193

VORTEX TUBE

by

THETTU RAGHULINGA REDDY

B. E. (Mechanical), Andhra University, Waltair, India, 1962

A MASTER'S REPORT

325

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Mechanical Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1966

Approved by:

Wilson Tripp
Major Professor

TABLE OF CONTENTS

NOMENCLATURE

a = Specific availability, ft. - lbf. per lb.

A = Total availability, ft. - lbf.

c = Velocity of sound, ft./sec.

C.O.P. = Coefficient of performance

c_p = Specific heat of constant pressure, BTU per lbm. ^oR

c_v = Specific heat of constant volume, BTU per lbm. ^oR

$\frac{D}{D\theta}$ = $u \frac{d}{dr}$

h = Specific enthalpy, BTU/lbm.

J = Mechanical equivalent of heat, 778.26 ft. - lbf. per BTU

k = Ratio of Specific heat $\frac{c_p}{c_v}$

M = Mach number

p = Pressure, lbf. per sq. in ab.

q = Velocity in hodograph plane, ft./sec.

R = Gas constant, ft. - lbf. per lbm. ^oR

r = Radial distance, ft.

s = Specific entrophy, BTU per lbm ^oR

T = Temperature, ^oR

t = Static temperature, ^oR

u = $q \cos \theta = \frac{\partial\varphi}{\partial y} = \frac{\rho_o}{\rho} \psi_y$ = ft. per sec.

v = $q \sin \theta = \frac{\partial\varphi}{\partial x} = \frac{\rho_o}{\rho} \psi_x$ = ft. per sec.

u,v,w = Velocity components in x,y,z, plane, ft.per sec.

W = Total rate of flow, lbm. per sec.

x,y,z = Coordinate axis

z = Direction of tube axis

α = Velocity angle in hodograph plane

Γ = Circulation, ft^2 per sec.

ε = Eddy diffusivity, ft^2 per sec.

η = Efficiency

θ = Time, sec.

μ = Cold fraction ratio of mass rate of cold flow to the total mass rate flow

ν = Kinematic viscosity, ft^2 per sec.

ρ = Density, lbm per $ft.^3$

τ = Turbulent shear stress;

Φ = $\rho\varepsilon\left(\frac{dv}{dr} - \frac{v}{r}\right)^2$ Dissipation function, lbf. per sec. $ft.^2$

φ = Potenti. function, $ft.^2$ per sec.

ψ = Stream function, $ft.^2$ per sec.

ψ_q = $\frac{\partial\psi}{\partial q}$ ft. per sec.

ψ_{qq} = $\frac{\partial^2\psi}{\partial\psi^2}$ $sec.^{-1}$

SUBSCRIPTS

0 = Stagnation condition

1 = State of gas supplied to the vortex tube

2 = State at end of reversible adiabatic expansion from 1 to p_c

b = Body being cooled at fixed temperature

c = State of cold gas leaving cold tube

d = Dead state environmental temperature and pressure

f = Free stream conditions

j = State of jet issuing from the nozzle

m = Manifold condition

s = Static condition.

t = total condition.

INTRODUCTION

The Vortex, Hilsch, or Ranque tube, as it is called in different names, is a remarkably simple device which produces hot and cold gas streams simultaneously from a source of compressed air. It is a cylindrical tube in which a stream of compressed gas enters tangentially and separates into two streams of gases at different temperatures. The hot stream leaves through the periphery at one end and the cold stream leaves through the center of the other end (Figure 4). The phenomenon is known as "Ranque - Hilsch effect," which occurs when a gas expands in a centrifugal field, as the one observed in cyclonic seperators. George Joseph Ranque [11] *, a French metallurgist invented this device, and later filed for a French patent docket in 1931. He also filed for a similar patent in the United States of America in 1932, which was issued in 1934 [34] . Even though Ranque did not actually give any name to his device, it came to be known as vortex tube or Ranque or Hilsch tube in the literature.

There are two basic designs of the Vortex tube in practice, namely, uniflow and counterflow. As these names suggest, in the uniflow design the cold and hot streams flow in the same direction, whereas in the counter flow the hot and cold streams flow in the opposite directions. (Figure 2 and 3) The isometric illustration of counter flow vortex tube is shown in Figure 4.

Ranque also described a refrigerating unit having its own compressor driven by an electric motor. The air from the atmosphere is sucked into the rotor, compressed and passed through an uniflow vortex tube without cooling,

---

*Numbers in parentheses refer to references at the end of the report.

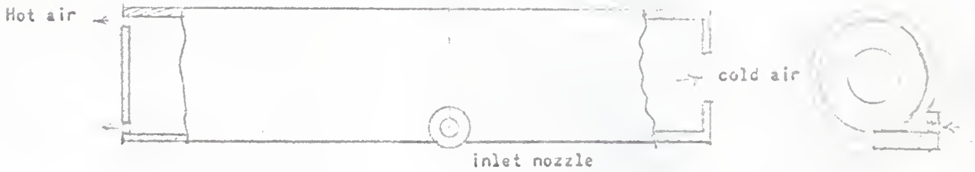


Fig. 1 Vortex tube as used by Hilsch

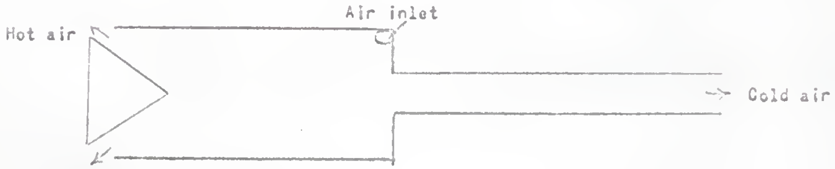


Fig. 2. Counterflow Vortex Tube

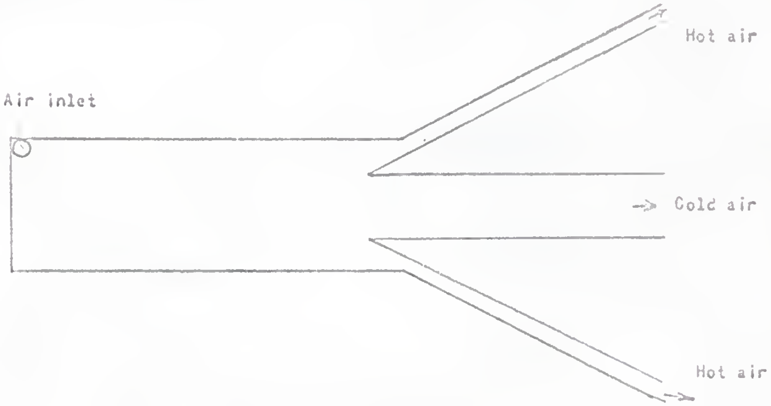


Fig. 3 Uniflow Vortex Tube

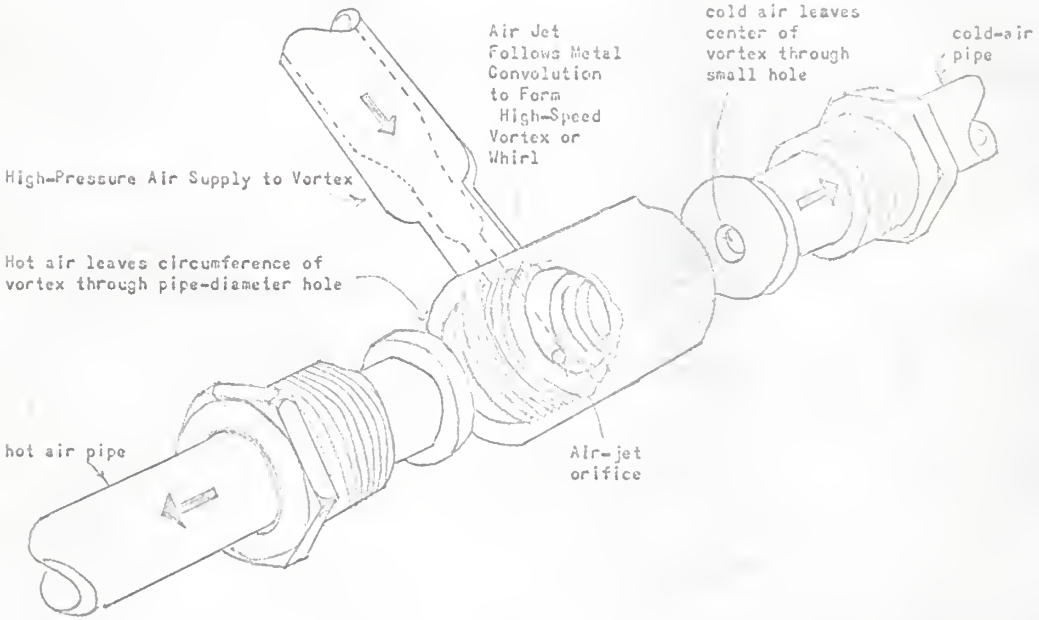


Fig. 4 Artist's illustration of the Vortex Tube
(Source Ref. 45)

where it is separated into two streams, viz hot and cold air. The hot air is discharged into the atmosphere and the cold air is used for refrigeration. Because of its low efficiency the vortex tube has limited applications as a refrigerator.

The invention of Ranque virtually remained unused for more than a decade. In the year 1945, Rudolf Hilsch, physicist in the University of Erlangen, Germany [16] made detailed investigation on the mechanism of the vortex tube and he is rightly known as the rediscoverer of vortex tube. However, this device gained wide publicity and popularity with Milton's publication under the title "Maxwellian Demon at Work" [27] .

It may be necessary to explain the principle involved in the separation of hot and cold streams of gas in a greater detail with the aid of analytical and experimental data. An assessment of the efficiency of the vortex tube, applying basic and nonbasic criteria, will be helpful in applications of the vortex tube for various purposes.

MECHANISM OF THE VORTEX TUBE

Several theories have been proposed to explain the working mechanism of the vortex tube since its invention by Ranque in the year 1931. According to Ranque [34] , the principle of working the apparatus involves the separation of hot and cold fluids from a current of a compressible fluid under pressure by causing the fluid to flow with a gyratory helical motion along a surface of revolution, and dividing the said fluid into two coaxial sheets moving along each other. This movement results in the compression of the outer sheet by the inner sheet, and by the action of the centrifugal force. A rise in the temperature of the outer sheet and a fall in the temperature of the inner sheet are caused. Hilsch [16] could not agree with this explanation of Ranque on the phenomenon of vortex tube. He felt that this device is not suitable to adopt for a domestic refrigerating unit, because of its low efficiency, but can be applied to produce large scale cooling of mines and liquification of gases.

Milton's article on vortex tube, "Maxwellian Demon at Work" [27] aroused the interest of engineers and scientists in the theory and application of the principle of the vortex tube. Rudkin [9] suggested that as the gas is spinning rapidly about the axis of the tube, the gas near the wall is compressed more than the gas in the center. Molecules of the gas having less than average kenetic energy would be unable to penetrate the zone of highly compressed air and would be forced toward the center of the tube.

Roebuck, [9] observed that under the conditions of rapid rotation there is a large pressure gradient in the radially inward direction. This causes some of the gas in the outer layer to move radially inward, and the work done by the gas as it expands toward the center, increases the angular velocity of the outer layer. Also, a part of this expansion is used to do work to overcome the effect of friction between the inner and outer layers. The gas that reaches the central layer has smaller kinetic energy.

Armand Foā [9] believed that when the valve at the end of hot flow tube of a counter flow vortex tube is fully opened, the air which is injected peripherally with high velocity is subjected to centrifugal force. Therefore, a ring of gas is formed under pressure at the inlet section with a hollow central core under a partial vacuum, resulting in the suction of atmospheric air into the apparatus from the cold tube. The external ring of gas will move toward the periphery of the hot tube from where it escapes. Some air from the periphery moves through the cold tube orifice. Because of the partial vacuum at the nozzle core zone, air is sucked in again, which expands, becomes cool, and comes in contact with the out-going air through the orifice, which in turn, is cooled. This process is repeated over and over until an equilibrium is reached between the temperatures of the air seeping in and the air escaping through the orifice. But the experiments of Hartnett and Eckert [13] and Lay [23] showed that there is a partial vacuum inside the hot tube.

Taylor [9] explained the theory of mechanism of the vortex tube through thermodynamics. The compressed gas, which is initially at a pressure p_1 and a temperature T_1, expands through a nozzle to a pressure P_J and a velocity V_J. The expansion from p_1 to P_J is irreversible. The kinetic energy of mass motion of the gas issuing from the jet is $\frac{V_J^2}{2}$ per unit mass. So there is a corresponding decrease in internal energy of the air.

$$c_p \, (T_1 - T_J) = \frac{V_J^2}{2} \tag{1}$$

where c_p = specific heat of constant pressure.

The jet issuing tangentially from the nozzle at the periphery of the tube spirals towards the center forming a miniature tornado. Due to the high velocity of the jet, there is an appreciable centrifugation. This is the reason for the suction near the cold tube orifice. Cooling through Joul-Thomson effect, as previously thought of, is not correct, because it has been observed that hydrogen at room temperature is cooled although its Joule-Thomson coefficient is negative. Some of the gas at the periphery expands because of the inwardly directed pressure gradient and moves toward the center, loosing both velocity and temperature. The temperature of the gas remaining at the periphery goes up due to the loss of kinetic energy of the inwardly moving gas and the transfer of this kinetic energy loss to the outer layers. Thus, Taylor explains the energy separation as super position of the two effects.

In succeeding chapters the theories on the vortex tube are presented which are based on the latest experimental and analytical data.

ANALYTICAL STUDY

In the steady flow of a fluid without viscosity and heat conductivity there is obviously no possibility for any energy transfer from one stream tube to another, since the pressure forces can deliver no work. Consequently, the total energy, or in the case of gases, the total temperature remains constant with respect to time. Even though there is no energy flow through the tube wall, the total temperature of one layer of the gas may be different than the total temperature of the adjacent layers in fluids with viscosity and conductivity. For instance, in the boundary layer on an insulated flat plate in a high velocity gas flow, the gas near the wall will have a lower total temperature (lower total energy) than the oncoming free stream;

while the flow away from the wall will be at a higher total temperature than the free stream. Many other rectilinear motions exhibit such an energy separation. Study of the energy separation between the layers in the vortex tube, with the help of rectilinear motions, may give better understanding of the phenomenon.

Flow in the vortex tube has been observed by many engineers and scientists as a spiral flow. Assuming a spiral and axial flows, Lay [24] explains the flow pattern through the hodograph equation,

$$q^2 \psi_{qq} + q (1 - M^2) \psi_q + (1 + M^2) \psi_{\alpha\alpha} = 0 \qquad (2)$$

For spiral flow in a plane the solution of the above equation is the combination of the solutions of vortex and sink flow. That is,

$$\psi = \frac{\varphi}{2\pi} \alpha + \frac{\Gamma}{2\pi} J \qquad (3)$$

$$R = \frac{2\pi c_o r}{\sqrt{\varphi^2 + \Gamma^2}} \qquad (4)$$

$$x = \frac{\varphi}{2\pi c_o} \frac{\cos \alpha}{(\rho/\rho_o) (\frac{q}{c_o})} + \frac{\Gamma \sin \alpha}{2\pi c_o (\frac{q}{c_o})} \qquad (5)$$

$$y = \frac{\varphi}{2\pi c_o} \frac{\sin \alpha}{(\rho/\rho_o) (\frac{q}{c_o})} - \frac{\Gamma \cos \alpha}{2\pi c_o (\frac{q}{c_o})} \qquad (6)$$

For three dimensional flow the velocity potential becomes,

$$\varphi + w t$$

Deissler and Perlmutter [8] assumed a turbulent vortex with axial and radial flows. The dimensionless velocity is obtained from the compressible Navier-Stokes equation. The velocity distribution is in fair agreement with the experimental results. Assuming uniform axial mass velocity, the calculated magnitudes of the individual terms in the momentum equation, reveal the fact that the inertial force (due to tangential velocity) is positive throughout the section, and the shear force is zero at the axis, goes to a negative maximum, and comes back to zero. The inertial force tends to accelerate the particle, in the flow, in order to maintain constant angular momentum, while the shear forces tend to slow it down.

The energy equation and its individual terms are calculated to find out the real cause of the energy separation. Considering incompressible flow, the energy equation can be written as,

$$\rho \ c_v \frac{Dt}{D\theta} = \rho \ c_v \ \varepsilon \frac{1}{r} \frac{d}{dr} \left(r \frac{dt}{dr}\right) + \Phi \qquad (7)$$

where Φ is the dissipation function and equal to, $\rho \ \varepsilon \left(\frac{dv}{dr} - \frac{v}{r}\right)^2$. So,

$$\rho \ c_v \frac{Dt}{D\theta} = \rho \ c_v \ \varepsilon \frac{1}{r} \frac{d}{dr} \left(r \frac{dt}{dr}\right) + \rho \ \varepsilon \left(\frac{dv}{dr} - \frac{v}{r}\right)^2$$

where

$\rho \ c_v \frac{Dt}{D\theta}$ = rate of change of internal energy per unit volume.

$\rho \ c_v \ \varepsilon \frac{1}{r} \frac{d}{dr} \left(r \frac{dt}{dr}\right)$ = rate of heat transfer into the element by conduction.

$\rho \ \varepsilon \left(\frac{dv}{dr} - \frac{v}{r}\right)^2$ = turbulent dissipation.

Deissler and Perlmutter [8] have also given the magnitudes of various terms in the case of uniform and axial flow. The static temperature change is positive for all radial positions, which shows that there is no energy separation for an incompressible flow.

If the flow is compressible the energy equation [8] for perfect gas can be written as,

$$\rho \ c_p \frac{D}{D\theta} \left(t - \frac{v^2}{2c_p}\right) = \frac{1}{r} \frac{d}{dr} \left(r \ \rho \ c_p \ \varepsilon \frac{dt}{dr}\right) - \frac{1}{r} \frac{d}{dr} \left(\frac{r\rho\varepsilon}{\rho} \frac{dp}{dr}\right) + \frac{1}{r} \frac{d}{dr} (r \ vT) \qquad (8)$$

where,

$\rho\, c_v \frac{D}{D\theta} (t - \frac{v^2}{2c_p})$ = rate of change of total temperature of a fluid element,

$\frac{1}{r} \frac{d}{dr} (r\, \rho c_p\, t\, \frac{dt}{dr})$ = turbulent heat transfer into the fluid element by temperature gradients,

$- \frac{1}{r} \frac{d}{dr} (\frac{r \rho \varepsilon}{\rho} \frac{dp}{dr})$ = turbulent heat transfer in the fluid element by pressure gradients (by expansion and contraction of eddies),

$\frac{1}{r} \frac{d}{dr} (r\, v\, T)$ = turbulent shear work done on the element.

The calculated values of each of the above terms in dimensionless form for uniform axial mass flow are given by the authors [8] . The contribution is the total temperature change with respect to time by turbulent conduction due to temperature gradients and to pressure gradients are individually very large, especially near the center of the vortex where they completely cancel each other. That is to say, the conduction of heat into the core region by temperature gradients is the same as that of the heat conduction by pressure gradients (by expansion and contraction of eddies). Therefore, the energy separation is due to shear work. Comparing Figure 3 and Figure 13 [8] , it is evident that the positive shear work corresponds to inviscid velocity profile ($v \propto \frac{1}{r}$). As the fluid element moves to smaller radii, the viscous or turbulent effects cause the velocity profile to depart from the inviscid flow. In that region the shear work done on the element becomes negative due to the slowing down tendency of the turbulent viscosity and the total temperature drops, thus the energy separation is dependent on the tangential velocity profile. The total effect is that the fluid in the core region does shear work on the fluid in the outer region as it expands while traveling toward the center. Thus, the energy is transferred from the core region to the annular region with a total temperature separation. Very small additional energy transfer is effected by the expansion and contraction of eddies in the radial pressure gradients.

EXPERIMENTAL INVESTIGATION

A perusal of the literature on the vortex tube indicates that many authors have carried out different types of experimental investigations. The curiosity of knowing the phenomenon made them study the tube thoroughly. Some investigators have modified the apparatus to obtain the maximum temperature difference between the inlet temperature and the temperature of cold flow stream.

MacGee [26] conducted experiments to investigate the fluid action on the counter flow vortex tube with different inlet nozzles. His observations on flow patterns reveal that the larger the number of nozzles, the better the flow is centered. When colored water is injected into the tube, and when the flow of air is high, it is observed that the water formed a spiral moving in the direction of the hot end of the tube. No appreciable noise is noticed. This gives the clue that the energy separation is taking place only in the hot tube. At low flow rates of air, the injected water spreads uniformly over the inner surface of the tube wall, and a low pitched whistle will be heard. As the hot end of the tube is progressively closed, the whistle increases in pitch.

Eckert and Hartnett [10 & 13] made a thorough experimental investigation on a three-inch counterflow vortex tube. Figure 5 and Figure 6 show

Distance of measurement ports from nozzle

A - 1; B - 3; c - 6; D - 12; E - 18.

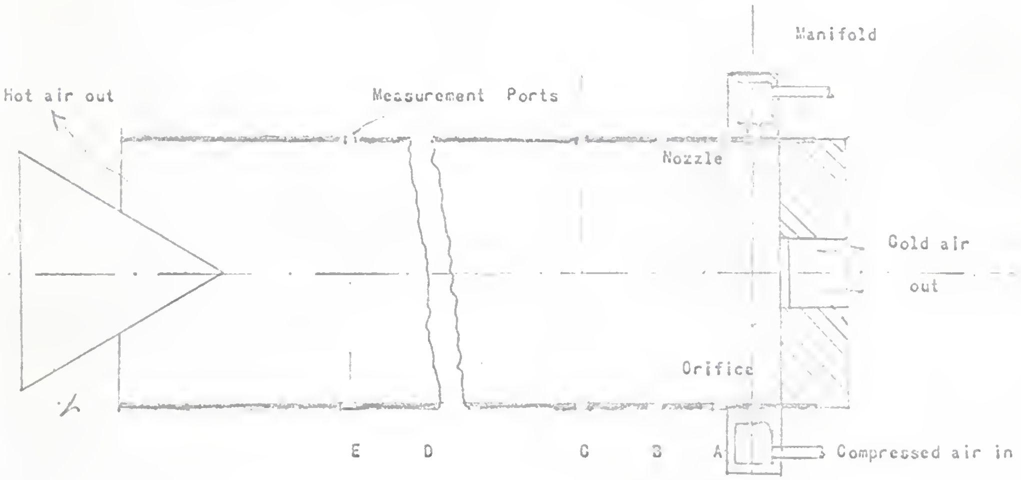


Fig. 5 Three-In-Diam Vortex Tube (Source Ref. 13)

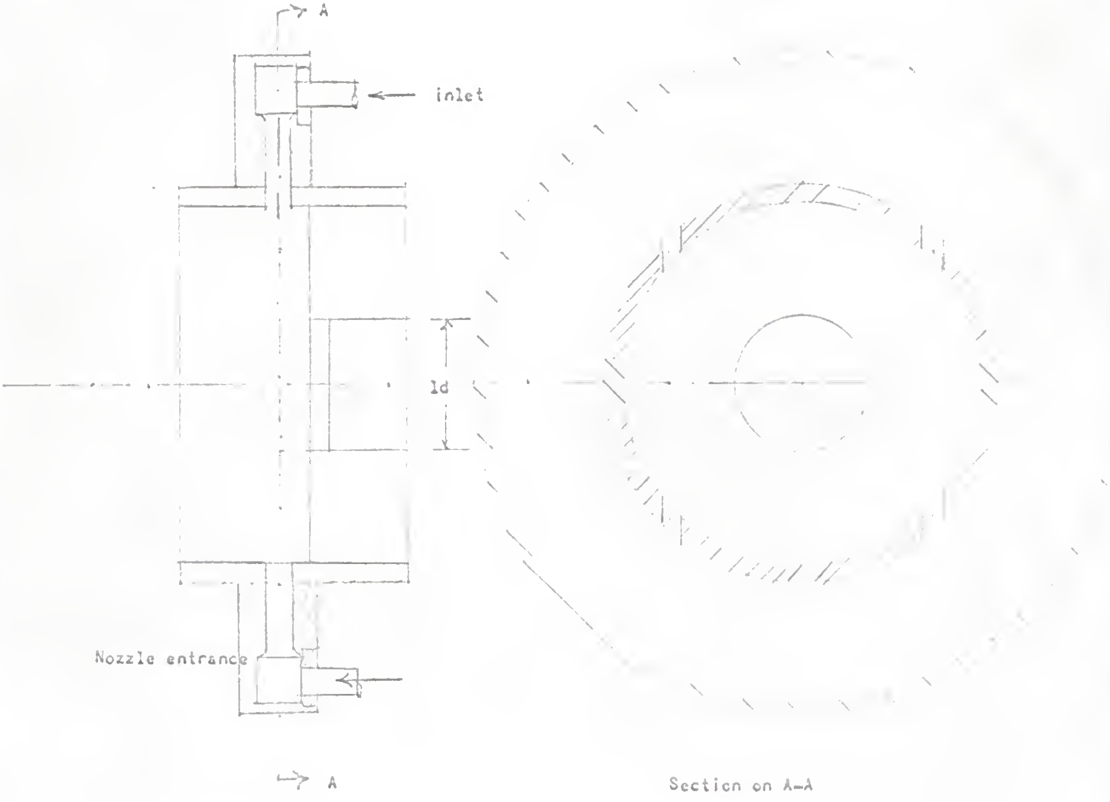


Fig. 6 Detail of Entrance of Vortex Tube (Source Ref. 13)

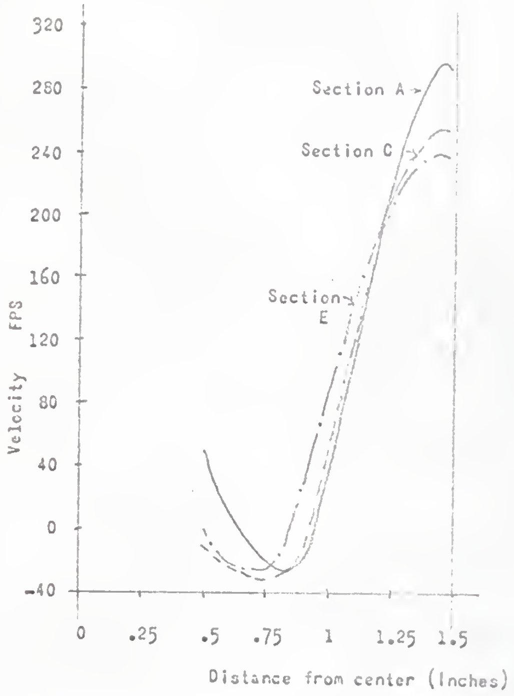


Fig. 7 Axial Velocity For 20-PSIG Inlet Pressure in 3" Vortex Tube (Source Ref. 13)

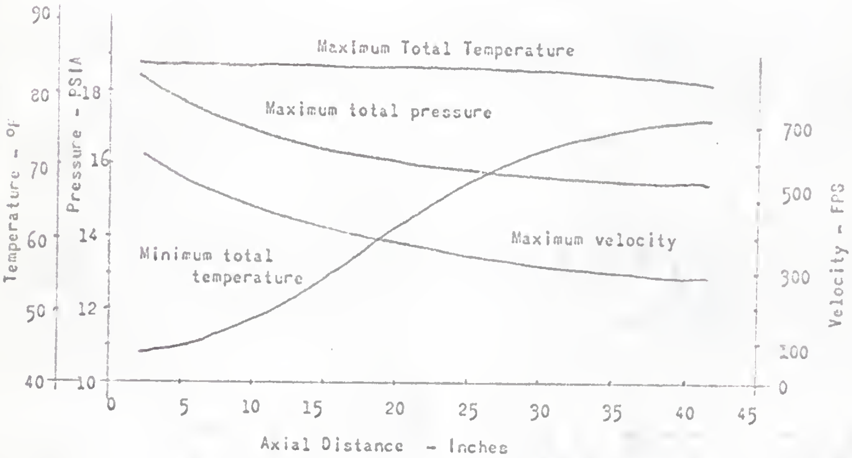


Fig. 8 Axial variation of pressure, temperature, velocity with inlet pressure = 10psig (Source Ref. 23)

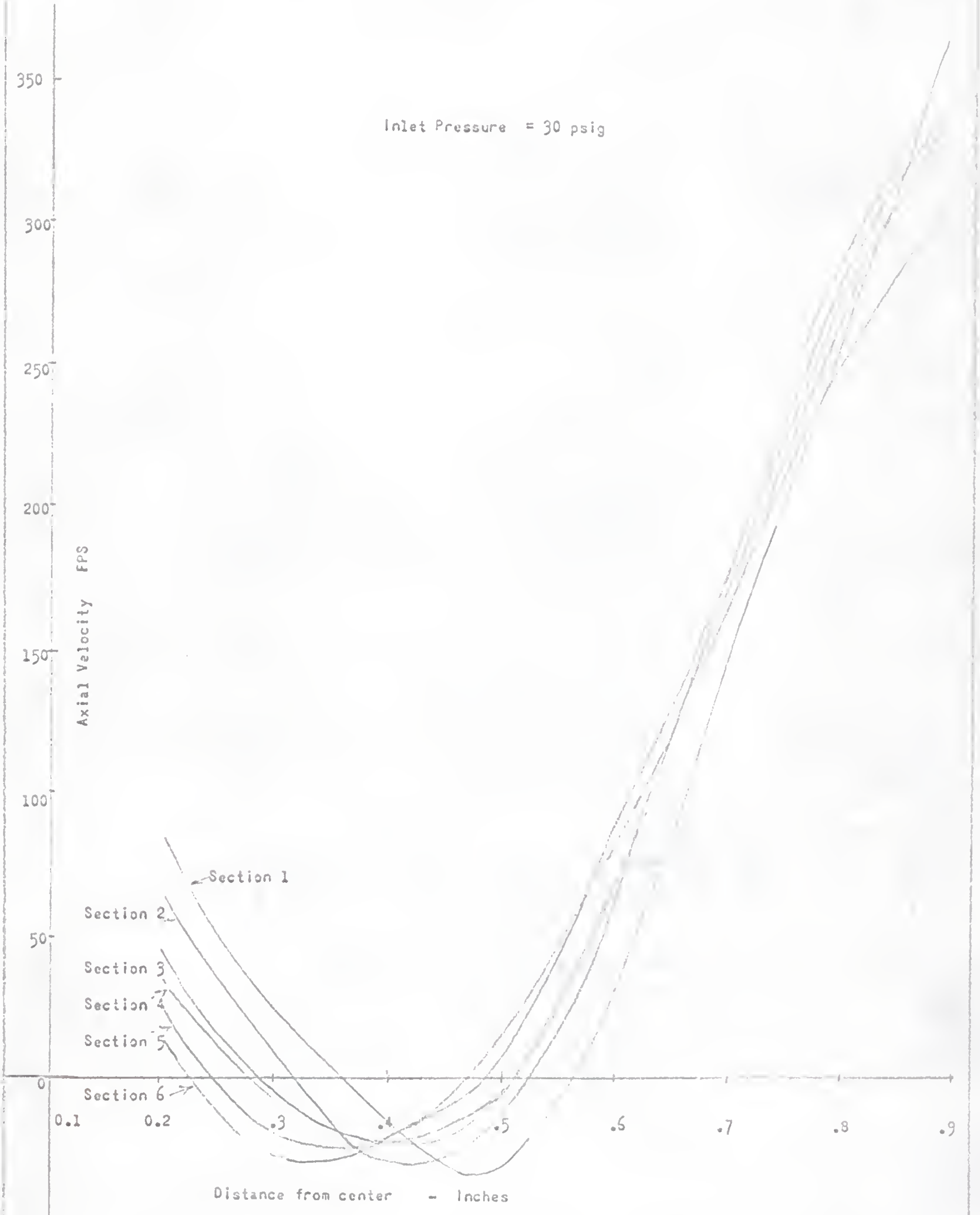


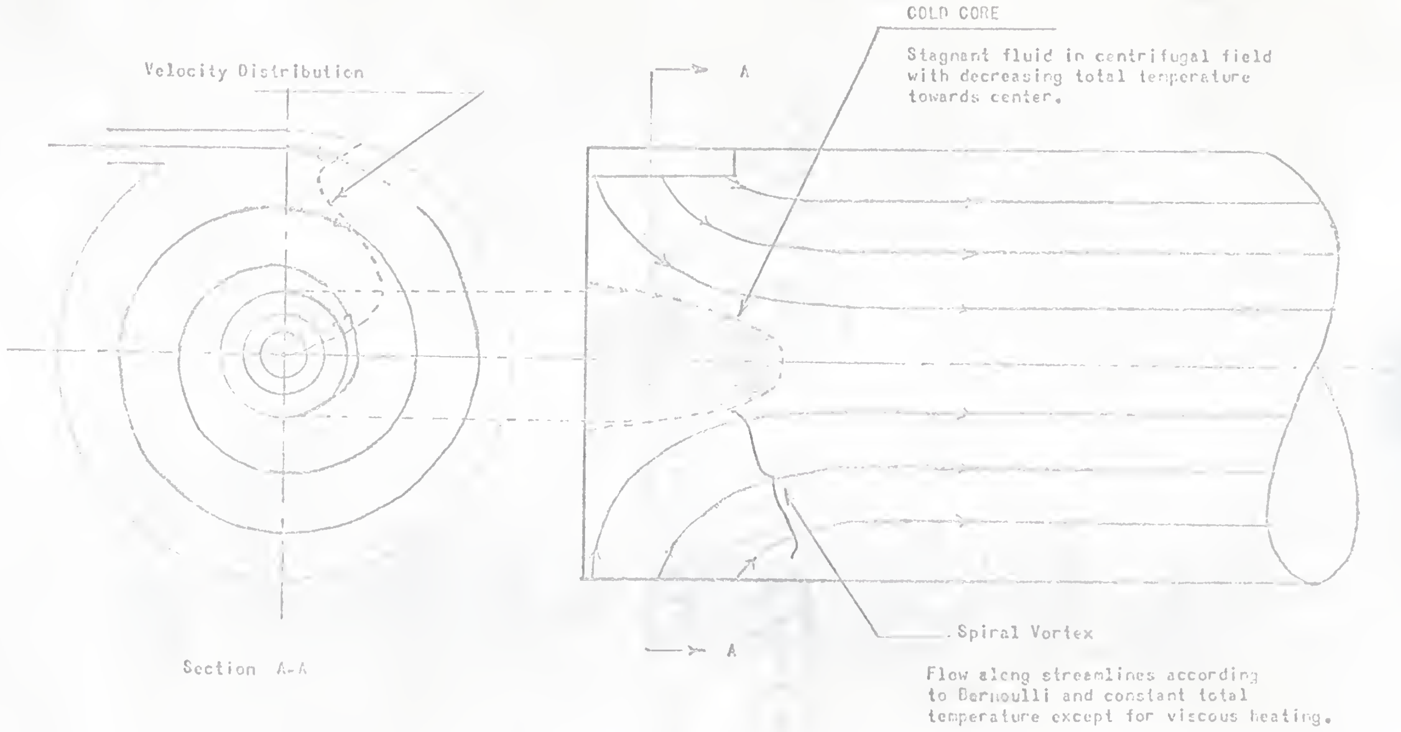
Fig. 9 Axial Velocity for 30-PSIG Inlet Pressure in 2" Uniflow Vortex Tube (Source Ref. 23)

the details of their apparatus. The measured flow conditions at various locations in the tube are plotted in Figure 7, 12, and 12a. The flow pattern observed in a five-inch diameter vortex tube is shown in Figure 14.

Lay [23] conducted systematic experimental study on a two-inch diameter uniflow vortex tube. The results are shown in Figure 8, 9, and 11. His observations on the flow of gas after injecting water are similar to those of MacGee.

The two experimental results of Eckert and Hartnett [10 & 13] and Lay [23] are in close agreement as to the velocity, pressure, and temperature distributions, which confirm the fact that energy separation is taking place only in the hot tube. The axial velocity distribution in both cases are shown in Figure 7 and 9. The predominant axial velocities are concentrated in a small annular region near the wall of the vortex tube. The axial velocities fall off sharply toward the center of the tube and a reverse flow occurs in the central portion. In the case of the counter flow vortex tube, the cold stream is at the central portion of the tube and the direction of flow is opposite to that of hot streams. Partly it can be said that the 'reverse flow' is the cause for the cold air to flow in the reverse direction. The temperature distribution indicates that the minimum temperatures are near the axis with the axial flow in the same direction as that of hot gas stream. So uniflow vortex tube obtains the minimum temperatures of the cold gas stream.

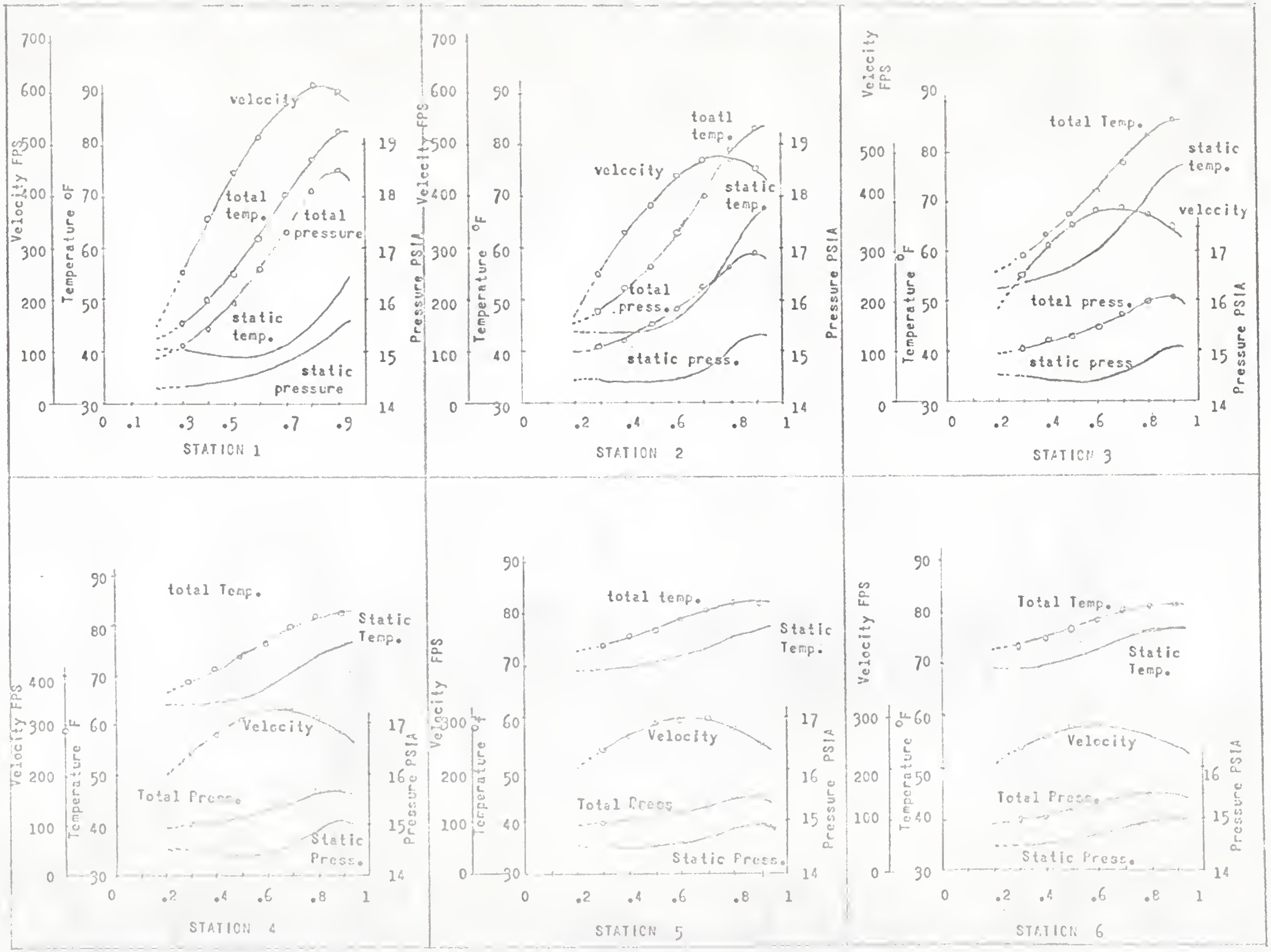
The reverse axial flow in the tube extends in an annular space throughout the full length of the tube, the inner radius of this annulus is approximately 20 per cent of the tube radius while the outer radius of the annulus is 40 per cent of the tube radius. The type of axial flow existing in the vortex tube (in the hot tube) is shown in Figure 9. The position of maximum



Stagnant fluid in centrifugal field with decreasing total temperature towards center.

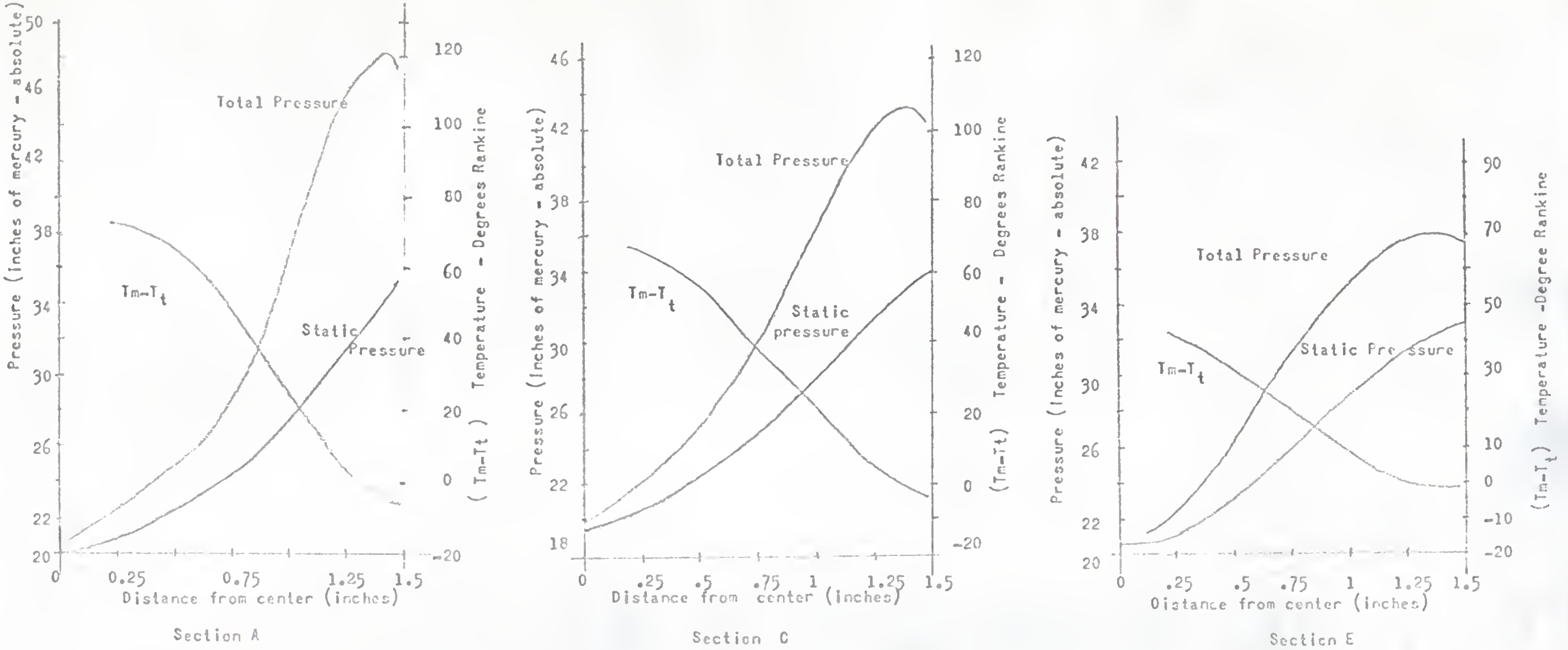
Flow along streamlines according to Bernoulli and constant total temperature except for viscous heating.

Fig. 10 Flow in a Vortex Tube (Centrifuge flow). Source Ref. 31



INLET PRESSURE 20 PSIG

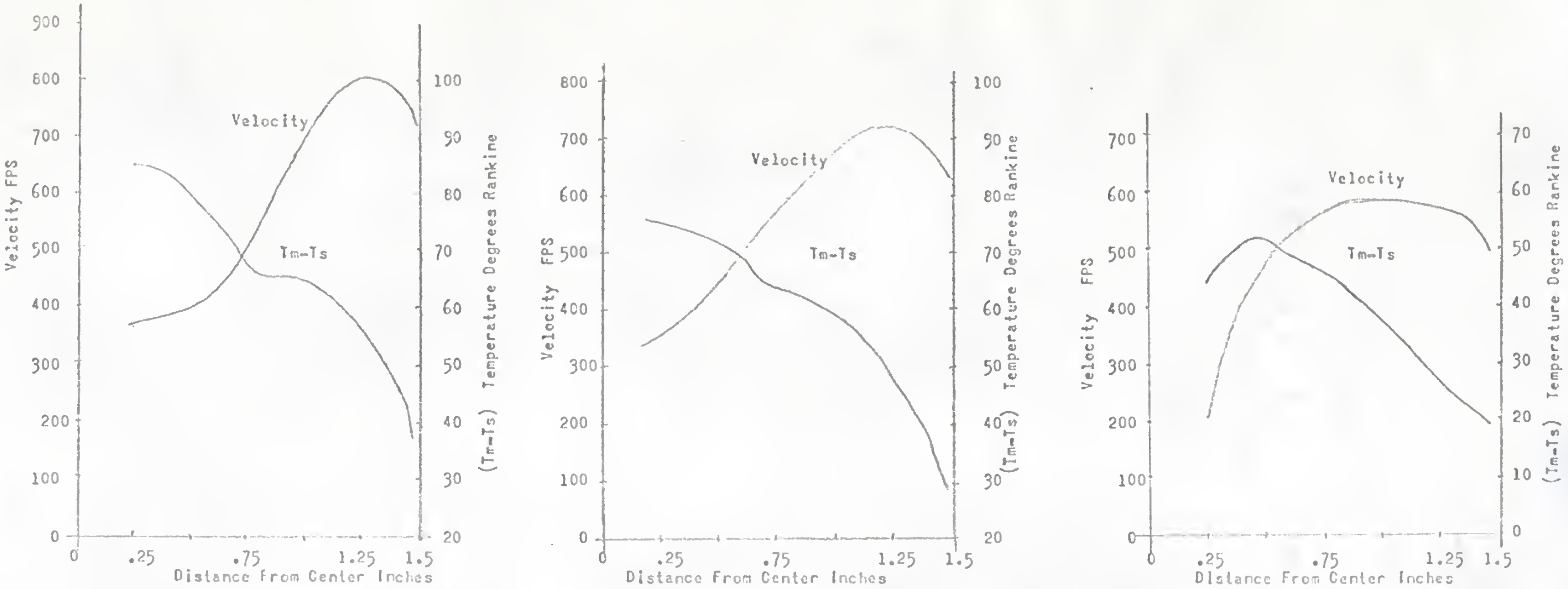
Fig. 11 Velocity, Temperatures and Pressures in 2" Vortex Tube (Uniflow). Source Ref. 23



INLET PRESSURE 20 PSIG

Fig. 12a Static and Total Pressures and Total Temperatures in 3" counterflow Vortex-tube

(Source Ref. 13)



Section A

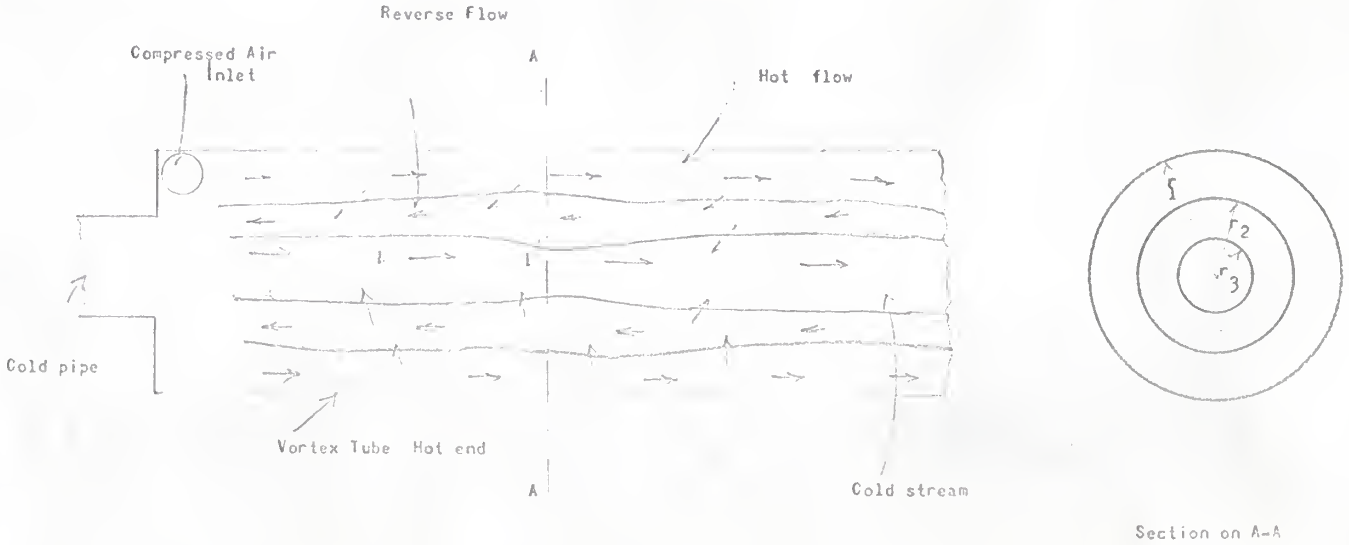
Section C

Section E

INLET PRESSURE IS AT 20 PSIG

Fig. 12b Velocity and Static Temperatures in 3" counter-flow Vortex-Tube

(Source Ref. 13)



$\frac{r_2}{r_1} = 0.4$ $\frac{r_3}{r1} = 0.2$

Fig. 13 Vortex Tube with reverse Flow Region Between Hot and Cold Streams

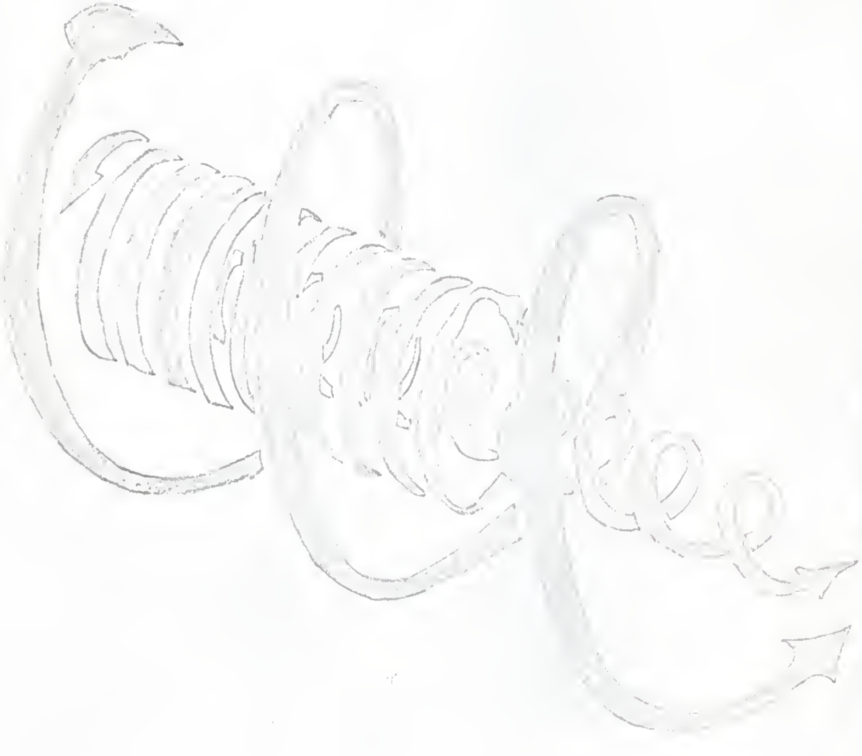


Fig. 14, Flow Pattern observed in 5" dia. Vortex Tube
(Source Ref. 10)

deviation takes place at an intermediate plane between nozzle and exit planes. The increase of inlet pressure makes the plane of maximum deviation of reverse flow to move away from the nozzle plane. The total temperature decreases rapidly with decrease in radius near the nozzle plane. As the flow moves away, the total temperature tends to become constant across the tube. The total temperatures in the reverse flow region, when compared with the maximum temperatures, are fairly low near the nozzle plane and approach the value of the hot end temperature away from the nozzle plane. The reverse flow is in between the cold core and the hot annulus in the tube. It appears that the energy is transferred through this zone. For maximum energy separation, the reverse flow length should be such that the static and total temperatures are equal at the end of this length. Scheper [39] in his theory on the vortex tube has explained a method to calculate the tube length. Figure 15 is a reproduction of the model used by Scheper. Figure 8 shows the maximum temperature, pressure, and velocity in the axial direction. The maximum temperatures and minimum temperatures are both approaching the same limit as the flow moves away from the nozzle plane. To get maximum temperature difference between the cold and hot streams of gas, a proper length of hot tube has to be chosen. It is seen that reverse flow mixes the flow of hot and cold streams, which are in the same direction and this mixing tends to limit the temperature difference between the hot and cold streams.

The characteristics of Fisher's [20, 21, & 22] , SS-8 (3/8 inch tube diameter) vortex tube are presented in Figures 16, 17, 18, 19, and 20. Figure 16 shows the maximum and minimum temperatures obtained with different hot tube lengths. With 13 inches of hot tube, at 100 psig inlet pressure, a minimum temperature of -30°F is obtained. Figure 17 shows the total hot

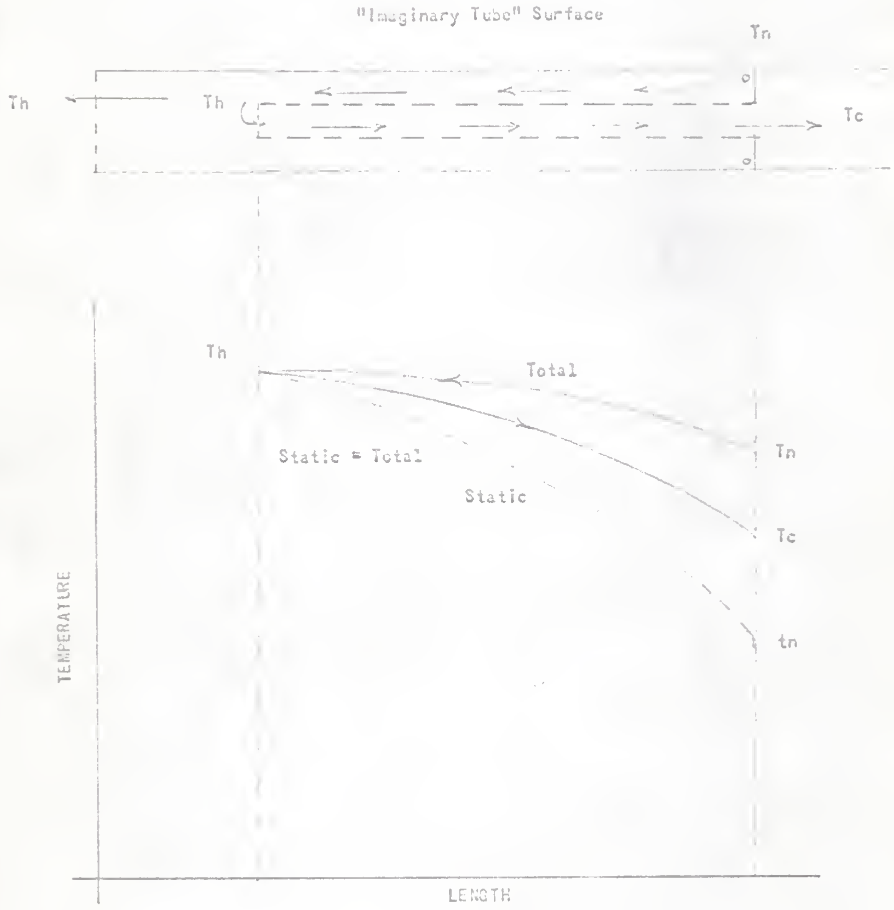


Fig. 15 Flow pattern and temperature nomenclature for theoritical derivation.

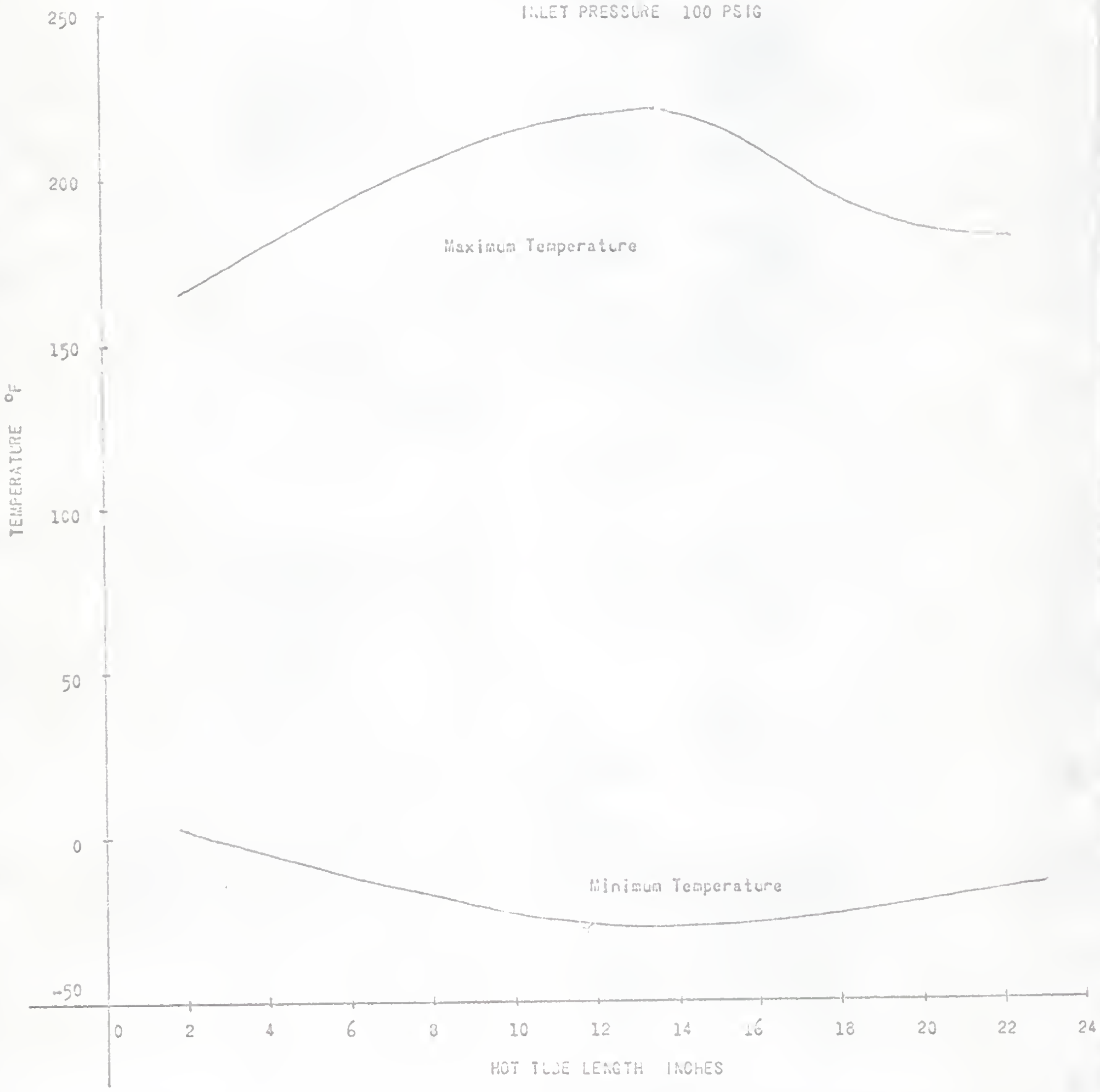


Fig. 16 Maximum and Minimum Temperatures with different hot tube length

(Ref. 20)

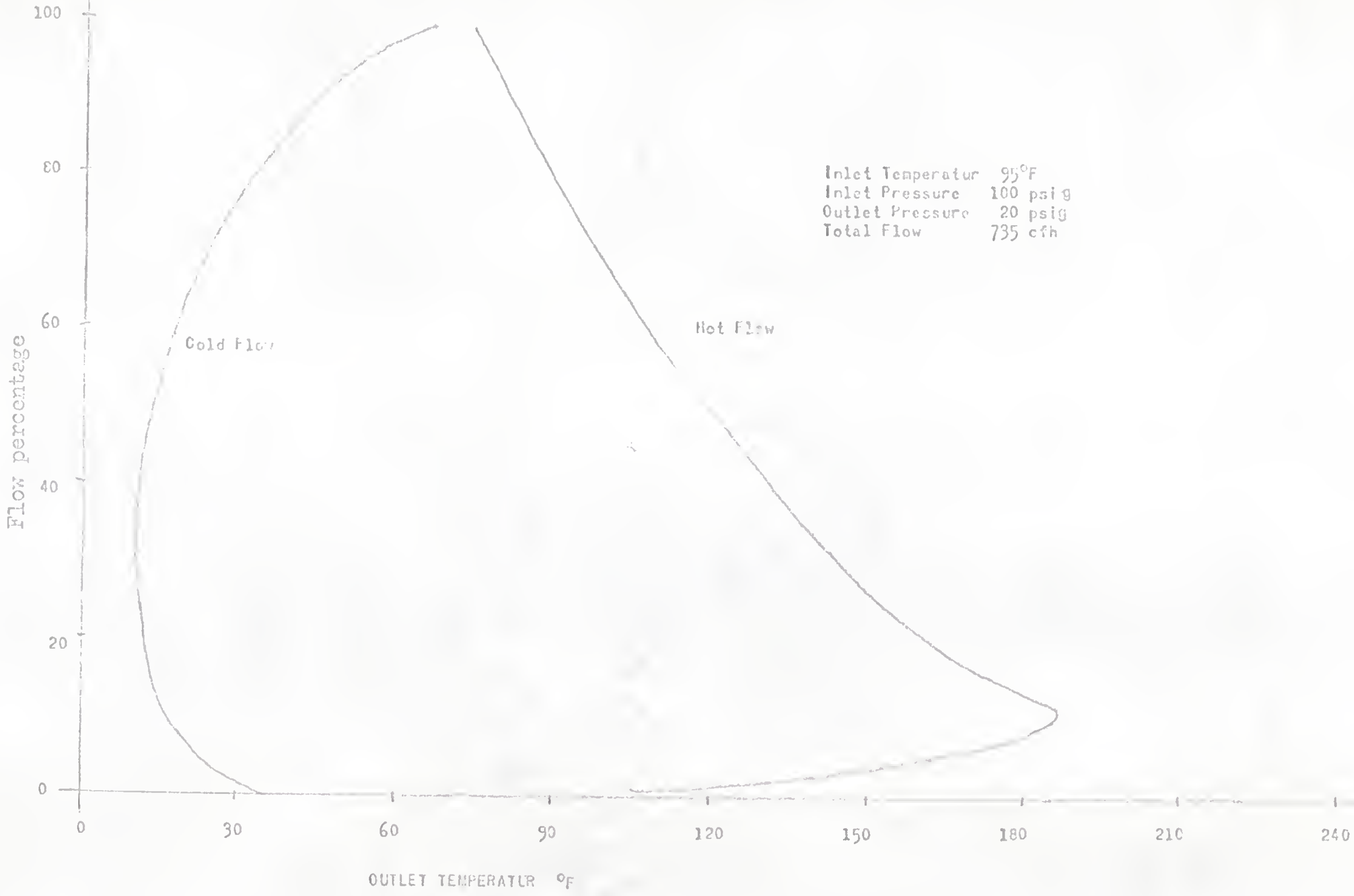


Fig. 17 (Source Ref. 21)

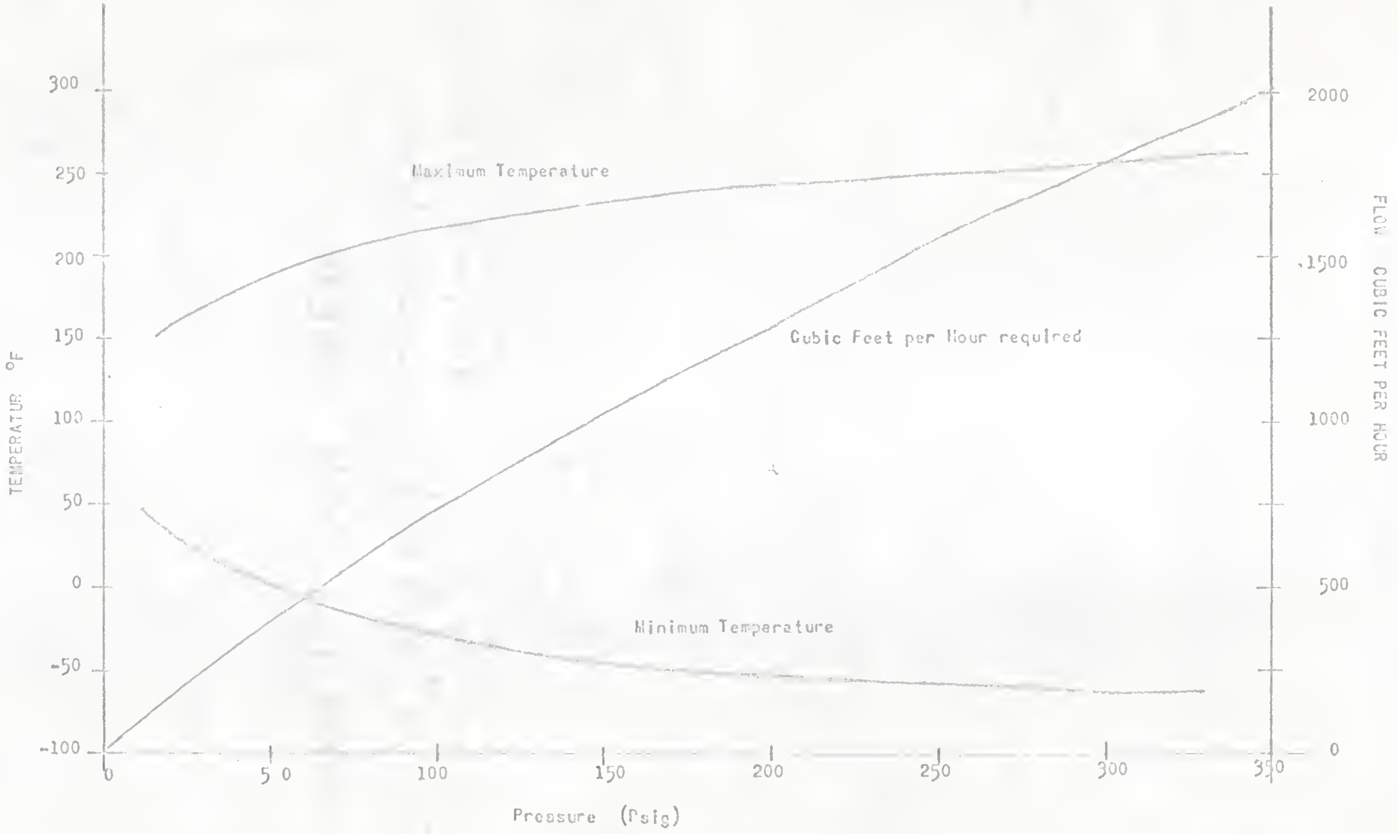


Fig. 18 Operation Characteristics of Fisher's Vortex Tube
(Source Ref. 20)

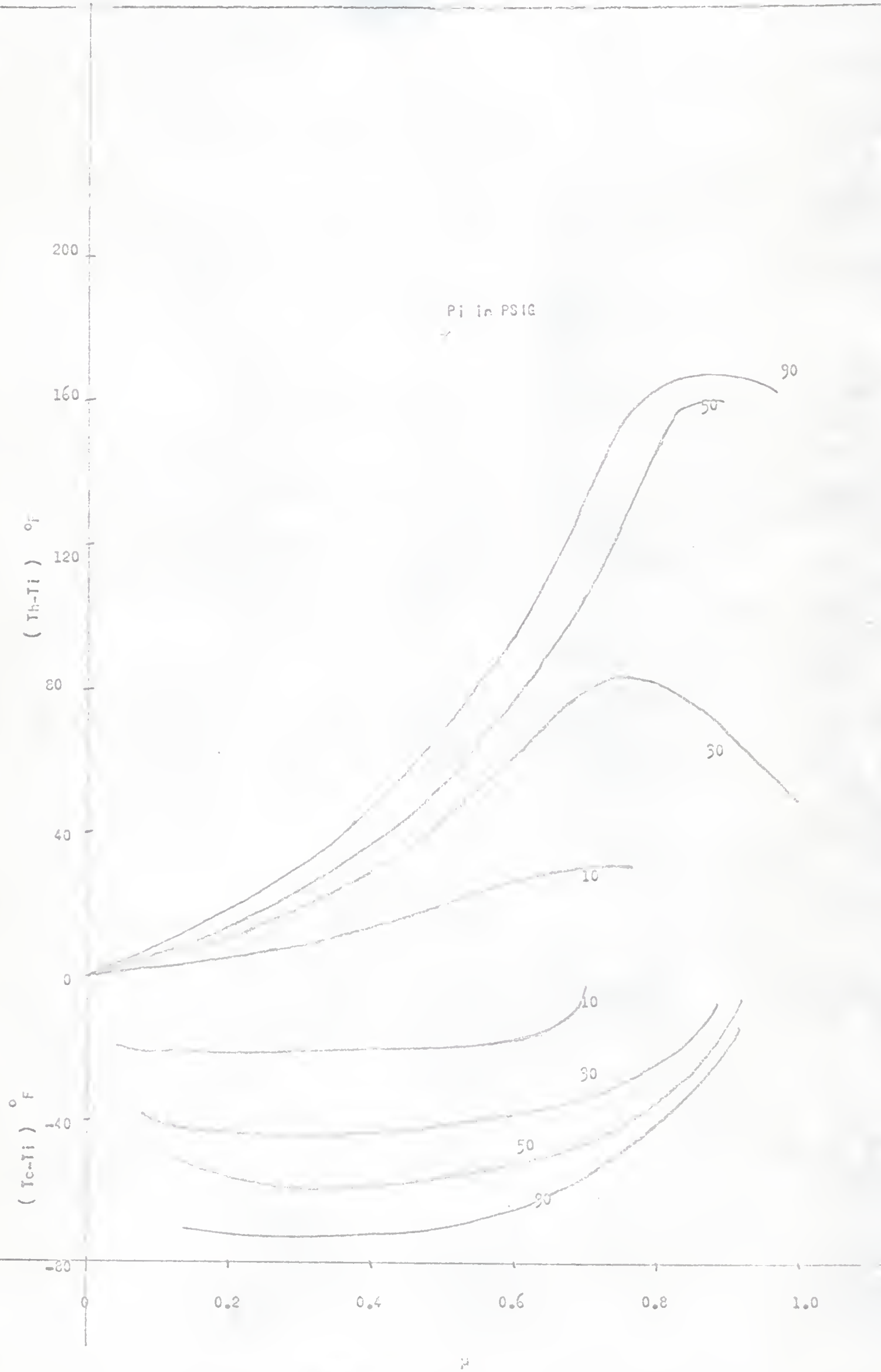


Fig. 19 (Source Fig. 22)

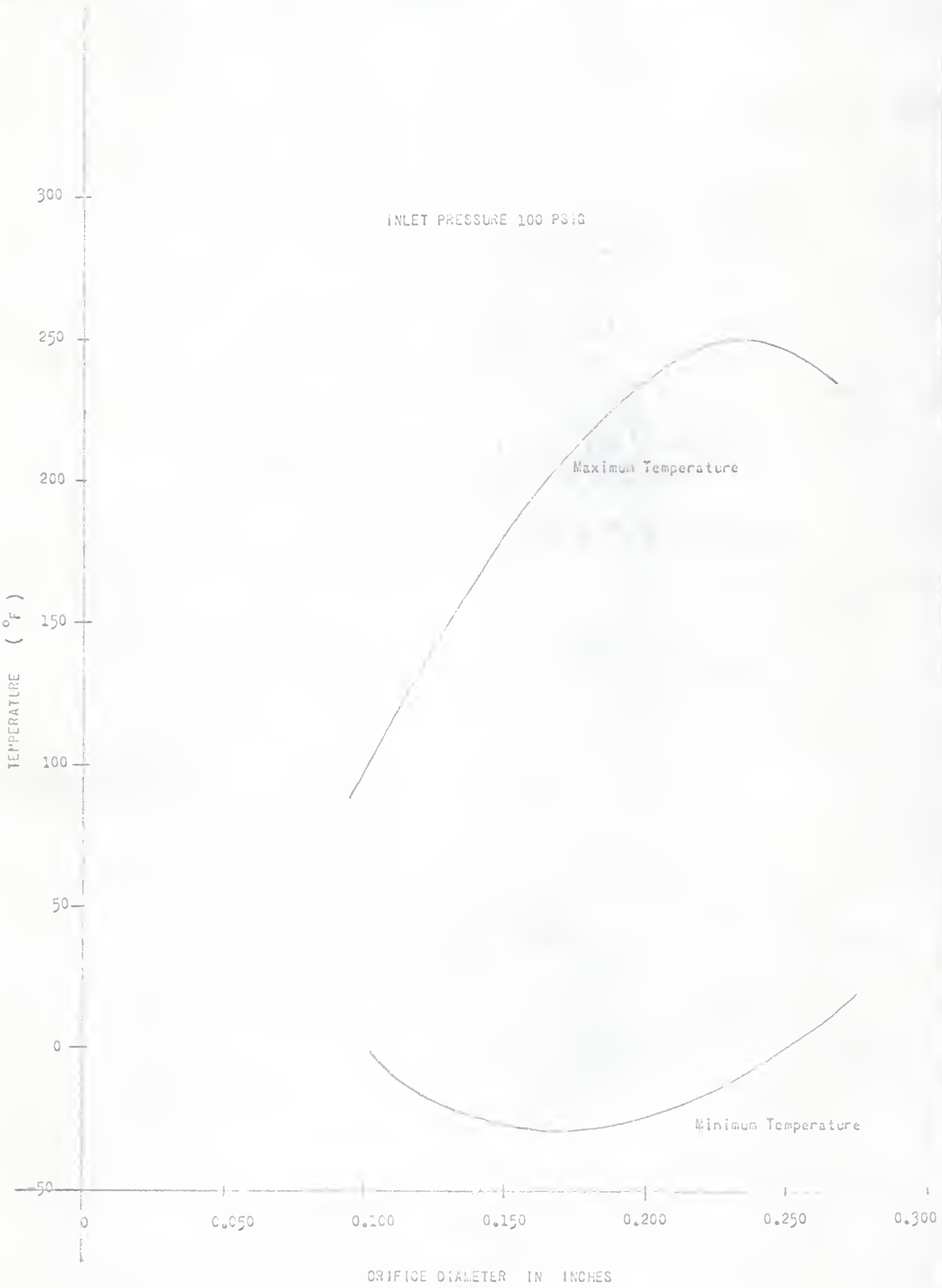


Fig. 20 (Source Ref. 20)

and cold flow variations and temperatures of the streams for a 100 psig inlet pressure. Figure 18 shows the operation characteristics of Fisher's SS-8 vortex tube. It is obvious that for a given geometry the increase of inlet pressure lowers the minimum temperature, which is however, limited to a certain minimum value. The increase of inlet pressure beyond this range does not result in further reduction in the temperature of the cold stream. The temperature depression at different cold flow rates, μ with different inlet pressures in the same vortex tube, are measured and presented in Figure 19. From the minimum temperature curve in Figure 20, it is evident that the orifice dimension is an important parameter in designing a vortex tube. The minimum temperature is obtained when the orifice diameter is approximately 0.17 inches, and the maximum temperature is obtained when the orifice, diameter is approximately 0.225 inches. The orifice diameter chosen for minimum cold stream will not give maximum temperature of the hot stream and vice versa.

The phenomenon of energy separation can be summarized as follows. The flow in the vortex tube is highly turbulent with a spiral flow pattern. The pressure, temperature, and velocity profiles are of spiral pattern, the center being at the axis of the tube. For energy separation, the fluid must be compressible. A part of the fluid from the inner periphery of the tube follows a spiral path, whose center is on the axis of the tube, passes through the hot-flow region, reverse-flow region, and cold-flow region as mentioned previously, and reaches the core region. Since the velocity profile is inviscid near the wall, the shear (turbulent shear) work done on the element near the wall is positive and hence the element gets heated up.. Near the core region (including reverse and cold flows regions) the velocity profile is no longer inviscid and the shear work (turbulent shear work) done on the element is negative, due to the slowing down tendency of the turbulent viscosity, therefore, the element cools down. The effect is that the fluid in the core region does shear work on the fluid in the outer region through turbulent mechanism, resulting in the total temperature separation. Counter flow always tends to minimize the separation by conducting heat from the outer layer to the inner core, thus making the difference between maximum and minimum total temperatures smaller as the distance from the nozzle plane increases. Savion and Ragsdale [37] , working on temperature and pressure measurements in a confined vortex tube, observed that considerable energy separation can take place in a vortex contained between two flat disks without any attached tube. The flow emerged from an opening at the center with a diameter of the order of the plate spacing. Most of the energy separation takes place near the opening.

Hartnett and Eckert [13] reported an interesting result. They compared the maximum difference in total temperatures encountered at 20 psig inlet pressure in the 3-inch counter flow vortex tube with the difference in total temperature encountered in a boundary layer on a flat plate in a flow with a velocity equal to the maximum velocity in the tube, and obtained the same energy separation in both cases.

VORTEX TUBE EFFICIENCY

The vortex tube process of heating or cooling is irreversible. Even if the wall is at adiabatic conditions the entropy change is positive, due to irreversibilities. The hot gas from the tube is not used in a refrigeration device. But the hot gas contains a good amount of thermodynamic availability due to its temperature and pressure. There is usually no way to use these availabilities directly in the application. If additional equipment is added to recover the availability of these hot gases, the simplicity of the vortex tube would be lost.

In the domestic refrigerator, the heat is pumped out from the body at a constant temperature. The Carnot refrigerator is taken in these studies as an ideal machine, and the compression machine is taken as a practical machine, for the purpose of comparison with the domestic refrigerator. The vortex tube, in which there is only a simple stream of cold gas, is not suitable for this purpose, and its use as a refrigerator, as previously stated, is limited because of high irreversibility and low efficiency.

The use of vortex tube as an air conditioner requires an isothermal compressor and an adiabatic expander. By the use of a simple compressor machine, the efficiency of the vortex tube is only about one-half.

Considering the basic and non-basic criteria the efficiency of the vortex tube can be analyzed in detail. Fulton [11] states,

> "If a machine which is ideal for one purpose is used as a criterion for a machine serving the other purpose, the criterion is not basic."

Non-basic Criteria -

Carnot criterion - The coefficient of performance for the vortex tube is defined as equal to the heat taken out of the cold gas divided by the work of reversible isothermal compression.

$$\text{C.O.P.} = \frac{\mu\ J\ c_p\ (T_1 - T_c)}{R\ T_1 \ln \frac{p_1}{p_c}} \qquad (9)$$

where p_1 and T_1 are the pressure and temperature of the supplied gas, P_c and T_c are the pressure and temperature of the cold gas leaving the tube, c_p is the specific heat at the constant pressure, R is the gas constant and 'μ' is the cold fraction defined by ratio of mass rate of cold flow to the total mass rate flow. The coefficient of performance according to Fisher Governors [22] data is 10.12 per cent and it is obtained at 0.4 μ, 50 psig inlet pressure and 74^0 inlet temperature.

The coefficient of performance of a Carnot refrigerator working between T_c and T_1 is equal to $\frac{T_c}{T_1 - T_c}$.

Dividing equation (9) by the Carnot coefficient of performance,

$$\text{Efficiency (Carnot)} = \frac{\mu J c_p (T_1 - T_c)^2}{R T_1 T_c \ln \frac{p_1}{p_c}} \qquad (10)$$

Dividing the numerator and denominator by T_1^2,

$$\text{Efficiency (Carnot)} = \frac{\mu J c_p \left(1 - \frac{T_c}{T_1}\right)^2}{R \frac{T_c}{T_1} \ln \frac{p_1}{p_c}} \qquad (11)$$

Let T_2 be the temperature that would be reached in a reversible adiabatic expansion from P_1, T_1 to P_c, then

$$\frac{P_1}{P_c} = \left[\frac{T_1}{T_2}\right]^{\frac{k}{k-1}} \qquad (12)$$

Taking the logarithm on both sides,

$$\ln \frac{P_1}{P_c} = \frac{k}{k-1} \ln \frac{T_1}{T_2} \qquad (13)$$

For a perfect gas,

$$C_p - C_v = \frac{R}{J} \qquad (14)$$

and

$$\frac{C_p}{C_v} = k \qquad (15)$$

Combining equations (14) and (15),

$$\frac{J\, C_p}{R} = \frac{k}{k-1} \tag{16}$$

Substituting equations (13) and (16) in equation (11),

$$\text{Efficiency (Carnot)} = \mu \, \frac{\left(1 - \frac{T_c}{T_1}\right)^2}{\frac{T_c}{T_1} \ln \frac{T_1}{T_2}} \tag{17}$$

Fulton obtained [11] a maximum efficiency of 2.8% at 11 atmosphere inlet pressure with Hilsch's data. Initially it was believed that the efficiency of the vortex tube would be over 10 per cent. With Fisher Governor's data at 50 psig inlet pressure, 74°F inlet temperature and 0.4 μ, an efficiency of 1.012 per cent is obtained.

Criterion for refrigeration at fixed temperature - Let cold air be used to remove heat from a body at a fixed temperature, say T_b. Then, cold air can be used from T_c up to T_b. So, the efficiency becomes,

$$\text{Efficiency (fixed temperature)} = \frac{\mu\, J C_p\, (T_b - T_c)\,(T_1 - T_b)}{R\, T_1\, T_b \ln \frac{p_1}{p_c}} \tag{18}$$

The efficiency becomes zero if the body temperature T_b is equal to T_c. This implies that the cold air is not utilized. Under no circumstances is T_b equal to T_1, to make the efficiency zero, as the Carnot work in zero for this condition. The efficiency expected is only about one-half.

Turbine Criterion: Consider that a vortex tube is compared with a reversible adiabatic turbine producing the same amount of cold gas μw at the same temperature T_c, at the expense of turbine work. This turbine pressure ratio will be smaller than the vortex tube. The compressor used for this purpose will have to handle only the flow μW instead of W. The turbine efficiency then is the ratio of the work of the turbine's compressor to the work of the vortex tube compressor, which is,

$$\text{Turbine Efficiency} = \mu \frac{J\, C_p \ln \frac{T_1}{T_c}}{R \ln \frac{P_1}{P_c}}$$

$$= \mu \frac{\ln \frac{T_1}{T_c}}{\ln \frac{T_1}{T_2}} \qquad (19)$$

The efficiency in this criterion is about 13 per cent at 11 atmospheres with 0.7 fraction of cold air as can be seen with Hilsch's data [11 & 16] . Fisher Governor data [22] at 0.4 μ, with 50 psig inlet pressure and 74°F inlet temperature, gives a turbine efficiency of 10.62 per cent.

Hilsch's criterion: - Fulton [11] reported that the efficiency given by Hilsch is nothing but the coefficient of performance, and he checked this statement with the graphs given by Hilsch.

Basic criterion: -

The basic criterion requires that the vortex tube be compared with a reversible producer of a cold gas, the latter being a reversible isothermal compressor followed by a reversible adiabatic expander. Under the light of the availability concept, this can be discussed as follows:

The availability of a system is defined as the minimum amount of work required to bring the system from the dead state to the given state. In other words, it is the maximum amount of work that can be obtained in reducing the given system to the dead state.

Under steady flow conditions, in the absence of change in electricity, magnetism, capillarity, gravitational position and velocity, availability is given by,

$$a = J \left[(h - h_d) - T_d (s - s_d)\right] \qquad (20)$$

in which the subscript, d, refers to dead state, namely the environment at P_d (Keenan 19).

For a perfect gas,

$$(h - h_d) = C_p (T - T_d) \qquad (21)$$

$$s - s_d = C_p \ln \frac{T}{T_d} - \frac{R}{J} \ln \frac{P}{P_d} \qquad (22)$$

Substituting the values of equations (21) and (22) in the equation (20) and simplifying, the expression for availability becomes,

$$a = J C_p T_d \left[(\frac{T}{T_d} - 1) - \ln \frac{T}{T_d} + \frac{k-1}{k} \ln \frac{P}{P_d} \right] \qquad (23)$$

Considering small temperature drops, the pressure drop is very small, so the ratio $\frac{P}{P_d}$ is nearly equal to unity, and so the logarithm of $\frac{P}{P_d}$ may be neglected

$$a = J C_p T_d \left[(\frac{T}{T_d} - 1) - \ln \frac{T}{T_d} \right] \qquad (24)$$

Further, expanding the term $\ln \frac{T}{T_d}$ for $2 < \frac{T}{T_d} < 0$,

$$\ln \frac{T}{T_d} = (\frac{T}{T_d} - 1) - \frac{1}{2} (\frac{T}{T_d} - 1)^2 + \frac{1}{3} (\frac{T}{T_d} - 1)^3 \qquad (25)$$

Substituting in equation (24) the availability for small temperature drops is

$$a_T = J\, C_p\, T_d \left[\frac{1}{2} \left(\frac{T}{T_d} - 1\right)^2 - \frac{1}{3} \left(\frac{T}{T_c} - 1\right)^3 + \cdots \right] \qquad (26)$$

Defining the efficiency of the vortex tube, as a producer of cold gas, as equal to the ratio of the work required by a reversible machine producing the same amount of cold gas at the same temperature and work required by the vortex tube using a reversible isothermal compressor,

$$\text{Efficiency} = \frac{A_c}{A_1} = \frac{W_c\, a_c}{W_1\, a_1} \qquad (27)$$

$$= \mu \frac{a_c}{a_1}$$

$$= \mu \frac{\frac{T_c}{T_d} - 1 - \ln \frac{T_c}{T_d}}{\frac{k-1}{k} \ln \frac{P_1}{P_c}} \qquad (28)$$

Fulton [11] evaluated the efficiency curves for various pressures based on Hilsch's data [16], and obtained efficiencies in the neighborhood of one per cent. He also mentioned that the efficiency may reach two per cent for large vortex tubes. If a means is found to use the hot gas to produce work the efficiency ranges between 10 and 20 per cent.

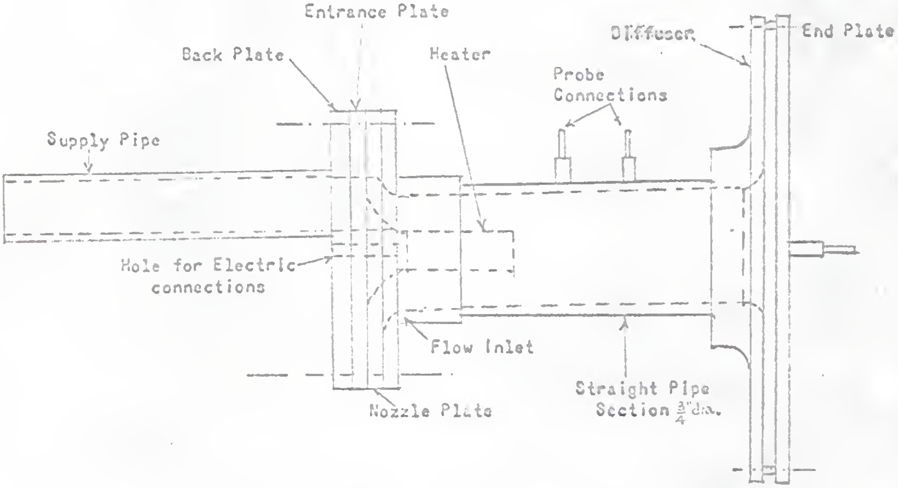


Fig. 21 Vortex Tube Cooler (Source Ref. 4)

APPLICATIONS OF THE VORTEX TUBE

Ranque might have thought of using vortex tube as a domestic refrigerating unit. He also devised a unit which had its own compressor. He did not perhaps notice some drawbacks in the cycle. It had low efficiency, which he later realized, and abandoned the idea of building the unit altogether. But its simplicity and easy operation caused many engineers to make use of the phenomenon. The result is the vortex tube thermometer, the cooler and other special applications. They are discussed below in detail.

If a large quantity of compressed air is available, it is very easy to get cooling and heating with this apparatus. Fisher Governor Company designed and improved a vortex tube for various laboratory and industrial uses, such as a temporary and convenient source of refrigeration, for cooling electronic equipment, for junction points in pyrometers and for cooling small die blocks. It is capable of producing sub-zero temperatures and high temperatures in excess of 250°F.

Vortex tube cooling device: - An improved vortex tube cooler is shown in Figure 21. Blatt and Trush [4] designed this for the first time, and studied its effect on hot bodies. This differs from the conventional vortex tube in not having the cold air outlet. The comparison between the cooler and conventional vortex tube with turbine analogy is clearly shown in Figure 22. The diffuser portion at the gas exit recovers most of the kinetic energy in the outer region of the tube, thus increasing the exhaust pressure.

The differences in the method of operation have the effect that the decrease in the stagnation temperature from the inlet of the nozzle to the central region is in a much higher percentage of the isentropic static

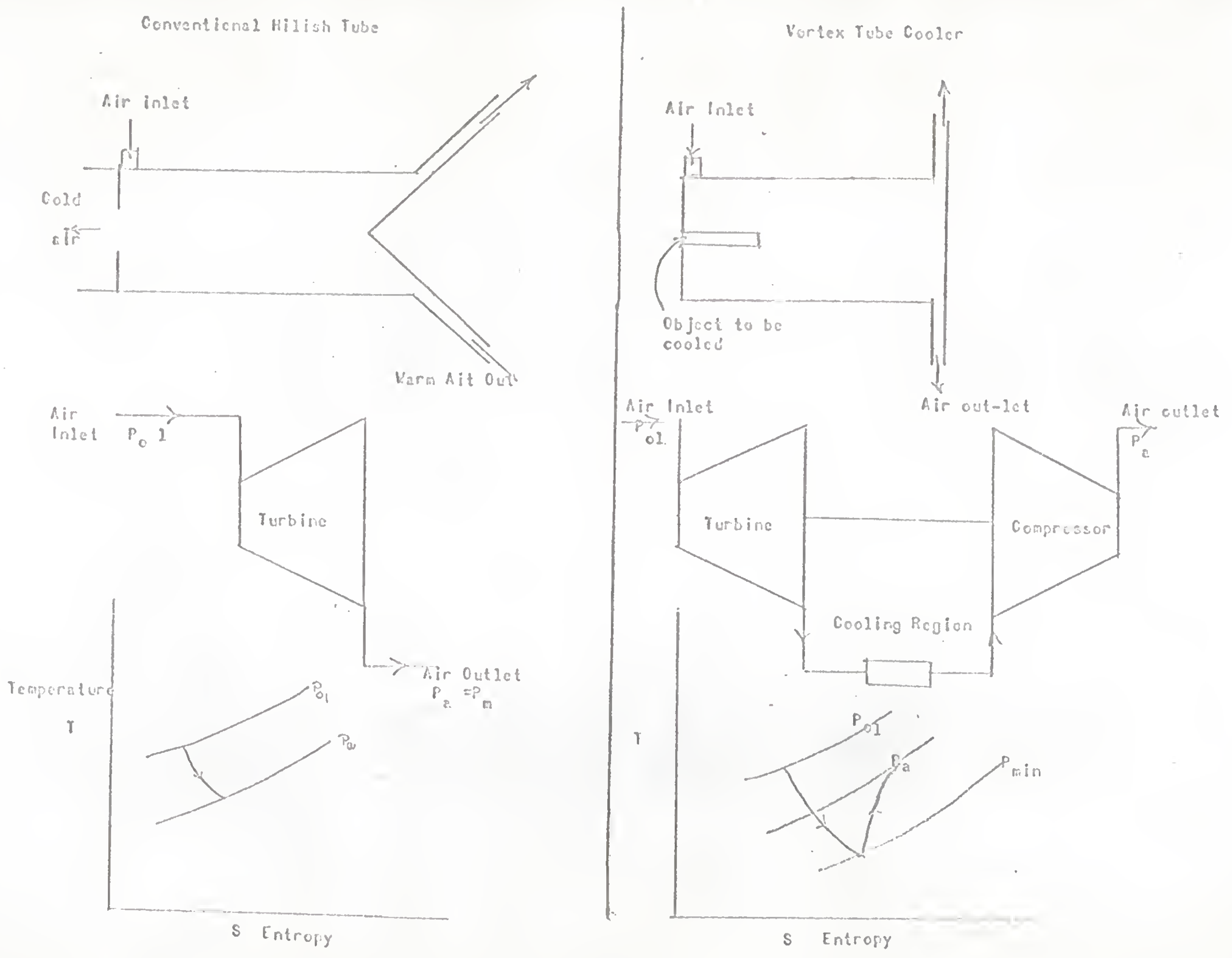
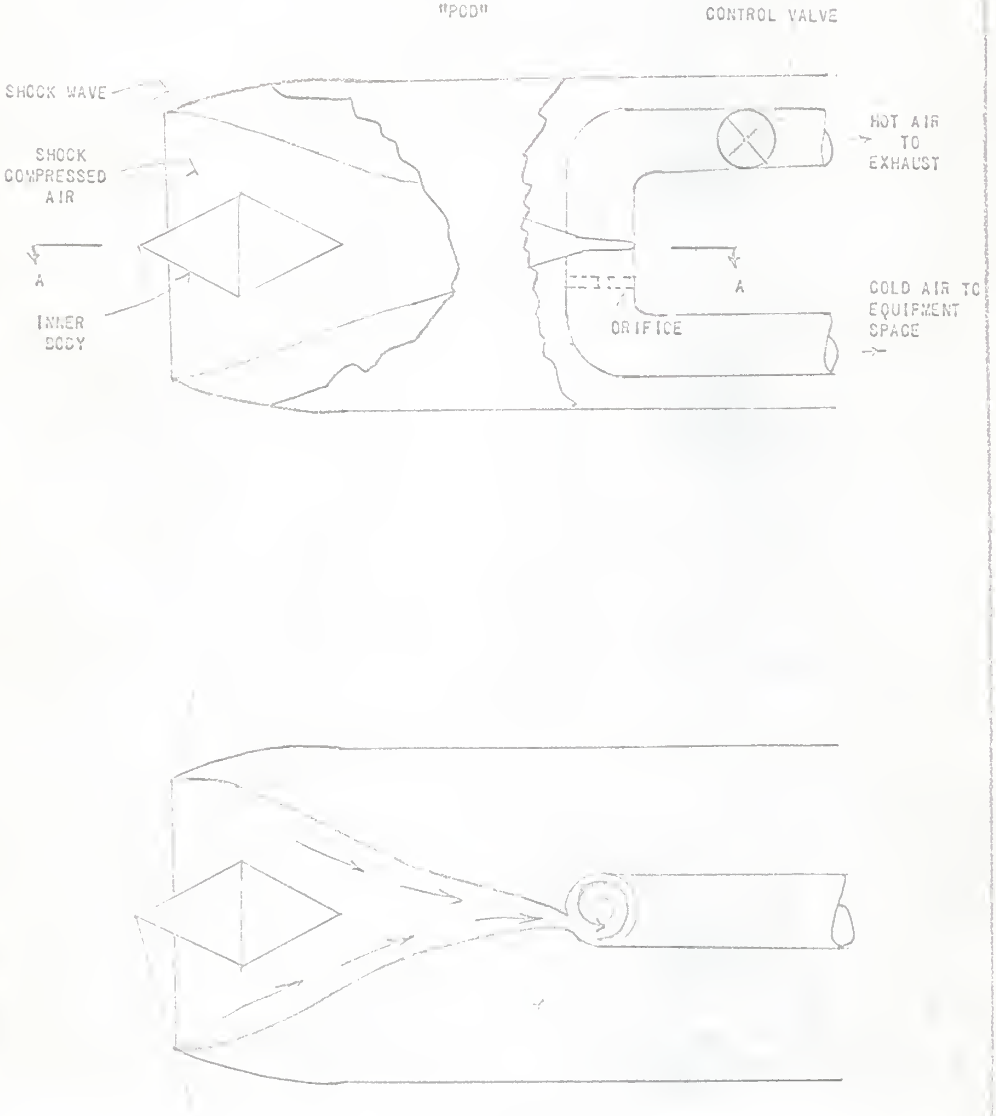


Fig. 22 Turbine analogy (Source Ref. 4)



SECTION ON A-A

Fig. 23 Vortex tube cooler for high speed air craft
(Source Ref. 44)

temperature drop between inlet and exhaust pressure than that obtained in the conventional vortex tube. In some cases the measured temperature drop is even less than this isentropic temperature drop.

According to Blatt and Trusch [4] the maximum temperature drop is recovered at the axis in the range of Z = 1-1.5d, where Z = distance from the edge of the cooler and d = pipe diameter of the cooler. Using a vaned diffuser they obtained a final temperature of - 108 F with an absolute pressure of 4.8 psia at the center of the cooler. The air enters the cooler at a pressure of 45 psia and a temperature of 70^0F. Further investigation has to be performed to find out the phenomenon for choking at flow rates that are less "than that corresponding to an Mach number of unity over the cross section." The air flows only in the outer region of the pipe. There is less turbulence in the tube, so the energy separation, as explained previously, will not take place. Thus, no temperature drop can be obtained.

Vortex tube air craft cooler for supersonic craft: - Figure 23 shows a proposed set-up of vortex tube as a cooler for equipment in a supersonic aircraft. The equipment was devised and studied by engineers of the Heat Transfer Laboratories of the Hallicrafters Company [44] . They used the vortex tube phenomenon to separate cold and hot streams in a compressed air shock.

Equipment to be cooled will be housed in a "POD" attached to the aircraft. The inlet of the "POD" is similar to that of a ram jet engine. A shock stands at the entrance as shown in Figure 23. The air after the shock expands in the vortex tube. With an ambient air temperature of 60^0F, the stagnated tube inlet air temperature of an aircraft at Mach 2, sea level, is 450^0F. According to Hallicrafter [44] , the Maxwell-Boltzmann distribution of air molecule velocities under these conditions shows that 65 per cent of the

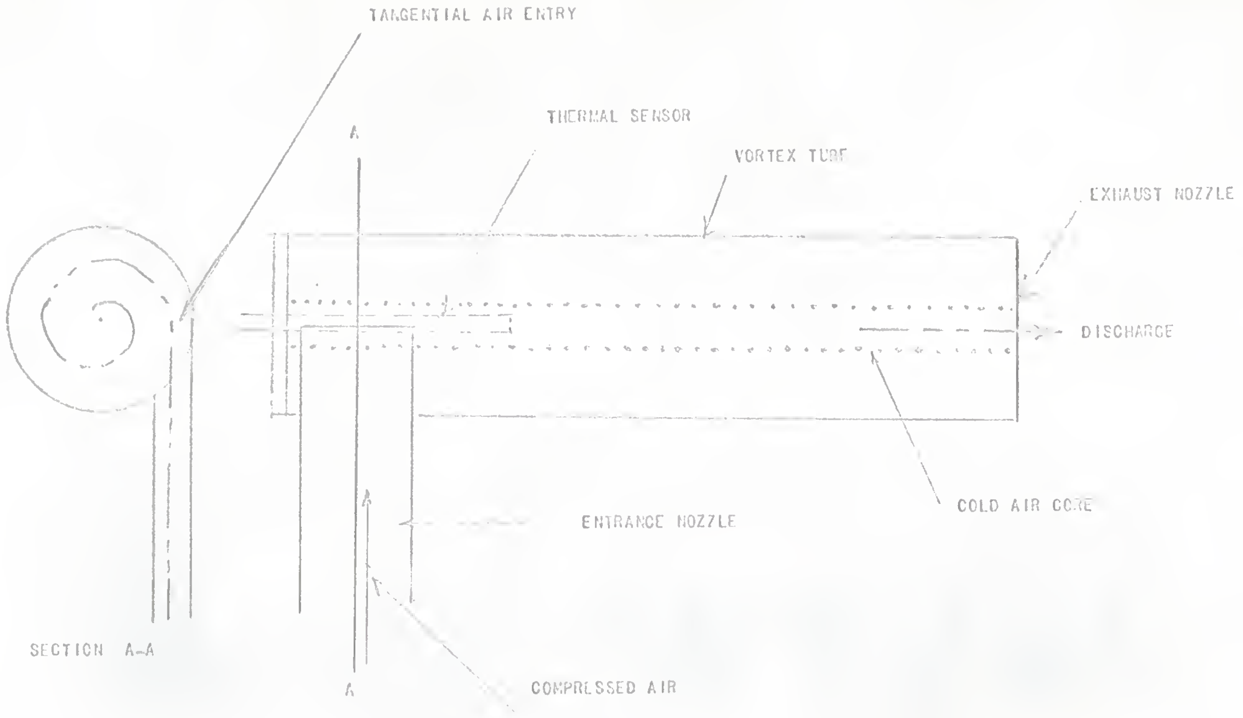


Fig. 24 Vortex Tube Thermometer

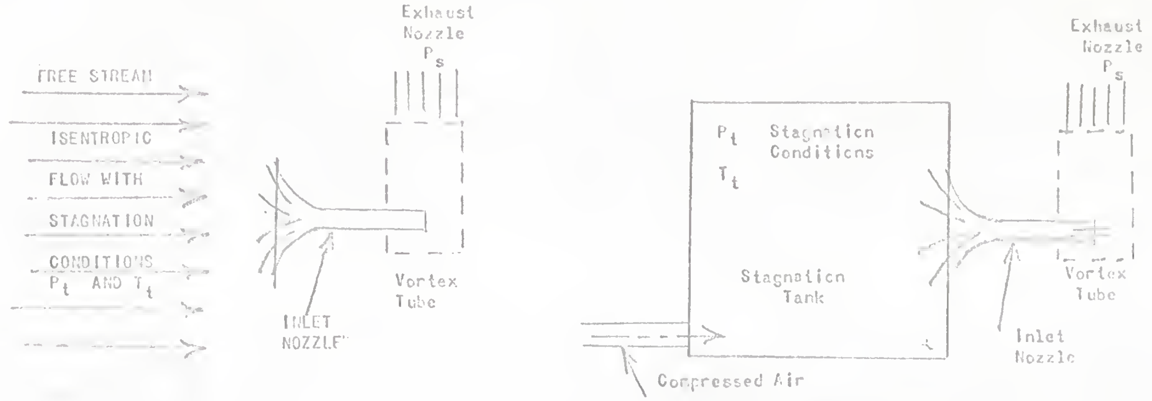
air molecules accepted by the tube have a mean velocity corresponding to an air temperature of 131°F. Under ideal conditions, when the total flow of air is 1.56 lbm per minute at 475°F, 1.00 lbm per minute of cold air is produced at 131°F.

In spite of the low efficiency of the unit its simplicity promises sufficient savings in space, weight, power, and cost over mechanical types of refrigerating equipments, and is a good device to be used in modern aircrafts.

Vortex tube as a centrifuge: - The tangential velocities in the vortex tube are sonic. One may feel like using the vortex tube as a centrifuge. After careful study of the vortex tube, one may realize that these high velocities are over a limited range. The tangential component of velocity increases with decrease in radius. This can be seen in Figures 11 and 12b. According to Corr [7] the centrifugal acceleration is approximately one million times greater than gravity for a velocity of 100 fps at the wall of $\frac{3}{4}$" vortex tube, and at such a high velocity it is expected that there would be separation of water particles from water vapor. But Norton, as reported by Corr, analyzed some samples of hot and cold streams with the mass spectrograph and could not find such separation of water particles. The reason for this is that the flow in the vortex tube is one having highly turbulent mixing [8] . Thus, no centrifuging effect is here.

Vortex tube thermometer:. - The first vortex tube thermometer was designed and studied by the Cornell Aeronotics laboratory [2] . Figure 25 shows the thermal sensor with a vortex tube used as a thermometer to measure the free stream air temperature. The principle involved is as follows:

Let the nozzle of the vortex tube point in the direction of motion of airplane such that the air is forced through the nozzle. If the aircraft



TRUE FLIGHT

LABORATORY SIMULATION

Fig. 25 (Source Ref. 2)

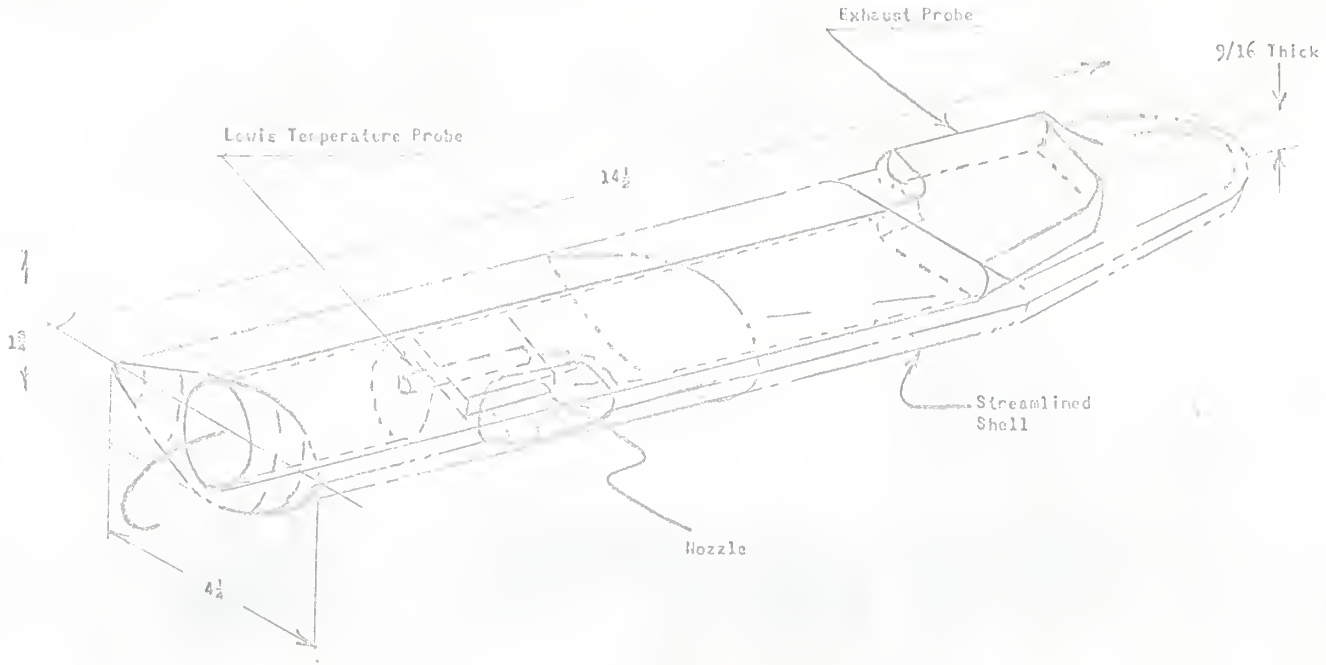


Fig. 26 Vortex Tube Thermometer Flight Model (Source Ref. 2)

is flying at Mach number, M, the stagnation temperature, T_s, is related to M and the free stream temperature, T_f, by the following equation:

$$T_s = T_f \left(1 + \frac{k-1}{2} M^2\right) \tag{29}$$

A thermocouple located at the entrance of the nozzle would read the stagnation temperature, T_s, for unity recovery factor. Therefore, to compensate for the aerodynamic heating of the stagnation probe, the thermocouple is located in the cold portion. Then the temperature read by the thermocouple will be the true air temperature. Figure 26 is the set-up for the test simulation. The actual scale model of the equipment is shown in Figure 26. This design has been tested and proved to be fairly successful for sonic and supersonic speeds [2] .

Vortex Tube Heater: - In the preceeding pages, the utilization of the cold gas stream for cooling purpose has been discussed. The hot gas is not used in any of these apparatus. A Vortex Tube Heater can be used to heat the base of a supersonic aircraft, increase the base pressure, and in turn, reduce the base pressure drag on the air foil. Bowyer and Carter [6] reported that in supersonic aircraft burning fuel at the base of an air foil requires less total fuel to overcome drag, then is required when fuel is used only in the engines. After thorough experimentation it can be found that hot air from the vortex tube can be used for this purpose. Fairly cool gases from the later stages of the main turbo engine may be expanded in a vortex tube. The hot gases coming from the tube are utilized for heating the base, and the cool gases are circulated over the hot tube to prevent it from melting.

Another use of the Vortex Tube Heater is to obtain higher propellent temperatures in a nuclear rocket than could be obtained by the nuclear reactor alone, which is limited in temperature by the reactor materials. The walls of the vortex tube could be cooled by the cold stream to prevent them from melting, and this cold stream could be reheated in the reactor.

SUMMARY AND SUGGESTIONS

The flow in the vortex tube is spiral and highly turbulent. There is a reverse axial flow between hot and cold spiral flows in the tube. Energy separation in the tube is due to turbulent shear work done on the element which is positive on the outer layers and negative on the inner layers. The reduction in the energy separation is due to the extended reverse flow away from the nozzle plane.

The efficiency of the vortex tube have been discussed with basic and non-basic criteria to show the real efficiency of the instrument. Applications of hot air and cold air were discussed in detail.

There is need for further work before the vortex tube could be used for supersonic aircrafts and rockets in this space age. The reverse flow effect also has to be analyzed more thoroughly.

ACKNOWLEDGMENT

The author would like to express his deep sense of gratitude to Dr. Wilson Tripp who has given valuable and patient guidance and encouragement throughout the work on this report. The courses "Advanced Thermodynamics" and "Gas Dynamics", taught by him were very helpful in pursuing this problem. The author also is grateful to Dr. J.M. Bowyer, Jr. whose help was readily available and who made the problem of the vortex tube more understandable. Grateful thanks are due to Mr. M. V. Reddi, graduate student in Agronomy Department, for his suggestions in drafting the report.

REFERENCES

1. Bellamy, John C., "Requirements for Vortex Thermometers," Symposium 'The Vortex Tube as a True Free Air Thermometer', Armour Research Foundation, Chicago, Illinois, 1955.

2. Beneke, Jack, "Cornell Aeronautical Laboratory Vortex Free Air Thermometer", Symposium, 'The Vortex Tube as a True Free Air Thermometer', Armour Research Foundation, Chicago, Illinois, 1955.

3. Blaber, M. P., "A Simply Constructed Vortex Tube for Producing Hot and Cold Air Streams," Journal of Scientific Instruments, Vol. 27, No. 6, (1950), pp. 168-169.

4. Blatt, Thomas A. and Raymond B. Trush, "An Experimental Investigation of an Improved Vortex Cooling Device," ASME paper No..62-WA-200.

5. Blows Hot and Cold, Fortune, Dec., 1946, pp. 180-183

6. Bowyer, J. M., Jr. and W. V. Carter, "Separated Flow Behind a Rearward-Facing Step With and Without Combustion," AIAA Journal, Vol. 3, No. 1 (1965), pp. 181-183.

7. Corr, J.E., "The Vortex Tube," The General Electric Data Folder No. 45289, July, 1948. Research Lab., General Electric Co., Schenectady, New York.

8. Deissler, R. G. and M. Perlmutter, "Analysis of the Flow and Energy Separation in a Turbulent Vortex," International Journal of Heat and Mass Transfer, Vol. 1, 1960, pp. 173-191.

9. Agan, Demon, "Industrial & Engineering Chemistry," Dec., 1946,pp.5-14.

10. Eckert, E. R. G. and J. P. Hartnett, "Investigation of the Energy Distribution in a High Velocity Vortex Type Flow," University of Minnesota Heat Transfer Laboratory, Technical Report No. 3., June, 1955.

11. Fulton, C. D., "Comments on the Vortex Tube," Refrigerating Engineering October, 1951, p. 984.

12. "Ranque's Tube" Refrigerating Engineering, May,1950 pp. 473-479.

13. Hartnett, J. P. and E. R. G. Eckert, "Experimental Study of the Energy Disbribution in a High Velocity Vortex Type Flow," ASME Trans., May, 1957, pp. 751-758.

14. Hedge, Jack C., "Performance of the ARF Vortex Free Thermometer at Subsonic and Supersonic Velocities," Symposium, 'The Vortex Tube as a True Free air Thermometer," Armour Res. Found.,Chicago,Ill.,1955.

15. Heffner, F. E., "Performance Characteristics of a Water-Jacketed Vortex Tube," ASHRAE Journal, September, 1959, pp.44-47 & 71.

16. Hilsch, R., "Use of the Expansion of Gas in a Centrifugal Field as a Cooling Process," Review of Scientific Instruments, Feb., 1947, pp. 108-113.

17. Home made "Maxwell's Demon" Blows Hot and Cold, Popular Science, Nov. 1947, pp. 190-192.

18. Katz, R., "The Vortex Tube and the Tornado," Geofisica Pura E Aplicata - Milano, Vol. 47, 1960/III, pp. 191-194.

19. Keenan, J. H., "Thermodynamics," John Wiley & Sons, Inc., New York, 1941, Chap. XVI.

20. Laboratory Report No. 2, Fisher Governor Company, Marshaltown, Iowa, Feb., 1956.

21. Laboratory Report No. 3., Fisher Governor Company, Marshaltown, Iowa Aug., 1956.

22. Laboratory Report No. 4., The Rice Institute Report, Fisher Governor Company, Marshaltown, Iowa.

23. Lay, J. E., "An Experimental and Analytical Study of Vortex Flow Temperature Separation by Superposition of Spiral and Axial Flows, Part 1," ASME Paper No. 58-SA-71.

24. "An Experimental and Analytical Study of Vortex Flow Temperature Separation by Superposition of Spiral and Axial Flows, Part 2," ASME Paper No. 58-A-90.

25. Loeb, Leonard B., "Kinetic Theory of Gases," McGraw-Hill Book Co., Inc. 1927.

26. MacGee, Roy, Jr., "Fluid Action in Vortex Tube," Refrigerating Engineering, Octo., 1950, pp. 974-975,

27. Maxwelliam Demon at Work, Industrial & Engineering Chemistry, May, 1946, p. 5.

28. Maxwell's Demon Comes to Life, Popular Science, May 1947, pp.144-146.

29. Nicklas, J. P., "The Vortex Tube as an Acoustic Generator with Application to True Air-Speed Measurement," Symposium, 'The Vortex Tube as a True Free Air Thermometer,' Armour Research Foundation, Chicago, Illinois, 1955.

30. Packer, L. S. and H. C. Box, "Vortex Tube Free Air Thermometry," ASME Paper No. 55-A-22.

31. Pengelley, Demond C., "Thermal Phenomena in a Vortex," Symposium, 'The Vortex Tube as a True Free Air Thermometer,' Armour Research Foundation, Chicago, Illinois, 1955.

32. Plank, R., "The Centrifugal Jet" Refrigerating Engineering, Vol. 57, pp. 448-449.

33. "The Vortex Tube," Refrigerating Engineering, Vol. 59, pp. 52-53.

34. Ranque, G. P., "Method and Apparatus for Obtaining from a Fluid Under Pressure Two Currents of Fluids at Different Temperatures," Official Gazette, United States Patent No. 1, 952, 281 (March,1934).

35. Roebuck, J. R., "A Novel Form of Refrigerator" Journal of Applied Physics, May, 1945, pp. 285-295.

36. Ruskin, R. E. and R. M. Schecter, "Operational Characteristics of NRL Axial Flow Vortex Thermometer," Symposium, 'The Vortex Tube as a Free Air Thermometer,' Armour Research Foundation, Chicago, Illinois, 1955.

37. Savino, J. M. and R. T. Ragsdale, "Some Temperature and Pressure Measurements in a Confined Vortex Fields," ASME Paper No.60-SA-4.

38. Schecter, R. M. and R. E. Ruskin, "The Internal Characteristics and Probe Variables of the NRL Axial Flow Vortex Thermometer," Symposium 'The Vortex Tube as a True Free Air Temperature,' Armour Research Foundation, Chicago, Illinois, 1955.

39. Scheper, G. W. Jr., "The Vortex Tube - Internal Flow Data and a Heat Transfer Theory," Refrigerating Engineering,Vol.59,pp.985-989.

40. Shiparo, A. H., "The Dynamics and Thermodynamics of Compressible Fluid Flow," Vol. I and II, The Ronald Press Company,New York,N.Y.,1953.

41. Stiefelmaier, C. A., "NRL Vortex Thermometer Installation and Reliability Experience," Symposium,'The Vortex Tube as a True Free Air Thermometer,' Armour Research Foundation, Chicago, Illinois, 1955.

42. Takahama, H. and K. Kawashima, "An Experimental Study of Vortex Tubes," Nagoya University, Research Reports, Nagoya,Japan,Vol.12(2); 227-245.

43. Tolman, R. C., "Statistical Mechanics with Applications to Physics and Chemistry," American Chemical Society Monograph Series No. 32, 1927.

44. Vortex Tube Cooling for Supersonic Craft, ASHRAE Jour., Feb.,1961,pp99.

45. Webster, D. S., "An Analysis of the Hilsch Vortex Tube," Refrigerating Engineering, Vol. 58, Feb., pp.163-171.

VORTEX TUBE

by

THETTU RAGHULINGA REDDY

B. E. (Mechanical), Andhra University, Waltair, India, 1962

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Mechanical Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1966

The vortex tube, as invented by Ranque, is a simple device without any moving parts. Compressed air enters the tube tangentially through a simple nozzle. Cold air is recovered from one end of the tube at the central region and hot air from the periphery of the other end.

In this report an attempt has been made to discuss various theories on the vortex tube that have been put forth by several investigators, and the mechanism of energy separation has been explained in all its aspects. The experimental results available are made use of in support of the conclusions drawn. The energy separation is due to turbulent shear work done on or by a fluid element in a compressible vortex. The efficiency of the vortex tube is compared with the basic and nonbasic criteria. The various uses and applications of vortex tube are discussed.