SALES FORECASTING FOR A CENTRALIZED MEAT CUTTING FACILITY

by

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CHAPTER I

INTRODUCTION AND DEFINITION OF THE PROBLEM

The subject of managerial control and allocation of resources presents myriad problems to any modern, mass-production oriented industry.

Some of the same basic problems exist for all industries, whether the industry be involved in the production of steel, automobiles, refrigerators, or razors. Sales must be forecast with reasonable accuracy so that production schedules can be planned; orders for raw materials made; inventories built up or depleted as necessary; and work force estimates made. In short, management needs a tool which will provide it with a legitimate margin of safety with which they can anticipate the requirements of their operation at some time in the future.

Many forecasting and data analysis techniques are available; from the continuation-of-trend method, through moving averages, to the sophisticated exponential smoothing and spectral analysis. The specific problem under consideration here is the application of spectral analysis to forecasting the demand of meat in a retail supermarket. All of the data used in this study is actual data obtained from Falleys Markets. Falleys supermarket chain, of Topeka, Kansas has been a pioneer in the establishment of a centralized meat-cutting facility. This concept removes the butcher-shops from each store and places the function and the work force under one supervisor in one facility.

The problems which have not been solved by the move are:

1. An accurate method of forecasting (predicting) demand for individual items;

2. Analysis of inventory requirements by item;

3. Determination of advertisement effect on sales (by item), on customer volume, total sales, and total meat sales.

The production decisions are presently made as the result of analysis of daily orders from individual stores, to which managerial intuition and judgement is applied. From a strict management viewpoint the existing method would be more acceptable _if_ the items in question were not perishable goods. Non-perishable goods, such as canned goods, paper goods, and cleaning supplies, to name a few, are controllable, inasmuch as monetary and quantity control can be accomplished through adjustments in ordering and store restocking schedules. Fresh meat products have limited shelf lives (i.e. the time during which they retain their original properties without spoilage or deterioration), are relatively expensive to purchase, and have a fairly low margin of profit (10-15 percent).

Because of these factors a method must be developed which will enable the meat market manager to accurately forecast his demand for individual items, as well as evaluate the effect that special sales have on the items.

The hypothesis which this study attempts to prove or disprove is:

1) That there are bi-weekly, semi-monthly, and monthly cycles in the data corresponding to the characteristics of the general population which might get paid on a weekly, bi-weekly, semi-monthly, or monthly basis. The Fally management informed this author that there was a definite weekly cycle.

2) That spectral analysis can be successfully used to uncover the

cycles in the data and, when combined with regression analysis, can provide an accurate model for forecasting purposes.

CHAPTER II

TIME SERIES

2.1 GENERAL DEFINITION

Time series are sequences, discrete or continuous, of quantitative data assigned to specific moments in time. They are observed and analyzed with respect to the statistics of their distribution in time. They may be simple and consist of a single numerically given observation at each sequence; or they may be multiple (or complex), in which case they will consist of a number of separate quantities tabulated according to a time common to all.

An example of a simple, discrete time series is the closing price of General Motors on the New York Stock Exchange, tabulated daily. A simple, continuous time series would be daily temperature measurements.

An example of a multiple, discrete time series is the closing price of all stocks on the New York Stock Exchange, tabulated daily. A multiple, continuous time series might be an infinite number of slightly different radar frequencies.

The fields in which time series analysis is applied are roughly divided into two areas: (1) economics, sociology, and short-time biological data; (2) astronomical, meteorological, geophysical, and physical data.

In the first category the time series are relatively short, discrete, and hinder drawing concrete conclusions based on a high degree of accuracy, i.e. the results may be accurate and significant within a very liberal error (confidence band). Additionally, since this type of time series is usually subject to human control, decisions, policy changes, etc., effects the statistical nature of the series and assume much importance.

The second category is characterized by long runs of accurate data taken under comparatively stable, uniform external conditions. Because of the length of the data and the stability of conditions, decisions, policies, etc., exert very little influence on the performance of the phenomena.

In this study we are primarily concerned with simple discrete data such as dollar sales per store per day, customer count per store per day, meat-dollar sales per store per day, and units sold per item per store per day.

2.2 CLASSIFICATION OF ECONOMIC TIME SERIES

2.2.1 Instantaneously recorded series are discrete series that can be thought of as being the values of a continuous time series at a specific point in time. The daily noon temperature reading in Glasgow, Montana is an example of such a series. Economic series which fall within this classification are price series, interest rate series, amount of equipment, debt, financial assets, etc.

2.2.2 Accumulated series are those series which represent the sum (or "accumulation") of a variable since the previous reading was taken. Examples are national income, production and sales data, and volume of transaction data.

The methods for analysis of both types are essentially the same, although certain problems arise which affect each one differently, e.g. the number of working days or shopping days per week, month, or year can greatly influence an accumulated series, but have a slight effect on an instantaneously recorded series.

Granger [8] suggests a more subtle classification of economic series. It is:

1. Series arising from micro-variables (those variables unable to affect economy-wide variables such as aggregate output, overall price index, and national income.)
2. Series arising from macro-variables (all non-micro variables).

Granger further states that "when we are considering the problem of how to analyze a <u>single</u> series the classification is not important: but when considering direction of casuality the classification is of considerable importance since the problem for macro-variables is more difficult than for micro-variables." [8]

2.3 TRENDS

One of the major problems throughout economic time series analysis is how to recognize, and what to do with, the various trends which occur.

The first and most simple type of trend is the trend in mean. We can say that a series has a trend in mean if the series is exhibiting an oscillation about a continually increasing/decreasing value. Part of the difficulty in discovering a trend in mean is that if the data is not over a long enough period we may only be looking at a seasonal component of a cycle. Where cycles are extremely long we may encounter analytical difficulties, no matter how much data we have.

The second trend is the trend in variance, which can be described as the extent to which the amplitude of oscillation about the mean is changing with time. This type of trend is often found in price and production data.

A rather subtle type of trend is one in which the relative importance of a particular component, or a change in correlation between the current value and the previous value in the series, takes place.

2.4 CLASSIFICATION OF TIME SERIES MOVEMENTS

The characteristic movements of time series may be classified into three main types (or "components").

Long-term, or secular, movements refer to the general direction which a graph of the time series illustrates. This would be described as "growing", "expanding", "contracting", "unchanging", etc. See Figure 1(a).

Cyclical movements, or cyclical variations, refer to long term oscillations about a trend line or curve. These cycles may or may not be regular, or periodic (i.e. they may not follow exactly the same patterns time after time). See Figure 1(b).

Seasonal movements, or seasonal variations, refer to the identical (or nearly identical) behavior which a time series exhibits during corresponding units of time (days, weeks, months, or years), e.g. Christmas season shopping, sales of shotguns in the Fall, fishing rods in the Spring. See Figure 1(c).

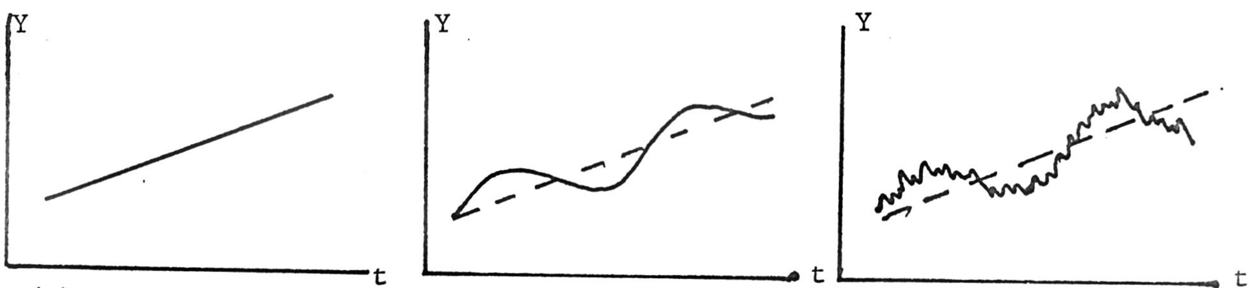
2.5 STATIONARY AND NON-STATIONARY TIME SERIES

Two of the most important assumptions which are made about time series are:

1. That the corresponding stochastic process is stationary; and
2. That a stationary stochastic process can be described by the lower moments of its probability distribution. i.e. mean, variance, covariance function, and the Fourier transform of the covariance function, the power spectrum.

Qualitatively speaking a series is said to be stationary if it contains no trends, and hence is in statistical equilibrium.

Obviously, non-stationarity is characterized by a series exhibiting definite trends. It should be pointed out that the methods of analyzing non-stationary time series are essentially the same as for stationary series, but with the additions of techniques for removing or filtering out the non-stationary components. This filtering results in a series which can then be treated as a stationary one.



(a) Long-Term Trend

(b) Long-Term Trend and Cyclical Movement

(c) Long-term Trend, Cyclical and Seasonal Movements

Figure 1

SAMPLES OF TRENDS

2.6 THE AUTOCOVARIANCE FUNCTION

One assumption which we must make in statistical analysis is that the data points represented by X(t) (t=1,2,...,N) are independent, since they were generated by an independent source. We can then say that if the probability distribution f(x) associated with the data is normal, then the data set can be characterized by its mean, variance, and autocovariance function.

In mathematical terms,

the mean, $$\mu = E[X] = \int_{-\infty}^{\infty} x f_X(x)\,dx$$

and the variance, $$\sigma^2 = E\left[(X - E[X])^2\right] = \int_{-\infty}^{\infty} (x-\mu)^2 f_X(x)\,dx \ .$$

The mean measures the "center of gravity" of the distribution, and the variance is a measure of the spread of the data points about the mean value.

Generally speaking, neighboring values of a time series, $(x_{t-1}, x_{t-2}, \ldots)$ are found to be correlated. In this sense, they cannot logically be considered to be independent (i.e. for only a purely random series could they be independent). So, when we have a stationary, normal series we can also describe its autocovariance function, (acvf), which is

$$\mathrm{acvf}(u) = E\left[(X(t) - \mu)(X(t+u) - \mu)\right] \ .$$

The acvf can be estimated as

$$C(u) = \frac{1}{N} \sum_{t=1}^{N-u} (x_t - \bar{x})(x_{t+u} - \bar{x}) \ .$$

We consider that $X(t)$ and x_t are essentially equivalent, (for a discussion of the use of the divisor, N, see Section 3.3) where $\bar{x} = \frac{1}{N}\sum_{t=1}^{N} x_t$ is the mean of the time series and N is the total number of data points in the series. We use the autocovariance function, $C(u)$, to provide information about the degree of linear relationship or product moment correlation between the random variable $X(t)$ with the past and future behavior of the time series.

If we wish to compare series with different scales it is convenient to normalize the acvf by dividing by $C(0)$. The result is called the sample autocorrelation function (acf), which we define as

$$r(u) = \frac{C(u)}{C(0)} .$$

A plot of $r(u)$ against u is called a correlogram. (see Figure 2).

The acvf and the acf are used to provide an indication of the manner in which the dependence in the series damps out with the lag or separation, u, between points in the series.

2.7 THE SPECTRUM

If our model is $x(t) = a \cos\omega_1 t + \varepsilon_t$, where ε_t is the error term, and a random independent series, then $\sigma_x^2 = \frac{1}{2} a^2 + \sigma_\varepsilon^2$. Removing the periodic term, ω_1, will reduce σ_x^2 to σ_ε^2. In a series which contains no periodic terms we cannot reduce the variance. Thus if $x(t)$ can be regarded as a mixture of cosine waves, its variance can be decomposed into components of variance $\frac{1}{2} a_i^2$, at the different frequencies, ω_i.

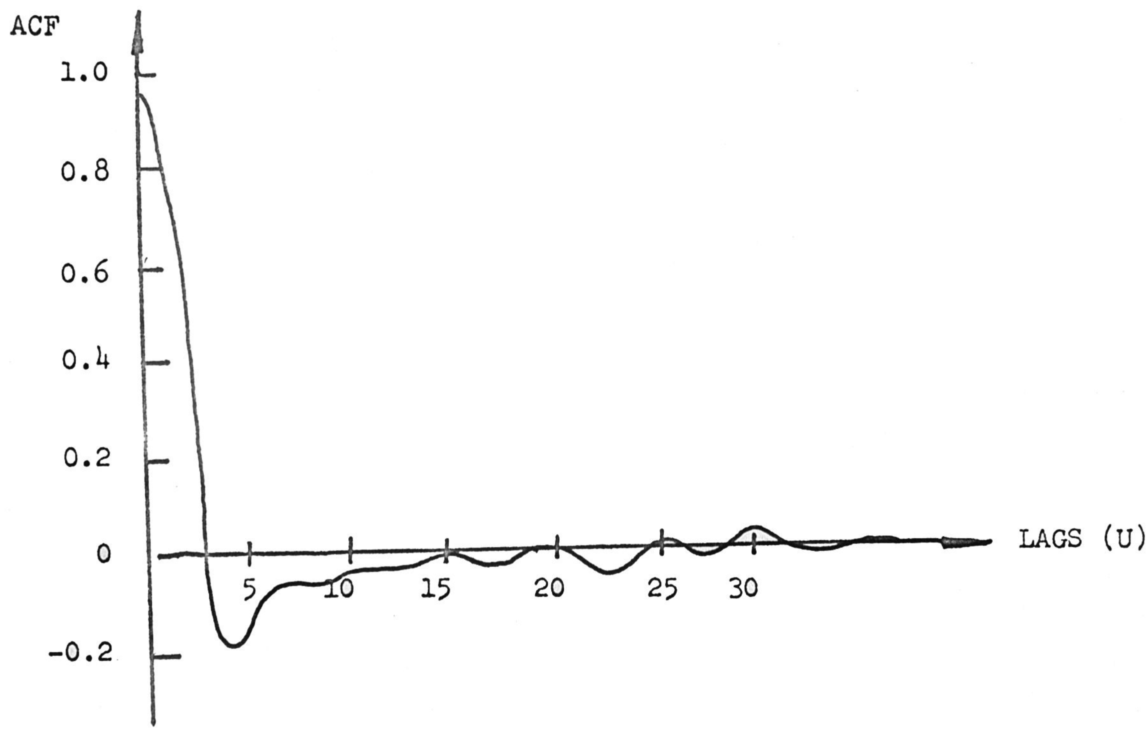


Figure 2

SAMPLE AUTOCORRELATION
FUNCTION, N=100

It will be shown in Chapter III that if x_t is a stationary stochastic process the variance is decomposed into contributions at a continuous range of frequencies, such that $\sigma^2 = C(0) = \int_{-\infty}^{\infty} \Gamma(f)df$, where $\Gamma(f)$ is the power spectrum of the stochastic process. It should also be pointed out that the spectrum and the autocovariance function are related according to the Fourier transform, where

$$\Gamma(f) = \int_{-\infty}^{\infty} C(u) \cos 2\pi f(u) du \ .$$

An important relationship which has been developed is that knowledge of the acvf is equivalent to knowledge of the spectrum of the process.

In the analysis of a series of finite length it is preferable to use the spectrum rather than the acvf function. The reasons are:

1. That estimates of the spectrum at neighboring frequencies are approximately independent;
2. Because of 1, the interpretation of the spectrum is easier than that of the acvf;
3. The physical properties of the spectrum (peaks, troughs, bandwidth, etc.) are of importance to the analysis.

2.8 OBJECTIVES OF TIME SERIES ANALYSIS

Time series problems (and the implications for utilization of spectral analysis) can be classified into three major categories (1) model-building, (2) frequency response studies, and (3) a combination of (1) and (2). For purpose of this study (3) will be considered as the primary problem. Model buildings fall into three basic categories:

1. exploratory and sophisticated models,
2. empirical and physical models,
3. parametric and non-parametric models.

Some of the most common uses for time series models are

1. prediction and forecasting,
2. estimation of transfer functions.
3. filtering and control
4. simulation and optimization
5. generating new physical theories.

The use of spectral analysis in frequency response studies is one of its most important applications. Through analysis of the spectrum of a stochastic process one can determine what cycles are present, frequencies of the cycles, periods, and relative contribution to the total variance of the system. Frequency response studies are usually made of physical systems such as radio frequencies, runways (vibration tests), and shock tests of airplane landing gear.

2.9 MODERN TIME SERIES ANALYSIS

Some significant early work was done by LaGrange [8], Buys-Ballot [3], and E. T. Whittaker [19], in the discovery of hidden periodicities. Perhaps the best known result of the pioneers in the field was the development of the periodogram method by Schuster in 1898-1906. The Schuster periodogram consists of the following functions.

$$I_N(\omega) = \frac{1}{N}\left[\left(\sum_{j=1}^{N} x_j \cos \frac{2\pi}{\omega} j\right)^2 + \left(\sum_{j=1}^{N} x_j \sin \frac{2\pi}{\omega} j\right)^2\right] , \qquad (2.9.1)$$

with data X_t, $t=1,2,\ldots N$. $I_N(\omega)$ will peak at $\omega = \omega_o$ if the data contains a periodic of frequency ω_o and there will be subsidiary peaks at

$$\omega = \omega_o + \frac{2\omega_o}{n} .$$

Davis [5] provides many examples of estimated periodograms for random and economic series. The best known periodogram is that of Beveridge [2] from a long series of European wheat prices. (1500-1869). It (the Beveridge periodogram) indicates a possibly significant peak corresponding to a cycle of 15 years. The original series was non-stationary; but was detrended by forming a series y(t) from the original series x(t) by

$$Y_t = \frac{x_t}{\sum_{j=-15}^{j=+15} x_{t+j}} . \qquad (2.9.2)$$

This transformation successfully removed the trends in mean and variance. The Beveridge periodogram may be found in Kendall's work [12].

Whittaker and Robinson [19] proved that the brightness or magnitude of a variable star at midnight on 600 successive days could be fitted by the sum of two periodic terms with periods of 24 and 29 days. Their equation was

$$x_t = 17 + 10\sin\frac{2\pi(t+3)}{29} + 7\sin\frac{2\pi(t-1)}{24} . \qquad (2.9.3)$$

Yule [9], [10], [11] noted that a time series generated by

$$x_t = \sum_{j=0}^{k} b_j \varepsilon_{t-j} \qquad (2.9.4)$$

or

$$\varepsilon_t = x_t + \sum_{j=1}^{k} a_j x_{t-j} \tag{2.9.5}$$

had an appearance similar to a naturally occuring series. The model, (2.9.4) is the weighted sum of a random series, called the "moving average process". Model (2.9.5) is an example of an autoregressive process, in which the value of x(t) is assumed to have been formed from a linear sum of past values of the series together with an unconnected independent term from the past.

These models invariably produce smooth series which do not emphasize the variance of the series with any accuracy at all.

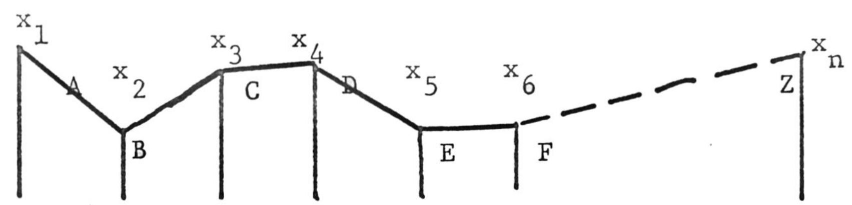


Figure 3
SAMPLE PERIODGRAM

The reason for the smooth periodogram is as follows.

If the data plotted in Figure 3 has a mean of zero, the length of the line ABC...Z will be a reflection of the "smoothness" of the data. The square of the length of ABC...Z is given by

$$k = \sum_{t=1}^{N-1} (x_t - x_{t-1})^2 + (N-1)J^2 \tag{2.9.6}$$

where J is the distance between readings. Further,

$$E[k] = 2\sigma_x^2(1-\rho) + (N-1)J^2 \tag{2.9.7}$$

where ρ is the correlation between x_{t-1} and x_t (assumed to be the same for all t.) We can see from (2.9.7) that the closer ρ becomes to 1 the smaller E[k] becomes. With the moving average or auto-regressive model, ρ can be made large and positive by the choice of a_j and b_j.

Equation (2.9.6) can also be solved as

$$x_t = \sum_{j=0}^{k} c_j \theta_j^{t-T} + \sum_{j=0}^{\infty} b_j \varepsilon_{t-j} \tag{2.9.8}$$

where X_t is the sum of a function of time plus an infinite moving average, assuming that the series started at T.

For many years the linear cyclic model and the moving average model, were considered as the only possible alternatives in time series analysis. Hence, the problem for analyst was only one of determining which one among the three was the best to fit.

In recent years the problems of fitting have become more complex and the above mentioned two models are no longer the only alternatives. There is now the auto-regressive model, which can be shown as

$$x_t + \sum_{j=1}^{k} a_j x_{t-j} = \sum_{j=0}^{M} b_j \varepsilon_{t-j} , \tag{2.9.9}$$

and using the auto-regressive model and the spectral analysis discussed in Chapter III, the series can be considered as having been generated by more complicated and unspecified mechanism.

The spectral theory used in this study is largely the result of theoretical work by Kolmogoroff [13], Wiener [20], and Cramer [4], and is basically generalizations of Fourier analysis. A leading individual in the transition from theoretical results to practical methods is J. S. Tukey, and his co-workers at Bell Telephone Laboratories.

In order to improve the sampling properties in spectrum estimation, Tukey [16] and Bartlett [1] suggested an approach that emphasized the use of the weighted sample autocovariance function. Their work also revealed that spectral analysis has considerably simpler sampling properties than does the corresponding time-domain analysis.

Some of the most recently developed techniques, principally by Hannan [9], [10] permit the derivation of asymptotically efficient coefficient estimators with relatively simple sampling properties. Wallis [17], [18] has used some of the new Hannon-developed techniques to study inventory problems.

CHAPTER III

SPECTRAL THEORY

As we have stated earlier, a stationary stochastic process can be described by its autocovariance function. The power spectrum (hereafter called the "spectrum") is the Fourier transform of the autocovariance function. A plot of the spectrum shows how the variance of the stochastic process is distributed with frequency.

3.1 AN ANALOGY

Consider the total amount of sound (or moise) that we are receiving on a radio receiver. If we have pure noise we cannot distinguish any single, intelligible sound. But in an ideal situation with the receiver perfectly tuned we would get a pure tone with no residual noise due to other nearby frequencies or random signals. In a real situation we are able to tune the receiver to a primary frequency, but we also get amounts of static and random signals.

Figure 4-a represents the "white noise" or a purely random signal. No one frequency exhibits any greater amplitude (variance) than any of the others. Figure 4-b is the ideal situation which shows those precise frequencies on which signals are being transmitted. Figure 4-c shows the actual situation in which the primary frequencies have greater amplitude than the other frequencies. Notice, however, that the entire range of frequencies does exhibit some amplitude. This would be the case in actual practice, and the type of spectrum with which we will be working in this study.

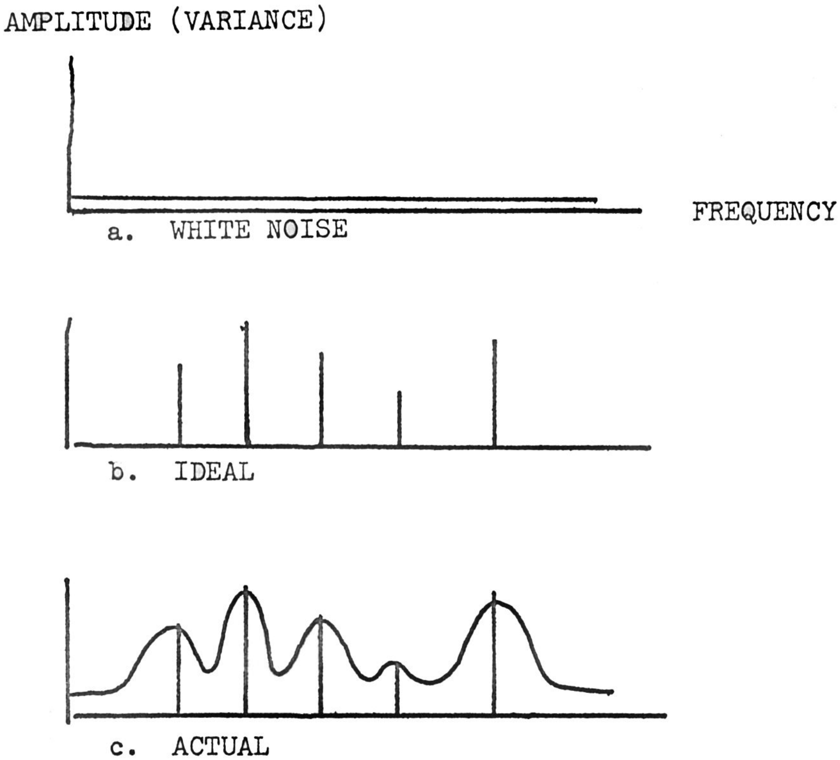


Figure 4

EXAMPLES OF SPECTRA

Part of the problem becomes a matter of discrimination and identification of the primary frequencies. This is discussed at length in this chapter.

3.2 POWER SPECTRA

If we have a discrete, stationary process $X_t (t=1,2,...N)$,

$$E[X_t] = 0, \quad E[X_t \bar{X}_t] = \sigma^2, \qquad E[X_t X_{t-\tau}] = \mu_\tau, \tag{3.2.1}$$

where $\bar{X}$ is the conjugate of X.

Consider the generating process where

$$X_t = \sum_{j=1}^{k} a_j e^{i\omega_j t} \tag{3.2.2}$$

and $\{\omega_j, j=1,2,...k\}$ is a set of real numbers with $|\omega_j| \leq \pi$ and $\{a_j, j=1,2,...k\}$ is a set of independent random variables with all j,

$$E[a_j] = 0$$

$$E[a_j a_j] = \sigma_j^2,$$

$$E[a_j a_k] = 0, \qquad j \neq k .$$

Any term $a_j e^{i\omega_j t} = a_j(\cos\omega_j t + i\sin\omega_j t)$ of this generating process is a periodic function, with period $T = \frac{2\pi}{\omega_j}$. The angular frequency, measured in radians per time unit is given by $2\pi\omega_j$. The angular frequency ω_j is used to describe the periodicity of the function, and will hereafter be called frequency.

As the a_j's take different values, the different series that may be generated by (3.2.2) arise. For any particular series the a_j's are constant throughout the time span (of the series) and the totality of the a_j's determine their distribution.

We stated before that $E[X_t] = 0$, and $E[X_t \bar{X}_{t-\tau}] = \sum_{j=1}^{k} \sigma_j^2 e^{i\omega_j} = \mu$. It follows that $\mu_\tau = \int_{-\pi}^{\pi} e^{i\omega\tau} dF(\omega)$, where $F(\omega)$ is a step function with steps of size σ_j^2 at $\omega = \omega_j$ $(j=1,2,\ldots k)$.

The process described by equation (3.2.2) is a linear cyclic process with the mean removed, as each sample is a sum of periodic functions.

The work of Cramer [4], Kolmogoroff [13], and Wiener [20] has given the following important results:

1. The sequence of autocovariances, μ_τ, for a stationary process can always be represented in the form

$$\mu_\tau = \int_{-\pi}^{\pi} e^{i\omega\tau} dF(\omega) \qquad (3.2.3)$$

   where $F(\omega)/\mu_o$ is a distribution function (it is monotonically increasing and bounded), $F(-\pi) = 0$, $F(\pi) = \mu_o = \sigma^2$

2. $X_t = \int_{-\pi}^{\pi} e^{it\omega} dz(\omega)$, where $z(\omega)$ is a complex random function called a process of non-correlated increments.

Equations (3.2.3) is called the spectral representation of the covariance function and $F(\omega)$ is the power spectral distribution function. Equation (3.2.4) is the Cramer representation of a stationary process, and the equality must be understood to hold as a limit in the mean square.

For real process, (3.2.3) becomes

$$\mu_t = \int_{-\pi}^{\pi} \cos\tau\omega d\Gamma(\omega), \qquad (3.2.4)$$

where

$$d\Gamma(\omega) = 2dF(\omega), \qquad 0 < \omega < \pi ,$$

$$d\Gamma(0) = dF(0), \qquad d\Gamma(\pi) = dF(\pi) ,$$

and equation (3.2.4) becomes

$$X_t = \int_0^{\pi} \cos t\omega du(\omega) + \int_0^{\pi} \sin t\omega dv(\omega) .$$

An important fact to note is that in spectral representation the variances of equation (3.2.2) and (3.2.4) are compared as

$$\sigma_x^2 = \sum_{j=1}^{k} |a_j|^2$$

and

$$\mu_o = \sigma_x^2 = \int_{-\pi}^{\pi} dF(\omega) .$$

In the first case a_j^2 is the contribution to the total variance of the component with frequency ω_j. In the second case the contribution to the total variance by the frequencies in the range $[\omega, \omega + d\omega)$ is $dF(\omega)$.

Since $F(\omega)$ is monotonically increasing, the decomposition is

$$F(\omega) = F_1(\omega) + F_2(\omega) + F_3(\omega).$$

Thus it can be shown that any stationary process can be decomposed into $X_t = X_1(t) + X_2(t)$, where $X_1(t)$ has an absolutely continuous power spectral distribution function. Equation (3.2.3) then becomes

$$\mu'_\tau = \int_{-\pi}^{\pi} e^{it\omega} f(\omega)d\omega \ .$$

Cramer's representation takes the following form for real processes

$$X_t = \int_0^{\pi} \cos t\omega du(\omega) + \int_0^{\pi} \sin t\omega dv(\omega) \ . \qquad (3.2.5)$$

If we have a series of finite length which has been generated by the real process, the series can be fitted by a finite Fourier series

$$X_t(N) = \sum_{j=0}^{N} a_j \cos\omega_j t + \sum_{j=1}^{N} b_j \sin\omega_j t$$

where $\omega = \frac{2\pi_j}{N}$, and the a_j's and b_j's are chosen to make $X(N) = X$ at $t = 1,2,\ldots N$. As $N \rightarrow \infty$, the first thing that we notice is that $\omega_{j+1} - \omega_j \rightarrow 0$, and the series becomes an integral of the form

$$X_t = \int_0^{\pi} a(\omega)\cos\omega t d\omega + \int_0^{\pi} b(\omega)\sin\omega t d\omega \ . \qquad (3.2.6)$$

This "evolution" from the sum of sine terms to an integral of a sine function over a band of periods is one of the most important advantages of spectral analysis over regular Fourier analysis. Through the use of an integral of a sine function we can mathematically approach and solve the problem of regularity and non-regularity which occurs in most economic data.

This means that the infinite sequence $[x_t, t=1,\ldots\infty]$ can be exactly fitted by the mathematical function on the right hand side of equation (3.2.6) if the functions $a(\omega)$, $b(\omega)$ are properly chosen. If $\{x_t\}$ contains a periodic element of period T (and a frequency of $\omega = \frac{2\pi}{T}$) then $a(\omega)$, $b(\omega)$ will have sharp peaks at $\omega = \omega_1$, but if $\{x_t\}$ contains no periodic elements, $a(\omega)$, $b(\omega)$ will be smooth.

One of the primary purposes of spectral analysis is to estimate those regularities or frequencies which appear to be most important. We then use these estimates to predict the future values of the variable. The measure of importance which we attach to the component is dependent upon its relative contribution to the overall variance. For a variable containing a periodic term, the importance of the term is the amount of variance reduction observed when the term is removed. This is actually our definition of a variable containing a periodic term, i.e. a frequency which, if subtracted from the variable will reduce the variance by a finite amount. If our model is $X_t = a\cos\omega_1 t + \varepsilon_t$ where ε_t is a random independent series then $\sigma^2 = \frac{1}{2} a^2 + \sigma_\varepsilon^2$. By removing the periodic term with frequency ω_1 the variance becomes $\sigma^2 = \sigma_\varepsilon^2$.

The important frequencies will correspond to high, sharp peaks in the estimated spectrum and the height of the peak will provide us with an estimate of the amplitude (relative contribution to the overall variance).

3.3 ESTIMATION OF THE SPECTRA

All spectral estimates are of the form

$$\hat{f}(\omega_j) = \frac{1}{2\pi}\left\{\lambda_o C_o + 2\sum_{k=1}^{M} \lambda_k C_k \cos\omega_j k\right\}, \tag{3.3.1}$$

$\omega_j = \frac{\pi j}{M}$, (j=1,2,...,M) and the estimated covariances are of the form

$$C_k = \frac{1}{N}\left\{\sum_{t=1}^{N-k} x_t x_{t+k} - \frac{1}{N}\sum_{t=1+k}^{N} x_t \sum_{t=1}^{N-k} x_t\right\} \qquad (3.3.2)$$

with data (x_t, t=1,2,...N), and the weights λ_k are usually dependent upon M. The weights are called spectral windows, or "averaging kernels", and will be discussed at length in this chapter.

There is presently substantial controversy as to whether C_k should use the divisor N-k or just N. Parzen [15] and Tukey [16] use the divisor N, while Granger [8] uses N-k. The spectral values obtained in this study were obtained using N, per Parzen [15]. Parzen [15] shows that using N-k we obtain an unbiased estimator of the sample covariance; while we obtain a biased estimator using N alone. The biased estimator has two desirable features:

1. We are estimating a positive definite function,

2. It provides us with non-negatives values of the spectrum.

Furthermore, in most cases the biased estimate has a lower mean square error than the unbiased estimator. However, the use of the N per Parzen was done as a matter of speed in computation and convenience <u>after</u> test results were compared using both divisors. The difference between the results was found to be negligible.

For purposes of computation the Tukey-Hanning window was used in this study. It is

$$\lambda_k = \frac{1}{2}\left(1 + \cos\frac{\pi k}{M}\right).$$

The actual formulae used to estimate the spectrum are: (3.3.2)

and

$$L_j = \frac{1}{2\pi}\left(C_o + 2\sum_{k=1}^{M-1} C_k \cos\frac{\pi kj}{M} + C_M \cos \pi j\right) \qquad (3.3.3)$$

where $L_{-1} = L_{+1}$, $L_{M+1} = L_{M-1}$ and

$$\bar{C}_{xx} = 0.25L_{j-1} + 0.50L_j + 0.25L_{j+1} . \qquad (3.3.4)$$

The L_j's are called the raw estimates of the spectrum, and the $\bar{C}_{xx}$'s are the smoothed estimates. An alternative which could have been used in spectral estimation is the "Parzen window", which is

$$\lambda_k = 1 - \frac{6k^2}{M^2}\left(1 - \frac{k}{M}\right), \qquad 0 \leq k \leq M/2 ,$$

$$= 2\left(1 - \frac{k}{M}\right)^3, \qquad \frac{M}{2} \leq k \leq M .$$

If we use the Parzen window the spectral estimating formula becomes

$$\bar{C}_{xxj} = \frac{C_0}{2\pi} + \frac{1}{\pi}\sum_{k=1}^{M/2}\left[1 - \frac{6k^2}{M^2}\left(1 - \frac{k}{M}\right)\right] C_k \cos\frac{\pi kj}{M}$$

$$+ \frac{2}{\pi}\sum_{k=\frac{M}{2}+1}^{M} (1 - k/M)^3 C_k \cos\frac{\pi kj}{M} . \qquad (3.3.5)$$

In the case of equation (3.3.5) the estimated covariance is computed as

$$C_k = \frac{1}{N}\sum_{t=0}^{N-k} (x_t - \bar{x})(x_{t+k} - \bar{x}),$$

where

$$\bar{x} = \frac{1}{N} \sum_{t=0}^{N} x_t \ .$$

This is contrasted to the computation of covariances in equation (3.3.2). The parameter M which appears in the formulae is the truncation point, or number of lags used. It may be considered to represent the number of frequency bands for which the spectrum is estimated. As M increases, the variance increases, and vice versa.

The spectrum should be plotted on a logarithmic scale in order to show more detail in the spectrum over a wider range.

3.4 SPECTRAL WINDOWS (AVERAGING KERNELS)

The development of a "window" is due to the need for a mathematical device to isolate certain frequency bands under inspection, while providing a true picture of them. The degree of goodness and clarity of the bands is called resolution. The window performs the function of concentrating the power of the frequency under inspection about a main point of interest. This gives the greatest weight to the corresponding spectrum ordinate.

There are many windows described in the literature, the four primary ones being the rectangular window, Barlett window, Parzen window, and the Tukey-Hanning window. The two most commonly used, Parzen and Tukey-Hanning window, are shown in Table 1. Figure 5 shows the Parzen and Tukey-Hanning windows for M equal to 6 lags.

Notice the following details in Figure 5.

1. The side lobes of both windows are small compared to the main lobes.

2. The Parzen window is non-negative everywhere, while the Tukey-Hanning window is not. [Note: since we are estimating a function, not a point, our interest is in the overall shape of the spectrum. Therefore the matter of non-negative versus negative estimates of the windows is insignificant. If negative estimates do appear they should be judged in comparison with the other point estimates of the spectrum]

The degree of concentration, or focusing power, on the frequency of interest is measured by bandwidth. We will define bandwidth as the width of a rectangle whose height and area correspond to those of the averaging kernel at the frequency of interest. It is

$$B_M = \frac{1}{K_M(0)} \int_{-\pi}^{\pi} K_M(\omega)\,d\omega$$

and is measured in radians.

Table 1 lists the properties of the spectral windows. The bias is inversely proportional to the number of lags in the Bartlett window, and inversely proportional to the square of the lags (M^2) with the Parzen and Tukey windows. In all cases the bias is reduced as M increases.

For the same truncation point (number of lags) M, the Parzen window has a larger bias than the Tukey window, since the Parzen window is wider than the Tukey window. However the Parzen window has a smaller variance than the Tukey window for the same M.

3.5 BANDWIDTH OF SPECTRAL WINDOWS

In section 3.4 we defined bandwidth as "the width ...at the frequency of interest."

TABLE 1

BANDWIDTH, VARIANCE, AND EQUIVALENT DEGREES OF FREEDOM FOR THREE AVERAGING KERNELS[a]

Averaging Kernel	(1) Bandwidth	(2) Variance/$g^2(\lambda)$	(3) E.D.F.
Unit	$\pi/(M+1/2)$	2.00 M/T	T/M
Tukey-Hanning	$2\pi/M$	0.75 M/T	2.7T/M
Parzen	$8\pi/(3M)$	0.54 M/T	3.7T/M

[a]At frequencies zero and $\pm\pi$, the variance is doubled and the EDF is halved. The variance and EDF are for a normal process.

TABLE 2

PROPERTIES OF SPECTRAL WINDOWS

Description	Spectral window	Variance ratio I/T	Degrees of freedom	Standardized bandwidth b_1
rectangular	$2M\,\dfrac{\sin 2\pi fM}{2\pi fM}$	$2\,\dfrac{M}{T}$	$\dfrac{T}{M}$	0.5
Bartlett	$M\left(\dfrac{\sin\pi fM}{\pi fM}\right)^2$	$0.667\,\dfrac{M}{T}$	$3\,\dfrac{T}{M}$	1.5
Tukey	$M\left(\dfrac{\sin 2\pi fM}{2\pi fM} \times \dfrac{1}{1-(2fM)^2}\right)$	$0.75\,\dfrac{M}{T}$	$2.667\,\dfrac{T}{M}$	1.333
Parzen	$3/4M\left(\dfrac{\sin(\pi fM/2)}{\pi fM/2}\right)^4$	$0.539\,\dfrac{M}{T}$	$3.71\,\dfrac{T}{M}$	1.86

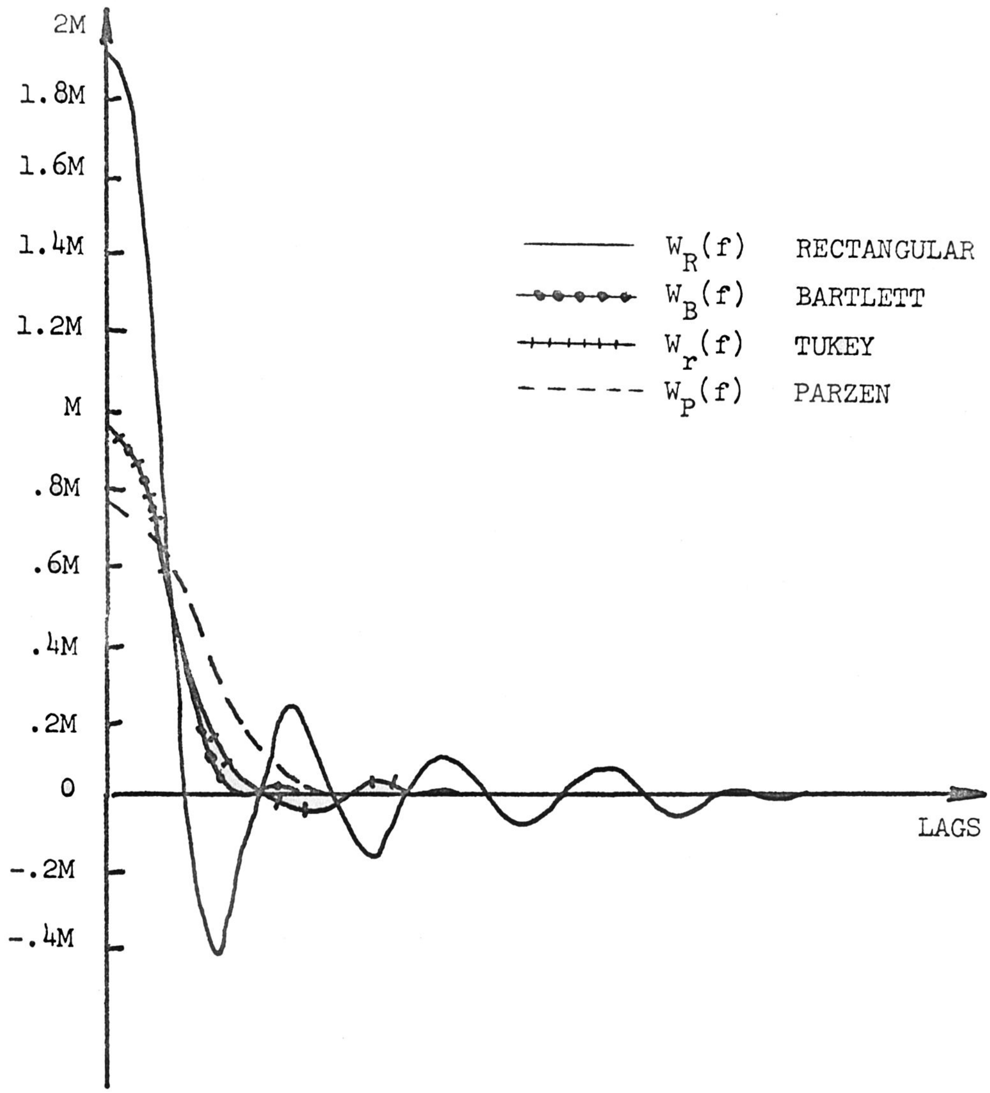


Figure 5

SOME COMMON SPECTRAL
WINDOWS

One useful characteristic of a spectral window is that

$$I = \int_{-\infty}^{\infty} \omega^2(u)du$$

where $\omega(u)$ is the particular spectral lag window, and I is the integrated spectrum, since $\frac{I}{T}$ provides a measure of the reduction in variance due to smoothing by the spectral window. To obtain a small error variance we must choose $\omega(u)$ so that I is small. This can be accomplished by using a small M. Furthermore, in order to obtain a good estimate of the peak of the spectrum, the width of the spectral window must be the same order as the width of the peak.

If we consider that the bandwidth in the frequency domain is rectangular with width, b, then the bandwidth becomes

$$b = \frac{1}{I} = \frac{1}{\int_{-\infty}^{\infty} \omega^2(u)du} = \frac{1}{\int_{-\infty}^{\infty} \omega^2(f)df} \quad . \tag{3.3.6}$$

So the bandwidths for the rectangular window in Table 2 is $\frac{M}{2}$, and is $\frac{3M}{2}$ for the Bartlett window.

For convenience and ease of computation we define a "standardized bandwidth", b_1, corresponding to M = 1, so that

$$b = \frac{b_1}{M} = \frac{1}{\int_{-\infty}^{\infty} \omega^2(u)du} \tag{3.3.7}$$

The standardized bandwidths are shown in Table 2.

Since we have said that the variance is inversely proportional to the bandwidth (equations (3.3.6), (3.3.7)) it follows that

$$\text{(variance) x (bandwidth) = constant.} \tag{3.3.8}$$

3.6 SPECTRAL CONFIDENCE LIMITS

If we assume that the spectrum is smooth with respect to the spectral window then the expected value of the smoothed spectral estimator ($\bar{C}_{xx}(f)$) approximates the spectrum, $\Gamma_{xx}(f)$, such that

$$E[\bar{C}_{xx}(f)] \simeq \Gamma_{xx}(f),$$

and

$$\mathrm{Var}[\bar{C}_{xx}(f)] \simeq \frac{\Gamma_{xx}^2(f)}{T} \int_{-\infty}^{\infty} \omega^2(u)\,du \tag{3.3.9}$$

with degrees of freedom,

$$v = \frac{2T}{\int_{-\infty}^{\infty} \omega^2(u)\,du} = \frac{2T}{I} \tag{3.3.10}$$

and has a chi-square distribution. Thus, we can say that the degrees of freedom of the smoothed spectral estimate depend on the window, $\omega(u)$. The corresponding degrees of freedom for each window is shown in Table 2.

It follows then that

$$\mathrm{PR}\left\{x_v(\tfrac{\alpha}{2}) < v\,\frac{\bar{C}_{xx}(f)}{\Gamma_{xx}(f)} \leq x_v(1 - \tfrac{\alpha}{2})\right\} = 1 - \alpha,$$

where

$$PR\left\{x_v^2 \leq x_v \left(\frac{\alpha}{2}\right)\right\} = \frac{\alpha}{2} \ .$$

Then the interval between

$$\frac{v\bar{C}_{xx}(f)}{x_v\left(1 - \left(\frac{\alpha}{2}\right)\right)}$$

and

$$\frac{v\bar{C}_{xx}(f)}{x_v\left(\frac{\alpha}{2}\right)}$$

is a 100 (1 − α)% confidence interval for $\Gamma_{xx}(f)$. It should be noted that the confidence limits are accurate only when the spectral window is sufficiently narrow so that no appreciable bias exists.

Since we are plotting the spectrum on a logarithmic scale we must also compute the confidence limits as logarithmic values. The confidence limits for log $\Gamma_{xx}(f)$ are

$$\text{Log } \bar{C}_{xx}(f) + \log \frac{v}{x_v\left(1 - \left(\frac{\alpha}{2}\right)\right)} \text{ , and Log } \bar{C}_{xx}(f) + \log \frac{v}{x_v\left(\frac{\alpha}{2}\right)} \ . \qquad (3.3.11)$$

The number of degrees of freedom, v, of the smoothed estimator is

$$v = \frac{2T}{I} = 2 \left(\frac{T}{M}\right) b_1 \ . \qquad (3.3.12)$$

3.7 TRANSFER FUNCTIONS

If we find, by a visual inspection, that the behavior of the spectrum is bad or erratic, we may want to digitally filter the data to improve the spectral estimates at the primary frequencies. The behavior may be due to leakage from the spectral window(s), or causing false peaks where there is insufficient power to justify them. When we apply some sort of a digital filter $F[\cdot]$ to a stationary process $\{X_t\}$, so that

$$F[X_t] = \sum_{j=-M}^{M} a_j X_{t-j} \quad , \qquad (3.3.13)$$

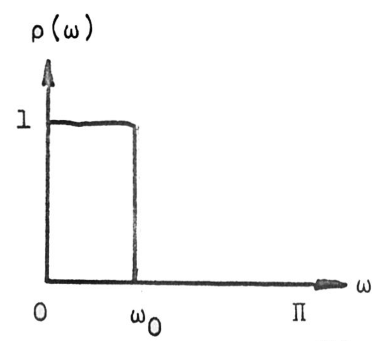
where X_t is a set of imput data, the transfer function is

$$s(\omega) = \sum_{j=-M}^{M} a_j e^{ij\omega} \quad . \qquad (3.3.14)$$

Considering only symmetric, cosine filters where $a_j = a_{-j}$ the transfer function becomes

$$s(\omega) = a_o + 2 \sum_{j=1}^{M} a_j \cos j\omega \quad . \qquad (3.3.15)$$

When we wish to estimate the trend component of a series by a means of filters we want to eliminate all frequencies except those near zero. The perfect transfer function would resemble the one shown in Figure 6.



$$\rho(\omega) = 1, \; 0 \le \omega \le \theta$$
$$= 0, \; \theta \le \omega \le \Pi \quad .$$

Figure 6

PERFECT TRANSFER FUNCTION

For finite M the coefficients a_j of (3.3.15) cannot be chosen so that $s(\omega)$ has the desired shape, so some kind of approximation must be used. We choose the a_j's so that $s(\omega)$ is the truncated Fourier series of the function $\rho(\omega)$, i.e.

$$a_j = \frac{\sin j\omega_o}{j\omega} \quad , \quad j = 1,2, \ldots . M \tag{3.3.16}$$

$$a_o = \frac{\omega_o}{2\pi} \quad .$$

This type of filter provides a reasonable approximation to $s(\omega)$ if M is large and ω_o is not too small. For a small ω_o and M not large the transfer function may be shaped as in Figure 7, with large side peaks which are important in the analysis.

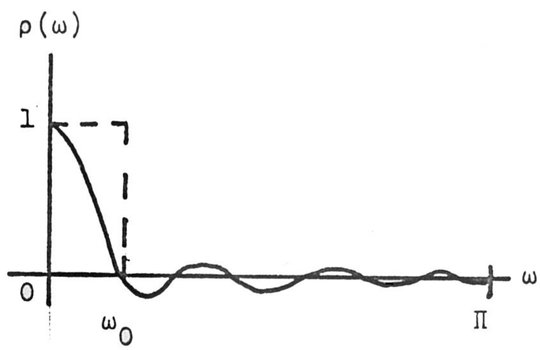


Figure 7

ACTUAL TRANSFER FUNCTION

As cos $j\omega$ is flat near $\omega = 0$ the usual shape of $s(\omega)$, as defined by (3.3.15), will be of a major, first peak, followed by a series of smaller side peaks. The problem of approximating $\rho(\omega)$ then becomes one of controlling the size of the side peaks.

3.8 NYQUIST FREQUENCY, ALIASING

Although economic data such as stock prices and commodity prices are recorded continuously, most economic data is, like the data used for this study, recorded periodically. The periodic sampling infers that the data is collected at equally spaced points in time. This creates a problem in correctly identifying the sources of mean square variation in the data. This problem is called "aliasing."

If our time series x(t) contains a frequency of $\frac{2\pi}{k}$, where k is the length of time between recordings (or the "period"), then x{t}, (a sample of X(t)) will contain no information about this frequency. The highest frequency about which we have any information is $\frac{\pi}{k}$ and it is known as the "Nyquist frequency". The power of this spectrum is recorded at $\omega = \pi$ in the power spectrum.

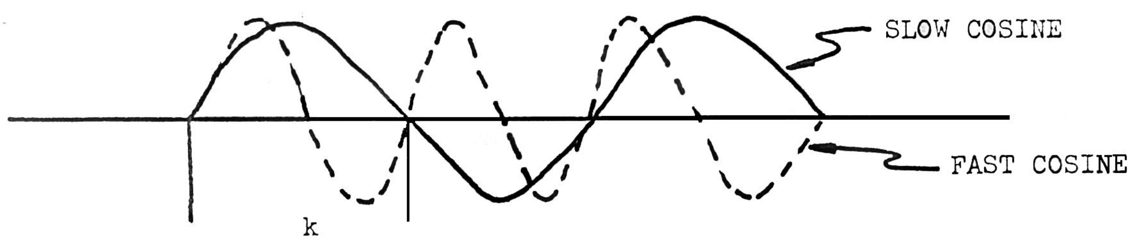


Figure 8

NYQUIST FREQUENCY, "ALIASING"

A fast cosine wave sample at regular intervals appears the same as a slow cosine wave. We can see from Figure 8 that the apparent contribution of any frequency is the result of superimposing the contributions of many frequencies which now becomes aliases of one another. If ω_o is the Nyquist frequency, then ω, $2\omega_o - \omega$, $2\omega_o + \omega$, $4\omega_o - \omega$, $4\omega_o + \omega$, ..., are confounded and are aliases of one another.

CHAPTER IV

PROCEDURES

4.1 THE DATA

The data used in this study was obtained from actual daily sales data of seven stores in Falleys supermarket chain. There are four (4) distinct types of data, as follows:

1. Total sales per store per day, in dollars,
2. Total customer count per store per day,
3. Total meat sales per store per day, in dollars, and
4. Shipments to the store per item per day, in units.

The data in (4) reflects production and shipment data, rather than the actual number of each item sold per store per day. The documents were coded each day to indicate if the item was on sale, and the relative prominence of the item in the advertising. In addition, when the item was on sale, the sale price was included, but the normal price was never recorded.

Falley's submitted the documents to KSU, where the data was transcribed onto punched cards. The data was arranged by item by day, and by store by day. The time series were visually inspected for errors and missing data.

Plots of the raw data were obtained and inspected for evidence of trends and cycles. The behavior of the plots of (1), (2), and (3) was found to contain a periodic term of seven days duration with no growth pattern. Therefore, where missing data was discovered, the figures for the same day in the previous week was substituted.

4.2 THE MATRIX APPROACH TO REGRESSION ANALYSIS

We define Y as the "vector of observation", X as the "matrix of independent variables:, β as the "vector of parameters to be estimated", and ε as the "vector of errors". Then from the store data the basic matrices might look like this for a linear model:

$$Y = \begin{pmatrix} 680 \\ 1190 \\ 450 \\ 526 \\ \cdot \\ \cdot \\ \cdot \\ 942 \\ 831 \end{pmatrix}, \quad X = \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ 1 & 307 \\ 1 & 308 \end{pmatrix}, \quad \beta = \begin{pmatrix} \beta_0 \\ \\ \beta_1 \end{pmatrix}, \quad \varepsilon = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \cdot \\ \cdot \\ \cdot \\ \varepsilon_{307} \\ \varepsilon_{308} \end{pmatrix}$$

Note that: Y is a 308 x 1 Vector
X is a 308 x 2 Vector
β is a 2 x 1 Vector
ε is a 308 x 1 Vector.

Writing the matrix equation $Y = X\beta + \varepsilon$

implies that $608 = \beta_0 + 1\ \beta_1 + \varepsilon_1$

$$\vdots$$

$831 = \beta_0 + 308\beta_1 + \varepsilon_{308}$, for each of the 308

observations. We can write the transpose of the matrices as:

$$\varepsilon' = (\varepsilon_1, \varepsilon_2, \varepsilon_3 \ . \ . \ . \ \varepsilon_{308}),$$

$$\varepsilon'\varepsilon = (\varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2 \ . \ . \ . + \varepsilon_{308}^2), \quad \text{and}$$

$$Y'Y = (Y_1^2 + Y_2^2 + Y_3^2 \ . \ . \ . + Y_{308}^2) \ .$$

Additionally,

$$X'X = \begin{pmatrix} 1 & 1 & 1 \ldots\ldots 1 \\ \\ 1 & 2 & 3 \ldots 308 \end{pmatrix} \times \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ 1 & 308 \end{pmatrix} = \begin{pmatrix} n & \sum x_i \\ \\ \sum x_i & \sum x_i^2 \end{pmatrix} ,$$

and

$$X'Y = \begin{pmatrix} 1 & 1 & 1 \ldots\ldots 1 \\ \\ 1 & 2 & 3 \ldots 308 \end{pmatrix} \times \begin{pmatrix} 680 \\ 1190 \\ \cdot \\ \cdot \\ \cdot \\ 831 \end{pmatrix} = \begin{pmatrix} \sum y_i \\ \\ \sum x_i y_i \end{pmatrix} .$$

The inverse of X'X can be shown as

$$(X'X)^{-1} = \begin{pmatrix} \dfrac{\sum x_i^2}{n\sum (x_i - \bar{x})^2} & \dfrac{-\bar{x}}{\sum (x_i - \bar{x})^2} \\ \\ \dfrac{-\bar{x}}{\sum (x_i - \bar{x})^2} & \dfrac{1}{\sum (x_i - \bar{x})^2} \end{pmatrix}$$

An alternate form is to write it as

$$(X'X)^{-1} = \frac{1}{n\sum (x_i - \bar{x})^2} \begin{pmatrix} \sum x_i^2 & -\sum x_i \\ \\ -\sum x_i & n \end{pmatrix} .$$

By postmultiplication we obtain:

$$(X'X)^{-1} \ (X'X)b = (X'X)^{-1} \ X'Y,$$

that is,

$$b = (X'X)^{-1} \ X'Y \ .$$

The effects of linear regression are additive and the solutions can always be written in this form, provided that X'X is non-singular and that the regression problem is properly expressed.

The matrix approach to a linear model can be summarized as follows:

1. The data is expressed as $Y = X\beta + \varepsilon$, and
2. The least squares estimates of (β_0, β_1), i.e. of

$$\beta = \begin{pmatrix} \beta_0 \\ \\ \beta_1 \end{pmatrix}, \quad \text{are given by} \begin{pmatrix} b_0 \\ \\ b_1 \end{pmatrix} = b = (X'X)^{-1} \ X'Y$$

3. The fitted values, $\hat{Y}$, are obtained by evaluating $Y = Xb$

4.3 ANALYSIS OF VARIANCE IN MATRIX TERMS

The general form of the analysis of variance is:

$$SS(b_1|b_0) = b_1 \left| \sum x_i y_i - \frac{(\sum x_i)(\sum y_i)}{n} \right|$$

$$= b_1 \left| \sum x_i y_i - n\bar{x}\bar{y} \right| \quad ,$$

where $SS(b_1|b_0)$ is the sum of squares due to regression.

Each of these sums of squares has 1 degree of freedom. Now, if we let

$SS(b_0)$ be the sum of squares about the mean $\left(SS(b_0) = \frac{(\sum Y_i)^2}{n} \right)$, then

$$SS(b_1|b_0) + SS(b_0) = (b_0, b_1) \begin{vmatrix} \sum y_i \\ \sum x_i y_i \end{vmatrix}$$

$$= b'X'Y$$

in matrix terms, with 2 degrees of freedom. The analysis of variance table in matrix terms appears as follows:

Source	Sum of Squares	Degree of Freedom	Mean Square
$b' = (b_0, b_1)$	$b'X'Y$	2	
Residual	$Y'Y-b'X'Y$	n-2	s^2
Total (uncorrected)	$Y'Y$	n	

4.4 DEVELOPMENT OF THE DATA MATRIX

A hypothesized linear model might look as follows:

$$X = \begin{array}{cc} x_0 & x_1 \\ \end{array}$$

$$X = \begin{vmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ 1 & 308 \end{vmatrix} \qquad Y = \begin{vmatrix} y_1 \\ y_2 \\ y_3 \\ \cdot \\ \cdot \\ \cdot \\ y_{308} \end{vmatrix} .$$

After regression analysis and spectral analysis of the regression residuals we might want to add cyclic components to the models. The data matrix would be expanded to look like this:

$$
\begin{array}{c} \\ X = \end{array}
\begin{array}{cccc}
x_0 & x_1 & x_2 & x_3 \\
\left| \begin{array}{c} 1 \\ 1 \\ \cdot \\ \cdot \\ \cdot \\ 1 \end{array} \right. &
\begin{array}{c} 1 \\ 2 \\ \cdot \\ \cdot \\ \cdot \\ 308 \end{array} &
\begin{array}{c} \mathrm{Sin}(1x\theta) \\ \mathrm{Sin}(2x\theta) \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{Sin}(308x\theta) \end{array} &
\left. \begin{array}{c} \mathrm{Cos}(1x\theta) \\ \mathrm{Cos}(2x\theta) \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{Cos}(308x\theta) \end{array} \right|
\end{array}
\qquad
Y = \left| \begin{array}{c} y_1 \\ y_2 \\ \cdot \\ \cdot \\ \cdot \\ y_{308} \end{array} \right| ,
$$

where θ is the cyclic component measured in radians. If several cyclic components are discovered through the spectral analysis (i.e. weekly, biweekly, monthly) the data matrix would be expanded further

$$
\begin{array}{c} \\ X = \end{array}
\begin{array}{cccccc}
x_0 & x_1 & x_2 & x_3 & x_4 & x_5 \\
\left| \begin{array}{c} 1 \\ 1 \\ \cdot \\ \cdot \\ \cdot \\ 1 \end{array} \right. &
\begin{array}{c} 1 \\ 2 \\ \cdot \\ \cdot \\ \cdot \\ 308 \end{array} &
\begin{array}{c} \mathrm{Sin}(1x\theta_1) \\ \mathrm{Sin}(2x\theta_1) \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{Sin}(308x\theta_1) \end{array} &
\begin{array}{c} \mathrm{Cos}(1x\theta_1) \\ \mathrm{Cos}(2x\theta_1) \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{Cos}(308x\theta_1) \end{array} &
\begin{array}{c} \mathrm{Sin}(1x\theta_2) \\ \mathrm{Sin}(2x\theta_2) \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{Sin}(308x\theta_2) \end{array} &
\left. \begin{array}{c} \mathrm{Cos}(1x\theta_2) \\ \mathrm{Cos}(2x\theta_2) \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{Cos}(308x\theta_2) \end{array} \right|
\end{array}
\quad
Y = \left| \begin{array}{c} y_1 \\ y_2 \\ \cdot \\ \cdot \\ \cdot \\ y_{308} \end{array} \right|
$$

Another method of developing the data matrix is through the use of an "01" dummy variable combination. This is accomplished by a nx7 matrix where each of the independent variables represented a potential "zero-not zero" condition. The data matrix would appear as follows:

$$
\begin{array}{c} \\ X = \end{array}
\begin{array}{c}
\begin{array}{ccccccc} x_0 & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{array} \\
\left| \begin{array}{ccccccc}
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot
\end{array} \right|
\end{array}
\qquad
Y = \left| \begin{array}{c} y_1 \\ y_2 \\ y_3 \\ \cdot \\ \cdot \\ \cdot \\ y_{308} \end{array} \right|
$$

In the gross sales data the x-matrix, above, would appear as a 308x7 matrix in its completed form, the 7x7 modules being repeated until the last observations.

4.5 SYSTEMATIC APPROACH TO SPECTRAL ANALYSIS.

All of the detail work concerned with spectral analysis was accomplished through the use of the system described by Figure 9. The statistical analysis was all done by a computer, the IBM 360/50.

The computer programs used to analyze the data were written in Fortran IV, and in Waterloo Fortran (because of the excellent diagnostics available.) A complete set of the programs may be found in Appendix I.

4.6 MODELS USED

Five different models were used and compared to determine which one best described the behavior of the data and should be used to forecast future behavior. These models were:

1. Dummy variables
2. Purely linear
3. Linear with 1 frequency (the primary frequency)
4. Linear with 2 frequencies (primary and 1st harmonic)
5. Linear with 3 frequencies (primary, 1st and 2nd harmonic).

A forecast model was developed for each regression model in order to demonstrate the behavior of the forecast residuals and to show how the addition or subtraction of terms to the model affected the behavior of the residuals. A summary of the results may be seen in TABLE 3, in Chapter V.

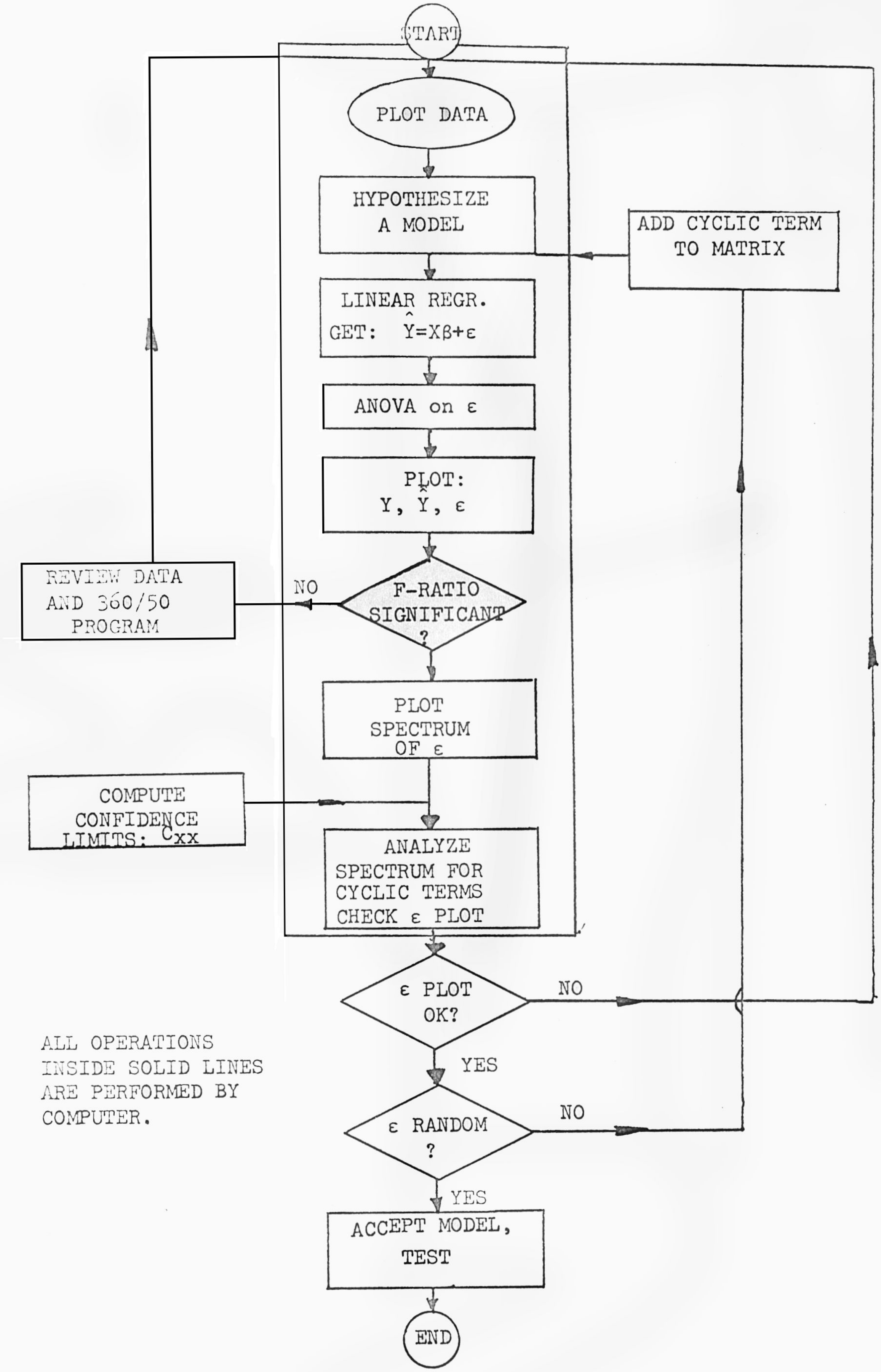


Figure 9.

ʼRT OF SYSTEM USED IN SPECTRAL ANALYSIS

4.7 CALCULATION OF CONFIDENCE LIMITS AND BANDWIDTH

Confidence limits and bandwidths for the spectrums are calculated according to sections 3.5 and 3.6, respectively. Confidence limits are calculated as

$$\text{Log } \bar{C}_{xx}(f) + \log \frac{v}{x_v\left(\frac{\alpha}{2}\right)} \qquad \text{(Upper Confidence Limit)}$$

$$\text{Log } \bar{C}_{xx}(f) + \log \frac{v}{x_v\left[1-\left(\frac{\alpha}{2}\right)\right]} \qquad \text{(Lower Confidence Limit).}$$

The number of degrees of freedom is computed by the formula

$$v = \frac{2T}{I} = 2\left(\frac{T}{M}\right)b_1$$

where T is the length of the time series, M is the number of lags, and b_1 is the standardized bandwidth, which in this case is 1.33 for the Tukey-Hanning window.

Bandwidth is computed as:

$$b = \frac{b_1}{M}$$

Where b_1 is the standardized bandwidth (1.33) and M is the number of lags.

4.7.1 For the gross store data (total sales, customer count, meat sales) the parameters mentioned above are calculated as follows:

1. Degrees of freedom:

$$v = 2\left(\frac{T}{M}\right)b_1 = 2\ b_1\left(\frac{T}{M}\right) = 2.66\ \left(\frac{308}{65}\right) = 12.631.$$

Note: M represents the number of lags required for optimum resolution of a spectrum. It is computed as $M = \frac{b_1}{a\Delta}$, where $a\Delta$ is the incremental value between successive plots of the spectrum.

Hence,

$$a = \frac{\pi}{k} = \frac{\pi}{155} = 0.0203. \quad \text{So, } M = \frac{1.33}{.0203} = 65.$$

2. Confidence limits:

   Upper confidence limits: $\bar{C}_{xx}(f) + 2.57$

   Lower confidence limits: $\bar{C}_{xx}(f) - 0.50$

3. Bandwidth:

$$b = \frac{b_1}{M} = \frac{1.33}{65} = 0.0203$$

4.7.2 Values are computed for the item data (pork steak, round steak, and whole fryers) as follows:

1. Number of lags:

$$M = \frac{b_1}{a\Delta} = \frac{1.33}{\pi/80} \simeq 34.$$

2. Degrees of freedom:

$$v = 2.66 \times \left(\frac{162}{34}\right) = 12.661 \simeq 13.$$

3. Confidence limits:

   Upper confidence limits: $\bar{C}_{xx}(f) = 2.50$.

   Lower confidence limits: $\bar{C}_{xx}(f) = 0.54$.

All of the confidence limits are computed for 0.95 probability.

CHAPTER V

RESULTS

The original hypothesis was that frequencies would be found indicating the presence of weekly, bi-weekly, semi-monthly, and monthly cycles. The management of Falley's had stated that a weekly cycle was definitely in evidence, based on their analysis.

The primary approach used for analysis of the data included regression analysis and spectral analysis. Fourier analysis was used to complement the spectral analysis and validate the frequencies which appeared. Periodograms were analyzed for all of the store data. An explanation of the Fourier technique which was used can be seen in Appendix II.

Detailed analysis of the data was accomplished in two phases:

(1) Analysis of the store data (total sales, customer count, meat sales) for Store number Seven;

(2) Item data for Store number Seven.

The discussion of results is separated to correspond to the two phases.

5.1 ANALYSIS OF THE STORE DATA.

The spectral analysis disclosed only a strong cycle of seven days duration. This weekly cycle was found in all three sets of data (total sales, customer volume, meat sales) and substantiated by the periodogram from the Fourier analysis (see Figure 11). Because of the exceptional strength at the frequency of the cycle, $\frac{2\pi}{7}$ radians, there were definite harmonics also present which corresponded to periods of 3 1/2 days and 1 3/4 days (1st and 2nd harmonics, respectively).

FIGURE 10

REPRESENTATIVE SPECTRA
OF TOTAL SALES

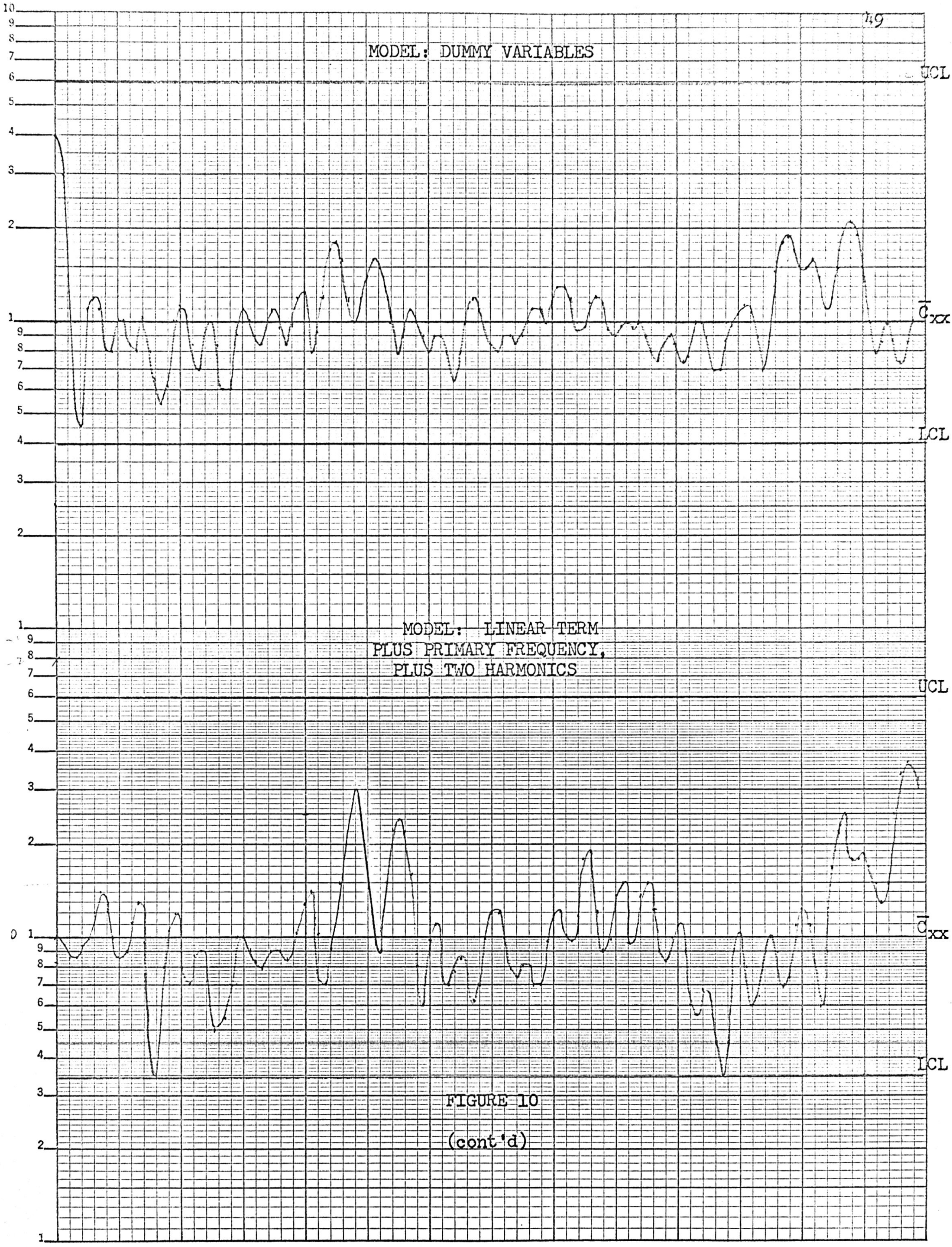
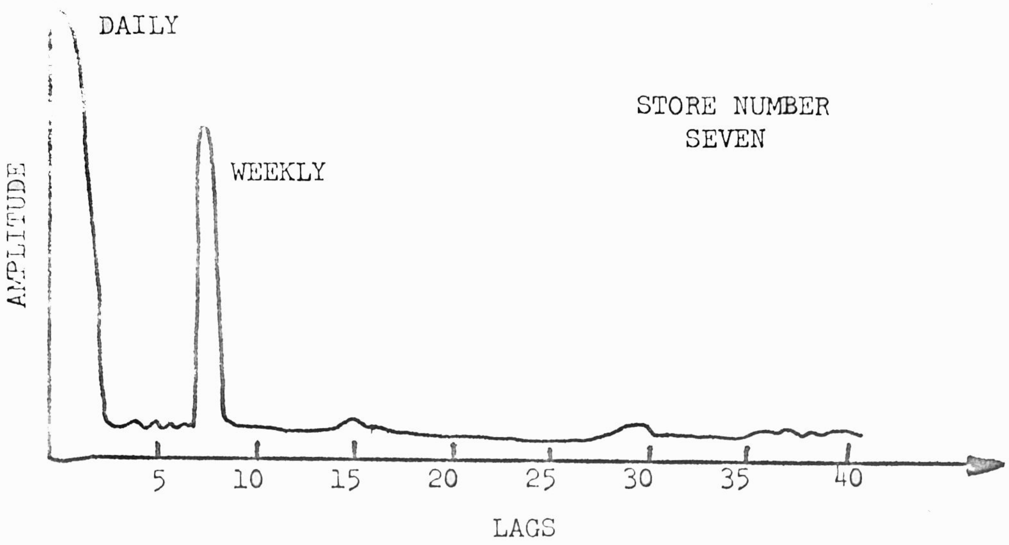


FIGURE 10

(cont'd)



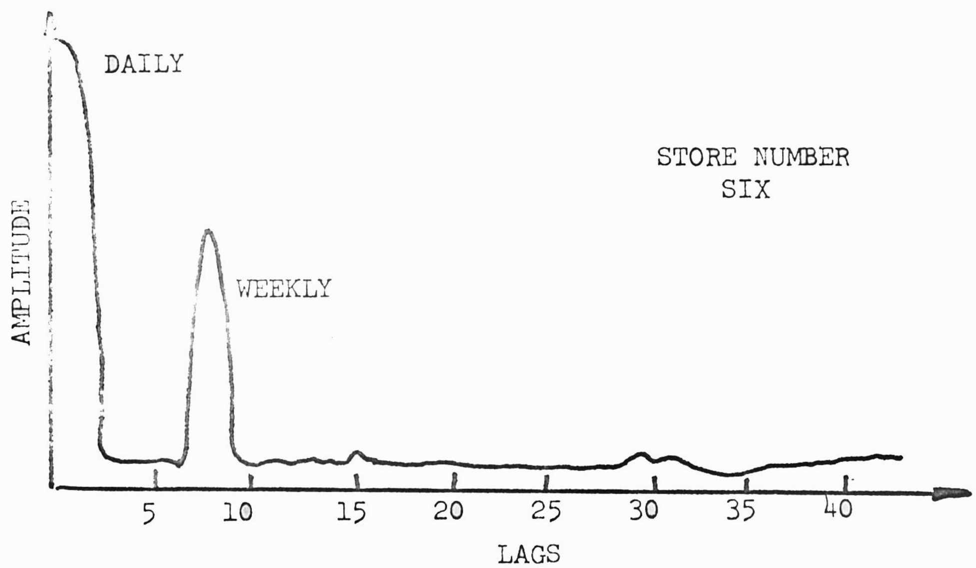


Figure 11

REPRESENTATIVE PERIODOGRAMS OF STORE DATA, USING TOTAL DOLLAR SALES

There appears to be no growth trend in the data. Regression analysis reveals that the growth component, β_1, is very small (10^{-2} to 10^{-3}) for each set of data and varies minutely from model to model.

If we examine the spectra for evidence of the biweekly and monthly cycles we find that there is none. This invalidates a major portion of the original hypothesis; that is, that bi-weekly and monthly frequencies do exist.

A disproportionately large amount of time and effort was spent in analyzing the gross store data. This was done because the data exhibited more of the characteristics of true time series. (i.e. randomness, sufficient amount of data, relatively stable conditions) which could be tied in later with advertising sales effect, through the use of cross-correlation and cross spectral analysis.

In order to determine whether or not the values for β_1 are actually insignificant, a t-test was performed on the β_1 values for total sales, customer count, and meat sales. The results of the t-test indicate that all of the β_1 values are definitely insignificant. See Appendix IV for a detailed explanation of the t-test.

The linear regressions which were performed using the various models described in Section 4.5, gave increasingly better results (with respect to the correlation coefficient, r, and the coefficient of determination, r^2) as terms were added. A summary of the statistics can be seen at TABLE 3.

It is important to note that in all three instances the best values of r and r^2 were obtained through the use of the model with linear term plus three frequencies and the model using dummy variables. There was little difference between the results of the two models.

TABLE 3

SUMMARY OF REGRESSION STATISTICS

STORE #7

TOTAL SALES

reg.	R	R^2	F-RATIO	Growth Component
01	.9107	.8295	2846.0	———
linear	.135	.018	2251.9	0.017
+1 freq	.764	.583	2853.1	0.017
+2 freq	.858	.736	2275.3	0.017
+3 freq	.920	.847	2638.7	0.017

CUSTOMER COUNT

reg.	R	R^2	F-RATIO	Growth Component
01	.627	.393	2854.5	———
linear	.246	.061	9236.2	0.0057
+1 freq	.540	.291	6154.4	0.0057
+2 freq	.604	.365	3419.3	0.0057
+3 freq	.672	.452	2631.6	0.0057

MEAT SALES

reg.	R	R^2	F-RATIO	Growth Component
01	.904	.817	1640.9	———
linear	.100	.100	1298.0	0.0036
+1 freq	.775	.601	1832.7	0.0036
+2 freq	.861	.741	1440.5	0.0036
+3 freq	.927	.860	1813.1	0.0036

FREQUENCIES	1 =	.89759	Primary = 7 days
(in Radians)	2 =	1.79518	1st Harmonic = 3 1/2 days
	3 =	2.69277	2nd Harmonic = 1 3/4 days

In all cases the model with only the linear terms (β_0, β_1) provided very poor results, as one might expect after determining the presence of strong cyclic terms. There was a significant increase in accuracy where the terms associated with the primary frequency were added to the linear terms. The increase in accuracy is not great between the model using the linear term plus the primary and the model with the linear term, primary frequency, and 1st harmonic; nor is it great between the latter model and the model with the linear term, primary frequency, and first and second harmonic.

Figure 10 shows a representative set of spectra for total dollar sales, with each model used. By inspection, the effect of additional terms becomes obvious. One can see the three peaks in the linear model; two peaks in the linear plus one frequency model; one peak in the linear plus two frequencies model; and no peaks in the linear plus three frequencies, or in the model using dummy variables. As stated before, the peaks are not significant and indicative of a frequency unless they protrude above the confidence limits.

The error variance σ_ε^2 decreases as each parameter is added to the regression (see Figure 12). As one would expect, a point is reached at which the addition of parameters begins to show less significant decreases in σ_ε^2. This "point" is different for each time series. From FIGURE 12 one can determine that σ_ε^2:

(1) for customer count, decreases very slightly after 2 parameters;

(2) for total sales, decreases substantially up to 7 parameters (the dummy variable regression) and slightly from 7 to 8 parameters (linear term plus primary frequency, plus two harmonics);

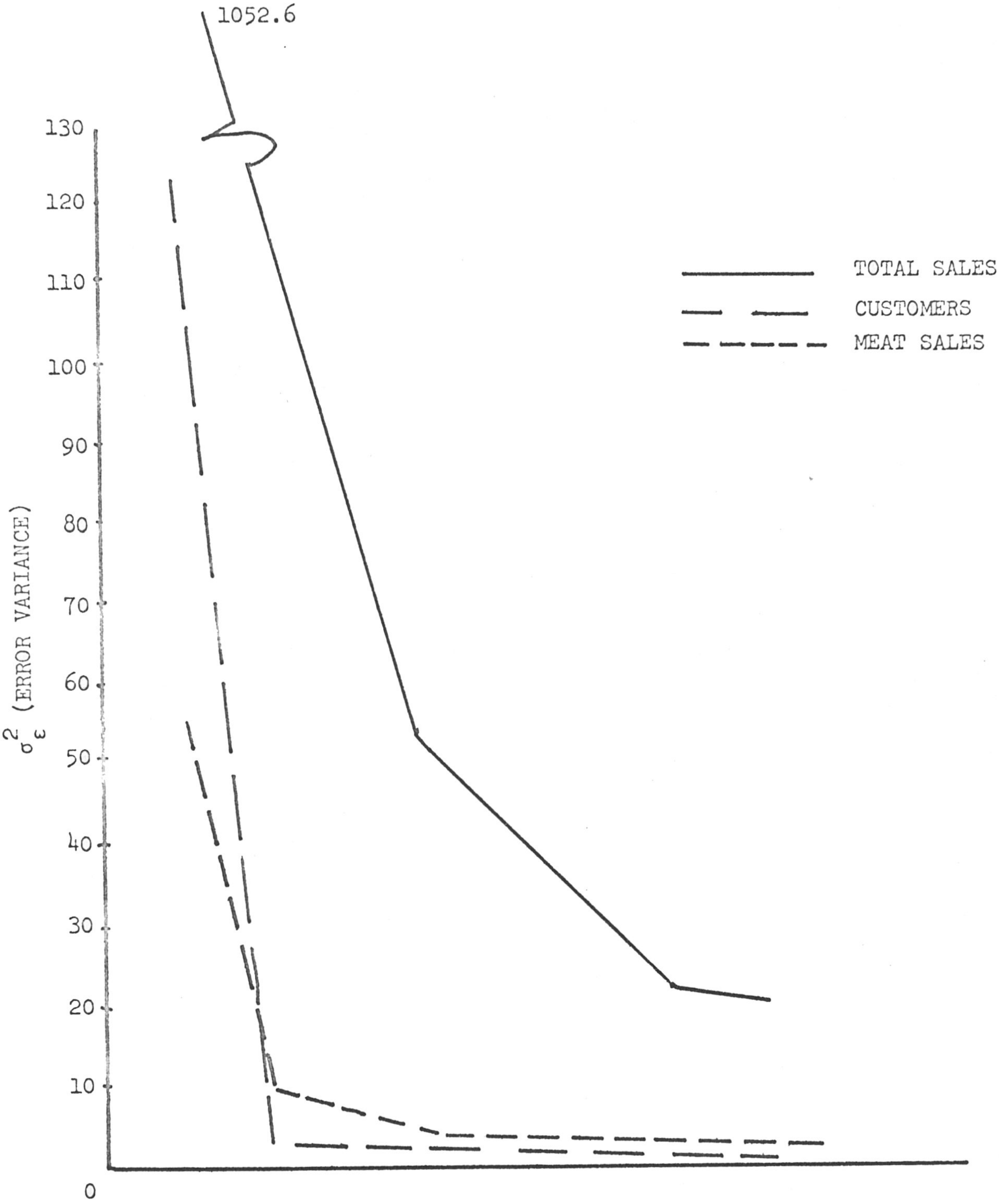


Figure 12

PLOT OF RESIDUAL MEAN SQUARE
VS. NUMBER OF VARIABLES
FOR STORE #7

(3) for meat sales, behaves similarly to the customer count data.

It is important to notice that the results obtained from the regression using dummy variables are almost as good as the results of the regression which used the linear term plus the primary frequency plus two harmonics. Only one series of regression computations was required for the dummy variable results, while four series were required to obtained comparable results using the other method (adding 1 frequency each time).

5.2 ANALYSIS OF ITEM DATA

Spectral analysis was performed on the data for whole fryers, round steak, and pork steak. A linear regression and a regression using dummy variables was accomplished with each set of data, not including the variables relating to the sale (advertising). Additionally, a regression using dummy variables plus the sale variables was performed on the round steak data. A summary of the results of the spectral analyses can be seen at Table 4.

In none of the cases did spectral analysis of the regression residuals disclose any significant frequencies. This was substantiated by Fourier analysis. The only significant results obtained from the item data was accomplished through the addition of the advertising variables to the regression using dummy variables. The results of that regression are astounding when compared to the results of the other regressions. As seen from Table 4 the coefficient of correlation is 0.0589 for the 01 regression without advertising variables, 0.1305 for the linear regression, and 0.9449 for the 01 regression using the ad variables!

TABLE 4

SUMMARY OF REGRESSION STATISTICS
FOR ITEM DATA

STORE #7

ROUND STEAK

reg.	R	R^2	F-RATIO	GROWTH COMPONENT
01	.0589	.0034	8.19	————
linear	.1305	.0170	44.811	0.005565
01+sales	.9449	.8927	336.648	————
			PORK STEAK	
01	.1438	.02068	11.882	————
linear	.0642	.00413	57.94	0.001640
			WHOLE FRYER	
01	.3315	.1098	10.661	————
linear	.0935	.0087	32.656	-0.006184

Degrees of freedom for all items: 162

The spectra for all of the regressions display a very similar shape which is indicative of a strong autocorrelation tendency in the data (see Figure 13). However, in all cases, the spectrum is well within the confidence limits, thus emphasizing the fact that there is nothing significant in the behavior of the residuals.

5.3 FORECASTS.

All of the models for gross sales data obtained through regression and spectral analysis were used to forecast behavior for an additional twelve weeks. As one would expect, the accuracy of the forecast increases as the model incorporates additional terms. See Appendix III for a representative plot which includes the actual data, forecasted data, and residuals.

The residuals were examined for evidence of trends which would indicate whether there were any long-term time effects influencing the data. Examples of possible time effects are shown in Figure 14 as follows:

1. Figure 14-(a) no assignable cause
2. Figure 14-(b) variance not constant, and it increases with time. A weighted least squares analysis should have been made.
3. Figure 14-(c) a linear term in time should have been used.
4. Figure 14-(d) quadratic terms in time should have been included in the model.

Combinations and variations (such as opposite slopes) of these defects can and do occur.

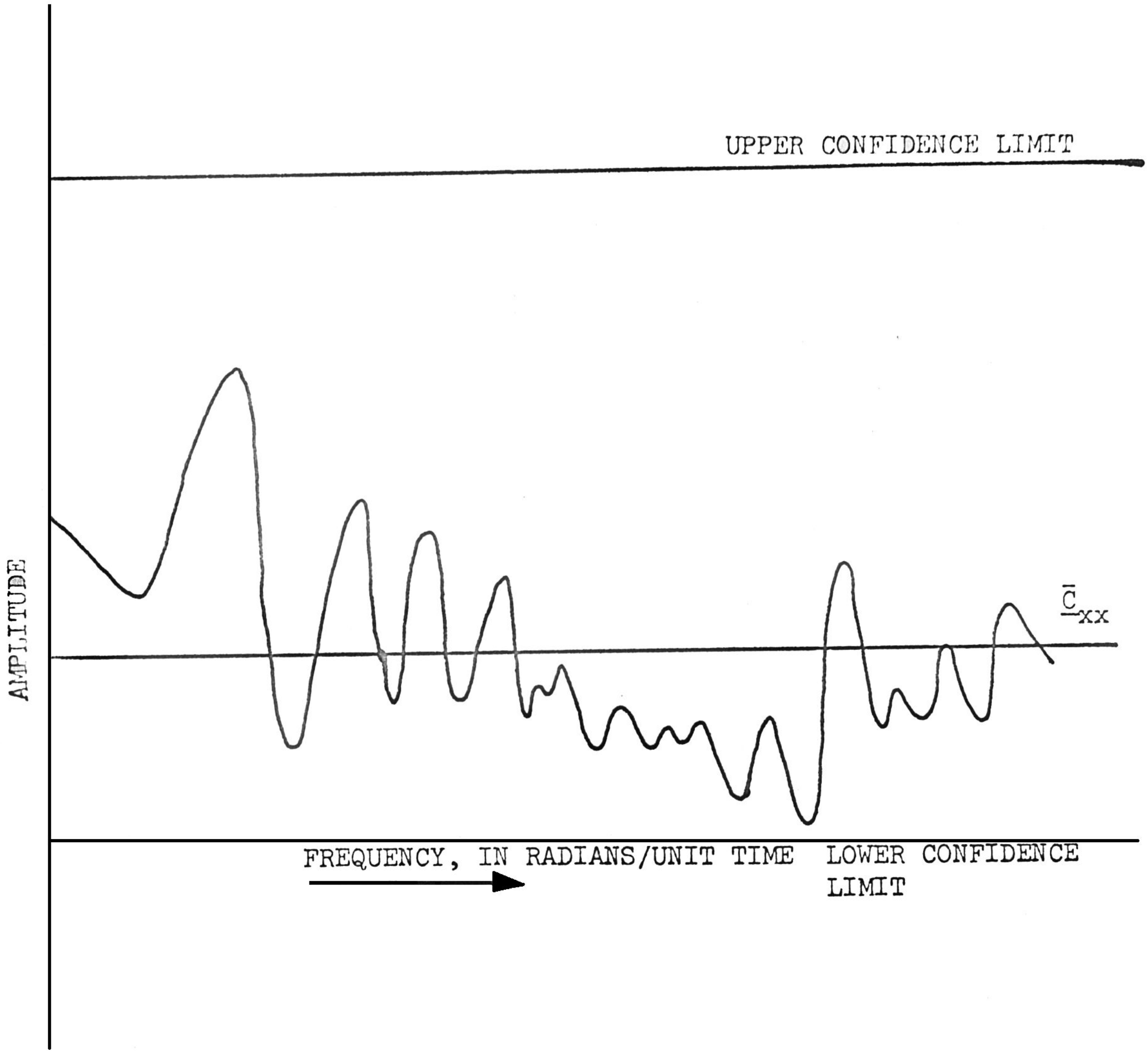


Figure 13

A REPRESENTATIVE SPECTRUM
OF THE ITEM DATA

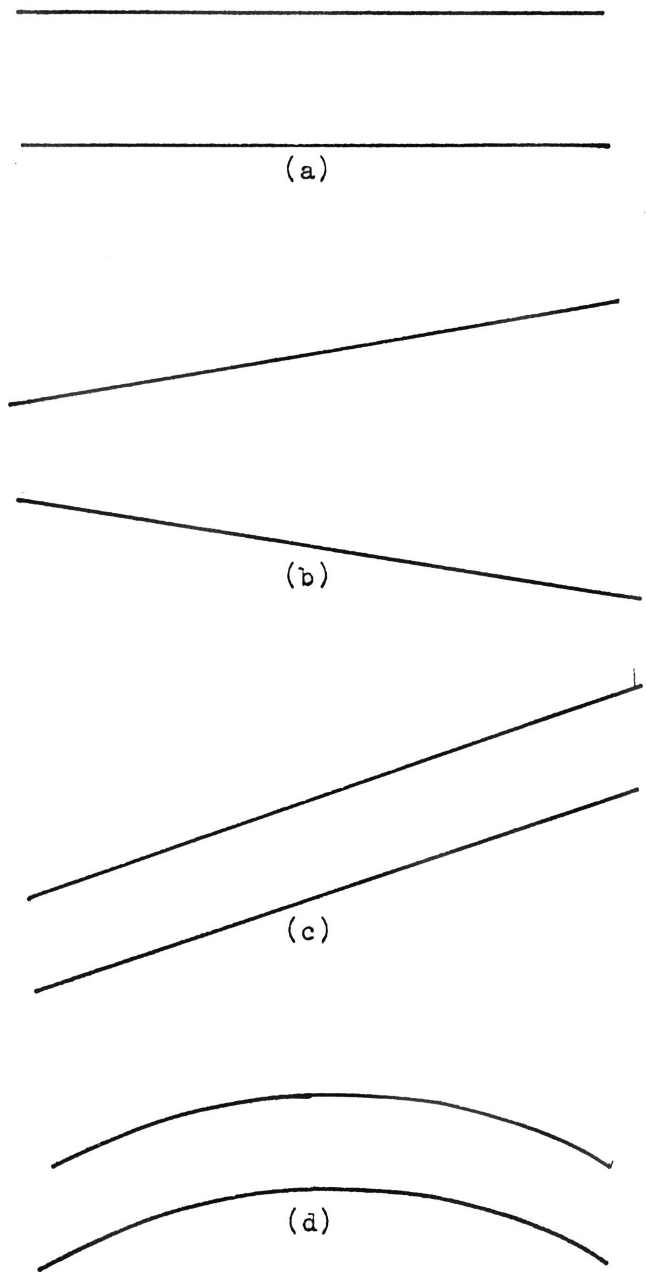


Figure 14

BEHAVIOR OF RESIDUALS

The general shape taken by the residuals obtained from forecasts using the various models corresponds to Figure 14-(a). This indicates inclusion of the necessary basic parameters in the model, as well as stable behavior of the data.

Another indication of the relative goodness of the forecast is the mean absolute error, (MAE) ($\sum|\varepsilon|N$). From TABLE 5 one can observe the decrease in MAE as terms are added, up to the dummy variable model. At that point there is a very slight increase in the MAE. The error for the total forecast, expressed as a percentage, decreases substantially from the linear model to the model with a linear term plus the primary frequency. It also shows an increase between the linear term plus three frequency model and the model using dummy variables.

As one would expect the relative decrease in MAE between the linear model and the linear model plus 3 frequencies is greatest where there is the greatest variability (meat sales, coefficient of variation = .4829) and least where there is the least variability (customer count, coefficient of deviation = .1993). The error variance, σ_ε^2, also decreases as the terms are added.

No forecasts were made for the item data because there was not sufficient data available to provide for a legitimate forecast.

TABLE 5

SALES DATA STATISTICS

1. FORECAST SUMMARY

MODEL	AVE $\|\varepsilon\|$ / FORECAST AS A %		ERROR FOR TOTAL FORECAST AS A %
		TOTAL SALES	
linear	28.2209		9.5842
+1 freq	21.7719		1.0101
+2 freq	18.0735		0.9511
+3 freq	13.1176		0.9421
Dummy Var.	13.2682		2.6228
		CUSTOMER COUNT	
linear	15.039		8.7442
+1 freq	14.2552		0.8176
+2 freq	13.5368		0.7960
+3 freq	12.6493		0.7936
Dummy Var.	12.6589		1.33
		MEAT SALES	
linear	42.5452		10.4313
+1 freq	31.5245		2.0592
+2 freq	25.3043		1.9853
+3 freq	17.2686		1.9727
Dummy Var.	17.2739		2.7789

2. DISTRIBUTION STATISTICS

	MEAN	STD. DEVIATION	COEFFICIENT OF VARIATION
TOTAL SALES	$ 3126.62	$ 1159.70	.3709
CUSTOMER COUNT	1111.86	223.46	.1993
MEAT SALES	696.90	336.54	.4829

CHAPTER VI

CONCLUSIONS

6.1 STORE DATA

There is very little difference between the results of the model using the dummy variables and the model using the linear term plus three frequencies. The more accepted model in the industry is the one using dummy variables. In the opinion of this author, it is easier to develop, explain, and use. In this case the difference in the results between the two models does not justify the time and expense of implementing the spectral technique, in spite of the fact that good results were eventually obtained from the spectral analyses. If the primary intent of this study had been to identify and analyze frequency response s, such as in the case of communications engineering studies, spectral analysis would be justified.

A logical explanation of the results of the approach using regression with dummy variables is that the cycle (in this case, weekly) is repeating itself and each value of the model (β_1, β_2,...β_7) reflects the same proportionate part of each week's business. There is also a logical explanation for the appearance of the linear term plus the primary frequency: the linear term is present because of the nature of the regression analysis, and the primary frequency appears because there is overwhelming evidence to indicate the presence of a legitimate weekly cycle. The only explanations of the presence of the two harmonics are

(1) That the primary is so strong that the harmonics were a natural result, and

(2) That there is substantial leakage due to using the Tukey-Hanning window.

Jenkins and Watts [11], in their discussion of window carpentry, state that the price which must be paid for optimal resolution and narrow bandwidth (such as we have used in this study) is that we get large side peaks (frequencies). The effect of these extra frequencies is to permit values of $\Gamma_{xx}(g)$ at frequencies away from the primary frequency to make large contributions to the bias of the primary frequency. They further state that the rectangular window, the Bartlett window, and the Tukey-Hanning window cause considerable trouble if the spectrum has a narrow peak (which is certainly the case for the spectra in this study). If side lobes are to be minimized, the Parzen window should be used, even though it is a wider window and requires more autocovariance terms to achieve a given bandwidth.

6.2 ITEM DATA

The only technique which produced any positive results is the regression using dummy variables, including the advertising variables. Spectral analysis, in this case, does not contribute to the development of a model. It does provide an accurate measure of the noiseness and unpredictability of the data under normal linear regression procedures.

As previously mentioned the data is stratified and exhibits great variability, hence great instability. If we use the Coefficient of Variation, $\frac{\sigma_x}{\mu_x}$, as a measure of the variability of the item data we obtain values close to 1.00, indicating an extreme amount of variability (over almost the entire range of the distribution). This is in contrast with values of .36, .20, and .48 for total dollar sales, customer count, and meat sales, respectively.

One important conclusion which should be emphasized is that spectral analysis makes a substantial contribution in the identification of assignable causes. Through examination of the spectrum one can determine those variables which should have been included in the regression. For example, examination of a spectrum of the residuals from a linear regression of gross sales data reveals the presence of the primary frequency and the two harmonics. On the other hand, examination of the spectrum of the residuals from the regression using dummy variables reveals no more significant variables.

6.3 FORECASTS

The main conclusions reached by this author regarding forecasts of data using models developed through regression and spectral analysis are:

(1) Given data which is basically Gaussian, accurate models can be developed and used successfully,

(2) Regression models developed through the use of dummy variables are as accurate in predicting behavior of stable time series as are models developed through linear regression and addition of cyclic terms, and

(3) The gross sales data used in this study is definitely predictable. This fact is not to startling to this author, but it joggles the professional pride of many store managers who have been wringing their hands and nodding their heads

in anguish as they decry the incredible inpredictability of their business.

The results of this study have definitely convinced this author that "it can be done!" The implications for the retail grocery business, as well as for military commissaries, is considerable.

CHAPTER VII

RECOMMENDATIONS FOR FUTURE WORK

7.1 CROSS SPECTRAL ANALYSIS

It seems almost a natural step to progress from spectral analysis to cross-spectral analysis. Through the use of cross spectral techniques relationships can be determined between different variables, such as meat sales and hamburger sales, customer count and the size of the advertisement in the daily paper, dollar sales and customer count, etc.

Further, lead-lag relationships between variables can be developed which incorporate phase angle and phase shifts. The true value of this type of statistical analysis is not fully realized until cross-spectral analysis is incorporated into the analysts box of tools.

7.2 ANALYSIS USING OTHER SPECTRAL WINDOWS

It would be quite interesting to evaluate the results of spectral analysis on the gross sales data, using different spectral windows. As mentioned in Section 6.1, one of the possible reasons for the harmonics in the spectrums could be that the Tukey-Hanning window allows too much leakage. Perhaps the use of a Parzen window or Bartlett window would provide considerably different results?

7.3 DETERMINING EFFECT OF ADVERTISING

There were some variables in the item data which were not used. One of the most important of these was a variable describing the classification of advertising, based on the prominence of the item in the daily advertising. Analysis of this relationship could be accomplished using regression techniques and spectral analysis or cross-spectral analysis.

7.4 IMPLICATIONS FOR STORE MANAGEMENT

The accuracy and reliability of the forecasts form an excellent basis for detailed analysis of daily customer behavior. It is this author's contention that consistent, predictable patterns will be uncovered in customer arrival data.

From an integration of daily predicted total volume and hourly arrival patterns accurate schedules can be derived well in advance for the efficient accomplishment of store functions such as stocking, warehousing and cashiering. GPSS/360 (General Purpose Simulation System) could be used to develop, evaluate, and test the effectiveness of such an integrated management system.

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APPENDIX I

COMPUTER PROGRAMS

CONTENTS

```
      REAL Y(1500)/1500*0.0/,T(400)/400*0.0/
      REAL X(2800)/2800*0.0/,Z(100)/100*0.0/,XX(99)/99*0.0/,XXI(20)/20*0
     1.0/,XY(500)/500*0.0/,B(10)/10*0.0/,EST(400)/400*0.0/,XTRAN(2500)/
     22500*0.0/,BXY(99)/99*0.0/,YY(400)/400*0.0/,REST(999)/999*0.0/,PLT(
     31500)/1500*0.0/,THETA(3)/3*0.0/,U(400)/400*0.0/,V(400)/400*0.0/
    1 FORMAT(6I5)
    2 FORMAT(   10X,F7.2,F7.0,F9.2)
    3 FORMAT(  6F12.6
    8 FORMAT(  6X,6HACTUAL,6X,9HESTIMATED,5X,8HRESIDUAL)
    9 FORMAT(   3F14.7)
   20 FORMAT( 20X,20HANALYSIS OF VARIANCE/6X,6HSOURCE,7X,14HSUM OF SQUAR
     1ES,12X,2HDF,10X,2HMS/8X,1HB,11X,E14.7,17X,1I3,6X,G14.7/6X,8HRESIDU
     2AL,6X,E14.7,15X,I3,6X,G14.7/6X,5HTOTAL,9X,E14.7,15X,I3,6X,G14.7/8X
     3,14HTHE F-RATIO IS,G14.7/8X,2HR=,G14.7/8X,5HR**2=,G14.7)
   22 FORMAT(   10X,I5,3X,F7.2,F7.0,F9.2)
   30 FORMAT( 12X,3HRAW,8X,9HESTIMATED,7X,8HRESIDUAL)
   31 FORMAT( 9X,F10.5,4X,F10.5,4X,F10.5)
   55 FORMAT(3F10.7)
   66 FORMAT(3F8.6)
  100 FORMAT('          VALUES OF T(I)'//)
  101 FORMAT(' DETERMINANT OF MATRIX INVERSE  ',E20.5)
  102 FORMAT(1H ,8E14.7)
  103 FORMAT(' BETA VALUES ARE  ', 4F12.5)
  104 FORMAT(' RESULTS OF Y TIMES Y-TRANSPOSED..', F12.5)
  108 FORMAT('  X-PRIME X')
  109 FORMAT('  X-PRIME X INVERSE'/)
  188 FORMAT('  6F12.5)
  198 FORMAT('   X-PRIME X INVERSE MATRIX'/)
  199 FORMAT('  VALUES OF THE RAW DATA'//)
      READ(1,66)(THETA(I), I=1,3)
      READ(1,1)N,NP,NQ,NVAR,LAGS,NOSTOR
      WRITE(3,199)
      DO 95 I=1,N
      READ(1,2)(Y(I+N*(j-1)),J=1,NVAR)
      WRITE(3,22)I,(Y(I+N*(J-1)),J=1,NVAR)
   95 CONTINUE
      DO 6 I=1,N
      Y(I)=Y(I)/100.0
      Y(I+N)=Y(I+N)/100.0
      Y(I+2*N)=Y(I+2*N)/100.0
    6 CONTINUE
      DO 555 IJ=1,3
      GO TO (39,35,37),IJ
   35 CONTINUE
      DO 36 JJ=1,N
   36 Y(JJ)=Y(JJ+N)
      GO TO 39
   37 CONTINUE
      DO 38 JJ=1,N
```

```
   38 Y(JJ)=Y(JJ+2*N)
   39 CONTINUE
C
C
      INTEGER M(100)/100*0/,M1(100)/100*0/
      SUMA=0.0
      CSUM=0.0
      YSUM=0.0
      BSUM=0.0
      RESID=0.0
      D=0.0
      J1=1
      R=0.
      NS=NP-2
      NB=N-NS
      DO 888 I=1,N
      J=I-154
      T(I)=J*THETA(1)
      U(I)=J*THETA(2)
      V(I)=J*THETA(3)
      X(I)=1.0
      X(I+N)=J
      X(I+2*N)=SIN(T(I))
      X(I+3*N)=COS(T(I))
      X(I+4*N)=SIN(U(I))
      X(I+5*N)=COS(U(I))
      X(I+6*N)=SIN(V(I))
      X(I+7*N)=COS(V(I))
  888 CONTINUE
      DO 4 I=1,N
    4 SUMA=SUMA+Y(I)
      CSUM=(SUMA**2)/N
C
C        MATRIX REGRESSION
C
      CALL GTPRD(X,X,XX,N,NP,NP)
      WRITE(3,108)
      KK=1
      DO 44 I=1,NP
      KKK=NP*I
      WRITE(3,102)(XX(J), J=KK,KKK)
   44 KK=KK+NP
      CALL MINV(XX,NP,D,M,M1)
      WRITE(3,109)
      KK=1
      DO 45 I=1,NP
      KKK=NP*I
      WRITE(3,102)(XX(J), J=KK,KKK)
   45 KK=KK+NP
      WRITE(3,101)D
      CALL GMTRA(X,XTRAN,N,NP)
```

```
      CALL GMPRD(XTRAN,Y,XY,NP,N,1)
      CALL GMPRD(XX,XY,B,NP,NP,1)
      CALL GTPRD(Y,Y,YY,N,J1,J1)
      WRITE(3,104) YY(1)
      CALL GTPRD(B,XY,BXY,NP,J1,J1)
C
C        ANALYSIS OF VARIANCE
C
      YSUM=YY(1)
      BSUM=BXY(1)
      RESID=YSUM-BSUM
      SMEAN1=BSUM/NS
      SMEAN2=RESID/NB
      SMEAN3=YSUM/N
      COEF2=(BSUM-CSUM)/(YSUM-CSUM)
      COEF1=SQRT(COEF2)
      F=SMEAN1/SMEAN2
      DO 10 I=1,N
      EST(I)=B(1)+(B(2)*X(I+N))+(B(3)*X(I+2*N))+(B(4)*X(I+3*N))+(B(5)*X(
     1I+4*N))+(B(6)*X(I+5*N))+(B(7)*X(I+6*N))+(B(8)*X(I+7*N))
      REST(I)=Y(I)-EST(I)
      PLT(I)=I
      PLT(I+N)=REST(I)
      PLT(I+2*N)=Y(I)
      PLT(I+3*N)=EST(I)
      REST(I+N)=I
   10 CONTINUE
      CALL PLOT(9,PLT,N,4,N,0,NOSTOR)
      WRITE(3,20)BSUM,NS,SMEAN1,RESID,NB,SMEAN2,YSUM,N,SMEAN3,F,COEF1,CO
     1EF2
      WRITE(3,3)(B(I),I=1,NP)
      NO=IJ
      CALL PARZNI(REST,N,NQ,LAGS,NOSTOR,NO)
  555 CONTINUE
      STOP
      END
```

```
      SUBROUTINE PARZN1(X,N,NQ,M,NOSTOR,NO)
C
C           THIS PROCEDURE COMPUTES THE AUTO AND CROSS CORRELATION
C           FUNCTIONS,R1(),R2(),CI(),AND CT(), FOR I=1,2,...M+1.The
C           FUNCTION AT LAG M IS STORED AT I=M+1. THE TIME SERIES ARE OF
C           EQUAL LENGTH, N , AND BOTH ARE STORED IN THE ARRAY Y(),
C           ONE BEGINNING AT L1, AND THE OTHER AT L2.  THE AUTO COR-
C           RELATION FUNCTIONS ARE NORMALIZED TO HAVE A VALUE 1 AT THE
C           ORIGIN, AND THE CROSS CORRELATION FUNCTIONS ARE ALSO CON-
C           SISTENTLY NORMALIZED.  THE NORMALIZING FACTORS ARE D1,D2,
C           and D3. THE FUNCTIONS ARE ADDED INTO THE ARRAYS R1(), R2(),
C           CI(),AND CT(), TO ALLOW POOLING OF COVARIANCES.
C
    1 FORMAT(2I4)
    2 FORMAT(4X,F5.0,4X,F5.0)
    3 FORMAT( 4F12.5)
   33 FORMAT( '       R1            R2              CI            CT'')
      DIMENSTION X(1)
      REAL R1(300)/300*0.0/,R2(300)/300*0.0/,CI(300)/300*0.0/,CT(300)/30
     10*0.0/,RPLT(400)/400*0.0/
      DATA D1,D2,D3/0.0,0.0,0.0/,SUM1,SUM2,SUM3,SUM4/0.0,0.0,0.0.0.0/
      MM=0
      DO 5 I=1,N
      D1=D1+X(I)**2
      D2=D2+X(I+N)**2
    5 CONTINUE
      D3=SQRT(D1*D2)
      MM=M+1
      DO 7 KK=1,MM
      KK1=KK-1
      NM=N+KK-1
      NK=N-KK+1
      SUM1=0.0
      SUM2=0.0
      SUM3=0.0
      SUM4=0.0
      DO 6 JL=1,NK
      SUM1=SUM1+X(JL)*X(JL+KK1)
      SUM2=SUM2+X(N+JL)*X(NM+JL)
      SUM3=SUM3+X(JL)*X(NM+JL)
    6 SUM4=SUM4+X(N+JL)*X(KK1+JL)
      R1(KK)=SUM1/D1
      R2(KK)=SUM2/D2
      CI(KK)=SUM3/D3
    7 CT(KK)=SUM4/D3
      WRITE(3,33)
      WRITE(3,3)(R1(KK),R2(KK),CI(KK),CT(KK), KK=1,M)
      DO 8 I=1,M
      RPLT(I)=I
```

```
8 RPLT(I+M)=R1(I)
  DO 9 I=1,M
9 RPST(I+M)=R2(I)
  CALL PARZN2(R1,R2,CI,CT,NQ,M,NOSTOR,NO)
  RETURN
  END
```

```
      SUBROUTINE PARZN2(R1,R2,RE,R,O,NQ,M,NOSTOR,NO)
C
C         THIS PROCEDURE COMPUTES N+1 POINTS OF TWO ESTIMATED SPECTRAL
C         DENSITY FUNCTIONS AND OF THE CO-SPECTRUM AND QUADATURE
C         SPECTRUM FROM R1(I) AND R2(I), WHICH ARE THE AUTO-CORRE-
C         LATION FUNCTION OF TWO TIME SERIES.  THE TRUNCATION POINT IS
C         M, THE WEIGHTING KERNEL USED IS OMEGA(I).  THE SINES AND
C         COSINES NEEDED ARE COMPUTED RECURSIVELY.
C
C
      DIMENSION R1(1),R2(1),RE(1),RO(1)
      REAL OMEGA(200)/200*0.0/,T1(200)/200*0.0/,T2(200)/200*0.0/,TE(200)
     1/200*0.0/,TO(200)/200*0.0/,F1(200)/200*0.0/,F2(200)/200*0.0/,CO(20
     20)/200*0.0/,QU(200)/200*0.0/
      DATA PI/3.1416/,PIV/0.3183/
    1 FORMAT(4F10.5)
    2 FORMAT('  RESULTS OF PARZEN TRANSFORM'//'        F1          F2
     1 CO        QU'/ (4F10.5))
        THIS DO LOOP IS USED TO DEVELOP WEIGHTING VALUES, USING THE TUKEY
        WINDOW.  SEE GRANGER, PAGE 60.
      DO 3 K=2,M
    3 OMEGA(K)=1.5*(L.0+COS(PI*K)/M)
      OMEGA(1)=0.0
      C=NQ
      C1=COS(PI/C)
      C3=C1
      D1-C1
      C2=SIN(PI/C)
      D4=C2
      D6=2.0*C1
      P1=0.5*R1(1)
      P2=0.5*RE(1)
      P4-0.5*R2(1)
      K=M+L
      DO 4 I=2,K
      A=OMEGA(L)
      P1=P1+R1(I)*A
      P2=P2+RE(I)*A
    4 P4=P4+R2(I)*A
      F1(1)=P1*PIV
      F2(1)=P4*PIV
      CO(1)=P2*PIV
      QU(1)=0.0
      DO 6 I=1,NQ
      U11=0.0
      U12=U11
      U13=U11
      U14=U11
      U21=U11
      U22=U11
```

```
      U23=U11
      U24=U11
      DO 5 J=2,K
      JJ=M+3-J
      A=OMEGA(JJ)
      U31=D6*U21-U11+R1(JJ)*A
      U32=D6*U22-U12+RE(JJ)*A
      U33=D6*U23-U13+RO(JJ)*A
      U34=D6*U24-U14+R2(JJ)*A
      U11-U21
      U21=U31
      U12=U22
      U22=U32
      U13=U23
      U23=U33
      U14=U24
      U24=U34
    5 CONTINUE
      F1(I+1)=(D1*U21-U11+R1(1)*0.5)*PIV
      F2(I+1)=(D1*U22-U14+R2(1)*0.5)*PIV
      CO(I+1)=(D1*U22-U12+RE(1)*0.5)*PIV
      QU(I+1)=D4*U23*PIV
      D1=(C1*C3)-(C2*D4)
      D4=(D4*C1)+(C3*C2)
      C3=D1
      D6=2.0*D1
    6 CONTINUE
      WRITE(3,2)(F1(I),F2(I),CO(I),QU(I), I=1,NQ)
      CALL SPLOT(NQ,F1,NO,NOSTOR)
      RETURN
      END
```

```
      SUBROUTINE PLOT(NO,A,N,M,NL,NS,NOSTOR)
C
      DIMENSION OUT(101),YPR(11),ANG(9),A(1)
C
    1 FORMAT(1H1,60X,7H CHART ,I3,//)
    2 FORMAT(1H ,F11.4,5H+    ,101A1)
    3 FORMAT(1H )
    4 FORMAT(10H *+0156789)
    5 FORMAT( 10A1)
    7 FORMAT(1H ,16X,101H.         .         .         .         .
     1   .         .         .         .         .         .)
    8 FORMAT(1H0,9X,11F10.4//)
    9 FORMAT(1H ,16X,101A1)
  200 FORMAT('  PLOT OF SPECTRUM OF TOTAL SALES'//)
  201 FORMAT('  PLOT OF SPECTRUM OF CUSOTMER COUNT'//)
  202 FORMAT('  PLOT OF SPECTRUM OF MEAT SALES')
  203 FORMAT('     PLOT OF AUTO-CORRELATION FUNCTION, R1')
  204 FORMAT( '     PLOT OF AUTO CORRELATION FUNCTION, R2')
  205 FORMAT('  SPECTRUM OF TOTAL SALES')
  206 FORMAT('  SPECTURM OF CUSTOMER COUNT')
  207 FORMAT('  SPECTRUM OF MEAT SALES')
  208 FORMAT('  STORE NUMBER ', I3//)
  209 FORMAT('  PLOT OF RESIDUALS VS. ACTUAL AND ESTIMATED'//)
C
C     ...............................................................
C
      NLL=NL
C
      IF(NS)16,16,10
C
C        SORT BASE VARIABLE IN ASCENDING ORDER
C
   10 DO 15 I=1,N
      DO 14 J=1,N
      IF(A(I)-A(J))14,14,11
   11 L=I-N
      LL=J-N
      DO 12 K=1,M
      L=L+N
      LL=LL+N
      F=A(L)
      A(L)=A(LL)
   12 A(LL)=F
   14 CONTINUE
   15 CONTINUE
C
C        TEST NLL
C
   16 IF(NLL)20,18,20
   18 NLL=50
```

```
C
C          PRINT TITLE
C
   20 WRITE(3,1)NO
      GO TO (91,92,93,94.95,96,97,98,99),NO
   91 WRITE(3,200)
      GO TO 21
   92 WRITE(3,201)
      GO TO 21
   93 WRITE(3,202)
      GO TO 21
   94 WRITE(3,203)
      GO TO 21
   95 WRITE(3,204)
      GO TO 21
   96 WRITE(3,205)
      GO TO 21
   97 WRITE(3,206)
      GO TO 21
   98 WRITE(3.207)
      GO TO 21
   99 WRITE(3.209)
   21 CONTINUE
      WRITE(3,208)NOSTOR
C
C          FIND BASE VARIABLE PRINT POSITION
C
      XB=A(1)
      L=1
      I=1
   45 F=I-1
      XPR=XB+F*XSCAL
      IF(A(L)-XPR)51,51,70
C
C          FIND CROSS VARIABLES
C
   51 DO 55 IX=1,101
   55 OUT(IX)=BLANK
   57 DO 60 J=1,MY
      LL=L+J*N
      JP=((A(LL)-YMIN)/YSCAL)+1.0
      OUT(JP)=ANG(J)
   60 CONTINUE
C
C          PRINT LINE AND CLEAR, OR SKIP
C
      WRITE(3,2)XPR,(OUT(IZ),IZ=1,101)
      L=L+1
      GO TO 80
   70 WRITE(3,3)
   80 I=I+1
```

```
      IF(I-NLL)45,84,86
   84 XPR=A(N)
      GO TO 51
C
C         PRINT CROSS VARIABLES NUMBERS
C
   86 WRITE(3,7)
      YPR(1)+YMIN
      DO 90 KN=1,9
   90 YPR(KN+1)=YPR(KN)+YSCAL*10.0
      YPR(11)=YMAX
      WRITE(3,8)(YPR(IR),IR=1,11)
      RETURN
      END
```

```
C            THIS PROGRAM IS USED TO CALCULATE THE FOURIER COEFFICIENTS.
C
      REAL X(1500)/1500*0.0/,A(100)/100*0.0/,B(100)/100*0.0/,R(100)/100*
     10.0/
    1 FORMAT(2I4)
    2 FORMAT(3F20.7,2I10)
   32 FORMAT(10X,F7.2,F8.0,F8.2)
      PI=3.1415927
      READ(1,1)ND,LIM
      READ(1,32)(X(I),X(I+ND),X(I+2*ND), I=1,ND)
      IND=ND
      DO 55 KJ=1,3
      GO TO(79,75,77),KJ
   75 CONTINUE
      DO 76 JJ=1,ND
   76 X(JJ)=X(JJ+ND)
      GO TO 79
   77 CONTINUE
      DO 78 JK=1,ND
   78 X(JK)=X(JK+2*ND)
   79 CONTINUE
C          CALCULATE THE FOURIER COEFFICIENTS
      DO 50 N=1,LIM
      A(N)=0.0
      B(N)=0.0
      TOU=N
      NR=N
      MIS=N
   66 NR=NR+MIS
      IF(ND-NR)22,44,66
   22 NR=NR-MIS
   44 CONTINUE
      DO 40 I=1,NR
      F=I-1
      B(N)=B(N)+SIN(2.*PI*F/TOU)*X(I)
      A(N)=A(N)+COS(2.*PI*F/TOU)*X(I)
   40 CONTINUE
      ANR=NR
      B(N)=(2./ANR)*B(N)
      A(N)=(2./ANR)*A(N)
      R(N)=SQRT(A(N)**2+B(N)**2)
      WRITE(3,2)A(N),B(N),R(N),N,NR
   50 CONTINUE
      CALL RPLOT(R,LIM)
   55CONTINUE
      STOP
      END
```

APPENDIX II

FOURIER ANALYSIS

Fourier Analysis was used to approximate and isolate the possible periodic and non-periodic functions in the store data. The periodic functions used in Fourier analysis are sine and cosine functions. They have the important properties that an approximation consisting of a given number of terms achieves the minimum square error between the singal and the approximation. They are also orthogonal, so the coefficients may be determined independently of one another.

The finite Fourier series may be represented as

$$\tilde{s}(t) = A_o + 2 \sum_{m=1}^{n-1} \left\{ A_m \cos 2\ mf_1 t + B_m \sin 2\ mf_1 t \right\} + A_m \cos 2\ mf_1 t$$

It (the series, $\tilde{s}(t)$) contains 2N constants, the A_m and B_m, which can be determined so that the discrete and continuous values coincide at points $t = r\Delta$, that is $\tilde{s}(t) = s_r$. Therefore, we can say that the function $\tilde{s}(t)$ provides an approximation to the original continuous function s(t) in the interval $-T/2 \leq t < T/2$, where T is the period of the function.

For the purpose of analysis we obtained values for the coefficients and the amplitude. Phase representation was not considered in the scope of this study, although it can be computed quite easily. The coefficients are calculated as

$$A_m = \frac{1}{N} \sum_{r=-M}^{M-1} s_r \cos \frac{2\pi mr}{N}$$

$$B_m = \frac{1}{N} \sum_{r=-M}^{M=1} s_r \sin \frac{2\pi mr}{N} \quad ,$$

for $m = 0, 1, \ldots n$. A_0 is the mean, or average value, of s_r. The amplitude is calculated as

$$R_m = \sqrt{A_m^2 + B_m^2}$$

and the phase is calculated as

$$\phi_m = \arctan - \frac{B_m}{A_m}$$

where

$$A_m = R_m \cos\phi_m , \quad B_m = -R_m \sin\phi_m \ .$$

APPENDIX III

PLOT OF FORECASTS FOR

TOTAL SALES

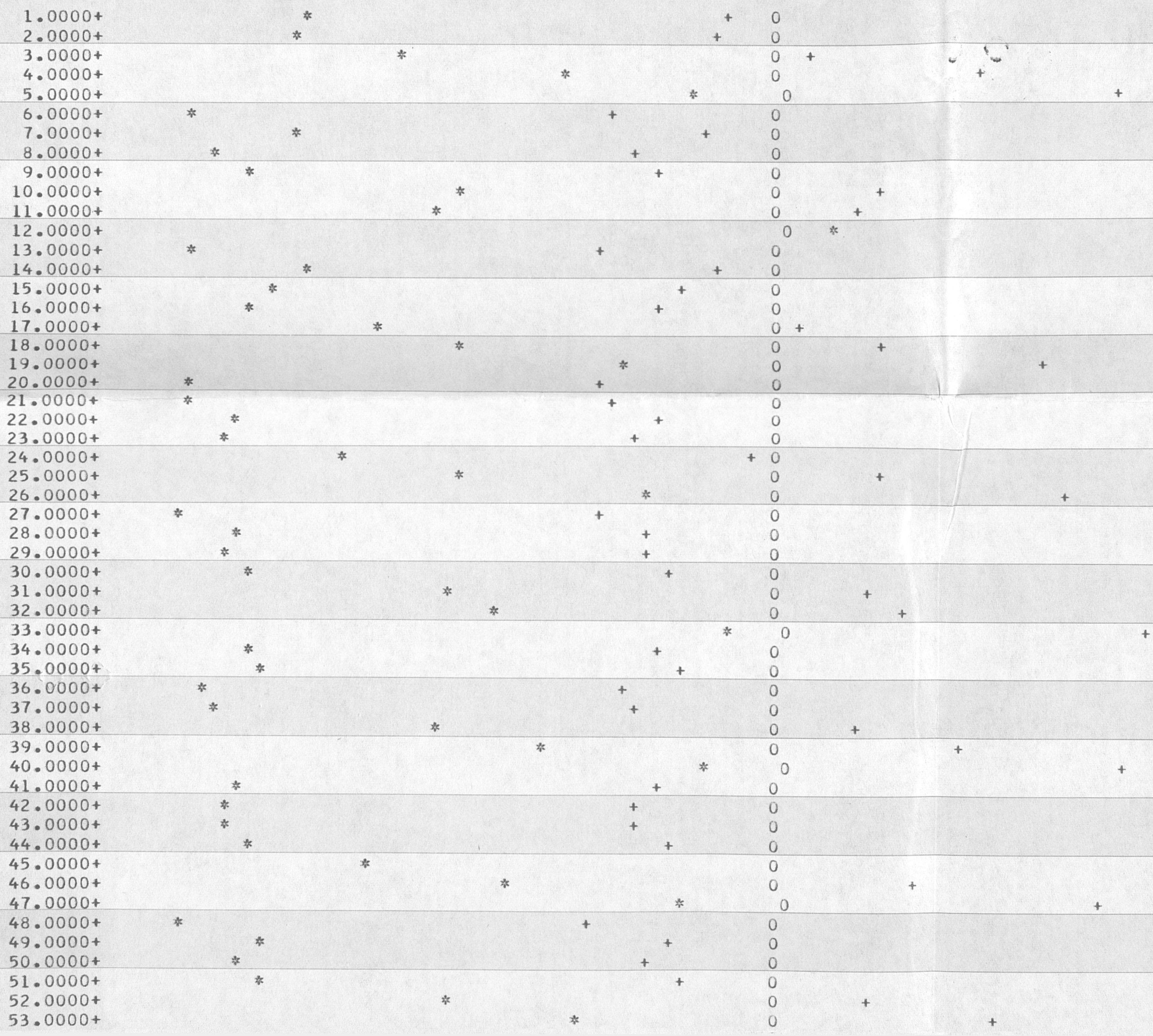
KEY: 0 FORECAST DATA
+ ACTUAL DATA
* RESIDUALS

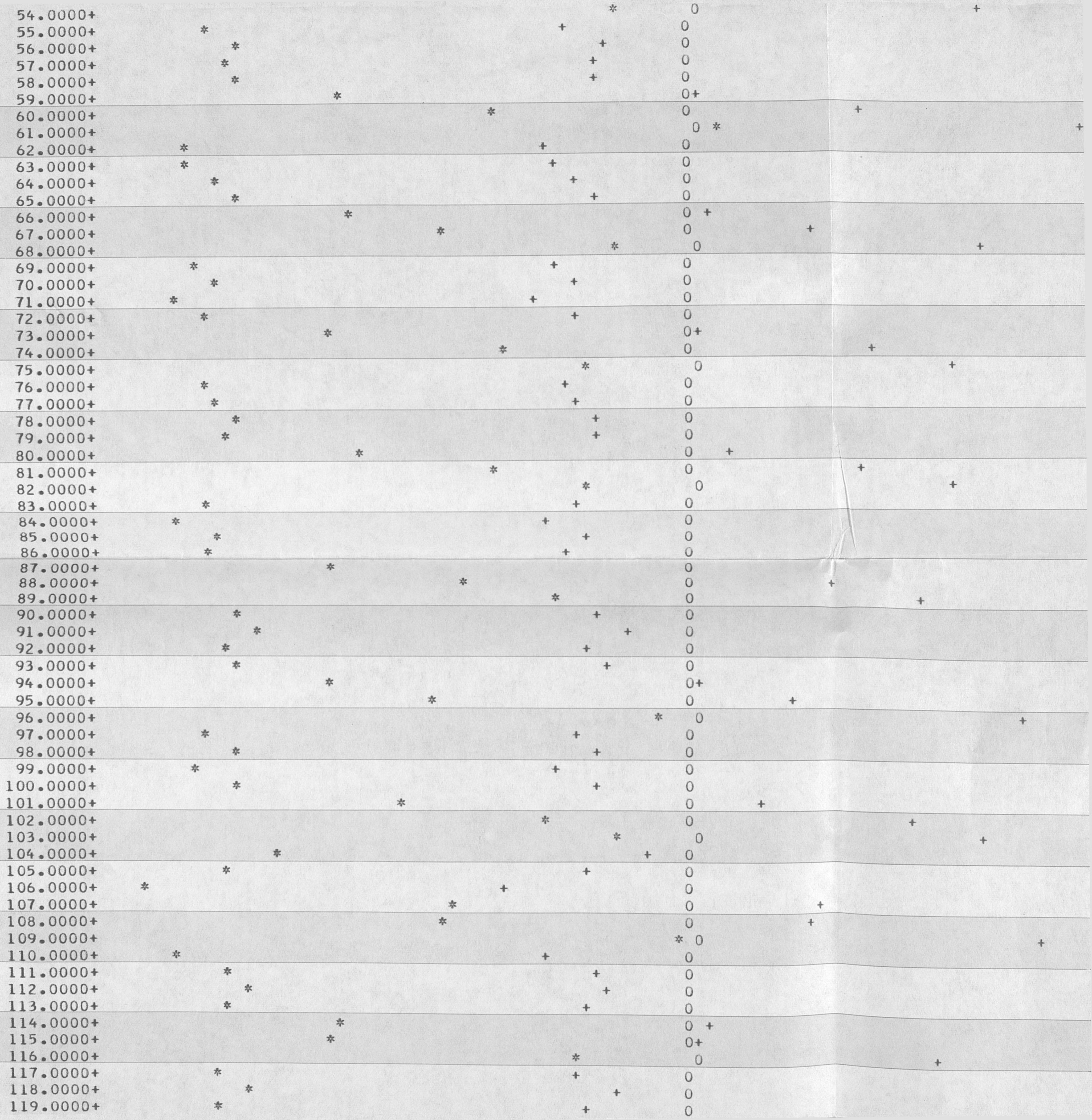
Page 85-87
Chart #1
Forecast, using Linear Term Only

CHART 1

FORECAST, USING TOTAL SALES DATA

RESIDUALS FROM FORECAST, USING LINEAR TERM ONLY
STORE NUMBER 7

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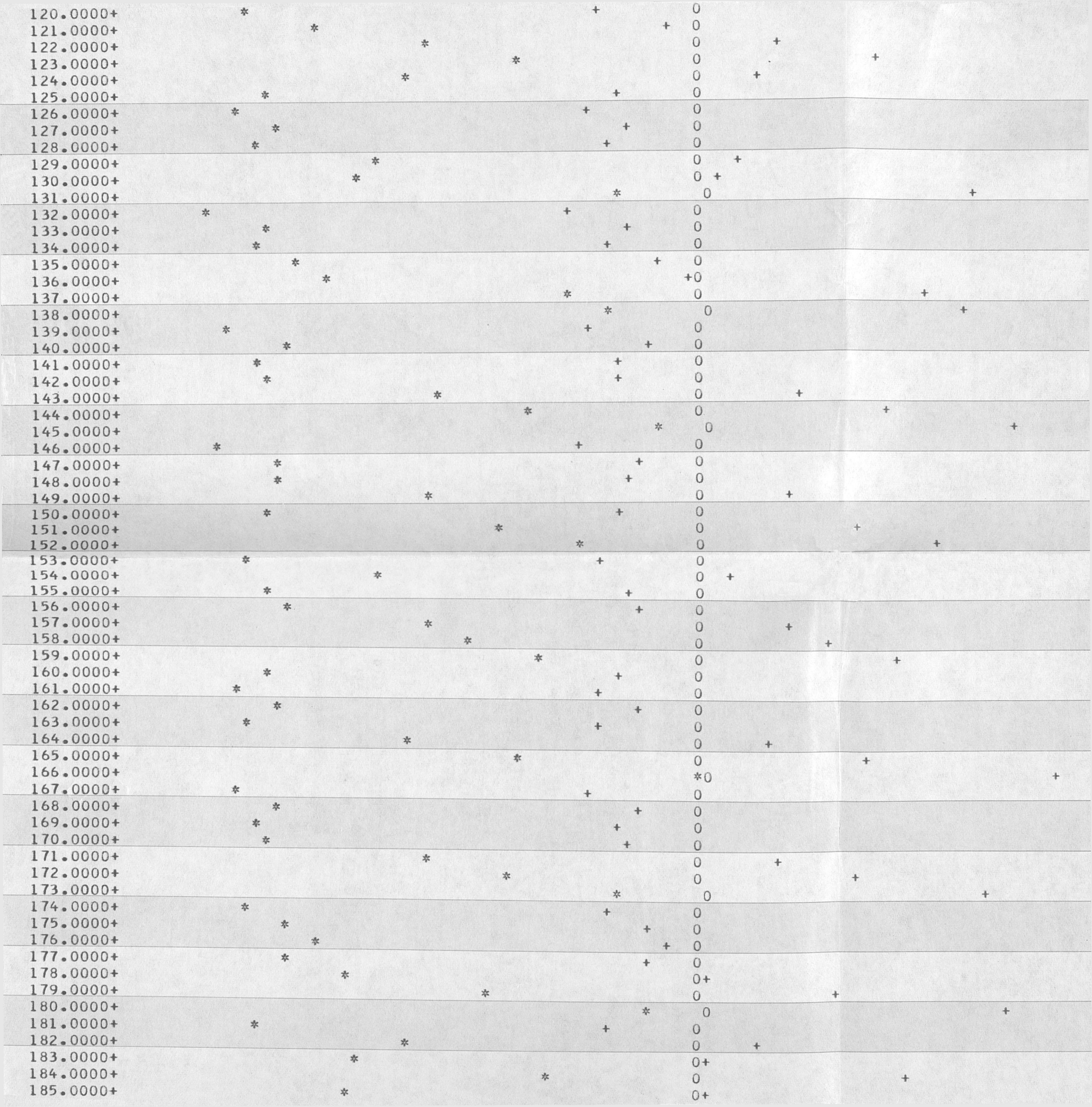
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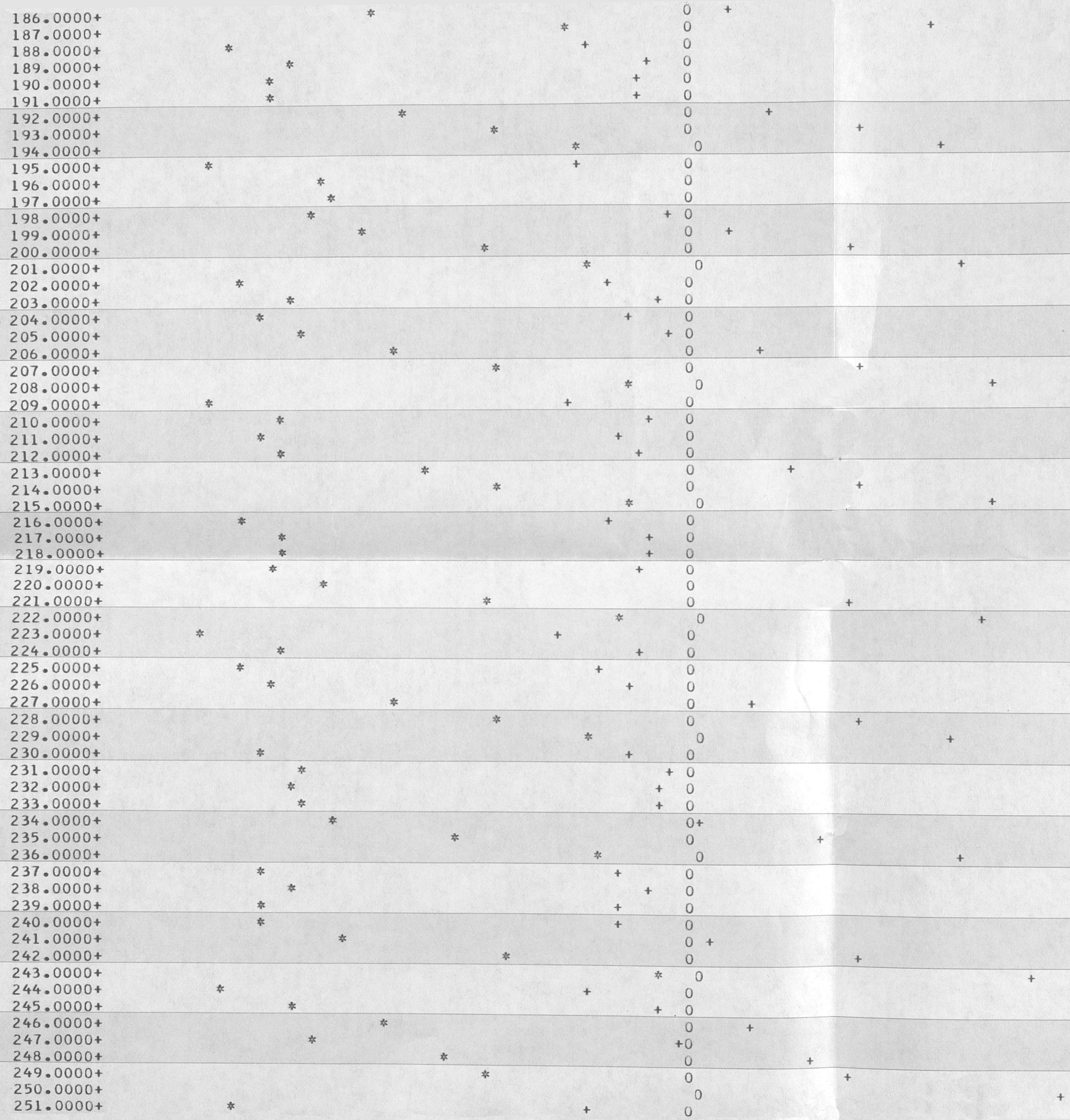
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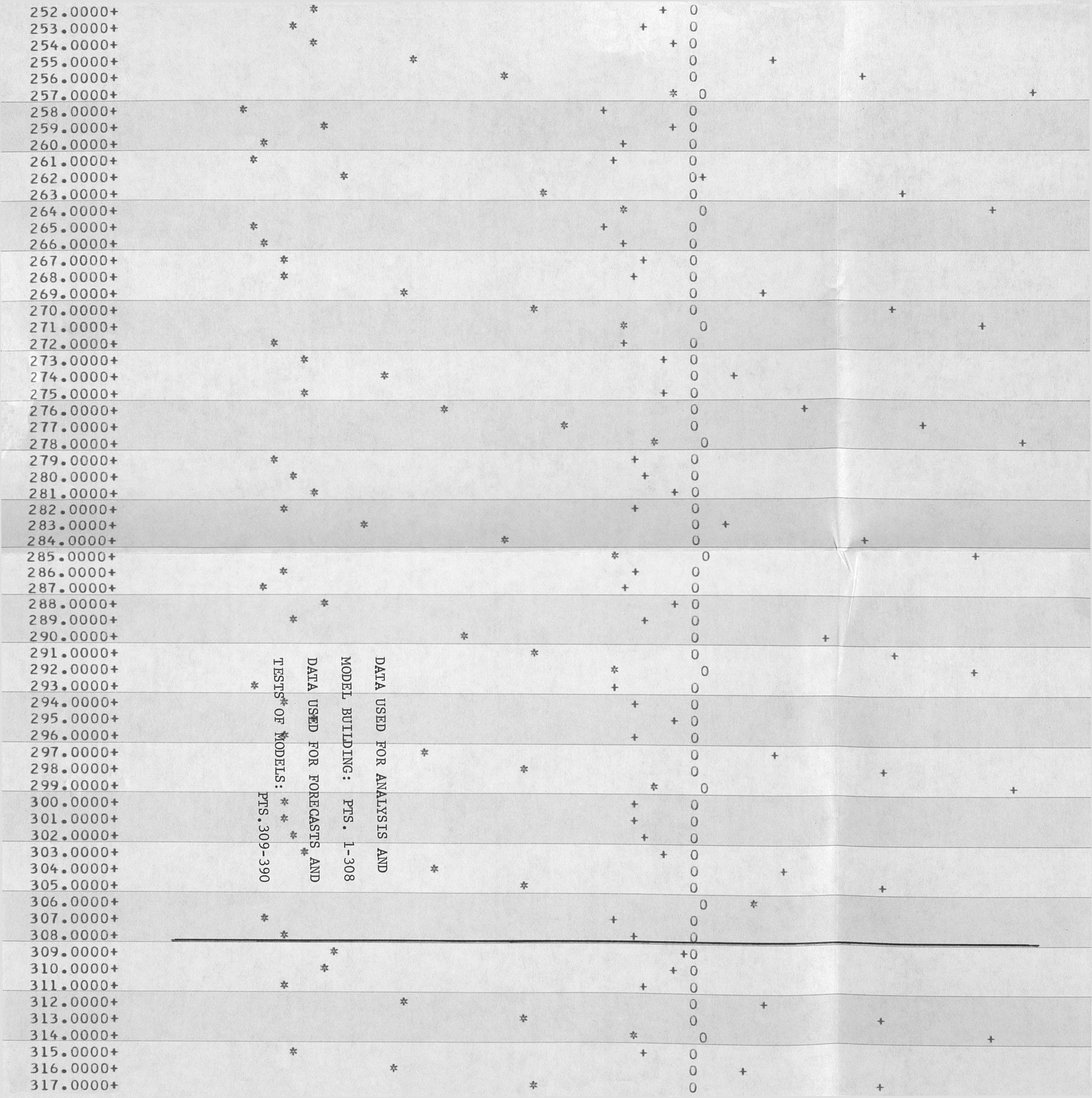
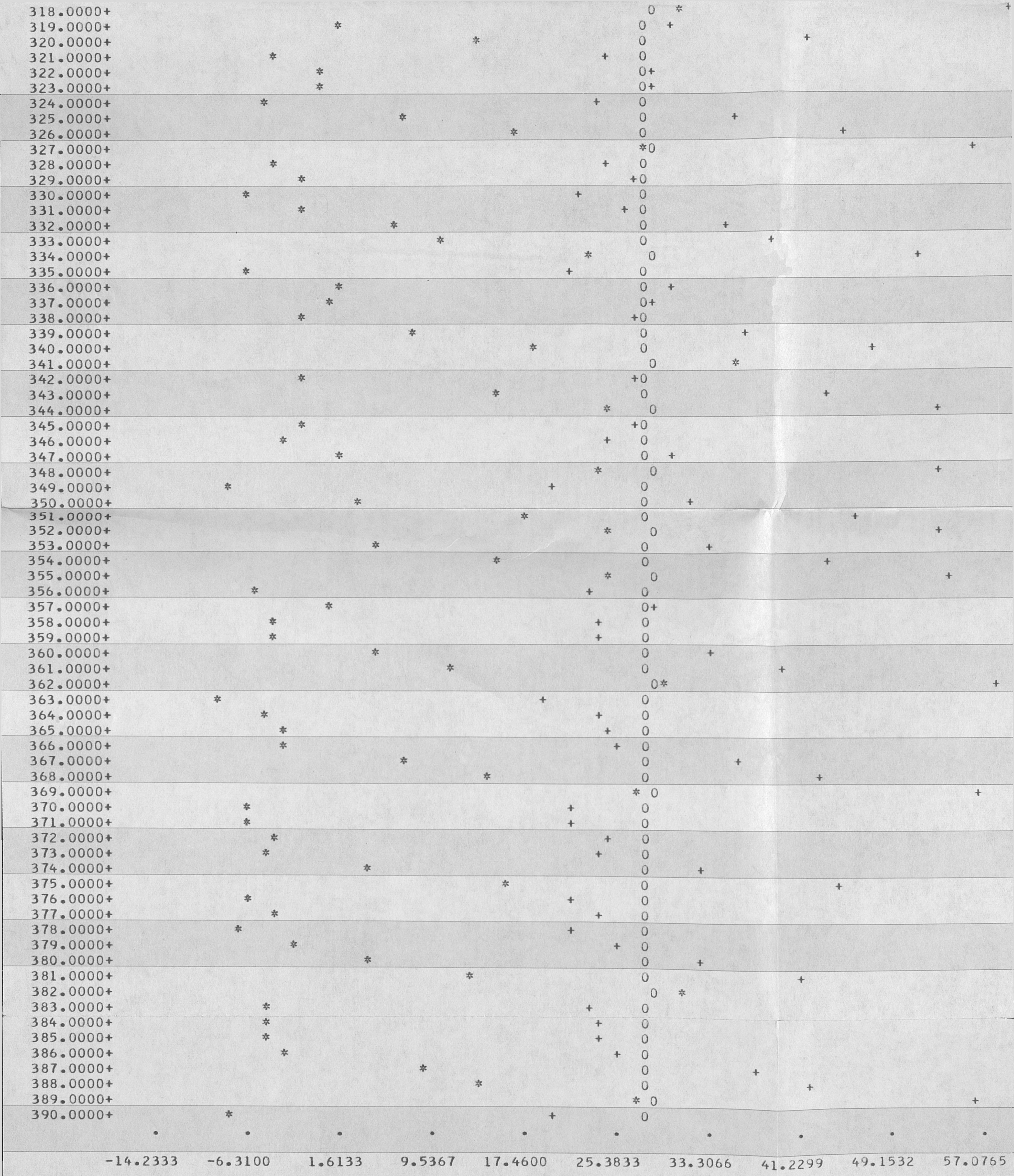

DATA USED FOR ANALYSIS AND
MODEL BUILDING: PTS. 1-308
DATA USED FOR FORECASTS AND
TESTS OF MODELS: PTS.309-390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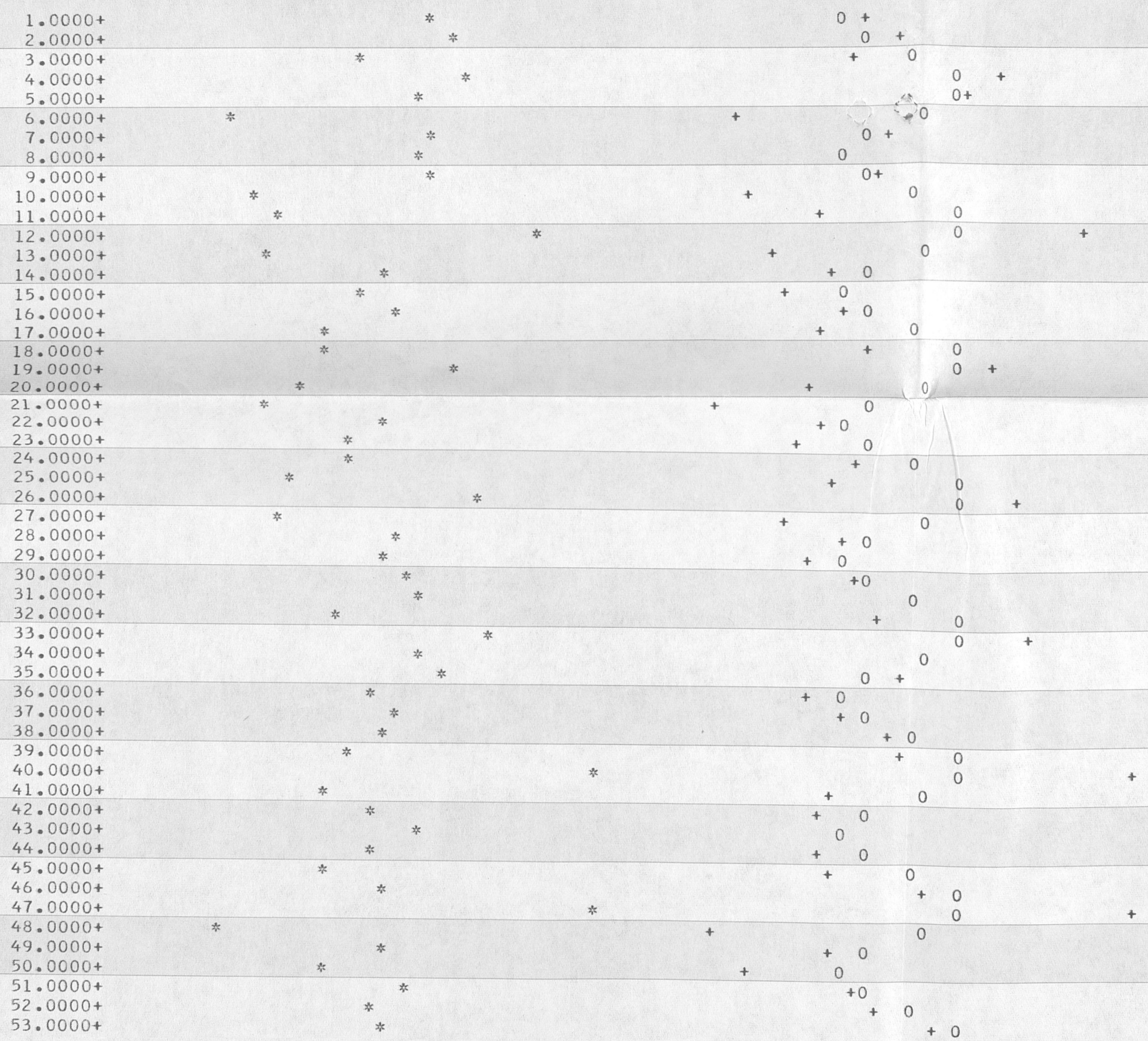
381.0000+
382.0000+
383.0000+
384.0000+
385.0000+
386.0000+
387.0000+
388.0000+
389.0000+
390.0000+
-14.2333
-6.3100
1.6133
9.5367
17.4600
25.3833
33.3066
41.2299
49.1532
57.0765

Page 88-90
Chart #2
Forecast, using Linear
Term, Plus 1 Frequency.

CHART 2

FORECAST, USING CUSTOMER COUNT DATA

RESIDUALS FROM FORECAST, USING LINEAR TERM, PLUS 1 FREQUENCY
STORE NUMBER 7



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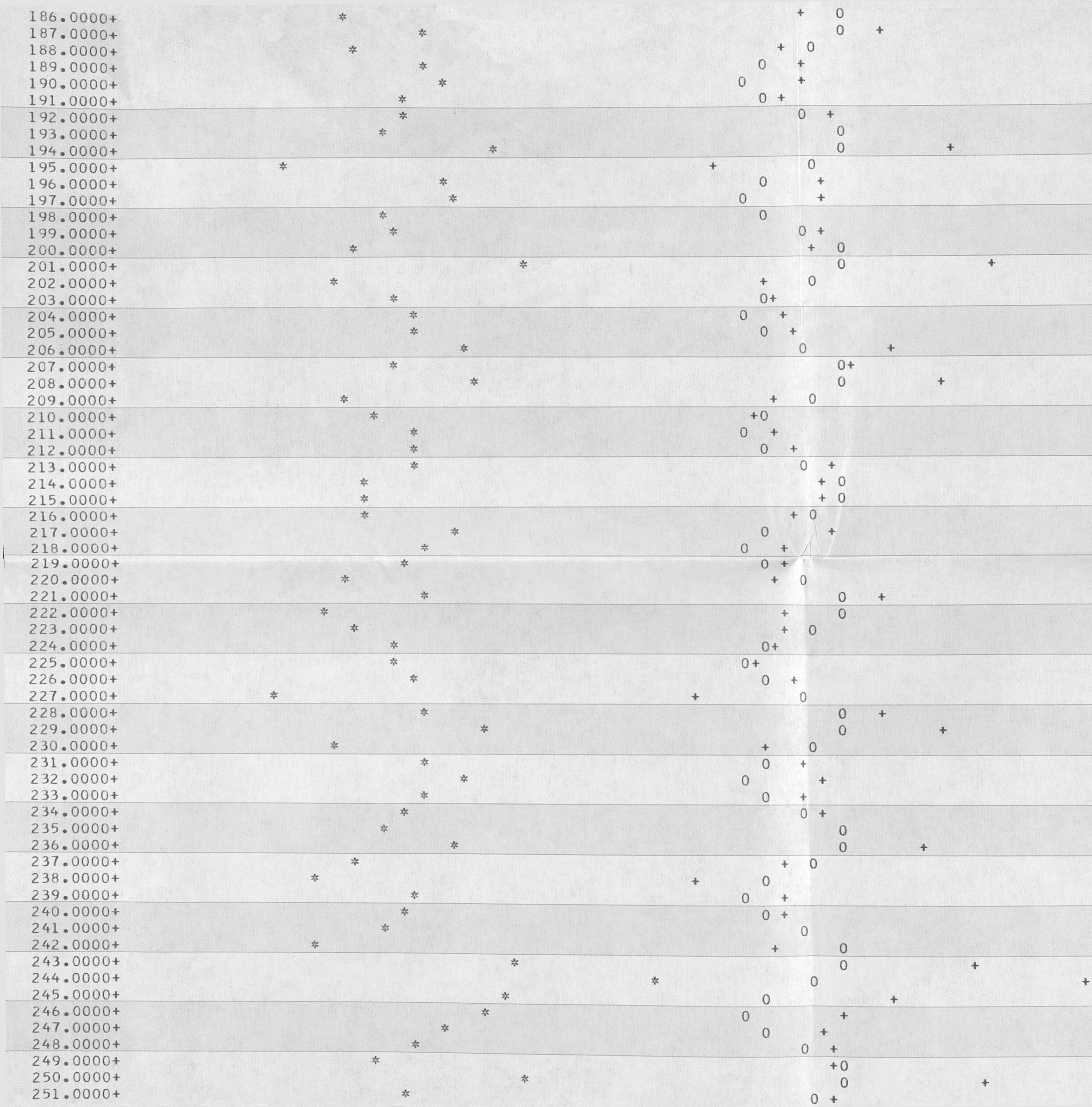
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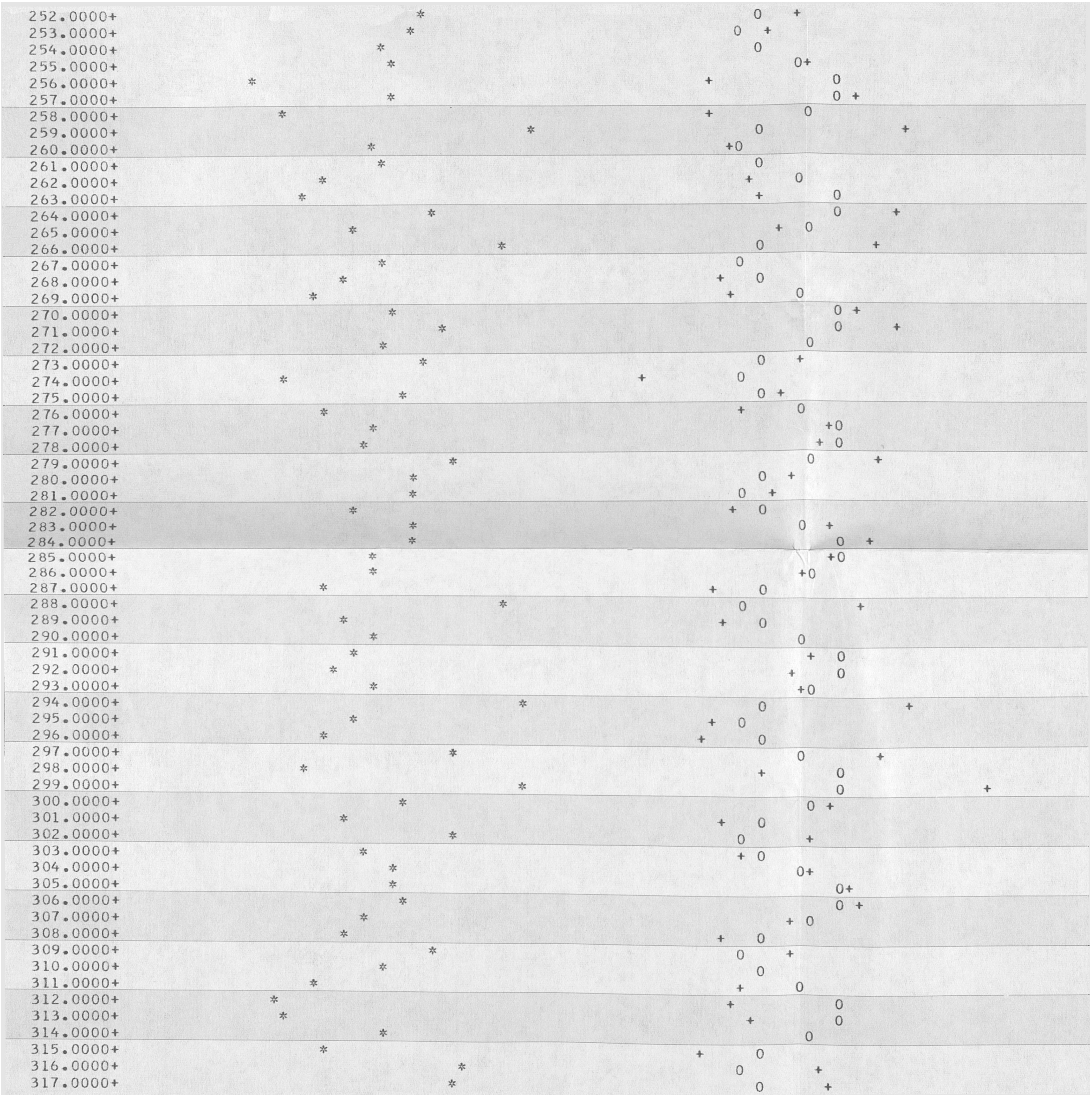
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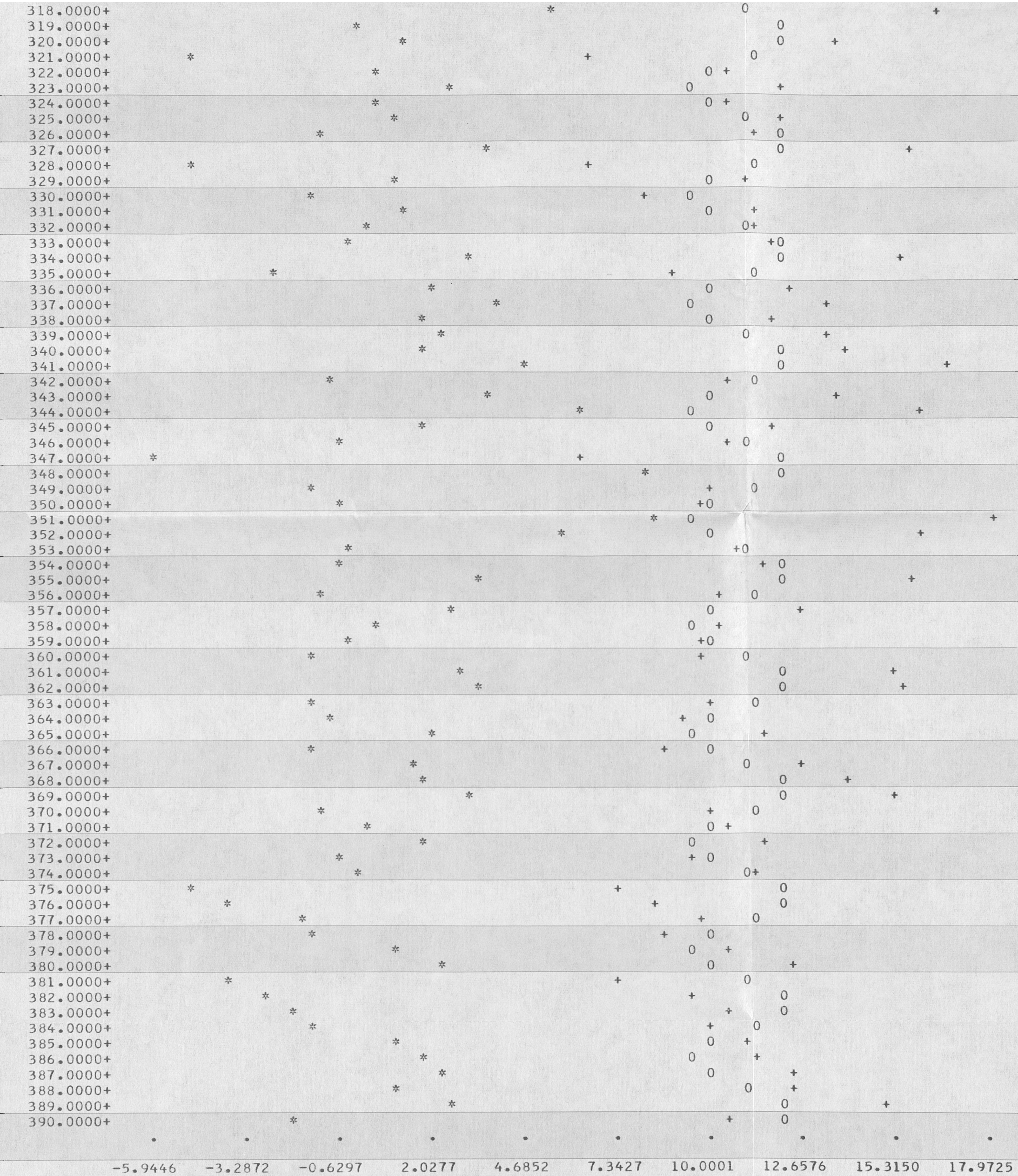
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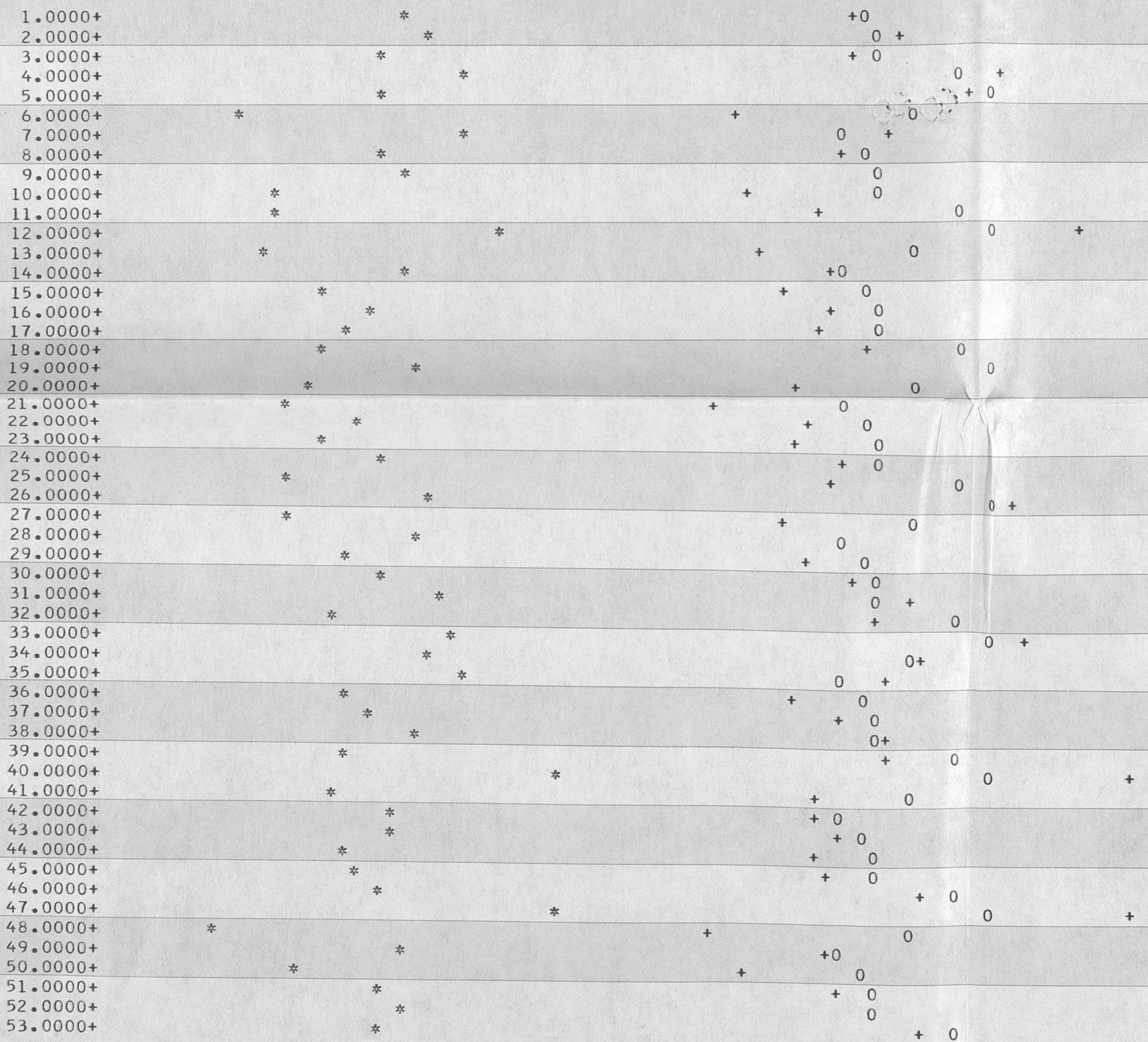
7.3427
10.0001
12.6576
15.3150
17.9725

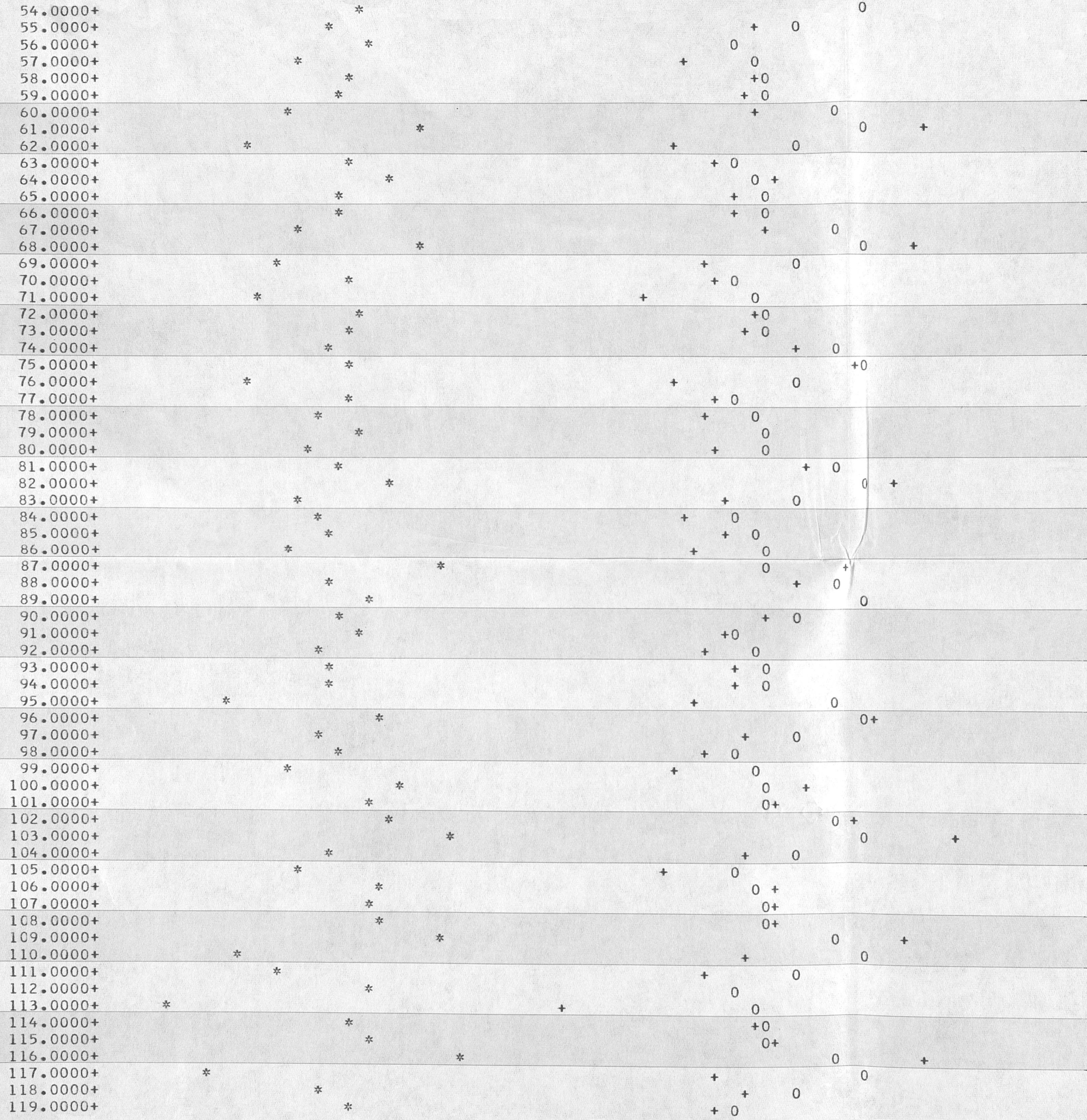
Page 91-93
Chart #3
Forecast, using Linear
Term Plus 2 Frequencies

CHART 3

FORECAST, USING CUSTOMER COUNT DATA

RESIDUALS FROM FORECAST, USING LINEAR TERM, PLUS 2 FREQUENCIES
STORE NUMBER 7

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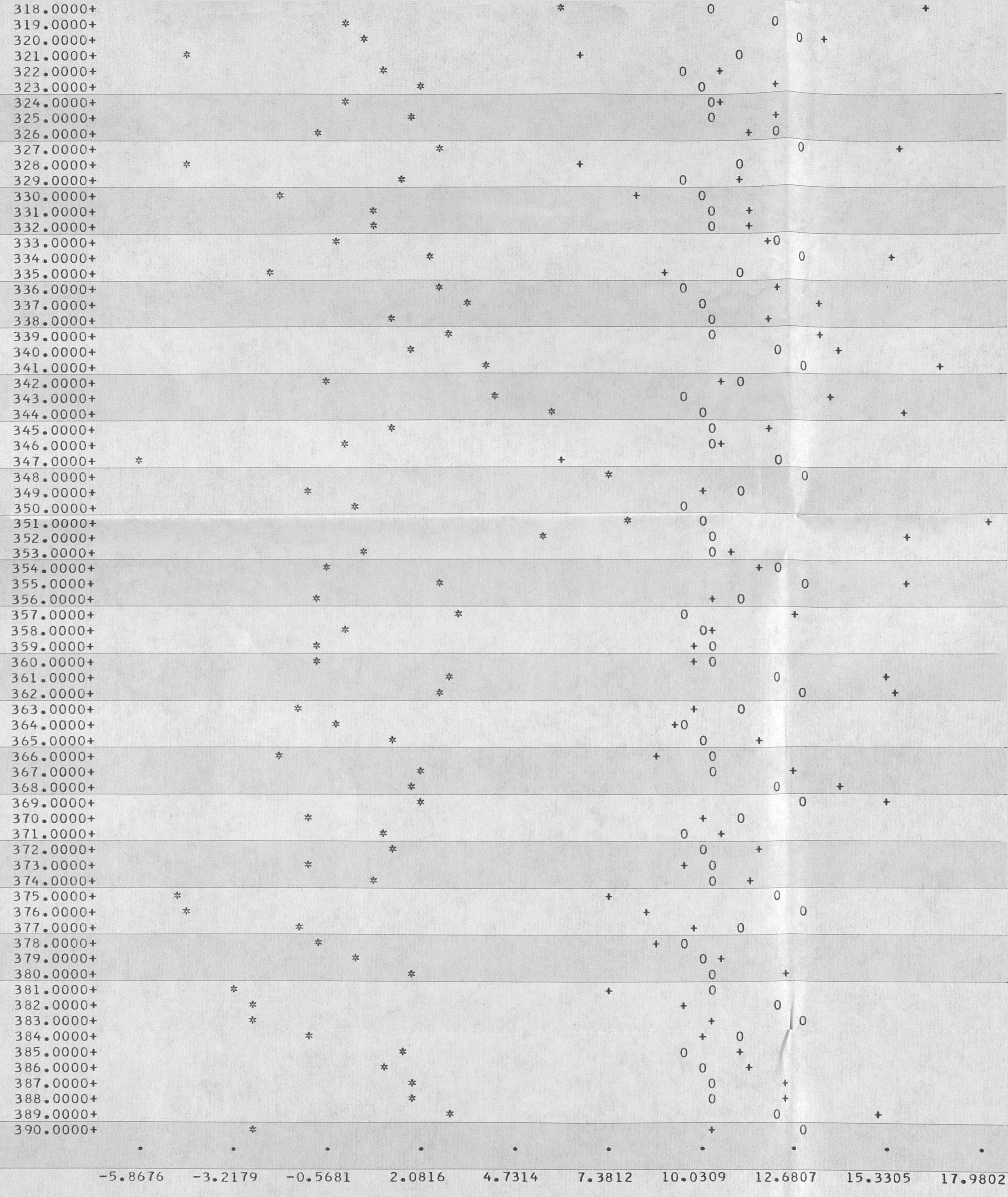
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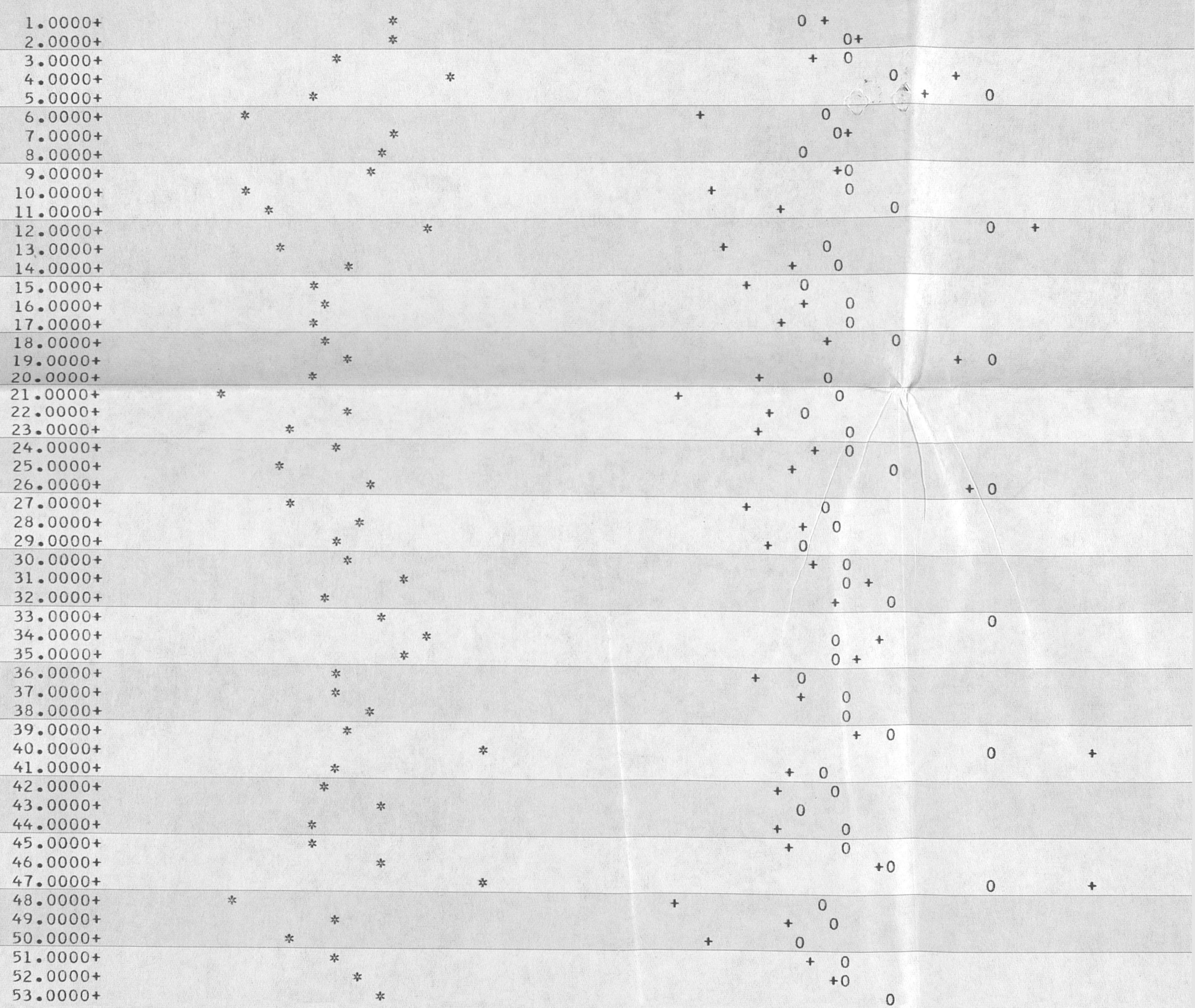
17.9802

Page 94-96
Chart #4
Forecast, using Linear
Term Plus 3 Frequencies.

FORECAST, USING CUSTOMER COUNT DATA

RESIDUALS FROM FORECAST, USING LINEAR TERM, PLUS 3 FREQUENCIES
STORE NUMBER 7



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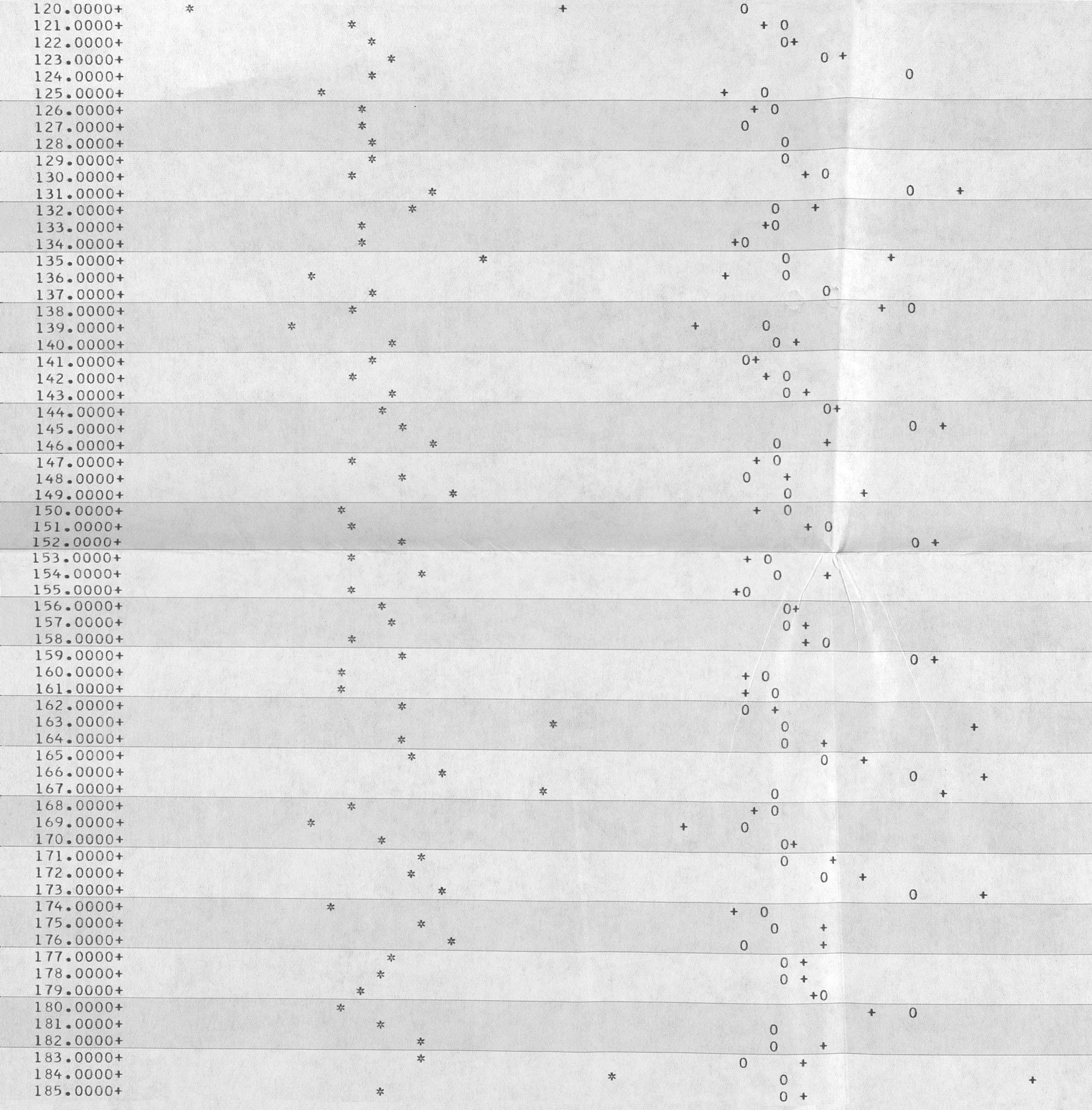
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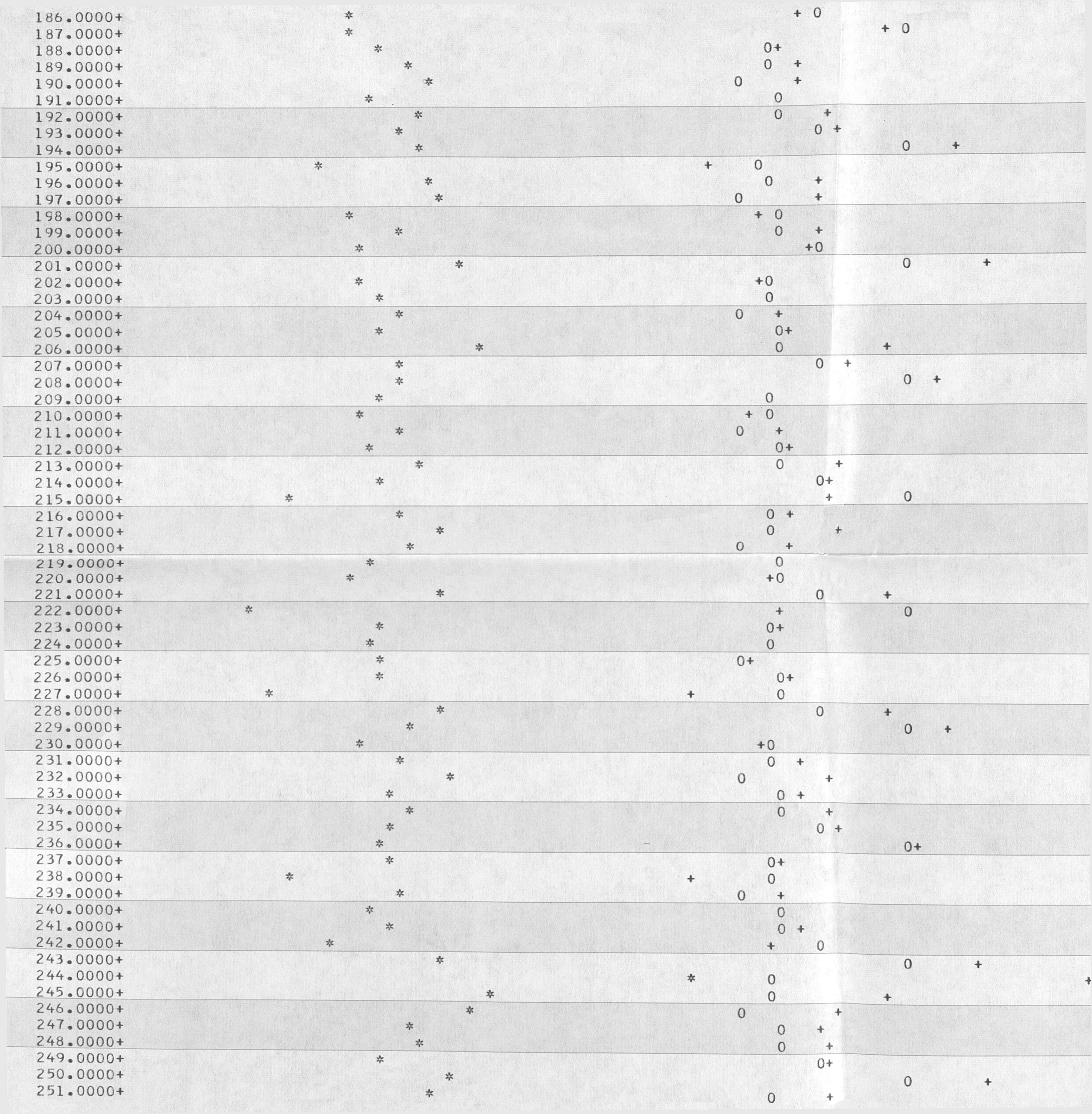
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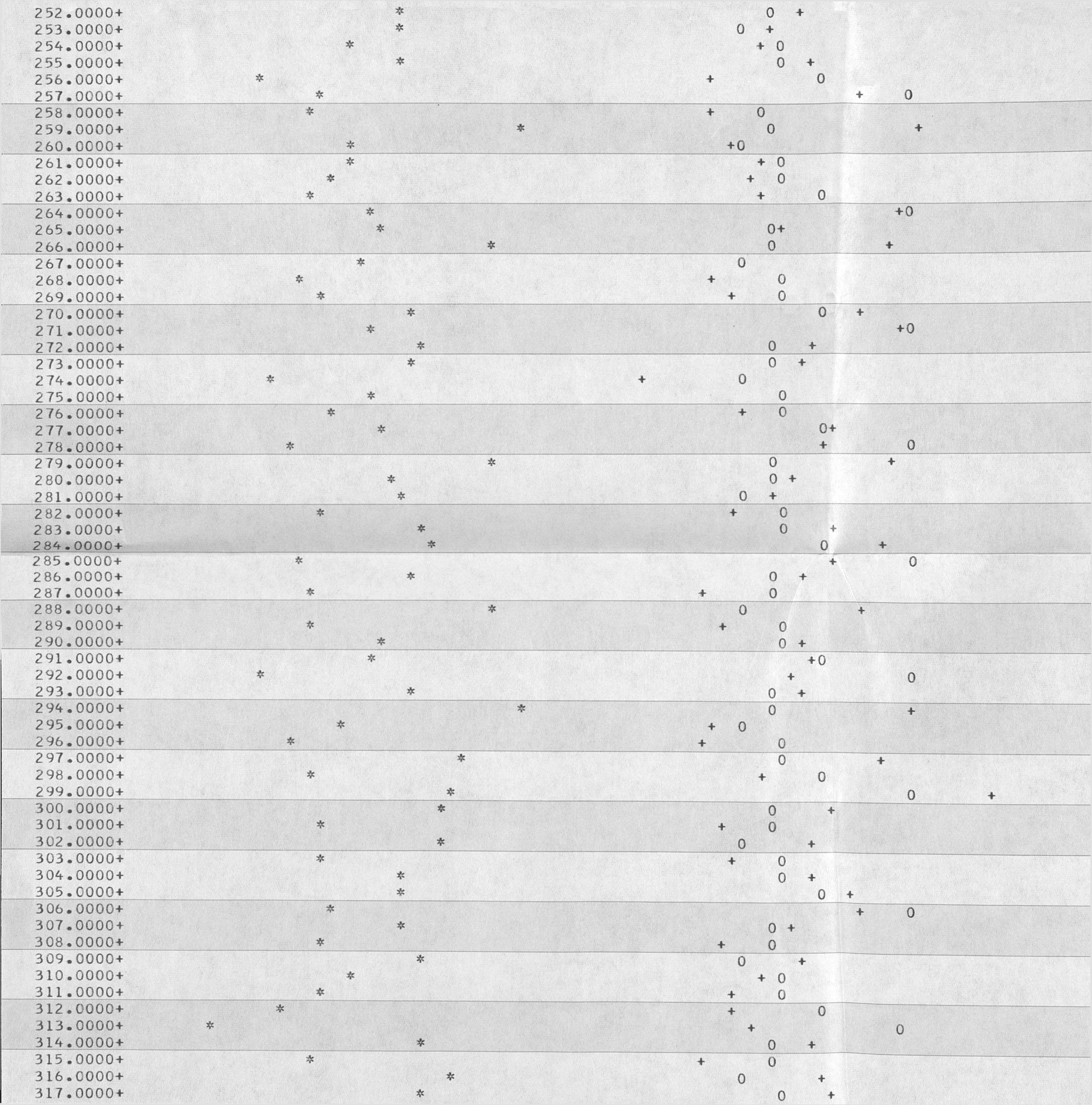
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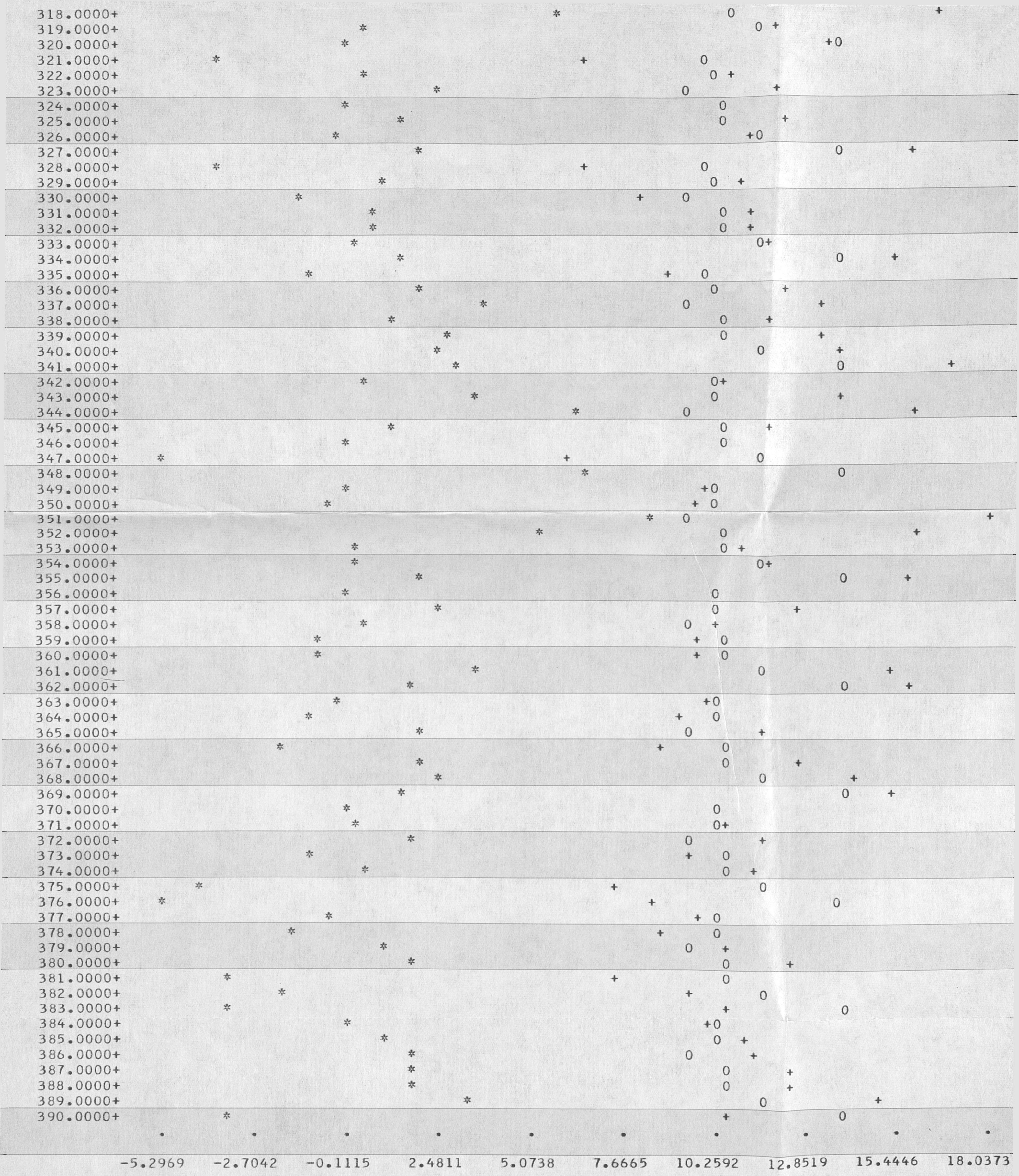
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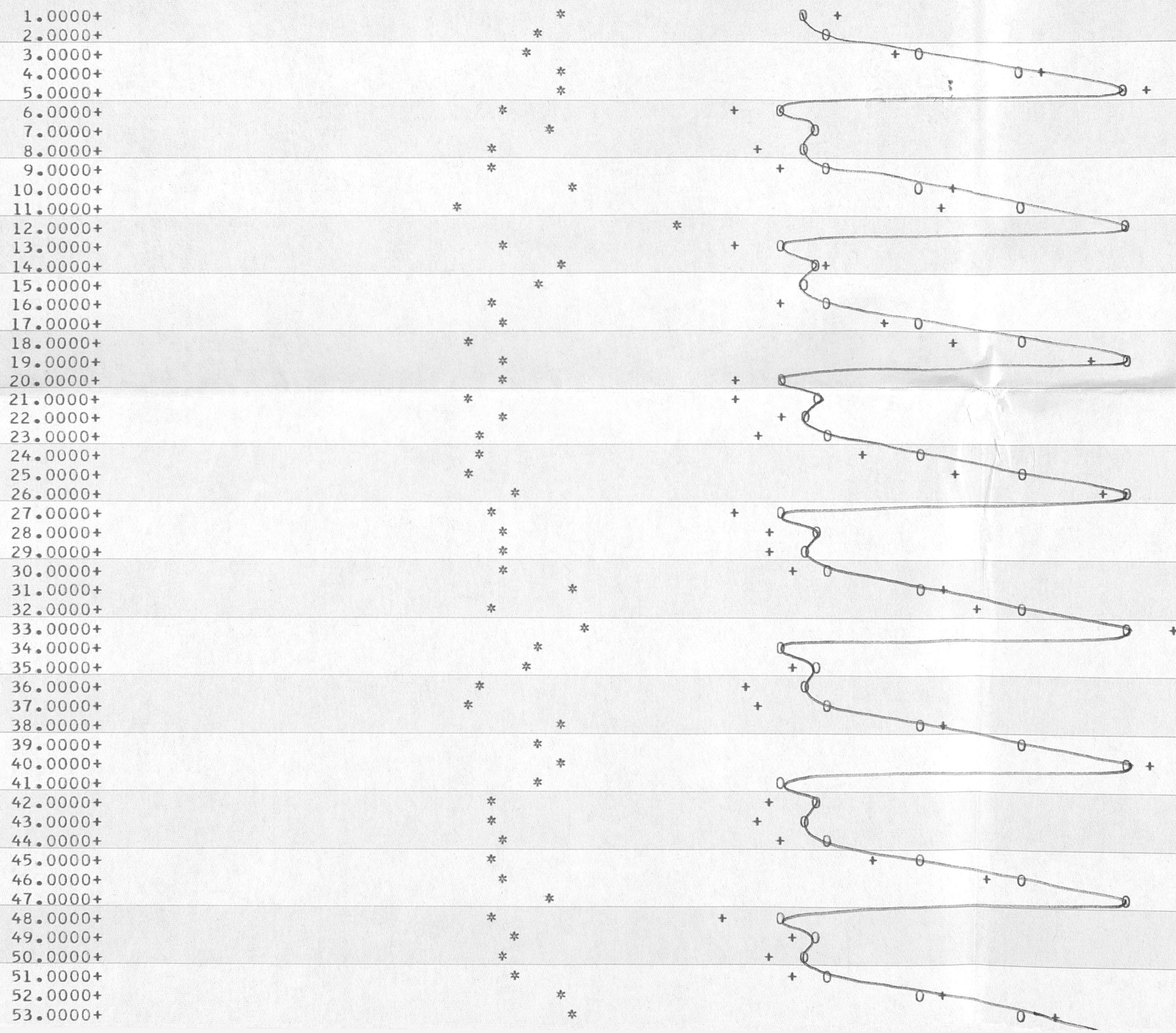
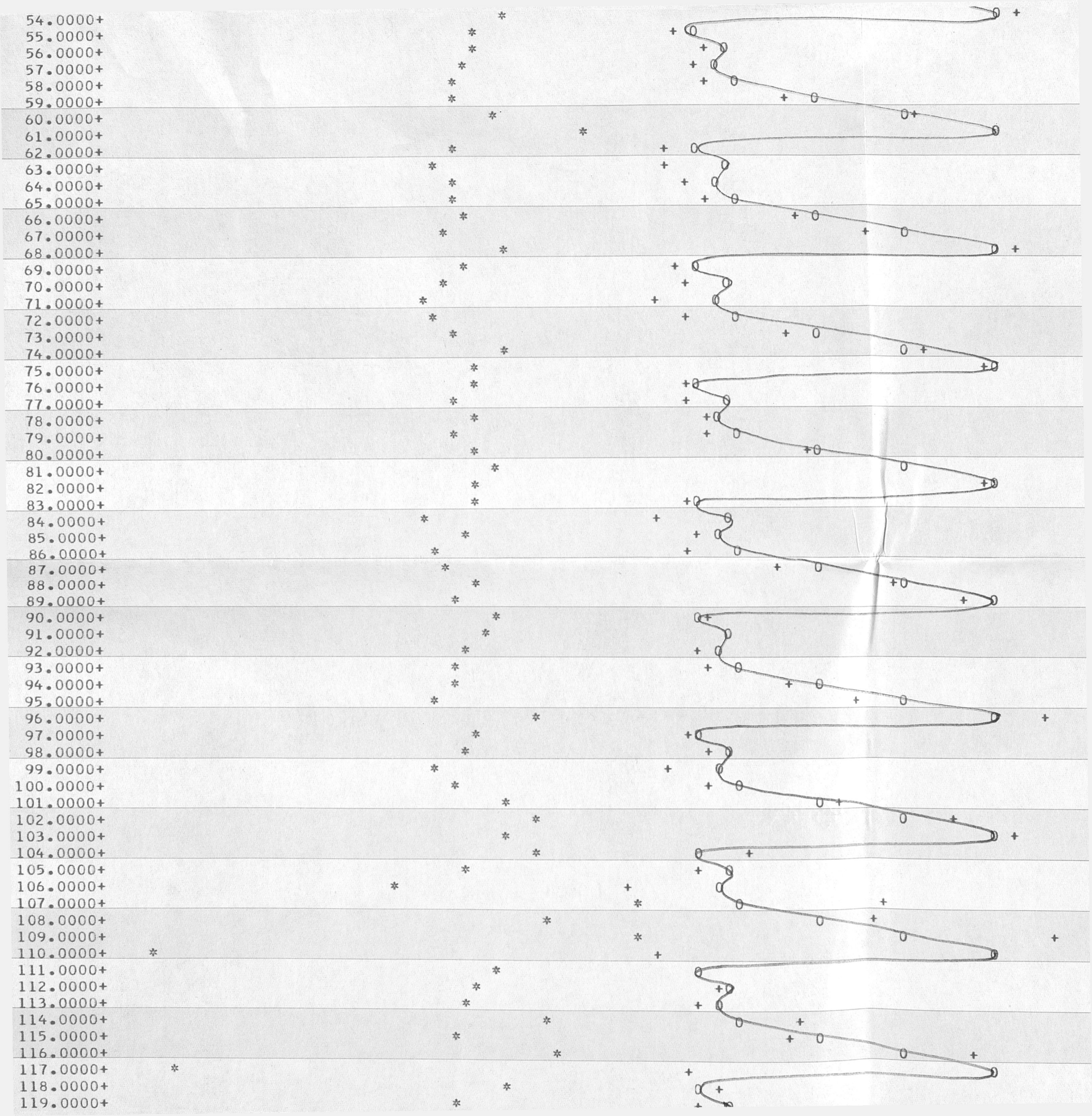
Pages 97-99
Chart #5
Forecast, using Dummy Variables.

CHART 5

FORECAST, USING TOTAL SALES DATA

RESIDUALS FROM FORECAST, USING DUMMY VARIABLES
STORE NUMBER 7

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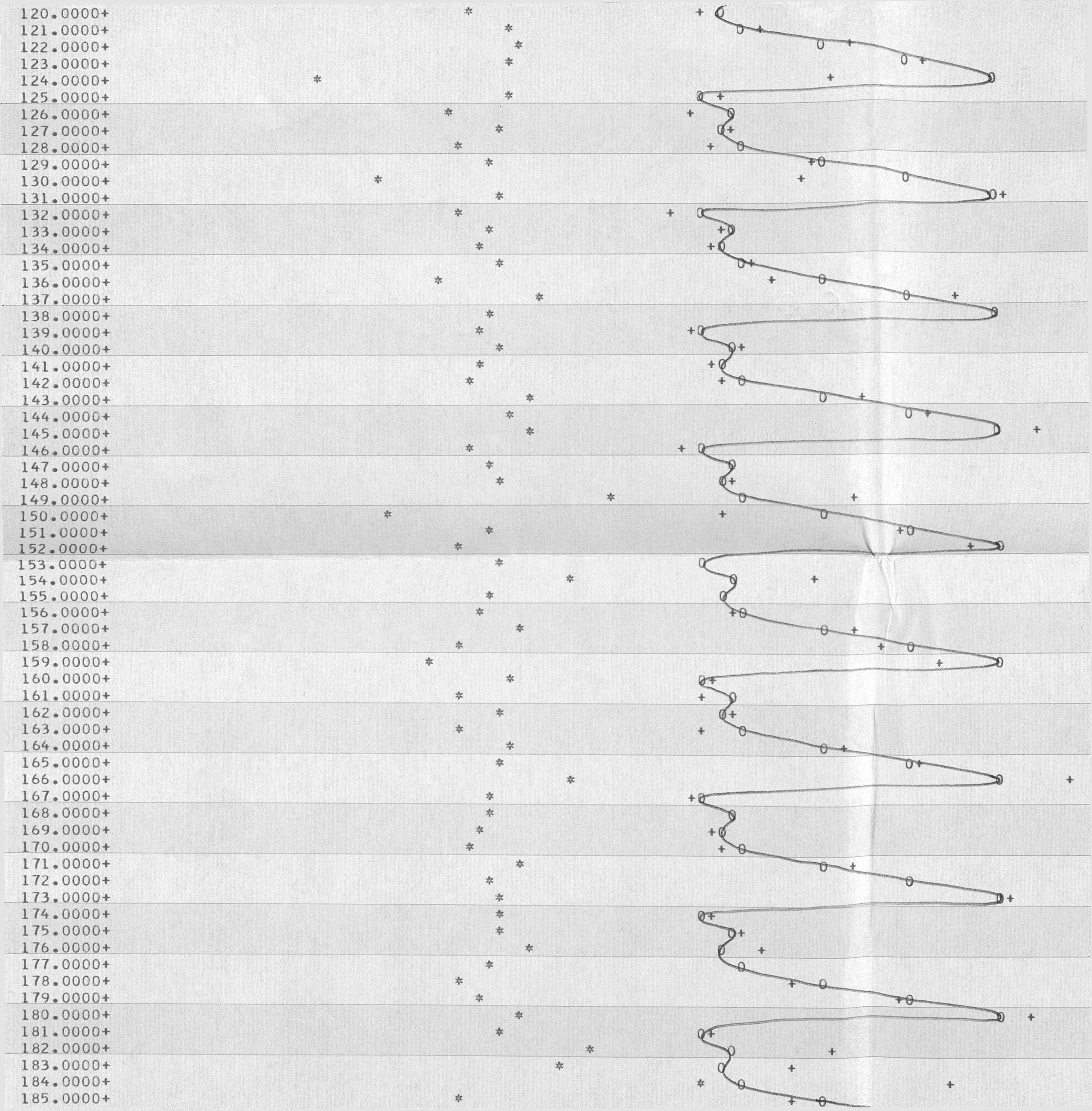
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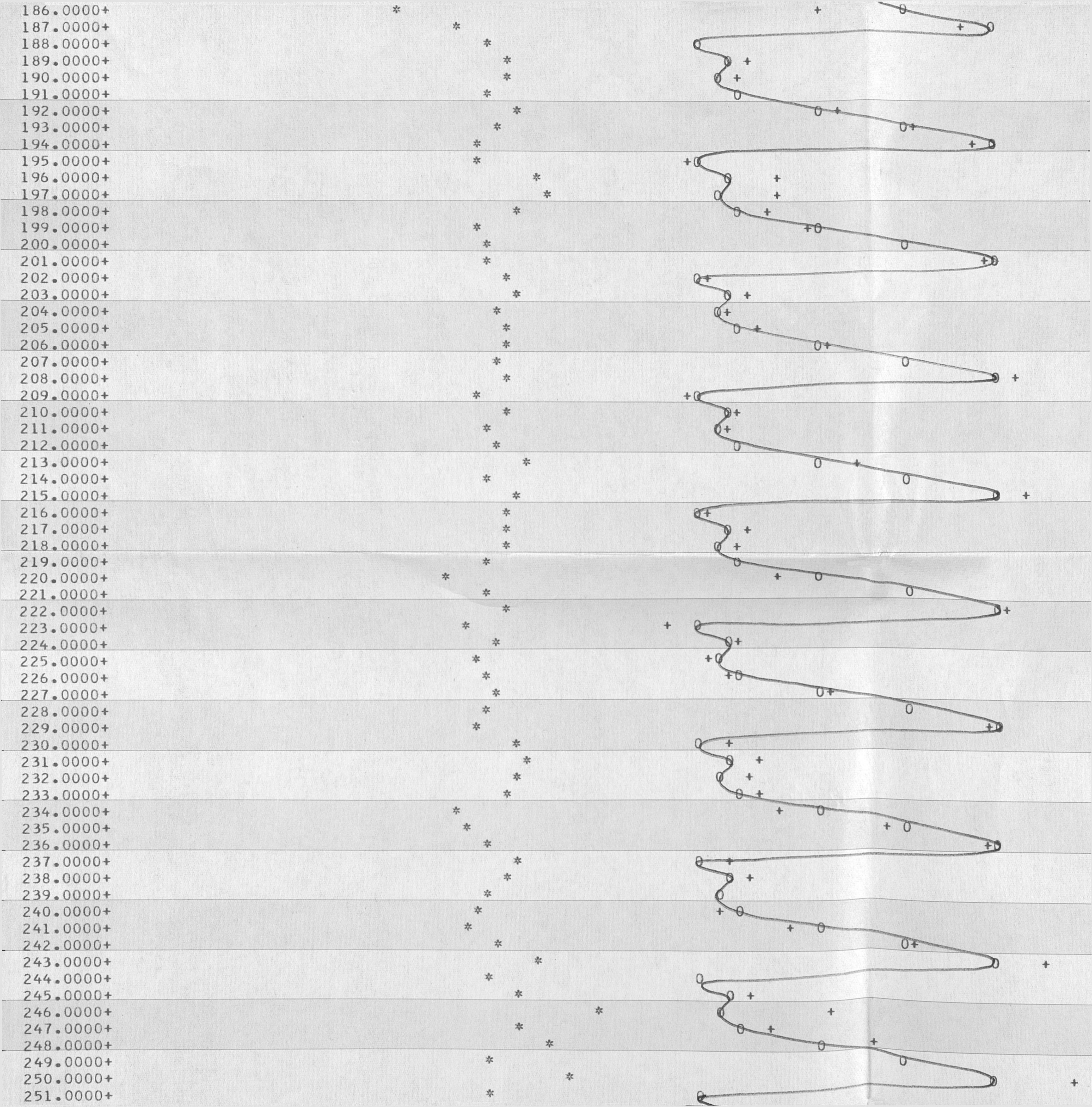
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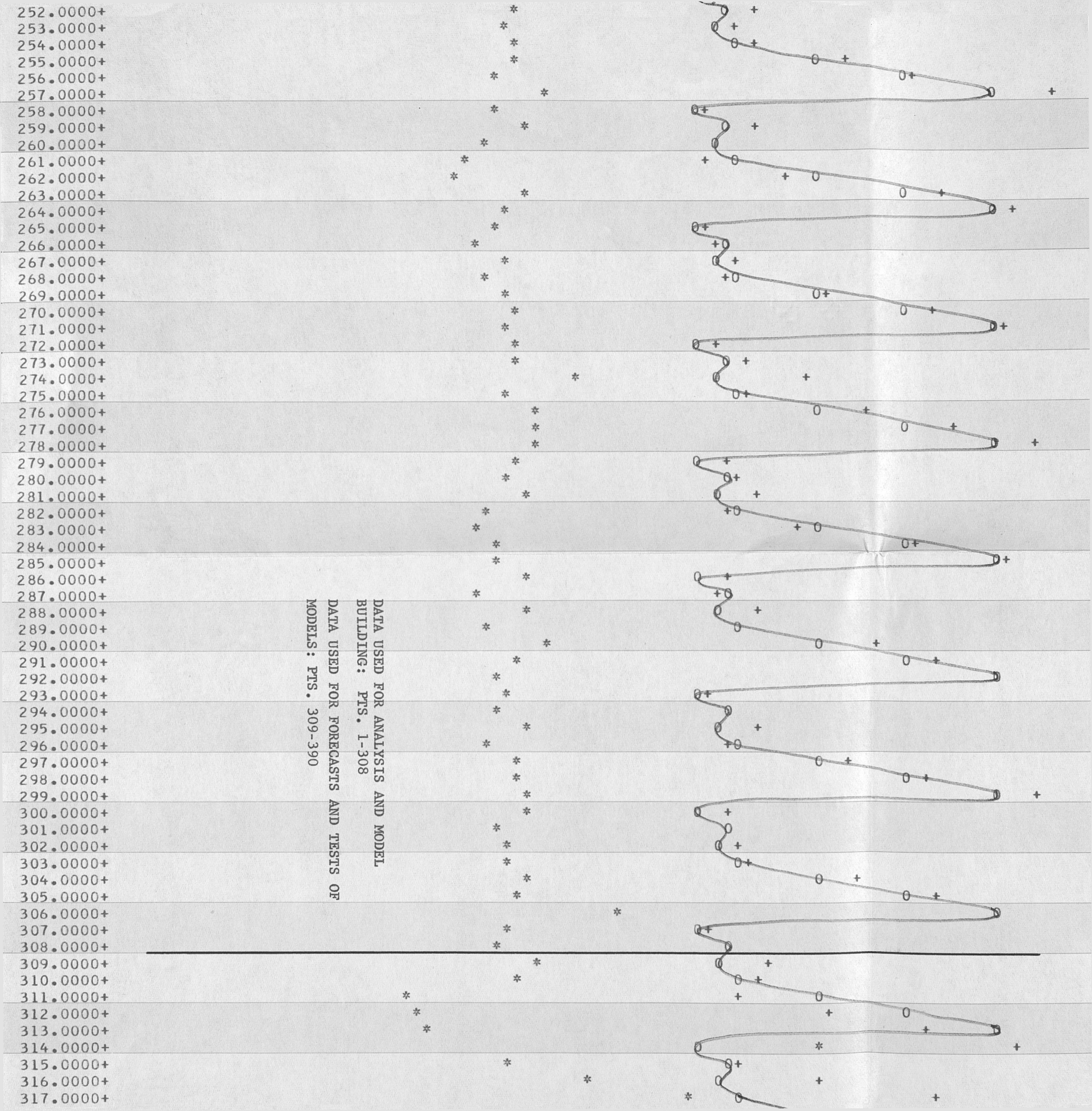
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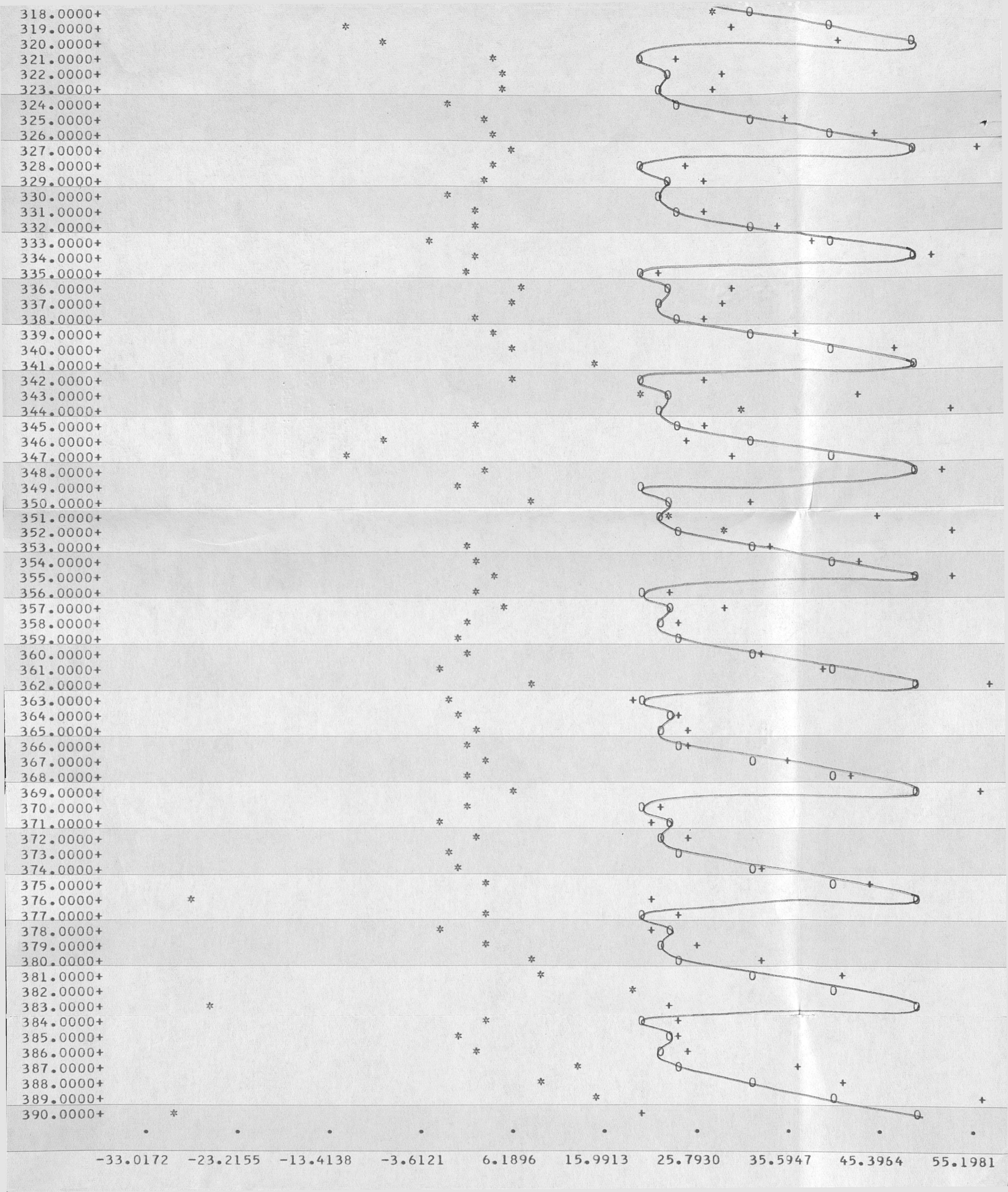
DATA USED FOR ANALYSIS AND MODEL BUILDING: PTS. 1-308
DATA USED FOR FORECASTS AND TESTS OF MODELS: PTS. 309-390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APPENDIX IV

SIGNIFICANCE TEST FOR β_1

It is stated several times in this report that the growth component, β_1, for the model is almost non-existant and definitely insignificant for mathematical considerations. Therefore, to support the contention of insignificance, a t-test was performed. Development of the t-test is as follows:

1. The standard error of b_1 is the square root of the variance, that is

$$s.e.(b_1) = \frac{\sigma}{\left\{\sum (X_i-\bar{X})^2\right\}^{1/2}} \quad .$$

2. If we assume that the variations of the observation about the lines are normal, it can be shown that we can assign $100(1-\alpha)\%$ confidence limits for β_1 by calculating

$$b_1 \pm \frac{t(n-np,\ 1-1/2\alpha)s}{\left\{\sum X_i-\bar{X})^2\right\}^{1/2}}$$

where $t(n-np,\ 1-1/2\alpha)$ is the $(1-1/2\alpha)$ percentage points of a t-distribution with $(n-np)$ degrees of freedom (the number of degrees on which s^2 is based). N is the number of data points, and np is the number of parameters involved. The test will be a two-sided test conducted at the $100(1-\alpha)\%$ level in this form.

The variance of b_1 is given as:

$$V(b_1) = \frac{\sigma^2}{\sum(X_i-\bar{X})^2} \quad ,$$

and is the (n,n) element of the $(X'X)^{-1}$ matrix of dimensions n x n. So we can say that the standard error of b_1 is

$$s.e.(b_1) = V(b_1) = \frac{\sigma}{\left\{\sum(X_i-\bar{X})^2\right\}^{1/2}}$$

Now we can state the hypothesis

$$H_0: B_1 = 0 \qquad\qquad H_a: B_1 \neq 0$$

and evaluate

$$t = b_1 \;/\; s.e.(b_1).$$

So, if we want to test the significance, or, in this case, substantiate the insignificance of B_1 we proceed as follows: H_0: $B_1=0$; H_a: $B_1 \neq 0$

1. Determine the critical value for $t(\infty, 0.975)$, which is 1.960.
2. Determine the (n,n) element for the $(X'X)^{-1}$ matrix. It is: .00649, and $\sqrt{0.00649} = .081 = s.e.(b_1)$
3. Calculate a t value for each set of data, and compare t against the determined critical value.

    a. Total Sales

    $t = b_1 \;/\; s.e.(b_1) =$

    $$= \frac{.0168}{.081} = 0.2067$$

    $|t| < 1.960$. So we do not reject the hypothesis

    H_0: $\beta_1 = 0$

b. Customer count

$t = b_1 \ / \ s.e.(b_1)$

$= \frac{.005639}{.081} = 0.0448$

$|t| < 1.960$. So we do not reject the hypothesis

$H_0 \colon \beta_1 = 0$.

c. Meat Sales

$t = b_1 \ / \ s.e.(b_1)$

$= \frac{.005629}{.081} = 0.0694$

$|t| < 1.960$. So we do not reject the hypothesis

$H_0 \colon \beta_1 = 0$.

Now, after completing t-test of all of the β_1 values, we can say that β_1 is very definitely insignificant. In fact, only the β_1 value for Total Sales could find a significance level anywhere on the t-distribution table, that being at $t(\infty, 0.30)$.

APPENDIX V

ORIGINAL TIME SERIES OF GROSS SALES DATA

DATA PT. #	TOTAL SALES $	CUSTOMER COUNT	MEAT SALES $
1	2292.74	955	413.56
2	1938.74	999	384.90
3	2135.48	1071	401.11
4	3133.90	1072	743.45
5	2980.08	1124	920.79
6	5063.00	1537	1251.41
7	1746.82	1140	308.74
8	2242.46	1013	455.36
9	2117.33	948	391.69
10	2495.85	1356	546.44
11	2697.00	915	592.61
12	4616.18	1166	1085.99
13	4966.76	1337	1195.95
14	1923.61	819	311.84
15	2397.91	1080	469.56
16	2172.41	976	423.69
17	2194.69	1021	437.56
18	3609.08	1125	837.14
19	4292.04	1196	1057.81
20	5389.61	1472	1303.53
21	1844.76	1174	292.30
22	2324.30	986	488.11
23	2295.32	1056	548.90
24	3544.23	1274	796.75
25	2217.85	999	459.06
26	4044.55	1107	986.50
27	4689.88	1466	1230.60
28	2055.62	969	435.99
29	3083.86	1161	542.77
30	2246.84	924	401.40
31	2344.24	1091	432.48
32	3511.29	1124	792.13
33	3842.39	1107	926.08
34	4390.05	1462	998.49
35	2181.69	949	430.09
36	1992.29	962	378.76
37	2312.75	1050	418.47
38	2058.78	1550	371.67
39	3381.66	1158	789.84
40	4196.42	1267	1042.00
41	5691.02	1585	1408.94
42	1969.99	1471	192.70
43	2305.75	977	430.25
44	2178.49	812	429.00
45	2244.66	1103	398.91
46	3473.49	1207	711.10
47	4118.49	1271	890.85
48	5077.34	1579	1113.18
49	2091.30	926	398.92
50	2400.68	1171	486.10
51	2586.87	1169	473.63
52	2408.52	1125	462.94
53	2903.76	1104	708.04
54	3953.15	1151	993.26
55	5279.42	1306	1305.29
56	2137.13	1048	375.65
57	3311.09	1174	752.51
58	2921.04	1112	584.01

59	4445.79	1704	1034.64
60	2903.76	1104	708.04
61	3168.14	1118	389.00
62	4692.08	1338	1094.45
63	2034.44	1064	395.02
64	2520.91	1128	451.20
65	2401.35	1115	458.30
66	2396.04	1070	446.73
67	3443.97	1197	752.68
68	4192.43	1232	994.98
69	4820.96	1522	1123.43
70	1923.45	886	344.34
71	2806.08	1179	636.11
72	2845.57	1163	524.07
73	2683.78	1012	538.45
74	3130.38	1168	716.11
75	4050.02	1138	1008.55
76	4920.43	1619	1284.09
77	2150.71	1013	369.35
78	2539.98	1047	521.16
79	2327.59	1055	447.36
80	2633.30	1100	580.58
81	3359.47	1356	792.96
82	4140.88	1252	963.94
83	5185.49	1482	1330.29
84	1902.59	1044	364.88
85	2466.24	992	517.02
86	2288.36	1053	389.68
87	2444.35	1080	511.77
88	3590.03	1209	897.49
89	4127.95	1186	992.96
90	5228.37	1185	1303.36
91	2171.71	1102	352.97
92	2482.91	1212	496.13
93	2460.18	1079	449.10
94	2405.45	1072	476.24
95	2805.96	1035	653.44
96	4059.88	1340	1006.89
97	5103.86	1070	1333.05
98	1783.74	1064	416.45
99	2421.46	1038	524.02
100	2128.51	996	418.44
101	2346.47	1099	468.66
102	3317.79	824	732.16
103	4126.39	1338	980.22
104	4870.02	1498	1193.29
105	2324.17	1010	392.79
106	2652.21	1121	530.63
107	2566.14	1184	496.64
108	2592.92	1128	504.74
109	2866.94	1183	630.32
110	3872.11	1222	918.38
111	4927.36	1430	1218.79
112	2286.94	1071	415.60
113	2502.72	826	502.53
114	2263.52	1055	465.36
115	2284.72	1073	457.72
116	2969.74	1122	714.16
117	4197.85	1048	1058.26

118	5482.48	1583	1410.00
119	2009.20	1867	364.67
120	2558.98	1350	518.53
121	3287.84	1228	643.64
122	2706.65	1176	494.97
123	3769.28	1207	902.67
124	4084.06	1193	953.74
125	5750.61	1598	1349.19
126	2024.67	1188	385.89
127	2613.00	1118	538.56
128	2433.67	1037	467.30
129	2641.44	1013	545.92
130	3464.37	1146	646.45
131	4157.32	884	1051.60
132	5530.59	1272	1403.73
133	2092.55	881	420.83
134	2668.85	1416	525.95
135	2258.63	930	456.74
136	2168.43	1019	455.03
137	2900.25	984	687.59
138	4466.77	1020	1079.35
139	5154.47	1380	1247.01
140	2101.39	1054	408.31
141	2230.70	1342	427.50
142	2391.65	954	470.81
143	2357.01	895	447.50
144	3358.61	944	805.57
145	4413.75	1268	1240.17
146	5117.64	1385	1335.38
147	2272.94	1147	449.09
148	2541.85	1125	493.74
149	3147.49	702	641.93
150	2578.90	1060	555.17
151	3658.04	962	862.63
152	4624.74	1199	1101.83
153	5400.36	1179	1310.65
154	2335.73	1341	363.64
155	2455.01	1093	490.81
156	2661.89	1049	575.81
157	2366.74	945	490.42
158	3040.80	1198	773.73
159	4176.27	1314	1025.65
160	5051.54	1201	1302.33
161	2358.15	1125	446.82
162	2221.54	869	421.24
163	2671.85	1281	499.59
164	2428.63	911	528.64
165	3809.80	1107	1006.63
166	4392.92	1153	1162.60
167	5011.04	1101	1398.30
168	2160.95	1120	409.22
169	2372.09	1409	554.80
170	2623.80	889	551.27
171	2376.67	866	496.50
172	3472.45	1320	843.65
173	4335.67	1014	1062.64
174	5345.37	1614	1337.58
175	2352.82	1198	418.74
176	2340.69	906	490.71

177	2457.40	1144	472.39
178	2544.42	948	477.14
179	3562.89	1151	741.80
180	4348.02	1254	973.63
181	6179.83	1277	1569.97
182	2182.22	1998	346.71
183	2340.69	906	490.71
184	2752.64	1105	603.35
185	2672.95	1007	478.62
186	2393.12	947	470.92
187	3360.23	935	777.71
188	4321.72	978	1055.09
189	5198.16	1148	1323.66
190	2442.10	866	493.07
191	3236.13	1167	836.25
192	4359.78	1191	1144.78
193	6009.26	1683	1269.75
194	3058.21	1221	661.46
195	4235.54	1380	1045.94
196	2476.99	669	471.93
197	2902.82	1079	610.94
198	2856.23	1233	514.41
199	2409.48	1063	484.86
200	3612.92	1239	835.93
201	4561.51	1136	1159.42
202	5666.89	1599	1362.74
203	2490.48	669	442.81
204	2703.22	1126	436.67
205	2239.78	839	349.93
206	2681.76	1149	476.16
207	3491.90	1148	764.46
208	3898.77	1202	843.41
209	5223.73	1560	1154.90
210	2217.85	912	367.00
211	3031.60	1241	619.47
212	2933.00	1362	509.05
213	2702.65	1206	489.30
214	3672.58	1361	907.29
215	4757.98	1414	1306.06
216	6499.99	1711	1763.42
217	2716.27	1076	569.08
218	4374.19	1389	996.27
219	5371.97	1612	1215.99
220	2702.65	1206	489.30
221	2519.81	1069	535.52
222	3048.29	629	763.01
223	5317.90	2063	1261.48
224	2057.92	1017	354.05
225	3179.64	976	785.49
226	4603.17	1826	1070.74
227	5371.97	1612	1215.99
228	3376.18	1106	725.43
229	4360.91	1182	1105.00
230	5398.07	1590	1293.79
231	2338.01	1036	445.40
232	2919.10	1285	551.45
233	2457.66	1031	503.68
234	2459.03	979	524.49
235	3331.48	994	727.34

236	3973.80	1534	1069.23
237	5810.67	1576	1495.53
238	1975.86	1001	308.43
239	2418.49	939	470.66
240	2528.08	1167	530.70
241	2559.88	885	555.98
242	3604.89	1285	806.62
243	4297.13	1421	1002.19
244	5628.87	1544	1251.07
245	2215.16	1026	404.93
246	2183.99	1060	441.34
247	2493.08	1163	422.82
248	2387.62	962	482.71
249	3292.90	1142	780.52
250	4457.59	744	1053.83
251	2184.81	863	449.21
252	2435.04	985	483.48
253	2157.18	877	411.79
254	2590.58	1062	505.18
255	3297.04	1252	718.33
256	4159.61	751	978.82
257	6024.21	973	1365.49
258	2362.21	1056	407.25
259	2389.96	1016	467.95
260	2387.63	1126	444.94
261	2569.78	1141	514.21
262	3728.29	1242	871.33
263	4210.69	1250	993.07
264	5631.30	1509	1329.93
265	2042.51	1056	324.15
266	2487.59	1003	467.68
267	2399.36	1088	455.81
268	3067.50	985	764.35
269	4251.80	1330	1045.88
270	5159.64	1247	1336.33
271	1678.08	722	209.89
272	2370.64	1058	606.58
273	1833.45	958	339.62
274	2032.48	1034	402.85
275	3508.64	752	860.70
276	3365.72	901	999.44
277	6084.96	1508	1514.11
278	1664.43	788	252.74
279	2455.54	944	566.44
280	2209.61	825	475.23
281	2019.09	950	457.34
282	2935.92	912	704.61
283	3510.19	1024	882.36
284	4645.16	1312	1153.44
285	1661.05	869	313.41
286	1684.21	667	357.88
287	1993.01	896	446.41
288	1879.82	860	355.11
289	2694.66	976	601.32
290	3503.10	941	838.72
291	4760.39	1360	1161.67
292	1610.96	829	277.10
293	1964.20	969	381.92
294	1920.44	878	351.74

295	2096.43	985	445.15
296	3457.53	1124	847.46
297	3720.74	1033	924.63
298	5373.45	1389	1151.15
299	2018.95	1141	376.72
300	2153.89	1080	429.84
301	1774.50	868	338.37
302	1832.62	953	335.05
303	3381.26	1058	867.52
304	4054.58	1081	1028.67
305	5196.11	1628	1205.87
306	1996.80	923	366.93
307	1894.53	903	347.85
308	1869.02	964	300.65
309	2086.29	904	391.56
310	2835.16	929	817.96
311	3803.00	1154	974.26
312	5017.76	1628	1353.50
313	1585.96	681	205.52
314	2111.56	936	441.65
315	1952.80	749	386.44
316	2159.17	976	465.16
317	3411.49	1031	806.77
318	4345.57	1160	1135.37
319	5142.73	1295	1254.50
320	1873.38	1017	319.02
321	2158.16	973	426.58
322	2073.35	837	376.28
323	2129.08	1013	386.58
324	2927.33	983	686.13
325	4163.38	1011	975.77
326	5889.01	1463	1421.63
327	1737.14	794	257.12
328	1759.00	899	403.98
329	1977.55	1071	337.89
330	2101.42	975	379.26
331	3001.68	972	696.35
332	3739.96	1036	907.74
333	5138.95	1448	1223.00
334	1808.08	882	317.51
335	1944.63	899	413.43
336	1641.75	731	291.40
337	1930.66	1029	375.44
338	2901.45	998	614.53
339	4269.69	1126	992.44
340	4868.08	1276	1183.50
341	1899.38	795	337.82
342	1955.32	904	405.31
343	2113.08	888	407.49
344	2091.85	1033	458.47
345	3094.46	908	728.85
346	4140.43	1143	1137.98
347	4878.13	1375	1241.17
348	1911.75	932	351.25
349	1667.25	831	339.22
350	2010.69	934	361.88
351	1893.98	857	319.61
352	2835.10	1244	570.51
353	3952.48	1132	967.39

354	4674.81	1315	1061.95
355	2120.33	1050	380.90
356	2304.56	939	473.18
357	2042.46	895	396.73
358	2148.47	963	451.89
359	2877.57	961	640.27
360	3635.97	867	686.12
361	5457.41	1346	1421.30
362	1914.36	989	308.11
363	2108.10	877	503.97
364	1783.09	815	311.02
365	2101.47	1142	394.22
366	3407.31	1060	880.00
367	4592.99	1269	1177.42
368	5140.70	1547	1314.84
369	2488.32	1003	502.18
370	2052.57	774	447.96
371	1373.17	1063	429.10
372	3857.01	1057	669.63
373	3769.00	1079	855.27
374	5600.27	1413	1352.18
375	1668.18	981	296.41
376	2085.00	885	445.10
377	2189.63	953	370.89
378	2042.45	502	395.94
379	2972.75	1005	748.99
380	2873.61	1061	944.04
381	4805.89	1477	1241.66
382	1954.19	901	376.17
383	2236.48	990	440.31
384	2002.44	915	343.86
385	2042.45	502	395.94
386	2613.01	1025	493.03
387	3482.48	1087	747.02
388	4271.29	1213	937.47
389	3309.12	1399	1241.35
390	2200.78	898	394.97

SALES FORECASTING FOR A CENTRALIZED MEAT CUTTING FACILITY

by

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B. A., UNIVERSITY OF TEXAS AT EL PASO, 1964

AN ABSTRACT OF A MASTER'S THESIS

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The subject of this thesis is the applicability of spectral analysis to the development of forecasting models for retail grocery store data. The time series data is of two types:

1. Daily gross sales; total dollar sales, customer count, and meat sales.
2. Daily item sales from the product mix of the store's meat market.

The original hypothesis was that spectral analysis would reveal the presence of weekly, bi-weekly, semi-monthly, and monthly cycles. The store management had emphasized the presence of a weekly cycle, but was unsure of other cycles. They had no accurate methods for forecasting.

Spectral analysis revealed only one primary cycle, of seven days, plus the first and second harmonics, corresponding to 3 1/2 and 1 3/4 days, respectively.

The results obtained from spectral analysis were compared to results obtained from models using conventional matrix regression with dummy variables. The major conclusion reached is that in this case the results of both techniques differ slightly. Further, it is the opinion of this author that in cases where the data is Gaussian the matrix regression with the dummy variable approach is perferable to spectral analysis.