

THE EFFECTS OF INTERNAL PARAMETERS
ON THE BREAKDOWN POTENTIALS OF LONG
LOW PRESSURE ALTERNATING CURRENT
ARCS AND GLOWS

by

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A THESIS

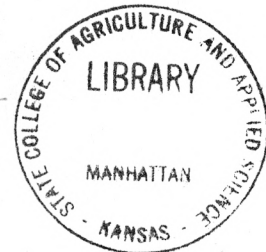
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INTRODUCTION

In 1889 Paschen (5) experimentally discovered that when a difference of potential was applied between two electrodes with a gas between them, the potential at which a discharge occurred was a continuous function of the product of the gaseous pressure and distance between the electrodes. This work with D.C. discharge was performed with electrode spacing distances of from .01 to 1.5 cm, using spherical electrodes, pressures of from 4 mm to 760 mm of mercury, and CO_2 , H_2 , and air as gases. This phenomenon was later theoretically explained by Townsend (7).

In order that one may more adequately describe work in this field, an experimentally determined plot of the current as a function of the voltage in a typical gaseous discharge tube, such as a gas filled phototube or a fluorescent lamp (4), is shown in Plate I.

The current indicated by the portion OA of the curve of Plate I is due to primary ionization of the gas and emission from the electrodes by such natural processes as radioactivity, cosmic rays, x-rays, photoelectric effect, etc. As the potential is increased from 0 to A, these electrons and ions move with a greater velocity and the current increases. The current in this portion is limited primarily by diffusion and recombination with other ions before reaching the electrodes.

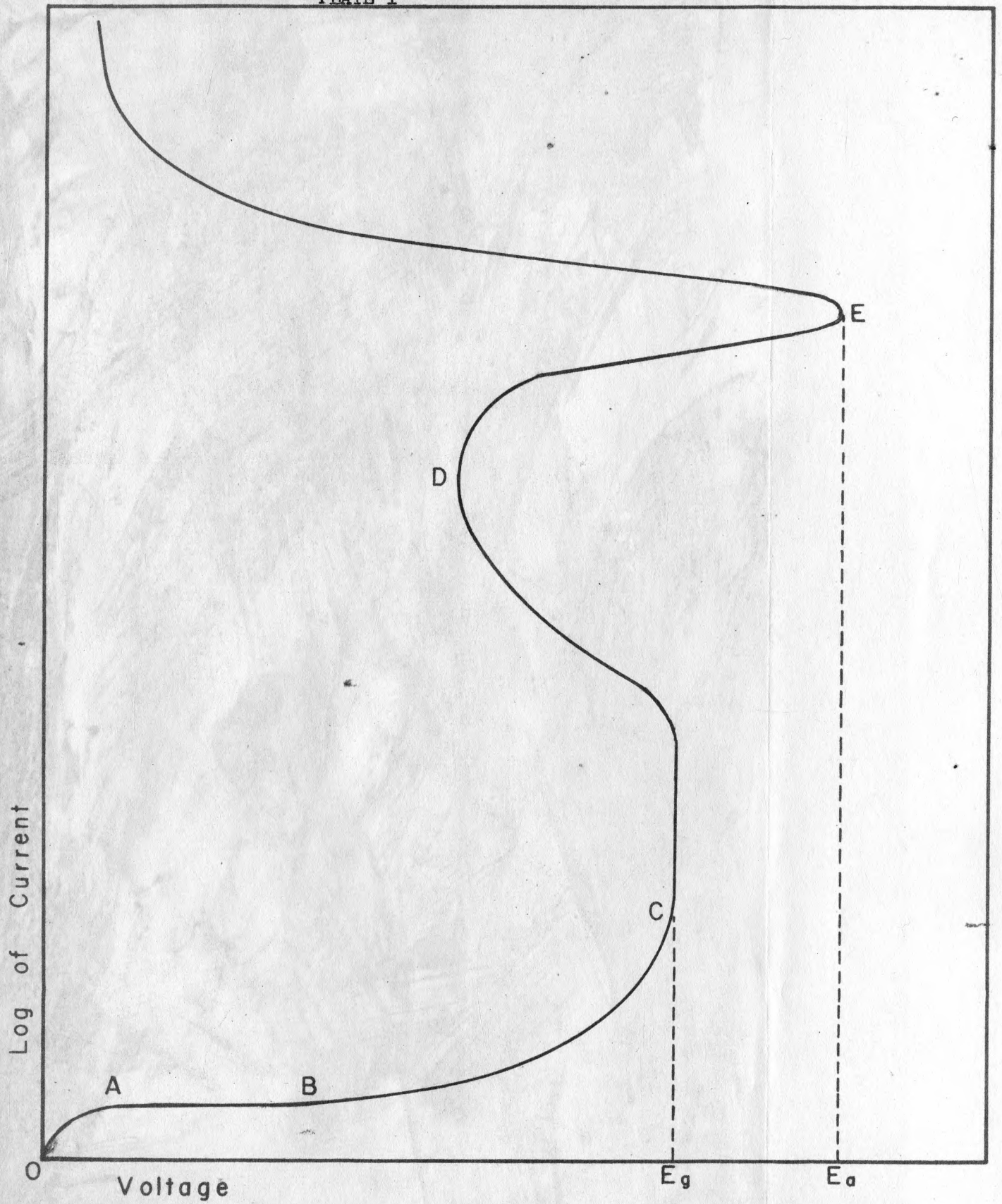
From A to B an increase in the potential difference between the electrodes causes no appreciable increase in the current. In this region the potential is large enough so that essentially all of the primary electrons and ions reach the electrodes before recombination can occur.

In the portion from B to C an amplification of the electrons and ions occurs. In this region the electrons have enough energy to liberate other

EXPLANATION OF PLATE I

Graph of the current as a function of voltage in a typical gaseous discharge tube.

PLATE I



electrons upon collision with neutral atoms. Therefore, the potential at B must depend on the ionization potential of the gas. According to the theory of Townsend if n_0 is the number of electrons leaving the cathode per second, α is a parameter usually known as the first Townsend coefficient, and x is the distance from the cathode, then the number of electrons crossing a point x distance from the cathode per second is given by n where

$$n = n_0 e^{\alpha x}.$$

Or, multiplying both sides of the equation by the electronic charge and evaluating at x equal to the interelectrode distance, d , we obtain an expression relating the total current, I , the current due to external ionization at the cathode, I_0 , and the gap length, d :

$$I = I_0 e^{\alpha d}.$$

The discharge characterized by this portion of the curve is known as a non-self maintained discharge since it depends upon the presence of the primary ionization.

At C, a drop in voltage and increase in current occurs across the tube. The region CD is known as the glow discharge region; and the voltage, E_g , is the potential difference that must be applied across the electrodes to "break" the gas into the glow region. Hence it shall be called the glow breakdown voltage.

Again according to Townsend (Maxfield and Benedict, 2), another mechanism seems to play an important role in this region. This is the mechanism of electron emission from the cathode due to positive ion bombardment, to photoemission produced by radiation from the excited atoms, to photoionization of the gas atoms, to the presence of metastable atoms near the cathode, etc. Breakdown occurs when

$$\gamma = \frac{1}{e^{\alpha d} - 1}.$$

Here α is the first Townsend coefficient, γ the second Townsend coefficient, and d the interelectrode distance.

As the potential across the tube is increased still further, the point E is reached. At this point the voltage again decreases while the current increases. The tube is now in the arc discharge region and this voltage, E_a , will be called the arc breakdown voltage.

The Townsend theory is unable to explain adequately the mechanism responsible for the arc breakdown, especially at large interelectrode distances. However, the streamer theory, developed independently by Raether and Loeb (Loeb and Meek, 1), seems effective in describing the initiation of a discharge between such electrodes, particularly in those instances when voltage overloads are present.

According to this theory an electron emitted from the cathode may cause a streamer to form. This electron forms a large progeny of electrons and ions as it crosses the gap from cathode to anode. In a typical case it may form 10^6 to 10^8 ion pairs in its journey. The electrons that are formed by this method move toward the anode rather rapidly, forming avalanches of their own. However, these occur later, are smaller, and are usually not instrumental in forming the streamer. During the time that it takes for the electrons to cross the gap, the positive ions, because of their larger mass, remain essentially stationary. This itself does not constitute breakdown. The positive ion filament extending across the gap, and with the greatest number of positive ions at the anode, is not a conductive filament.

However, in addition to the ion pairs formed there are about 5 to 10 times as many excited atoms formed in the gas due to the initial electron. These excited atoms have a life of about 10^{-8} seconds -- a short time compared to the time that it takes for the positive ions to cross the gap. These atoms

emit ultra-violet radiation and this radiation results in photoionization of the gas because of impurities and two step processes. The photoelectrons produce their own avalanches and more excited atoms, which in turn produce more photoelectrons.

Furthermore, the photoelectrons produced near the anode are in the vicinity of a very strong positive space charge and are drawn into the positive ion region. This region now becomes conductive, starting at the anode and extending toward the cathode as the positive ions move toward the cathode. The process of photoionization causes free electrons to be formed constantly and allows the positive streamer to be self-propagating in its advance toward the cathode. When it reaches the cathode, a conductive strip exists across the gap and breakdown has occurred.

Penning and Addink (6), as early as 1934, found that a slight impurity of argon in neon produced a considerably lower breakdown potential in the mixture than occurred in either of these gases alone. Their work was performed, for the most part, at higher pressures and shorter distances than the work reported in this paper.

In 1935 Varney, et al (8) investigated slight deviations from Paschen's law for cases in which the mechanism of the arc depended on the concentration of the ions rather than on their total number. They explain these deviations on the basis of the Townsend theory, as being due to the effects of strong space charges in the gas on Townsend's coefficients. Their investigations were performed for plane parallel electrodes, short distances (in the order of 10 cm and less), low pressures (a few mm of mercury), and pure molecular gases.

In this paper an attempt was made to study the effects of pressure and distance upon the breakdown potentials of long, low pressure alternating

current arc and glow discharges using a mixture of argon and mercury vapor with barium oxide coated electrodes. This work differed from most of the previous work in this field in that the distances used were much larger, the gas consisted of a mixture of mercury at a relatively constant vapor pressure and argon at a variable pressure. The electrodes were coiled-coil barium oxide coated and the potential differences were alternating. Also, external capacitors existed to distort the applied potentials.

A secondary purpose of this paper was the correlation of the results of previous workers with those obtained on this more complicated system.

EXPERIMENTAL APPARATUS AND PROCEDURE

A photograph of the equipment used to obtain the results used in this paper is shown on Plate II. All of the glass in the system was pyrex-brand glass.

The system was evacuated by means of a mercury diffusion pump backed up by a Cenco-Megavac vacuum fore pump.

Since all of the work was to be done in the presence of mercury vapor, there was no reason to trap it out of the vacuum system. Hence the best attainable vacuum was about 2×10^{-3} mm of Hg.; this corresponded to the vapor pressure of mercury at room temperature (80°F.).

The gauge used to read these pressures was a Thermocouple gauge, type 05-0100, manufactured by the National Research Corporation.

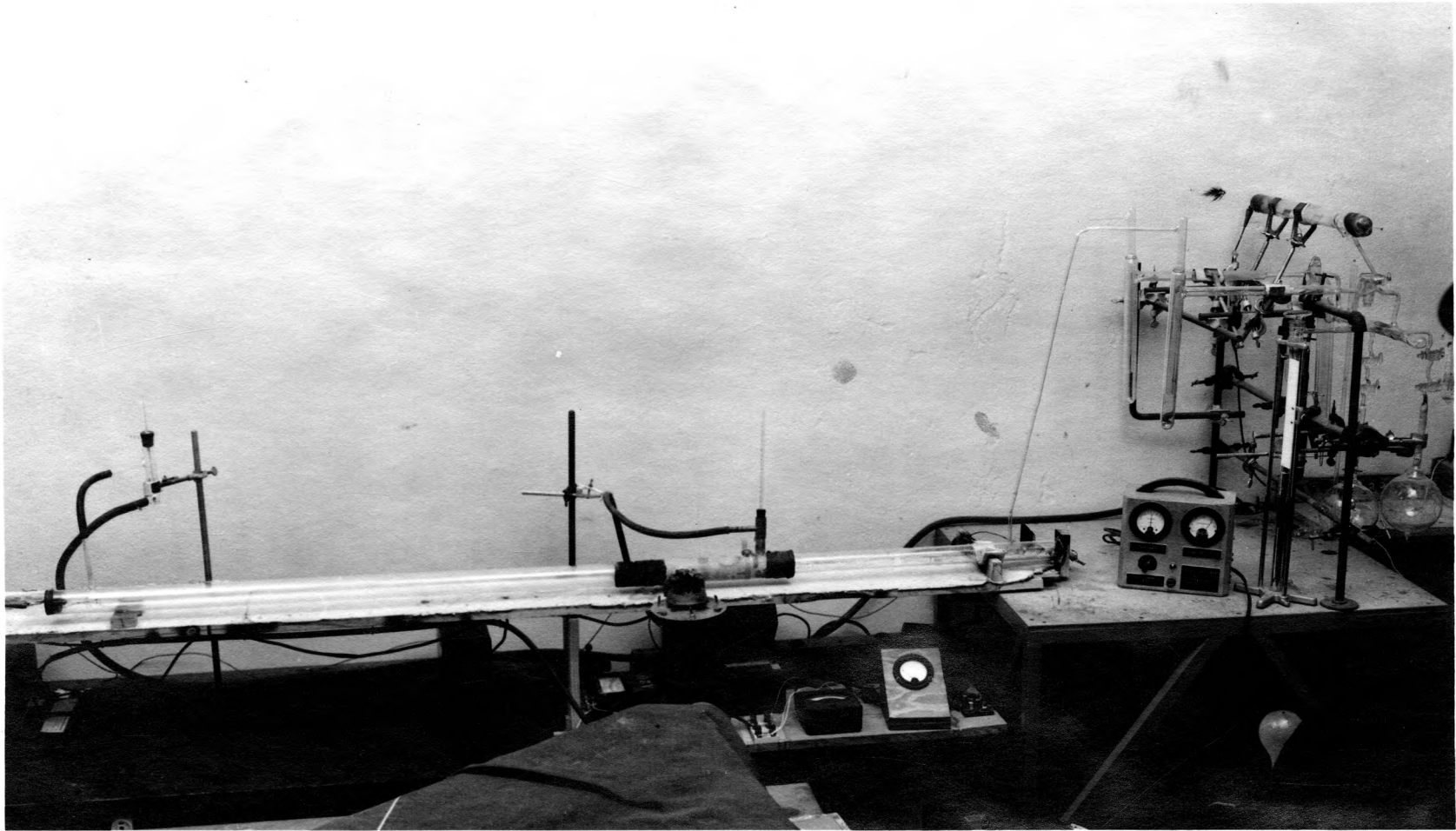
All the valves used in the system were pyrex-brand standard taper stopcocks lubricated with Lubri Seal stopcock grease.

The discharge tube itself consisted of a 40 mm outside diameter, eight foot long pyrex tube sealed off on the ends. At each end a commercially prepared copper-tungsten-molybdenum "weld" was sealed through the pyrex wall.

EXPLANATION OF PLATE II

View of the equipment used in this work.

PLATE II



Prior to the sealing through the glass of the "welds" an electrode was attached to each of them, directly in the case of the grounded electrode and to a movable glass holder in the case of the high-voltage one. The electrodes were coiled-coil oxide-coated tungsten electrodes obtained from new commercial 40 watt instant starting fluorescent lamps. These electrodes were then spot welded to the "welds", care being exercised not to contaminate them by touching them with the fingers during this process.

The inter-electrode distance was made variable by means of the movable high voltage electrode. The arrangement is shown in Plate III. The movable electrode was supported by passing its lead wire through a hollow glass cylinder with the ends sealed off. This glass cylinder was of such a size that it could just slide within the discharge tube. This cylinder had within it a glass-jacketed rod of iron. Thus the glass cylinder could be moved by an external magnet.

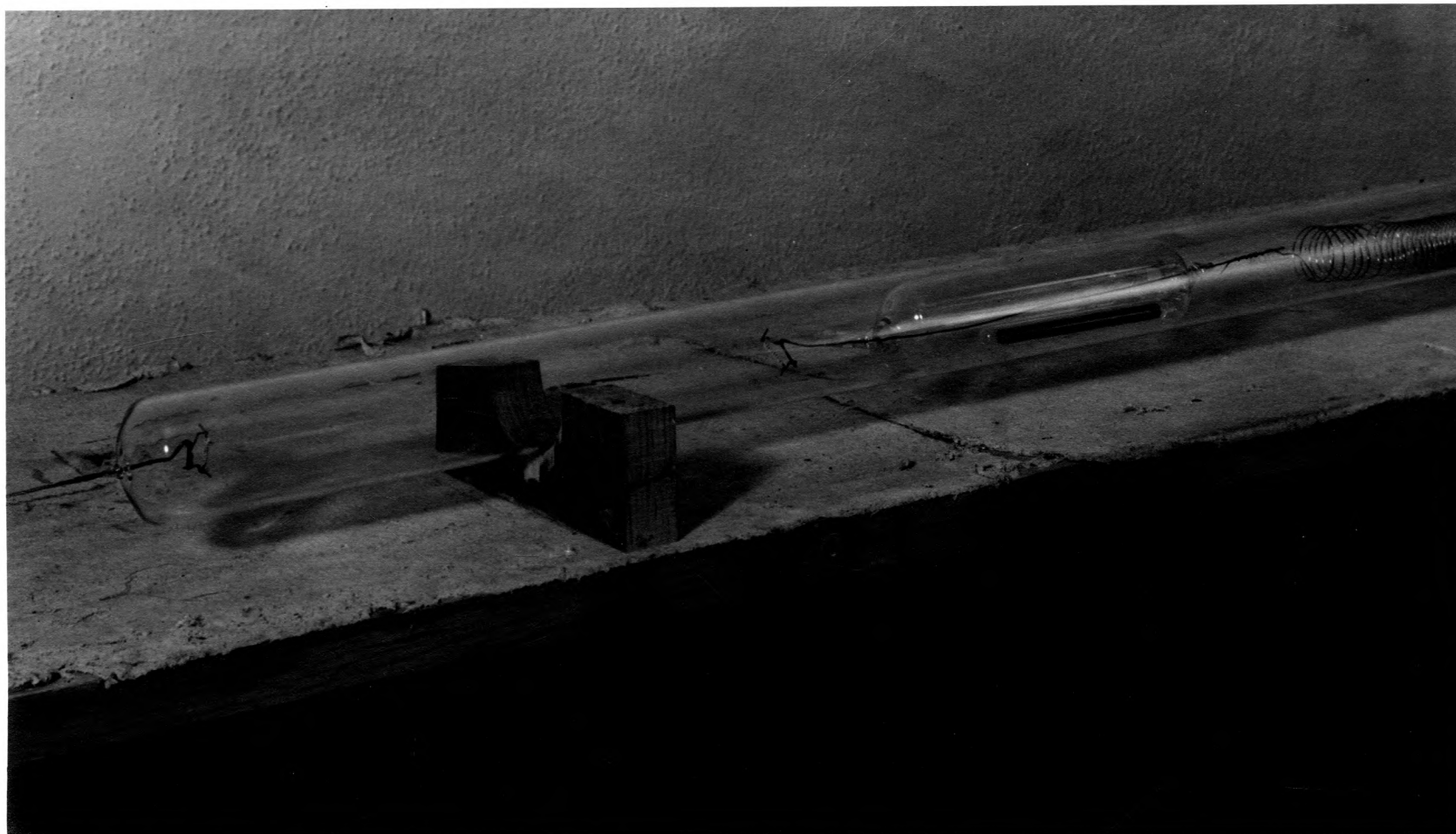
Electrical contact was established between the movable electrode and the molybdenum "weld" by means of a coiled nichrome wire spring. The restitutional force of the spring was sufficiently small to allow the movable electrode to remain where it was placed after the magnet was removed. The resistance of the nichrome wire was about 30 ohms.

It was of the utmost importance to exclude all impurities in this work and for this reason outgassing became quite important. The discharge tube was outgassed by evacuation and subsequent baking at 400° C for one hour, still under vacuum. Baking could not be used on the rest of the system as it contained greased stopcocks. However, it was outgassed by the use of a tesla discharge coil until it reached stability. The entire system was then "flushed out" several times with argon gas.

EXPLANATION OF PLATE III

Close up view of tube showing electrodes and moving mechanism.

PLATE III



When the system was thoroughly outgassed, a water jacket was placed around the discharge tube. This water jacket consisted of a tube somewhat larger than the discharge tube with water flowing through it. The purpose of this was to carry off the heat generated by the discharge and maintain reasonably constant temperature at the walls of the discharge tube. It, also, acted as an electrical ground in starting the lamp. A constant temperature was necessary so that the pressure of the mercury vapor within the tube remained constant. Thermometers were located at the water inlet and outlet so that the increase in temperature could be determined. The inlet temperature was 22°C and the outlet temperature was 22.7°C . The maximum temperature variation for each was 0.5°C .

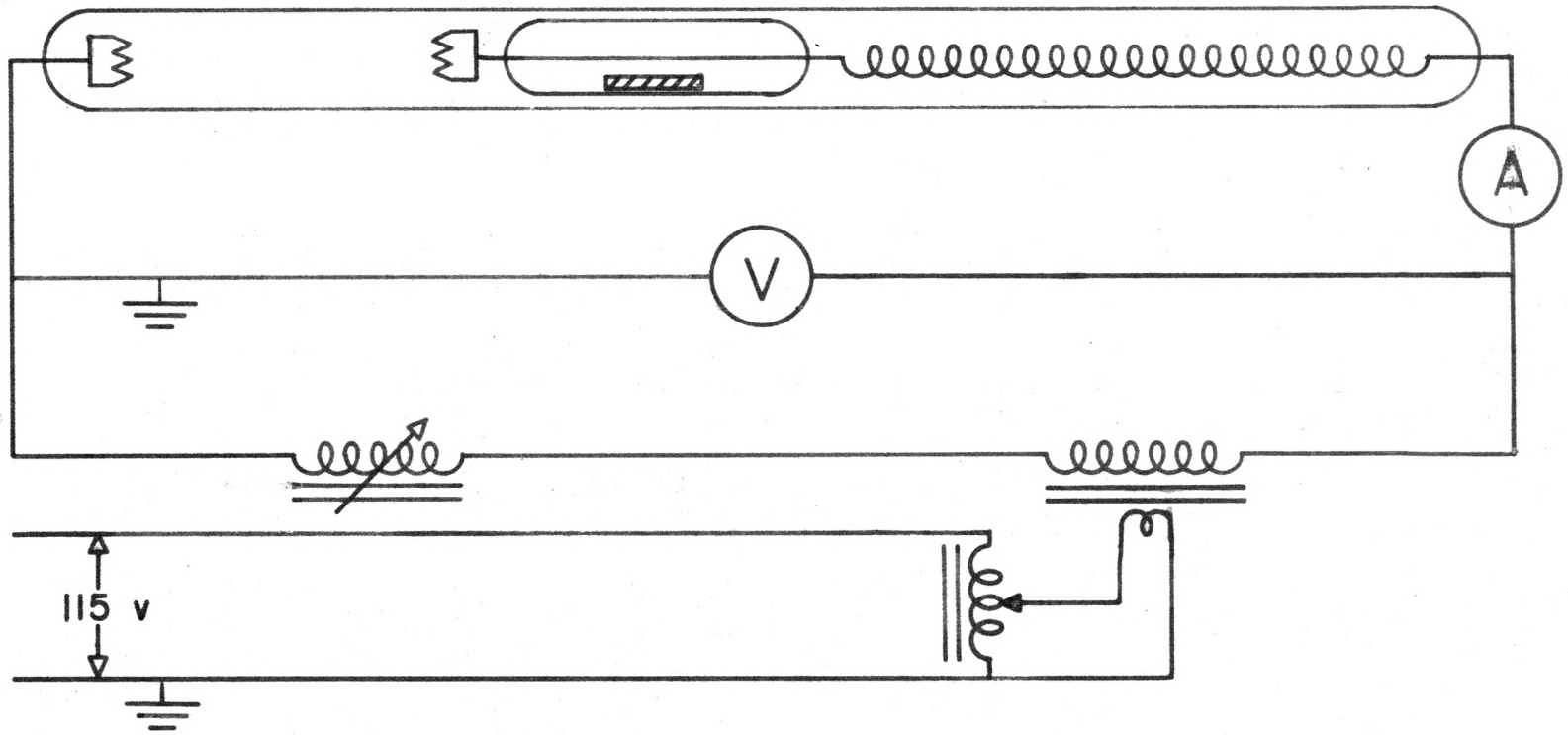
The discharge gas consisted of argon and mercury vapor. Only mass-spectroscopically pure argon gas was used. The mercury vapor came from a few drops of triply distilled mercury introduced into the discharge tube. According to the manufacturer's specifications this mercury contained less than 0.01 percent impurities.

An electrical circuit diagram is shown on Plate IV. A variable auto-transformer gave alternating voltages of from 0 to 135 volts. This variable output voltage was increased uniformly by a motor which turned the control knob at the rate of $2\frac{1}{2}$ volts per second (one revolution per minute). The auto-transformer output voltage was impressed across a transformer with a 9.5 to 1 voltage ratio. The output of this transformer was then impressed across the series circuit consisting of the ballasts, ammeter, and discharge tube. There was an A.C. RMS voltmeter across the tube and also a high resistance voltage divider so that the voltage wave form across the tube could be viewed on the screen of an oscilloscope.

EXPLANATION OF PLATE IV

Schematic diagram of electrical circuit used.

PLATE IV



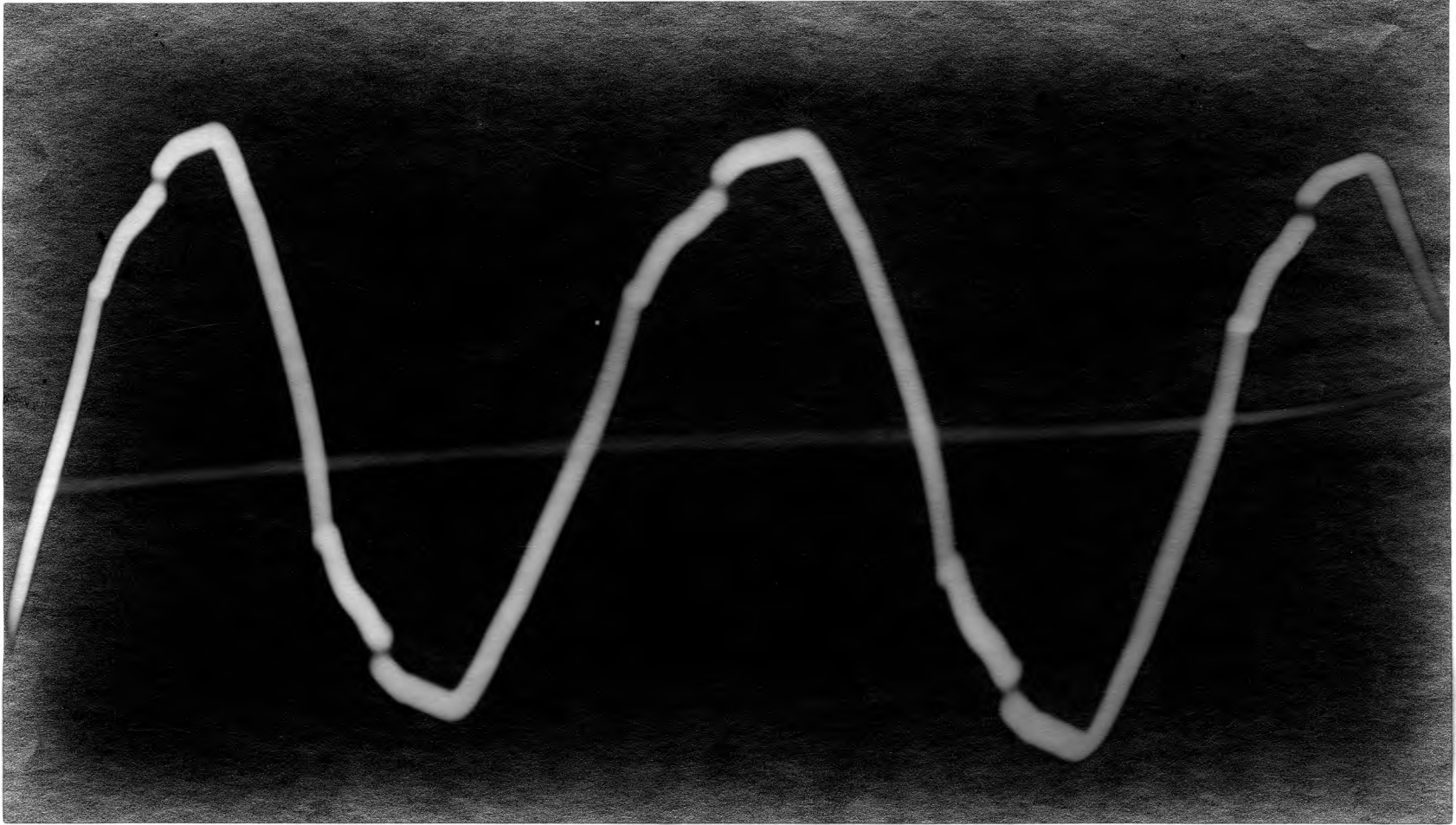
The voltage just before the glow breakdown had the form of a sine wave. Therefore the actual instantaneous peak potential difference between the electrodes at the glow breakdown point was the meter voltage multiplied by 1.414. However, after glow breakdown, and just before arc breakdown, the voltage form departed from that of a sine wave. This wave form is shown in Plate V. The actual peak voltage necessary to cause breakdown is, therefore, slightly different than 1.414 times the meter voltage. However, in this paper all of the voltages given are meter voltages read from a root-mean-square meter.

The procedure for taking the data was as follows: After thorough outgassing, the stopcock to the pumps was closed and argon at a pressure of 15 mm of Hg was introduced into the system from the argon flask. The movable electrode was set at 80 cm. After aging of the gas, the voltage across the discharge tube was increased uniformly by the motor driven auto-transformer. At the glow and arc breakdown voltages, the voltmeter would drop and these voltages were recorded. The measurements were repeated several times at the same pressure and distance. Then the distance was decreased in steps of 10 cm and the experiment repeated at electrode distances down to 10 cm. This process was then repeated at argon pressures of 10, 8, 6, 4, and 2 mm of Hg. After this the system was pumped out and argon at a pressure of 15 mm of Hg again was introduced into the system, and the whole set of readings repeated.

EXPLANATION OF PLATE V

Voltage wave form across the tube just before arc breakdown.

PLATE V



EXPERIMENTAL RESULTS

The experimental results of this work are presented in Plates VI and VII.

Plate VI shows how the glow breakdown voltages varied with $p \times d$ (the product of the pressure and distance) provided the pressure remained constant. The curves shown are each an average of two separate trials. Each point of each trial was an average of at least three individual readings. Therefore, each experimental point of plate VI represents an average of at least six individual measurements. The deviation from the mean was of the order of 10 volts, although in a few cases (only for high $p \times d$) it ran as high as 30 volts.

Plate VII shows how the arc breakdown voltage varied with $p \times d$, the pressure being held constant for each curve. The deviations from the mean were of the same magnitude in this case as in the case of the glow breakdown.

DISCUSSION OF RESULTS

Glow Breakdown

Paschen's law for the least complex type of glow tube, would indicate that all of the points of Plate VI should lie, within experimental limits, along the same curve. Therefore it seems that Paschen's law is not completely valid under the conditions at which this work was performed.

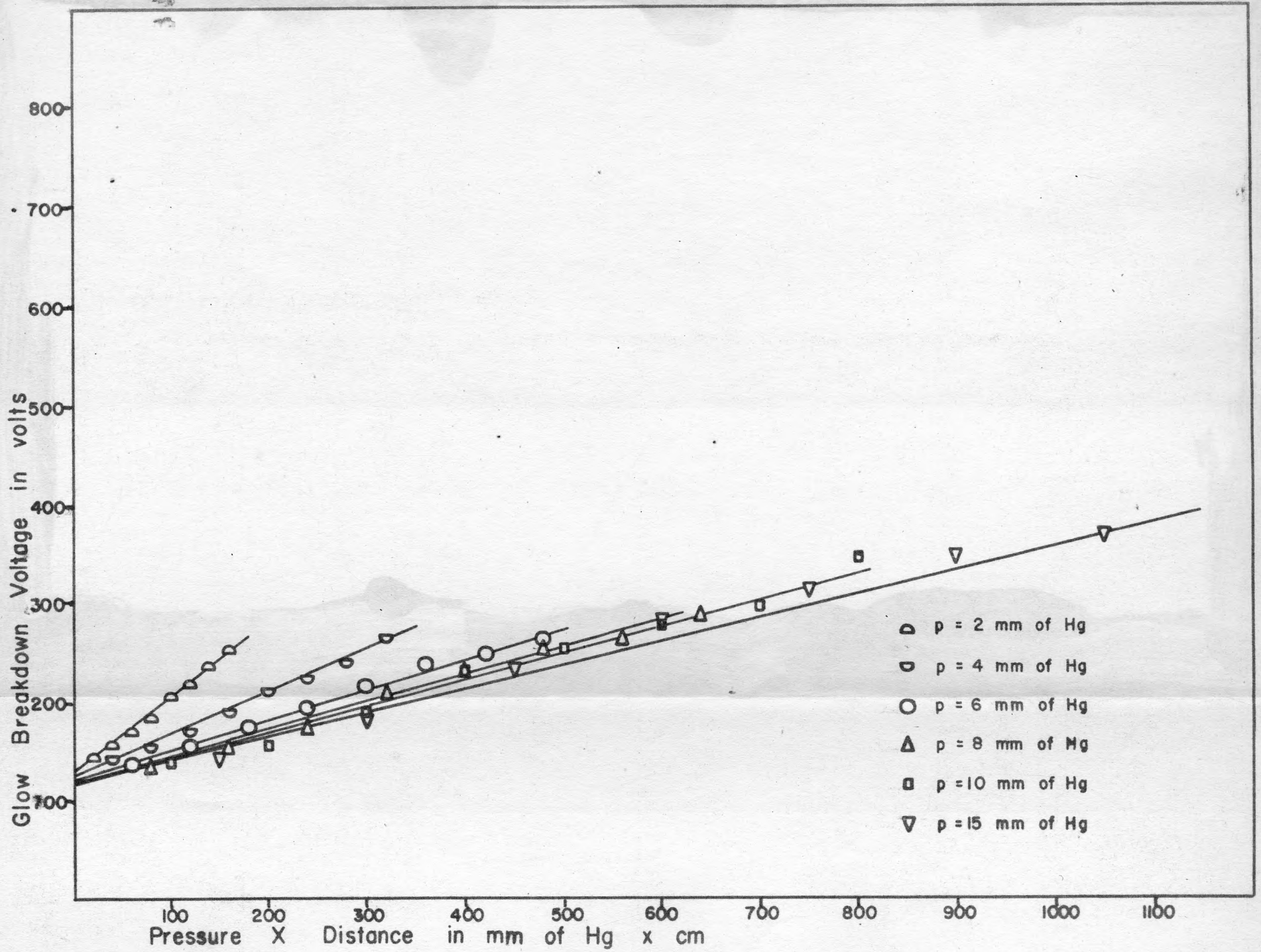
It was observed that these curves closely approximate a family of straight lines with a common intercept. The equation of such a family would be

$$E_g = a + b \times (p \times d)$$

EXPLANATION OF PLATE VI

Graph showing glow breakdown voltage as a function of pressure times distance with pressure held constant for each curve.

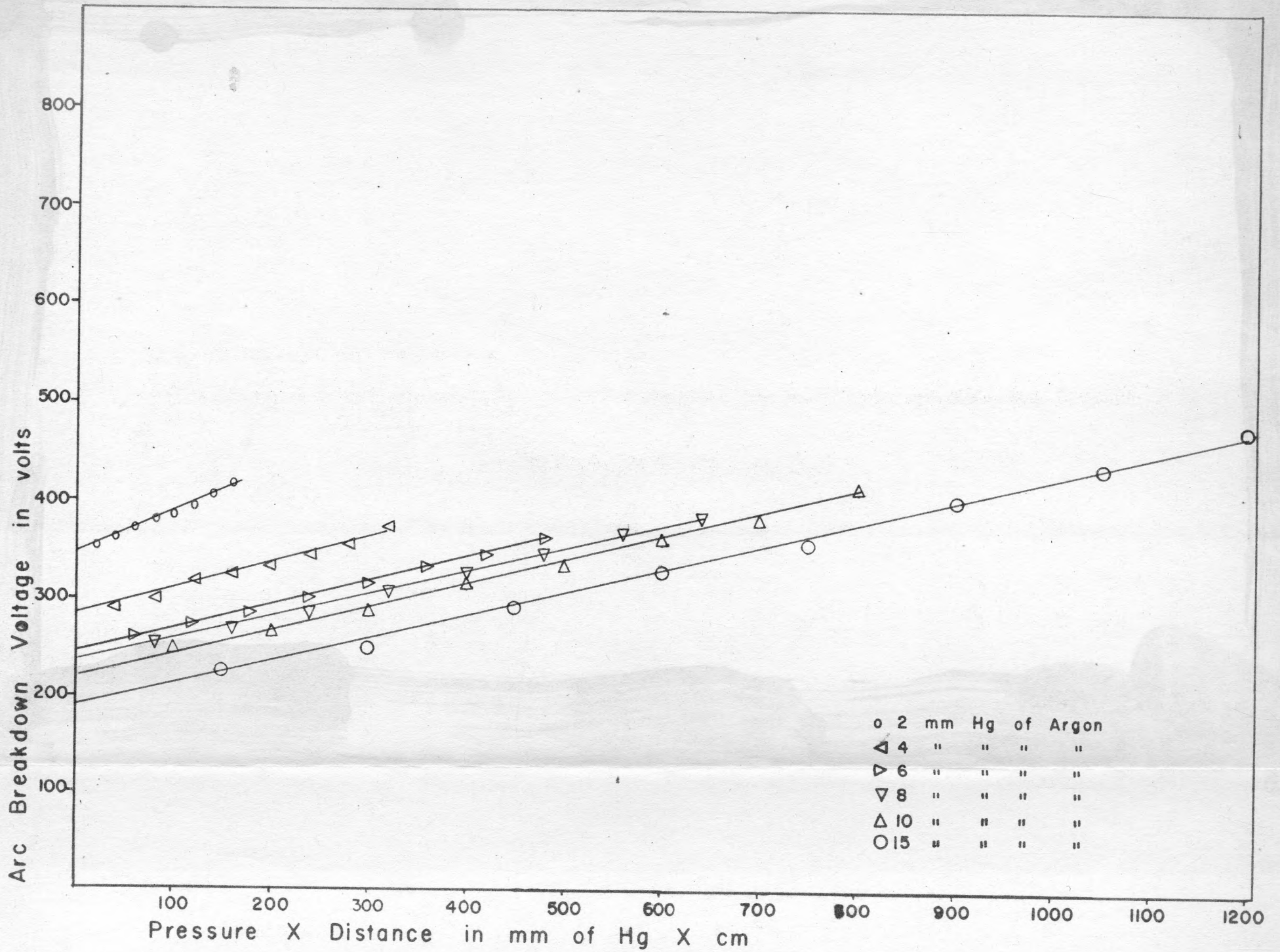
PLATE VI



EXPLANATION OF PLATE VII

Graph showing arc breakdown voltage as a function of pressure times distance with pressure held constant for each curve.

PLATE VII



where a is a constant for the family and b a constant for each line of the family.

The constant a could be read to two significant figures, directly from Plate VI. It was found to a rough approximation, to be 120 volts.

The constant b , the slope of each of these curves, has different values. Since the individual curves were plotted with the pressure held constant, b must be a function of pressure.

Plate VIII shows how b varies with pressure. On the same plate the equation

$$b = 1.9 + \frac{.68}{p-1}$$

is plotted. This equation seemed to represent the experimental data closely.

On this basis the equation for glow breakdown voltage as a function of $p \times d$ could be written as:

$$V_g = 120 + (1.9 + \frac{.68}{p-1}) p \times d.$$

Obviously this empirical relationship becomes nonsensical at pressures lower than those measured in this experiment. However, this objection is not too serious if one keeps in mind that the equation is meant only to describe the situation at pressures of from 2 to 15 mm of Hg.

Further interpretation of these results could be made in terms of the manner in which such an elongated gaseous discharge tube is observed to break into a glow. Initial currents are observed to flow between the high voltage electrode and the grounded water shield (2 cm distant) at voltages of the order of 120 volts. Previous work (McFarland, 3) has shown that this discharge spreads down the tube as the applied potential difference is increased until the discharge exists between the two electrodes.

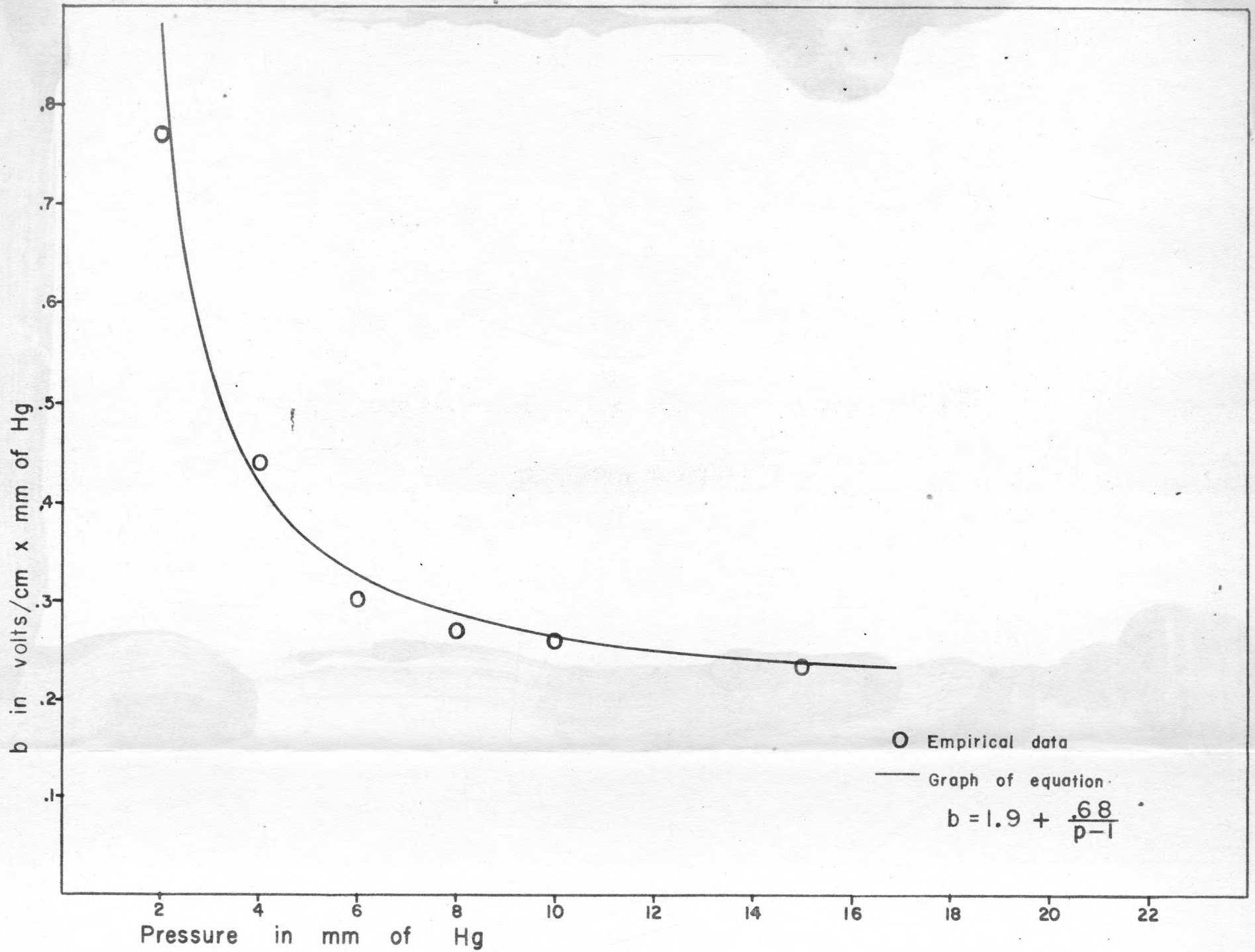
In terms of the relationship

$$E_g = a + b \times (p \times d)$$

EXPLANATION OF PLATE VIII

Graph comparing experimental data with the equation $b = 1.9 + \frac{.68}{p-1}$.

PLATE VIII



a may well be considered to be only relatively constant. In all probability a itself is a function of $p \times d'$ where d' remains constant and is the 2 cm distance mentioned above.

Thus a more general relationship,

$$E_g = f(p \times d') + \frac{(1.9 p - 1.22)}{p - 1} \times (p \times d)$$

may exist where $f(p \times d')$ is approximately 120 volts for the values of p and d' used in this experiment, d' is the constant distance between the high voltage electrode and the nearest grounded surface, and d is the distance between the electrodes. The results shown in Plate VI are somewhat unreliable in this region but would tend to indicate that $f(p \times d')$ decreases with increased pressure. Further work will be done in an attempt to clarify the details of this function, but it seems entirely possible that this may be closely related to the Paschen's law function.

Arc Breakdown

The curve of Plate VII indicates a dependence of arc breakdown potential on $p \times d$ under the experimental conditions of this work. An attempt was made to obtain a mathematical expression for the arc breakdown voltage.

To a first approximation the curves of Plate VII were linear and parallel. Thus it was assumed that the arc breakdown voltage consisted of the sum of two terms. The first term was an indication of the linear dependence of the breakdown voltage upon the product of the pressure and arc length. The second was a corrective term needed to shift the curves into juxtaposition. Written mathematically, this becomes

$$E_a = f_1(p \times d) + f_2$$

Here E_a is the arc breakdown voltage, $f_1(p \times d)$ a "Paschen's law" function (linear in this range of $p \times d$), and f_2 the corrective function.

An attempt to identify the function f_2 led to the conclusion that f_2 was a function of p . This meant that:

$$E_a = f_1(p \times d) + f_2(p).$$

Several plots of arc breakdown voltage as functions of pressure are shown in Plate IX. Here $p \times d$ was held constant for each of the curves. These curves show the dependence of E_a on pressure for constant values of $p \times d$. Certainly their most striking feature is the indication that for a given $p \times d$, the arc breakdown potential decreases with increasing pressure.

Again, curves of Plate X exhibit this pressure distance relationship in still another manner. Here d is held constant; and with increasing pressure, the striking potential is plotted versus $p \times d$. Particularly at small electrode distances, E_a is decreased by increasing p . However, at greater arc distances, E_a is shown to increase with p .

A possible explanation of this effect is as follows:

The potential across the glow discharge, the immediate precursor of the arc, may be divided so that

$$E_a = E_k + E_p$$

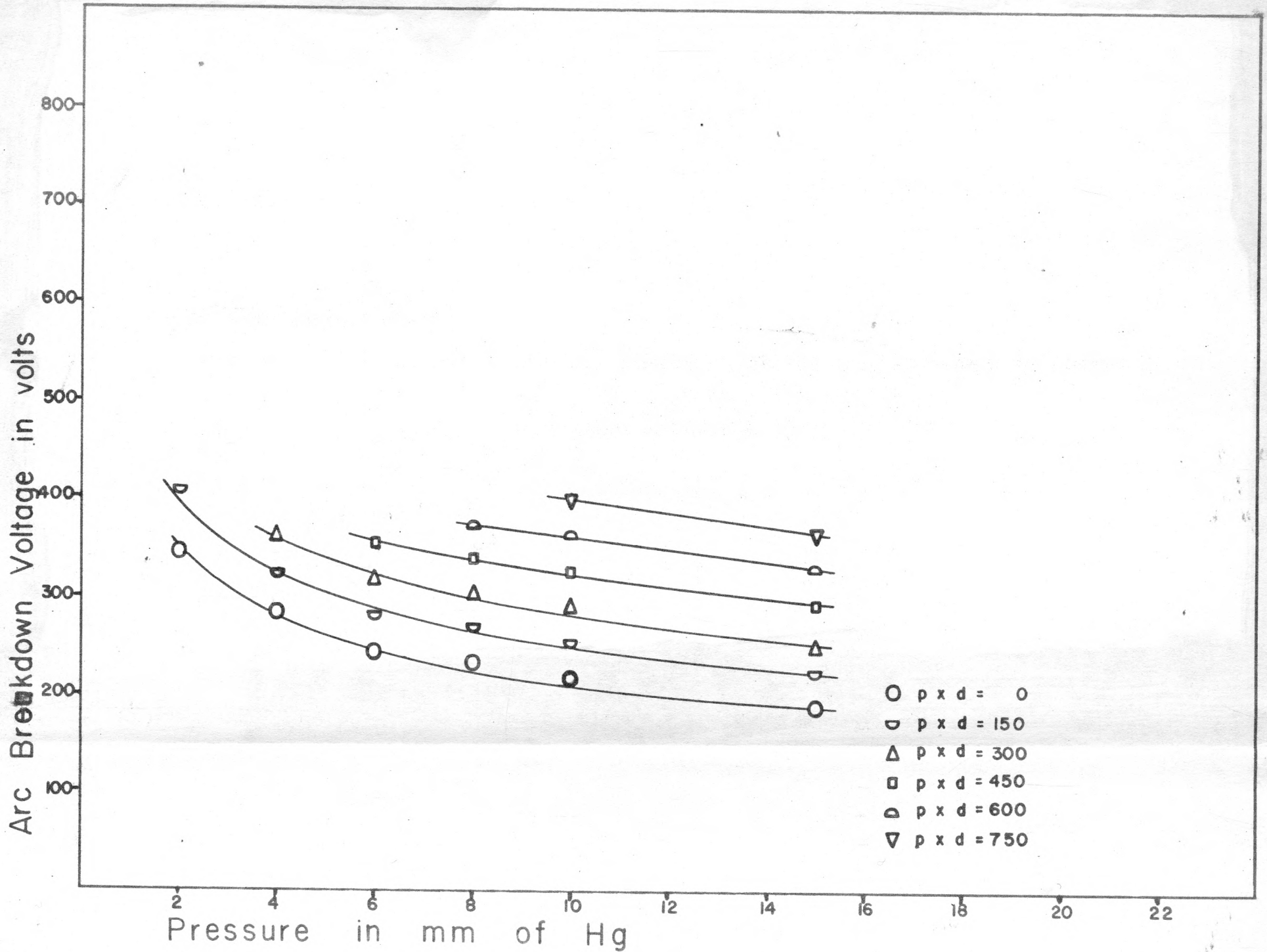
where E_k is the cathode drop and E_p is the drop across the positive column. Since the impedance of an arc is largely resistive and the current the same throughout, the power expended in these two parts would be proportional to these potential drops. The energy due to E_k would be expended largely in producing electron emission through ion bombardment and high field effects. The energy expended in the positive column would result in ion and excited atom formation. These in turn would affect the electronic emission at the cathode.

An increase in pressure would result in decreased mean free electronic paths in the positive column, and in a lower probability of production of

EXPLANATION OF PLATE IX

Graph showing arc breakdown voltage as a function of pressure with pressure times distance held constant for each curve.

PLATE IX



EXPLANATION OF PLATE X

Graph showing arc breakdown voltage as a function of pressure times distance with distance held constant for each curve.

excited and ionized atoms. To restore this probability both E_p and E_a would have to increase. This was observed to happen at large values of d and the larger values of $p \times d$.

In order to produce a competitive effect resulting in lowering E_a with increasing p , one must consider what happens in the cathode drop distance, d_k . It is known that in a glow discharge the potential drop at the cathode, E_k , is independent of the pressure and is slightly less than the minimum breakdown potential for the gas and electrodes used. Further, it is known that the length across which the cathode drop is applied, d_k , adjusts itself so that $p \times d_k$ remains constant and equal to the value of $p \times d$ corresponding to a minimum breakdown potential for the glow discharge involved. Also the electron current density, J , due to E_k is given by (4):

$$J = K \frac{E_k^{3/2}}{d_k^2} = K' E_k^{3/2} p^2.$$

Thus the effect of increasing pressure is to increase the cathode emission. If a given level of emission is assumed to be necessary to cause a glow to change to an arc, increasing the pressure would increase the emission due to E_k and reduce the necessity for contribution to cathode emission from those activities controlled by E_p . This would lead to a lowered E_p and E_a .

To return then to:

$$E_a = f_1(p \times d) + f_2(p),$$

$f_1(p \times d)$ must correspond to a linear dependence on $p \times d$ while $f_2(p)$ would appear to be an inverse function of p^2 . Actually, because of the complexity of the dependence of breakdown potential on pressure, the function $f_2(p)$ has not been adequately evaluated at this time. Its variance with pressure, however, may be observed in Plate IX.

A very rough fit of the present data may be had with the following relationship:

$$E_a = 190 + .235(p \times d) + 80 \left(\frac{1}{p}\right)^{3/2}.$$

CONCLUSION

As stated in the introduction, an attempt was made to describe how the breakdown voltages varied with pressure, distance, and the product of the two.

In the case of the glow breakdown the following approximate equation was presented as a possible description:

$$E_g = 120 + (1.9 + \frac{.68}{p-1}) pxd \text{ volts}$$

The 120 volt term is presumably a function of the pressure and the dimensions of the discharge tube used but for the conditions of this experiment was a constant.

For arc breakdown a corrective term was postulated which was to be added on to a "Paschen's law" term. This term, f_2 , was found to be a function of the pressure and was illustrated by any one of the curves of Plate IX. The "Paschen's law" function, $f_1(p \times d)$, was considered a linear function of $p \times d$ in this range of pressure and distance. Or

$$E_a = A \times (p \times d) + f_2(p).$$

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The objective of this experiment was to describe how the glow and arc breakdown potentials varied with pressure and electrode distance in a gaseous discharge tube. This was accomplished by using a discharge tube with a movable electrode so that distances of from 10 cm to 80 cm could be obtained. The tube was attached to a glass system so that argon pressures of from 2 mm to 15 mm of mercury could be used. Also, a few microns of pressure of mercury vapor was in the tube because of the presence of a few drops of mercury in it. Data was obtained by increasing the alternating voltage across the tube uniformly until breakdown occurred. The temperature was held constant throughout the experiment by surrounding the tube with a water jacket.

The results of the experimental data were observed to deviate slightly from the results expected by applying Paschen's law. This was explained by noting the great differences between Paschen's experimental conditions and the experimental conditions of the work reported in this paper. (Paschen used direct currents and distances of 1.5 cm and less).

In the case of the glow breakdown the following equation was suggested because it fit the empirical data:

$$E_g = f(p \times d') + (1.9 + \frac{68}{p-1}) p \times d$$

It was thought that discharge started between the ungrounded electrode and the grounded wall of the tube a distance d' away. Here $f(p \times d')$ was considered to be the "Paschen's law" function relating the pressure and the distance d' and was taken to be approximately constant in this experimental range and set equal to about 120 volts.

To describe the arc breakdown the following equation was suggested:

$$E_a = f_1(p \times d) + f_2(p).$$

Here $f_1(p \times d)$ was assumed to be the "Paschen's law" term and $f_2(p)$ a corrective term. This equation was supported by the experimental data and the graph of

the function $f_2(p)$ was illustrated empirically. Physically, the potential just before breakdown was considered to consist of two terms. Or

$$E_a = E_k + E_p$$

where E_k was the cathode fall potential and E_p was the positive column potential.

The energy expended due to E_k produced electron emission through ion bombardment and high field effects while that due to E_p produced ion and excited atom formation.

It was thought that increased pressure caused increased current density and increased the emission at the cathodes due to E_k . This would cut down the necessity of the contribution due to E_p and hence lead to lower E_a . On this basis the term $f_1(p \times d)$ was found to reduce to a constant $A \times (p \times d)$ while the function $f_2(p)$ was illustrated graphically.

