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REPRESENTATION ON THE TEMPERATURE-ENTROPY AND
PRESSURE-VOLUME PLANES OF THE FLOW OF ROTARY SHAFT WORK
REQUIRED FOR GAS TURBINES, GAS COMPRESSORS AND VAPOR COMPRESSORS

by

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HSIEH TIEN SUN

Diploma, Taipei Institute of Technology
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Approved by:

Wilson Tripp
Major Professor

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NOMENCLATURE

C.O.P. = coefficient of performance

c_v = specific heat at constant volume, Btu/lbm-R

c_p = specific heat at constant pressure, Btu/lbm-R

e_c = efficiency of compression; defined in Appendix B

e_e = efficiency of expansion; defined in Appendix A

F = mechanical energy converted to thermal energy by friction, ft-lbf/lbm

g = gravitational acceleration, ft/sec^2

h = specific enthalpy, Btu/lbm

J = 778.161 ft-lbf/Btu

k = ratio of specific heats

M = mass flow, lbm

n = polytropic exponent in pv^n = a const.

n' = polytropic exponent for the reversible path that has the same flow of rotary shaft work as the irreversible path

n" = polytropic exponent for the reversible path that has the same heat flow out as the irreversible diabatic path

p = pressure, psia or psfa

Q = heat flow, Btu

R = gas constant = 53.342 ft-lbf/lbm-R for air

s = specific entropy, Btu/lbm-R

S = entropy, Btu/R

t = temperature, F

T = temperature, R

u = specific internal energy, Btu/lbm

v = specific volume, ft^3/lbm

V = volume, ft^3

V = velocity, ft/sec

W = work, Btu

W_s = rotary shaft work, Btu

x = quality

z = elevation, ft

φ = ratio of compression work and heat flow, defined in Section 1.7

η = compression efficiency, defined in Section 2.2

Δ = symbol for finite difference

Subscripts

adiab = adiabatic

diab = diabatic

in = input

irrev = irreversible

isen = isentropic

i = intermediate state along the path

i,r = intermediate state along the isentropic path

i' = intermediate state along the reversible path that has the same flow of rotary shaft work as the irreversible path

o = output

rev = reversible

I, II = first and second stage of the two-stage turbine or the two-stage compressor

INTRODUCTION

In this study of gas power cycles and vapor refrigeration cycles, gas turbines, gas compressors and vapor compressors are to be considered. If a fluid at an initial condition (p_1, v_1, T_1) undergoes a change through a turbine or a compressor to a final state condition (p_2, v_2, T_2), at least two of the three variables pressure, volume, and temperature must change. During the change, work must be accomplished by or on the thermodynamic system, the internal energy of the system may increase or decrease, and heat may be either added or removed. Gas turbines develop power as a result of gas expansion and, by means of power input, compressors are used to compress gases and vapors. Work depends upon the types of expansions or compressions. In a frictionless or reversible process, work input or work output may be represented by areas on the pressure-volume plane and the temperature-entropy plane.

As frictional forces always occur in actual cases, reversible processes are merely ideal ones. Unfortunately, it is not possible to represent accurately an irreversible process on either a pressure-volume or a temperature-entropy plane; only an approximate path may be drawn. Work cannot be judged from areas on those planes when these planes contain only lines which are approximations of irreversible processes. It is necessary, therefore, to find an equivalent reversible path which has an equal amount of work as the actual one. By comparing the reversible path of an ideal process and the equivalent reversible path the work lost in expansion, or the extra work required for compression, may be represented by an area on the plane.

The flow of rotary work has been represented on the pressure-volume plane for a long period of time and by a large number of authors. However, the representation of the flow of rotary shaft work involved in turbines and compressors on the temperature-entropy plane has, to the knowledge of the author, not been used except in a limited and rather complex manner in one instance.*

The main purpose of this report is to develop a suitable technique for the accurate representation of the flow of rotary shaft work, by means of areas on the temperature-entropy plane and to correlate these areas with those on the pressure-volume plane which also represent rotary shaft work. The development of this technique will give another use to the temperature-entropy diagram (whose main use now is the representation of heat flow) and will, it is hoped, encourage further use of the entropy concept and Second Law analysis.

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* Page 49,50, and 51 of reference (1).

PART I

GAS POWER CYCLES

SINGLE-STAGE GAS TURBINES

1.1 Reversible adiabatic gas turbine

The gas turbine expands gases from a given state of higher pressure and temperature to a new state of lower pressure and temperature. If there is no friction and heat flow in the expansion process, it is defined as a reversible adiabatic process.

As shown in Figure 1.1, hot air expands according to the reversible adiabatic process from 1 to 2. The flow of rotary shaft work done by the turbine is represented on the pressure-volume diagram by area (a).

It can be proved as follows for perfect gases that the heat added at a constant pressure of p_1 from 3 to 1, and represented on the temperature-entropy diagram by area (a), equals the flow of rotary shaft work represented on the pressure-volume diagram.

The Bernoulli equation is,

$$\int_1^2 \frac{p\,dv}{J} = \frac{p_2v_2 - p_1v_1}{J} + \frac{V_2^2 - V_1^2}{2gJ} + \frac{z_2 - z_1}{J} + W_{s,o} + \frac{F}{J}$$

or in its equivalent form,

$$-\int_1^2 \frac{v\,dp}{J} = \frac{V_2^2 - V_1^2}{2gJ} + \frac{z_2 - z_1}{J} + W_{s,o} + \frac{F}{J}$$

Neglecting the kinetic energy $\frac{V_2^2 - V_1^2}{2gJ}$, the difference in elevation $\frac{z_2 - z_1}{J}$, and the friction $\frac{F}{J}$, the flow of rotary shaft work out for the reversible turbine becomes

$$W_{s,o} = -\int_1^2 \frac{v\,dp}{J} \quad .$$

This is represented by the area (a) on the pressure-volume diagram.

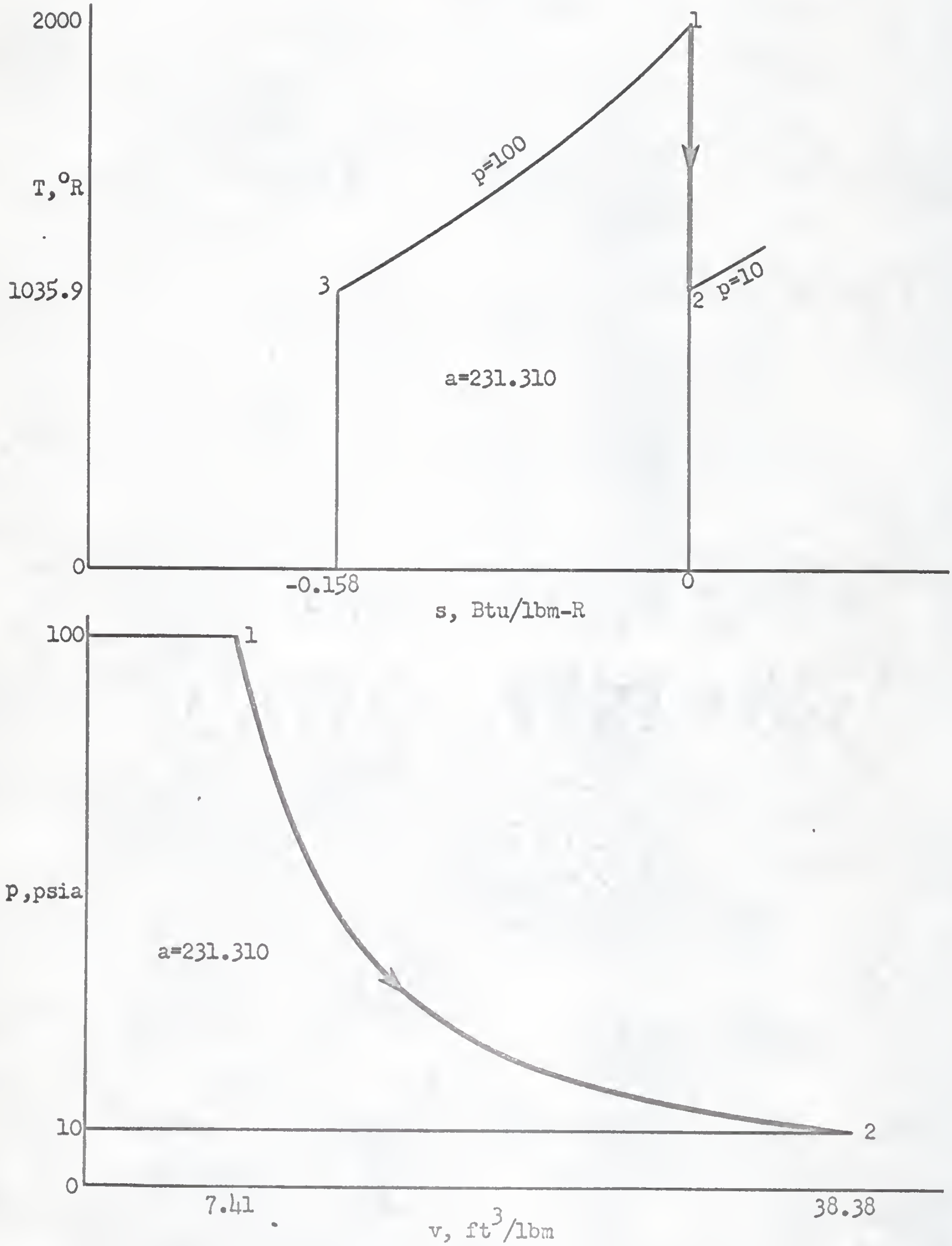

FIG. 1.1 TEMPERATURE-ENTROPY AND PRESSURE-VOLUME DIAGRAMS FOR REVERSIBLE ADIABATIC GAS TURBINE

Consider the closed cycle 1-2-3-1 on the temperature-entropy diagram, where $T_3 = T_2$.

$$W_{s,o,cycle} = W_{s,o,1-2} - W_{s,in,2-3}$$

or

$$W_{s,o,1-2} = W_{s,o,cycle} + W_{s,in,2-3} \qquad (1.1)$$

From the First Law of Thermodynamics, the cyclic work may be written as,

$$W_{s,o,cycle} = Q_{in,3-1} - Q_{o,2-3} \qquad (1.2)$$

The steady flow equation is,

$$h_2 + W_{s,in,2-3} = h_3 + Q_{o,2-3}$$

Since T_2 and T_3 are chosen equal, $h_2 = h_3$ and

$$W_{s,in,2-3} = Q_{o,2-3} \qquad (1.3)$$

Substituting equations (1.2) and (1.3) in (1.1),

$$W_{s,o,1-2} = Q_{in,3-1} \qquad \text{Q.E.D.}$$

Assume one pound of air expands under reversible adiabatic conditions from an initial pressure and temperature of 100 psia and 2000 R to a final pressure of 10 psia. For air k equals 1.40.

The temperature of the air after reversible adiabatic expansion equals,

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{(k-1)/k} = 2000 \left(\frac{10}{100}\right)^{(1.4-1)/1.4} = 1035.894 \text{ R}$$

The gas turbine cycle may be analyzed by use of enthalpy values and the First Law steady flow energy equation. Neglecting the kinetic energy and potential energy, the rotary shaft work output for the adiabatic turbine is,

$$W_{s,o} = h_1 - h_2 = h_1 - h_3 = Q_{in,3-1} = -\int_1^2 \frac{vdp}{J}$$

$$= c_p\ (T_1 - T_2)$$

$$= 0.239922\ (2000 - 1035.894)$$

$$= 231.310 \text{ Btu}$$

1.2 Irreversible adiabatic gas turbine

Actual turbines have irreversible expansion processes owing to friction. If the efficiency of expansion is 0.8 for an irreversible adiabatic gas turbine, the value of n equals

$$n = 1 + e_e\ (k - 1)^{*} = 1 + 0.8\ (1.4 - 1) = 1.32$$

The data are the same as those for the reversible adiabatic gas turbine.

$M = 1$ lb of air; $p_1 = 100$ psia; $T_1 = 2000$ R; $p_2 = 10$ psia

The temperature of the air after expansion equals

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{(n-1)/n} = 2000 \left(\frac{10}{100}\right)^{(1.32 - 1)/1.32} = 1144.473 \text{ R}$$

The flow of rotary shaft work output is

$$W_{s,o} = h_1 - h_2 = c_p\ (T_1 - T_2) = 0.239922\ (2000 - 1144.473) = 205.259 \text{ Btu}$$

The ideal expansion work under the process 1 to 2 is,

$$\int_1^2 \frac{pdv}{J} = \frac{R}{(n - 1)\ J}\ (T_1 - T_2) = \frac{0.068549}{1.32 - 1}\ (2000 - 1144.473) = 183.267 \text{ Btu}$$

The efficiency of expansion is defined as

$$e_e = \frac{W_{o,1-2}}{W_{o,ideal,1-2}} = \frac{\int_1^2 \frac{pdv}{J} - \frac{F}{J}}{\int_1^2 \frac{pdv}{J}} \qquad (1.4)$$

* See part A in Appendix.

Simplifying equation (1.4)

$$\frac{F}{J} = (1 - e_e) \int_1^2 \frac{pdv}{J} = (1 - 0.8)\ 183.267 = 36.653 \text{ Btu}$$

In Figure 1.2, area (a+b+c) on the pressure-volume diagram equals

$$-\int_1^2 \frac{vdp}{J} = \frac{nR}{(n-1)J} (T_1 - T_2) \qquad (1.5)$$

$$= n\int_1^2 \frac{pdv}{J} = 1.32\ (183.267) = 241.912 \text{ Btu}$$

The Bernoulli equation gives,

$$W_{s,o} = -\int_1^2 \frac{vdp}{J} - \frac{F}{J} = 241.912 - 36.653 = 205.259 \text{ Btu}$$

This answer checks with the one calculated from the steady flow equation.

Now assume there is a reversible expansion process which has the equal amount of rotary shaft work. The new exponent of polytropic expansion n' may be found by applying equation (1.5).

$$W_{s,o} = \int_1^{2'} \frac{vdp}{J} = \frac{n'R}{(n'-1)J} (T_1 - T_{2'}) = \frac{n'R}{(n'-1)J}\ T_1 \left[1 - \left(\frac{p_2}{p_1}\right)^{(n'-1)/n'}\right]$$

$$205.259 = \frac{n'}{n'-1}\ 0.068549\ (2000) \left[1 - \left(\frac{10}{100}\right)^{(n'-1)/n'}\right]$$

By trial and error,

$n' = 1.6812$

In Figure 1.2, the reversible path is represented by solid lines (————), the actual path by a sequence of circles (——o——o——), and the equivalent reversible path by broken lines (—— —— ——). The path from 1 to 2' is the equivalent path for the irreversible adiabatic expansion from 1 to 2.

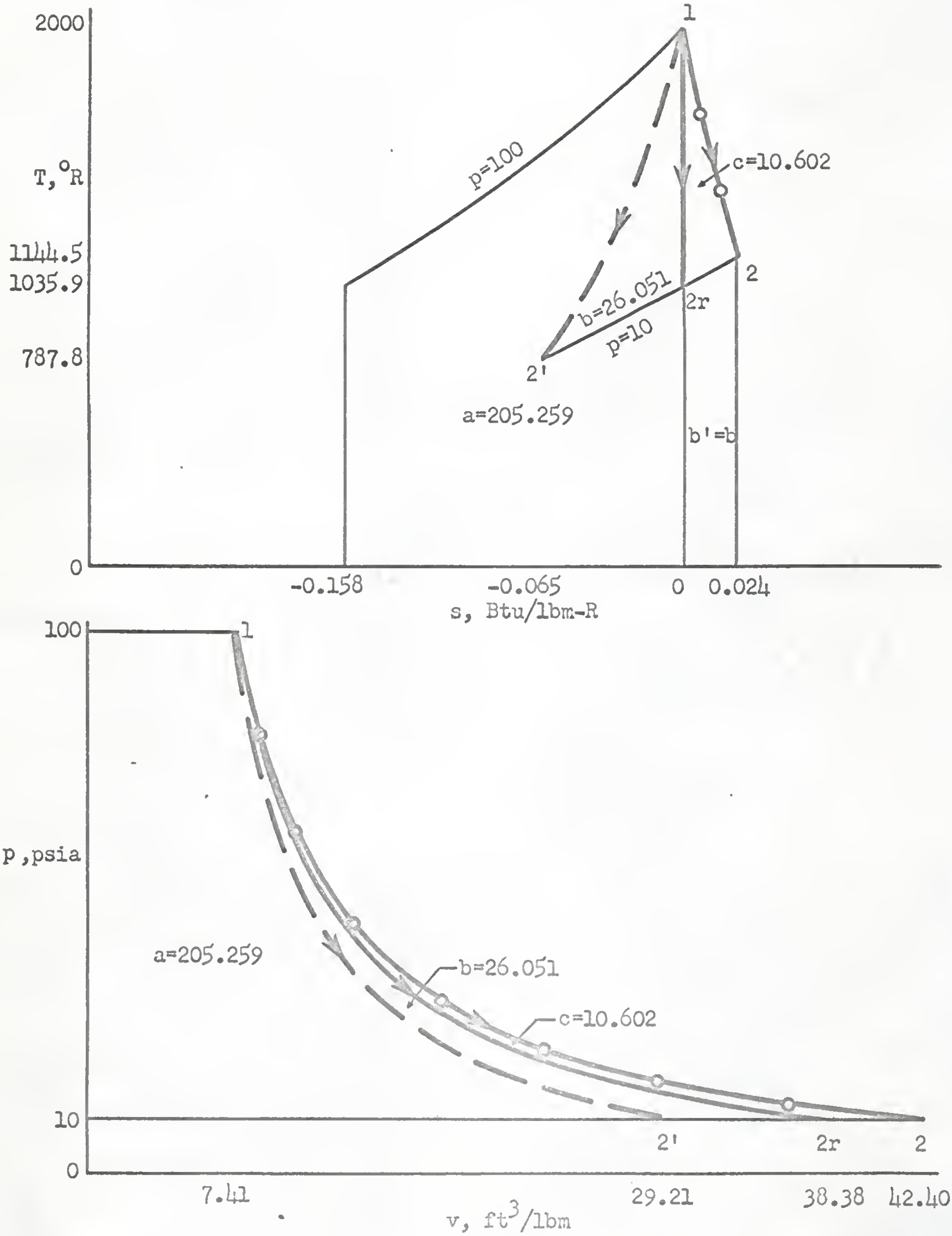

FIG. 1.2 TEMPERATURE-ENTROPY AND PRESSURE-VOLUME DIAGRAMS FOR IRREVERSIBLE ADIABATIC GAS TURBINE

Area (a) represents the actual flow of rotary shaft work for the irreversible adiabatic gas turbine. In comparison with the reversible adiabatic turbine, area (b) represents the penalty for irreversibility.

The expansion path from 1 to 2 is an actual path. If it is assumed to be a reversible one, the area under the expansion path 1-2 on the temperature-entropy plane represents the heat input which also equals the work due to friction in the irreversible case.

The First Law non-flow equation may be written,

$$Q_{in,1-2,rev} = u_2 - u_1 + \int_1^2 \frac{pdv}{J} = c_v (T_2 - T_1) + \int_1^2 \frac{pdv}{J}$$

$$= 0.171373 (1144.473 - 2000) + 183.267$$

$$= 36.653 \text{ Btu}$$

Area (b') under the constant pressure line 2r-2 on the temperature-entropy diagram represents the heat input from 2r to 2 at a constant pressure.

$$Q_{in,2r-2} = c_p (T_2 - T_{2r}) = 0.239922(1144.473 - 1035.894)$$

$$= 26.051 \text{ Btu}$$

Area (a+b) represents the expansion rotary shaft work for the reversible turbine. Area (b) may be calculated by subtracting the area (a).

$$(b) = (a+b) - (a)$$

$$= 231.310 - 205.259$$

$$= 26.051 \text{ Btu}$$

Since area (b) equals area (b'), it may be stated that area (b) is the non-recoverable part of work due to friction. Area (c) is the recoverable part of work due to friction and equals,

$$(c) = (c + b') - (b') = 36.653 - 26.051 = 10.602 \text{ Btu}$$

Area (c) may be calculated from the pressure-volume diagram also, since area (a+b+c) represents the rotary shaft work from 1 to 2.

(c) = (a+b+c) - (a+b) = 241.912 - 231.310 = 10.602 Btu

The work lost in this case is 11.26 per cent.

SINGLE-STAGE GAS COMPRESSORS

1.3 Reversible adiabatic gas compressor

Compressors are used to compress air and other gases. As shown in Figure 1.3, air is compressed according to the reversible adiabatic process from 1 to 2. The work done on the gas by the compressor is represented on the pressure-volume diagram by area (a).

$$W_{s,in} = \int_1^2 \frac{vdp}{J}$$

Consider the closed cycle 1-2-3-1 on the temperature-entropy diagram.

$$W_{s,in,cycle} = W_{s,in,1-2} - W_{s,o,3-1}$$

or

$$W_{s,in,1-2} = W_{s,in,cycle} + W_{s,o,3-1} \qquad (1.6)$$

From the First Law of Thermodynamics, the cyclic work can be written as,

$$W_{s,in,cycle} = Q_{o,2-3} - Q_{in,3-1} \qquad (1.7)$$

The steady flow equation is,

$$h_3 + Q_{in,3-1} = h_1 + W_{s,o,3-1}$$

Since T_1 and T_3 are chosen equal, $h_1 = h_3$ and

$$Q_{in,3-1} = W_{s,o,3-1} \qquad (1.8)$$

Substituting equations (1.7) and (1.8) in (1.6),

$$W_{s,in,1-2} = Q_{o,2-3} \qquad \text{Q.E.D.}$$

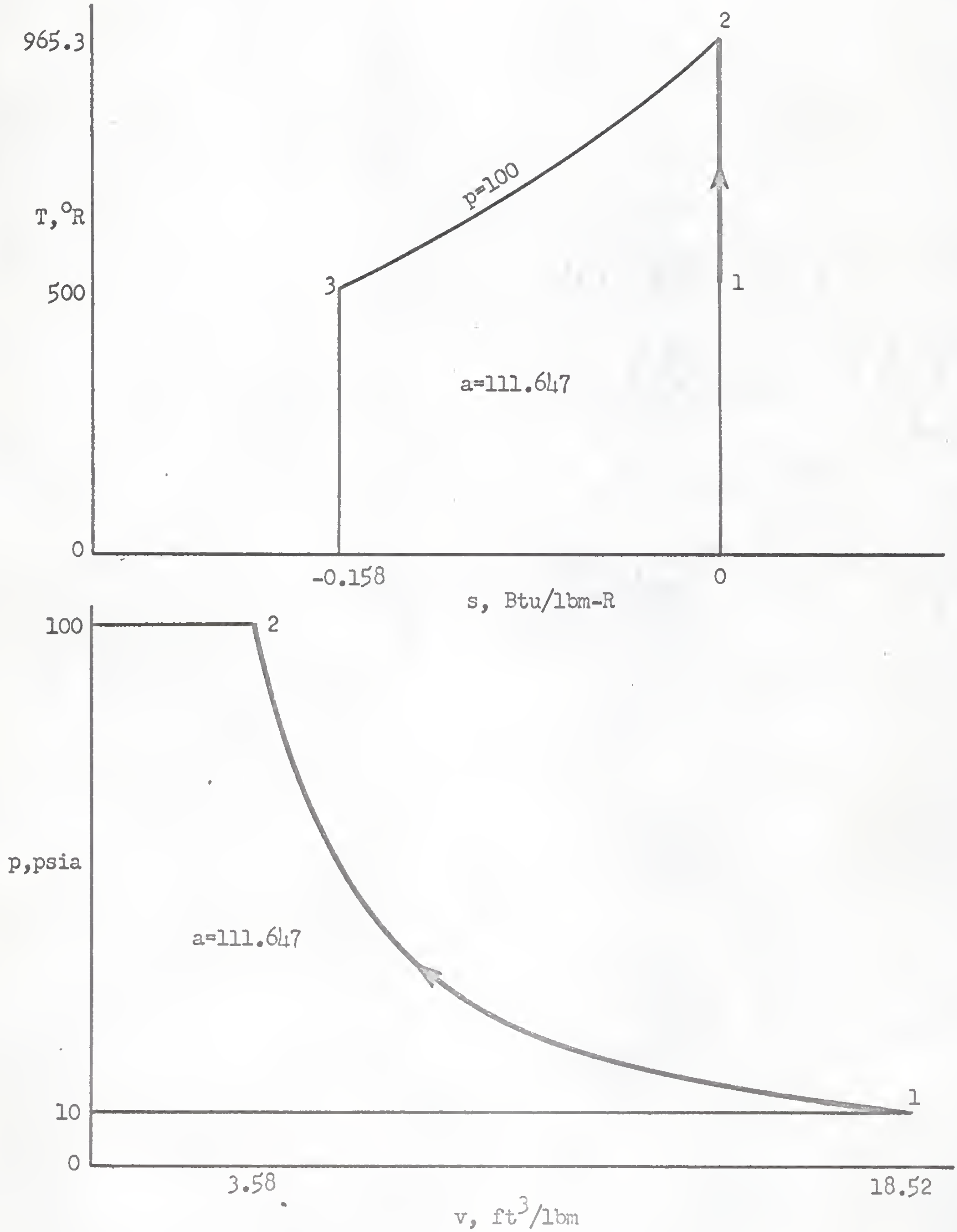

FIG. 1.3 TEMPERATURE-ENTROPY AND PRESSURE-VOLUME DIAGRAMS FOR REVERSIBLE ADIABATIC GAS COMPRESSOR

This proves that the area (a) on the temperature-entropy diagram represents the equal amount of rotary shaft work.

Assume one pound of air is compressed under reversible adiabatic conditions from an initial pressure and temperature of 10 psia and 500 R to a final pressure of 100 psia.

The temperature of the air after reversible adiabatic compression is,

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{(k-1)/k} = 500 \left(\frac{100}{10}\right)^{(1.4-1)/1.4} = 965.349 \text{ R}$$

From the steady flow energy equation, the rotary shaft work input for the reversible adiabatic gas compressor is,

$$W_{s,in} = h_2 - h_1 = c_p (T_2 - T_1) = 0.239922 (965.349 - 500)$$

$$= 111.647 \text{ Btu}$$

1.4 Irreversible adiabatic gas compressor

Let the efficiency of compression be 0.8 for an irreversible adiabatic gas compressor, the value of n equals

$$n = 1 + \frac{1}{e_c} (k-1) \text{*} = 1 + \frac{1}{0.8} (1.4-1) = 1.5$$

The data are the same as those for the reversible adiabatic gas compressor.

M = 1 lb of air; p_1 = 10 psia; T_1 = 500 R; p_2 = 100 psia

The temperature of the air after compression equals

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{(n-1)/n} = 500 \left(\frac{100}{10}\right)^{(1.5-1)/1.5} = 1077.217 \text{ R}$$

The rotary shaft work input is,

$$W_{s,in} = h_2 - h_1 = c_p (T_2 - T_1) = 0.239922 (1077.217 - 500) = 138.487 \text{ Btu}$$

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*See part B in Appendix.

The ideal compression work under the process 1 to 2 is,

$$\int_1^2 \frac{pdv}{J} = \frac{R}{(n-1)J}(T_1 - T_2) = \frac{0.068549}{1.5-1}(500 - 1077.217)$$

$$= -79.135 \text{ Btu}$$

The efficiency of compression is defined as,

$$e_c = \frac{W_{in,ideal,1-2}}{W_{in,\,1-2}}$$

$$= \frac{-\int_1^2 \frac{pdv}{J}}{-\int_1^2 \frac{pdv}{J} + \frac{F}{J}} \qquad (1.9)$$

Simplifying equation (1.9)

$$\frac{F}{J} = (1 - \frac{1}{e_c})\int_1^2 \frac{pdv}{J} = (1 - \frac{1}{0.8})(-79.135) = 19.784 \text{ Btu}$$

In Figure 1.4, the area (a+b) on the pressure-volume diagram equals,

$$\int_1^2 \frac{vdp}{J} = -\frac{nR}{(n-1)J}(T_1 - T_2) = -n\int_1^2 \frac{pdv}{J} = -1.5\,(-79.135) = 118.703 \text{ Btu}$$

The Bernoulli equation gives,

$$W_{s,in} = \int_1^2 \frac{vdp}{J} + \frac{F}{J} = 118.703 + 19.784 = 138.487 \text{ Btu}$$

This answer checks with the one calculated from the steady flow equation.

Now we assume there is a reversible compression process which has the equal amount of the flow of rotary shaft work. The new exponent of polytropic compression n' may be found by applying equation (1.5).

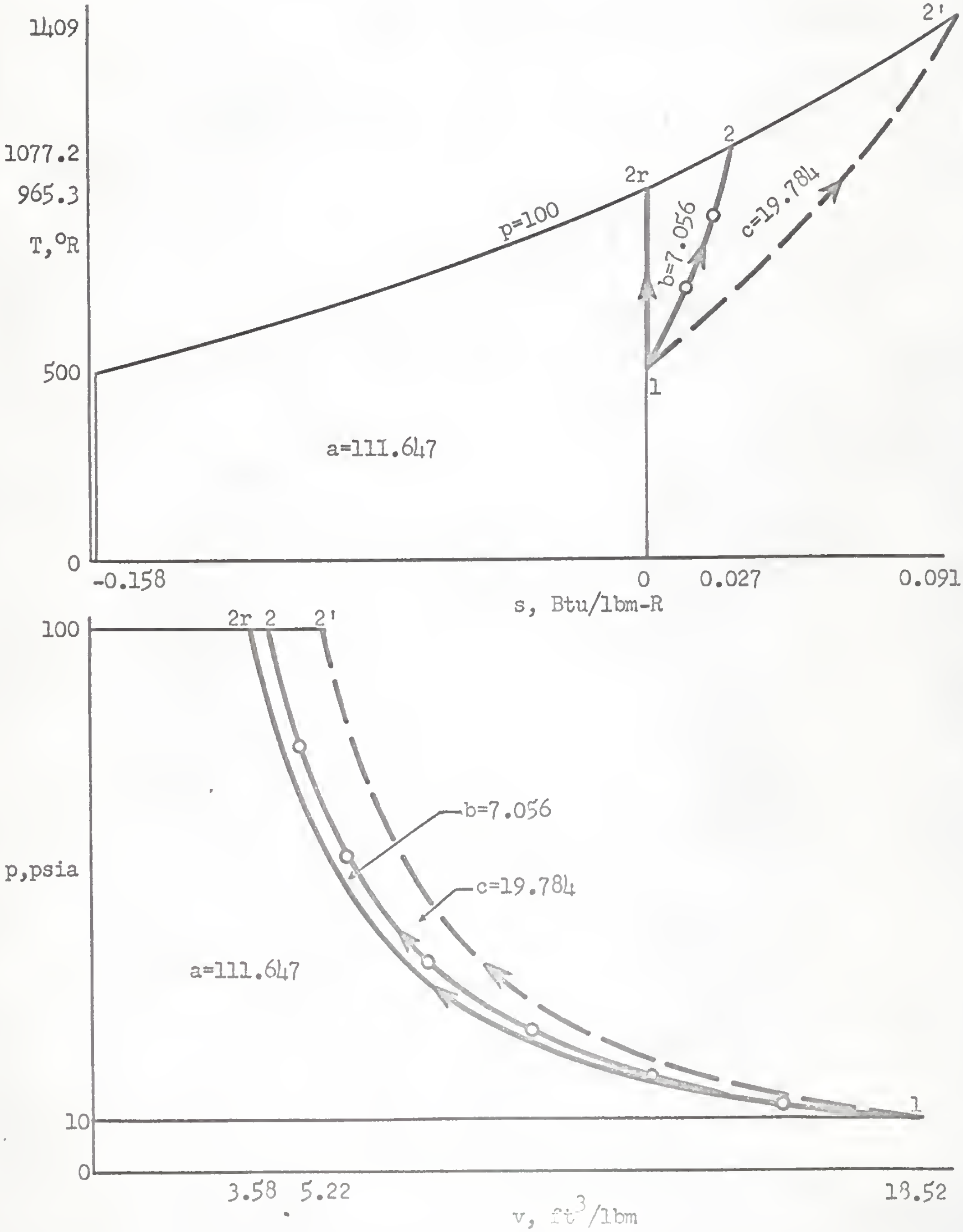

FIG. 1.4 TEMPERATURE-ENTROPY AND PRESSURE-VOLUME DIAGRAMS FOR IRREVERSIBLE ADIABATIC GAS COMPRESSOR

$$W_{s,in} = \int_1^{2'} \frac{vdp}{J} = \frac{-n'\ R}{(n'-1)J}(T_1 - T_2) = \frac{n'\ R}{(n'-1)J} T_1 \left[\left(\frac{p_2}{p_1}\right)^{(n'-1)/n'} -1 \right]$$

$$138.487 = \frac{n'}{n'-1}\ 0.068549\ (500) \left[\left(\frac{100}{10}\right)^{(n'-1)/n'} -1 \right]$$

By trial and error,

$$n' = 1.818$$

The path from 1 to 2' is the equivalent reversible path for the irreversible adiabatic compression from 1 to 2.

Area (a+b+c) represents the actual flow of rotary shaft work for the irreversible adiabatic gas compressor.

The temperature of the air after compression equals

$$T_{2'} = T_1 \left(\frac{p_2}{p_1}\right)^{(n'-1)/n'} = 500 \left(\frac{100}{10}\right)^{(1.818-1)/1.818}$$

$$= 1409.013\ R$$

In comparison with the reversible adiabatic gas compressor, area (b+c) represents the extra work required for compression.

Areas which represent the heat flow or rotary shaft work may be calculated from the temperature-entropy diagram.

In Figure 1.5, the heat flow under the constant pressure line from 2r to 2 is represented by area (b+d+f).

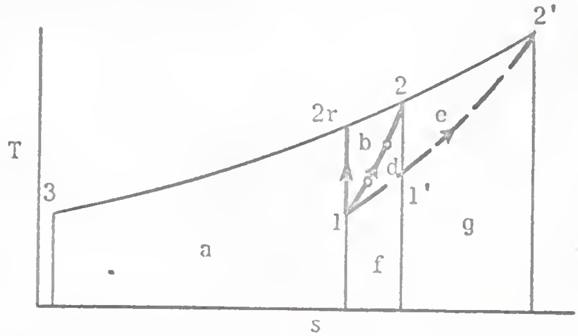

Figure 1.5 Temperature-entropy diagram.

$$Q_{in} = (b+d+f) = c_p\ (T_2 - T_{2r}) = 0.239922\ (1077.217 - 965.349)$$

$$= 26.840 \text{ Btu}$$

The heat input under the reversible path from 1 to 2 is represented by area (d+f).

$$Q_{in} = (d+f) = u_2 - u_1 + \int_1^2 \frac{pdv}{J} = c_v\ (T_2 - T_1) + \int_1^2 \frac{pdv}{J}$$

$$= 0.171373\ (1077.217 - 500) - 79.135 = 19.784 \text{ Btu}$$

$$(b) = (b+d+f) - (d+f) = 26.840 - 19.784 = 7.056 \text{ Btu}$$

The heat input under the constant pressure line from 2 to 2' is represented by area (e+g).

$$Q_{in} = (e+g) = c_p\ (T_{2'} - T_2) = 0.239922\ (1409.013 - 1077.217)$$

$$= 79.605 \text{ Btu}$$

The entropy change from 1 to 1' is equal to the entropy change from 2r to 2.

$$S_{1'} - S_1 = S_2 - S_{2r} = c_p \ln \frac{T_2}{T_{2r}} = c_p \ln \frac{T_{1'}}{T_1} - \frac{R}{J} \ln \frac{p_{1'}}{p_1}$$

or

$$c_p \ln \frac{T_2}{T_{2r}} = c_p \ln \frac{T_{1'}}{T_1} - \frac{R}{J} \ln \left(\frac{T_{1'}}{T_1}\right)^{n'/(n'-1)}$$

In this equation only $T_{1'}$ is unknown.

Solving for $T_{1'}$

$T_{1'} = 675.189$ R

The heat input under the line from 1 to 1' is,

$$Q_{in} = (f) = c_v\ (T_{1'} - T_1) + \int_1^{1'} \frac{pdv}{J} = c_v (T_{1'} - T_1) - \frac{R}{(n'-1)J}(T_{1'} - T_1)$$

$$= 0.171373\ (675.189 - 500) - \frac{0.068549}{(1.818-1)}\ (675.189 - 500)$$

$$= 15.342 \text{ Btu}$$

$(d) = (d+f) - (f) = 19.784 - 15.342 = 4.442$ Btu

The heat input under the line from 1' to 2' is represented by area (g) on the temperature-entropy diagram.

$$Q_{in} = c_v\,(T_{2'} - T_{1'}) + \int_{1'}^{2'} \frac{pdv}{J} = c_v(T_{2'} - T_{1'}) - \frac{R}{(n-1)J}\,(T_{2'} - T_{1'})$$

$$= 0.171373\,(1409.013 - 675.189) - \frac{0.068549}{(1.818-1)}(1409.013 - 675.189)$$

$$= 64.263 \text{ Btu}$$

$(e) = (d+g) - (g) = 79.605 - 64.263 = 15.342$ Btu

Therefore, area (e) equals area (f).

The area (c) in Figure 1.4 is represented by area (e+d) here.

$(c) = (e+d) = (d+f) = 4.442 + 15.342 = 19.784$ Btu

In Figure 1.4, area (b) represents the heat that must be rejected when following the irreversible isentropic path 1 to 2r. Therefore, area (b) is called the irreversible isentropic friction factor.*

Area (c) represents the part of work due to friction which changes mechanical energy into thermal energy. It is called the irreversible adiabatic friction factor.*

Both areas (b) and (c) are non-recoverable parts of work, and represent the double penalty for the irreversible adiabatic compressor. The extra work in this case is 24.04 per cent.

1.5 Reversible isothermal gas compressor

Assume one pound of air is compressed under reversible isothermal conditions from an initial pressure and temperature of 10 psia and 500 R to a final pressure of 100 psia.

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*See part C in Appendix.

In Figure 1.6, area (a) represents the flow of rotary shaft work input, which is,

$$W_{s,in} = \frac{R}{J} T_1 \ln \frac{p_2}{p_1} = 0.068549\ (500) \ln \frac{100}{10} = 78.920 \text{ Btu}$$

1.6 Irreversible isothermal gas compressor

Let the efficiency of compression be 0.8 for an irreversible isothermal gas compressor.

The data are the same as those for the reversible isothermal gas compressor.

$M = 1$ lb of air; $p_1 = 10$ psia; $T_1 = T_2 = 500$ R; $p_2 = 100$ psia

In the isothermal process, the flow of rotary shaft work equals the work of compression. This may be proved from the steady flow equation and the non-flow equation.

$$h_1 + W_{s,in} = h_2 + Q_o$$

Since $T_1 = T_2$, $h_1 = h_2$ and

$$\therefore W_{s,in} = Q_o \tag{1.10}$$

$$W_{in} = u_2 - u_1 + Q_o$$

Since $T_1 = T_2$, $u_1 = u_2$ and

$$\therefore W_{in} = Q_o \tag{1.11}$$

Equations (1.10) and (1.11) give

$$W_{s,in} = W_{in} \qquad \text{Q.E.D.}$$

The rotary shaft work input for the irreversible gas compressor equals,

$$W_{s,in} = \frac{W_{s,in,ideal}}{e_c} = \frac{78.920}{0.8} = 98.650 \text{ Btu}$$

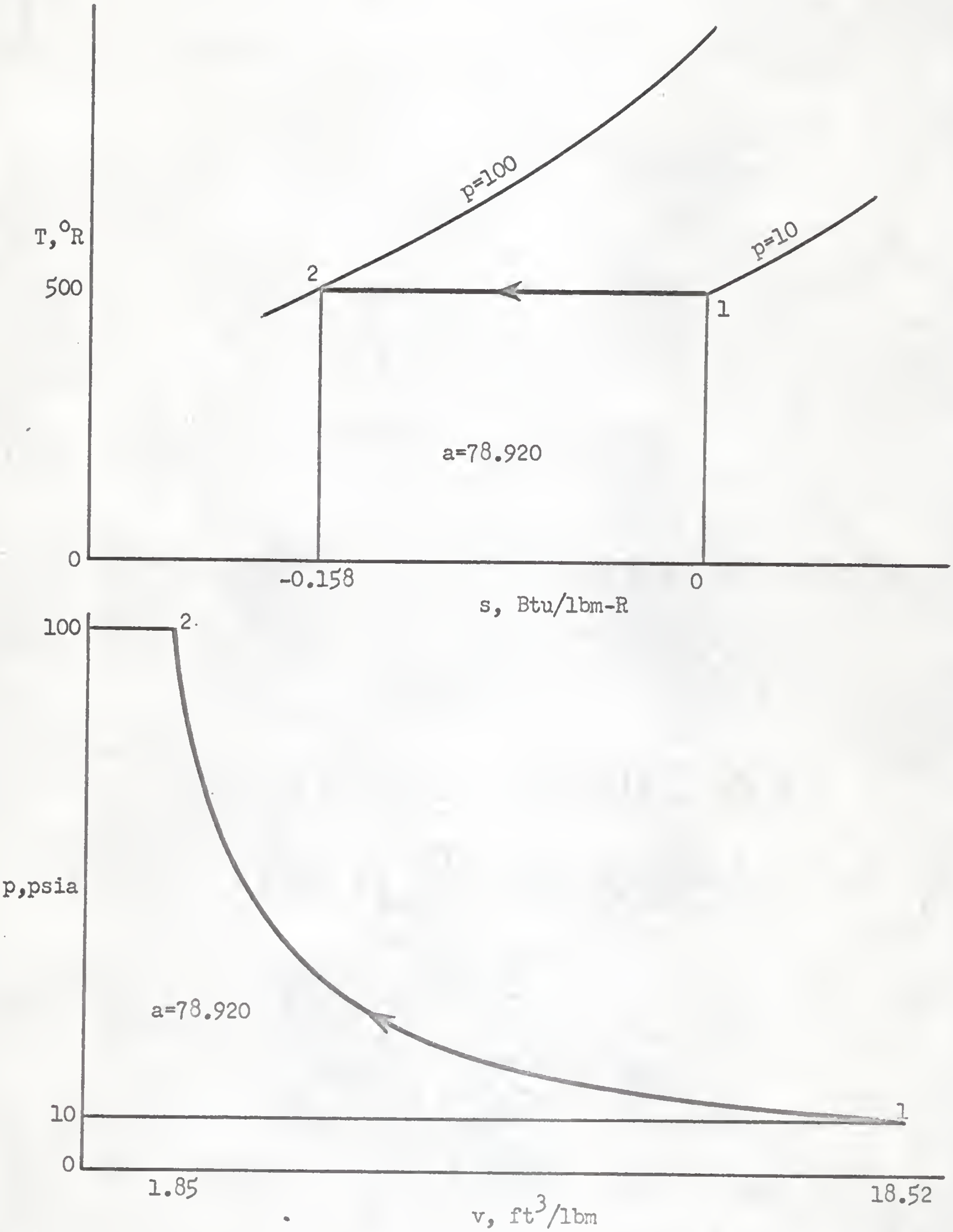

FIG. 1.6 TEMPERATURE-ENTROPY AND PRESSURE-VOLUME DIAGRAMS FOR REVERSIBLE ISOTHERMAL GAS COMPRESSOR

The equivalent reversible path for the irreversible one may be found from the equation (1.5).

$$W_{s,in} = \int_1^{2'} \frac{vdp}{J} = \frac{n'R}{(n'-1)J} T_1 \left[\left(\frac{p_2}{p_1}\right)^{(n'-1)/n'} -1 \right]$$

$$98.650 = \frac{n'}{n'-1}(0.068549)\ 500 \left[\left(\frac{100}{10}\right)^{(n'-1)/n'} -1 \right]$$

By trial and error,

$$n' = 1.230$$

The temperature of the air after compression equals

$$T_{2'} = T_1 \left(\frac{p_2}{p_1}\right)^{(n'-1)/n'} = 500 \left(\frac{100}{10}\right)^{(1.23-1)/1.23} = 769.063 \text{ R}$$

The work due to friction is,

$$\frac{F}{J} = (1 - \frac{1}{e_c}) \int_1^2 \frac{pdv}{J} = (1 - \frac{1}{e_c}) (- \int_1^2 \frac{vdp}{J})$$

$$= (1 - \frac{1}{e_c}) (-\frac{W_{s,in,rev}}{J}) = (1 - \frac{1}{0.8}) (- 78.920) = 19.730 \text{ Btu}$$

In Figure 1.7, area (a+b) represents the rotary shaft work input, which equals,

$$W_{s,in} = \frac{W_{s,in,rev}}{J} + \frac{F}{J} = 78.920 + 19.730 = 98.650 \text{ Btu}$$

The answer checks with the one calculated from the efficiency equation.

Area (b) represents the extra work due to friction. In this case there is a 25 per cent increase in the work required.

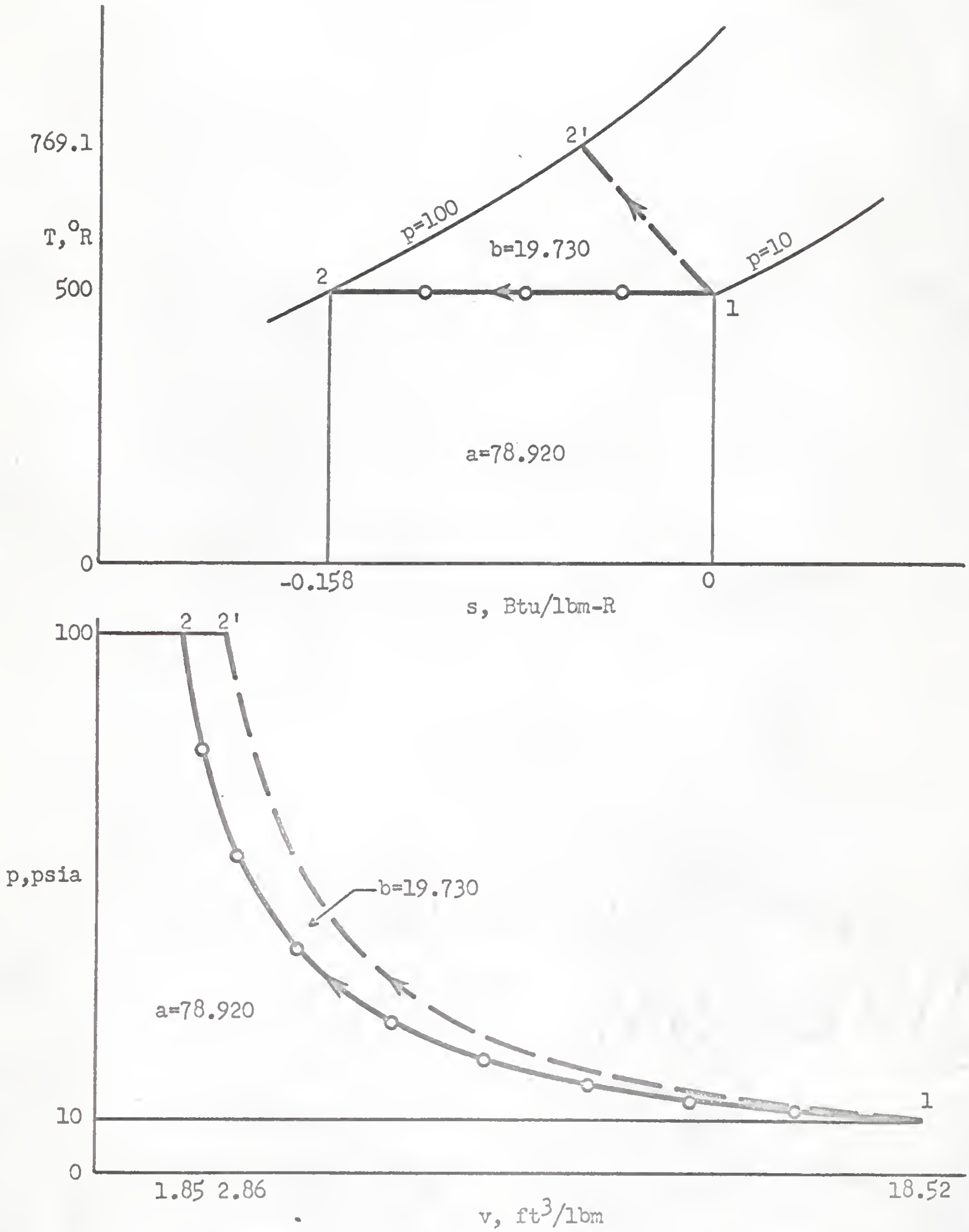

FIG. 1.7 TEMPERATURE-ENTROPY AND PRESSURE-VOLUME DIAGRAMS FOR IRREVERSIBLE ISOTHERMAL GAS COMPRESSOR

1.7 Reversible diabatic gas compressor

The reversible diabatic gas compressor means that there is heat flow during the compression process. The compression work and heat flow ratio φ is defined as,

$$\varphi = \frac{\frac{1}{e_c}\int_1^2 \frac{pdv}{J}}{Q_{in}}$$

Assume one pound of air is compressed under reversible diabatic conditions from an initial pressure and temperature of 10 psia and 500 R to a final pressure of 100 psia. If φ = 5, e_c = 1, the value of n equals,

$$n = 1 + \frac{1}{e_c}\ (k-1)\ (1 - \frac{1}{\varphi})^{*} = 1 + (1.4 - 1)\ (1 - \frac{1}{5}) = 1.32$$

The temperature of the air after compression equals,

$$T_2 = T_1\ (\frac{p_2}{p_1})^{(n-1)/n} = 500\ (\frac{100}{10})^{(1.32-1)/1.32} = 873.764\ R$$

The rotary shaft work input is,

$$W_{s,in} = \int_1^2 \frac{vdp}{J} = \frac{nR}{(n-1)J}\ (T_2 - T_1) = \frac{1.32}{1.32-1}\ (0.068549)\ (873.764 - 500)$$

$$= 105.687\ \text{Btu}$$

The heat flow may be calculated from the steady flow equation.

$$h_1 + W_{s,in,1-2} = h_2 + Q_{o,1-2}$$

or

$$Q_{o,1-2} = W_{s,in,1-2} + h_1 - h_2 =$$

$$W_{s,in,1-2} + c_p\ (T_1 - T_2) = 105.687 + 0.239922\ (500 - 873.764)$$

$$= 16.013\ \text{Btu}$$

---

*See part D in Appendix.

The compression work is,

$$\int_1^2 \frac{pdv}{J} = \frac{R}{(n-1)J} (T_1 - T_2) = \frac{0.068549}{1.32-1} (500 - 873.764)$$

$$= -80.066 \text{ Btu}$$

The work may be calculated from the non-flow equation also.

$$Q_{in,1-2} = u_2 - u_1 + \int_1^2 \frac{pdv}{J}$$

or

$$\int_1^2 \frac{pdv}{J} = u_1 - u_2 - Q_{o,1-2} = c_p (T_1 - T_2) - Q_{o,1-2}$$

$$= 0.171373 (500 - 873.764) - 16.013 = - 80.066 \text{ Btu}$$

In Figure 1.8, area (a) represents the rotary shaft work input for the reversible diabatic gas compressor.

Area (a+b) represents the rotary shaft work input (111.647 Btu) for the reversible adiabatic gas compressor. (See Section 1.3)

Area (b) represents the work saved in this case, which is 5.960 Btu (5.338%).

1.8 Irreversible diabatic isentropic gas compressor

If the efficiency of compression $e_c = 0.8$, $\varphi = 5$, the value of n equals,

$$n = 1 + \frac{1}{e_c} (k-1) (1 - \frac{1}{\varphi}) = 1 + \frac{1}{0.8} (1.4-1) (1 - \frac{1}{5}) = 1.4$$

For this value of n, the compression follows the isentropic path.

The data are the same as those for the reversible diabatic gas compressor.

$M = 1$ lb of air; $p_1 = 10$ psia; $T_1 = 500$ R; $p_2 =$ psia

The temperature of the air after compression equals,

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{(n-1)/n} = 500 \left(\frac{100}{10}\right)^{(1.4-1)/1.4} = 965.349 \text{ R}$$

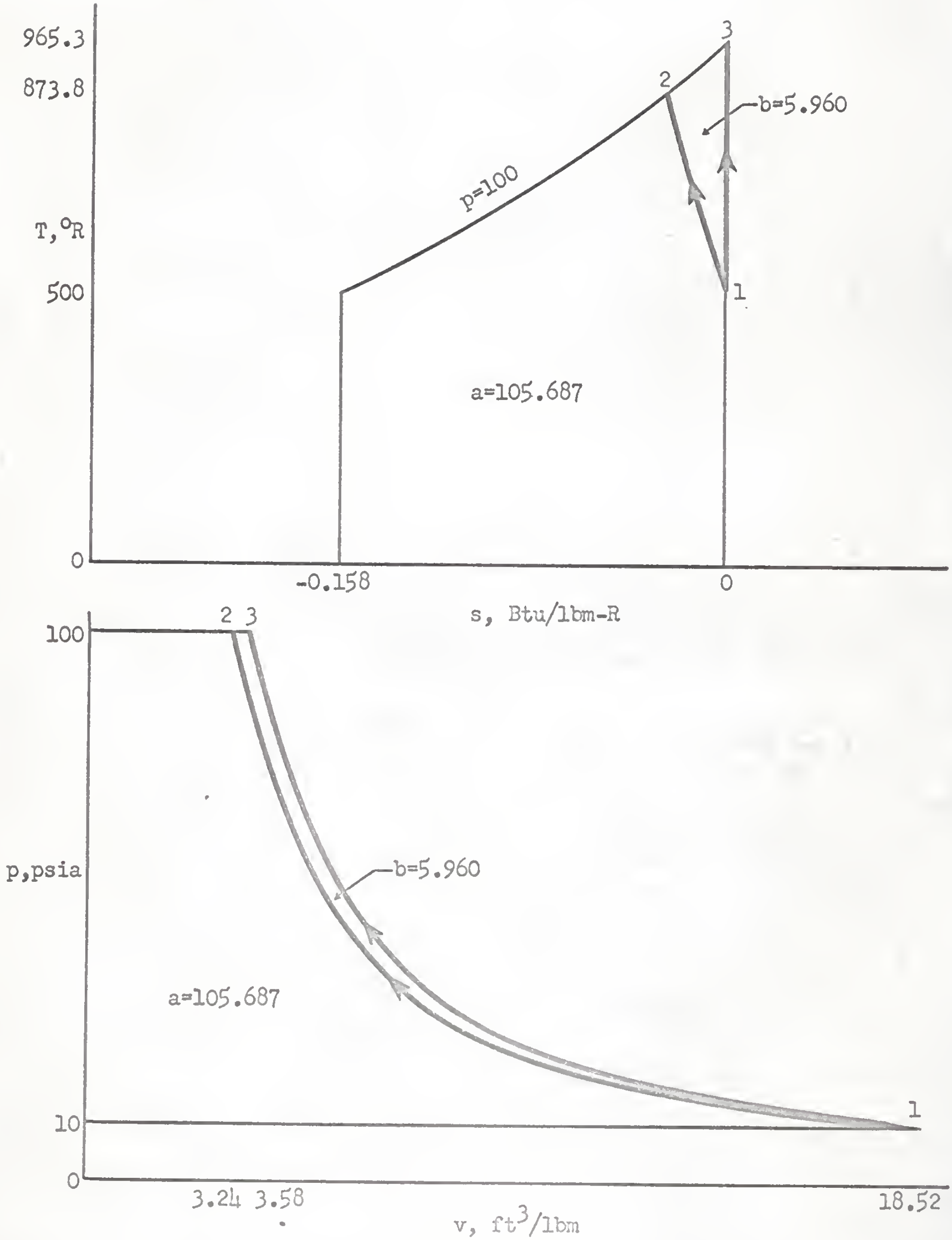

FIG. 1.8 TEMPERATURE ENTROPY AND PRESSURE VOLUME DIAGRAMS FOR REVERSIBLE DIABATIC GAS COMPRESSOR

The change of internal energy is,

$$u_2 - u_1 = c_v\ (T_2 - T_1) = 0.171373\ (965.349 - 500) = 79.748\ \text{Btu}$$

$$\int_1^2 \frac{pdv}{J} = \frac{R}{(n-1)J}\ (T_1 - T_2) = \frac{0.068549}{1.4-1}\ (500 - 965.349)$$

$$= -79.748\ \text{Btu}$$

The heat flow out from 1 to 2 (See Figure 1.9) is,

$$Q_o = u_1 - u_2 - \frac{1}{e_c}\int_1^2 \frac{pdv}{J} = -\ 79.748 + \frac{79.748}{0.8}$$

$$= 19.937\ \text{Btu}$$

The work due to firction is,

$$\frac{F}{J} = (\ 1 - \frac{1}{e_c})\int_1^2 \frac{pdv}{J} = (1 - \frac{1}{0.8})\ (-79.748) = 19.937\ \text{Btu}$$

Therefore, the work due to friction is equal to the heat out.

$$\frac{F}{J} = Q_o = 19.937\ \text{Btu}$$

The rotary shaft work input equals,

$$W_{s,in} = \int_1^2 \frac{vdp}{J} + \frac{F}{J} = -\ n\int_1^2 \frac{pdv}{J} + \frac{F}{J} = (-1.4)(-79.748) + 19.937$$

$$= 131.584\ \text{Btu}$$

The equivalent path for the irreversible diabatic isentropic compression may be found by applying the equation (1.5).

$$W_{s,in} = \int_1^{2'} \frac{vdp}{J.} = \frac{n'R}{(n'-1)J}\ T_1 \left[(\frac{p_2}{p_1})^{(n'-1)/n'} - 1 \right]$$

$$131.584 = \frac{n'}{n'-1}\ (0.068549)\ (500) \left[(\frac{100}{10})^{(n'-1)/n'} - 1 \right]$$

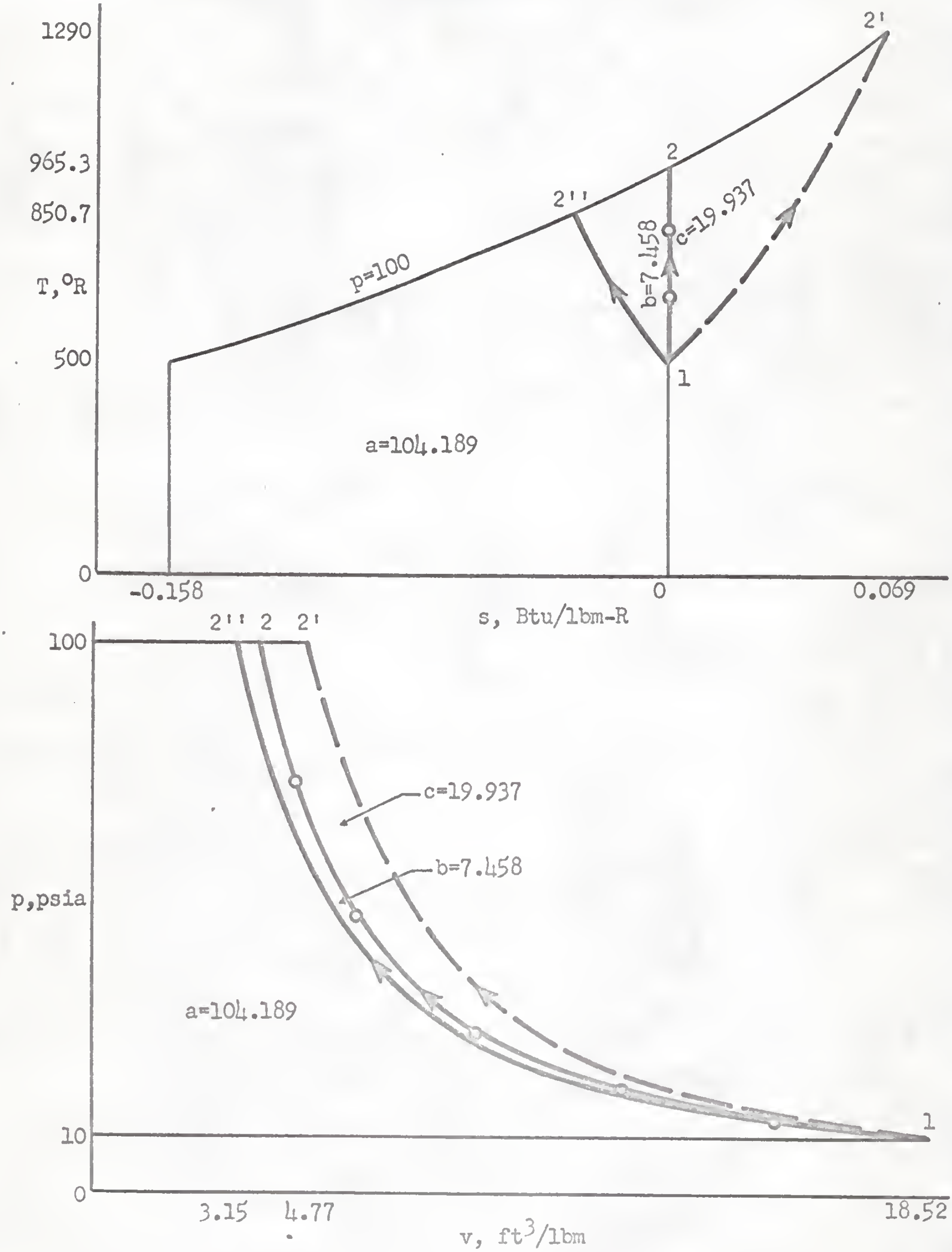

FIG. 1.9 TEMPERATURE-ENTROPY AND PRESSURE-VOLUME DIAGRAMS FOR IRREVERSIBLE DIABATIC ISENTROPIC GAS COMPRESSOR

By trial and error,

$n' = 1.6999$

Now assume there is a reversible path from 1 to 2'' (See Figure 1.9) which has the equal amount of heat flow out from 1 to 2 and the same pressure as p_2. The exponent of polytropic compression n'' may be found by the following manner.

$$W_{s,in} = \int_1^{2''} \frac{vdp}{J} = \frac{n''R}{(n''-1)J} T_1 \left[\left(\frac{p_2}{p1}\right)^{(n''-1)/n''} -1 \right]$$

$$= h_{2''} - h_1 + Q_{o,1-2''} = c_p \; (T_{2''} - T_1) + Q_{o,1-2}$$

$$= c_p \; T_1 \; \left(\frac{T_{2''}}{T_1} -1\right) + Q_{o,1-2} = c_p \; T_1 \left[\left(\frac{p_2}{p_1}\right)^{(n''-1)/n''} -1 \right] + Q_{o,1-2}$$

$$= \frac{k \; R}{(k - 1) \; J} \; T_1 \left[\left(\frac{p_2}{p_1}\right)^{(n''-1)/n''} -1 \right] + Q_{o,1-2}$$

or

$$\frac{n'' \; R}{(n''-1)J} T_1 \left[\left(\frac{p_2}{p_1}\right)^{(n''-1)/n''} -1 \right] = \frac{k \; R}{(k-1)J} T_1 \left[\left(\frac{p_2}{p_1}\right)^{(n''-1)/n''} -1 \right] + Q_{o,1-2}$$

$$\frac{n''}{n''-1} (0.068549) \; (500) \left[\left(\frac{100}{10}\right)^{(n''-1)/n''} -1 \right] = \frac{1.4}{1.4-1} (0.068549) \times (500) \left[\left(\frac{100}{10}\right)^{(n''-1)/n''} -1 \right] + 19.937$$

By trial and error,

$n'' = 1.3005$

In Figure 1.9, area (a) represents the rotary shaft work input from 1 to 2", which is

$$\int_1^{2''} \frac{vdp}{J} = \frac{n''R}{(n''-1)J} T_1 \left[\left(\frac{p_2}{p_1}\right)^{(n''-1)/n''} -1 \right]$$

$$= \frac{1.3005}{(1.3005-1)} (0.068549) (500) \left[\left(\frac{100}{10}\right)^{(1.3005-1)/1.3005} -1 \right]$$

$$= 104.189 \text{ Btu}$$

Area (b) represents the extra heat that must be rejected when following the irreversible diabatic path 1 to 2". Therefore, area (b) is called the irreversible diabatic friction factor.*

Area (c) represents that part of work due to friction which changes mechanical energy into thermal energy. It is called the irreversible diabatic isentropic friction factor.*

Both areas (b) and (c) are non-recoverable parts of work, and represent the doublt penalty for the irreversible diabatic isentropic compressor.

Area (a+b+c) represents the rotary shaft work input for this case.

1.9 Irreversible diabatic gas compressor

If the efficiency of compression $e_c = 0.8$, and $\varphi = 2.5$, the value of n equals,

$$n = 1 + \frac{1}{e_c} (k-1) (1 - \frac{1}{\varphi}) = 1 + \frac{1}{0.8} (1.4-1) (1 - \frac{1}{2.5}) = 1.30$$

The data are the same as those for the reversible diabatic gas compressor.

$M = 1$ lb of air; $p_1 = 10$ psia; $T_1 = 500$ R; $p_2 = 100$ psia

The temperature of the air after compression equals

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{(n-1)/n} = 500 \left(\frac{100}{10}\right)^{(1.3-1)/1.3} = 850.627 \text{ R}$$

---

*See part E in Appendix.

The change of internal energy is,

$$u_2 - u_1 = c_v\ (T_2 - T_1) = 0.171373\ (850.627 - 500) = 60.088\ \text{Btu}$$

$$\int_1^2 \frac{pdv}{J} = \frac{R}{(n-1)J}\ (T_1 - T_2) = \frac{0.068549}{(1.3-1)}\ (500 - 850.627)$$

$$= -80.117\ \text{Btu}$$

The heat flow out from 1 to 2 (See Figure 1.10) is,

$$Q_o = u_1 - u_2 - \frac{1}{e_c}\int_1^2 \frac{pdv}{J} = -60.088 + \frac{80.117}{0.8} = 40.058\ \text{Btu}$$

The work due to friction is,

$$\frac{F}{J} = \left(1 - \frac{1}{e_c} \right) \int_1^2 \frac{pdv}{J} = \left(1 - \frac{1}{0.8}\right)\ (-80.117) = 20.029\ \text{Btu}$$

The rotary shaft work input may be calculated from the steady flow equation.

$$W_{s,in,1-2} = h_2 - h_1 + Q_{o,1-2} = c_p\ (T_2 - T_1\ ) + Q_{o,1-2}$$

$$= 0.239922\ (850.627 - 500) + 40.058$$

$$= 124.181\ \text{Btu}$$

$$\int_1^2 \frac{vdp}{J} = -n \int_1^2 \frac{pdv}{J} = (-1.3)\ (-80.117) = 104.152\ \text{Btu}$$

$$W_{s,in,1-2} = \int_1^2 \frac{vdp}{J} + \frac{F}{J} = 104.152 + 20.029 = 124.181\ \text{Btu}$$

This answer checks with that calculated from the steady flow equation.

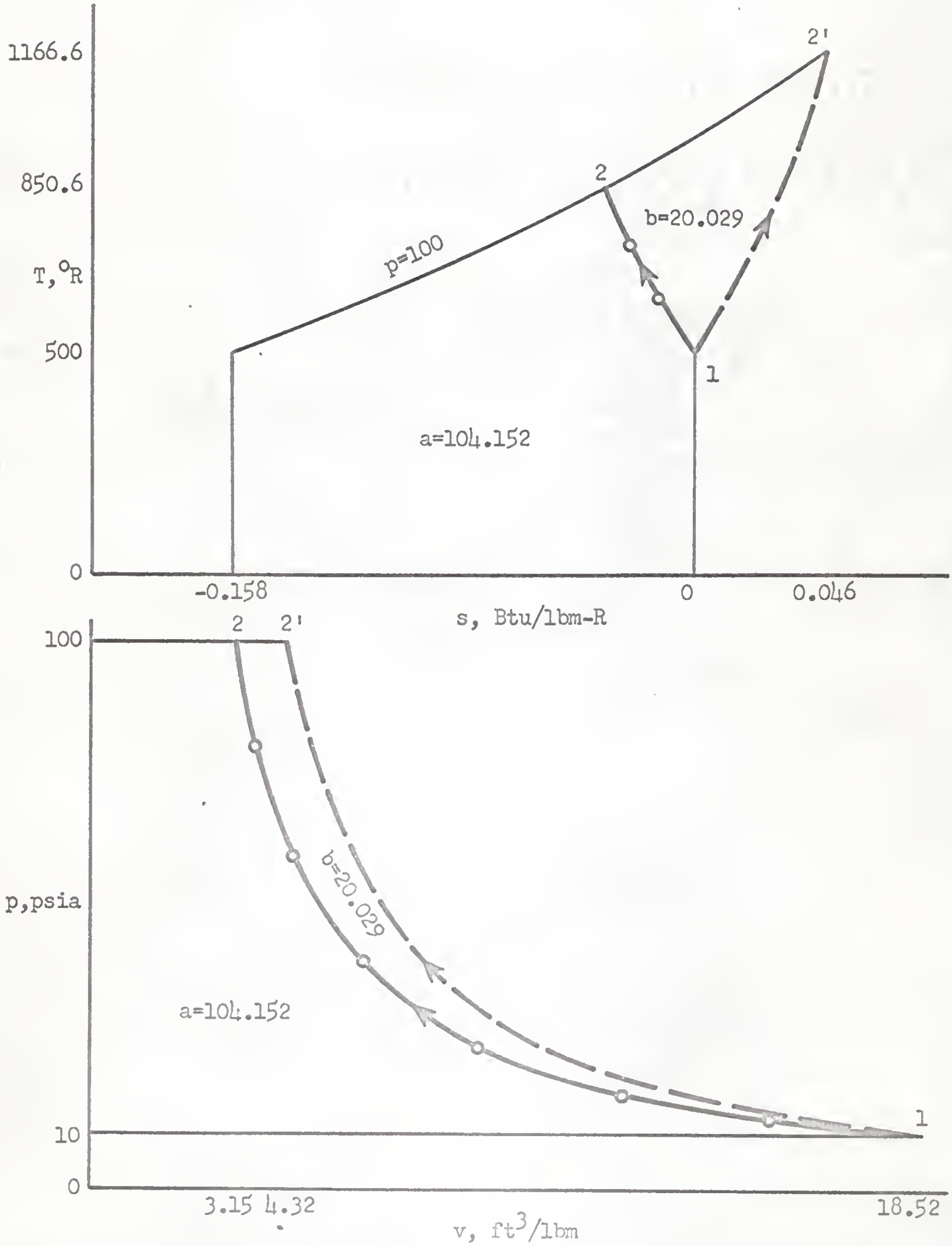

FIG. 1.10 TEMPERATURE-ENTROPY AND PRESSURE-VOLUME DIAGRAMS FOR IRREVERSIBLE DIABATIC GAS COMPRESSOR

The equivalent reversible path for the irreversible diabatic compression may be found by applying the equation (1.5).

$$W_{s,in} = \int_1^{2'} \frac{vdp}{J} = \frac{n'R}{(n'-1)J} T_1 \left[\left(\frac{p_2}{p_1}\right)^{(n'-1)/n'} -1 \right]$$

$$124.181 = \frac{n'}{n'-1} (0.068549)(500) \left[\left(\frac{100}{10}\right)^{(n'-1)/n'} -1 \right]$$

By trial and error,

$n' = 1.5821$

In Figure 1.10, area (a+b) represents the rotary shaft work input for this case. Area (b) represents the part of work due to friction.

MULTI-STAGE GAS TURBINES

1.10 Reversible adiabatic two-stage reheat gas turbine

One pound of air in the first stage of the turbine expands under reversible adiabatic conditions from an initial pressure p_1 = 100 psia and temperature T_1 = 2000 R to an intermediate pressure $p_i = p_2 = p_3$. The temperature T_2 is reheated to $T_3 = T_1$ by means of a combustion chamber. The air in the second stage then expands under reversible adiabatic conditions from the pressure p_i and temperature T_3 to a final pressure p_4 = 10 psia (See Figure 1.11).

The intermediate pressure is selected to give the maximum amount of rotary shaft work output for the entire expansion.

$$p_i = \sqrt{p_1 p_4}\,*$$
$$= 100\ (10)$$
$$= 31.623 \text{ psia}$$

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*See part F in Appendix.

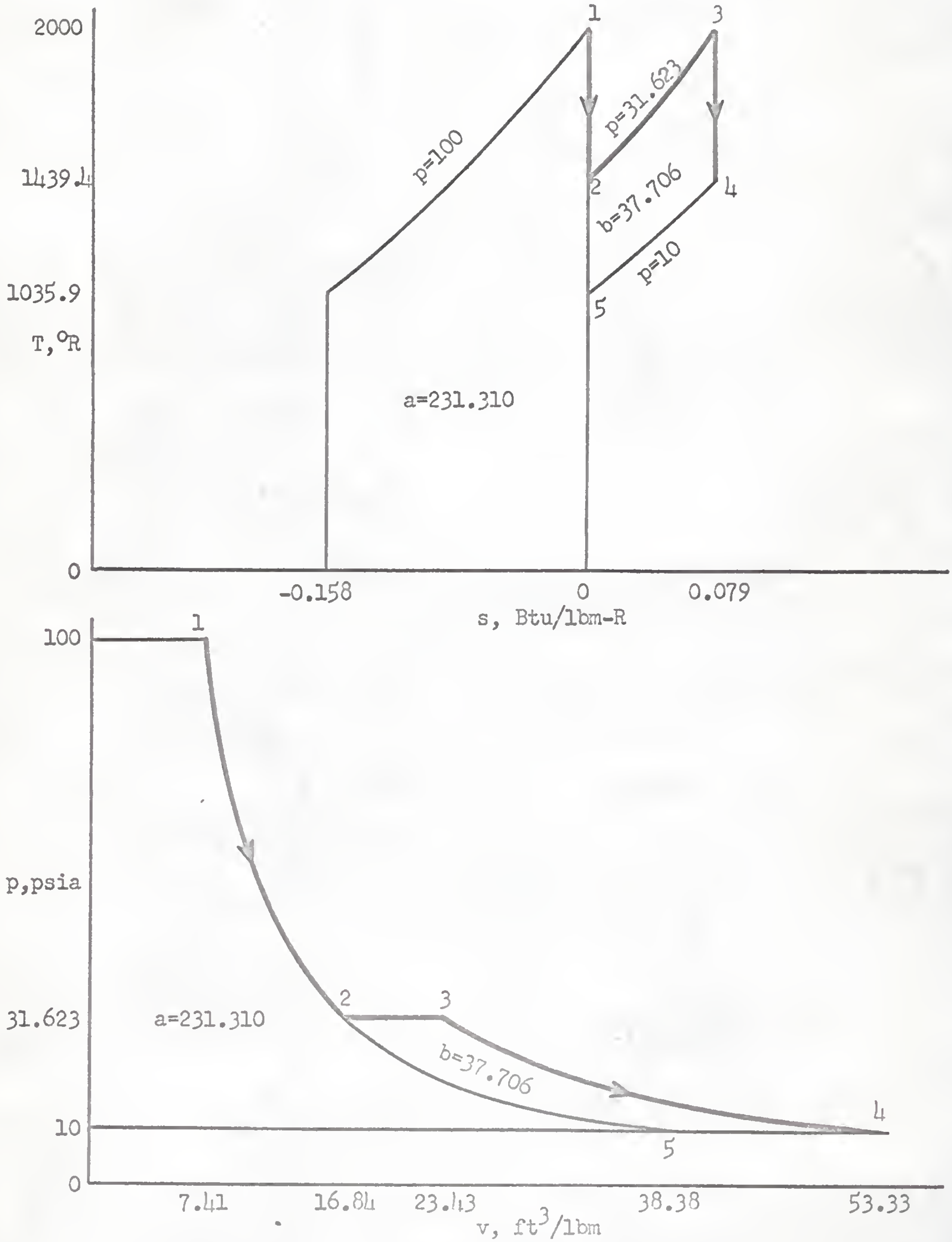

FIG. 1.11 TEMPERATURE-ENTROPY AND PRESSURE-VOLUME DIAGRAMS FOR REVERSIBLE ADIABATIC TWO-STAGE REHEAT GAS TURBINE

The temperature of the air after the first stage expansion is,

$$T_2 = T_1 \left(\frac{p_i}{p_1}\right)^{(k-1)/k} = 2000 \left(\frac{31.623}{100}\right)^{(1.4-1)/1.4} = 1439.371\ R$$

Since $T_3 = T_1$, the final temperature $T_4 = T_2$.

The rotary shaft work output is the same for both stages of the turbine.

$$W_{s,o,I} = W_{s,o,II} = h_1 - h_2 = c_p\ (T_1 - T_2)$$

$$= 1.239922\ (2000 - 1439.371) = 134.508\ \text{Btu}$$

The total rotary shaft work output is

$$W_{s,o,I} + W_{s,o,II} = 2\ (134.508) = 269.016\ \text{Btu}$$

For the single-stage gas turbine, the rotary shaft work output was found to be 231.310 Btu (See Section 1.1).

The increase in rotary shaft work for reheating versus no reheating is represented by area (b) in Figure 1.11.

$$(b) = 269.016 - 231.310 = 37.706\ \text{Btu}$$

or

$$(b) = c_p\ T_1 \left[1 - 2\left(\frac{p_5}{p_1}\right)^{(k-1)/2k} + \left(\frac{p_5}{p_1}\right)^{(k-1)/k}\right]^{*}$$

$$= 0.239922\ (2000) \left[1 - 2\left(\frac{10}{100}\right)^{(1.4-1)/2(1.4)} + \left(\frac{10}{100}\right)^{(1.4-1)/1.4}\right]$$

$$= 37.706\ \text{Btu}$$

$$\text{increase} = \frac{37.706}{231.310} = 0.1630 \text{ or } 16.30 \text{ per cent}$$

Area (a+b) represents the total shaft work output for this case, while area (a) represents that for the single-stage gas turbine.

---

*See part G in Appendix.

1.11 Irreversible adiabatic two-stage reheat gas turbine

If the efficiency of expansion is 0.8 for an irreversible adiabatic two-stage reheat gas turbine, the value of n for each stage equals

$$n = 1 + e_e\ (k-1) = 1 + 0.8\ (1.4-1) = 1.32$$

The data are the same as those for the reversible adiabatic reheat gas turbine.

M = 1 lb of air; p_1 = 100 psia; $T_1 = T_3$ = 2000 R;

$p_i = p_2 = p_3 = \sqrt{p_1 p_4}$ = 31.623; p_4 = 10 psia

The temperature of the air after the first stage expansion is,

$$T_2 = T_1\left(\frac{p_i}{p_1}\right)^{(n-1)/n} = 2000\left(\frac{31.623}{100}\right)^{(1.32-1)/1.32} = 1512.927\ \text{R}$$

Since $T_3 = T_1$, the final temperature $T_4 = T_2$.

The rotary shaft work output is,

$$W_{s,o,I} = W_{s,o,II} = h_1 - h_2 = c_p\ (T_1 - T_2)$$

$$= 0.239922\ (2000 - 1512.927) = 116.861\ \text{Btu}$$

The total rotary shaft work output is,

$$W_{s,o,I} + W_{s,o,II} = 2(116.861) = 233.722\ \text{Btu}$$

The equivalent path for each irreversible adiabatic expansion process may be found by applying equation (1.5).

$$W_{s,o,I} = -\int_1^{2'} \frac{vdp}{J} = \frac{n'R}{(n'-1)J}\,T_1\left[1 - \left(\frac{p_i}{p_1}\right)^{(n'-1)/n'}\right]$$

$$116.861 = \frac{n'}{n'-1}\,(0.068549)\ 2000\left[1 - \left(\frac{31.623}{100}\right)^{(n'-1)/n'}\right]$$

By trial and error

$$n' = 2.2284$$

For the single-stage irreversible adiabatic gas turbine, the rotary shaft work output was found to be 205.259 Btu (See Section 1.2).

The increase in rotary shaft work output for reheating versus no reheating is represented by area (e-b) in Figure 1.12.

$$(e-b) = 233.722 - 205.259 = 28.463 \text{ Btu}$$

$$\text{increase} = \frac{28.463}{205.259} = 0.1387 \text{ or } 13.87 \text{ per cent}$$

Area (a+d+e) represents the rotary shaft work output for the irreversible adiabatic two-stage reheat gas turbine. Area (a+b+d) represents that for the single-stage gas turbine.

MULTI-STAGE GAS COMPRESSORS

1.12 Reversible adiabatic two-stage compressor

A two-stage gas compressor is to be used to compress one pound of air at a pressure of 10 psia. The air in the first stage of the compressor is compressed under reversible adiabatic conditions from an initial pressure p_1 = 10 psia and temperature T_1 = 500 R to an intermediate pressure $p_i = p_2 = p_3$. The temperature T_2 is cooled by means of the intercooler to $T_3 = T_1$. The air in the second stage is then compressed under reversible adiabatic from the pressure p_i and temperature T_3 to a final pressure p_4 = 100 psia. (See Figure 1.13).

The intermediate pressure is selected to give the minimum amount of rotary shaft work input for the entire compression process.

$$p_i = \sqrt{p_1 p_4} = \sqrt{10\ (100)} = 31.623 \text{ psia}$$

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*See part H in Appendix.

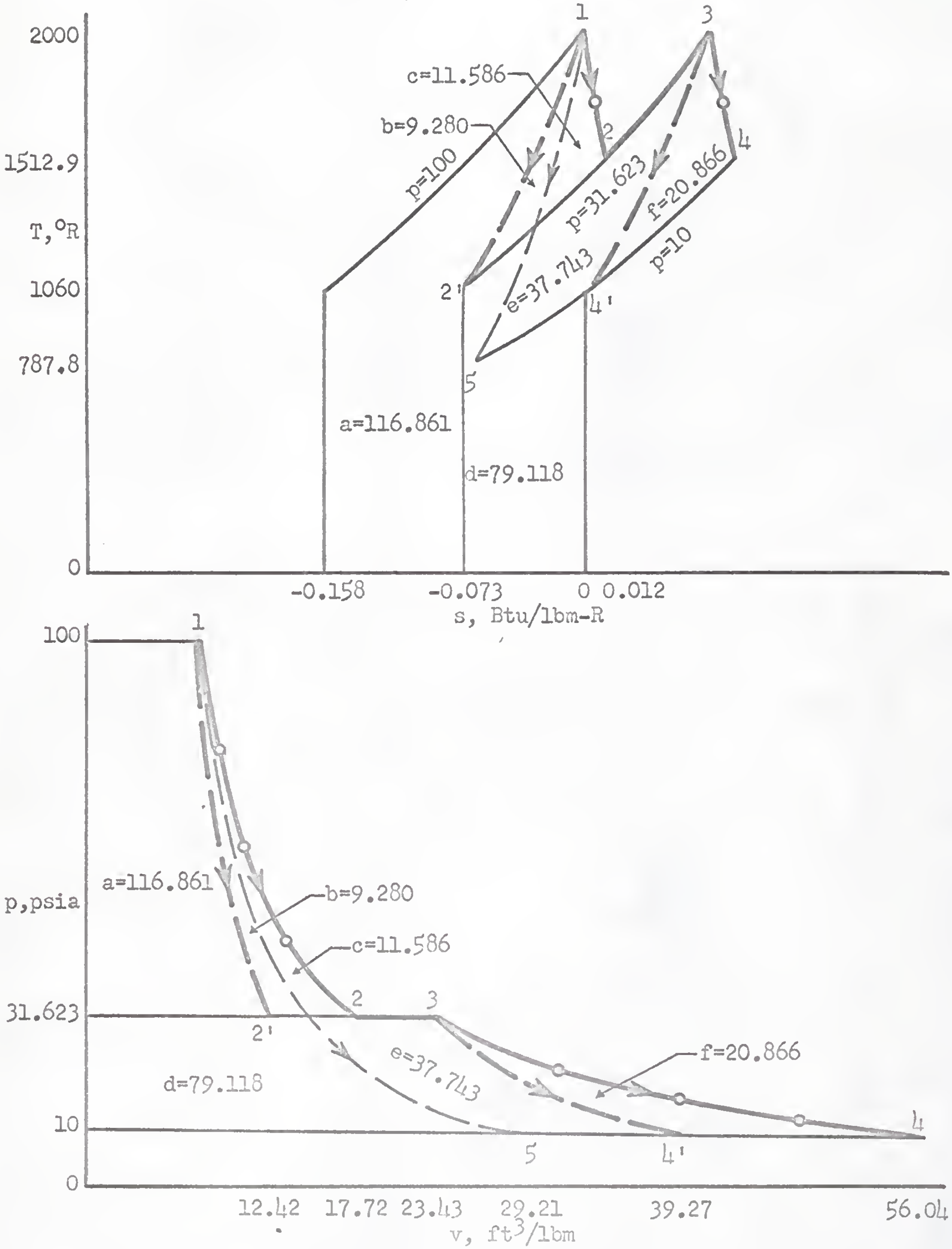

FIG. 1.12 TEMPERATURE-ENTROPY AND PRESSURE-VOLUME DIAGRAMS FOR IRREVERSIBLE ADIABATIC TWO-STAGE REHEAT GAS TURBINE

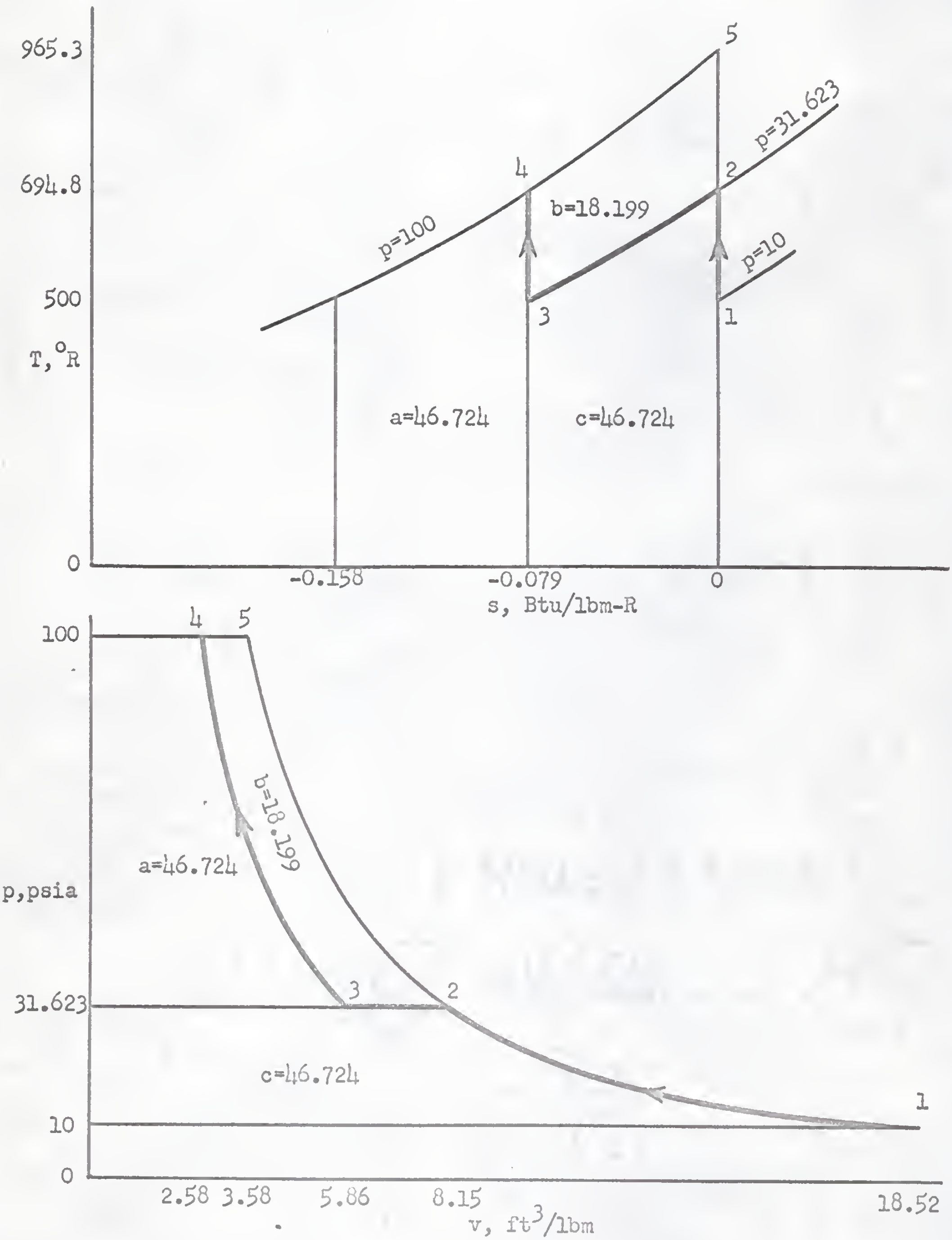

FIG. 1.13 TEMPERATURE-ENTROPY AND PRESSURE-VOLUME DIAGRAMS FOR REVERSIBLE ADIABATIC TWO-STAGE GAS COMPRESSOR

The temperature of the air after the first stage compression is

$$T_2 = T_1 \left(\frac{p_i}{p_1}\right)^{(k-1)/k} = 500 \left(\frac{31.623}{10}\right)^{(1.4-1)/1.4} = 694.745 \text{ R}$$

Since $T_3 = T_1$, the final temperature $T_4 = T_2$.

The rotary shaft work input is the same for both stages.

$$W_{s,in,I} = W_{s,in,II} = h_2 - h_1 = c_p (T_2 - T_1)$$

$$= 0.239922 (694.745 - 500) = 46.724 \text{ Btu}$$

The total rotary shaft work input is,

$$W_{s,in,I} + W_{s,in,II} = 2(46.724) = 93.448 \text{ Btu}$$

For the single-stage gas compressor, the rotary shaft work input was found to be 111.647 Btu (see Section 1.3).

The saving in rotary shaft work input is represented by area (b) in Figure 1.13.

$$(b) = 111.647 - 93.448 = 18.199 \text{ Btu}$$

$$\text{Saving} = \frac{18.199}{111.647} = 0.1630 \text{ or } 16.30 \text{ per cent}$$

This percentage of saving may be found by the following manner.

$$\text{Saving} = 1 - \frac{2\left[\left(\frac{p_5}{p_1}\right)^{(k-1)/2k} - 1\right]^*}{\left(\frac{p_5}{p_1}\right)^{(k-1)/k} - 1}$$

$$= 1 - \frac{2\left[\left(\frac{100}{10}\right)^{(1.4-1)/2(1.4)} - 1\right]}{\left(\frac{100}{10}\right)^{(1.4-1)/1.4} - 1} = 0.1630 \text{ or } 16.30 \text{ per cent}$$

---

*See part I in Appendix.

Area (a+c) represents the total rotary shaft work input for this case, while area (a+b+c) represents that for the single-stage gas compressor.

1.13 Irreversible adiabatic two-stage gas compressor

If the efficiency of compression is 0.8 for an irreversible adiabatic two-stage gas compressor, the value of n for each stage equals,

$$n = 1 + \frac{1}{e_c}(k-1) = 1 + \frac{1}{0.8}(1.4-1) = 1.5$$

The data are the same as those for the reversible adiabatic two-stage gas compressor.

$$M = 1 \text{ lb of air};\ p_1 = 10 \text{ psia};\ T_1 = T_3 = 500 \text{ R};\ p_i = p_2 = p_3$$

$$= \sqrt{p_1 p_4} = 31.623 \text{ psia};\ p_4 = 100 \text{ psia}$$

The temperature of the air after the first stage compression is,

$$T_2 = T_1 \left(\frac{p_i}{p_1}\right)^{(n-1)/n} = 500 \left(\frac{31.623}{10}\right)^{(1.5-1)/1.5} = 733.900 \text{ R}$$

Since $T_3 = T_1$, the final temperature $T_4 = T_2$.

The rotary shaft work input equals

$$W_{s,in,I} = W_{s,in,II} = h_2 - h_1 = c_p (T_2 - T_1)$$

$$= 0.239922 \ (733.900 - 500) = 56.118 \text{ Btu}$$

The total rotary shaft work input is,

$$W_{s,in,I} + W_{s,in,II} = 2(56.118) = 112.236 \text{ Btu}$$

The equivalent path for each irreversible adiabatic compression process may be found by applying equation (1.5).

$$W_{s,in,I} = \int_1^{2'} \frac{vdp}{J} = \frac{n'R}{(n'-1)J} T_1 \left[\left(\frac{p_i}{p_1}\right)^{(n'-1)/n'} -1 \right]$$

$$56.118 = \frac{n'}{n'-1} (0.068549)\ 500 \left[\left(\frac{31.623}{10}\right)^{(n'-1)/n'} -1 \right]$$

By trial and error

$$n'' = 2.3789$$

For the single-stage irreversible adiabatic gas compressor, the rotary shaft work input was found to be 138.487 Btu (see Section 1.4).

The saving in rotary shaft work input is represented by area (c+d-g) in Figure 1.14.

$$(c+d-g) = 138.487 - 112.236 = 26.251 \text{ Btu}$$

$$\text{Saving} = \frac{26.251}{138.487} = 0.1896 \text{ or } 18.96 \text{ per cent}$$

Area (a+b+e+f+g) represents the rotary shaft work input for the irreversible adiabatic two-stage gas compressor. Area (a→f) represents that for the single-stage gas compressor.

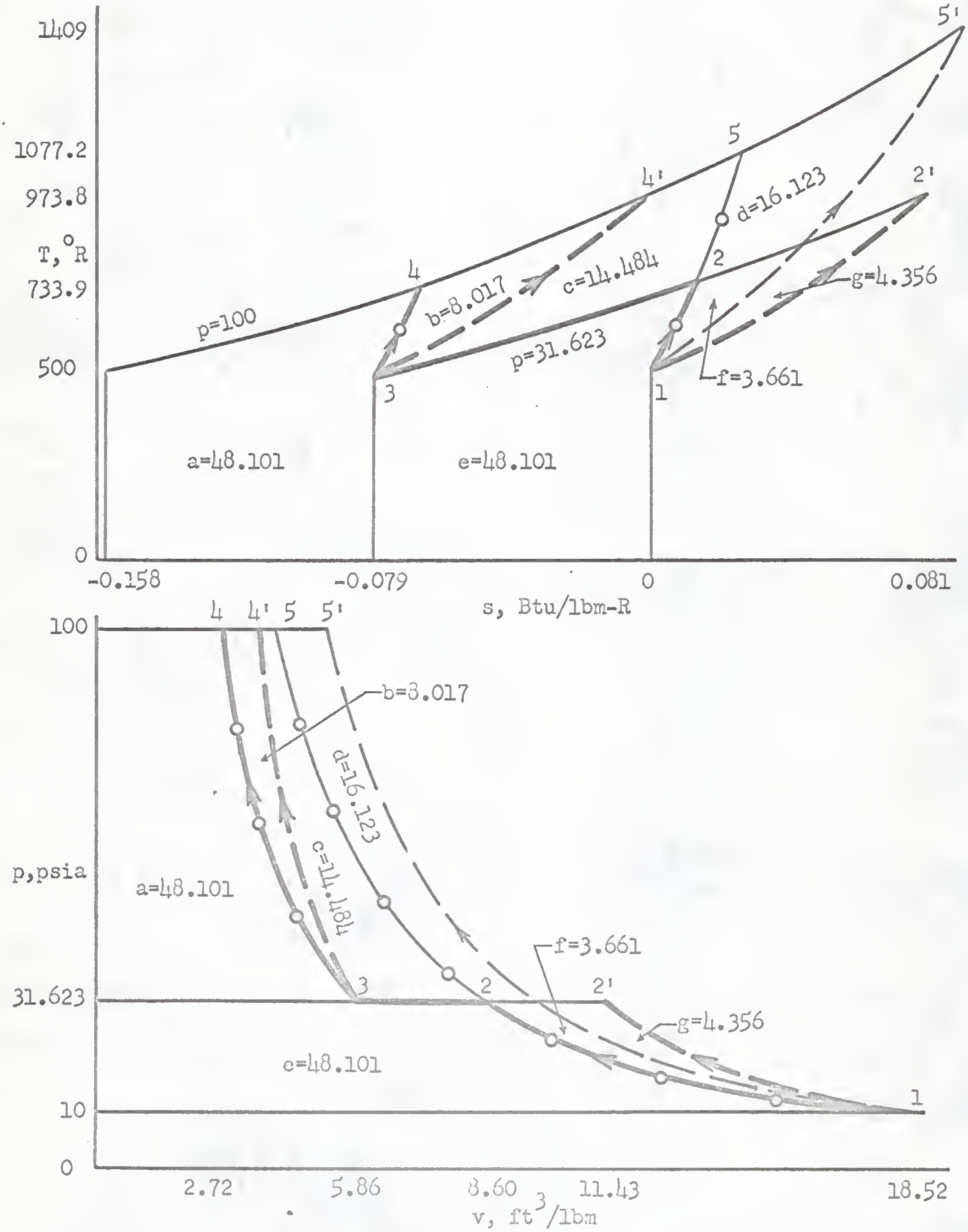

FIG. 1.14 TEMPERATURE-ENTROPY AND PRESSURE-VOLUME DIAGRAMS FOR IRREVERSIBLE ADIABATIC TWO-STAGE GAS COMPRESSOR

PART II

VAPOR REFRIGERATION CYCLES

SINGLE-STAGE VAPOR COMPRESSORS

2.1 Reversible adiabatic vapor compressor

In vapor refrigeration cycles, Freon-12 is commonly used as a refrigerant. Assume one pound of dry and saturated vapor Freon-12 is compressed under reversible adiabatic conditions from an initial temperature of 400 R to a final pressure of 222.124 psia. The properties of the refrigerant at each state in the equipment diagram (see Figure 2.1) and the temperature-entropy diagram (see Figure 2.2) are summarized in the following table.

State	t F	p psia	h Btu/lb_m	s $Btu/lb_m R$	x
1 (sat.vap.)	-59.70	5.405	70.727	0.17708	1
2	188.17	222.124	99.651	0.17708	*
1'	140.30	222.124	70.727	0.12953	0.605
3 (sat.liq.)	140.30	222.124	41.242	0.08033	0
4	-59.70	5.405	41.242	0.10354	0.606

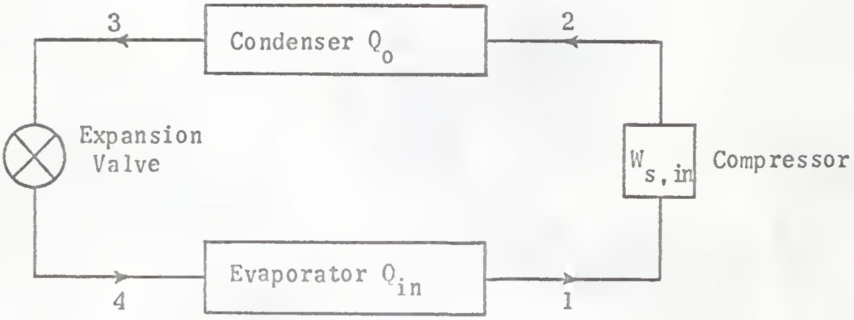

Figure 2.1 Equipment diagram for the single compression, single expansion refrigeration cycle.

*Superheated vapor.

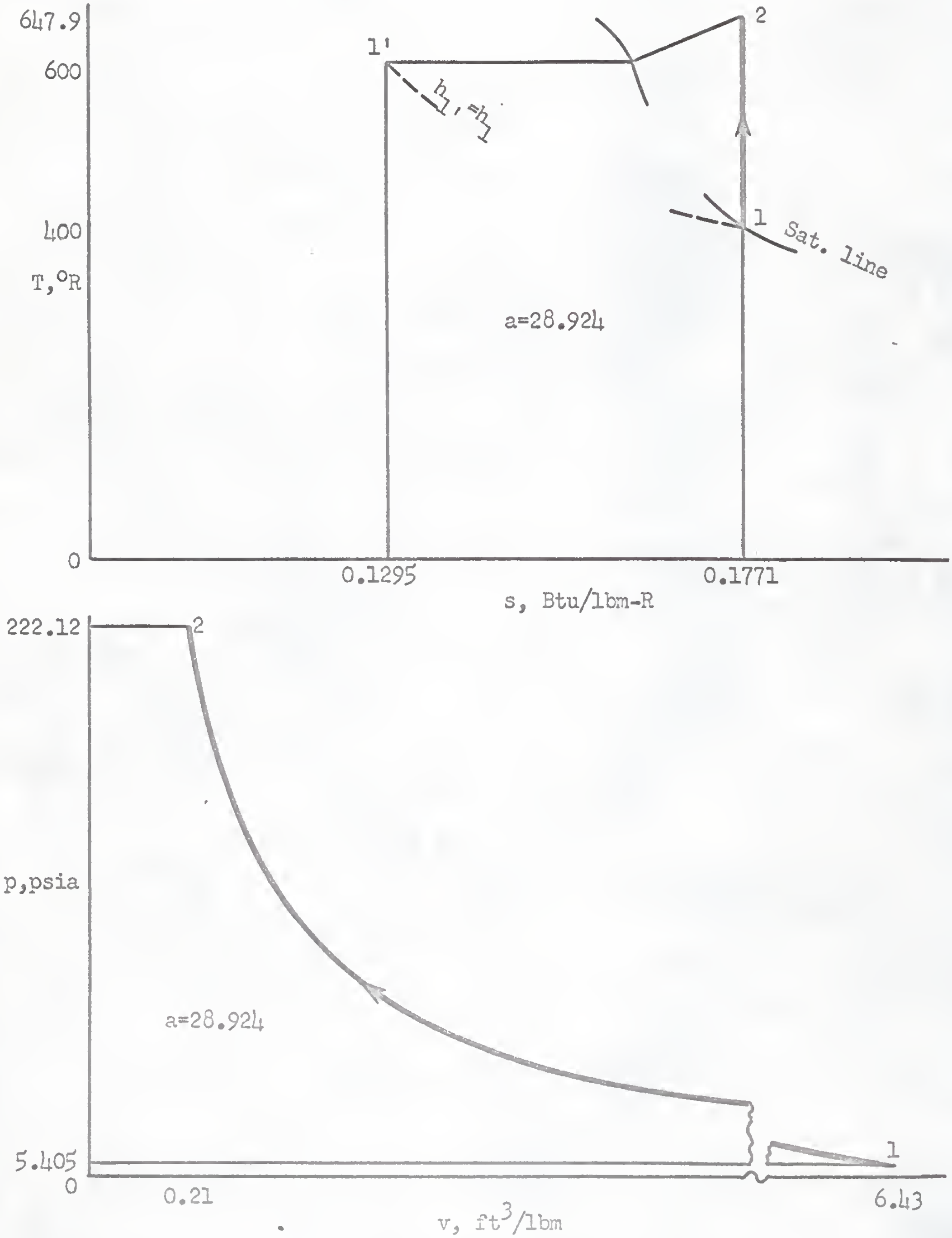

FIG. 2.2 TEMPERATURE-ENTROPY AND PRESSURE-VOLUME DIAGRAMS FOR REVERSIBLE ADIABATIC VAPOR COMPRESSOR

In Figure 2.2, if a constant enthalpy line 1-1' is drawn on the temperature-entropy diagram, it can be proved that the area under the constant pressure line from 2 to 1' equals the rotary shaft work input from 1 to 2 on the pressure-volume diagram.

Consider the closed cycle 1-2-1'-1 on the temperature-entropy diagram.

$$W_{s,in,cycle} = W_{s,in,1-2} - W_{s,o,1'-1}$$

or

$$W_{s,in,1-2} = W_{s,in,cycle} + W_{s,o,1'-1} \quad (2.1)$$

From the First Law of Thermodynamics, the cyclic work may be written as,

$$W_{s,in,cycle} = Q_{o,2-1'} - Q_{in,1'-1} \quad (2.2)$$

The steady flow equation is,

$$h_{1'} + Q_{in,1'-1} = h_1 + W_{s,o,1'-1}$$

Since $h_{1'} = h_1$, and

$$\therefore Q_{in,1'-1} = W_{s,o,1'-1} \quad (2.3)$$

Substituting equations (2.2) and (2.3) into equation (2.1)

$$W_{s,in,1-2} = Q_{o,2-1'} \qquad \text{Q.E.D.}$$

Area (a) represents the rotary shaft work input on both temperature-entropy and pressure-volume planes, which is equal to

$$W_{s,in,1-2} = h_2 - h_1 = 99.651 - 70.727 = 28.924 \text{ Btu}$$

The coefficient of performance (C.O.P.) of the refrigeration cycle shown by the equipment diagram of Figure 2.1 is,

$$\text{C.O.P.} = \frac{\text{Refrigeration Load}}{\text{Rotary Shaft Work Input}}$$

$$= \frac{h_1 - h_4}{W_{s,in,1-2}} = \frac{70.727 - 41.242}{28.924} = 1.0194$$

2.2 Irreversible adiabatic vapor compressor

One pound of dry and saturated vapor Freon-12 is compressed under irreversible adiabatic conditions from an initial temperature of 400 R to a final pressure of 222.124 psia. The efficiency of compression, $\eta_c = 0.8$ and it is defined as

$$\eta_c = \frac{h_{2r} - h_1}{h_2 - h_1} \tag{2.4}$$

The actual path from 1 to 2 (see Figure 2.3) was plotted by chosen intermediate pressures (p_i) along that path. The enthalpy h_i was determined by equation (2.4).

$$h_i = h_1 + \frac{h_{ir} - h_1}{\eta_c}$$

Other properties were found in the Freon-12 table.

The properties of the refrigerant for some end states are shown in the following table.

State	t F	p psia	h Btu/lb_m	s $Btu/lb_m R$
1 (sat.vap.)	-59.70	5.405	70.727	0.17708
2r	188.17	222.124	99.651	0.17708
2	226.76	222.124	106.882	0.18799
2'	330.37	222.124	125.838	0.21367

The rotary shaft work input for the irreversible adiabatic vapor compressor is

$$W_{s,in,1-2} = h_2 - h_1 = 106.882 - 70.726 = 36.155 \text{ Btu}$$

In Figure 2.3, the reversible adiabatic compression is represented by the path 1-2r.

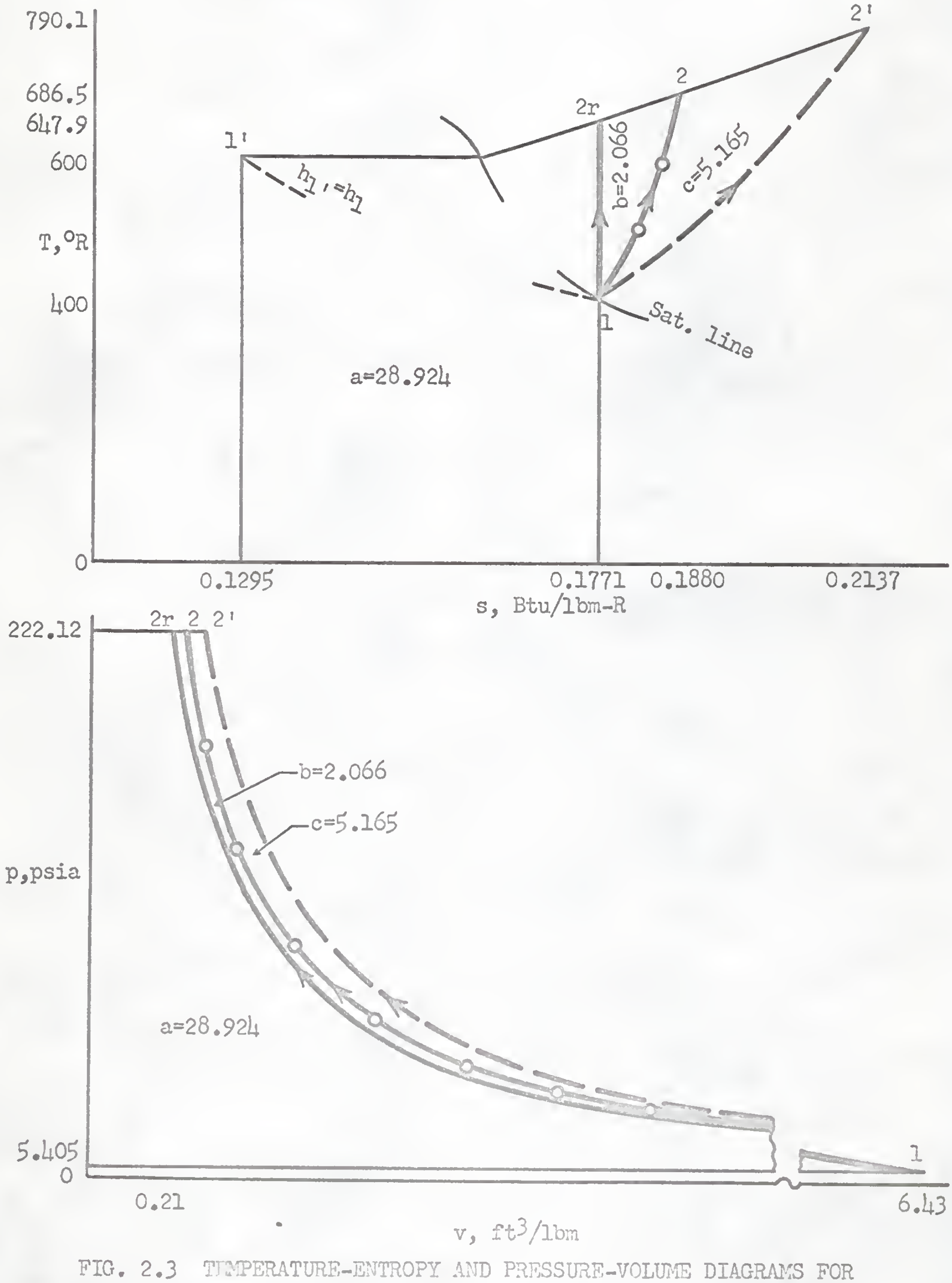

FIG. 2.3 TEMPERATURE-ENTROPY AND PRESSURE-VOLUME DIAGRAMS FOR IRREVERSIBLE ADIABATIC VAPOR COMPRESSOR

The equivalent reversible adiabatic path 1-2' which represents the same rotary shaft work input in this irreversible case was found by the following procedures.

1. Choose several suitable intermediate pressures between 5.405 psia and 222.124 psia.

2. Select a ratio of $\frac{h_{ir} - h_1}{h_{i'} - h_1}$ and keep it constant along the whole path to find the enthalpy value $h_{i'}$.

3. Find the corresponding properties of the refrigerant from the Freon-12 table for given p_i and $h_{i'}$.

4. Plot the pressure-volume diagram and apply Simpson's Rule to find $\int_1^{2'} \frac{pdv}{J}$.

5. Calculate the heat flow from 1 to 2' by the non-flow equation.

$$Q_{in,1-2'} = u_{2'} - u_1 + \int_1^{2} \frac{pdv}{J} .$$

6. Repeat the procedures until by trial and error find a ratio that gives $W_{s,in,1-2'} = h_{2'} - h_1 - Q_{in,1-2'} = 36.155$ Btu. The ratio was found to be 0.5251 in this case.

In Figure 2.3, area (a+b+c) represents the rotary shaft work input (36.155 Btu) for this irreversible adiabatic vapor compressor. Area (a) represents the rotary shaft work input (28.924 Btu) for the reversible case.

Area (b+c) represents the extra work input due to friction. Both two parts are non-recoverable.

(b+c) = (a+b+c) - (a) = 36.155 - 28.924 = 7.231 Btu

As compared with the reversible adiabatic vapor compressor, 25 per cent extra work input is needed.

Area (b) represents the irreversible isentropic friction factor and area (c) represents the irreversible adiabatic friction factor. (See Section 1.2)

The coefficient of performance (C.O.P.) is,

$$\text{C.O.P.} = \frac{h_1 - h_4}{W_{s,in,1-2}} = \frac{70.727 - 41.242}{36.155} = 0.8155$$

This is a 20 per cent decrease in the coefficient of performance.

2.3 Reversible adiabatic Freon-12 expansion engine

One pound of saturated liquid Freon-12 expands under reversible adiabatic conditions from an initial temperature of 600 R to a final temperature of 400 R. The constant pressure line from 3 to 3', in which $h_{3'} = h_{4'}$, was drawn on the temperature-entropy diagram (see Figure 2.4).

The properties of the refrigerant are summarized in the following table.

	t	p	h	s	x
State	F	psia	Btu/lb_m	Btu/lb_m	
1 (sat.vap.)	-59.70	5.405	70.727	0.17708	1
2	188.17	222.124	99.651	0.17708	
3 (sat.liq.)	140.30	222.124	41.242	0.08033	0
3'	103.84	222.124	32.027	0.06485	*
4	-59.70	5.405	41.242	0.10337	0.606
4'	-59.70	5.405	32.027	0.08033	0.483

In Figure 2.4, the area (a) under the constant pressure line from 3 to 3' represents the rotary shaft work output for the engine, which is equal to,

$$W_{s,o,3-4'} = h_3 - h_{4'} = h_3 - h_{3'} = 41.242 - 32.027 = 9.215 \text{ Btu}$$

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*Compressed liquid.

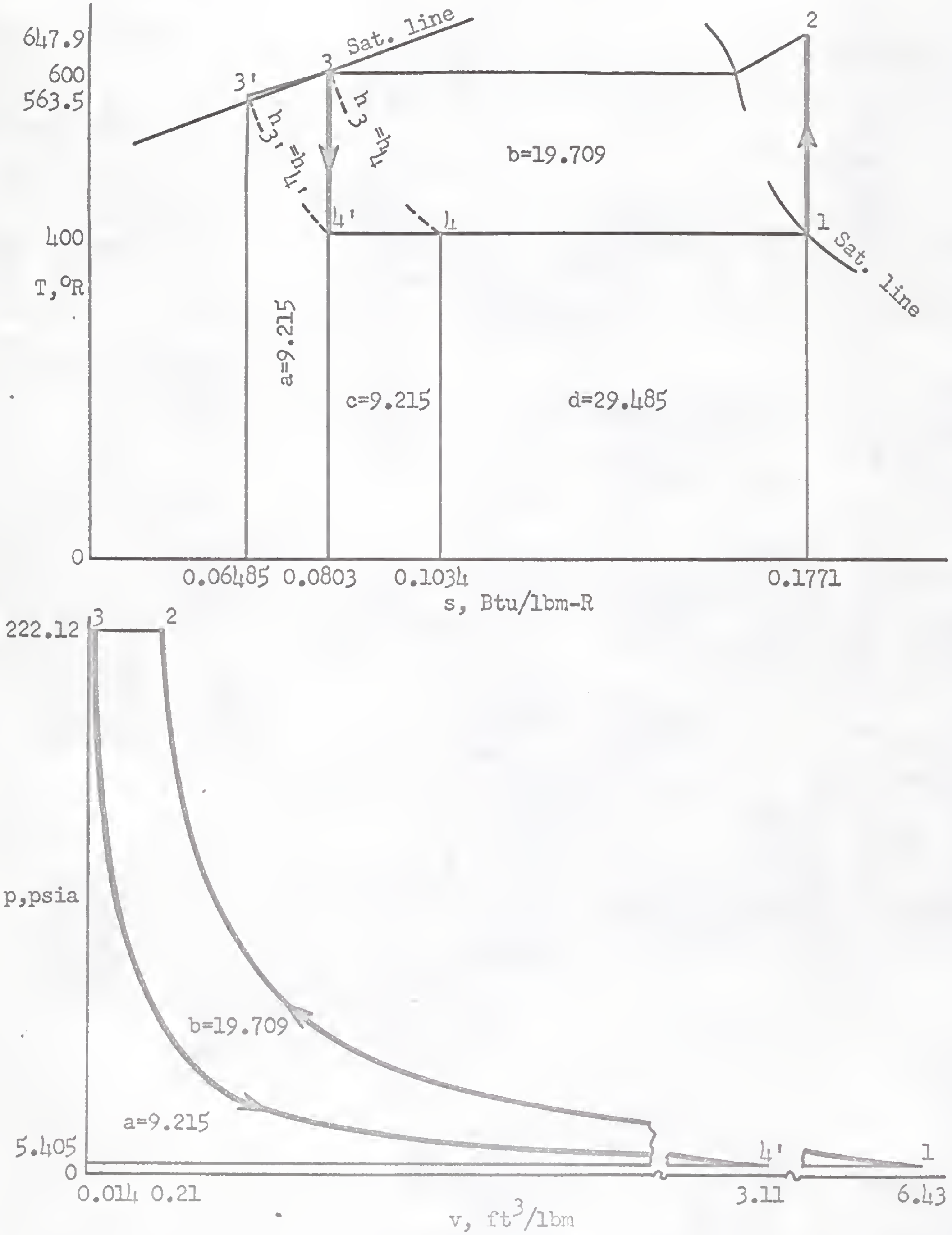

FIG. 2.4 TEMPERATURE-ENTROPY AND PRESSURE-VOLUME DIAGRAMS FOR REVERSIBLE ADIABATIC FREON-12 EXPANSION ENGINE

The net rotary shaft work input for the refrigeration cycle with the equipment of an expansion engine is the difference between the rotary shaft work input for the adiabatic reversible vapor compressor and the rotary shaft work output for the expansion engine.

$$W_{s,in,net} = (h_2 - h_1) - (h_3 - h_{4'}) =$$

$$= (99.651 - 70.727) - (41.242 - 32.027) = 19.709 \text{ Btu}$$

This amount of rotary shaft work is represented by area (b). Area (c+d) represents the refrigeration load.

$$Q_{in,4'-1} = h_1 - h_{4'} = 70.727 - 32.027 = 38.700 \text{ Btu}$$

Area (d) represents the refrigeration load for the refrigeration cycle with an expansion valve.

Area (c) represents the increase in refrigeration load.

The coefficient of performance is,

$$\text{C.O.P.} = \frac{Q_{in,4'-1}}{W_{s,in,net}} = \frac{38.700}{19.709} = 1.9636$$

An increase in coefficient of performance is $\frac{1.9636 - 1.0194}{1.0194} = 92.62$ per cent.

MULTI-STAGE VAPOR COMPRESSORS

2.4 Reversible adiabatic two-stage vapor compressor with flash intercooler.

One pound of dry and saturated vapor Freon-12 is compressed under reversible adiabatic conditions in the first stage compressor from an initial temperature $T_1 = 400$ R to an intermediate pressure $p_2 = p_3 = p_6 = p_7 = 56$ psia. The equipment diagram is shown in Figure 2.5. The superheated vapor flows into a flash intercooler at a constant pressure p_2, in which it becomes saturated. The saturated vapor leaving the flash intercooler with 1.71215 pounds in mass is compressed under reversible adiabatic conditions in the second stage compressor from p_3 = 56 psia to a final pressure p_4 = 222.124 psia.

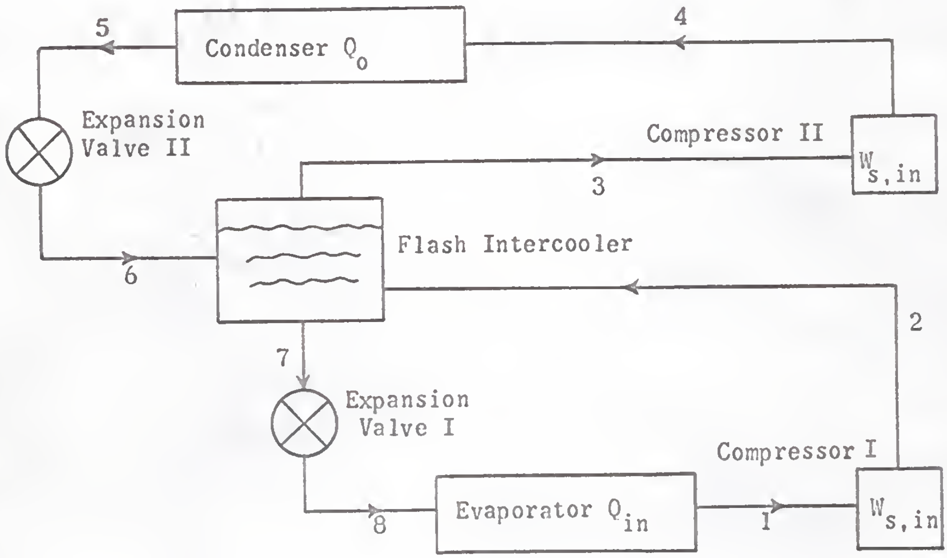

Figure 2.5 Equipment diagram for the dual compression, dual expansion refrigeration cycle.

The properties of the refrigerant are summarized in the following table.

State	t F	p psia	h Btu/lb_m	s $Btu/lb_m R$	x
1 (sat.vap.)	-59.70	5.405	70.727	0.17708	1
2	82.45	56	87.913	0.17708	
3 (sat.vap.)	44.61	56	81.899	0.16559	1
4	151.82	222.124	92.419	0.16559	
5 (sat.liq.)	140.30	222.124	41.242	0.08033	0
6	44.61	56	41.242	0.08497	0.361
7 (sat.liq.)	44.61	56	18.301	0.03948	0
8	-59.70	5.405	18.301	0.04602	0.300

The mass in the second stage compressor was found by the energy balance on the flash intercooler (see Figure 2.5).

$$M_I h_2 + M_{II} h_6 = M_I h_7 + M_{II} h_3$$

$$M_I(h_2 - h_7) = M_{II}(h_3 - h_6)$$

$$M_{II} = \frac{h_2 - h_7}{h_3 - h_6} M_I = \frac{87.913 - 18.301}{81.899 - 41.242}(1) = 1.71215$$

The intermediate pressure was found by trial and error on the basis of the best coefficient of performance, which is,

$$\text{C.O.P.} = \frac{M_I(h_1 - h_8)}{M_I(h_2 - h_1) + M_{II}(h_4 - h_3)}$$

$$= \frac{70.727 - 18.301}{(87.913 - 70.727) + 1.71215\ (92.419 - 81.899)}$$

$$= 1.4894$$

In Figure 2.6, area (a) represents the rotary shaft work input for the first stage compressor.

$$W_{s,in,I} = M_I(h_2 - h_1) = 1\ (87.913 - 70.727) = 17.186 \text{ Btu}$$

Area (b) represents the rotary shaft work input for the second state compressor.

$$W_{s,in,II} = M_{II}(h_4 - h_3) = 1.71215\ (92.419 - 81.899) = 18.012 \text{ Btu}$$

The total rotary shaft work input equals 35.198 Btu.

2.5 Comparison of reversible adiabatic two-stage and single-stage vapor compressors

In this case, one pound of refrigerant in the reversible adiabatic single-stage vapor compressor in which the refrigeration load Q_{in} = 29.485 Btu (see Section 2.1) is compared with the reversible adiabatic two-stage vapor compressor that has the best coefficient of performance (1.4894) as found in

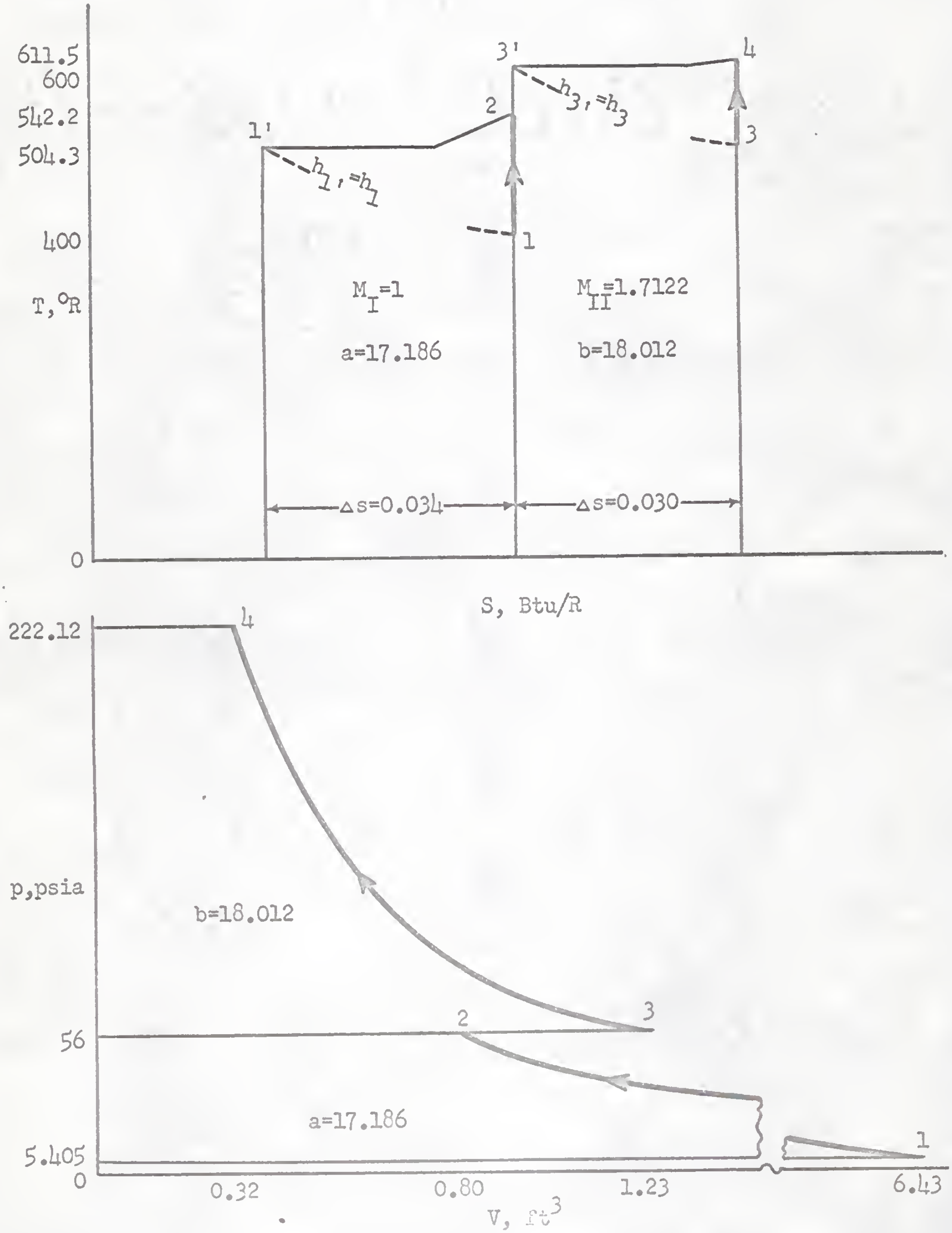

FIG. 2.6 TEMPERATURE-ENTROPY AND PRESSURE-VOLUME DIAGRAMS FOR REVERSIBLE ADIABATIC TWO-STAGE VAPOR COMPRESSOR

the previous case and the same refrigeration load as the single-stage compressor.

Since the ratio of M_{II} and M_I was found to be 1.71215 in Section 2.4, the mass of the refrigerant in each stage of the two-stage compressor may be calculated in the following manner.

$$Q_{in} = M_I(h_1 - h_8)$$

or

$$M_I = \frac{Q_{in}}{h_1 - h_8} = \frac{29.485}{70.727 - 18.301} = 0.56242 \text{ lb}$$

$$M_{II} = 1.71215\ M_I = 1.71215\ (0.56242) = 0.96294 \text{ lb}$$

In Figure 2.7, area (a+b+c+d) represents the rotary shaft work input for the single-stage vapor compressor, which equals 28.924 Btu (see Section 2.1).

Area (c) represents the rotary shaft work input for the first stage compressor of the two-stage vapor compressor.

$$W_{s,in,1_2 - 2_2} = M_I(h_{2_2} - h_{1_2}) = 0.56242\ (87.913 - 70.727) = 9.666 \text{ Btu}$$

Area (d) represents the rotary shaft work input for the second stage compressor.

$$W_{s,in,3_2-4_2} = M_{II}(h_{4_2} - h_{3_2}) = 0.96242\ (92.419 - 81.899) = 10.130 \text{ Btu}$$

The total rotary shaft work input for the two-stage vapor compressor equals 19.796 Btu.

Area (a+b) represents the saving in rotary shaft work input, which is

$$(a+b) = (a+b+c+d) - (c+d) = 28.924 - 19.796 = 9.128 \text{ Btu}$$

$$\text{Saving} = \frac{9.128}{28.924} = 0.3156 \text{ or } 31.56 \text{ per cent}$$

The coefficient of performance for the single-stage vapor compressor is 1.0194, while the two-stage vapor compressor is 1.4894. A 46.11 per cent increase in coefficient of performance is obtained.

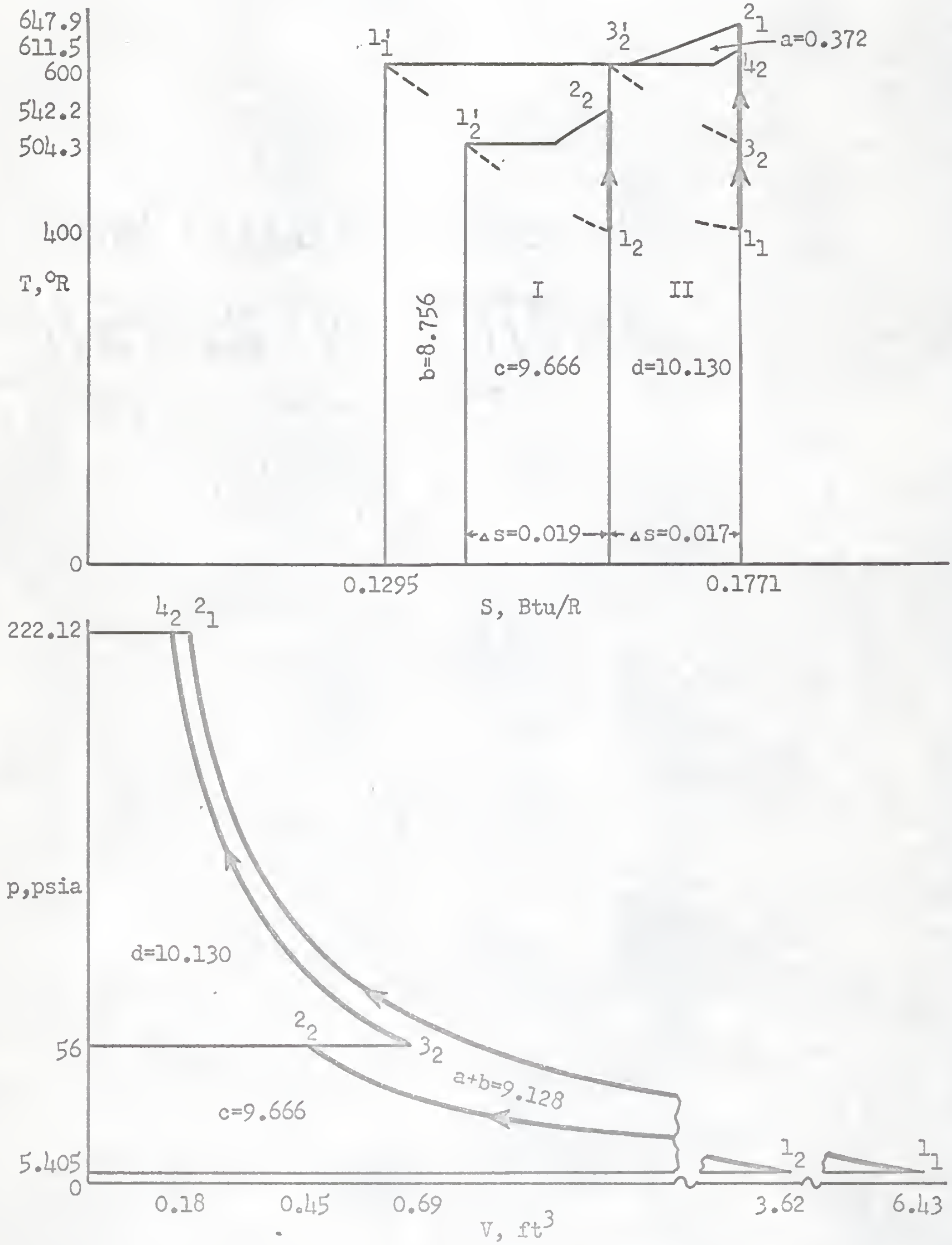

FIG. 2.7 TEMPERATURE-ENTROPY AND PRESSURE-VOLUME DIAGRAMS FOR COMPARISON OF REVERSIBLE ADIABATIC TWO-STAGE AND SINGLE-STAGE VAPOR COMPRESSORS

REFERENCES

1. E. A. Bruges, "Available Energy and Second Law Analysis", Academic Press, Inc., Publishers, New York, 1959.

2. "Thermodynamics Properties of Freon-12 Refrigerant," E.I. DuPont de Nemours & Company, Inc., Wilmington 98, Delaware, 1956.

APPENDIX

A. Polytropic exponent for the irreversible adiabatic expansion of a perfect gas (Section 1.2).

$$dQ = du + dW$$

$$0 = c_v dT + e_e \frac{pdv}{J} \qquad (1)$$

$$c_v = \frac{1}{k-1} \frac{R}{J} \qquad (2)$$

Differentiating the ideal gas law results in

$$dT = (pdv + vdp)/R \qquad (3)$$

Substituting the values for c_v and dT from relations (2) and (3) in (1) gives

$$0 = \frac{1}{k-1}(pdv + vdp) + e_e\, pdv$$

$$0 = \frac{dp}{p} + \left[1 + e_e(k-1)\right] \frac{dv}{v}$$

Let $n = 1 + e_e(k-1)$; pv^n = a const.

B. Polytropic exponent for the irreversible adiabatic compression of a perfect gas (Section 1.4).

$$dQ = du + dW$$

$$0 = c_v dT + \frac{1}{e_c} \frac{pdv}{J} \qquad (4)$$

$$c_v = \frac{1}{k-1} \frac{R}{J} \qquad (2)$$

$$dT = (pdv + vdp)/R \qquad (3)$$

Substituting the values for c_v and dT from relations (2) and (3) in (4) gives

$$0 = \frac{1}{k-1}(pdv + vdp) + \frac{1}{e_c} pdv$$

$$0 = \frac{dp}{p} + \left[1 + \frac{1}{e_c}(k-1)\right] \frac{dv}{v}$$

Let $n = 1 + \frac{1}{e_c}(k-1)$; pv^n = a const.

C. Friction factors for the irreversible adiabatic gas compression of Section (1.4).

1. Irreversible isentropic friction factor.

This is represented by area (b) in Figure 1.4.

Consider that the path 1-2 (Figure 1.4) represents the process for a reversible diabatic compression (Section 1.4).

$$W_{s,in,rev,1-2} = 118.703 \text{ Btu} \qquad \text{area (a+b)}$$

Now consider the case of an irreversible isentropic compression in which the path 1-2r of Figure 1.4 is followed, and where the rotary shaft work input is equal to the rotary shaft input for the reversible diabatic path 1-2; i.e.,

$$W_{s,in,irrev,\ 1-2r} = 118.703 \text{ Btu} \qquad \text{area (a+b)}$$

From the steady flow equation

$$Q_{o,irrev,\ 1-2r} = W_{s,in,irrev,1-2r} - (h_{2r} - h_1)$$

From Section 1.4

$$h_{2r} - h_1 = W_{s,in,rev,1-2r} = 111.647 \text{ Btu} \qquad \text{area (a)}$$

$$Q_{o,irrev,1-2r} = 118.703 - 111.647 = 7.056 \text{ Btu} \qquad \text{area (b)}$$

$$W_{s,in,irrev,1-2r} = \int_1^{2r} \frac{vdp}{J} + \left(\frac{F}{J}\right)_{irrev,isen}$$

From Section 1.4

$$\int_1^{2r} \frac{vdp}{J} = W_{s,in,rev,1-2r} = 111.647 \text{ Btu} \qquad \text{area (a)}$$

$$\therefore \left(\frac{F}{J}\right)_{irrev,\ isen} = 118.703 - 111.647 = 7.056 \text{ Btu}$$

$$= Q_{o,irrev,1-2r} \qquad \text{area (b)}$$

If the isentropic process 1-2r is reversible the work input equals 111.647 Btu (represented by area (a)), and no heat is rejected. When the isentropic process 1-2r is irreversible, and the work input equals that required for the reversible diabatic process 1-2, an additional amount of rotary shaft work input (7.056) is required to overcome friction. This additional amount of work input must be rejected as heat in order that the process by one of constant entropy.

However, in the actual irreversible adiabatic compression no heat is rejected, and it is for this reason that the actual path is 1-2. Thus, it is the above extra work input of 7.056 Btu, required to overcome friction in the irreversible isentropic case, that causes the path to be shifted from 1-2r to 1-2. The term $(F/J)_{irrev,isen}$ (7.056 Btu in this Section) for the irreversible isentropic compression is called the "irreversible isentropic friction factor."

2. Irreversible adiabatic friction factor.

This is represented by area (c) in Figure 1.4.

When the process 1-2 of Figure 1.4 is reversible and diabatic, the work input, $W_{s,in,rev,1-2} = 118.703$ Btu, and, from Section 1.4, the heat input, $Q_{in,rev,1-2} = h_2 - h_1 - W_{s,in,rev,1-2} = 138.487 - 118.703 = 19.784$ Btu.

In the irreversible adiabatic process 1-2, there is no heat influx, and the work input is increased by 19.784 Btu to $W_{s,in,irrev,1-2} = 138.487$ Btu. The extra work input in this case is that necessary to overcome the friction, $(F/J)_{irrev,adiab} = 19.784$ Btu, represented by area (c) of Section 1.4. It is this term $(F/J)_{irrev,adiab}$ for the irreversible adiabatic compression that is called the "irreversible adiabatic friction factor."

Summarizing:

$$W_{s,in,irrev,1-2} = W_{s,in,rev,1-2r} + (F/J)_{irrev,isen} + (F/J)_{irrev,\ adiab}$$

$$138.487 = 111.647 + 7.056 + 19.784$$

area (a+b+c) = area (a) + area (b) + area (c)

where $(F/J)_{irrev,isen} + (F/J)_{irrev,\ adiab}$ is the double penalty for inefficient adiabatic compressor design.

D. Polytropic exponent for the reversible or irreversible diabatic compression of a perfect gas (Section 1.7)

$$dQ = du + dW$$

$$dQ = c_v dT + \frac{1}{e_c}\ \frac{pdv}{J} \qquad (5)$$

$$dQ = \frac{pdv}{e_e \varphi J} \qquad (6)$$

$$c_v = \frac{1}{k-1}\ \frac{R}{J} \qquad (2)$$

$$dT = (pdv + vdp)/R \qquad (3)$$

Substituting the values for dQ, c_v, and dT from relations (6), (2), and (3) in (5) gives

$$0 = \frac{1}{k-1}\ (pdv + vdp) + (\frac{1}{e_c} - \frac{1}{e_c \varphi})\ pdv;\ 0 = \frac{dp}{p} + \left[1 + \frac{1}{e_c}(k-1)(1-\frac{1}{\varphi})\right]\frac{dv}{v}$$

Let $n = 1 + \frac{1}{e_c}\ (k-1)\ (1 - \frac{1}{\varphi})$; pv^n = a const.

E. Friction factors for the irreversible diabatic gas compression of Section 1.8.

1. Irreversible diabatic friction factor.

This is represented by area (b) in Figure 1.9.

Consider the path 1-2 represents the process for a reversible adiabatic compression (Section 1.8).

$$W_{s,in,rev,1-2} = 111.647 \text{ Btu} \qquad \text{area (a+b)}$$

Now consider the case of an irreversible diabatic compression in which the path 1-2" is followed, and where the rotary shaft work input is equal to the rotary shaft work input for the reversible adiabatic path 1-2; i.e.,

$$W_{s,in,irrev,1-2''} = 111.647 \text{ Btu} \qquad \text{area (a+b)}$$

$$Q_{o,irrev,1-2''} = W_{s,in,irrev,1-2''} - (h_{2''} - h_1)$$

From Section 1.8

$$h_{2''} - h_1 = W_{s,in,rev,1-2''} - Q_{o,rev,1-2''}$$

$$Q_{o,irrev,1-2''} = W_{s,in,irrev,1-2''} - (W_{s,in,rev,1-2''} - Q_{o,rev,1-2''})$$

$$= 111.647 - (104.189 - 19.937)$$

$$= 27.395 \text{ Btu} \qquad \text{area (b+c)}$$

Since $Q_{o,rev,1-2''}$ was assumed to be 19,937 Btu (area (c)), the additional heat that must be rejected in this irreversible case is represented by area (b).

$$W_{s,in,irrev,1-2''} = \int_1^{2''} vdp/J + (F/J)_{irrev,diab}$$

From Section 1.8

$$\int_1^{2''} vdp/J = W_{s,in,rev,1-2''} = 104.189 \text{ Btu} \qquad \text{area (a)}$$

$$\therefore (F/J)_{irrev,\ diab} = 111.647 - 104.189 = 7.458 \text{ Btu} \qquad \text{area (b)}$$

If the diabatic process 1-2" is reversible, the work input is represented by area (a), the heat efflux is 19.937 Btu. When the diabatic process 1-2" is irreversible, and the work input equals that required for the reversible adiabatic process 1-2, an additional amount of rotary shaft work input (represented by area (b)) is required to overcome friction. This additional amount of work must be rejected as heat in order to keep the diabatic compression process.

However, in the actual irreversible diabatic isentropic compression the heat efflux is 19.937 Btu, and it is for this reason that the actual path is 1-2. Thus, it is the above extra work input (b), required to overcome friction in the irreversible diabatic isentropic case, that causes the path to be shifted from 1-2" to 1-2. The term $(F/J)_{irrev,diab}$ (7.458 Btu in this Section) for the irreversible diabatic compression is called the "Irreversible diabatic friction factor."

2. Irreversible diabatic isentropic friction factor

This is represented by area (c) in Figure 1.9.

When the process 1-2 is reversible and adiabatic, the work input, $W_{s,in,rev,1-2}$ = 111.647 Btu, and, from Section 1.8, there is no heat flow.

In the irreversible diabatic isentropic process 1-2, there is heat efflux (represented by area (c)), and the work input increased by 19.937 Btu to $W_{s,in,irrev,1-2}$ = 131.584 Btu. The extra work input in this case is that necessary to overcome the friction, $(F/J)_{irrev,diab,isen}$ = 19.937 Btu, represented by area (c) of Section 1.8. It is this term $(F/J)_{irrev,diab,isen}$ for the irreversible diabatic isentropic compression that is called the "irreversible diabatic isentropic friction factor."

Summarizing:

$$W_{s,in,irrev,1-2} = W_{s,in,rev,1-2''} + (F/J)_{irrev,diab} + (F/J)_{irrev,diab,isen}$$

$$131.584 = 104.189 + 7.458 + 19.937$$

$$\text{area } (a+b+c) = \text{area } (a) + \text{area } (b) + \text{area } (c)$$

where $(F/J)_{irrev,diab} + (F/J)_{irrev,diab,isen}$ is the double penalty for inefficient diabatic compressor design.

F. The intermediate pressure p_i for the reversible adiabatic two-stage reheat gas turbine (Section 1.10).

$$W_{s,o} = W_{s,oI} + W_{s,oII}$$

$$= \frac{k}{k-1} \frac{R}{J} T_1 \left[1 - \left(\frac{p_i}{p_1}\right)^{(k-1)/k} \right] + \frac{k}{k-1} \frac{R}{J} T_3 \left[1 - \left(\frac{p_4}{p_i}\right)^{(k-1)/k} \right]$$

Since $T_3 = T_1$, the equation may be simplified as follows.

$$W_{s,o} = \frac{k}{k-1} \frac{R}{J} T_1 \left[2 - \left(\frac{p_i}{p_1}\right)^{(k-1)/k} - \left(\frac{p_4}{p_i}\right)^{(k-1)/k} \right]$$

Differentiating with respect to p_i and equating to zero.

$$\frac{dW_{s,o}}{dp_i} = 0 = \frac{\left(\frac{k-1}{k}\right)\left(p_i^{-1/k}\right)}{p^{(k-1)/k}} + \left(\frac{k-1}{k}\right)\left(p_4^{(k-1)/k}\right)\left(p_i^{(1-2k)/k}\right)$$

solving for p_i gives

$$p_i = \sqrt{p_1 \, p_4}$$

G. The increase in rotary shaft work for the reversible adiabatic two-stage reheat gas turbine (Section 1.10).

$$(b) = (a+b) - (a)$$

$$= 2c_p (T_1 - T_2) - c_p (T_1 - T_5)$$

$$= 2c_p T_1 \left(1 - \frac{T_2}{T_1}\right) - c_p T_1 \left(1 - \frac{T_5}{T_1} \right)$$

$$= 2c_p T_1 \left[1 - \left(\frac{p_2}{p_1}\right)^{(k-1)/k} \right] - c_p T_1 \left[1 - \left(\frac{p_5}{p_1}\right)^{(k-1)/k} \right]$$

$$= c_p T_1 \left[1 - 2 \left(\frac{p_2}{p_1}\right)^{(k-1)/k} + \left(\frac{p_5}{p_1}\right)^{(k-1)/k} \right] \qquad (7)$$

From

$$p_i = \sqrt{p_1 p_4}\ ;\ \frac{p_i}{p_1} = \frac{p_4}{p_i}$$

$$\left(\frac{p_i}{p_1}\right)^2 = \left(\frac{p_4}{p_i}\right)\left(\frac{p_i}{p_1}\right) = \frac{p_4}{p_1}$$

$$\frac{p_i}{p_1} = \left(\frac{p_4}{p_1}\right)^{1/2}$$

Since $p_2 = p_i$ and $p_5 = p_4$,

$$\therefore\ \frac{p_2}{p_1} = \left(\frac{p_5}{p_1}\right)^{1/2} \qquad (8)$$

Substituting the value for p_2/p_1 from relation (8) in (7) gives

$$(b) = c_p T_1 \left[1 - 2\left(\frac{p_5}{p_1}\right)^{(k-1)/2k} + \left(\frac{p_5}{p_1}\right)^{(k-1)/k} \right]$$

H. The intermediate pressure p_i for the reversible adiabatic two-stage gas compressor (Section 1.12).

$$W_{s,in} = W_{s,in,I} + W_{s,in,II}$$

$$= \frac{k}{k-1}\frac{R}{J}\ T_1 \left[\left(\frac{p_i}{p_1}\right)^{(k-1)/k} - 1 \right] + \frac{k}{k-1}\frac{R}{J} T_3 \left[\left(\frac{p_4}{p_i}\right)^{(k-1)/k} - 1 \right]$$

Since $T_3 = T_1$, the equation may be simplified as follows.

$$W_{s,in} = \frac{k}{k-1}\ \frac{R}{J} T_1 \left[\left(\frac{p_i}{p_1}\right)^{(k-1)/k} + \left(\frac{p_4}{p_i}\right)^{(k-1)/k} - 2 \right]$$

Differentiating with respect to p_i and equating to zero.

$$\frac{dW_{s,in}}{dp_i} = 0 = \frac{\frac{k-1}{k}\left(p_i^{-1/k}\right)}{p_i^{(k-1)/k}} - \left(\frac{k-1}{k}\right)\left(p_4^{(k-1)/k}\right)\left(p_i^{(1-2k)/k}\right)$$

Solving for p_i gives

$$p_i = \sqrt{p_1\ p_4}$$

I. The saving in rotary shaft work input for the reversible adiabatic two-stage gas compressor (Section 1.12).

$$(b) = (a+b+c) - (a+c) = c_p (T_5 - T_1) - 2c_p (T_2 - T_1)$$

$$= c_p T_1 \left(\frac{T_5}{T_1} - 1\right) - 2c_p T_1 \left(\frac{T_2}{T_1} - 1\right)$$

$$= c_p T_1 \left[\left(\frac{p_5}{p_1}\right)^{(k-1)/k} - 1\right] - 2c_p T_1 \left[\left(\frac{p_2}{p_1}\right)^{(k-1)/k} - 1\right]$$

$$= c_p T_1 \left[\left(\frac{p_5}{p_1}\right)^{(k-1)/k} - 2\left(\frac{p_2}{p_1}\right)^{(k-1)/k} + 1\right]$$

$$\frac{p_2}{p_1} = \left(\frac{p_5}{p_1}\right)^{1/2} \qquad \text{(See part G)}$$

$$(b) = c_p T_1 \left[\left(\frac{p_5}{p_1}\right)^{(k-1)/k} - 2\left(\frac{p_5}{p_1}\right)^{(k-1)/2k} + 1\right]$$

The percentage of saving is

$$\text{Saving} = \frac{(b)}{(a+b+c)} = \frac{c_p T_1 \left[\left(\frac{p_5}{p_1}\right)^{(k-1)/k} - 2\left(\frac{p_5}{p_1}\right)^{(k-1)/2k} + 1\right]}{c_p T_1 \left[\left(\frac{p_5}{p_1}\right)^{(k-1)/k} - 1\right]}$$

$$= 1 - \frac{2\left[\left(\frac{p_5}{p_1}\right)^{(k-1)/2k} - 1\right]}{\left(\frac{p_5}{p_1}\right)^{(k-1)/k} - 1}$$

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REPRESENTATION ON THE TEMPERATURE-ENTROPY AND PRESSURE-VOLUME PLANES OF THE FLOW OF ROTARY SHAFT WORK REQUIRED FOR GAS TURBINES, GAS COMPRESSORS AND VAPOR COMPRESSORS

by

HSIEH TIEN SUN

Diploma, Taipei Institute of Technology
Taipei, Taiwan, China, 1959

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In order to encourage the increased use of the temperature-entropy diagram in its relation to refrigeration cycles, the flow of rotary shaft work for several cases is investigated and this work is represented by areas on the temperature-entropy and pressure-volume planes. The conditions studied include : (1) reversible and irreversible, adiabatic and diabatic, and single-stage and two-stage processes in gas turbines and compressors, (2) reversible and irreversible adiabatic, single-stage, vapor compression, (3) single-stage, reversible, adiabatic vapor expansion, and (4) reversible, two-stage, vapor compression.

Numerical analyses of all cases investigated have been made, and the results of these analyses are represented, accurately and to scale, by areas on the temperature-entropy and pressure-volume planes. Comparisons of these areas describe, pictorially, the penalties in rotary shaft work that are the consequence of irreversible processes, and the advantage of two-stage operation over single-stage action.