STRUCTURAL ANALYSIS OF A CABLE STIFFENED-GIRDER BRIDGE

by

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SYNOPSIS

This report presents two possible methods of structrual analysis for cable-stiffened girder bridges. One is to solve a reduced base structure, then apply these results to obtain a complete solution and the other is to solve the complete structure as one problem. In this report the reduced structure is solved first by direct moment distribution and then by the displacement method. The problem is also solved by the force method considering the entire structure as one problem.

The method which takes advantage of the reduced structure consists of three stages, the first stage is the use of some method, in this case, direct moment distribution or the displacement method, to find the influence lines for the base structure which is, in this case, a continuous beam on elastic supports. The second stage is the use of the reciprocal theorem to find influence lines for the cable tensions. In the final stage, the influence lines for the base structure and for the cable tensions are combined to find the influence lines for the complete structure.

The entire structure is also solved by the force method utilizing the digital computer to facilitate computation.

Numerical examples are given to illustrate the procedures for finding the influence lines for the structure. Since the direct moment distribution method, the displacement method and the force method are used to solve the same problem, a comparison of the results of the various methods can be made. The comparison shows that the solutions agree closely with each other. Various influence lines for the structure are also drawn.

INTRODUCTION

The two essential conditions for economy in bridge design are an efficient utilization of material and maximum reduction of the dead weight of the structure.

These conditions for economy in bridge construction are met by using the cable-stiffened girder in conjunction with either a deck of orthotropic steel plate or of prestressed concrete (1)[1].

Cable-stiffened bridges occupy a middle position between unstiffened girder bridges and suspension bridges. The steel plate or prestressed concrete deck is usually light in weight and straight cables may be used to prestress the concrete girders or steel box girders. Bridges of this type may be advantageously used for spans exceeding the capacity of unstiffened girders. An advantage of this type bridge compared with a suspension bridge is its greater rigidity. Two of the possible types of cable-stiffened bridges are: (1) cables intersecting at the top of a tower system, Fig. 1(a); and, (2) parallel-cable system, Fig. 1(b).

There are many possible methods of analysis that can be employed in analyzing this type of structure. However many of these methods involve laborious procedures in solving the resulting simultaneous equations for these high-order statically indeterminate bridges.

This study takes a beam on elastic supports as a base structure and solves the symmetrical and anti-symmetrical load conditions on the beam by direct moment distribution, and then analyzes the influence lines for the

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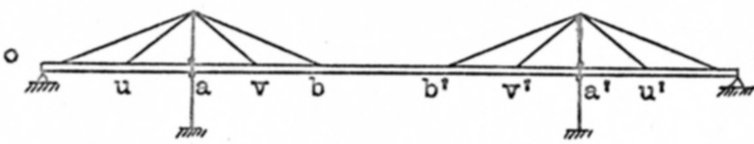
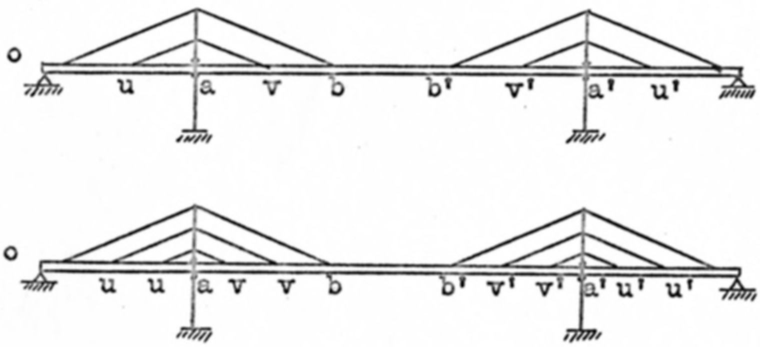
[1]Numbers in parentheses refer to references listed in the Bibliography.



(a) Cables intersecting at the top of a tower system.



(b) Parallel-cable systems.

Fig. 1. Types of cable-stiffened bridges.

bridge by the reciprocal theorem. This procedure is found to be a greatly simplified method of structural analysis for the cable-stiffened girder bridge. The end results have been checked through the use of the stiffness and flexibility methods with a digital computer for ease of computation.

In comparing the moment distribution method with the displacement and force methods, the later are of course much simpler whenever a program and a computer are available. However, the moment distribution method and the slide rule are familiar to most practicing engineers and therefore, this hand-computing method will have a place in many design offices.

EVOLUTION OF CABLE-STIFFENED GIRDER BRIDGE CONSTRUCTION

Cable-stiffened girder bridges were developed after World War II. It was a time of extreme shortages of construction materials to rebuild the German bridges which had been destroyed in the war. The only hope for engineers was to use their intelligence to design new types of construction which would be safe, durable, economical in cost and in quantity of construction materials, and pleasing architecturally all at the same time. Soon various kinds of orthotropic steel plate deck bridges, and cable-stiffened girder bridges were designed and to date have been very successful bridge structures.

In 1957, the Duesseldorf-North Bridge at Duesseldorf, Germany, was opened to traffic (1). This is a continuous girder bridge stiffened by a parallel-cable system. The span lengths are 354-853-354 ft. The bridge cross section consists of a steel plate deck 9/16 in. thick, acting in conjunction with the girders. The longitudinal ribs, spaced 16 in. o.c., are made of 8 x 4 x 7/16 in. angles, spaced 6.1 ft. apart, running continuously through rectangular cutouts in the floor beams.

In 1960, the Severin bridge across the Rhine at Cologne, Germany, was completed and opened to traffic after three years construction (2). The bridge is unique in its use of an asymmetric suspension system carried by a single A-shaped steel tower. To save weight, the design incorporates two continuous steel box girders. The steel bridge deck is a stiffened orthotropic plate that serves as the top flange of the girders and of the floor beams, and also supports the roadway. Spanning two highways, two harbor basins, a wharfage area, and the river, the seven spans of the bridge have a total length of 2,226 ft. The two suspension spans are 990 ft. and 495 ft. long. Each span is supported by six sets of cables that fan out from the tops of the towers. Each of the suspenders consists of a rectangular or square arrangement of steel wire cable.

In 1962, the five and one-half mile long bridge spanning the end of Venezuelas' Lake Maracaibo was opened to traffic (3). The bridge is one of the world's greatest prestressed concrete structures. It includes five 771 ft. cable-stiffened continuous girder spans, which clear the lake's shipping channels by 148 ft. The main girders of these spans are six 620 ft. long continuous prestressed beams. The continuous beam is prestressed internally by steel cables and externally by the horizontal component of the oblique tension members. The latter are prestressed to compensate for the tension caused by the dead load deflection of the cantilever beams. This means that the only lowering of this support is that caused by moving loads. Maximum design force in the cable is 6,586 tons; minimum force is 6,027 tons.

THE RELATIONSHIPS BETWEEN THE SUSPENSION POINT DEFLECTIONS AND THE OBLIQUE CABLE TENSIONS

The towers of suspension bridges are usually hinge supported at the bottom in order to eliminate bending moments in the towers. In Fig. 2(a), a tower column OO_1O_2 has a hinge at O, and has a roller at O_2. Since the summation of the horizontal forces at O_2 must be equal to zero, we observe that the left and right cable tension at that joint are equal, provided the

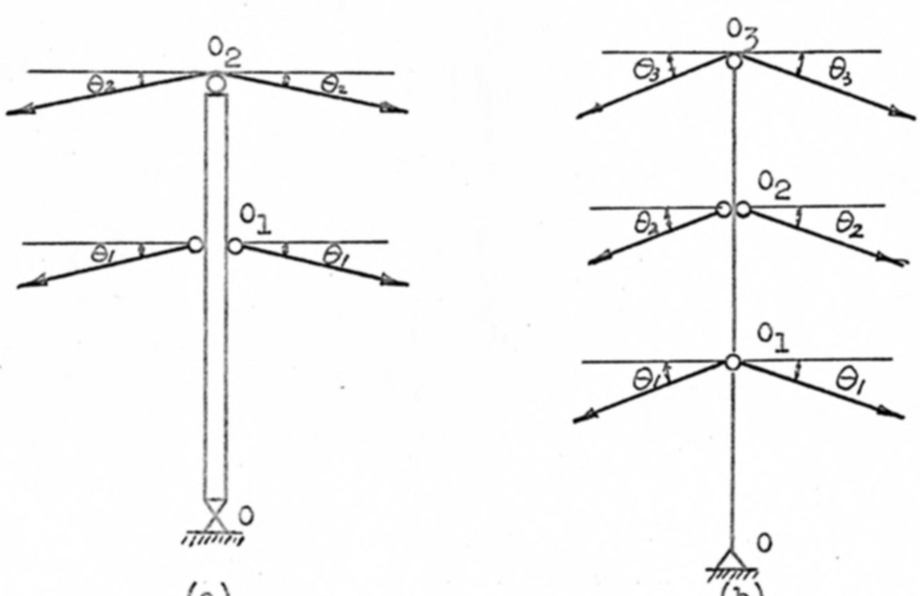


Fig. 2. Tower columns.

angles of the cables θ_2 are equal. Further, since the two subcables are connected to the tower column at point O_1 with an equal angle, the moment at O is zero, we observe that the tension forces are equal at that joint.

Similarly, in Fig. 2(b), two cables pass through rollers O_1 and O_3, while the other two cables are connected to the tower column at O_2. Since there is a hinge at the bottom of the tower column, we observe that the cable tensions are equal at each joint.

In Fig. 3, OB is a tower column, uB and vB represent two cables on the left and right of the tower. If we assume small deflections, that is, that

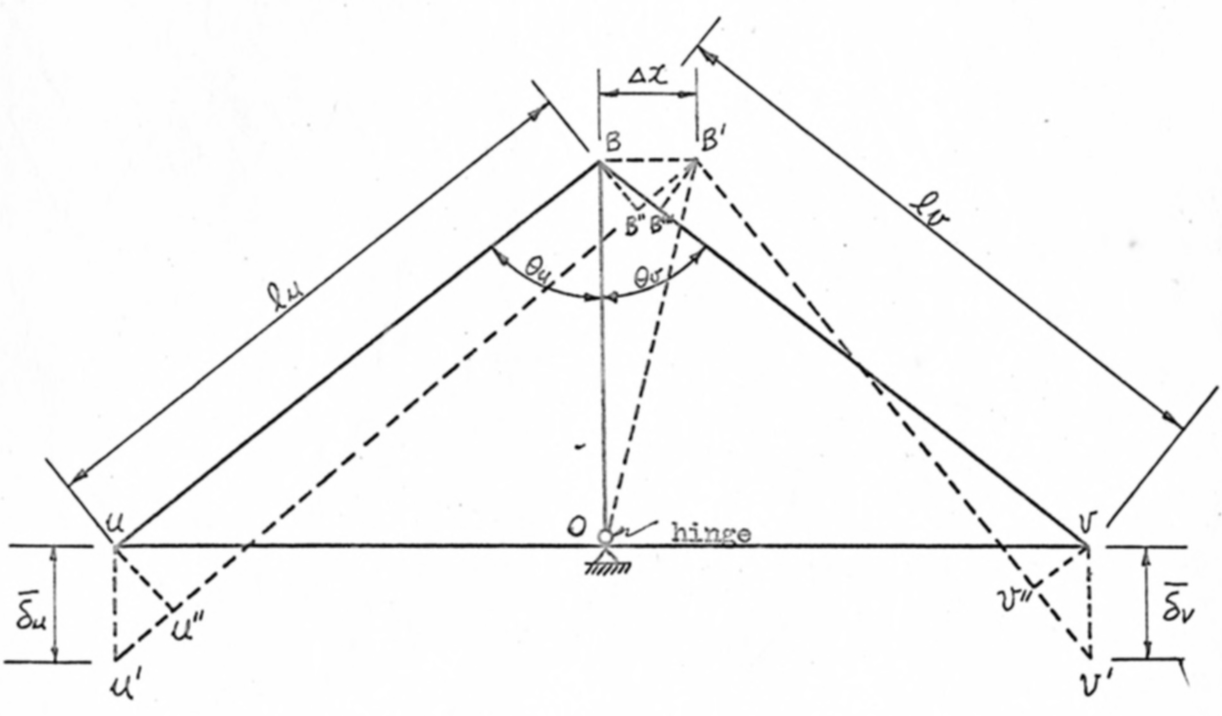


Fig. 3. Tower-Cable Displacement Diagram.

the angles between the cables and column does not change as the deformation takes place and that the horizontal deformations of points u and v are negligible we may then deduce several geometrical relations as follows:

Let $\bar{\delta}u$ and $\bar{\delta}v$ be the final vertical displacements at points u and v (assume downward displacement is positive), and let

Δx = the horizontal displacement of point B of the tower column,

θu, θv = the angles between the cables and the column,

ℓu, ℓv = the original length of the cables,

$\Delta \ell u$, $\Delta \ell v$ = the elongations of the cables,

Ac = the cross sectional areas of the cables,

Ec = the modulus of elasticity of the cables,

Tu, Tv = the tensions in the cables (when $\theta u = \theta v$, $Tu = Tv = T$).

Considering the left hand side of the tower column, we observe that

$$\Delta \ell u = B'u' - B''u'' = B'u' - Bu = B'B'' + u'u'', \text{ because } B''u'' \doteqdot Bu.$$

Thus, $\Delta \ell u = \Delta x \sin\theta_u + \delta u \cos\theta_u$. (1)

On the other hand, considering the right side of the tower, we find

$$\Delta \ell v = B'v' - Bv$$

Since $B'v' = B'v'' + v''v'$, $Bv = BB''' + B''' v$, and $B'''v = B'v'$,

the elongation of the cable Bv becomes

$$\Delta \ell v = B'v'' + v''v' - BB''' - B'v''$$

$$= v''v' - BB'''$$

$$= \delta v \cos\theta v - \Delta x \sin\theta v. \quad (2)$$

Adding equations (1) and (2), we obtain

$$\Delta \ell u + \Delta \ell v = \Delta x(\sin\theta u - \sin\theta v) + \delta u \cos\theta u + \delta v \cos\theta v.$$

When $\theta u = \theta v = \theta$, and $\ell u = \ell v = \ell$,

we have $Tu = Tv = T$, and $\Delta \ell u = \Delta \ell v = \Delta \ell$.

Therefore, $2\Delta \ell = (\delta u + \delta v) \cos\theta$.

But $\Delta \ell = \frac{T\ell}{E_c A_c}$.

Therefore, $\frac{2T\ell}{E_c A_c} = (\delta u + \delta v) \cos\theta$. (3)

From Fig. 4, since $\delta o = 0$, and assuming that the final position of point

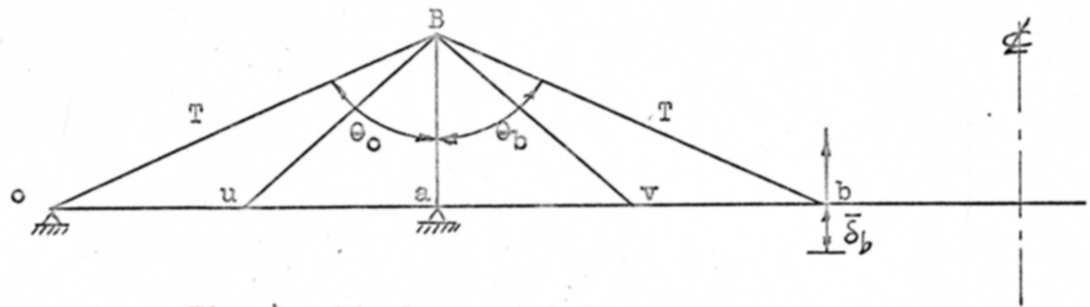


Fig. 4. Displacement-Deformation Diagram

b is $\bar{\delta}$b downward from its original position, and that the vertical component of cable tension is T', we find that T' = Tcosθ, whenever $\theta_o = \theta_b = \theta$.
Substituting into equation (3), we obtain

$$\frac{2T\ell}{E_c A_c \cos\theta} = \bar{\delta}_b,$$

or $$T' = \frac{\bar{\delta}_b E_c A_c}{2\ell}\cos^2\theta.$$

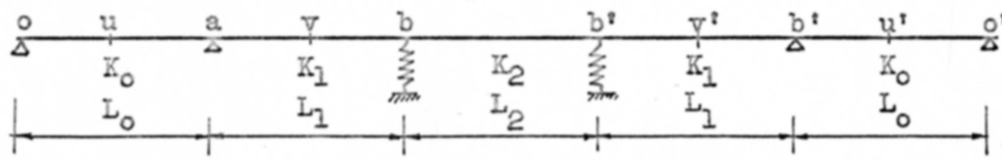
Letting C_b = Stiffness of the cable, oBb, and $\bar{\delta}$b = Unity, we have

$$T' = \frac{E_c A_c}{2\ell}\cos^2\theta. \tag{4}$$

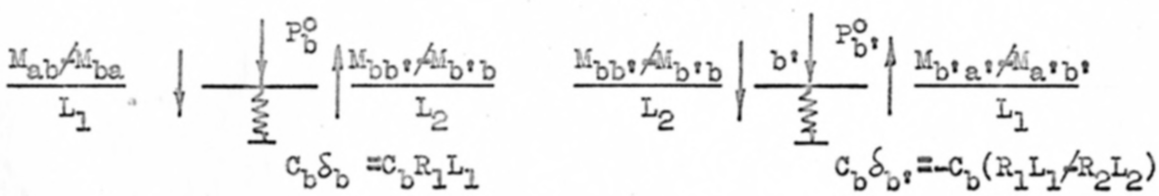
CHOICE OF THE BASE STRUCTURE AND ITS SLOPE DEFLECTION EQUATIONS

Among the general types of cable stiffened girder bridges, as shown in Fig. 1, type (a) is a statically indeterminate structure of the tenth degree. If we want to solve a structure, such as type (a), by the deflection method, we have to solve eight equations for eight unknowns for each load. Obviously, this involves tedious and laborious procedures. Therefore, choosing a base structure, and then solving for the influence lines for the base structure are necessary steps for simplifying the solution.

Generally when solving statically indeterminate structures the choice of a base structure would be a statically determinate structure (4). However, in this case, the choice of a four degree statically indeterminate structure as a base structure is very advantageous (Fig. 5).



(a) Selected base structure.



(b) Free bodies of the elastic supports.

Fig. 5. Base Structure and Supports

The reasons for choosing this type of base structure are that the base structure has a deformation pattern which very closely resembles that of the actual structure and, since one of the cables begins at support o, passes through the top of the tower and is connected to the girder at b, we observe that the deformation of point b is the only source of tension in the cable. If $\delta b = 1$, the corresponsing vertical component force of the cable would be the stiffness of the elastic supports.

Fig. 5(a) shows the symmetrical base structure. Assume that:

θ_a, θ_b, θ_b', θ_a' = the angle changes at a, b, b' and a' due to any load.

R_1, R_2 = the rigid body rotations of beam ab and bb'.

K_o, K_1, K_2, K_1, K_o = the stiffnesses of beams oa, ab, bb', b'a' and a'o'.

M_{mn}^F, $M_{mn}^{F}{}'$ = the fixed end moments at m of beam mn. $M_{mn}^{F}{}'$ is the fixed end moment at m while n is a hinge support.

P_b^o, $P_{b'}^{o}$ = reactions at elastic support b and b'.

$$m = \frac{L_1}{L_2},\ n = \frac{EK_1}{EK_2},\ C_b^o = C_{b'}^{o} = \frac{C_b L_1^2}{EK_1}.$$

From the conditions of equilibrium at each joint, we have the equations:

$\Sigma M_a = 0$, $\Sigma M_b = 0$, $\Sigma M_{b'} = 0$, $\Sigma M_{a'} = 0$, $\Sigma V_b = 0$ and $\Sigma V_{b'} = 0$.

Therefore, the slope deflection equations in terms of θ_a, θ_b, $\theta_{b'}$, $\theta_{a'}$, R_1 and R_2 would be as follows:

$$\begin{bmatrix} 3K_o+4K_1 & 2K_1 & 0 & 0 & -6K_1 & 0 \\ 2K_1 & 4(K_1+K_2) & 2K_2 & 0 & -6K_1 & -6K_2 \\ 0 & 2K_2 & 4(K_1+K_2) & 2K_1 & 6K_1 & -6(K_2-\frac{K_1}{m}) \\ 0 & 0 & 0 & 3K_1+3K_o & 6K_1 & \frac{6K_1}{m} \\ 6K_1 & 6\frac{n-m}{n}K_1 & -\frac{6m}{n}K_1 & 0 & (12+C_b^o)K_1 & 12mK_2 \\ 0 & 6mK_2 & 6(mK_2-K_1) & 0 & C_b^oK_1+12K_1 & 12mK_2+\frac{C_b^oK_1}{m}+\frac{12K_1}{m} \end{bmatrix} \begin{Bmatrix} \theta_a \\ \theta_b \\ \theta_{b'} \\ \theta_{a'} \\ R_1 \\ R_2 \end{Bmatrix}$$

$$= \frac{-1}{E} \begin{Bmatrix} M_{ao}^F + M_{ab}^F \\ M_{ba}^F + M_{bb'}^F \\ M_{b'b}^F + M_{b'a'}^F \\ M_{a'b'}^F + M_{a'o'}^F \\ P_b^oL_1 + M_{ab}^F + M_{ba}^F - m(M_{bb'}^F + M_{b'b}^F) \\ P_{b'}^oL_1 + M_{b'a'}^F + M_{a'b'}^F + m(M_{bb'}^F + M_{b'b}^F) \end{Bmatrix} \qquad (5)$$

However, solving the equations for different loadings is still laborious. If we use the conventional moment distribution method in solving the base structure, because of the slow convergence, it would take a great deal of time. Therefore, a much simplified procedure is desired.

DIRECT MOMENT DISTRIBUTION FOR THE BASE STRUCTURE

The beam in Fig. 5(a) is a symmetrical continuous beam which has elastic supports at b and b'. The moments at hinges o and o' are zero and the modified stiffnesses for members oa and o'a' are 3EK which will be denoted as K'. Thus, the ends a and a' can be taken as elastic springs with stiffnesses of K'. Let u_a denote the moment distribution factors at a and a' of the members ab and a'b', and suppose that there are consecutively symmetrical and anti-symmetrical moments and concentrated loads acting on b and b' of the beam as shown in Figs. 6 to 9. M_{ab} and M_{ba} can then be found for each case:

(1) Symmetrical loads

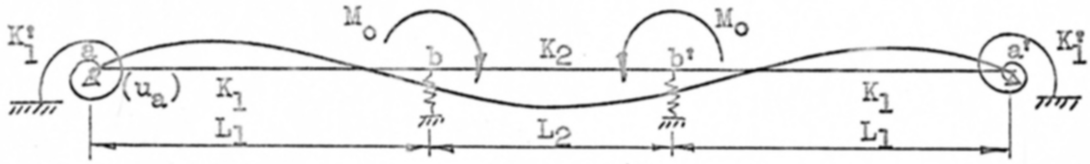
(A) Symmetrical moment M_o at middle supports b and b'.

As shown in Fig. 6(a), a symmetrical moment may cause the beam aa' to have a final rotation θ_b and a final vertical translation Δ_b when the beam is in its deformed position. By the method of superposition, the deformations can be taken as the summation of Figs. 6(b), (c), (d).

In Fig. 6(b) we assume that the member is fixed against translation but has a resisting moment at b due to the rotation θ_b. Taking the end a as the fixed end and carrying over half of the resisting moment to the end a, then releasing the unbalanced moment at the end a and carrying over half of that moment to the end b, which is fixed during that time since it must maintain its final rotation θ_b, we can express the sum of the moments at each end as

$$M'_{ab} = 2EK_1 (1 - u_a)\theta_b \text{ and } M'_{ba} = EK_1 (4 - u_a)\theta_b.$$

On the other hand, assuming that the member ab is fixed against rotation but has a vertical deflection Δ_b as shown in Fig. 6(c), we have by the same



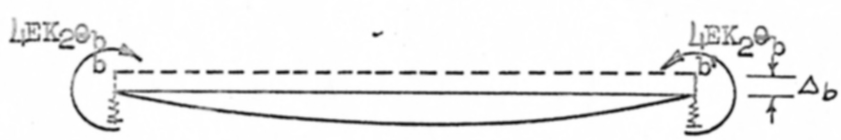
(a) Symmetrical moment acting at middle supports.

F.E.M.		$4EK_1\theta_b$
C.O.	$2EK_1\theta_b$	
Distr.	$-2EK_1u_a\theta_b$	
C.O.		$-EK_1u_a\theta_b$
Total	$2EK_1(1-u_a)\theta_b$	$EK_1(4-u_a)\theta_b$

(b) M_{ab} and M_{ba} due to the final rotation, θ_b.

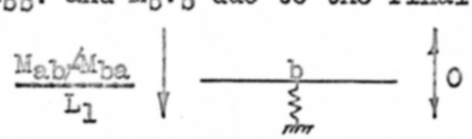
F.E.M.	$-6EK_1R_1$	$-6EK_1R_1$
Distr.	$6EK_1U_aR_1$	
C.O.		$3EK_1u_aR_1$
Total	$-6EK_1(1-u_a)R_1$	$3EK_1(u_a-2)R_1$

(c) M_{ab} and M_{ba} due to the final deflection, $_b$.



F.E.M.	$4EK_2\theta_b$	$-4EK_2\theta_b$
C.O.	$-2EK_2\theta_b$	$2EK_2\theta_b$
Total	$2EK_2\theta_b$	$2EK_2\theta_b$

(d) $M_{bb'}$ and $M_{b'b}$ due to the final rotation, θ_b.



(e) Free body of support b.

Fig. 6. Derivation of direct moment distribution factors for symmetrical moment at the middle supports of the base structure.

reasoning as the previous analysis,

$$M''_{ab} = 6EK_1(u_a - 1)R_1 \text{ and } M''_{ba} = 3EK_1(u_a - 2)R_1.$$

The total moments at each end are

$$M_{ab} = 2EK_1(1 - u_a)(\theta_b - 3R_1) \tag{6}$$

and
$$M_{ba} = EK_1[(4 - u_a)\theta_b + 3(u_a - 2)R_1]. \tag{7}$$

Because of the symmetrical moment of Fig. 6(d), we have

$$M_{bb'} = -M_{b'b} = 2EK_2\theta_b. \tag{8}$$

Furthermore, at support b, as shown in Fig. 6(e), we have

$$\frac{M_{ab} + M_{ba}}{L_1} = C_bL_1K_1 = C^o_b\frac{EK_1}{L_1}R_1$$

or
$$M_{ab} + M_{ba} = C^o_bEK_1R_1. \tag{9}$$

Substituting equations (6) and (7) in equation (9), we have

$$2(1 - u_a)(\theta_b - 3R_1) + (4 - u_a)\theta_b + 3(u_a - 2)R_1 = C^o_bR_1,$$

from which

$$R_1 = \frac{3(2 - u_a)}{12 - 9u_a + C^o_b}\theta_b. \tag{10}$$

Substituting the value of R_1 into equations (6) and (7), we have

$$M_{ab} = 2EK_1(1 - u_a)\frac{C^o_b - 6}{12 - 9u_a + C^o_b}\theta_b, \tag{11}$$

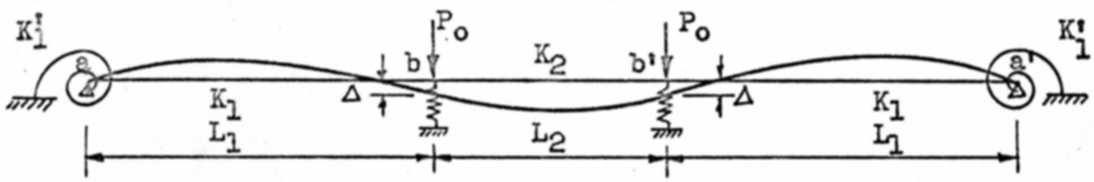
and
$$M_{ba} = EK_1\frac{12(1 - u_a) + C^o_b(4 - u_a)}{3(4 - 3u_a) + C^o_b}\theta_b. \tag{12}$$

The sign conventions of this paper take clockwise moments, rotations and translations as positive and counterclockwise moments, rotations and translations as negative.

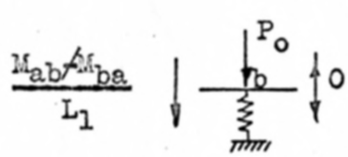
The stiffness S_{ba} is defined as $\frac{M_b}{\theta_b}$ and the carry-over factor C_{ba} as the ratio $\frac{M_{ab}}{M_{ba}}$. Therefore, taking $\theta_b = 1$ in equations (11) and (12), we obtain the modified stiffnesses and carry-over factors for member ab. Then direct moment distribution can be carried out in one cycle for any loading which results in symmetrical fixed-end moments.

(B) Concentrated load at middle supports b and b'.

Consider the member bb' of Fig. 7(a). We observe there are no rotations at points b and b' due to the concentrated loads. Further, we observe that there is a symmetrical vertical deflection. Therefore, the fixed-end moments $M^F_{bb'} = M^F_{b'b} = 0$.



(a) Symmetrical concentrated load at middle supports.



(b) Free body of elastic support b.

Fig. 7. Derivation of direct moment distribution factors for symmetrical concentrated load acting at middle supports of the base structure.

However, there are moments M_{ab} and M_{ba} due to the final deflection, Δ_b, these moments have been found in section (A), and are shown in Fig. 6(c).

Considering the free body of Fig. 7(b), we have $\Sigma V = 0$, that is,

$$\frac{M_{ab} + M_{ba}}{L_1} + P_o = C_b L_1 R_1 = C_o^b \frac{EK_1}{L_1} R_1. \qquad (13)$$

But $\quad M_{ab} = 6EK_1R_1\ (u_a - 1)$

and $\quad M_{ba} = 3EK_1R_1\ (u_a - 2).$

Substituting in equation (13), we obtain

$$R_1 = \frac{P_oL_1}{EK_1(C_b^o - 9u_a + 12)}. \tag{14}$$

Substituting the value of R_1 into equation (7), the final end moment M_{ab} due to the deflection Δ_b is

$$M_{ab}^{\Delta} = -\frac{6(1 - u_a)}{C_b^o - 9u_a + 12} P_oL_1. \tag{15}$$

At the same time, the fixed-end moment ba is

$$M_{ba}^{F\Delta} = \frac{3(u_a - 2)}{C_b^o - 9u_a + 12} P_oL_1. \tag{16}$$

If we consider the fixed-end moments $M_{ba}^{F\Delta}$ and $M_{ab}^{F\Delta}$ as external symmetrical loads, we can obtain the moments due to the concentrated load P_o by use of the modified distribution factors and carry-over factors derived for symmetrical moment acting at point b and b' in case (A).

(2) Anti-symmetrical moment at middle supports b and b'.

(C) Anti-symmetrical moment at middle supports b and b'.

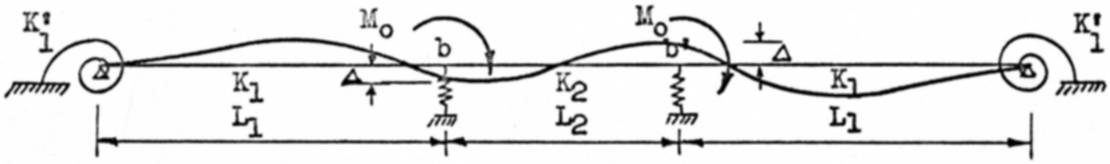
Consider the final deformations of member aa', θ_b and Δ_b due to the anti-symmetrical moment of Fig. 8. The end moments for member ab and b'a' are the same as those given by equations (6) and (7). However, some additional consideration must be given member bb'.

If the translation angle for member ab is R_1 as shown in Fig. 8, the translation angle for member bb' should be $-2mR_1$, where $m = \frac{L_1}{L_2}$. The minus sign denotes a counterclockwise translation. Thus, the slope-deflection equation is

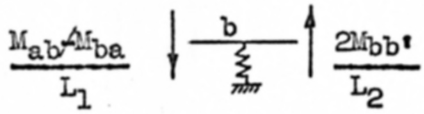
$$M_{bb'} = 2EK_2(2\theta_b + \theta_b' + 6mR_1), \text{ where } \theta_b = \theta_b .$$

From which

$$M_{bb'} = 6EK_2(\theta_b + 2mR_1). \tag{17}$$



(a) Anti-symmetrical moment at middle supports.



(b) Free body of elastic support b.

Fig. 8. Derivation of direct moment distribution factors for anti-symmetrical moment acting at middle supports of the base structure.

Since $\Sigma V = 0$ at the elastic support, then

$$\frac{M_{ab} - M_{ba}}{L_1} - \frac{2M_{bb'}}{L_2} = C_b L_1 R_1 = C_b^o \frac{EK_1}{L_1} R_1. \tag{18}$$

Substituting equations (6) and (7) in equation (18), we have

$$\frac{EK_1}{L_1}\left[2(1 - u_a)(\theta_b - 3R_1) + (4 - u_a)\theta_b - 3(u_a - 2)R_1\right] - \frac{12EK_2(\theta_b + 2mR_1)}{L_2} =$$

$$C_b^o \frac{EK_1}{L_1} R_1.$$

Since $\frac{EK_1}{EK_2} = n$, and $\frac{L_1}{L_2} = m$, we have

$$3n\left[(2 - u_a)\theta_b - (4 - 3u_a)R_1\right] - 12m(\theta_b + 2mR_1) = C_b^o n R_1,$$

from which

$$R_1 = \frac{3\left[n(2 - u_a) - 4m\right]}{n(C_b^o + 12 - 9u_a) + 24m^2}\,\theta_b. \tag{19}$$

Substituting equation (19) into equation (6), we have

$$M_{ab} = 2EK_1(1 - u_a)\frac{n(C_b^o - 6) + 12m(3 + 2m)}{n(C_b^o + 12 - 9u_a) + 24m^2}\,\theta_b. \tag{20}$$

Again, substituting equation (19) into equation (7), we obtain

$$M_{ba} = EK_1\,\frac{24m(4m + 3) + 4n(C_b^o + 3) - u_a(nC_b^o + 12n + 36m + 24m^2)}{n(C_b^o + 12 - 9u_a) + 24m^2}\,\theta_b. \tag{21}$$

Substituting the expression for R_1 into equation (6), we obtain

$$M_{bb'} = M_{b'b} = 6EK_2n\frac{C_b^o + 12(1 + m) - 3(3 + 2m)u_a}{n(C_b^o + 12 - 9u_a) + 24m^2}\,\theta_b. \tag{22}$$

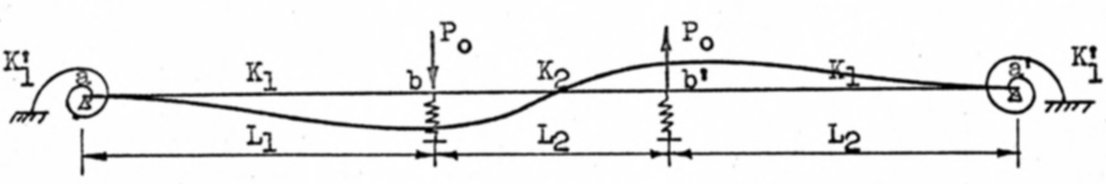
When θ_b=1 in equations (21) and (22), M_{ba} and $M_{bb'}$ are the stiffnesses of member ba and bb' at the end b. Further, the carry-over factor from b toward a for member ba is

$$C_{ba} = \frac{M_{ab}}{M_{ba}}.$$

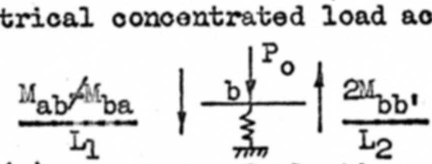
(D) Anti-symmetrical concentrated load at middle supports b and b'.

Consider the vertical deflections at b and b' in Fig. 9(a). In this case we have M_{ab}=$6EK_1R_1(u_a - 1)$ and M_{ba}=$3EK_1R_1(u_a - 2)$ as in equations (6) and (7). In addition, the slope-deflection equation for member bb' is

$$M_{bb'} = 6EK_2(2mR_1). \tag{23}$$



(a) Anti-symmetrical concentrated load acting at middle supports.



(b) Free body of elastic support b.

Fig. 9. Derivation of direct moment distribution factors for anti-symmetrical concentrated load acting at middle supports of the base structure.

Since $\Sigma V = 0$ at the elastic support in Fig. 9(b), we have

$$\frac{M_{ab} + M_{ba}}{L_1} + P_o - \frac{2M_{bb'}}{L_2} = C_b^o \frac{EK_1}{L_1} R_1. \qquad (24)$$

Substituting equations (6), (7) and (23) into equation (24), and simplifying, we have

$$R_1 = \frac{P_o L_1}{EK_2 \left[C_b^o n + 24m^2 + 3n(4 - 3u_a) \right]}. \qquad (25)$$

Substituting equation (25) into equations (6), (7) and (23), we have

$$M_{ab}^{\Delta} = \frac{6n(u_a - 1)\, P_o L_1}{C_b^o n + 24m^2 + 3n(4 - 3u_a)}, \qquad (26)$$

$$M_{ba}^{F\Delta} = - \frac{3n(2 - u_a)\, P_o L_1}{C_b^o n + 24m^2 + 3n(4 - 3u_a)} \qquad (27)$$

and

$$M_{bb'}^{F\Delta} = - \frac{12 m P_o L_1}{C_b^o n + 3n(4 - 3u_a) + 24m^2}. \qquad (28)$$

If we take $M_{ba}^{F\Delta} + M_{bb'}^{F\Delta} = M_b$, the anti-symmetrical moment at the middle supports, we can find the end moments directly by use of the modified distribution factors and carry-over factors for the anti-symmetrical moment acting at point b and b' as in case (C).

INFLUENCE COEFFICIENTS FOR THE BASE STRUCTURE

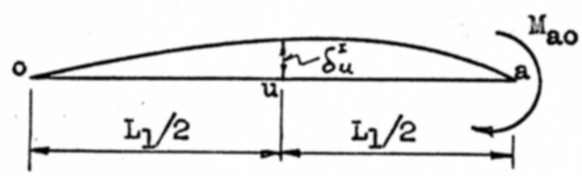
Points u, v, u' and v' are located on the base structure as shown in Fig. 5. The vertical displacements of these points can be determined as follows:

(1) The vertical displacement of point u: the total displacement at point u due to a unit load in span oa can be separated into two parts. One part is

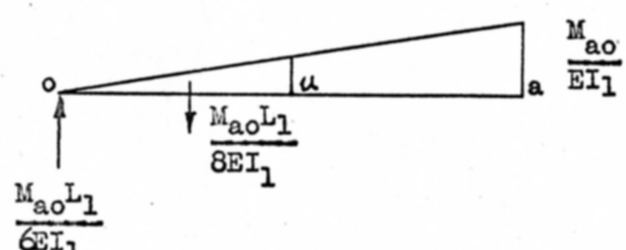
the deflection due to the end moment M_{ao} of member oa, and the other part is due to the unit load on the beam oa.

The vertical deflection due to M_{ao}, as shown in Fig. 10, can be formulated as below:

$$\delta_u^I = -\left(\frac{M_{ao}L_1}{6EI_1} \times \frac{L_1}{2} - \frac{M_{ao}L_1}{8EI_1} \times \frac{L_1}{2} \times \frac{1}{3}\right) = -\frac{M_{ao}L_1}{16EI_1} \tag{29}$$



(a) The deflection curve due to M_{ao}.



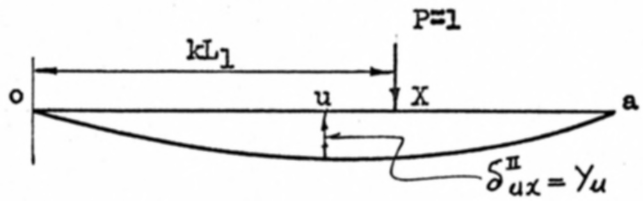
(b) Elastic weight on conjugate beam ao.

Fig. 10. The vertical displacement of point u due to M_{ao}.

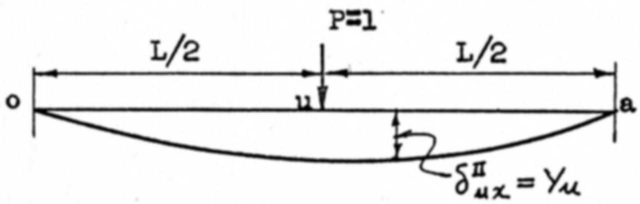
The vertical deflection of point u, which is due to the unit load on the span oa, as shown in Fig. 11, can be found through numerical analysis (5):

In Fig. 11, let δ_{ux} denote the displacement of u in the given direction due to a unit load at x acting in the direction P_x. Then the deflection due to P_x can be written as

$$y_u = \delta_{ux}P_x.$$



(a) Deflection curve due to P=1 moving on span oa.



(b) Deflection curve due to P=1 at point u.

Fig. 11. The vertical displacement of point u due to the unit load moving on the span oa.

Since in this case $P_x=1$, we have $y_u=\delta_{ux}$. By the reciprocal theorem, we have

$$y_u = \delta_{ux} = \delta_{xu},$$

which states that the deflection of point u due to the unit load at x is equal to the deflection of x due to the unit load at u. Therefore, the ordinates of the deflection curve due to $P_u=1$ are the influence coefficients for point u.

Conditions of symmetry are used in order to reduce the computations required by half. The boundary conditions are $y\big|_{k=0}=0$ and $y'\big|_{k=.5}=0$. The y_u values are obtained by numerical solutions and are shown in Fig. 12.

From Fig. 12, since P=1

$$\delta^{II}_{u} = u_k \frac{1 \times L_1^3}{EI_1}. \tag{30}$$

Where u_k = the coefficient of y_u in Fig. 12.

The total deflection is then

$$\delta_u = \delta^{I}_{u} + \delta^{II}_{u} = -\frac{M_{ao}L_1^2}{16EI_1} + u_k\frac{L_1^3}{EI_1}. \tag{31}$$

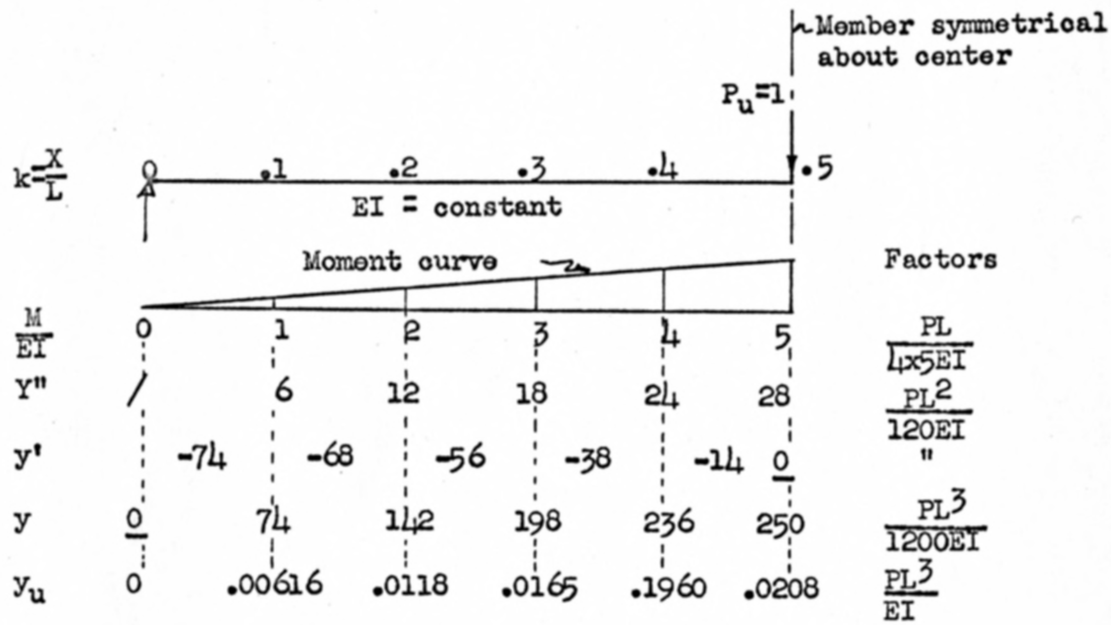


Fig. 12. Numerical solution for the deflection influence coefficients of point u.

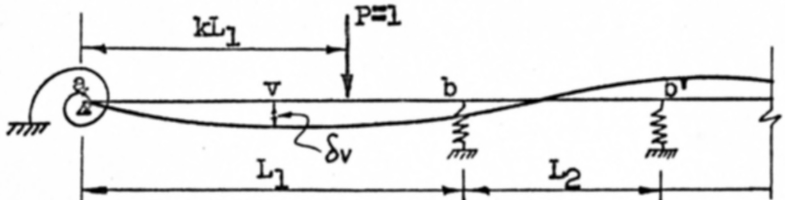
(2) The vertical displacement of point v: the vertical deflection of point v due to a unit load in span ab is made up of (A) the deflection due to the end moments M_{ab} and M_{ba} of member ab, (B) the deflection due to the unit load which moves on the span ab, and (C) the deflection due to the settlement of elastic support b. The analysis for each case is shown in Fig. 13 and the development is as follows:

(A) The deflection due to the end moments M_{ab} and M_{ba} of member ab: By the conjugate beam method and equation (29), we have

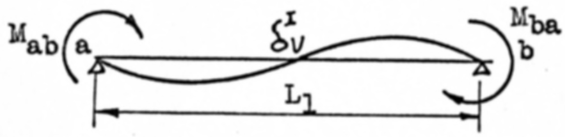
$$\delta_v^{I} = \frac{L_1^2}{16EI_1}(M_{ab} - M_{ba}) \ . \tag{32}$$

(B) The deflection due to the unit load moving on the span ab: This is the same case as in equation (30). That is

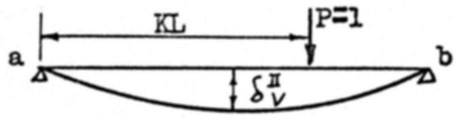
$$\delta_v^{II} = u_k \frac{L_1^3}{EI_1} \ .$$



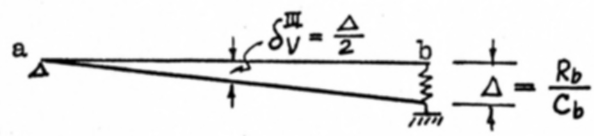
(a) The unit load moving on the base structure.



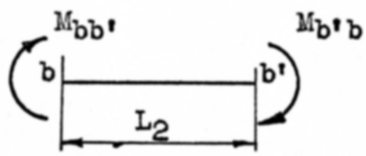
(b) Deflection curve due to end moments.



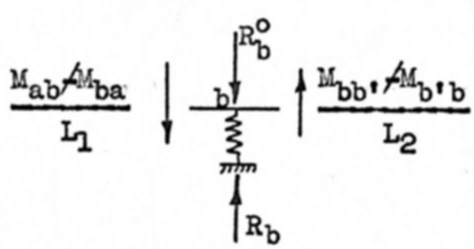
(c) Deflection curve due to the unit load on span ab.



(d) Deflection of point v due to the elastic support settlement.



(e) Free body of beam bb'.



(f) Free body of the elastic support.

Fig. 13. The vertical displacement of point v.

(C) The deflection due to the settlement of elastic support b: From Fig. 13(d), we observe that

$$\delta_v^{III} = \tfrac{1}{2}\Delta = \tfrac{1}{2}\frac{R_b}{C_b} = \tfrac{1}{2}\frac{R_b L_1^3}{2C_b^o EI_1} = \frac{L_1^3}{2C_b^o EI_1}\left(R_b^o + \frac{M_{ab} + M_{ba}}{L_1} - \frac{M_{bb'} + M_{b'b}}{L_2}\right). \quad (33)$$

Therefore, the total deflection of point v is

$$EI\delta_v = EI(\delta_v^I + \delta_v^{II} + \delta_v^{III}) = \frac{L_1^2}{16}(M_{ab} + M_{ba}) + u_k L_1^3 + \frac{L_1^3}{2C_b^o}\left(R_b^o + \frac{M_{ab} + M_{ba}}{L_1} - \frac{M_{b'b} - M_{ba}}{L_2}\right) \quad (34)$$

By the symmetry of the structure, $\delta_u = \delta_{u'}$ and $\delta_v = \delta_{v'}$.

SUBCABLE TENSION AND END MOMENT INFLUENCE LINES FOR THE STRUCTURE

(1) Influence lines for subcables.

Let the unit load P=1 be at a distance x from left support o, as shown in Fig. 14, and let the corresponding deflections of u and v be δ_{ux} and δ_{vx}. In addition, let the vertical components of the subcable tensions be T_1' and T_2', and let $\bar{\delta}_u$ and $\bar{\delta}_v$ be the final vertical displacements. Taking the cable tensions as external loads, the total deflections of u and v due to the unit load are

$$\bar{\delta}_u = \delta_{ux} - (\delta_{uu} + \delta_{uv})T_1' - (\delta_{uv'} + \delta_{uu'})T_2' \,, \quad (35)$$

and $$\bar{\delta}_v = \delta_{vx} - (\delta_{vu} + \delta_{vv})T_1' - (\delta_{vv'} + \delta_{vu'})T_2' \,. \quad (36)$$

By the theorem of reciprocal deflections $\delta_{uv} = \delta_{vu}$ and $\delta_{uv'} = \delta_{v'u}$, and because of the symmetry of the structure, $\delta_{uv'} = \delta_{u'v}$. Thus equations (35) and (36) become:

$$(\delta_{uu} + \delta_{uv})T_1' + (\delta_{uv'} + \delta_{uu'})T_2' = \delta_{ux} - \bar{\delta}_u, \quad (37)$$

and $$(\delta_{uv} + \delta_{vv})T_1' + (\delta_{vv'} + \delta_{uv'})T_2' = \delta_{vx} - \bar{\delta}_v. \quad (38)$$

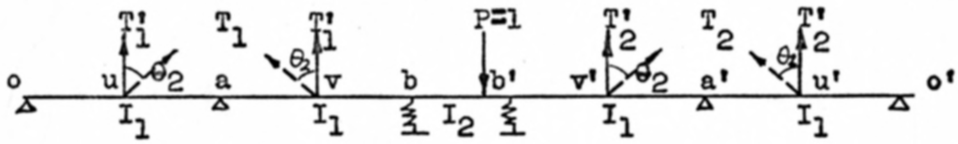


Fig. 14. Subcable positions on the base structure.

Adding equations (37) and (38), and substituting equation (3), we obtain

$$(\delta_{uu} + 2\delta_{uv} + \delta_{vv})T_1' + (\delta_{uu'} + 2\delta_{uv'} + \delta_{vv'})T_2' + \frac{2T\ell_2}{E_oA_o\cos\theta_2} = \delta_{ux} + \delta_{vx}.$$

Substituting $T_1 = \dfrac{T_1'}{\cos\theta_2}$, we have

$$(\delta_{uu} + 2\delta_{uv} + \delta_{vv} + \frac{2\ell_2}{E_oA_c\cos^2\theta_2})T_1' + (\delta_{uu'} + 2\delta_{uv'} + \delta_{vv'})T_2' = \delta_{ux} + \delta_{vx}. \quad (39)$$

Similarly, letting $\bar{\delta}_{u'}$ and $\bar{\delta}_{v'}$ be the final displaced positions of points u' and v' on the structure due to the unit load, we have

$$\bar{\delta}_{u'} = \delta_{u'x} - (\delta_{u'u} + \delta_{u'v})T_1' - (\delta_{u'v'} + \delta_{u'u'})T_2' , \quad (40)$$

and $$\bar{\delta}_{v'} = \delta_{v'x} - (\delta_{v'u} + \delta_{vv'})T_1' - (\delta_{v'v'} + \delta_{v'u'})T_2' \quad (41)$$

Again adding equations (40) and (41), and substituting equation (3), we have

$$(\delta_{uu'} + 2\delta_{uv'} + \delta_{vv'})T_1' + (\delta_{uu} + 2\delta_{uv} + \delta_{vv} \frac{2\ell_2}{E_oA_{o2}\cos^2\theta_2})T_2' = \delta_{ux} + \delta_{vx}. \quad (42)$$

Let $$\delta_{uv} + 2\delta_{uv} + \delta_{vv} + \frac{2\ell_2}{E_oA_{o2}\cos^2\theta_2} = \alpha \quad (43)$$

and $$\delta_{uu'} + 2\delta_{uv'} + \delta_{vv'} = \beta. \quad (44)$$

Substituting equations (43) and (44) into equations (39) and (42), we have

$$\alpha T_1' + \beta T_2' = \delta_{ux} + \delta_{vx} \quad (45)$$

and $$\beta T_1' + \alpha T_2' = \delta u'x + \delta v'x. \quad (46)$$

Solving equations (45) and (46) simultaneously for T_1', we have

$$T_1' = \frac{\alpha(\delta_{ux} + \delta_{vx}) - (\delta_{u'x} + \delta_{v'x})}{\alpha^2 - \beta^2}, \tag{47}$$

and the subcable tensions may be written as

$$T_1 = T_1' \sec\theta_2 . \tag{48}$$

(2) The end moment influence lines: Taking the subcable tensions as external loads as shown in Fig. 14, and using the influence lines for M_{ab} and M_{ba} of the base structure and the influence coefficients for points u and v, we can obtain the influence lines for the end moments of the structure. This procedure is illustrated in Example 1, step 4.

COMPUTER SOLUTIONS FOR INFLUENCE LINES FOR CABLE-STIFFENED GIRDER BRIDGES

(1) One way to analyze the structure by computer is to solve the chosen base structure, shown in Fig. 5, subjected to the symmetrical and anti-symmetrical loads at the middle supports, and then solve for the influence lines for the base structure by using the theorem of reciprocal deflections. These procedures will be demonstrated in Example 2.

(2) Another way to analyze the structure is to solve for the influence lines directly by the computer.

The structure of Fig. 15 is statically indeterminate to the sixth degree.

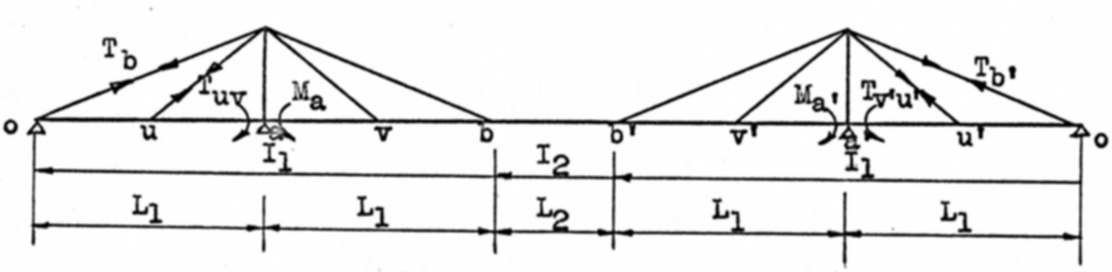


Fig. 15. The cable-stiffened girder bridge showing the chosen redundant constraints.

In order to reduce it to a statically determinate structure, the constraints M_a, $M_{a'}$, T_{uv}, T_b, $T_{b'}$, and $T_{v'u'}$ are removed. The deformations in the primary structure due to the external loads and the redundant constraint conditions are then found.

The details of the analysis procedure for the structure will be demonstrated in Example 3.

NUMERICAL EXAMPLES

Example 1

Given: A cable-stiffened girder bridge and its base structure as shown in Fig. 16. The towers of the bridge are hinge supported, which allows the tower columns to rotate about their bases. While the inside cables are connected to the top of the tower column from each side, the outside cables are continuous over the top of the tower.

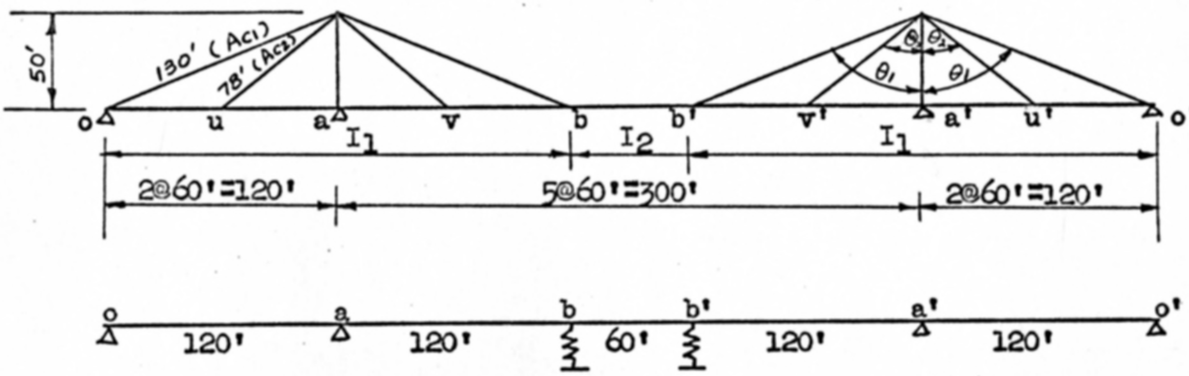
Required: Influence lines for moment at a, b, u and v and for the cable tensions.

Solution:

Step 1. Determine the modified stiffnesses and carry-over factors and the end moments for the base structure due to the symmetrical and anti-symmetrical loads. Use the equations that were derived in the previous sections and compute as follows:

From equation (4) $$C_b = \frac{E_c A_{c1}}{2\ell} \cos^2\theta_1 = \frac{23 \times 10^8}{260} \times \frac{25}{169} = 1308.6 \text{ kips per ft,}$$

and $$C_b^o = \frac{C_b L_1^3}{EI_1} = \frac{1308.6 \times (120)^3 \times 144 \times 1000}{30 \times 10^6 \times 73000} = 148.7.$$



$E_c = 23 \times 10^6$ psi.

$E = 30 \times 10^6$ psi.

$A_{c1} = 100$ sq. in.

$A_{c2} = 7.5$ sq. in.

$I_1 = 73{,}000$ in^4.

$I_2 = 50{,}000$ in^4.

$2\ell_1 = 2 \times 130 = 260$ ft.

$2\ell_2 = 2 \times 78 = 156$ ft.

$E_cA_{c1} = 23 \times 10^8$ lb.

$E_cA_{c2} = 17.25 \times 10^7$ lb.

$\cos^2\theta_1 = \frac{25}{169}$.

$\cos^2\theta_2 = \frac{225}{549}$.

$m = \frac{L_1}{L_2} = 2.$

$n = \frac{K_1}{K_2} = 0.730.$

Fig. 16. Example 1.

(A) Symmetrical moment $M_o=1$ action at middle supports b and b':

From equation (11)

$$M_{ba} = \frac{12(1 - \frac{4}{7}) + 148.7(4 - \frac{4}{7})}{3(4 - \frac{12}{7}) + 148.7} EK_1\theta_b = 3.310EK_1\theta_b.$$

From equation (8)

$$M_{bb'} = 2EK_2\theta_b = \frac{2EK_1\theta_b}{n} = \frac{2}{0.730}EK_1\theta_b = 2.740EK_1\theta_b\ .$$

When $\theta = 1$, $S_{ba} = 3.31EK_1$ and $S_{bb'} = 2.74EK_1$.

Therefore,

$$u_{ba} = \frac{3.31}{3.31 + 2.74} = 0.547,$$

$$u_{bb'} = \frac{2.74}{3.31 + 2.74} = 0.453$$

and $C_{ba} = \dfrac{0.787}{3.31} = 0.238$.

(B) Symmetrical concentrated load P_o=1 acting at middle supports b and b':

From equation (15)

$$M^{\Delta}_{ab} = -\frac{6(1 - \frac{4}{7})}{148.7 + 12 - \frac{36}{7}} \times 1 \times 120 = -1.983.$$

From equation (16)

$$M^{F\Delta}_{ba} = -\frac{3(2 - \frac{4}{7})}{148.7 + 12 - \frac{36}{7}} \times 1 \times 120 = -3.306.$$

(C) Anti-symmetrical moment M_o=1 acting at middle supports b and b':

From equation (20)

$$M_{ab} = 2 \times \frac{3}{7} \times \frac{12 \times 2 \times 7 + 0.73(148.7 - 6)}{0.73(148.7 + 12 - 9 \times \frac{4}{7}) + 24 \times 4} EK_1\theta_b = \frac{6}{7}$$

$$\times \frac{168 + 104.2}{209.55} EK_1\theta_b = 1.113EK_1\theta_b.$$

From equation (21)

$$M_{ba} = \frac{48(3 + 8) + 2.92(3 + 148.7) - \frac{4}{7}(108.6 + 8.75 + 72 + 96)}{209.55} EK_1\theta_b$$

$$= 3.857EK_1\theta_b.$$

Again, from equation (22)

$$M_{bb'} = 6 \times 0.73 \frac{148.7 + 12(1 + 2) - 3(3 + 4)\frac{4}{7}}{209.55} \times \frac{K_1}{0.73} E\theta_b = 4.945EK_1\theta_b.$$

Therefore,

$$u_{ba} = \frac{3.857}{3.857 + 4.945} = 0.438,$$

$$u_{bb'} = \frac{4.945}{3.857 + 4.945} = 0.562$$

and $$C_{ba} = \frac{1.113}{3.857} = 0.289.$$

(D) Anti-symmetrical concentrated load P_o=1 acting at middle supports b and b':

From equation (26)

$$M_{ab}^{\Delta} = - \frac{6 \times 0.73(1 - \frac{4}{7}) \times 1 \times 120}{148.7 \times 0.73 + 3 \times 0.73(4 - \frac{12}{7}) + 24 \times 4} = - 1.074.$$

From equation (27)

$$M_{ba}^{F\Delta} = - \frac{3 \times 0.73(2 - \frac{4}{7}) \times 1 \times 120}{209.55} = - 1.792.$$

From equation (28)

$$M_{bb'}^{F\Delta} = \frac{12 \times 2 \times 1 \times 120}{209.55} = 13.744.$$

Step 2. The second step in the solution is to use the results of Step 1 and draw the M_{ab} and M_{ba} influence lines for the base structure. Fig. 17 shows the moments in the base structure due to the indicated loads acting at the middle supports. Next assume that there is a fixed-end moment F=100 at each end of each member in turn, and use the direct moment distribution factors of Fig. 18 to obtain the end moments for the base structure due to each load case, as shown in Fig. 19.

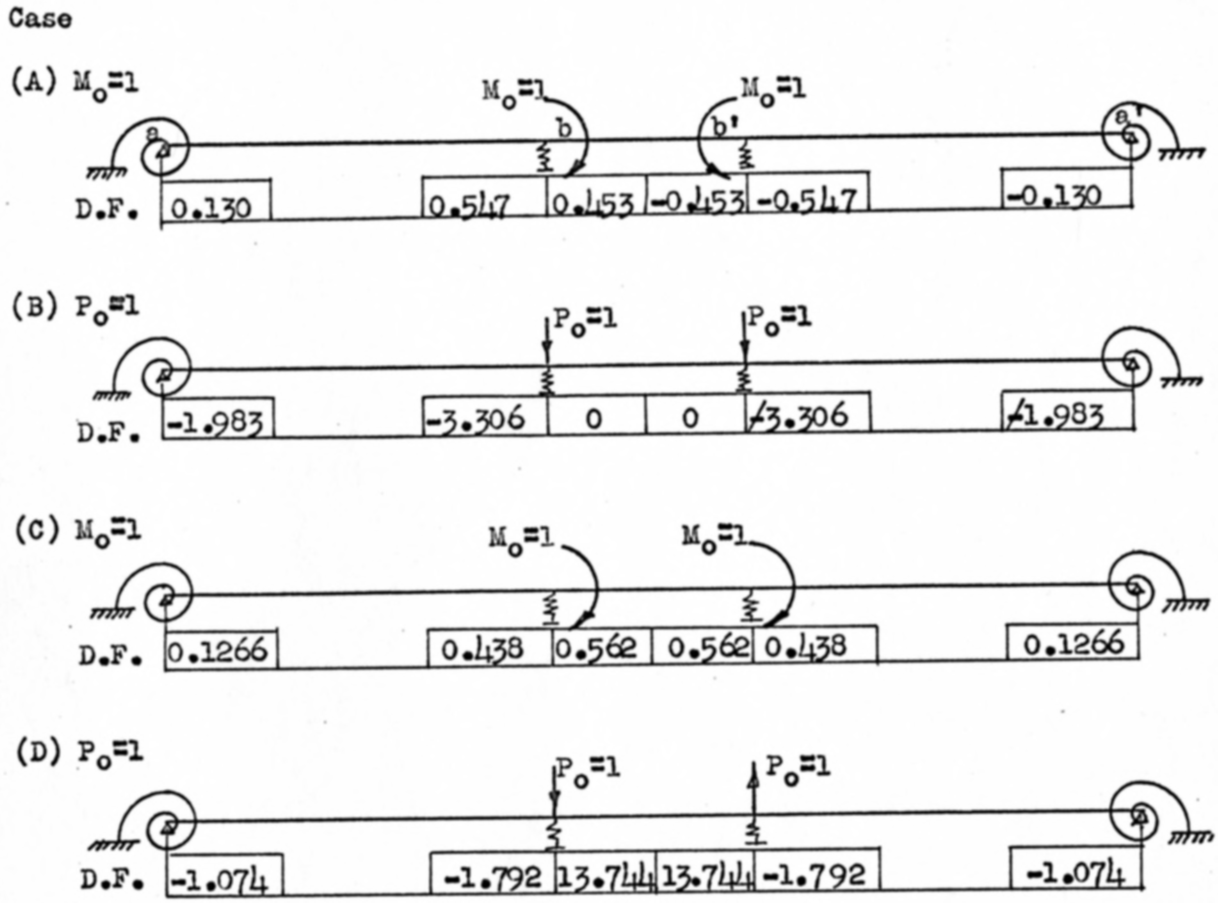


Fig. 17. Summary of distribution factors for the reduced base structure.

From Fig. 19, we have

$$M_{ab} = -0.520F'_{ao} + 0.463F_{ab} - 0.146F_{ba} - 0.085F_{bb'} + 0.045F_{b'b} + 0.0026F_{b'a'} - 0.001F_{a'b'} + 0.003F'_{a'o'} - 2.070P_o^b + 0.517P_o^{b'}.$$

$$M_{ba} = -0.115F'_{ao} - 0.150F_{ab} + 0.472F_{ba} - 0.375F_{bb'} + 0.172F_{b'b} + 0.032F_{b'a'} - 0.019F_{a'b'} + 0.0041F'_{a'o'} - 4.263P_o^b + 2.765P_o^{b'}.$$

Computation for influence lines: Assume clockwise moment is positive.

Let F'_{ao} denote the F.E.M. of ao when support o is a hinge, and let k be the ratio of the distance of the unit load from the left support to the total

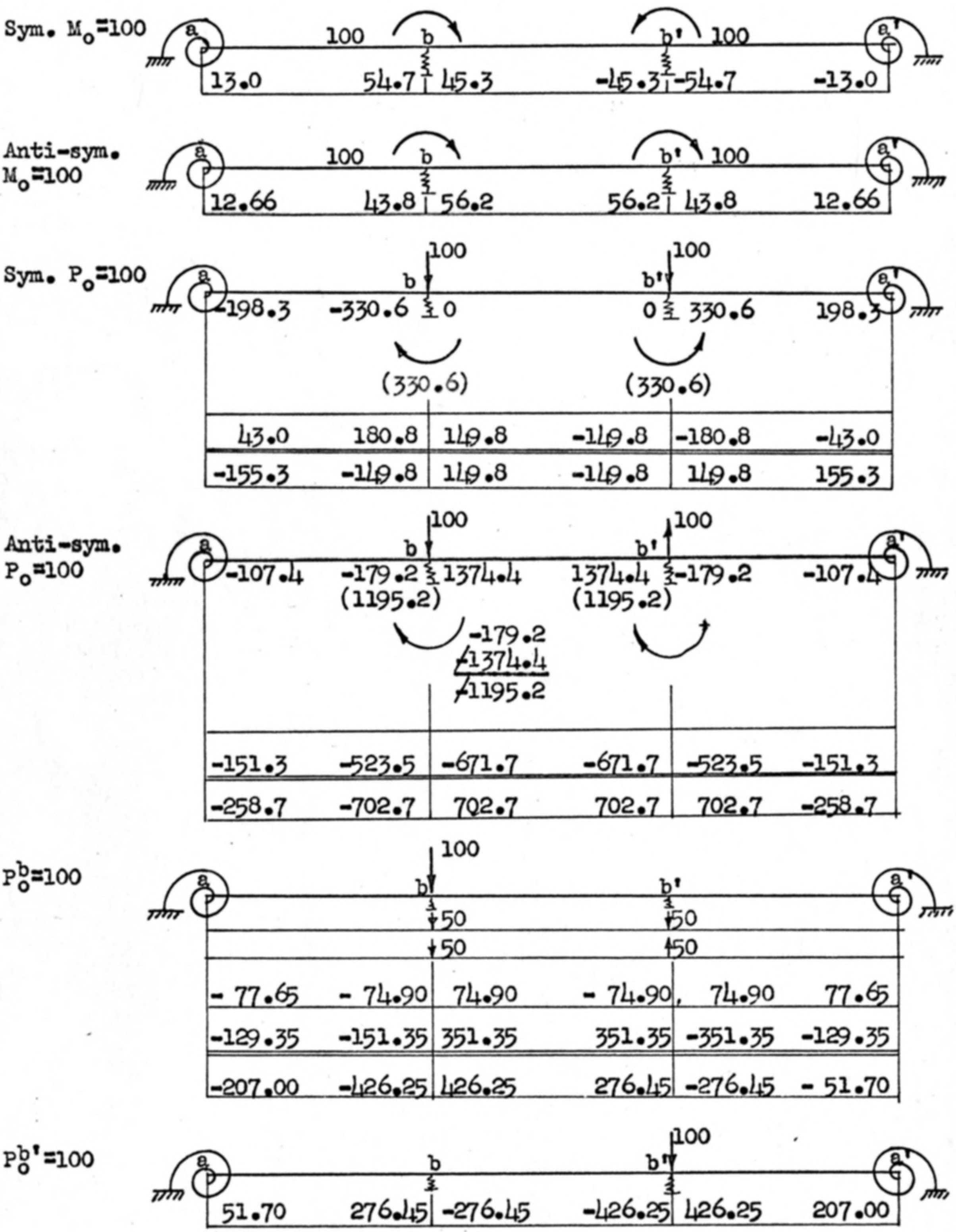


Fig. 18. End moments for the members due to the indicated loadings.

(A) F'_{ao}=100

	.428	a .572			b			b'			a
F.E.M.	+100										
Distr.	- 42.86	-57.14		-28.57							
S.M.D.	- 1.86	+ 1.86		+ 7.82	+ 6.47	14.29	-6.47	-7.82		-1.86	
A.S.M.D.	- 1.81	+ 1.81		+ 6.26	+ 8.03	14.29	+8.03	+6.26		+1.81	
P_o^b D.	- 1.48	+ 1.48		+ 3.04	- 3.04		-1.97	+1.97		+0.37	
Total	+51.99	-51.99		-11.45	+11.45		-0.41	+0.41		+0.32	-0.32

F.E.M. = Fixed-end moment.

S.M.D. = Symmetrical moment distribution.

A.S.M.D. = Anti-symmetrical moment distribution.

$P_oD.$ = End moment due to concentrated load at support b.

where $P_o^b = \frac{1}{120}(-57.14 - 28.57) = -0.714 \uparrow$

(B) F_{ab}=100

		a			b			b'		a'
F.E.M.		+100								
Distr.		- 57.14		-28.57						
S.M.D.		+ 1.86		+ 7.82	+ 6.47	14.29	-6.47	-7.82		-1.86
A.S.M.D.		+ 1.81		+ 6.26	+ 8.03	14.29	+8.03	+6.26		+1.81
P_o^b D.		- 0.25		- 0.51	- 0.51		+0.33	-0.33		-0.06
Total		+ 46.28		-15.00	+15.00		+1.89	-1.89		-0.11

$$P_o^b = \frac{1}{120}(100 - 57.14 - 28.5) = -0.119 \downarrow$$

Fig. 19a. The end moments for the base structure due to fixed-end moment F'_o=100 and F_{ab}=100.

(C) $F_{ba}=100$

	a	b	b		b'	b'	a'
F.E.M.		+100					
S.M.D.	- 6.50	- 27.35	-22.65	50	+22.65	+27.35	+6.50
A.S.M.D.	- 6.33	- 21.90	-28.10	50	-28.10	-21.90	-6.33
P_o^b D.	- 1.72	- 3.55	+ 3.55		+ 2.30	- 2.30	-0.43
Total	-14.55	+ 47.20	-47.20		- 3.15	+ 3.15	-0.25

$$P_o^b = \frac{1}{120}(-6.50 + 100 - 27.35) = 0.833 \downarrow$$

(D) $F_{bb'}=100$

	a	b	b		b'	b'	a'
F.E.M.		+100					
S.M.D.	-6.50	- 27.35	-22.65	50	+22.65	+27.35	+6.50
A.S.M.D.	-6.33	- 21.90	-28.10	50	-28.10	-21.90	-6.33
P_o^b	+4.31	+ 11.71	-11.71		-11.71	+11.71	+4.31
Total	-8.52	- 37.54	+37.54		-17.16	+17.16	+4.18

$$P_o^b = \frac{-100}{60} = -1.667 \uparrow$$

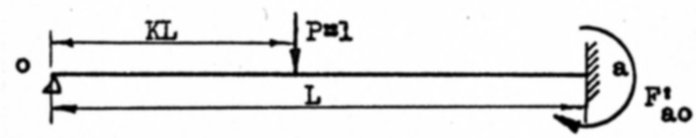
Fig. 19b. The end moments for the base structure due to fixed-end moment $F_{ba}=100$ and $F_{bb'}=100$.

length of the beam, as shown in Fig. 20, we then have

$$F_{left} = \frac{1 \times (L - kL)^2(kL)}{L^2} = k(1 - k)^2L,$$

and $$F_{right} = \frac{1 \times (L - kL)(kL)^2}{L^2} = k^2(1 - k)L.$$

Varying k from .1 to .9 by tenths, we obtain the F.E.M. for the left and right supports respectively due to the unit load at each load point on the span as shown in Table 2.



F.E.M.	$-F_{oa}$	F_{ao}
Distr.	F_{oa}	
C.O.		$+\frac{1}{2}F_{oa}$
Total	0	$F'_{ao} = F_{ao} + \frac{1}{2}F_{oa}$

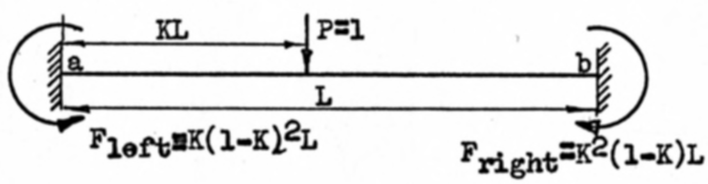


Fig. 20. Fixed-end moments due to unit load on the member.

Substituting the length of each span, we obtain the F.E.M. due to the unit load at each load point in turn for each support as shown in Table 3.

Step 3. The third step in the solution is to find the vertical displacement influence coefficients for points u and v of the base structure. From equation (31), the deflection of point u is

$$\delta_u = \frac{-(120)^2}{16EI_1} M_{ao} + \frac{(120)^3}{EI_1} u_k,$$

Table 1. Influence coefficients for M_{ab} and M_{ba} due to fixed-end moments.

Loaded Span	M_{ab}	M_{ba}
oa	$-0.520F'_{ao}$	$-0.115F'_{ao}$
ab	$+0.463F_{ab} - 0.146F_{ba} - 2.070P_o^b$	$-0.150F_{ab} + 0.472F_{ba} - 4.263P_o^b$
bb'	$-0.085F_{bb'} + 0.045F_{b'b} - 2.070P_o^b$ $+0.517P_o^{b'}$	$-0.375F_{bb'} + 0.172F_{b'b} - 4.263P_o^b$ $+2.765P_o^{b'}$
b'a'	$+0.26F_{b'a'} - 0.001F_{a'b'} + 0.517P_o^b$	$+0.032F_{b'a'} - 0.019F_{a'b'} + 2.765P_o^{b'}$
a'o'	$+0.003F_{a'o'}$	$+0.0041F_{a'o'}$

Table 2. Fixed-end moment coefficients.

k	F_{left}	F_{right}
0	0	0
.1	0.081L	0.009L
.3	0.147L	0.063L
.5	0.125L	0.125L
.7	0.063L	0.147L
.9	0.009L	0.081L
1.0	0	0

Substituting the F.E.M.s corresponding to the various k values of Table 3 into Table 1, we obtain the influence lines for M_{ab} and M_{ba} as shown in Table 5. The influence lines are shown in Fig. 21.

Table 3. Fixed-end moments due to unit load on the base structure assuming that supports u, u', v, v', b and b' are rigid.

k	F'_{ao}	F_{ab}	F_{ba}	$F_{bb'}$	$F_{b'b}$	$F_{b'a'}$	$F_{a'b'}$	$F'_{a'o'}$
0	0	0	0	0	0	0	0	0
.1	5.94	- 9.72	1.08	-4.86	0.54	- 9.72	1.08	-10.26
.3	16.38	-17.64	7.56	-8.82	3.78	-17.64	7.56	-21.42
.5	22.50	-15.00	15.00	-7.51	7.50	-15.00	15.00	-22.50
.7	21.42	- 7.56	17.64	-3.78	8.820	- 7.56	17.64	-16.38
.9	10.26	- 1.08	9.72	-0.54	4.86	- 1.08	9.72	- 5.94
1.0	0	0	0	0	0	0	0	0

that is

$$EI_1\delta_u = 900M_{oa} + 1,728,000u_k. \tag{49}$$

From equation (34), the deflection of point v is

$$EI_1\delta_v = \frac{(120)^2}{16}(M_{ab} - M_{ba}) + 1,728,000u_k + \frac{(120)^3}{2 \times 148.7}\left(R_b^o + \frac{M_{ab} + M_{ba}}{120} - \frac{M_{b'b} - M_{ba}}{60}\right) = 948.4M_{ab} - 754.7M_{ba} + 5810.35R_b^o - 96.8M_{b'b} + 1,728,000u_k. \tag{50}$$

The calculations for equations (49) and (50) are shown in Table 5. And the influence lines are shown in Fig. 22.

Step 4. The fourth step in the solution is to use the results of the previous steps to find the cable tension and the girder end moment influence lines.

Since the structure of Fig. 16 can be looked at as in Fig. 23, we can use the results of the previous steps to find the main influence lines for the structure as follows:

Table 4. M_{ab} and M_{ba} influence line calculations.

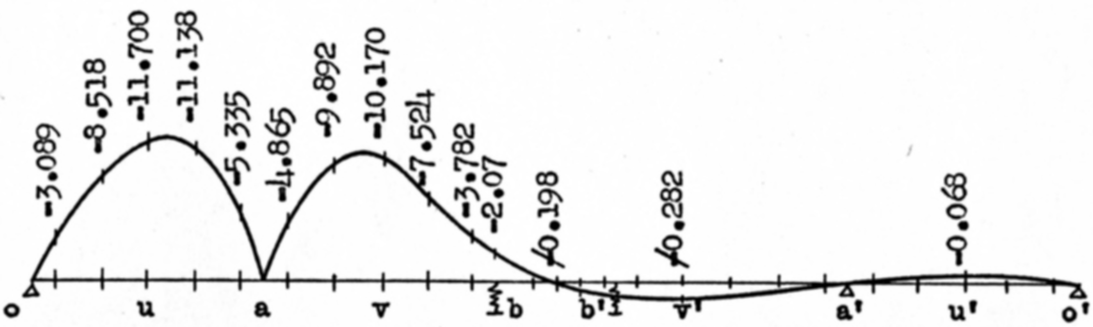
	k	Load at oa	Load at ab				Load at bb'	
		$-.520F'_{ao}$	$+.463F_{ab}$	$-.146F_{ba}$	$-2.07P_o^b$	M_{ab}	$-.085F_{bb'}$	$+.045F_{b'b}$
M_{ab} Influence Line	0	0	0	0	0	0	0	0
	.1	- 3.089	-4.500	-0.158	-0.207	- 4.865	0.413	0.024
	.3	- 8.518	-8.167	-1.104	-0.621	- 9.892	+ 0.750	0.170
	.5	-11.700	-6.945	-2.190	-1.035	-10.170	+ 0.638	0.336
	.7	-11.138	-3.500	-2.575	-1.449	- 7.524	+ 0.321	0.397
	.9	- 5.335	-0.500	-1.419	-1.863	- 3.782	+ 0.046	0.219
	1.0	0	0	0	-2.07	- 2.07	0	0

	k	Load at oa	Load at ab				Load at bb'	
		$-.115F_{ao}$	$-.150F_{ab}$	$+.472F_{ba}$	$-4.263P_o^b$	M_{ba}	$-.375F_{bb'}$	$+.172F_{b'b}$
M_{ba} Influence Line	0	0	0	0	0	0	0	0
	.1	- 0.683	+1.458	+0.510	-0.426	1.542	1.823	0.093
	.3	- 1.884	+2.646	+3.568	-1.279	4.935	3.308	0.650
	.5	- 2.588	+2.250	+7.080	-2.132	7.198	2.813	1.290
	.7	- 2.463	+1.134	+8.326	-2.984	6.476	1.418	1.517
	.9	- 1.180	+0.162	+4.588	-3.837	0.913	0.203	0.836
	1.0	0	0	0	-4.263	-4.263	0	0

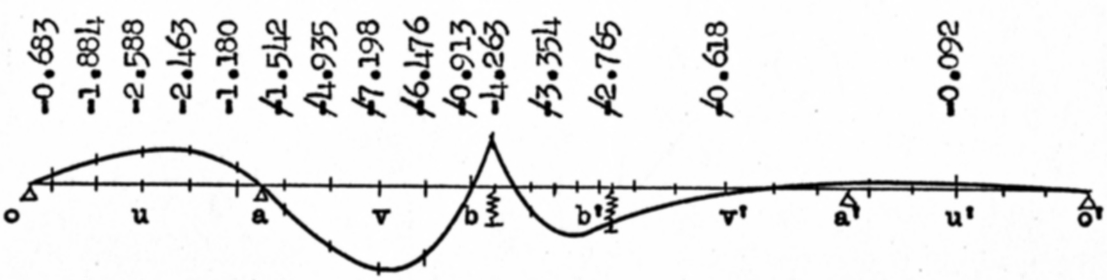
Table 4. (Contd.)

	k	Load at bb' (Contd.)		Load at b'a'					Load at a'o'
		$-2.07P_o^b$	$+.517P_o^{b'}$	M_{ab}	$.0026F_{b'a'}$	$-.001F_{a'b'}$	$+.517P_o^{b'}$	M_{ab}	$+.003F_{a'o'}$
	0	-2.07	0	-2.07	0	0	0.517	.517	0
	.1	-1.863	0.052	-1.374	.025	- .0010	0.465	0.480	- .030
	.3	-1.449	0.155	-0.374	.046	- .008	0.362	0.400	- .064
M_{ab} Influence Line	.5	-1.035	0.259	0.198	.038	- .015	0.259	0.282	.068
	.7	-0.621	0.362	0.459	.020	- .018	0.155	0.157	-0.049
	.9	-0.207	0.465	0.523	.003	- .010	0.052	0.045	-0.017
	1.0	-0	0.517	0.517	0	0	0	0	0

	k	Load at bb' (Contd.)		Load at b'a'					Load at a'o'
		$+4.263P_o^b$	$+2.765P_o^{b'}$	M_{ba}	$+.032F_{b'b}$	$-.019F_{a'b}$	$+2.765P_o^{b'}$	M_{ba}	$+.0041F_{a'o'}$
	0	-4.263	0	-4.263	0	0	2.765	2.765	0
	.1	-3.837	0.277	-1.644	- .311	- .020	2.489	2.158	- .042
	.3	-2.984	0.830	1.804	- .564	- .144	1.935	1.227	- .088
M_{ba} Influence Line	.5	-2.132	1.383	3.354	- .480	- .285	1.383	0.618	- .092
	.7	-1.279	1.935	3.591	- .245	- .335	.830	0.250	- .067
	.9	-0.426	2.489	3.102	.035	- .185	.277	0.057	- .024
	1.0	0	2.765	2.765	0	0	0	0	0



(a) Influence line for M_{ab}.



(b) Influence line for M_{ba}.

Fig. 21. Influence line for M_{ab} and M_{ba}.

Table 5. Influence coefficient computations for $EI\delta_x$, $EI\delta_y$, T_1' and T_1.

k	(1)	(2)	(3)	$EI\delta_x$				$EI\delta_y$								T_1'					T_1
	M_{ab}	M_{ba}	$M_{a'b}$	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)
				η_{4k}	1728×10^3	$900M_{ab}$	$EI\delta_x$	η_{4k}	$1728\times10^3\eta_{4k}$	$948.4M_{ab}$	$-754.7M_{ba}$	$-96.8M_{a'b}$	$5810.35\eta_k$	$EI\delta_y$	$EI(\delta_x/\xi)$	$EI(\delta_y/\xi_y)$	68620.3x(15)	249.1x(16)	(17)x(18)	(19) / 4,160,869x10³	T_1=[illegible]T_1' / 1.56
0	0	0	0	0	0	0	0			0	0	0	0	0	0	0	0	0	0	0	0
.1	-3.089	-0.683	-0.024	0.00616	10,644.5	-2,780.1	7,864.4			-2,929.6	515.5	+2.3		-2,411.8	5,162.6	52.8	351,803x10³	13x10³	351,816x10³	0.0846	0.1318
.3	-8.518	-1.884	-0.067	0.01650	28,512.0	-7,666.2	20,845.8			-8,078.4	1,421.9	6.5		-6,650.0	14,395.8	142.4	915,917x10³	36x10³	915,953x10³	0.2200	0.3432
.5	-11.700	-2.588	-0.092	0.02083	35,994.2	-10,530.0	25,464.2			-11,096.3	1,953.2	8.9		-9,134.2	16,330.0	194.2	1,053,616x10³	48x10³	1,053,664x10³	0.2531	0.3948
.7	-11.198	-2.465	-0.088	0.01650	28,512.0	-10,024.2	18,487.8			-10,565.3	1,858.8	8.5		-8,696.0	9,791.8	186.5	651,770x10³	47x10³	651,817x10³	0.1518	0.2368
.9	-5.335	-1.180	-0.042	0.00616	10,644.5	-4,801.5	5,843.0			-5,059.7	890.5	4.1		-4,169.2	1,673.8	90.4	107,994x10³	23x10³	108,017x10³	0.0259	0.0404
a	0	0	0	0		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
.1	-4.865	+1.512	0.057			-4,378.5	-4,378.5	0.00616	10,644.5	-4,614.0	-1,163.7	-5.5	581.0	+5,142.3	1,063.8	-109.0	68,657x10³	-27x10³	68,630x10³	0.0165	0.0258
.3	-9.892	+4.035	0.250			-8,902.8	-8,902.8	0.0165	28,512.0	-9,381.6	-3,724.4	-24.2	1,743.1	+17,124.9	8,222.1	-376.2	530,192x10³	-94x10³	530,098x10³	0.1274	0.1987
.5	-10.170	7.198	0.618			-9,153.0	-9,153.0	0.02083	35,994.2	-9,645.2	-5,432.3	-99.8	2,905.2	+23,762.1	14,609.1	-642.0	942,584x10³	-160x10³	942,424x10³	0.2265	0.3535
.7	-7.584	6.476	1.227			-6,771.6	-6,771.6	0.01650	28,512.0	-7,135.8	-4,807.4	-118.8	4,067.2	+20,437.2	13,665.6	-813.5	881,709x10³	-203x10³	881,506x10³	0.2118	0.3304
.9	-3.782	0.913	2.158			-3,403.8	-3,403.8	0.00616	10,644.5	-3,586.2	-689.0	-208.9	5,229.3	+11,389.1	7,985.3	-829.8	515,214x10³	-207x10³	515,007x10³	0.1237	0.1930
b	-2.070	-4.263	2.765			-1,863.0	-1,863.0			-1,963.2	+3,217.2	-267.7	5,810.4	+6,796.7	4,933.7	-718.4	318,304x10³	-179x10³	318,125x10³	0.0764	0.1192
.1	-1.374	-1.644	3.102			-1,236.6	-1,236.6			-1,303.1	+1,240.7	-300.3	5,229.3	+4,866.6	3,690.0	-694.3	234,209x10³	-158x10³	234,051x10³	0.0562	0.0877
.3	-0.374	+1.804	3.591			-336.6	-336.6			-354.7	-1,361.5	-347.6	4,067.2	+2,003.4	1,666.8	-352.1	107,542x10³	-88x10³	107,454x10³	0.0258	0.0402
.5	+0.198	+3.354	+3.354			-178.2	-178.2			+187.8	-2,531.2	-324.7	2,905.2	+237.1	58.9	58.9	3,800x10³	15x10³	3,815x10³	0.0092	0.0144
.7	0.469	3.591	1.804			443.1	443.1			495.3	-2,710.1	-174.6	1,743.1	-706.3	-352.1	1,666.8	-22,718x10³	416x10³	-22,302x10³	-0.0054	-0.0084
.9	0.523	3.102	-1.644			470.7	470.7			496.0	-2,341.1	+459.1	581.0	-1,105.0	-694.3	3,690.0	-40,925x10³	905x10³	-40,020x10³	-0.0096	-0.0150
b'	0.517	2.765	-4.263			465.3	465.3			490.3	-2,086.7	+412.7	0	-1,183.7	-718.4	4,933.7	-46,351x10³	1,230x10³	-45,121x10³	-0.0108	-0.0168
.1	0.480	2.158	0.913			432.0	432.0			455.2	-1,628.6	-88.4		-1,261.8	-829.8	7,985.3	-53,539x10³	1,992x10³	-51,547x10³	-0.0124	-0.0193
.3	0.400	1.227	6.476			360.0	360.0			379.4	-926.0	-626.9		-1,173.5	-813.5	13,665.6	-52,187x10³	3,108x10³	-49,079x10³	-0.0118	-0.0184
.5	0.282	0.618	7.198			253.8	253.8			267.4	-166.4	-696.8		-895.8	-642.0	14,609.1	-41,422x10³	3,644x10³	-37,778x10³	-0.0091	-0.0142
.7	0.157	0.250	4.035			141.3	141.3			148.9	-188.7	-477.7		-517.5	-376.2	8,222.1	-24,273x10³	2,051x10³	-22,222x10³	-0.0053	-0.0083
.9	0.045	0.057	1.512			40.5	40.5			42.8	-43.0	-146.3		-146.5	-109.0	1,063.8	-7,033x10³	265x10³	-6,768x10³	-0.0016	-0.0025
a'	0	0	0			0	0			0	0	0		0	0	0	0	0	0	0	0
.1	-0.030	-0.042	-1.180			-27.0	-27.0			-28.5	+31.7	+114.2		+117.4	+90.4	1,673.8	+5,833x10³	417x10³	6,250x10³	+0.0015	+0.0023
.3	-0.064	-0.088	-2.465			-57.6	-57.6			-60.7	+66.4	+238.4		+244.1	+186.5	9,791.8	+12,033x10³	2,442x10³	14,475x10³	+0.0035	+0.0055
.5	-0.068	-0.092	-2.588			-61.2	-61.2			-64.5	+69.4	+250.5		+255.4	+194.2	16,330.0	+12,530x10³	4,073x10³	16,603x10³	+0.0040	+0.0062
.7	-0.049	-0.067	-1.884			-44.1	-44.1			-46.5	+50.6	+182.4		+186.5	+142.4	14,3958	+9,188x10³	3,540x10³	12,728x10³	+0.0031	+0.0048
.9	-0.017	-0.024	-0.683			-15.3	-15.3			-16.1	+18.1	+66.1		+68.1	+52.8	5,162.6	+3,107x10³	1,609x10³	4,716x10³	+0.0012	+0.0019
a'	0	0	0			0	0			0	0	0		0	0	0	0	0	0	0	0

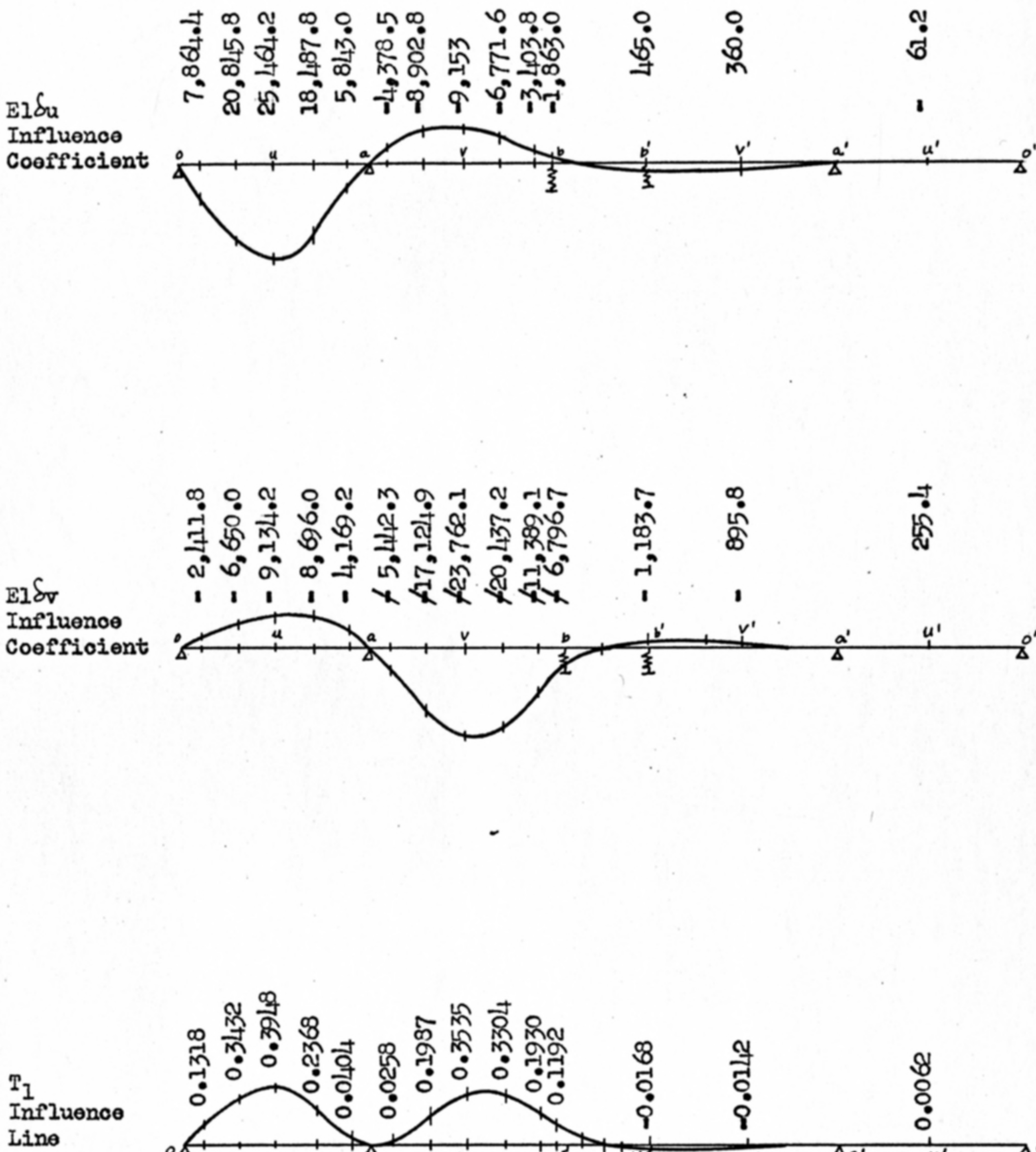


Fig. 22. Influence lines for EIδu, EIδv and cable tension T_1.

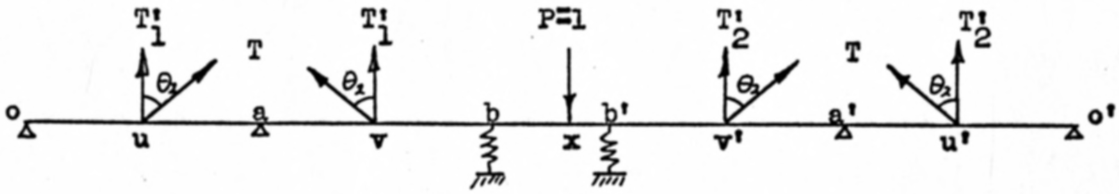


Fig. 23. Simplified structure of Example 1.

(1) Cable tension influence line.

From equation (48), we have

$$T_1 = T'_1 \sec\theta_2.$$

(2) Influence line for elastic supports of the base structure.

From Fig. 13(f), and $\Sigma V = 0$, we have

$$[R_b]^o = R^o_b + \frac{M^o_{ab} + M^o_{ba}}{120} + \frac{M^o_{ba} - M^o_{b'b}}{60}.$$

Where $[R_b]^o$ = The reaction of the elastic support b of the base structure, and

M^o_{mn} = The end moment at m of the member mn of the base structure.

(3) The end moment influence lines for the cable-stiffened girders.

(A) $\overline{M}_{ao}$ influence line: ($\overline{M}_{ao} = -\overline{M}_{ab}$)

From Fig. 23, the end moment at a of member ao can be written

$$\overline{M}_{ao} = M^o_{ao} - [(m_{ao})^o_u + (m_{ao})^o_v]T'_1 - [(m_{ao})^o_{v'} + (m_{ao})^o_{u'}]T'_2$$

$$= M^o_{ao} - [11.7 + 10.17]T'_1 - [-0.282 + 0.068]T'_2 = M^o_{ao} - 21.87T'_1 + 0.214T'_2.$$

Where $(m_{pq})^o_x$ = The influence line coordinate of point x at the end p of the member pq of the base structure.

(B) $\overline{M}_{ba}$ influence line ($\overline{M}_{ba} = -\overline{M}_{bb'}$).

$$\bar{M}_{ba} = M^o_{ba} - [(m_{ba})^o_u + (m_{ba})^o_v]T'_1 - [(m_{ba})^o_{v'} + (m_{ba})^o_{u'}]T'_2$$

$$= M^o_{ba} - [-2.588 + 7.198]T'_1 - [0.618 - 0.092]T'_2$$

$$= M^o_{ba} - 4.61T'_1 - 0.526T'_2.$$

(C) $\bar{R}_b$ influence line: The vertical component of the cable tension at b can be written as

$$\bar{R}_b = [R^o_b] - [(R_b)^o_u + (R_b)^o_v]T'_1 - [(R_b)^o_{v'} + (R_b)^o_{u'}]T'_2$$

$$= [R^o_b] - [-0.161 + 0.581]T'_1 - [-0.102 + 0.040]T'_2$$

$$= [R^o_b] - 0.420T'_1 + 0.062T'_2.$$

The calculations for the influence lines for $\bar{M}_{ao}$, $\bar{M}_{ba}$ and $\bar{R}_b$ are shown in Table 6.

(D) $\bar{M}_{uo}$ influence line: There are three kinds of influence coefficients which contribute to M_{uo} as shown in Fig. 24. The analysis for each case is as follows:

(a) The unit load is between o and u:

$$\bar{M}_{uo} = \tfrac{1}{2}M_{ao} - m_p + m_{T'_1} = \tfrac{1}{2}M_{ao} - \frac{kL}{L} \times 60 + \frac{T'_1 \times 120}{4}$$

$$= \tfrac{1}{2}M_{ao} - 60k + 30T'_1.$$

(b) The unit load is between u and a:

$$\bar{M}_{uo} = \tfrac{1}{2}M_{ao} - 60(1 - k) + 30T'_1.$$

(c) The unit load is out of span oa:

$$\bar{M}_{uo} = \tfrac{1}{2}M_{ao} + 30T'_1.$$

(d) $\bar{M}_{va}$ influence line.

Table 6. $\overline{M}_{ao}$, $\overline{M}_{ba}$, $\overline{R}_b$ and T_b influence line computations.

	$\overline{M}_{ao}$				$\overline{M}_{ba}$				$[R_b^o]$						$\overline{R}_b$			T_b
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
k	M_{ao}^o	$-21.87T_1'$	$+0.214T_2'$	M_{ao}	M_{ba}^o	$-4.61T_1'$	$-0.526T_2'$	M_{ba}	R_b^o	$M_{ab}^o+M_{ba}^o$	$M_{ba}^o-M_{b'b}^o$	$\frac{(10)}{120}$	$+\frac{(11)}{60}$	$[R_b^o]$	$-0.420T_1'$	$0.062T_2'$	$R_b=(14)+(15)+(16)$	$(17)\times2.6$
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
.1	3.089	-1.842	0.0000	1.247	-0.683	-0.352	-0.000	-1.035		-3.772	-0.659	-0.031	-0.011	-0.042	-0.035	0.000	-0.077	-0.200
.3	8.518	-4.796	0.001	3.723	-1.884	-0.915	-0.002	-2.801		-10.402	-1.817	-0.087	-0.030	-0.117	-0.092	0.000	-0.209	-0.543
u .5	11.700	-5.518	0.001	6.183	-2.588	-1.053	-0.002	-3.643		-14.288	-2.496	-0.119	-0.042	-0.161	-0.106	0.000	-0.267	-0.690
.7	11.138	-3.309	0.001	7.830	-2.463	-0.631	-0.002	-3.096		-13.601	-2.375	-0.113	-0.040	-0.153	-0.064	0.000	-0.217	-0.564
.9	5.335	-0.565	0.000	4.770	-1.180	-0.108	-0.001	-1.289		-6.515	-1.138	-0.054	-0.019	-0.073	-0.011	0.000	-0.084	-0.218
a	0	0	0	0	0	0	0	0		0	0	0	0	0	0	0	0	0
.1	4.865	-0.360	-0.000	4.505	+1.542	-0.076	0.001	+1.467	0.1	-3.323	1.485	-0.028	0.025	0.097	-0.007	-0.000	0.098	+0.234
.3	9.892	-2.777	-0.001	7.114	+4.935	-0.530	0.003	+4.408	0.3	-4.957	4.685	-0.041	0.078	0.337	-0.054	-0.000	0.283	+0.735
v .5	10.170	-4.950	-0.002	5.218	+7.198	-1.043	0.005	+6.160	0.5	-2.972	6.380	-0.025	0.106	0.581	-0.095	-0.001	0.485	1.260
.7	7.524	-4.617	-0.003	+2.904	6.476	-0.881	0.006	+5.601	0.7	-1.048	5.249	-0.009	0.087	0.778	-0.089	-0.001	0.688	1.789
.9	3.782	-2.697	-0.003	+1.082	0.913	-0.515	0.007	+0.405	0.9	-3.917	-1.245	-0.033	-0.021	0.846	-0.052	-0.001	0.793	2.062
b	2.070	-1.666	-0.002	+0.402	-4.263	-0.318	0.006	-4.575	1.0	-6.333	-7.028	-0.053	-0.117	0.830	-0.032	-0.001	0.797	2.072
.1	1.374	-1.225	-0.002	+0.147	-1.644	-0.234	0.005	-1.873	0.9	-3.018	-4.746	-0.025	-0.079	0.796	-0.024	-0.001	0.771	2.005
.3	0.374	-0.562	-0.001	-0.184	+1.804	-0.107	+0.003	+1.700	0.7	+1.430	-1.787	0.012	-0.030	0.682	-0.011	-0.001	0.670	1.742
.5	-0.198	-0.201	0.002	-0.397	+3.354	-0.038	-0.005	+3.311	0.5	3.552	0	0.030	0	0.530	0.004	0.000	0.534	1.388
.7	-0.459	0.118	0.006	-0.347	3.591	+0.022	-0.014	3.599	0.3	4.050	1.787	0.034	0.030	0.374	0.002	0.002	0.378	0.983
.9	-0.523	0.209	0.012	-0.302	3.102	+0.040	-0.030	3.112	0.1	3.625	4.746	0.030	0.079	0.209	0.004	0.003	0.216	0.562
b'	-0.517	0.235	0.016	-0.266	2.765	+0.045	-0.040	2.770	0	3.282	7.028	0.027	0.117	0.144	0.005	0.004	0.153	0.398
.1	-0.480	0.270	0.026	-0.184	2.158	+0.052	-0.065	2.145		2.638	1.245	0.022	0.021	0.043	0.005	0.008	0.056	0.146
.3	-0.400	0.257	0.047	-0.096	1.227	+0.049	-0.111	1.165		1.627	-5.249	0.014	0.087	-0.073	0.005	0.013	-0.055	-0.143
v' .5	-0.282	0.198	0.048	-0.036	0.618	+0.038	-0.119	0.537		0.900	-6.580	0.008	-0.110	-0.102	0.004	0.014	-0.084	-0.218

Table 6. (Contd.)

	$\overline{M}_{ao}$				$\overline{M}_{ba}$				$[R^p_b]$						$\overline{R}_b$			T_b
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
k	M^o_{ao}	$-21.87T'_1$	$+0.214T'_2$	M_{ao}	M^o_{ba}	$-4.61T'_1$	$-0.526T'_2$	M_{ba}	R^o_b	$M^o_{ab}+M^o_{ba}$	$M^o_{ba}-M^o_{b'b}$	$\frac{(10)}{120}$	$+\frac{(11)}{60}$	$[R^p_b]$	$-0.420T'_1$	$0.062T'_2$	$R_b=(14)+(15)+(16)$	$(17)\times 2.6$
.7	- 0.157	0.116	0.027	-0.014	0.250	+0.022	-0.067	0.207		0.407	-4.685	0.003	-0.078	-0.075	0.002	0.008	-0.065	-0.169
.9	- 0.045	0.035	0.004	-0.006	0.057	+0.007	-0.010	0.054		0.102	-1.485	0.001	-0.025	-0.024	0.001	0.001	-0.022	-0.057
a'	0	0	0	0	0	0	0	0		0	0	0	0	0	0	0	0	0
.1	+ 0.030	-0.033	0.006	+0.003	-0.042	-0.006	-0.014	-0.062		- 0.072	+1.138	0	0.019	0.019	-0.001	0.002	0.020	0.052
.3	0.064	-0.076	0.032	+0.020	-0.088	-0.015	-0.080	-0.183		- 0.152	+2.375	-0.001	0.040	0.039	-0.001	0.009	0.047	0.122
u' .5	0.068	-0.087	0.054	+0.035	-0.092	-0.017	-0.133	-0.242		- 0.160	2.496	-0.001	0.041	0.040	-0.002	0.017	0.055	0.143
.7	0.049	-0.068	0.047	+0.028	-0.067	-0.013	-0.116	-0.196		- 0.116	1.817	-0.001	0.030	0.029	-0.001	0.014	0.042	0.109
.9	0.017	-0.026	+0.018	+0.009	-0.024	-0.005	-0.044	-0.073		- 0.041	0.659	0	0.011	0.001	-0.001	+0.005	0.016	0.042
o'	0	0	0	0	0	0	0	0		0	0	0	0	0	0	0	0	0

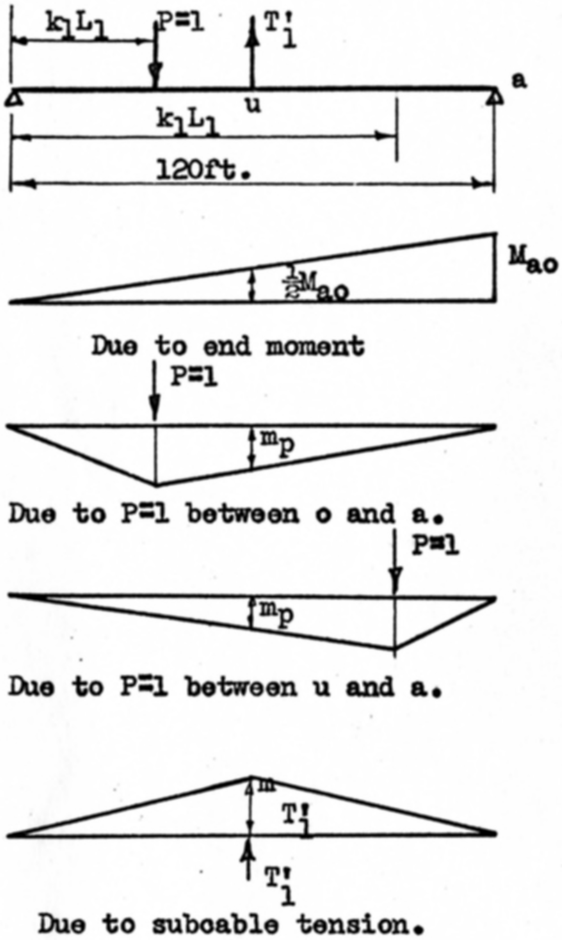


Fig. 24. Moment at point u due to the unit load on the structure.

There are also three kinds of influence coefficients which contribute to M_{va} as shown in Fig. 25. The analysis for each case is as follows:

(a) The unit load is between a and v:

$$\overline{M}_{va} = -\tfrac{1}{2}M_{ab} + \tfrac{1}{2}M_{ba} - 60k + 30T_1' \, .$$

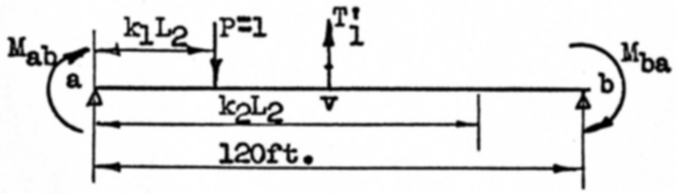
(b) The unit load is between v and b:

$$\overline{M}_{va} = -\tfrac{1}{2}M_{ab} + \tfrac{1}{2}M_{ba} - 60(1 - k) + 30T_1' \, .$$

(c) The unit load is out of span ab:

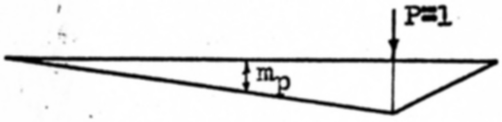
$$\overline{M}_{va} = \tfrac{1}{2}(M_{ba} - M_{ab}) + 30T_1' \, .$$

The calculations for the influence lines for $\overline{M}_{uo}$ and $\overline{M}_{va}$ are shown in Table 7 and all influence lines are drawn in Fig. 26.

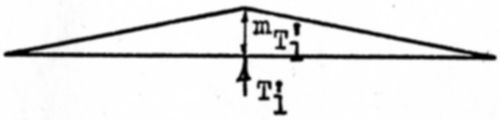


Due to end moments.

Due to P=1 between a and v.



Due to P=1 between v and b.



Due to subcable tension.

Fig. 25. Moment at point v due to the unit load on the structure.

Table 7. $\overline{M}_{uo}$ and $\overline{M}_{va}$ influence line computations.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
k	$\frac{1}{2}\overline{M}_{ao}$ $=-\frac{1}{2}\overline{M}_{ab}$	$\frac{1}{2}\overline{M}_{ba}$	$30T'_1$	-60k (P=1 Between ou)	-60(1-k) (P=1 Between va)	-60k (P=1 Between av)	-60(1-k) (P=1 Between vb)	$\overline{M}_{uo}$= (2)+(4) +(5)+(6)	$\overline{M}_{va}$= -(-2)+(3) +(4)+(7)+(8)
0	0	0	0	0	0	0	0	0	0
.1	0.624	-0.518	2.535	- 6.000				- 2.841	2.641
.3	1.862	-1.401	6.600	-18.000				- 9.538	7.061
u .5	3.092	-1.822	7.593	-30.00				-20.315	8.863
.7	3.915	-1.548	4.554		-18.000			- 9.531	6.921
.9	2.385	-0.645	0.777		- 6.000			- 2.838	2.517
a	0	0	0		0			0	0
.1	2.253	0.734	0.495			- 6.000		2.748	- 2.518
.3	3.557	2.204	3.822			-18.000		7.379	- 8.417
v .5	2.609	3.080	6.786			-30.000		9.395	-17.525
.7	1.452	2.801	6.354				-18.000	7.806	- 7.393
.9	0.541	0.203	3.711				- 6.000	4.252	- 1.545
b	0.201	-2.288	2.290				0	2.491	0.203
.1	0.074	-0.937	1.686					1.760	0.823
.3	-0.095	0.850	0.774					0.869	1.529

Table 7. (Contd.)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
k	$\frac{1}{2}\overline{M}_{ao}$ $=-\frac{1}{2}\overline{M}_{ab}$	$\frac{1}{2}\overline{M}_{ba}$	$30T'_1$	-60k (P=1 Between ou)	-60(1-k) (P=1 Between va)	-60k (P=1 Between av)	-60(1-k) (P=1 Between vb)	$\overline{M}_{uo}=$ (2)+(4) +(5)+(6)	$\overline{M}_{va}=$ -(-2)+(3) +(4)+(7)+(8)
.5	-0.149	1.656	0.276					0.127	1.783
.7	-0.179	1.799	-0.162					- 0.341	1.458
.9	-0.151	1.556	-0.288					- 0.439	1.117
b'	-0.133	1.385	-0.324					- 0.457	0.929
.1	-0.092	1.073	-0.372					- 0.464	0.609
.3	-0.048	0.583	-0.354					- 0.402	0.181
.5	-0.018	0.269	-0.273					- 0.291	- 0.022
.7	-0.007	0.104	-0.159					- 0.166	- 0.052
.9	-0.003	0.027	-0.048					- 0.051	- 0.024
a'	0	0	0					0	0
.1	+0.001	-0.031	0.045					0.044	0.015
.3	+0.010	-0.092	0.105					0.115	0.023
.5	0.018	-0.121	0.120					0.138	0.017
.7	0.014	-0.098	0.093					0.107	0.009
.9	0.005	-0.037	0.036					0.041	0.004
o	0	0	0					0	0

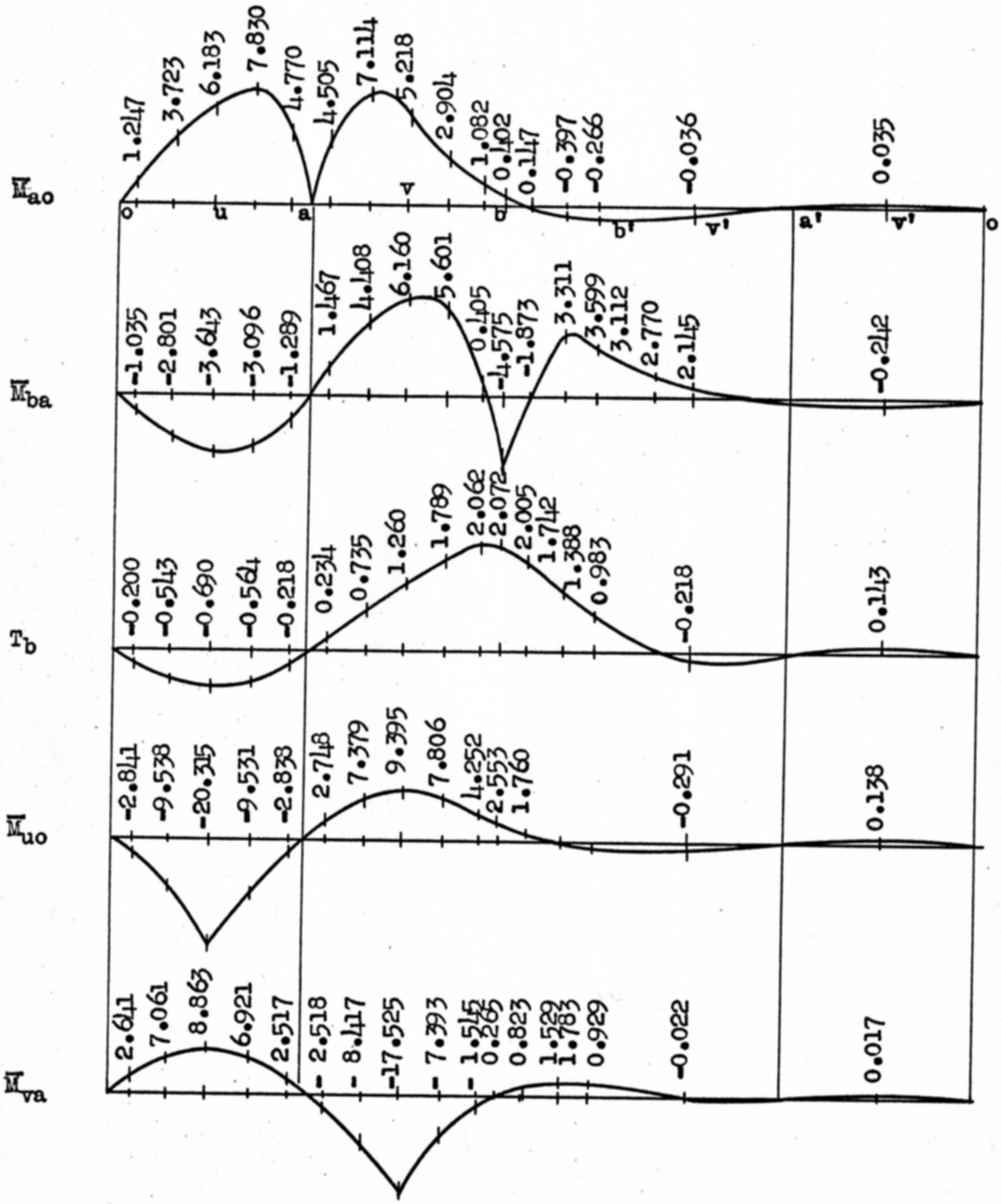


Fig. 26. Influence lines for the cable-stiffened girder bridge of Example 1.

Example 2

Required:

Draw the influence lines for the base structure of Example 1 using the electronic computer.

Solution:

The continuous beam is considered as made up of five members: oa, ab, bb', b'a' and a'o' and labeled as shown in Fig. 27.

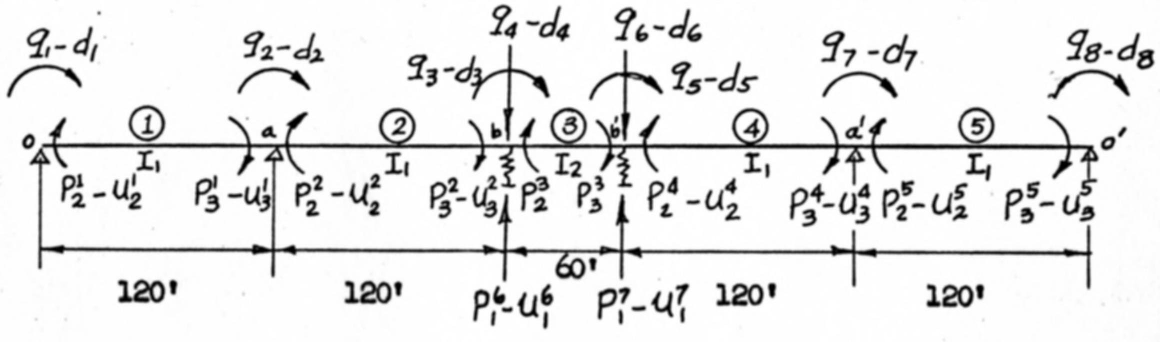


Fig. 27. The displacement and deformation relations.

By the Displacement Method (6), the transformation matrix $\left[A/r\right]$ which transforms the load point and redundant point displacement d and   into the member deformations u is defined by equation (51).

$$\{u\} = \left[A/r\right] \cdot \left\{\frac{d}{\Delta}\right\} .$$

From Fig. 27 the displacement-deformation transformation matrix is

$$\{u\} = \begin{bmatrix} 1 & . & . & . & . & . & . & . \\ . & 1 & . & . & . & . & . & . \\ . & . & . & -.00833 & . & . & . & . \\ . & . & 1 & -.00833 & . & . & . & . \\ . & . & 1 & .01667 & . & -.01667 & . & . \\ . & . & . & .01667 & 1 & -.01667 & . & . \\ . & . & . & . & 1 & .00833 & . & . \\ . & . & . & . & . & .00833 & 1 & . \\ . & . & . & . & . & . & 1 & . \\ . & . & . & . & . & . & . & 1 \\ . & . & . & 1 & . & . & . & . \\ . & . & . & . & . & 1 & . & . \end{bmatrix} \{d\}$$

The elements a_{ij} of the displacement-deformation matrix $\left[A/r\right]$ are defined as the deformations of member i produced by a unit joint displacement at j, while all other joint displacements are held at zero. The dots in the matrix indicate zeroes.

The transformation matrix K, which transforms joint deformations into end member forces is defined as

$$\{P\} = [K]\{u\} . \tag{52}$$

From Fig. 27, the force-deformation transformation matrix can be expressed as

$$K^1 = K^2 = K^4 = K^5 = \begin{bmatrix} \frac{4EI}{L} & \frac{2EI}{L} \\ \frac{2EI}{L} & \frac{4EI}{L} \end{bmatrix} = \frac{EI_1}{60}\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix} ,$$

$$K^3 = \frac{EI_2}{60}\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix} \text{, and } K^6 = K^7 = \frac{A_c E_c}{\ell} = [1308.67] .$$

For the entire structure, we have

$$K = \begin{bmatrix} K^1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & K^2 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & K^3 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & K^4 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & K^5 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & K^6 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & K^7 \end{bmatrix} ,$$

which is a diagonal matrix of the submatrixes K^s. The superscripts on the elements of the K matrix identify the member to which these elements apply.

The load matrix q for symmetrical and anti-symmetrical loads is

$$\begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \\ q_7 \\ q_8 \end{Bmatrix} = \begin{bmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ 100 & \cdot & 100 & \cdot \\ \cdot & 100 & \cdot & 100 \\ 100 & \cdot & -100 & \cdot \\ \cdot & 100 & \cdot & -100 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{bmatrix}$$

The dots in the matrix indicate zeroes.

Using the displacement method program, available in the Civil Engineering Department, we obtain a set of answers which are shown in Fig. 28. These values compare favorably with the ones found by hand computations, which are shown in Fig. 18, page 31. The influence lines for the base structure may then be found by use of the second step of Example 1.

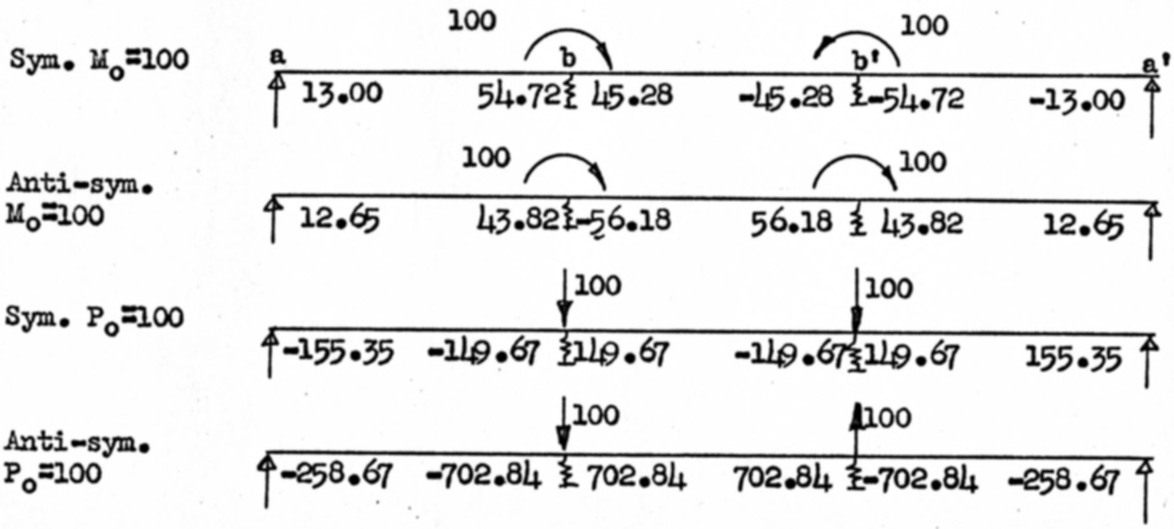


Fig. 28. Computer solutions for end moments due to indicated loadings.

Example 3

Required:

Find the influence lines for the cable stiffened girder bridge, Fig. 15 (the same structure as described in Example 1), by the Force Method.

Solution:

Step 1. Remove the constraints M_a, $M_{a'}$, T_{uv}, T_b, $T_{b'}$ and $T_{v'u'}$ to make the structure statically determinate, as shown in Fig. 29.

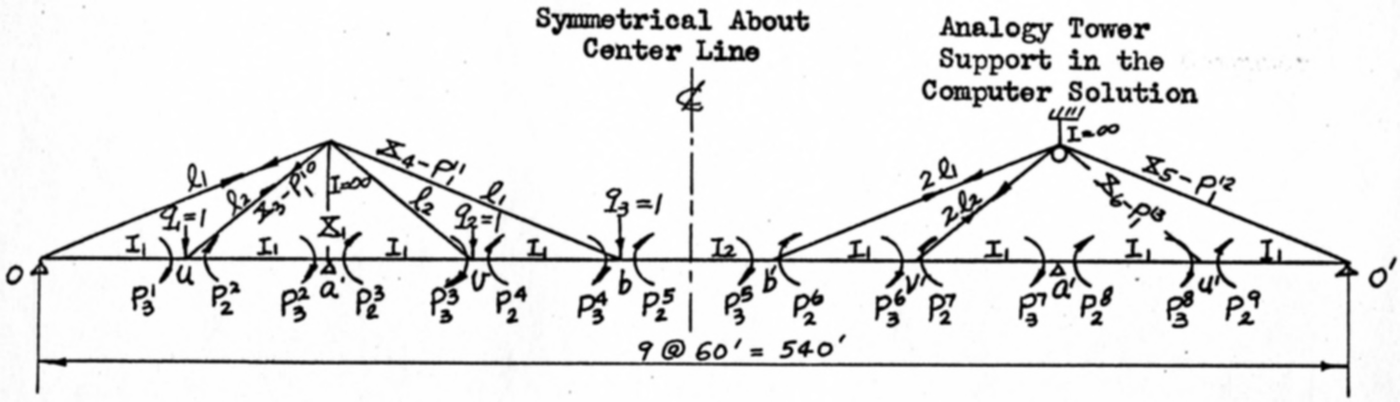


Fig. 29. Example 3.

Step 2. Determine the load (redundant)-force matrix $[\alpha \mid \beta]$ which transforms the loads q and redundants X into the internal force P. The total force on any member of the primary structure is due to the superposition of forces due to the external loads and the redundants and is given by equation (53)

$$\{P\} = [\alpha \mid \beta] \cdot \left\{\frac{q}{X}\right\} \tag{53}$$

In terms of the notation indicated in Fig. 29, equation (53) may be written as follows:

$$
\begin{Bmatrix} P^1_3 \\ P^2_2 \\ P^2_3 \\ P^3_2 \\ P^3_3 \\ P^4_2 \\ P^4_3 \\ P^5_2 \\ P^5_3 \\ P^6_2 \\ P^6_3 \\ P^7_2 \\ P^7_3 \\ P^8_2 \\ P^8_3 \\ P^9_2 \\ P^{10}_1 \\ P^{11}_1 \\ P^{12}_1 \\ P^{13}_1 \end{Bmatrix}
=
\begin{bmatrix}
-30 & 0 & 0 & +0.5 & 0 & +19.2 & 0 & 0 & 0 \\
+30 & 0 & 0 & -0.5 & 0 & -19.2 & 0 & 0 & 0 \\
0 & 0 & 0 & +1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & -48 & -36 & +0.8 & 0.2 & +30.7 & +13.85 & +9.24 & +7.68 \\
0 & +48 & +36 & -0.8 & -0.2 & -30.7 & -13.85 & -9.24 & -7.68 \\
0 & -36 & -72 & +0.6 & +0.4 & +23.1 & +27.7 & +18.5 & +15.4 \\
0 & +36 & +72 & -0.6 & -0.4 & -23.1 & -27.7 & -18.5 & -15.4 \\
0 & -24 & -48 & +0.4 & +0.6 & +15.4 & +18.5 & +27.7 & +23.2 \\
0 & +24 & +48 & -0.4 & -0.6 & -15.4 & -18.5 & -27.7 & -23.2 \\
0 & -12 & -24 & +0.2 & +0.8 & +7.68 & +9.24 & +13.85 & +30.7 \\
0 & +12 & +24 & -0.2 & -0.8 & -7.68 & -9.24 & -13.85 & -30.7 \\
0 & 0 & 0 & 0 & +1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & +0.5 & 0 & 0 & 0 & +19.2 \\
0 & 0 & 0 & 0 & -0.5 & 0 & 0 & 0 & -19.2 \\
0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1
\end{bmatrix}
\left\{ \begin{array}{c} q_1 \\ q_2 \\ q_3 \\ \hline X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \end{array} \right\}
\qquad (54)
$$

Step 3. Determine the deformation-displacement matrix $[C]$, which transforms the member deformations $\{u\}$ into joint displacements $\{d\}$.

It is defined in equation (55).

$$\{d\} = [C]\,\{u\} \qquad (55)$$

The deformation-displacement matrix can be shown to be the load-force matrix transposed (6). Thus we have

$$\{d\} = [C]\{u\} = [\alpha \mid \beta]^T\{u\} .$$

Step 4. Determine the force-deformation matrix ϕ, which transforms the deformations $\{u\}$ into the internal forces $\{P\}$. That is

$$\{u\} = [\phi]\{P\} . \tag{56}$$

From Fig. 24, the force-deformation matrix is determined by combining the individual ϕ^s submatrixes for each member. They are as follows:

$$\phi_3^1 = \phi_2^9 = \left[\frac{L}{3EI}\right] = \frac{1}{EI_1}\left[\ 20\ \right],$$

$$\phi^2 = \phi^3 = \phi^4 = \phi^6 = \phi^7 = \phi^8 = \begin{bmatrix} \frac{L}{3EI} & \frac{-L}{6EI} \\ \frac{-L}{6EI} & \frac{L}{3EI} \end{bmatrix} = \frac{1}{EI_1}\begin{bmatrix} 20 & -10 \\ -10 & 20 \end{bmatrix},$$

$$\phi^5 = \frac{1}{EI_2}\begin{bmatrix} 20 & -10 \\ -10 & 20 \end{bmatrix}, \quad \phi_1^{10} = \phi_1^{13} = \frac{2\ell}{A_cE_c} = \frac{1}{EI_1}[1370] ,$$

and $\phi_1^{11} = \phi_1^{12} = \frac{1}{EI_1}[1720]$.

Therefore, the entire ϕ matrix for the structure is

$$\phi = \begin{bmatrix}
\phi_3^1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \phi^2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \phi^3 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \phi^4 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \phi^5 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \phi^6 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \phi^7 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \phi^8 & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \phi_2^9 & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \phi_1^{10} & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \phi_1^{11} & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \phi_1^{12} & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \phi_1^{13}
\end{bmatrix} \tag{57}$$

The dots in the ϕ matrix represent zero elements.

Step 5. Write the external load and redundant force matrix $\left\{\frac{q}{X}\right\}$. Since we are going to check the internal stresses due to unit loads at u, v and b acting on the structure separately, the q matrix is

$$q = \begin{bmatrix} 1 & \cdot & \cdot \\ \cdot & 1 & \cdot \\ \cdot & \cdot & 1 \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix} \tag{58}$$

Using the flexibility method program, available in the Civil Engineering Department, we obtain the answers shown in Fig. 31.

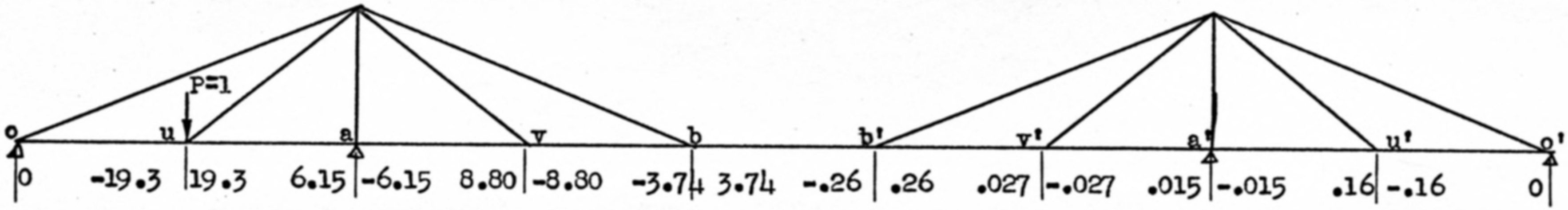
If we put the unit load at each point from k = 0.1 to 0.9 on each span, we obtain the internal stresses for the structure due to each position of the unit load. Then if we select in turn the stresses for which the influence lines are to be drawn, we obtain the influence lines for the structure.

However, in this study, we calculated the influence coefficients at only three points, namely u, v and b of the structure. Table 8 shows the agreement between the computer solutions and hand computations. Therefore, we can see that the computer solutions would also result in the same influence lines for the structure as those shown in Fig. 26.

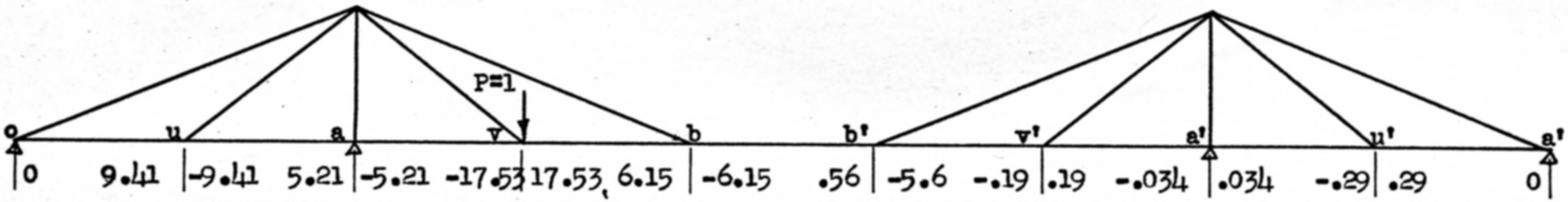
CONCLUSIONS

The structural analysis for the cable-stiffened girder bridge can be performed either by hand computation or computer solutions.

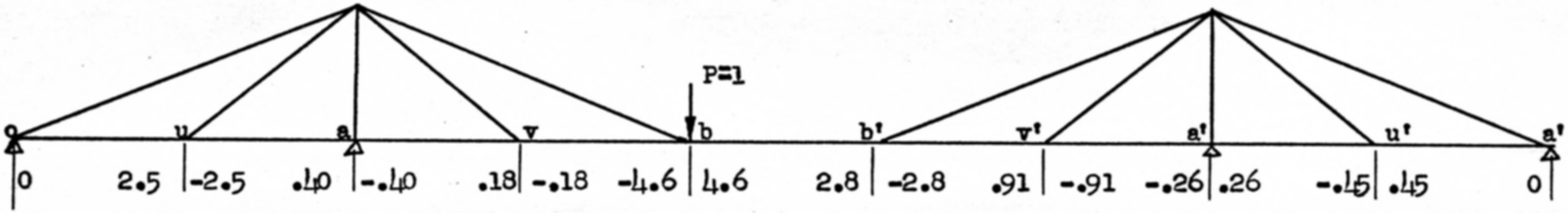
In order to save time in analyzing the structure for moving loads, influence lines for the structure should be drawn. Taking advantage of the symmetry of the structure, and taking all possible loads as a linear combination of



(a) Influence coefficients due to P=1 at point u.



(b) Influence coefficients due to P=1 at point v.



(c) Influence coefficients due to P=1 at point b.

Fig. 31. Computer solutions for the influence coefficients of internal forces of the structure due to unit load at points u, v and b.

Table 8. Comparison between computer solutions and hand computations.

	Internal Stress	Load at u Computer Sol.	Load at u Hand Cal.	Load at v Computer Sol.	Load at v Hand Cal.	Load at b Computer Sol.	Load at b Hand Cal.
$M_{u'}$	M_{uo}	-19.305	-19.315	9.407	9.395	2.504	2.491
	M_{ua}	19.305		- 9.407		-2.504	
M_{ao}	M_{au}	6.154	6.183	5.208	5.218	0.395	0.402
	M_{av}	- 6.154		- 5.208		-0.395	
M_{va}	M_{va}	8.804	8.863*	-17.533	-17.525	0.182	0.203
	M_{vb}	- 8.804		+17.533		-0.182	
M_{ba}	M_{bv}	- 3.740	- 3.643	6.149	6.160	-4.626	-4.575
	$M_{bb'}$	+ 3.740		- 6.149		+0.426	
	$M_{b'b}$	- 0.257		0.559		2.807	
	$M_{b'v'}$	0.257		- 0.559		-2.807	
	$M_{v'b'}$	0.027		- 0.019		0.908	
	$M_{v'a'}$	- 0.027		0.019		-0.908	
	$M_{a'v'}$	+ 0.0154		- 0.034		-0.261	
	$M_{a'u'}$	+ 0.0154		+ 0.034		+0.261	
	$M_{u'a'}$	0.164		- 0.291		-0.453	
	$M_{u'o'}$	- 0.164		+ 0.291		+0.453	
	T_1	0.397	0.395	0.354	0.354	0.120	0.120
	T_b	- 0.697	- 0.690	1.269	1.260	2.072	2.072
	T'_b	+ 0.140		- 0.220		0.399	
	T_2	+ 0.008		- 0.014		-0.017	

*Maximum difference .6%.

symmetrical and anti-symmetrical loads at joints, the solutions for the selected base structure can be found. By the use of the reciprocal theorem and tables for calculations, the influence lines can readily be obtained. On the other hand, the influence lines also can be found by digital computer methods whenever a program and a computer are available.

In this study, the structural analyses by direct moment distribution and by computer solution, have been shown to agree closely. Therefore, we see that it could be advantageous to use one method to analyze the structure and then to check the answers by the other approach. Thus we can be sure that we have the correct internal forces for the design work.

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NOTATIONS

A_c	:	Cross-sectional area of cable.
$[A/r]$	:	Displacement-deformation matrix.
$[C]$	:	Deformation-displacement matrix.
C_b	:	Spring constant of elastic support.
C_b^o	:	$C_b L_1^2/EK_1$.
C_{mn}	:	Carry-over factor from m to n for member mn.
$\{d\}$	:	Deformation matrix.
E	:	Modulus of elasticity of member.
E_c	:	Modulus of elasticity of cable.
F_{mn}	:	Fixed-end moment of member mn at point m.
K_s	:	Stiffness of member.
$[K^s]$	:	Stiffness matrix for members.
K'	:	Modified stiffness of spring.
k	:	Length ratio between the span length and the distance of the unit load from the left support of the span.
L	:	Length of member.
ℓ	:	Length of cable.
M_{mn}	:	End moment of member mn at point m.
M_o	:	Symmetrical or anti-symmetrical moment at elastic support.
m	:	Member length ratio L_1/L_2.
P_o	:	Symmetrical or anti-symmetrical concentrated load at elastic support.
P_b^o, $P_{b'}^o$	:	Reactions at elastic supports b and b'.
P_u, P_x	:	Unit load at point u or x.
$\{P\}$	:	Internal force matrix.
$\{q\}$	:	External load matrix.

R	:	Rigid rotation of members.
T	:	Tension in the cable.
T'	:	The vertical components of the cable tensions.
$\{u\}$	:	Column matrix of the displacement.
u_a	:	Distribution factor at point a of member ab.
u_k, η_{ak}	:	The coefficient of y_u in Fig. 12.
δu, δv	:	Deflection of point u or v due to the unit load on the structure.
θ_1, θ_2	:	Angles between tower column and outside and inside cables separately.
θ_m	:	Angle change of a member at point m.
$[\alpha \mid \beta]$	:	Load-force matrix.
$[\phi]$	:	Force-deformation matrix.

ABBREVIATIONS

ft.	:	Foot (feet).
in.	:	Inch(es).
K	:	Kip(s)=1000 pounds.
Ksi.	:	Kip(s) per square inch.
lb.	:	Pound(s).
Psi.	:	Pound(s) per square inch.
sq.	:	Square.

APPENDIX A

```
COMPUTER OUTPUT FOR EXAMPLE 2

(A) SYMMETRICAL MOMENT MO=100 AT MIDDLE SUPPORTS B AND B1

DET.         .59821637E+42

NODAL DISPLACEMENTS - LOADING NO  1

      .17091681E-04      -.34183363E-04       .13041347E-03       .43104095E-03
     -.13041348E-03       .43104099E-03       .34183359E-04      -.17 91679E-04

INTERNAL STRESSES - LOADING NO  1

     -.30000000E-06      -.12996799E+02       .12996796E+02       .54717517E+02
      .45282447E+02      -.45282460E+02      -.54717522E+02      -.12996799E+02
      .12996797E+02       .30000000E-06       .56406018E+00       .56406023E+00


(B) SYM. CONCENTRATED LOAD PO=100 AT MIDDLE SUPPORTS B AND B1

DET.         .59821637E+42

NODAL DISPLACEMENTS - LOADING NO  1

     -.20430159E-03       .40860320E-03       .43104106E-03       .74475885E-01
     -.43104085E-03       .74475890E-01      -.40860328E-03       .20430164E-03

INTERNAL STRESSES - LOADING NO  1

      .10000000E-04       .15535433E+03      -.15535435E+03      -.14966697E+03
      .14966710E+03      -.14966690E+03       .14966710E+03       .15535440E+03
     -.15535436E+03       .00000000E-99       .97459143E+02       .97459149E+02
```

(C) ANTI-SYMMETRICAL MOMENT MO=100 AT MIDDLE SUPPORTS.

DET. .59821637E+42

NODAL DISPLACEMENTS - LOADING NO 1

.16637767E-04	-.33275536E-04	.89675813E-04	-.10719718E-02
.89675811E-04	.10719719E-02	-.33275535E-04	.16637767E-04

INTERNAL STRESSES - LOADING NO 1

-.50000000E-06	-.12651635E+02	.12651638E+02	.43816387E+02
.56183613E+02	.56183612E+02	.43816386E+02	.12651638E+02
-.12651635E+02	-.20000000E-06	-.14027822E+01	.14 27824E+01

(D) ANTI-SYMMETRICAL CONCENTRATED LOAD PO=100 AT MIDDLE SUPPORTS.

DET. .59821637E+42

NODAL DISPLACEMENTS - LOADING NO 1

-.34017243E-03	.68034490E-03	-.10719720E-02	.52390370E-01
-.10719719E-02	-.52390376E-01	.68034491E-03	-.34 17245E-03

INTERNAL STRESSES - LOADING NO 1

.10000000E-04	.25867278E+03	-.25867282E+03	-.70283644E+03
.70283646E+03	.70283650E+03	-.70283642E+03	-.25867282E+03
.25867279E+03	.10000000E-04	.68558038E+02	-.68558046E+02

APPENDIX B

COMPUTER OUTPUT FOR EXAMPLE 3

DET. .47090495E+22

NODAL DISPLACEMENTS - LOADING NO 1 (P=1 ACTING AT POINT U)

.21319588E+05	-.12851045E+05	-.31234157E+04

NODAL DISPLACEMENTS - LOADING NO 2 (P=1 ACTING AT POINT V)

-.12851057E+05	.20474458E+05	.56835368E+04

NODAL DISPLACEMENTS - LOADING NO 3 (P=1 ACTING AT POINT B)

-.31234147E+04	.56834845E+04	.92644501E+04

INTERNAL STRESSES - LOADING NO, 1

-.19304747E+02	.19304747E+02	.61536927E+01	-.61536927E+01
.88041940E+01	-.88041940E+01	-.37393178E+01	.37393178E+01
-.25681934E+00	.25681934E+00	.26577590E-01	-.26577590E-01
.15365026E-01	-.15365026E-01	.16375530E+00	-.16375530E+00
.39679207E+00	-.69732891E+00	.13985140E+00	.81287914E-02

INTERNAL STRESSES - LOADING NO 2

.94072334E+01	-.94072334E+01	.52079235E+01	-.52 79235E+01
-.17533257E+02	.17533257E+02	.61494728E+01	-.61494728E+01
.55888618E+00	-.55888618E+00	-.18822100E-01	.18822100E-01
-.34123084E-01	.34123084E-01	-.29093490E+00	.29 93490E+00
.35433707E+00	.12689790E+01	-.22042529E+00	-.14264238E-01

INTERNAL STRESSES - LOADING NO 3

.25040144E+01	-.25040144E+01	.39532482E+00	-.39532482E+00
.18153530E+00	-.18153530E+00	-.46256464E+01	.46256464E+01
.28056034E+01	-.28056034E+01	.90848060E+00	-.90848060E+00
-.26099824E+00	.26099824E+00	-.45296363E+00	.45296363E+00
.12012250E+00	.20704686E+01	.39856181E+00	-.16795027E-01

STRUCTURAL ANALYSIS OF A CABLE STIFFENED-GIRDER BRIDGE

by

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requirements for the degree

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1966

ABSTRACT

A cable-stiffened girder bridge is a recent innovation in the design of long span bridges. It has the following advantages over conventional long span bridge designs: (1) Light in weight, (2) Efficient utilization of material and (3) Greater rigidity than suspension bridges.

There are many possible approaches to the analysis of these high-order statically indeterminate structures. However, many of the methods suitable involve laborious procedures for solving the resulting simultaneous equations.

This study presents two possible methods of structural analysis for cable-stiffened girder bridges. One is to solve a reduced base structure, then apply these results to obtain a complete solution and the other is to solve the complete structure as one problem.

The method which takes advantage of the reduced structure consists of three stages. The first stage is the use of some method, in this case, direct moment distribution or the displacement method, to find the influence lines for the base structure which is, in this case, a continuous beam on elastic supports. The second stage is the use of the reciprocal theorem to find influence lines for the cable tensions. In the final stage, the influence lines for the base structure and for the cable tensions are combined to find the influence lines for the complete structure.

Numerical examples are given to illustrate the procedures for finding the influence lines for the structure. Since the direct moment distribution method, the displacement method and the force method are used to solve the same problem, a comparison of the results of the various methods can be made. The comparison shows that the solutions agree closely with each other. Various influence lines for the structure are also drawn.