

ANALYSIS OF REINFORCED CONCRETE FOLDED PLATES
BY THE SLOPE-DEFLECTION METHOD

by

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TABLE OF CONTENTS

SYNOPSIS 1

INTRODUCTION 2

BASIC ASSUMPTION 4

SLOPE DEFLECTION 5

SLAB SYSTEM 6

DEEP BEAM SYSTEM 11

JOINT DEFLECTIONS AND DEEP BEAM ROTATION 17

CONCLUSIONS 19

APPENDIX I - PROBLEMS 22

APPENDIX II - MOMENT AND DEFLECTION COEFFICIENTS 41

APPENDIX III - NOTATION 42

APPENDIX VI - BIBLIOGRAPHY 44

LIST OF FIGURES

FIG. 1.	SIMPLY-SUPPORTED FOLDED PLATE STRUCTURE	5
FIG. 2.	ELEMENTS OF A FOLDED PLATE STRUCTURE	6
FIG. 3.	CONCEPTS OF THE ANALYSIS	7
FIG. 4.	REACTIONS AND MOMENTS ACTING AT HINGED JOINTS OF SLAB SYSTEM ..	7
FIG. 5.	SLOPE DEFLECTION TERMS	10
FIG. 6.	JOINT LOADS AND THEIR COMPONENTS	11
FIG. 7.	FORCES AND STRESSES OF A DEEP BEAM	12
FIG. 8.	FREE BODY DIAGRAM OF A TYPICAL DEEP BEAM	14
FIG. 9.	EQUILIBRIUM OF HORIZONTAL FORCES	15
FIG. 10.	BEAM DISPLACEMENTS	17
FIG. 11.	WILLIOT DIAGRAM - DEFLECTION OF JOINT	19
FIG. 12.	DIMENSIONS AND LOADING OF A SIMPLY-SUPPORTED FOLDED PLATE	22
FIG. 13.	THE RESULTS OF THE ANALYSIS OF THE FOLDED PLATE	35

LIST OF TABLES

TAELE A. ANALYSIS OF A SIMPLY SUPPORTED, ONE SPAN, FOLDED PLATE	33
TAELE B. ANALYSIS OF A CONTINUOUS, TWO SPAN, FOLDED PLATE	38

SYNOPSIS

The slope-deflection formulas are derived for computation of the transverse moments at joints, longitudinal stresses of plates (called deep beams), rotations of deep beams at joints and vertical deflections in folded plate structures. The slope-deflection formulas are written in terms of the change of angle θ , at joints between the deep beams. This change of angle, θ , is a consequence of: deep beam rotation of adjacent slabs caused by unequal settlement of joints (chord rotation) and caused by transverse moments. The structure is treated as a continuous one-way slab that is assumed to be supported by the joints of the deep beam structure. The reactions from these imaginary supports will act as forces on the deep beam structure. The slopes and stresses at the joints must satisfy the continuity of a slab and deep beam system. The variation of the slope deflections (θ) along each joint is adjusted to fit a similar function along the structure. The computation procedure is performed in a tabular form which enables easy checking at all stages. Such a table can also be used for calculations for any folded plate with a similar cross section.

INTRODUCTION

Folded plates have gained increasing popularity because of their economy and appearance. Simultaneously, analysis of folded plates has been developed by many authors.¹ Most of them have tried to approach the real solution and to simplify the procedures of analysis.

In general, the behavior of a folded plate is divided into interdependent behaviors in the transverse and longitudinal directions. The structure, in the transverse direction, is treated as a continuous slab. The transverse strips are assumed to be supported at the joints on rigid or flexible supports. The longitudinal plates are treated as deep beams with forces acting on the joints of the plates.

The method of analysis in this report focuses on the slope-deflection method. The slope-deflection means the change of angle, θ , at joints in the transverse direction. Two examples, a one-span and a two-span structure, will be presented. The basic assumptions and properties of folded plates will be demonstrated in the slope-deflection method.

The object of folded plate analysis is to solve for the longitudinal stresses, transverse moments, and the deflections of the structure. The continuous one-way slab is assumed to be rigidly supported by the joints of the deep beam structures [Fig. 3(a)]. The one-way slab is analyzed as an

¹Arnim Defries-Skene and A. C. Scordelis. "Direct Stiffness Solution for Folded Plates." ASCE Vol. 90, No. ST4, August, 1964. p. 15-47. A. C. Scordelis, "Matrix Formulation of the Folded Plate Equations." ASCE Vol. 86, No. ST10, October, 1960. p. 1-22. David Yitzhaki and Max Reiss. "Analysis of Folded Plates." ASCE Vol. 88, No. ST5, October, 1962. p. 107-142. Portland Cement Association. "Analysis of Folded Plates." 1958. Portland Cement Association. "Direct Solution of Folded Plate Concrete Roofs." 1960.

ordinary continuous beam, by moment-distribution, for finding the moments and reactions at joints. The complete stress solution is obtained by allowing the longitudinal joints to deflect. This is achieved by applying the joint support reactions to the combined slab and deep beam system, which is then free to deflect [Fig. 3(b)]. These reactions are considered to be acting on the hinged joints. The joint moments then must provide along the joints to secure the continuity of the slab in the transverse direction. The values of the joint moments are solved by the slope-deflection equations (page 8). The deep beam structure will be considered as one which is supported by the end diaphragms. The method of general beam theory will be used in solving for the fiber stresses and shear stresses of the deep beams. An additional computation required in this problem is that of the shearing stress exerted by the adjacent deep beams.

The slope deflections of each joint in the longitudinal direction can be expressed similarly along the structure, that is, the corresponding slope deflections at every transverse strip will be proportional to the maximum slope deflections of the joints in the longitudinal direction.

The final values of the stresses and deflections are found by combining the effects of the hinge reactions and joint moments.

Notation. - The letter symbols adopted for use in this report are defined where they first appear and are arranged alphabetically in Appendix III.

BASIC ASSUMPTIONS AND PROPERTIES

Folded plates are treated as combined slab and beam structures. The fundamental properties and the assumptions of the folded plates are stated below to clarify the application of the analysis.

(a) The material is elastic, isotropic and homogeneous.

(b) The structure is formed by rectangular plates monolithically connected along the joints and supported by transverse diaphragms.

(c) Slab action is defined by the behavior of transverse one-way slabs, spanning between longitudinal joints.

(d) Longitudinal stresses developed in each plate due to longitudinal plate action can be calculated by the elementary beam theory. Therefore, the longitudinal stress varies linearly over the depth of each plate. The shearing strain and the lateral strain of the plates are neglected.

(e) Stiffness of the deep beam structures are caused by the resistance of the plates to forces acting in their planes.

(f) Supporting diaphragms are infinitely stiff parallel to their own plane, but perfectly flexible normal to their plane.

(g) The length to height ratio, L/h , of the deep beams is at least 2 (Fig. 1).

SLOPE DEFLECTION

The general shape of a one-span folded plate structure is shown in Fig. 1. The structure in the transverse direction will be solved as a one-way slab for the moments and reactions at joints, caused by the external loading. The reactions solved from the one-way slab will be considered as the forces acting upon the deep beam structures. The elements of a folded plate are shown in Fig. 2.

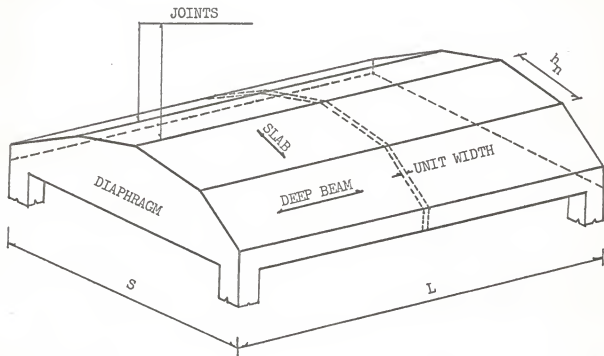


FIG. 1. - SIMPLY-SUPPORTED FOLDED PLATE STRUCTURE

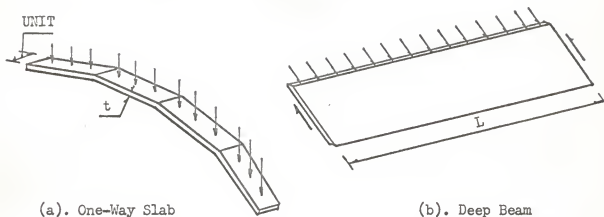


FIG. 2. - ELEMENTS OF A FOLDED PLATE STRUCTURE

A SLAB SYSTEM

A slab strip of unit width [Fig. 2(a)] with the external load is analyzed as a continuous structure. The joints of the deep beams are assumed to be supported by imaginary supports along the longitudinal direction so that the external load is carried entirely by the one-way slab system. The rigidly supported joint moments may be readily obtained by moment distribution and the joint reactions may then be found by equations of static [Fig. 3(a)]. The reactions thus calculated will be applied as loads to the joints of the deep beam structure. For applying these reactions to the deep beams, the joints are assumed to be hinged and the reactions and joint moments are applied separately to them (Fig. 4). The reactions are split into components in the plane of the deep beam members. The joints will be displaced under these reactions. The structure is monolithically connected along the joints so that there are unknown moments at the joints to maintain the continuity of the structure. Solving for these moments, unit moments are applied at each joint for finding the influence coefficients of the rotations at each joint to satisfy the slope deflection equations of all joints.

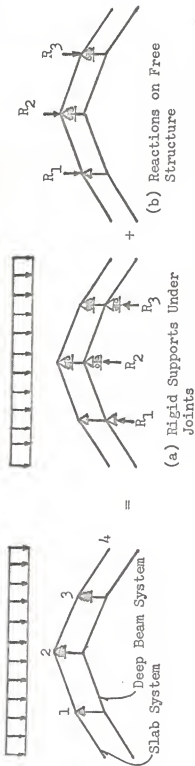


FIG. 3. CONCEPT OF THE ANALYSIS

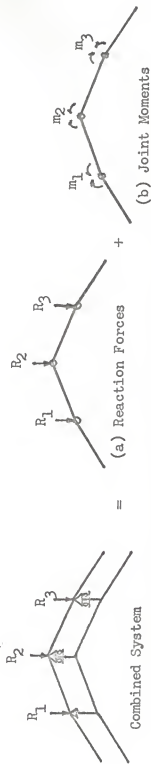


FIG. 4. REACTIONS AND MOMENTS ACTING AT HINGED JOINTS OF SLAB SYSTEM

Assume that m_1 , m_2 , and m_3 are unknown moments at joint 1, 2, and 3. The rotations of the slab structure due to external loads, and joint moments are:

$$\theta_1 = m_1 \theta_{11} + m_2 \theta_{12} + m_3 \theta_{13} + \theta_{10} \text{ ----- (1a)}$$

$$\theta_2 = m_1 \theta_{21} + m_2 \theta_{22} + m_3 \theta_{23} + \theta_{20} \text{ ----- (1b)}$$

$$\theta_3 = m_1 \theta_{31} + m_2 \theta_{32} + m_3 \theta_{33} + \theta_{30} \text{ ----- (1c)}$$

where θ_1 , θ_2 , and θ_3 denote the angle changes at joints 1, 2, and 3 due to loads and moments acting on the structure. Angles θ_{10} , θ_{20} , and θ_{30} denote the angle change at joints 1, 2, and 3 due to external loads and hinged supports. Angles θ_{11} , θ_{22} , and θ_{33} denote the angle changes at joints 1, 2, and 3 due to a unit moment acting at the corresponding joints. Angles θ_{21} and θ_{31} denote the angle changes at joints 2 and 3 due to a unit moment acting at joint 1. Similar notation applies for angles θ_{12} , θ_{32} , θ_{13} , and θ_{23} .

Since the slab is a continuous structure,

$$\theta_1 = \theta_2 = \theta_3 = 0 \text{ ----- (2)}$$

Equations (1) are subject to the condition of Eq. (2) so we can solve Eqs. (1) by using a successive approximation method such as a matrix form using a digital computer. From moments m_1 , m_2 , and m_3 which are found by solving Eqs. (1), it is easy to solve for the stresses and deflections at the joints of the structure due to these moments by proportion of the assumed joint moments to the obtained moments.

The variation of the slope deflections along the joint can be expressed in the form $\theta_m(f(x))$, in which θ_m is the maximum slope deflection, and $f(x)$

is the function defining the curve of variation in the longitudinal direction. The variation of the joint moments will be chosen proportional to the same $f(x)$, that is, in form $m(f(x))$.

The reactions which are obtained from the slab system will apply as the loads to the beam system. The form of the deflection curve of a folded plate due to these reactions depends mainly on its support conditions. For the simply supported structure, for example, when the reactions have the same longitudinal distribution, the deflection curve is nearly a sine curve² (other cases are shown in Appendix II). For nonharmonic loading, the difference between the distribution of the load and the deflection curve may be overcome by introducing extra-harmonic joint loads, such as a form of Fourier sine series.³ The displacement of the structure computed from the longitudinal loading must be compatible with the displacement obtained from the transverse loading. Several important load and support conditions are shown in Appendix II.

The slope deflection terms at a joint, as previously defined, are illustrated in Fig. 5. It will be noted that:

- (1) Part of the rotation of joint n , θ'_n , is due to the unequal deflections of joints in adjacent slabs.
- (2) Part of the rotation of joint n , θ''_n , is due to the joint moments.

²David Yitzhaki and Max Reiss. "Analysis of Folded Plates." ASCE Vol. 88, No. ST5, October, 1962.

³"Direct Solution of Folded Plate Concrete Roofs," Portland Cement Association. 1960. The reactions of simply-supported folded plates are approximated by a Fourier Series in form of

$$W_1 = \frac{4W}{\pi} \left(\sin \frac{\pi x}{L} + \frac{1}{3} \sin \frac{3\pi x}{L} + \frac{1}{5} \sin \frac{5\pi x}{L} + \dots \right)$$

Where W is the uniform load of the reactions; W_1 is the sum of partial sinusoidal loads. For the design, only the first term of the Fourier Series needs to be used.

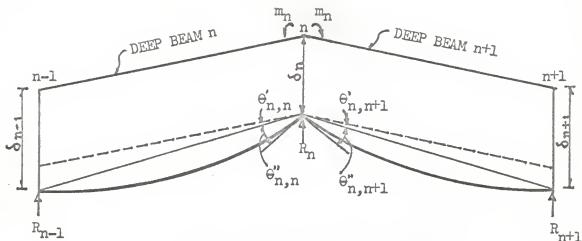


FIG. 5. SLOPE DEFLECTION TERMS

The calculation of these two cases is shown in the following computations (reference to Fig. 5):

$$\theta'_n = \theta'_{n,n} + \theta'_{n,n+1} = \frac{\delta_{n-1} - \delta_n}{S_n} - \frac{\delta_n - \delta_{n+1}}{S_{n+1}} \quad (3)$$

$$\theta''_{n-1,n} = \frac{h_n}{6EI_n} m_n \quad (4a)$$

$$\theta''_{n,n} = \frac{h_n}{3EI_n} m_n \quad (4b)$$

$$\theta''_{n,n+1} = \frac{h_n}{3EI_{n+1}} m_n \quad (4c)$$

$$\theta''_{n+1,n+1} = \frac{h_n}{6EI_{n+1}} m_n \quad (4d)$$

Part of the rotation θ'' caused by the bending of the slabs by the external load (loading $m = 0$) will be separated from this analysis.

DEEP BEAM SYSTEM

The deep beams are supported at the ends by the diaphragms and the joint loads which are the reactions of the one-way slab acts along the joints of the beams. The load function is assumed similar along all joints. Loads that do not vary similarly can be resolved into similarly varying components and each of the components analyzed separately. The joint loads, R , can be resolved into components parallel to the adjacent deep beams. The joint loads and their components are shown in Fig. 6.

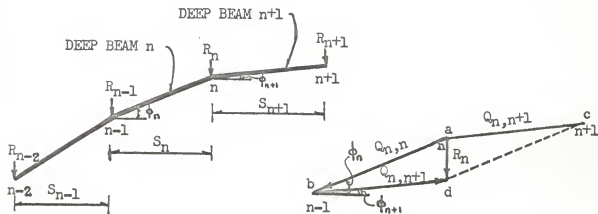


FIG. 6. JOINT LOADS AND THEIR COMPONENTS.

Let $Q_{n,n}$ and $Q_{n,n+1}$ be the components of the load R_n at joint n acting on deep beams n and $n+1$. From the forces of polygon $a-b-c-d$ in Fig. 6, it is seen that

$$R_n = (Q_{n,n}) \sin \phi_n - (Q_{n,n+1}) \sin \phi_{n+1} \quad (5)$$

$$\text{but } (Q_{n,n}) \cos \phi_n = (Q_{n,n+1}) \cos \phi_{n+1} = H_n \quad (6)$$

$$\therefore R_n = H_n (\tan \phi_n - \tan \phi_{n+1}) \text{ ----- (7)}$$

Let $\tan \phi_n - \tan \phi_{n+1} = d_n$ so that

$$H_n = R_n / d_n \text{ ----- (8)}$$

$$Q_{n,n} = R_n / (d_n \cos \phi_n) \text{ ----- (9)}$$

$$Q_{n,n+1} = R_n / (d_n \cos \phi_{n+1})$$

where ϕ_n is the inclined angle of the deep beam n (Fig. 6).

The forces act on the joints of deep beam n must be equal to the sum of $Q_{n,n}$ and $Q_{n-1,n}$. If the resultant of $Q_{n,n}$ and $Q_{n-1,n}$ equals P_n , then

$$P_n = Q_{n,n} - Q_{n-1,n} = (H_n - H_{n-1}) / \cos \phi_n \text{ ----- (10)}$$

The variation of the P -forces is similar to that of the R -loads. The free body of a deep beam is shown in Fig. 7.

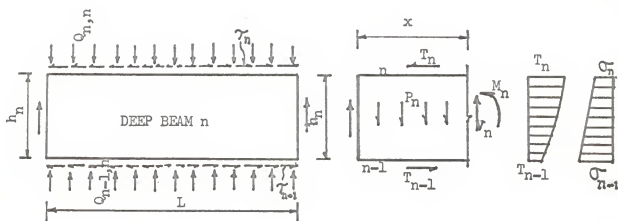


FIG. 7. FORCES AND STRESSES OF A DEEP BEAM.

From Fig. 7 it is easy to see that the deep beam analysis needs some modification of the existing shearing stresses at the edges and ends of the beams. The principle of superposition will be applied to the analysis of the deep beam structure. First, the free edges of a beam are analyzed. Second, the shearing stresses acting on the edges are considered. The fiber stress (σ) and shear stress (T) are calculated in the following steps:

Free Edge. —

$$\sigma_{n-1,n} = \frac{M_n}{Z_n} \text{-----} \quad (11a)$$

$$\sigma_{n,n} = -\frac{M_n}{Z_n} \text{-----} \quad (11b)$$

$$\sigma_{n,n+1} = \frac{M_{n+1}}{Z_{n+1}} \text{-----} \quad (11c)$$

Where Z_n is the section modulus of deep beam n .

Fiber Stresses. — The fiber stresses of a beam are solved by the general beam theory. Then the shear stresses along the joints of adjacent deep beams balance the unequal fiber stresses at joints which were solved by the condition of free edges. The relationship of these stresses is shown in Fig. 8.

The formulas obtained by the elementary beam theory⁴ are

$$\sigma_n = \frac{-M_n}{Z_n} + \frac{4T_n}{A_n} + \frac{2T_{n-1}}{A_n} = \sigma_{n,n} + \frac{4T_n}{A_n} + \frac{2T_{n-1}}{A_n} \text{-----} \quad (12a)$$

⁴"Matrix Formulation of the Folded Plate Equations" by A. C. Scordelis, ASCE, Vol. 86, No. ST 10, October, 1960, page 6. "Analysis of Folded Plates" by David Yitzhaki and Max Reiss, ASCE, Vol. 88, No. ST 5, October, 1962, page 117. "Advanced Reinforced Concrete" by Clarence W. Dunham, McGraw-Hill Book Company. 1964, page 329.

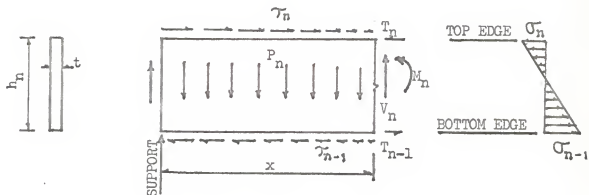


FIG. 8. FREE BODY DIAGRAM OF A TYPICAL DEEP BEAM.

$$\sigma_n = \sigma_{n,n+1} - \frac{4T_n}{A_{n+1}} - \frac{2T_{n+1}}{A_{n+1}} \quad (12b)$$

Where A_n is section area of deep beam n .

The equations expressed above have one unknown quantity, T , the shearing stress.—If we study these equations, we will find that the influence of the shearing stress on one edge of a deep beam will carry $-\frac{1}{2}$ of the value to the far edge of the deep beam. This can be expressed by the Hardy Cross method of moment-distribution. The first two terms when combined and solved in terms of the free edge fiber stress show that

$$T_n = (\sigma_{n,n+1} - \sigma_{n,n}) \frac{A_n A_{n+1}}{4(A_n + A_{n+1})} \quad (13)$$

$$\begin{aligned}\sigma_n &= \sigma_{n,n} + (\sigma_{n,n+1} - \sigma_{n,n}) \frac{A_{n+1}}{A_n + A_{n+1}} \\ &= \sigma_{n,n+1} - (\sigma_{n,n+1} - \sigma_{n,n}) \frac{A_n}{A_n + A_{n+1}}\end{aligned}\quad (14)$$

It is obvious from the above equation for σ_n that the stress-distribution factors are in inverse proportion to the area of the deep beams.

$$r_{n,n} = \frac{A_{n+1}}{A_n + A_{n+1}} \quad (15a)$$

$$r_{n,n+1} = \frac{A_n}{A_n + A_{n+1}} \quad (15b)$$

where r is the distribution factor. After stresses at joints are distributed, half of the distributed stresses must be carried over to the adjacent joints. The process continues until the required precision is obtained.

Shearing Stresses.—The equilibrium of the horizontal forces of the deep beams is shown in Fig. 9.

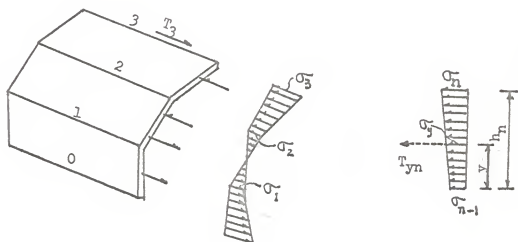


FIG. 9. EQUILIBRIUM OF HORIZONTAL FORCES.

The resultants of the shearing stresses at a section of a folded plate can be calculated by equilibrium of the horizontal forces (Fig. 8): that is,

$$T = \int \sigma \, dA \quad \text{-----} \quad (16)$$

The T-values at the joints can be obtained successively by semi-graphic integration; these become

$$T_1 = \frac{\sigma_0 + \sigma_1}{2} A_1 \quad \text{-----} \quad (17a)$$

$$T_2 = T_1 + \frac{\sigma_1 + \sigma_2}{2} A_2 \quad \text{-----} \quad (17b)$$

$$T_3 = T_2 + \frac{\sigma_2 + \sigma_3}{2} A_3 \quad \text{-----} \quad (17c)$$

At an intermediate point at a distance y on plate n from joint n-1,

$$T_{yn} = T_{n-1} + \left(\frac{\sigma_{n-1} + \sigma_y}{2} \right) t y \quad \text{-----} \quad (18)$$

Where t is the thickness of beam n.

From Fig. 7 the resultants of shearing stresses acting along the edges are

$$T_{n-1} = \int_0^x \tau_{n-1} t \, dx \quad \text{-----} \quad (19a)$$

and

$$T_n = \int_0^x \tau_n t \, dx \quad \text{-----} \quad (19b)$$

Where τ_n is the unit shearing stress of the edge n. The shear stresses can be expressed as

$$\tau_x = \frac{dT}{t dx} \quad \text{-----} \quad (20)$$

Where dx is a small distance in the longitudinal direction. From the equilibrium of the horizontal forces on the deep beam, the shear stresses are functions of fiber stresses so that maximum shear stress occurs where moment is maximum. It is easier to show the shearing stress in terms of the maximum value. This shearing stress is

$$\tau_x = \frac{T_{\max}}{M_{\max}} \frac{dM_x}{dx} = \frac{T_{\max}}{M_{\max}} \frac{V_x}{t} \quad (21)$$

Where dM_x is the change in bending moment between two cross sections a distance dx in the longitudinal direction.

JOINT DEFLECTIONS AND DEEP BEAM ROTATIONS

The vertical deflection δ at each interior joint produced by a given set of R loads can be expressed in terms of the deep beam displacements A of the two deep beams adjacent to the joint. A Williot diagram for a joint displacement is used to find the vertical deflection at that joint. The deep beam displacements are shown in Fig. 10.

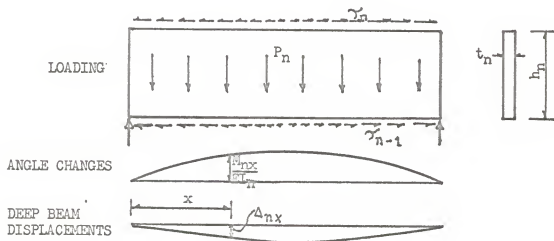


FIG. 10. DEEP BEAM DISPLACEMENTS.

The deflection curve of a deep beam can be expressed by the differential equation $EI \frac{d^2}{dx^2} = M$. By replacing $\frac{M}{I} = \frac{\sigma}{y}$ by $(\frac{\sigma_{n-1} - \sigma_n}{h_n})$, the differential equation can be expressed in terms of fiber stresses which were solved in the previous section (pages 13-15) at joint n as

$$\frac{d^2 \Delta_n}{dx^2} = (\sigma_{n-1} - \sigma_n) / E h_n \text{ ----- (22)}$$

Integration of this equation will give the displacement curve of the deep beam. Also, the deflection curve can be expressed in terms of the angle changes shown in Fig. 10, as $\Delta_{nx} = (\frac{M_{nx}}{EI_n})(\frac{1}{C_v})$, in which C_v is a constant that depends on the load variation and support conditions. The significant deflection of a folded plate is usually considered to be the maximum deflection. The maximum deflection $(\Delta_n)_{max}$ can be expressed as

$$(\Delta_n)_{max} = \frac{\sigma_{n-1} - \sigma_n}{E h_n} \frac{L^2}{C_v} \text{ ----- (23)}$$

Several common cases of C_v are listed on Appendix II. The Eq. 23 shows that the maximum deflection can be expressed in terms of the difference of the maximum edge stresses at the point of maximum moment.

A Williot diagram of a joint displacement is shown in Fig. 11. The displacement Δ_{n+1} of deep beam $n+1$, is parallel to the deep beam itself. On the Williot diagram, the new location of the joint n is denoted as n' . The vertical deflection of joint n , δ_n , can be found by using triangular relations.

$$\delta_n (\tan \phi_n + \tan \phi_{n+1}) = \Delta_n / \cos \phi_n - \Delta_{n+1} / \cos \phi_{n+1}$$

$$\text{but } \cos \phi_n = \frac{S_n}{h_n}, \cos \phi_{n+1} = \frac{S_{n+1}}{h_{n+1}}$$

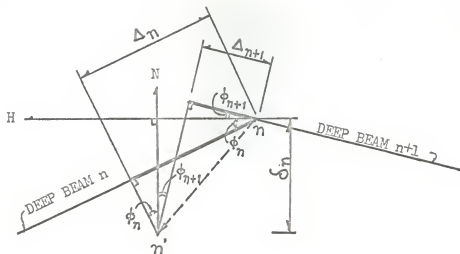


FIG. 11. WILLIOT DIAGRAM FOR THE DEFLECTION OF A JOINT.

$$\delta_n (\tan \phi_n + \tan \phi_{n+1}) = \left[\left(\frac{\sigma_{n-1} - \sigma_n}{S_n} \right) - \left(\frac{\sigma_n - \sigma_{n+1}}{S_{n+1}} \right) \right] \frac{L^2}{EC_v}$$

$$\text{or } \delta_n = \left[\left(\frac{\sigma_{n-1} - \sigma_n}{S_n} \right) - \left(\frac{\sigma_n - \sigma_{n+1}}{S_{n+1}} \right) \right] \frac{L^2}{EC_v} / (\tan \phi_n + \tan \phi_{n+1}) \quad (24)$$

The rotations of the joints due to the deflections are shown in Fig. 5.

After the joint moments are found by using Eqs. 1, actual fiber stresses σ , vertical deflections δ , and joint reactions can be found by using the superposition equations. These equations are shown below.

$$\sigma_i = \sigma_{i0} + \sum m_k \sigma_{ij} \quad (25a)$$

$$\delta_i = \delta_{i0} + \sum m_k \delta_{ij} \quad (25b)$$

$$R_i = R_{i0} + \sum m_k R_{ij} \quad (25c)$$

Where i , j and k are the subscripts of the number of the joints.

CONCLUSION

As shown in the two design solutions in Appendix I, the slope-deflection method used in this report is a practical method. The joints of the deep beams are assumed to be rigidly supported so that the external load is carried entirely by the one-way slab system. The complete stress solution is obtained by allowing the longitudinal joints to deflect. The slope deflections of the joints are introduced from these deflections. To eliminate these slope deflections the transverse moments are introduced at each joint of the structure to satisfy the slope deflection equations. The procedures of the analysis have been presented for determination of the longitudinal stresses, transverse moments and vertical deflections in the case of a simply-supported folded plate structure. The method can be applied, however, with proper modifications to cases involving other end conditions. The deflection curves of all joints in the longitudinal direction (due to the reactions of the one-way slab on unyielding supports) may be assumed in similar form along the structure. The computations are functions only of the geometry and dimensions of the structure cross section and modulus of elasticity. Thus, they need not be recomputed for each different loading condition. The simplifications that have been made arise from the use of forces, rotations and displacements; these concepts emerge as a most powerful aid of structural analysis.

The method is practical because of the convenience of performing the analysis in a table that reflects all calculations and enables easy checking in all stages. Such a table can also be used for calculations for any plate with a similar cross section. Design tables prepared for typical cross sections will greatly reduce the required computational work.

ACKNOWLEDGMENT

The writer wishes to express his sincere gratitude to Professor V. H. Rosebraugh for his kind guidance and valuable suggestions in completing the report in the most systematic manner.

APPENDIX I. PROBLEMS

To illustrate the slope deflection method, two simple problems are solved.

Example 1: The dimensions and loadings are stated as follows: single span; $L = 60$ ft.; plate thickness $t = 4$ in. for all plates; loading, live load 40 psf., dead load is the weight of the plate itself. The shape and all dimensions of the folded plate are shown in Fig. 12. The function of joint loads in the longitudinal direction will be considered in two cases, (a) uniformly distributed joint loads, (b) sine curve loads.

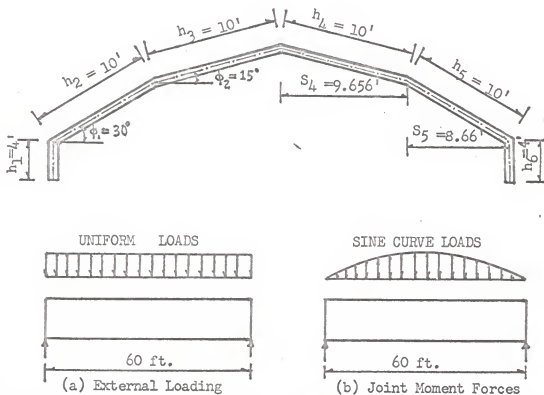


FIG. 12. DIMENSIONS AND LOADING OF A SIMPLY-SUPPORTED FOLDED PLATE.

In view of the symmetry of the structure, only half of the folded plate will be dealt with in the calculations. As indicated (page 4), the torsional resistance is neglected in this analysis. The analysis of this problem will follow the steps of previous discussions and the results of each step will lead to the tabulation on page 33.

SLAB SYSTEM

Loading:

Live load = 40.0 psf

Dead load

Plate 1 and 6 — $150 \times \frac{4}{12} \times 4 = 200.0$ lb. per ft.

Plate 2, 3, 4 and 5 — $150 \times \frac{4}{12} = 50.0$ psf

Moment-distribution Factors: Joints 1 and 5 are considered to be simply supported. For joints 2 and 4 distribution factors are

$$K_{21} = \frac{3/4}{1 + 3/4} = 3/7$$

$$K_{23} = \frac{1}{1 + 3/4} = 4/7$$

For joint 3 distribution factors are $K_{32} = K_{34} = 1/2$

The fixed-end moments are

$$M_{21}^F = - \frac{(90 \times \cos 30^\circ) \times 10^2}{8} = - 975 \text{ ft.-lb.}$$

$$M_{23}^F = \frac{(90 \times \cos 15^\circ) \times 10^2}{12} = 725 \text{ ft.-lb.}$$

The moment distribution computations are as follows:

	2		3		4																												
	<table style="width: 100%; border-collapse: collapse;"> <tr><td style="border: 1px solid black; padding: 2px;">3/7</td><td style="border: 1px solid black; padding: 2px;">4/7</td></tr> <tr><td style="border: 1px solid black; padding: 2px;">-975</td><td style="border: 1px solid black; padding: 2px;">725</td></tr> <tr><td style="border: 1px solid black; padding: 2px;">107</td><td style="border: 1px solid black; padding: 2px;">143</td></tr> <tr><td style="border: 1px solid black; padding: 2px;">-868</td><td style="border: 1px solid black; padding: 2px;">868</td></tr> </table>	3/7	4/7	-975	725	107	143	-868	868	←	½	→	<table style="width: 100%; border-collapse: collapse;"> <tr><td style="border: 1px solid black; padding: 2px;">1/2</td><td style="border: 1px solid black; padding: 2px;">1/2</td></tr> <tr><td style="border: 1px solid black; padding: 2px;">-725</td><td style="border: 1px solid black; padding: 2px;">725</td></tr> <tr><td style="border: 1px solid black; padding: 2px;">72</td><td style="border: 1px solid black; padding: 2px;">-72</td></tr> <tr><td style="border: 1px solid black; padding: 2px;">-653</td><td style="border: 1px solid black; padding: 2px;">653</td></tr> </table>	1/2	1/2	-725	725	72	-72	-653	653	←	½	→	<table style="width: 100%; border-collapse: collapse;"> <tr><td style="border: 1px solid black; padding: 2px;">4/7</td><td style="border: 1px solid black; padding: 2px;">3/7</td></tr> <tr><td style="border: 1px solid black; padding: 2px;">-725</td><td style="border: 1px solid black; padding: 2px;">975</td></tr> <tr><td style="border: 1px solid black; padding: 2px;">-143</td><td style="border: 1px solid black; padding: 2px;">107</td></tr> <tr><td style="border: 1px solid black; padding: 2px;">-868</td><td style="border: 1px solid black; padding: 2px;">868</td></tr> </table>	4/7	3/7	-725	975	-143	107	-868	868
3/7	4/7																																
-975	725																																
107	143																																
-868	868																																
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-725	725																																
72	-72																																
-653	653																																
4/7	3/7																																
-725	975																																
-143	107																																
-868	868																																

Reactions at joints:

$$R_1^1 = 200 + \frac{90 \times 10}{2} - \frac{868}{8.66} = 550 \text{ lb. per ft.}$$

$$R_2^1 = 900 + \frac{868 - 653}{9.656} = 922 \text{ lb. per ft.}$$

$$R_3^1 = 900 - 2 \times \frac{868 - 653}{9.656} = 856 \text{ lb. per ft.}$$

$$R_4^1 = R_2^1 = 922 \text{ lb. per ft.}$$

$$R_5^1 = R_3^1 = 856 \text{ lb. per ft.}$$

The joint moments due to external loading are assumed to be zero. Now we consider the joint moments by the superposition method. The joint moment is applied separately at each joint for finding the reaction at each joint.

The moments are as follows:

(1) at joints 1 and 5 the moments are zero.

(2) at joints 2 and 4, it is assumed that $m_2 = m_4 = 1000 \frac{\sin \frac{\pi x}{L}}{L}$ (lb).

The reactions due to moments m_2 and m_4 are

$$R_1^2 = \frac{1000}{8.66} = 115 \text{ lb. per ft.}$$

$$R_2^2 = -1000 \left(\frac{1}{8.66} + \frac{1}{9.656} \right) = -219 \text{ lb. per ft.}$$

$$R_3^2 = 2 \left(\frac{1000}{9.656} \right) = 208 \text{ lb. per ft.}$$

$$R_5^2 = R_1^2$$

$$R_4^2 = R_2^2$$

(3) at joint 3, it is assumed that $m_3 = 1000 \frac{\sin \pi x}{L}$ lb.

$$R_1^3 = R_5^3 = 0$$

$$R_2^3 = R_5^3 = \frac{1000}{9.656} = 104 \text{ lb. per ft.}$$

$$R_3^3 = -2 \left(\frac{1000}{9.656} \right) = -208 \text{ lb. per ft.}$$

The results obtained by solution of the slab system will be applied to the deep beam system.

DEEP BEAM SYSTEM

H: The components of reaction R, are computed by using equation (8) from page 12.

$$\tan 30^\circ = 0.5774, \quad \tan 15^\circ = 0.1763$$

(1) due to R^1

$$H_1 = 0$$

$$H_2 = 922 / (0.5774 - 0.1763) = 2305 \text{ lb. per ft.}$$

$$H_3 = 856 / (0.1763 + 0.1763) = 2428 \text{ lb. per ft.}$$

(2) due to R^2

$$H_1 = 0$$

$$H_2 = -219 / 0.4011 = -546 \text{ lb. per ft.}$$

$$H_3 = 208 / 0.3526 = 580 \text{ lb. per ft.}$$

(3) due to R^3

$$H_1 = 0$$

$$H_2 = 104/0.4011 = 259 \text{ lb. per ft.}$$

$$H_3 = -208/0.3526 = -580 \text{ lb. per ft.}$$

P: The loadings of the plate, using Eqs. (10) from page 12 with $\cos 30^\circ = 0.8660$ and $\cos 15^\circ = 0.9656$, are:

(1) due to H in case (1)

$$P_1 = R_1^1 = 550 \text{ lb. per ft.}$$

$$P_2 = 2305/0.866 = 2660 \text{ lb. per ft.}$$

$$P_3 = (2428 - 2305)/0.9656 = 127 \text{ lb. per ft.}$$

$$P_4 = P_2$$

$$P_5 = P_1$$

(2) due to H in case (2)

$$P_1 = R_1^2 = 115 \text{ lb. per ft.}$$

$$P_2 = -546/0.866 = -630 \text{ lb. per ft.}$$

$$P_3 = (580 + 546)/0.9656 = 1165 \text{ lb. per ft.}$$

(3) due to H in case (3)

$$P_1 = 0$$

$$P_2 = 259/0.866 = 299 \text{ lb. per ft.}$$

$$P_3 = (-580 - 259)/0.9656 = -868 \text{ lb. per ft.}$$

σ : (a) Fiber stresses at free edges are computed by using equation (11) from page 13; (b) Combine the shear stresses at edges by using equations (14) from page 14.

$$(a) (1) \sigma_{0,1} = \frac{6 \times 550 \times (60)^2}{8 \times (4^2 \times 4/12)} = 278,200 \text{ psf or } 278.2 \text{ ksf}$$

$$\sigma_{1,2} = \frac{6 \times 2660 \times (60)^2}{8 \times (10^2 \times 4/12)} = 215,400 \text{ psf or } 215.4 \text{ ksf}$$

$$\sigma_{2,3} = \frac{6 \times 127 \times (60)^2}{8 \times (10^2 \times 4/12)} = 10,300 \text{ psf or } 10.3 \text{ ksf}$$

$$(2) \sigma_{0,1} = -\sigma_{1,0} = \frac{115}{550} \times 278.2 \times \frac{8.00}{9.87} = 47.1 \text{ ksf}$$

$$\sigma_{1,2} = -\sigma_{2,1} = -\frac{630}{2660} \times 215.4 \times \frac{8.00}{9.87} = -41.3 \text{ ksf}$$

$$\sigma_{2,3} = -\sigma_{3,2} = \frac{1165}{127} \times 10.3 \times \frac{8.00}{9.87} = 76.5 \text{ ksf}$$

$$(3) \sigma_{0,1} = 0$$

$$\sigma_{1,2} = \frac{299}{2660} \times 215.4 \times \frac{8.00}{9.87} = 19.6 \text{ ksf}$$

$$\sigma_{2,3} = -\frac{868}{127} \times 10.3 \times \frac{8.00}{9.87} = -57.0 \text{ ksf}$$

(b) (1) Find the distribution factors by Eq. (15) from page 15.

$$\text{joint 1, } r_{1,0} = \frac{40/12}{(16/12 + (40/12))} = 0.714$$

$$r_{1,1} = \frac{16/12}{(16/12 + (40/12))} = 0.286$$

(2) Stress-distribution by using Eq. (14) from page 15.

0	$\frac{1}{2}$	1	$\frac{1}{2}$	2	$\frac{1}{2}$	3
	0.714	0.286		0.500	0.500	
Case 1						
278.2	-278.2	215.2		-215.2	10.3	-10.3
-176.3	—	—	40.0	70.5	—	—
				—		40.0
<u>14.3</u>	—	—		- 5.7		
116.2	-278.2	175.4		-146.1	10.3	29.7
	<u>323.8</u>	<u>-129.8</u>		<u>78.2</u>	<u>-78.2</u>	
	45.6	45.6		-67.9	-67.9	
Case 2						
47.1	-47.1	-41.3		41.3	76.5	-76.5
- 2.1	—	—	8.9	0.8	—	—
				—		8.9
<u>3.2</u>	—	—		- 1.4		
48.2	-47.1	-50.2		40.9	76.5	-67.6
	<u>- 2.2</u>	<u>0.9</u>		<u>17.8</u>	<u>-17.8</u>	
	-49.3	-49.3		58.7	58.7	
Case 3						
0	0	19.6		-19.6	-57.0	57.0
- 7.0	—	—	10.4	2.8	—	—
				—		-10.4
<u>- 3.7</u>	—	—		1.4		
-10.7	0	30.0		-15.4	-57.0	46.6
	<u>21.4</u>	<u>- 8.6</u>		<u>-20.8</u>	<u>20.8</u>	
	21.4	21.4		-36.2	-36.2	

Elihu Geer's a -method⁵ is used in shortening the stress-distribution calculation; $a = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times 0.286 = 0.036$. After stress-distribution calculations have been completed, the joint stresses of adjacent plates agree with each other due to this balancing procedure. The assumed moments at

⁵Civil Engineering. January 1951, p. 53 by Elihu Geer.

at joints 2, 3 and 4 can be found from Eqs. (1) for zero rotation at the joints. The rotations of the joints are explained on pages 10 and 11, giving Eqs. (3) and (4) which show that the deflections of the joints in adjacent slabs have to be found before the joint rotations can be obtained.

JOINT DEFLECTIONS

Use Eqs. (23) and (24) from pages 18 and 19 for solving the vertical deflections of joints. The coefficient of C_v is given on Appendix II, and is

$$C_v = 9.6 \text{ for uniform load.}$$

$$= 9.87 \text{ for sine curve load.}$$

$$(1) \delta_1 = \Delta_1 = \frac{116.2}{E k} = \frac{45.6}{C_v} \frac{L^2}{EC_v} = 17.65 \left(\frac{L^2}{EC_v} \right)$$

$$\delta_2 = \frac{45.6 + 67.9}{8.66} - \frac{67.9 + 29.7}{9.656} \frac{1}{0.5774 + 0.1763} \left(\frac{L^2}{EC_v} \right) = 30.8 \times \left(\frac{L^2}{EC_v} \right)$$

$$\delta_3 = \frac{67.9 + 29.7}{9.656} - \frac{29.7 + 67.9}{9.656} \frac{1}{2 \times 0.1763} \frac{L^2}{EC_v} = -57.3 \left(\frac{L^2}{EC_v} \right)$$

$$(2) \delta_1 = \Delta_1 = \frac{48.2 + 49.3}{4} \frac{L^2}{EC_v} = 24.4 \frac{L^2}{EC_v}$$

$$\delta_2 = \frac{49.3 + 58.7}{8.66} - \frac{58.7 + 67.6}{9.656} \frac{1}{0.7537} \frac{L^2}{EC_v} = -34.0 \frac{L^2}{EC_v}$$

$$\delta_3 = \frac{58.7 + 67.6}{9.656} - \frac{67.6 + 58.7}{9.656} \frac{1}{0.3526} \frac{L^2}{EC_v} = 74.1 \frac{L^2}{EC_v}$$

$$(3) \delta_1 = \Delta_1 = \frac{-10.7 + 21.4}{E k} \frac{L^2}{C_v} = -8.0 \frac{L^2}{EC_v}$$

$$\delta_2 = \frac{21.4 + 36.2}{8.66} - \frac{-36.2 - 46.6}{9.656} \frac{1}{0.7537} \frac{L^2}{EC_V} = 20.2 \frac{L^2}{EC_V}$$

$$\delta_3 = \frac{-36.2 - 46.6}{9.656} - \frac{46.6 + 36.2}{9.656} \frac{1}{0.3526} \frac{L^2}{EC_V} = -47.0 \frac{L^2}{EC_V}$$

JOINT ROTATIONS

$$\theta^i : (1) \theta_1^i = -\frac{17.65 - 30.8}{8.66} \frac{L^2}{EC_V} = 13.15 \frac{L^2}{EC_V}$$

$$\theta_2^i = \frac{17.65 - 30.8}{8.66} - \frac{30.8 + 57.3}{9.656} \frac{L^2}{EC_V} = -10.66 \frac{L^2}{EC_V}$$

$$\theta_3^i = \frac{30.8 + 57.3}{9.656} - \frac{-57.3 - 30.8}{9.656} \frac{L^2}{EC_V} = 18.25 \frac{L^2}{EC_V}$$

$$(2) \theta_1^i = -\frac{24.4 + 34.0}{8.66} \frac{L^2}{EC_V} = -6.7 \frac{L^2}{EC_V}$$

$$\theta_2^i = \frac{24.4 + 34.0}{8.66} - \frac{-34.0 - 74.1}{9.656} \frac{L^2}{EC_V} = 17.9 \frac{L^2}{EC_V}$$

$$\theta_3^i = \frac{-34.0 - 74.1}{9.656} - \frac{74.1 + 34.0}{8.66} \frac{L^2}{EC_V} = -22.4 \frac{L^2}{EC_V}$$

$$(3) \theta_1^i = -\frac{-8.0 - 20.2}{8.66} \frac{L^2}{EC_V} = 3.3 \frac{L^2}{EC_V}$$

$$\theta_2^i = \frac{-8.2 - 20.2}{8.66} - \frac{20.2 + 47.0}{9.656} \frac{L^2}{EC_V} = -10.2 \frac{L^2}{EC_V}$$

$$\theta_3^i = \frac{20.2 + 47.0}{9.656} - \frac{-47.0 - 20.2}{9.656} \frac{L^2}{EC_V} = 13.9 \frac{L^2}{EC_V}$$

θ^n : (1) Joint 1 is a hinged support, that is, it has no joint moment.

$$(2) \theta_{22}'' = \theta_{2,1}'' + \theta_{2,3}'' = \frac{1000h_2}{3EI_2} + \frac{1000h_3}{3EI_3} = \frac{2000h}{3EI}$$

$$\theta_{23}'' = \theta_{3,2}'' + \theta_{3,4}'' = \frac{1000h_3}{6EI_3} + \frac{1000h_4}{6EI_4} = \frac{1000h}{3EI}$$

$$(3) \theta_{32}'' = \frac{1000h_2}{6EI_2}$$

$$\theta_{33}'' = \frac{1000h_3}{3EI_3} + \frac{1000h_4}{3EI_4} = \frac{2000h}{3EI}$$

Solve the joint moments by using Eqs. (1) from page 8.

$$-10.66 \frac{L^2}{EC_{v1}} + 17.9 \frac{L^2}{EC_{v2}} m_2 + \frac{2000h}{3EI} m_2 - 10.2 \frac{L^2}{EC_{v2}} m_3 + \frac{1000h}{6EI} m_3 = 0 \quad \text{---(A)}$$

$$18.25 \frac{L^2}{EC_{v1}} - 22.4 \frac{L^2}{EC_{v2}} m_2 + \frac{1000h}{3EI} m_2 + 13.9 \frac{L^2}{EC_{v2}} m_3 + \frac{2000h}{3EI} m_3 = 0 \quad \text{---(B)}$$

where $I = \frac{1}{12} \times \left(\frac{4}{12}\right)^3 = 1/324 \text{ ft.}^4$; $L = 60 \text{ ft.}$

$$\frac{L^2}{EC_{v1}} = \frac{60 \times 60}{E \times 9.60} = 375/E; \quad \frac{L^2}{EC_{v2}} = \frac{60 \times 60}{E \times 9.87} = 365/E$$

$$\frac{1000h}{EI} = \frac{1000 \times 10}{1000E (1/324)} = 3240/E$$

$$-10.66 (375/E) + 17.9 m_2 (365/E) + 6480 m_3/3E - 10.2 m_3 \times (365/E) + 3240 m_3/6E = 0$$

$$\text{or } -4000 + 8590 m_2 - 3180 m_3 = 0 \quad \text{---(A')}$$

$$6850 - 7100 m_2 + 7230 m_3 = 0 \quad \text{---(B')}$$

Solving equations (A') and (B'), it is found that

$$m_2 = 0.204 \text{ (kips) or } 204 \text{ (lb).}$$

$$m_3 = -0.770 \text{ (kips) or } -770 \text{ (lb).}$$

Final Values:

By using Eqs. (25) from page 19, values of σ and δ are

$$\sigma_0 = 116.2 + \frac{48.2}{1000} (204) + \frac{-10.7}{1000} (-770) = 134.27 \text{ kips/ft.}^2$$

$$\sigma_1 = 45.6 + \frac{-49.3}{1000} (204) + \frac{21.4}{1000} (-770) = 19.05 \text{ kips/ft.}^2$$

$$\sigma_2 = -67.9 + \frac{58.7}{1000} (204) + \frac{-36.2}{1000} (-770) = -27.73 \text{ kips/ft.}^2$$

$$\sigma_3 = 29.7 + \frac{-67.6}{1000} (204) + \frac{46.6}{1000} (-770) = -20.00 \text{ kips/ft.}^2$$

$$\delta_1 = 17.65 \frac{L^2}{EC_{v1}} + 24.4 \frac{L^2}{EC_{v2}} 0.204 - (-80) \frac{L^2}{EC_{v2}} (-0.770) = 29.29 \frac{L^2}{EC_{v2}}$$

$$\delta_2 = 30.8 \frac{L^2}{EC_{v1}} + (-34.0) \frac{L^2}{EC_{v2}} 0.204 + 20.2 \frac{L^2}{EC_{v2}} (-0.770) = 9.22 \frac{L^2}{EC_{v2}}$$

$$\delta_3 = -57.3 \frac{L^2}{EC_{v1}} + 74.1 \frac{L^2}{EC_{v2}} 0.204 + (-47.0) \frac{L^2}{EC_{v2}} (-0.770) = -7.59 \frac{L^2}{EC_{v2}}$$

Example 2: A continuous two-span folded plate has the same cross section as that in the first example, with the spans ($L_1 = L_2 = 60$ ft.) continuous over the middle diaphragm. The loading conditions are the same as in the first example. Each span is treated as the middle diaphragm.

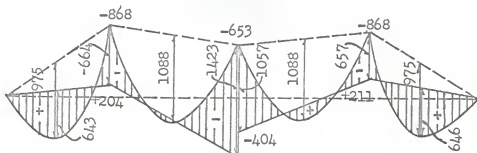
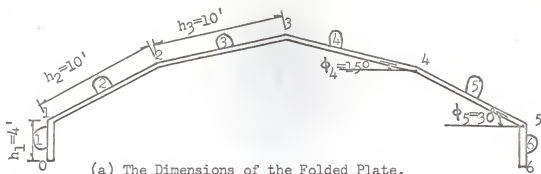
The stresses and deflections will be calculated at point $x = 3L/8$ from the outer support, where they have approximately the maximum values. From Appendix II, the coefficients are $C_m = 14.28$ and $C_v = 12.99$ for a uniform load, $C_m = 17.82$ and $C_v = 13.33$ for a normal load. The procedures of calculation are almost the same as in example 1. The loads R_0 of the reactions of the one-way slab on unyielding supports are the same and lead to the same P-forces. Some data needs to be modified, such as values for σ , δ , and θ . For loading I the coefficient of the proportion of the maximum moments of these two examples is $8/14.28$ (Appendix II); for the other loading the coefficient is

TABLE A. -- ANALYSIS OF A SIMPLY SUPPORTED, ONE SPAN, FOLDED PLATE.

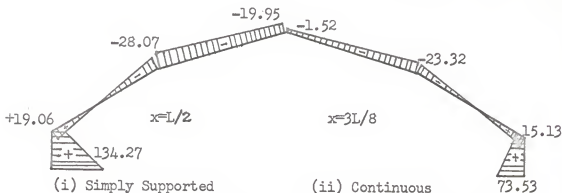
	Joint	R lb./ft. ²	H lb./ft. ²	F = ΔH/cos lb./ft. ²	$\sigma_{m-1,n}$ k/ft. ²	σ k/ft. ²	Cδ k/ft. ²	ce' k/ft. ⁴	ce'' k/ft. ⁴	ce k/ft. ⁴
	0					116.2	$c = \frac{I^2}{9.6E}$			
Loading I External load only	1	550.0	0	550.0	278.2	45.6	17.65		0	
	2	922.0	2305.0	2660.0	215.4	-67.9	30.8	-10.66	0	-10.66
	3	856.0	2428.0	127.0	10.3	29.7	-57.3	18.25	0	18.25
	0					48.2	$c = \frac{I^2}{9.67E}$			
Loading II $M_2 = M_4 = 1000$ (lb) $(M_2 = 204$ (lb) actual)	1	115.0	0	115.0	47.1	-49.3	24.4		0	
	2	-219.0	-546.0	-630.0	-41.3	58.7	-34.0	17.9	5.9	23.8
	3	208.0	580.0	1165.0	76.5	-67.6	74.1	-22.4	3.0	-19.4
	0					-10.7	$c = \frac{I^2}{9.67E}$			
Loading III $M_2 = 1000$ (lb) $(M_2 = 770$ (lb) actual)	1	0	0	0	0	21.4	-8.0		0	
	2	104.0	259.0	299.0	19.6	-36.2	20.2	-10.2	1.5	-8.7
	3	-208.0	-580.0	-868.0	-57.0	46.6	-47.0	13.9	5.9	19.8

TABLE A. (CONCL.)

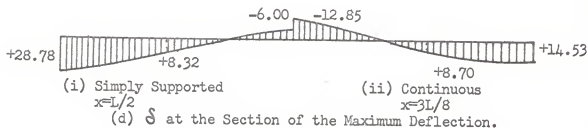
	Joint	R lb./ft ²	H lb./ft ²	$P = \frac{\Delta H}{\cos}$ lb./ft ²	$\sigma_{n-1,n}$ k/ft ²	σ^- k/ft ²	C δ k/ft ²	C θ^1 k/ft ⁴	C θ^{11} k/ft ⁴	C θ k/ft ⁴
Actual Loading	0					134.27		$C = \frac{L^2}{9.87E}$		
	1	573.4	2068.0			19.06	28.78			
	2	797.4	2756.2			-28.07	8.32		0	
	3	1058.4				-19.95	-6.00		0	



(b) Moments at the Section of the Maximum Deflection.



(c) σ at the Section of the Maximum Deflection.



(d) δ at the Section of the Maximum Deflection.

FIG. 13. THE RESULTS OF THE ANALYSIS OF THE FOLDED PLATE.

9.87/17.83. In calculation of $C\theta^n$, C_v must be considered and the multiplier is 12.99/9.6 (Appendix II). The results of the analysis are presented in Table B.

In this problem we are going to solve the joint moments m first. We will use Eqs. (1) and the data which are shown in Table B. The equilibrium equations are

$$-5.97 \frac{I^2}{EC_{v1}} + 17.9 \frac{I^2}{EC_{v2}} m_2 + \frac{2000h}{3EI} m_2 - 3.65 \frac{I^2}{EC_{v2}} m_3 + \frac{1000h}{6EI} \times m_3 = 0 \quad \text{---(C)}$$

$$10.2 \frac{I^2}{EC_{v1}} - 8.3 \frac{I^2}{EC_{v2}} m_2 + \frac{1000h}{3EI} m_2 + 15.7 \frac{I^2}{EC_{v2}} m_3 + \frac{2000h}{3EI} \times m_3 = 0 \quad \text{---(D)}$$

where $I = 1/324 \text{ ft.}^4$

$$C_{v1} = 12.99$$

$$C_{v2} = 13.33$$

$$L = 60 \text{ ft.}$$

$$\frac{I^2}{C_{v1}} = \frac{60 \times 60}{12.99} = 275.0$$

$$\frac{I^2}{C_{v2}} = \frac{60 \times 60}{13.33} = 270.0$$

$$-5.97 (275/E) + 17.9 (270/E) m_2 + 6480 m_2/3E - 3.65 (270/E) \\ + 3240 m_3/6E = 0$$

$$10.2 (275/E) - 8.3 (270/E) m_2 + 3240 m_2/3E + 15.7 m_3 (270/E) \\ + 6480 m_3/3E = 0$$

Simplifying the above equations,

$$-1654 + 6990 m_2 - 445 m_3 = 0 \quad \text{---(C')}$$

$$2826 - 1160 m_2 + 6400 m_3 = 0 \quad \text{---(D')}$$

Solve the equations (C') and (D') to get the values of m_2 and m_3 which are

$$m_2 = 0.211(\text{kips}) \text{ or } 211(\text{lb})$$

$$m_3 = -0.404(\text{kips}) \text{ or } -404(\text{lb})$$

Final Values:

Using Eq. (25),

$$\sigma_0 = 65.5 + \frac{26.7}{1000} (211) + \frac{-5.93}{1000} (-404) = 73.53 \text{ kips/ft.}^2$$

$$\sigma_1 = 25.7 + \frac{-27.3}{1000} (211) + \frac{11.9}{1000} (-404) = 15.13 \text{ kips/ft.}^2$$

$$\sigma_2 = -38.3 + \frac{32.7}{1000} (211) + \frac{-20.0}{1000} (-404) = -23.30 \text{ kips/ft.}^2$$

$$\sigma_3 = 16.8 + \frac{-37.5}{1000} (211) + \frac{25.8}{1000} (-404) = -1.52 \text{ kips/ft.}^2$$

$$\delta_1 = 9.9 \frac{L^2}{EC_{v1}} + 13.5 \frac{L^2}{EC_{v2}} 0.211 - 4.4 \frac{L^2}{EC_{v2}} (-0.404) = 14.83 \frac{L^2}{EC_{v2}}$$

$$\delta_2 = 17.2 \frac{L^2}{EC_{v1}} - 18.8 \frac{L^2}{EC_{v2}} 0.211 + 11.2 \frac{L^2}{EC_{v2}} (-0.404) = 9.21 \frac{L^2}{EC_{v2}}$$

$$\delta_3 = -32.1 \frac{L^2}{EC_{v1}} + 41.0 \frac{L^2}{EC_{v2}} 0.211 - 26.0 \frac{L^2}{EC_{v2}} (-0.404) = -13.85 \frac{L^2}{EC_{v2}}$$

The calculated values are listed in Table A. They can be explained as follows:

1. Record the joint loads of every loading in column 1.
2. Calculate the joint reactions by moment-distribution and static equilibrium (Col. 3), the H-forces by Eq. 8 (Col. 4), the P-forces by Eq. 10 (Col. 5), and the free edge stresses by Eqs. 11 (Col. 6).
3. Distribute the stresses in a separate table as demonstrated in page 28. Enter the computed stresses in Col. 7.
4. Calculate the C-values ($C = \frac{L^2}{C_{vE}}$) from the joint stresses by Eqs. 21 and 22 (Col. 8).

TABLE B. - ANALYSIS OF A CONTINUOUS, TWO SPANS, FOLDED PLATE.

	Joint	R lb/ft ²	σ_1^* kips/ft ²	σ kips/ft ²	C_5 kips/ft ²	C_6' kips/ft ²	C_6'' kips/ft ²	C_6 kips/ft ²
Loading I External load only	0		116.2	65.5	$C = I^2/12.99E$			
	1	550.0	45.6	25.7	9.9			
	2	856.0	-67.9	-38.3	17.2	-5.97	0	-5.97
	3	922.0	29.7	16.8	-32.1	10.20	0	10.20
Loading II $M_2=M_4=1000$ (lb) $(M_2=211$ (lb) actual)	0		48.2	26.7	$C = I^2/13.33E$			
	1	115.0	-49.3	-27.3	13.5			
	2	-219.0	58.7	32.7	-18.8	9.90	8.0	17.90
	3	208.0	-67.6	-37.5	41.0	-12.40	4.1	-8.30
Loading III $M_2=1000$ (lb) $(M_2=-404$ (lb) actual)	0		-10.7	-5.93	$C = I^2/13.33E$			
	1	0	21.4	11.9	-4.4			
	2	104.0	-36.2	-20.0	11.2	-5.65	2.0	-3.65
	3	-208.0	46.6	25.8	-26.0	7.70	8.0	15.70

* σ_1^* : the values from Table A.

TABLE B. (CONCL.)

	Joint	R lb/ft ²	σ_1^* kips/ft ²	σ kips/ft ²	C ₃ kips/ft ²	C _{9'} kips/ft ²	C _{9''} kips/ft ²	C ₉ kips/ft ²
	0			73.53	$C = 12/13.33E$			
Actual	1			15.13	14.53			
Loading	2	767.8		-23.32	8.70			
	3	1045.9		- 1.52	-12.85			

* σ_1^* : the values from Table A.

5. Calculate the slopes (deflection) $C\theta'$ caused by the joint deflections by Eq. 3 (Col. 9) and the slopes (deflections) $C\theta''$ caused by the bending of the slab by Eqs. 4 (Col. 10). Combine the slopes (deflection) $\theta = \theta' + \theta''$ (Col. 11).

6. Form the slope-deflection equations, Eqs. 1. Solve them for the m -values.

7. Calculate the final σ and δ by superposition method.

The final values of σ and δ of the examples 1 and 2 are shown respectively in the left and the right half of the span in Fig. 13. This shows the section where the maximum moment and the maximum deflection occurred. The unit of the moments is ft.-lb., of the stresses kips per square foot, of the deflection inches. The coefficient C times the ordinate of stresses and deflections is $\frac{L^2}{EC_v}$.

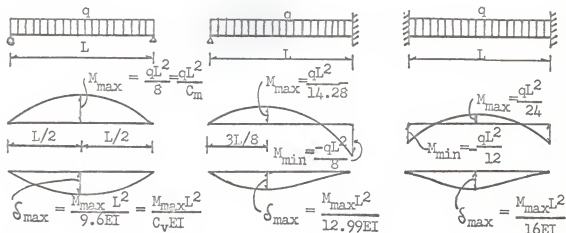
8. In the Fig. 13(b) shows the maximum positive moment occurs at section between the joints, the maximum negative moment occurs at the joints.

9. In the Fig. 13(c) shows the maximum positive stress occurs at the edge of the folded plates so the edge beams need be enforced.

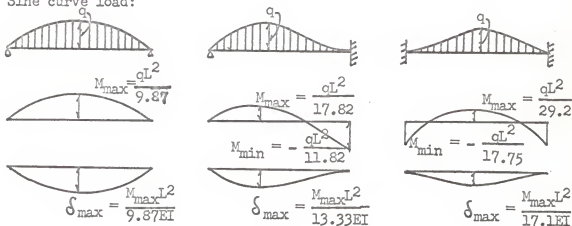
10. In the Fig. 13(d) shows the maximum downward deflection occurs at the edge of the folded plates, if the edge beams are enforced the deflection will be reduced.

APPENDIX II. - MOMENT AND DEFLECTION COEFFICIENTS.

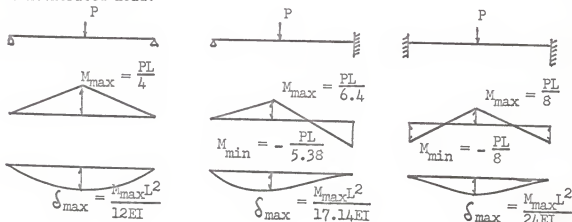
1. Uniform load:



2. Sine curve load:



3. Concentrated load:



APPENDIX III - NOTATION

A	= section area of a plate (deep beam)	
C	= L^2/EC_v	
C_m	= moment constant, $M_{\max} = q_{\max}L^2/C_m$	see Appendix I
C_v	= deflection constant, $\delta_{\max} = M_{\max}L^2/(C_vEI)$	
E	= modulus of elasticity.	
H	= horizontal component of P.	
h	= plate depth between longitudinal edges.	
I	= moment of inertia of slab per unit width.	
K	= moment distribution factor.	
L	= longitudinal length.	
M	= deep beam moment caused by P.	
m	= transverse slab moment.	
$n-1, n, n+1$	= subscripts denoting longitudinal joints and deep beams, where deep beam n is situated between joint n-1 and n.	
P_n	= deep beam load parallel to a deep beam.	
$Q_{n, n+1}$	= compressive force at joint n parallel to plate n+1.	
q	= load per unit length (or area) of a beam (or plate).	
R	= vertical joint force equals to the reaction of the slab.	
r	= stress-distribution factor.	
s	= horizontal projection of deep beam depth.	
T	= resultant of shear stress along a deep beam edge.	
V	= shear force across a deep beam.	
$w_{n, n+1}$	= deflection of joint n normal to deep beam n+1.	
x	= distance from a support along length of deep beam.	
y	= distance from joint n across deep beam n+1.	
Z	= section modulus of a deep beam $th^2/6$.	

- Δ = deep beam displacement parallel to a deep beam.
- δ = vertical joint deflection.
- θ = slope deflection, $\theta' + \theta''$
- θ' = slope deflection at a joint caused by joint deflections.
- θ'' = slope deflection at a joint caused by slab bending.
- $\theta_{n,n+1}$ = slope deflection at joint n produced by m_{n+1} .
- σ_n = longitudinal stress at edge n at a deep beam section.
- $\sigma_{n,n+1}$ = free-edge stress at edge n of deep beam n+1.
- τ = shearing stress.

APPENDIX VI - BIBLIOGRAPHY

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ANALYSIS OF REINFORCED CONCRETE FOLDED PLATES
BY THE SLOPE-DEFLECTION METHOD

by

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A method of analysis of folded plate structures using the slope-deflection method is presented. Two examples of application of the method are included.

The folded plate is treated as a one-way slab in the transverse direction and as a deep beam structure in the longitudinal direction. First, the one-way slab is considered to be supported at the joints of the deep beam structures and to be acted upon by the external loading. The one-way slab is analyzed as an ordinary continuous beam, by using the method of moment-distribution, for finding the moments and reactions at the joints. Since there are no supports at the joints of the folded plate, the reactions at the joints will apply as forces to the structure (after these reactions have been solved, the external loading does not need to be considered). Second, the deep beam structure is considered to be acted upon by the reactions at the joints. The slab can be analyzed in two steps. In the first step, the reactions are considered to be acting on the hinged joints. In the second step, the joint moments caused by the deflections of the deep beams under the action of the external loading will be considered. The rotations of two adjacent deep beams will be different at their common edges. Joint moments m are introduced at each joint in order to secure the continuity of the structure at the joints. From these moments we can find the reactions at the joints. The continuity of the slab at a joint is maintained if the changes in slope of adjacent slabs, due to the external load and the joint moment acting simultaneously are reduced to zero. The slope deflection equations are

$$\theta_1 = \theta_{10} + \theta_{11} m_1 + \theta_{12} m_2 + \theta_{13} m_3 = 0$$

$$\theta_2 = \theta_{20} + \theta_{21} m_1 + \theta_{22} m_2 + \theta_{23} m_3 = 0$$

$$\theta_3 = \theta_{30} + \theta_{31} m_1 + \theta_{32} m_2 + \theta_{33} m_3 = 0$$

The actual joint moments can be solved from these equations.

The deep beam, which is supported at the end diaphragms, is solved by the general beam theory. When the shears at the joints of adjacent slabs are considered, the equations are set up so that the stresses in adjacent plates must be equal at their common edge. The reactions which are found from the one-way slab analysis, are applied to the deep beam structures in order to find the rotations, stresses, and deflections in the longitudinal direction. After the calculation processes are completed for the hinged reactions forces, they can be applied for the case of the joint moment loads by multiplying by a coefficient. The variation of the longitudinal forces at the joints depends on the support conditions at the ends. The deep beams are monolithically joined so that the stresses and deflections of adjacent deep beams must be equal at a joint.

The final stresses and deflections of the structure are calculated at the section where maximum deflection occurs.

The calculations of the examples are set in tabular form for easy checking and can be used for other structures with similar dimensions.