TWO RC NOTCH FILTERS

by

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TERMINOLOGY

A notch filter is a network whose output frequency spectrum contains a single minimum in its frequency response. It is oftentimes required that the notch amplitude be zero; this results in a zero notch filter.

An equal amplitude notch filter is one with a notch and equal amplitude at zero frequency and infinite frequency in its frequency response curve.

An unequal amplitude notch filter is one with a notch and unequal amplitudes at zero and infinite frequencies in its frequency response curve.

INTRODUCTION

Notch networks continue to play an important role in conjunction with feedback amplifier problems. In the past, notch networks with equal amplitudes at zero and infinite frequencies have been considered. The purpose of this report is to exhibit a notch network with unequal amplitude at zero and infinite frequencies.

PREVIOUS WORK

The parallel-T resistance-capacitance circuits were first described in Tuttle's (10) and Scott's (8) papers. Tuttle discussed the resonant conditions of bridge-T and parallel-T networks in general, and considered the asymmetrical parallel-T resistance-capacitance network as a single case. Scott applied the selective behavior of parallel-T resistance-capacitance networks to feedback amplifiers for emphasizing a given frequency. The conditions for zero transmission of the network (it is sometimes called the null network) were investigated and a number of arrangements for the network were suggested by Tuttle.

Hastings (6) made further investigation on the stability of the amplifier with a parallel-T network in it, and used the amplifier as an oscillator.

Stanton (9) made use of the delta-to-wye conversion concept to simplify the network. He suggested a number of considerations in designing the network.

Gitzendanner (2) and Givens and Saby (3) gave a detailed analysis of the resistance-capacitance twin-T filter. Augustadt (1) and Purington (7) gave practical applications of this kind of RC filter.

EQUAL AMPLITUDE NOTCH NETWORK

The parallel-T resistance-capacitance network in Fig. 1 is bisected into half sections, as shown in Fig. 2. Consider only a half section. It can be disintegrated into still simpler parallel component parts as exhibited in Figs. 3a and 3b.

Let (A_{1a}, A_{1b}) and (B_{1a}, B_{1b}) be the short-circuit input impedance and open-circuit impedance of these component parts with respective subscript.

$$A_{1a} = 1$$
$$A_{1b} = 1/s$$
$$B_{1a} = 1 + 1/s$$
$$B_{1b} = 1 + 1/s.$$

Let A_1 = the short-circuit input impedance of the half section

B_1 = the open-circuit input impedance of the half section.

$$A_1 = A_{1a} \| A_{1b} = \frac{1}{1 + s}$$

$$B_1 = B_{1a} \| B_{1b} = \frac{1 + s}{2s}$$

Use of Bartlett's representation theorem for symmetric network yields the transfer function T_1.

$$T_1 = \frac{B_1 - A_1}{B_1 + A_1} = \frac{1 + s^2}{1 + 4s + s^2}$$

Since $(s^2 + 1)$ is a factor of the numerator, there is a zero notch at $\omega = 1$ in the frequency response curve for this transfer function.

Substituting $s = j\omega$ in the expression for T_1 yields

$$T_1(j\omega) = \frac{1 - \omega^2}{(1 - \omega^2) + j\,4\omega}$$

$$\left| T_1(j\omega) \right|^2 = \frac{\omega^8 + 12\omega^6 - 26\omega^4 + 12\omega^2 + 1}{\omega^8 + 28\omega^6 + 198\omega^4 + 28\omega^2 + 1}$$

A graph of $\left| T_1(j\omega) \right|^2$ versus ω on log-log paper is shown in Fig. 4. The data for this graph were obtained with program No. 2 in the appendix.

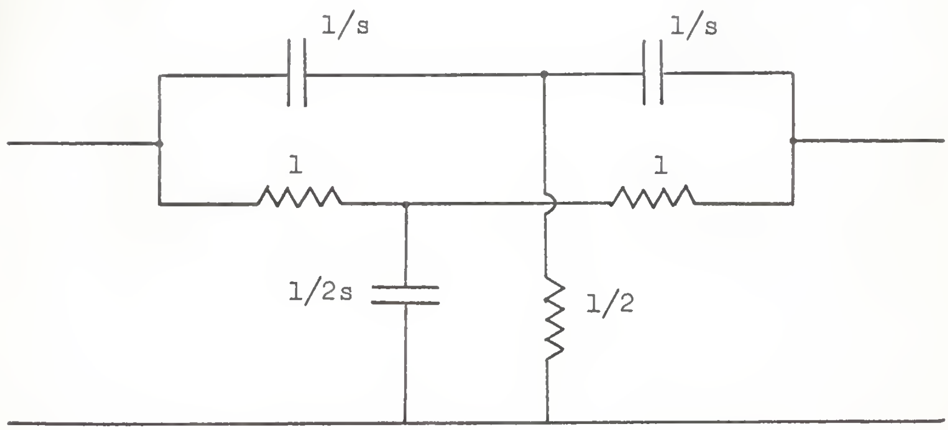

Fig. 1. The parallel-T resistance-capacitance network.

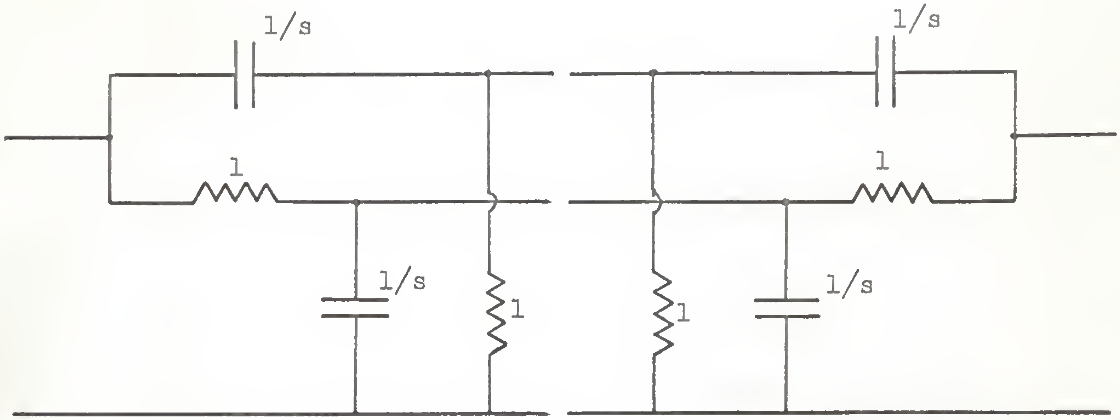

Fig. 2. The half section of the RC network in Fig. 1.

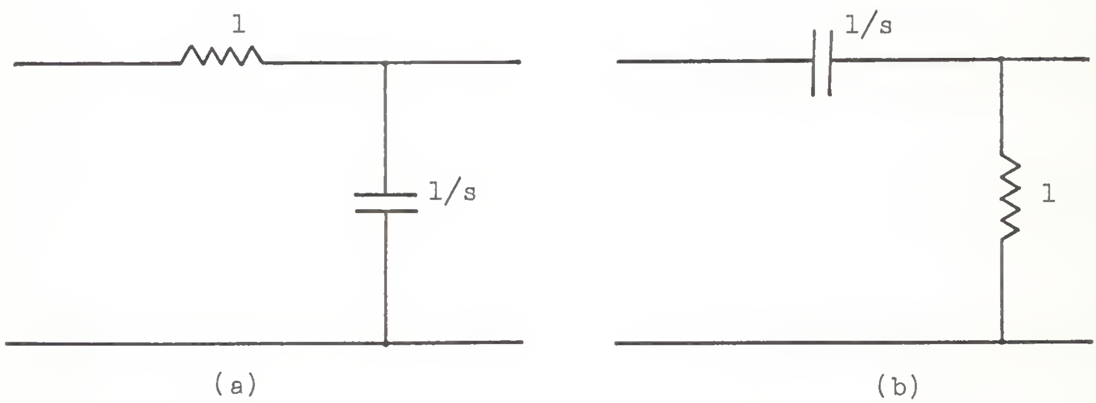

Fig. 3. Component T's of the half section.

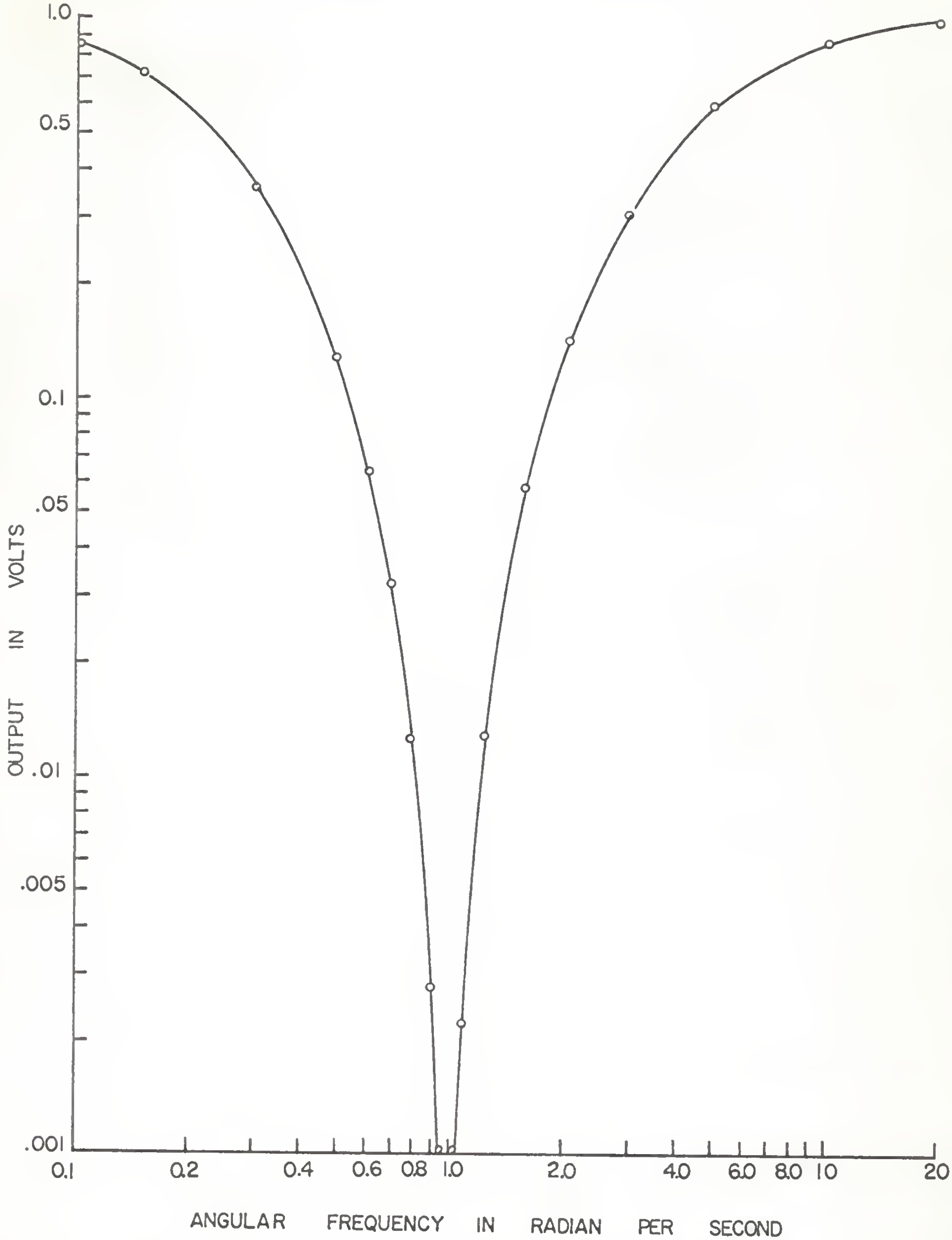

Fig. 4. Frequency response for the equal amplitude notch network.

UNEQUAL AMPLITUDE RC NOTCH FILTER

Theorem 1. The network of Fig. 5 has an unequal amplitude zero notch spectrum if $\lambda = 1$. Its output transfer function is $(1 + s^2)/(1 + 6s + 3s^2)$.

Proof. A half section of the network is considered.

Let A_2 = mid short-circuit input impedance

B_2 = mid open-circuit input impedance.

Considering the component parts of the half section, for short circuit, the impedances as indicated in Figs. 6a and 6b are

$$A_{21} = \frac{2 + 2\lambda + \lambda s}{2 + \lambda s}$$

$$A_{22} = \frac{(4\lambda + \lambda s)(2 + 2\lambda + \lambda s)}{(2 + 6\lambda + 4\lambda^2)s + (2\lambda + \lambda^2)s^2}$$

$$A_2 = A_{21} \,\|\, A_{22}$$

$$A_2 = \frac{(2 + 2\lambda + \lambda s)(4\lambda + \lambda s)}{(2\lambda + 2\lambda^2)s^2 + (2 + 8\lambda + 8\lambda^2)s + 8\lambda}$$

For mid open-circuit impedance calculations, component parts are identical; their open-circuit impedances B are the same (see Fig. 7).

$$2B = 1 + \frac{4\lambda s + 8\lambda}{2\lambda s^2 + 4\lambda s + 2s} = \frac{2\lambda s^2 + 2s + 8\lambda s + 8\lambda}{2\lambda s^2 + 4\lambda s + 2s}$$

$$B_2 = \frac{\lambda s^2 + (4\lambda + 1)s + 4\lambda}{2\lambda s^2 + (4\lambda + 2)s}$$

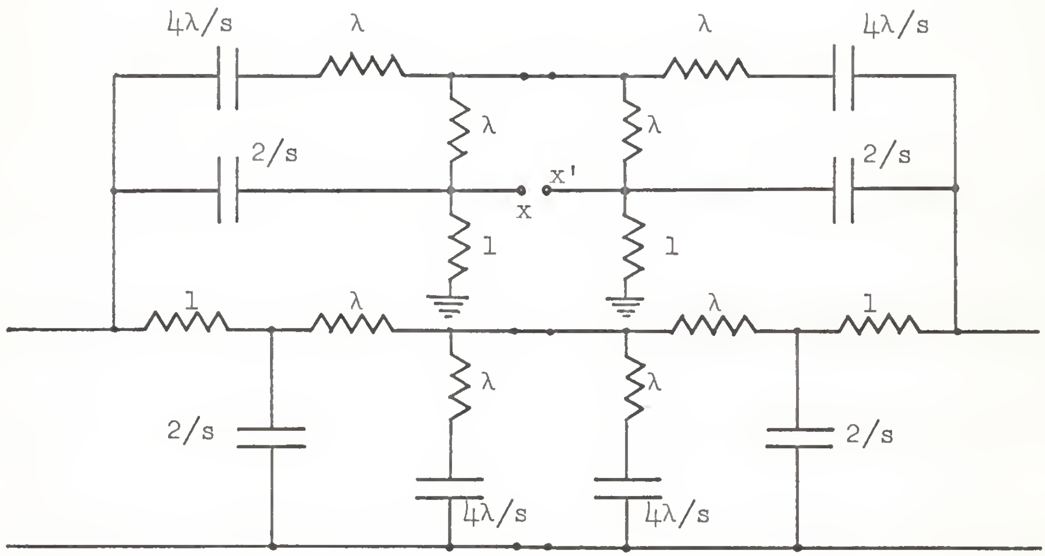

Fig. 5. An unequal amplitude RC notch filter in half sections.

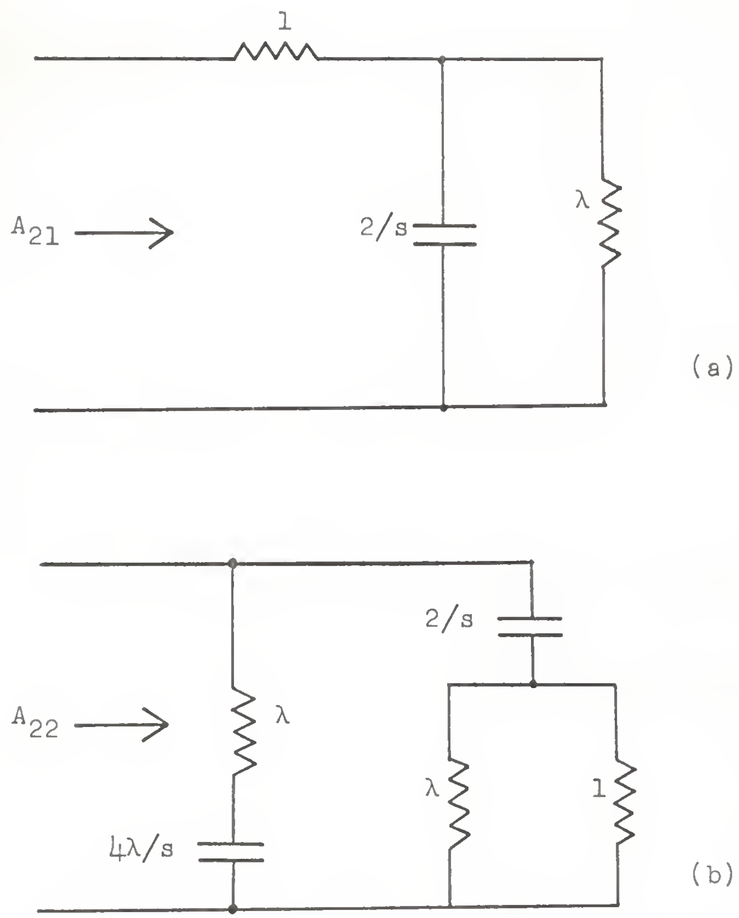

Fig. 6. Short-circuited components of the half section.

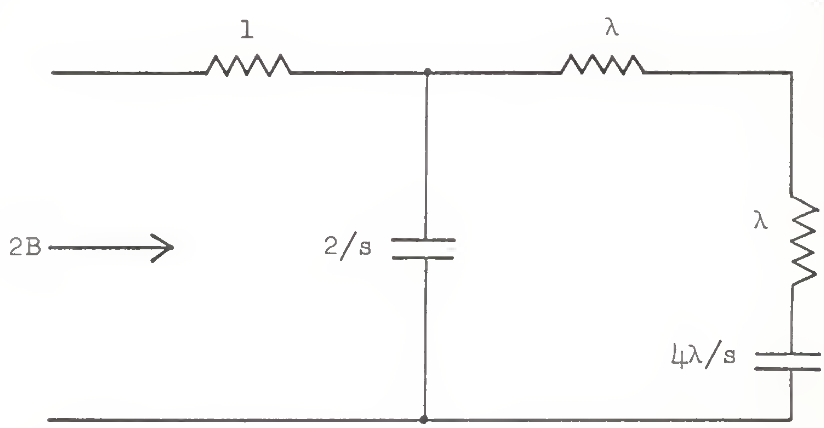

Fig. 7. Open-circuited component of the half section.

$$B_2 \pm A_2 = \frac{4\lambda + s + 4\lambda s + \lambda s^2}{2s + 4\lambda s + 2\lambda s^2}$$

$$\pm \frac{(2 + 2\lambda + \lambda s)(4\lambda + s\lambda)}{8\lambda + (2 + 8\lambda + 8\lambda^2)s + (2\lambda + 2\lambda^2)s^2}$$

Let $D = (2s + 4\lambda s + 2\lambda s^2)(8\lambda + 2s + 8\lambda s + 8\lambda^2 s + 2\lambda s^2 + 2\lambda^2 s^2)$.

$$B_2 \pm A_2 = \frac{1}{D}\Big[(\lambda s^2 + 4\lambda s + s + 4\lambda)(2\lambda^2 s^2 + 2\lambda s^2 + 8\lambda^2 s + 8\lambda s + 2s + 8\lambda)$$
$$\pm (2\lambda s^2 + 4\lambda s + 2s)(\lambda^2 s^2 + 6\lambda^2 s + 2\lambda s + 8\lambda^2 + 8\lambda)\Big]$$

$$B_2 - A_2 = \frac{1}{D}\Big[2\lambda^2 s^4 + (12\lambda^2 + 4\lambda)s^3 + (20\lambda^2 + 12\lambda + 2)s^2 + 16\lambda^2 s + 32\lambda^2\Big]$$

$$B_2 + A_2 = \frac{1}{D}\Big[(4\lambda^3 + 2\lambda^2)s^4 + (32\lambda^3 + 24\lambda^2 + 4\lambda)s^3$$
$$+ (80\lambda^3 + 92\lambda^2 + 20\lambda + 2)s^2$$
$$+ (64\lambda^3 + 112\lambda^2 + 32\lambda)s + 32\lambda^2\Big]$$

Use of Bartlett's representation theorem for a symmetric network yields the transfer function T_2 with λ undetermined.

$$T_2 = \frac{B_2 - A_2}{B_2 + B_2} = \frac{(2\lambda^2)s^4 + (12\lambda^2 + 4\lambda)s^3 + (20\lambda^2 + 12\lambda + 2)s^2 + 16\lambda^2 s + 32\lambda^2}{(4\lambda^3 + 2\lambda^2)s^4 + (32\lambda^3 + 24\lambda^2 + 4\lambda)s^3 + (80\lambda^3 + 92\lambda^2 + 20\lambda + 2)s^2 + (64\lambda^3 + 112\lambda^2 + 32\lambda)s + 32\lambda_2{}^2}$$

Choose λ so that the numerator polynomial has $(s^2 + \omega^2)$ for a factor. To determine this unique value for λ, the Routh array is formed with even and odd power terms of $(B_2 - A_2)$ separated.

$$B_2 - A_2 = (2\lambda^2)s^4 + (12\lambda^2 + 4\lambda)s^3 + (20\lambda^2 + 12\lambda + 2)s^2 + 16\lambda^2 s + 32\lambda^2 \quad (1)$$

Routh Array

$$\begin{array}{c|ccc} s^4 & 2\lambda^2 & (20\lambda^2 + 12\lambda + 2) & 32\lambda^2 \\ s^3 & (12\lambda^2 + 4\lambda) & 16\lambda^2 & 0 \end{array}$$

Consider a polynomial of the same form, say

$$as^4 + bs^3 + cs^2 + ds + e = P_e(s) + P_0(s)$$

where $P_e(s)$ = even power polynomial

$P_0(s)$ = odd power polynomial

The Routh Array

$$\begin{array}{c|ccc} s^4 & a & c & e \\ s^3 & b & d & \\ s^2 & c - \dfrac{ad}{b} & e & \\ s^1 & d - \dfrac{b^2 e}{bc - ad} & 0 & \end{array}$$

In order to have a factor of the form $(s^2 + \omega^2)$ we must have a row of zeros at the end, and all the coefficients in the array must be positive. Therefore

$$d = \frac{b^2\, e}{bc - ad}$$

Returning to our original polynomial in the numerator of the transfer function T_2,

$$B_2 - A_2 = (2\lambda^2)s^4 + (12\lambda^2 + 4\lambda)s^3 + (20\lambda^2 + 12\lambda + 2)s^2 + (16\lambda^2)s + 32\lambda^2$$

$$= s^4 + (6 + \frac{2}{\lambda})s^3 + (10 + \frac{6}{\lambda} + \frac{1}{\lambda^2})s^2 + 8s + 16 \qquad (2)$$

It will be convenient to put this polynomial into the form

$$a_2s^4 + b_2s^3 + c_2s^2 + d_2s + e \qquad (3)$$

Comparing coefficients of equations (2) and (3), we have

$$a_2 = 1$$

$$b_2 = (6 + \frac{2}{\lambda}) = 2(3 + \frac{1}{\lambda})$$

$$c_2 = (10 + \frac{6}{\lambda} + \frac{1}{\lambda^2})$$

$$d_2 = 8$$

$$e_2 = 16$$

Substituting these values into the equation

$$d_2 = \frac{b_2{}^2\ e_2}{b_2c_2 - a_2d_2}$$

one has

$$d_2 = 8 = \frac{64(3 + \frac{1}{\lambda})^2}{2(3 + \frac{1}{\lambda})(10 + \frac{6}{\lambda} + \frac{1}{\lambda^2}) - 8}$$

After simplification, the equation becomes

$$10\lambda^3 - 4\lambda^2 - 5\lambda - 1 = 0$$

or

$$\lambda^3 - 0.4\lambda^2 - 0.5\lambda - 0.1 = 0$$

The roots (obtained with program No. 1 in the appendix) are:

$$\lambda_1 = 1.00 \qquad \text{success}$$

$$\left.\begin{array}{l}\lambda_2 = -0.3 + j\,0.1 \\ \lambda_3 = -0.3 - j\,0.1\end{array}\right\} \text{failure}$$

Substitute $\lambda = 1.00$ into coefficients of equation (1).

$$\left.\begin{array}{l} a = 2\lambda^2 = 2 \\ b = 12\lambda^2 + 4\lambda = 16 \\ c = 20\lambda^2 + 12\lambda + 2 = 34 \\ d = 16\lambda^2 = 16 \\ e = 32\lambda^2 = 32 \end{array}\right\} \text{All } (+)$$

$$c - \frac{ad}{b} = 34 - \frac{2(16)}{16} = 34 - 2 = 32 \ (+)$$

$$d - \frac{b^2\,e}{bc - ad} = 16 - \frac{(16)^2 32}{16 \times 34 - 2(16)} = 0$$

Hence $d - \dfrac{b^2\,e}{bc - ad} = 0$

$$c - \frac{ad}{b} > 0$$

$$e > 0$$

Therefore, with $\lambda = 1$, all coefficients in the array are positive and coefficients in the last row are all zeros.

We have a row of zeros; hence we have a common factor $(c - \frac{ad}{b})s^2 + e$ for the numerator polynomial. Therefore

$$(c - \frac{ad}{b})s^2 + e = 32(s^2 + 1)$$

$$T_2 = \frac{B_2 - A_2}{B_2 + B_2} = \frac{(2\lambda^2)s^4 + (12\lambda^2 + 4\lambda)s^3 + (20\lambda^2 + 12\lambda + 2)s^2 + 16\lambda^2 s + 32\lambda^2}{(4\lambda^3 + 2\lambda^2)s \;\; + (32\lambda^3 + 24\lambda^2 + 4\lambda)s^3 + (80\lambda^3 + 92\lambda^2 + 20\lambda + 2)s^2 + (64\lambda^3 + 112\lambda^2 + 32\lambda)s + 32\lambda_2^{\,2}}$$

For $\lambda = 1$ the transfer function T_2 becomes,

$$T_2 = \frac{2s^4 + 16s^3 + 34s^2 + 16s + 32}{6s^4 + 60s^3 + 194s^2 + 208s + 32}$$

Repeating for the numerator and denominator polynomials the Routh array to check for common factors,

$$\begin{array}{c|ccccc}
s^5 & 2 & 16 & 34 & 16 & 32 \\
s^5 & 6 & 30 & 194 & 208 & 32 \\
s^4 & -4 & -\frac{92}{3} & -\frac{160}{3} & \frac{64}{3} & 0 \\
s^4 & 14 & 114 & 240 & 32 & 0 \\
s^3 & \frac{80}{42} & \frac{640}{42} & \frac{1280}{42} & 0 & 0 \\
s^3 & 2 & 16 & 32 & 0 & 0 \\
s^2 & 0 & 0 & 0 & 0 & 0
\end{array}$$

In this Routh array we have a row of zeros. This indicates a common factor in the numerator and denominator with $\lambda = 1$. The common factor is

$$(2s^2 + 16s + 32)$$

or $\quad 2(s + 4)^2$

With $\lambda = 1$,

$$T_2 = \frac{B_2 - A_2}{B_2 + A_2} = \frac{(s \quad + 8s \quad + 17s \quad + 8s + 16)}{(3s^4 + 30s^3 + 97s^2 + 104s + 16)}$$

Factoring both numerator and denominator, and knowing that $(s^2 + 8s + 16)$ is a common factor, yields

$$T_2 = \frac{(s^2 + 8s + 16)(s^2 + 1)}{(s^2 + 8s + 16)(3s^2 + 6s + 1)}$$

The required transfer function is

$$T_2 = \frac{1 + s^2}{1 + 6s + 3s^2}$$

Furthermore, substituting $s = j\omega$ yields

$$T_2(j\omega) = \frac{(3\omega^4 - 4\omega^2 + 1) - j(6\omega - 6\omega^3)}{9\omega^4 + 30\omega^2 + 1}$$

$$\left| T_2(j\omega) \right|^2 = \frac{9\omega^8 + 12\omega^6 - 50\omega^4 + 28\omega^2 + 1}{81\omega^8 + 540\omega^6 + 918\omega^4 + 60\omega^2 + 1}$$

Figure 8 shows the relationship $\left|T_2(j\omega)\right|^2$ versus ω. The data for this graph were obtained with program No. 3 in the appendix.

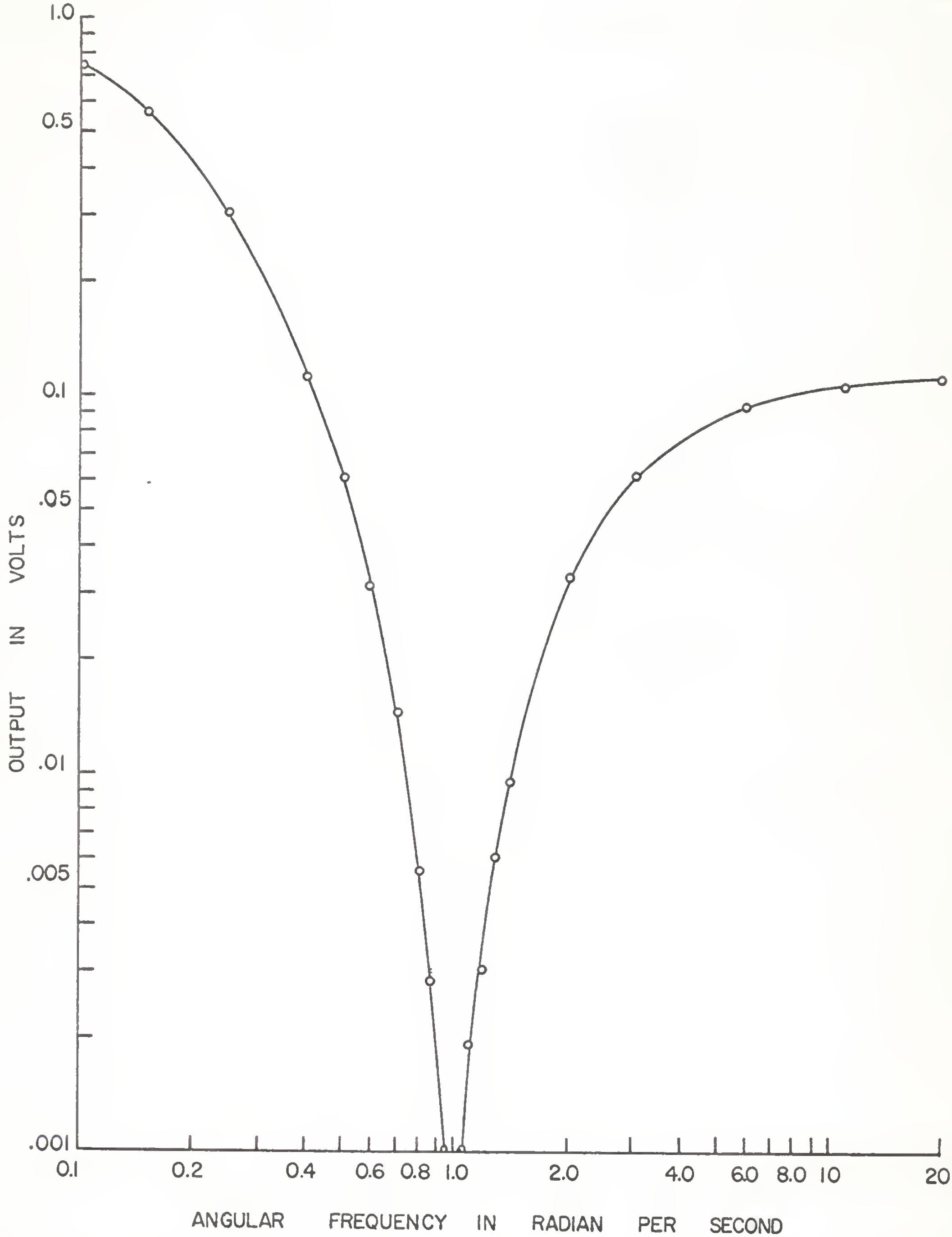

Fig. 8. Frequency response for the unequal amplitude notch network.

EQUAL AMPLITUDE RC NETWORK WITH CONNECTION OF TERMINALS X, X'

Theorem 2. Connection of X, X' terminals in the network of Fig. 9 results in a zero notch network with equal amplitude characteristics.

Proof. Connect X, X' of Fig. 4 and obtain the half section representation of the network after connection as shown in Fig. 9. Considering the short-circuited half section of the RC network of equal amplitude, we have the short-circuit impedance as indicated in Fig. 10.

$$A_{31} = \frac{2\mu s + 8\mu}{\mu s^2 + 4\mu s + 2s}$$

$$A_{32} = \frac{\mu s + 2\mu + 2}{\mu s + 2}$$

The mid short-circuit input impedance A of the half section is

$$A_3 = A_{31} \| A_{32}$$

$$= \frac{(2\mu^2)s^2 + (12\mu^2 + 4\mu)s + 16(\mu^2 + \mu)}{\mu^2 s^3 + (8\mu^2 + 4\mu)s^2 + (16\mu^2 + 16\mu + 4)s + 16\mu}$$

The mid open-circuit impedance of the half section is the same as that of the previous case before connecting X, X', namely,

$$B_3 = \frac{\mu s^2 + (4\mu + 1)s + 4\mu}{2\mu s^2 + (4\mu + 2)s}$$

Use of Bartlett's representation theorem for symmetric networks yields the transfer function

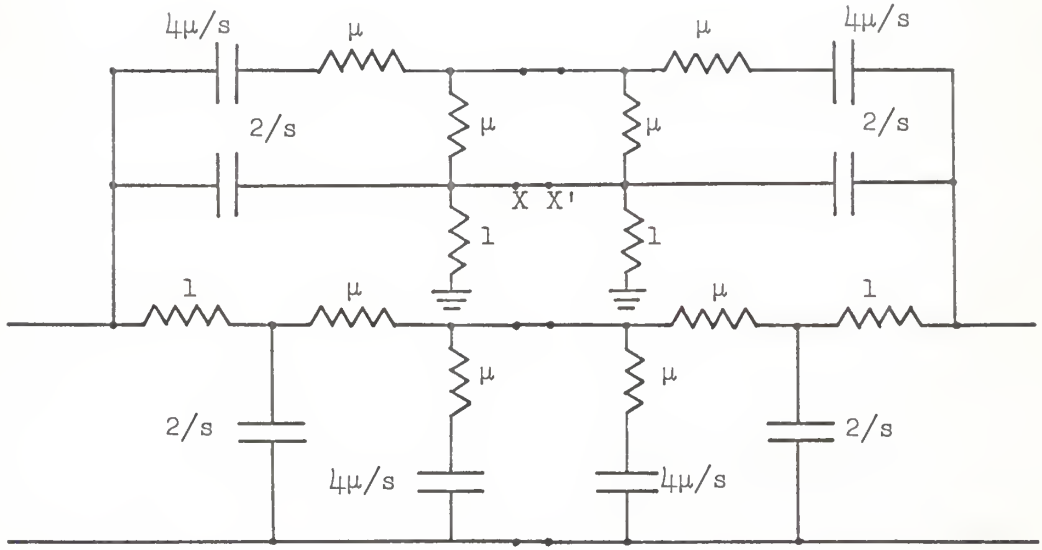

Fig. 9. Equal amplitude RC network with terminals X, X' connected.

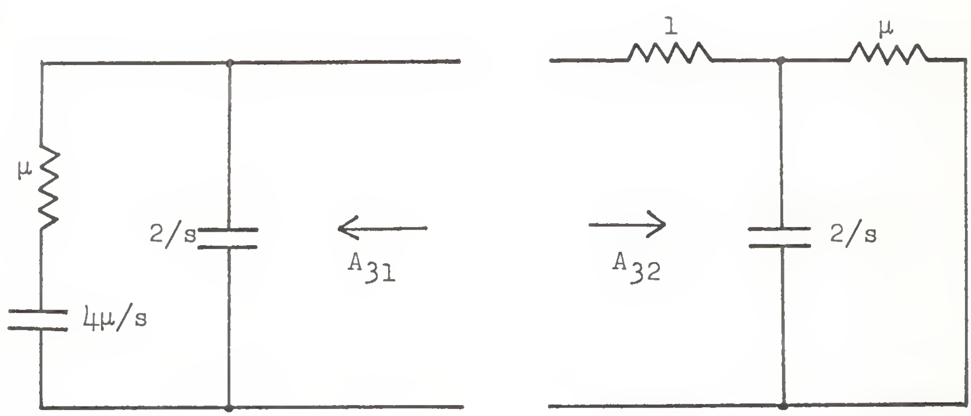

Fig. 10. Short-circuited half section of equal amplitude network.

$$T_3 = \frac{B_3 - A_3}{B_3 + A_3}$$

$$= \frac{\begin{array}{l}\mu^3 s^5 + (5\mu^2 + 8\mu^3)s^4 + (8\mu + 28\mu^2 + 20\mu^3)s^3 \\ + (4 + 24\mu + 40\mu^2 + 16\mu^3)s^2 + (32\mu^2)s + 64\mu^2\end{array}}{\begin{array}{l}\mu^3 s^5 + (5\mu^2 + 16\mu^3)s^4 + (8\mu + 52\mu^2 + 84\mu^3)s^3 \\ + (4 + 40\mu + 184\mu^2 + 176\mu^3)s^2 + (64\mu + 224\mu^2 \\ \qquad\qquad + 128\mu^3)s + 64\mu^2\end{array}}$$

In order for this transfer function to have a zero notch in its frequency response, the value of μ must be so chosen that the polynomial in the numerator has a factor of the form $(s^2 + \omega^2)$.

This unique value of μ is found by splitting the numerator into even and odd polynomials which are then subjected to a Routh array calculation. First let us consider a polynomial of the same degree, say

$$(as^5 + bs^4 + cs^3 + ds^2 + es + f) \tag{4}$$

Forming the Routh array as described above, one obtains

$$
\begin{array}{c|ccc}
s^5 & a & c & e \\
s^4 & b & d & f \\
s^3 & \dfrac{bc - ad}{b} = P & \dfrac{be - af}{b} = Q & 0 \\
s^2 & \dfrac{Pd - Qb}{P} = R & f & 0 \\
s^1 & \dfrac{RQ - Pf}{R} & 0 & 0
\end{array}
$$

In order to find a common factor in these polynomials there must be a row of zeros at the end; i.e.,

$$Q = \frac{P}{R} \cdot f \,, \qquad Q(Pd - Qb) = P^2 \cdot f$$

$$(bc - ad) \quad (be - af)d - (bc - ad)f \; = (be - af)^2 \cdot b \qquad (5)$$

and the factor is $(d - \frac{Q}{P} b)s^2 + f = 0$.

$$\left(d - \frac{be - af}{bc - ad} b\right)s^2 + f = 0 \qquad (6)$$

Referring to the original polynomial, one obtains

$$\mu^3 s^5 + (5\mu^2 + 8\mu^3)s^4 + (8\mu + 28\mu^2 + 20\mu^3)s^3$$
$$+ (4 + 24\mu + 40\mu^2 + 16\mu^3)s^2 + 32\mu^2 s + 64\mu^2 \qquad (7)$$

If $x = 1/\mu$, then the above polynomial becomes

$$s^5 + (5x + 8)s^4 + (8x^2 + 28x + 20)s^3 + (4x^3 + 24x^2 + 40x + 16)s^2$$
$$+ 32xs + 64x \qquad (8)$$

On comparing coefficients of polynomials in equations (4) and (8), one identifies

$a_3 = 1$

$b_3 = 5x + 8$

$c_3 = 8x^2 + 28x + 20$

$d_3 = 4x^3 + 24x^2 + 40x + 16$

$e_3 = 32x$

$f_3 = 64x$

Substitution of these values into equation (5) yields

$$4(9x^3 + 45x^2 + 71x + 36) \quad 128x(5x^4 + 36x^3 + 86x^2 + 80x + 24)$$
$$- 128x(18x^3 + 90x^2 + 142x + 72)$$
$$= 512x(250x^4 + 1000x^3 + 1320x^2 + 576x)$$

$$(9x^3 + 45x^2 + 71x + 36)(5x^4 + 18x^3 - 4x^2 - 62x - 48)$$
$$= (250x^4 + 1000x^3 + 1320x^2 + 576x)$$

$$(45x^7 + 387x^6 + 1129x^5 + 720x^4 - 2858x^3 - 6706x^2 - 5640x - 1728)$$
$$= 250x^4 + 1000x^3 + 1320x^2 + 576x$$

Therefore

$$45x^7 + 387x^6 + 1129x^5 + 470x^4 - 3858x^3 - 8026x^2 - 6216x - 1728$$
$$= 0$$

Since $x = 1/\mu$, one has

$$1728\mu^7 + 6216\mu^6 + 8026\mu^5 + 3858\mu^4 - 470\mu^3 - 1129\mu^2$$
$$- 387\mu - 45 = 0$$

The roots of this equation (obtained with program No. 1 in the appendix) are:

$\mu_1 = -0.8407152$

$\mu_2 = -1.2191628$

$\mu_3 = -0.625000$

$\mu_4 = -0.8303191$

$$\mu_5, \mu_6 = -0.2879760 \pm j\,0.1272471$$

$$\mu_7 = 0.4939271$$

Since there is no purely imaginary term in the original equation, only the positive root $\mu_7 = 0.4939271$ is ideal in our case. With $\mu = 0.4939$ approximately, the coefficients of the original polynomial (7) are

$$a = \mu^3 = 0.1205$$

$$b = 5\mu^2 + 8\mu^3 = 2.1838$$

$$c = 8\mu + 28\mu^2 + 20\mu^3 = 13.1924$$

$$d = 4 + 24\mu + 40\mu^2 + 16\mu^3 = 27.54$$

$$e = 32\mu^2 = 7.8068$$

$$f = 64\mu^2 = 15.6137$$

Substituting these values into equation (6), one obtains

$$\left[27.54 - \left(\frac{15.1671}{25.491}\right)(2.1838)\right] s^2 + 15.6137 = 0$$

$$s^2 + 0.595 = 0$$

a factor of the numerator polynomial. The presence of this factor, which is of the form $(s^2 + \omega^2)$, indicates that there is a notch in the frequency response. If normalization of the zero notch frequency to one radian per second is desired, then one can replace s by $\sqrt{0.595}\,s$.

For $\mu = 0.4939271$, the transfer function becomes

$$T_3 = \frac{0.1205s^5 + 2.1838s^4 + 13.1924s^3 + 27.54s^2 + 7.806s + 15.6137}{0.1205s^5 + 3.1478s^4 + 26.857s^3 + 89.854s^2 + 101.6833s + 15.6137}$$

which factors into (see program No. 1 in the appendix),

$$T_3 = \frac{(s \pm j.7713683)(s + 5.4572264 \pm j\,0.6550105)(s + 7.2084062}{(s + 0.1809557)(s + 13.0390631)(s + 1.8777198)(s + 4.4439298)(s + 6.5811530)}$$

Substitution of $s = j\omega$ yields

$$T_3(j\omega) = \frac{(2.1834\omega^4 - 27.54\omega^2 + 15.6137) + j(0.1205\omega^5 - 13,1924\omega^3 + 7.806\omega)}{(3.1478\omega^4 - 89.854\omega^2 + 15.6137) + j(0.1205\omega^5 - 26.8574\omega^3 + 101.6833\omega)}$$

$$\left|T_3(j\omega)\right|^2 = \frac{0.01452\omega^{10} + 1.58792\omega^8 + 55.659\omega^6 + 620.6739\omega^4 - 799.069\omega^2 + 243.7876}{0.01452\omega^{10} + 3.43604\omega^8 + 180.1407\omega^6 + 2710.1409\omega^4 + 7533.5867\omega^2 + 243.7876}$$

For the graph of $\left|T_3(j\omega)\right|^2$ versus ω on log-log paper, refer to Fig. 11.

The data for this graph were obtained with the program No. 4 in the appendix.

The connection of the terminals X, X' have essentially changed the RC notch network into an equal amplitude network, as shown in the frequency response curve in Fig. 11. The notch frequency has also been shifted to $\omega = .771$ (less than $\omega = 1$) and the minimum notch value is zero.

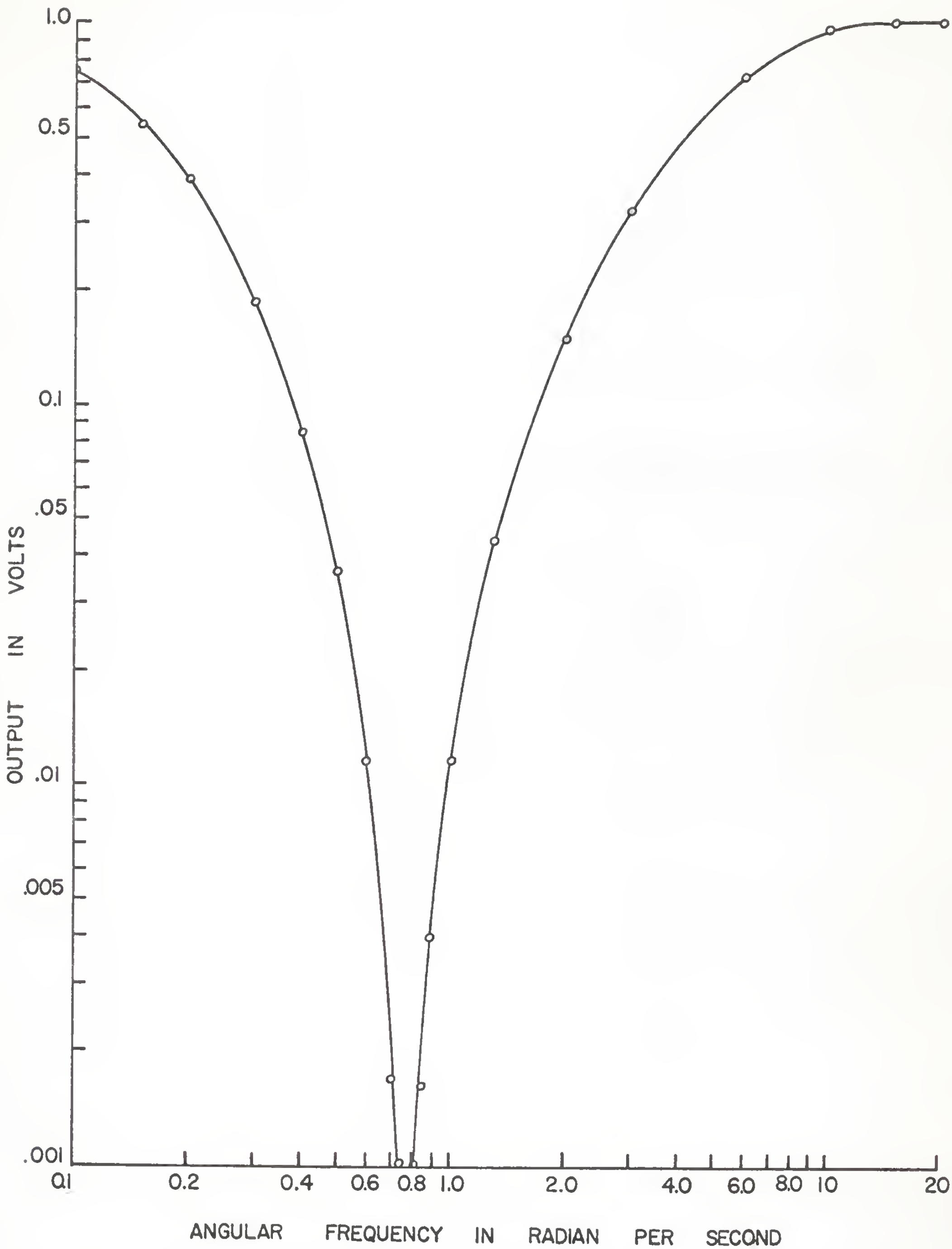

Fig. 11. Frequency response for equal amplitude network with terminals X, X' connected.

COMPARISON OF NOTCH FILTER Q's

The factor Q defines the sharpness of the notch curve. A bandwidth ω is defined by the points at which the curve has dropped to 70.7 per cent of its peak value. The explicit formula is

$$Q = \frac{\omega_0}{(\Delta \omega_0)_{3db}}$$

where ω_0 is the notch frequency.

In other words, this factor Q is the reciprocal of the bandwidth in units of the notch frequency.

Case 1. Equal amplitude network.

From Fig. 4, the curve for equal amplitude standard twin-tee network, one obtains

$$Q_1 = \frac{1}{6.5 - 0.16} = 0.158$$

Case 2. Unequal amplitude network.

Since the magnitude of the curve (Fig. 8) at zero and infinite frequencies is unequal, the bandwidth is undefined, so Q_2 for this network is undetermined.

$$Q_3 = \frac{1}{6.0 - 0.1} = 0.132$$

As Q_3 is smaller than Q_1, the notch for the standard network (Fig. 4) is sharper than that of the new network. The smaller Q_3 for the new network also indicates that a wider bandwidth is achieved and this may be useful for certain applications. The bandwidth is increased by 16 per cent.

SUMMARY

An RC notch filter is a filter with a notch in its frequency response curve. Three cases of such filters have been exhibited.

1. An equal amplitude RC network.
2. An unequal amplitude RC network.
3. An equal amplitude RC network resulted from the connection between terminals X, X' (see Fig. 5).

Connection of terminals X, X' in Fig. 9 has changed the nature of the original network with a zero magnitude notch at a lower frequency than $\omega = 1$.

The graph for the frequency response in each case clearly illustrates its particular amplitude qualification of the network. Measurement of Q's indicates that a wider bandwidth can be obtained with the new network.

A notch filter cascaded with a low-pass RC filter will have a frequency response like that of the unequal amplitude case. This type of filter may be particularly useful in averaging nonstationary random signals and in smoothing commutated signals before sampling. Design data for this type of unequal amplitude network are given by Hansen (5).

ACKNOWLEDGMENT

The author expresses his thanks to his major professor, Dr. Charles A. Halijak, for his counsel and encouragement throughout the preparation of this report.

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APPENDICES

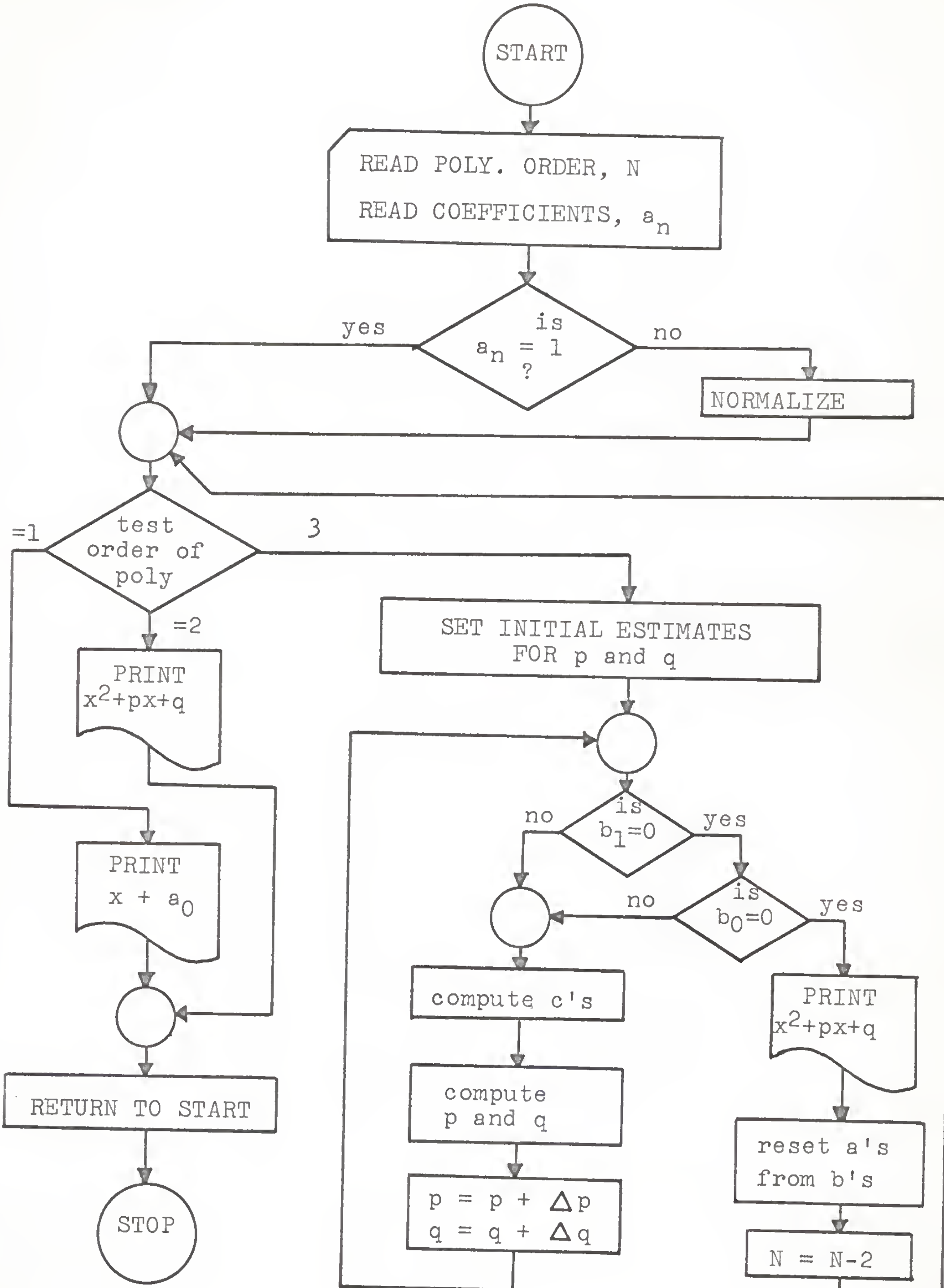

Fig. 12. Block diagram of Bairstow's polynomial factoring computer program.

PROGRAM 1. BAIRSTOW METHOD OF POLYNOMIAL FACTOR

```
MCN$$     JOB  BAIRSTOW METHOD OF POLYNOMIAL FACTOR
MCN$$     ASGN MJB,12
MCN$$     ASGN MGO,16
MCN$$     MODE GO
MCN$$     EXEQ FORTRAN,,,18,3,,,BAIRSTOW
      DIMENSION A(20),B(20),C(20),PI(5),QI(5)
      EQUIVALENCE (A1,A(1)),(B1,B(1)),(B2,B(2)),(B3,B(3)),(C1,C(1)),
     1(C2,C(2)),(C3,C(3)),(C4,C(4))
1     FORMAT(I2)
2     FORMAT(E14.8)
3     FORMAT(18HKMETHOD HAS FAILED)
4     FORMAT(1X,F16.8)
5     FORMAT(4HSX =,F11.7,25H ,                         X =,F11.7)
6     FORMAT(4HSX =,F11.7,4H +J ,F11.7,3X,7H ,  X =,F11.7,4H - J,F11.7)
7     FORMAT(4HSX =,F11.7)
8     FORMAT(39H1COEFFICIENTS IN DECREASING POWERS OF X/)
9     FORMAT(1HK,60X,61HITERATION    B(1)           B(0)             P
     1             Q/)
10    FORMAT(61X,I5,3X,1PE14.7,3(1PE15.7))
C     SET 5 INITIAL GUESSES.
      PI(1)=4.
      QI(1)=3.
      PI(2)=2.
      QI(2)=1.
      PI(3)=0.
      QI(3)=-1.
      PI(4)=2.
      QI(4)=2.
      PI(5)=-2.
      QI(5)=2.
C     READ ORDER N.  N=0 TO TERMINATE PROGRAM.
12    READ (1,1)N
      IF(N.EQ.0) STOP
      N=N+1
      WRITE(3,8)
      DO 14 I=1,N
      J=N+1-I
C     READ COEFFICIENTS IN DECREASING POWERS.
      READ(1,2)A(J)
14    WRITE(3,4)A(J)
      IF(A(N).EQ.1.) GO TO 16
      DO 15 I=1,N
C     NORMALIZE COEFFICIENTS.
15    A(I)=A(I)/A(N)
16    I=1
      WRITE(3,9)
17    IF(N-3) 130,120,18
```

```
18    L=N-2
      ITRY=1
C     SET INITIAL GUESS FOR P AND Q.
20    P=PI(ITRY)
      Q=QI(ITRY)
      ITCNT=1
C     CALCULATE BS.
25    B(N)=1.
      B(N-1)=A(N-1)-P
      DO 30 K=2,L
      M=N-K
30    B(M)=A(M)-P*B(M+1)-Q*B(M+2)
      B1=A1-Q*B3
      WRITE(3,10)ITCNT,B2,B1,P,Q
C     CHECK ACCURACY OF GUESS.
      IF(ABS(B2).GE..00000001) GO TO 45
      IF(ABS(B1).LT..00000001) GO TO 60
45    ITCNT=ITCNT+1
      IF(ITCNT.GT.25) GO TO 150
C     CALCULATE CS FOR CORRECTION OF P AND Q.
      C(N)=1.
      C(N-1)=B(N-1)-P
      DO 50 K=2,L
      M=N-K
50    C(M)=B(M)-P*C(M+1)-Q*C(M+2)
      C1=B1-Q*C3
      DENOM=C3*C3+C4*(B2-C2)
      IF(DENOM.EQ.0.) GO TO 55
C     CALCULATE DELTA P AND DELTA Q.
      DELTP=(B2*(C3+P*C4)-C4*B1)/DENOM
      DELTQ=(C3*B1-(B2*(C2-B2+P*C3)))/DENOM
      GO TO 57
55    DELTP=.1
      DELTQ=.1
C     CORRECT P AND Q.
57    P=P+DELTP
      Q=Q+DELTQ
      GO TO 25
C     ROUTINE TO FACTOR QUADRATIC.
60    DSCRM=P*P-4.*Q
      IF(DSCRM.LT.0.) GO TO 110
      ROOT1=(-P+SQRT(DSCRM))*.5
      ROOT2=(-P-SQRT(DSCRM))*.5
      WRITE(3,5)ROOT1,ROOT2
80    N=N-2
      DO 90 J=1,N
90    A(J)=B(J+2)
      GO TO (17,12),I
110   REAL=-P*.5
      CXPT=SQRT(-DSCRM)*.5
      WRITE(3,6)REAL,CXPT,REAL,CXPT
```

```
      GO TO 80
C     FACTOR LAST QUADRATIC REMAINING.
120   I=2
      P=A(2)
      Q=A(1)
      GO TO 60
C     REMOVE LAST LINEAR FACTOR.
130   ROOT=-A(1)
      WRITE(3,7)ROOT
      GO TO 12
C     PREPARE FOR ANOTHER INITIAL GUESS.
150   ITRY=ITRY+1
      IF(ITRY.LE.5) GO TO 20
      WRITE(3,3)
      GO TO 12
      END
     MON$$      EXEQ LINKLOAD
                CALL BAIRSTOW
     MON$$      EXEQ BAIRSTOW,MJB
```

COEFFICIENTS IN DECREASING POWERS OF X

1.0000000
-.4000000
-.5000000
-.1000000

ITERATION	B(1)	B(0)	P	C
1	1.4100000E 01	1.3100000E 01	4.0000000E 00	3.0000000E 00
2	3.5358303E 00	3.1876117E 00	2.1196196E 00	1.3048048E 00
3	8.6664880E-01	7.2894613E-01	1.1886798E 00	5.2178298E-01
4	1.8213953E-01	1.3600028E-01	7.6190179E-01	2.0311552E-01
5	2.0324777E-02	1.3134158E-02	6.1933679E-01	1.1098800E-01
6	3.6113516E-04	2.0553958E-04	6.0033323E-01	1.0017215E-01
7	1.1098226E-07	5.7337018E-08	6.0000009E-01	1.0000004E-01
8	9.8060000E-15	4.6980000E-15	6.0000000E-01	1.0000000E-01

X = -.3000000 +J .1000000 , X = -.3000000 - J .1000000

X = 1.0000000

COEFFICIENTS IN DECREASING POWERS OF X

1.00000000
8.00000000
17.00000000
8.00000000
16.00000000

ITERATION	B(1)	B(0)	P	Q
1	4.0000000E 00	2.2000000E 01	4.0000000E 00	3.0000000E 00
2	4.9920000E 00	2.0256000E 01	3.2000000E 00	-1.4000000E 00
3	4.1411712E 00	1.7938247E 01	4.7515386E 00	2.3794039E 00
4	1.3697562E 01	5.7182235E 01	2.8527815E 00	-5.3629362E 00
5	5.0472371E 00	2.1073821E 01	3.6925611E 00	-1.7707817E 00
6	4.5285167E 00	1.8631804E 01	4.7065366E 00	2.5366896E 00
7	1.0799677E 01	4.4097310E 01	2.9964120E 00	-4.3912653E 00
8	4.4020663E 00	1.7972643E 01	3.8244128E 00	-9.8067844E-01
9	6.7038499E 00	2.7183807E 01	5.2495898E 00	4.8618119E 00
10	5.1168715E 00	2.0737231E 01	3.6301104E 00	-1.6811064E 00
11	4.7105707E 00	1.9049691E 01	4.6910003E 00	2.6349023E 00
12	9.7024721E 00	3.9178279E 01	3.0545356E 00	-3.9596763E 00
13	4.2025158E 00	1.6969077E 01	3.8844047E 00	-6.0047901E-01
14	8.9333331E 00	3.5985311E 01	5.7517016E 00	6.9457871E 00
15	6.5041643E 00	2.6198888E 01	3.3858077E 00	-2.5783520E 00
16	3.9551670E 00	1.5926604E 01	4.2730355E 00	1.0012443E 00
17	2.0553677E 02	8.2695067E 02	-7.2586788E-01	-1.9107548E 01
18	6.0756904E 01	2.4445056E 02	1.2051611E 00	-1.1337993E 01
19	1.8435007E 01	7.4174683E 01	2.4911873E 00	-6.1629325E 00
20	6.3646371E 00	2.5610532E 01	3.4022860E 00	-2.4948643E 00
21	3.9853509E 00	1.6032667E 01	4.2992230E 00	1.1187347E 00
22	1.1903559E 02	4.7855739E 02	2.3481015E-01	-1.5218252E 01
23	3.5412965E 01	1.4237259E 02	1.8407743E 00	-8.7614509E 00
24	1.1131251E 01	4.4753567E 01	2.9270126E 00	-4.3934068E 00
25	4.5088960E 00	1.8128644E 01	3.7645144E 00	-1.0237693E 00
1	-6.0000000E 00	1.2000000E 01	2.0000000E 00	1.0000000E 00
2	3.0729600E 05	3.9078000E 05	-6.4000000E 01	-8.3000000E 01
3	9.1017512E 04	1.1574950E 05	-4.1517061E 01	-5.4406263E 01
4	2.6945993E 04	3.4272944E 04	-2.6539422E 01	-3.5352559E 01
5	7.9691030E 03	1.0140950E 04	-1.6570982E 01	-2.2659762E 01
6	2.3510587E 03	2.9967246E 03	-9.9509804E 00	-1.4205091E 01
7	6.8944885E 02	8.8370343E 02	-5.5784107E 00	-8.5647736E 00
8	1.9885278E 02	2.5982275E 02	-2.7326970E 00	-4.7714233E 00
9	5.4215643E 01	7.5852957E 01	-9.6765017E-01	-2.1507915E 00
10	1.1391578E 01	2.0607039E 01	-7.3270136E-02	-2.5510275E-01
11	3.8398621E-01	2.3449674E 00	4.6143035E-02	8.6605527E-01
12	4.8924319E-03	-3.4637623E-02	-1.2119481E-03	1.0016618E 00
13	7.7143681E-06	-1.4808063E-05	-8.1031117E-07	1.0000005E 00
14	4.3532800E-12	-3.9466000E-12	-3.3519965E-13	1.0000000E 00

X = .0000000 +J 1.0000000 , X = .0000000 - J 1.0000000

X = -4.0000000 +J .0000011 , X = -4.0000000 - J .0000011

COEFFICIENTS IN DECREASING POWERS OF X

3.00000000
30.00000000
97.00000000
104.00000000
16.00000000

ITERATION	B(1)	B(0)	P	C
1	-4.6666666E 00	-1.0666666E 01	4.0000000E 00	3.0000000E 00
2	9.3166641E 02	2.8777490E 03	-4.5454545E 00	-2.3545454E 01
3	2.7218000E 02	8.4162260E 02	-1.3486234E 00	-1.3646039E 01
4	7.7835478E 01	2.4170036E 02	7.1800119E-01	-7.1927945E 00
5	2.0808743E 01	6.5926528E 01	1.9795261E 00	-3.0985764E 00
6	3.8847125E 00	1.4527399E 01	2.5455281E 00	-6.5607062E-01
7	4.6929866E-01	-4.1991853E 00	1.6548037E 00	5.2076368E-01
8	2.4293044E-01	-3.2379164E-01	1.9229069E 00	3.4372879E-01
9	2.0055946E-02	-6.3314911E-03	1.9949520E 00	3.3309297E-01
10	1.0362436E-04	-1.1886662E-06	1.9999765E 00	3.3333041E-01
11	2.3423312E-09	2.3634202E-10	1.9999999E 00	3.3333333E-01

X = -.1835034 ,   X = -1.8164965

X = -4.0000000 +J   .0000331   , X = -4.0000000 - J   .0000331

COEFFICIENTS IN DECREASING POWERS OF X

```
  1728.00000000
  6216.00000000
  8026.00000000
  3858.00000000
  -470.00000000
 -1129.00000000
  -387.00000000
   -45.00000000
```

ITERATION	B(1)	B(0)	P	Q
1	2.5516493E 02	2.5516666E 02	4.0000000E 00	3.0000000E 00
2	8.5047573E 01	8.5048580E 01	3.5882410E 00	2.5882274E 00
3	2.8278617E 01	2.8279376E 01	3.2493719E 00	2.2483288E 00
4	9.3659466E 00	9.3666170E 00	2.9691878E 00	1.9690778E 00
5	3.0811422E 00	3.0817772E 00	2.7416270E 00	1.7413638E 00
6	1.0010796E 00	1.0016964E 00	2.5585424E 00	1.5579311E 00
7	3.1735996E-01	3.1796227E-01	2.4145130E 00	1.4131231E 00
8	9.5611802E-02	9.6189801E-02	2.3054379E 00	1.3023607E 00
9	2.6037218E-02	2.6547970E-02	2.2271172E 00	1.2206385E 00
10	6.1348586E-03	6.4923267E-03	2.1723425E 00	1.1600930E 00
11	1.3589314E-03	1.5316265E-03	2.1320685E 00	1.1126898E 00
12	3.2829560E-04	3.8972107E-04	2.1025099E 00	1.0769171E 00
13	8.6394747E-05	1.0475993E-04	2.0826327E 00	1.0527089E 00
14	2.2709849E-05	2.7668554E-05	2.0704955E 00	1.0379131E 00
15	5.4614688E-06	6.6580952E-06	2.0638427E 00	1.0298023E 00
16	9.9498795E-07	1.2130497E-06	2.0608174E 00	1.0261139E 00
17	7.6251968E-08	9.2963562E-08	2.0599560E 00	1.0250637E 00
18	6.1461388E-10	7.4931442E-10	2.0598781E 00	1.0249688E 00

X = -.8407152 ,   X = -1.2191628

ITERATION	B(1)	B(0)	P	Q
1	6.6331926E 01	6.6282194E 01	4.0000000E 00	3.0000000E 00
2	2.0985952E 01	2.0960182E 01	3.2885995E 00	2.2901126E 00
3	6.6475735E 00	6.6306000E 00	2.7556262E 00	1.7596950E 00
4	2.1123527E 00	2.0997791E 00	2.3563408E 00	1.3647972E 00
5	6.7528783E-01	6.6602664E-01	2.0575268E 00	1.0730043E 00
6	2.1745269E-01	2.1118117E-01	1.8352038E 00	8.6059297E-01
7	6.9893640E-02	6.6183101E-02	1.6729546E 00	7.1024444E-01
8	2.1535603E-02	1.9720913E-02	1.5604481E 00	6.0951110E-01
9	5.6149627E-03	4.9646639E-03	1.4918120E 00	5.4990858E-01
10	8.6548880E-04	7.4400123E-04	1.4615873E 00	5.2421460E-01
11	3.1648285E-05	2.6743915E-05	1.4555469E 00	5.1913975E-01
12	4.4733096E-08	3.7473264E-08	1.4553195E 00	5.1894979E-01
13	8.6829300E-14	7.2421800E-14	1.4553192E 00	5.1894952E-01

X = -.6250000 ,   X = -.8303191

ITERATION	B(1)	B(0)	P	Q
1	1.2486544E 01	1.1704966E 01	4.0000000E 00	3.0000000E 00
2	3.1754156E 00	2.8922202E 00	2.2180303E 00	1.3769530E 00
3	8.2107099E-01	6.9763416E-01	1.3119007E 00	6.0704764E-01
4	2.0919558E-01	1.5618776E-01	8.5452167E-01	2.6556333E-01
5	4.3572025E-02	2.7225183E-02	6.4578254E-01	1.3513650E-01
6	4.0179961E-03	2.0979988E-03	5.8239487E-01	1.0203860E-01
7	4.0722268E-05	1.8452199E-05	5.7601347E-01	9.9147043E-02
8	3.7752014E-09	1.5373884E-09	5.7595203E-01	9.9122021E-02

X = -.2879760 +J .1272471 , X = -.2879760 - J .1272471

X = .4939271

ERROR NO. 861 AT LOCATION 07019

COEFFICIENTS IN DECREASING POWERS OF X

.12050000
2.1838000
13.1924000
27.5400000
7.8060000
15.6137000

ITERATION	B(1)	B(0)	P	Q
1	-3.0077178E 01	1.7090705E 02	4.0000000E 00	3.0000000E 00
2	1.1956114E 01	1.5190224E 02	4.0674059E 00	-9.0116484E-01
3	2.5144495E 01	1.5911167E 02	5.9449913E 00	2.9202899E 00
4	8.8951078E 01	4.8260030E 02	3.2132181E 00	-4.3360223E 00
5	5.5134991E 01	2.5540366E 02	7.0914867E 00	-4.8404184E 00
6	2.0561624E 01	1.2774057E 02	7.3752274E 00	4.3091363E-01
7	5.7761998E 01	5.5506772E 02	1.4139591E 01	5.0678223E 01
8	2.6104901E 01	2.1828214E 02	1.3454913E 01	4.5235954E 01
9	1.3013826E 01	9.9853764E 01	1.2913701E 01	4.1180744E 01
10	1.5870869E 01	1.1597879E 02	1.2184571E 01	3.5891361E 01
11	2.3572171E 01	1.6816217E 02	1.3125133E 01	4.2638991E 01
12	1.3038626E 01	9.3681740E 01	1.2558525E 01	3.8562505E 01
13	1.2511636E 04	9.0217871E 04	2.0836356E 01	9.8250850E 01
14	3.9103930E 03	2.8196658E 04	1.8269257E 01	7.9740365E 01
15	1.2129884E 03	8.7464255E 03	1.6389940E 01	6.6189424E 01
16	3.7289636E 02	2.6887716E 03	1.5033622E 01	5.6409885E 01
17	1.1434451E 02	8.2480927E 02	1.4070042E 01	4.9462498E 01
18	3.6580510E 01	2.6373516E 02	1.3383979E 01	4.4516379E 01
19	1.4843426E 01	1.0700746E 02	1.2640253E 01	4.0596739E 01
20	2.2003915E 01	1.5861611E 02	1.1938221E 01	3.4094478E 01
21	1.4879180E 01	1.0725275E 02	1.2842013E 01	4.0609304E 01
22	2.1835862E 01	1.5740105E 02	1.1945226E 01	3.4144925E 01
23	1.4967244E 01	1.0789056E 02	1.2847355E 01	4.0647842E 01
24	2.1334935E 01	1.5379105E 02	1.1966092E 01	3.4295336E 01
25	1.5259255E 01	1.0994477E 02	1.2863925E 01	4.0767277E 01
1	-1.3136514E 02	6.9619087E 01	2.0000000E 00	1.0000000E 00
2	2.3285138E 15	2.3867778E 15	-6.9412596E 03	-7.1159944E 03
3	7.3675631E 14	7.5519140E 14	-5.2046203E 03	-5.3359007E 03
4	2.3311428E 14	2.3894726E 14	-3.9021407E 03	-4.0008306E 03
5	7.3758807E 13	7.5604397E 13	-2.9252812E 03	-2.9995282E 03
6	2.3337742E 13	2.3921698E 13	-2.1926368E 03	-2.2485515E 03
7	7.3842045E 12	7.5689717E 12	-1.6431537E 03	-1.6853193E 03
8	2.3364067E 12	2.3948682E 12	-1.2310418E 03	-1.2628955E 03
9	7.3925275E 11	7.5775030E 11	-9.2195827E 02	-9.4607815E 02
10	2.3390345E 11	2.3975638E 11	-6.9014622E 02	-7.0846570E 02
11	7.4008275E 10	7.5860107E 10	-5.1628799E 02	-5.3025719E 02
12	2.3416510E 10	2.4002437E 10	-3.8589537E 02	-3.9660190E 02
13	7.4090338E 09	7.5944224E 09	-2.8810234E 02	-2.9636189E 02
14	2.3442102E 09	2.4026670E 09	-2.1475947E 02	-2.2118384E 02
15	7.4169198E 08	7.6025064E 08	-1.5975488E 02	-1.6480290E 02
16	2.3465852E 08	2.4053022E 08	-1.1850486E 02	-1.2252069E 02
17	7.4237509E 07	7.6095154E 07	-8.7571961E 01	-9.0413699E 01
18	2.3483557E 07	2.4071239E 07	-6.4378492E 01	-6.7039677E 01
19	7.4271051E 06	7.6130223E 06	-4.6991790E 01	-4.9217442E 01
20	2.3481152E 06	2.4069455E 06	-3.3963182E 01	-3.5861709E 01
21	7.4187267E 05	7.6051097E 05	-2.4207356E 01	-2.5859173E 01
22	2.3409422E 05	2.4002596E 05	-1.6912036E 01	-1.8375413E 01
23	7.3686137E 04	7.5602938E 04	-1.1470975E 01	-1.2784635E 01
24	2.3079195E 04	2.3726171E 04	-7.4336844E 00	-8.6161385E 00
25	7.1510910E 03	7.3995288E 03	-4.4705859E 00	-5.5127917E 00
1	1.7526058E 02	3.7624481E 02	.0000000E-00	-1.0000000E 00
2	6.4614562E 00	4.6781414E 01	7.7380189E-02	3.8832193E-01
3	1.1390247E-01	-1.0400127E 00	-2.7675254E-03	5.9917912E-01
4	4.0660141E-04	-9.8298203E-04	-4.1404832E-05	5.9501270E-01
5	1.0956333E-09	-1.3863300E-09	-3.7578999E-05	5.9500914E-01

X = .0000187 +J .7713683 , X = .0000187 - J .7713683

ITERATION	B(1)	B(0)	P	Q
1	4.9394733E 01	1.7539996E 02	4.0000000E 00	3.0000000E 00
2	1.2799982E 01	4.7150649E 01	7.5775665E 00	1.6179531E 01
3	3.4216800E 00	1.3520671E 01	9.4273448E 00	2.3488677E 01
4	8.9846873E-01	3.9008550E 00	1.0375220E 01	2.7604242E 01
5	1.7832343E-01	8.4796814E-01	1.0797504E 01	2.9612296E 01
6	1.2035931E-02	6.1354132E-02	1.0907212E 01	3.0171543E 01
7	5.2021255E-05	2.7884998E-04	1.0914425E 01	3.0210205E 01
8	7.5636090E-10	4.2337520E-09	1.0914452E 01	3.0210359E 01

X = -5.4572264 +J .6550105 , X = -5.4572264 - J .6550105

X = -7.2084062

```
COEFFICIENTS IN DECREASING POWERS OF X

      .12050000
     3.14780000
    26.85740000
    89.85400000
   101.68330000
    15.61370000
```

ITERATION	B(1)	B(0)	P	Q
1	-1.6529460E 02	-3.3164896E 02	4.0000000E 00	3.0000000E 00
2	1.2078236E 02	5.7524509E 02	3.2884592E 00	-1.5382244E 00
3	8.8883933E 05	-3.2746530E 05	3.7227092E 01	-1.3975429E 01
4	2.7993397E 05	-1.0297707E 05	2.9622558E 01	-1.1092575E 01
5	8.7783133E 04	-3.2122291E 04	2.3971362E 01	-9.0630695E 00
6	2.7285798E 04	-9.8001524E 03	1.9805801E 01	-7.6723939E 00
7	8.3252708E 03	-2.7694584E 03	1.6779613E 01	-6.9776843E 00
8	2.4488232E 03	-4.6260590E 02	1.4610223E 01	-7.6865905E 00
9	9.9927833E 02	3.8212917E 03	1.2123757E 01	-2.4952476E 01
10	1.6083190E 02	1.0260705E 03	1.2483029E 01	-9.0282033E 00
11	1.8563857E 01	2.4713015E 02	1.2911176E 01	-1.6591183E 00
12	3.2163326E 00	4.1828703E 01	1.3154963E 01	1.5110546E 00
13	1.7651830E-01	2.3021977E 00	1.3216219E 01	2.3099485E 00
14	6.5168309E-04	8.4971583E-03	1.3220004E 01	2.3593101E 00
15	8.9909620E-09	1.1723512E-07	1.3220018E 01	2.3594936E 00
16	-1.2100000E-13	-2.2000000E-14	1.3220018E 01	2.3594936E 00

```
X =  -.1819557 ,            X =-13.0390631
```

ITERATION	B(1)	B(0)	P	Q
1	1.1336988E 01	2.8207727E 01	4.0000000E 00	3.0000000E 00
2	2.4303433E 00	5.7583700E 00	5.5589558E 00	6.6937351E 00
3	3.7275715E-01	8.1848710E-01	6.1694942E 00	8.0343340E 00
4	2.0439723E-02	4.1767977E-02	6.3124618E 00	8.3264840E 00
5	9.3709682E-05	1.6374799E-04	6.3216111E 00	8.3443814E 00
6	1.4832561E-09	2.8388921E-09	6.3216496E 00	8.3444551E 00

```
X = -1.8777193 ,            X = -4.4439299
X = -6.5811530

    ERROR NO. 961  AT LOCATION      07019
```

ERROR

PROGRAM 2. EQUAL AMPLITUDE RC NOTCH NETWORK

```
C   C  EQUAL AMPLITUDE RC NETWORK
     1 READ, W1, W2, DELW
       PUNCH 4
     4 FORMAT(11X, 5HOMEGA10X,4HEVAL/)
     3 W = W1
       X = 1.*W**8+28.*W**6+198.*W**4+28.*W**2+1.
       EVAL=(1.*W**8+12.*W**6-26.*W**4+12.*W**2+1.)/ X
       PUNCH 5 , W   , EVAL
     5 FORMAT ( 5X, F12.6 , 5X, F12.6)
       W1 = W1 + DELW
       IF ( W2 - W ) 2,3,3
     2 GO TO 1
       END
```

C C DATA FOR GRAPH OF VALUE VS OMEGA - EQUAL AMPLITUDE RC NETWORK

OMEGA	EVAL
.000000	1.000000
.050000	.961353
.100000	.859661
.150000	.726341
.200000	.590164
.250000	.467775
.300000	.365107
.350000	.282053
.400000	.216071
.450000	.164088
.500000	.123288
.550000	.091337
.600000	.066390
.650000	.047016
.700000	.032111
.750000	.020824
.800000	.012498
.850000	.006617
.900000	.002778
.950000	.000658
1.000000	0.000000
1.050000	.000595
1.100000	.002273
1.150000	.004891
1.200000	.008333
1.250000	.012498
1.300000	.017303
1.350000	.022674
1.400000	.028549
1.450000	.034873
1.500000	.041597
1.550000	.048680
1.600000	.056082
1.650000	.063770
1.700000	.071711
1.750000	.079880
1.800000	.088249
1.850000	.096795
1.900000	.105496
1.950000	.114333
2.000000	.123288
2.050000	.132342

OMEGA	EVAL
.000000	1.000000
10.000000	.859661
20.000000	.961353
30.000000	.982495
40.000000	.990087
50.000000	.993636
60.000000	.995573
70.000000	.996744
80.000000	.997506
90.000000	.998028
100.000000	.998402
110.000000	.998679

OMEGA	EVAL
100.000000	.998402
200.000000	.999600
300.000000	.999822
400.000000	.999900
500.000000	.999936
600.000000	.999956
700.000000	.999967
800.000000	.999975
900.000000	.999980
1000.000000	.999984
1100.000000	.999987
1200.000000	.999989
1300.000000	.999991
1400.000000	.999992
1500.000000	.999993
1600.000000	.999994
1700.000000	.999994
1800.000000	.999995
1900.000000	.999996
2000.000000	.999996
2100.000000	.999996
2200.000000	.999997
2300.000000	.999997
2400.000000	.999997
2500.000000	.999997
2600.000000	.999998
2700.000000	.999998
2800.000000	.999998
2900.000000	.999998
3000.000000	.999998
3100.000000	.999998
3200.000000	.999998
3300.000000	.999999

3400.000000	.999999
3500.000000	.999999
3600.000000	.999999
3700.000000	.999999
3800.000000	.999999
3900.000000	.999999
4000.000000	.999999
4100.000000	.999999
4200.000000	.999999
4300.000000	.999999
4400.000000	.999999
4500.000000	.999999
4600.000000	.999999
4700.000000	.999999
4800.000000	.999999
4900.000000	.999999
5000.000000	.999999
5100.000000	.999999
5200.000000	.999999
5300.000000	.999999
5400.000000	.999999
5500.000000	.999999
5600.000000	.999999
5700.000000	1.000000
5800.000000	1.000000
5900.000000	1.000000
6000.000000	1.000000
6100.000000	1.000000
6200.000000	1.000000
6300.000000	1.000000
6400.000000	1.000000
6500.000000	1.000000
6600.000000	1.000000
6700.000000	1.000000
6800.000000	1.000000
6900.000000	1.000000
7000.000000	1.000000
7100.000000	1.000000
7200.000000	1.000000
7300.000000	1.000000
7400.000000	1.000000
7500.000000	1.000000
7600.000000	1.000000
7700.000000	1.000000
7800.000000	1.000000
7900.000000	1.000000
8000.000000	1.000000

PROGRAM 3. UNEQUAL AMPLITUDE RC NOTCH NETWORK

```
C   C  UNEQUAL AMPLITUDE RC NETWORK
     1 READ, W1, W2, DELW
       PUNCH 4
     4 FORMAT(11X, 5HOMEGA10X,4HEVAL/)
     3 W = W1
       X = 81.*W**8+540.*W**6+918.*W**4+60.*W**2+1.
       EVAL=(9.*W**8+12.*W**6-50.*W**4+28.*W**2+1.)/ X
       PUNCH 5 , W  , EVAL
     5 FORMAT ( 5X, F12.6 , 5X, F12.6)
       W1 = W1 + DELW
       IF ( W2 - W ) 2,3,3
     2 GO TO 1
       END
```

C C DATA FOR GRAPH OF VALUE VS OMEGA - UNEQUAL AMPLITUDE RC NETWORK

OMEGA	EVAL
.000000	1.000000
.050000	.925539
.100000	.753401
.150000	.568904
.200000	.416185
.250000	.302013
.300000	.219486
.350000	.160083
.400000	.117007
.450000	.085438
.500000	.062069
.550000	.044640
.600000	.031589
.650000	.021824
.700000	.014563
.750000	.009237
.800000	.005426
.850000	.002813
.900000	.001157
.950000	.000268
1.000000	0.000000
1.050000	.000233
1.100000	.000874
1.150000	.001844
1.200000	.003080
1.250000	.004530
1.300000	.006151
1.350000	.007906
1.400000	.009765
1.450000	.011703
1.500000	.013699
1.550000	.015733
1.600000	.017792
1.650000	.019862
1.700000	.021932
1.750000	.023995
1.800000	.026041
1.850000	.028066
1.900000	.030064
1.950000	.032030
2.000000	.033962
2.050000	.035857

OMEGA	EVAL
1.000000	0.000000
11.000000	.106352
21.000000	.109778
31.000000	.110497
41.000000	.110759
51.000000	.110884
61.000000	.110952
71.000000	.110994
81.000000	.111021
91.000000	.111040
101.000000	.111053

OMEGA	EVAL
100.000000	.111052
200.000000	.111096
300.000000	.111105
400.000000	.111107
500.000000	.111109
600.000000	.111109
700.000000	.111110
800.000000	.111110
900.000000	.111110
1000.000000	.111111
1100.000000	.111111
1200.000000	.111111
1300.000000	.111111
1400.000000	.111111
1500.000000	.111111
1600.000000	.111111
1700.000000	.111111
1800.000000	.111111
1900.000000	.111111
2000.000000	.111111
2100.000000	.111111
2200.000000	.111111
2300.000000	.111111
2400.000000	.111111
2500.000000	.111111
2600.000000	.111111
2700.000000	.111111
2800.000000	.111111
2900.000000	.111111
3000.000000	.111111
3100.000000	.111111
3200.000000	.111111
3300.000000	.111111
3400.000000	.111111
3500.000000	.111111
3600.000000	.111111
3700.000000	.111111
3800.000000	.111111

3900.000000	.111111
4000.000000	.111111
4100.000000	.111111
4200.000000	.111111
4300.000000	.111111
4400.000000	.111111
4500.000000	.111111
4600.000000	.111111
4700.000000	.111111
4800.000000	.111111
4900.000000	.111111
5000.000000	.111111
5100.000000	.111111
5200.000000	.111111
5300.000000	.111111
5400.000000	.111111
5500.000000	.111111
5600.000000	.111111
5700.000000	.111111
5800.000000	.111111
5900.000000	.111111
6000.000000	.111111
6100.000000	.111111
6200.000000	.111111
6300.000000	.111111
6400.000000	.111111
6500.000000	.111111
6600.000000	.111111
6700.000000	.111111
6800.000000	.111111
6900.000000	.111111
7000.000000	.111111
7100.000000	.111111
7200.000000	.111111
7300.000000	.111111
7400.000000	.111111
7500.000000	.111111
7600.000000	.111111
7700.000000	.111111
7800.000000	.111111
7900.000000	.111111
8000.000000	.111111
8100.000000	.111111
8200.000000	.111111
8300.000000	.111111
8400.000000	.111111
8500.000000	.111111
8600.000000	.111111
8700.000000	.111111

8800.000000	.111111
8900.000000	.111111
9000.000000	.111111
9100.000000	.111111
9200.000000	.111111
9300.000000	.111111
9400.000000	.111111
9500.000000	.111111
9600.000000	.111111
9700.000000	.111111
9800.000000	.111111
9900.000000	.111111
10000.000000	.111111
10100.000000	.111111

PROGRAM 4. EQUAL AMPLITUDE RC NOTCH NETWORK WITH CONNECTION X, X′

```
C   C   EQUAL AMPLITUDE RC NETWORK    ( WITH CONNECTION X, X′ )
      1 READ, W1, W2, DELW
        PUNCH 4
      3 W = W1
      4 FORMAT(11X, 5HOMEGA10X,4HEVAL/)
        Y = 0.01452*W**10+1.58792*W**8+55.659*W**6
        Z = 620.6739*W**4-799.069*W**2+243.7876
        X = 0.01452*W**10+3.43604*W**8+180.1407*W**6
        P =  2710.1409*W**4+7533.5867*W**2+243.7876
        EVAL = (Y + Z )/ ( X + P )
        PUNCH 5 , W  , EVAL
      5 FORMAT ( 5X, F12.6 , 5X, F12.6)
        W1 = W1 + DELW
        IF ( W2 - W ) 2,3,3
      2 GO TO 1
        END
```

C C DATA FOR GRAPH OF VALUE VS OMEGA - EQUAL AMPLITUDE RC NETWORK

OMEGA	EVAL
.000000	1.000000
.050000	.920634
.100000	.738456
.150000	.545313
.200000	.387315
.250000	.270637
.300000	.187457
.350000	.128612
.400000	.086919
.450000	.057329
.500000	.036396
.550000	.021767
.600000	.011819
.650000	.005408
.700000	.001716
.750000	.000142
.800000	.000236
.850000	.001657
.900000	.004140
.950000	.007474
1.000000	.013212

OMEGA	EVAL
1.000000	.013212
2.000000	.154531
3.000000	.334853
4.000000	.485036
5.000000	.603284
6.000000	.701436
7.000000	.773125
8.000000	.832461
9.000000	.876527
10.000000	.913462

OMEGA	EVAL
10.000000	.913462
20.000000	.995837
30.000000	.997543
40.000000	.998122
50.000000	.998963
60.000000	.999017
70.000000	.999109
80.000000	.999120
90.000000	.999163
100.000000	.999179

OMEGA	EVAL
100.000000	.999179
200.000000	.999187
300.000000	.999199
400.000000	.999205
500.000000	.999491
600.000000	.999647
700.000000	.999740
800.000000	.999801
900.000000	.999925
1000.000000	.999947

TWO RC NOTCH FILTERS

by

THOMAS WING KAI CHENG

B. S., Kansas State University, 1965

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Electrical Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1966

The notch RC network with equal amplitude at zero and infinite frequencies has been adequately covered in literature. Its chief applications have been in feedback amplifier problems.

The purpose of this report is to exhibit an RC notch network with unequal amplitude at zero and infinite frequencies. Further investigation was made with terminals X, X' connected. The resulting network became an equal amplitude network, with the notch frequency slightly less than that of the original case and the notch minimum value equal to zero.

Investigation of the Q's for each network shows that the new network (Fig. 9) has a smaller Q, and hence a wider bandwidth than the standard twin-tee network.