

RADIAL FLOW BETWEEN TWO PARALLEL DISCS

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TABLE OF CONTENTS

INTRODUCTION	1
THEORETICAL ANALYSIS	3
RESULTS	14
DISCUSSION OF RESULTS	21
RECOMMENDATIONS FOR FUTURE STUDY	22
ACKNOWLEDGEMENT	23
BIBLIOGRAPHY	24

INTRODUCTION

The laminar flow of a viscous fluid between two infinite parallel plates, provided the flow remains laminar, is termed a Poiseuille flow for which an exact solution for flow characteristics may be obtained. In a laminar flow, all fluid elements move in ordered parallel layers, neither crossing one another's path nor mixing with one another. A thin layer of real fluid adheres to the solid walls of the plates therefore the velocity all along the wall is zero. It is reasonable to assume that the velocity distribution at any cross section is symmetrical about the center line, so that all points equi-distant from the plates have the same velocity.

The radial flow between two parallel discs of ideal fluid takes place because of a pressure drop ΔP between the inner and outer radii r_1 and r_2 , and represents a simple elementary inviscid flow.

It is the purpose of this report to study the laminar flow of a viscous fluid between two infinite parallel discs, as in Figure 1, as an extension of the two previous problems.

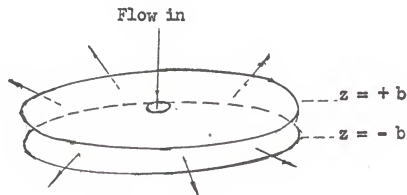


Figure 1

Such a flow could represent, in an idealized form, an example, a thrust bearing whose proper design would require knowledge and study of the hydro-

dynamic properties of the fluid between two discs, such as the velocity distribution, pressure distribution and volumetric rate of flow.

It was assumed in the study that the flow remained laminar throughout. The region considered was $r_1 < r < r_2$, and it was assumed that the flow was steady, incompressible and Newtonian. By using the equation of continuity and the equation of motion for the flow system, the flow was studied for two different flow regimes which were: 1. Creeping flow wherein the inertial terms could be neglected. 2. A laminar flow with sufficient velocity so that the inertial terms must be included, which yielded a non-linear differential equation. For the first case, a linearized differential equation resulted, and an exact solution was obtained. In the second case, a perturbation solution was employed to approximate a solution to the resulting nonlinear differential equation.

A literature survey reflected very few publications describing research for the flow considered in this report. No experimental data could be found in the literature.

THEORETICAL ANALYSIS

In the analysis that flows, the following notation will be used:

r	= radius to any point, feet
r_1	= inner radius, feet
r_2	= outer radius, feet
V_r	= radial component of velocity, feet/second
V_θ	= peripheral component of velocity, feet/second
V_z	= z-component of velocity, feet/second
ρ	= density, slugs/feet ³
P	= pressure, pounds/feet ²
P_1	= pressure at r_1 , pounds/feet ²
μ	= absolute viscosity, pound-second/feet ²
Q	= volumetric flow, cubic feet/second
η	= $\frac{z}{b}$ ratio of depth
C, C_1, C_2	= integration constants
P	= $\frac{b^2 \Delta P}{\mu \ln r_2/r_1}$
B	= $\frac{\frac{1}{2} \rho (1/r_1^2 - 1/r_2^2) b^2}{\mu \ln (r_2/r_1)}$

Assume that a radial flow between parallel discs is steady, laminar, incompressible, Newtonian, and consider the region $r_1 < r < r_2$.

From Navier Stokes equations

$$V_r \frac{\partial V_r}{\partial r} - \frac{V_\theta^2}{r} + V_z \frac{\partial V_r}{\partial z} = \frac{1}{\rho} \frac{\partial P}{\partial r} + \frac{\mu}{\rho P} \left(\frac{\partial^2 V_r}{\partial r^2} + \frac{1}{r} \frac{\partial V_r}{\partial r} - \frac{V_r}{r^2} + \frac{\partial^2 V_r}{\partial z^2} \right) \quad (1)$$

$$V_r \frac{\partial V_z}{\partial r} + V_z \frac{\partial V_z}{\partial z} = -\frac{1}{\rho} \frac{\partial P}{\partial z} + \frac{\mu}{\rho} \left(\frac{\partial^2 V_z}{\partial r^2} + \frac{1}{r} \frac{\partial V_z}{\partial r} + \frac{\partial^2 V_z}{\partial z^2} \right) \quad (2)$$

The flow is assumed to be radial then $V_\theta = 0$, and $V_z = 0$; and the flow produced by a pressure difference between the inner and outer radius.

Since $V_\theta = V_z = 0$, Equation (1) reduces to

$$-\frac{1}{\rho} \frac{dP}{dz} = 0 \quad \text{or} \quad \frac{dP}{dz} = 0$$

which implies pressure is independent of z , hence pressure is function of r only.

The continuity equation for the flow described is

$$\frac{\partial V_r}{\partial r} + \frac{V_r}{r} + \frac{\partial V_z}{\partial z} = 0$$

but since $V_z = 0$

$$\frac{\partial V_r}{\partial r} + \frac{V_r}{r} = 0$$

Differentiation with respect to r yields

$$\frac{\partial}{\partial r} \left(\frac{\partial V_r}{\partial r} + \frac{V_r}{r} \right) = \frac{\partial^2 V_r}{\partial r^2} + \frac{1}{r} \frac{\partial V_r}{\partial r} - \frac{V_r}{r^2} = 0$$

Substitution into Equation (1) yields

$$V_r \frac{\partial V_r}{\partial r} = -\frac{1}{\rho} \frac{\partial P}{\partial r} + \frac{\mu}{\rho} \frac{\partial^2 V_r}{\partial z^2} \quad (3)$$

but since

$$\frac{\partial V_r}{\partial r} = -\frac{V_r}{r}$$

then

$$\frac{r}{dr} = - \frac{V_r}{dV_r}$$

Integration of this expression yields

$$\ln r = - \ln V_r + \ln \bar{\phi}(z)$$

or

$$V_r = \frac{\bar{\phi}(z)}{r}$$

where $\bar{\phi}(z)$ is the constant of integration.

If the above expression for V_r is substituted into Equation (2), one obtains

$$\frac{\bar{\phi}(z)}{r} \left(\frac{-\bar{\phi}(z)}{r^2} \right) = - \frac{1}{\rho} \frac{dP}{dr} + \frac{\mu}{\rho} \frac{\bar{\phi}''(z)}{r}$$

or

$$\frac{\mu}{\rho} \bar{\phi}''(z) \frac{dr}{r} + \frac{\bar{\phi}^2(z)}{r^3} dr = \frac{1}{\rho} dP$$

which may be integrated with respect to r

$$\int_{r_1}^{r_2} \frac{\mu}{\rho} \bar{\phi}''(z) \frac{dr}{r} + \int_{r_1}^{r_2} \frac{\bar{\phi}^2(z)}{r^3} dr = \frac{1}{\rho} \int_{P_1}^{P_2} dP$$

$$\frac{\mu}{\rho} \bar{\phi}''(z) \ln \frac{r_2}{r_1} - \frac{1}{2} \bar{\phi}^2(z) \left(\frac{1}{r_2^2} - \frac{1}{r_1^2} \right) = - \frac{\Delta P}{\rho}$$

to yield the above non-linear differential equation.

If the flow is creeping flow, fluid inertia is small as compared to the viscous shear, so that the inertial term may be neglected.

When the non-linear term is omitted, the resulting equation is

$$\mu \ln \frac{r_2}{r_1} \bar{\phi}''(z) = - \Delta P$$

or

$$\bar{\xi}''(z) = -\frac{\Delta P}{\mu} \frac{1}{\ln \frac{r_2}{r_1}}$$

Integration with respect to z yields

$$\bar{\xi}'(z) = -\frac{\Delta P}{\mu} \frac{1}{\ln \frac{r_2}{r_1}} z + C_1$$

and a second integration gives

$$\bar{\xi}(z) = -\frac{\Delta P}{2\mu} \frac{1}{\ln \frac{r_2}{r_1}} z^2 + C_1 z + C_2$$

Boundary conditions are, at $z = \pm b$ the velocity is zero, so $\bar{\xi}'(z) = 0$ is satisfied and the coefficient $C_1 = 0$ and $C_2 = \frac{\Delta P}{2\mu} \frac{b^2}{\ln \frac{r_2}{r_1}}$

or

$$\bar{\xi}(z) = \frac{b^2 \Delta P}{2\mu \ln \frac{r_2}{r_1}} \left[1 - \left(\frac{z}{b} \right)^2 \right]$$

and

$$V_r(r, z) = \frac{b^2 \Delta P}{2r\mu \ln \frac{r_2}{r_1}} \left[1 - \left(\frac{z}{b} \right)^2 \right]$$

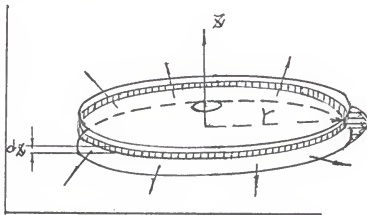


Figure 3

the rate of flow, Q , is then

$$\begin{aligned}
 Q &= 2 \int_0^b v \cdot 2\pi r \, dz \\
 &= 2 \int_0^b \frac{2\pi r}{2\pi\mu \ln \frac{r_2}{r_1}} \frac{b^2 \Delta P}{r_2} \left[1 - \left(\frac{z}{b}\right)^2 \right] dz \\
 &= \frac{4\pi b^3 \Delta P}{\ln \frac{r_2}{r_1}} \int_0^1 (1 - \eta^2) d\eta, \quad \eta = \frac{z}{b}
 \end{aligned}$$

or

$$Q = \frac{4\pi b^3 \Delta P}{3\mu \ln \frac{r_2}{r_1}}$$

Finally since

$$\Delta P = \frac{3Q\mu \ln \frac{r_2}{r_1}}{4\pi b^3}$$

then

$$P(r) = P_1 - \frac{3Q\mu \ln \frac{r}{r_1}}{4\pi b^3}$$

Perturbation Solution

A laminar flow with sufficient velocity so that the inertial term must be included gives rise to a non-linear differential equation. Saaty (2)* has indicated that the principle of superposing solutions to obtain the general solution of such a system does not apply, as it does with a linear system.

* Bracketed numbers refer to corresponding reference in the bibliography.

As it is usually not possible to write down the general solution of such a non-linear system, or even to obtain an exact particular solution, one frequently resorts to careful approximations whose analysis reveals the characteristic properties of the system containing a parameter ϵ (introduced for convenience). The parameter ϵ is a constant whose value will be set anywhere between 0 and 1. Its purpose is to control the size or magnitude of the perturbation. As the perturbation is increased from 0 to its full value it is customary to suppose that the values of $\bar{\phi}_n$ will vary in some smooth manner from their starting point $\bar{\phi}_n^0$. There is no reason to expect $\bar{\phi}_n$ to deviate in an exactly linear manner from the "starting point" $\bar{\phi}_n^0$. We must allow for some curvature. As in Figure 3 we approximate the true curve with a linear term ϵ with a coefficient $\bar{\phi}_n^1$, plus a second degree term in ϵ^2 which has different - and here smaller - coefficient $\bar{\phi}_n^2$.

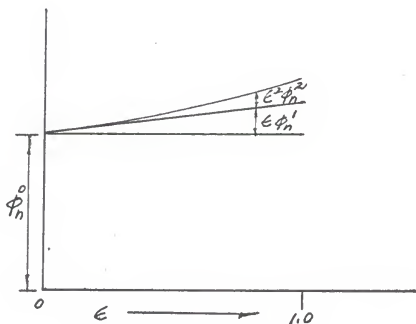


Figure 3

If the curvature is sharper it may be necessary to synthesize the true curve with terms dependent upon ϵ^3 , ϵ^4 , etc. We shall be concerned here only with "first order" approximations. This means that we shall restrict ourselves to perturbation in which, even when the perturbation is "on" at full density ($\epsilon = 1$), the square terms are in all cases small when compared with the linear terms.

By introducing a parameter ϵ , the non-linear equation may be written

$$\bar{\phi}''(z) - \epsilon \frac{\frac{1}{2} \rho \bar{\phi}^2(z) \left(\frac{1}{r_2} - \frac{1}{r_1} \right)}{\ln \frac{r_2}{r_1}} + \frac{\Delta P}{\mu \ln \frac{r_2}{r_1}} = 0 \quad (4)$$

it is assumed that, for $\epsilon \neq 0$

$$\bar{\phi}(z) = \bar{\phi}_0(\eta) + \epsilon \bar{\phi}_1(\eta) + \epsilon^2 \bar{\phi}_2(\eta) + \dots \quad (5)$$

where

$$\eta = \frac{z}{b}$$

$$\bar{\phi}''(z) = \frac{1}{b^2} \left[\bar{\phi}''_0(\eta) + \epsilon \bar{\phi}''_1(\eta) + \epsilon^2 \bar{\phi}''_2(\eta) + \dots \right] \quad (6)$$

substitution of the two series (5), (6) into Equation (4) and letting

$$P = \frac{b^2 \Delta P}{\mu \ln \frac{r_2}{r_1}} \quad B = \frac{\frac{1}{2} \rho b^2 \left(\frac{1}{r_1} - \frac{1}{r_2} \right)}{\mu \ln \frac{r_2}{r_1}}$$

one obtains

$$\left[\bar{\phi}''_0(\eta) + P \right] + \epsilon \left[\bar{\phi}''_1(\eta) + B \bar{\phi}_0^2(\eta) \right] + \epsilon^2 \left[\bar{\phi}''_2(\eta) + 2B \bar{\phi}_0(\eta) \bar{\phi}_1(\eta) + \dots \right] = 0$$

Equating coefficients of like powers of ϵ yields

$$\bar{\phi}''_0(\eta) + P = 0 \quad (7)$$

$$\bar{\Phi}_1''(\eta) + B\bar{\Phi}_0^2(\eta) = 0 \quad (8)$$

$$\bar{\Phi}_2''(\eta) + 2B\bar{\Phi}_0(\eta) \bar{\Phi}_1(\eta) = 0 \quad (9)$$

These differential equations may be solved as follows

$$1. \quad \bar{\Phi}_0''(\eta) = -P \quad \bar{\Phi}_0' = -P\eta + C_1$$

$$\bar{\Phi}_0 = -\frac{1}{2}P\eta + C_1\eta + C_2$$

Boundary conditions

$$\eta = 1 \quad \bar{\Phi}_0 = 0 \quad C_2 = \frac{1}{2}P$$

$$\eta = -1 \quad \bar{\Phi}_0 = 0 \quad C_1 = 0$$

$$\bar{\Phi}_0 = \frac{1}{2}P(1 - \eta^2)$$

$$2. \quad \bar{\Phi}_1''(\eta) = -B\bar{\Phi}_0^2(\eta)$$

$$= -\frac{B}{4}P^2(1 - 2\eta^2 + \eta^4)$$

$$\bar{\Phi}_1'(\eta) = -\frac{B}{4}P^2\left(\eta - \frac{2}{3}\eta^3 + \frac{1}{5}\eta^5\right) + C_1$$

$$\bar{\Phi}_1(\eta) = -\frac{B}{4}P^2\left(\frac{1}{2}\eta^2 - \frac{1}{6}\eta^4 + \frac{1}{30}\eta^6\right) + C_1\eta + C_2$$

Boundary conditions

$$\eta = 1 \quad \bar{\Phi}_1(\eta) = 0$$

$$\eta = -1 \quad \bar{\Phi}_1(\eta) = 0$$

$$0 = -\frac{B}{4}P^2\left(\frac{1}{2} - \frac{1}{6} + \frac{1}{30}\right) + C_1 + C_2$$

$$0 = -\frac{B}{4}P^2\left(\frac{1}{2} - \frac{1}{6} + \frac{1}{30}\right) - C_1 + C_2$$

$$C_2 = \frac{B}{4}P^2\frac{11}{30} \quad C_1 = 0$$

$$\bar{\Phi}_1(\eta) = \frac{B}{4}P^2\left(\frac{11}{30} - \frac{1}{2}\eta^2 + \frac{1}{6}\eta^4 - \frac{1}{30}\eta^6\right)$$

$$3. \quad \bar{\psi}''_2(\eta) = -2B\bar{\psi}_0(\eta) \bar{\psi}'_1(\eta)$$

$$= -2B \left[\frac{1}{2} P (1 - \eta^2) \right] \left[\frac{B}{4} P^2 \left(\frac{11}{30} - \frac{1}{2} \eta^2 + \frac{1}{6} \eta^4 - \frac{1}{30} \eta^6 \right) \right]$$

$$= -\frac{B^2 P^3}{4} \left(\frac{11}{30} - \frac{1}{2} \eta^2 + \frac{1}{6} \eta^4 - \frac{1}{30} \eta^6 \right. \\ \left. - \frac{11}{30} \eta^2 + \frac{1}{2} \eta^4 - \frac{1}{6} \eta^6 + \frac{1}{30} \eta^8 \right)$$

$$= -\frac{B^2 P^3}{4} \left(\frac{11}{30} - \frac{26}{30} \eta^2 + \frac{2}{3} \eta^4 - \frac{1}{3} \eta^6 + \frac{1}{30} \eta^8 \right)$$

$$\bar{\psi}'_2(\eta) = -\frac{B^2 P^3}{4} \left(\frac{11}{30} \eta - \frac{26}{90} \eta^3 + \frac{2}{15} \eta^5 - \frac{1}{35} \eta^7 + \frac{1}{270} \eta^9 \right) + C_1$$

$$\bar{\psi}_2(\eta) = -\frac{B^2 P^3}{4} \left(\frac{11}{60} \eta^2 - \frac{13}{180} \eta^4 + \frac{1}{45} \eta^6 - \frac{1}{280} \eta^8 + \frac{1}{2700} \eta^{10} \right) \\ + C_1 \eta + C_2$$

Boundary conditions

$$\eta = 1 \quad \bar{\psi}_2(\eta) = 0$$

$$\eta = -1 \quad \bar{\psi}_2(\eta) = 0$$

$$C_2 = \frac{B^2 P^3}{4} \left(\frac{11}{60} - \frac{13}{180} + \frac{1}{45} - \frac{1}{280} + \frac{1}{2700} \right)$$

$$= \frac{B^2 P^3}{4} \left(\frac{4919}{2700 \times 14} \right) \quad C_1 = 0$$

$$\bar{\psi}_3(\eta) = \frac{B^2 P^3}{4} \left(\frac{4919}{2700 \times 14} - \frac{11}{60} \eta^2 + \frac{13}{180} \eta^4 - \frac{1}{45} \eta^6 + \frac{1}{180} \eta^8 - \frac{1}{2700} \eta^{10} \right)$$

The solution of $\bar{\psi}(z)$ may be expressed as

$$\bar{\psi} = \bar{\psi}_0(\eta) + \epsilon \bar{\psi}_1(\eta) + \epsilon^2 \bar{\psi}_2(\eta) + \dots$$

Taking the perturbation at full density ($\epsilon = 1$) then $\bar{\psi}(z)$ becomes

$$\bar{\psi} = \bar{\psi}_0(\eta) + \bar{\psi}_1(\eta) + \bar{\psi}_2(\eta) + \dots$$

The expression for $V_x(r, z)$ is then

$$\begin{aligned}
 V(r, z) &= \frac{\bar{\phi}(z)}{r} \\
 &= \frac{1}{r} [\bar{\phi}_0(\eta) + \bar{\phi}_1(\eta) + \bar{\phi}_2(\eta) + \dots] \\
 &= \frac{1}{r} \left[\frac{1}{2} P (1 - \eta^2) + \frac{B}{4} P^2 \left(\frac{11}{30} - \frac{1}{2} \eta^2 + \frac{1}{6} \eta^4 - \frac{1}{30} \eta^6 \right) \right. \\
 &\quad \left. + \frac{B^2 P^3}{4} \left(\frac{4919}{2700 \times 14} - \frac{11}{60} \eta^2 + \frac{13}{180} \eta^4 - \frac{1}{45} \eta^6 + \frac{1}{280} \eta^8 \right. \right. \\
 &\quad \left. \left. - \frac{1}{2700} \eta^{10} \right) + \dots \right]
 \end{aligned}$$

The volumetric rate of flow, Q, will be given by

$$\begin{aligned}
 Q &= 2\pi r \cdot b \int_{-1}^1 \frac{\bar{\phi}(z)}{r} d\eta \\
 &= 2\pi b \int_{-1}^1 \bar{\phi}_0(\eta) + \bar{\phi}_1(\eta) + \bar{\phi}_2(\eta) + \dots d\eta \\
 \int_{-1}^1 \bar{\phi}_0(\eta) d\eta &= \int_{-1}^1 \frac{1}{2} P (1 - \eta^2) d\eta \\
 &= \frac{1}{2} \times 2P \left(1 - \frac{1}{3} \eta^3 \right)_0^1 = \frac{2}{3} P \\
 \int_{-1}^1 \bar{\phi}_1(\eta) d\eta &= 2 \int_0^1 \frac{B}{4} P^2 \left(\frac{11}{30} - \frac{1}{2} \eta^2 + \frac{1}{6} \eta^4 - \frac{1}{30} \eta^6 \right) d\eta \\
 &= \frac{2BP^2}{4} \left(\frac{11}{30} \eta - \frac{1}{6} \eta^3 + \frac{1}{30} \eta^5 - \frac{1}{210} \eta^7 \right)_0^1 \\
 &= \frac{2BP^2}{4} \left(\frac{11}{30} - \frac{1}{6} + \frac{1}{30} - \frac{1}{210} \right) \\
 &= \frac{2BP^2}{4} \cdot \frac{48}{210} = 2BP^2 \cdot \frac{2}{35} \\
 \int_{-1}^1 \bar{\phi}_2(\eta) d\eta &= 2 \times \frac{B^2 P^3}{4} \int_0^1 \left(\frac{4919}{2700 \times 14} - \frac{11}{60} \eta^2 + \frac{13}{180} \eta^4 - \frac{1}{45} \eta^6 \right. \\
 &\quad \left. + \frac{1}{280} \eta^8 - \frac{1}{2700} \eta^{10} \right) d\eta
 \end{aligned}$$

$$\begin{aligned}
&= \frac{2B^2P^3}{4} \left(\frac{4919}{2700 \times 14} \pi - \frac{11}{60 \times 3} \pi^3 + \frac{13}{180 \times 5} \pi^5 \right. \\
&\quad \left. - \frac{1}{45 \times 7} \pi^7 + \frac{1}{280 \times 9} \pi^9 - \frac{1}{2700 \times 11} \pi^{11} \right)_0^1 \\
&= \frac{2B^2P^3}{4} \left(\frac{4919}{2700 \times 14} - \frac{11}{60 \times 3} + \frac{13}{180 \times 5} - \frac{1}{45 \times 7} \right. \\
&\quad \left. + \frac{1}{280 \times 9} - \frac{1}{2700 \times 11} \right) \\
&= \frac{2B^2P^3}{4} \cdot \frac{2 \times 1048}{51975}
\end{aligned}$$

At last then,

$$Q' = 4\pi b \left(\frac{1}{3} P + \frac{2}{35} BF^2 + \frac{1048}{51975} B^2P^3 + \dots \right)$$

RESULTS

The following, Table 1, gives the results that were obtained from the analysis. In the perturbation solution, the first term corresponds to the creeping flow. The higher terms account for the deviations from creeping flow when ΔP is increased. Since the deviation of the curve is not great, only second degree terms were retained.

To illustrate the differences that might result from utilization of each of the derived results, solutions were obtained for the following assumed numerical data for the linearized as well as the nonlinear case. The data used are as in Table 2. In Tables 3, 4, and 5 are tabulated the calculated quantities indicated, while Table 6 gives the results for creeping flow. Figure 4 and Figure 5 display the results graphically.

Table 1. Derived Results

Flow	Mathematical Method	Velocity V_r	Volumetric Rate of Flow	Pressure Distribution
Creeping Flow	Linear Analysis	$\frac{1}{r} \phi_0$	$4\pi b \left(\frac{1}{2} P\right)$	$P_1 - \frac{3\mu \ln \frac{r}{r_1}}{4\pi b^3}$
Ordinary Flow	Perturbation Method	$\frac{1}{r}(\phi_0 + \phi_1 + \phi_2 + \dots)$	$4\pi b \left(\frac{1}{2} P + \frac{2}{35} B P^2 + \frac{1048}{51975} B^2 P^3 + \dots\right)$	

$$B = \frac{\frac{1}{2} \rho \left(\frac{1}{r_1^2} - \frac{1}{r_2^2} \right) b^2}{\mu \ln \frac{r_2}{r_1}}$$

where $P = \frac{b^2 \Delta P}{r_2 \ln \frac{r_2}{r_1}}$

$$\phi_0 = \frac{1}{2} P (1 - \eta^2)$$

$$\phi_1 = \frac{B}{4} P^2 \left(\frac{11}{30} - \frac{1}{2} \eta^2 + \frac{1}{6} \eta^4 - \frac{1}{30} \eta^6 \right)$$

$$\phi_2 = \frac{B^2 P^3}{4} \left(\frac{4919}{37800} - \frac{11}{60} \eta^2 + \frac{13}{180} \eta^4 - \frac{1}{45} \eta^6 + \frac{1}{280} \eta^8 - \frac{1}{2700} \eta^{10} \right)$$

$$\eta = \frac{r}{b}$$

Table 2. Assumed Numerical Data

b	r ₁	r ₂	μ	ln $\frac{r_2}{r_1}$	ρ	$P = \frac{b^2 \Delta P}{\ln \frac{r_2}{r_1}}$	$B = \frac{\frac{1}{2} \rho \left(\frac{1}{r_1^2} - \frac{1}{r_2^2} \right) b^2}{\ln \frac{r_2}{r_1}}$
0.01'	$\frac{3}{12}$ '	1'	2×10^{-3}	1.388	0.985	0.036 ΔP	0.0266
			$\frac{\text{lb-sec}}{\text{ft}^2}$			$\frac{\text{lb-sec}^2}{\text{ft}^4}$	

Table 3. Data of Volumetric Flow

P	P = 0.036 ΔP	Q/4πrb	B	$\frac{2}{35} B P^2$	$\frac{1048}{51975} B^2 P^3$	Q'/4πrb
1 $\frac{\text{lb.}}{\text{ft.}^2}$	0.036	0.012	0.0266	≐ 0	≐ 0	0.012,000
2 $\frac{\text{lb.}}{\text{ft.}^2}$	0.072	0.024	0.0266	0.000007	≐ 0	0.24,007
5 $\frac{\text{lb.}}{\text{ft.}^2}$	0.18	0.03	0.0266	0.000025	≐ 0	0.060,025
10 $\frac{\text{lb.}}{\text{ft.}^2}$	0.36	0.12	0.0266	0.0001	≐ 0	0.120,100

Table 4. Perturbation Solution Calculation

	$\eta = 0$	$\eta = \pm \frac{1}{4}$	$\eta = \pm \frac{1}{2}$	$\eta = \pm \frac{3}{4}$	$\eta = \pm 1$
$1 - \eta^2$	1	0.93	0.75	0.437	0
$\frac{11}{30} - \frac{1}{2} \eta^2 + \frac{1}{6} \eta^4 - \frac{1}{30} \eta^6$	0.3666	0.3353	0.252	0.1326	0
$\frac{4919}{37800} - \frac{11}{60} \eta^2 + \frac{13}{180} \eta^4$ $- \frac{1}{45} \eta^6 + \frac{1}{280} \eta^8 - \frac{1}{2700} \eta^{10}$	0.1301	0.129	0.0906	0.0498	0

Table 5. Perturbation Solution Velocity Profile

η	$\bar{\varphi}_0 = \frac{1}{2} P(1 - \eta^2)$	$\bar{\varphi}_1 = \frac{B^2 P^2}{4} \left(\frac{11}{30} - \frac{1}{2} \eta^2 + \frac{1}{6} \eta^4 - \frac{1}{30} \eta^6 \right)$	$\bar{\varphi}_2 = \frac{B^2 P^3}{4} \left(\frac{4919}{37800} - \frac{11}{60} \eta^2 + \frac{13}{180} \eta^4 - \frac{1}{45} \eta^6 + \frac{1}{280} \eta^8 - \frac{1}{2700} \eta^{10} \right)$
0	0.018	0.000008 x 0.3666	$\doteq 0$
$\pm \frac{1}{4}$	0.18 x 0.93 = 0.01674	0.000008 x 0.3353	$\doteq 0$
$\pm \frac{1}{2}$	0.018 x 0.75 = 0.0135	0.000008 x 0.252	$\doteq 0$
$\pm \frac{3}{4}$	0.018 x 0.4375 = 0.007875	0.000008 x 0.1326	$\doteq 0$

Table 6. Creeping Flow Velocity Profile

$V_r = \frac{1}{r} \dot{\phi}_0$					
r	$\eta = 0$	$\eta = \pm \frac{1}{4}$	$\eta = \pm \frac{1}{2}$	$\eta = \pm \frac{3}{4}$	$\eta = \pm 1$
r_1	$\frac{0.018}{r_1}$	$\frac{0.01674}{r_1}$	$\frac{0.0135}{r_1}$	$\frac{0.0079}{r_1}$	0
$1.5 r_1$	$\frac{0.0012}{r_1}$	$\frac{0.00112}{r_1}$	$\frac{0.0009}{r_1}$	$\frac{0.0005}{r_1}$	0
$2.0 r_1$	$\frac{0.0009}{r_1}$	$\frac{0.00837}{r_1}$	$\frac{0.00625}{r_1}$	$\frac{0.00395}{r_1}$	0
$4.0 r_1$	$\frac{0.0045}{r_1}$	$\frac{0.00414}{r_1}$	$\frac{0.00313}{r_1}$	$\frac{0.00198}{r_1}$	0

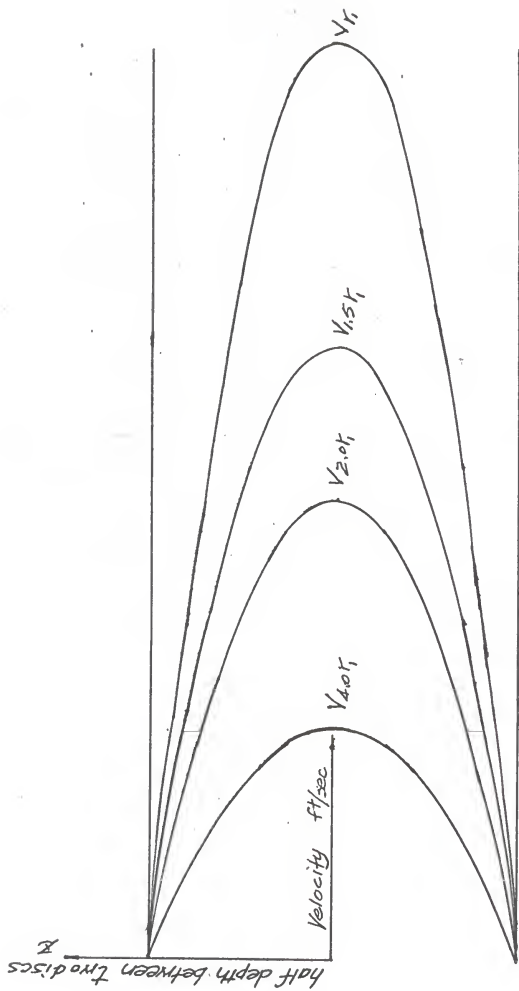


Figure 5. Velocity Profile Between Two Discs

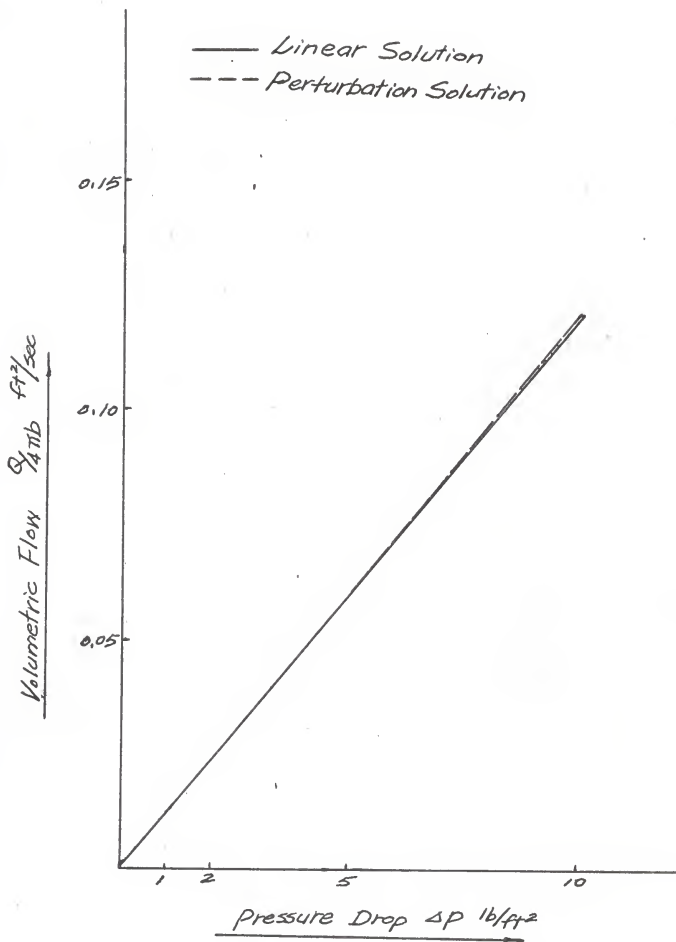


Figure 4. Volumetric Rate of Flow v.s. Pressure Drop

DISCUSSION OF RESULTS

Consideration of the numerical data shown in Figure 4 indicates that between the linearized and perturbation solution, the deviation is very small, and further, that the deviation appeared to be linear. Because of the small deviation it would appear, at least for the range of pressure studied, that the effect of the nonlinear term may be neglected. Figure 5 shows the velocity profile for several different radii. No variation from the profile shown, which are for the linear case, could be found if the nonlinear effects were included.

RECOMMENDATIONS FOR FUTURE STUDY

In the problem solved in this report it was assumed that the flow was steady, incompressible, and Newtonian, and the flow was considered only for the region $r_1 < r < r_2$, r_1 being the radius where the flow became fully developed laminar flow. Those areas wherein future study should be done include: 1. an experimental study to determine the inlet distance required to produce a fully developed laminar flow. 2. a study to determine the Reynold's number for transition. 3. experimental verification of the velocity profiles and pressure distribution. 4. extension of the analysis to include non-Newtonian fluids and turbulent flow.

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BIBLIOGRAPHY

1. Bird, R. Byron, Warren E. Stewart, and Edwin N. Lightfoot, "Transport Phenomena," John Wiley and Sons, Inc., 1960, p. 114, p. 122.
2. Saaty, Thomas L., and Joseph Bram, "Nonlinear Mathematics," McGraw-Hill Book Company, 1964, p. 199-203.
3. Sherwin, Chalmers W., "Introduction to Quantum Mechanics" Holt, Rinehart and Winston, Inc., 1960, p. 164-174.
4. Kaufmann, Walter, "Fluid Mechanics," McGraw-Hill Book Company, 1963.
5. Eskionazi, Salamon, "Principles of Fluid Mechanics," Ally and Bacon, Inc., p. 25.

RADIAL FLOW BETWEEN TWO PARALLEL DISCS

by

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AN ABSTRACT OF A MASTER'S REPORT

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Radial, incompressible, Newtonian flow between two parallel discs is considered for a region $r_1 < r < r_2$ wherein the flow is fully developed and laminar. Expressions for the rate of flow, pressure distribution, and velocity profile are presented. The analysis includes a study of creeping flow, as well as an attempt to approximate the solution to nonlinear problem by a perturbation technique. Areas for future research are identified.