# Some properties of non-octave-repeating scales, and why composers might care

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**Some Properties of Non-Octave-Repeating Scales,**
**and Why Composers Might Care**

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Paper Presentation
(Oral presentation version)

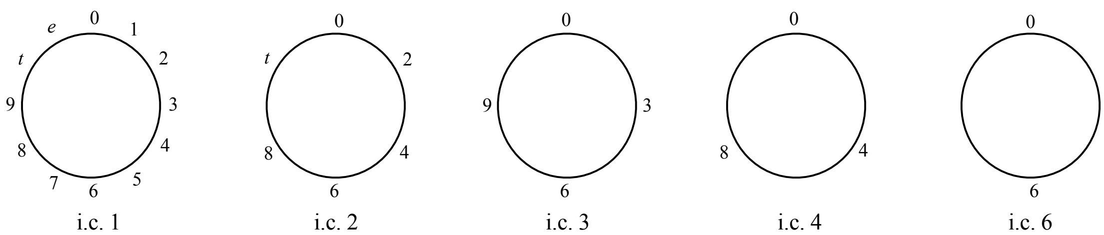
This paper focuses on the family of scales can be generated using interval patterns that repeat at some modular interval other than the octave. These scales and the compositional syntaxes one might build from them have some very interesting properties. Of particular interest are various hybrids of pitch and pitch-class interval structures, common tones under transposition, and the possibilities for modulations of different nearness or distantness.

I'll begin by discussing the derivation of these scales and some of their properties. I will offer several comparisons and contrasts to more familiar octave-repeating scales and the compositional systems they have sparked. Later, I'll address the second part of the title of this paper "and Why Composers Might Care." Or, perhaps more precisely, I'll address the issue of why *this particular* composer cares, and leave it as an open question whether others might wish to care as well.

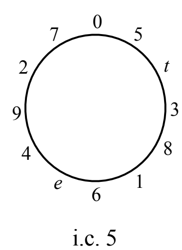
## I. Interval Cycles and the Generation of Scales

Example 1 shows graphs of the five interval cycles that are closed within an octave. (One reads these graphs by following the circle clockwise.) One can think of these graphs not only as representing the aggregate in pitch-*class* space (what the circle typically represents in these types of graphs), but also as representing an octave in pitch space, divided into equal increments of 1, 2, 3, 4, and 6 semitones, respectively, and repeating in higher octaves if one continues the ascending pattern beyond the first octave in pitch space.

Example 2 shows a graph of interval cycle 5, which, like the cycles in Example 1 generates the total chromatic and repeats itself in pitch-class space. But, unlike the interval cycles in Example 1, interval cycle 5 does not close within a single octave of pitch space. Indeed, it takes five octaves in pitch space for a cycle of ascending five-semitone intervals to repeat itself. (If you wish to see this notated on the staff, please refer to Example 5). Unlike interval cycle 1, which generates the total aggregate within one octave of pitch-space, interval cycle 5 generates the total aggregate within five octaves of pitch-space, or, of course, in pitch-class space. Therefore, we will say that the modular interval of this interval cycle is five octaves.




Example 1: Interval cycles that close within a single octave in pitch-space

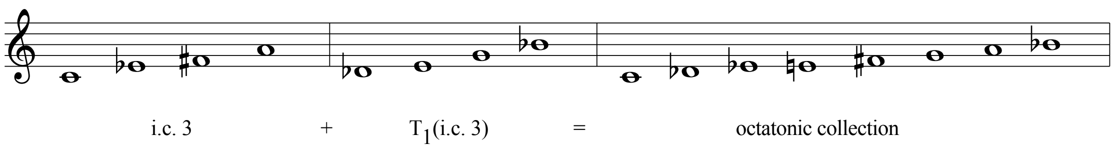



Example 2: Interval cycle 5

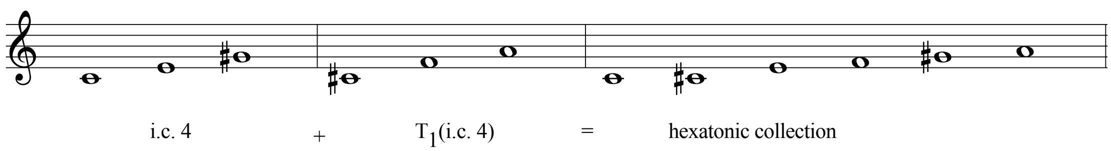
The remaining interval cycles involving intervals smaller than an octave can be read in the graphs of Examples 1 and 2 by reading the graphs of interval cycles 1, 2, 3, 4, and 5 counter-clockwise rather than clockwise, which will represent interval cycles 11, 10, 9, 8, and 7, respectively.

Many of the symmetrical scales familiar in Western practice can be thought of as deriving from these interval cycles. Interval cycle 1 generates the chromatic scale, while interval cycle 2 generates the whole-tone scale. Example 3 shows how two interval cycle 3s, a semitone apart, combine to generate the octatonic scale. Example 4 shows the same derivation formula for interval cycle 4 and the hexatonic scale.

The familiar asymmetrical scales in western practice, the diatonic and pentatonic scales, can be thought of as derived by interval cycle 5 in pitch-class space. Any seven contiguous pitch-classes in the graph on Example 2 comprise a diatonic collection, while any five contiguous pitch-classes in this cycle comprise a pentatonic collection. (If one reorders these pitch classes and places them within an octave in pitch space, one gets the familiar scale representations.)

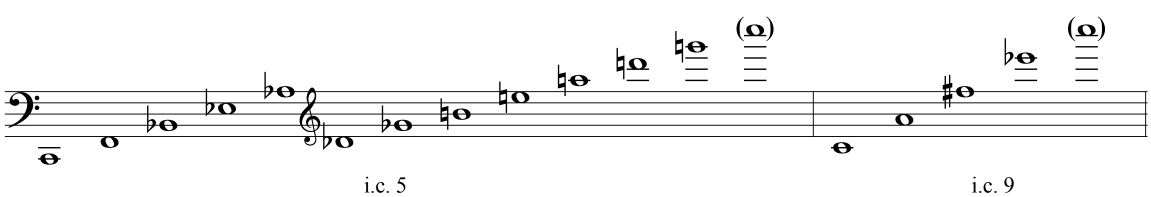



Example 3: Derivation of octatonic collection from two i.c. 3s




Example 4: Derivation of hexatonic collection from two i.c. 4s

Example 5 gives an example of interval cycles 5 and 9 represented in pitch space. Interval cycle 5 has a modular interval of five octaves, and it generates the aggregate upon completing its cycle. Interval cycle 9 has a modular interval of three octaves, and does not generate the aggregate.




Example 5: interval cycles 5 and 9 in pitch space

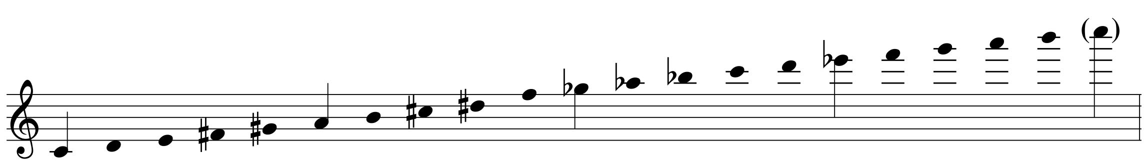
To summarize, pitch interval cycles 1, 2, 3, 4, and 6 are closed within a single octave in pitch-space, while pitch interval cycles 5, 7, 8, 9, 10, and 11 are not. Obviously, any pitch interval cycle of greater than 12 will also not be closed within a single octave of pitch space. All interval cycles that are not closed within a single octave of pitch-space can generate scales that I refer to as non-octave-repeating scales.

## II. Non-octave-repeating Scales

Example 6 gives an example of a non-octave-repeating scale based on interval cycle 5. Each 5-semitone interval in the cycle is filled, in this case, with intervals of 2, 2, and 1 semitones, respectively. (The naming convention gives the generative interval cycle, and then the partitioning of each instance of the generative interval. The name for this scale could be read “interval cycle 5, partitioned 2, 2, 1.”) Note that this scale repeats its pattern every 5 semitones and every 5 octaves, but not in every octave as familiar scales do. In colloquial terminology associated with diatonic tonal practice, one could describe this scale as “moving one position toward the flat side” with each repeating 5-semitone segment. The stemmed notes do not represent any type of musical priority, as in a Schenkerian graph: they simply highlight, visually, the notes of the generating interval 5 cycle.

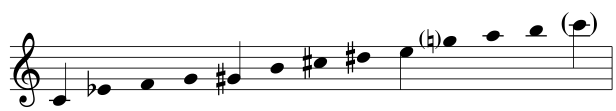
Example 6: Non-octave-repeating scale i.c. 5 <2,2,1>

Example 7 gives an example of a non-octave-repeating scale i.c. 9 <2,2,2,2,1>. Note that this scale repeats its pattern every 9 semitones and every 3 octaves, but not in every octave as familiar scales do. While the scale in Example 6 had a familiar diatonic quality to it, this one does not—it is strongly suggestive of whole-tone music, since, in historical Western practice, one rarely encounters four consecutive whole-steps in a scale, except in the whole-tone scale. (For those who are now racking their brains trying to come up with an example from outside the whole-tone collection, it would be scale degrees three through seven of the melodic minor scale, sometimes known in a different ordering as the lydian/mixolydian scale.)



Example 7: Non-octave-repeating scale i.c. 9 <2,2,2,2,1>

It is interesting to examine the underlying pitch-class content of these non-octave repeating scales. The scales in Examples 6 and 7 both include the total aggregate within their modular spans. (In Example 6, the modular interval is five octaves. In Example 7, it is three octaves.) Not all non-octave repeating scales exhibit this property, however. Example 8 gives a non-octave-repeating scale i.c. 8 <3,2,2,1>. This scale does not contain the aggregate within its four-octave modular span: there are no instances of pitch classes 2, 6, or 10.

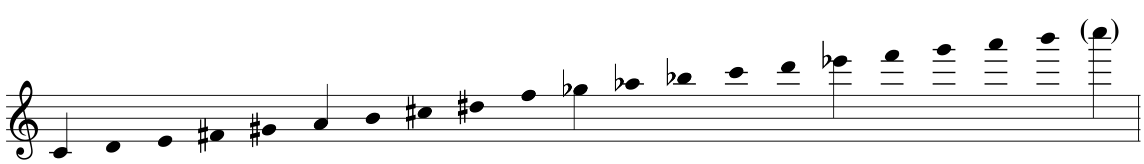


Example 8: Non-octave-repeating scale i.c. 8 <3,2,2,1>

Examples 9 through 11 reproduce the scales in Examples 6 through 8, with pitch-class distribution charts added.

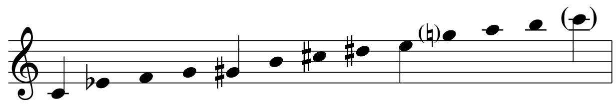
| pitch class | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | *t* | *e* |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| instances | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |

Example 9: Non-octave-repeating scale i.c. 5 <2,2,1>, with pitch-class distribution table



| pitch class | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | *t* | *e* |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| instances | 1 | 2 | 2 | 1 | 2 | 2 | 1 | 2 | 2 | 1 | 2 | 2 |

Example 10: Non-octave-repeating scale i.c. 9 <2,2,2,2,1>,
with pitch-class distribution table



| pitch class | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | *t* | *e* |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| instances | 1 | 1 | 0 | 2 | 1 | 1 | 0 | 2 | 1 | 1 | 0 | 2 |

Example 11: Non-octave-repeating scale i.c. 8 <3,2,2,1>,
with pitch-class distribution table

This intersection between pitch and pitch-class structures yields some interesting deep-structural properties to be explored. In examples 10 and 11, we see uneven distribution of pitch-classes. In example 10, the generative interval cycle is i.c. 9. In pitch-class terms, the four notes of the generating cycle (the four stemmed notes in example 10) are a set of prime form [0, 3, 6, 9]. The four pitch classes that appear only once in this scale are also a set of prime form [0, 3, 6, 9], and, of course, the eight pitch classes that appear twice in this scale are a set of prime form [0, 1, 3, 4, 6, 7, 9, *t*], which is the complement of [0, 3, 6, 9]. So, while this set displays a major intersection with the whole-tone scale on its surface, its deep-structural pitch-class properties could be said to reflect the octatonic collection, because the pitch classes that one would presumably hear most in music based on this scale all belong to a single octatonic collection.

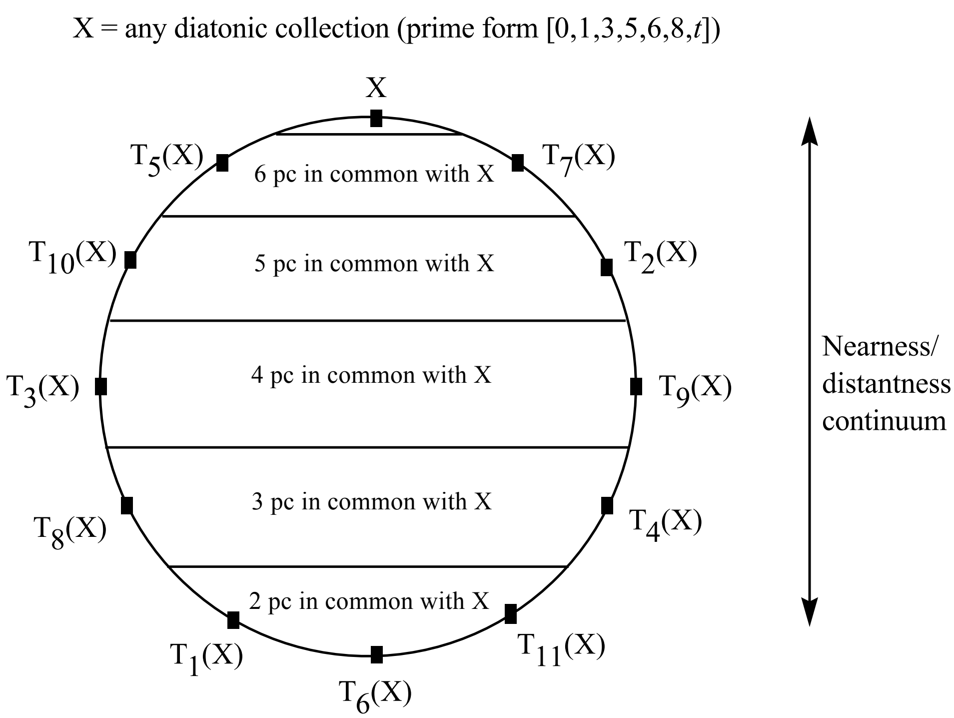
Likewise, in Example 11, the generative interval cycle is i.c. 8. In pitch-class terms, the three notes of the generating cycle (the three stemmed notes in example 11) are a set of prime form [0, 4, 8]. The three pitch classes that do not appear at all in this scale are also a set of prime form [0, 4, 8], as are the three pitch classes that appear more frequently than any of the others. The nine pitch classes that appear in this scale are, of course, a set of prime form [0, 1, 2, 4, 5, 6, 8, 9, t], which is the complement of [0, 4, 8].

So, in pitch-class terms, we find the generative set embedded in the resulting scale in various ways, which suggest richer deep structures in some scales than in others.

I would pause at this point to remind you that we, as listeners, are constantly perceiving and understanding the pitch domain of musical structures as a hybrid of pitch and pitch-class relationships. A somewhat famous example of this is Elliott Carter's song, Anaphora, from the cycle *A Mirror on Which to Dwell*. In this song, each pitch-class is fixed in exactly one specific location in pitch space. Thus, pitch class relationships literally do not exist in this song—there is only one possible C, on possible C-sharp, and so on. But, one must hear pitch class relationships in order to understand what relates the various note groupings in the song to one another. (Which we have no problem doing, of course, since that is how we normally listen to music.) This is a deep-structural modeling of Elizabeth Bishop's poem "Anaphora," which describes a particular day, but is really talking about how all days are subject to the same cycles. (At the risk of belaboring the obvious, in this analogy, pitch space is like a specific day, while pitch-class space is like the class of all days.)

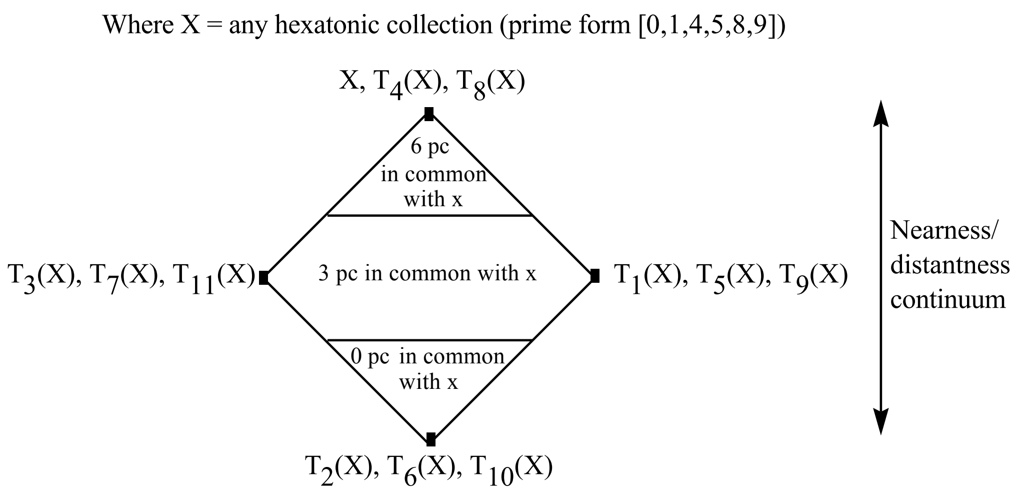
## III. Common Tones Under Transposition, and Modulation Systems

In traditional Western practice, one of the great compositional appeals of the diatonic collection is its robust set of modulation possibilities. Please refer to Example 12.




Example 12: Common pitch classes under transposition for the diatonic collection

For any given diatonic collection, there are two transpositions with 6 of the 7 pitch classes in common, two transpositions with 5 of the 7 pitch classes in common, two with 4 pitch classes in common, two with 3 pitch classes in common, and three transpositions with 2 of the 7 pitch classes in common. Thus, the familiar array of incrementally more distant modulation relationships as one travels in either direction around the circle of fifths. This system allows for modulations with five different levels of intersection in terms of pitch-class content, or put another way, five different degrees along the nearness/distantness continuum in modulation space.

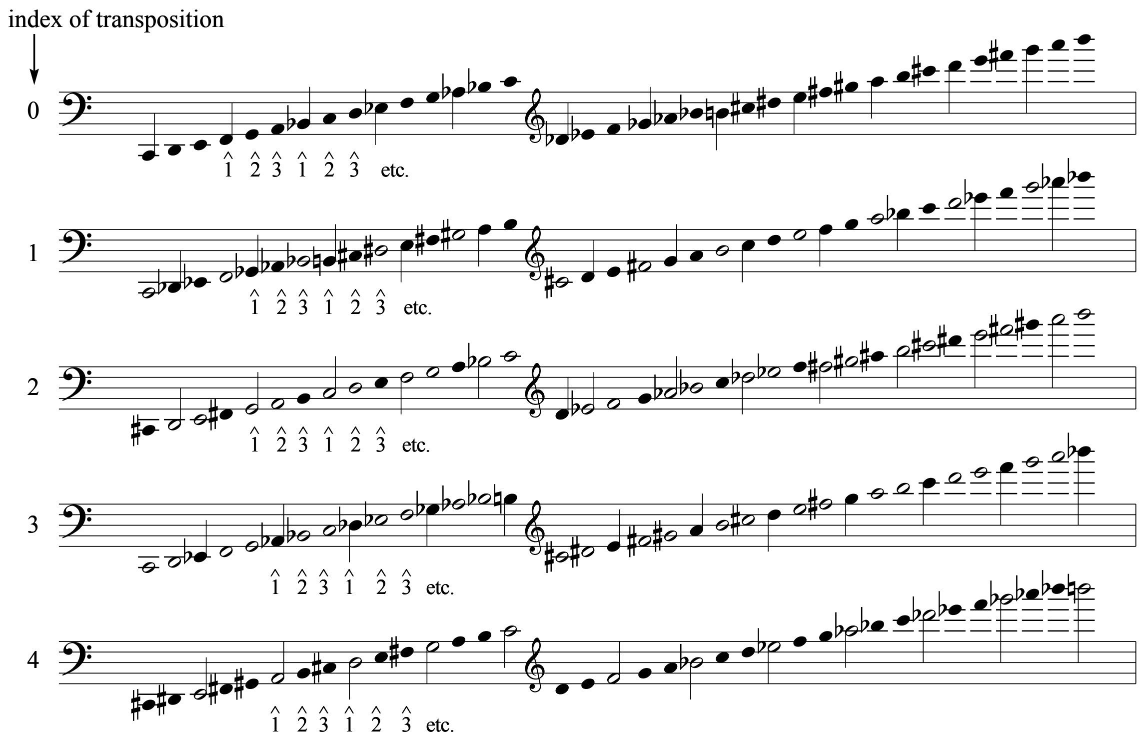



Example 13: Common pitch classes under transposition for the hexatonic collection

By contrast, many of the symmetrical collections common in the music of the Twentieth Century allow for very limited modulation possibilities. The aggregate or total chromatic collection can only be transposed onto itself, of course. The whole-tone collection can only be transposed onto itself or onto its complement (thus having no pitch classes in common). All transpositions of the octatonic collection map the original collection either onto itself or onto a set with 4 of the 8 pitch classes in common. The hexatonic collection is the only one of these collections that has a meaningful modulation continuum, but it is quite limited, with only two non-trivial common-tone relationships possible. This is illustrated in Example 13.

But this method of determining pitch *classes* in common will clearly not be informative in looking at common tones under transposition in the non-octave-repeating scales. The scales in Examples 8 and 9, for example, contain all twelve pitch classes, and therefore will have 12 pitch *classes* in common in any transposition of the scale. But these scales are structures in pitch space (as opposed to pitch class space), so we need to examine common pitches (rather than pitch classes), in order to describe the nearness/distantness continuum in modulation space.

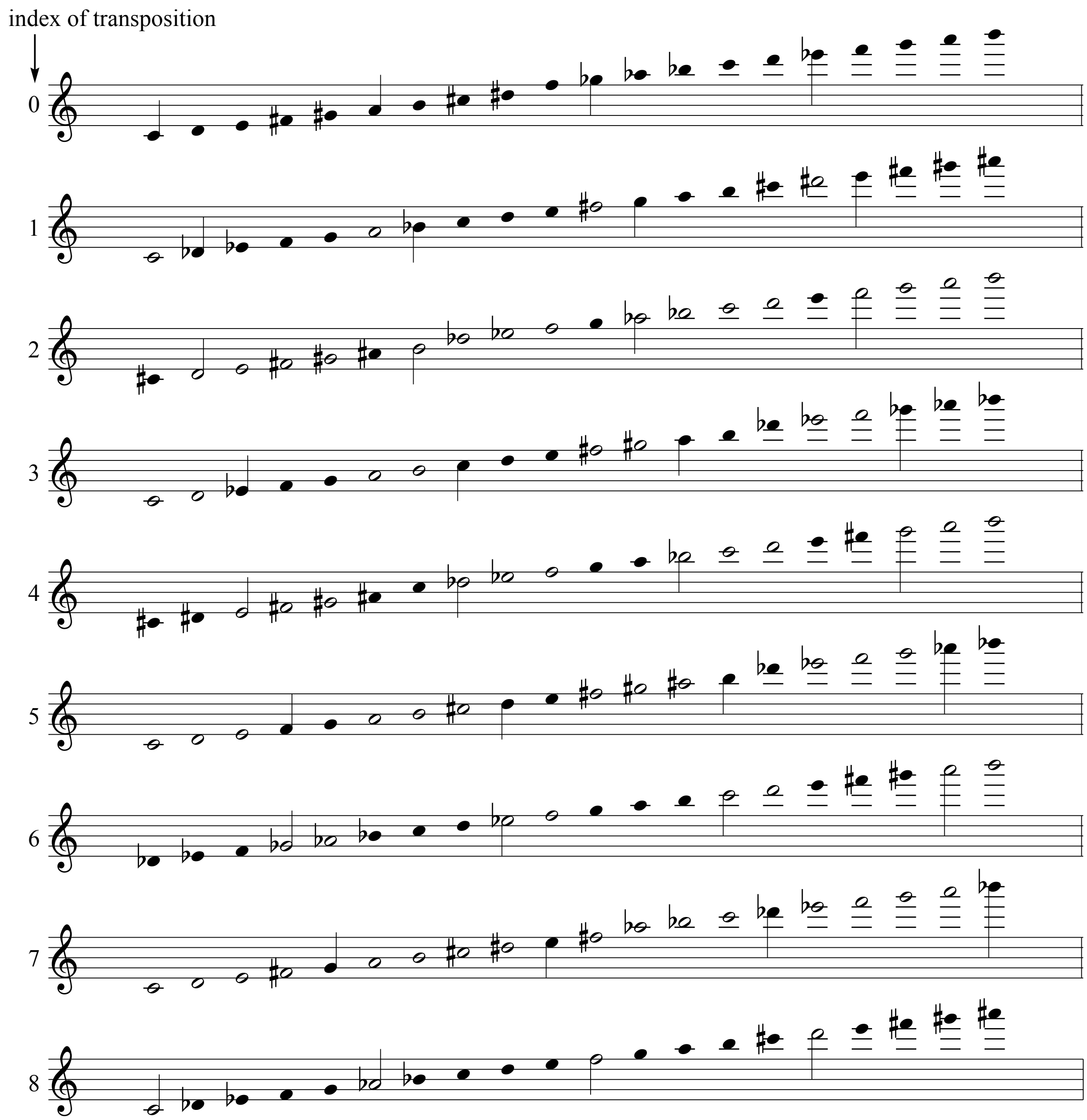
Example 14 gives the i.c. 5 <2, 2, 1> scale in 5 different transpositions. If one were to transpose the scale by 5 semitones, it would map onto itself. The scale degree numbers (one through 3, in repeating cycles) in Example 14 help illuminate the simple transpositional relationship between the scales: each scale has been extended at the bottom, and truncated at the top as needed to correspond with the same subset of pitch space as the original scale. Thus each scale in Example 14 contains 36 notes, beginning no lower than pitch $C_2$, and ending no higher than pitch $B_6$. The open note-heads represent a common pitch with the $T_0$ transposition of the scale.




(open noteheads = pitches in common with the $T_0$ form of the scale)

| Index of transposition | Number of pitches in common with $T_0$ form within the modular interval (36 notes, for this scale). |
|---|---|
| 1 | 12 |
| 2 | 24 |
| 3 | 24 |
| 4 | 12 |

Example 14: Common pitches under transposition for
the non-octave-repeating scale i.c. 5 <2,2,1>



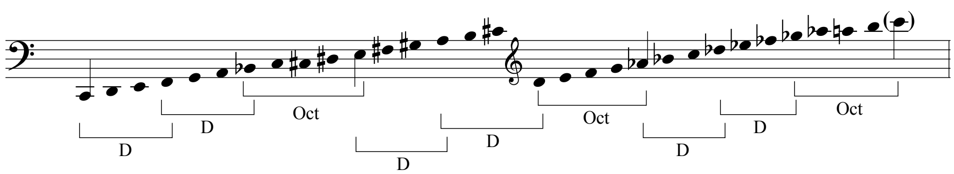

(open noteheads = pitches in common with the $T_0$ form of the scale)

| Index of transposition | Number of pitches in common with $T_0$ form within the modular interval (20 notes, for this scale). |
|---|---|
| 1 | 4 |
| 2 | 16 |
| 3 | 8 |
| 4 | 12 |
| 5 | 12 |
| 6 | 8 |
| 7 | 16 |
| 8 | 4 |

Example 15: Common pitches under transposition for
the non-octave-repeating scale i.c. 9 <2,2,2,2,1>

The table gives the number of common pitches within this 36 note span for each transposition of the scale. In this case, there are two possible numbers of common pitches: either 12 of 36 pitches in common or 24 of 36 pitches in common.

Example 15 gives the same information for the i.c. 9 <2,2,2,2,1> scale. In this case, there are four possible numbers of common pitches: 4, 8, 12, or 16 out of 20. Thus one can imagine that a piece of music composed using the various transpositions of this scale would have a robust nearness/distantness continuum in modulation space: almost as robust as that of the diatonic collection.



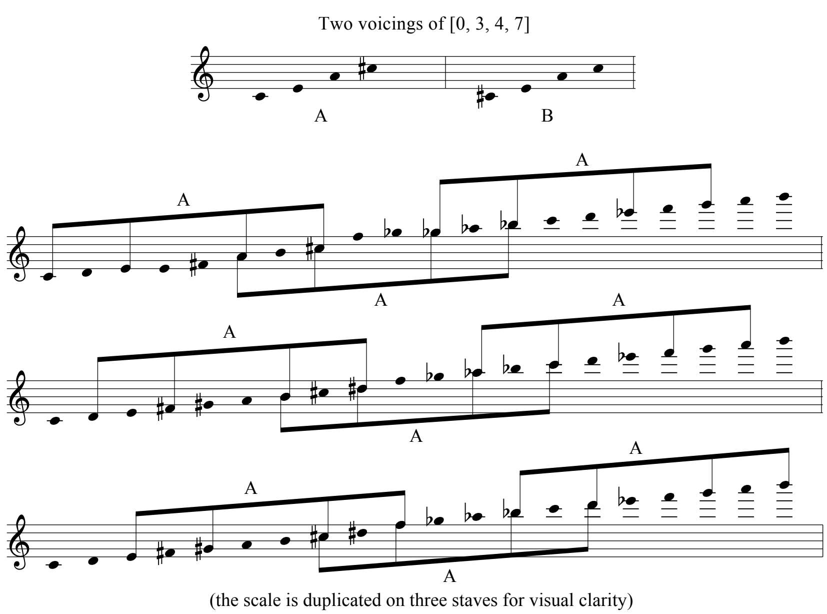

D = diatonic tetrachord  Oct = Octatonic pentachord (spanning tritone)

| Index of transposition | Number of pitches in common with $T_0$ form within the modular interval (30 notes, for this scale). |
|---|---|
| 1 | 12 |
| 2 | 18 |
| 3 | 24 |
| 4 | 12 |
| 5 | 24 |
| 6 | 18 |
| 7 | 15 |
| 8 | 24 |
| 9 | 15 |
| 10 | 18 |
| 11 | 24 |
| 12 | 12 |
| 13 | 24 |
| 14 | 18 |
| 15 | 12 |

Example 16: Common pitches under transposition for
the non-octave-repeating scale i.c. 16 <2,2,1, 2,2,1, 2,1,2,1>

Example 16 gives a favorite scale of mine, i.c. 16 <2,2,1 | 2,2,1 | 2,1,2,1>. The annotations indicate the partition scheme as I generally image it: two diatonic tetrachords followed by an octatonic segment spanning a tritone. The table gives the common pitches for each of the 16 transpositions of this scale that contains 30 notes within its modular span of four octaves. I have spared you the enumeration of all 16 transpositions, primarily because there is no way to make that fit on an 8.5 X 11 page at a size that those of us over 40 could ever hope to read. The table indicates the robust continuum of nearness/distantness in modulation space, from a low of 12 pitches out of 30 (40%) to a high of 24 out of 30 (80%).

Example 17 barely scratches the surface of the topic of pitch-scale "filtering" of pitch-class sets. Scales such as these, which contain all or most of the 12 pitch classes, but limit which octaves in which they appear, allow some voicings of pitch-class sets in pitch-space, while excluding others. The example shows two possible voicings of the pitch-class set [0,3,4,7]. While numerous examples of voicing A can be found in the given scale, i.c. 9 <2,2,2,2,1>, voicing B does not occur in this scale.




Example 17: "Filtering" effect of i.c. 9 <2,2,2,2,1> on voicings of p.c. set class [0,3,4,7]

## IV. Why Composers Might Care

You have no doubt inferred some answers to that question from the preceding discussion of some of the properties of non-octave-repeating scales. Unlike conventional octave-repeating scales, many non-octave-repeating scales include the total chromatic (or a large subset of it) without having to transpose the scale, or, put another way, without having to modulate. But, the possibility of modulation systems still exists, as it does not, in any meaningful way, in music based on the chromatic scale.

What follows will be a more personal exploration of that question.

I often tell composition students that they need to know at least two versions of music history: the official version and what might be called the chutzpah version, where all musical events of the past lead inexorably to one's own music. We appropriate everything we covet about the way other composers' music works, and eliminate anything we consider flaws. (I did mention that this is the chutzpah version, right?)

I would like to add that one of my basic premises of compositional strategy is that I am fundamentally bored with the false dichotomy between tonal and atonal music. First, one is an old way of writing music, while the other is an even older way. Second, it is a fallacy to imagine that these two categories are mutually exclusive, and that they encompass all musical expressions. I personally have no interest in writing diatonic tonal music, but I am very interested in writing music that can and does modulate, particularly using scales with a robust nearness/distantness continuum in modulation space. I also am very fond of the musical concept of a scale: as a grid over which pitch relations are understood (how, for example, scale degree 1 relates to scale degree 2, and so on), and as a source of melody, both in thematic form and in what might be called generic scalar melodic motion, which is, in my opinion, by no means unimportant in the grand melodic scheme of things. (Simply put, I tend to miss the scale as a melodic construct in music that lacks this feature.) Also, the designation whereby some pitches "belong to" the operative scale at any given moment, while others do not, allows for rich musical experiences. We must somehow account for the outliers when we hear them—are they decorative? Fleeting anomalies? Or do they signal a modulation? It can be challenging to suggest these possibilities in unrestricted chromatic music.

I would like to think that the music I have composed using non-octave-repeating scales is both like and unlike tonal music, and also both like and unlike atonal music. The connections to traditional tonality come not in the designation of certain notes as tonic or tonic-like in function (although that does sometimes happen in my music), but more so from the deep-structural properties of modulation systems.

The filtering effect as illustrated in Example 17 (which serves to limit the possible voicings of pitch-class sets in pitch space) strikes me as a feature more so than a bug. Apparently I stand with Stravinsky when he said "the more art is controlled, limited, worked over, the more it is free."

The existence of a huge selection of different non-octave-repeating scales (including different partitions of the same interval cycles) makes it easy for composers to tailor these scale systems to their general preferences and the specific needs of a piece or movement. To give one quick example, I like the i.c. 16 <2,2,1 | 2,2,1 | 2,1,2,1> scale system from Example 16 in part because it embeds one of my favorite harmonies, which is stacked 4ths in which two out of three of the fourths are perfect fourths and one out of three is a tritone. (Two perfect fourths and a tritone add up to 16 semitones, the generating interval of i.c. 16.)

I'll close with two musical examples. The first is a movement entitled "Sweetly Singing," from my piece *Glancing Spirals*, for violin, clarinet, and piano. This movement uses the i.c. 9 <2,2,2,2,1> scale that has been used in several examples in this paper. Second is an excerpt from a somewhat iffy recording of my recent piece for saxophone and piano, *Intensity 8.5*, which uses the i.c. 16 <2,2,1 | 2,2,1 | 2,1,2,1> scale system from example 16. In both cases, I hope that you'll be able to hear some of the compositional strategies we have discussed, including what I believe are fairly obvious modulations in places.

[The examples played can be found at the following links:

http://www.youtube.com/watch?v=cavVayZ9s1I

http://www.youtube.com/watch?v=FYZdir7ChUY&feature=relmfu ]

Thank you.