ANALYSIS OF REINFORCED CONCRETE FOLDED PLATES BY MINIMUM ENERGY PRINCIPLE

by

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B. S., S. V. V. (University), Anand, 1964

A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Civil Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1965

Approved by:

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ANALYSIS OF REINFORCED CONCRETE FOLDED PLATES BY MINIMUM ENERGY PRINCIPLE

by Vinu K. Patel[a]

Synopsis

At the present time, a number of methods of analysis of folded plate structures are available that are based on rigorous theory and that are practically applicable without the aid of high-speed computers. The method presented here is based on the principle of minimum total potential energy. In this method the stress resultants of minor importance such as membrane shear, longitudinal bending moment in slab, slab twisting moment, etc. are considered. These are in addition to the stress resultants ordinarily considered (longitudinal direct stresses and transverse slab moments). The method of analysis is developed in detail for simply supported structures. Strain-energy and potential energy expressions for the stress resultants, considering the elastic curve distribution and sine wave distribution of deflection, are developed. To illustrate the method, an example is solved.

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INTRODUCTION

The problem of carrying roof loads over long spans has frequently occupied the attention of structural engineers and architects, and many structural forms have been developed. It is generally recognized that the shell roofs provide an efficient solution to the problem. These structures translate the applied external loads into compressive and tensile forces and shears in the plane of their surface. These forces and shears are called membrane stresses. The measure of the structural economy of the system depends on the degree to which membrane stresses are dominant over the out-of-plane flexural stresses. The shell structure of reinforced concrete is economical of material, but its cost may be high because of the elaborate false-work required, and due to the difficulty in placing concrete over the shell structure.

The disadvantages of shell structure can be overcome, retaining the other advantages, by folded plate structure. Folded plates structures, sometimes called prismatic shells, are the types of roofs consisting of a series of flat plates, mutually supporting each other along their longitudinal edges, that frame into transverse end diaphrams. Folded plates have been used extensively in the construction of long-span roof systems because of economy and their interesting and pleasing architectural appearance. The materials required for folded plate construction are usually less than needed for flat slab, slab and beams, or other conventional systems and are little more than required for continuous curved shells, with the advantage of utilizing simple forms. These structures have a deep corrugated form somewhat similar to that of multiple-barrel cylindrical shells, except that plane elements are used, intersecting in "folded lines" in the direction parallel to the span. Folded plate structures represent an attempt to simplify forming and still retain the advantages of shell construction, but are not ideal shells, because flexure action may have a considerable influence on their design. Folded plates may be simply supported at their ends or they may be continuous over transverse diaphrams.

There are various theories of analyzing the folded plate structure, some of which are exact and some are inexact. The inexact theories are relatively simpler and easier to follow as compared to exact theories. The first man to develop the principle of folded plate construction was G. Ehlers of Germany. He published the first technical paper[1] on this subject in 1930. In his method of analysis he considered the various plate elements as beams supported at the cross-and end diaphrams. Along the longitudinal edges, the plates were assumed to be connected by hinged joints, that do not slide longitudinally and that are considered capable of transferring edge shears between the contiguous plate elements. Thus he neglected entirely the connecting moments transmitted between the plates due to the rigidity of joint (as construction is monolithic). The uniform loads on the plates were transferred to the line loads, P, acting at the joints, as shown in Figure 1. These loads P were then resolved into two components, P_{cd} and P_{cb}, parallel to the two adjacent plates as shown in Figure 2.

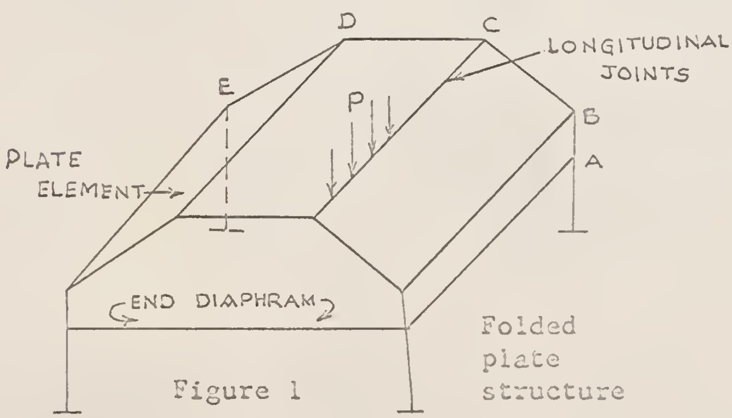
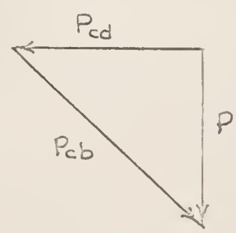


Figure 1

Folded plate structure



Resolution of loads at ridges

Figure 2

[1] "Ein neues konstruktionprinzip", by G. Ehlers, Bauingenieur, Vol. 9, 1930, p. 125.

The plates, acting as beams between the diaphrams, carried the loads P. As he assumed that there was no longitudinal relative displaeement between the two plates at longitudinal edges, there must exist the shear stresses along the edges. The condition of equal longitudinal strains at edges was used to determine the magnitude and distribution of shear stresses. Thus in his theory G. Ehlers neglected two things.

1. Connecting moments at longitudinal edges.
2. Relative displacements between the joints.

In 1932, E. Gruber published his method in which he considered the effeet of the rigidity of the joints, i.e., conneeting moments acting along the common edges of the plates and the relative displacements between the joints. This method consists in solving simultaneous differential equations, whieh is a tedious job. He showed that the maximum longitudinal stresses on a cross section and maximum deflections for a roof with hinged plates were about twice as great as those for the rigidly connected plates. Thus he concluded that the influence of the rigid eonnections should not be neglected as it had been, aecording to practice.

Later the theory was further developed and expanded in many respects by H. Craemer[2, 3], Mr. Gruber[4] and others. With the exeeption of Mr. Gruber[4] all writers have made the simplifying assumption of negleeting the effect of the relative defleetions of the joints. The method most commonly used in the U.S.A. was introduced by George Winter and Pei[5], and was later modified

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[2] "Der hentige stand der Theorie der Sehneibentraeger and Faltwerke. in Eisenbenton" by H. Craemer, Beton and Eisen, Vol. 36, 1937, p. 269.
[3] Ibid., p. 297.
[4] "Berechnug prismatischer scheiben werke" by E. Gruber, Memoirs, International Assn. of Bridges and Struetural Engg., Vol. 1, 1932, p. 225.
[5] "Hipped plate eonstruetion" by G. Winter and Pei, Journal ACI, Jan., 1947.

by Ibrahim Gaafar[6], to include the effect of joint displacements.

John E. Goldberg, and H. Leve[8] have developed the solution using the theory of elasticity. The method developes a solution for the stresses in a folded plate structure by combining the equations of the classical plate theory for loads normal to the plane of the plates together with the elasticity equations defining the plane stress problem for loads in the plane of the plates. This method requires extensive computation and so becomes practical when programmed for a digital computer.

At the present time many methods of analyzing folded plate structure are available. The design methods referred to are those presented by I. Gaafar and D. Yitzhaki,[7] which overcome objections to previous methods by taking into account the slab reactions induced by relative displacements of the ridges.

The method presented here is an alternate method of solving folded plate roofs, which is based on the principle of minimum total potential energy. In this method, appropriate expressions for the deflection curves are introduced and used to develop the potential of the external forces and the strain energy of the deflected structure. The total potential energy of the deformed structure is obtained by summing the potential energy of the external loads and internal forces. The potential energy of the internal forces is the strain energy of the deformed structure. The principle of minimum total potential energy is applied to evaluate the deflection coefficients. The stresses are then calculated with the help of deflections. The method is developed for structures simply supported at the ends.

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[6] "Hipped plate analysis considering joint displacements" I. Gaafar, Transactions, ASCE., Vol. 119, 1959, pp. 743-789.

[7] "The design of prismatic and cylindrical shell roofs" by D. Yitzhaki, Haifa Science publishers, Haifa, Isreal, 1958.

[8] "Theory of prismatic folded plate structures" by J. E. Goldberg, E. H. L. Leve I.A.B.S.E. (Zurich), No. 87, 1957, p. 54.

DEFINITIONS

The following definitions are used as a basis for the discussion in this report:

(1) A plate is an individual planar element of the structure.

(2) The length of a plate is the dimension between transverse supports. (Figure 3, "L").

(3) The width of a plate is the dimension between longitudinal edges. (Figure 3, "h").

(4) The height of the structure is the vertical dimension of the upper and lower extremes of a transverse cross section. (Figure 3, "H").

(5) The work stored in the elastically distorted body in the form of energy is defined as strain energy, or elastic energy. ("The Analysis of Structures" by N. J. Hoff, p. 122).

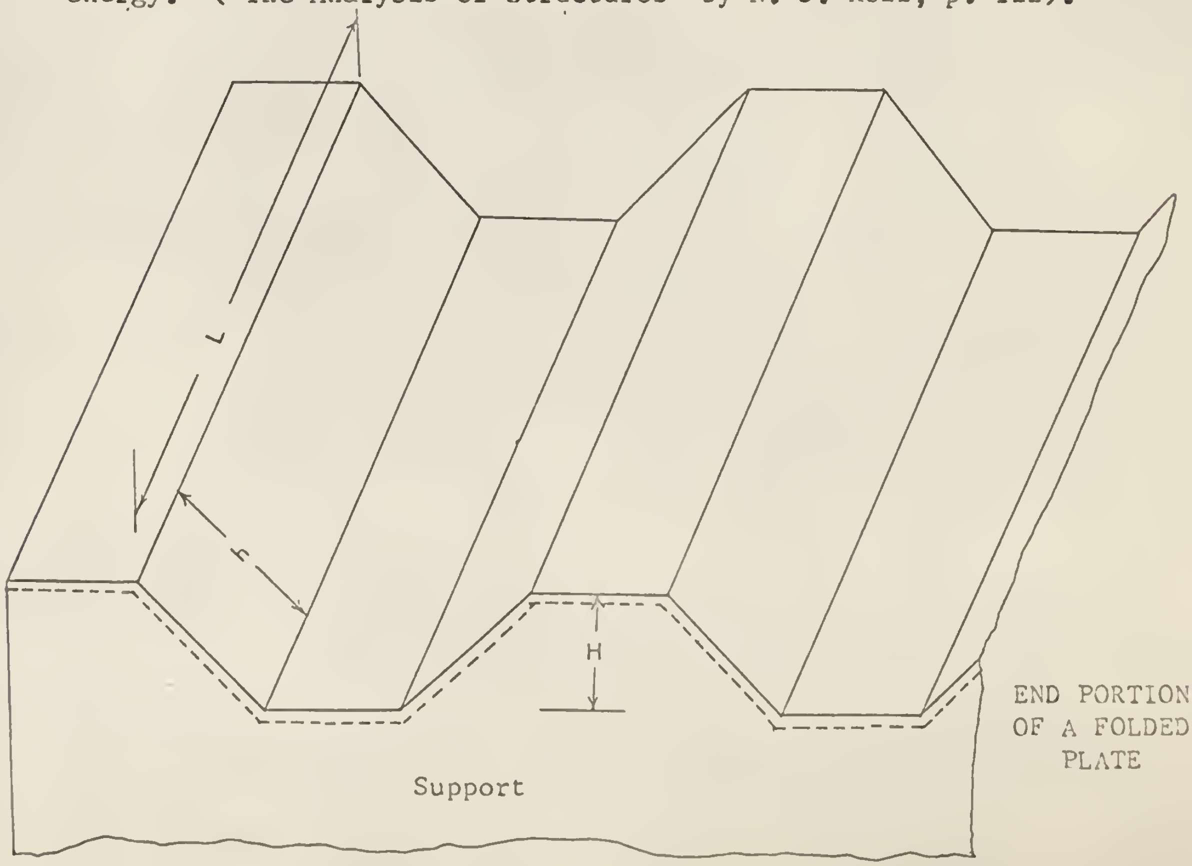


Figure 3

(6) Potential energy of a system is defined as the negative of the work done by forces acting on the system. ("Energy Methods in Applied Mechanics" by H. L. Langhaar, p. 18).

(7) Total potential of the system is the summation of strain energy and potential energy of the system. ("The Analysis of Structures" by N. J. Hoff, p. 139.).

Advantages of folded plate structure over shell structure: Following are the advantages of a folded plate structure over a shell structure:

(1) The formwork required is relatively simpler in a folded plate structure as it involves only straight planks, while in case of a shell structure the formwork is complicated as it involves curved members.

(2) Formwork can be removed at the end of seven days, if not earlier, because of their greater rigidity; This results in quicker turnover which, in turn, cuts down the construction time.

(3) The design involves only simple calculations which do not call for a knowledge of higher mathematics, while calculations involved in shell structures are complicated and laborious.

(4) As folded plate construction involves only straight planks, movable formwork can be employed in its construction with great ease, while in shell construction movable formwork cannot be employed with ease as it involves curved planks.

(5) Folded plate construction requires simple rectangular diaphrams as against complicated transverses required for a shell structure.

(6) Their light-reflecting geometry and pleasing outlines make them comparable with shells in their aesthetic appearance.

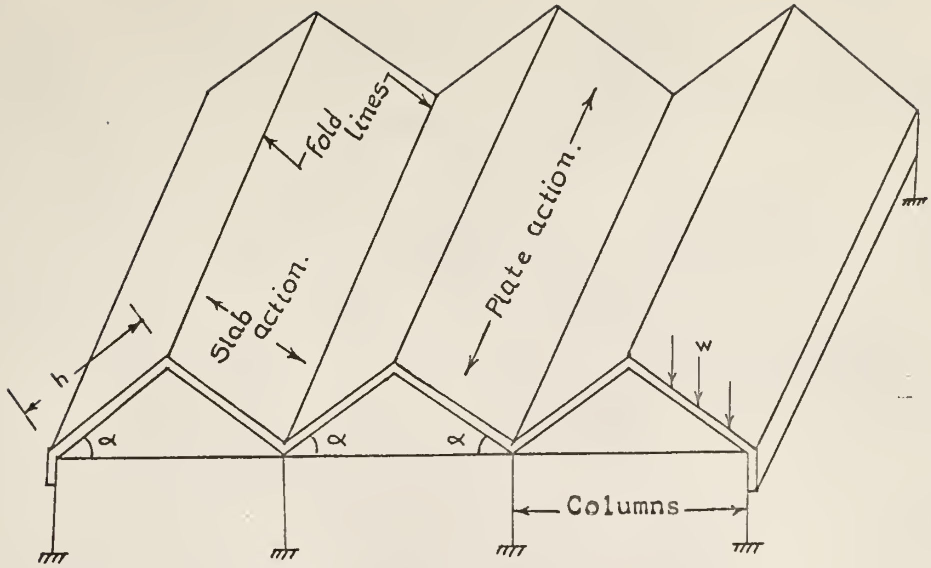
STRUCTURAL ACTION OF A FOLDED PLATE

The action of a folded plate structure in carrying externally applied loads is conveniently separated into two parts. One in the transverse direction and second in the longitudinal direction. In the transverse direction between fold lines, loads are carried by slab action, i.e., the loads applied to the surface are carried by the bending strength of the surface. In the longitudinal direction, the reaction of all such transverse beam strips is applied as a line loading along the fold lines. The structural action of the plate units in resisting this line loading is same as that of inclined deep girders, laterally braced by adjacent plates, and spanning between end walls of the structure. Thus any load applied on a folded plate structure is carried on supports (end diaphrams).

The key to the structural behavior of such structures is in the capacity of the fold lines to serve as lines of support for the transverse beam strips. To study the structural action of the folded plate structure, let us consider a simple folded plate building as shown in Figure 4.

Let the structure be loaded by a uniformly distributed load of w lbs/sq.ft. Let α be the angle made by inclined plates with the horizontal as shown in Figure 4.

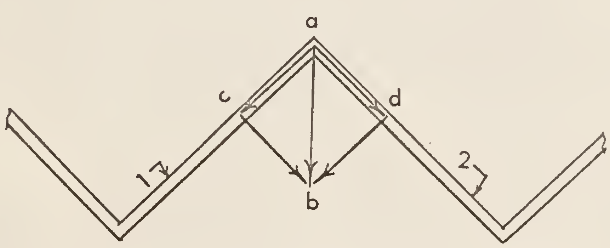
The component normal to the surface of the plate will be carried to fold lines by slab action and the component in the plane of the plate will be carried by plate action. The transverse slab strip will deliver reactions to the fold lines. The reactions from all such transverse slab strips cause



FOLDED PLATE BUILDING

Figure 4

a line loading along the entire length of the fold line. Let P be the resultant of all the line loads acting at the fold line of plates 1 and 2 per foot length of the plate. The direction of the fold line deflection, ab, will depend on the relative stiffness of the plates at that fold line, i.e., plates 1 and 2. The deflection of each plate can be resolved into components parallel and perpendicular to the plate, for example ac and cb parallel and perpendicular to plate 1 and ad and bd parallel and perpendicular to plate 2 as shown in Figure 5. Each plate has negligible resistance to deflection normal to its

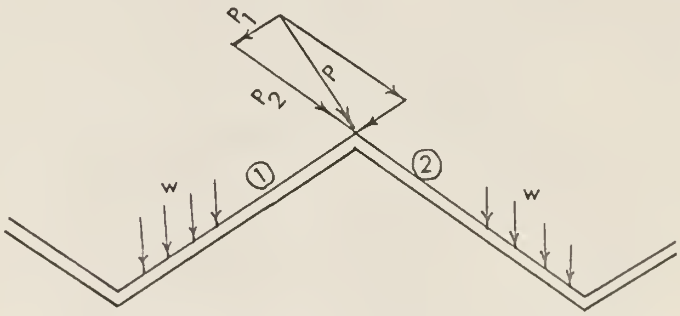


DEFLECTION OF PLATES AT A RIDGE

Figure 5

plane compared to its great resistance to deflection in its own plane. It is seen that a fold line cannot deflect in any direction without causing in-plane deflection of one or both of the adjacent plates.

While any plate is extremely flexible out of its plane, it is extremely stiff in its own plane, just as a deep beam is stiff. The load in any direction is resolved into components parallel to the two adjacent plates. The load P at a fold line is resolved into two components P_1 and P_2 parallel to plates 1 and 2 as shown in Figure 6. The load P_1 in addition to W.h.Sin α lbs/ft. length will be carried to the end diaphrams by plate action. A similar situation is obtained at any other fold line. For satisfactory structural behavior, it is only necessary that the plates do not meet at too flat an angle.



RESOLUTION OF LOADS AT A RIDGE

Figure 6

Reinforced concrete folded plates are designed transversely as continuous slabs with top steel at fold lines and bottom steel for positive moment between the fold lines. These transverse slab strips deliver fold line loads which are resolved as described, into components in the plane of the two adjacent plates. The plates span as deep beams between end walls, with principal tensile reinforcement at the lower fold line or valley.

There are two design complications which result from reinforced concrete folded plate structures. One design complication is that relatively slight differential displacements of the fold lines relative to one another can have a substantial influence on slab moments, and on the reactions delivered to the fold lines. This reaction difference requires a corrective deflection analysis. A second characteristic design feature is that the primary analysis may result in longitudinal plate stresses differing on either side of a fold line. This will cause strains also to differ on either side of a fold line. This strain incompatability cannot exist in actual structure because the stress and strain on one side of a fold line must be the same as on the other. The result is that longitudinal shears are caused along the fold line, acting equal and in opposite directions on two adjacent plates, which restore compatability. These edge shears modify the longitudinal stresses across the entire width of each plate.

PRINCIPLE OF MINIMUM TOTAL POTENTIAL

Statement. The principle states that the actual configuration of an elastic structure deformed by loading is such that the total potential energy, which consists of the potential energy of the applied loads and strain energy of the deformed structure, is a minimum.

We know that the principle of virtual displacements establishes the vanishing of the sum of the external and internal work for any virtual displacement as the necessary and sufficient condition of equilibrium of a system of mass points.

$$\delta we + \delta wi = o$$

where

δwe = External work of the external loads acting on the system.

δwi = Internal work of the internal forces.

The external work δwe is $\Sigma P \cdot \delta P$, if the external loads P are all concentrated forces and the displacements δP are those of their points of attack in the directions of the forces corresponding to the virtual displacement. When the elastic body is under the action of distributed loads, the external work can be calculated by integration rather than by summation. If for the sake of simplicity the principle of virtual displacements is written for the case of concentrated external loads, it becomes

$$\delta we + \delta wi = \Sigma P \cdot \delta P - \delta U = 0 \quad \ldots\ldots \quad (1)$$

because

$$\delta wi = -\delta U$$

Equation (1) is a necessary and sufficient condition of equilibrium, provided the variation sign δ is understood to imply any arbitrary displacement.

The change in the strain energy has to be calculated on the assumption that the forces remain unchanged during the variation of the state of strain. It is convenient to define potential V of the external forces in such a manner that the work done by the forces during a variation of the state of deformations be equal to $-\delta V$. In the form of an equation

$$-\delta V = \Sigma P \cdot \delta P \quad \ldots\ldots \quad (2)$$

when all the external loads are concentrated forces. Substitution of $-\delta V$ in equation (1) yields

$$-\delta U - \delta V = 0$$

or

$$\delta (U + V) = 0 \quad \ldots\ldots \quad (3)$$

The expression $U + V$ is known as the total potential of the system. Let it now be assumed that the total potential $U + V$ is a function of one single displacement parameter q. Then the elastic body is in equilibrium if

$\delta(U + V) = d(U + V)/dq \cdot \delta q = o$ (Proof by Taylor's Theorem & considering the terms of first order in virtual displacement)

that is, if

$$d(U + V)/dq = o \quad \ldots \quad (4)$$

since $\delta q \neq o$ by assumption.

When U + V is a function of two independent displacement parameters q_1 and q_2, the body is in equilibrium for

$$\partial(U + V)/\partial q_1 = o \quad \ldots \quad (5)$$

$$\partial(U + V)/\partial q_2 = o \quad \ldots \quad (6)$$

If the function U + V is plotted against the independent variable q, eq.(4) requires that the curve has a horizontal tangent. This is so when U + V is a maximum or minimum or when it has an inflection point. In each case the function is said to have a stationary value. Therefore, the total potential has a stationary value when an elastic body is in equilibrium. The stationary value always corresponds to a minimum when the equilibrium is stable. Therefore, the total potential is a minimum when an elastic body is in equilibrium configuration.

This principle is of great importance in structural analysis. Methods of calculating stresses by its use are known as energy methods.

RAYLEIGH-RITZ METHOD

The simplest procedure for obtaining an approximate deflection function was devised by Lord Rayleigh. It consists of assuming arbitrarily a reasonable deflected shape involving an undetermined coefficient and equating the first derivative of total potential with respect to an undetermined coefficient.

This will give the undetermined constant, which when substituted in the assumed deflection function gives the approximate deflection function. The assumed deflection function should be such that it satisfies the boundary conditions. Rayleigh's method has often been criticized because it provides no information about the accuracy of the approximation. The most convenient shapes one can assume in the Rayleigh method for the deflections functions are those represented by trigonometric functions and polynomials.

A natural extension of the Rayleigh method, denoted as the Rayleigh-Ritz, or often simply as the Rayleigh method, makes use of a more complex expression for the deflected shape. We can choose a function involving n undetermined coefficients. We have to adjust the 'n' undetermined constants in such a manner as to approximate best the true deflected shape. The best approximation is derived by the principle of the minimum of the total potential. We know that the true deflected shape differs from all other geometrically possible shapes inasmuch as it corresponds to the minimum value of the total potential. The total potential must therefore be made as small as possible by a suitable choice of the constants. This can also be expressed by saying that the total potential must be minimized with respect to the n undetermined coefficients. The 'n' undetermined coefficients can be obtained by putting partial derivatives of total potential with respect to each constant equal to zero and solving these n equations simultaneously. Substituting these n constants in an assumed deflection function we get an approximate deflection function. The deflection function obtained by this method is better than that obtained by the Rayleigh method.

The number of undetermined constants to be taken into an approximate deflection function depends on the degree of accuracy required. The Rayleigh-Ritz method gives an exact solution when an infinite trigonometric series is used in it and all the terms are considered in the calculation. It is seen from this discussion that Rayleigh-Ritz method replaces by a much simpler procedure the task of minimizing the total potential of a system with the aid of the variational calculus and of solving the differential equations so obtained. The solution obtained by the Rayleigh-Ritz method is approximate when a finite number of terms are considered, and often a very small number of terms suffice to obtain a satisfactory solution. The solution is exact when all the terms of an infinite series are taken into account.

In short, following are the steps to obtain a deflection function:

(1) Assume the deflection function such that

   (a) The boundary conditions of deflection are fulfilled

   (b) The shape of the deflection curve is generally in accord with the expected deflected shape, and

   (c) The actual shape and amplitude of the curve is defined by a set of undetermined coefficients.

(2) Find the strain energy of the system corresponding to an assumed deflection function.

(3) Find the potential of the loads corresponding to an assumed deflection function.

(4) Obtain the total potential as the summation of the strain energy and the potential of the loads. The total potential is a function of the undetermined constants.

(5) Set the partial derivatives of the total potential with respect to each undetermined constant equal to zero.

(6) Solve as many equations as there are unknowns simultaneously and obtain all the undetermined constants.

(7) Substitute these constants in the assumed deflection function and obtain the deflection function.

Once the deflection function is obtained, stresses and deflections can be calculated easily.

GENERAL THEORY

General. The theory presented herein is based on the following assumptions.

(1) The material is homogeneous, uncracked and elastic.

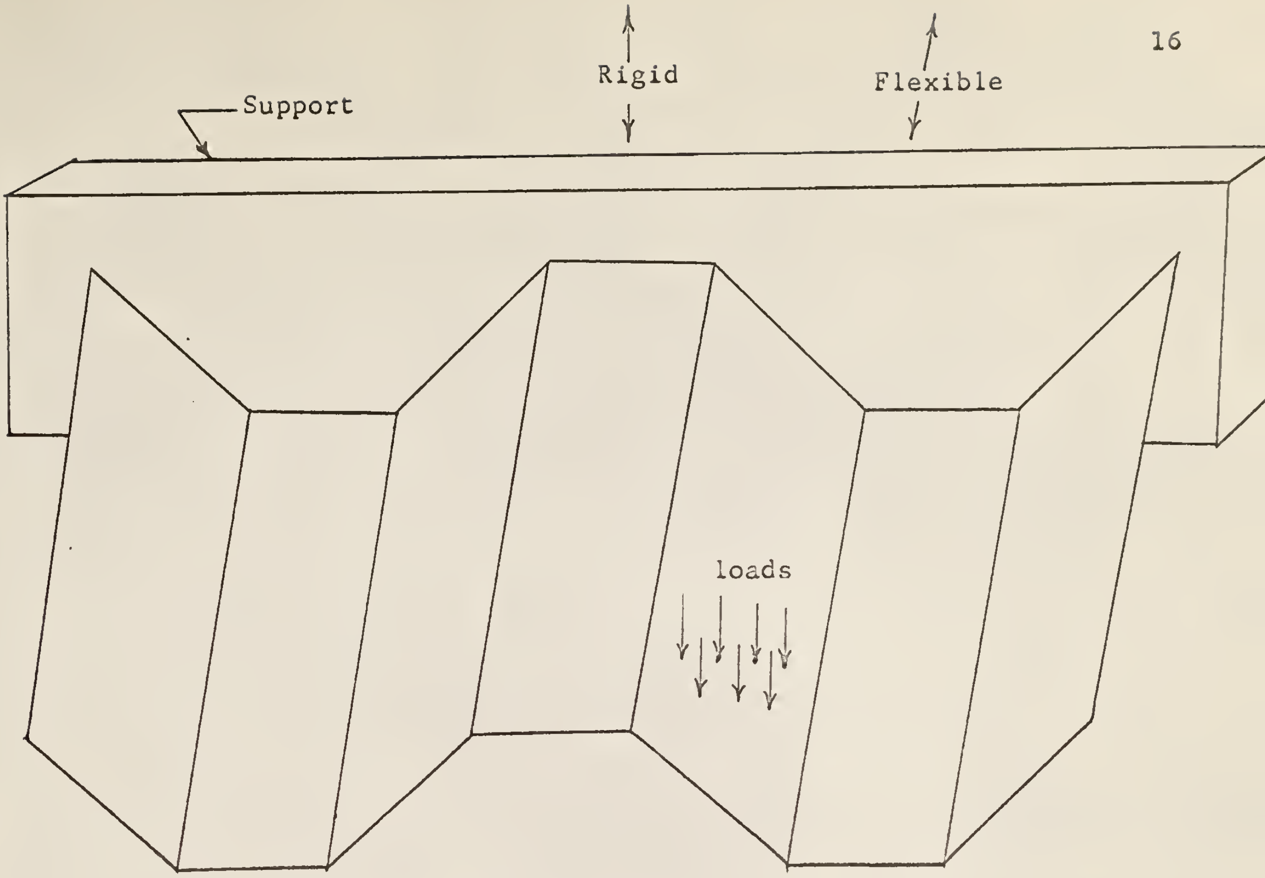
(2) Longitudinal edge joints are fully monolithic and continuous; there is no relative rotation or translation of two adjoining plates at their common boundary.

(3) The principle of superposition holds, that is, the structure may be analyzed separately for the effects of its redundants and various external loadings and the results combined algebraically.

(4) The longitudinal strain due to plate action varies linearly across the width of the plate (plane section remains plane).

(5) The supports are infinitely stiff edge beams in the plane of the loads but are completely flexible in the plane of the plates. For Figure 7, this means that the end support is very stiff vertically but can distort horizontally as the deformations of the plates may require.

The assumptions involved in developing strain-energy expressions will be presented subsequently. First, the structure with non-yielding supports (actually non-existant) at the ridges is subjected to the actual loading and the ridge reactions are computed. Second, these ridge reactions are

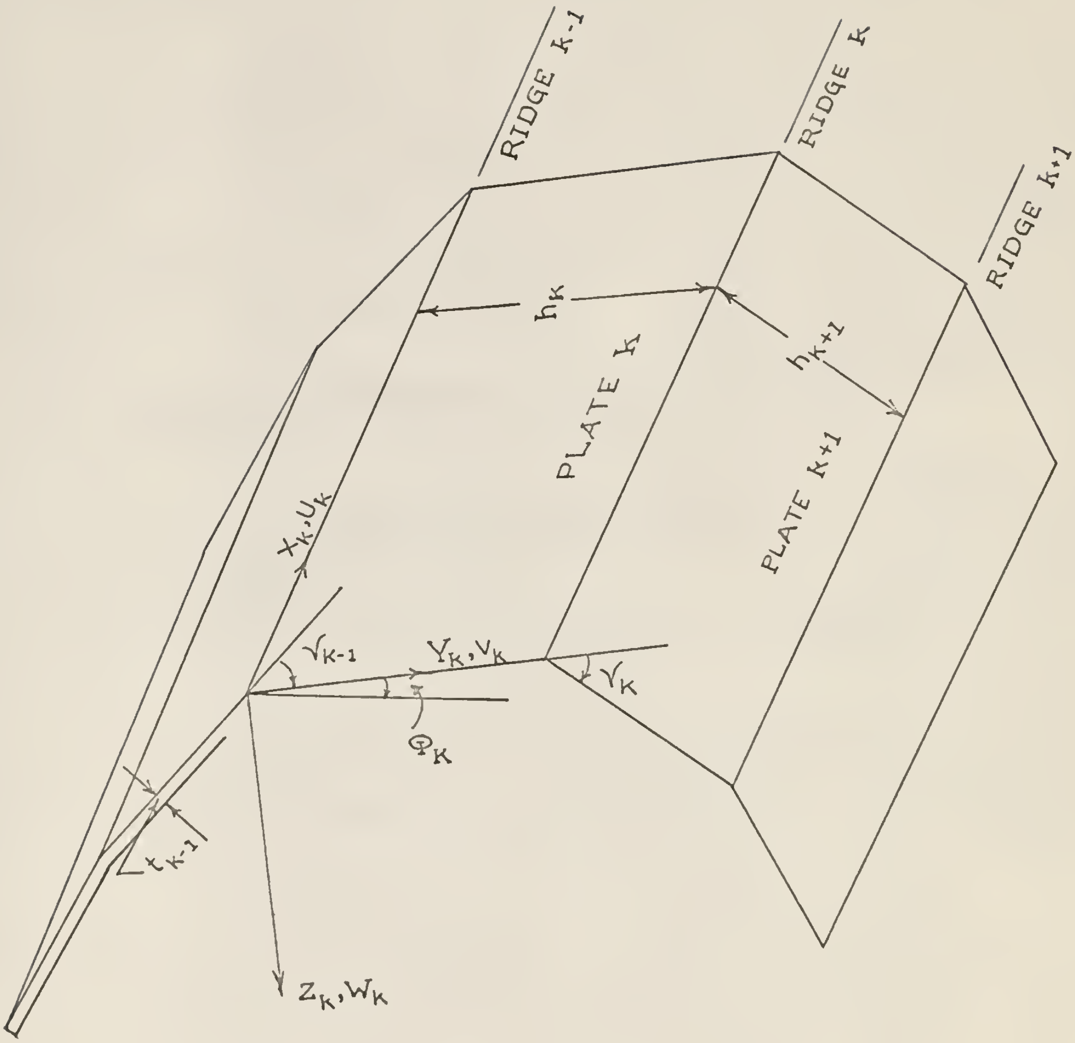


SUPPORT CONDITION

Figure 7

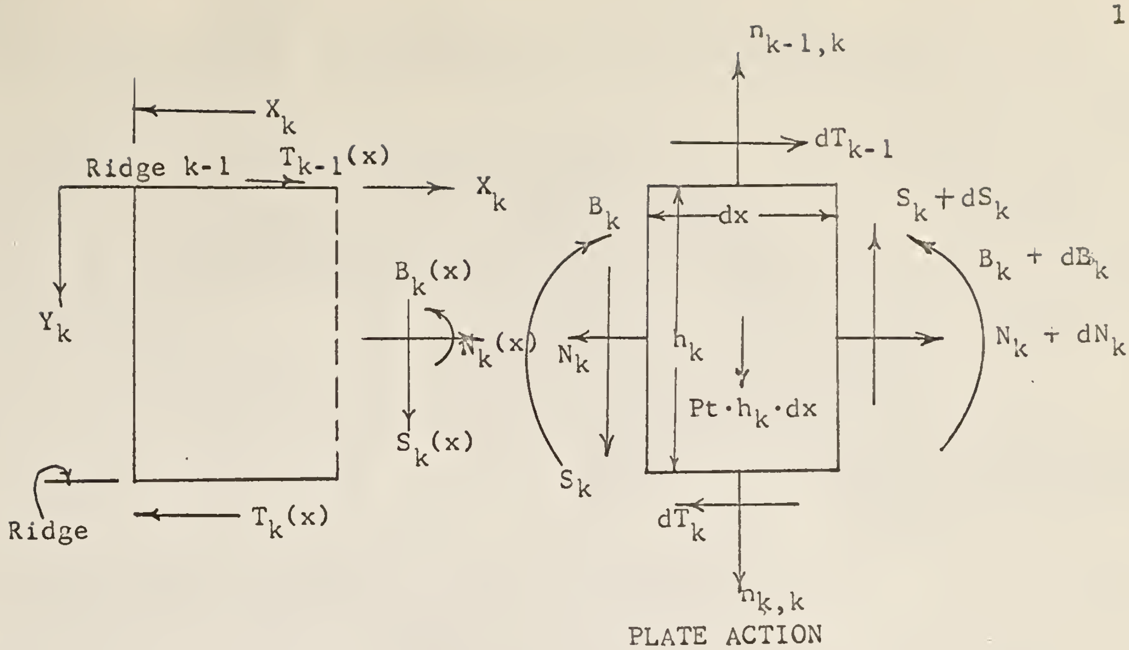
applied to the actual folded plate structure. Solution of the first loading on the structure represents a conventional analysis of a continuous slab over non-yielding supports.

Forces Acting on a Folded Plate. The forces acting on a folded plate are shown in Figure 9. Figure 8 shows folded plate notation.

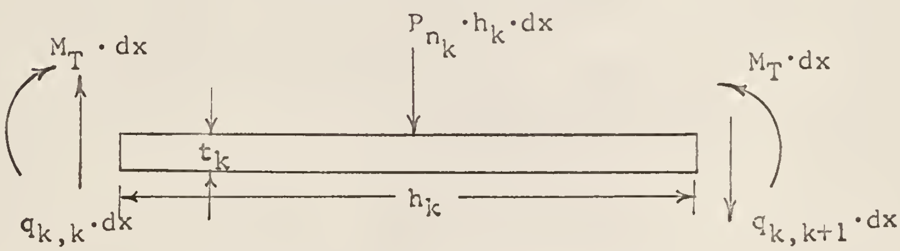


FOLDED PLATE NOTATION

Figure 8



PLAN OF A PLATE ELEMENT



ELEVATION OF A PLATE ELEMENT

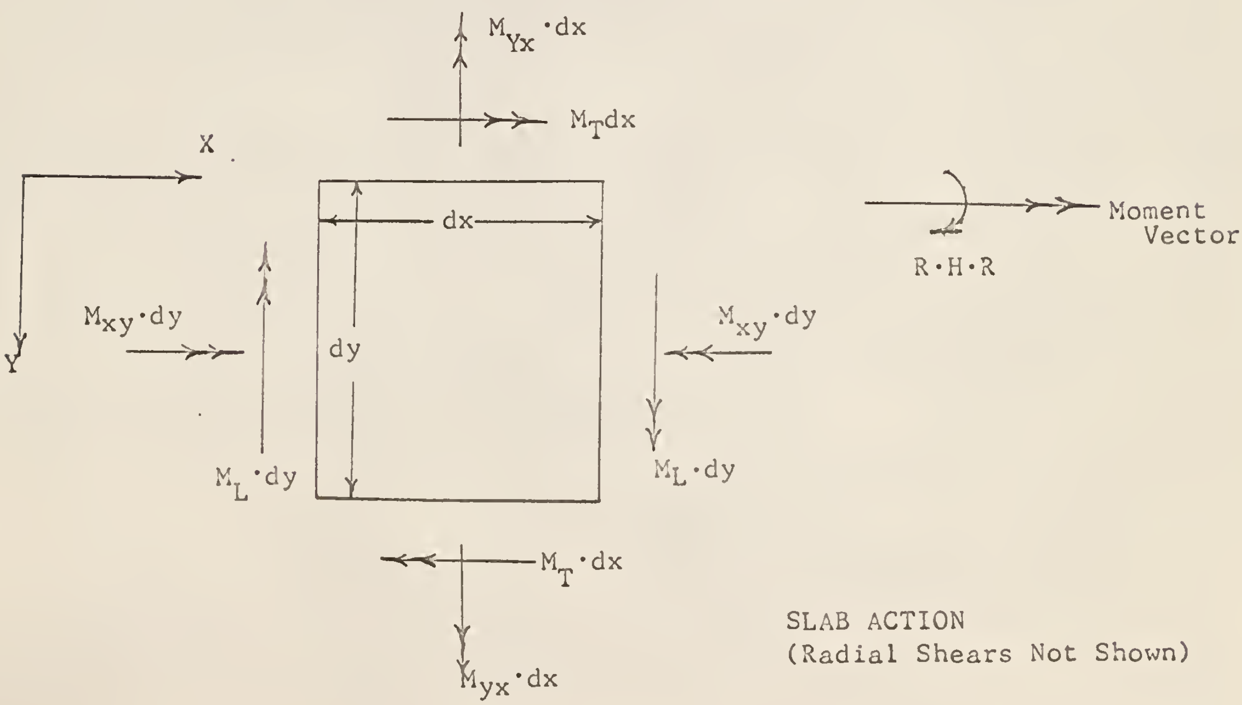


Figure 9

<u>Deflection Curves and Deflection Expressions of Edges</u>. Complete specification of the displacement of any point on the folded plate structure would involve three components in each plate; that is, in the longitudinal (u), transverse (v), and normal directions (W), with respect to each plate. However, in the development of energy expressions herein, two basic characteristics of folded plate action are utilized so as to deal only with the transverse deflection component, v, in each plate. Thus, the requirement that the longitudinal strains at each ridge must be the same for each adjoining plate is used to obviate explicit consideration of longitudinal displacement. Another fact, that the normal deflections of each plate at the ridges can be expressed in terms of the transverse deflections of these plates, will be used. To express the normal deflections of the plates in terms of the transverse deflections of the plates the geometry of the deflected folded plate structure will be considered. The vertical and the horizontal displacement of any ridge can be expressed in terms of the transverse deflections in the plates.

The transverse deflection of the k^{th} plate, v_k, that is consistent with the previous assumptions may be obtained to a precise degree of accuracy if v_k is expressed in terms of a complete set of functions each multiplied by a coefficient, as yet not determined. Thus for a simply supported structure,

$$v_k = K_1 \cdot \text{Sin}\, \pi x/L + K_2 \cdot \text{Sin}\, 2\pi x/L + \text{---} \quad \ldots\ldots\ldots\ldots \quad (7)$$

The Rayleigh-Ritz method could be used to evaluate all the coefficients K_i. Once the deflection is known, the stress can be evaluated easily. The actual longitudinal distribution of deflection of a folded plate structure corresponding to a particular loading is closely approximated by a linear combination of (a) the elastic curve corresponding to the loading and the end support conditions of the structure and (b) the normal curve in accordance with the end-support conditions. For symmetric loading of a simple span structure, the shape of the elastic curve is very nearly the same as the normal curve for the simple span, which is a half wave of a sine curve. Experience with the Rayleigh-Ritz method[9,10] has shown that the values of the maximum deflections obtained are relatively insensitive to small variations in the assumed deflection curve. The use of either the simple beam elastic curve or a sine wave as the shape of the transverse deflection curve for the plate should, therefore, yield almost equal values of the transverse slab moments that are linearly dependent on these deflections. However, this insensitivity is not characteristic of the longitudinal stresses as these are dependent on the plate moment and the central load, both of which vary as the second derivative of the displacement. A linear combination of the elastic curve and a sine-wave distribution could be used as the assumed deflection curve to take this into account where greater accuracy is desired, but this refinement is not included herein.

The displacements of the ridges can be expressed in terms of the displacements of the adjoining plates on the basis of geometric considerations.[11] In deriving these relationships, the following assumptions are made:

(a) The width of each plate remains unchanged (normal strains in transverse direction are neglected).

(b) The slope of each plate with respect to its original direction is very small.

Referring to Fig. 10 the displacements normal to plate K at ridges

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[9] "Elasticity in Engineering". by E. E. Sechler, John Wiley & Sons, Inc., New York, N. Y., 1952, pp. 195-199.

[10] "Energy Methods in Applied Mechanics" by H. L. Langhaar, John Wiley & Sons, Inc., New York, N. Y., 1962.

[11] "The Design of Prismatic and Cylindrical Shell Roofs", by D. Yitzhaki, Haifa Science Publishers, Haifa, Israel, 1958.

K-1 and K, respectively, are

$$W_{k-1,k} = v_k/\tan \gamma_{k-1} - v_{k-1}/\sin \gamma_{k-1} \quad \ldots\ldots\ldots\ldots\ldots \quad (8)$$

and

$$W_{k,k} = \frac{v_{k+1}}{\text{Sin } \gamma_k} - \frac{v_k}{\tan \gamma_k} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

The vertical displacement at ridge k is

$$\delta_k = v_{k+1}(\cos\varphi_k/\sin\gamma_k) - v_k(\cos\varphi_{k+1}/\sin\gamma_k) \quad \ldots\ldots\ldots \quad (10)$$

The above expressions 8, 9, and 10 are true when the stress resultants of minor importance are not included in the analysis. When the effect of the stress resultants of relatively minor importance is included in the analysis, the preceding displacement expressions must be revised to account for the additional transverse displacement in the plane of each plate, s_k, due to membrane shear. For this case, the foregoing equations become

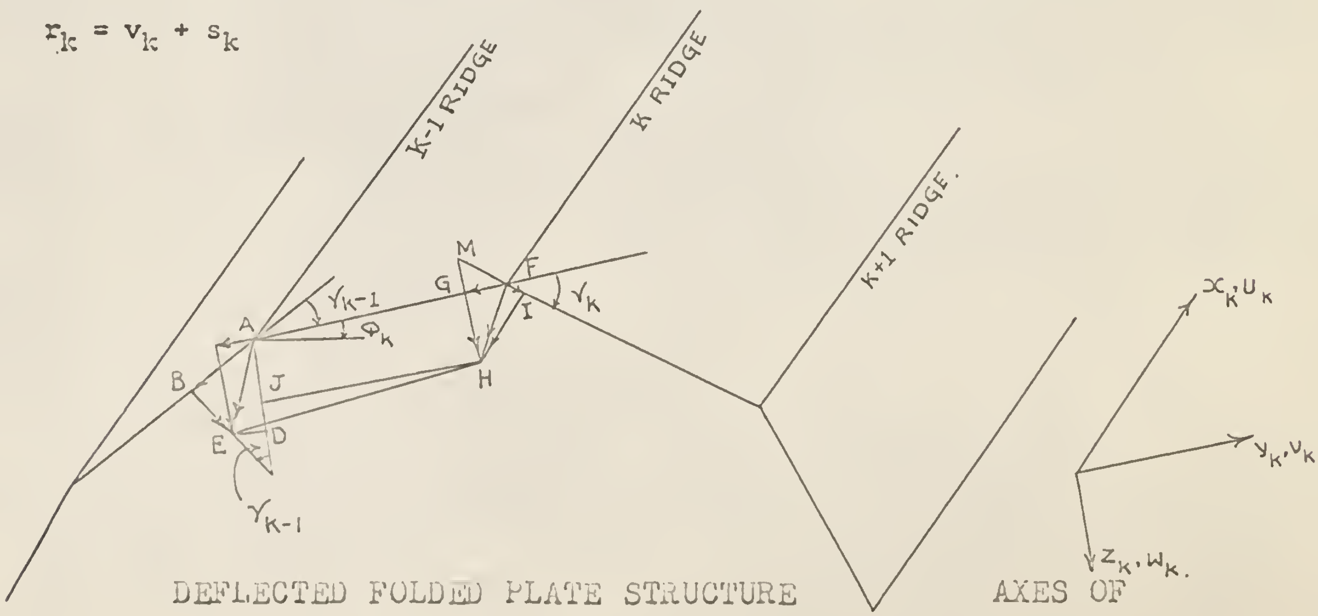
$$W_{k-1,k} = r_k/\tan \gamma_{k-1} - r_{k-1}/\text{Sin } \gamma_{k-1} \quad \ldots\ldots\ldots\ldots \quad (11)$$

$$W_{k,k} = r_{k+1}/\text{Sin } \gamma_k - r_k/\tan \gamma_k \quad \ldots\ldots\ldots\ldots\ldots \quad (12)$$

$$\delta_k = r_{k+1}(\text{Cos}\varphi_k/\text{Sin } \gamma_k) - r_k(\text{Cos}\varphi_{k+1}/\text{Sin } \gamma_k) \quad \ldots\ldots \quad (13)$$

in which

$$r_k = v_k + s_k$$



DEFLECTED FOLDED PLATE STRUCTURE

AXES OF REFERENCE

Figure 10

E & H are the respective positions of points A and F in the deflected structure.

$AB = v_{k-1}$ $AD = W_{k-1,k}$ Axes x_k, y_k, z_k are the axes of the reference.

$GF = v_k$ $FI = v_{k+1}$

We can prove the above expressions 8, 9 and 10 using the geometry in Figure 10.

DERIVATIONS OF STRAIN ENERGY AND POTENTIAL ENERGY EXPRESSIONS CONSIDERING SINE-WAVE DISTRIBUTION OF TRANSVERSE DEFLECTION

For the uniform longitudinal ridge loading: The energy expressions are derived for the case of uniform vertical loads, R_k, along each ridge. The sine wave transverse deflection of the k^{th} plate is taken as

$$v_k(x) = v_k \cdot \sin \pi \cdot x/L \quad \ldots \quad (14)$$

The potential of the distributed external load, R_k, at the k^{th} ridge, V_k, is evaluated by

$$V_k = -\int_0^L R_k(x) \cdot \delta_{k(x)} \cdot dx \quad \ldots \quad (15)$$

For sine-wave distribution of deflection and R_k constant,

$$V_k = -\int_0^L R_k \cdot \delta_k \cdot \sin \pi x/L \cdot dx = -2 R_k \cdot \delta_k \cdot L/\pi \quad \ldots \quad (16)$$

The potential of all the ridge loads is obtained from

$$V = \sum_{k=1}^{n} V_k = \sum_{k=1}^{n} - 2L \cdot R_k \cdot \delta_k/\pi = - 0.6366L \sum_{k=1}^{n} R_k \cdot \delta_k \quad (17)$$

The strain energy of the longitudinal bending of the k^{th} plate, U_{b_k}, is evaluated from

$$U_{b_k} = \int_0^L EI_k \left[v_k'' (x) \right]^2 /2 \cdot dx$$ Primes (") denote second derivative of $v_k(x)$ with respect to x

For sine wave distribution of deflection

$$U_{b_k} = \frac{1}{2}\int_0^L EI_k\left[-v_k\cdot\pi^2/L^2 \quad \sin \pi\cdot x/L\right]^2 \cdot dx$$

$$U_{b_k} = \frac{EI_k v_k^2 \cdot\pi^4}{2L^4}\int_0^L \sin^2\frac{\pi x}{L}\cdot dx$$

$$= \frac{EI_k v_k^2 \pi^4}{2L^4}\cdot\frac{L}{2} = \frac{EI_k v_k^2 \pi^4}{4L^3} = \frac{24\cdot 352}{L^3} EI_k v_k^2 \quad \ldots\ldots \quad (18)$$

The expression for the total strain energy of the longitudinal bending of all the plates is

$$U_b = \sum_{k=1}^{n} U_{b_k} = 24\cdot 352\,\frac{E}{L^3}\sum_{k=1}^{n} I_k v_k^2 \quad \ldots\ldots\ldots\ldots \quad (19)$$

The strain energy due to the central load, N_k, resulting from the difference in the longitudinal shear at the k_{th} and $k-1^{th}$ ridge is now considered. The longitudinal distribution of these shears and, consequently, of the central loads, N_k, has been shown [12] to be the same as the longitudinal distribution of moment due to the external loads on a beam with similar supports. The strain energy due to the central loading is, in general,

$$U_{N_k} = \int_0^L \frac{N_k^2(x)\cdot dx}{2A_k E} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (20)$$

For the sine-wave distribution of the deflection,

$$N_k(x) = N_k \sin\frac{\pi x}{L}$$

$$U_{N_k} = \int_0^L (N_k^2 \sin^2\frac{\pi x}{L} / 2A_k E)\cdot dx = \frac{LN_k^2}{4EA_k} \quad \ldots\ldots\ldots\ldots \quad (21)$$

and expression for the total strain energy is

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[12] "The Design of Prismatic and Cylindrical Shell Roofs," by D. Yitzhaki, Haifa Science Publishers, Haifa, Israel, 1958, p. 35.

$$U_N = \sum_{k=1}^{n} \frac{LN_k^2}{4EA_k} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (22)$$

To evaluate the foregoing expressions for U_N in terms of the undetermined coefficients, v_k, use is made of the fact that the longitudinal stress at each ridge must be the same for each adjoining plate. Thus, for the folded plate structure with n plates, there exists a set of N-1 equations

$$\frac{N_k}{A_k} - \frac{Ev_k''(x)h_k}{2} = \frac{N_{k+1}}{A_{k+1}} + \frac{Ev_{k+1}''(x)h_{k+1}}{2} \quad \ldots\ldots\ldots \quad (23)$$

The final equation required to express each N_k in terms of the various v_k values is obtained from equilibrium considerations. For the usual case of no applied longitudinal load,

$$\sum_{k=1}^{n} N_k = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (24)$$

For the sine-wave distribution of the deflection, the foregoing equations become

$$\frac{N_k}{A_k} - \frac{N_{k+1}}{A_{k+1}} = \frac{Ev_{k+1}''(x)h_{k+1}}{2} + \frac{Ev_k''(x)h_k}{2} \quad \ldots\ldots\ldots \quad (25)$$

Now we have,

$$v_k(x) = v_k \sin \pi x/L$$

$$\therefore \quad v_k''(x) = -v_k \frac{\pi^2}{L^2} \sin \frac{\pi x}{L}$$

at $x=L/2$

$$v_k''(L/2) = -\frac{v_k \pi^2}{L^2}$$

Putting the value of $v_k''(L/2)$ in equation (25) we get,

$$\frac{N_k}{A_k} - \frac{N_{k+1}}{A_{k+1}} = -4.935\ E/L^2 \left[h_k v_k + h_{k+1} \cdot v_{k+1} \right] \quad \ldots\ldots\ldots \quad (26)$$

The strain-energy expression corresponding to the transverse bending of the slab (slab action) is now developed. This strain-energy is evaluated as the external work done by the ridge reactions generated by the slab bending in moving through the assumed deflections at the ridges. These ridge reactions may be obtained either by the slope deflection equations or by the moment distribution. In the latter method, one plate is deflected at a time and the ridge reactions corresponding to that deflection obtained. The total ridge reaction is then obtained as the sum of the ridge reactions due to each of the plate deflections. Because the ridge reactions, R_{S_k}, depend linearly on the deflections, the longitudinal distribution of these reactions will be the same as the deflection distribution. The general strain energy due to the transverse slab bending is

$$U_{T_k} = \int_0^L \tfrac{1}{2} R_{S_k}(x)\delta_k(x) \cdot dx. \quad \ldots\ldots\ldots\ldots \quad (27)$$

For the sine-wave distribution

$$R_{S_k}(x) = R_{S_k} \cdot \sin \frac{\pi x}{L}$$

and

$$\delta_k(x) = \delta_k \cdot \sin \frac{\pi x}{L}$$

Therefore,

$$U_{T_k} = \int_0^L \tfrac{1}{2} \cdot R_{S_k} \cdot \mathrm{Sin}\, \frac{\pi x}{L} \cdot \delta_k \cdot \mathrm{Sin}\, \frac{\pi x}{L} \cdot dx$$

$$= \tfrac{1}{2} \int_0^L R_{S_k} \delta_k \cdot \mathrm{Sin}^2\, \frac{\pi x}{L} \cdot dx$$

$$= \tfrac{1}{4} \cdot L \cdot R_{S_k} \cdot \delta_k \quad \ldots\ldots\ldots\ldots\ldots \quad (28)$$

and

$$U = \frac{1}{4} \sum_{k=1}^{n} R_{S_k} \cdot \delta_k \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (29)$$

The foregoing components of the total potential, namely, the potential of the applied loads and the strain energies of the longitudinal bending, the longitudinal central load, and the transverse slab bending, correspond to the stress resultants ordinarily considered in the analysis of the folded plate structure. However, for certain dimensions of the structure, the effects of twisting moment, the longitudinal slab moment, the torsional resistance of the edge members, and the transverse shear, become more important. These will now be investigated. Because their effects are minor, approximate expressions will be developed in the interest of simplicity. For the same reason, only the sine wave distribution of the deflection will be considered in developing the strain energy expressions for these stress resultants.

The general expression for the strain energy due to the twisting moment in plate K is

$$U_{M_k} = \int_0^{h_k} \int_0^{L} \frac{(M_{xy})^2 \cdot dx \cdot dy}{2GJ} \quad \ldots\ldots\ldots\ldots\ldots \quad (30)$$

We know that,

$$M_{xy} = D(1 - \mu) \frac{\partial^2 W_k}{\partial x \partial y}$$

where

$$D = \frac{Eh^3}{12(1-\mu^2)}$$

From mechanics of materials we know that,

$$E = 2\ G(1+\mu)$$

Putting the values of D, and E in the equation of M_{xy}, we get,

$$M_{xy} = \frac{2\,G(1+\mu)\cdot h^3\,(1-\mu)}{12(1-\mu^2)} \cdot \frac{\partial^2 W_k}{\partial x \partial y}$$

$$= \frac{2\,Gh^3}{12}\;\frac{\partial^2 W_k}{\partial x \partial y} \qquad \text{but } h^3/12 = J$$

$$= 2\,GJ\;\frac{\partial^2 W_k}{\partial x \partial y}$$

Putting the value of M_{xy} in Equation (30), we get,

$$U_{M_k} = \int_0^{h_k}\int_0^{L} \frac{\left(2\,GJ \cdot \frac{\partial^2 W_k}{\partial x \partial y}\right)^2}{2\,GJ} \cdot dx \cdot dy$$

$$U_{M_k} = \int_0^{h_k}\int_0^{L} 2\,GJ\left(\frac{\partial^2 W_k}{\partial x \partial y}\right)^2 \cdot dx \cdot dy \quad \ldots\ldots\ldots\ldots \quad (31)$$

The deflection normal to the plate, W, has been defined along the longitudinal edges as

$$W_k(x,o) = W_{k-1,k} \cdot \sin\frac{\pi x}{L} \quad \text{for } y = o \quad \ldots\ldots\ldots\ldots \quad (32a)$$

and

$$W_k(x,h_k) = W_{k,k} \cdot \sin\frac{\pi x}{L} \quad \text{for } y = h_k \quad \ldots\ldots\ldots\ldots \quad (32b)$$

The distribution of W throughout the plate is required for evaluation of the strain energy, but it is not known. The strain energy is not sensitive to changes in the transverse distribution of W, once the values of W along the edges are fixed. If a transverse section of the deformed plate is taken to be a straight line between the longitudinal edges, that is, for a pinned condition along the edges of the plates,

$$W_k(x,y) = W_{k-1,k} \cdot \sin\frac{\pi x}{L} + (W_{k,k} - W_{k-1,k})\,\frac{y}{h_k} \cdot \sin\frac{\pi x}{L} \;.\;. \quad (33a)$$

$$\partial^2 W_k/\partial x \partial y = \frac{\pi}{L} \frac{(W_{k,k} - W_{k-1,k})}{h_k} \cdot \cos \frac{\pi x}{L}$$

Putting the value of $\frac{\partial^2 W_k}{\partial x \partial y}$ in equation (31) and integrating, we get,

$$U_{M_k} = \frac{GJ\,\pi^2}{Lh_k} (W_{k,k} - W_{k-1,k})^2 \quad \ldots\ldots\ldots\ldots\ldots \quad (33b)$$

If we had assumed the edges to be fixed we would have obtained approximately the same result. The total strain energy of the twisting moment of all the plates is

$$U_{M_k} = \sum_{k=1}^{n} \frac{GJ\,\pi^2}{Lh_k} (W_{k,k} - W_{k-1,k})^2 \quad \ldots\ldots\ldots\ldots\ldots \quad (34)$$

In performing actual computations, it is desirable to express U_M in terms of U_B. Comparison of equations (18) and 33b) yields

$$U_{M_k} = 0.4016\, \frac{Gt_k^2\, L^2\, (W_{k,k} - W_{k-1,k})^2}{Eh_k^4\;\; V_k^2}\, U_{b_k} \quad \ldots\ldots\ldots\ldots \quad (35)$$

Where the exterior edges of the folded plate structure are free, the normal deflection, W, along these exterior edges is not defined by the plate deflections, v. Consequently, the effect of the twisting stresses in these edge members cannot be evaluated by the preceding method. To take account of the twisting strain energy in such edge members, the previous method of determining the strain energy due to the transverse bending of the slabs will be revised to include the torsional resistance of the edge members. For sine wave longitudinal deflection, this torsional resistance may be accounted for[13] using for the edge member stiffness factor, K_{10}, applied at the first interior edge the following value,

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[13] "The Design of Prismatic and Cylindrical Shell Roofs," by D. Yitzhaki, Haifa Science Publishers, Haifa, Israel, 1958.

$$K_{10} = \frac{\pi^2 G\ J_t\ \Theta_1}{L^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (36)$$

The moment distribution method or the slope deflection equations can then be used to compute the ridge reactions corresponding to the transverse slab bending combined with the torsion of the edge members. With these reactions, eq. 29 is used to compute the strain energy.

We know from classical plate theory that,

$$M_{L_k} = D\left[\frac{\partial^2 W_k}{\partial x^2} + \mu \frac{\partial^2 W_k}{\partial y^2}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (37a)$$

where $D = E \cdot J/(1-\mu^2)$

In developing the strain-energy expression for slab bending in the longitudinal direction, the effect of Poisson's ratio is neglected and the following expression is used:

$$U_{L_k} = \int_0^{h_k} \int_0^{L} \frac{EJ}{2}\left[\frac{\partial^2 W_k(x)}{\partial x^2}\right]^2 dx\, dy \quad \ldots\ldots\ldots\ldots \quad (38)$$

As noted previously, W, is defined along the ridges, but the distribution of W throughout the plate is not known. U_L will be evaluated on the assumption that W varies linearly between its values at the ridges.

$$W_k(x,y) = W_{k-1,k} \sin \frac{\pi x}{L} + (W_{k,k} - W_{k-1,k}) \frac{y}{h_k} \sin \frac{\pi x}{L} \quad \ldots \quad (39a)$$

Taking the second partial derivative of $W_k(x,y)$ in equation (32a), inserting in eq. (38) and integrating it, we get,

$$U_{L_k} = \frac{\pi^4 E J h_k}{4L^3} \cdot \bar{W}_k^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (39b)$$

$$U_L = \frac{\pi^4 EJ}{4L^3} \sum_{k=1}^{n} h_k \bar{W}_k^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (40)$$

in which

$$\bar{W}_k^2 = \frac{W_{k-1,k}^2 + W_{k,k}^2 + W_{k-1,k} \cdot W_{k,k}}{3} \quad \ldots \ldots \ldots \ldots \ldots \quad (41a)$$

In performing an actual computation, it is often desirable to express U_L in terms of U_b. Comparison of Eqs. (18) and (39b) yields

$$U_{L_k} = \frac{t_k^2 \bar{W}_k^2}{h_k^2 v_k^2} U_{b_k} \quad \ldots \ldots \ldots \ldots \ldots \quad (42)$$

The effect of the shearing stresses in the plane of the plates (membrane shear), is considered in two categories, that is, strain energy of shear and additional deflection in the plane of the plates due to shearing strains. The precise expression for strain energy depends on the transverse variation of shearing stress across each plate. As an average value

$$U_{S_k} = \int_o^L \frac{S_k^2(x)}{2GA_k} dx \quad \ldots \ldots \ldots \ldots \ldots \quad (43a)$$

For equilibrium of the plate element shown in Figure (9)

$$S_k(x) = B_k^1(x) + \frac{h_k}{2} (T_{k-1}^1(x) + T_k^1(x)).^* \quad \ldots \ldots \ldots \ldots \quad (43b)$$

We have already seen that B_k and T_k have the same longitudinal distribution.

$$T_{k-1}(x) = T_{k-1}(\tfrac{L}{2}) \text{ Sin } \frac{\pi x}{L} \quad \ldots \ldots \ldots \ldots \ldots \quad (44b)$$

$$T_k(x) = T_k(\tfrac{L}{2}) \text{ Sin } \pi x/L \quad \ldots \ldots \ldots \ldots \ldots \quad (44c)$$

and

$$B_k(x) = B_k(\tfrac{L}{2}) \text{ Sin } \frac{\pi x}{L} \quad \ldots \ldots \ldots \ldots \ldots \quad (44d)$$

therefore

$$T_{k-1}^1(x) = T_{k-1} \cdot \frac{\pi}{L} \text{ Cos } \frac{\pi x}{L} \quad \ldots \ldots \ldots \ldots \ldots \quad (44e)$$

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* Superscript 1 denotes first derivative

$$T_k^1(x) = T_k \cdot \frac{\pi}{L} \cdot \text{Cos}\,\frac{\pi x}{L} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (44f)$$

$$B_k^1(x) = B_k(\tfrac{L}{2})\ \text{Cos}\,\frac{\pi x}{L} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (44g)$$

$$B_k(L/2) = -EI_k\, v_k''\,(\tfrac{L}{2})$$

$$= EI_k\,\frac{\pi^2}{L^2} \cdot v_k \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (44h)$$

Putting $B_k(L/2)$ from eq. (44h) in 44g), we get,

$$B_k^1(x) = EI_k\,\frac{\pi^3}{L^3}\,v_k \cdot \text{Cos}\,\frac{\pi x}{L} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (44j)$$

Putting the values found in eq. (43b), we get,

$$S_k(x) = \text{Cos}\,\frac{\pi x}{L}\left[\frac{\pi^3}{L^3}\,EI_k v_k + \frac{\pi}{2L} \cdot h_k(T_{k-1} + T_k)\right] \quad \ldots\ldots\ldots \quad (45a)$$

Putting value of $S_k(x)$ in Equations (43a) and integrating it, we get,

$$U_{S_k} = \frac{\pi^2}{4GLA_k}\left[\frac{\pi^2}{L^2}\,EI_k v_k + \frac{h_k}{2}\,(T_{k-1} + T_k)\right]^2 \quad \ldots\ldots\ldots \quad (46a)$$

and

$$U_S = \sum_{k=1}^{n} \frac{\pi^2}{4GLA_k}\left[\frac{\pi^2}{L^2}\,EI_k v_k + \frac{h_k}{2}\,(T_{k-1} + T_k)\right]^2 \quad \ldots\ldots\ldots \quad (46b)$$

The additional deflection due to membrane shear strain, s_k, is obtained by integration of the expression for unit strain.

$$E_{xy} = \frac{S_k(x)}{A_k G} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (47a)$$

and

$$s_k(x) = \int_0^x \frac{S_k(x)}{A_k G}\,dx \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (47b)$$

We have already found $S_k(x)$ [Eq. (45a)] for a sine-wave curve for $v_k(x)$. Putting this value in the equation below, we get, $s_k(L/2)$.

$$s_k(L/2) = \int_0^{L/2} \frac{\cos\frac{\pi x}{L}\left[\frac{\pi^3}{L^3} EI_k v_k + \frac{\pi}{2L} h_k\ (T_{k-1} + T_k)\right]}{A_k G} dx \quad . . . \quad (47c)$$

$$= \frac{1}{GA_k}\left[\frac{\pi^2}{L^2} EI_k v_k + \frac{h_k}{2}\ (T_{k-1} + T_k)\right] \quad . . . . \quad (48)$$

The total transverse deflection, r_k, is the sum of v_k and S_k. This total deflection, r_k, must be used in an equation in determining deflections normal to each of the slabs and ridge deflections for subsequent use in the computations for the slab bending and twisting energies, U_L, U_T, and U_M and potential of the external loads.

DERIVATIONS OF STRAIN ENERGY AND POTENTIAL ENERGY EXPRESSIONS CONSIDERING THE ELASTIC CURVE DISTRIBUTION OF TRANSVERSE DEFLECTION

For Uniform Longitudinal Ridge Loading: In the following paragraphs, energy expressions are derived for the case of uniform vertical loads, R_k, along each ridge. The elastic curve deflection is taken as

$$v_k(x) = v_k\ \frac{16}{5}\left(\frac{x^4}{L^4} - \frac{2x^3}{L^3} + \frac{x}{L}\right) \quad . . . . \quad (49)$$

The potential of the distributed external load, R_k, at the k^{th} ridge, V_k, is evaluated by

$$V_k = - \int_0^L R_k(x)\ \delta_k(x) \cdot dx \quad . . . . \quad (50a)$$

With the elastic-curve distribution of deflection

$$V_k = - \int_0^L \frac{16}{5} R_k \delta_k \left[x^4/L^4 - 2x^3/L^3 + x/L\right] dx$$

$$= -16/25\ L \cdot R_k \delta_k \quad . . . . \quad (50b)$$

and

$$V = \sum_{k=1}^{n} V_k = \sum_{k=1}^{n} -16/25 \cdot L \cdot R_k \delta_k = -0.64L \sum_{k=1}^{n} R_k \delta_k \quad . . . \quad (50c)$$

The strain energy of the longitudinal bending of the k^{th} plate, U_{b_k}, is evaluated from

$$U_{b_k} = \int_0^L EI_k \left[v_k''(x)\right]^2/2 \cdot dx \qquad (51a)$$

Putting the second derivative of $v_k(x)$ in eq. (51a) and integrating it, we get,

$$U_{b_k} = 24.576 \ \ EI_k v_k^2/L^3 \qquad (51c)$$

and

$$U_b = \sum_{k=1}^{n} U_{b_k} = 24.576 \ \ E/L^3 \sum_{k=1}^{n} I_k v_k^2 \qquad (52)$$

The strain-energy expression due to the longitudinal central load, N_k, resulting from the difference in the longitudinal shears at the k^{th} and $(k-1)^{th}$ ridges will now be derived. The longitudinal distribution of these shears and, consequently, of the central loads, N_k, has been shown to be the same as the longitudinal distribution of moment due to the external loads on a beam with similar support conditions. The strain energy due to the central loading is, in general,

$$U_{N_k} = \int_0^L N_k^2(x)dx \ / \ 2A_kE \qquad (53a)$$

For the elastic curve distribution of deflection,

$$N_k(x) = N_k \cdot 4x(L-x) \ / \ L^2 \qquad (53b)$$

$$U_{N_k} = \int_0^L \frac{16x^2 \ (L-x)^2 \ N_k^2}{L^4 \cdot 2A_kE} dx = \frac{LN_k^2}{3 \cdot 75 \ EA_k} \qquad (53c)$$

and

$$U_N = \sum_{k=1}^{n} LN_k^2 / 3{\cdot}75\ EA_k \quad \ldots\ldots\ldots\ldots \quad (54)$$

To evaluate the foregoing expressions for U_N in terms of the undetermined coefficients, v_k, use is made of the fact that the longitudinal stress at each ridge must be the same for each adjoining plate. Thus, for the folded plate structure with n plates, there exists a set of n-1 equations.

$$\frac{N_k}{A_k} - \frac{E\, v_k''(x)\, h_k}{2} = \frac{N_{k+1}}{A_{k+1}} + \frac{E\, v_{k+1}''\, h_{k+1}}{2} \quad \ldots\ldots\ldots \quad (55)$$

The final equation required to express each N_k in terms of the various v_k values is obtained from equilibrium considerations. For the usual case of no applied longitudinal load

$$\sum_{k=1}^{n} N_k = 0 \quad \ldots\ldots\ldots\ldots \quad (56a)$$

For the elastic-curve distribution, equation (55) becomes

$$\frac{N_k}{A_k} - \frac{N_{k+1}}{A_{k+1}} = -4{\cdot}8\ \frac{E}{L^2}\left[h_k v_k + h_{k+1}\, v_{k+1}\right] \quad \ldots\ldots\ldots \quad (56b)$$

The strain-energy expression corresponding to the transverse bending of the slabs (slab action) is now derived. This strain energy is evaluated as the external work done by the ridge reactions generated by the slab bending in moving through the assumed deflections at the ridges. These ridge reactions are either obtained by moment distribution method or slope deflection equations. Because the ridge reactions, R_{S_k}, depend linearly on the deflections, the longitudinal distribution of these reactions will be the same as the deflection distribution. Therefore

$$R_{S_k}(x) = R_{S_k}\left[x^4/L^4 - 2x^3/L^3 + x/L\right]\frac{16}{5} \quad \ldots\ldots\ldots\ldots \quad (57a)$$

and

$$\delta_k(x) = \delta_k\left[x^4/L^4 - 2x^3/L^3 + x/L\right]\frac{16}{5} \quad \ldots\ldots\ldots\ldots \quad (57b)$$

The general strain energy due to transverse slab bending is

$$U_{T_k} = \int_0^L \tfrac{1}{2} R_{S_k}(x)\, \delta_k(x)\, dx \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (57c)$$

$$U_{T_k} = \int_0^L \tfrac{1}{2} R_{S_k}\, \delta_k (16/5)^2 \left[x^4/L^4 - 2x^3/L^3 + x/L\right]^2 dx$$

$$= 0{\cdot}252\, L \cdot R_{S_k}\, \delta_k \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (57d)$$

and

$$U_T = 0{\cdot}252\, L \sum_{k=1}^{n} R_{S_k}\, \delta_k \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (58)$$

The expression for U_T can be readily converted to an expression involving v_k, using expression of δ_k in terms of v_k.

EVALUATION OF DEFLECTIONS AND STRESS RESULTANTS

Deflection. Using the expressions of the preceding section, the total potential energy, U + V, is expressed in terms of the undetermined coefficients; the term U includes either the strain energies associated with only the principal stress resultants or the strain energies of all the stress resultants, depending on the degree of accuracy desired. The principle of minimum total potential energy is then applied to get the set of 'n' linear equations involving the n undetermined coefficients, v_k:

$$\frac{\partial}{\partial v_k}(U + V) = 0,\ k = 1, 2, 3, \text{-----}\ n \quad \ldots\ldots\ldots\ldots \quad (59)$$

Solving these equations simultaneously, the coefficients, v_k, representing the midspan transverse deflections, are evaluated. The vertical deflections of ridges are obtained using equation (10) or (13) as applicable. The values obtained for v_k will be practically the same whether the transverse deflection curve is taken to be the elastic curve or the normal curve.

Transverse Slab Moment. In the calculations for strain energy associated with this stress resultant, the transverse slab moments at the center of each ridge have been expressed in terms of the coefficients, v_k. Consequently, with the determination of v_k, the previously obtained expressions are used to evaluate the transverse slab moments.

Longitudinal Plate Bending Moment. The bending moment at the center of the span, B_k, is obtained from the deflection curve by

$$B_k(L/2) = -EI_k v_k'' (L/2) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (60)$$

For sine-wave deflection

$$B_k(L/2) = \pi^2/L^2 \; EI_k v_k \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (61)$$

The value of v_k'' depends on the shape of the deflection curve. Sufficient accurate and conservative values of longitudinal plate bending moments are obtained from the sine-wave distribution.

Longitudinal Plate Central Load. In computing the strain energy due to this stress resultant, the longitudinal central loads, N_k, have been expressed in terms of v_k. Once v_k values are found, N_k values can be evaluated easily.

Longitudinal Plate Shear. The longitudinal shears at the horizontal edges of each plate can be obtained from the longitudinal central loads, N_k, on the basis of equilibrium considerations. Beginning with the exterior plate and taking each adjoining plate in turn, these shears are evaluated by setting the sum of longitudinal forces acting on each plate equal to zero:

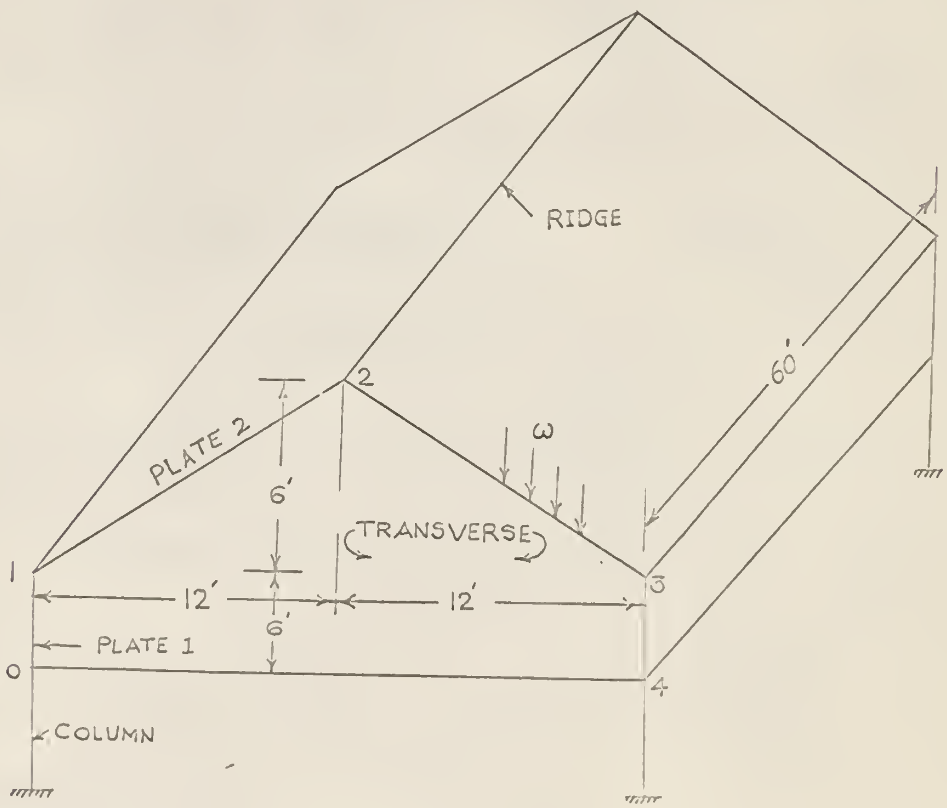
$$N_k = T_{k-1} - T_k \quad \dots \dots \dots \dots \dots \dots \dots \quad (62)$$

<u>Transverse Plate Shear</u>. The transverse plate shear is obtained by equation (45a) for sine-wave distribution of transverse deflection.

PROBLEM

To illustrate the method, a simple problem shown in Figure 11 is solved.

<u>Statement of the Problem</u>. Consider a simple span of 60 ft. in which we shall arbitrarily assume a column spacing along the ends of this longer span of 24 ft. (Fig. 11)



STATEMENT OF THE PROBLEM

Figure 11

Ridge	Plate	'h' in ft.	't' in ft.	'A' in sq.ft.	'I' in ft.^{4}	φ°	γ°
0							
	1	6.0	0.375	2.25	6.75	90°	
1							63.4°
	2	13.42	0.375	5.03	75.5	26.6°	
2							53.2°

DIMENSIONS

The data for this sample problem are as follows:

Slab thickness--------------$4\frac{1}{2}$ inches

Span between transverses----60'-0"

Column spacing--------------24'-0"

LOADING:

Roofing---5p.s.f.

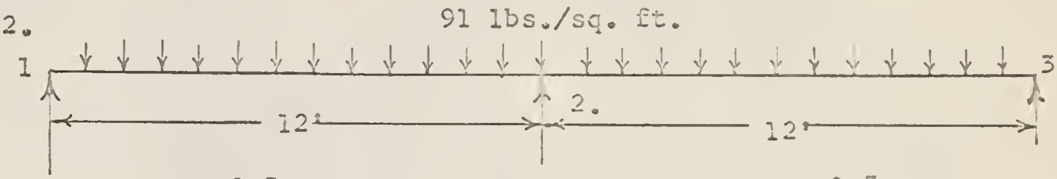
Snow load, insulation, acoustics, etc----------30p.s.f.

Dead load of slab------------$\dfrac{4.5 \times 150}{12}$ = 56p.s.f.

Total Load--------91p.s.f.

For most folded plate structures, sufficiently accurate results are obtained when analysis is based only on the effects of the applied loads, of longitudinal bending and central load in the plane of each plate (membrane action), and of transverse slab bending. Only for relatively short and thick structures are the normally minor effects of membrane shear, slab twisting moments, etc. considered. First of all a continuous one-way slab analysis, with all ridges supported, is made and slab bending moments and ridge reactions computed. These ridge reactions are then considered as loads applied to the folded plate structure, as there are no supports at ridges. The final slab moments are obtained by superposition of the previously obtained values and those obtained subsequently. Because of the symmetry of the structure and the loading, only one-half the structure has to be investigated.

Primary Slab Moment. For the purpose of analysis, assume fictitious supports at joints 1, 2, 3, and calculate the moments and reactions. These moments will be termed as primary moments. It is assumed that the relative displacement of joints is not present. A transverse strip one foot wide is considered, treating it as continuous slab supported at the joints by non-yielding supports. The moment distribution for this condition is performed in Figure 12.



C. O. Factor	0.5 →		← 0.5	
Distr. Factor		0.5	0.5	
F. E. M_s	-1092	+1092	-1092	+1092
Balance 1,3	+1092			-1092
C. O.		+ 546	- 546	
Final	0.00	+1638	-1638	+0.00

SLAB MOMENT DUE TO EXTERNAL LOAD

Figure 12

Reactions are as follows:

Total reaction at 1 = 409.5 lbs.

Total reaction at 2 = 1365.0 lbs.

Total reaction at 3 = 409.5 lbs.

These reactions were obtained on the assumption that fictitious supports exist at ridges 1, 2, and 3 and hence, on removal, they give equal and opposite reactions. These reactions are placed on the plates as loads as shown in Figure 13.

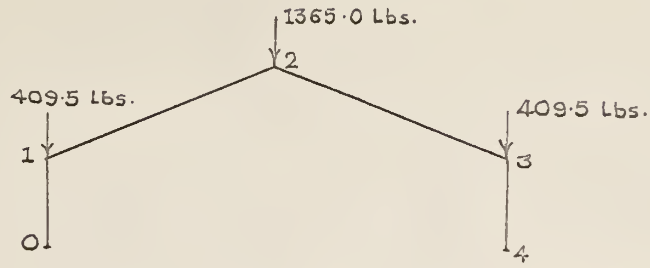


PLATE SHOWING FORCES OPPOSITE TO REACTIONS DUE TO SLAB ACTION
Figure 13

After finding the reactions the following "Ordinary Procedure" is followed.

Ordinary Procedure. In the following analysis, only the energy of the applied loads, of plate bending and central loading, and of slab transverse bending is considered. Energy expressions obtained by use of the sine-wave deflection curve are used as practically equal energy is obtained from elastic curve and sine-wave of transverse deflection.

(a) Ridge Deflections: Use Equation (10) to express the vertical deflection of the ridges in terms of the coefficients, v_k:

$$\delta_1 = -v_1 \quad \ldots \quad (63a)$$

$$\delta_2 = 2.24\, v_2 \quad \ldots \quad (63b)$$

(b) Strain Energy of Plate Longitudinal Bending: Use the equation for strain energy of longitudinal bending Equation (19) to express U_b in terms of v_k:

$$U_b = \frac{24.352E}{(60)^3}\left[6.75\, v_1^2 + 75.5\, v_2^2\right] \quad \ldots \quad (64a)$$

E = Modulus of elasticity of concrete = 3×10^6 p.s.i.

$$U_b = \frac{24 \cdot 352 E}{3600 \times 60} \left[6 \cdot 75\, v_1^2 + 75 \cdot 5\, v_2^2 \right]$$

$$= 1 \cdot 125 \times 10^{-4} E \left[6 \cdot 75\, v_1^2 + 75 \cdot 5\, v_2^2 \right]$$

$$\frac{\partial U_b}{\partial v_1} = 15 \cdot 2 \times 10^{-4}\, E\, v_1 \text{ and } \frac{\partial U_b}{\partial v_2} = 170 \times 10^{-4}\, E\, v_2 \;.\;.\;.\;.\;. \quad (64b)$$

(c) Strain Energy of Plate Longitudinal Central Loading: Apply Equation (26) to enforce the equality of longitudinal stress on each side of ridge 1.

$$\frac{N_1}{2 \cdot 25} - \frac{N_2}{5 \cdot 03} = - \frac{4 \cdot 935}{(60)^2} E \left[6\, v_1 + 13 \cdot 42\, v_2 \right] \;.\;.\;.\;.\;.\;.\;.\;.\;. \quad (65a)$$

Apply the condition of equilibrium of longitudinal forces:

$$N_1 + N_2 = 0 \;. \quad (65b)$$

Solve the preceding equations simultaneously to express N_k in terms of v_k,

$$N_1 = -10^{-4} E \left[1 \cdot 28\, v_1 + 2 \cdot 86\, v_2 \right]$$

$$N_2 = +10^{-4} E \left[1 \cdot 28\, v_1 + 2 \cdot 86\, v_2 \right]$$

Use the equation for strain energy of longitudinal central loading Equation (22) to express U_N in terms of v_k.

$$U_N = 15 \times 10^{-8} E \left[\frac{(1 \cdot 28\, v_1 + 2 \cdot 86\, v_2)^2}{2 \cdot 25} + \frac{(1 \cdot 28\, v_1 + 2 \cdot 86\, v_2)^2}{5 \cdot 03} \right]$$

$$\frac{\partial U_N}{\partial v_1} = 15 \times 10^{-8} E \left[2 \cdot 11\, v_1 + 4 \cdot 71\, v_2 \right] \;.\;.\;.\;.\;.\;.\;.\;.\;.\;. \quad (65c)$$

and

$$\frac{\partial U_N}{\partial v_2} = 15 \times 10^{-8} E \left[4 \cdot 71\, v_1 + 10 \cdot 52\, v_2 \right] \;.\;.\;.\;.\;.\;.\;.\;.\;.\;. \quad (65d)$$

(d) Potential of Loads: Use the equation for potential of uniform loads Equation (17) and previous results to express V in terms of v_k.

$$V = -0{\cdot}6366\ L \sum_{k=1}^{n} R_k \cdot \delta_k \quad \ldots \quad (67a)$$

$$V = -0{\cdot}6366 \times 60 \left[409{\cdot}5\ \delta_1 + 682{\cdot}5\ \delta_2\right] \quad \ldots \quad (67b)$$

Using equations (63b) and 63c), we get,

$$V = 15680\ v_1 - 58250\ v_2 \quad \ldots \quad (67c)$$

$$\frac{\partial V}{\partial v_1} = 15680 \quad \ldots \quad (67d)$$

and

$$\frac{\partial V}{\partial v_2} = -58250 \quad \ldots \quad (67e)$$

(e) Strain Energy of Slab Transverse Bending: Evaluate the slab ridge reactions in terms of the ridge deflections. Slope deflection equations are used to evaluate the ridge reactions

$$M_{10} = 0$$

$$M_{12} = \frac{2EJ}{h_2}\left[2\Theta_1 + \Theta_2 - \frac{3\delta_{12}}{h_2 \cos\Theta_2}\right] = \frac{2EJ}{13{\cdot}42}\left[2\Theta_1 + \Theta_2 - 0{\cdot}25\ \delta_{12}\right] \quad (68a)$$

$$M_{22} = \frac{2EJ}{h_2}\left[2\Theta_2 + \Theta_1 - \frac{3\ \delta_{12}}{h_2 \cos\Theta_2}\right] = \frac{2EJ}{13{\cdot}42}\left[2\Theta_2 + \Theta_1 - 0{\cdot}25\ \delta_{12}\right] \quad (68b)$$

$$M_{23} = \frac{2EJ}{h_3}\left[2\Theta_2 + \Theta_3 - \frac{3\ \delta_{23}}{h_3 \cos\Theta_3}\right] = \frac{2EJ}{13{\cdot}42}\left[2\Theta_2 + \Theta_3 - 0{\cdot}25\ \delta_{23}\right] \quad (68c)$$

By symmetry

$$\left.\begin{array}{l}\Theta_1 = -\Theta_3 \\ \delta_{12} = \delta_{23}\end{array}\right] \quad \ldots \quad (68d)$$

Applying equilibrium at each joint,

$$M_{10} = -M_{12} \quad \ldots \quad (68e)$$

$$M_{22} = -M_{23} \quad \ldots \quad (68f)$$

The foregoing system of equations is solved simultaneously to give

$$\Theta_2 = \delta_{12/8} \quad \ldots \quad (68g)$$

$$\Theta_1 = \delta_{12/16} \quad \ldots \quad (68h)$$

and

$$M_{22} = \frac{2EJ}{13\cdot 42}\left[2\Theta_2 + \Theta_1 - 0\cdot 25\ \delta_{12}\right] = \frac{2EJ}{13\cdot 42}\left[\frac{1}{16}\ \delta_{12}\right] \quad \ldots \quad (68i)$$

Putting J = 0.006 ft.^{4} in equation (68i), we get,

$$M_{22} = 0\cdot 56 \times 10^{-4}\ E\ \delta_{12} \quad \ldots \quad (68j)$$

Now

$$\delta_{12} = \delta_2 - \delta_1$$

Investigating slab 2 as a free body, gives,

$$R_{S_1} = 0\cdot 047 \times 10^{-4}\ E\ (2\cdot 24\ v_2 + v_1)$$

and

$$R_{S_2} = -0\cdot 047 \times 10^{-4}\ E\ (2\cdot 24\ v_2 + v_1)$$

Use the equation for strain energy of slab transverse bending Equation (24) to express U_T in terms of v_k:

$$U_T = 15 \times 0\cdot 047 \times 10^{-4}\ E\ (-4\cdot 48\ v_1 v_2 - v_1^2 - 5\cdot 02\ v_2^2) \quad \ldots \quad (68k)$$

$$\frac{\partial U_T}{\partial v_1} = 15 \times 0\cdot 047 \times 10^{-4}\ E\ (-4\cdot 48\ v_2 - 2v_1) \quad \ldots \quad (68l)$$

and

$$\frac{\partial U_T}{\partial v_2} = 15 \times 0\cdot 047 \times 10^{-4}\ E\ (-4\cdot 48\ v_1 - 10\cdot 04\ v_2). \quad \ldots \quad (68m)$$

(f) Principle of Minimum Total Potential Energy: Use the minimum principle by applying Equation (59) to the sum of V, U_b, U_N, and U_T.

$$\partial(U + V)/\partial v_1 = \frac{\partial U_T}{\partial v_1} + \frac{\partial U_N}{\partial v_1} + \frac{\partial U_b}{\partial v_1} + \frac{\partial V}{\partial v_1} = 0 \quad \ldots \quad (69a)$$

$$\partial(U + V)/\partial v_2 = \frac{\partial U_T}{\partial v_2} + \frac{\partial U_N}{\partial v_2} + \frac{\partial U_b}{\partial v_2} + \frac{\partial V}{\partial v_2} = 0 \quad \ldots\ldots\ldots \quad (69b)$$

Putting the values of various derivatives as found in the above equation, we get,

$$v_1 - 0{\cdot}23v_2 = -0{\cdot}0315 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (69c)$$

and

$$-v_1 + 51{\cdot}6v_2 = 0{\cdot}426 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (69d)$$

Solving the above equations simultaneously,

$v_1 = -0.0286$ ft.

$v_2 = +0.0126$ ft.

(g) Ridge Deflections: Use Equations in paragraph (a) to find ridge deflections.

$\delta_1 = -v_1$

$\delta_1 = +0{\cdot}0286$

$\delta_2 = +2{\cdot}24v_2$

$\delta_2 = 0.0282$ ft.

(h) Transverse slab moment at the center of the Span: Use Equation (68j) to find moment in the slab in the transverse direction:

$$M_{22} = +0{\cdot}56 \times 10^{-4}\ E(\delta_2 - \delta_1)$$

$$= 0{\cdot}56 \times 10^{-4} \times 3 \times 10^{6} \times 144(0.0282 - 0.0286)$$

$$\doteq 0$$

Total moment in the slab in the transverse direction = Primary moment + 0

= 1638 ft. lbs/ft. .... (69)

(i) Longitudinal moment in the plate in its plane at the center of the span:

$B_k = -EI_k v_k''(L/2)$ . . . . (70a)

For sine-wave deflection curve

$B_k = EI_k v_k(L/2)\ \pi^2/L^2$

$B_1 = +EI_1 v_1(L/2)\ \pi^2/L^2$

$= -9.86/3600 \left[3 \times 10^6 \times 144 \times 6.75 \times 0.0286\right]$

$= -229000$ ft. lbs. . . . . (70b)

Similarly $B_2 = EI_2 v_2(L/2)\ \pi^2/L^2$

$= +1132{,}000$ ft. lbs. . . . . (70c)

(j) Longitudinal central load in the plane of the plate at the center of the span: Use the equations in paragraph (c) to compute the longitudinal central loads in the plates:

$N_1 = -10^{-4} \times 3 \times 10^6 \times 144 \left[1.28(-0.0286) + 2.86(0.0126)\right]$

$= 35$ lbs. . . . . (71a)

$N_2 = -N_1 = -35$ lbs. . . . . (72b)

(k) Longitudinal plate shear between end and center of the span: Values ot T_k are obtained from the values of N_k using equation (62)

$T_o = 0$ . . . . (73a)

$T_o - T_1 = N_1$

$-T_1 = 35$ lbs. . . . . (73b)

$T_1 = -35$ lbs. .

$T_1 - T_2 = N_2$

$-35 - T_2 = -35$

$T_2 = 0$ . . . . (73c)

(l) Transverse plate shear at the end of the span: Use equation (45a) to find shear at the end of the span.

$$S_k(o) = \frac{\cos \pi \times 0}{L}\left[\frac{\pi^3}{L^3} I_k V_k E + \frac{\pi}{2L} h_k (T_{k-1} + T_k)\right]$$

$$S_1 = \left[\pi^3/(60)^3\right] \times \left[6\cdot 75 \times (-0\cdot 0286) \times 3 \times 10^6 \times 144 + \pi/(2 \times 60)\right] \times 6(-35)$$

$$= -4105 \text{ lbs.} \quad \ldots \quad (74a)$$

Similarly,

$$S_2 = \left[\pi^3/(60)^3\right] \times \left[75\cdot 5\ (0.0126) \times 3 \times 10^6 \times 144 + \pi/(2 \times 60)\right](-35-0)$$

$$= +22000 \text{ lbs.} \quad \ldots \quad (74b)$$

CONCLUSIONS

The method presented herein is an alternate method to the Gaafar and Yitzhaki methods. The principle of minimum total potential energy using the Rayleigh-Ritz method with sine-wave taken as the transverse deflection curve affords a sufficiently accurate method of analyzing folded plate structures. It has the inherent advantage of minimizing the complexity of physical arguments involved in the analysis. The ordinary procedure described herein is used for structures in which the effects of twisting moments, membrane shear, longitudinal slab bending moments, etc. are neglible compared to major stress resultants. We are required to consider the effects of minor stress resultants where the dimensions and edge conditions of the structures are such that it is necessary to take into account these minor stress resultants in analysis. This can be done by considering the energy of these stress resultants.

Although this method does not furnish results as accurate as those obtained by J. E. Goldberg and H. L. Leve, it furnishes adequately accurate results. The method presented herein can be extended to a folded plate configuration other than the simple span structure. This method reduces the computational effort.

Among available methods, the Winter and Pei procedure is the simplest. It is applicable to short folded plate structures for which joint displacements can be ignored without much loss of accuracy. For thick and short folded plate structures, the stress resultants of minor importance should be considered. For long span folded plate structures ordinary procedure will give the same results as others. The method presented herein is applicable to any type of loadings.

ACKNOWLEDGMENT

The writer wishes to express his sincere gratitude to Prof. V. M. Rosebraugh for his kind guidance and assistance in completing the report in the most systematic manner.

APPENDIX - I. EXPLANATION OF TERMS

A = Cross-sectional area of plate

B = Bending moment in plate in its plane

E = Modulus of elasticity

G = Shearing modulus of elasticity

h = Width of plate or slab

I = Moment of inertia of plate

J = Moment of inertia of slab per unit length

J_t = $C_t \cdot h \cdot t^3$, where value of C_t depends on ratio of h to t

K_{10} = Torsional resistance of edge slab

k = Subscript referring to ridge number or plate number

L = Span of structure

M_L, M_T, M_{xy} = Longitudinal bending moment, transverse bending moment, and twisting moment in slab per unit length respectively

N = Central load in plane of plate

R = Distributed load applied at ridge

R_S = Distributed ridge reaction due to slab transverse bending

r = Deflection in y - direction due to combined plate shear and bending

s = Deflection in y - direction due to plate shear

T = Longitudinal shear in plate at horizontal edge

t = Thickness of plate or slab

U = Total strain energy

U_B, U_N, U_S = Strain energy of plate bending, central load in plane of plate, and shear in plate in its plane respectively

U_L, U_M, U_T = Strain energy of slab bending moment in longitudinal direction, twisting moment in slab, and bending moment in slab in transverse direction respectively.

u = Deflection in x - direction

V = Potential of applied loads

v = Deflection in y - direction due to plate bending

W = Deflection in z - direction (normal to slab)

$W_{k-1,k}$ = Deflection at ridge k-1 normal to slab k

$W_{k,k}$ = Deflection at ridge k normal to slab k

x,y,z = Rectangular co-ordinate axes for plate or slab

γ = Deflection angle between successive plate

δ = Vertical deflection of ridge

δ_{12} = Vertical deflection of ridge 2 relative to ridge 1

E_{xy} = Unit shear strain

Θ = Angular rotation at ridge

μ = Poisson's ratio

φ = Inclination angle of plate with horizontal

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ANALYSIS OF REINFORCED CONCRETE FOLDED PLATES BY MINIMUM ENERGY PRINCIPLE

by

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B. S., S. V. V. (University), Anand, 1964

AN ABSTRACT OF
A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Civil Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1965

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Major Professor

A method of analysis of folded plate structures using the principle of minimum total potential energy is presented.

Strain energy and potential energy expressions are derived considering the elastic curve as the transverse deflection curve. In this method of analysis we can include the effects of minor stress resultants such as longitudinal slab bending moment, twisting moments in the slab, membrane shear, etc. Thus the method can be applied to short and thick folded plate structures. For long span folded plate structures, "ordinary procedure", described herein, gives sufficiently accurate results. To start with the solution, fictitious supports at the ridges and valleys are assumed, and a one foot wide strip of the slab is analyzed as a continuous beam. The moments obtained are termed as primary moments. The reactions are obtained by statics. The forces equal and opposite to these reactions are then applied to folded plate structures as the external loads. The transverse deflection function of the plate is assumed to be a half sine-wave. The total potential energy corresponding to this assumed deflection is obtained, using the derived expressions here. The principle of minimum total potential energy is then applied to evaluate the undetermined constants in the assumed deflection function. Substitution of these constants in the deflection function, gives the transverse deflection function. Using this deflection function and the expressions derived, stress resultants are calculated. Total slab moment in the transverse direction is obtained by summing the primary moment and the moment obtained by the procedure described above.

A simple problem is outlined by "ordinary procedure".