achieved using either the gas-filled hollow-core fiber reference with modest RF (scheme 2 below), two optical references (scheme 3), or no RF references (scheme 4). We estimate the comb stability for four different locking schemes: scheme 1, f_0 and $f_{\rm rep}$ locked to an RF reference; 2, f_0 locked to RF while a single comb tooth was locked to an optical reference; 3, $f_{\rm rep}$ locked to RF and a single comb tooth to optical reference; and 4, two comb teeth locked to two optical references. The comb instability calculations for these four locking schemes are summarized in Table 1. The degradation of the comb stability is investigated when different RF references are used, as we look 100 nm and 500 nm away from 1539.4 nm, shown in the last two columns of Table 1. In this calculation, we used the following fiber comb parameters: repetition rate is 90 MHz, f_0 is 30 MHz, and one stabilized comb tooth is at the reference wavelength $\lambda_{\rm r1}$ = c/ $f_{\rm r1}^{\rm (opt)}$ = 1539.4 nm (case 2-4). For the optical reference, we assume the fractional instability at 100 ms is 10^{-12} as measured in our experiment.

Table 1. Calculation of Comb Optical Instability at 100 ms within 100 and 500 nm of 1539 nm When Various Locking Schemes and RF and Optical References Are Employed

	Locking parameters	$\delta f_{\rm RF}/f_{\rm RF}$		$\delta f_{\rm opt}/f_{\rm opt}$	$\lambda_{\rm r1}$ (nm)	$\Delta\lambda_{\rm r}$ [a] (nm)	Comb tooth instability about 1539.4 nm	
							± 100 nm	± 500 nm
(1)	$f_0^{\rm (RF)} + f_{\rm rep}^{\rm (RF)}$	Quartz	1×10^{-9}	–––	–––	–––	1×10^{-9}	1×10^{-9}
		GPS-Rb	2×10^{-11}				2×10^{-11}	2×10^{-11}
		H maser	1×10^{-13}				1×10^{-13}	1×10^{-13}
(2)	$f_0^{\rm (RF)} + f_{\rm r1}^{\rm (opt)}$	Quartz	1×10^{-9}	1×10^{-12}	1539.4	–––	1×10^{-12}	1×10^{-12}
		GPS-Rb	2×10^{-11}					
		H maser	1×10^{-13}					
(3)	$f_{\rm rep}^{\rm (RF)} + f_{\rm r1}^{\rm (opt)}$	Quartz	1×10^{-9}	1×10^{-12}	1539.4	–––	7×10^{-11}	3×10^{-10}
		GPS-Rb	2×10^{-11}				2×10^{-12}	7×10^{-12}
		H maser	1×10^{-13}				9×10^{-13}	7×10^{-13}
(4)	$f_{\rm r1}^{\rm (opt)} + f_{\rm r2}^{\rm (opt)}$	–––	–––	1×10^{-12}	1539.4	3 nm	7×10^{-11}	3×10^{-10}
						30 nm	5×10^{-12}	3×10^{-11}
						300 nm	2×10^{-12}	3×10^{-12}

[a] wavelength separation between two stabilized comb teeth $\Delta\lambda_{\rm r} = \lambda_{\rm r1} - \lambda_{\rm r2}$

In scheme 1, the instability of the $m^{\rm th}$ comb tooth can be expressed as Eq. (11) of [5] in terms of the instability of f_0 and $f_{\rm rep}$. The microwave instability of $f_{\rm rep}$ is transferred to the optical regime (ν_m) due to the large m number ~10^6 for a sub-100 MHz fiber laser. In this case, the f_0 fractional instability is essentially irrelevant. Within ± 500 nm around the stabilized comb tooth, the comb instability is dominated by the RF instability of the repetition rate.

In scheme 2, if f_0 is locked to an RF reference while one comb tooth is locked to an optical reference, the stability of the $m^{\rm th}$ comb tooth can be expressed as:

$$\sigma_{\nu_m} = \frac{\delta\nu_m}{\nu_m} = \left(1 - \frac{m}{n_r}\right)\frac{\delta f_0}{\nu_m} + \frac{m}{n_r}\frac{\delta\nu_r}{\nu_m} \qquad (1)$$

where $\nu_{\rm r}$ is the optical frequency of the stabilized comb tooth and $n_{\rm r}$ is the mode number of that tooth. In this equation, the first term is the uncertainty induced by the f_0 instability. As

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shown in Table 1, the f_0 instability (δf_0) is irrelevant for the resulting comb instability, and the comb instability is dominated by the second term, which depends on two parameters: the instability of the comb tooth locked to the optical reference ($\delta\nu_r$), and how far the comb tooth of interest is away from the stabilized tooth. Based to our calculation, within ±500 nm from the stabilized comb tooth at 1539.43 nm, the uncertainty of the stabilized optical tooth is still the dominant factor.

Scheme 3 involves f_{rep} locked to an RF reference while one comb tooth is locked to an optical reference. An advantage of this locking scheme is that the generation of carrier-envelope offset frequency f_0 can be avoided, which reduces the system complexity. However, a high performance RF reference is required such that the multiplied RF instability does not dominate. The instability of the comb tooth of interest can be expressed by Eq. (5) of [5] in terms of the fractional instability of the repetition rate and the locked single tooth. This calculation shows that the optical instability depends critically on the employed RF reference.

In scheme 4, two comb teeth are stabilized to two separate optical references at wavelengths λ_{r1} and λ_{r2}, which can be two transitions from the same or different types of atoms/molecules, or high finesse optical cavities. By using gas-filled photonic microcells [26] as the optical references, the system can likely be made simpler and more portable without the integration of RF references. For simplicity, we assume that both feedback loops are identical and independent from each other. Therefore, the stability of the m^{th} comb tooth can be expressed as:

$$\sigma_{\nu_m} = \frac{\delta\nu_m}{\nu_m} = \frac{m - n_{r2}}{n_{r1} - n_{r2}} \times \frac{\nu_{r1}}{\nu_m} \times \frac{\delta\nu_{r1}}{\nu_{r1}} + \frac{n_{r1} - m}{n_{r1} - n_{r2}} \times \frac{\nu_{r2}}{\nu_m} \times \frac{\delta\nu_{r2}}{\nu_{r2}} \qquad (2)$$

In this case, besides the instability of the two stabilized comb teeth ($\delta\nu_{r1}/\nu_{r1}$ and $\delta\nu_{r2}/\nu_{r2}$, respectively), another key parameter that determines the overall comb instability is the wavelength separation between the two stabilized comb teeth. As an example, Table 1 shows the comb instability when two stabilized comb teeth are separated by 3 nm, 30 nm and 300 nm. If the two locked teeth are separated by only 3 nm, the comb stability degrades by almost two orders of magnitude 500 nm away from the locked tooth at 1539.4 nm. In contrast, if the separation is 300 nm, the degradation is only a factor of three even for comb teeth almost 500 nm away from one of the locked teeth. Therefore, the two stabilized comb teeth need to be at least hundreds of nanometers apart to avoid fast degradation in comb stability, which may be achievable with 2 separate gas-filled fiber references based on different gasses. This result is consistent with the conclusions obtained in [5].

6. Summary

We have demonstrated the first direct sub-Doppler spectroscopy with a single tooth from an optically-referenced fiber comb. To do this, a single comb tooth was amplified from 20 nW to 40 mW with high fidelity, sufficient to perform saturated absorption spectroscopy on an overtone transition in acetylene near 1540 nm directly with the amplified comb tooth. No intermediate cw laser was required, nor did a cw laser need to be phase-locked to the comb.

The resulting optical frequency comb exhibits high short-term stability (6×10^{-12} at 100 ms) exceeding that of the GPS-disciplined Rb oscillator by an order of magnitude; thus the stability of the comb is equal to that of a cw fiber laser locked to the reference. Long-term drift is attributed to technical noise and should be readily reduced to the level of a cw laser locked to the fiber reference (shown in green diamonds in Fig. 6(b), the corrected data from [21]). Calculations indicate that f_{rep} can be read out as a source of stable RF, a factor 10 better than that of a quartz oscillator at 100 ms, when f_0 is stabilized to a modest RF reference (quartz oscillator) or a second optical reference at least 300 nm away from the first. Thus this work is a significant advance towards an all-fiber metrology system for moderate accuracy and good short-term instability in the near-IR and RF regimes without reliance on GPS.

This result demonstrates the viability of direct stabilization of a sub-100 MHz repetition rate fiber comb to a gas-filled hollow-core fiber toward an all-fiber metrology system. With some modifications, this system can be made more portable. The gas-filled hollow fiber, here mounted between two vacuum chambers, can be replaced with a sealed photonic microcell [26]. Furthermore, the 89 MHz rep rate comb with 9 GHz single stage filtering cavity may in the future be replaced with a GHz repetition rate comb based on one of many technologies [6–10, 12].

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