A REDUCTION OF OVERDETERMINED SATELLITE TRACKING DATA

by

THOMAS G. SWANEY

B. S., University of Missouri, 1960

A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Applied Mechanics

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1965

Approved by: Philip G. Kessler

Major Professor

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NOTATION

$\{\ \}$	Column vector
$\lfloor\ \rfloor$	Row vector
δ_{ij}	Kronecker delta
N	Number of observing stations
$\{S\}$	Satellite position vector
E	Sum of the squares of the errors
$\{P_i\}$	Position vector of the i^{th} observation station
(I)	Identity matrix
$\{\hat{s}_i\}$	Unit sighting vector from the i^{th} station
ρ	Radius vector
$d\rho$	Radius of the error sphere
Ω	Angular velocity of the earth
K	Gravitational constant -- 1.54165×10^{-6}

INTRODUCTION

This report is a continuation of a paper by Kirmser and Wakabayashi [1] on the determination of satellite position vectors.

In satellite photography, a number of cameras located at different positions and fixed to the earth yield plates which show sectionally continuous satellite traces. This experimental data may then be reduced to unit sighting vectors.

The plates may have simultaneous timing marks, but generally they will not, although the plates do overlap in time. In the latter case it is necessary to use some point-matching technique to determine corresponding unit sighting vectors.

The purpose of this report is to determine the position vectors of a portion of the orbit of satellite 1960 Iota one, Revolution No. 121, from experimental data that has already been reduced to unit sighting vectors. The accuracy of the satellite locations and the effect of small variations in the orbit parameters will be examined.

DETERMINATION OF SATELLITE POSITIONS

In general, simultaneous sighting data from N stations leads to inconsistent sets of equations (Fig. 1). In [1] it is shown that the inconsistent sets of equations can be solved in the least-square sense by finding the point S, such that E, which is defined by (1), is a minimum.

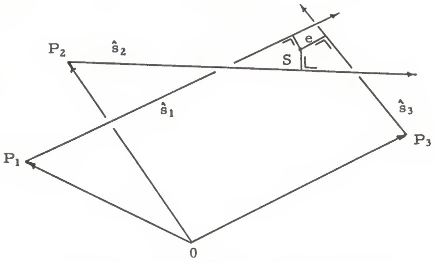

Fig. 1. The geometry of simultaneous sightings from three stations.

$$E = \sum_{i=1}^{N} \lfloor e_i^T \rfloor \{e_i\} = \sum_{i=1}^{N} \lfloor P_i^T - S^T \rfloor (M_i)^T (M_i) \{P_i - S\} \tag{1}$$

where
$$(M_i) = (I - \{\hat{s}_i\} \lfloor \hat{s}_i \rfloor). \tag{1'}$$

E is minimized with respect to S by setting $\frac{\partial E}{\partial S} = 0$.

This gives
$$\{S\} = A^{-1} \sum_{i=1}^{N} (M_i)\{P_i\} \tag{2}$$

where
$$A = \sum_{i=1}^{N} (M_i). \tag{2'}$$

This theory has been extended [1] to include the case when the sighting data from the N stations is continuous, or consists of many discrete unsynchronized observations (Fig. 2). Formulas for E and S will now be functions of some parameters

ξ_j, $j = 1, 2, \ldots, N$ and the point-matching problem becomes finding a set ξ_j, $j = 1, 2, \ldots, N-1$ which minimizes

$$E = \sum_{i=1}^{N} \lfloor P_i^T - S^T \rfloor (M_i) \{P_i - S\} = E(\xi_1, \xi_2, \ldots, \xi_N). \quad (3)$$

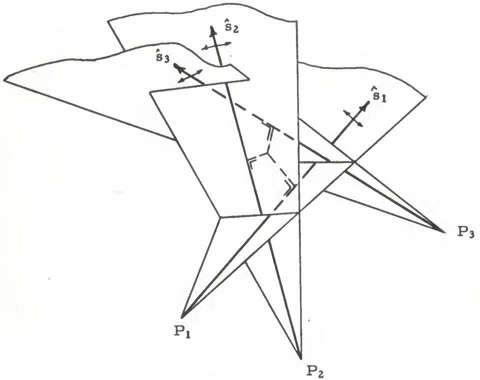

Fig. 2. Point-matching for continuous data from N stations.

This leads to a set of non-linear algebraic equations in the parameters $\xi_1, \ldots, \xi_{N-1}$ of the form

$$\lfloor P_K^T - S^T \rfloor (M_K') \{ P_K - S \} = 0. \quad K = 1, 2, \ldots, N-1 \quad (4)$$

Equation (4) may be solved using a multi-dimensional Newton's method -- a successive approximation technique in which the initial values of the parameters are estimated and the corrections to these initial values are determined from the matrix equation

$$\{\Delta \xi\} = (\alpha_{ij})^{-1} \{\beta\} \tag{5}$$

where

$$\{\Delta\xi\} = \begin{Bmatrix} \Delta\xi_1 \\ \Delta\xi_2 \\ \vdots \\ \Delta\xi_{N-1} \end{Bmatrix}, \quad \{\beta\} = \begin{Bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_{N-1} \end{Bmatrix},$$

and

$$(\alpha_{ij})^{-1} = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1} \\ \alpha_{21} & \cdots & & \vdots \\ \vdots & & & \vdots \\ \alpha_{N-1,1} & & \cdots & \alpha_{N-1,N-1} \end{pmatrix}^{-1}$$

In these formulas,

$$\alpha_{ij} = \lfloor P_i^T - S_o^T \rfloor (2M_i' A^{-1} M_j' - \delta_{ij} M_i'') \{P_j - S_o\}$$

where

$$\{S_o\} = A^{-1} \sum_{i=1}^{N} (M_i(\xi_{io}))\{P_i\},$$

$$M_i' = \frac{dM_i}{d\xi_i}, \quad M_i'' = \frac{d^2 M_i}{d\xi_i^2},$$

and

$$\beta_K = \lfloor P_K^T - S_o^T \rfloor (M_K') \{P_K - S_o\}.$$

Because of the iterative nature of this technique, it is ideally suited for solving by a computer.

A program was written to solve the system of non-linear algebraic equations. In addition, the program, written in Fortran II for the IBM 1620, computed E and S using the adjusted parameters. The program, Appendix I, was designed to continue adjusting the parameters for a given set of data until all of the parameter corrections became smaller than an arbitrarily selected value. This arbitrary value could be adjusted to improve the convergence any time the program was in the computer.

The physical identity of the parameters is entirely arbitrary. The parameter used may be the observation number, one co-ordinate of a sighting vector, or the time of the observation. In fact, the parameters used may vary from station to station.

The time of the observation was chosen as the parameter with the express purpose of determining the timing error between the various stations.

The program was applied to a set of sighting vectors from four stations which were located at Manhattan, Kansas; Oakley, Kansas; St. Louis, Missouri; and Menlo Park, California. (The experimental data may be found in Appendix II.) The Manhattan station was the reference station during the adjustment of the parameters. That is, the initial value of the parameter of the Manhattan station was not adjusted, whereas the others were corrected to satisfy (4). The value of the

Manhattan parameter was used as the initial estimate of the parameters at the other stations.

As stated earlier, the program computed E and S for each trial. Since E is the sum of the errors squared, the radius of an error sphere may be determined by

$$d\rho = \left(\frac{E}{N}\right)^{\frac{1}{2}} . \tag{6}$$

Hence around each point in space determined by S there is an error sphere of radius $d\rho$. Combining these error spheres over the time interval of observation gives an irregular tube which approximates the orbit in the least square sense. Portions of the data output for various times are shown below in Table I.

Table I. Portions of data output.

Time	$\{S\}$	$\lvert S\rvert$	E	$d\rho$
192.000	-.15618584 -.93124562 .81713672	1.2487292	0.00000196	0.0008078
200.000	-.14986779 -.93515921 .81496110	1.2494577	0.00000001	0.0000527
218.000	-.13411623 -.94365799 .80819492	1.2496626	0.00000003	0.0001062
240.000	-.11553407 -.95309533 .79952632	1.2493923	0.00000038	0.0003095

Time is in seconds, and lengths are in earth radii.

The smallest error sphere corresponds to time 200.000. The radius of this sphere is found using (6). Here

$$d\rho = \left(\frac{0.00000001}{3}\right)^{\frac{1}{2}} = 0.0000527 \text{ earth radii,}$$

which is about 0.2 miles.

Since time was used as the parameter, the total computed change in parameters at each station is an indication of the timing error between stations. The variations in time may be seen below in Table II.

Table II. Adjusted parameters (time).

Time	Manhattan	Menlo Park	Oakley	St. Louis
192.000	192.000	192.633	192.514	
200.000	200.000	199.556	199.653	
208.000	208.000	207.965	208.140	
218.000	218.000	217.816	218.073	
230.000	230.000	229.713	229.994	
234.000	234.000	233.967	234.085	235.055
240.000	240.000	239.845	239.481	239.928
242.000	242.000	242.536	241.476	240.864

EFFECTS OF VARIATION IN MEASURED ORBIT PARAMETERS

Using the previously computed sequence of satellite position vectors and corresponding error spheres, the effect of variations of the measured parameters, ρ and θ, on some of the other orbit parameters may be analyzed.

The sequence of satellite vectors is first corrected for the rotation of the earth during the elapsing times of observation, which will affect an instantaneous set of position vectors. The vectors are corrected using the transformation

$$\begin{Bmatrix} \xi \\ \eta \\ \zeta \end{Bmatrix}_i = \begin{pmatrix} \cos\omega_i & -\sin\omega_i & 0 \\ \sin\omega_i & \cos\omega_i & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}_i, \tag{7}$$

where $\omega_i = \Omega \times (t_i - t_1)$ and Ω is the angular velocity of the earth in radians per second. $\lfloor x, y, z \rfloor_i$ is the satellite position vector at time t_i.

The determination of the best orbit plane was calculated using the least square technique described by Kirmser and Wakabayashi [2]. The spherical co-ordinates of a unit normal to the orbit plane are

$$\theta' = \sin^{-1} n$$

and

$$\phi' = \tan^{-1}\left(\frac{\frac{m}{n}}{\frac{\ell}{n}}\right)$$

where

$$\begin{Bmatrix} \frac{\ell}{n} \\ \frac{m}{n} \end{Bmatrix} = \frac{\begin{pmatrix} \eta\cdot\eta & -\eta\cdot\xi \\ -\xi\cdot\eta & \xi\cdot\xi \end{pmatrix}\begin{Bmatrix} \xi\cdot\zeta \\ \eta\cdot\zeta \end{Bmatrix}}{(\xi\cdot\eta)^2 - (\xi\cdot\xi)(\eta\cdot\eta)}$$

and

$$n = \frac{1}{\sqrt{1 + \left(\frac{\ell}{n}\right)^2 + \left(\frac{\ell}{m}\right)^2}}$$

The position vectors determined in (7) are transformed into the orbit plane co-ordinate system with the transformation

$$\begin{Bmatrix} \xi^* \\ \eta^* \\ \zeta^* \end{Bmatrix}_i = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sin\theta' & \cos\theta' \\ 0 & -\cos\theta' & \sin\theta' \end{pmatrix} \begin{pmatrix} -\sin\phi' & \cos\phi' & 0 \\ -\cos\phi' & -\sin\phi' & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{Bmatrix} \xi \\ \eta \\ \zeta \end{Bmatrix}_i .$$

The basis of this new co-ordinate system is the ascending node vector, the normal to the orbit plane (orbit vector), and their mutual perpendicular.

Geometrical Approach

The effect of variations in the measurable parameters may be considered through geometry alone. The equation of a focal ellipse in polar co-ordinates, Fig. 3, is

$$\rho = e\,(p + \rho \cos(\theta - \omega)) \tag{8}$$

where

ρ = radius vector,

e = eccentricity,

θ = angle with respect to the ξ^* axis,

p = distance from the focus to the directrix,

and ω = angle from the ξ^* axis to the major axis.

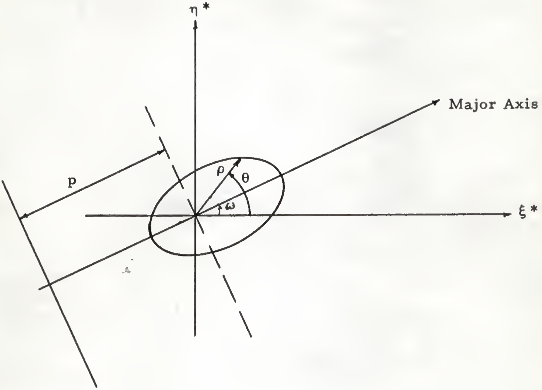

Fig. 3. Focal ellipse in the orbit plane.

Equation (8) may be rewritten as

$$ep + e\rho\cos(\theta - \omega) - \rho = 0 = f(e, p, \rho, \theta, \omega).$$

Taking the derivative of this expression gives the matrix equation

$$\left\lfloor \frac{\partial f}{\partial \rho} \;\; \frac{\partial f}{\partial \theta} \right\rfloor \begin{Bmatrix} d\rho \\ d\theta \end{Bmatrix} = \left\lfloor -\frac{\partial f}{\partial e} \;\; -\frac{\partial f}{\partial \rho} \;\; -\frac{\partial f}{\partial \omega} \right\rfloor \begin{Bmatrix} de \\ dp \\ d\omega \end{Bmatrix} \tag{9}$$

where

$$\frac{\partial f}{\partial \rho} = e\cos(\theta - \omega) - 1, \quad \frac{\partial f}{\partial \theta} = -e\rho\sin(\theta - \omega),$$

$$\frac{\partial f}{\partial e} = p + \rho\cos(\theta - \omega), \quad \frac{\partial f}{\partial p} = e$$

and

$$\frac{\partial f}{\partial \omega} = e \rho \sin (\theta - \omega)$$

Since it is necessary to invert the coefficient matrix on the right hand side of (9) to solve for the unknown variations, three distinct sets of data are used to form a square coefficient matrix. Then (9) becomes

$$(B)^{-1} (A) \begin{Bmatrix} d\rho \\ d\theta \end{Bmatrix} = \begin{Bmatrix} de \\ dp \\ d\omega \end{Bmatrix} \tag{9'}$$

where B^{-1} is a 3 x 3 matrix, and A is a 3 x 2 matrix.

Since the experimental data covered only a small portion of the orbit the columns of B are almost the same, element by element, and B tends to be singular. Therefore the three distinct sets of data used in forming the matrix equation (9') are picked from the first, middle, and last sets of sighting data to reduce this tendency toward singularity.

However, the matrix B is still very nearly singular, and for small changes in ρ and θ, (9') gives quite large values of de, dp, and dω. Because of these large scale changes in the orbit parameters due to small measurement errors, the orbit parameters cannot be satisfactorily determined by geometry alone.

Dynamical Approach

Dynamical principles may also be used in considering the parameter variations. Parabolas are passed through three well-spaced points (ξ_i^*, η_i^*). The parabolas are

$$\begin{aligned} \xi_i^* &= \alpha_1 + \alpha_2 \tau_i + \alpha_3 \tau_i^2 \\ \eta_i^* &= \beta_1 + \beta_2 \tau_i + \beta_3 \tau_i^2 \end{aligned} \tag{10}$$

where $\tau_i = t_i - t_1$.

The radius, velocity, and angles shown in Fig. 4 are given by the set

$$|\rho_1| = \sqrt{\alpha_1^2 + \beta_1^2}, \tag{11}$$

$$|\dot{\rho}_1| = \sqrt{\alpha_2^2 + \beta_2^2}, \tag{12}$$

$$\theta_1 = \tan^{-1} \frac{\beta_1}{\alpha_1}, \tag{13}$$

and

$$\phi_1 = \tan^{-1} \frac{e \sin(\omega - \theta_1)}{1 - e \cos(\omega - \theta_1)} \tag{14}$$

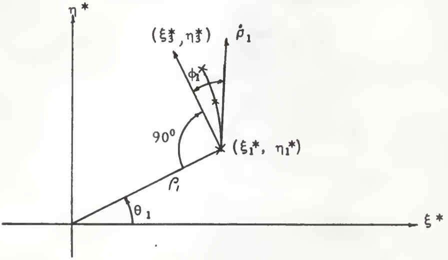

Fig. 4. Vectors in the orbit plane.

Since in central force motion the angular momentum is a constant, we have

$$C = | \rho_1 \times \dot{\rho}_1 | = | \rho_1 | \, | \dot{\rho}_1 | \cos \phi_1. \tag{15}$$

Also

$$2E = | \dot{\rho}_1 |^2 - \frac{K}{| \rho_1 |} \tag{16}$$

where E is the total energy of the system and K is the gravitational constant.

With these expressions some of the orbit parameters are given by

$$ep = \frac{C^2}{K} \tag{17}$$

and
$$e = \sqrt{1 + \frac{2EC}{K^2}} \tag{18}$$

From (15) and (17)

$$d(ep) = \frac{2C}{K} \, dC$$

and
$$dC = |\dot{\rho}_1| \cos \phi_1 \, d\rho_1 + | \rho_1 | \cos \phi_1 \, d\dot{\rho}_1 - |\rho_1| \, |\dot{\rho}_1| \sin \phi_1 \, d\phi_1.$$

Now as
$$\dot{\rho}_1^2 = \alpha_2^2 + \beta_2^2 ,$$
$$|\dot{\rho}_1| \, d\dot{\rho}_1 = \alpha_2 \, d\alpha_2 + \beta_2 \, d\beta_2.$$

Also
$$\xi_i = \alpha_1 + \alpha_2 \tau_i + \alpha_3 \tau_i^2 ,$$

so that
$$\dot{\xi}_i \big|_{\tau_1} = \alpha_2$$

and
$$d\dot{\xi}_1 = d\alpha_2 = \frac{2d\rho_1}{t_2 - t_1} = d\beta_2.$$

This gives
$$d\dot{\rho}_1 = \frac{2(\alpha_2 + \beta_2) \, d\rho_1}{(t_2 - t_1) \, |\dot{\rho}_1|}$$

From (14)

$$d\phi_1 = \frac{\cos^2 \phi_1}{(1 - e \cos(\omega - \theta_1))^2} \{ \sin(\omega - \theta_1) \, de + (e \cos(\omega - \theta_1) - e^2) \, d\omega + (-e \cos(\omega - \theta_1) + e^2) \, d\theta_1 \}.$$

Combining the above gives the matrix equation

$$\left\lfloor \frac{K}{2C} \qquad -m \sin(\theta_1 - \omega) \qquad m(e \cos(\theta_1 - \omega) - e^2 \right\rfloor \begin{Bmatrix} d(ep) \\ de \\ d\omega \end{Bmatrix} =$$

$$\left\lfloor \dot{\rho}_1 \cos\phi_1 + \frac{2\rho_1 \cos\phi_1 (\alpha_2 + \beta_2)}{\dot{\rho}_1 (t_2 - t_1)} \right.$$

$$\left. m(e \cos(\theta_1 - \omega) - e^2 \right\rfloor \begin{Bmatrix} d\rho \\ d\theta \end{Bmatrix} \tag{19}$$

where

$$m = \frac{\rho_1 \dot{\rho}_1 \sin\phi_1 \cos^2\phi_1}{(1 - e\cos(\theta_1 - \omega))^2} .$$

Once again it is necessary to invert the coefficient matrix of the unknown orbit parameter variations. Since it is a 1 x 3 matrix, three sets of data must be considered to form a 3 x 3 matrix, or a square matrix, which may be inverted. (As before, well-spaced sets of data are used since the coefficient matrix tends to be singular.) This gives the matrix equation

$$(A) \begin{Bmatrix} d(ep) \\ de \\ d\omega \end{Bmatrix} = (B) \begin{Bmatrix} d\rho \\ d\theta \end{Bmatrix} . \tag{19'}$$

from which

$$\begin{Bmatrix} d(ep) \\ de \\ d\omega \end{Bmatrix} = (A)^{-1} (B) \begin{Bmatrix} d\rho \\ d\theta \end{Bmatrix} . \tag{20}$$

The coefficient matrix A is nearly singular, as it was in the geometrical investigation. Although not as bad as in the first investigation, it nevertheless showed that for small variations in the measured parameters ρ and θ, there were large changes in the other orbit parameters.

LIMITATIONS ON ERROR SPHERE SIZE

From the previous work, the size of the maximum allowable error sphere for specific error tolerances in the various orbit parameters may be calculated.

From (19) it is seen that the coefficient matrix of the measurable parameters is a 1 x 2 matrix. Two sets of distinct data are considered so that this matrix is built-up into a 2 x 2 matrix. Then it may be inverted and (19) is written as

$$\begin{Bmatrix} d\rho \\ d\theta \end{Bmatrix} = (B)^{-1} \; (A) \begin{Bmatrix} d\,(ep) \\ de \\ d\omega \end{Bmatrix}. \tag{21}$$

For an error allowance of 0.1 per cent in the orbit parameters ep, e, and ω it is found from (21) that the error sphere will have a radius of approximately 1×10^{-5} earth radii. The smallest error sphere computed from the set of experimental data had a radius of 5×10^{-5} earth radii or 0.2 miles, which is about five times larger than the error sphere required for the specified error allowance.

Typical values of ep, e, and ω were obtained from sample calculations in [2].

The necessary increase in accuracy could perhaps be attained from an analysis of the techniques used in obtaining and reducing the experimental data as detailed by Kirmser, Wakabayashi, and Creech [3].

DISCUSSION

This report shows that the given sighting data did not include enough of the orbit to determine the orbit parameters, but that the various satellite positions could be located quite accurately. If data were available from another set of observation stations, located at least one third of the way around the earth from central USA, the orbit parameters could be satisfactorily determined. This second set of stations would remove the singularity of the coefficient matrix that "blew up" both the geometrical and dynamical analyses. It would be best to have three sets of stations located at the third points of the orbit. The sighting data used in this report covered less than 5° (central angle) of the satellite orbit.

Several other points should be made. The eccentricity of this particular orbit was in the neighborhood of 0.02 which is almost a circle. For a circle $p = \infty$, hence with an orbit approximating a circle, p will be quite large. Also, as an orbit approaches a circle the determination of ω becomes meaningless, for any axis of a circle is a major axis.

As stated earlier, time was used as the parameter to determine the timing error between stations. However, the timing error values must be viewed with caution, for this timing error will include other errors such as errors in the location of the observer's positions, observational errors, computational errors because of the parameter technique used, and errors in the reduction of photographic glass plate negatives [3].

ACKNOWLEDGMENT

The author wishes to acknowledge his indebtedness to Dr. Philip G. Kirmser, professor and head, Department of Applied Mechanics, for his guidance in writing this paper and for making available the sighting data used in this paper. A considerable amount of work on this topic had already been accomplished with the conversion of the photographs to the sighting vectors which were used as the initial data in this report.

The author would also like to thank Mr. Norton Goodwin, Program Director of the Independent Tracking Coordination Program, for providing Phototrack negatives from which the fundamental position data was obtained.

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APPENDICES

APPENDIX I

The appendix contains the IBM 1620 programs, written in FORTRAN II language, and the corresponding flow charts, which were used to convert the experimental data to satellite position vectors as explained on pp 1-7 of this report.

The programs are -

I. Program for normalization of the experimental sighting vectors and calculation of the elements of the 3 x 3 matrix, defined in (1'), for each normalized sighting vecotr. The input is the set of sighting vectors, their corresponding times of observation, and the station from which they were observed. The stations, or observer locations, were numbered 1 through N with the number N being given to the reference station. The output are the elements of the M matrix. This output makes up a portion of the input for program II.

II. Program for solving the system of nonlinear equations (4) by an iterative technique. The input includes the number of stations, the initial estimate of the parameter (time), the observer's position vectors, and the output of program I. A typical set of input data is shown after the listing of program II.

Since derivatives of the M matrix are required, a parabola is passed through the corresponding elements of three M matrices to form a new matrix whose elements are functions of time. Therefore it is necessary to read in three M matrices for each participating station. The three matrices chosen should bracket the initial parameter (time) estimate. For the M matrix input the cards must be in the following specified order:

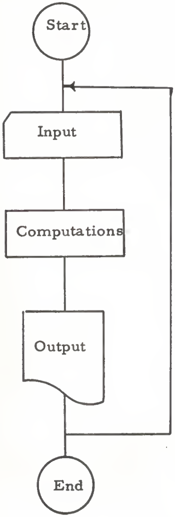
1. The row one cards, from each of the three M matrices, arranged from lowest to highest time.
2. The row two cards from each of the three M matrices.
3. The row three cards from each of the three M matrices.

This is done for station one, then two, and up to station number N so that we have nine cards for station one, then nine cards for station two, etc.

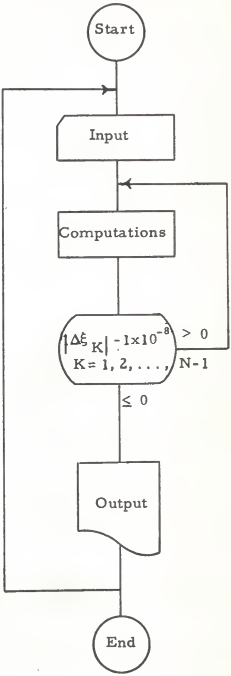
The output of program II includes satellite position vectors, the final value of the parameter for each observer, and the radius of the error sphere.

FLOW CHARTS

PROGRAM I



PROGRAM II



```
C      NORMALIZATION  DETERMINATION OF MATRICES
*16 5
    1 DIMENSION AM(3,3)
    4 FORMAT (3F14.8, F8.3,I2)
```

Read in Sighting Vectors (X, Y, Z), Time (T), and Corresponding Station (ISTA)

```
    5 READ 4,X,Y,Z,T,ISTA
```

Normalization of the Sighting Vectors

```
      A= SQRTF (X*X+Y*Y+Z*Z)
      B= X/A
      C=Y/A
      D=Z/A
```

Computation of the 3x3 M Matrix

```
      AM(1,1)=1.0-B*B
      AM(1,2)=-B*C
      AM(1,3)=-B*D
      AM(2,1)=AM(1,2)
      AM(2,2)=1.0-C*C
      AM(2,3)=-C*D
      AM(3,1)=AM(1,3)
      AM(3,2)=AM(2,3)
      AM(3,3)=1.0-D*D
      DO 9  I=1, 3
```

Read out the M Matrix (I -- Row Number)

```
    9 PUNCH 10, AM(I,1), AM(I,2), AM(I,3), T, ISTA,  I
   10 FORMAT (3F14.10,F8.3,2I2 )
```

Return to 5 for a New Set of Sighting Vectors

```
      GO TO 5.
   50 END
```

PROGRAM II

```
*16 5 MAIN PROGRAM
      DIMENSION  AAM(3), CMA(3,3,4), CMB(3,3,4), CMC(3,3,4)
      DIMENSION T(5), AM(3,3,4), AMP(3,3,4), AMPP(3,3,4)
      DIMENSION AI(3,3), SSS(3,1,4), SS(3,1), S(3,1)
      DIMENSION BBB(3,1,4), BBBB(3,1,4), B(4), AA(3,3,4)
      DIMENSION D(4,4), AB(1,3,4), ABB(4,4), AE(4,5)
      DIMENSION P(3,3,4),A(3,3),BB(1,3,4),AAA(1,3,4)
      DIMENSION EB(4),DT(4) ,EA(3,1,4)
      DIMENSIONX(9),G(9),Y(9),C(9),U(9),H(9),F(9)
    2 FORMAT (F5.3)
    3 FORMAT (3F 14.8)
    4 FORMAT (3F14.10,F8.3,2I2)
    5 FORMAT ( 3E16.10,3I2)
   14 FORMAT (I2, F8.3)
   20 FORMAT (3E16.10, 3I2)
```

Read in Number of Stations and Initial Parameter Estimates

```
  201 READ14,N,T(5)
```

Read in Step Factor

```
      READ 2, DEL
      NI=N-1
      AN=N
      DO114 K=1,N
```

Read in Observer Position Vectors (K-Station Number)

```
  114 READ 3, P(1,1,K), P(1,2,K), P(1,3,K)
      NU=0
  205 DO 6 I=1,3
```

Read in M Matrix (Program I Output)

```
    6 READ 4, X(I), X(I+3), X(I+6),T(I), ISTA, L
      NU =NU + 1
      DO 13 I=1,7,3
```

Pass Parabola Through Corresponding Elements of M Matrix, Then M Becomes a Function of Time

```
      U(I)=X(I)/((T(1)-T(2))*(T(1)-T(3)))
      H(I)=X(I+1)/((T(2)-T(1))*(T(2)-T(3)))
      F(I)=X(I+2)/((T(3)-T(1))*(T(3)-T(2)))
      G(I)=T(2)*T(3)*U(I)+T(1)*T(3)*H(I)+T(1)*T(2)*F(I)
      Y(I)=-((T(2)+T(3))*U(I)+(T(1)+T(3))*H(I)+(T(1)+T(2))*F(I))
   13 C(I)=U(I)+H(I)+F(I)
      DO 12 I=1,3
```

```
      MU = 3*I-2
      CMA (L,I,ISTA) = G(MU)
      CMB (L,I,ISTA) = Y(MU)
   12 CMC (L,I,ISTA) = C(MU)
      IF (NU-3*N)    205,215,215
  215 DO 15 K=1,N
```

Set All Parameters Equal to Initial Estimate

```
   15 T(K)=T(5)
      NO = 0
   16 DO 17 K=1,N
      DO 17 I=1,3
      DO 17 J=1,3
```

Expressions for M, M', and M'' as Functions of Time

```
      AM(I,J,K)=CMA(I,J,K)+CMB(I,J,K)*T(K)+CMC(I,J,K)*T(K)**2
      AMP(I,J,K)=CMB(I,J,K)+2.*(CMC(I,J,K)*T(K))
   17 AMPP(I,J,K)=2.*(CMC(I,J,K))
      DO 18 I=1,3
      DO 18 J=1,3
      A(I,J)=0.0
      DO 18 K=1,N
```

Computation of A and A^{-1}

```
   18 A(I,J)=A(I,J)+AM(I,J,K)
      AD=(A(1,1)*A(2,2)*A(3,3)+A(1,2)*A(2,3)*A(3,1)
     1+A(1,3)*A(3,2)*A(2,1)-A(1,3)*A(2,2)*A(3,1)-A(2,3)
     2*A(3,2)*A(1,1)-A(3,3)*A(2,1)*A(1,2))
      AI(1,1)=(A(2,2)*A(3,3)-A(2,3)*A(3,2))/AD
      AI(1,2)=-(A(1,2)*A(3,3)-A(1,3)*A(3,2))/AD
      AI(1,3)=(A(1,2)*A(2,3)-A(1,3)*A(2,2))/AD
      AI(2,1)=-(A(2,1)*A(3,3)-A(2,3)*A(3,1))/AD
      AI(2,2)= (A(1,1)*A(3,3)-A(1,3)*A(3,1))/AD
      AI(2,3)=-(A(1,1)*A(2,3)-A(1,3)*A(2,1))/AD
      AI(3,1)= (A(2,1)*A(3,2)-A(2,2)*A(3,1))/AD
      AI(3,2)=-(A(1,1)*A(3,2)-A(1,2)*A(3,1))/AD
      AI(3,3)= (A(1,1) *A(2,2)-A(1,2)*A(2,1))/AD
```

Computation of $(M_K)\ \{P_K\}$

```
      DO220 K=1,N
      DO220 I=1,3
      SSS(I,1,K)=0.0
      DO220 J=1,3
  220 SSS(I,1,K)=SSS(I,1,K)+AM(I,J,K)*P(1,J,K)
```

Computation of $\Sigma(M_K)\ \{P_K\}$

```
      DO 25 I=1,3
      SS(I,1)=0.0
      DO 25 K=1,N
   25 SS(I,1)=SS(I,1)+SSS(I,1,K)
```

Computation of $S = A^{-1} \Sigma(M_K) \{P_K\}$

```
      DO 30 I=1,3
      S(I,1)=0.0
      DO 30 J=1,3
30    S(I,1)= S(I,1)+ AI(I,J)*SS(J,1)
```

Computation of $\lfloor P_K^T - S^T \rfloor$

```
      DO 35 K=1,N
      DO 35 J=1,3
35    BB(1,J,K)=P(1,J,K)-S(J,1)
```

Computation of Satellite Distance

```
31    SDS = 0.0
      DO 32  I=1,3
32    SDS = SDS+S(I,1)*S(I,1)
      SD = SQRTF(SDS)
```

Computation of $\{P_K - S\}$

```
      DO 40 K=1,N
      DO 40 I=1,3
40    BBB(I,1,K)=P(1,I,K)-S(I,1)
```

Computation of $(M_K') \{P_K - S\}$

```
      DO 45 K=1,NI
      DO 45 I=1,3
      BBBB(I,1,K)=0.0
      DO 45 J=1,3
45    BBBB(I,1,K)=BBBB(I,1,K)+AMP(I,J,K)*BBB(J,1,K)
```

Computation of $\lfloor P_K^T - S \rfloor (M_K') \{P_K - S\}$

```
      DO 50 K=1,NI
      B(K)=0.0
      DO 50 I=1,3
50    B(K)=B(K)+BB(1,I,K)*BBBB(I,1,K)
```

Computation of $2(M_K') \ A^{-1}$

```
      DO 55 K=1,NI
      DO 55 I=1,3
      DO 55 J=1,3
      AA(I,J,K)=0.0
      DO 55 L=1,3
55    AA(I,J,K)=AA(I,J,K)+2.*(AMP(I,L,K)*AI(L,J))
```

Computation of $\lfloor P_K^T - S^T \rfloor (2M_K' A^{-1})$

```
      DO 60 K=1,NI
      DO 60 J=1,3
      AAA(1,J,K)=0.0
      DO 60 I=1,3
60    AAA(1,J,K)=AAA(1,J,K)+BB(1,I,K)*AA(I,J,K)
```

Computation of $\lfloor P_K^T - S^T \rfloor (2M_K' A^{-1})(M_{KK}')\{P_{KK} - S\}$

```
      DO 65 K=1,NI
      DO 65 KK=1,NI
      D(K,KK)=0.0
      DO 65 I=1,3
   65 D(K,KK)=D(K,KK)+AAA(1,I,K)*BBBB(I,1,KK)
```

Computation of $\lfloor P_K^T - S^T \rfloor (M''_K)$

```
      DO 70 K=1,NI
      DO 70 J=1,3
      AB(1,J,K)=0.0
      DO 70 I=1,3
   70 AB(1,J,K)=AB(1,J,K)+BB(1,I,K)*AMPP(I,J,K)
```

Computation of $\lfloor P_K^T - S^T \rfloor (M_K'')\{P_{KK} - S\}$

```
      DO 75 K=1,NI
      DO 75 KK=1,NI
      ABB(K,KK)=0.0
      DO 75 I=1,3
   75 ABB(K,KK)=ABB(K,KK)+AB(1,I,K)*BBB(I,1,KK)
```

Computation of $\lfloor P_K^T - S^T \rfloor (2M_K' A^{-1} \quad M'_{KK} - M''_{KK} \quad \delta_{K,KK})\{P_{KK} - S\}$

```
      DO 90 K=1,NI
      DO 90 KK=1,NI
      IF(K-KK) 80,85,80
   80 AF(K,KK)=D(K,KK)
      GO TO 90
   85 AF(K,KK)=D(K,KK)-ABB(K,KK)
   90 CONTINUE
```

Computation of $E = \Sigma \lfloor P_K^T - S^T \rfloor (M_K)\{P_K - S\}$

```
   95 DO 100  K=1,N
      DO 100  I=1,3
      EA(I,1,K) = 0.0
      DO 100  J=1,3
  100 EA(I,1,K) = EA(I,1,K)+AM(I,J,K)*BBB(J,1,K)
      DO 105  K=1,N
      FB(K) = 0.0
      DO 105  I=1,3
  105 FB(K) = FB(K)+BB(1,I,K)*EA(I,1,K)
      E = 0.0
      DO 110  K=1,N
  110 E = E+EB(K)
```

Solution of N-1 Equations $(\alpha_{ij})\{\Delta\xi\} = \{\beta\}$

```
      DO 135 I=1,NI
  135 AE(I,N)=B(I)
      DO 125 K=1,NI
```

```
      DO 111 I=1,NI
      DO 111 J=1,N
  111 D(I,J) = AE(I,J)
      DO 115 I=1,NI
      D(I,NI) = AE(I,K)
  115 D(I,K) = AE(I,NI)
      DO 120 M=2,NI
      DO 120 I=M,NI
      DO 120 J=M,N
  120 D(I,J) = D(M-1,M-1)*D(I,J) - D(I,M-1)*D(M-1,J)
```

Change in Parameter for Each Station $\{\Delta\xi\}$

```
  124 DT(K) = D(NI,N)/D(NI,NI)
  125 CONTINUE
      NC = NC+1
      DO 199 K=1,NI
```

Comparison of $\Delta\xi_K$ with a small number. If all $\Delta\xi_K$ are smaller than the number, process output and return to the start of the program for a new set of data. If not, continue iteration using the newly adjusted parameters.

```
      IF (ABSF(DT(K))-1.0E-08) 199,199,170
  199 CONTINUE
      EP = ABSF (E)
      DR = SQRTF(EP/AN)
  159 PRINT 167, T(5)
  160 PRINT 161, E
  161 FORMAT (10H ERROR IS E14.8)
  162 PRINT 163, NC, SD
  163 FORMAT (7H TRIAL I2, 4H SD E14.8)
  164 FORMAT (3E14.8, F8.3)
      PUNCH 165, T(5), E, SD, DR
      PUNCH 164, S(1,1), S(2,1), S(3,1), T(5)
      PUNCH 167, (DT(K), K=1,NI)
      DO 168 K= 1,N
  168 PUNCH 169 , T(K), K
  165 FORMAT (4E14.8)
  167 FORMAT (E14.8)
  169 FORMAT (E14.8, I2)
      GO TO 201
  170 DO 175 K=1,NI
```

May Print Out Latest Set of Parameter Changes

```
      IF (SENSE SWITCH 1 ) 171, 175
  171 PRINT 167, DT(K)
  175 T(K)=T(K) + DEL * DT(K)
      IF (NC-4) 177, 176, 176
  176 DEL=1.0
```

May Change Del, the Step Factor, to Improve Convergence

```
177 IF (SENSE SWITCH 2) 178, 200
178 ACCEPT 2, DEL
200 GO TO 16
    END
```

TYPICAL INPUT FOR PROGRAM II

```
3 240.000

.5

     -.00583085     -.78295593     +.62000437
     -.42302268     -.67287305     +.60483238
     -.08897742     -.77102630     +.62851819

     .8330888353   -.2557116668    .2714099689 239.000 1 1
     .8348237517   -.2551634671    .2697863234 240.000 1 1
     .8605358999   -.2452303308    .2446956266 250.000 1 1
    -.2557116668    .6082439621    .4158061905 239.000 1 2
    -.2551634671    .6058247136    .4167646038 240.000 1 2
    -.2452303308    .5687928640    .4302669249 250.000 1 2
     .2714099689    .4158061905    .5586672025 239.000 1 3
     .2697863234    .4167646038    .5593515346 240.000 1 3
     .2446956266    .4302669249    .5706712360 250.000 1 3
     .5544217278    .4074522400   -.2846417882 238.000 2 1
     .5521084997    .4086015304   -.2834245818 240.000 2 1
     .5497642085    .4097567495   -.2821753527 242.000 2 1
     .4074522400    .6274115272    .2602863323 238.000 2 2
     .4086015304    .6272418418    .2585619906 240.000 2 2
     .4097567495    .6270829707    .2568060059 242.000 2 2
    -.2846417882    .2602863323    .8181667449 238.000 2 3
    -.2834245818    .2585619906    .8206496583 240.000 2 3
    -.2821753527    .2568060059    .8231528207 242.000 2 3
     .9869032149   -.0823980010    .0783315304 236.770 3 1
     .9891528973   -.0755812718    .0708301794 240.000 3 1
     .9904694570   -.0711930733    .0661154896 241.820 3 1
    -.0823980010    .4815956309    .4928203005 236.770 3 2
    -.0755812718    .4733590289    .4935359460 240.000 3 2
    -.0711930733    .4681883562    .4938821373 241.820 3 2
     .0783315304    .4928203005    .5315011540 236.770 3 3
     .0708301794    .4935359460    .5374880736 240.000 3 3
     .0661154896    .4938821373    .5413421666 241.820 3 3
```

APPENDIX II

This appendix contains the observers' position vectors and the sequence of sighting vectors and corresponding times of observation from the observing stations. The coordinate system has the x and y axis on the equatorial plane and the z axis pointing north. The coordinate system is rigidly attached to the earth.

The format of the sighting vectors is the output of the IBM 650.

MANHATTAN, KANSAS
- .08897742 - .77102630 .62851819

-x	-y	z	time	
3606108750	5401477050	7566182350	291596805E00000-00+	+
3450909750	5565203250	7542987650	291650005E000000000	+
3299326050	5712655550	7515516450	291701005E000000000	+
3240095250	5761555050	7503766050	291720905E000000000	+
3175985550	5813913750	7490761950	291741905E000000000	+
3112495950	5865182450	7477434750	291763205E000000000	+
3075795250	5894625050	7469463250	291775105E000000000	+
3059967950	5907229850	7466004950	291780405E000000000	+
2934895550	6005797150	7437525650	291800605E000000000	+
2936142850	6004528750	7437733850	291821205E000000000	+
2875740650	6051799450	7425331050	291840805E00000000+	+
2815871050	6097785250	7408636550	291860205E000000000	+
2748182650	6149291950	7391460950	291882005E00000*000	+
2689151050	6193727750	7376056350	291900805E000000000	+
2631331650	6236897050	7360516350	291919305E00000000-	+
2567975550	6263637750	7345116650	291939105E000000000	+
2502681650	6351344950	7324661150	291960005E000000000	+
2440527150	6376213750	7306691150	291979405E00000000-	+
2421220150	6390141950	7300943650	291985605E00000000-	+
2398760750	6406120050	7294352050	291992405E000000000	+
2336506950	6446910650	7276189950	292012605E000000000	+
2277548750	6469799750	7259151350	292029605E000000000	+
2257808050	6503526150	7253630150	292035905E00000-00+	+
2208178350	6537926550	7237365350	292051205E000000000	+
2145201650	6560933350	7217300750	292070005E00000-0*+	+
2098904350	6612277450	7202248950	292084405E000000000	+
2074947050	6626286650	7194470250	292092005E000000000	+
2019735650	6679604950	7162463750	292110805E00000000-	+
1944902250	6714407450	7150610850	292131505E00000000+	+
1872363250	6761529550	7125720650	292143505E000000000	+
1812050750	6800140950	7104542950	292171605E00000000+	+
1711544050	6863000950	7068351550	292201505E000000000	+
1648567150	6902751950	7045156050	292220405E00000000+	+
1582900550	6944170150	7019453150	292240605E000000000	+
1518804550	6960444650	6995021550	292259205E000000000	+
1380145950	7065233050	6941015250	292300005E000000000	+
1311641950	7104911050	6913741350	292320405E00000000+	+
1240270650	7146538850	6884464250	292341605E000000000	+
1144411950	7200030650	6844697650	292367705E00000000+	+
1041494350	7257003750	6800823350	292400005E000000000	+
976245054R	7292342150	6772426450	292418205E000000000	+
906457604R	7350135550	6741455450	292439705E000000000	+

MENLO PARK, CALIFORNIA

- .42302268 - .67287305 .60483238

x	-y	z	time	
6206477950	6037204950	5003176450	291900005E000000 00+	+
6228458650	6040653050	4971601150	291920005E000000 0*+	+
6249832950	6044745250	4939701250	291940005E000000000	+
6270789550	6048816550	4906054950	291960005E000000000-	+
6291581950	6052502150	4876802650	291980005E000000000	+
6312286550	6056371150	4845139650	292000005E000000 0*+	+
6333192950	6059765450	4813513950	292020005F000000000	+
6353398150	6063221650	4782434150	292040005E000000 00+	+
6373667150	6066541550	4751151850	292060005E000000000	+
6393670550	6069771550	4720047750	292080005E000000000+	+
6413600950	6072811450	4688995350	292100005E000000*000	+
6433574350	6075762750	4657707350	292120005E000000000+	+
6453359950	6078269150	4626915050	292140005E000000000	+
6472623550	6081154150	4596167450	292160005E000000000	+
6491786450	6083641050	4565475550	292180005E000000000	+
6510922050	6086206050	4534971050	292200005E000000000+	+
6529800550	6088639150	4504461850	292220005E000000000	+
6548485650	6090989950	4474056750	292240005E000000000	+
6566784350	6093256050	4444049350	292260005E000000000	+
6585461950	6095306950	4413494350	292280005E000000000	+
6603992750	6097218750	4383061150	292300005E000000*000	+
6621687550	6099239350	4353451250	292320005E000000000	+
6639670650	6100791150	4323786250	292340005E000000 00+	+
6657470550	6102416150	4294015050	292360005E000000-000	+
6675165050	6104002650	4264191150	292380005E000000*00+	+
6692469750	6105592450	4234977550	292400005E000000000	+
6709961250	6106693550	4205320250	292420005E000000+000	+
6726845550	6107968250	4176393550	292440005E000000000	+
6744222250	6108989850	4146773250	292460005E000000000	+
6760949550	6110192050	4117657150	292480005E00000000*+	+
6777733450	6111019650	4088736250	292500005E000000000	+
6794872050	6111771750	4059059050	292520005E000000 00-	+
6811358750	6112466750	4030277050	292540005E000000*000	+
6828006350	6113246650	4000618750	292560005E000000000	+
6844026050	6113734050	3972599950	292580005E000000000	+
6884392950	6114648150	3900799250	292630005E000000000	+
6900131450	6114793150	3872661450	292650005E000000000	+
6915753450	6114731150	3844768750	292670005E000000-000	+
6931257450	6114699250	3816498550	292690005E000000000	+
6946611850	6115003650	3788314650	292710005E000000*00+	+
6961928050	6114784450	3760444150	292730005E000000000	+
6977055950	6114787750	3732299950	292750005E000000000	+
6992430950	6114254050	3704297550	292770005E000000000	+
7007279450	6114008250	3676538550	292790005E000000 00+	+
7021995350	6113578550	3649074250	292810005E000000*00-	+
7036761250	6113040050	3621427750	292830005E000000000-	+
7051390650	6112515050	3595751250	292850005E000000000	+
7065786550	6111688050	3566438650	292870005E000000000	+
7077280950	6114066550	3535396250	292890005E000000000	+
7094261850	6110531750	3512130650	292910005E000000 00+	+

ST. LOUIS, MISSOURI

- .00583085 - .78295593 .62000437

-x	-y	z	time	
4253075850	6105233950	6681128750	292330005E000000-00+	+
4224681750	6131843850	6674770350	292340005E0000000000	+
4194667150	6159265150	6666450050	292350005E00000000*+	+
4172602450	6179620050	6663452250	292360005E0000000000	+
4143334850	6206316350	6656906750	292370005E0000000000	+
4117007650	6230545550	6650606350	292380005E000000*000	+
4085476750	6259042450	6643289050	292390005E0000000000	+
4064187650	6278337650	6638135350	292400005E0000000000	+
3734489250	6500030550	6552316350	292500005E0000000000	+
3446221650	6600855050	6470651950	292600005E000000-00-	+
3413283550	6626672450	6461117150	292610005E000000 00+	+
3365352750	6848420150	6452792250	292620005E0000000000	+
3352326950	6874174350	6442641350	292630005E000000000C	+
3326616850	6894741950	6433984150	292640005E0000000000	+
3297733850	6916998250	6424958650	292650005E0000000000	+
3266598150	6941172150	6414785450	292660005E0000000000	+
3237083750	6963672250	6405354750	292670005E0000000000	+
3207324750	6986520350	6395447750	292680005E000000000+	+
2882080750	7225677750	6283565850	292770005E0000000000	+
2244366250	7647997450	6039116950	292960005E0000000000	+

OAKLEY, KANSAS
- .14625620   - .76195594   .62900467

-x	-y	z	time	
1496394950	6016745950	7645991550	291595405E0000000000	+
1444475050	6050826750	7629495350	291610005E000000 000	+
1319667450	6131586650	7706563050	291648605E0000000000	+
1263716550	6107282750	7709661450	291663405E0000000000	+
1140969650	6244552850	7726836650	291702005E000000-000	+
1066611650	6270413550	7707191350	291716005E000000 00+	+
952662714R	6360250050	7657654550	291757605E0000000000	+
891571264R	6397188850	7634203950	291773305E0000000000	+
764246044R	6472568850	7584311850	291812805E0000000000	+
709097484R	6503887850	7562845150	291830005E0000000000	+
578367624R	6577984050	7509701750	291869405E0000000000	+
515878674R	6613567450	7462952350	291886705E0000000000	+
104752444R	6631705250	7301837050	292008005E0000000000	+
582190244Q	6655174550	7280329650	292021605E0000000000	+
803032694H	6923656150	7215022350	292063005E0000000000	+
130662094I	6940558650	7190301650	292077605E0000000000	+
268248344I	7014438750	7122196950	292118605E0000000000	+
315536494I	7057219550	7097745950	292132405E0000000000	+
453164094I	7099919250	7027502350	292173605E0000000000	+
501346024I	7121447250	7002403850	292188205E000000000+	+
642001554I	7164174450	6926432250	292230005E0000000000	+
694662794I	7207447050	6897110950	292246605E0000000000	+
831998864I	7265822150	6820235350	292267805E0000000000	+
876939574I	7284652350	6794470650	292302405E0000000000	+
1032706050	7337003750	6715794350	292344705E0000000000	+
1078782950	7357397150	6666175150	292358805E0000000000	+
1214554450	7412523050	6601466350	292401505E0000000000	+
1267095150	7453993150	6567359350	292417405E0000000000	+
1390961050	7401156550	6480259950	292457205E0000000000	+
1441465850	7500251750	6455129350	292472205E0000000000	+

A REDUCTION OF OVERDETERMINED SATELLITE TRACKING DATA

by

THOMAS G. SWANEY

B. S., University of Missouri, 1960

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Applied Mechanics

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1965

In this report a sequence of discrete unsynchronized satellite sighting vectors from N observation stations located on the earth was reduced to a set of satellite position vectors. The effect of errors in the observation parameters on orbit parameters is discussed.

The theory used leads to a set of N-1 non-linear algebraic equations in terms of a set of parameters. This system is solved using a multi-dimensional Newton's method. Since the method is of an iterative nature a program for computation was written for the IBM 1620.

The smallest error sphere obtained about a satellite position vector was approximately 0.2 miles. It was found that for a variance of 0.1 per cent in orbit parameters the error sphere must be about 0.04 miles, or one fifth as large as the smallest error sphere obtained from the experimental data.

Experimental position data obtained from segments of orbits subtending central angles of 5^0 are ill-conditioned for the computation of orbit parameters. The need for super accurate data could be avoided by locating an additional pair of observation stations a sizable fraction of the earth's circumference away from the first pair.