# AN INVESTIGATION OF THE AVAILABILITY OF POTENTIAL ENERGY AND ITS RELATION TO POWER CYCLES RESULTING FROM CHANGES IN ELEVATION IN A STANDARD ATMOSPHERE

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by

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# INTRODUCTION

In a steady-flow process, the available part of potential energy is equal to the potential energy, $gz/g_cJ$ BTU/lbm, but in a non-flow process it is less than the potential energy. In a footnote in Keenan's Thermodynamics (page 297), he mentioned that if a piece of fluid is lowered in a medium, the amount of rotary shaft work that can be realized is equal to the decrease in potential energy minus the work done by the buoyant force of the medium; i.e. the available part of potential energy equals $(z_1 - z_o)(1 - \frac{v_1}{v_a})$, in which $v_a$ denotes the specific volume of the medium and $v_1$ denotes the specific voluma of the piece of fluid. This equation can apply only to a constant specific volume medium. However, below the tropopause the specific volume of the atmosphere air changes with the altitude according to the relation

$$\frac{\rho_o}{\rho_{S.L.}} = (\frac{T_o}{T_{S.L}})^{4.260} = (1 - 0.000006871z)^{4.260}$$

This density-temperature relationship can be derived by either a differential element force method or a thermodynamics steady-flow analysis method. From the second method it can be shown that $pv^n = c$ holds for the standard atmosphere up to the tropopause and n equals 1.2347.

From the equations of the pressure, temperature and density relations, the equations for the available part of potential energy can be obtained.

Several numerical examples are presented to show the detail of calculations required to obtain the net rotary shaft work in non-flow processes, non-flow cycles and steady-flow cycles with change in elevations. Several equations for net rotary shaft work are presented.

In a non-flow cycle or steady-flow cycle in which elevation changes are a part of the cycle and the processes are adiabatic during the elevation changes, the net rotary shaft work equals the net rotary shaft work of a Carnot Cycle working between the same temperature and pressure limits as those of the atmosphere at the two prescribed elevations. This is so because, in the non-flow cycle, the work done by the atmosphere at high altitude plus the work done by the buoyant force during ascent equals the work done on the atmosphere at sea level plus the work done against the buoyant force during descent. In the case of a steady-flow cycle, no work is done by the atmosphere on the working fluid of the cycle, and no work is done by the working fluid on the atmosphere.

This relation can be also explained in the following example; one cubic foot of vacuum is created at sea level in a container of negligible weight. Then it is brought to 20,000 ft height. The rotary shaft work input at sea level required to create the vacuum is 2.72 BTU. The rotary shaft work output produced by the buoyant force of the atmosphere is 1.47 BTU. The rotary shaft work output produced by the availability of this vacuum at 20,000 ft height is 1.25 BTU. The net rotary shaft work output for these three processes, which starts from the sea-level dead state and ends at the dead state at 20,000 ft height is $1.47 + 1.25 - 2.72 = 0$. The details of these relations are presented in this report.

# NOMENCLATURE

AEH : Available part of enthalpy, BTU per lbm.

AEPE : Available part of potential energy, BTU per lbm.

$AEQ_{in}$ : Available part of heat in, BTU per lbm.

$AEQ_{o}$ : Available part of heat out, BTU per lbm.

AEU : Available part of internal energy, BTU per lbm.

$c_p$ : Specific heat at constant pressure, BTU per (lbm)($^{o}$F).

$c_v$ : Specific heat at constant volume, BTU par (lbm)($^{o}$F).

F : Friction loss, ft-lbf per lbm.

g : Acceleration of gravity, ft per sec$^2$.

$g_c$ : Defined by ma/F = 32.2 lbm-ft per (lbf)(sec$^2$).

h : Enthalpy, BTU per lbm.

J : Mechanical equivalent of heat, 77B.16 ft-lbf per BTU.

p : Absolute pressure, psia or psfa; $p_{S.L}$ for tha atmospheric pressure at sea level, 14.696 psia or 2116.2 psfa. $p_{oz}$ for the atmospheric pressure at altitude z.

Q : Heat BTU per lbm; $Q_{in}$, heat in; $Q_o$, heat out.

s : Entropy, BTU per (lbm)($^{o}$R).

T : Absolute temperature, $^{o}$R; $T_{S.L}$ for the atmospheric temperature at sea level. $T_{oz}$ for the atmospheric temperature at altitude z.

t : Temperature, $^{o}$F.

u : Internal energy, BTU per lbm.

$UEQ_{in}$ : Unavailable part of heat in, BTU per lbm.

$UEQ_{o}$ : Unavailable part of heat out, BTU per lbm.

v : Specific volume, cu ft per lbm.

$V$ : Total volume, cu ft; velocity, ft per second.

$W$ : Piston work, BTU per lbm; $W_o$ for work output; $W_{in}$ for work input.

$W_{rs}$ : Rotary shaft work, BTU per lbm; $W_{rso}$ for work output; $W_{rsin}$ for work input.

$W_{on\ atm.}$ : Work done on the atmosphere, BTU per lbm.

$W_{by\ atm.}$ : Work done by the atmosphere, BTU per lbm.

$z$ or $Z$ : Altitude, ft.

$\rho$ : Density, lbm per cu ft; $\rho_{S.L}$ for the atmospheric density at sea level. $\rho_{oz}$ for the atmospheric density at altitude z.

# THE STANDARD ATMOSPHERE

## A. General Description

The atmosphere may be thought of consisting of four layers; troposphere, stratosphere, ionosphere and exosphere.

The height of the troposphere varies from about 5 miles at the poles to approximately ten miles at the equator. The stratosphere extends from the upper limits of the troposphere, the tropopsuse, to approximately fifty to seventy miles above the earth. The temperature in this region remains nearly constant at 392.78 °R or -66.92 °F. The ionosphere is characterized by the presence of ions and free electrons. The exosphere ranges from 300 to 600 miles.

The standard atmosphere is an assumed standard which has been derived from an average of the seasonal variations at latitude 40° N in the United States.

(1) The sea-level standard conditions are:

$$P_{S.L} = 760 \text{ cm Hg} = 29.921'' \text{ Hg} = 2116.2 \text{ lb/ft}^2$$
$$= 14.696 \text{ psia}$$

$$t_{S.L} = 59^{\circ}F \quad \text{or} \quad T_{S.L} = 518.7^{\circ}R$$

$$g = 32.174 \text{ ft/sec}^2$$

(2) $pv = RT$ is assumed to hold for the atmosphere air as well as the following constants.

$$R = 53.342 \text{ ft-lb/lb}^{\circ}F = 0.068549 \text{ 8tu/lb}^{\circ}F$$

$$C_p = 0.23992 \text{ 8tu/lb}^{\circ}F$$

$$C_v = 0.17137 \text{ 8tu/lb}^{\circ}F$$

$$k = 1.4$$

(3) The variation of temperature with altitude is linear up to the stratosphere and is given by the equation:*

$$t_{oz}\,^{o}F = 59 - 0.003564z$$

(4) The troposphere extends up to 35,332 ft.

B. Derivations of the Expressions for Temperature, Pressure and Density as Functions of Altitude by Means of a Balance of Forces.

Assume that the value of g does not change with altitude. Consider a unit element of the atmosphere as shown in Fig. 1.

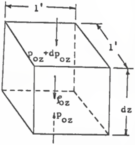

Fig. 1

$$p_{oz} - (p_{oz} + dp_{oz}) - \rho_{oz}\frac{g}{g_c}dz = 0$$

$$\rho_{oz} = \frac{p_{oz}}{RT_{oz}}$$

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[*] From the "NACA Standard Atmosphere" the atmosphere temperature at 35,000 ft is -65.75 F. Because the variation of temperature with altitude is assumed to be linear, therefore

$$t_{oz}\,^{o}F = 59 - \frac{59 - (-65.74)}{35,000}z = 59 - 0.003564z$$

$$\frac{dp_{oz}}{p_{oz}} = -\frac{dz}{RT_{oz}} = -\frac{dz}{53.342(T_{S.L} - 0.003564z)}$$

$$\int_{p_{S.L}}^{p_{oz}} \frac{dp_{oz}}{p_{oz}} = \int_{0}^{z} 5.260\,\frac{d(T_{S.L} - 0.003564z)}{T_{S.L} - 0.003564z}$$

$$\frac{p_{oz}}{p_{S.L}} = \left(\frac{T_{oz}}{T_{S.L}}\right)^{5.260} = (1 - 0.000006871z)^{5.260} \quad \cdot\cdot\cdot\cdot \quad (1)$$

$$\frac{\rho_{oz}}{\rho_{S.L}} = \frac{p_{oz}T_{S.L}}{p_{S.L}T_{oz}} = \left(\frac{T_{oz}}{T_{S.L}}\right)^{4.260} = (1 - 0.000006871z)^{4.260} \quad (2)$$

To obtain the expressions for the pressure and density ratios above the tropopause we use the differential equation

$$\frac{dp_{oz}}{p_{oz}} = -\frac{dz}{53.342T_{oz}}$$

The integration is performed in two parts;

$$\int_{p_{S.L}}^{p_{oz}} \frac{dp_{oz}}{p_{oz}} = 5.260\int_{0}^{35332} \frac{d(T_{S.L} - 0.003564z)}{T_{S.L} - 0.003564z} + \int_{35332}^{z} \frac{dz}{53.342 \times 392.78}$$

$$\ln \frac{p_{oz}}{p_{S.L}} = -\left(1.4627 + \frac{z - 35332}{20952}\right)$$

or

$$\frac{p_{oz}}{p_{S.L}} = \mathrm{Exp}\left(0.2236 - \frac{z}{20952}\right) \quad \cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot \quad (3)$$

$$\frac{\rho_{oz}}{\rho_{S.L}} = \frac{p_{oz}T_{S.L}}{p_{S.L}T_{oz}} = 1.3206\ \mathrm{Exp.}\left(0.2236 - \frac{z}{20952}\right) \quad \cdot\cdot \quad (4)$$

C. Derivations of the Expression for the Relation Between Temperature and Pressure by Means of Thermodynamics Relation:

$$p\,v^n = c\,, \qquad \left(\frac{p}{p_1}\right)^{\frac{n-1}{n}} = \frac{T}{T_1}$$

$$dp = \frac{n}{n-1}\,\frac{p}{T}\,dT$$

Assume that the atmosphere flows very slowly with negligible velocity change inside a pipe as shown in Fig. 2. From Bernoulli's equation:

$$\int_{S.L}^{z} -\frac{vdp}{J} = W_{rso} + \frac{g\,z}{g_c J} + \frac{F}{J} + \frac{V_z^2 - V_{S.L}^2}{2g_c J}$$

$W_{rso} = 0$ and $F = 0$ in this case.

$$\frac{g}{g_c}\,dz = -\,vdp$$

$$= -\,v\,\frac{n}{n-1}\,\frac{p}{T}\,dT$$

$$= \frac{n}{n-1}\,R\,(-\,dT)$$

But $T_{oz} = T_{S.L} - 0.003564z$

$$dT_{oz} = -\,0.003564\;dz$$

$$1 = \frac{n \times 53.342 \times 0.003564}{n-1}$$

or

$$n = 1.2347$$

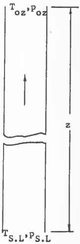

Fig. 2

From the result it follows that n is constant below the stratosphere and equals 1.2347.

Above the stratosphere, the temperature is constant, therefore n equals 1.0.

Below the stratosphere:

$$\frac{P_{oz}}{P_{S.L}} = \left[\frac{T_{oz}}{T_{S.L}}\right]^{\frac{n}{n-1}} = \left[\frac{T_{oz}}{T_{S.L}}\right]^{\frac{1.2347}{0.2347}} = \left(\frac{T_{oz}}{T_{S.L}}\right)^{5.260}$$

# AVAILABLE PART OF ENERGY

## A. Available Part of Enthalpy (AEH).

Consider that one pound of a perfect gas is flowing with negligible velocity at $p_1$ and $T_1$ as shown in Fig. 3. The dead state of the gas is attained when it has negligible velocity and is at the same pressure and temperature as the atmosphere, $p_{oz}$ and $T_{oz}$. The maximum amount of rotary shaft work that can be obtained when the gas is brought to the dead state is

$$AEH \;=\; C_p(T_1 - T_{oz}) - T_{oz}(s_1 - s_{oz}) \quad \cdots\cdots\cdots \quad (4)$$

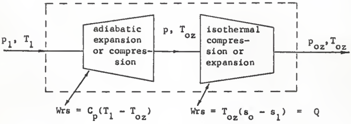

Fig. 3.

The shaded areas in Fig. 4 and Fig. 5 are the available parts of enthalpy. When the gas changes from $p_1$, $T_1$ to $p_2$, $T_2$, the change in the available part of enthalpy is

$$AEH_2 - AEH_1 \;=\; c_p(T_2 - T_1) - T_{oz}(s_2 - s_1) \quad \cdots\cdots \quad (5)$$

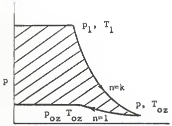

Fig. 4. $s_0 < s_1$

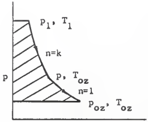

Fig. 5. $s_1 < s_0$

B. Available Part of Internal Energy (AEU)

One pound of a perfect gas is in a cylinder at state $p_1$ and $T_1$ as shown in Fig. 6. The maximum amount of rotary shaft work that can be obtained when the gas is brought to the dead state is given by

$$AEU = C_v(T_1 - T_{oz}) - T_{oz}(s_1 - s_{oz}) - \frac{P_{oz}}{J}(v_{oz} - v_1) \quad \cdot \cdot \cdot \; (6)$$

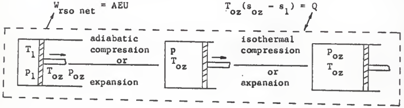

Fig. 6.

The last term, $\frac{P_{oz}}{J}(v_{oz} - v_1)$, is the work done on the atmosphere, and is energy which is wholly unavailable. The change in the available part of internal energy from $p_1$, $T_1$ to $p_2$, $T_2$ is

$$AEU_2 - AEU_1 = C_v(T_2 - T_1) - T_{oz}(s_2 - s_1) + \frac{p_{oz}}{J}(v_2 - v_1) \quad \cdot\ \cdot \quad (7)$$

The shaded area in Fig. 7 and Fig. 8, are the available parts of internal energy.

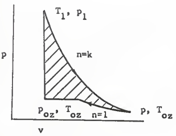

Fig. 7. $s_1 > s_{oz}$

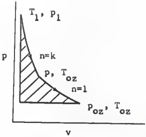

Fig. 8. $s_1 < s_{oz}$

C. Available Energy of a Vacuum, $AE_{vac}$.

$$AE_{vac} = \frac{p_{oz}V}{J} \quad \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \quad (8)$$

D. Available Part of Potential Energy, AEPE, For Constant Density.

There are two forces acting on the system: the gravity force, $\rho Vg/g_c$, and the buoyant force, $\rho_{oz}Vg/g_c$.

$$\text{Net downward force} = V(\rho - \rho_{oz})\frac{g}{g_c} = m\frac{g}{g_c}\left(1 - \frac{\rho_{oz}}{\rho}\right)$$

$$= \left(1 - \frac{\rho_{oz}}{\rho}\right)\frac{g}{g_c} \text{ per lbm.}$$

Below the tropopause

$$\rho_{oz} = \rho_{S.L}(1 - 0.000006817z)^{4.260} .$$

We can assume g is constant; therefore,

$$\text{AEPE} = \frac{1}{J}\frac{g}{g_c}\int_0^z \left[1 - \frac{\rho_{S.L}}{\rho}(1 - 0.000006871z)^{4.260}\right]dz$$

$$= \frac{1}{J}\frac{g}{g_c}\, z - \frac{\rho_{S.L}}{\rho \times 5.260 \times 0.000006871}\left[1 - (1 - 0.000006871z)^{5.260}\right] \quad ,$$

$$\rho_{S.L} = \frac{2116.2}{53.342 \times 518.7} = 0.076483 \quad \text{lbm/ft}^3 \ ,$$

$$v_{S.L} = \frac{1}{\rho_{S.L}} = 13.074 \ \text{ft}^3/\text{lbm} \ ,$$

$$\text{AEPE} = \frac{1}{J}\frac{g}{g_c}\, z - \frac{2116.2}{\rho\, J}\left[1 - (1 - 0.000006871z)^{5.260}\right] \ , \quad \cdots \quad (9)$$

or

$$\text{AEPE} = \frac{z\,g}{Jg_c} - 2.7195\, v \left[1 - \frac{P_{oz}}{P_{S.L}}\right] \quad \text{Btu/lbm} \ , \quad \cdots\cdots\cdots \ (10)$$

AEPE above the tropopause:

$$\text{AEPE} = \frac{1}{J}\frac{g}{g_c}\int_0^{35332} \left[1 - \frac{\rho_{S.L}}{\rho}(1 - 0.000006871z)^{4.260}\right]dz$$

$$+ \frac{1}{J}\frac{g}{g_c}\int_{35332}^{z} \left[1 - \frac{\rho_{S.L}}{\rho} \times 1.3206 \ \text{Exp}\left(0.2236 - \frac{z}{20952}\right)\right]dz$$

$$= \frac{z}{J}\frac{g}{g_c} - \frac{2116.2}{J\rho}\frac{g}{g_c}\left(- 0.75723^{5.260} + 1 - e^{0.2236 - \frac{z}{20952}} + e^{-1.4627}\right)$$

$$= \frac{z}{J}\frac{g}{g_c} - 2.719\, v \left[1 - e^{0.2236 - \frac{z}{20952}}\right]$$

or

$$\text{AEPE} = \frac{z}{J}\frac{g}{g_c} - 2.7195\, v \left[1 - \frac{p_{oz}}{p_{S.L}}\right]$$

or

$$\text{AEPE} = \frac{z}{J}\frac{g}{g_c} - \frac{v}{J}\left[p_{S.L} - p_{oz}\right] \quad \cdots\cdots\cdots\cdots \quad (11)$$

The equations of the available part of potential energy in the stratosphere and in the troposphere are the same despite the difference in the equations for the density of the atmosphere. The decrease in the available part of potential energy from elevation (1) to (2) is

$$\text{AEPE}_1 - \text{AEPE}_2 = \frac{g}{Jg_c}(z_1 - z_2) - \frac{v}{J}(p_{oz2} - p_{oz1}) \quad \cdots\cdots \quad (12)$$

This means that the work done against the buoyant force per pound mass of fluid is equal to the product of the specific volume and the difference in the atmospheric pressures.

Therefore at elevations $z_1$ and $z_2$ it can be shown* that equation (12), the equation for the available part of potential energy, not only can be applied to the standard atmosphere but also can be applied to the atmosphere at any latitude.

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*For any atmosphere:

$$\text{AEPE} = \frac{g\ z}{g_c J} - \frac{g\ v_1}{g_c J}\int_0^z \frac{dz}{v_{oz}}, \quad \text{where } v_{oz} = f(z)$$

For steady flow: $$W_{rso} = h_1 - h_{S.L} - T_{S.L}(s_1 - s_{S.L}) + \frac{g\ z}{g_c J} \quad \cdots \quad (A)$$

For non-flow: $$W_{rso} = \frac{v_1(p_1 - p_{oz})}{J} + \frac{g\ z}{g_c J} - \frac{g}{g_c J} v_1 \int_0^z \frac{dz}{v_{oz}}$$

$$+ u_1 - u_{S.L} - T_{S.L}(s_1 - s_{S.L}) - \frac{p_{S.L}}{J}(v_{S.L} - v_1) \quad (B)$$

According to the Second Law, equations (A) and (B) are equal, therefore

$$\frac{g\ v_1}{g_c J}\int_0^z \frac{dz}{v_{oz}} = \frac{v_1}{J}(p_{S.L} - p_{oz}) \quad \text{or} \quad \text{AEPE} = \frac{g\ z}{g_c J} - \frac{v_1}{J}(p_{S.L} - p_{oz})$$

## NUMERICAL EXAMPLE I -- AVAILABLE PART OF POTENTIAL ENERGY IN NON-FLOW PROCESSES

One pound of air is at $p_1$ = 100 psia, $T_1$ = 1000°R and $Z_1$ = 20,000 ft. The problem is to determine the maximum amount of rotary shaft work that can be produced when the one pound of air initially at state (1) is brought to the sea-level dead state. Four different methods of bringing the air to the sea-level dead state are presented; in the last method (case D) more rotary shaft work is obtained than in each of the first three cases.

Case A:

The one pound of air is brought to sea level by an adiabatic, constant-volume process, then is expanded adiabatically to sea-level temperature, and finally is compressed isothermally to the dead state as shown in Fig. 10. The atmospheric pressure and temperature at 20,000 ft height are 6.75 psia and 447.5°R.

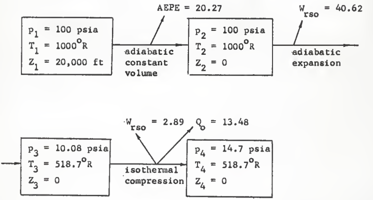

Fig. 10. Case A

$$p_3 = p_2\left[\frac{T_3}{T_2}\right]^{\frac{k}{k-1}} = 100\left[\frac{518.7}{1000}\right]^{3.5} = 10.08 \text{ psia.}$$

$$v_1 = \frac{RT_1}{p_1} = \frac{1000 \times 53.342}{100 \times 144} = 3.704 \text{ ft}^3/\text{lbm}$$

$$\text{AEPE} = \frac{z_1}{J} - 2.7195\, v_1\left[1 - \frac{p_o}{p_{S.L}}\right]$$

$$= \frac{20{,}000}{778.16} - 2.7195 \times 3.074 \left[1 - \frac{6.75}{14.696}\right]$$

$$= 20.27 \text{ Btu/lbm.}$$

$$W_{o\ 2-3} = C_v(T_2 - T_3) = 0.17137(1000 - 518.7) = 82.48 \text{ Btu/lbm.}$$

$$v_3 = \frac{53.342 \times 518.7}{10.08 \times 144} = 19.09 \text{ ft}^3/\text{lbm.}$$

$$W_{on\ atm\ 2-3} = \frac{p_{S.L}}{J}\left[v_3 - v_2\right]$$

$$= 2.7195[19.09 - 3.704] = 41.86 \text{ Btu/lbm.}$$

$$W_{rso\ 2-3} = 82.48 - 41.86 = 40.62 \text{ Btu/lbm.}$$

$$W_{in\ 3-4} = Q_{o\ 3-4} = UEQ_{o\ 3-4} = T_{S.L}\Delta s = T_{S.L}\frac{R}{J}\ln\frac{p_4}{p_3}$$

$$= 518.7 \times 0.068549 \ln\frac{14.7}{10.08} = 13.48 \text{ Btu/lbm.}$$

$$W_{by\ atm\ 3-4} = 2.7195[19.09 - 13.074] = 16.37 \text{ Btu/lbm.}$$

$$W_{rso\ 3-4} = -13.48 + 16.37 = +2.89 \text{ Btu/lbm.}$$

$$AEU_2 - AEU_4 = W_{rso\ 2-3} + W_{rso\ 3-4}$$

$$= 40.62 + 2.89 = 43.51 \text{ Btu/lbm.}$$

$$AEPE + AEU = 20.27 + 43.51 = 63.78 \text{ Btu/lbm.}$$

The original potential energy of the air is 20,000/778.16 = 25.70 Btu/lbm. The sum of this figure and $AEU_{1-S.L}$ is 69.21. However, in this case, the work done by the buoyant forces on the one pound of air causes production of only 20.27 Btu/lbm of rotary shaft work during the descent of the system. Thus the total amount of rotary shaft work is 63.78 Btu/lbm, a loss of 5.43 Btu/lbm.

Case B:

Let the one pound of air of state (1) expand to $p_o$ and $T_o$ at 20,000 ft, then let the one pound of air be at same pressure and temperature as the atmosphere during descent to sea level as shown in Fig. 11. In this case no rotary shaft work will be realized during the descent of the air because the buoyant force and the weight force cancel each other.

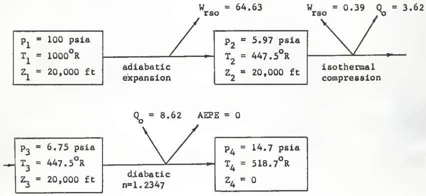

Fig. 11. Case B.

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}}$$

$$p_2 = 100\left(\frac{447.5}{1000}\right)^{\frac{1.4}{1.4-1}} = 5.97 \text{ psia.}$$

$$W_{o\ 1-2} = C_v(T_1 - T_2) = 0.17137(1000 - 447.5) = 94.68 \text{ Btu/lbm.}$$

$$v_1 = \frac{RT_1}{p_1} = \frac{53.342 \times 1000}{100 \times 144} = 3.704 \text{ ft}^3\text{/lbm.}$$

$$v_2 = \frac{RT_2}{p_2} = \frac{53.342 \times 447.5}{5.97 \times 144} = 27.76 \text{ ft}^3\text{/lb}$$

$$W_{on\ atmo\ 1-2} = \frac{p_o}{J}(v_2 - v_1) = \frac{6.75 \times 144}{778.16}(27.76 - 3.704) = 30.06 \text{ Btu/lbm}$$

$$W_{rso\ 1-2} = 94.68 - 30.05 = 64.63 \text{ Btu/lbm.}$$

$$W_{in\ 2-3} = T_o \frac{R}{J} \ln \frac{p_3}{p_2} = 447.5 \times 0.068549 \ln \frac{6.75}{5.97} = 3.62 \text{ Btu/lbm}$$

$$v_3 = \frac{RT_3}{p_3} = \frac{53.342 \times 447.5}{6.75 \times 144} = 24.55 \text{ ft}^3\text{/lbm}$$

$$W_{by\ atm\ 2-3} = \frac{p_o}{J}(v_2 - v_3) = \frac{6.75 \times 144}{778.16}(27.76 - 24.55) = 4.01 \text{ Btu/lbm}$$

$$W_{rso\ 2-3} = 4.01 - 3.62 = 0.39 \text{ Btu/lbm.}$$

$$W_{rso\ net\ 1-3} = 64.63 + 0.39 = 65.02 \text{ Btu/lbm.}$$

$$\Delta U_{3-4} = C_v(T_4 - T_3) = 0.17137(518.7 - 447.5) = 12.20 \text{ Btu/lbm}$$

$$W_{in\ 3-4} = \frac{R(T_4 - T_3)}{(n-1)J} = \frac{0.068549(518.7 - 447.5)}{1.2347 - 1} = 20.82 \ \text{Btu/lbm}$$

$$Q_{o\ 3-4} = 20.82 - 12.20 = 8.62 \ \text{Btu/lbm.}$$

$$UEQ_{o\ 2-3} = \frac{R}{J} T_{S.L} \ln \frac{p_3}{p_2} = 0.068549 \times 518.7 \ln \frac{6.75}{5.97} = 4.20 \ \text{Btu/lbm}$$

$$AEQ_{o\ 2-3} = Q_o - UEQ_o = 3.62 - 4.20 = -0.58 \ \text{Btu/lbm.}$$

$$s_4 - s_3 = c_p \ln \frac{T_4}{T_3} - \frac{R}{J} \ln \frac{p_4}{p_3} = 0.23992 \ln \frac{518.7}{447.5} - 0.068549$$

$$\ln \frac{14.696}{6.75} = 0.01790 \ \text{Btu/lbm}^{\circ}\text{R}$$

$$UEQ_{o\ 3-4} = T_o \Delta s = 518.7 \times 0.01790 = 9.29 \ \text{Btu/lbm.}$$

$$AEQ_{o\ 3-4} = Q_o - UEQ_o = 8.62 - 9.29 = -0.67 \ \text{Btu/lbm.}$$

$$UEQ_{o\ 2-3} + UEQ_{o\ 3-4} = 4.20 + 9.29 = 13.49 \ \text{Btu/lbm.}$$

$$AEQ_{o\ 2-3} + AEQ_{o\ 3-4} = -0.58 - 0.67 = -1.25 \ \text{Btu/lbm.}$$

$$(AEU + AEPE)_{\text{Case B}} - (AEU + AEPE)_{\text{Case A}} = 65.02 - 63.78 = 1.24 \ \text{Btu/lbm.}$$

$$(AEQ_o)_{\text{Case A}} - (AEQ_o)_{\text{Case B}} = 0 - (-1.25) = 1.25 \ \text{Btu/lbm.}$$

The reason that case B developed more rotary shaft work than case A is that the available part of the heat rejected in case B is less than in case A.

Case C.

Let the one pound of air of state (1) expand to $p_{S.L}$ and $T_{S.L}$ at 20,000 ft altitude, then hold the volume constant during descent to sea level, as shown in Fig. 12.

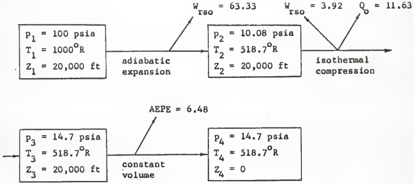

Fig. 12. Case C.

$$\begin{aligned} AEU_1 - AEU_3 &= 0.17137(1000 - 518.7) - 447.5(0.23992 \ln \frac{1000}{518.7} \\ &\quad - 0.068549 \ln \frac{100}{14.7}) + \frac{6.75 \times 144}{778.16} (\frac{53.342 \times 1000}{100.0 \times 144} \\ &\quad - \frac{53.342 \times 518.7}{14.696 \times 144} = 82.48 - 11.36 - 11.71 = 59.41 \text{ Btu/lbm.} \end{aligned}$$

$$\begin{aligned} AEPE &= \frac{z_3}{J} - 2.7195(1 - \frac{p_o}{p_{S.L}}) v_3 \\ &= \frac{20,000}{778.16} - 2.7195 \times (1 - \frac{6.75}{14.696}) \times 13.074 \\ &= 6.48 \text{ Btu/lbm.} \end{aligned}$$

$$AEU + AEPE = 59.41 + 6.48 = 65.89 \text{ Btu/lbm.}$$

Case D.

In this case the one pound of air of state (1) is expanded to $T_o$ at 20,000 ft and is compressed again isothermally to the pressure of state (1). Then it is brought to the sea level and is expanded to the dead state. These processes are shown in Fig. 13.

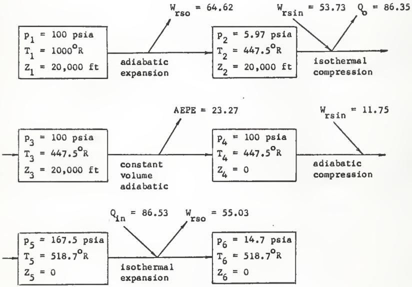

Fig. 13. Case D.

$$p_2 = p_1\left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}} = 100\left(\frac{447.5}{1000}\right)^{3.5} = 5.97 \text{ psia}$$

$$p_5 = p_4\left(\frac{T_5}{T_4}\right)^{\frac{k}{k-1}} = 100\left(\frac{518.7}{447.5}\right)^{3.5} = 167.5 \text{ psia}$$

$$AEU_1 - AEU_3 = C_v(T_1 - T_3) - T_o(C_p \ln \frac{T_1}{T_3} - \frac{R}{J} \ln \frac{p_1}{p_3}) + \frac{p_o}{J}(v_1 - v_3)$$

$$= 0.17137(1000 - 447.5) - 447.5(0.23992 \times \ln \frac{1000}{447.5})$$

$$+ \frac{972.6}{778.16}(\frac{53.342 \times 1000}{100 \times 144} - \frac{53.342 \times 447.5}{100 \times 144})$$

$$= 94.68 - 86.35 + 2.56 = 10.89 \text{ Btu/lbm.}$$

$$AEPE_{3-4} = \frac{Z_3}{J} - 2.7195(1 - \frac{p_o}{p_{S.L}})\ v_3$$

$$= \frac{20,000}{778.16} - 2.7195(\frac{6.75}{14.696})\ \frac{53.342 \times 447.5}{100 \times 144}$$

$$= 23.27 \text{ Btu/lbm.}$$

$$AEU_4 - AEU_6 = 0.17137(447.5 - 518.7) - 518.7(0.23992 \times \ln \frac{447.5}{518.7}$$

$$- 0.06855 \ln \frac{100}{14.7}) + \frac{2116.2}{778.16}(\frac{53.342 \times 447.5}{100 \times 144} - 13.074)$$

$$= -12.20 + 86.53 - 31.05 = 43.28 \text{ Btu/lbm.}$$

$$AEU + AEPE = 10.89 + 23.27 + 43.28 = 77.44 \text{ Btu/lbm.}$$

First Modification of Case D

In this case after the air has been brought to the state (3) in the same manner as in case D, the temperature of the air is kept equal to that of the atmosphere during descent, while the volume remains constant. This process is shown in Fig. 14.

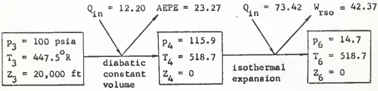

Fig. 14.

$$p_4 = p_3 \frac{T_4}{T_3} = 100 \times \frac{518.7}{447.5} = 115.9 \text{ psia.}$$

$$AEU_4 - AEU_6 = T_o \frac{R}{J} \ln \frac{p_4}{p_6} + \frac{p_{S.L}}{J}(v_4 - v_6)$$

$$= 518.7 \times 0.068549 \ln \frac{115.9}{14.7} + 2.7195\left(\frac{53.342 \times 518.7}{115.9 \times 144} - 13.074\right)$$

$$= 73.42 - 31.05 = 42.37 \text{ Btu/lbm.}$$

$$AEU_{adiab.} - AEU_{diab.} = 43.28 - 42.37 = 0.91 \text{ Btu/lbm.}$$

The available part of the internal energy is smaller by 0.91 Btu/lbm when the process during descent is diabatic instead of adiabatic. The reason is because the diabatic case has an inflow of negative available energy as heat flows from the atmosphere to the system during descent. This flow of negative available energy can be determined in the flowing manner:

$$\Delta s_{3-4} = C_v \ln \frac{T_4}{T_3} = 0.17137 \ln \frac{518.7}{447.5} = 0.025286 \text{ Btu/lbm}^{\circ}\text{R}$$

$$UEQ_{in} = T_o \Delta s = 518.7 \times 0.025286 = 13.11 \text{ Btu/lbm.}$$

$$Q_{in} = C_v(T_4 - T_3) = 0.17137(518.7 - 447.5) = 12.20 \text{ Btu/lbm.}$$

$$AEQ_{in} = Q_{in} - UEQ_{in} = 12.20 - 13.11 = -0.91 \text{ Btu/lbm.}$$

Second Modification of Case D

In this case, after the air is brought to state (4) in the same manner as in case D, it is first expanded at constant pressure to the sea-level temperature, and then is expanded isothermally to the dead state, as shown in Fig. 15.

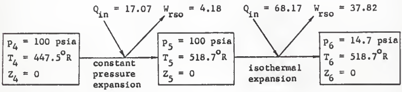

Fig. 15.

$$W_{o\ 4-5} = \frac{P_4}{J}(v_5 - v_4) = \frac{R}{J}(T_5 - T_4) = \frac{53.342}{778.16}(518.7 - 447.5) = 4.88 \text{ Btu/lbm.}$$

$$W_{o\ 5-6} = \frac{R}{J} \ln \frac{P_5}{P_6} = 0.068549 \ln \frac{100}{14.7} = 68.17 \text{ Btu/lbm.}$$

$$W_{on\ atm.\ 4-6} = \frac{P_o}{J}(v_6 - v_5) = 2.7195(13.074 - \frac{53.342 \times 447.5}{100 \times 144}) = 31.05 \text{ Btu/lbm.}$$

$$W_{rso\ 4-6} = 4.88 + 68.17 - 31.05 = 42.00 \text{ Btu/lbm.}$$

$$AEU_{4-6} - W_{rso\ 4-6} = 43.28 - 42.00 = 1.28 \text{ Btu/lbm.}$$

This difference between the change in the available part of internal energy and the production of rotary shaft work for the process 4-6 can be explained in the following manner:

$$\Delta s_{4-5} = C_p \ln \frac{T_5}{T_4} = 0.23992 \ln \frac{518.7}{447.5} = 0.035397 \text{ Btu/lbm.}$$

$$UEQ_{in\ 4-5} = T_o \Delta s = 518.7 \times 0.035397 = 18.36 \text{ Btu/lbm.}$$

$$Q_{in\ 4-5} = C_p(T_5 - T_4) = 0.23992(518.7 - 447.5) = 17.08 \text{ Btu/lbm.}$$

$$AEQ_{in\ 4-5} = Q_{in} - UEQ_{in} = 17.08 - 18.36 = -1.28 \text{ Btu/lbm.}$$

It is this negative available part of the heat flow in (-1.28 Btu/lbm) during the process 4-5 which is the reason that the production of rotary shaft work during the process 4-6 is less by 1.28 Btu/lbm than the decrease in the available part of internal energy during the process 4-6.

Summary for the Non-flow Processes of Case (A) to (D)

| | $W_{rso}$ net Btu/lbm | Heat rejected to the atmosphere at elevations above sea level Btu/lbm |
|---|---|---|
| Case A | 63.78 | 0 |
| Case B | 65.02 | 3.62 at 20,000 ft<br>8.62 during descent |
| Case C | 65.89 | 11.36 at 20,000 ft |
| Case D | 77.44 | 86.35 at 20,000 ft |

Case A: Work done against buoyant force during descent

$= 25.70 - 20.27 = 5.43$ Btu/lbm.

$W_{on\ atm.\ 2-3} = 41.86$ Btu/lbm.

$W_{by\ atm.\ 3-4} = 16.37$ Btu/lbm.

$\Sigma$ work done on atm. $= 41.86 - 16.37 + 5.43 = 30.92$ Btu/lbm.

Case B: $W_{\text{on atm. } 1-2} = 30.06$ Btu/lbm.

$W_{\text{by atm. } 2-3} = 4.01$ Btu/lbm.

Work done against buoyant force = 25.70 Btu/lbm.

$W_{\text{by atm. } 3-4} = 20.82$ Btu/lbm.

$\Sigma$ work done on atm. = $30.06 + 25.7 - 4.01 - 20.82 = 30.93$ Btu/lbm.

Case C: $W_{\text{on atm. } 1-3} = 11.71$ Btu/lbm.

Work done against buoyant force = $25.70 - 6.48 = 19.22$ Btu/lbm.

$\Sigma$ work done on atm. = $11.71 + 19.22 = 30.93$ Btu/lbm.

Case D: $W_{\text{by atm. } 1-3} = 2.56$ Btu/lbm.

Work done against buoyant force = $25.70 - 23.27 = 2.43$ Btu/lbm.

$W_{\text{on atm. } 4-6} = 31.05$ Btu/lbm.

$\Sigma$ work done on atm. = $31.05 + 2.43 - 2.56 = 30.92$ Btu/lbm.

From the previous calculations it follows that the greater the heat rejected to the atmosphere above sea level, the greater is the production of net rotary shaft work.

Furthermore, when the system changes from state (1) at high altitude to the dead state at sea-level, the summation of work done on the atmosphere is constant and is independent of the process.

Derivation of the Equation of $\Sigma W_{rso}$ for One Pound of Ideal Gas at $p_1$, $T_1$, $z_1$ which undergoes the Processes As Shown in Fig. 16, which is case D, the case that produces more rotary shaft work than the other three.

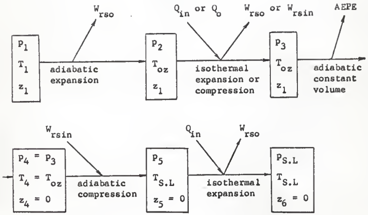

Fig. 16.

$$v_3 = \frac{RT_{oz}}{p_3}$$

$$s_1 - s_3 = C_p \ln \frac{T_1}{T_{oz}} - \frac{R}{J} \ln \frac{p_1}{p_3}$$

$$W_{in\ 1-3} = T_{oz}(s_1 - s_3) - C_v(T_1 - T_{oz})$$

$$W_{by\ atm.\ 1-3} = \frac{p_{oz}}{J}(v_1 - v_3)$$

$$W_{rs\ in\ 1-3} = T_{oz}(s_1 - s_3) - C_v(T_1 - T_{oz}) - \frac{p_{oz}}{J}(v_1 - v_3)$$

$$AEPE_{3-4} = \frac{g\ z}{g_c J} - \frac{p_{S.L}}{J} v_3 (1 - \frac{p_{oz}}{p_{S.L}}) = \frac{g\ z}{g_c J} - \frac{v_3}{J}(p_{S.L} - p_{oz})$$

$$s_{S.L} - s_3 = C_p \ln \frac{T_{S.L}}{T_{oz}} - \frac{R}{J} \ln \frac{p_{S.L}}{p_3}$$

$$W_{o\ 4-6} = T_{S.L}(s_{S.L} - s_3) - C_v(T_{S.L} - T_{oz})$$

$$W_{on\ atm.\ 4-6} = \frac{p_{S.L}}{J}(v_{S.L} - v_3)$$

$$W_{rso\ 4-6} = T_{S.L}(s_{S.L} - s_3) - C_v(T_{S.L} - T_{oz}) - \frac{p_{S.L}}{J}(v_{S.L} - v_3)$$

$$\begin{aligned}
\Sigma W_{rso} &= \frac{g\ z}{g_c J} - \frac{v_3}{J}(p_{S.L} - p_{oz}) + T_{S.L}(s_{S.L} - T_{oz}) \\
&\quad - C_v(T_{S.L} - T_{oz}) - \frac{p_{S.L}}{J}(v_{S.L} - v_3) \\
&\quad - T_{oz}(s_1 - s_3) + C_v(T_1 - T_{oz}) \\
&\quad - \frac{p_{oz}}{J}(v_1 - v_3) \\
&= \frac{g\ z}{g_c J} + T_{S.L}(s_{S.L} - s_3) + C_v(T_1 - T_{S.L}) \\
&\quad + \frac{1}{J}(p_{oz}v_1 - p_{S.L}v_{S.L}) - T_{oz}(a_1 - s_3)
\end{aligned}$$

or

$$\begin{aligned}
\Sigma W_{rso} &= \frac{g\ z}{g_c J} + T_{S.L}(s_{S.L} - s_1) + (T_{S.L} - T_{oz})(s_1 - s_3) \\
&\quad + C_v(T_1 - T_{S.L}) + \frac{1}{J}(p_{oz}v_1 - p_{S.L}v_{S.L}) \quad \cdots\cdots (13)
\end{aligned}$$

The greater is $p_3$, the greater is $(a_1 - s_3)$ and the greater is $\Sigma W_{rso}$. In Fig. 27 the production of rotary shaft work is plotted versus altitude for the cases in which $p_3$ has the values of 50 psia, 100 psia, and 200 psia, and for which $p_1 = p_{oz}$ and $T_1 = T_{oz}$.

## NUMERICAL EXAMPLE II -- POWER PRODUCTION IN NON-FLOW CYCLES WITH CHANGES IN ELEVATION

### Non-Flow Cycle (1)

Consider that one pound of air in a cylinder completes the simple non-flow air cycle shown in Fig. 17.

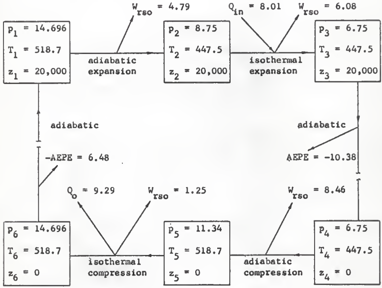

Fig. 17. Non-flow cycle

$AEPE_{6-1} = -6.48$ Btu/lbm (See III Case C)

$$AEU_{1-3} = 0.17137(518.7 - 447.5) - 447.5(0.23992 \ln \frac{518.7}{447.5} - 0.068549 \ln \frac{14.696}{6.75}) + \frac{6.75 \times 144}{778.16}(13.074 - \frac{53.342 \times 447.5}{6.75 \times 144})$$

$$= 12.20 + 8.01 - 14.34 = 5.87 \text{ Btu/lbm.}$$

$$W_{rso\ 6-3} = -6.48 + 5.87 = -0.61 \text{ Btu/lbm.}$$

$$AEPE_{3-4} = \frac{20,000}{778.16} - 2.7195 \times 24.546\left(1 - \frac{6.75}{14.696}\right)$$

$$= -10.38 \text{ Btu/lbm.}$$

$$AEU_{4-6} = 0.17137(447.5 - 518.7) - 518.7\left(0.23992 \ln \frac{447.5}{518.7}\right.$$

$$\left. - 0.068549 \ln \frac{6.75}{14.696}\right) - 2.7195(13.074 - 24.546)$$

$$= -12.20 - 9.29 + 31.20 = 9.71 \text{ Btu/lbm.}$$

$$W_{rso\ 3-6} = 9.71 - 10.38 = -0.67 \text{ Btu/lbm.}$$

$$W_{rso\ cycle} = (9.71 + 5.87) - (10.38 + 6.48) = -1.28 \text{ Btu/lbm.}$$

General Equation for $W_{rso\ cycle}$ for the Process Shown in Fig. 17.

The available part of the internal energy of the system when its pressure and temperature are the same as those of the atmosphere at elevation z, referred to a dead state whose pressure and temperature are the same as the atmosphere at sea-level is

$$AEU_{oz} - AEU_{S.L} = C_v(T_{oz} - T_{S.L}) - T_{S.L}(s_{oz} - s_{S.L})$$

$$- \frac{P_{S.L}}{J}(v_{S.L} - v_{oz}) \quad \cdots\cdots\cdots\cdots \quad (A)$$

The available part of the potential energy of the system whose state properties are: (1) elevation z, (2) pressure and temperature equal to those of the atmosphere at elevation z, referred to a dead state whose state properties are: (1) sea-level elevation, (2) pressure and temperature equal to those of the atmosphere at sea-level is

$$\mathrm{AEPE}_{z - \mathrm{S.L}} = \frac{z}{J}\frac{g}{g_c} - \frac{p_{\mathrm{S.L}}}{J} v_{oz}(1 - \frac{p_{oz}}{p_{\mathrm{S.L}}}) \quad \cdots\cdots\cdots \quad (B)$$

The available part of the internal energy of the system when its pressure and temperature are the same as those of the atmosphere at sea-level referred to a dead state whose pressure and temperature are the same as the atmosphere at elevation z is

$$\mathrm{AEU}_{\mathrm{S.L}} - \mathrm{AEU}_{oz} = C_v(T_{\mathrm{S.L}} - T_{oz}) - T_{oz}(s_{\mathrm{S.L}} - s_z) - \frac{p_{oz}}{J}(v_z - v_{\mathrm{S.L}}) \quad (C)$$

The available part of the potential energy of the system whose state properties ara: (1) sea-leval elevation, (2) pressure and temperature equal to those of the atmosphere at sea-level, referred to a dead state whose state properties ara: (1) elevation z, (2) pressure and temperature equal to those of the atmosphere at elevation z is

$$\mathrm{AEPE}_{\mathrm{S.L} - z} = -\frac{z}{J}\frac{g}{g_c} + \frac{p_{oz}}{J} v_{\mathrm{S.L}}(1 - \frac{p_{oz}}{p_{\mathrm{S.L}}}) \quad \cdots\cdots \quad (D)$$

$$W_{\mathrm{rso\ cycle}} = A + B + C + D = -(T_{\mathrm{S.L}} - T_{oz})(s_{oz} - s_{\mathrm{S.L}}) \quad (14)$$

If z = 20,000 ft

$$W_{\mathrm{rso\ cycla}} = -(518.7 - 447.5)(0.23992 \ln \frac{447.5}{518.7} - 0.068549 \ln \frac{6.75}{14.696})$$

$$= -1.28 \quad \mathrm{Btu/lbm.} \qquad \mathrm{Q.E.D.}$$

This means that the $W_{\mathrm{rsin}}$ required to raise tha one pound of air from sea level to altitude z plus tha $W_{\mathrm{rsin}}$ required to lower it from z to sea level exceeds the $W_{\mathrm{rso}}$ produced by $\mathrm{AEU}_z - \mathrm{AEU}_{\mathrm{S.L}}$ plus $\mathrm{AEU}_{\mathrm{S.L}} - \mathrm{AEU}_z$ by

$(T_{S.L} - T_{oz})(s_z - s_{S.L})$. The thermodynamic cycle is as shown in Fig. 18.

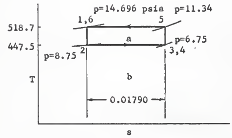

Fig. 18. T-s diagram for non-flow cycle (1).

The process 3 - 5 - 6 gives $AEU_z - AEU_{S.L}$

The process 1 - 2 - 3 gives $AEU_{S.L} - AEU_z$.

Area (a) represents $(T_{S.L} - T_{oz})(s_z - s_{S.L})$.

Non-Flow Cycle (2)

If the air at (6) is expanded isothermally to (5) while at sea level and then raised to z, and if the air at (3) is compressed isothermally to (2) while at z and then lowered to sea level, the cycle will then go in the opposite direction from that shown in Fig. 18. The result will be a production of $W_{rso}$ from the cycle which is greater than the $W_{rsin}$ required to raise and lower the one pound of air by the factor $(T_{S.L} - T_{oz})(s_z - s_{S.L})$. This is demonstrated by the following set of computations. The T-s diagram is shown in Fig. 19.

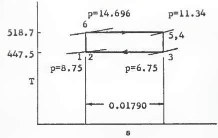

Fig. 19. T-s diagram for non-flow cycle (2).

$$p_5 = 6.75\left(\frac{518.7}{447.5}\right)^{3.5} = 11.34 \text{ psia.}$$

$$p_2 = 14.696\left(\frac{447.5}{518.7}\right)^{3.5} = 8.75 \text{ psia.}$$

$$v_5 = \frac{53.342 \times 518.7}{11.34 \times 144} = 16.95 \text{ ft}^3/\text{lbm.}$$

$$v_1 = 13.074 \text{ ft}^3/\text{lbm.}$$

$$v_2 = \frac{53.342 \times 447.5}{8.75 \times 144} = 18.93 \text{ ft}^3/\text{lbm.}$$

$$v_3 = 24.546 \text{ ft}^3/\text{lbm.}$$

$$W_{o\ 6-5} = Q_{in\ 6-5} = 518.7(0.01790) = 9.29 \text{ Btu/lbm.}$$

$$W_{on\ atm.\ 6-5} = 2.7159(16.95 - 13.074) = 10.54 \text{ Btu/lbm.}$$

$$W_{rsin\ 6-5} = 10.54 - 9.29 = 1.25 \text{ Btu/lbm.}$$

$W_{rsin}$ needed to raise the one pound of air to z

$= 20{,}000 \text{ ft} = 25.70 - 2.7195 \times 16.95 \times (1 - \frac{6.75}{14.696})$

$= 0.83$ Btu/lbm.

$W_{o\ 4-3} = 0.17137(518.7 - 447.5) = 12.20$ Btu/lbm.

$W_{on\ atm\ 4-3} = \frac{6.75 \times 144}{778.16}(24.546 - 16.95) = 9.47$ Btu/lbm.

$W_{rso\ 4-3} = 12.20 - 9.47 = 2.73$ Btu/lbm.

$W_{in\ 3-2} = 447.5(0.01790) = 8.00$ Btu/lbm.

$W_{by\ atm\ 3-2} = \frac{6.75 \times 144}{778.16}(24.546 - 18.93) = 7.01$ Btu/lbm.

$W_{rsin\ 3-2} = 8.00 - 7.01 = 0.99$ Btu/lbm.

$W_{rsin}$ needed to lower the one pound of air to sea level

$= - 25.70 + 2.7195 \times 18.95(1 - \frac{6.75}{14.696}) = 2.10$ Btu/lbm.

$W_{in\ 1-6} = 0.17137(518.7 - 447.5) = 12.20$ Btu/lbm.

$W_{by\ atm\ 1-6} = 2.7195(18.95 - 13.074) = 15.92$ Btu/lbm.

$W_{rso\ 1-6} = 15.92 - 12.20 = 3.72$ Btu/lbm.

Net $W_{rso}$ in thermo. cycle $= (2.73 + 3.72) - (1.25 + 0.99)$

$= 4.21$ Btu/lbm.

$W_{rsin}$ needed to raise and lower $= 0.83 + 2.10 = 2.93$ Btu/lbm.

$$\text{Net } W_{rso} \text{ produced} = 4.21 - 2.93 = 1.28 \text{ Btu/lbm.}$$

$$= (T_{S.L} - T_{oz})(s_{oz} - s_{S.L}) \quad \cdots\cdots \quad (15)$$

Hence the lower is $p_5$ and the greater is $p_2$, the greater will be $W_{rso\ net}$ and it will equal $(T_{S.L} - T_{oz})(s_5 - s_2)$.

From the derivation of dquation (14) it is very interasting to note that

$$\Sigma W_{on\ atm} = \Sigma W_{by\ atm} + \Sigma AEPE \quad \cdots\cdots\cdots\cdots \quad (16)$$

For non-flow cycle (1)

$$14.34 = 31.20 + (-10.38 - 6.48)$$

$$= 14.34$$

For non-flow cycle (2) — power producing cycle

$$W_{on\ atm} = W_{by\ atm} + AEPE_{cycle}.$$

$$10.54 + 9.47 = 7.01 + 15.92 + (-0.83 - 2.10)$$

$$20.01 = 20.00$$

Therefore the above two cycles are Carnot cycles despite the influence of $W_{by\ atm.}$, $W_{on\ atm.}$ and the buoyant force.

Equation (16) can also be illustrated in tha following manner:

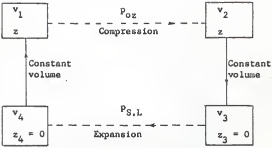

Fig. 20. Non-flow cycle.

In the above non-flow cycle (Fig. 20).

$$v_4 = v_1 . \qquad v_2 = v_3 .$$

$$W_{\text{by atm}} = \frac{p_{oz}}{J}(v_1 - v_2)$$

$$W_{\text{on atm}} = \frac{p_{S.L}}{J}(v_4 - v_3) = \frac{p_{S.L}}{J}(v_1 - v_2)$$

$$\Delta AEPE_{2-3} = \frac{z}{J}\frac{g}{g_c} - \frac{v_2}{J}(p_{S.L} - p_{oz})$$

$$\Delta AEPE_{4-1} = -\frac{z}{J}\frac{g}{g_c} + \frac{v_1}{J}(p_{S.L} - p_{oz})$$

$$\Sigma AEPE = \frac{1}{J}(v_1 - v_2)(p_{S.L} - p_{oz})$$

$$\Sigma W_{\text{on atm}} = \Sigma W_{\text{by atm}} + \Sigma AEPE$$

$$\frac{p_{S.L}}{J}(v_1 - v_2) = \frac{p_{oz}}{J}(v_1 - v_2) + \frac{1}{J}(v_1 - v_2) \times (p_{S.L} - p_{oz})$$

$$= \frac{p_{S.L}}{J}(v_1 - v_2)$$

Q.E.D.

## NUMERICAL EXAMPLE III -- POWER PRODUCTION IN STEADY-FLOW CYCLES WITH CHANGES IN ELEVATION

The atmosphere temperature at high altitude is much less than the sea-level temperature. We can use the atmosphere at high altitude as a heat sink and the sea-level atmosphere as a heat source to construct a power cycle. It is very interesting to see the relations between various kinds of steady-flow cycles in which there are changes in elevation in the cycles.

Four cases are given which have the following identical conditions: (1) the flow starts at sea level and goes to an altitude of 20,000 feet, (2) the pressure and temperature of the system at the start of the upflow are the same as the atmospheric air at sea level, and (3) at the start of the downflow the system has a pressure of 100 psia and a temperature which is the same as that of the atmosphere at 20,000 feet.

### Steady-Flow Cycle (1)

The upward flow and the downward flow are adiabatic processes. The schematic diagram, and the p-v and T-S diagrams are shown in Figs. 21, 22 and 23.

$$C_p T_1 = C_p T_2 + \frac{z_2 g}{J g_c} \quad \text{(negligible velocity change)}$$

$$0.23992 \times 518.7 = 0.23992 \times T_2 + \frac{20,000}{778.16}$$

$$T_2 = 411.6^{o}R$$

$$p_2 = p_1 (\frac{T_2}{T_1})^{\frac{k}{k-1}} = 14.696(\frac{411.6}{518.7})^{3.5} = 6.54 \text{ psia}$$

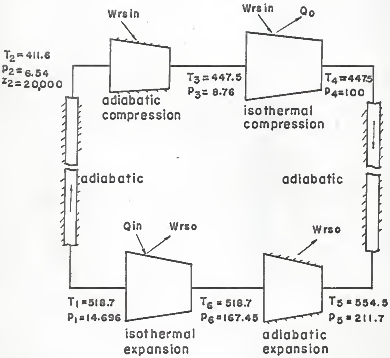

Fig.21. Steady flow cycle I.

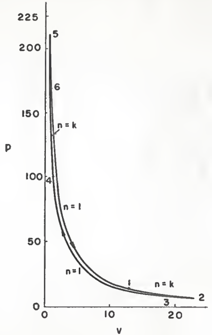

Fig. 22. p-v diagram for steady-flow cycle l.

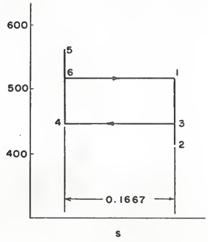

Fig. 23. T-s diagram for steady-flow cycle l.

$$P_3 = P_2\left(\frac{T_3}{T_2}\right)^{\frac{k}{k-1}} = 6.54\left(\frac{447.5}{411.6}\right)^{3.5} = 8.76 \text{ psia}$$

$$W_{rsin\ 2\text{-}3} = C_p\,(T_3 - T_2) = 0.23992(447.5 - 411.6) = 8.62 \text{ Btu/lbm.}$$

$$W_{rsin\ 3\text{-}4} = \frac{R}{J}\,T_o \ln\frac{P_4}{P_2} = 0.68549 \times 447.5 \ln\frac{100}{8.76} = 74.67 \text{ Btu/lbm.}$$

$$C_p\,T_4 + \frac{z_4 g}{J g_c} = C_p\,T_5$$

$$T_5 = 447.5 + \frac{20{,}000}{778.16} = 554.6^\circ R$$

$$P_5 = 100\left(\frac{554.6}{447.5}\right)^{3.5} = 211.70 \text{ psia}$$

$$W_{rso\ 5\text{-}6} = C_p(T_5 - T_6) = 0.23992(554.6 - 518.7) = 8.62 \text{ Btu/lbm.}$$

$$P_6 = 211.7\left(\frac{518.7}{554.6}\right)^{3.5} = 167.45 \text{ psia}$$

$$W_{rso\ 6\text{-}1} = \frac{R}{J}\,T_{S.L} \ln\frac{P_6}{P_{S.L}} = 0.068549 \times 518.7 \ln\frac{167.45}{14.696}$$

$$= 86.55 \text{ Btu/lbm}$$

Cycle net work = 86.55 + 8.62 − 8.62 − 74.6 = 11.88 Btu/lbm

$$\text{Carnot cycle efficiency} = \frac{518.7 - 447.5}{518.7} = 0.1373$$

$$\text{Cycle efficiency} = \frac{W_{rso\ net}}{Q_{in}} = \frac{11.88}{86.55} = 0.1373$$

$$s_{6-1} = \frac{86.55}{518.7} = 0.1669 \quad \text{Btu/lbm}^{\circ}\text{R}$$

$$s_{3-4} = \frac{74.67}{447.5} = 0.1669 \quad \text{Btu/lbm}^{\circ}\text{R}$$

Steady-Flow Cycle (2)

Diabatic processes are used in both the upward flow and downward flow instead of adiabatic processes. In these diabatic processes the pressure and temperature of the system at any altitude are the same as those of the atmosphere at that altitude. The schematic diagram, p-v and T-s diagrams ara shown in Figs. 24, 25 and 26.

$$C_p\, T_1 + Q_{in} = C_p\, T_2 + \frac{z_2 g}{J g_c} \quad \text{in which } T_2 = T_{oz} = 447.5^{\circ}\text{R}$$

$$0.23992 \times 518.7 + Q_{in} = 0.23992 \times 447.5 + \frac{20{,}000}{778.16}$$

$$Q_{in} = 8.62 \quad \text{Btu/lbm}$$

$$p_4 = p_3 \left(\frac{T_4}{T_3}\right)^{\frac{n}{n-1}} = 100\left(\frac{518.7}{447.5}\right)^{\frac{1.2347}{1.2347-1}} = 217.8 \quad \text{psia}$$

$$W_{rsin\ 2-3} = 0.68549\ T_o \ln \frac{p_3}{p_2} = 0.068549 \times 447.5 \ln \frac{100}{6.75}$$

$$= 82.67 \quad \text{Btu/lbm}$$

$$W_{rso\ 4-1} = 0.068549 \times 518.7 \ln \frac{217.8}{14.696} = 95.84 \quad \text{Btu/lbm}$$

$$\text{Cycle work net} = 95.84 - 82.67 = 13.17 \quad \text{Btu/lbm}$$

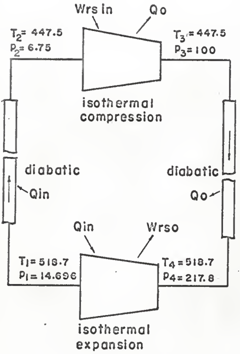

Fig. 24 · Steady flow cycle 2.

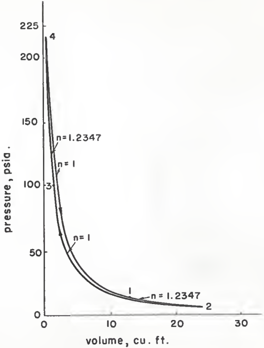

Fig.25. p-v diagram for steady-flow cycle 2.

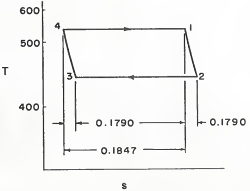

Fig.26. T-s diagram for steady-flow cycle 2.

$$s_1 - s_2 = C_p \ln \frac{T_1}{T_2} - \frac{R}{J} \ln \frac{P_1}{P_2} = 0.23992 \ln \frac{518.7}{447.5}$$

$$- 0.068549 \ln \frac{14.696}{6.75}$$

$$= - 0.01790 \quad \text{Btu/lbm}^{\circ}\text{R}$$

$$s_3 - s_4 = 0.23992 \ln \frac{447.5}{518.7} - 0.068549 \ln \frac{100.0}{217.8}$$

$$= + 0.01790 \quad \text{Btu/lbm}^{\circ}\text{R}$$

$$s_2 - s_3 = \frac{82.67}{447.5} = 0.1847 \quad \text{Btu/lbm}^{\circ}\text{R}$$

$$s_1 - s_4 = \frac{95.84}{518.7} = 0.1847 \quad \text{Btu/lbm}^{\circ}\text{R}$$

$$\text{Carnot cycle efficiency} = \frac{518.7 - 447.6}{518.7} = 0.1373$$

$$\text{The cycle efficiency} = \frac{Q_{in} - Q_o}{Q_{in}} = \frac{95.84 - 82.67}{95.84} = 0.1373$$

Steady-Flow Cycle (3)

Let the upward flow be the adiabatic process of cycle (1) and the downward flow be the diabatic process of cycle (2). As compared with cycle 1 and cycle 2 it is obvious that the cycle net work equals

$- 8.62 - 74.67 + 95.84 = 12.55$ Btu/lbm.

Steady-Flow Cycle (4)

Let the upward flow be the diabatic process of cycle (2) and the downward flow be the adiabatic process of cycle (1). As compared with cycle 1 and cycle 2, the cycle net work equals $-82.67 + 8.62 + 86.55 = 12.50$ Btu/lbm.

Summary for the Above Four Steady-Flow Cycles

cycle 1 $W_{rso\ net}$ = 11.88 Btu/lbm, adiab. up and down.

cycle 2 $W_{rso\ net}$ = 13.17 Btu/lbm, diab. up and down.

cycle 3 $W_{rso\ net}$ = 12.55 Btu/lbm, adiab. up, diab. down.

cycle 4 $W_{rso\ net}$ = 12.50 Btu/lbm, diab. up, adiab. down.

In cycle 2, the $Q_{in}$ in the diabatic upward flow is 8.62 Btu/lbm,

$\Delta s = 0.01790$ Btu/lbm°R, therefore

$UEQ_{in} = T_{oz}\Delta s = 447.5 \times 0.1790 = 8.00$ Btu/lbm

$AEQ_{in} = Q_{in} - UEQ_{in} = 8.62 - 8.00 = 0.62$ Btu/lbm

$W_{rso\ net\ 2} - W_{rso\ net\ 3} = 13.17 - 12.55 = 0.62$ Btu/lbm

The $Q_o$ in the diabatic downward flow is 8.62 Btu/lbm,

$\Delta s = -0.1790$ Btu/lbm°R, therefore

$UEQ_o = 518.7 \times 0.01790 = 9.29$ Btu/lbm

$AEQ_o = Q_o - UEQ_o = 8.62 - 9.29 = -0.67$ Btu/lbm

$W_{rso\ net\ 2} - W_{rso\ net\ 4} = 13.17 - 12.50 = 0.67$ Btu/lbm

From the previous calculations it follows that cycle (2) is the best cycle, because during the upward flow process there is 0.62 Btu/lbm of available part of heat flow into the system, and during the downward flow process there is 0.67 Btu/lbm of negative available part of heat flow out. Therefore the net rotary shaft work produced by cycle (2) is greater than the net rotary shaft work produced by cycle (1) by 0.62 + 0.67 = 1.29 Btu/lbm.

Derivation of the Formula For $W_{rso}$ In Cycle 2; Diabatic Flow Up and Down, Below the Tropopause:

$$W_{rso\ net} = \frac{R}{J} T_4 \ln \frac{p_4}{p_1} - \frac{R}{J} T_2 \ln \frac{p_3}{p_2}$$

in which $p_2 = p_{oz}$ , $T_2 = T_{oz}$ , $T_4 = T_{S.L}$

$$\frac{p_2}{p_1} = (\frac{T_2}{T_1})^{\frac{n}{n-1}} = (\frac{T_3}{T_4})^{\frac{n}{n-1}} = \frac{p_3}{p_4}$$

$$\frac{p_4}{p_1} = \frac{p_3}{p_2}$$

$$W_{rso\ net} = \frac{R}{J}(518.7 - T_{oz})\ln \frac{p_3}{p_{oz}} \quad \cdots\cdots\cdots\cdots \quad (17)$$

In the stratosphere, $pv^n = C$. $n = 1$. $T$ = constant = 392.78°R. It is obvious that $\frac{p_4}{p_1} = \frac{p_3}{p_2}$ still holds above tropopause. Therefore:

$$W_{rso\ net} = \frac{R}{J}(T_4 - T_2)\ln \frac{p_3}{p_2} = \frac{R}{J}(518.7 - 392.78)\ln \frac{p_3}{p_2}$$

$$= \frac{R}{J} \times 125.92 \ln \frac{p_3}{p_{oz}} \quad \cdots\cdots\cdots\cdots \quad (18)$$

The $W_{rso\ net}$ versus height and $p_3$ is shown in Fig. 27.

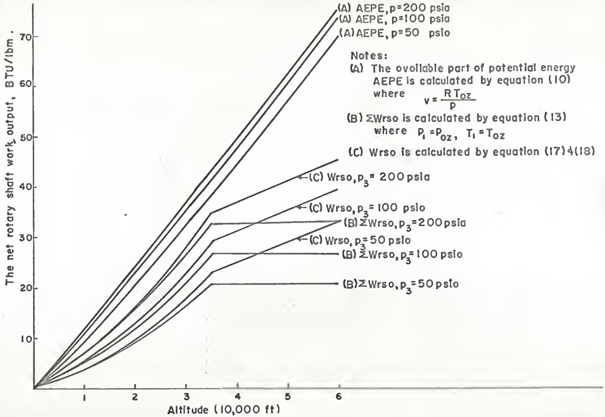

Fig. 27. The net rotary shaft work versus altitude and pressure.

# CONCLUSIONS

(1) The equation $pv^n = c$ holds for the standard atmosphere. Below the tropopause n equals 1.2347. In the stratosphere n equals 1.

(2) The available part of potential energy for non-flow processes can be expressed by this equation:

$$\text{AEPE} = \frac{z}{J}\frac{g}{g_c} - 2.7195v\left[1 - \frac{P_{oz}}{P_{S.L}}\right]$$

The smaller is the specific volume during descent, the greater is the available part of potential energy, but it can not be greater than $\frac{z}{J}\frac{g}{g_c}$. This equation holds for any atmosphere.

(3) In non-flow processes, the greater the heat rejected to the atmosphere above sea-level, the greater is the production of net rotary shaft work. Furthermore, when the system changes from state (1) at high altitude to the dead state at sea-level, the summation of work done on the atmosphere is constant and is independent of the process.

(4) In a non-flow cycle or a steady-flow cycle, in which elevation changes are a part of the cycle, and the processes are adiabatic during the elevation changes, the net rotary shaft work equals the net rotary shaft work of a Carnot Cycle working between the sama temperature and pressure limits as those of the atmosphere at the two prescribed elevations. For both non-flow and steady-flow cycles, the greater the pressure before the fluid descends to sea level and the smaller the pressure at sea level before the fluid rises, tha greater the production rotary shaft work.

(5) In steady-flow processes, if diabatic processes are used in both the upward flow and downward flow, the cycle efficiency equals the cycle efficiency of a Carnot Cycle working between the same temperature limits.

# ACKNOWLEDGMENT

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# AN INVESTIGATION OF THE AVAILABILITY OF POTENTIAL ENERGY AND ITS RELATION TO POWER CYCLES RESULTING FROM CHANGES IN ELEVATION IN A STANDARD ATMOSPHERE

by

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Diploma, Taiwan Provincial Taipei Institute of Technology, 1959

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AN ABSTRACT OF A MASTER'S REPORT

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requirements for the degree

MASTER OF SCIENCE

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KANSAS STATE UNIVERSITY
Manhattan, Kansas

1967

This report deals with the available part of potential energy, available part of internal energy and available part of enthalpy as it is related to the NACA Standard Atmosphere.

The equations of the temperature, pressure and density ratio relationships and the equations of the available part of potential energy are derived for the elevation change from sea level to the stratosphere.

Several numerical examples are presented to show the detailed calculations required to obtain net rotary shaft work in non-flow processes, non-flow cycles and steady flow cycles with change in elevations. Several equations for net rotary shaft work are presented.

In a non-flow cycle or a steady-flow cycle, in which elevation changes are a part of the cycle and the processes are adiabatic during the elevation changes, the net rotary shaft work equals the net rotary shaft work of a Carnot Cycle working between the same temperature and pressure limits as those of the atmosphere at the two prescribed elevations. This is so because, in a non-flow cycle, the work done by the atmosphere at high altitude plus the work done by the buoyant force during ascent equals the work done on the atmosphere at sea level plus the work done against the buoyant force during descent. In the case of a steady-flow cycle, no work is done by the atmosphere on the working fluid of the cycle, and no work is done by the working fluid on the atmosphere.