TESTING METHODS OF AGGREGATING LINEAR PROGRAMMING PROBLEMS

by

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CHAPTER I

INTRODUCTION

This study was conducted in order to delve more deeply into the unanswered problem of the estimation of supply functions by the linear programming method. While it was beyond the scope of this paper to estimate supply functions, it must be recognized that the selection of representative farms has a fundamental role to play in this area. New and better methods are needed. Better methods of selecting representative farms could result in a saving of time and money. Also, a more important result of better methods of selection would be methods which provide more accurate results.

Description of the Problem

In the past, representative farms have been selected by the researcher using judgments based upon his knowledge and previous studies. This study was conducted with this problem in mind. In this study there was a total population to test the results against. At the start of this study there were individual data on 49 separate farms.[1] These 49 farms were originally selected by stratified random sampling, but for the purposes of this study they were

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[1]Paul L. Kelley and Dale A. Knight, "Short-run Elasticities of Supply for Milk," Journal of Farm Economics, vol. 47, pp. 93-104 (February 1965).

treated as the total finite population.

These 49 farms were individually set up for linear programming with the maximization of net revenue, or profit, the objective and programmed with the "results" being thought of as the maximum net revenue (or weighted total output) of the 49 farms subject to their constraints. This procedure is sometimes referred to in this paper as "micro-programming". Later, by the two methods of selecting representative farms that are described later in this paper, the 49 farms were divided into 9 groups with 1 representative farm for each group. These representative farms were set up to be programmed, and they were programmed. The results for these representative farms were multiplied by the number of farms in their group and summed. This sum is supposed to represent the total profit of the 49 farms. This procedure is sometimes referred to in this paper as "macro-programming". This estimated total profit can then be compared with the true sum of the 49 individual farms' profits.

A supply function can be generated by changing the price of one of the goods that the farm produces. The price of the good would be increased or decreased, and the farm re-programmed. The schedule of amounts produced by changing the price would give the supply function. In this study alternative prices were not considered. One set of prices was used for each farm. Moreover, overall profit was considered rather than specific outputs for each good. However, overall profit is a weighted sum of many outputs. Therefore, the comparisons made in this study are believed to be relevant to the problem of using representative farms in estimation of supply functions.

It is expected that a bias will result from the representative farm approach to the problem. The representative farms are used for the farms in the strata, and the expected effect of averaging a set of farms is to enlarge or contract the set of feasible solutions. The enlargement or contraction of this set will be expected to result in a bias in profit. It is desired that the bias be a minimum.

Statement of Hypothesis

In this study two such methods of selecting representative farms for aggregation of linear programming problems were tried and tested against the method of analyzing the total parent population. The two methods may be called (1) the conventional method and (2) the homogeneous restriction method (both methods explained in detail in chapter III). It was desired that those two methods be illustrated and tested empirically against the total population. The main hypothesis was that when there exists linear programming problems involving a large number of farms, the homogeneous restriction method would provide a reasonably good grouping of farms as well as a problem of manageable size. The best results, in regard to the testing of these methods, would be the representative farm selection which gave the smallest profit bias because the principal thing of interest was the minimization of aggregation bias (given the degree of aggregation), which is the difference between the average profit for the entire group of farms when micro-programmed, and the weighted average profit of the 9 representative farms when macro-programmed.

It would also be of interest to find out which method of selection and construction of representative farms gave the least squared difference between the profits of the actual farms of the particular groups and the average profit of the groups, but time did not permit making this latter comparison.

Review of Literature

In the following section a review of some of the past studies dealing with the use of representative farms is presented. Only the highlights and main conclusions will be covered.

Dr. Hartley outlined his procedure in a paper presented before the North Central Farm Management Research Committee in March, 1962.[1] Hartley described a method of selecting representative farms. He maintained that the selection and construction of representative farms could be accomplished by separating out the farms in the production region into groups or domains in a manner so that the farms within a domain had a greater degree of homogeneity in their production characteristics. The resources of the representative farms (P_o or b column) were taken as the arithmetic means of all the resources of all the farms in the domain. Arithmetic means would also be computed for the net revenues (c vector) and for the cost coefficients (A matrix). The representative farm was thus so constructed; then the linear programming technique could be used for

[1]H. O. Hartley, "Total Supply Functions Estimated From Farm Survey," A paper presented before the North Central Farm Management Research Committee (March 1962).

an optimal solution. The linear programming solution of each representative farm could then be multiplied by the number of farms in its domain, and these products summed to obtain the aggregation production in the region.

In an unpublished PhD dissertation Dahab performed an empirical test of a method of representative farm selection.[1] Dahab was interested in testing the method described by Hartley for the selection and construction of representative farms. From a population of 49 farms Dahab constructed 9 representative farms for the estimation of milk supply functions. The 49 farms were divided into 9 strata by designating the 49 farms according to the level of milk production (three levels) and geographical area (three areas). A representative farm was constructed for each stratum. The representative farm was the arithmetic mean of the farms in the stratum. The linear programming results of the 9 representative farms were tested against the previous results of micro-programming of the same 49 farms by Kelley and Knight.[2] Dahab found that this method of selecting representative farms resulted in very small biases. Dahab was also interested in the amount of saving in time and money that would result in the use of representative farms, and he found that it took approximately 85% less time and considerably less money.

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[1]M. G. Abou-el-Dahab, "An Aggregation Procedure for Deriving Representative Firms in Estimating Supply Functions," PhD Dissertation, Kansas State University, 1965.

[2]Kelley and Knight, loc. cit.

Sheehy and Alexander discussed the selection of representative farms and the aggregation bias resulting from the methods of selecting representative farms in a recent paper.[1] They defined aggregation bias as was done above in a previous section of the paper only using the output of one commodity -milk- rather than overall profit. They compared the bias resulting from two methods of selecting representative farms. These two methods were (1) the conventional method, in which farms were classified on the basis of absolute levels of certain resources, and (2) the homogeneous restriction method, in which use was made of the level of resources on sample farms and the productivity of these resources. The homogeneous restriction method of grouping farms seems to have been based upon grouping of similar constraints together, with these similar constraints being the "most limiting" constraint. The conclusion reached by this study was that the homogeneous restriction method reduced bias by more than the conventional method, and that even though the homogeneous restriction method involved more work it gave enough better results to warrant its use.

Frick and Andrews in a study similar to the one in this paper used a total population of 51 farms to test methods of selecting representative farms.[2] They used four methods of summing farm

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[1]S. J. Sheehy and R. H. Alexander, "Selection of Representative Benchmark Farms for Supply Estimation," Journal of Farm Economics, vol. 47, pp. 681-95 (August 1965).

[2]G. E. Frick and R. A. Andrews, "Aggregation Bias and Four Methods of Summing Farm Supply Functions," Journal of Farm Economics, vol. 47, pp. 696-700 (August 1965).

supply functions. The four methods used were (1) taking the mean of all resources and using this as one representative farm, (2) selection of six representative farms by the number of stanchions, (3) a homogeneous restriction method of combining the most limiting resources, and (4) a method based upon the potential number of stanchions filled by milk cows. The third method was very similar to the method developed by Sheehy and Alexander. The number of representative farms used by each method were: Method (1), 1 farm; Method (2), 6 farms; Method (3), 5 farms; and Method (4), 6 farms. The conclusion reached by this study was that Method (3), the homogeneous restriction method, resulted in the smallest bias of any of the methods tested.

A theoretical result obtained by Day[1] may also be briefly mentioned. Day proved that in the special case where individual farms in a domain have (1) identical A-matrices, (2) proportional b-vectors, and (3) proportional c-vectors, programming of representative farms creates no bias.

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[1]R. H. Day, "On Aggregating Linear Programming Models of Production," Journal of Farm Economics, vol. 47, pp. 93-104 (February 1965).

CHAPTER II

LINEAR PROGRAMMING MODEL FOR EACH INDIVIDUAL FARM

Definitions of Resources and Constraints

The model used for each of the forty-nine farms[1] and for each of the eighteen representative farms (nine representative farms for each of the two models of aggregating linear programming problems) was a coefficient matrix consisting of eleven rows and twenty-eight columns, a row vector (called "c") and also a column vector (called "b" or "P_0"). The P_0 column vector contained the amounts of the various resources available. The c row vector defined the objective function. The objective function was to be maximized. The objective function had a coefficient in each of the twenty-eight columns. The twenty-eight columns were for seven different crops grown on four different classes of land. Originally, there were six classes of land, but class C land was combined with class B land, and class F land was combined with class E land, thus making four classes. The seven crops were: wheat, oats, corn, barley, alfalfa, milo, and soybeans. In regard to the rows of the coefficient matrix, the first four rows were labor coefficients, the wheat allotment was the next row, the four classes of land were the next four rows, the tenth row was the soybean restriction, and the last row was the

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[1]Kelley and Knight, loc. cit.

alfalfa restriction.

The twenty-eight columns of the coefficient matrix denote activities in detail as follows:

Activity Number	Activity Name	Unit
P_1	Wheat production and selling on owned land A.	1 Acre
P_2	Wheat production and selling on owned land B.	1 Acre
P_3	Wheat production and selling on rented land D.	1 Acre
P_4	Wheat production and selling on rented land E.	1 Acre
P_5	Oats production and selling on owned land A.	1 Acre
P_6	Oats production and selling on owned land B.	1 Acre
P_7	Oats production and selling on rented land D.	1 Acre
P_8	Oats production and selling on rented land E.	1 Acre
P_9	Corn production and selling on owned land A.	1 Acre
P_{10}	Corn production and selling on owned land B.	1 Acre
P_{11}	Corn production and selling on rented land D.	1 Acre
P_{12}	Corn production and selling on rented land E.	1 Acre
P_{13}	Barley production and selling on owned land A.	1 Acre

Activity Number	Activity Name	Unit
P_{14}	Barley production and selling on owned land B.	1 Acre
P_{15}	Barley production and selling on rented land D.	1 Acre
P_{16}	Barley production and selling on rented land E.	1 Acre
P_{17}	Alfalfa hay production and selling on owned land A.	1 Acre
P_{18}	Alfalfa hay production and selling on owned land B.	1 Acre
P_{19}	Alfalfa hay production and selling on rented land D.	1 Acre
P_{20}	Alfalfa hay production and selling on rented land E.	1 Acre
P_{21}	Milo production and selling on owned land A.	1 Acre
P_{22}	Milo production and selling on owned land B.	1 Acre
P_{23}	Milo production and selling on rented land D.	1 Acre
P_{24}	Milo production and selling on rented land E.	1 Acre
P_{25}	Soybeans production and selling on owned land A.	1 Acre
P_{26}	Soybeans production and selling on owned land B.	1 Acre
P_{27}	Soybeans production and selling on rented land D.	1 Acre
P_{28}	Soybeans production and selling on rented land E.	1 Acre

The rows of the simplex tableau are represented by R_i, where i goes to 11, as follows:

The criterion, or (c) row vector (dollars) which represents net revenue before fixed costs and will appear in the first row of the P_o column in the final tableau.

R. 1 The total labor limit for April through May. The labor limit is measured in hours available during the time period.

R. 2 The total labor limit for June through July. The labor limit is measured in hours available during the time period.

R. 3 The total labor limit for August through October. The labor limit is measured in hours available during the time period.

R. 4 The total labor limit for November through March. The labor limit is measured in hours available during the time period.

R. 5 The wheat allotment or the amount of cropland that can be planted to wheat. The wheat allotment is measured in acres of land.

R. 6 Class A owned land. The amount of class A owned land available is measured in acres. Class A owned land is the highest grade of owned land.

R. 7 Class B owned land. The amount of class B owned land available is measured in acres. Class B owned land consists of all other owned land that is not class A owned land.

R. 8 Class D rented land. The amount of class D rented land available is measured in acres. Class D rented land is the highest grade of rented land.

R. 9 Class E rented land. The amount of class E rented land available is measured in acres. Class E rented land consists of all other rented land that is not class D rented land.

R. 10 The soybeans restriction or the amount of cropland that can be planted to soybeans. The soybeans restriction is measured in acres.

R. 11 The alfalfa hay restrictions or the amount of cropland that can be planted to alfalfa hay. The alfalfa hay restriction is measured in acres.

Derivation of Coefficients in the Objective Function

There were 28 coefficients in the objective function for each individual farm and for each representative farm. The 28 coefficients of the objective function represented the amount of net revenue that would result in the use of one acre of the activity (28 activities). Net revenue was the difference between total revenue and the variable cost per acre. Fixed costs were not subtracted because there would be no change in the solution due to them. Fixed costs would have been incurred whether or not anything had been produced.

Method of Arriving at Alfalfa Constraint

An alfalfa constraint was introduced into the study because when a sample farm was programmed for a trial run nearly all the land was used for alfalfa. It seemed that this was not a good solution because of the lack of markets for alfalfa production if all the farmers in the area were to behave in this manner. Even if the optimal solution for all farmers (or some particular farmer) was all alfalfa, it was thought that farmers do not always behave in the optimal manner, and observation shows that very few farmers plant a large portion of their farm to alfalfa.

The alfalfa constraint was arrived at by going back to the original data[1] on each of the forty-nine farms and checking on whether or not the farmer had any alfalfa planted. The amount of alfalfa that each farmer had planted in the year of the survey was used as his alfalfa constraint. For the farmers who did not have any alfalfa planted their alfalfa restrictions were computed as a percentage of their total cropland. This percentage was arrived at by summing the total cropland (owned and rented cropland) of all farmers who had alfalfa planted, summing the amount of alfalfa they had planted, and then dividing the total alfalfa planted by the total amount of cropland. The percentage arrived at by this process was 15%; that is, 15% of all cropland for those farmers who grew alfalfa was planted to alfalfa. Thus the alfalfa constraint for a farmer who did not have any alfalfa and had say 100 acres of cropland was 15 acres of alfalfa. There were seven out of the forty-nine farms that did not grow alfalfa at the time of the survey.

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[1] Ibid.

Method for Filling Blank Spaces in Matrix

The procedure used of filling in the C_j elements and the cost coefficients for farms not having particular activities was as follows: From other farms in their group[1] average data were computed. When no farms in an area had data, the procedure was as follows: For missing data in Area I the data from Area II

TABLE 1

COUNTIES IN THREE AREAS OF TOPEKA MILKSHED[2]

Area I (Northeast)	Area II (Southeast)	Area III (West)
Brown	Osage	Clay
Doniphan	Franklin	Riley
Jackson	Coffey	Dickinson
Atchinson	Linn	Geary
Shawnee	Anderson	Morris
Jefferson		Marion
Douglas		Chase
Leavenworth		Greenwood
Wyandotte		Lyon
		Pottawatomie
		Wabaunsee
		Washington
		Marshall

[1]The designation as to the group comes from the conventional method of aggregating linear programming problems.

[2]Dahab, *loc. cit.*, p. 34.

were used if possible; if this was not possible, Area III data were used. For missing data in Area II the data from Area I were used if possible, if this was not possible the data from Area III were used. For missing data in Area III the data from Area I and Area II were randomly used. The three areas are described in Table 1.

Operations Performed on the Data

The operations that were performed on the data may be grouped into three distinct phases. The first phase of operations concerned individual farms: setting up of the data and linear programming them. The second phase of operations was the setting up of the representative farms for the conventional method of aggregation. These farms had to be selected, constructed, and linear programmed. The third phase of operations dealt with the homogeneous restriction method of aggregation. For this method the data, after the selection and construction of the representative farms, had to go through the same process as for the conventional method.

In addition to the operations involved in construction of the representative farms, a total of 49 programming problems were run for the individual farms and 9 programming problems were run for each of the two methods of aggregation.

CHAPTER III

THE TWO METHODS OF AGGREGATION

The Conventional Method

The conventional method of aggregating linear programming problems that was used in this study was the method used by Dahab[1] and developed by Hartley.[2] The 9 representative farms were selected by dividing the population of 49 farms into 9 strata by using 3 levels of production and 3 geographical areas. The 9 representative farms were constructed by taking the arithmetic mean of the A, b, and c matrices for the farms in each stratum. The levels of production were the pounds of milk per day that the farms produced, and the geographical areas were areas of the Topeka milkshed.[3]

TABLE 2

NUMBER OF SAMPLE FARMS IN EACH CELL[4]

Daily Milk Production	Area of Production			
	Northeast	Southeast	West	Total
Less than 200 lbs. (Small)	3	3	4	10
200-499 lbs. (Medium)	11	6	8	25
500 lbs. or more (Large)	5	4	5	14
Total	19	13	17	49

[1]Dahab, loc. cit.

[2]Hartley, loc. cit.

[3]Kelley and Knight, loc. cit.

[4]Dahab, loc. cit., p. 22.

TABLE 3

RESULTS OF STRATIFICATION BY THE CONVENTIONAL METHOD

STRATUM			PROFIT - DOLLARS PER FARM		
Identification Number	Characteristic[a]	Number of Farms	Representative Farm	True Average	Bias
1	Northeast small	3	$4595	$4667	$-72
2	Southeast small	3	5725	6034	-309
3	West small	4	2637	2623	50
4	Northeast medium	11	5035	5008	23
5	Southeast medium	6	5577	5698	-121
6	West medium	8	5274	5654	-380
7	Northeast large	5	11018	11416	-398
8	Southeast large	4	6778	7097	-319
9	West large	5	6794	6727	67

[a]See Table 2 for precise definition.

The objective of this study was to minimize aggregation bias. The aggregation bias is defined as the difference between the stratum average net revenue when farms are micro-programmed and the net revenue of the representative farm. The results of the conventional aggregation are shown in Table 3. The aggregation biases for each stratum are in the last column.

THE HOMOGENEOUS RESTRICTION METHOD

The homogeneous restriction method of aggregating linear programming problems was based upon the idea of grouping those farms

TABLE 4

GROUPING OF FARMS INTO STRATA BY THE HOMOGENEOUS RESTRICTION METHOD

Stratum	Rows of b Vector Having 0 Elements	Number of Farms in Stratum
1 A	6, 7	2
2 A	6, 8, 10	6
3 A	6, 9, 10 6, 10	2
4 A	7 7, 8 7, 8, 9, 10 7, 9, 10	4
5 A	8 8, 9 9	8
6 A	8, 9, 10	13
7 A	8, 10 10	6
8 A	9, 10	3
9 A	None	5

together that had the severest restrictions in the same resource or resources. On most farms the resources that were the most restrictive were the four classes of land and the soybeans restriction. The wheat and alfalfa restrictions were also of importance, but there was not an instance of there being a complete absence of either. Nine different groups of farms were selected in regard to the severest limitation of the four classes of land and the soybeans restriction. By "severest limitation" is meant that the amount of the resource available to the farm was zero. Farms were grouped so that farms in the same group had zeros in the same, or nearly the same resources. Specifically, the grouping into 9 strata is defined in Table 4. There is admittedly

TABLE 5

RESULTS OF STRATIFICATION BY THE HOMOGENEOUS RESTRICTION METHOD

Stratum	Representative Farm	True Average of Farms in Stratum	Bias
1 A	$5210	$5261	$ -51
2 A	3065	3575	-510
3 A	4251	4165	86
4 A	5385	5483	-98
5 A	6232	6531	-299
6 A	6644	6682	-38
7 A	6543	6908	-365
8 A	7177	7148	29
9 A	5889	6392	-503

some arbitrariness in the classification. For example, the farm with a zero in row 9 alone of the b vector, which was placed in Stratum 5A, might have been placed in Stratum 7A. It is also recognized that numerical values of the given elements, other than zeros, played no part in this classification.

The idea was, following Day,[1] Sheehy, and Alexander,[2] that when constraints are more homogeneous within groups, the aggregation bias in activity levels and profits is less.

The representative farms were constructed as before by taking the arithmetic means of the data for farms in each stratum. The aggregation biases that resulted from this method of representative farm selection and construction are shown in Table 5.

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[1]Day, loc. cit.

[2]Sheehy and Alexander, loc. cit.

CHAPTER IV

ANALYSIS OF RESULTS AND CONCLUSIONS

Statement of Results

The main result of this study was that both methods of aggregation slightly underestimated the actual net revenue of the total populations of the 49 farms. The average net revenue per farm for the micro-programmed farms was $6045. For the macro-programmed farms the conventional method gave an average net revenue of $5895, and the homogeneous restriction method gave an average net revenue of $5823. See Tables 6 and 7. The conventional method underestimated the true average by $150 per farm or 2.5%. The homogeneous restriction method underestimated the true average by $222 per farm or 3.7%. The conventional method underestimated the true average by 1.2% less than did the homogeneous restriction method.

Analysis of Results

It was found that the weighted average absolute bias (neglecting the sign of the bias) was $184 per farm for the conventional method (see Table 6) and $233 per farm for the homogeneous restriction method (see Table 7). In both cases the weighted average absolute bias was greater than the average bias with algebraic signs taken into account. The greater difference between the two biases for the conventional method ($34 over $11) was because it had more positive and negative deviation from the strata averages than did the homogeneous restriction method; or in other words the homogeneous method was more consistent in its underestimation. The homogeneous restriction method overestimated the stratum average twice in nine strata and conventional method overestimated three times in nine strata.

TABLE 6

WEIGHTED BIASES FOR THE CONVENTIONAL RESTRICTION METHOD

Stratum	Number of Farms	Representative Farms	True Average	Bias	Absolute Weighted Bias
1	3	$4595	$4667	$-72	$216
2	3	5725	6034	-309	927
3	4	2673	2623	50	200
4	11	5035	5008	27	297
5	6	5577	5698	-121	726
6	8	5274	5654	-380	3040
7	5	11018	11416	-398	1990
8	4	6778	7097	-319	1276
9	5	6794	6727	67	335
Total	49				
Weighted Total		$288,864	$296,203	-$7343	$9007
Average (weighted)		$ 5895	$ 6045	-$ 150	$ 184

Limitations of the Study

The original data were constructed and used for dairy production. The original data matrices contained dairy, transfer, and production and selling activities, but for this study the matrices were reduced to only the production and selling activities for crops. Thus these farms in the Topeka milkshed were probably more geared for milk pro-

TABLE 7

WEIGHTED BIASES FOR THE HOMOGENEOUS RESTRICTION METHOD

Stratum	Number of Farms	Representative Farms	True Average	Bias	Absolute Weighted Bias
1 A	2	$5210	$5261	$-51	$102
2 A	6	3065	3575	-510	3060
3 A	2	4251	4163	86	172
4 A	4	5385	5483	-98	392
5 A	8	6232	6531	-299	2392
6 A	13	6644	6682	-38	494
7 A	6	6543	6908	-365	2190
8 A	3	7177	7148	29	87
9 A	5	5889	6392	-503	2515
Total	49				
Weighted Total		$285,320	$296,203	-$10,886	$11,404
Average (weighted)		$ 5823	$ 6045	-$ 222	$ 233

duction that the growing of crops for revenue. For the conventional method the study was restricted to the original breakdown of 9 strata with milk production and geographical area the determinants while in this study the milk production was taken out, as the interest was on crop production for immediate sale. In regard to the homogeneous restriction method, it was limited to 9 strata for comparison purposes with the conventional method and 9 strata may not have been the optimum number of strata for the homogeneous method. Also, it would be hoped that farmers act in a rational and optimum manner, and there is a good possibility that they do not act rationally or in an optimum manner. In this problem only the physical variables have been measured while many non-physical variables may have a great deal to do with the problem.

The profit motive may not be the driving force for all people. Many farmers have personal biases against the growing of some crops and have other crops which they regard as favorites.

Summary and Conclusions

This study was made to present and evaluate two methods for the selection and construction of representative farms for linear programming, with the idea in mind that these methods could be used for the estimation of supply functions. The objective of the study was to test these two methods of aggregating linear programming problems to see how well they were able to estimate the net revenue of the total farms. An ideal method would minimize the aggregation bias, given the degree of aggregation.

In conclusion, it was found that both methods of aggregating linear programming problems underestimated the true average net revenue per farm. The conventional method was 1.2% closer to the true value than was the homogeneous restriction method, but both methods were within 4% of the true value. By the use of either method it was found that the number of operations could be reduced from 49 to 9. This result would give rise to a considerable reduction in machine time and a substantial savings in money.

It would have to be concluded that the conventional method produced the best results, but 9 strata may not have been the optimal number of strata for the homogeneous restriction method. Also, the selection of strata for the homogeneous restriction method took much less time than for the conventional method, but in this instance the conventional method was superior.

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Recently linear programming has been used in the estimation of supply functions, and the use of the linear programming technique has required a large number of sample farms. The representative farm approach has been used to reduce the number of farms for programming. At this time it was desired to test methods of aggregating linear programming problems and this study was done in order to test two methods of aggregation, even though no supply functions were to be estimated by this study. The two methods of aggregation to be tested were the conventional method and the homogeneous restriction method. It was hoped that these two methods of aggregation would reduce the amount of computational time and money and minimize the aggregation bias subject to the degree of aggregation.

Data on 49 sample farms were available from the 1957-58 survey of the Topeka, Kansas milkshed. These 49 farms were treated as the total population. The milk production and transfer activities were taken out of the original farm data leaving only crop production and selling activities. Nine representative farms were selected and constructed for each of the two methods of aggregation. The nine representative farms were selected for the conventional method by using three levels of production and three geographical areas. The nine representative farms were selected for the homogeneous restriction method by grouping the farms together in regard to their "most limiting" resource. The construction of the representative farms for the two methods was accomplished by taking the arithmetic means of the given numerical data of the farms within each group. These representative farms were then programmed in order to determine the net revenue of each of the representative farms. The estimated total net revenue of the group was determined by multiplying the representative farm by the number of farms in the group, and the estimated total net revenue of the 49 farms was found by adding the net revenue of the groups together.

The results that were obtained from this study indicate that the conventional method did a better job of estimating the net revenue than did the homogeneous restriction method. The micro-programming of the 49 farms gave a net revenue of $6045 per farm, while the macro-programming by the conventional method gave net revenue of $5895 per farm and the macro-programming by the homogeneous restriction method gave $5823. Thus the conventional method underestimated the per farm net revenue by $150 or 2.5% and the homogeneous restriction method underestimated the per farm net revenue by $222 or 3.7%. The weighted absolute bias per farm was $184 for the conventional method and $233 by the homogeneous restriction method.

The results seem to indicate that both methods of aggregation give only small biases and both methods also give a considerable saving in computational time and money.