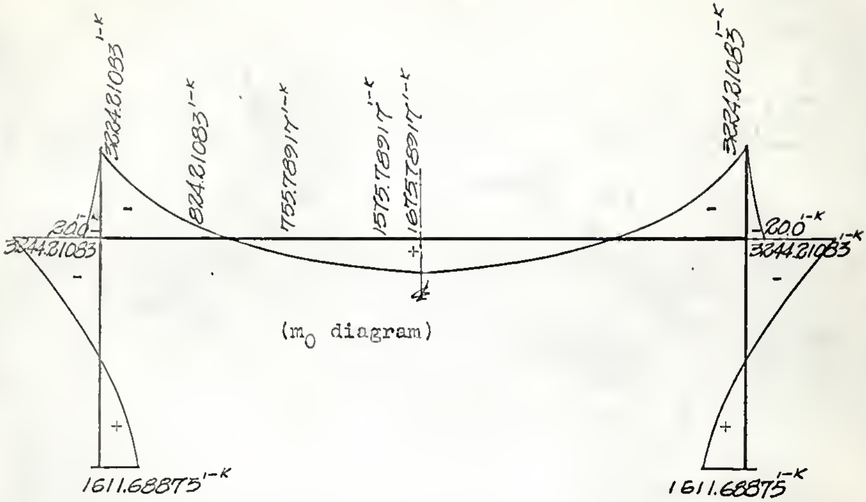
(c)

Fig. 17

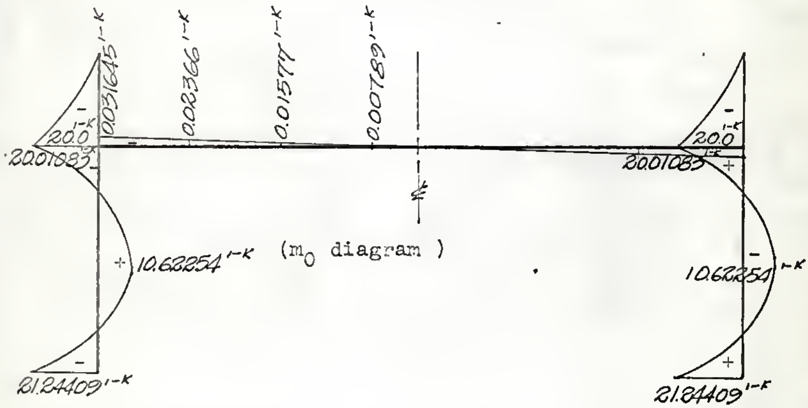
The moment diagrams both for external load and unit loads on the base structure are shown in Fig. 18 and Fig. 19.

Deflections of the base structure:

The multiplying and integrating of the moment areas have been done by Computer, and the program and data output for this problem is shown in Appendix B, the program 2. The inversion of the influence coefficient matrix f_{ij} and the final solution of the cable stresses X_1, X_2, X_3 due to wind load and dead load of the frame are also done by the Computer; the answer is shown in Appendix B, program 3. and the output of problem 2.

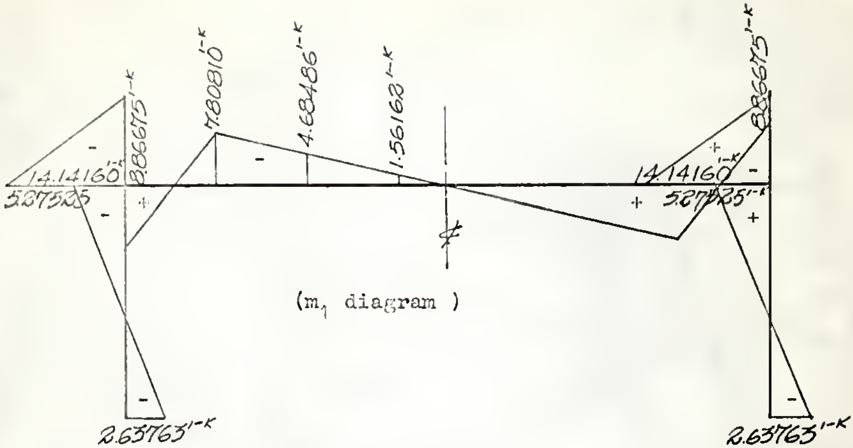


(a) Moment diagram of base structure due to external loads (symmetrical loading condition)

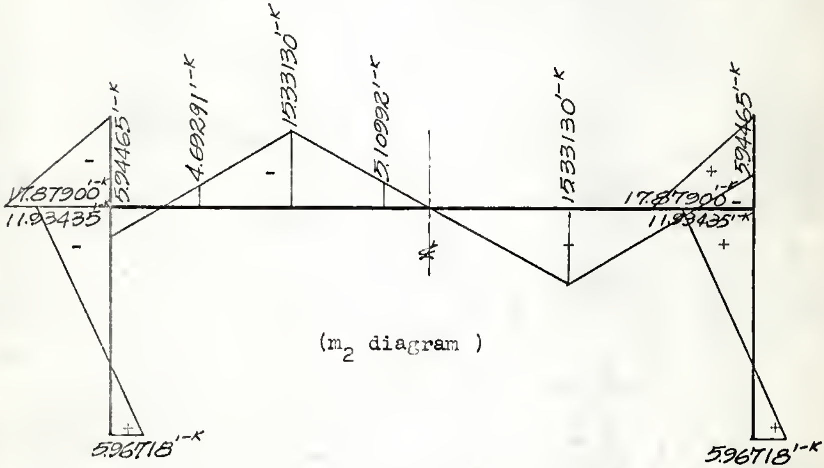


(b) Moment diagram of the base structure due to external loads (anti-symmetrical loadind condition)

Fig. 18

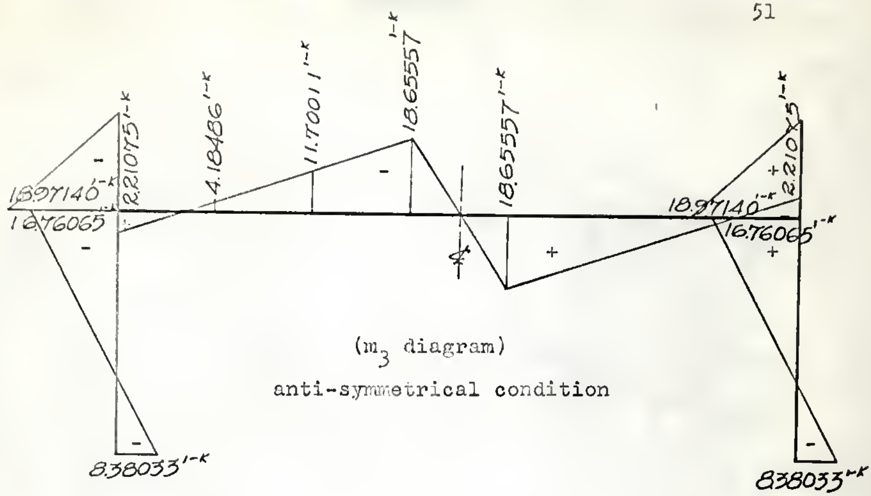


(a) Moment diagram of base structure due to unit load on cable 1. (anti-symmetrical condition)



(b) Moment diagram of base structure due to unit load on cable 2. (antisymmetrical condition)

Fig. 19



(c) Moment diagram due to unit load on cable 3.

Fig. 19 continue

Deflections of the base structure: (symmetrical load condition)

$$u_1 = \int_s \frac{m_0 m_1}{EI} ds = -11776.1340/E$$

$$u_2 = \int_s \frac{m_0 m_2}{EI} ds = -23875.7760/E$$

$$u_3 = \int_s \frac{m_0 m_3}{EI} ds = -27264.7400/E$$

$$f_{11} = \int_s \frac{m_1 m_1}{EI} ds + \frac{L_1}{A_1 E} = 35.60180 + 1018.1785 = 1053.78030/E$$

$$f_{12} = \int_s \frac{m_1 m_2}{EI} ds = 56.02362/E$$

$$f_{13} = \int_s \frac{m_1 m_3}{EI} ds = 51.87099/E$$

$$f_{21} = \int_s \frac{m_2 m_1}{EI} ds = 56.02362/E$$

$$f_{22} = \int_s \frac{m_2^2}{EI} ds + \frac{L_2}{A_2 E} = 107.83205 + 1610.6911 = 1718.52315/E$$

$$f_{23} = \int_s \frac{m_2 m_3}{EI} ds = 107.27478/E$$

$$f_{31} = \int_s \frac{m_3 m_1}{EI} ds = 51.87090/E$$

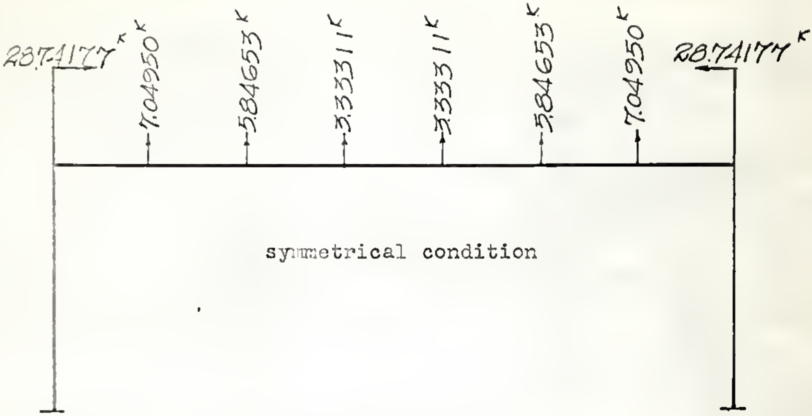
$$f_{32} = \int_s \frac{m_3 m_2}{EI} ds = 107.27478/E$$

$$f_{33} = \int_s \frac{m_3 m_3}{EI} ds + \frac{L_3}{A_3 E} = 129.05937 + 2276.9258 = 2405.98517/E$$

Those influence coefficients can be put into a matrix equation as

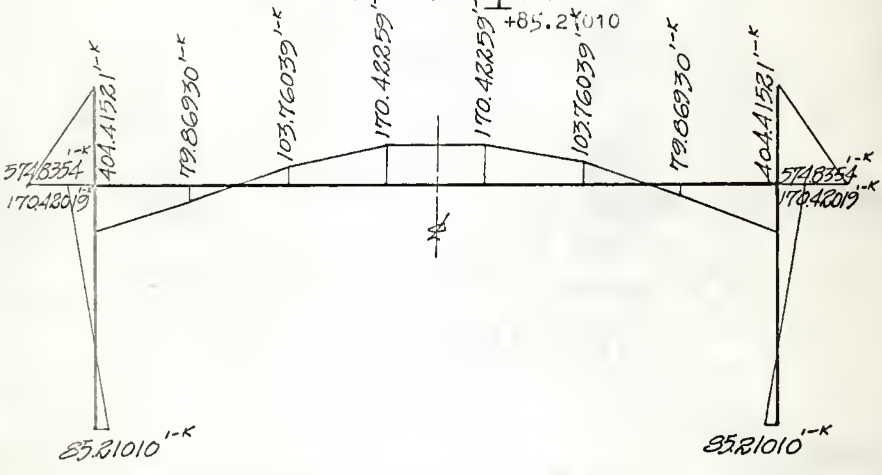
$$\begin{bmatrix} 1053.78030 & 56.02362 & 51.87099 \\ 56.02362 & 1718.52315 & 107.27478 \\ 51.87099 & 107.27478 & 2405.98517 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = - \begin{bmatrix} -11776.13400 \\ -23875.77600 \\ -27264.74000 \end{bmatrix}$$

The inversion of the matrix and the solution for the cable stressed have been done by the Computer (see Appendix b), answer as follows:



(a) Loads on the frame due to cable stresses

.0	0.987	0.013
-574.83540	0	+402.17057
0	+170.42019	+2.24464
-574.8354	+170.42019	+404.41521
		+85.21010



(b) Moment diagram due to cable stresses

Fig. 20

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 0.0009514941 & -0.0000298211 & -0.0000191838 \\ -0.0000298211 & 0.0005844536 & -0.0000191838 \\ -0.0000191838 & -0.0000254159 & 0.0004171769 \end{bmatrix} \begin{bmatrix} 11776.1340 \\ 23875.7760 \\ 27264.7400 \end{bmatrix}$$

$$= \begin{bmatrix} 9.96988 \\ 13.08007 \\ 10.54149 \end{bmatrix}$$

Deflections of the base structure: (anti-symmetrical condition)

$$u_1 = \int_s \frac{m_0 m_1}{EI} ds = -31.86760/E$$

$$u_2 = \int_s \frac{m_0 m_2}{EI} ds = -83.58460/E$$

$$u_3 = \int_s \frac{m_0 m_3}{EI} ds = -122.40455/E$$

$$f_{11} = \int_s \frac{m_1 m_1}{EI} ds + \frac{L_1}{A_1 E} = 59.32237 + 1018.1785 = 1077.50087/E$$

$$f_{12} = \int_s \frac{m_1 m_2}{EI} ds = 104.96799/E$$

$$f_{13} = \int_s \frac{m_1 m_3}{EI} ds = 112.89622/E$$

$$f_{21} = \int_s \frac{m_2 m_1}{EI} ds = 104.96799/E$$

$$f_{22} = \int_s \frac{m_2 m_2}{EI} ds + \frac{L_2}{A_2 E} = 229.23666 + 1610.6911 = 1839.92776/E$$

$$f_{23} = \int_s \frac{m_2 m_3}{EI} ds = 273.73558/E$$

$$f_{31} = \int_s \frac{m_3 m_1}{EI} ds = 122.89622/E$$

$$f_{32} = \int_s \frac{m_3 m_2}{EI} ds = 273.73558/E$$

$$f_{33} = \int_s \frac{m_3 m_3}{EI} ds + \frac{L_3}{A_3 E} = 419.98676 + 2276.8258 = 2696.91256/E$$

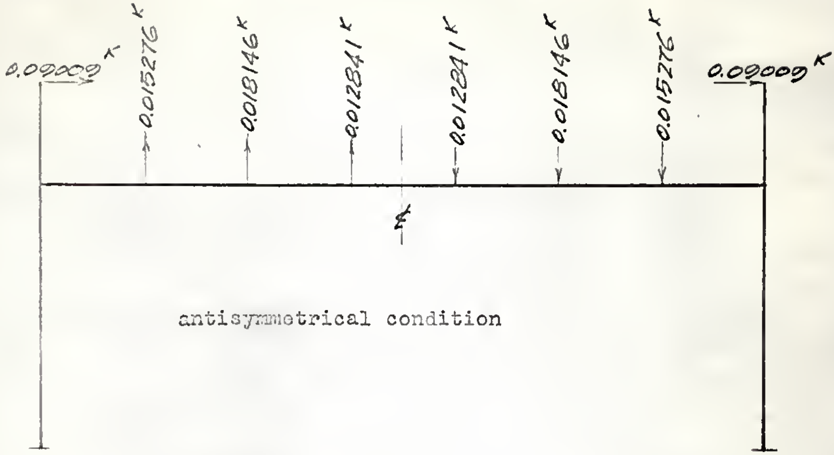
put into matrix equation thus

$$\begin{bmatrix} 1077.50087 & 104.96799 & 112.89622 \\ 104.96799 & 1839.92776 & 273.73558 \\ 122.89622 & 273.73558 & 2696.91256 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = - \begin{bmatrix} -31.86760 \\ -83.58460 \\ -122.40455 \end{bmatrix}$$

The solution from Computer output gives,

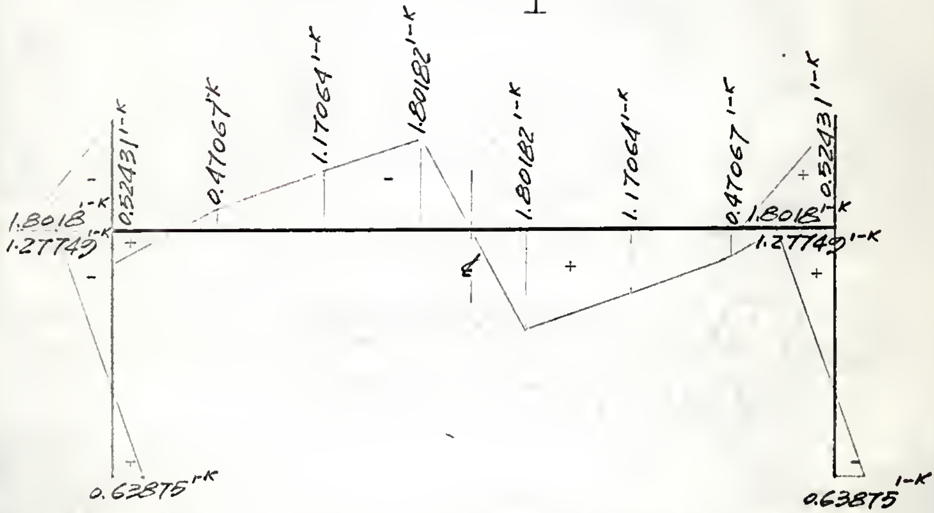
$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 0.0009363737 & -0.0000483181 & -0.0000342935 \\ -0.0000483181 & 0.0005543259 & -0.0000342935 \\ -0.0000342935 & -0.0000542412 & 0.0003777354 \end{bmatrix} \begin{bmatrix} 31.86760 \\ 83.58460 \\ 122.40455 \end{bmatrix}$$

$$= \begin{bmatrix} 0.0216036 \\ 0.0405956 \\ 0.0406099 \end{bmatrix}$$



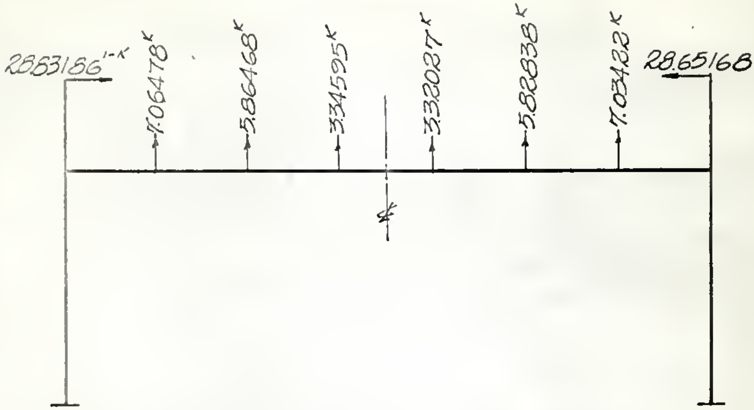
(a) Loads on frame due to cable stresses

0	0.962	0.038
-1.8018	0	+0.47385
0	+1.27749	+0.05046
-1.8018	+1.27749	+0.52431

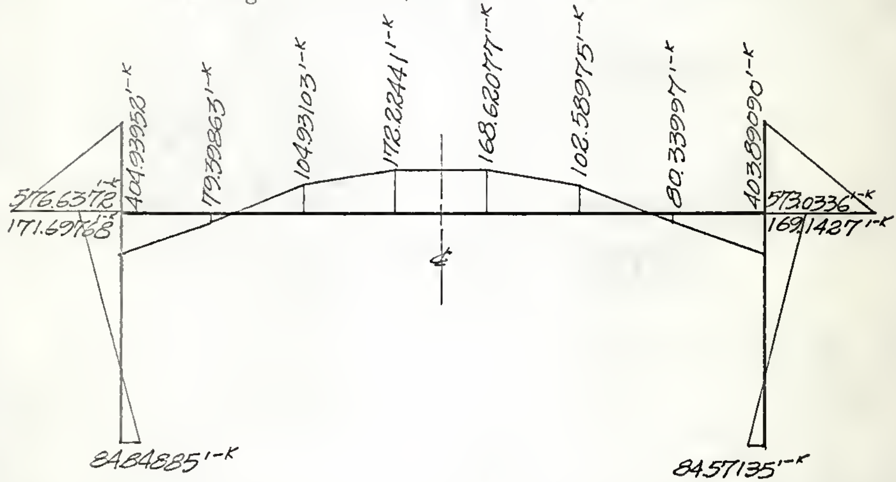


(b) Moment diagram for antisymmetrical cable stresses

Fig. 21

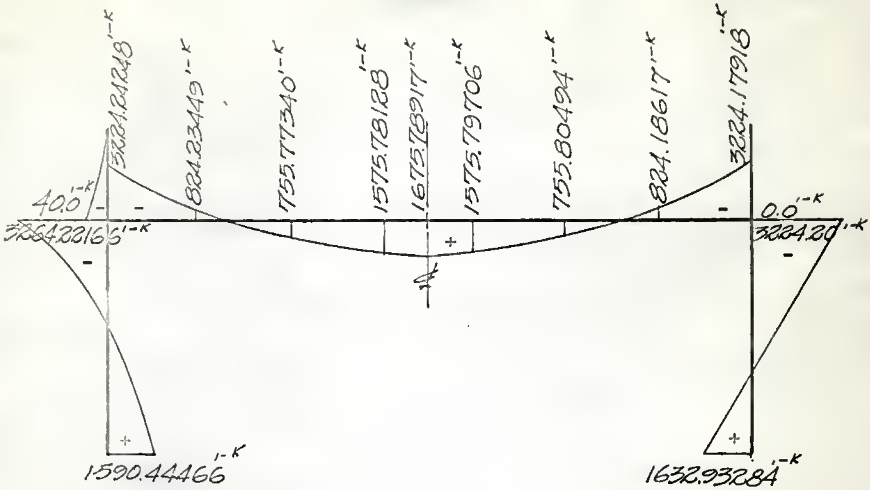


(a) Loading on the frame due to cable stresses
(summation of symmetrical and anti-symmetrical
loading conditions)

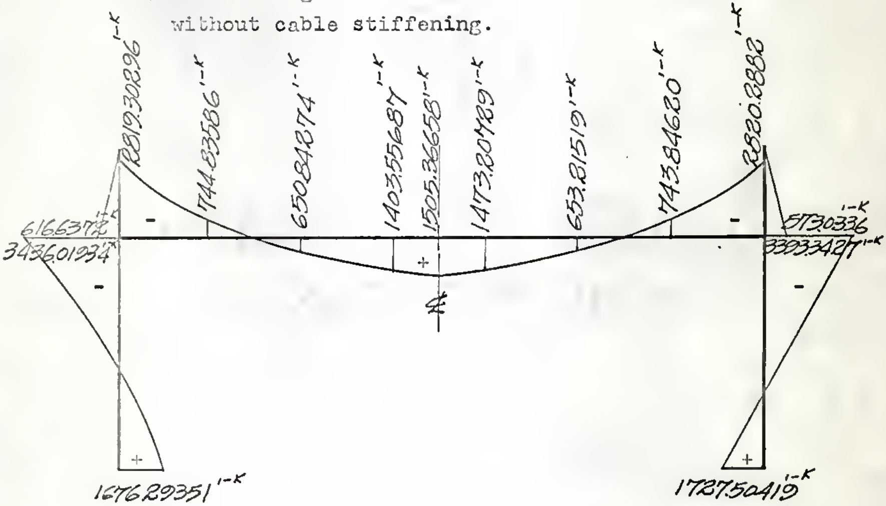


(b) Moment diagram of the frame due to cable stresses.

Fig. 22



(a) Moment diagram of the frame due to external loads without cable stiffening.



(b) Final Moment diagram of the frame due to external loads with cable stiffening.

Fig. 23

APPLICATION OF HAUNCH MEMBER THEOREM TO CABLE STIFFENED STRUCTURE ANALYSIS

In concerning the elastic behavior of a reinforced concrete member at low stresses up to about f'c/2, the concrete is seen to behave nearly elastic, that is, stresses and strains are quite closely proportional, because the compression strain in the concrete at any given load is equal to the compression strain in the steel.

$$\varepsilon_c = f_c/E_c = \varepsilon_s = f_s/E_s \quad \text{so that,}$$

$$f_s = \frac{E_s}{E_c} f_c = nf_c, \quad \text{where } n = E_s/E_c \text{ is known as the modular ratio.}$$

Let A_c = the net area of concrete and A_s = area of reinforcing bars. Assume a concrete beam is under axial load P, that

$$P = f_cA_c + f_sA_s = f_cA_c + nf_cA_s = f_c(A_c + nA_s) \qquad (33)$$

The term $(A_c + nA_s)$ can be interpreted as the area of a fictious concrete cross section, called transformed area, which is subject to the particular concrete stress f_c result in the same axial load P as the section composed of both steel and concrete. This transformed area consists of the actual concrete area plus n times the area of reinforcement.

If a beam is under bending moment, it is assumed that compression will occur on the top of the section, and the stresses are carried by concrete and compression steel, if any. There is also tension in the bottom of the beam section, and the stresses are carried by steels.

It is assumed that the concrete is not capable of resisting any tension, so the shape of the transformed section as shown in Fig. 24 (b) is acting by internal force as the stresses shown in Fig. 24 (c). Let A_t = total transformed areas, and I_t = moment of inertia of the transformed area. The maximum stress in concrete is

$$f_{cmax} = \frac{P}{A_t} + \frac{Mc_c}{I_t}$$

and the tension stress in steel is

$$f_s = n(\frac{P}{A_t} - \frac{Mc_s}{I_t}$$

where P is compression force of the section and M is the bending moment. Therefore, once the transformed section has been obtained, the usual methods of analysis of elastic homogeneous beam can be applied.

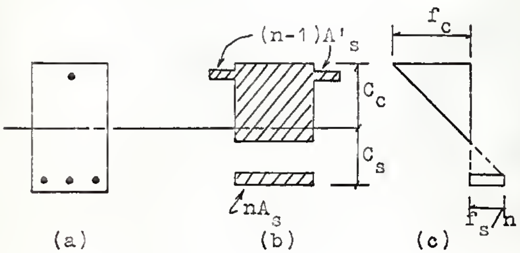


Fig. 24

The transformed section method is usually applied in the reinforced concrete members. The author wants to use this method in the cable stiffened structures. It is advantageous to increase moment of inertia at the ends of cable stiffened member without increasing concrete quantities.

In Fig. 25 is shown a cable stiffened rigid frame, the cables on this structure really work as the reinforcing steel to the concrete beam, because the cables just take part of the tension from the reinforcing steel in the beam, so that the concrete and steel section is reduced due to cable stiffening. If the cable sections are transformed into a fictious concrete section, the beam is really stiffened as a haunched member, and this structure can be analyzed as a haunch member structure.

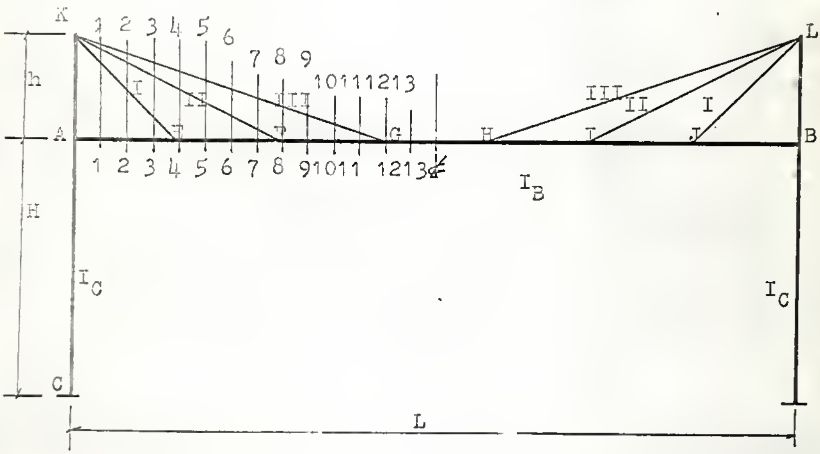


Fig. 25

It is necessary to illustrate some fundamental theory of haunch members in order to apply to the cable stiffed structures analysis. A haunch member structure can be analyzed by general moment distribution method after the stiffness, carry-over factor, and fixed end moment have been determined.

Now a cable stiffened structure as shown in Fig. 25 is taken into consideration. This is a long span rigid frame with cables hanging on the beam top. It is a rigid frame of uniform section, span L, the lower column height is H and the upper column height is h. The beam AB is hanged by six cables equally distanced at points E, F, G, H, I, J, and three of the cables on each side are built in the top of the upper columns at K and L respectively. The moment of inertia of the beam is I_B and column is I_C.

Assume the beam AB is divided into equal span L as shown in Fig. 25, the sections 1-1, 2-2, . . .n-n. Because it is considered that both the cables and the beam taking the loads acting on the beam, so the transformed areas on each section of the beam can be divided into four groups on each side of the frame. That is, the sections from A to E of the transformed areas consist of the actual beam sectional area and of the cables; the sections from E to F consist of beam area plus two fictitious concrete areas of two cables; the sections from F to G consist of the beam areas and one fictitious concrete area of cable; and the sections from G to H is the beam cross sectional areas, because there is no cable through these sections.

Let cable KE = LJ = cable 1, cable KF = LI = cable 2 and cable KG = LH = cable 3, so the transformed areas on each sectional group will be,

from A to E, $A_1 = A_c + n(A_{s1} + A_{s2} + A_{s3})$

where A_{s1}, A_{s2}, A_{s3} are the cross sectional areas of each cable.

From E to F, $A_2 = A_c + n(A_{s2} + A_{s3})$.

From F to G, $A_3 = A_c + n(A_{s3})$.

From G to H, $A_4 = A_c$.

In calculation of moment of inertia for each section, it is necessary to locate the axis of center of gravity of those sections because the positions of the cables on each section are varied and the distances between the central axis of the cables and beam of each section are varied, and proportional to the slope of the cables. For the sake of simplicity, the central axis of beam AB is taken as the central axis of each transformed section, because the section of the cables is so small in comparison to the concrete beam section, so that the error in location of the central of gravity of the transformed sections is small and neglegible.

The moment of inertia of the transformed sections will be,

$$I_x = I_B + nA_s h_x^2 \qquad (34)$$

where the subscript x is the location of the section number counted from A. For example, in section 1-1, the moment of inertia of this section is $I_1 = I_B + n(A_{s1}h_{11}^2 + A_{s2}h_{12}^2 + A_{s3}h_{13}^2)$, where h_{12} means the vertical distance of cable 1 to the beam axis at section 1, and so do h_{12} and h_{13}. For further examples, the moment of inertia of section 5-5 is $I_5 = I_B + n(A_{s2}h_{52}^2 + A_{s3}h_{53}^2)$ and in section 10-10 that $I_{10} = I_B + n(A_{s3}h_{103}^2)$. Now the location h_x for each cable at each section of the frame shown in Fig. 25 can be written in matrix form as,

$$\{h_x\} = \begin{bmatrix} 1 & 1 & 1 \\ 3/4 & 7/8 & 11/12 \\ 2/4 & 6/8 & 10/12 \\ 1/4 & 5/8 & 9/12 \\ 0 & 4/8 & 8/12 \\ 0 & 3/8 & 7/12 \\ 0 & 2/8 & 6/12 \\ 0 & 1/8 & 5/12 \\ 0 & 0 & 4/12 \\ 0 & 0 & 3/12 \\ 0 & 0 & 2/12 \\ 0 & 0 & 1/12 \end{bmatrix} \begin{Bmatrix} h \\ h \\ h \end{Bmatrix} = \begin{bmatrix} 4 & 8 & 12 \\ 3 & 7 & 11 \\ 2 & 6 & 10 \\ 1 & 5 & 9 \\ 0 & 4 & 8 \\ 0 & 3 & 7 \\ 0 & 2 & 6 \\ 0 & 1 & 5 \\ 0 & 0 & 4 \\ 0 & 0 & 3 \\ 0 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} h/4 \\ h/8 \\ h/12 \end{Bmatrix}$$

The moment of inertia for each section can be found by equation (34) and the variable moment of inertias can be expressed as

$$\begin{bmatrix} I_1 \\ \cdot \\ \cdot \\ \cdot \\ I_n \end{bmatrix} = \begin{bmatrix} (h_{ij})^2 \end{bmatrix} \begin{bmatrix} nA_{s1} \\ nA_{s2} \\ nA_{s3} \end{bmatrix} + \begin{bmatrix} I_B \\ \cdot \\ \cdot \\ \cdot \\ I_B \end{bmatrix} \qquad (35)$$

A diagram can be ploted for the moment of inertia in each section as shown in Fig. 36. It is obvious that the moments of inertia near the supports are appreciably larger than the central portion of the beam, which has the same properties as the ordinary haunch members.

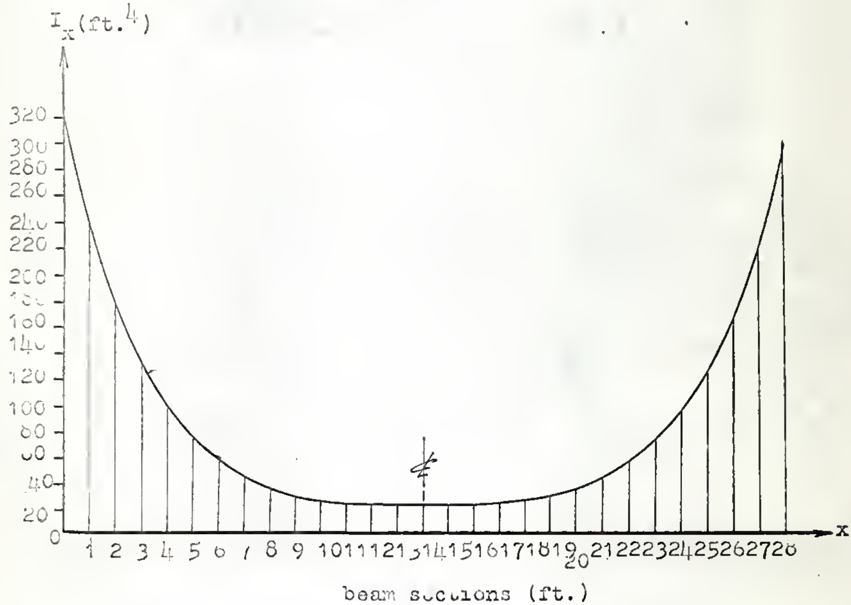


Fig. 26 I_x diagram

The bending moment in a rigid frame with variable moment of inertia will undergo the following change:

(1) the negative moment at the end AB becomes larger than for beam with constant I.

(2) the length of the region subjected to negative moments becomes larger.

(3) the positive bending moment in the center becomes smaller than for the beam with constant I.

(4) the increase of the moment of inertia at the fixed end has the effect of putting more bending moment near the supports.

From the characteristics of a haunch member as discribed above, for the cable stiffened structure, the increased negative moment produced at each end shall be taken by the cables, because the tension forces at each cable attachment form a moment couple with respect to the ends A and B. In other words, the fixed end moments of beam AB is really reduced due to cable stiffening, but in this case the bending moment of columns at points A and B shall be increased due to the increased tension forces of cables at the top of the columns. If this structure can be modified by extending cables from points K and L to the ground E and F as shown in Fig. 27, then both the moments of beam and columns can be reduced by cable stiffening. This kind of arrangement is better than the one shown in Fig. 25 unless it is architecturally displeasing.

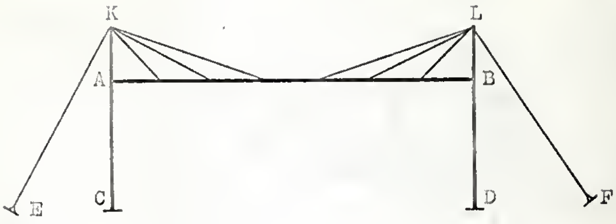
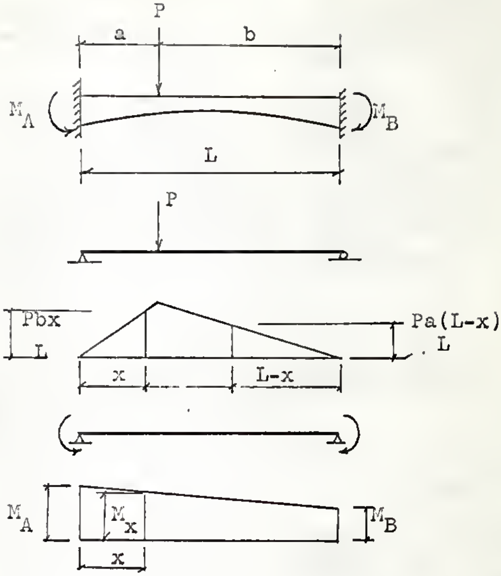


Fig. 27

In order to show the analysis of cable stiffened rigid frame as a general rigid frame with variable moment of inertia, it is necessary to derive some formulas for beam with variable moments of inertia.

(1) Fixed end moment:

(a) For concentrated load condition general expressions for the fixed end moments of this kind beam as shown in Fig. 28 can be found by stating that the changes in slope and vertical deflection between the ends of the beam must be zero, that is



$$\int_0^L \frac{M dx}{EI} = 0$$

Fig. 28

$$\int_0^L \frac{M x dx}{EI} = 0$$

Where both M and I are function of x. The bending moment of this beam can be expressed in two parts, that is, the portion due to the concentrate load acting on a simply supported beam, and the part caused by two end moments. So the integration across the beam must be made in two steps due to the discontinuity of the moment curve for the simply supported beam.

$$\int_0^a \frac{Pbx}{LI}dx + \int_a^L \frac{Pa(L-x)}{LI}dx - \int_0^L \left[M_A - \frac{x}{L}(M_A-M_B)\frac{dx}{I}\right] = 0 \qquad (36a)$$

$$\int_0^a \frac{Pbx^2}{LI}dx + \int_a^L \frac{Pax(L-x)}{LI}dx - \int_0^L \left[M_A - \frac{x}{L}(M_A-M_B)\frac{xdx}{I}\right] = 0 \qquad (36b)$$

Equation (36a) can be written in the following form by adding and substracting $\int_0^L \frac{xdx}{LI}dx$, thus

$$\frac{Pb}{L}\int_0^L \frac{xdx}{I} - P\int_0^L \frac{(x-a)}{LI}dx = M_A\int_0^L \frac{dx}{I} - \frac{M_A-M_B}{L}\int_0^L \frac{xdx}{I} \qquad (37a)$$

Likewise the equation (36b) can be expressed thus:

$$\frac{Pb}{L}\int_0^L \frac{x^2dx}{I} - P\int_0^L \frac{x(x-a)}{I}dx = M_A\int_0^L \frac{xdx}{I} - \frac{M_A-M_B}{L}\int_0^L \frac{x^2dx}{I} \qquad (37b)$$

Eliminating M_A-M_B from equation (37a) and (37b) and solving for M_A gives,

$$M_A = \frac{\int_0^L \frac{xdx}{I}\int_a^L \frac{x(x-a)}{I}dx - \int_0^L \frac{x^2dx}{I}\int_0^L \frac{(x-a)}{I}dx}{\int_0^L \frac{dx}{I}\int_0^L \frac{x^2dx}{I} - \left(\int_0^L \frac{xdx}{I}\right)^2} \qquad (38)$$

A similar expression for M_B can be obtained by changing a to b.

(b) For a uniform load condition; in case a load w is uniformly distributed over the entire span of a beam with variable moment of inertia, the fixed end moment at the left end can be found in a similar manner, thus

$$M_A = \frac{w}{2}x \frac{\int_0^L \frac{xdx}{I} \int_0^L \frac{x^2dx}{I} - \left[\int_0^L \frac{x^2dx}{I}\right]^2}{\int_0^L \frac{dx}{I} \int_0^L \frac{x^2dx}{I} - \left[\int_0^L \frac{xdx}{I}\right]^2} \tag{39}$$

M_B can be found from equation (39) by performing the integration from the opposite direction.

(2) Carry-over factor:

The carry-over factor is the ratio of the moment induced at the fixed end of a member to the moment causing rotation at the other simply supported end. In Fig. 29 is shown a beam with variable moment of inertia subjected to the action of a moment M at the simply supported end A and resisted by a moment of magnitude CM at the fixed end B. When deflection at A is zero, then

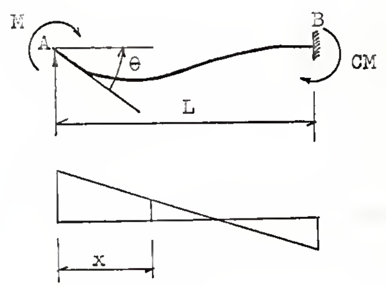


Fig. 29

$$\int_0^L \frac{(1-x/L)M - \frac{x}{L}CM}{EI} x dx = 0 \qquad (40)$$

Integrating and solving for C gives the carry-over factor from A to B is

$$C_{AB} = \frac{L\int_0^L \frac{xdx}{I} - \int_0^L \frac{x^2dx}{I}}{\int_0^L \frac{x^2dx}{I}} \qquad (41)$$

The carry-over factor from B to A can be found by equating the deflection at B to zero, as A is fixed, which gives

$$C_{BA} = \frac{L\int_0^L \frac{xdx}{I} - \int_0^L \frac{x^2dx}{I}}{\int_0^L \frac{dx}{I} - 2L\int_0^L \frac{xdx}{I} \int_0^L \frac{x^2dx}{I} + \int_0^L \frac{x^2dx}{I}} \qquad (42)$$

(3) Stiffness:

The stiffness factor at one end of a beam is the moment required to produce unit rotation at one end, which is assumed simply supported while the other end is fixed. According to the principle of moment area,

$$\int_0^L \frac{M}{EI}\left[1 - \frac{x}{L}(1 + C)\right] dx = \theta \qquad (43)$$

when $\theta = 1$, K may be substituted for M and solve equation (43) and gives

$$K = \frac{E}{\int_0^L \frac{dx}{I} - \frac{1+C}{L}\int_0^L \frac{x\,dx}{I}} \tag{44}$$

If the value of the carry-over factor C in equation (41) is substituted in equation (44), the expression becomes

$$K_{AB} = E\,\frac{\int_0^L \frac{x^2dx}{I}}{\int_0^L \frac{dx}{I}\int_0^L \frac{x^2dx}{I} - \left[\int_0^L \frac{x\,dx}{I}\right]^2} \tag{45}$$

Similarly, the stiffness K_{BA} can be found by substituting C_{BA} to equation (44) and gives

$$K_{BA} = E\,\frac{\int_0^L \frac{dx}{I} - 2\int_0^L \frac{x\,dx}{I} + \int_0^L \frac{x^2dx}{I}}{\int_0^L \frac{dx}{I}\int_0^L \frac{x^2dx}{I} - \left(\int_0^L \frac{x\,dx}{I}\right)^2} \tag{46}$$

(4) Lateral stiffness:

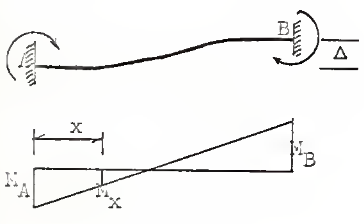


Fig. 30

The lateral stiffness is the moment produced at one end of a member fixed against rotation, if the other, also fixed against rotation, is displaced laterally with respect to the first end. In Fig. 30 is shown a beam with variable moment of inertia, in which the stiffness K_{AB}, K_{BA} and carry-over factor C_{AB}, C_{BA} have already been determined. The end A of the beam has been displaced an amount Δ . It is desired to find fixed end moments M_A and M_B. Because there are no forces acting on the beam, so the moment at a point, at a distance x from the end, is

$$M_x = M_A - \frac{x}{L}(M_A + M_B) \tag{47}$$

According to the principle of moment area, the parallel tangents to the elastic curve at the end can be expressed as

$$\int_0^L (M_A - \frac{x}{L}(M_A + M_B)) \frac{dx}{I} = 0 \tag{48}$$

The vertical displacement $-\Delta$ between the ends can be expressed as

$$\int_0^L (M_A - \frac{x}{L}(M_A + M_B) \frac{xdx}{EI} = -\Delta \tag{49}$$

Equation (48) and equation (49) can be written thus:

$$M_A L \int_0^L \frac{dx}{I} - (M_A + M_B) \int_0^L \frac{xdx}{I} = 0 \tag{50}$$

$$M_A L \int_0^L \frac{xdx}{I} - (M_A + M_B) \int_0^L \frac{x^2 dx}{I} = -E\Delta \tag{51}$$

Solving for M_A gives,

$$M_A = E\Delta \frac{\int_0^L \frac{x\,dx}{I}}{\int_0^L \frac{dx}{I} \int_0^L \frac{x^2 dx}{I} - \left[\int_0^L \frac{x\,dx}{I} \right]^2} \tag{52}$$

and also can be written:

$$M_A = \Delta \frac{E \int_0^L \frac{x^2 dx}{I}}{\int_0^L \frac{dx}{I} \int_0^L \frac{x^2 dx}{I} - \left[\int_0^L \frac{x\,dx}{I} \right]^2} \times \frac{\int_0^L \frac{x\,dx}{I}}{\int_0^L \frac{x^2 dx}{I}} \tag{53}$$

where the first fraction in equation (53) is the stiffness and the last fraction is equal to (1+C)÷ 1, therefore

$$M_A = \frac{\Delta}{L} K_{AB}(1 + C_{AB}) \tag{54}$$

Similarly

$$M_B = \frac{\Delta}{L} K_{BA}(1 + C_{BA}) \tag{55}$$

The above derived formulas are the necessary fcators for solving frames with variable moment of inertia. A numerical example will be given to describe the method of applying haunch member theories to solve cable stiffened structures.

NUMERICAL EXAMPLE 3

Find the fixed end moment of the cable stiffened rigid frame as shown in example 1. Consider the horizontal cable stiffened girder as a haunched member. The frame is uniform loaded with 2.25 k/ft.

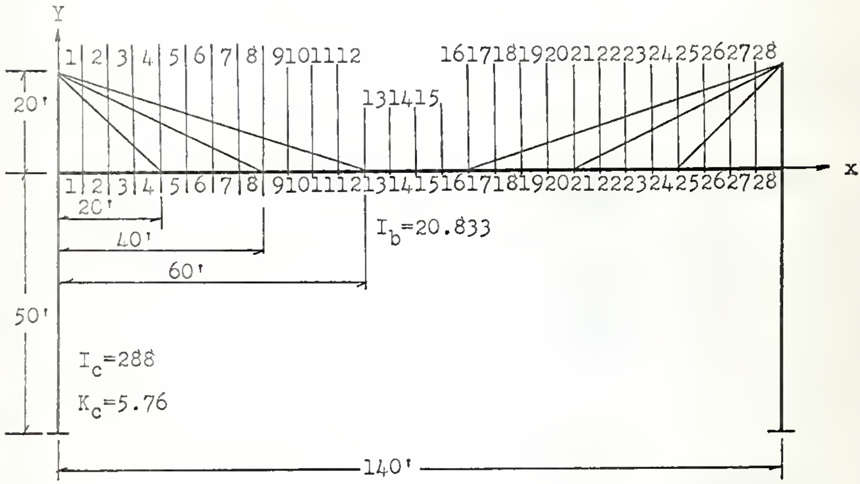


Fig. 31

In the frame as shown in Fig. 31, the horizontal cable stiffened girder is divided into 28 equal intervals, as shown in the figure, with sections from 1-1 to 28-28, each interval being 5 feet long, the frame is 2'wide.

The first step is to locate the new axis of each section for the transformed areas. For the uniform horizontal girder, the central axes are all the same throughout the sections, and are loeated at the half way of girder depth. Due to the additional transform area of cables, the central axis for each seetion will move a little amount upward from the central axis of the girder. The eecentricity for locating the new axis will show in the following table.

The seeond step is to ealculate the moment of inertia in respeet to the new axis for each section. It is simple without mathmatic expression.

The third step is to caleulate the carry-over factor and stiffness. First, a unit moment=1 is applied at seetion 1, then at eaeh section, and the corresponding moment due to the unit moment can be found by proportion, so that the diagram for moment divided by moment of inertia, (M/I) can be plotted as shown in Fig. 32.

The slopes of the M/I loading diagram ean be found by summation of those ordinates from section 1 to 28, and take moment in respeet to the left end of section 1. Let the slope θ_0 be the reaction of the girder loaded with M/I diagram at point 0, and θ_{28} be the reaction of the girder, for the same loading condition, at end point 28, and get the relationship between stiffness S and carry-over factor C as:

$$Sx\theta_0 - CSx\theta_{28} = 1$$

$$Sx\theta_{28} - CSx\theta_0 = 0$$

The results of the above simultaneous equations gives the earry-over factor and stiffness. From the computer answer,

TABLE 1

Section No.	Transform Section Areas (ft^2)	Essentricity (ft)	Moment of Inertias (ft^4)
28	10.75006	1.39545270	299.923540
27	10.75006	1.18225850	222.664900
26	10.75006	0.96906436	161.444290
25	10.75006	0.75587025	116.261710
24	10.75006	0.54267611	87.117154
23	10.50004	0.45638405	66.740130
22	10.50004	0.35717016	50.746025
21	10.50004	0.25795627	39.059457
20	10.50004	0.15874237	31.680424
19	10.25002	0.12196085	26.931048
18	10.25002	0.08130727	23.543246
17	10.25002	0.04065369	21.510563
16	10.00000	0.00	20.833333
15	10.00000	0.00	20.833333
14	10.00000	0.00	20.833333
13	10.00000	0.00	20.833333
12	10.25002	0.04065369	21.510563
11	10.25002	0.08130727	23.543246
10	10.25002	0.12196085	26.931048
9	10.50004	0.15874237	31.680424
8	10.50004	0.25795627	39.059457
7	10.50004	0.35717016	50.746025
6	10.50004	0.45638405	66.740130
5	10.75006	0.54267611	87.117154
4	10.75006	0.75587025	116.261710
3	10.75006	0.96906436	161.444290
2	10.75006	1.18225850	222.664900
1	10.75006	1.39545270	299.923540

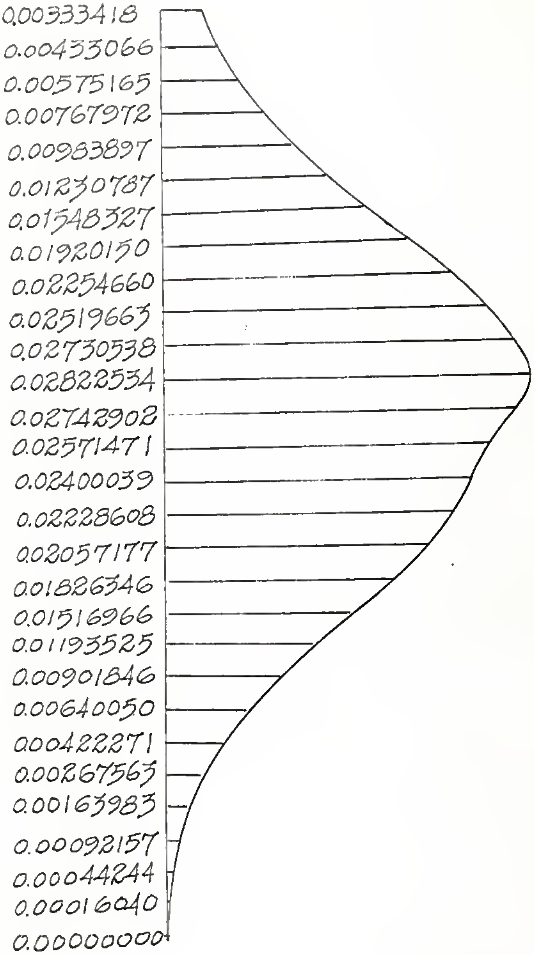


Fig. 32 M/I diagram

there are:

carry-over factor = 0.73432514   stiffness = 2.03819740

Because the frame is symmetrical, so the carry-over factors are same for both ends.

The fourth step is to calculate the fixed end moment influence line. This is done by the conjugate beam method. First cut the fixed end O and put a moment and a reaction there. This cantilever beam is loaded with a moment and an upward force at point O, so the M/I loading diagram for this cantrilever beam can be found as described in step 2. The reaction of the fixed end for the M/I loading is the summation of the loading diagram. So the slope theta is the loading diagram, as shown in Fig. 33.

The deflection of the cantilever is the fixed end moments for the M/I loadings. The parts of this deflection curve enclosed by a straight line through both ends of the curve is the fixed end moment influence line of the girder, which is the same for both ends due to symmetrical condition. The deflection of the cantilever and the fixed end moment influence line of the girder are shown in Fig. 34.

The fifth step is to calculate the fixed end moment due to external loads, which is the integration of the moment influence areas times the uniform loads, and it gives FEM = 3996.9413260875 kips-ft.

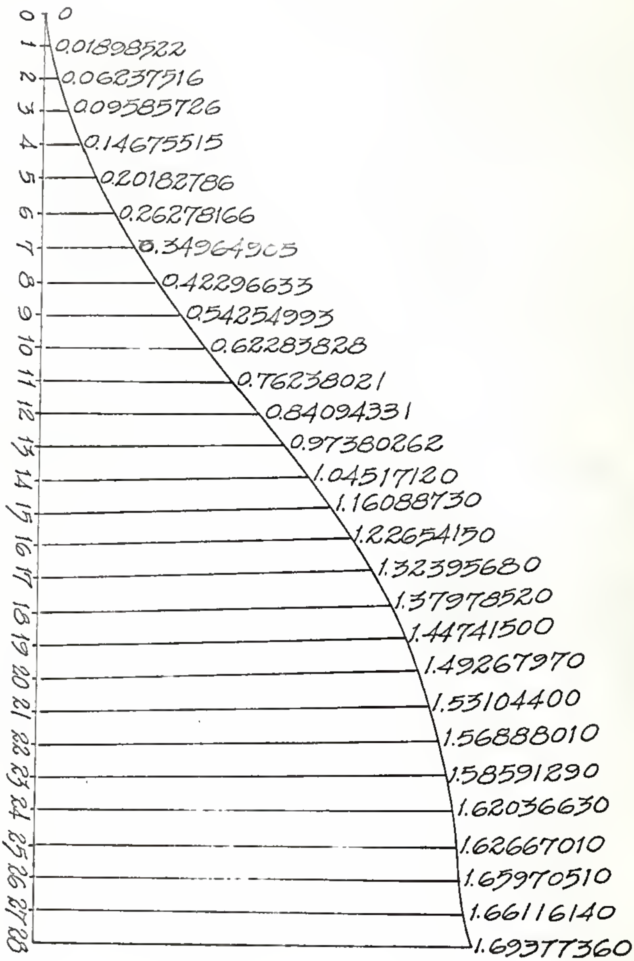
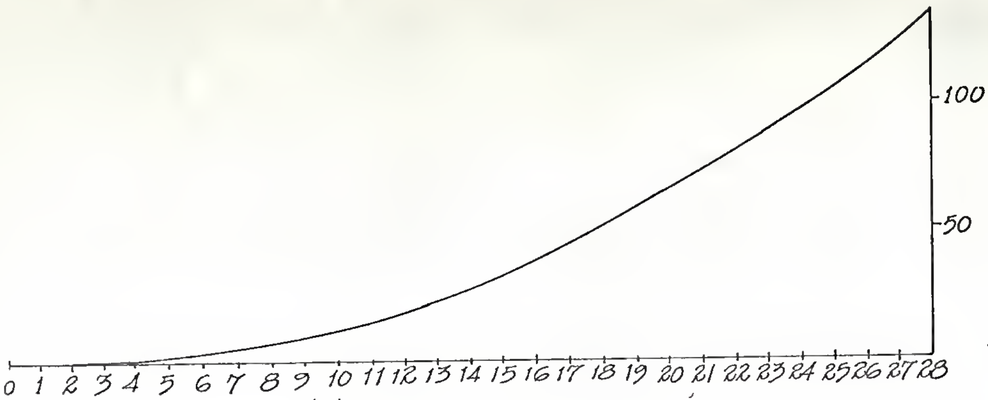
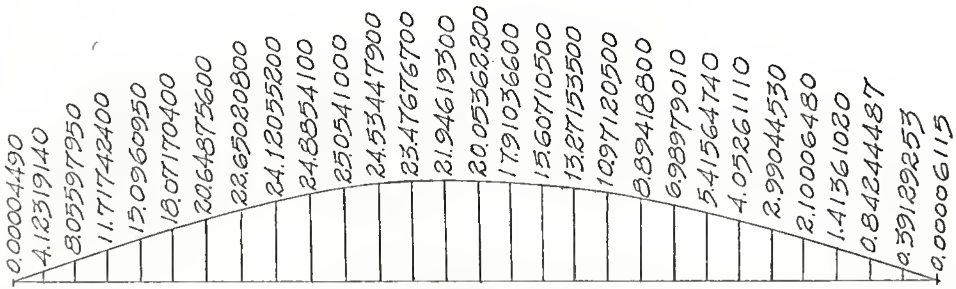


Fig. 33 Theta diagram (Cantilevel)



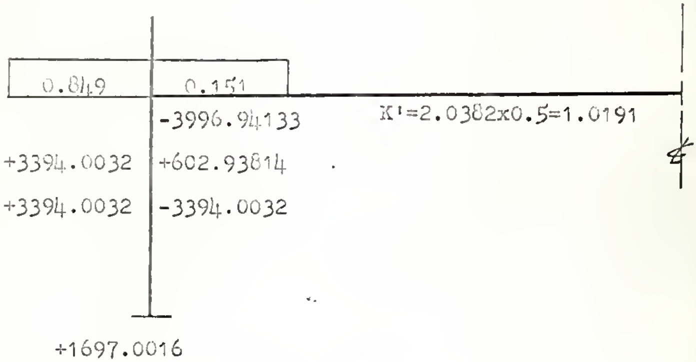
(a) Deflection curve



(b) Fixed end moment influence line

Fig. 34

The final step is to balance the fixed end moment by moment distribution, which gives:



(NOTE: These moments are in ft-kips)

The fixed end moments are somewhat larger than the foregoing examples, because the negative moment for a frame with variable moment of inertias is larger than for frame with constant moment of inertia. These results are the approximate answer, because this is an approximate analysis. The more smaller interval sections are divided, the more accurate the answer it gives. The moment of the girder end is the total moment. In designing the horizontal girder, the reinforcement for this moment, deduct the cable areas is the actual steel areas for the horizontal girder reinforcement because the cables take part of reinforcing function from the girder reinforcement.

This method is not suggested for actual design, because it is quite laborious to calculate section by section, and it is impossible to give high accuracy. The purpose of this example is to introduce one kind of method for cable stiffened structure analysis.

CONCLUSION

A cable stiffened structure usually is a composited structure. The cable can be used as stiffener for any kind of reinforced concrete rigid frame, steel frame or truss as well as timber structures. This kind of composite structure always combines different stress conditions in one structure: the cable takes tension only; the frame can take moment, shear and thrust; and the truss is assumed to take direct force and shear. Therefore, a cable stiffened structure always has a high degree of indeterminacy. In this condition, the best way is to use the principle of superposition to solve the simultaneous equation.

In this report the author presents a typical long spanned rigid frame with six cables hanging on the top, which has been analyzed by both superposition method and haunched member method. Checking the numerical answers from the numerical examples shows that the fixed end moments are somewhat different due to different approaches, but the method of superposition is more accurate than the haunched member method because the haunched member is calculated by taking small sections. If more smaller sections are divided, more accurate results can be obtained.

Three numerical examples have been presented in this report. The first one is calculated by superposition method and the frame is considered to be loaded with vertical dead and live loads only. The frame is analyzed in symmetrical condition and has a moment reduced by the cable about one-fourth of the original. Here the author has tried to control the fixed end moment to be reduced in an amount as desired. It is done by prestressing the cables in a definite vertical lengthening. The numerical results for this pretension work are satisfied. The second example shows the frame under lateral wind stresses, assumes no live load on the top, and analyzes it by superposing the symmetrical and antisymmetrical conditions. The results show that under normal dead load condition, the cables are still all in tension due to lateral wind stresses and the frame is little influenced by the live load, because the span is very large and the dead load of the structure is much larger than the live load. The third example shows that the fixed end moment of the frame is calculated by assuming the horizontal girder is haunched by cables, which gives the fixed end moment for haunch. After moment distribution, it is found that the girder end moment is larger and the column moment is smaller than in the first example.

This report has presented computer method for solving problems. The major part of the three examples are calculated by computer. Those examples also can be calculated by hand, but with much labor. It is evident that it is more convenient to solve simultaneous equations by computer in analyzing high degree inderterminate cable stiffened structures.

In conclusion, the cable stiffened structure is very useful in modern architectural structure design. There are many possible ways of structural analysis for cable stiffened structures. It is the author's opinion that each method in a different approach can get agreement in result, but the time consumed and percentage of accuracy should be taken into consideration. Most of the calculations in this study have been by computer rather than slide rule.

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APPENDIX A

NOTATIONS AND ABBREVIATIONS

APPENDIX A NOTATIONS AND ABBREVIATIONS

A_c	:	Cross sectional area of concrete girder
A_s	:	Cross sectional area of steel or cable
AE	:	External rigidity of cable
I_b	:	Moment of inertia of beam
I_c	:	Moment of inertia of column
I_t	:	Moment of inertia of the transformed area
EI	:	The flexural rigidity of concrete beam
η	:	coefficient for extensional rigidity of the effective straight cable
δT	:	The increment of cable tension
δA	:	A unit deflection at point A
ΔL	:	Elongation of cable
K	:	Stiffness of the frame
K'	:	The modified stiffness
L	:	The actual length of cable
X	:	The cable stress
U	:	The strain energy
m_0	:	Moment distribution due to the action of external load on the base structure
$m_n X_n$	:	The moment distribution due to the action of the arbitrary constant X_n alone
f_{ij}	:	The influence coefficient of deflection on base structure
u	:	The deflection on the base structure due to applied load

[F]	:	Flexibility matrix
[X]	:	Column matrix for cable stress
[u]	:	Column matrix for displacemont
[L]	:	Lower traingular matrix
M	:	Moment in kip-ft.
Δh	:	The vertical olongation for cable prestressing
C_{AB}	:	Carry over factor from A to B
$n = E_s/E_c$	:	The modular ratio ofconcrete and steel
nAs	:	Transform area for cablo

' or ft.	:	foot(feet)
" or in.	:	inch
k	:	kip
sq.	:	square
# or lb	:	pound
Ø	:	Diamoter of cable
e	:	angle
k/'	:	kip per foot

APPENDIX B

COMPUTER PROGRAM , DATA AND OUTPUT

```
PROGRAM 1 FOR FIRST EXAMPLE
C   C  BEAM DEFLECTION AT CABLE PTS. DUE TO VERTICAL LOADS    JIN-SIEN GUO
       DIMENSION  X(15),Y(15),Z(15),ZM(15)
       BI=20.833
       CI=288.0
       DO 200 LL=1,4
       READ,XA,XB,XBC
       READ,(X(I),I=1,5)
       DO 100 K=1,3
       READ,YA,YB,YBC
       A1=25.*(XA*YA+XB*YB+(XA+XB)*(YA+YB))/3.
       A2=20.*XBC*YBC/3.
       READ,(Y(I),I=1,5)
       DO 10 I=1,5
    10 Z(I)=X(I)*Y(I)
       DO 20 I=1,3
       J=I+1
    20 ZM(I)=(X(I)+X(J))*(Y(I)+Y(J))/4.
       A=Z(1)+2.*(Z(2)+Z(3))+4.*(ZM(1)+ZM(2)+ZM(3))+Z(4)
       A=A*10./3.
       A=A+5.*(Z(4)+Z(5))
       A=A/BI+(A1+A2)/CI
       PUNCH 30,K,A
   100 CONTINUE
   200 CONTINUE
    30 FORMAT(I5,F18.8)
       STOP
       END
```

```
DATA
+1813.61250  -3627.22500  +.000
-3627.2250  -927.2250  +672.7750  +1772.7750  +1885.2750
+ .99702  -1.99403  -14.14160
+12.14797  -1.99403  -1.99403  -1.99403  -1.99403
+2.52087  -5.04174  -17.87900
+12.83726  +3.89139  -5.04174  -5.04174  -5.04174
+4.01246  -8.02491  -18.97140
+10.94649  +4.62270  -1.70128  -8.02491  -8.02491
+ .99702  -1.99403  -14.14160
+12.14797  -1.99403  -1.99403  -1.99403  -1.99403
+ .99702  -1.99403  -14.14160
+12.14797  -1.99403  -1.99403  -1.99403  -1.99403
+2.52087  -5.04174  -17.87900
+12.83726  +3.89139  -5.04174  -5.04174  -5.04174
+4.01246  -8.02491  -18.97140
+10.94649  +4.62270  -1.70128  -8.02491  -8.02491
+2.52087  -5. 4174  -17.87900
+12.83726  +3.89139  -5.04174  -5.04174  -5.04174
+2.52087  -5. 4174  -17.87900
+12.83726  +3.89139  -5.04174  -5.04174  -5.04174
+4.01246  -8.02491  -18.97140
+10.94649  +4.62270  -1.70128  -8.02491  -8.02491
+ .99702  -1.99403  -14.14160
+12.14797  -1.99403  -1.99403  -1.99403  -1.99403
+4.01245  -8.02491  -18.97140
+10.94649  +4.62270  -1.70128  -8.02491  -8.02491
+4.01246  -8.02491  -18.97140
+10.94649  +4.62270  -1.70128  -8.02491  -8.02491
+2.52087  -5.04174  -17.87900
+12.83726  +3.89139  -5.04174  -5.04174  -5.04174
+ .99702  -1.99403  -14.14160
+12.14797  -1.99403  -1.99403  -1.99403  -1.99403
OUTPUT
C   C   BEAM DEFLECTION AT CABLE PTS. DUE TO VERTICAL LOADS    JIN-SIEN GUO
    1   -18070.64300000
    2   -31524.61300000
    3   -31746.20100000
    1        55.08971900
    2        72.75584700
    3        66.19554700
    1       125.3755900
    2       123.31475000
    3        72.75584700
    1       134.56568000
    2       123.31475000
    3        66.19554700
      STOP   END OF PROGRAM AT  STATEMENT 0030 + 01 LINES
```

```
PROGRAM 2 FOR SECOND EXAMPLE
C   C  BEAM DEFLECTION AT CABLE PTS DUE TO WIND LOADS       JIN-SIEN GUO
       DIMENSION X(15,4),Y(16,4),D(4,4),PX(15),PY(16)
       BI=20.833
       CI=288.0
       DO 200 LL=1,2
       IF(LL-1)11,11,12
    11 PUNCH 81
       GO TO 31
    12 PUNCH 82
    31 DO 100 N=1,4
       READ, W,TY
       READ, Y(1,N),Y(11,N)
       READ, X(1,N),X(3,N),X(5,N),X(7,N),X(8,N)
       H=0.0
       DY=(Y(11,N)-Y(1,N))/10.
       DO 10 I=2,10
       H=H+5.
       J=I-1
    10 Y(I,N)=Y(J,N)+DY+(50.-H)*W*H/2.
       H=0.0
       ST=TY/20.
       DO 20 I=1,5
       J=17-I
       Y(J,N)=-W*H**2/2.+ST*H
    20 H=H+5.
       DO 30 I=2,6,2
       J=I-1
       K=I+1
    30 X(I,N)=(X(J,N)+X(K,N))/2.
   100 CONTINUE
       DO 300 IT=1,4
       IF(IT-1)310,310,320
   310 IN=IT+1
       GO TO 330
   320 IN=IT
   330 DO 350 ID=IN,4
       DO 40 I=1,16
    40 PY(I)=Y(I,IT)*Y(I,ID)
       DO 45 I=1,8
    45 PX(I)=X(I,IT)*X(I,ID)
       A1=0.0
       DO 50 I=2,10,2
       J=I-1
       K=I+1
    50 A1=A1+PY(J)+4.*PY(I)+PY(K)
```

```
      DO 60 I=13,15,2
      J=I-1
      K=I+1
   60 A1=A1+PY(J)+4.*PY(I)+PY(K)
      A1=A1*5./(3.*CI)
      A2=0.0
      DO 70 I=2,6,2
      J=I-1
      K=I+1
   70 A2=A2+PX(I)+4.*PX(I)+PX(K)
      A2=(A2*10./3.+(PX(7)+PX(8))*5.)/BI
      D(IT,ID)=A1+A2
  350 PUNCH 80,IT,ID,D(IT,ID)
  300 CONTINUE
  200 CONTINUE
   80 FORMAT(2I5,11HDEFLECTION=,F16.8)
   81 FORMAT(19HSYMMETRICAL LOADING)
   82 FORMAT(23HANTISYMMETRICAL LOADING)
      STOP
      END
DATA
+ .1        -20.0
+1611.68875  -3244.21083
-3224.21083   -844.21083   +755.78917   1555.78917   1675.78917
  .0        -14.14160
+ .99702  -1.99403
+12.14797  -1.99403  -1.99403  -1.99403  -1.99403
  .0        -17.87900
+2.52087  -5.04174
+12.83726  +3.89139  -5.04174  -5.04174  -5.04174
  .0        -18.97140
+4.01245  8.02491
+10.94649  +4.62270  -1.70128  -8.02491  -8.02491
+ .1        -20.0
-21.23978  -20.03165
- .03165  -0.02366  -0.01577  -0.00789  0.00
  .0        -14.14160
+2.63763  -5.27525
+8.86675  -7.80810  -4.68486  -1.56162  0.00
  .0        -17.87900
+5.96718  -11.93435
+5.94465  -4.69291  -15.33130  -5.10992  0.00
  .0        -18.97140
+8.38033  -16.76065
+2.21075  -4.18486  -11.70011  -18.65557   .00
```

```
OUTPUT
C   C   BEAM DEFLECTION AT CABLE PTS DUE TO WIND LOADS       JIN-SIEN GUO
SYMMETRICAL LOADING
     1      2DEFLECTION= -11776.13400000
     1      3DEFLECTION= -23875.77600000
     1      4DEFLECTION= -27264.74000000
     2      2DEFLECTION=      35.60180000
     2      3DEFLECTION=      56.02361900
     2      4DEFLECTION=      51.87098700
     3      3DEFLECTION=     107.83205000
     3      4DEFLECTION=     107.27478000
     4      4DEFLECTION=     129.05937000
ANTISYMMETRICAL LOADING
     1      2DEFLECTION=     -31.86760300
     1      3DEFLECTION=     -83.58460200
     1      4DEFLECTION=    -122.40455000
     2      2DEFLECTION=      59.32236700
     2      3DEFLECTION=     104.96799000
     2      4DEFLECTION=     112.89622000
     3      3DEFLECTION=     229.23666000
     3      4DEFLECTION=     273.73558000
     4      4DEFLECTION=     419.98676000
        STOP   END OF PROGRAM AT  STATEMENT 0082 + 01 LINES
```

```
PROGRAM 3 FOR FIRST EXAMPLE
C   C   CALCULATION OF CABLE STRESSES BY MATRIX INVERSION     JIN-SIEN GUO
        DIMENSION A(20,20),B(39),C(39)
        READ,N
        NN=N-1
     10 READ,((A(I,J),J=1,N),I=1,N)
        A(1,1)=1./A(1,1)
        DO 110 M=1,NN
        K=M+1
     50 DO 60 I=1,M
        B(I)=0
        DO 60 J=1,M
     60 B(I)=B(I)+A(I,J)*A(J,K)
        D=0
        DO 70 I=1,M
     70 D=D+A(K,I)*B(I)
        D=-D+A(K,K)
        A(K,K)=1./D
        DO 80 I=1,M
     80 A(I,K)=-B(I)*A(K,K)
        DO 90 J=1,M
        C(J)=0
        DO 90 I=1,M
     90 C(J)=C(J)+A(K,I)*A(I,J)
        DO 100 J=1,M
    100 A(K,J)=-C(J)*A(K,K)
        DO 110 I=1,M
        DO 110 J=1,M
    110 A(I,J)=A(I,J)-B(I)*A(K,J)
        PUNCH 67
        DO 170 I=1,N
        DO 170 J=1,N
    170 PUNCH 66, A(I,J)
        READ,(B(I),I=1,N)
        DO 200 I=1,N
        C(I)=0.0
        DO 200 J=1,N
    200 C(I)=C(I)+A(I,J)*B(J)
        PUNCH 68
        DO 300 I=1,N
    300 PUNCH 66, C(I)
     66 FORMAT(F20.10)
     67 FORMAT(/14X,16HMATRIX INVERSION)
     68 FORMAT(/14X,14HCABLE STRESSES)
        STOP
        END
         3
1073.268219     72.755847     66.195547
  72.755847   1736.066690    123.314750
  66.195547    123.314750   2411.491480
1807[illegible].64300     31524.613[illegible]     31746.201[illegible]0
```

```
OUTPUT
C  C  CALCULATION OF CABLE STRESSES BY MATRIX INVERSION    JIN-SIEN GUO

          MATRIX INVERSION
      .0009357434
     -.0000375273
     -.0000237672
     -.0000375273
      .0005796196
     -.0000237672
     -.0000237672
     -.0000286095
      .0004167965

          CABLE STRESSES
   +14.9719330000
   +16.8396230000
   +11.9903150000
   STOP  END OF PROGRAM AT  STATEMENT 0068 + 01 LINES
```

```
PROGRAM 3 FOR SECOND EXAMPLE
C  C  CALCULATION OF CABLE STRESSES BY MATRIX INVERSION      JIN-SIEN GUO
      DIMENSION A(20,20),B(39),C(39)
      DO 350 LL=1,2
      IF (LL-1) 11,11,12
   11 PUNCH 81
      GO TO 10
   12 PUNCH 82
   10 READ,N
      NN=N-1
      READ,((A(I,J),J=1,N),I=1,N)
      A(1,1)=1./A(1,1)
      DO 110 M=1,NN
      K=M+1
    5 DO 60 I=1,M
      B(I)=0
      DO 60 J=1,M
   60 B(I)=B(I)+A(I,J)*A(J,K)
      D=0
      DO 70 I=1,M
   70 D=D+A(K,I)*B(I)
      D=-D+A(K,K)
      A(K,K)=1./D
      DO 80 I=1,M
   80 A(I,K)=-B(I)*A(K,K)
      DO 90 J=1,M
      C(J)=0
      DO 90 I=1,M
   90 C(J)=C(J)+A(K,I)*A(I,J)
      DO 100 J=1,M
  100 A(K,J)=-C(J)*A(K,K)
      DO 110 I=1,M
      DO 110 J=1,M
  110 A(I,J)=A(I,J)-B(I)*A(K,J)
      PUNCH 67
      DO 170 I=1,N
      DO 170 J=1,N
  170 PUNCH 66, A(I,J)
      READ,(B(I),I=1,N)
      DO 200 I=1,N
      C(I)=0.0
      DO 200 J=1,N
  200 C(I)=C(I)+A(I,J)*B(J)
      PUNCH 68
      DO 300 I=1,N
  300 PUNCH 66, C(I)
   66 FORMAT(F20.10)
   67 FORMAT(/14X,16HMATRIX INVERSION)
   68 FORMAT(/14X,14HCABLE STRESSES)
   81 FORMAT(33HPROBLEM 2 FOR SYMMETRICAL LOADING)
   82 FORMAT(38HPROBLEM 2 FOR ANTI-SYMMETRICAL LOADING)
  350 CONTINUE
      STOP
      END
```

```
DATA
   3
 1053.78030     56.02362      51.87099
   56.02362   1718.52315     107.27478
   51.87099    107.27478    2405.98517
-11776.1340  -23875.7760   -27264.7400
   3
+1077.50087   +104.96799    +112.89622
 +104.96799  +1839.92776    +273.73556
 +112.89622   +273.73556   +2696.91256
  -31.86760    -83.58460    -122.40455
OUTPUT
C  C  CALCULATION OF CABLE STRESSES BY MATRIX INVERSION     JIN-SIEN GUO
PROBLEM 2 FOR SYMMETRICAL LOADING

              MATRIX INVERSION
         .0009514941
        -.0000298211
        -.0000191838
        -.0000298211
         .0005844536
        -.0000191838
        -.0000191838
        -.0000254159
         .0004171769

              CABLE STRESSES
      -9.9698810000
     -13.0800680000
     -1[illegible].5414860000
PROBLEM 2 FOR ANTI-SYMMETRICAL LOADING

              MATRIX INVERSION
         .0009363737
        -.0000483181
        -.0000342935
        -.0000483181
         .0005543259
        -.0000342935
        -.0000342935
        -.0000542412
         .0003777354

              CABLE STRESSES
        -.0216036460
        -.0405956390
        -.0406099480
     STOP  END OF PROGRAM AT  STATEMENT 0082 + 01 LINES
```

PROGRAM 4 FOR THIRD EXAMPLE

```
C   C   CALCULATION OF HUNCH MEMBER DUE TO CABLE STIFFENING    JIN-SIEN GUO
        DIMENSION A(30),B(30),C(15),DT(15),BM(30),SI(30),T(30),P(15,3)
        DIMENSION P2(15,3),P1(15,4)
        READ,H,AS,DX,AC,CI
        AS=AS*9.
        DP=H/4.
        DO 10 I=2,12
        DO 10 J=1,3
     10 P(I,J)=0.0
        DO 20 J=1,3
     20 P(1,J)=H
        DO 30 I=2,4
        J=I-1
     30 P(I,1)=P(J,1)-DP
        DP=H/8.
        DO 40 I=2,8
        J=I-1
     40 P(I,2)=P(J,2)-DP
        DP=H/12.
        DO 50 I=2,12
        J=I-1
     50 P(I,3)=P(J,3)-DP
        DO 60 I=1,12
        DO 60 J=1,4
     60 P1(I,J)=1.0
        DO 61 I=6,12
     61 P1(I,1)=0.0
        DO 62 I=10,12
     62 P1(I,2)=0.0
        DO 65 I=1,3
     65 B(I)=AS
        B(4)=AC
        DO 66 I=1,12
        A(I)=0.0
        DO 66 J=1,4
     66 A(I)=A(I)+P1(I,J)*B(J)
        DO 67 I=1,12
        C(I)=0.
        DO 67 J=1,3
     67 C(I)=C(I)+P(I,J)*B(J)
        DO 68 I=1,12
     68 DT(I)=C(I)/A(I)
        DO 70 I=1,12
        DO 70 J=1,3
     70 P2(I,J)=P(I,J)**2
        DO 71 K=1,12
        SI(K)=CI
        DO 71 J=1,3
     71 SI(K)=SI(K)+P2(K,J)*B(J)
```

```
      DO 72 K=1,12
   72 SI(K)=SI(K)-A(K)*(DT(K)**2)
      PUNCH 100
      PUNCH 101,(SI(K),K=1,12)
      PUNCH 102
      PUNCH 101,(DT(I),I=1,12)
      PUNCH 103
      PUNCH 101,(A(I),I=1,12)
      DO 73 K=13,17
   73 SI(K)=CI
      DO 75 I=1,12
      J=30-I
   75 SI(J)=SI(I)
      DM=1./28.
      BM(1)=1.0
      DO 76 I=2,29
      J=I-1
   76 BM(I)=BM(J)-DM
      DO 77 I=1,29
   77 BM(I)=BM(I)/SI(I)
      PUNCH 104
      PUNCH 101,(BM(I),I=1,29)
      AA=0.0
      DO 78 I=2,28,2
   78 AA=AA+BM(I)
      BB=0.0
      DO 79 I=3,27,2
   79 BB=BB+BM(I)
      AA=(BM(29)+4.*AA+2.*BB)*DX/3.
      X=0
      DO 80 I=1,29
      T(I)=BM(I)*X
   80 X=X+DX
      BB=0.0
      DO 81 I=2,28,2
   81 BB=BB+T(I)
      CC=0.0
      DO 82 I=3,27,2
   82 CC=CC+T(I)
      BB=(T(1)+T(29)+4.0*BB+2.*CC)*DX/3.
      TB=BB/140.
      TA=AA-TB
      CF=TB/TA
      ST=TA/(TA**2-TB**2)
      PUNCH 105
      PUNCH 106, CF,ST
      DO 83 I=1,27,2
      J=I+1
      K=I+2
      T(I)=(5.*BM(I)+8.*BM(J)-BM(K))*DX/12.
```

```
      A(1)=0.0
      DO 84 I=2,29
      J=I-1
   84 A(I)=A(J)+T(J)
      PUNCH 107
      PUNCH 101,(A(I),I=1,29)
      DO 85 I=1,27,2
      J=I+1
      K=I+2
      T(I)=(5.*A(I)+8.*A(J)-A(K))*DX/12.
   85 T(J)=(5.*A(K)+8.*A(J)-A(I))*DX/12.
      B(1)=0.0
      DO 86 I=2,29
      J=I-1
   86 B(I)=B(J)+T(J)
      PUNCH 108
      PUNCH 101,(B(I),I=1,29)
      DM=B(29)/28.
      DY=0
      DO 87 I=1,29
      B(I)=DY-B(I)
   87 DY=DY+DM
      DO 88 I=1,29
      J=30-I
   88 A(J)=B(I)
      DO 89 I=1,29
   89 T(I)=ST*(B(I)-CF*A(I))
      PUNCH 109
      PUNCH 101,(T(I),I=1,29)
  100 FORMAT(/15X,14HMOMENT INERTIA)
  101 FORMAT(10X,F20.8)
  102 FORMAT(/16X,12HESSENTRICITY)
  103 FORMAT(/16X,12HSECTION AREA)
  104 FORMAT(/20X,3HM/I)
  105 FORMAT(/3X,17HCARRY-OVER-FACTOR,7X,9HSTIFFNESS)
  106 FORMAT(2F20.8)
  107 FORMAT(/11X,17HTHETA(CANTILEVEL))
  108 FORMAT(/11X,18HDEFLTN(CANTILEVEL))
  109 FORMAT(/10X,18HFEM INFLUENCE LINE)
      STOP
      END
DATA
20.0    0.02778    5.0    10.0    20.833
```

OUTPUT

C C CALCULATION OF HUNCH MEMBER DUE TO CABLE STIFFENING JIN-SIEN CUO

MOMENT INERTIA

299.92354000
222.66490000
161.44429000
116.26171000
87.11715400
66.74013000
50.74602500
39.05945700
31.68042400
26.93104800
23.54324600
21.51056300

ESSENTRICITY

1.39545270
1.18225850
.96906436
.75587025
.54267611
.45638405
.35717016
.25795627
.15874237
.12196085
.08130727
.04065369

SECTION AREA

10.75006000
10.75006000
10.75006000
10.75006000
10.75006000
10.50004000
10.50004000
10.50004000
10.50004000
10.25002000
10.25002000
10.25002000

M/I

.00333418
.00433066
.00575165
.00767972
.00983897
.01231787
.01548327
.01920150
.02254660
.02519663
.02730538
.02822534

```
                    .02742902
                    .02571471
                    .02400039
                    .02228608
                    .02057177
                    .01826346
                    .01516966
                    .01193525
                    .00901864
                    .00641050
                    .00422271
                    .00267563
                    .00163983
                    .00092157
                    .00044244
                    .00016040
                    .00000000

CARRY-OVER-FACTOR          STIFFNESS
       .73432514             2.03819740

      THETA(CANTILEVEL)
                    .00000000
                    .01898522
                    .06237516
                    .09585726
                    .14675515
                    .20182786
                    .26278166
                    .34964905
                    .42296633
                    .54254993
                    .62283828
                    .76238021
                    .84094331
                    .97380262
                   1.04517120
                   1.16088730
                   1.22654150
                   1.32395680
                   1.37978520
                   1.44741500
                   1.49267970
                   1.53104400
                   1.56888010
                   1.58591290
                   1.62036630
                   1.62667010
                   1.65970510
                   1.66116140
                   1.69377360

      DEFLTN(CANTILEVEL)
                    .00000000
                    .03729443
                    .23052676
                    .61885126
                   1.21812570
```

```
          2.08713270
          3.24620610
          4.78292880
          6.72011320
          9.15027700
         12.08012100
         15.56857500
         19.60229100
         24.16477700
         29.23783200
         34.77383700
         40.76326800
         47.15684100
         53.93352300
         61.01084200
         68.37039700
         75.92992600
         83.67995600
         91.55967900
         99.56811800
        107.67457000
        115.87937000
        124.16855000
        132.54290000

  FEM INFLUENCE LINE
           .00004490
          4.12319140
          8.05597950
         11.71742400
         15.09609500
         18.07170400
         20.64875600
         22.65020800
         24.12055200
         24.88541000
         25.05424200
         24.53447900
         23.47676700
         21.94619300
         20.05362200
         17.91036600
         15.60710500
         13.27153500
         10.97120500
          8.89418800
          6.98979010
          5.41564740
          4.05261110
          2.99044530
          2.10006480
          1.41361020
           .84244487
           .39129253
          -.00006115
STOP  END OF PROGRAM AT  STATEMENT 0109 + 01 LINES
```

STRESS ANALYSIS OF CABLE STIFFENED STRUCTURES

by

JIN-SIEN GUO

Diploma, Taipei Institute of Technology, 1958

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

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MASTER OF SCIENCE

College of Architecture and Design

Architectural Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1968

A cable stiffened structure is a recent innovation in the architectural structure design. It has the following advantages over common type of structural design: (1) reducing the dead load of the structure, (2) efficient utilization of materials and (3) larger span or space can be built.

The cable stiffened structures generally are high degree inderminate structures. There are many possible approaches to the analysis of this type of structure. In this study, a typical long span rigid frame with cable stiffening is chosen to illustrate some method of stress analysis. Two possible methods which have never been applied to the cable stiffened structural analysis are introduced in this analysis.

The first method is the application of the principle of superposition. It is based on Castigliano's theorem that the deflection at each cable attachment can be superposed by the deflections due to external load and due to cable stress. So a linear equation for deflections with functions of cable stress can be formed for each cable attachment. The number of cables it has is equal to the number of equations which can be formed. Therefore, cable stress can be found by solving the simultaneous equations, the digital computer can be utilized.

The second method is applying the haunch member theory to the cable stiffened structural analysis. Because the cable stiffened horizontal girder of the long span rigid frame can be regarded as a haunched beam. The girder is divided into many small equal interval sections. In combining the girder section with the transformed section of cables, the moment of inertia for each section of the girder can be found and a frame with variable moment of inertia is formed. Therefore, the cable stiffened rigid frame can be analyzed as a haunched frame.

Numerical examples are given to illustrate the methods. There are examples with same dimension and loading conditions for both methods. A comparison of the results of the two methods shows the result from the haunched frame is about 8% smaller than the result worked by the method of superposition. This is because of the haunch member is calculated section by section, error is introduced from these smaller sections. The smaller the section is divided, the more accurate result is given.

Larger space and light weight of construction are needed in modern architectural design. Cable stiffening enables this kind of structure to be built. The cable stiffened rigid frame can be designed for many purposes, such as the construction of large spaced field house, auditorium, factory and hangar.