A STUDY OF THE ATMOSPHERIC WIND GRADIENT IN THE LAYER NEAR THE GROUND

by

NEIL PARKER WOODRUFF

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INTRODUCTION

A sensitive anemometer placed anywhere within a few feet of the surface of the earth shows that the motion of the air consists for the most part of a succession of gusts and lulls accompanied by rapid and irregular alterations in direction. This feature, immediately obvious to anyone watching smoke from a chimney or ripples passing over a field of Kansas wheat in summer, is more than a meteorological curiosity. The unsteadiness of the wind has much to do with the shape of life as we know it, for it is this property which largely controls such apparently dissimilar phenomena as the warming of the atmosphere near the surface of the earth, the evaporation of water from land and sea, the scattering of pollen and the lighter seeds, and the removal of pollution from the air above the great industrial cities.

The systematic study of atmospheric turbulence, i.e., of the eddying of the wind as a diffusing process, is of recent origin and all major developments have occurred within the last 30 or 40 years. To a large extent the subject has advanced with the study of aerodynamics and any systematic account must include some basic background of fluid motion theory.

A study of atmospheric turbulence, particularly as it pertains to the gustiness and unsteadiness of the wind near the ground, becomes of even more importance in regions where intensive agriculture is carried on. This is especially true in areas having arid and semi-arid climates where periods of drought are often accompanied by serious wind erosion. Fundamental knowledge of the many factors which affect the wind is basic to a solution of these problems.

Some of the factors which are of particular importance from the agricultural standpoint are: (1) effects of the immediate ground surface on the average shearing force of the wind, (2) effects of the shearing forces associated with gustiness of the wind, and (3) effects of atmospheric stability on the shearing force of the wind.

An evaluation of the effects of these factors presents a complicated problem. For example, consider the problem of determining the average shearing force of the wind over any common agricultural area. Physically, it is impossible to measure the shear of the wind with any degree of accuracy; therefore it is necessary to approach the problem in a round-about manner by use of mathematical equations based on theory.

There have been several theories postulated with the object of solving this problem. Nearly all of them are based on the assumption that the shearing force of the wind is associated with the velocity gradient as determined by measuring the velocity of the wind at various heights above the ground. Using this basic assumption, Prandtl (Brunt (3)) and von Kärmän (24) have developed logarithmic laws and equations relating the shear to the velocity of the wind and the height above the ground. It has been found that these equations give reasonable values for the average shear of the wind.

The development of methods and procedures for determining the average shear is but one of the problems common to research on wind phenomena. There is also the problem of determining whether shear is constant for surfaces of varying roughness. Bagnold (1) has assumed that the shearing force of the wind is due to meteorological forces higher up and is not greatly affected by local conditions near the surface. Sheppard (18) has indicated increased shear with increasing roughness. Another problem of primary importance is that of determining extremes in the shearing forces caused by fluctuations of the wind. Still another problem is the possible effect of atmospheric stability on the wind gradient. Several investigators have shown that there is considerable variation in the wind structure, depending upon the condition of the atmosphere at the time measurements are made. Since the relationship of wind velocity and height is involved in nearly all analytical and theoretical considerations of atmospheric turbulence, it is important that the true basic relationships for all conditions be known.

The problems mentioned here are a few of those related to the complex phenomena known as "atmospheric turbulence." They are the problems which have a direct bearing upon wind erosion. This study was initiated to obtain information on those phases of the problem concerned with wind erosion in an agricultural area. It was not designed to secure information on large scale meteorological phenomena.

The results reported here are not conclusive in many instances, and a definite need exists for further rigorous investigation. They do, however, present leads for possible future lines of investigation of a complex problem which is not understood thoroughly.

OBJECTIVES OF THE STUDY

The objectives of this study are: (1) to review and summarize previous work on the subject, (2) to determine the effect of various crops and tillage methods on the average shearing force of the wind, (3) to study and describe the velocity fluctuations of the wind, (4) to determine the possible effect of various crops and tillage methods on velocity fluctuations and associated shear, and (5) to determine the effect of atmospheric stability on the wind gradient near the ground and to describe these effects in terms of functional relationships for each of three atmospheric conditions, i.e. stable, unstable, and inversional.

THEORY

General Atmospheric Conditions

The condition of the atmosphere in the layer near the ground is usually described or defined in terms of the diurnal variation of the temperature gradient. Perhaps the best way to describe and define the various possible atmospheric conditions is to consider a typical day. Soon after the sun rises, the incoming radiation starts to raise the temperature of the ground, and the air at the lower levels becomes warmer than that at greater heights. As the sun ascends, the rate of decrease of temperature with height, or lapse rate, increases, reaching a maximum at approximately 1:00 p.m. As the sun descends, the lapse rate decreases until approximately an hour before sunset at which time there is a very short period of zero lapse or an isothermal condition. After sunset, the ground loses heat rapidly because of the escape of long wave radiation to space; the air near the ground becomes colder than the air above, and the so-called inversion appears in which temperature increases with height. This condition persists throughout the night until dawn, at which time the whole cycle begins again.

The variation of the temperature gradient indicates and limits the turbulence present in the atmosphere. A large lapse rate implies that warmer and, therefore, less dense air lies below colder and, therefore, denser air. Such a disposition is favorable to the formation of convection currents, and any mass of air displaced slightly upward will continue to rise because of the difference in density between it and its surroundings. This condition is favorable to the growth of turbulence and is known as a state of instability. Large lapse rates are usually accompanied by vigorous oscillations of the wind. An inversion, on the other hand, means colder and denser air lying below warmer and less dense air. It represents a condition of stability in which disturbances tend to be damped out.

There are two criteria commonly used as a measure of the stability or instability of the atmosphere. One of the criteria defines the condition of the atmosphere in terms of the adiabatic lapse rate, Γ, which is equal to 0.0055° F. per foot and is the amount of decrease of the temperature per foot of rise in height above the ground surface. The atmosphere is said to be stable, unstable, or in neutral equilibrium if the rate of increase or decrease of the temperature with height is as follows:

$$\frac{dt}{dz} < \Gamma = \text{stable}$$

$$\frac{dt}{dz} > \Gamma = \text{unstable}$$

$$\frac{dt}{dz} = \Gamma = \text{neutral}$$

The second criterion defines atmospheric conditions in terms of a parameter known as Richardson's number. This number is

$$R_i = \frac{g}{T} \frac{\left[\frac{dt}{dz} - \frac{dt}{dzA}\right]}{\left(\frac{du}{dz}\right)^2} \tag{1}$$

where $\frac{du}{dz}$ = vertical gradient of wind velocity in feet per second per foot.

$\frac{dt}{dz}$ = vertical temperature gradient in degrees per foot.

$\frac{dt}{dzA}$ = adiabatic lapse rate = 0.0055° F. per foot.

g = gravitational constant = 32.2 feet per second per second.

T = average temperature at arbitrary height in degrees Rankine.

The atmosphere is said to be stable, unstable, or in neutral equilbrium as follows:

$$R_i = + = \text{stable}$$

$$R_i = - = \text{unstable}$$

$$R_i = 0 = \text{neutral}$$

The Velocity Profile

It has been found that, in general, the relationship between wind velocity and height may be expressed by using either a power or logarithmic law. The law applied depends upon the condition of the atmosphere at the time measurements are made and, to some extent, on the preference of the investigator.

There are several modifications of both the log and power laws. All of them are based, however, on Prandtl's (Brunt (3)) mixing length, ℓ, which is the mean length traversed by turbulent elements before mixing completely with their surroundings. Prandtl's theory (Brunt (3)) begins with the Reynolds' equation for horizontal shearing stress,

$$\tau = -\rho u' w' \tag{2}$$

where τ is the horizontal shearing stress, ρ is the mass density of the fluid, and u' and w' are the fluctuation eddy velocities in the X and Z planes, respectively. The mixing length concept is then introduced to formulate an expression for shear which is

$$\tau = -\rho \, \ell^2 \left(\frac{du}{dz}\right)^2 \tag{3}$$

Von Kårmån (24) then made the assumption of an identical pattern of turbulent flow throughout the field and related ℓ with the field of mean velocity by

$$\ell = k \frac{\frac{du}{dz}}{\frac{d^2u}{dz^2}} \qquad (4)$$

where u is velocity at height z, k is a dimensionless constant, and τ and ρ have the same definitions as for equation (2).

Equations (2), (3), and (4) have been the basis for almost all the expressions for the shearing force of the wind.

There are two ways by which equations (3) and (4) may be solved for the shearing force of the wind. One way is by substitution of equation (4) into equation (3) to obtain the following expression for shear:

$$\sqrt{\frac{\tau}{\rho}} = -k \frac{\left(\frac{du}{dz}\right)^2}{\frac{d^2u}{dz^2}} \qquad (5)$$

Integration of this expression yields the following basic equation for velocity and shear:

$$u = \frac{1}{k}\sqrt{\frac{\tau}{\rho}}\ \ln\frac{z}{H} + C \qquad (6)$$

where H is a linear height of roughness, C is a constant of integration and the other symbols have the same meaning given previously. Prandtl (Brunt (3)) suggested that this equation could be extended to the atmosphere.

Nikuradse's experiments (Rouse (16)) in artificially roughened pipes to determine the constants in this basic equation resulted in a value of k = 0.4 and C = 8.5. Sheppard (18) verified this value of k for the atmosphere in 1947 by measuring the drag of the wind by observing the deflection of a horizontal plate floating in oil. Substitution of Nikuradse's values of k and C into equation (5) and converting to the $\log_{10}$ yields,

$$u = 5.75 \sqrt{\frac{\tau}{\rho}} \log_{10} \frac{z}{H} + 8.5 \tag{7}$$

In place of H, a value known as z_o, the roughness parameter, or the value of z where u = 0 and usually found to be $\frac{1}{30}$ H, can be substituted into the equation, thus eliminating the constant and giving

$$u = 5.75 \sqrt{\frac{\tau}{\rho}} \log_{10} \frac{z}{z_o} \tag{8}$$

Equation (8) is the logarithmic law for the velocity gradient.

This same result can also be obtained by using the two basic equations (equations (3) and (4)) in conjunction with the equation expressing the relationship of velocity u and height z. This equation, if the data fit the logarithmic law, usually takes the form:

$$z = z_o e^{nu} \tag{9}$$

or one of the modified forms,

$$z = z_o e^{nu^P} \tag{10}$$

or,

$$z - c = z_o e^{nu} \tag{10a}$$

where z_o is the y intercept or point of zero velocity, u is velocity at elevation z, n is the slope of the curve, c is a constant related to base of measurement, and P is the exponent of the velocity u.

The resultant expression for shear, corresponding to equation (9) is the same as equation (8). The expression for equation (10) is:

$$u = 5.75\, P \sqrt{\frac{\tau}{\rho}} \log_{10} \frac{z}{z_o} + \left(\frac{1}{P} - 1\right) \tag{11}$$

and the expression for (10a) is:

$$u = 5.75 \sqrt{\frac{\tau}{\rho}} \log_{10} \frac{z - c}{z_o} \tag{12}$$

where the symbols are defined as before. Actual procedure and steps in the derivation of equations (8), (11), and (12) will be found in Appendices II and III.

The straight logarithmic relationship for shear (equation (8)) adapts itself well to simplification if an expression for the velocity at 30 times z_o is set up as follows:

$$u_{30z_o} = 5.75 \sqrt{\frac{\tau}{\rho}} \log_{10} \frac{30z_o}{z_o} \tag{13}$$

which gives

$$u_{30z_o} = 8.5 \sqrt{\frac{\tau}{\rho}} \tag{13a}$$

Introduction of proper conversion factors into (13a) will give the following expression for shear, where τ is in pounds per acre and u_{30z_o} is in miles per hour:

$$\tau = 3.03 \, (u_{30z_o})^2 \tag{14}$$

Sheppard (18) also suggested that τ might be expressed as a function of a drag coefficient similar to that used in aerodynamics. The basic drag relationship is generally written as:

$$\tau = \tfrac{1}{2} \, C_d \, A \, \rho \, u^2 \tag{15}$$

in which C_d is a variable coefficient of drag, A represents the projected area of the body on a plane normal to the direction of motion, ρ is fluid density, and u is velocity of fluid flow. If A is considered as being unity, equation (15) may be written as:

$$C_d = \frac{2\tau}{\rho u^2} \tag{16}$$

where C_d will necessarily be related to the level at which u is measured. Substitution of u as expressed in equation (8) in equation (16) will give,

$$C_d = \frac{0.0605}{(\log \frac{z}{z_o})^2} \tag{17}$$

Insertion of conversion factors into equation (16) gives the following expression for shear in pounds per acre, where u is the velocity in miles per hour at an elevation equal to z:

$$\tau = 109\ C_d\ u^2 \tag{18}$$

C_d may be evaluated from equation (17).

If the basic expression for the relationship of velocity u and height z fits the power law, i.e.,

$$u = u_1\ (\frac{z}{z_1})^n \tag{19}$$

where u is wind velocity at height z and u_1 is wind velocity at the constant reference height z_1, then procedures similar to those used to derive equations (11) and (12) will give the following expression for shear:

$$\tau = \rho\ (\frac{k\ n\ u}{1 - n})^2 \tag{20}$$

(See Appendix I for derivation of this expression.)

Power law distribution of wind velocity with height implies that the shearing force of the wind is not constant with height, and therefore the shearing force can be evaluated at any height from equation (20).

REVIEW OF LITERATURE

Numerous investigations of the velocity gradient of the atmospheric wind in the layer near the ground have been made in an attempt to express the relationships between wind velocity and height above the surface. Most investigators have used either a power or a logarithmic law to express this relationship. Sverdrup (21) used the logarithmic law for conditions which were not stable and a power law for conditions where the air was convectively stable. Kepner et al (9) used a version of Sverdrup's power equation in analyzing data obtained over a citrus orchard at night when there was a definite temperature inversion resulting in atmospheric stability. They reported that these data would be better represented under these conditions with the power law than with the logarithmic law. Sutton (20) suggested that the power law might be used as an approximation of the logarithmic profile, where the logarithmic law might lead to considerable mathematical difficulties. However, he was of the opinion that for conditions of adiabatic temperature gradient the logarithmic profile is followed very closely provided vegetative cover is not too high. Zingg[1], while making observations with a wind tunnel, found the velocity-height relationship could be represented by either a logarithmic or power law, depending on the base level from which measurements were made. Thornthwaite and Kaser (23) say the logarithmic law is valid only in the morning and evening when the thermal structure of the lower atmosphere is adiabatic, and it is not applicable when the air is convectively unstable. Halstead (7) also agrees with Thornthwaite and Kaser in that he states that the logarithmic law gives a good fit only under conditions of neutral equilibrium and does not fit records covering pronounced heating and cooling. He modifies the logarithmic equation by adding or subtracting a so-called stability term given as:

$$\frac{C}{T_o} \propto P\ (z_2 - z_1)$$

where $\propto$ is the lapse rate and T_o is surface temperature. The logarithmic equation then takes the form:

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[1] Zingg, A. W., "Some characteristics of the expanding turbulent boundary layer in a wind tunnel designed for the study of soil erosion." Soil Conservation Service Research, Manhattan, Kansas. 1949. (Unpublished)

$$u = 5.75\sqrt{\frac{\tau}{\rho}}\ \frac{z}{z_0} \pm \frac{C\infty}{T_0} \qquad (21)$$

The logarithmic law has been used most frequently by a majority of investigators. However, it is not always used in the straight forward manner given in the sections on theory and in the appendices. Rossby and Montgomery (14) developed a modification of the logarithmic law. They start with the assumption that the effect of a rough surface is to modify the mixing length near the ground. A value of z_0, or distance above the surface at which the velocity is zero, was introduced. They referred to this value as a surface depth constant and obtained, by assuming that the ratio $\frac{\ell}{z + z_0}$ is proportional to the constant k,

$$\ell = k\ (z + z_0) \qquad (22)$$

By defining $U_* = \sqrt{\frac{\tau}{\rho}}$ as a friction velocity and using the relationship in equation (3), it was reasoned that the velocity profile must satisfy the first order differential equation,

$$\frac{d\bar{u}}{dz} = \frac{U_*}{k(z + z_0)} \qquad (23)$$

which, upon integration and setting $\bar{u} = 0$ when $z = 0$ will yield

$$u = \frac{1}{k} U_*\ \ln \frac{z + z_0}{z_0} \qquad (24)$$

Sheppard (18) used the Rossby and Montgomery interpretation in his determination of τ, ℓ, and k near the ground.

Sutton (20) further modified the above equation when he introduced a quantity $N = V_* z_0$, which he called the macroviscosity. He believed the modification was necessary because in equations (8) and (24) as $z \longrightarrow 0$ neither approaches the smooth surface form. His final equation for the velocity profile applicable to all states of flow is:

$$\frac{\bar{u}}{U_*} = \frac{1}{k} \ln \frac{U_* z}{N + \frac{V_*}{9}} \qquad (25)$$

where V_* is the kinematic viscosity of the fluid.

The power law although not as extensively used as the logarithmic law is actually the simplest expression for the mean wind profile. Some investigators believe it can be used only in a shallow layer and that the accuracy increases with increasing height. It has been found also by some investigators that the index of the power law can be correlated with the magnitude and sign of the temperature gradient. Wind tunnel investigations have indicated a power law profile having an index of $\frac{1}{7}$ for smooth boundaries at relatively high Reynolds' numbers. This profile is known as the "7th root profile." One of the modifications of the power law has been made by Sverdrup (21). He used a term z_r, a roughness parameter, and obtained an equation of the form:

$$u = u_1 \left(\frac{z + z_r}{z_1 + z_r}\right)^{\frac{1 + n}{4 n}} \qquad (26)$$

He used this equation for a convectively stable atmosphere and found that if the temperature and velocity-distributions are similar n will have a value of 3. Kepner et al (9) used this type of equation in their measurements over a citrus orchard and found that when the velocities were < 3 miles per hour, the constant was approximately $\frac{1}{3}$.

Research as to specific effects of various crops and tillage methods on the wind profile has not been too extensive. Sheppard (18) made measurements over several types of surfaces and reported various values of z_0, τ, k, and C_d for these surfaces. Rossby and Montgomery (15) made measurements over various types of grassland and over the sea. Landsberg (10) also made measurements over the short grass of a baseball field and over sand dunes.

Paeschke (13) in 1937 in an analysis of profile over various natural surfaces (snow, grassland, cultivated land) used a displacement "d" similar to the factor used in this study. He took this "d" to be equal to the average of the measured heights of the roughness elements. More recently, Deacon (Sutton (19)) concluded that in conditions of neutral stability the logarithmic law can represent the profile between heights of 1 and 13 meters over grass of various lengths with great accuracy, provided both z_0 and d are chosen independently to give the best fit.

Studies regarding frequency distributions of velocity fluctuations are also rather limited. Hesselberg and Bjorkdal (Sutton (19)) found the distribution of eddy velocities follows a law similar to the Maxwell law for molecular velocities. Best (Sutton (19)) examined this relation by plotting the log of the number of fluctuations per unit range against the quantity $(u-\bar{u})^2$. The results show a close fit to the normal error law of frequencies. Cramer (4) followed this procedure and obtained a reasonably good fit to a straight line which indicates a normal frequency distribution. Other studies on velocity frequencies were reported in June, 1951 at the symposium on atmospheric turbulence in the boundary layer held at Massachusetts Institute of Technology. Many of these investigations were in the instrumentation stage at the time of the meeting and undoubtedly more information will be forthcoming.

DESCRIPTION OF EXPERIMENTAL EQUIPMENT

The velocity measurements used in those parts of the study confined to the lower 6 feet of the atmosphere, i.e., effect of crops and tillage methods and studies of velocity fluctuations, were made with combinations of two types of equipment: (a) Pitot tubes and a multiple manometer, and (b) whirling cup-type anemometers and a weather station recorder.

Two designs of the Pitot tube-manometer combinations were used. The type shown in figure 1 of Plate I was the one used in the early experiments and consisted of six Pitot tubes connected to a multiple inclined manometer for determining velocities. The Pitot tubes were mounted on a single staff at any height from 1 to 36 inches. A clamping device on the manometer made it possible to take five simultaneous readings at 10-second intervals. This equipment did not record and, therefore, it was necessary for two men to read and record velocities and operate the clamping lever. The other design of the Pitot-tube manometer combination is shown in figure 2 of Plate I. Two of these models were developed during the past year to fulfill a need for taking simultaneous readings over two surfaces at the same time. These instruments can be operated by one man and are capable of obtaining velocities at five heights. Each unit has a manometer and an improved clamping device which consists of a gang of five valves operated simultaneously from one lever. Readings were taken at one-minute intervals with these instruments.

The anemometer-recorder combination is shown in Plate II. The four anemometers are the Friez three-cup type giving an electrical contact with the passage of each $\frac{1}{12}$ mile of wind. These indications are recorded by a re-built multiple weather station recorder. The anemometers are mounted on an "A" frame and are adjustable within a vertical range of one to six feet above the surface. The anemometers were calibrated in the wind tunnel before being used in the field. Electrical current for operation of the recorder was taken from a 6-volt storage battery.

Corrections for the variation in the general level of wind velocity from one test to another were determined by placing the cup anemometer and

EXPLANATION OF PLATE I

Fig. 1. Pitot tube-manometer combination for measuring wind velocity.

Fig. 2. Modified design of Pitot tube-manometer combination for measuring wind velocity.

EXPLANATION OF PLATE II

The anemometer recorder combination for measuring wind velocity.

recording equipment shown in Plate III at a 2-foot elevation at a central location. Readings from this equipment were used as a base for adjusting the velocities to a comparable level.

Plate IV is a photograph of the installation used to obtain data on the effects of atmospheric stability. This equipment was located in an open bluestem pasture. The height of the grass averaged approximately 8 inches. Five Friez three-conical cup-type anemometers were mounted on the pole at heights of 2, 6, 12, 20, and 31 feet. Wind velocities were recorded on a re-built weather station recorder located in a small instrument house near-by. Measurements of temperature and humidity were obtained from hygrothermographs placed in small instrument shelters at elevations of 1, 6, and 28 feet. Wind direction and barometric pressures were also obtained for each test period. The instrument shelter at the top of the pole in the photograph was on a pulley system to facilitate chart changing.

PROCEDURE

Data used in this study have been collected at intervals during the past three years. Measurements were made during selected relatively "high" winds in the spring of 1949 and 1950 over varying crop and tillage surfaces. All the data with the exception of March 20, 1950 were taken near Manhattan. The time of taking data is confined to a period from 1 p.m. to 4 p.m. for each of ten days. Wind velocity readings were taken at various heights over five different growing crops and four different tillage conditions. Sites were selected where the effect of topography was a minimum, and the measuring devices were placed at heights where the velocity distribution was considered to be in equilibrium with the surface. These data were used to study the

EXPLANATION OF PLATE III

Low cup anemometer and recorder at central location.

EXPLANATION OF PLATE IV

A view of the permanent pole installation for measuring wind velocity, wind direction, temperature, relative humidity, and barometric pressure.

effect of crops and tillage methods on average shear, distribution of velocity fluctuations of the wind, and the effect of crops and tillage methods on velocity fluctuations.

The study was later expanded to obtain information on the effect of atmospheric stability on the wind gradient. In this connection, wind velocity, temperature, and humidity measurements were made from the stationary pole. A given "wind" was measured three or four times during the day. The time-interval for each set of records varied, but averaged three hours. Readings were taken at various times during a given day, the limits being from 8:30 a.m. to 10:30 p.m. Average wind velocities for a 30-minute period were determined from the anemometer records by counting the number of indications per unit of time. The velocity in miles per hour was then read from a calibration curve for the instrument, which had been determined at an earlier date in the laboratory wind tunnel. Temperatures for a corresponding 30-minute period were plotted from the hygrothermograph records versus time. The average temperature for each 30-minute period and for each elevation was then read from the graph. One hundred and seventy-three 30-minute profiles were obtained and processed for analysis in the above manner.

The effects, if any, of barometric pressure and humidity are not considered in this study.

RESULTS

Effect of Crops and Tillage Methods on the Average Shearing Force of the Atmospheric Wind

Data obtained for purposes of analysis of the effect of crops and tillage methods on the average shear of the wind are summarized in Table 1. Dates, average wind velocities and crops and tillage conditions are listed. All

Table 1. Mean wind velocity in miles per hour over natural surfaces as measured at various heights above the ground.[1]

Date	Type of surface	Height above the ground in inches																														
		1	2	3	4	5	6	7	8	9	10	12	13	15	16	17	18	19	21	22	24	27	30	31	35	36	41	42	46	48	70	72
April 15, 1949	Wheat		1.5		4.3				6.3						8.3								10.7									
	Plowed ground	2.5	4.5		6.6				8.5						10.0								11.6									
	Plowed ground	3.0	5.1		7.1				8.6						10.1								11.8									
	Alfalfa	.85	1.2		4.0				7.2						9.4								11.8									
	Alfalfa	.40	.72		3.2				6.4						8.4								10.5									
May 3, 1949	Wheat						4.3	5.9		8.6			11.8						14.4							17.5						
	Plowed ground						12.7	13.4		14.5			16.1						17.5							19.4						
	Alfalfa											1.3	2.9	5.8				9.9				13.1						16.4				
	Short grass						11.3	12.0		12.9			14.6						15.8							17.5						
	Short grass		8.0	9.7		11.5				13.9						16.2								18.3								
February 27, 1950	Short grass											8.4									10.2									11.4		12.1
March 1, 1950	Short grass											10.7									11.7									13.0		13.9
March 6, 1950	Wheat										17.7										21.1									24.2		25.9
	Corn stalks										12.7										17.2									22.1		24.8
March 7, 1950	Lister ridges										16.8									21.4									25.1		27.3	
	Alfalfa stub.											18.6									22.5									25.9		27.8
March 20, 1950	Fallow ground	14.3						19.7										22.9						24.7								
	Fallow ground	12.2						17.4										20.8						23.0								
	Fallow ground	11.2						15.8										19.0						21.6								
March 24, 1950	Past. sorghum											12.5									14.9									16.5		17.4
	Stubble mulch											10.1									13.7			16.3								18.2
April 10, 1950	Wheat			5.6			12.4					17.5					20.1				22.0					23.7						
	Lister ridges			6.8			16.1					20.6					24.4				25.4					25.6						
April 23, 1950	Lister ridges		5.6			7.6					9.9						11.3				11.9				12.9							
	Wheat								2.9		6.5					9.6					11.3		12.4				13.7					

[1] Each velocity shown is an average of 20 to 50 individual readings, depending on the gustiness of the wind.

these data, as was stated previously, were obtained from selected relatively high winds with the time for taking data ranging from 1 p.m. to 4 p.m. These conditions would be favorable for unstable atmospheric conditions, indicating that the logarithmic law might be more applicable. For this reason and other reasons stated previously in this thesis, the average shear was determined by use of the logarithmic approach.

Several of the modifications of the logarithmic laws were applied to the data obtained in this study. When equations (24) and (25) were applied to the very rough surfaces encountered in the field it was found that values of z_o and $N = V_* z_o$ were small in comparison to the linear roughness. They could be disregarded, therefore, without appreciably changing the relationship. Conclusions were that it would be necessary to use an equation of the type given in equation (12). The equation may be used in this form, or an alternate method of obtaining the same result is to subtract the amount C or z_ℓ and then use the original von Kármán equation as defined by Nikuradse (Rouse (16)) (equation 8). The latter method was chosen since it offers a simpler mathematical solution.

Mathematical interpretation of equation (8) indicates that the velocity is proportional to the logarithm of the height. It therefore follows that if the velocity distribution with height is logarithmic in nature, a plot of velocity against log of height should yield a straight line on semi-log graph paper. Consequently, the first step in analyzing the data was to test the validity of this hypothesis. This was accomplished by determining an average velocity for the duration of each test for each height. The average velocity was then plotted against the logarithm of the height as measured from the ground. Figure 1 shows several of these curves for the various days.

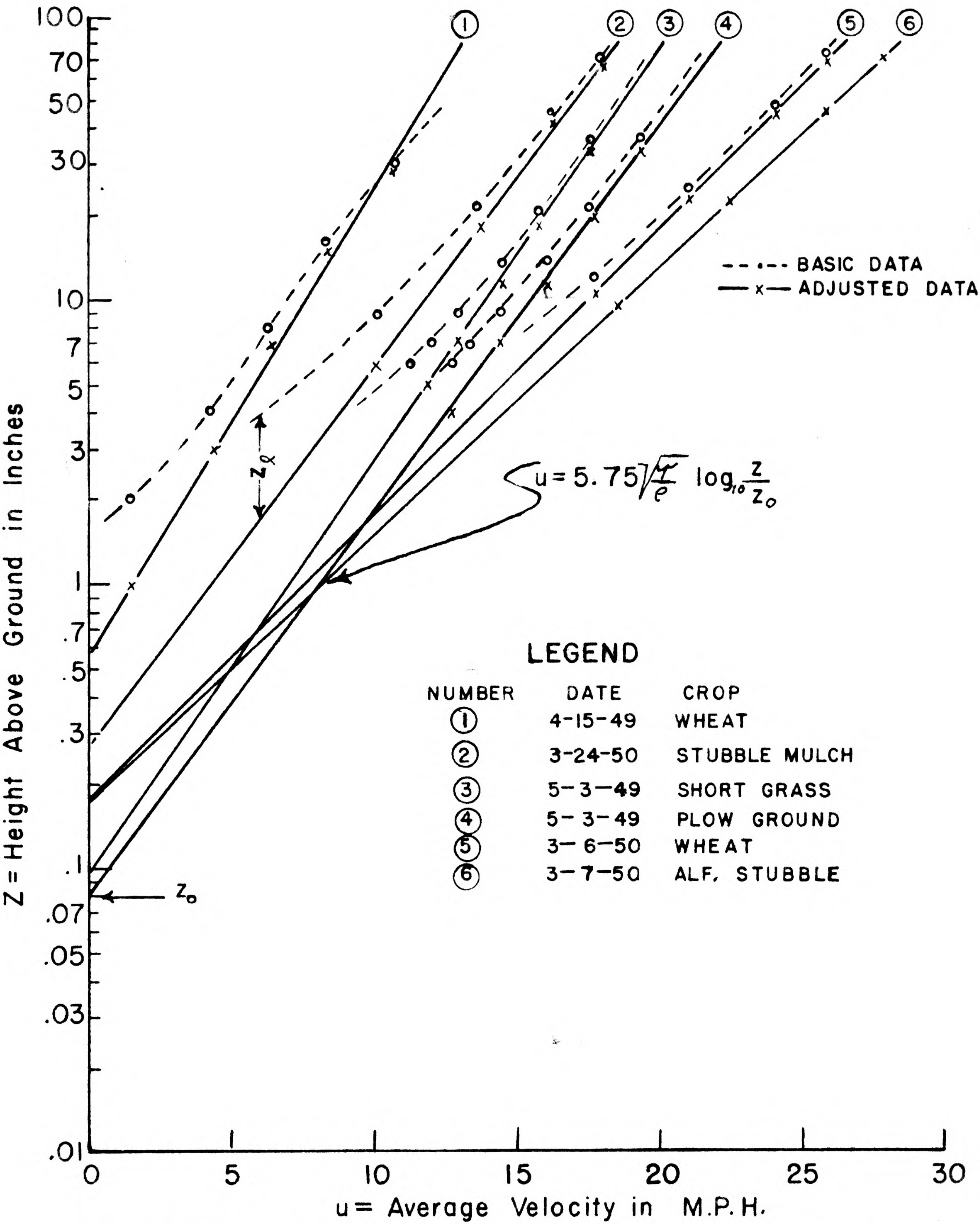


Figure 1.--Relationship of average velocity to the logarithm of the height over various rough surfaces.

It is evident from these graphs that a line connecting the points has a concave curvature. This systematic deviation, evident in most of the data, indicates that the effective height of the measuring devices might be different from the measured height. Zingg and Chepil (25), Kepner et al (9), Thornthwaite and Halstead (22), and Best (2) all have found this curvature in data secured during this period of the day. Sheppard (18) also indicated the necessity of making some adjustment in base of measurement when he stated that "for surfaces whose roughness elements are an appreciable fraction of the height at which the velocity is measured, a zero point displacement equal to a major fraction of the height of the roughness elements should be made in applying the logarithmic equation." In other words, the base of measurement should be some distance above the ground. The adjustment applied to the data in this study was obtained by trial and error methods, adjustment being made until a value was found that gave the best straight line on the semi-log paper.

Determination of the best straight line will graphically determine a value of z_0, or height above the base of measurement at which a point of zero velocity exists. Average shear may then be calculated using equation (14) or (18). A value for the drag coefficient C_d analogous to any height z may also be calculated from equation (17). Values of z_0, z_ℓ, τ, and a value of C_d at the 2-foot height as determined by the methods described are shown in Table 2.

The variation in the results obtained makes it difficult to draw conclusions or to compare them with the results obtained by other investigators. However, it was found that the results shown in Table 2 could be expressed in equation form as:

$$\tau_{(\text{lb./acre})} = 3.42\,(z_0)^{.48}(u_2)^2 \qquad (27)$$

Table 2. Values of z_ℓ, z_o, u_{30z_o}, u_2, C_d, and τ for various natural surfaces.

Date	Field surface	Crop density	Height of crop (in.)	z_ℓ (in.)	z_o (in.)	u_{30z_o} (mph)	u_2 (mph)	C_d	τ (lb./a.)
April 15, 1949	Wheat	Thin	3-4	1.0	0.56	8.6	9.5	0.0228	224
April 15, 1949	Plowed ground	--	2-5	.5	.20	7.9	11.0	.0140	189
April 15, 1949	Plowed ground	--	2-5	.2	.22	8.2	11.3	.0146	204
April 15, 1949	Alfalfa	Thick	4-6	2.0	.52	10.0	11.3	.0220	303
April 15, 1949	Alfalfa	Thick	4-6	2.0	.58	9.1	10.0	.0231	251
May 3, 1949	Wheat	Thin	8-10	4.0	.76	15.9	16.2	.0268	766
May 3, 1949	Plowed ground	--	2-5	2.0	.08	11.2	18.3	.0098	380
May 3, 1949	Alfalfa	Thick	15-18	12.0	.76	14.9	15.2	.0268	673
May 3, 1949	Short grass	Thick	1-2	2.0	.095	10.2	16.5	.0104	315
May 3, 1949	Short grass	Thick	1-2	.5	.16	11.8	17.5	.0128	422
February 27, 1950	Short grass	Thick	1-2	1.0	.084	6.2	10.1	.0099	116
March 1, 1950	Short grass	Thick	1-2	1.0	.084	7.2	11.7	.0099	157
March 6, 1950	Wheat	Thin	1-3	1.0	.18	14.8	21.2	.0134	665
March 6, 1950	Corn stalks	Thin	12-36	8.0	.60	17.9	19.3	.0235	971
March 7, 1950	Lister ridges	--	9-11	3.0	.26	16.6	22.2	.0157	835
March 7, 1950	Alfalfa stubble	Thin	1-3	0.0	.18	15.7	22.8	.0134	747
March 20, 1950	Fallow ground	--	1-4	-1.0	.095	14.5	23.5	.0104	637
March 20, 1950	Fallow ground	--	1-4	-1.0	.180	14.9	21.5	.0134	673
March 20, 1950	Fallow ground	--	1-4	-1.0	.180	13.8	19.9	.0134	577
March 24, 1950	Pastured sorghum	Thin	1-4	0.0	.20	10.6	14.9	.0140	340
March 24, 1950	Stubble mulch	Thin	4-8	3.0	.27	11.1	14.6	.0160	373
April 10, 1950	Wheat	Thick	4-6	1.5	.58	20.5	22.5	.0231	1273
April 10, 1950	Lister ridges	--	9-11	5.0	.30	20.4	26.2	.0167	1261
April 23, 1950	Lister ridges	--	9-11	3.0	.23	8.6	11.8	.0149	224
April 23, 1950	Wheat	Medium thick	7-10	6.6	.70	12.0	12.5	.0256	436

where z_0 is expressed in inches and u_2 is the velocity in miles per hour two feet above the base of measurement. The data also indicate that values of z_0 are roughly $\frac{1}{13}$ of the average crop height for the surfaces with vegetative cover on them. z_0 for the ridged or plowed ground seems to be approximately $\frac{1}{40}$ the average linear height of roughness. The value of $\frac{1}{13}$ of the crop height appears to be in good agreement with values obtained in the laboratory wind tunnel using simulated ridges. The average value of z_0 equal to 0.106 inches obtained for short grass compares well with the 0.22 cm. obtained by Landsberg (10) over a baseball court. However, all the values are much less than those obtained by Sheppard (18), although it is difficult to make a direct comparison since the average wind velocity used in his experiment was only 10 m.p.h. and it is not known from what base his measurements were made.

Values of C_d, the drag coefficient, compare rather well with data given by Sheppard (18), particularly on the short grass where he has a value ranging from 1.01×10^{-2} to 1.60×10^{-2} as compared to the 9.9×10^{-3} to 1.28×10^{-2} obtained in this study. His values for higher vegetation are slightly larger than those obtained in this study, i.e., 3.21×10^{-2} for 19-inch thick grass as compared to 2.68×10^{-2} for $16\frac{1}{2}$-inch alfalfa. Considering the entire group, however, there is very good agreement between the two sets of data.

Substitution of a value of z_0 equal to $\frac{H}{13}$ in equation (27) or a value of C_d in equation (18) indicates that the shearing force exerted by the wind increases with height of cover provided u_2 remains constant. However, the data indicate a decrease in velocity at the 2-foot elevation over the higher roughness. Sheppard's data indicate an increased shearing stress as the height of crop increases. Bagnold (1) states that the shearing force of the wind is due to meteorological forces higher up and that the shearing stress tends to approach a constant level over any surface. Since there appears to be some indication of a lesser velocity at the 2-foot level over the higher roughness in the data obtained in this study, it follows that the shear could tend to remain constant over any surface for a given wind. The results obtained on the subject are not conclusive. It appears that there is need for further study with emphasis being placed on obtaining velocity measurements over several surfaces simultaneously.

Distribution of the Velocity Fluctuations of the Wind

Determination of the average shear over field surfaces is a fundamental problem. From a soil erosion standpoint, the variation in the shearing force caused by atmospheric wind turbulence must be considered also. This turbulence is indicated by the irregular velocity fluctuations known as gusts. The problem of describing the fluctuations in the wind velocity is therefore presented.

The usual assumption made concerning any fluctuating quantity is that it fluctuates about a mean according to some statistical law. Kalinski (8) has determined that the velocity fluctuations in rivers are distributed according to the normal error law, i.e.,

$$Y_c = \frac{N\,i}{\sigma\sqrt{2\pi}}\, e^{\frac{-u_x^2}{2\sigma^2}} \qquad (28)$$

where Y_c is the frequency function, N is the number of frequencies, i is the class interval, σ is the standard deviation, and u_x is the deviation of the velocity from the mean. This approach was applied to the wind velocities obtained with the Pitot tube and manometer. The anemometer was not considered sensitive enough to give a true picture of small fluctuations but appears to respond well to those of 2- or 3-minutes' duration.

The data were tested for normal error distribution by first plotting ratios of $\frac{\sigma}{\bar{u}}$ versus height. The factor $\frac{\sigma}{\bar{u}}$ is associated with the intensity of gusts. This procedure was followed for records secured on each of five days. The results are plotted in Figure 2. From these graphs a group of data was selected from each day to study velocity fluctuations. The data were selected at a height where values of $\frac{\sigma}{\bar{u}}$ for the various surfaces were nearly equal. The velocity fluctuations at these levels were analyzed by obtaining ratios of $\frac{u}{\bar{u}}$, where u was the fluctuating velocity and $\bar{u}$ is the average velocity for that particular level. A frequency distribution was formed by arranging the ratios into groups according to magnitude. Theoretical frequencies were calculated as shown in Table 3. These theoretical frequencies were compared with observed frequencies by means of the chi-square, X^2, test of goodness of fit as given by:

$$X^2 = \sum \frac{(f - f_c)^2}{f_c} \qquad (29)$$

where f is an observed frequency in a class and f_c is the corresponding theoretical frequency. Table 4 summarizes the values of X^2, n, and P.

Table 4. Results of chi-square goodness of fit test.

Date	Degrees of freedom (n)	X^2	Probability (P)
April 15, 1949	3	1.014	0.79
May 3, 1949	5	1.499	.91
March 20, 1950	5	3.926	.53
April 10, 1950	2	2.805	.25
April 23, 1950	4	6.924	.15

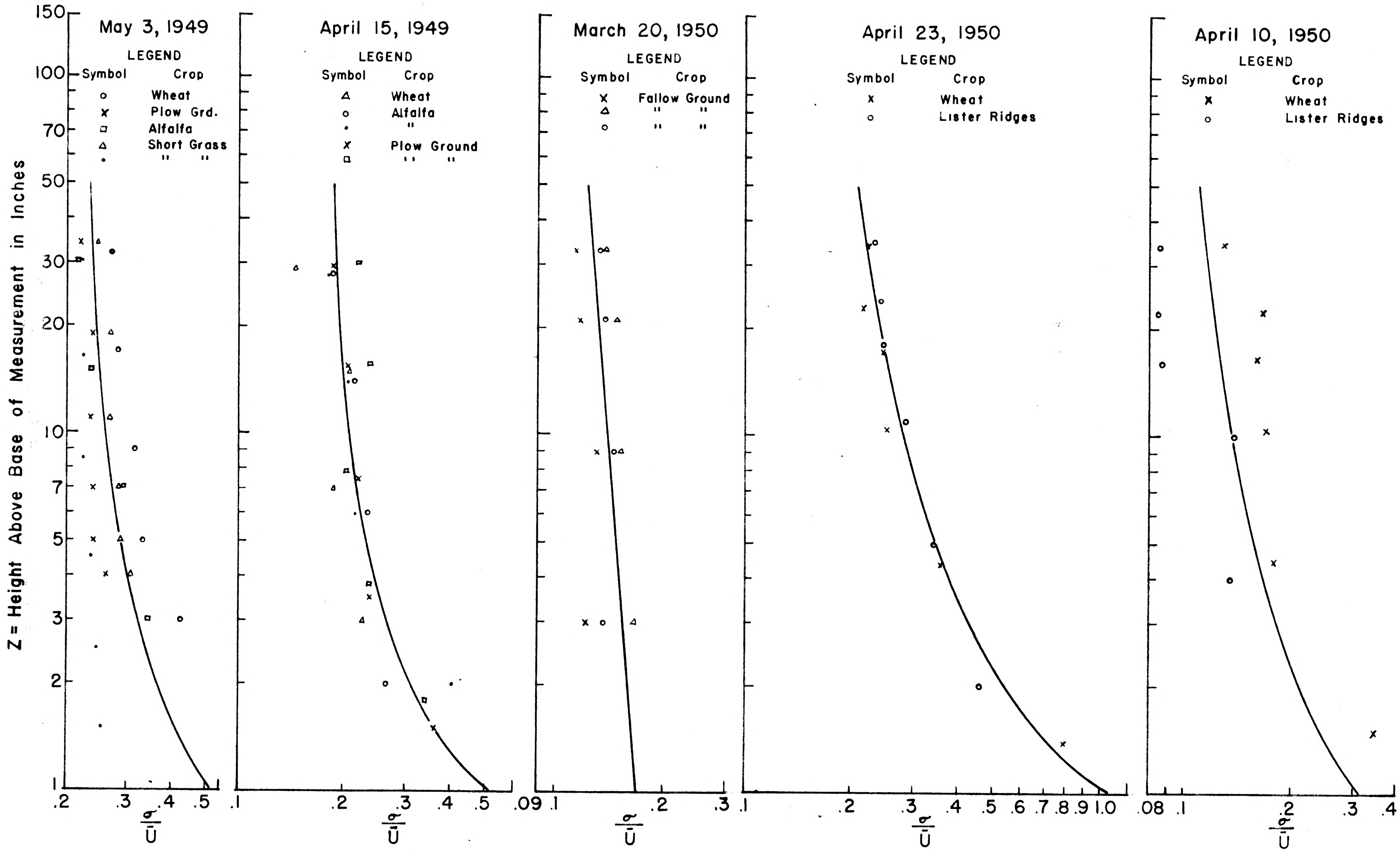


Figure 2.--A plot showing the relationship of the ratio of standard deviation to average wind velocity for various heights over different surfaces.

Table 3. Determination of theoretical frequencies of wind velocity fluctuations for five days.

Velocity ratio ($\frac{u}{\bar{\bar{u}}}$) classification	Class limits: Lower	Class limits: Upper	Deviation (x) from mean to limit	$\frac{x}{\sigma}$	Area between mean and limit (%)	Area in each class (%)	Expected frequency in each class	Observed frequency in each class
April 15, 1949. Standard deviation ($\sigma_{\frac{u}{\bar{\bar{u}}}}$) = 0.219 and N = 100								
< .4	--		--	--	50.0	0.3	0.3	---
.4-.6	0.4		0.6	2.74	49.7	3.1	3.1	2
.6-.8	.6		.4	1.83	46.6	14.7	14.7	15
	.8		.2	.91	31.9			
.8-1.0						31.9	31.9	35
		1.0	0	0	0			
1.0-1.2		1.2	.2	.91	31.9	31.9	31.9	30
1.2-1.4		1.4	.4	1.83	46.6	14.7	14.7	15
1.4-1.6		1.6	.6	2.74	49.7	3.1	3.1	3
> 1.6		--	--	--	50.0	.3	.3	0
May 3, 1949. Standard deviation ($\sigma_{\frac{u}{\bar{\bar{u}}}}$) = 0.27 and N = 250.								
< .2	--		--	--	50.0	.2	.5	0
.2-.4	.2		.8	2.96	49.8	1.1	2.75	2
.4-.6	.4		.6	2.22	48.7	5.6	14.0	13
.6-.8	.6		.4	1.48	43.1	16.1	40.3	45
	.8		.2	.74	27.0			
.8-1.0						27.0	67.5	68
		1.0	0	0	0			
1.0-1.2		1.2	.2	.74	27.0	27.0	67.5	66
1.2-1.4		1.4	.4	1.48	43.1	16.1	40.3	38
1.4-1.6		1.6	.6	2.22	48.7	5.6	14.0	14
1.6-1.8		1.8	.8	2.96	49.8	1.1	2.75	2
1.8-2.0		2.0	1.0	3.70	49.9	.1	.25	2
> 2.0		--	--	--	50.0	.1	.25	0
March 20, 1950. Standard deviation ($\sigma_{\frac{u}{\bar{\bar{u}}}}$) = 0.145 and N = 141.								
< .6	--		--	--	50.0	.3	.42	0
.6-.7	.6		.4	2.76	49.7	1.2	1.69	2
.7-.8	.7		.3	2.17	48.5	6.9	9.72	11
.8-.9	.8		.2	1.38	41.6	16.1	22.70	20
.9-1.0	.9		.1	.69	25.5	25.5	36.00	34
	1.0		0	0	0			
1.0-1.1						25.5	36.00	37
		1.1	.1	.69	25.5			
1.1-1.2		1.2	.2	1.38	41.6	16.1	22.70	20
1.2-1.3		1.3	.3	2.17	48.5	6.9	9.72	15
1.3-1.4		1.4	.4	2.76	49.7	1.2	1.69	2
> 1.4		--	--	--	50.0	.3	.42	0
April 10, 1950. Standard deviation ($\sigma_{\frac{u}{\bar{\bar{u}}}}$) = 0.158 and N = 85								
< .5	--		--	--	50.0	.1	.08	0
.5-.7	.5		.5	3.16	49.9	2.8	2.38	2
.7-.9	.7		.3	1.90	47.1	23.5	20.00	18
	.9		.1	.63	23.6			
.9-1.1						47.2	40.20	47
		1.1	.1	.63	23.6			
1.1-1.3		1.3	.3	1.90	47.1	23.5	20.00	15
1.3-1.5		1.5	.5	3.16	49.9	2.8	2.38	3
> 1.5		--	--	--	50.0	.1	.08	0
April 23, 1950. Standard deviation ($\sigma_{\frac{u}{\bar{\bar{u}}}}$) = 0.269 and N = 100.								
< .3	--		--	--	50.0	.5	.50	0
.3-.5	.3		.7	2.60	49.5	2.6	2.60	2
.5-.7	.5		.5	1.86	46.9	10.3	10.30	6
.7-.9	.7		.3	1.11	36.6	22.2	22.20	29
	.9		.1	.37	14.4			
.9-1.1						28.8	28.80	31
		1.1	.1	.37	14.4			
1.1-1.3		1.3	.3	1.11	36.6	22.2	22.20	20
1.3-1.5		1.5	.5	1.86	46.9	10.3	10.30	7
1.5-1.7		1.7	.7	2.60	49.5	2.6	2.60	5
> 1.7		--	--	--	50.0	.5	.50	0

Examination of Table 4 shows that the probability of wind velocity fluctuations having a normal distribution is very high for all the days except April 23, 1950. If the usual levels of significance of 5 and 1 percent are considered, all the data are well above these levels.

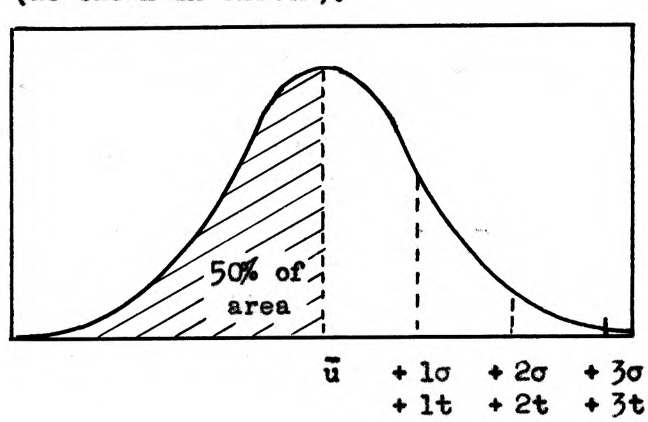
Since there is a good indication that the velocity fluctuations do have a normal distribution, the standard deviation of the velocity

$$\sigma = \sqrt{\frac{\Sigma u^2}{N} - \bar{u}^2}$$

will describe these fluctuations. The velocity equaled or exceeded any percent of the time may then be determined from the properties of the normal curve and a knowledge of values of $\frac{\sigma}{\bar{u}}$. This relationship may be expressed in the general equation form as

$$u_{\text{any \% of time}} = \bar{u} \pm t\sigma \qquad (30)$$

where $\bar{u}$ is the average velocity for a given height; t is the distance, in terms of σ, of any given ordinate measured from the mean of the normal curve (plus if to right and negative if to left of mean); and σ is the standard deviation. Values of t correspond to different percentages of the area of one-half the normal curve and are given in statistician's tables. $\bar{u}$ occurs at the center of the normal curve and, in terms of distance, represents one-half the total area (as shown in sketch).



It follows, therefore, that statistically the maximum velocity, occurring approximately 0.1 percent of the time, is found by the equation

$$u_{max} = \bar{u} + 3\sigma \tag{30a}$$

The procedure for determining the velocity greater than the average occurring 10 percent of the time would be to find the value of t corresponding to an area of 40 percent, or 1.285, giving

$$u_{10\%} = \bar{u} + 1.285\sigma \tag{30b}$$

This same procedure could be applied to determine the velocity level equaled or exceeded any given percent of the time. Values of σ in terms of $\bar{u}$ at some particular level may be obtained from Figure 2. It will be noted that the value of $\frac{\sigma}{\bar{u}}$ at the 1-inch level on March 20, 1950, is 0.17. Substitution of $\sigma = 0.17\ \bar{u}$ into equation (30a) will give

$$u_{max} = 1.51\ \bar{u} \tag{30c}$$

The momentary extreme values of shear associated with these maximum velocities may be determined by making use of the assumption that shear varies as the square of the velocity. Thus, if the maximum velocity on March 20 is equal to 1.51 $\bar{u}$, it follows that the maximum shear would be 2.28 τ, where τ is the average shear as determined in a previous section of this report.

Values of shear equaled or exceeded 10 percent and 0.1 percent of the time for the various days at the 1-inch level are shown in Table 5. It is evident from this table that there is considerable variation in the fluctuation of the shearing force from day to day. This fact could well explain why winds of the same average velocity move varying amounts of soil. It is also evident that the maximum shear and also the shear equaled or exceeded 10 percent of the time is several times the average. Considering an average for

Table 5. Values of shear equaled or exceeded 10% and 0.1% of the time.

Date	$\frac{\sigma}{\bar{u}}$ (1" level)[1/]	τ equaled or exceeded 10% of time	τ equaled or exceeded 0.1% of time
April 15, 1949	0.51	2.74 $\tau_{avg.}$	6.40 $\tau_{avg.}$
May 3, 1949	.52	2.78 $\tau_{avg.}$	6.55 $\tau_{avg.}$
March 20, 1950	.17	1.48 $\tau_{avg.}$	2.28 $\tau_{avg.}$
April 10, 1950	.31	1.98 $\tau_{avg.}$	3.72 $\tau_{avg.}$
April 23, 1950	1.10	5.83 $\tau_{avg.}$	18.80 $\tau_{avg.}$

[1/] Values of $\frac{\sigma}{\bar{u}}$ at 1-inch level are an average for all surfaces for one particular day.

the five days, the maximum shear would be 7.55 times the average; the shear expected to occur 10 percent of the time would be 2.96 times the average. It therefore seems probable that the determination of the shear associated with velocity fluctuations, or the magnitude of the gusts, will have a great deal of bearing upon the erosion of soil by wind. The average velocity or shear when considered alone will not necessarily provide an index of the erosive power of the wind.

Crop and Tillage Effects on Wind Velocity Fluctuations and Associated Shear

The fact that velocity readings were not taken simultaneously over various surfaces makes it difficult to discern possible differences in the magnitude of fluctuations associated with the various surfaces. When the preceding analysis is applied to the data taken over specific crops and tillage methods, little difference is apparent in the magnitude of fluctuations over various crops. It appears that major differences in the magnitude of the fluctuations are due to the variability in the wind from day to day. This would indicate that the effect of surface conditions on the maximum shearing force associated with these fluctuations is minor in importance.

While these results are not conclusive they do emphasize the need for obtaining velocities over several surfaces simultaneously. It is hoped that the new instruments shown in Figure 2 of Plate I, which were developed during the past year, will accomplish this. If they do not, there is apparently a need for a more sensitive automatic recording instrument to obtain the velocities associated with the gusts of the wind.

Temperature-Height Relationships

The data obtained in this part of the study were classified into three arbitrary categories as follows: (a) a stable condition, defined as one showing a small to medium lapse from the ground to the maximum elevation, usually obtained from approximately 7:00 a.m. to 10:30 a.m.; (b) an unstable condition, defined as an inversion overlying a fairly large lapse at the ground, usually obtained from 10:30 a.m. to 4:30 p.m.; and (c) an inversion, defined as a fairly constant increase in temperature with height, obtained from approximately 5:00 p.m. to 7:00 a.m.

These classifications are not as finite as those given previously, i.e., classifications based on the Richardson's number or the adiabatic lapse rate. However, the temperature profiles obtained in this study do not adapt themselves to application of the Richardson's number since the sign of R_i would depend upon the height at which it was calculated. On the other hand, the adiabatic lapse rate appears to be too finite for determination in the manner used in this study. The three categories used seemed to fit the situation with regard to the wind erosion problem, and it was found that all the data could be classified successfully into these groups.

Table 6 gives the number of winds sampled, the number of 30-minute profiles per wind, wind direction, and the average wind velocities and temperatures for each of the atmospheric conditions.

The three atmospheric conditions were defined in terms of temperature and height by the following procedure: All temperatures at each height were converted to ratios by using the temperature at the 1-foot elevation as the base. These values of $\frac{T}{T_1}$ were then plotted versus height. This gave a general indication of the type equation which would describe the relationship.

Table 6. A summary table of the winds and temperatures for stable, unstable, and inversion atmospheric conditions.

Date	Number of 30-minute samples per wind	Wind direction	Average wind velocity for test period, Height in feet: 2 (mph)	6 (mph)	12 (mph)	20 (mph)	31 (mph)	Average temperature for test period, Height in feet: 1 (°F.)	6 (°F.)	28 (°F.)
STABLE ATMOSPHERIC CONDITION (Approximate C.S.T. limits for occurrence: 7:00 a.m. - 10:30 a.m.)										
8-29-51	8	S.	6.7	9.0	10.3	11.2	11.5	89.4	88.8	88.3
9-11-51	3	S.W.	13.2	17.7	20.4	22.5	24.3	79.8	77.5	77.2
9-26-51	4	S.W.	11.9	15.2	17.3	18.9	20.7	78.5	78.1	77.5
10-13-51	7	S.	10.8	14.4	16.3	17.6	18.0	70.6	69.6	67.1
11- 8-51	3	S.S.W.	8.8	11.3	12.7	13.9	15.0	54.8	55.2	53.3
11-16-51	7	N.W.	9.9	14.3	17.0	18.6	20.7	34.6	33.2	29.9
12- 3-51	12	N.W.	11.4	15.0	17.3	18.8	21.2	56.2	52.1	51.2
12- 4-51	3	S.E.	12.4	15.6	17.4	18.7	19.4	46.9	46.6	46.5
12- 6-51	13	S.W.	22.8	27.6	30.8	33.6	36.4	49.9	49.6	49.5
Total	60									
UNSTABLE ATMOSPHERIC CONDITION (Approximate C.S.T. limits for occurrence: 10:30 a.m. - 4:30 p.m.)										
8-29-51	7	S.	6.7	9.2	10.8	11.8	10.6	97.5	94.8	97.1
8-30-51	13	S.S.W.	7.1	9.7	11.4	12.5	12.9	95.1	93.8	94.6
9-11-51	19	S.W.	13.2	17.8	20.6	22.7	24.2	85.1	82.2	83.6
9-26-51	13	N.W.	7.3	10.5	12.7	14.0	15.6	81.0	79.4	80.5
10-12-51	6	S.S.E.	12.6	16.2	18.8	20.2	21.0	76.7	75.8	76.6
11-21-51	3	S.W.	6.8	8.8	9.9	11.0	11.8	61.1	58.6	59.8
12- 3-51	5	N.W.	9.8	11.9	13.5	15.0	16.3	57.5	53.0	53.8
Total	66									
INVERSION (Approximate C.S.T. limits for occurrence: 5:00 p.m. - 7:00 a.m.)										
8-29-51	13	S.	7.3	10.1	12.0	13.2	14.1	88.4	90.1	91.1
10-12-51	9	S.S.E.	10.4	13.6	15.4	16.3	17.5	68.7	69.7	70.6
11-21-51	3	S.W.	2.7	3.4	4.3	5.0	5.7	55.4	55.7	56.5
12- 4-51	7	S.E.	10.4	12.7	13.9	15.1	15.7	50.3	47.7	48.0
12- 6-51	7	S.W.	16.7	20.1	22.4	24.4	26.5	45.0	45.6	45.7
1-14-52	8	W.	8.9	10.9	12.4	13.7	15.3	46.8	47.1	47.6
Total	47									

A mathematical expression for each condition was determined by a method of regression described by Milne (12). Figure 3 shows the average curve for each condition.

The average temperature profiles shown in Figure 3 indicate very little difference in the profile near the ground for a stable or an unstable atmospheric condition. However, at heights greater than 10 feet there is a marked contrast in the two profiles. It is apparent, therefore, that any measurements of the differences in the temperature profiles must be made to some height greater than 10 feet if one wishes to define or describe the atmospheric condition in these terms. The inversion condition would be more easily determined due to the increase of the temperature with height.

These results differ slightly from those of Sheppard (18). He made a rather broad survey of the temperature field over land and sea and reported that during the daylight hours temperature decreases roughly proportional to the logarithm of the height. This study has shown that the temperature decreases approximately proportional to the logarithm of the height for a stable condition, i.e., early morning, and increases proportional to the logarithm of the height in late evening during an inversion. However, during mid-day when an unstable condition is present, the temperature first decreases near the ground, then increases at higher elevations. The actual relationship may be expressed by a second degree polynomial type equation. Sheppard has not broken his results into as finite groupings as those measured here. It is probable that this fact accounts for the differences.

The small differences in the temperature profiles forestalls any simple means of measuring the atmospheric condition. Apparently, one must rely on temperature measurements made from permanent installations which extend to

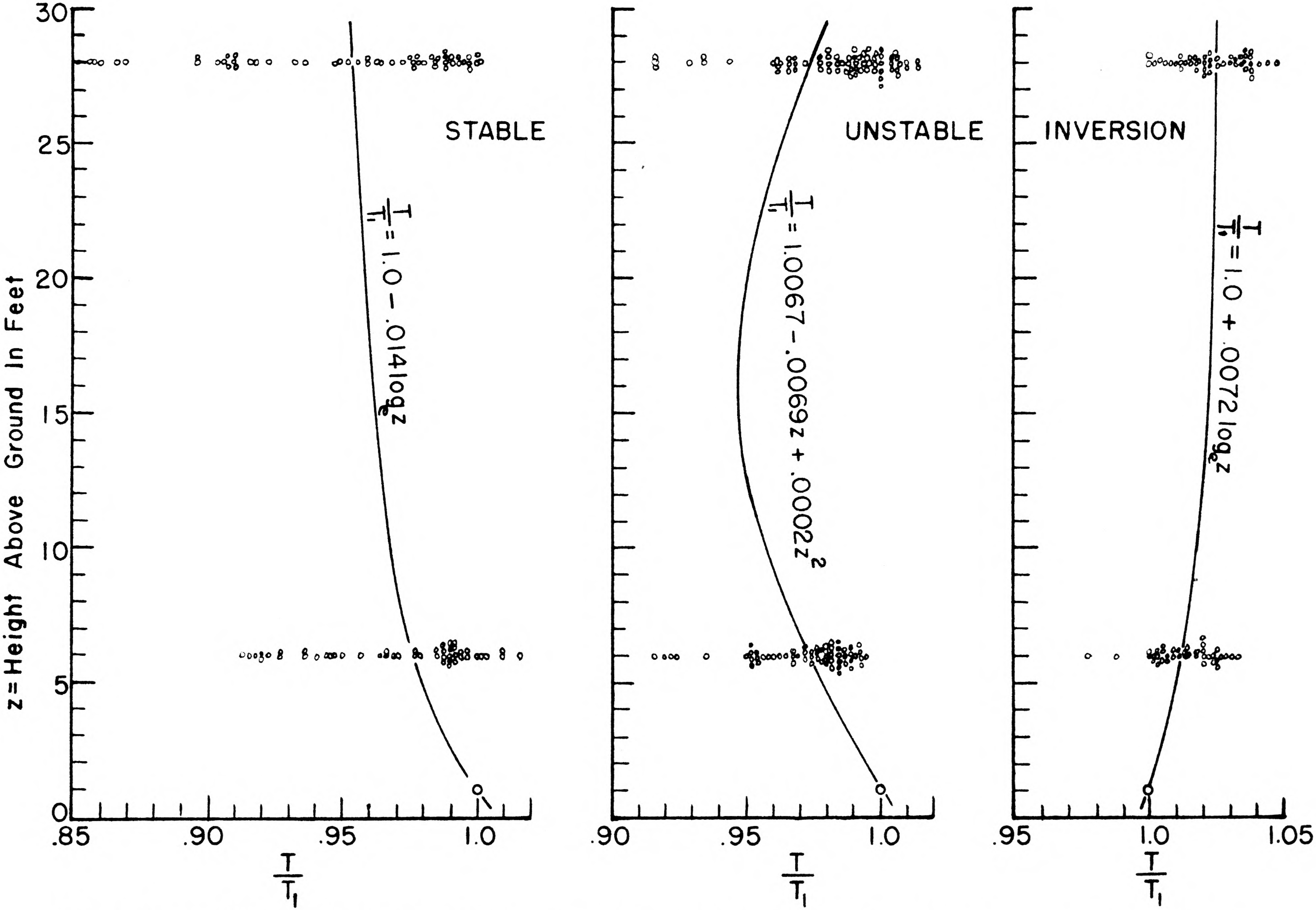


Figure 3.—A plot showing the functional relationship between the height z and a ratio of the temperature at the 1-foot height to the temperature at various other heights, $\frac{T}{T_1}$, for 3 atmospheric conditions.

considerable height. This is regrettable due to the fact that in wind erosion studies we are primarily interested in the wind profile in the layer near the ground. It would be very advantageous, therefore, to be able to confine temperature measurements to that level in order that the atmospheric condition might be known.

Wind Velocity-Height Relationships

Velocity-height relationships were established for each arbitrary condition, i.e., stable, unstable, and inversion, from velocity profiles corresponding to the temperature profiles used in the previous analysis. This procedure was as follows: The height of the vegetation was considered constant. This eliminated adjustment of base of measurement; therefore, all relationships were derived from a ground level base. All the actual field data, u in miles per hour versus z in feet, were plotted on both semi-log and log-log paper. Typical profiles are shown in Figures 4 and 5. It was evident that there was considerable variation in the shape of the wind profiles. Table 7 shows the percentage of the profiles which were straight, concave, or convex for each atmospheric condition on both types of plotting paper.

The shape of the velocity profile for each condition also was determined by another method. All velocities were converted to ratios by using the velocity at the 2-foot elevation for each profile as a base. This has the effect of putting data obtained over a wide range of conditions on a comparable basis. The ratios $\frac{u}{u_2}$ were plotted versus height on both types of plotting paper. In addition, semi-log plots of $z - c$ versus $\frac{u}{u_2}$ and z versus $(\frac{u}{u_2})^n$ were considered. The average curve for each condition was determined

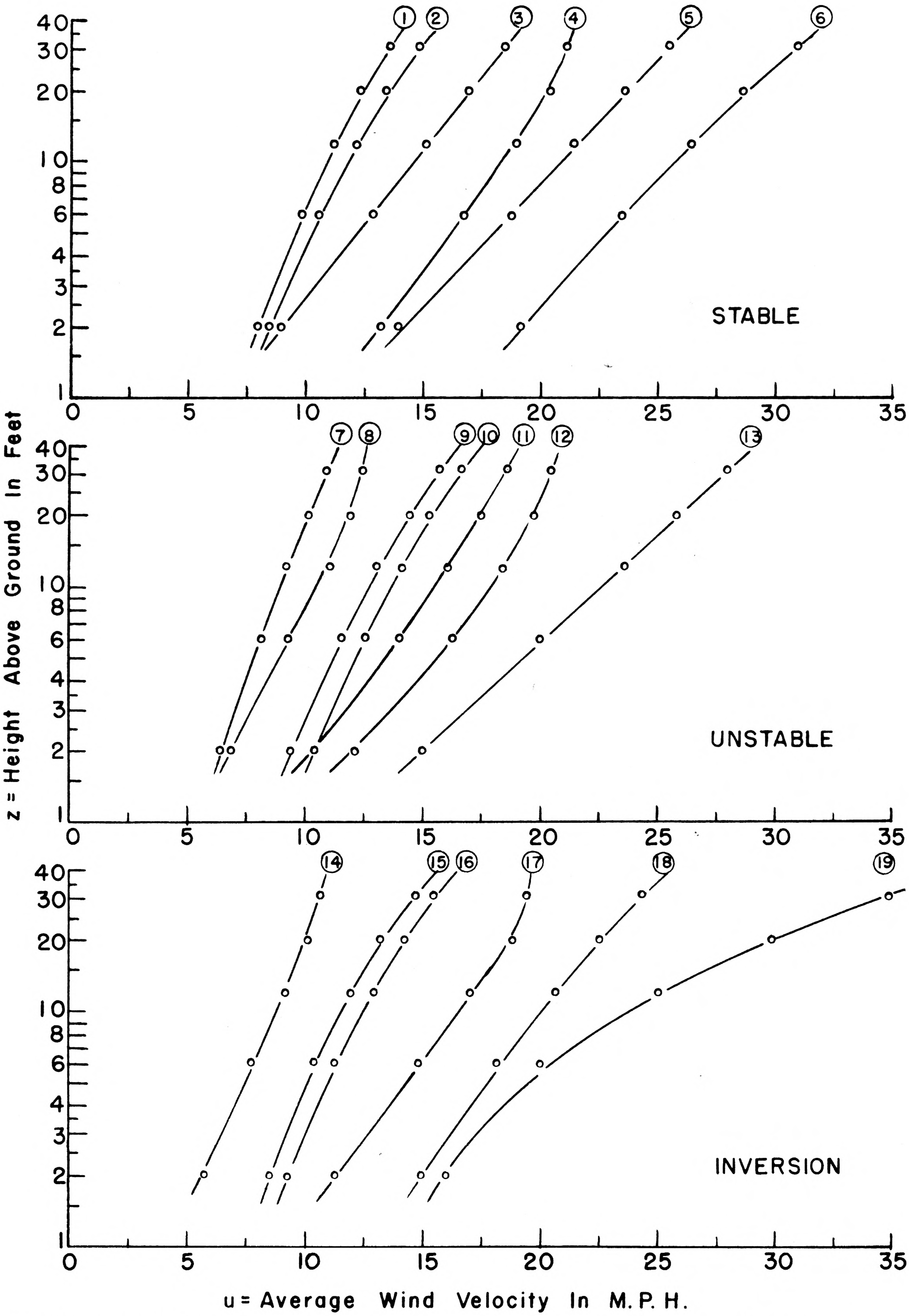


Figure 4.—Typical wind profiles showing the relationship of the average velocity for a 30-minute period to the logarithm of the height, for 3 atmospheric conditions. Each of these curves is also shown in Figure 5, the numbers on the curve identifies each profile for purposes of comparison with the same curve in Figure 5.

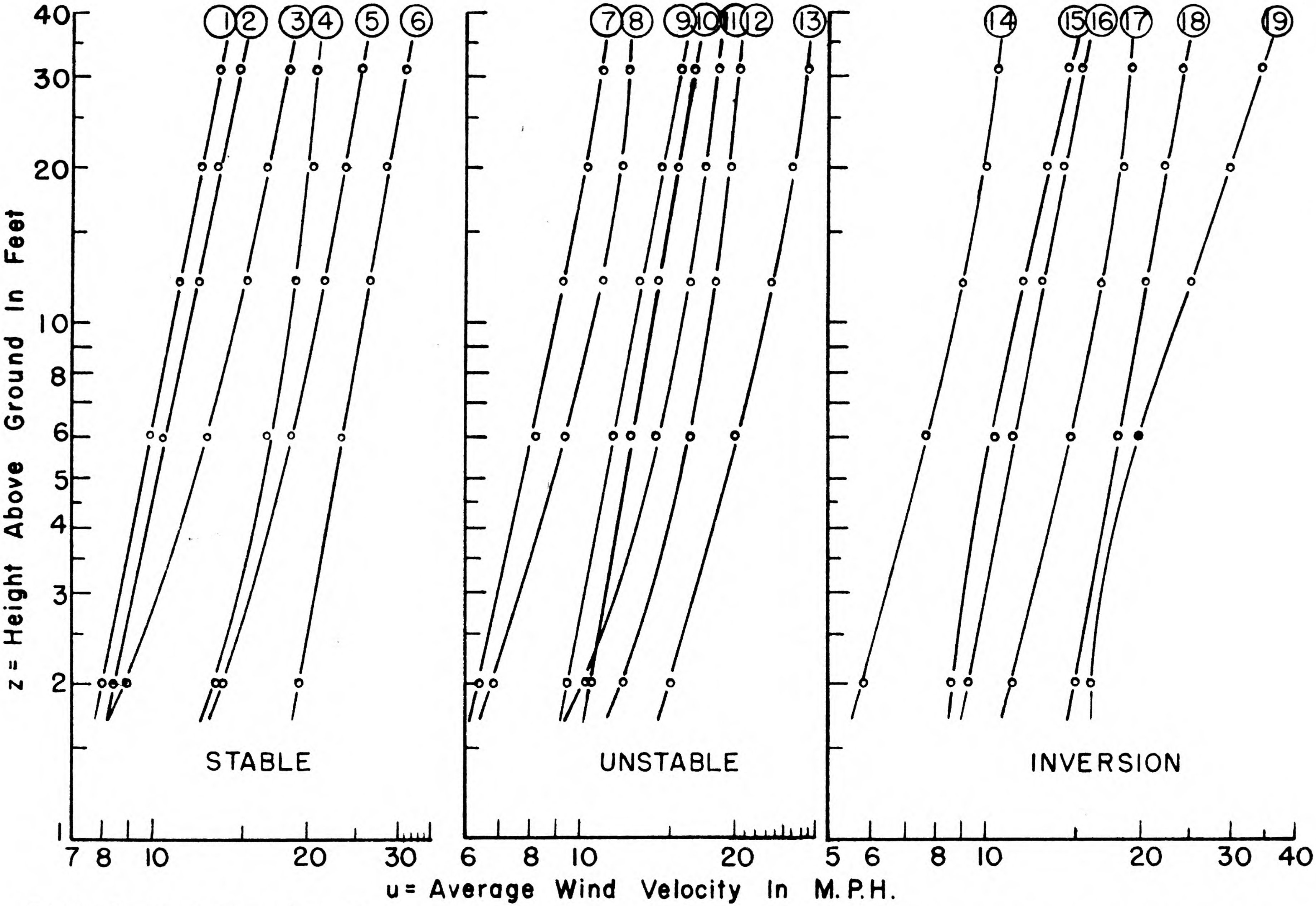


Figure 5.--Typical wind profiles showing the relationship of the logarithm of the average velocity for a 30-minute period to the logarithm of the height, for 3 atmospheric conditions. Each curve is also shown in Figure 4, the numbers on the curves identifies each profile for purposes of comparison with Figure 4.

Table 7. Variation in shape of the wind profiles for three atmospheric conditions on semi-log and log-log paper.

Shape or nature of curve	Atmospheric condition					
	Stable		Unstable		Inversion	
	Semi-log plotting (%)	Log-log plotting (%)	Semi-log plotting (%)	Log-log plotting (%)	Semi-log plotting (%)	Log-log plotting (%)
Straight line	28.4	48.4	19.7	12.0	12.8	36.2
Concave	30.0	51.6	63.6	88.0	40.5	51.0
Convex	41.6	0	16.7	0	46.7	12.8

from the plot giving the best straight line. The slope of the curve and constants necessary to write the equation were determined graphically. These equations are:

Atmospheric condition	Average relationship of $\frac{u}{u_2}$ and height z	
Stable	$z = 0.052\ 10^{1.585 \frac{u}{u_2}}$	(31)
Unstable	$z = 0.124\ 10^{1.21 \frac{u}{u_2}^{1.1}}$	(32)
	$z = z_o\ 10^{1.47 \frac{u}{u_2}} + 0.45$	(32a)
Inversion	$z = 2.0 \frac{u}{u_2}^{4.75}$	(33)

Figure 6 is a plot of the $\frac{u}{u_2}$ ratios versus height. The average curve for each condition is also shown.

These equations imply the following generalized functional relationships for wind velocity u and height z:

Atmospheric condition	Average relationship of wind u and height z	
Stable	$z = z_o\ 10^{nu}$	(34)
Unstable	$z = z_o\ 10^{nu^p}$	(35)
	$z - c = z_o\ 10^{nu}$	(35a)
Inversion	$z = C\ u^n$	(36)

The shear expressions corresponding to the above conditions would be:

Stable

$$\tau = \frac{33.06\ \rho}{u^2}\left[\log_{10}\frac{z}{z_o}\right]^2 \qquad (37)$$

Unstable

$$\tau = \rho\left[\frac{k\,u}{p\ l\ n\ z + (\frac{1}{p} - 1)}\right]^2 \qquad (38)$$

$$\tau = \frac{33.06\ \rho}{u^2}\left[\log_{10}\frac{z - c}{z_o}\right]^2 \qquad (38a)$$

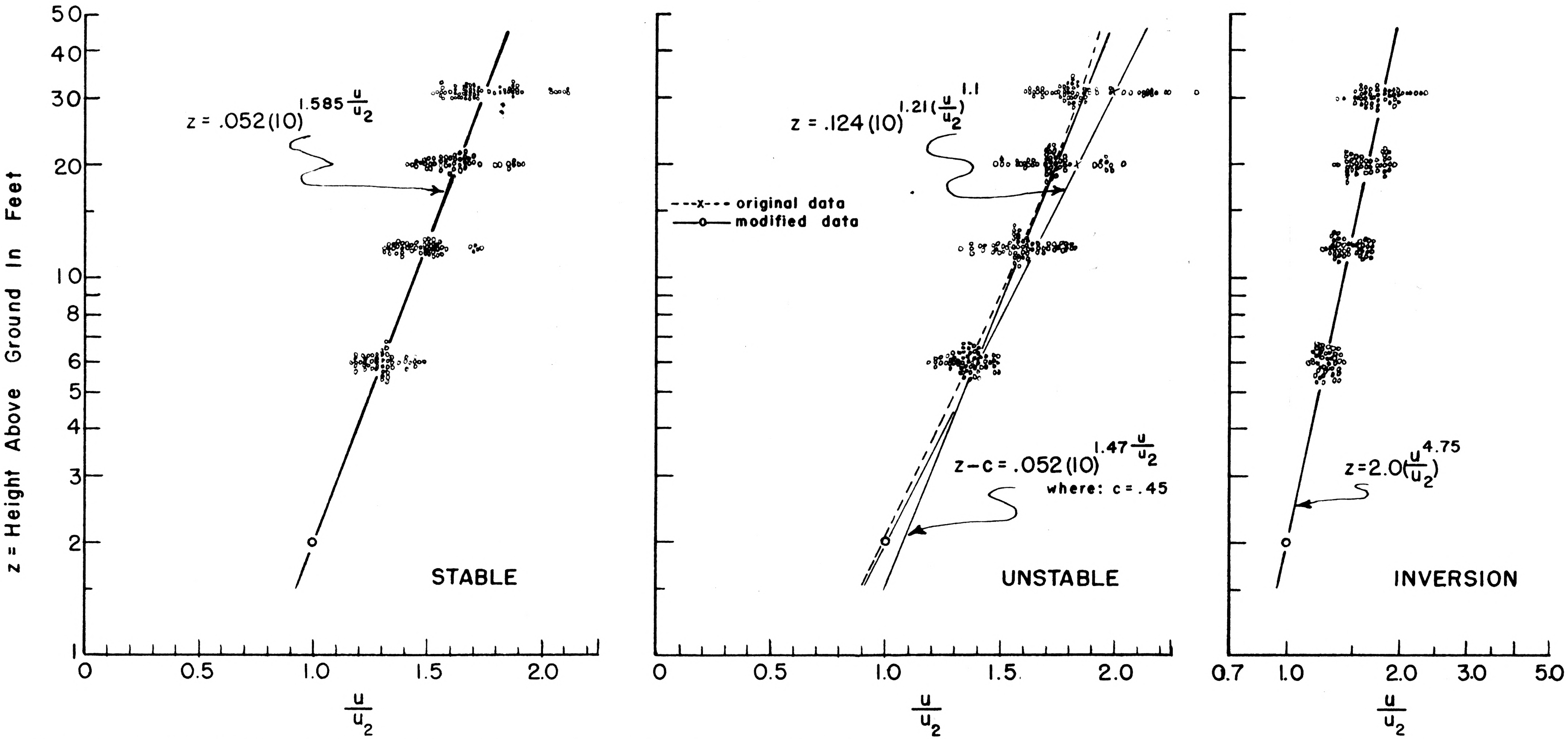


Figure 6.--A plot of the logarithm of the height z to a ratio of the average velocities $\frac{u}{u_2}$ (logarithm of z versus logarithm of ratio for inversion). The equation expresses the average functional relationship for each of the three atmospheric conditions.

Inversion $$\tau = \rho \left[\frac{k \rho u}{1 - n}\right]^2 \qquad (39)$$

The wind velocity-height relationships necessarily must be discussed from the standpoint of both the individual and the average wind profile. This is apparent when one observes the wide variation in type of profile which may be obtained under relatively the same temperature gradients. Much of this variation apparently occurs between winds on different days. It was evident that any group of profiles for a given wind could have characteristics of shape different from those of another wind even though the temperature gradients for the two cases were identical. This would indicate that some other variable or variables besides temperature must affect the distribution of wind velocity with height.

The data shown in Table 7 give a good indication of the different types of relationships which would be required to fit the individual profiles. These data may be summarized roughly as follows:

> Stable condition.--Approximately one-half of the profiles could be represented by a power function, one-fourth by a log function, and one-fourth by a modification of either the power or log function.
>
> Inversion condition.--Approximately three-eighths of the profiles could be represented by a power function, one-eighth by a log function, and the remaining one-half by a modification of either the log or power function.
>
> Unstable condition.--Approximately two-tenths of the profiles could be represented by a power function, one-tenth by a log function, and the remaining seven-tenths by a modification of either the log or power function.

It is evident that the individual profile data are variable in the extreme; however, there are indications that each atmospheric condition has its characteristic profile if it is considered from the standpoint of the average relationship based on all the data for any one classification.

Equations (34), (35), (35a), and (36) are the results of such an analysis. It will be noted that the stable condition is represented by a log function, the inversion by a power function, and the unstable condition by a modified log. Two forms of the modified log are given for the unstable condition; actually, either of these modifications will fit most of the data. Equation (35), which was derived by plotting the velocity $\frac{u}{u_2}$ to a power n versus the logarithm of the height z, fits most of the data, and particularly the average condition, better than does equation (35a). The difficulty encountered with the use of an equation of this type is the lack of a theory or basis for determining shear. If one assumes that the defining equations and theory given by Prandtl (3) would apply and derives equation (3) (see Appendix III), it will be found that the value of shear obtained will be approximately 12 times that obtained from equation (38a). This does not seem reasonable; and, since there seems to be some doubt as to the validity of applying the Prandtl theory to anything other than a straight logarithmic function, it was decided to use equation (35a) and its corresponding shear relationship (38a) as a matter of expediency. Ic is of some interest to note, however, that the value of the power, 1.1, which gives the best fit to the average data agrees with that suggested by Thornthwaite and Halstead (22) for an unstable condition.

It is evident that the results of this study are not conclusive; however, the indicated average relationships are in fair agreement with results reported by other investigators. Table 8, where these results are compared with those of six other investigators, illustrates this point.

Table 8. Types of relationships used by various investigators to describe the wind profile.

Investigators[1]	Atmospheric condition		
	Stable	Unstable	Inversion
Thornthwaite and Halstead (22)	log	power	Not logarithmic if there is pronounced heating or cooling.
Sverdrup (21)	power	log	---
Kepner et al (9)	log	---	power
Sutton (19)	Uses power as an approximation of log		
Deacon (19)	log	modified log	---
Best (19)	log	modified log	---
This study	log	modified log	power

[1]Number in parentheses following name refers to Literature Cited.

The table shows general agreement that a log relationship fits the stable condition; however, there is some disagreement in regard to the other two conditions. Perhaps this is due to lack of reported results for the unstable and inversion conditions.

The discrepancies which are present in the wind velocity-height relationships in the unstable atmospheric condition point to the need of an accurate means of measuring the shearing force of the wind. Until some method is perfected to measure accurately this force or until a new theory is propounded regarding atmospheric turbulence, it appears that equation (35a) is probably the best expression available for the wind profile in the unstable condition. The principle involved in this type of equation, that of subtracting an amount from the height z, was used in the analysis of the effect of crops and tillage methods on the shearing force of the wind. The value of the constant may be determined graphically as before or calculated by a method described by Lipka (11). It should be noted that the actual value of this constant would vary with the specific conditions encountered and would not be equal necessarily to the constant calculated for this study.

DISCUSSION OF RESULTS

The problem of studying and describing the atmospheric wind gradient in the layer near the ground is of a complicated nature. This study has attempted to describe the wind gradient and some of the various factors which affect it. The investigation was limited to those phases of the problem concerned with wind erosion in an agricultural area. It has not dealt with the finite meteorological phases of the problem. Many of the results are not conclusive; however, they do indicate new approaches to some aspects of the problem.

A summary is given of the more important previous work. The wide variation in these investigations and the differences in the results point to the lack of a definite knowledge of the complex problem.

Values of average shear and the roughness parameter z_o as determined by the logarithmic laws are presented. There is an indication that the shear tends to be constant over all of the surfaces for a given wind. These results are more in agreement with those of Bagnold (1), who believes that the shearing stress tends to approach a constant level over any surface, than those of Sheppard (18), whose data would indicate increased shearing stress as the height of vegetation increases. The values of z_o are only in fair agreement with other observers. In general, most investigations indicate the same order of magnitude of z_0, but, as might be expected, there is considerable spread in the actual values obtained. The value of $\frac{1}{40}$ obtained for the ridges is apparently low; however, there are no previous studies which can be used as a basis of comparison. The value of z_0 obtained for the grass and for the tall vegetation compares well with measurements by others.

The statistical method used to describe the wind velocity fluctuations is different from methods tried previously. However, the basic principal used is the same; and the results agree with those of Best (Sutton (19)) and Hesselberg and Bjorkdal (Sutton (19)), and more recently Cramer (4), i.e., the velocity fluctuations of the atmospheric wind may be described by the normal error law. The differences in the magnitude of the fluctuations from day to day indicate the importance of associating velocity fluctuations with determinations of the shearing force of the wind. It was noted that the shear was as much as $7\frac{1}{2}$ times the average for short periods and 3 times the average for approximately 10 percent of the time.

The results of the study to determine the effect of tillage and cropping conditions on the velocity fluctuations indicate that, in general, this effect would be of minor importance. However, there is a need for further study using simultaneous readings. The magnitude of the fluctuations from day to day are apparently more important than the effect of the immediate ground cover.

The investigations of the effect of atmospheric stability on the wind gradient shows considerable variation in results. There is a definite need for a continuation of the study, perhaps in a more open area of the state to assure equilibrium conditions with the wind profile and the surrounding terrain. The results as they have been interpreted are in fair agreement with previous studies and seem to corroborate the selection of the logarithmic type equation for determination of average shear over different crops and tillage methods during unstable periods.

Other investigators have reported a more definite relationship between the velocity profile and the temperature profile. Kepner, et al (9) showed similarities between the temperature and the wind velocity profiles. Also, they found it possible to apply the Richardson number to their data with fair results. Giblett (6) indicated that the index of z in the power type equation could be closely correlated with the magnitude and sign of the temperature gradient. He found the index to be about 0.01 in high lapse rates and 0.62 in large inversions. Frost (5) also gave a value of the index ranging from 0.145 to 0.77, and Scrase (17) used a value of 0.13 for his index over downland in conditions of small temperature gradient. This study has indicated no such definite relationship for any one condition of stability. Perhaps, if sufficient data were secured such relationships would become evident.

The equation found for the velocity profiles under various atmospheric stability conditions are not conclusive, and there is a definite need for further corroborative efforts along these lines. Also, there seems to be a need for a re-investigation of the whole theory of atmospheric turbulence. A method for actually measuring the shearing force of the wind needs to be perfected. These measurements should then be compared with the present theoretical considerations in order to test the validity of the present theory.

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APPENDIX I

Derivation of General Expression for Shear for Power Law Distribution of Wind Velocity with Height

The Prandtl (Brunt (3)) defining equations for the shearing stresses in turbulent flow are:

$$-\frac{\tau}{\rho} = \ell^2 \left(\frac{du}{dz}\right)^2 \qquad (1)$$

$$\ell = k \frac{\frac{du}{dz}}{\frac{d^2u}{dz^2}} \qquad (2)$$

where τ = shear in pounds per square foot

ρ = mass density in $\frac{\text{lbs. sec.}^2}{\text{ft.}^4}$

ℓ = Prandtl's mixing length

k = von Kärmän's constant, usually taken as 0.40[1]

The general expression for wind velocity u as a function of height z is:

$$u = u_1\left(\frac{z}{z_1}\right)^n \qquad (3)$$

where u_1 = wind velocity at the constant reference height z_1. Thus,

$$\frac{du}{dz} = \frac{u_1}{z_1^{\,n}}\, nz^{(n-1)} \qquad (4)$$

and

$$\frac{d^2u}{dz^2} = \frac{u_1}{z_1^{\,n}}\, n(n-1)\; z^{(n-2)} \qquad (5)$$

---

[1] Sheppard (18) reports that over short grass k will have a value of 0.61 for unstable, 0.40 for stable, and 0.22 for inversion atmospheric conditions.

By substitution of equations (4) and (5) into equation (2)

$$-\ell = k \frac{z}{1 - n} \tag{6}$$

By substitution of equations (4) and (6) in equation (1)

$$\sqrt{\frac{\tau}{\rho}} = \frac{k\ z}{1 - n} \frac{u_1}{z^n} z^{n-1}\ n \tag{7}$$

Then from equation (3) above

$$u_1 = \frac{u}{(\frac{z}{z_1})^n}$$

which, when substituted into equation (7) above, gives

$$\sqrt{\frac{\tau}{\rho}} = \frac{k\ n\ u}{1 - n}$$

and

$$\tau = \rho \left(\frac{k\ n\ u}{1 - n}\right)^2 \tag{8}$$

APPENDIX II

Derivation of General Expression for Shear for Logarithmic Law Distribution of Wind Velocity with Height

Prandtl (Brunt (3)) defining equations given in Appendix I apply to this derivation. The general expression for wind velocity u with height z may be written as:

$$u = \frac{1}{n} \ln \frac{z}{z_o} \tag{1}$$

Thus,

$$\frac{du}{dz} = \frac{1}{nz} \tag{2}$$

and

$$\frac{d^2u}{dz^2} = -\frac{1}{n}\frac{1}{z^2} \tag{3}$$

By substitution of equations (2) and (3) in equation (2) of Appendix I,

$$-\ell = kz \qquad (4)$$

By substitution of equations (2) and (4) into equation (1) of Appendix I,

$$\sqrt{\frac{\tau}{\rho}} = k\frac{1}{n} \qquad (5)$$

From equation (1) above

$$\frac{1}{n} = \frac{u}{\ln\frac{z}{z_o}}$$

Thus,

$$\sqrt{\frac{\tau}{\rho}} = \frac{k\,u}{\ln\frac{z}{z_o}} \qquad (6)$$

or, converted to $\log_{10}$ and using 0.40 for k,

$$u = 5.75\sqrt{\frac{\tau}{\rho}}\,\log_{10}\frac{z}{z_o} \qquad (7)$$

If the generalized expression for wind velocity u with height z is modified to the form:

$$z - c = z_o\, e^{nu} \qquad (1)$$

then the following derivation applies. Write equation (1) as

$$u = \frac{1}{n}\ln\frac{z - c}{z_o} \qquad (2)$$

Thus,

$$\frac{du}{dz} = \frac{1}{n(z - c)} \qquad (3)$$

and

$$\frac{d^2u}{dz^2} = -\frac{1}{n(z - c)^2} \qquad (4)$$

By substitution of equations (3) and (4) into equation (2) in Appendix I,

$$\ell = k\,(z - c) \qquad (5)$$

By substitution of equations (3) and (5) in equation (1) of Appendix I,

$$\sqrt{\frac{\tau}{\rho}} = k\,\frac{1}{n} \qquad (6)$$

From equation (2) above

$$\frac{1}{n} = \frac{u}{\ln \frac{z - c}{z_o}}$$

Thus,

$$\sqrt{\frac{\tau}{\rho}} = \frac{k\,u}{\ln \frac{z - c}{z_o}} \qquad (7)$$

or, converted to $\log_{10}$ and using 0.40 for the value of k,

$$u = 5.75\sqrt{\frac{\tau}{\rho}}\,\log_{10}\frac{z - c}{z_o} \qquad (8)$$

APPENDIX III

Derivation of General Expression for Shear for Modified Logarithmic Law Distribution of Wind Velocity with Height

This derivation is based on the assumption that the Prandtl (Brunt (3)) defining equations would apply to a modified wind velocity-height profile of the form:

$$z = z_o\, e^{nu^P} \qquad (1)$$

This assumption may or may not be valid. Assuming that it is, the following derivation applies: Write equation (1) as

$$u = \left(\frac{1}{n}\right)^{\frac{1}{P}} \left(\ln \frac{z}{z_o}\right)^{\frac{1}{P}} \qquad (2)$$

Thus,

$$\frac{du}{dz} = \left(\frac{1}{n}\right)^{\frac{1}{P}} \frac{1}{P} \left[\left(\ln \frac{z}{z_o}\right)^{\frac{1}{P} - 1} \frac{1}{z} \right] \qquad (3)$$

and,

$$\frac{d^2u}{dz^2} = -\left(\frac{1}{n}\right)^{\frac{1}{P}} \frac{1}{P} \frac{1}{z^2} \left(\ln \frac{z}{z_o}\right)^{\frac{1}{P} - 1} \left[1 + \left(\frac{1}{P} - 1\right)\left(\ln \frac{z}{z_o}\right)^{-1} \right] \qquad (4)$$

By substitution of equations (3) and (4) into equation (2) of Appendix I,

$$-\ell = \frac{k\, z}{\left[1 + \left(\frac{1}{P} - 1\right)\left(\ln \frac{z}{z_o}\right)^{-1} \right]} \qquad (5)$$

By substitution of equations (3) and (5) into equation (1) of Appendix I,

$$\sqrt{\frac{\tau}{\rho}} = \frac{k\left(\frac{1}{n}\right)^{\frac{1}{P}} \frac{1}{P} \left(\ln \frac{z}{z_o}\right)^{\frac{1}{P}}}{\ln \frac{z}{z_o} + \left(\frac{1}{P} - 1\right)} \qquad (6)$$

From equation (2) above

$$\left(\frac{1}{n}\right)^{\frac{1}{P}} = \frac{u}{\left(\ln \frac{z}{z_o}\right)^{\frac{1}{P}}} \qquad (7)$$

Thus,

$$\sqrt{\frac{\tau}{\rho}} = \frac{k \frac{1}{P} u}{\ln \frac{z}{z_o} + \left(\frac{1}{P} - 1\right)} \qquad (8)$$

or, converted to $\log_{10}$ and using $k = 0.40$,

$$u = 5.75\, P \sqrt{\frac{\tau}{\rho}} \log_{10} \frac{z}{z_o} + \left(\frac{1}{P} - 1\right) \qquad (9)$$

A STUDY OF THE ATMOSPHERIC WIND GRADIENT IN THE LAYER NEAR THE GROUND

by

NEIL PARKER WOODRUFF

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AN ABSTRACT OF A THESIS

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ABSTRACT

The objectives of this study were: (1) to review and summarize previous research on the atmospheric wind gradient near the ground, (2) to determine the effect of various crops and tillage methods on the average shearing force of the wind, (3) to study and describe the velocity fluctuations of the wind, (4) to determine the possible effect of various crops and tillage methods on velocity fluctuations and associated shear, and (5) to determine the effect of atmospheric stability on the wind gradient near the ground and to describe these effects in terms of functional relationships for each of three atmospheric conditions, i.e., stable, unstable, and inversional.

A summary is given of the more important previous research on the subject. A review of the basic theory of Prandtl (Brunt (3)), Nikuradse (Rouse (16)), and von Kármán (24) is included as well as some of the results and findings of researchers who have worked on specific phases of the problem. The wide variations and the differences in the results in these investigations point to the lack of a definite knowledge of the complex problem.

The experimental equipment used to measure wind velocities was a combination of Pitot tubes and a multiple manometer or a combination of whirling cup-type anemometers and weather station recorder. These measuring devices were mounted at various heights ranging from 1 inch to 31 feet, depending upon the phase of the problem being considered. Measurements of temperature and humidity were obtained from hygrothermographs placed at three different heights. Wind direction was obtained from directional vanes mounted on the anemometer poles.

The data to determine the effect of various crops and tillage methods on the average shearing force of the wind were obtained in mid-afternoon under

an unstable atmospheric condition. The resultant wind profile was found to be concave in nature. If a slight adjustment in base of measurement was made the data could be fitted to the logarithmic law of wind velocity and height. Values of shear and the roughness parameter, z_o, were calculated from these relationships. A value of z_o equal to $\frac{1}{13}$ the average crop height was found for surfaces having vegetative cover on them. z_o for the ridged or plowed surfaces was approximately $\frac{1}{40}$ the average linear height of roughness. The results indicated that the average shear tends to remain constant over all surfaces for a given wind.

The velocity fluctuations of the wind are studied by statistical methods. The velocity fluctuations about the average for a given period were fitted to a normal error distribution and measured for goodness of fit by the Chi-square test. It was found that the velocity fluctuations of the wind could be described by the normal error law with a high degree of probability. The properties of the normal curve and the standard deviation of the velocities were used to predict extremes in velocity fluctuations and corresponding shear. The differences in the magnitude of the fluctuations from day to day indicate the importance of associating velocity fluctuations with determinations of the shearing force of the wind. It was noted that the shear was as much as $7\frac{1}{2}$ times the average for short periods and 3 times the average approximately 10% of the time.

The preceeding methods were applied to specific crops and tillage conditions. In general, the difference in effect of one crop as compared to another was minor in importance. The magnitude of the fluctuations from day to day apparently was more important than the effect of the immediate ground cover.

The results of the study to determine the effect of atmospheric stability on the wind gradient are presented in two parts: (1) the temperature-height relationships and, (2) the wind velocity-height relationships under stable, unstable, and inversional conditions. The temperature decreases approximately proportional to the logarithm of the height for stable conditions, i.e. early morning, and increases proportional to the logarithm of the height in late evening during an inversion. However, during mid-day, when an unstable condition is present, the temperature first decreases near the ground then increases at higher elevations. The relationship may be expressed by a second degree polynomial equation. The wind velocity-height relationships indicate that under the average condition during a stable period the wind varies approximately as the logarithm of the height, and, therefore, can be expressed as a straight logarithmic function. The average condition during an inversional period shows that the logarithm of the wind velocity is approximately proportional to the logarithm of the height and can be expressed as a log log or power function. The average relationship during an unstable period can be expressed two ways: the wind velocity raised to a power varies as the logarithm of the height, or, the velocity is proportional to a modified logarithm of the height. Consideration of existing theory and the extreme values the power equation yields indicates that it is not valid; therefore, as a matter of expediency, the best expression for the unstable condition is probably the modified logarithmic equation.

This study has not been conclusive in all instances. This is particularly true in those parts of the study dealing with the effects of atmospheric stability on the wind gradient. A definite need for further corroborative efforts along these lines exists; also there seems to be a need for re-investigation of the whole theory of atmospheric turbulence. A method for actually measuring the shearing force of the wind needs to be developed.