APPLICATION AND ANALYSIS OF JUST IN TIME TEACHING METHODS IN A CALCULUS COURSE

by

REKHA NATARAJAN

B.S., Arizona State University, 2001
M.A., Arizona State University, 2002
B.S., Kansas State University, 2004

AN ABSTRACT OF A DISSERTATION

submitted in partial fulfillment of the
requirements for the degree

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Deparment of Mathematics
College of Arts and Sciences

KANSAS STATE UNIVERSITY
Manhattan, Kansas
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Abstract

"Just In Time Teaching" (JiTT) is a teaching practice that utilizes web based technology to collect information about students' background knowledge prior to attending lecture. Traditionally, students answer either multiple choice, short answer, or brief essay questions outside of class; based on student responses, instructors adjust their lectures "just-in-time." In this study, modified JiTT techniques in the form of online review modules were applied to a first semester calculus course at a large midwestern state university during the spring 2012 term. The review modules covered algebra concepts and skills relevant to the new material presented in calculus lecture (the "just-in-time" adjustment of the calculus lectures was not implemented in this teaching experiment). The reviews were part of the course grade. Instead of being administered purely "just-in-time," the reviews were assigned ahead of time as part of the online homework component of Calculus-I. While previous studies have investigated the use of traditional JiTT techniques in math courses and reported student satisfaction with such teaching tools, these studies have not addressed gains in student achievement with respect to specific calculus topics. The goal of this study was to investigate the latter, and to determine whether timing of the reviews plays a role in bettering student performance. Student progress on weekly Calculus-I online assignments was tracked in spring of 2012 and compared to student scores from weekly Calculus-I online assignments from spring 2011, when modified JiTT instruction was not available. For select Calculus-I online assignments during the spring 2012 term, we discovered that the review modules significantly increased the number of students receiving perfect scores, even when the reviews were not purely administered "just-in-time." Analysis of performance, success of review assignments, and future implications are also discussed.

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Dedication

This is dedicated to my boys, Advith and Ayavanth.

Preface

Just-in-Time Teaching is a teaching practice that utilizes web based technology to collect information about students' background knowledge prior to attending class, thus allowing instructors to tailor lectures and lessons accordingly in real time. We adapt a modified approach to JiTT in a first year calculus course, with hopes of bettering student performance. Student scores over two semesters are compared and analyzed, followed by implications and suggestions for future research.

Chapter 1

Introduction

1.1 Motivation

Success in mathematics courses plays a key role in student retention at the undergraduate level. Typical survey courses, such as college algebra, trigonometry, and calculus, serve as general math requirements for many majors, and the inability to pass these courses is linked to increased drop out rates nationwide. While doing well in an introductory level mathematics course is a critical element of successfully completing most college degrees, evidence, spanning over three decades, suggests that college freshman continue to be in dire need of mathematics remediation [35]. In the community college setting, authors Deil-Amen and Rosenbaum cite that roughly 80% of community colleges nationwide report offering remedial math courses as institutional credit, credit that does not even count towards actual degree completion [11]. For students who do seek extra help via remedial math courses, a closer look at enrollment statistics reveals that approximately 60-70 percent of all students that take remedial math courses either do not successfully complete the sequence of required courses or avoid math classes altogether and are unable to graduate [6].

The National Center for Educational Statistics, in a 2004 publication, revealed that approximately 33% of new college students lack necessary preparation and skills needed to be successful in college-level work, requiring additional academic support to fill in these deficiencies [14]. Developmental programs have become a fundamental component of various undergraduate programs nationwide, with the goal of improving basic skill competencies in the core subject areas of reading, writing, and mathematics [19]. Other parts of the world are noticing similar trends. In mathematics specifically, such deficiencies can be traced back to a study done at the University of Southern Queensland where researchers Cretchley, Jourdan, and Passmore observed that incoming university students lacked basic proficiency in the areas of algebra, functions, and trigonometry [10]. They reported that 48.5%, 37.4%, and 44.2% of students scored less than 50% in each of these three areas respectively (the study being done with a sample size of 206 students) [10].

Despite the need for remediation, the debate continues as to whether or not degree granting institutions should allocate funds for developmental courses and programs amidst tight budget constraints and limited resources [20]. One way to circumvent these limitations is to address the issue of remediation with online homework systems. In beginning level mathematics courses especially, automated self-assessment tools are becomingly increasingly popular. These tools not only identify strengths and weaknesses of the student, but also provide individualized opportunities for review and practice. Various systems, including ALEKS, MyMathLab, WebAssign, WebWork, and ThomsonNOW, for instance, are among some of the software packages that are being used throughout the country for review/remediation purposes.

As students continue to enroll in mathematics classes without understanding crucial pre-requisite material, instructors are faced with the burden of bringing students "up to speed" with relevant and necessary background knowledge. This task can be daunting, as there is not enough in-class time to spend on review material when syllabi need to focus on covering the new topics and ideas linked to the actual course itself. Expecting students to apply background knowledge to new concepts is natural in any math course. With regard to calculus, it appears as though many incoming students lack fluency in fundamental mathematical skills, never having internalized foundational ideas presented in the pre-requisite courses of college algebra/trigonometry and pre-calculus.

1.2 Research Questions

In light of the lingering dilemma surrounding students' lack of fluency in pre-requisite mathematical skills/concepts, assisting calculus students and helping them succeed and learn mathematics is of prime importance. In particular, helping first semester calculus students mend gaps in their understanding of college algebra and trigonometry is critical. With this goal in mind, this research investigates the following ideas:

1. Do modified "just-in-time" online teaching tools covering review topics in algebra lead to gains in student learning on specific calculus topics?

2. Does timing of these review assignments make a difference in terms of student achievement on calculus online homework assignments?

3. Do students with the "just-in-time" review intervention access optional additional feedback differently than students without the intervention?

The traditional "just-in-time-teaching" methodology referred to in research question (1) was developed and popularized by Gregory Novak and his contemporaries at IUPUI in the late 1990s and has gained prominence as an instructional tool in many disciplines throughout the country. More commonly referred to as "JiTT," this teaching practice utilizes web based technology to collect information about students' background knowledge prior to attending lecture, thus allowing the instructor to adjust lectures and lessons accordingly in real time. In the study discussed here, we focus on using the preparatory aspect of "just-in-time-teaching," which involves assigning students homework outside of class pertaining to a new topic they will see in an upcoming lecture. As one of the goals of the "just-in-time" approach focuses on making students more accountable for their own learning, the preparatory assignments were also created with this idea in mind.

Similar "just-in-time-teaching" techniques have been used in math classes across the continent, including universities in North Carolina, New York, West-Virginia and Canada.

While these studies report positive feedback from both instructors and students alike, none of the literature addresses actual improvement in scores on particular calculus topics after implementation of JiTT in the classroom. One of the main purposes of this teaching experiment is to determine if such JiTT tools do lead to specific gains in learning, as well as whether or not the "just-in-time" feature makes a difference in student achievement on various calculus topics.

1.3 The Teaching Experiment

To help answer these research questions and gain more insight on various aspects of student learning with regard to calculus and pre-requisite skills, review modules over a series of college algebra and trigonometry topics were administered in a calculus course prior to their application to new calculus concepts. Based on past research and instructor observations about student weaknesses in learning calculus, reviews covering the topics of rational expressions, composition of functions, order of operations and triangle trigonometry were created. Since each review assignment contains three randomly generated questions, it is very unlikely that any two students will have identical problems. Students have an unlimited number of chances up until the deadline to receive their best score on each assignment. The reviews have optional help links which students may access after having received instantaneous feedback on a problem set.

The teaching experiment took place at a midwestern research university during the spring 2012 semester. The university has a typical SAT/ACT profile of 980/21 and enrolled students typically meet Board of Regents' guidelines. Under these guidelines, students that are under the age of 21 from Kansas high schools with fewer than 24 transfer credits are admitted to the university upon meeting at least one of the following criteria: (a) An ACT score of at least 21, (b) Graduation in the top one-third of their class, (c) Completion of the pre-college curriculum with a minimum GPA of 2.0. The Pre-College curriculum includes 4 years of English, 3 years of mathematics including Algebra I/II and Geometry and courses beyond Algebra II, 3 years of natural sciences, and 3 years of social sciences [27].

To determine how successful the review modules were, two sets of students' Calculus-I online homework scores were compared in this study. Spring 2011 student scores served as a control group; this group of students did not have the modified "just-in-time-teaching" intervention, while the spring 2012 students had the intervention. Statistical analyses revealed that the reviews were indeed helpful in various ways, all of which will be discussed in detail in chapters four and five. In particular, the timing of the reviews did not seem to matter when considering student achievement topic-wise.

1.4 Limitations

One of the limitations encountered in this study involves course instruction; the calculus course was taught by a different instructor during each of the spring semesters. This was the only difference between semesters. Other features either remained constant or were very similar. In particular, the course was taught in the same format across both semesters. The lecture-recitation format comprised of two lectures and two recitations per week. The same textbook was used across both semesters, and both professors covered the same topics. The pace of the course was similar, and the format and content of the regular online homework assignments were the same as well. The book assignments covered the same material during both semesters. The online assignments administered during both semesters contained similar problems of the same format; the online homework system that was used randomly generates different problems for different students, however the problem prototype across both semesters were the same.

To verify that both populations of students were similar in terms of mathematical ability upon entering the course, average math ACT scores of both populations were considered. These scores were 24.9 and 24.6 in spring 2011 and spring 2012 respectively, and a two sample t-test assuming equal variances revealed that the mean ability of both groups of students as represented by their ACT math scores was the same with 99.95% confidence.

Therefore, both populations of students were similar in terms of mathematical strengths upon entering the course. Spring demographics from year to year tend to be similar at this particular midwestern university, with a combination of upper classmen and non-traditional students, but a lower percentage of freshmen than in fall semesters.

Note that data was collected during spring semesters, a semester when historically, student demographics at this particular university are much different than fall demographics. In particular, more freshmen are likely to enroll during the fall, and this group of students typically just finished high school and are more likely to have seen/worked on math more recently. The spring semester has a mixed population of students in terms of freshmen through seniors, and our results may have been different had we compared data from fall semesters.

Other limitations of this study are related to student population at the university as a whole. This university's demographics possibly differ from that of a large state university or community college on the east or west coast or perhaps the population of students at a private liberal arts and science college. In particular, the number of non-traditional students is not particularly high, and the number of evening classes offered are not as abundant as those found in a commuter college or junior college setting.

Finally, although average ACT math scores were compared between semesters to demonstrate similarity in overall mathematical ability, this study does not identify other student characteristics that may have contributed to the findings.

1.5 Summary

Strengthening mathematics achievement is a key factor in the ongoing effort to improve university retention rates. As suggested by previous research as well as anecdotal evidence and instructor observations, helping students learn mathematics and succeed in their courses is an area of concern. In particular, mending gaps in pre-requisite knowledge (without sacrificing class time do so) is critical for calculus students and instructors alike.

Past research indicates that the use of online tools via JiTT/similar instructional techniques have yielded positive results in terms of student and instructor feedback, and a subset of these studies has also reported some level of improvement in terms of student course grades in Calculus-I and subsequent courses. This research study adds to the existing body of literature with new results that address how modified JiTT teaching tools help improve learning with respect to specific calculus topics. This study also investigates issues related to the timing of the review tools in context of presenting new material.

Learning any new mathematical subject requires fluency in pre-requisite material, and an online tool that assists students with recalling previously learned skills is advantageous in various ways. User friendliness, multiple opportunities for skilled practice, automated grading, and instantaneous student feedback are among some of the advantages, while flexibility in implementation is another positive feature on the instructional side. Through this study, we hope to offer additional insight on all of these areas collectively, serving as a springboard for future research related to helping students re-activate and access background mathematical knowledge more efficiently and successfully.

Chapter 2

Literature Survey

This chapter discusses the history and development of Just-in-Time Teaching and provides a detailed account of similar teaching practices that have been/are being conducted in undergraduate mathematics courses across North America. Key points related to calculus readiness and success in calculus are presented, focusing on students' understanding of new calculus concepts/skills. The relevance of online teaching tools with respect to achievement in math courses is also outlined in this section. Ultimately, all of these ideas work together to create the foundation and motivation for the overall design of the teaching experiment.

2.1 Just-In-Time Teaching

> Giving students the chance to grapple with specific relevant material prior to a lecture enables them to learn more from the lecture [5].

This principle is one of the driving forces behind Just-in-Time Teaching (hereon referred to as JiTT), a teaching practice that utilizes web based technology to collect information about students' background knowledge prior to attending class, thus allowing instructors to tailor lectures and lessons accordingly in real time. This strategy, introduced by Gregor M. Novak and his collaborators, was initially used in introductory physics courses at Indiana University, Purdue, the U.S. Air Force Academy, and Davidson College [26]. The push for such a teaching methodology was motivated by a series of observations by Novak and his contemporaries. In particular, they noted that students were more focused on grades and less concerned about learning. As a reaction to these observations, these lecturers wanted to blend active learning with technology and make the learning process more student centered. One of their key goals was to increase accountability on the students' part both inside and outside of the physics classroom. Coined as "Just-in-Time Teaching," a JiTT framework was developed and implemented in physics classrooms at the above listed universities during the late 1990s. In addition to increasing student accountability for learning, other goals of JiTT [26] include:

1. Using class time when access to human help is available as efficiently as possible.

2. Providing an out-of-class routine that prepares students for lecture.

3. Promoting an atmosphere of team spirit.

2.1.1 Features of JiTT

To help achieve the goals mentioned above, one aspect of the JiTT framework utilizes preparatory assignments, usually via the web, which are set up ahead of time, prior to lecture. The preparatory exercises require that students answer questions based on readings, concepts or background knowledge. The assignments may be in the form of warm-up exercises, writing activities, or puzzles. Multiple choice, short answer, and true/false questions are among some possible formats for these assignments. Another purpose of assigning out of class work on a regular basis is to help students develop a habit of preparing for lecture ahead of time. A typical JiTT question in an introductory physics course might involve asking students to explain an idea or concept in their own words and submitting their summary online, such as in the following example:

> "Hamilton's Principle states that a dynamical system traces out a path that minimizes the time integrals of the difference between the kinetic and potential energies. Please explain this in your own words" [26].

Along with a series of questions, the preparatory assignment may have space for student comments and might ask whether the student sought extra help on the assignment. Students have to complete the questions a few hours before lecture begins, at which point instructors collect submitted answers and determine the areas where students have either understood/misunderstood key ideas. Based on student responses, instructors adjust their lectures accordingly. For instance, if student responses to a preparatory question illustrate mastery of a concept, lecturers might choose to spend less time on that given topic and move at a faster pace with the prescribed course content for a given day. On the other hand, if student responses reveal a lack of understanding, then instructors may decide to spend a little extra time during lecture addressing these misconceptions. The web-based features of JiTT really serve as a bridge between the learning that takes place outside and inside the classroom; in fact, Gavrin and Novak view JiTT as a continuous "feedback loop" between the students' work outside of class, their classroom tasks, and the instructor [26].

In addition to preparatory assignments, JiTT also incorporates more challenging exercises, called 'puzzles,' after students receive formal instruction on a topic. After having studied a chapter of material or taking a test, for example, instructors may introduce a puzzle via the web. The following question is an example of a puzzle that students were asked to solve in a mathematics classroom after a chapter test, which was assigned as extra credit at IUPUI [26]:

> You may have debated this with a friend: "Where is the best place to sit in a movie theater?" A movie theater has a screen that is positioned 10 feet off the floor and is 25 feet high. The first row of seats is placed 9 feet from the screen and the rows are 3 feet apart. The floor of the seating area is inclined at an angle of $\alpha = 20$ degrees above the horizontal and the distance up the incline that you sit is x. The theater has 21 rows of seats. Suppose you decide that the best place to sit is in the row where the angle θ subtended by the screen at your eyes is a maximum. Show that $\theta = \arccos \dfrac{a^2 + b^2 - 625}{2ab}$, where

$a^2 = (9 + x\cos\alpha)^2 + (31 - x\sin\alpha)^2$ and $b^2 = (9 + x\cos\alpha)^2 + (x\sin\alpha - 6)^2$. Use a graph of θ as a function of x to estimate the value of x that maximizes θ. In which row should you sit? What is the viewing angle θ in this row?

Puzzles such as this might also require that students explain how they arrived at their solution, in addition to presenting the solution itself. In this manner, JiTT assignments hone other skills relevant to communication and presentation, along with strengthening content knowledge. By placing an emphasis on student accountability, JiTT aims at bettering a student's learning experience within in the classroom setting. Table 2.1 summarizes key motivating factors and related strategies proposed by JiTT [26].

Table 2.1: *Why JiTT?*

JiTT Desired Goal	**Strategy**
Accountability	Base lectures on student responses
Increased Motivation	Include real life applications, create community
Better Study Habits	Incorporate daily work, De-emphasize cramming
Improved Analytic Thinking	Present open ended problems

2.2 Previous Studies

The JiTT methodology has been adapted across a vast number of disciplines, ranging from natural and social sciences, to statistics, calculus, and differential equations. JiTT and similar techniques have been applied to mathematics curriculum in various institutions across North America and Canada. While past research reports positive feedback from the perspectives of both students and instructors, none of the studies in the literature have reported any gains in learning with respect to different calculus topics upon implementing a JiTT/similar strategy. The studies outlined here have either reported positive student/instructor feedback or have reported partial student success in terms of final course grades. Many of these studies incorporated various web-based technology packages, all of whose details are presented in the following discussion.

2.2.1 JiTT in Differential Equations

At North Carolina State University in 2007, mathematics instructors and graduate teaching assistants used WebAssign to generate review problems from calculus to help with students' performance in a differential equations course [21]. (WebAssign was initially developed at North Carolina State and applied to various courses in the mathematical sciences on their campus). The researchers in this study found a new use for WebAssign in their differential equations course, namely that it would serve as an out of class, self-paced review tool for remedial topics that are relevant to solving differential equations problems. Two pilot sections of the differential equations course with the WebAssign intervention ran, one having 30 students and the other having 100 students. The two instructors ran the course in a similar format, while they handled technical and mathematical concerns in different ways. The instructor of the smaller class used email to resolve student difficulties, while the lecturer in the larger section used office hours and lecture time to answer questions. Other aspects of the course remained the same, including the number of tests, assignments and final exam. Through this pilot study, both instructors wanted to assess whether or not WebAssign reviews would benefit students.

Seven review assignments, each consisting of seven to ten questions, were administered over the course of the fall semester at North Carolina State University. The individual assignments were randomized, containing the same types of problems for all students, while each students' answers may have been different. These assignments were automatically scored. Surveys were conducted, asking students a variety of questions, including how often they used WebAssign before coming to class, whether they recognized review material and how it connected to new ideas, whether WebAssign was useful to them, and in general what kinds of study skills they had. The overall opinion of the students regarding the WebAssign reviews was positive. Most students felt that the reviews were helpful, in fact 61% of the smaller section and 79% of the larger section actually observed at least twice during the semester that the review skills appeared in the differential equations material. Roughly

90% of all the students felt that the reviews helped ‘brush up’ their skills. A formal analysis regarding student performance on actual problem sets was not published in this study [21].

At Indiana University East, a small commuter campus containing roughly 2500 students, JiTT was synchronized between a differential equations course and an applied physics class. The teaching of the differential equations concepts was aligned to associated physics ideas as and when they arose in physics class. This particular differential equations course had fewer than ten students enrolled, therefore no qualitative data was published regarding the effect of this different teaching style. The study documented anecdotal evidence supporting initial apprehension towards performing physics experiments in a math class, and student feedback revealed that the course, overall, was enjoyable [28].

2.2.2 Calculus I Courses

The University of Ontario Institute of Technology recently reported a project in which pre-calculus review was the focus [17]. Motivated by observations of first year calculus students struggling with fundamental pre-calculus notions including the concept of function and equation solving, an interactive, technology-based approach took center stage in an accelerated three-week review session. 289 calculus bound students participated in the review session prior to the start of their first semester course. The main strategy utilized during the review session involved “learning objects,” which are defined as “interactive web-based tools that support learning by enhancing, amplifying, and guiding the cognitive processes of learners” [17]. The researcher in the study chose learning objects primarily for their accessibility, user friendliness, graphical capabilities, and adaptive nature [17].

The learning objects consisted of three parts: a reading component, video clips, and an optional online mastery tool (for self-assessment purposes). The topics covered in each learning object included operations with functions, solving equations, linear functions, exponential/logarthmic functions, and trigonometric functions. Surveys were once again administered in this study and yielded positive results. Students spent more time on the online mastery tool in comparison to the video clips and reading components. Most students in the study found the learning objects as either useful or very useful [17]. An interesting outcome of the study was that student-perceived levels of knowledge gain came from the video clips and reading assignments rather than the online assessment tool. As in the North Carolina State study, a formal analysis of the learning objects' effect on student performance in Calculus-I topics was not reported in this study.

In another study at Colorado State University, researchers investigated the impact of ALEKS as a review tool in a Calculus for Physical Scientists course [30]. In one part of the study, final exams of students from two fall semesters were compared, each semester containing 70-90 students. This study found that the use of ALEKS did not help improve final exam scores overall, as the semester of students without the ALEKS intervention scored higher on the final exam than the intervention group. The researchers commented that these findings are likely due to differences in student population as well as differences in the final exam that was administered. They also suggested that the extra time that students spent on ALEKS may have infringed upon the time that would have been dedicated towards learning new course content. Student satisfaction with the review tool was positive, with students reporting that the inclusion of ALEKS was helpful and motivating [30].

At The Rochester Institute of Technology (RIT), a Calculus Pilot Project took place from 2001-2002. The reform focused on re-structuring the calculus program, emphasizing on innovative pedagogical techniques [22]. One of the outcomes of this reform movement was the incorporation of JiTT in the calculus classroom, which is still taking place today. As of spring 2011, RIT was running nine sections of JiTT calculus, in which individual instructors could use a format of their choice. Some instructors provide relevant skill sets prior to a lecture, while other instructors allow the lecture to motivate the need for various skills from algebra/pre-calculus. Students meet in lecture four hours a week and in workshop two hours a week; the JiTT material is covered during the workshop. Actual impact of the JiTT component in terms of student achievement at this institution has not yet been tracked, although success rates in this particular version of Calculus I have been high.

At Clarkson University, improving calculus retention has been approached from the angle of diagnostic testing and associated "correction tools" [36]. Incoming freshman (who have just completed high school) complete a twenty question diagnostic exam prior to enrolling in first semester calculus. The exam covers concept based questions, covering a wide of range of pre-requisite topics, including rules of exponents, equations of lines, triangle trigonometry, factoring, graphing, geometry and logarithms. Calculators are not permitted. Based on the student's performance, recommendations are made on a case by case basis. For those who qualify, there are two intervention programs designed to help students succeed in their calculus course. The first of these is the CU-Math program, a web-based math course with live online instructor support available. The course is divided into seven units, covering basic algebra fundamentals, solving equations, functions, geometry and trigonometry. Each unit begins with a pre-test identifying student strengths and weaknesses; the students have one week to study material and brush up identified weaknesses, and then they complete a quiz at the end of the week over the unit material and submit it via email. Solutions are posted the following Monday. During the week, students may work at their own pace in this online course. The second program is called "Springboard to Calculus," a nine day immersion course held in a small classroom setting with around twenty-five students. The Springboard course covers the same topics in CU-Math but in a condensed two week time period; the course is specifically assigned to selected freshmen that have already moved onto campus for the upcoming fall semester. Students that scored below the 13/20 mark on the diagnostic exam and subsequently enrolled in CU-Math or Springboard Calculus were tracked in Calculus-I and their final grades in Calculus-I were monitored to help investigate efficacy of these intervention programs. The study found that students who enrolled in and completed at least four weeks of the CU-Math curriculum benefitted most from the intervention (highest percentage of students earning a C or better grade in calculus), while students enrolling in a traditional Pre-calculus course as intervention did not receive as many passing grades in calculus. Springboard Calculus was somewhat effective, but did not have as much success as CU-Math at the four week mark. Assessment of student performance by topic was not covered in this study.

Student retention in undergraduate engineering degree programs and successful completion of mathematics courses go hand in hand; this is the motivation behind another study that took place at West Virginia University [15]. A version of “just-in-time” calculus has been in full swing in the mathematics and engineering departments on this campus. This version of calculus is an extended, two-semester sequence of Calculus-I that incorporates just-in-time review of algebra and trigonometry. Students who enroll in the extended Calculus-I course are identified as students that placed into traditional Calculus-I, but with gaps in pre-requisite areas. Researchers collected four semesters worth of course data, and analyzed students’ exit grades in Calculus-II for two groups of students, those students that enrolled in extended “just-in-time” Calculus-I prior to Calculus-II, and those who took a traditional one semester Calculus-I course prior to Calculus-II. This research sought answers regarding the benefits of such a “just-in-time” calculus course, and whether the slower pace prepares students for achieving success in Calculus-II. The researchers found that there was no significant difference in Calculus-II final grades between the students having a conventional Calculus-I course and the extended “just-in-time” Calculus-I course. In fact, both groups of students had relatively similar levels of preparation upon entering Calculus-II. The data did reveal that the extended Calculus-I course helped the weaker students strengthen their mathematical foundation, enabling them to perform as well in Calculus-II as their peers who only required conventional Calculus-I. This particular study did not report the effects of the “just-in-time” component on students’ understanding of individual topics within calculus itself.

2.2.3 Beyond Calculus I

The issue of prerequisite remediation has been studied in depth at the University of Virginia. In pilot sections of Calculus II, students were given “Pre-Diagnostic Quizzes (PDKs)” to monitor deficiencies in the areas of algebra, pre-calculus and calculus. Student who failed the initial PDK were required to complete a detailed tutorial, in which questions were generated by the ThomsonNOW learning environment. The effect of completing these tutorials on performance of actual Calculus-II course-work was not discussed in this study, however students reacted positively to the PDKs, in which 20 out of 34 students indicated that the tutorials “helped a lot” [4].

Along with ALEKS, ThomsonNOW, and WebAssign, Colorado College has experimented with JiTT methods in a Calculus III course via the WebWork program [34]. Note that all courses taught at Colorado College operate in a three week block format, where a student only enrolls in one 3 week class at a time. WebWork is an open source online homework system for math and science courses, supported by the MAA and NSF. In this course, the instructor used WebWork to build online assignments based on required reading assignments (one of the JiTT components discussed in section 2.1). The classroom lessons were based on responses to the online assignments. Student performance was not documented in this presentation, however the instructor observed that students were more engaged and excited about learning when it came to problems they were not initially successful with. The instructor commented that the program had its own limitations in terms of user friendliness [34].

2.3 Comparison of Past Studies and Implications

From the standpoint of implementation, we can place the various programs outlined here into one of two groups, those that focus on students prior to the onset of the course (we might call this a “crash course” review method), and those programs that assist students concurrently. In “crash-course review” mode, students participate in a targeted review session prior to the beginning of a course. The programs at Clarkson University and the University of Ontario utilized such techniques in addressing deficiencies in background material. The majority of the other programs, including those at the Rochester Institute of Technology, Colorado State University, and West Virginia university, used review strategies during the course itself; some in a JiTT format, and others using online diagnostic tests with follow up tutorials. A common strand running through all of these studies is the use of technology, as well as positive feedback from both students and instructors after implementation.

The past research discussed here has been successful in terms of student and instructor reactions. However, it would also be worthwhile to explore the effectiveness of JiTT in context of learning specific topics within a Calculus-I course. We are interested in exploring the use of modified JiTT preparatory assignments in a Calculus-I course, and if it is possible to design an effective online teaching tool that helps students demonstrate improved knowledge of specific calculus topics. We are also curious as to how much the 'just-in-time' factor can be relaxed when presenting such review assignments within a calculus course. In context of these ideas, we present some background information pertaining to web-based learning environments, along with what calculus readiness and understanding entails.

2.4 Advantages of a Web-Based Learning Environment

There is growing evidence supporting enhanced learning experiences via computer-based teaching tools in mathematics courses. In a study carried out in Hong Kong among lower secondary students (released in 2001), researchers discovered that students who completed computer based practice problems performed significantly better on assessments and retained more information than students who completed traditional paper-based homework [37]. Similarly, middle school students from Southeastern Texas participated in a study that investigated the mathematical achievement of students who either completed web-based modules or paper/pencil problems [25]. The study also found that achievement was significantly higher in the web based group compared to the control group who completed practice problems with paper and pencil. Both researchers in these studies note that immediate feedback and unlimited opportunities to practice different problems are advantages of web-based frameworks that possibly explain the significant difference in achievement amongst the web-based and paper-based groups.

During the fall semester of 2008, researchers at a large research institution in the southeastern part of the United States investigated how WebWork, when used as an instructional technological tool, helped enhance mathematics performance of students enrolled in two courses, Introduction to Engineering and Introduction to Engineering Technology [35]. Selected sections of these courses were assigned to either a control group or a treatment group; the main difference between the groups was the use of WebWork. This study revealed that the use of WebWork seemed to have helped decrease the gap in pre-requisite skills between incoming engineering and engineering technology majors. The research also suggested that WeBWorK may be a suitable means of mathematics remediation for students in need of such intervention.

In a study conducted at the University of North Dakota, researchers reported positive results from the application of an online homework system in first semester calculus that operated on an "attempt-feedback-reattempt" model [38]. Their online homework system was housed within Blackboard, an online course management system in which instructors may upload teaching materials in various formats, including animations, Power-point presentations, and other multi-media applications. The online homework created for this study included a 1000 question bank of multiple choice, true/false, which of the following apply, and fill in the blank formatted questions covering various calculus topics. Students would receive different problem sets in the sense that the system would choose different combinations of questions from the 1000 question bank. Upon comparing students who were required to do the online homework with students that were not required to do so, the researchers found that students completing the online homework assignments performed better on quizzes and exams, the differences being significant at levels of 93% and 70% respectively [38].

2.5 Success in Calculus

Helping students achieve success in calculus courses has been a topic of discussion for the past several decades. The calculus reform movement seen in the late 1980s and early 1990s, for example, was primarily fueled by the need to improve student pass rates in calculus and bump up student achievement in calculus courses across the country. The reform curricula steer away from a traditional lecture format and seek alternative ways, often interdisciplinary in their approach, of presenting calculus content. With these goals in mind, students might work in small groups, investigating various ideas via inquiry-based/exploratory approaches. Often times, technological tools such as computers and graphing utilities are used in the process.

2.5.1 Calculus Readiness and Understanding

Whether approaching the course from a traditional or reform point of view, the goal of calculus instruction centers around successful student outcomes related to understanding content and demonstrating proficiency in a foundational set of concepts and skills. This leads us to ask what exactly is involved in understanding first year calculus. Perhaps earning an above average grade in the course or doing well in subsequent courses signals understanding of first year calculus. In context of the reform movement, understanding calculus might include having the ability to apply calculus ideas to other disciplines. The literature suggests that a certain level of fluency in a variety of skills and concepts are relevant to calculus understanding. Sofronasa, et al., categorize these areas into four groups, as follows [12]:

(a) Mastery of the fundamental concepts and-or skills of the first-year calculus;

(b) Construction of connections and relationships between and among concepts and skills;

(c) Ability to use the ideas of the first-year calculus;

(d) A deep sense of the context and purpose of the calculus.

Ferrini-Mundi and Grahm highlight four key areas of understanding with respect to fundamental calculus concepts [13]:

(a) Functions

(b) Limits and Continuity

(c) Differentiation

(d) Integration

The fundamental skills cited by Sofronasa et al. cover a vast selection of problems, including derivative computations, techniques of integration, limit calculations, manipulating algebraic expressions, area/volume, and trigonometric manipulations. All of these skills rely heavily upon a student's background in precalculus/trigonometry and college algebra. Instructors have especially observed that handling algebraic manipulations and working with expressions containing radicals, factoring, and the zero product property are seemingly problematic areas for introductory calculus students [29]. Lack of fluency with these skills can be an obstacle when solving classic multi-step calculus problems, especially exercises involving limits where rationalizing a denominator is involved, differentiation (the chain/quotient rule and solving related rates problems), and integration (applications involving solids of revolution, for instance).

Ferrini-Mundi and Grahm list functions as one of the key conceptual notions of calculus; researchers from Arizona State University, the University of Arkansas, and Francis Marion University have piloted a Calculus Concept Readiness Instrument (CCR) also focusing on the concept of function [7]. In particular, several of the questions on the instrument are given in word problem format and assess student reasoning abilities regarding notions of function, function composition, and inverses of functions. The researchers reported that incorrect responses to various questions in the CCR were linked to "weak understandings or misconceptions," which, through student interviews, had previously been verified as (a) being incorrect notions viewed by pre-calculus students, and, (b) as being problematic areas for students learning calculus [7].

Ample evidence also suggests that success in pre-requisite courses is a good predictor of success in calculus [31]. In a study carried out at Clemson University, researchers wanted to determine whether or not students who complete prerequisite courses are successful in completing calculus. In the study, records of 5,645 students were analyzed and tracked over a four year period. The data indicated that student grades earned in a pre-calculus course and a readiness test administered at the start of the calculus course, when combined, turned out to be better predictors of success than Clemson's math placement exam alone [31].

Calculus success is also attributed to overcoming some of the inherently difficult concepts that students encounter for the first time in a calculus course, such as the limit concept. The University of Warwick's Mathematics Education Research Center faculty observe that the notion of limit leads to many cognitive difficulties, such as confusion with using terminology and understanding terms such as 'tends to,' 'approaches,' 'getting arbitrarily small,' etc. [33]. They also note that students face other obstacles, including difficulty when translating real-world problems into calculus terms, selecting and using appropriate representations (graphical vs. numerical vs. algebraic), internalizing new ideas in short time periods, preferring procedure over concept, and handling quantifiers in multiple quantified definitions [33]. In another study conducted by A.T. Morgan, research revealed that students had difficulties in interpreting derivatives when appearing as a rational expression, along with trouble applying the chain rule to differentiation [24]. How students relate their knowledge of function composition to the chain rule for derivatives has also been formally investigated in other research studies (see [9], for instance).

2.5.2 Student Attitude Towards Learning Calculus

Along with pre-requisite readiness, much can be said about the effect of student attitude and readiness on calculus success. Studies have shown that students coming into a math course with better attitudes towards the content are more inclined to try more persistently during the problem solving process [23]. More recently, Laura Pyzdrowski, et al., conducted a study investigating how attitudes and readiness factor into student achievement in introductory calculus courses at Northeastern State University [29]. 107 calculus students participated in the study, completing an inventory over attitudes towards mathematics and a readiness assessment. The attitude inventory consisted of 40 questions relating to self-confidence, value, enjoyment and motivation, while the readiness assessments contained 33 questions covering content from algebra, trigonometry and pre-calculus [29]. Researchers also conducted student interviews and tracked student performance in the calculus course itself. Based on these factors and results from the inventory, readiness assessment, student interviews and course performance, the researchers wanted to identify indicators of successful course completion, as defined by a letter grade of an A, B, or C in the course. Quantitative analyses revealed that high school GPA, with readiness test scores and the total scores on the attitude inventory were positively significantly correlated to course performance, with the strongest positive correlation being between attitude and course performance [29].

2.6 Summary

Many universities have approached the problem of student success rates in calculus (and other math courses) either through a targeted mini-review course aimed at a batch of entering students or through concurrent review methods mimicking the JiTT concept. Overall, student satisfaction has been high with these interventions, and instructors report positive feedback as well. These studies also use technology as the preferred method of assessment for tracking student deficiencies and addressing gaps in understanding. Several aspects that contribute to student success in calculus have been discussed, including mastery of fundamental skills/concepts, overcoming inherently difficult ideas, and other internal factors such as attitude and persistence in learning new ideas. Note that the teaching experiment presented here focuses on aspects related to Calculus-I students' pre-requisite knowledge and did not investigate issues related to student motivation.

Chapter 3

The Teaching Experiment

3.1 Purpose

Based on observations in introductory calculus courses, it is apparent that incoming freshman lack various mathematical skills and necessary background knowledge. As conveyed by past research, students specifically struggle with algebra and pre-calculus concepts during introductory calculus. In light of JiTT techniques, calculus readiness/success, and the documented advantages associated with web-based learning, four online review modules are administered in a Calculus-I course. These review modules focus on the topics of combining rational expressions, order of operations, composition of functions, and triangle trigonometry. Upon analysis of data pertaining to various aspects of student performance on both the online review modules and associated Calculus-I online homework assignments, we are interested in seeking answers to the following research questions, which have not been addressed in the literature thus far:

1. Do modified "just-in-time" online teaching tools covering review topics in algebra lead to gains in student learning on specific calculus topics?

2. Does timing of the review assignments make a difference in terms of student achievement on calculus online homework assignments?

3. Do students with the "just-in-time" review intervention access optional additional

feedback differently than students without the intervention?

3.2 Participants and Setting

We compare two batches of Calculus-I students from spring semesters of 2011 and 2012. The spring 2011 population consisted of an initial enrollment of 285 students, while spring 2012 contained an initial population of 345 students (these numbers are based on the number of students who completed the first online assignment). To verify that both populations of students were similar in terms of mathematical ability upon entering the course, average math ACT scores of both populations were considered. These scores were 24.9 and 24.6 in spring 2011 and spring 2012 respectively, and a two sample t-test assuming equal variances revealed that the mean ability of both groups of students as represented by their ACT math scores was the same with 99.95% confidence. Therefore, both batches of calculus students were definitely comparable.

The Calculus-I ("Analytic Geometry and Calculus I") course at this university follows a traditional lecture-recitation format where students meet in a large lecture setting twice a week and smaller breakout recitation sections twice a week. The topics covered in this course include limits, differentiation, applications of differentiation, integration, and a few applications of integration including computing area between curves and volume of solids of revolution during the spring 2011 semester . The syllabus in this course aligns well with other traditional Calculus-I courses at institutions with accredited engineering and reputable undergraduate mathematics programs.

During both semesters, the lecturers covered the same topics and assigned written homework from the same text. Online homework was a major component of the coursework as well and both groups of students used the same online homework system, whose details are described in Section 3.3. Both batches of students completed three tests and a final exam, and the course was taught in the same lecture/recitation format during both semesters.

3.3 Format of the JiTT Assignments

The review assignments that were developed for this teaching experiment follow the same format as the online homework sets that were already in place in the Calculus-I course. Both review and regular online homework assignments contain three or four problems. After logging in, the students may enter their answers and save their work, and they may come back and work on the problem set at any time before the deadline for the assignment. When students initially submit their answers for grading on the first problem set, the system instantly marks which responses are correct and incorrect, and the students have one free chance to correct their initial mistakes. After the second round of grading, students may choose to keep the score earned, or, they may log in again and try a different problem set. Students may do as many different problem sets as they wish until they are happy with the score. The system records the student's highest score attained up until the deadline for each individual assignment. All of the review assignments were written in PHP, with automatic, instantaneous grading routines written in PHP/Javascript/Java.

Numerous aspects related to individual student performance can be tracked, such as average time until one earns a perfect score, mean scores on an assignment, and how often the student accesses optional additional help. Appendix A contains each of the modified JiTT problem sets (reviews) that were newly created. It is worth noting that the instantaneous feedback feature of this online homework system aligns well with JiTT's goals. In particular, the system allows students to reach perfection on any given assignment, and they are immediately presented with worked out solutions if they haven't scored 100% on the problem set. Another notable feature of the online homework system used here is that the system supports a variety of question formats, including graphical, tabular, fill in the blank and which-of-the-following apply types of questions. The online homework system randomly generates different problems for every student; this feature differs from the online homework systems used in previous studies (see Chapter 2, Section 2.4 for instance) in that the problems do not come from a pre-determined question bank; the system used here randomly generates problems on the spot. In this manner, students truly complete different assignments so that copying solutions from each other is not an option. Figures 3.1 through 3.7 below illustrate a typical sequence of screenshots that a student cycles through on a given assignment; examples of the help links are included in these figures as well.

Figure 3.1: *Screen Shot 1: Initial Problem Set over Function Composition Review*

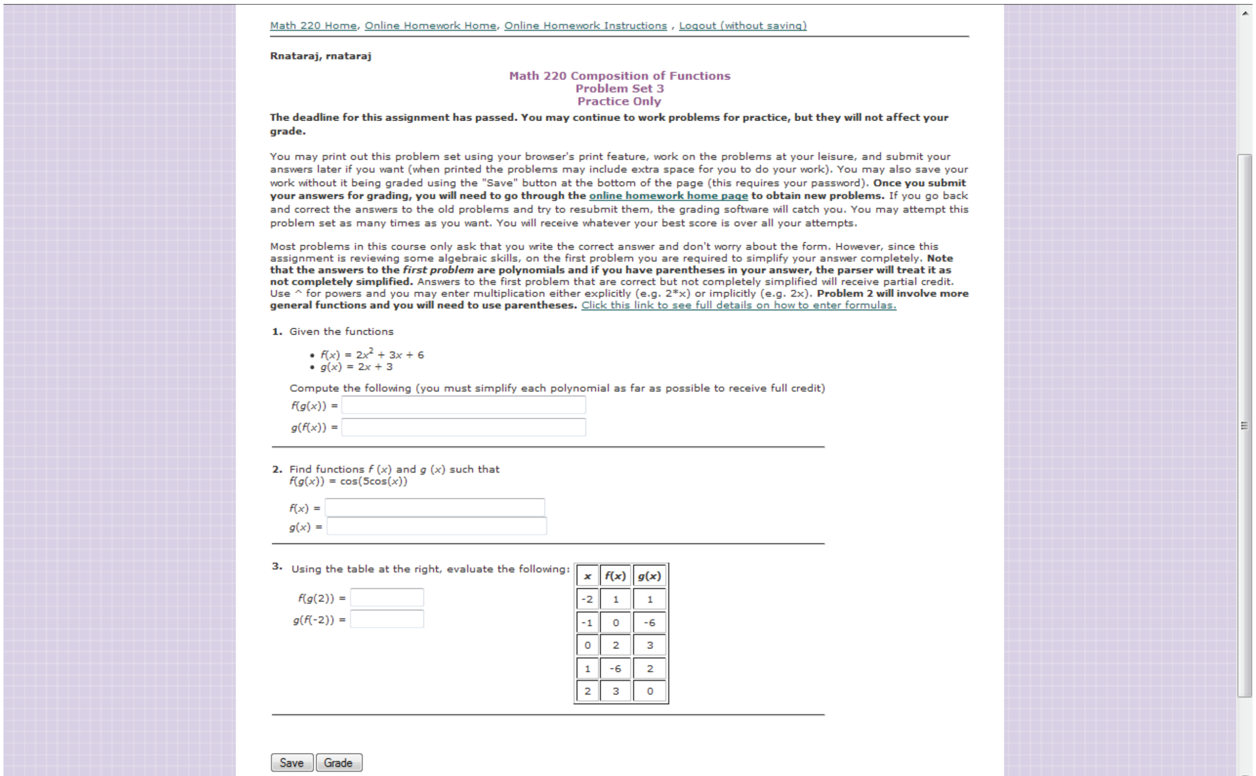
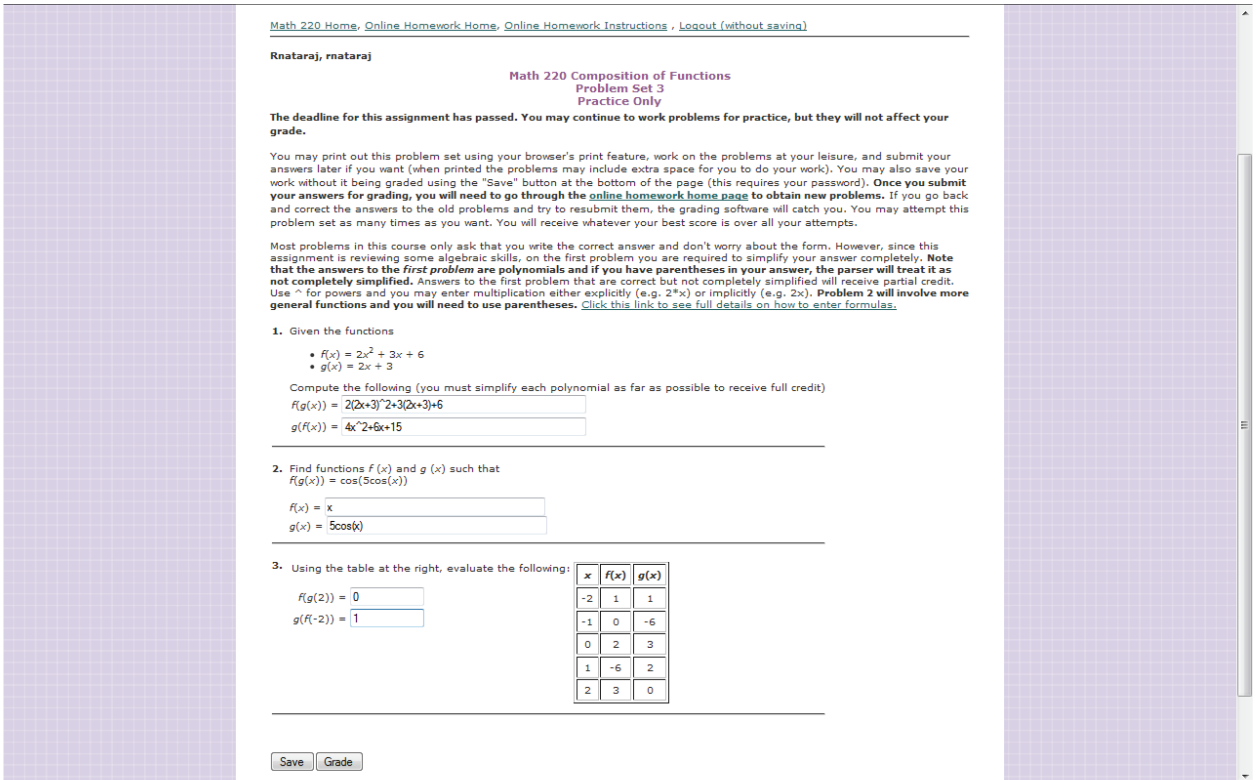


Figure 3.2: *Screen Shot 2: Sample Student Response – Function Composition Review*



Math 220 Home, Online Homework Home, Online Homework Instructions , Logout (without saving)

Rnataraj, rnataraj

Math 220 Composition of Functions
Problem Set 3
Practice Only

The deadline for this assignment has passed. You may continue to work problems for practice, but they will not affect your grade.

You may print out this problem set using your browser's print feature, work on the problems at your leisure, and submit your answers later if you want (when printed the problems may include extra space for you to do your work). You may also save your work without it being graded using the "Save" button at the bottom of the page (this requires your password). **Once you submit your answers for grading, you will need to go through the online homework home page to obtain new problems.** If you go back and correct the answers to the old problems and try to resubmit them, the grading software will catch you. You may attempt this problem set as many times as you want. You will receive whatever your best score is over all your attempts.

Most problems in this course only ask that you write the correct answer and don't worry about the form. However, since this assignment is reviewing some algebraic skills, on the first problem you are required to simplify your answer completely. **Note that the answers to the *first problem* are polynomials and if you have parentheses in your answer, the parser will treat it as not completely simplified.** Answers to the first problem that are correct but not completely simplified will receive partial credit. Use ^ for powers and you may enter multiplication either explicitly (e.g. 2*x) or implicitly (e.g. 2x). **Problem 2 will involve more general functions and you will need to use parentheses.** Click this link to see full details on how to enter formulas.

1. Given the functions

- $f(x) = 2x^2 + 3x + 6$
- $g(x) = 2x + 3$

Compute the following (you must simplify each polynomial as far as possible to receive full credit)

$f(g(x))$ = 2(2x+3)^2+3(2x+3)+6

$g(f(x))$ = 4x^2+6x+15

2. Find functions $f(x)$ and $g(x)$ such that
$f(g(x)) = \cos(5\cos(x))$

$f(x)$ = x

$g(x)$ = 5cos(x)

3. Using the table at the right, evaluate the following:

$f(g(2))$ = 0

$g(f(-2))$ = 1

x	$f(x)$	$g(x)$
-2	1	1
-1	0	-6
0	2	3
1	-6	2
2	3	0

Save Grade

Figure 3.3: *Screen Shot 3: First Round of Automatic Grading – Function Composition Review*

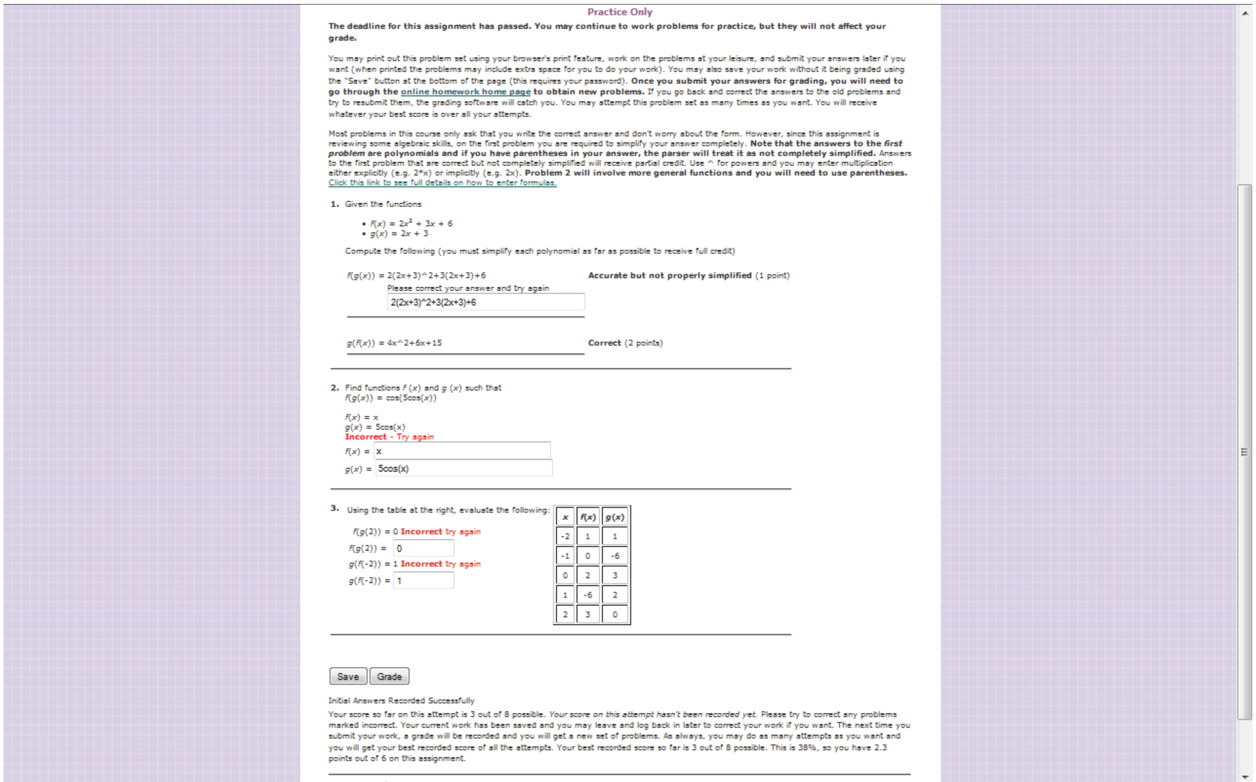
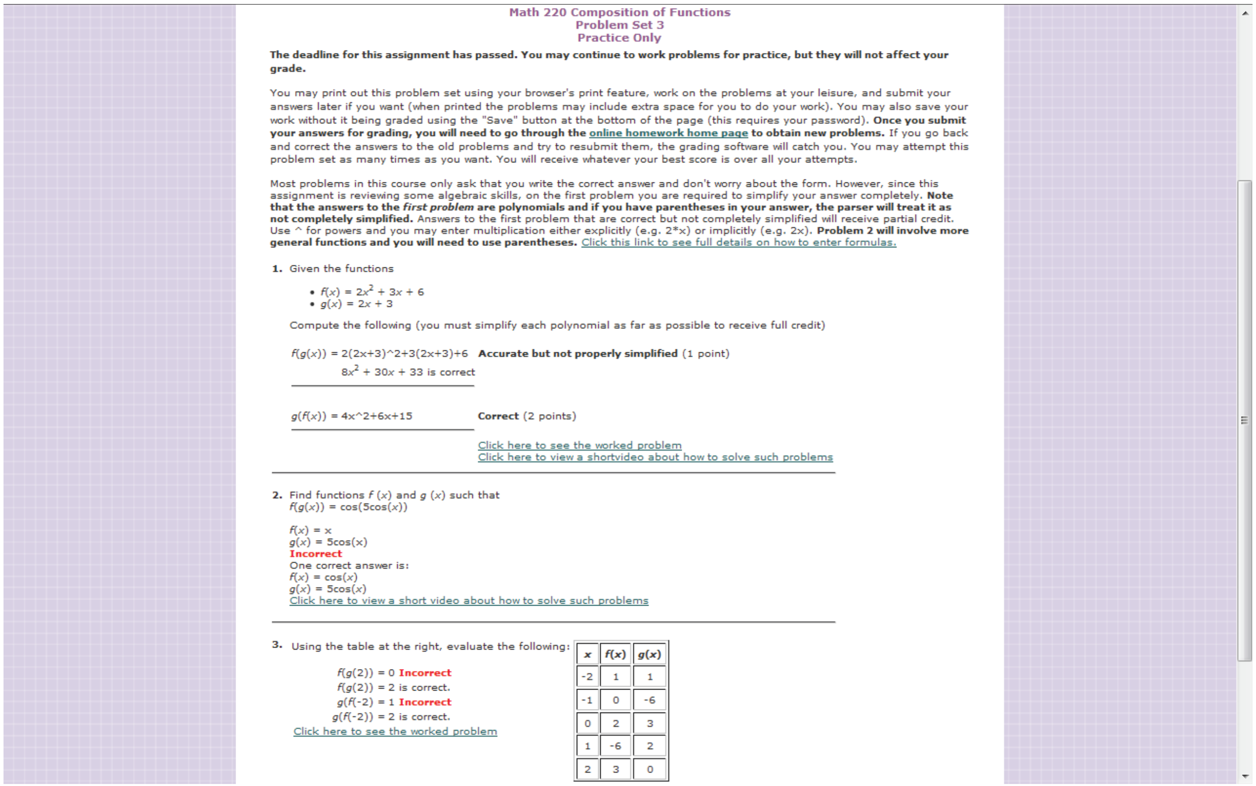


Figure 3.4: *Screen Shot 4: Second Round of Automatic Grading – Function Composition Review*



Math 220 Composition of Functions
Problem Set 3
Practice Only

The deadline for this assignment has passed. You may continue to work problems for practice, but they will not affect your grade.

You may print out this problem set using your browser's print feature, work on the problems at your leisure, and submit your answers later if you want (when printed the problems may include extra space for you to do your work). You may also save your work without it being graded using the "Save" button at the bottom of the page (this requires your password). **Once you submit your answers for grading, you will need to go through the online homework home page to obtain new problems.** If you go back and correct the answers to the old problems and try to resubmit them, the grading software will catch you. You may attempt this problem set as many times as you want. You will receive whatever your best score is over all your attempts.

Most problems in this course only ask that you write the correct answer and don't worry about the form. However, since this assignment is reviewing some algebraic skills, on the first problem you are required to simplify your answer completely. **Note that the answers to the *first problem* are polynomials and if you have parentheses in your answer, the parser will treat it as not completely simplified.** Answers to the first problem that are correct but not completely simplified will receive partial credit. Use ^ for powers and you may enter multiplication either explicitly (e.g. 2*x) or implicitly (e.g. 2x). **Problem 2 will involve more general functions and you will need to use parentheses.** Click this link to see full details on how to enter formulas.

1. Given the functions

- $f(x) = 2x^2 + 3x + 6$
- $g(x) = 2x + 3$

Compute the following (you must simplify each polynomial as far as possible to receive full credit)

$f(g(x))$ = 2(2x+3)^2+3(2x+3)+6 **Accurate but not properly simplified** (1 point)
$8x^2 + 30x + 33$ is correct

$g(f(x))$ = 4x^2+6x+15 **Correct** (2 points)

Click here to see the worked problem
Click here to view a shortvideo about how to solve such problems

2. Find functions $f(x)$ and $g(x)$ such that
$f(g(x)) = \cos(5\cos(x))$

$f(x)$ = x
$g(x)$ = 5cos(x)
Incorrect
One correct answer is:
$f(x) = \cos(x)$
$g(x) = 5\cos(x)$
Click here to view a short video about how to solve such problems

3. Using the table at the right, evaluate the following:

$f(g(2))$ = 0 **Incorrect**
$f(g(2)) = 2$ is correct.
$g(f(-2)$ = 1 **Incorrect**
$g(f(-2)) = 2$ is correct.
Click here to see the worked problem

x	$f(x)$	$g(x)$
-2	1	1
-1	0	-6
0	2	3
1	-6	2
2	3	0

Figure 3.5: *Screen Shot 5: Help Link – Function Composition Review*

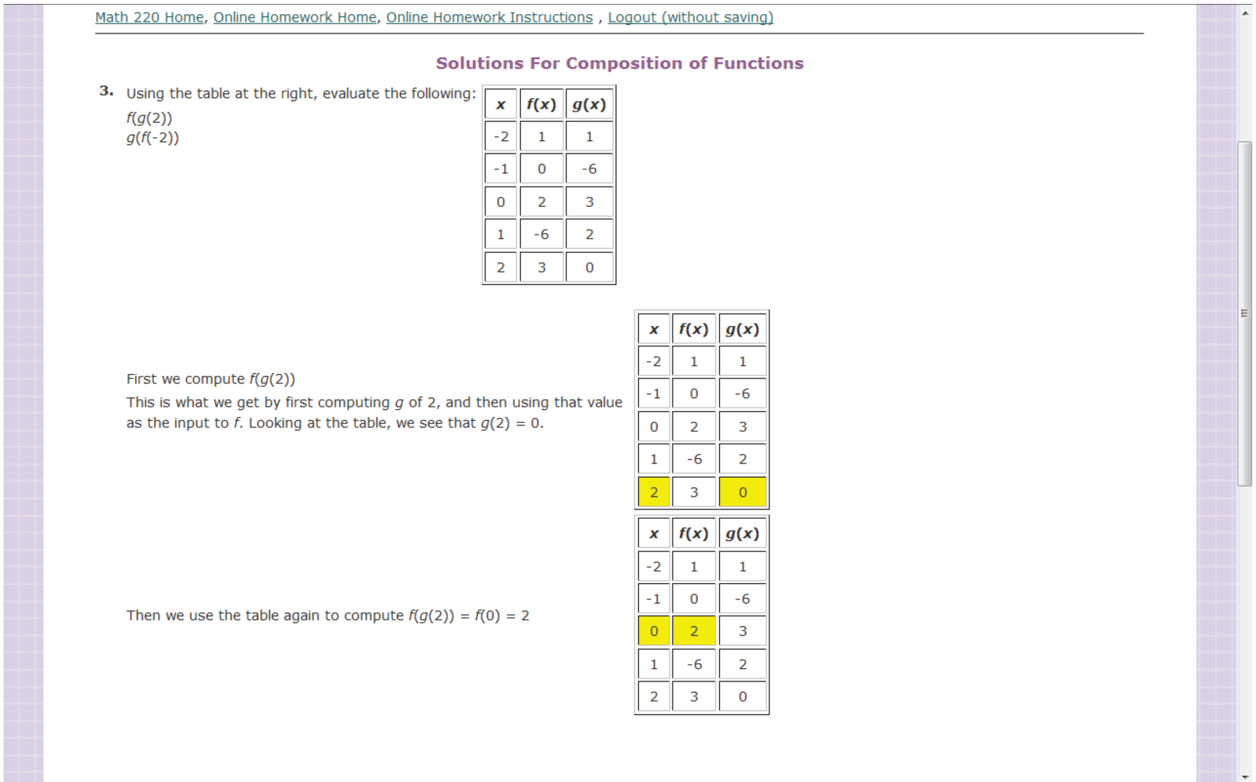


Figure 3.6: *Screen Shot 6: Video Help Link – Function Composition Review*

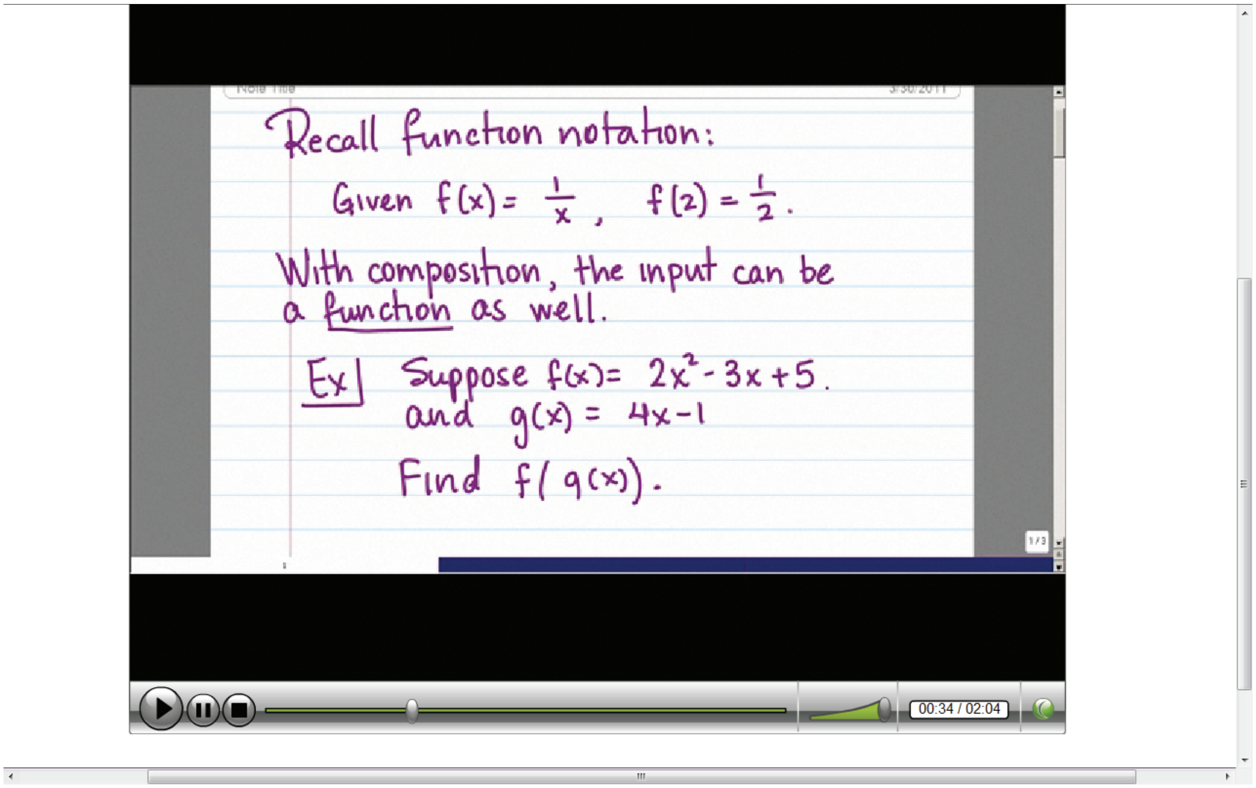
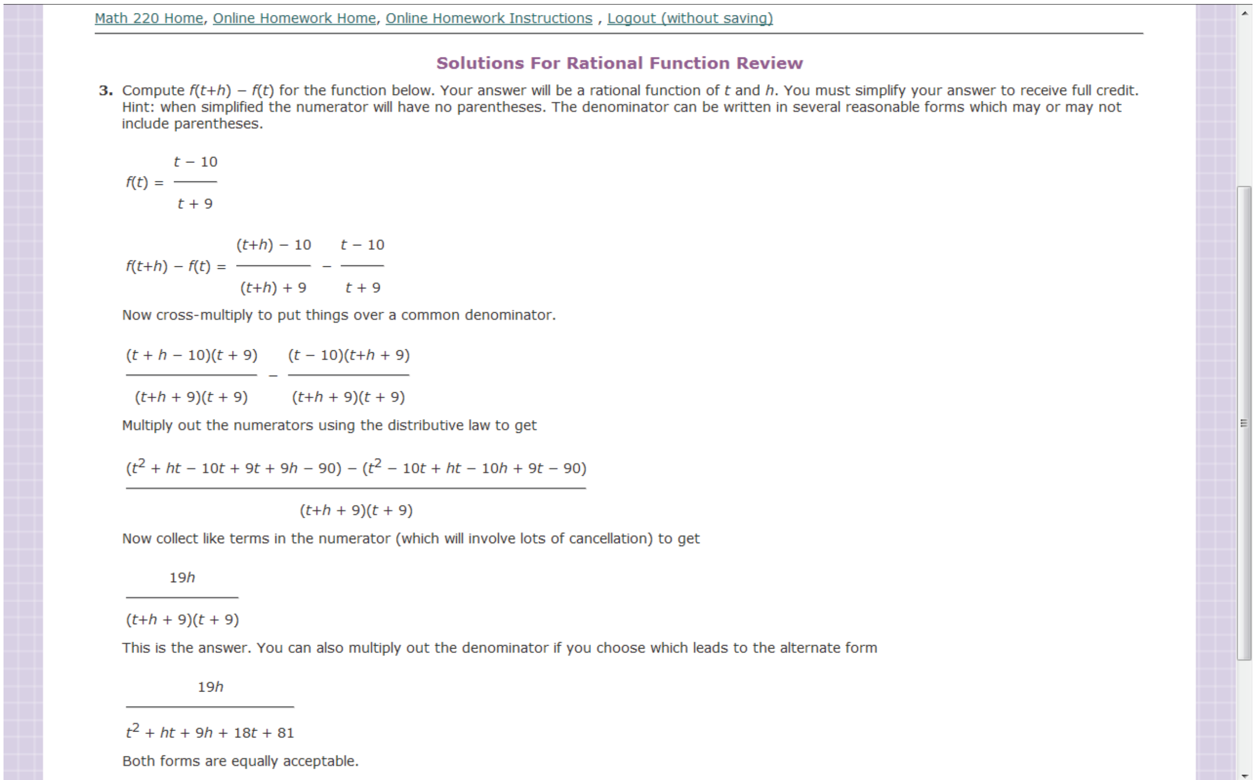


Figure 3.7: *Screen Shot 7: Help Link – Rational Expression Review*



Math 220 Home, Online Homework Home, Online Homework Instructions , Logout (without saving)

Solutions For Rational Function Review

3. Compute $f(t+h) - f(t)$ for the function below. Your answer will be a rational function of t and h. You must simplify your answer to receive full credit. Hint: when simplified the numerator will have no parentheses. The denominator can be written in several reasonable forms which may or may not include parentheses.

$$f(t) = \frac{t - 10}{t + 9}$$

$$f(t+h) - f(t) = \frac{(t+h) - 10}{(t+h) + 9} - \frac{t - 10}{t + 9}$$

Now cross-multiply to put things over a common denominator.

$$\frac{(t + h - 10)(t + 9)}{(t+h + 9)(t + 9)} - \frac{(t - 10)(t+h + 9)}{(t+h + 9)(t + 9)}$$

Multiply out the numerators using the distributive law to get

$$\frac{(t^2 + ht - 10t + 9t + 9h - 90) - (t^2 - 10t + ht - 10h + 9t - 90)}{(t+h + 9)(t + 9)}$$

Now collect like terms in the numerator (which will involve lots of cancellation) to get

$$\frac{19h}{(t+h + 9)(t + 9)}$$

This is the answer. You can also multiply out the denominator if you choose which leads to the alternate form

$$\frac{19h}{t^2 + ht + 9h + 18t + 81}$$

Both forms are equally acceptable.

3.4 Review Topics

The JiTT assignments used by Novak, as described in Chapter 2, are quite different than those designed for this teaching experiment. In particular, the preparatory assignments designed in this study are not strictly administered right before lecture, nor are they based on out of class reading. Instead, the preparatory assignments created here are solely administered online at some time during the semester, simply at some point before students need to apply the ideas when they arise in calculus. With syllabus constraints, this method of implementation was readily adaptable. Moreover, the reviews are designed in such a way that lecturers have the option of using them solely as review tools outside of class, or, they may use them in a pure JiTT manner by adjusting their lectures based on student responses.

3.4.1 Rational Expression Review

The first review assignment focuses on preparing students for computational problems involving limits and the definition of derivative. Choosing the topic of rational expressions was primarily motivated by instructors' past observations regarding obstacles that students face when learning the concept of the difference quotient in calculus. Rather than experiencing difficulty with the new idea of a difference quotient itself, students seemed to struggle more with computational skills, a struggle mainly due to a lack of fluency in manipulating rational expressions. The purpose of this review assignment is to help students make connections between new concepts and previously learned skills, connections that are one of the components of understanding first year calculus, as emphasized by Sofronosa et al. (see Chapter 2, section 2.5). In this particular assignment, the student must submit solutions to three questions. The first problem requires adding two rational expressions, possibly needing to find a least common denominator in the process. They must simplify the answer as far as possible. The second question covers compound fractions, and involves manipulating fractions using basic rules of algebra. The last exercise is a lengthier computation that one would see when applying the definition of derivative to a quadratic or rational function. The three problems have optional help links attached to them. The help links contain step by step explanations to the student's specific problem which may be viewed after having submitted answers for the second (and final) round of grading. This assignment was administered the week before the online assignment over limits.

3.4.2 Trigonometry

Recall from Chapter 2 that Carlson et al. incorporate trigonometric functions as part of the Calculus Concept Readiness Instrument [7]. Other reasons for needing a trigonometry review include the following. There are various calculus word problems that require manipulating the basic trigonometric functions. Related rates problems especially require fluency in trigonometric relationships. Secondly, students need to remember exact values of the trig functions (which most of them learn via the unit circle) – this is useful when computing definite integrals of certain trigonometric functions for instance. The review module designed here covers triangle trigonometry. The student must complete three exercises which involve finding missing angles and side lengths of triangles. The law of sines and cosines, applying the six basic trigonometric functions, and evaluating trig function values are the main ideas in this segment. All of these problems have optional help links available as well.

3.4.3 Composition of Functions

Past research suggests that a relationship exists between a student's understanding of the chain rule for derivatives and composition of functions [9]. Moreover, Ferrini and Mundi cite functions as being one of the key areas of understanding in calculus (see section 2.5), while Carlson et al. include function composition as part of their Calculus Concept Readiness Instrument [7]. In light of these past studies, the third assignment is more concept-oriented and the topic of function composition was chosen. This particular assignment takes students through three tasks. The first question is a straightforward computation of $f(g(x))$ given two functions $f(x)$ and $g(x)$. The student must simplify the composition as far as possible, according to the assignment directions. The next problem asks students to find an outside and inside function which upon composition yields a given function. The third question is purely conceptual, asking students to find function composition values from a table of values for two functions $f(x)$ and $g(x)$. All of these questions also have optional help links attached to them and a different added feature in the assignment involves a video explanation of how to do the problems as well. The video clip was created using both Camtasia recording software and Windows Journal. Camtasia is a software that captures all activity taking place on a computer screen; Windows Journal allows one to write on notebook or graph paper just as though one is taking notes in class. This assignment is administered roughly five weeks before the lecture on chain rule takes place.

3.4.4 Order of Operations

The need for an assignment on order of operations stems from observations of students' responses on the online quotient rule assignment. Previous semester logs of student work indicated difficulty in entering the correct answers, interpreting computer syntax, and using parentheses correctly. While the review assignment focuses on basic algebra, these are ideas that students must pay close attention to when submitting answers for the quotient rule assignment. This review module also contains three questions. The first question involves using a calculator to simplify a fraction in which the numerator contains a natural logarithm. The next exercise presents a fraction with various parameters and asks the student to select choices that are equivalent to the given fraction. The last problem displays an expression that one might run into when applying the quotient rule to a function involving exponentials; from a given list, students must select appropriate choices that are equivalent to the given expression. Like the other assignments, this assignment contains optional additional help. The review is given roughly two weeks prior to the lecture during which the quotient rule is covered.

3.4.5 Schedule of JiTT Assignments

During the spring 2012 semester, the modified JiTT assignments were assigned according to the schedule listed in Table 3.1.

Table 3.1: *Spring 2012 Schedule of Assignments*

Due Date	Assignment	Topic
1/23	Online Homework 1 Due	**Rational Expression Review**
1/30	Online Homework 2 Due	Limits
2/6	Online Homework 3 Due	**Trigonometry Review**
2/13	Online Homework 4 Due	**Composition of Functions Review**
2/20	Online Homework 5 Due	Differentiation
2/27	Online Homework 6 Due	**Order of Operations Review**
3/5	Online Homework 7 Due	Product Rule
3/12	Online Homework 8 Due	Quotient Rule
3/26	Online Homework 9 Due	Chain Rule
4/2	Online Homework 10 Due	Natural Logarithms
4/9	Online Homework 11 Due	Exponential Functions
4/16	Online Homework 12 Due	Finding Extreme Points
4/23	Online Homework 13 Due	Definite Integrals
4/30	Online Homework 14 Due	Integration with Substitution

Note that the first rational expression review is given "just-in-time," i.e., right before the concept is used, while the other reviews were given somewhat earlier in the semester. The difference in timing of these review assignments will help us address research question (2) discussed earlier. Students were required to do all of the online homework assignments for part of the course grade; the JiTT review assignments were mandatory and part of the course grade as well. Spring 2011 students did not have the modified JiTT review assignments; their schedule is presented below.

Table 3.2: *Spring 2011 Schedule of Assignments*

Due Date	Assignment	Topic
2/4	Online Homework 1 Due	Limits
2/11	Online Homework 2 Due	Differentiation
2/18	Online Homework 3 Due	Product Rule
2/25	Online Homework 4 Due	Quotient Rule
3/4	Online Homework 5 Due	Chain Rule
3/11	Online Homework 6 Due	Natural Logarithms
3/18	Online Homework 7 Due	Exponential Functions
4/1	Online Homework 8 Due	Finding Extreme Points
4/29	Online Homework 9 Due	Definite Integrals
5/6	Online Homework 10 Due	Integration with Substitution

3.5 Data Collection

In this teaching experiment, two sets of data are analyzed. The first data set comes from the spring 2011 calculus students' online homework scores, and serves as the control group. This group of students did not have the JiTT intervention. Recall that the students complete each of the online homework assignments individually; and while the topic of each assignment is the same, students actually work through different problem sets. The online homework system generates different problems for each student, but the problem prototypes for the students are the same.

The spring 2012 calculus students' online homework scores serve as the experimental group. These scores were tracked on the exact set of assignments as the spring 2011 population, in addition, the spring 2012 students completed four new JiTT review assignments on the topics of rational expressions, order of operations, triangle trigonometry, and composition of functions. From these two groups of students, the following data was collected.

1. The number of students who eventually completed the assignment and received a score.

2. Average time to achieving a perfect score on any given online assignment (for each individual student).

3. Inverse time to perfect for each assignment (for each individual student).

4. The average number of times help links were viewed for any given online assignment.

5. The average number of attempts for a given assignment.

6. The average score for each assignment.

7. The number of students who scored perfectly on any given assignment.

3.6 Data Analysis

To determine which type of statistical tests would help best answer the research questions at hand, both parametric and non-parametric tests were carefully considered. These two families of tests are typically used for data analysis/inferencing. Underlying assumptions and examples of these tests are summarized in this section.

3.6.1 Parametric Tests

Parametric tests assume that the given data set has some type of underlying distribution, such as a normal, poisson, or binomial distribution for instance [32]. Typical assumptions for applying parametric tests include (1) homogenous variance of data, (2) data being measured on an interval scale, and (3) subjects in the study being independently selected. Parametric tests make use of measures including mean and standard deviation, and are generally more robust than their non-parametric counterpart [32], i.e., these tests produce reliable results even when some underlying assumptions are violated. Typical parametric tests include a two-sample t-test; a t-test for paired samples, a one-way ANOVA test for more than two samples, and Pearson's R-test for a correlation test. The process of carrying out a parametric test can be summarized as follows [1]:

1. Identify null and alternate hypotheses;

2. Collect data and determine sample size;

3. Identify appropriate parametric test and calculate test statistic;

4. Calculate power of test and reject null hypothesis if $p < 1-\alpha$, where α is the confidence level.

3.6.2 Non-Parametric Tests

Nonparametric tests require fewer assumptions about the shape of the underlying population distribution. When at least one of the underlying assumptions for a parametric test is violated, the related non-parametric test is sometimes considered to be a better choice [32]. Applying a non-parametric test requires that the given data set be rank-ordered (an ordinal data set). One of the drawbacks of non-parametric tests is their lower power-efficiency [8]. Contrastingly, methods of measurement and calculation involved in non-parametric tests are much simpler [8]. Some examples of non- parametric tests include the Wilcoxon signed rank test, the Whitney-Mann-Wilcoxon (WMW) test, Kruskal-Wallis (KW) test and Friedman's chi-square test. Out of these tests, we will summarize details regarding the Mann-Whitney U-test, Wilcoxon rank-sum test, and the chi-square test, as these ultimately will be the tests applied to our raw data.

3.6.3 Mann-Whitney U-Test

The Mann-Whitney U-test is considered to be one of the most powerful non-parametric tests [2]. This test applies when parametric t-tests fail as a statistical measure for two independent groups (the experimental and control groups) [2]. Assumptions of this test include independent selection of subjects from the population and that the data set used is ordinal. As opposed to analyzing raw data directly, this test converts raw data into ranks for analytical purposes. The Mann-Whitney U-test gauges the extent to which the experimental and control groups are in fact similar. Under the null hypothesis, the assumption states that the two groups under consideration do not differ "systematically," while the alternative hypothesis states that the two sets of scores do differ "systematically" (this test was originally created to determine the probability that differences in observed group rankings are attributed to chance) [8]. The Mann-Whitney test uses what is called the U-statistic, which is computed as follows [8]. Let n_1 and n_2 be the sizes of the smaller and larger sample sizes, respectively. After ranking all the values from both samples from smallest to largest, the ranks from each of the samples must be totaled; call these sums R_1 and R_2. The U-statistic is computed for both sample sizes according to the following formulas:

$$U_1 = n_1 n_2 + \frac{n_1(n_1+1)}{2} - R_1 \tag{3.1}$$

$$U_2 = n_1 n_2 + \frac{n_2(n_2+1)}{2} - R_2 \tag{3.2}$$

A U-statistic table lists the critical U-value for the respective sample sizes; to reject the null hypothesis, $\min(U_1, U_2)$ must be lower than the critical U-value.

3.6.4 Wilcoxon Rank-Sum Test

Statistically equivalent to the Mann-Whitney U-test is the Wilcoxon rank-sum test; either test can be applied to a situation that warrants the use of these specific non-parametric tests [3]. The Wilcoxon rank-sum test, like the Mann-Whitney test, involves analyzing differences between two samples. This inferential test compares median values of two independent groups of scores [16]. Calculations involved in this test are carried out as follows [16]. With sample sizes of n_1 and n_2, observations are ranked; (just as in the Mann-Whitney U-test, these observations are drawn independently). One sums up the ranks for each of the samples, setting $N = n_1 + n_2$, so that the smallest rank receives a value of 1 and the largest rank receives a value of N. In the case of tied observations, the average of the ranks is assigned.

To carry out the Wilcoxson test, one totals the ranks of the group expected to have the smaller total. One compares this sum to the prescribed critical value based on n_1 and n_2 (found in a table of critical values). If the rank sum of the group expected to have the smaller total exceeds the critical value, then statistical significance is negated.

It is worth mentioning that the Wilcoxon test is valid for data for any distribution, regardless of normality. Moreover, the Wilcoxon test is more resistant to outliers than its parametric counterpart, the two-sample t-test.

3.6.5 Chi-Square Test

There are two types of chi-square tests, the first being the chi-square test for goodness of fit. This particular test is used when there is only one "categorical variable" under consideration [18]. This test investigates whether or not the distribution of data is "what is expected."

The other type of chi-square test checks for independence. In this version, data is viewed as two (or more) separate samples representing the different populations under comparison. This tests determines whether or not there is a relationship between two "categorical variables" [18]. The data are presented in matrix form with the different samples in rows and the categories of the variable in columns. Such a matrix is referred to as a contingency table. Being a non-parametric test, applying the chi-square test for independence requires certain assumptions, including independence of observations and nominal data (no inherent rank/order) [16]. Sample size does not matter.

Table 3.3: *Summary: Parametric vs. Non-Parametric Tests*

	Parametric	Non-Parametric
Distribution	Normal	Any
Variation	Homogenous	Any
Data Set	Interval	Ordinal
Central Measure	Mean	Median
Advantages	Robustness	Simple Calculations

After careful consideration of the data at hand and a thorough investigation of parametric vs. non-parametric statistical tests, it was decided that certain non-parametric tests would be appropriate for the analyses of interest in this experiment. To assess the extent to which the review assignments aided in student achievement on select calculus assignments during spring 2012, the chi-square test for independence and Wilcoxon rank-sum test were applied to the raw data that was collected. Results of these statistical tests appear in Chapter 4, while Chapter 5 contains a detailed interpretation of these results.

3.7 Surveys

The research questions in this study do not address student feedback/reactions. However, it is of interest to determine whether or not the student perceptions of the review tool used here align with student feedback from past studies. Therefore an attitude survey regarding the online review tool was administered at the end of the spring 2012 semester to the calculus students in the JiTT intervention group. As an incentive, completion of the survey was worth on bonus point of extra credit. As noted by past studies, students in various math courses report that JiTT methods and other similar review programs are beneficial and useful. To gain a sense of how students in this teaching experiment viewed the modified online JiTT tool, the online survey included the following items:

1. The review assignments helped me prepare for new calculus concepts.
2. I already knew the material covered in the review assignments.
3. I see value in the review material.
4. I was not comfortable with the material covered in the review assignments.
5. I enjoyed the review assignments.

The survey utilized a Likert scale in which choices A to E represented the following attitudes: A: strongly agree; B - agree; C - neutral; D - disagree; and E - strongly disagree.

Out of 343 students that were still enrolled by the end of the spring 2012 semester, 206 students responded to this survey. Actual survey results are discussed in Chapter 4 in full detail.

Chapter 4

Results

This chapter presents the raw data that was collected from the teaching experiment. Statistical results from the chi-square test for independence and Wilcoxon rank-sum test are also discussed in detail.

4.1 Data Collection from Spring 2011/Spring 2012

This section presents raw data covering various aspects of student performance on select online homework assignments during the spring 2011 and spring 2012 semesters in Calculus-I. During the spring 2011 semester, students actually completed ten online calculus assignments, while in the spring 2012 semester, students completed these ten assignments along with four additional just-in-time review assignments. The just-in-time review assignments that were designed to lead into online calculus assignments included the rational expressions review, composition of functions review, and order of operations review. The fourth just-in-time review covered triangle trigonometry and was geared towards preparing students for applications assigned through book homework in Calculus-I. There was no online calculus assignment directly covering trigonometric applications.

4.1.1 Raw Data: Review Assignments

The database connected to the online homework system used in this experiment tracks the number of students who have ever logged into and looked at the problem sets at least one time, as well as the number of students that actually completed each assignment. Table 4.1 on page 45 lists the number of students who eventually earned a score by each assignment's deadline on the four review modules during the spring 2012 semester.

Table 4.1: *Number of Students Completing Review Assignments*

Assignment	Spring 2012
Rational Expressions	321
Composition of Functions	313
Order of Operations	299
Trigonometry Review	322

Table 4.2 lists the average score earned on each of the four review modules during the spring 2012 semester. Each assignment was worth 10 points. Students have an unlimited number of chances to earn a score they are happy with, and the system retains the student's highest score attained before the deadline for each assignment.

Table 4.2: *Mean Score on Reviews*

Assignment	Spring 2012
Rational Expressions	8.71
Composition of Functions	9.42
Order of Operations	9.82
Trigonometry Review	9.51

Table 4.3 presents the average number of attempts on the four review modules during the spring 2012 semester.

Table 4.3: *Average Number of Attempts on Reviews*

Assignment	Spring 2012
Rational Expressions	2.57
Composition of Functions	2.06
Order of Operations	1.79
Trigonometry Review	1.52

Table 4.4 lists the average number of help views on the four review modules during the spring 2012 semester.

Table 4.4: *Average Number of Help Views on Reviews*

Assignment	Spring 2012
Rational Expressions	2.31
Composition of Functions	.83
Order of Operations	.29
Trigonometry Review	.81

4.1.2 Raw Data: Target Calculus Assignments

Table 4.5 lists the number of students who eventually earned a score, before each assignment's deadline, on the three calculus assignments that were paired up with a JiTT review module (referred to as 'target calculus assignments' from hereon). For comparison purposes, Table 4.5 also includes the number of students who completed the differentiation assignment, an online calculus homework assignment which did not have a modified JiTT review assignment associated with it.

Table 4.5: *Number of Students Completing Each Target Calculus Assignment*

Assignment	Spring 2011	Spring 2012
Limits	256/285=.898	330/341=.968
Chain Rule	230/245=.939	263/279=.943
Quotient Rule	223/248=.899	275/289=.952
Differentiation	260/278=.935	314/326=.963

The average score earned on the limits, chain rule, quotient rule, and differentiation online assignments are displayed in table 4.6 for both the spring 2011 and spring 2012 semesters. These assignments are also worth 10 points. Students have an unlimited number of chances to earn a score they are happy with; just as with the review assignments, the system retains the student's highest score attained before the deadline for each assignment.

Table 4.6: *Mean Score on Each Assignment*

Assignment	Spring 2011	Spring 2012
Limits	9.08	9.32
Chain Rule	7.52	7.72
Quotient Rule	9.39	9.66
Differentiation	9.55	9.55

Table 4.7 outlines the average number of times that students attempted each of the target calculus assignments during the spring 2011 and spring 2012 semesters.

Table 4.7: *Mean Number of Attempts on Each Target Assignment*

Assignment	Spring 2011	Spring 2012
Limits	2.38	2.18
Chain Rule	2.40	2.14
Quotient Rule	5.02	4.11
Differentiation	2.44	2.56

Upon finishing a problem set and earning a recorded score, students have the option to view help clips that present worked out solutions with detailed explanations of the student's specific problem set (recall that every student has different problems, but the same problem prototype, as discussed in Chapter 3 section 3.3). Table 4.8 tracks the number of times, on average, that students actually clicked on the help link for the assignments with JiTT intervention (in spring 2012) and without intervention (spring 2011).

Table 4.8: *Mean Number of Help Views*

Assignment	Spring 2011	Spring 2012
Limits	1.64	1.50
Chain Rule	1.31	1.00
Quotient Rule	2.75	1.96
Differentiation	1.48	1.23

Figure 4.1 lists the percentage of students (based on total number of students scoring on the assignment) viewing help links three or fewer times on target calculus assignments during

both semesters, while Table 4.9 presents the number of students who received a perfect score on the first try alone on the same four assignments, across both spring semesters.

Figure 4.1: *Percentage of Students Viewing Help 3 or Fewer Times*

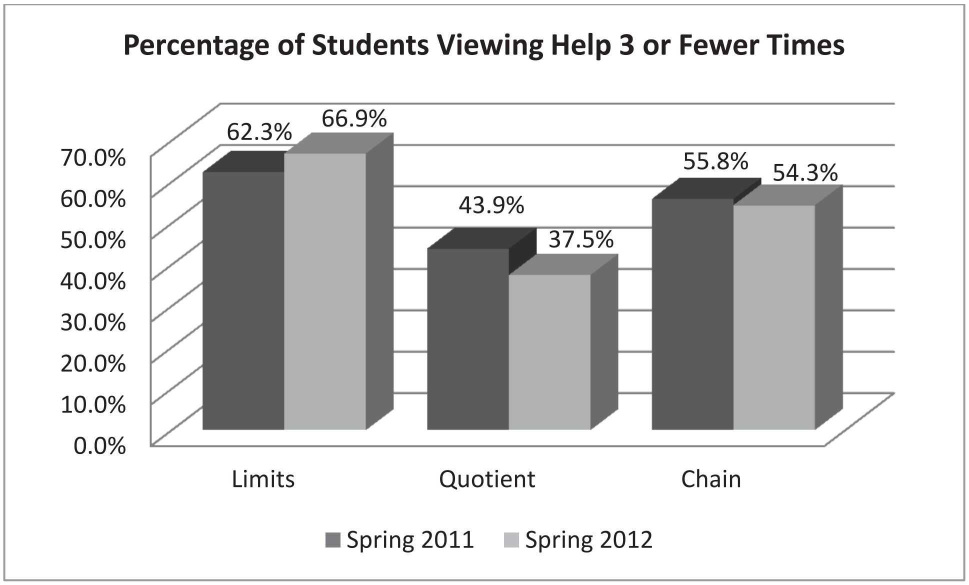


Table 4.9: *Percentage of Students Receiving a Perfect Score on Very First Attempt*

Assignment	Spring 2011	Spring 2012
Limits	76/256=.296	128/330=.388
Chain Rule	93/230=.40	138/263 = .524
Quotient Rule	33/223=.148	40/275=.145
Differentiation	88/260=.338	118/314=.375

4.2 Statistical Results: Bringing Students to Perfection

Table 4.10 outlines how many students eventually reached a perfect score on the limits, chain rule, quotient rule, and differentiation assignments. Note that the differentiation assignment did not have an online just-in-time review module associated to it, while the other three assignments did have just-in-time reviews.

Table 4.10: *Percentage of Students Receiving a Perfect Score on Assignments*

Assignment	Spring 2011	Spring 2012
Limits	172/256=.672	248/330=.752
Chain Rule	200/230=.870	245/263 = .932
Quotient Rule	126/223=.565	176/275=.64
Differentiation	235/260=.904	284/314=.904

4.2.1 Limits Assignment

We would like to determine if student performance on this assignment, as measured by perfection on the assignment, is independent of the semester in which the assignment was administered. Since the chi-square test for independence treats data as two separate samples that represent the different populations being compared, and since the data used here does not follow a normal distribution, this test is an appropriate choice. For the limits assignment, null and alternate hypotheses along with contingency tables from the chi-square test are presented below.

H_0: Student performance on the limits assignment (as measured by perfection on the assignment) and the semester in which the assignment was administered are independent.

H_1: Student performance on the limits assignment (as measured by perfection on the assignment) and the semester in which the assignment was administered are not independent.

It is important to understand the implication of H_0 within the context of JiTT, i.e., JiTT methods were only applied during the spring 2012 semester.

Table 4.11: *Chi-Square Test: Limits Assignment*

	Spring 2011	Spring 2012	Total
Perfect	172	248	420
Imperfect	84	82	166
Total	256	330	586

Table 4.12: *Chi-Square Test: Expected Values, Limits Assignment*

	Spring 2011	Spring 2012
Perfect	183.4812	236.5188
Imperfect	72.51877	93.48123
p-value= 0.033824		

4.2.2 Chain Rule Assignment

We would like to determine if student performance on this assignment, as measured by perfection on the assignment, is independent of the semester in which the assignment was administered. For the chain-rule assignment, null and alternate hypotheses along with results from the chi-square test are presented below.

H_0: Student performance on the chain rule assignment (as measured by perfection on the assignment) and the semester in which the assignment was administered are independent.

H_1: Student performance on the chain rule assignment (as measured by perfection on the assignment) and the semester in which the assignment was administered are not independent.

Table 4.13: *Chi-Square Test: Chain Rule Assignment*

	Spring 2011	Spring 2012	Total
Perfect	200	245	445
Imperfect	30	18	48
Total	230	263	493

Table 4.14: *Chi-Square Test: Expected Values, Chain Rule Assignment*

	Spring 2011	Spring 2012
Perfect	207.6065	237.3935
Imperfect	22.39351	25.60649
p-value= 0.02		

4.2.3 Quotient Rule Assignment

We would like to determine if student performance on this assignment, as measured by perfection on the assignment, is independent of the semester in which the assignment was administered. For the quotient rule assignment, null and alternate hypotheses, along with results from the chi-square test are presented below.

H_0: Student performance on the quotient rule assignment (as measured by perfection on the assignment) and the semester in which the assignment was administered are independent.

H_1: Student performance on the quotient rule assignment (as measured by perfection on the assignment) and the semester in which the assignment was administered are not independent.

Table 4.15: *Chi-Square Test: Quotient Rule Assignment*

	Spring 2011	Spring 2012	Total
Perfect	126	176	302
Imperfect	97	99	196
Total	223	275	498

Table 4.16: *Chi-Square Test: Expected Values, Quotient Rule Assignment*

	Spring 2011	Spring 2012
Perfect	135.2329	166.7671
Imperfect	87.76707	108.2329
p-value= 0.088534		

4.2.4 Differentiation Assignment

This particular assignment did not have a review assignment attached to it. Performance on this assignment was analyzed as another way of verifying that the control and experimental groups of students possessed similar mathematical ability. Null and alternate hypotheses, along with results from the chi-square test are presented below.

H_0: Student performance on the differentiation assignment (as measured by perfection on the assignment) and the semester in which the assignment was administered are independent.

H_1: Student performance on the differentiation assignment (as measured by perfection on the assignment) and the semester in which the assignment was administered are not independent.

Table 4.17: *Chi-Square Test: Differentiation Assignment*

	Spring 2011	Spring 2012	Total
Perfect	235	284	519
Imperfect	25	30	55
Total	260	314	574

Table 4.18: *Chi-Square Test: Expected Values, Differentiation Assignment*

	Spring 2011	Spring 2012
Perfect	235.0871	283.9129
Imperfect	24.91289	30.08711
p-value= 0.98		

Table 4.19 provides a summary of findings related to students achieving perfection on target calculus assignments during spring 2012.

Table 4.19: *Timing of Associated Reviews and Statistical Summary of Perfect Scores*

Timing of Review	Target Calculus Assignment	%Perfect 2011	%Perfect 2012	p-value
1 Week Before	Limits	67.2%	75.0%	.034
5 Weeks Before	Chain Rule	87.0%	93.2%	.02
2 Weeks Before	Quotient Rule	56.5%	64.0%	.089
No Review	Differentiation	90.4%	90.4%	.98

4.3 Statistical Results: Achieving Perfection More Quickly

The following statistical results help determine whether or not each just-in-time review helped improve speed to achieving a perfect score on the target calculus assignment that it was intended to help. To help answer this question, inverse-time-to-perfect data for each student was used:

$$\text{Inverse-time-to-perfect} := \frac{1}{\text{number of attempts until earning a perfect score}}. \tag{4.1}$$

This data was available for every student on every assignment for both of the spring 2011 and spring 2012 semesters. Given the nature of such a data set, it was not obvious as to which parametric test would help analyze differences in speed to perfect between the two student populations. However, by ranking students based on their inverse-time-to-perfect measurement for a given target calculus assignment, the Wilcoxon rank-sum test turned out to be an appropriate non-parametric test for the following reasons: (a) The data set in question is rank-ordered; (b) the underlying distribution of the data set is not normal; (c) both groups of scores are independent, and all online assignments were completed independently. The Wilcoxon rank-sum test, an inferential non-parametric test that compares medians of two independent groups of scores, gauges how similar the two sets of scores are. (The chi-square test for independence is not a preferred choice for this particular set of data because rank-ordered data requires comparing medians). In the data tables presented below, note that each sample size contains students who, throughout the course of their spring semester, completed at least five online assignments; n_1 and n_2 represent the spring 2011 and spring 2012 sample sizes of students who completed five or more online assignments that respective semester.

4.3.1 Rational Expression Review

To determine whether or not the rational expression review helped students reach a perfect score on the limits assignment more quickly than without having completed such a review assignment, the Wilcoxson rank-sum test was applied. The null hypothesis, H_0, and alternate hypothesis, H_1, are outlined below, while results from the Wilcoxon rank-rum test are presented in Table 4.20.

H_0: Rational review did not help speed up time to perfect on limits assignment

H_1: Rational review helped speed up time to perfect on limits assignment

Response variable used: Inverse time to perfect on the limits assignment.

Table 4.20: *Wilcoxon Rank-Sum Test: Limits Assignment*

n_1	n_2	W	p-value
155	244	19675	.2302

4.3.2 Composition of Functions Review

To determine whether or not the composition of functions review helped students reach a perfect score on the chain rule assignment more quickly than without having completed such a review assignment, the Wilcoxon rank-sum test was applied. The null hypothesis, H_0, and alternate hypothesis, H_1, are outlined below, while results from the Wilcoxon rank-sum test are presented in Table 4.21.

H_0: Function composition review did not help speed up time to perfect on chain rule assignment

H_1: Function composition review helped speed up time to perfect on chain rule assignment

Response variable used: Inverse time to perfect on the chain rule assignment.

Table 4.21: *Wilcoxon Rank-Sum Test: Chain Rule Assignment*

n_1	n_2	W	p-value
178	245	23083	.1288

4.3.3 Order of Operations Review

To determine whether or not the order of operations review helped students reach a perfect score on the quotient rule assignment more quickly than without having completed such a review assignment, the Wilcoxon rank-sum test was applied. The null hypothesis, H_0, and alternate hypothesis, H_1, are outlined below, while results from the Wilcoxon rank-sum test are presented in Table 4.22.

H_0: Order of operations review did not help speed up time to perfect on quotient rule assignment

H_1: Order of operations review helped speed up time to perfect on quotient rule assignment

Response variable used: Inverse time to perfect on the quotient rule assignment.

Table 4.22: *Wilcoxon Rank-Sum Test: Quotient Rule Assignment*

n_1	n_2	W	p-value
118	177	10492	.39

4.3.4 Student Feedback

At the end of the sprig 2012 semester, a student survey was administered asking students to rate the usefulness and value of the review assignments. Prior to administering the survey, students read a short paragraph that explained how online assignments during the spring 2012 semester of Calculus-I contained a subset of review assignments. Results from the survey are given below.

Table 4.23: *Spring 2012 Survey over JiTT Assignments*

Question	Str Agree	Agree	Neutral	Disagree	Str Disagree
Reviews helped me	30.6%	45.1%	16.5%	3.9%	3.9%
Already knew reviews	17.5%	36.4%	23.3%	15.0%	7.8%
Saw value in reviews	50%	34.5%	10.7%	2.4%	2.4%
Uncomfortable with reviews	8.3%	12.1%	23.8%	31.6%	9.2%
Enjoyed reviews	24.3%	27.2%	29.6%	9.7%	9.2%

Chapter 5

Discussion of Results

This chapter addresses each research question in context of the data reported in Chapter 4. Overall, results from the teaching experiment confirm that incorporating modified JiTT tools affects student achievement in Calculus-I positively. The relevance of the timing of these assignments with respect to our findings is also discussed. A detailed account of the individual calculus topics for which the JiTT assignments made a significant difference, including suggestions and ideas for future research, is also included in this chapter.

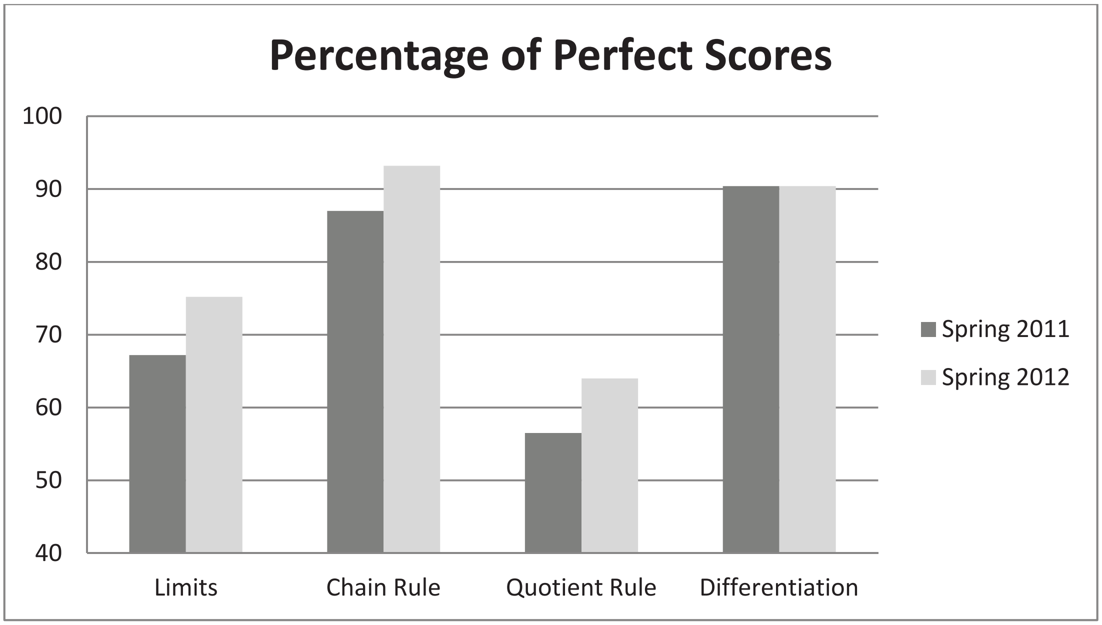
5.1 Research Question 1

Do modified "just-in-time" online teaching tools lead to gains in student learning?

As results from the individual chi-square tests for independence in Chapter 4 reveal, there is a statistically significant relationship between student performance on the limits assignment and the chain rule assignment and the semester in which these assignments were administered (chi-square with one degree of freedom, p=0.03 and p=.02 respectively).

Implications of the chi-square test for independence are certainly favorable. Higher percentages of perfect scores on the limits and chain rule assignments (summarized again in Figure 5.1) were attained during the spring 2012 semester, which was the semester during which the "just-in-time" modules were the only newly added content-specific components in the Calculus-I syllabus. As explained in Chapter 3, all other features related to course content during the spring 2011 and spring 2012 semesters, including the textbook, textbook assignments, online calculus assignments, syllabi, and number of exams were either identical or very comparable. It is safe to conclude that the just-in-time tools certainly contributed positively towards significantly raising the number of students that reach perfection on the limits and chain rule assignments.

Figure 5.1: *Percentage of Students Receiving a Perfect Score*



The differentiation assignment was a regular calculus online homework assignment that did not have a modified JiTT assignment attached to it. The chi-square test for independence for this assignment gave a p-value of virtually 1 (chi-square with one degree of freedom, $p = 0.98$ – See Table 4.18). Being an untargeted calculus assignment, this result provides additional support that the control and experimental groups of students are similar in terms of mathematical ability.

On the quotient rule assignment, although a greater percentage of students did receive a perfect score in spring 2012 compared to spring 2011, the p-value was not as low (chi-square with one degree of freedom, $p = 0.08$ – See Table 4.16). It is worth mentioning that the review assignment corresponding to the quotient rule assignment contained exercises pertaining to basic algebra. The review focused on order of operations as related to computer/calculator syntax, aiming to help students with translating written expressions into an acceptable format that the online homework system could interpret. In particular, past observations regarding students' struggles with entering proper syntax led us to hypothesize that misinterpretation/misunderstanding of parentheses might be a barrier in successfully completing the quotient rule assignment. While results from the chi-square test do not fall within a desired p-value range, we have reason to believe that the order of operations of review is certainly not harmful, as indicated by the higher percentage of perfect scores during the spring 2012 semester.

These results lead us to believe that there may also be other pre-requisite skills hindering students' performance on the quotient rule assignment. In particular, simplifying expressions that result from applying the quotient rule for derivatives often requires usage of the distributive property and rules of exponents. Perhaps incorrect manipulation of exponents and/or mismanagement of the distributive property may have contributed to overall imperfection on the online quotient rule assignment. Questions pertaining to these skills could be added to the existing problems within the order of operations review module.

Additionally, observe that the order of operations review pertains to basic skills, while the function composition review assignment was geared towards reviewing pre-requisite concepts. While our data shows that there was some improvement in spring 2012 student performance versus spring 2011 performance on the quotient rule assignment, the improvement did not reach a statistically significant level (but came close). Based on these observations, we may infer that the modified JiTT reviews seem to have the most impact when they are based on concepts as well as skills. For future assignments, it might be useful to modify some of the existing "just-in-time" reviews to include more conceptual questions, and perhaps create new reviews that emphasize conceptual thinking and reasoning.

In terms of improving speed and earning better scores at a faster rate (i.e., fewer attempts) on the calculus assignments, we would like to have seen smaller p-values in the

Wilcoxon rank-sum results. Based on the results generated by the statistical package R, we can conclude with roughly 77% confidence that the rational review assignment helped students achieve perfection at a faster rate (See Table 4.20). With slightly more confidence, in fact roughly 88% confidence, we may conclude that the composition of functions review assignment helped students achieve perfection more quickly on the chain rule assignment (see Table 4.21). The order of operations review assignment was less effective (p-value=.39) in terms of helping students get to a perfect score more quickly on the quotient rule assignment (see Table 4.22).

While the review modules certainly help with significantly bringing a larger percentage of students to perfection, the speed at which they get there is not necessarily much quicker than the students who did not have the online intervention during the spring 2011 semester. Note that student study habits might play a role in terms of the actual raw data collected regarding the "inverse-time-to-perfect" data used in the Wilcoxon rank-sum test. In particular, one of the features of the calculus online homework assignments are detailed help explanations. Students have an unlimited number of chances to earn a perfect score, so it is possible that some students deliberately answer questions incorrectly on the first attempt in order to view the help explanations, which contain step-by-step explanations followed by the correct answer. While this tactic contributes to learning the procedures involved in the problem sets, it may be possible that students do reach a perfect score more quickly at a statistically significant level. This observation is certainly worth investigating in future research via a qualitative study.

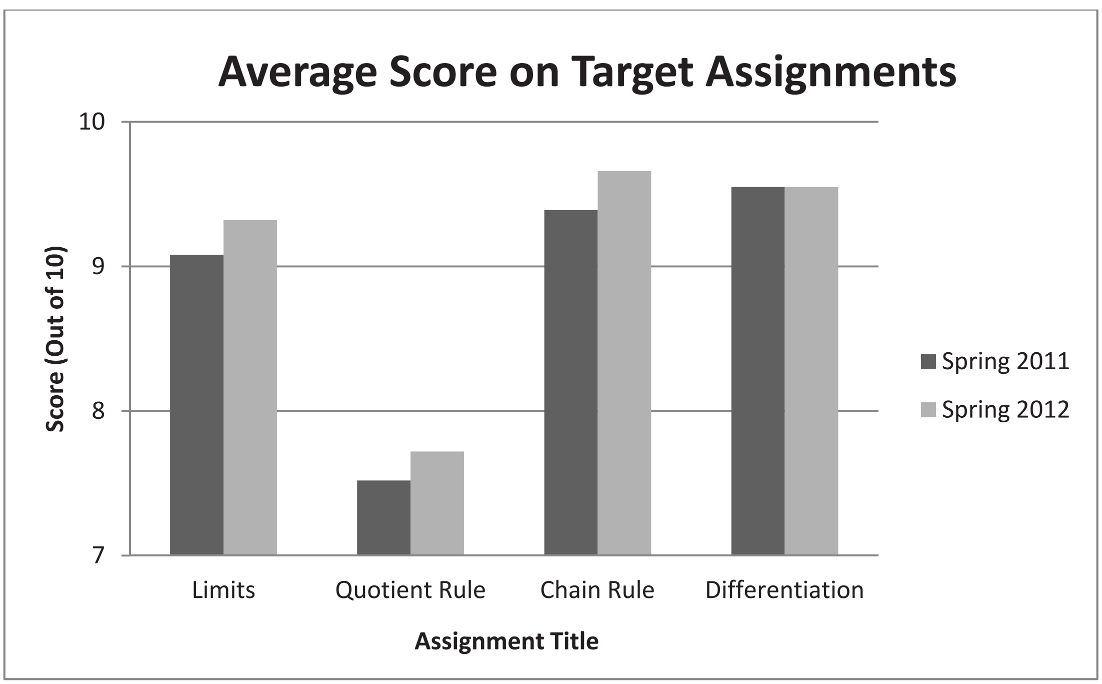
5.2 Research Question 2

Does timing of the review assignment make a difference in terms of student achievement on calculus online homework assignments?

As discussed in chapter two, traditional "Just-in-Time Teaching" methodology requires administering preparatory assignments right before students attend lecture. Ideally, reviews created in accordance with this instructional practice would have been administered no more than a few days prior to when the associated calculus topics were covered in lecture. In this study, the reviews were not implemented in a pure JiTT manner (which is why we have been referring to these reviews as "modified" JiTT assignments). In fact, one of the review assignments, namely the composition of functions review, was given five weeks prior to the chain rule online assignment. The other review assignments were given roughly one to two weeks before the associated online calculus homework was assigned (consult Table 4.19 for the exact timeline).

Several of our results affirm that timing of the JiTT reviews is not a crucial factor when it comes to improving student performance. Figure 5.2, for instance, re-caps average scores on select Calculus-I assignments, which were higher on the target calculus assignments (these were the online assignments with the JiTT intervention during the spring 2012 semester) and about the same on an assignment that had no intervention (the differentiation assignment).

Figure 5.2: *Average Score on Online Assignments with/without Intervention*



As long as students work through the review material at some point during the course prior to their application in calculus, the reviews seem to have a positive effect. In fact, the review assignment that was administered most ahead of time (five weeks ahead of time) was the composition of functions review, which corresponds to the chain rule assignment. (Recall from Table 4.19 that the chi-square test for independence for the chain rule assignment turned out to have the lowest p-value.) Rather than having completed the reviews "just-in-time," it seems that having seen the pre-requisite material at some point *during the course of the semester* did have a positive impact.

Since our results reveal that gains in learning occur regardless of the timing of the assignments, we have a flexible online tool that instructors can easily adapt without having to follow a pure "just-in-time" approach. While instructors have the option of using these reviews as a traditional "just-in-time" tool (adjusting their lectures according to student responses), this tool lends itself well to instructors who choose utilize the assignments purely for review purposes early on in the semester.

Another possible advantage of completing the review modules ahead of time rather than "just-in-time" is that if the review covered a topic that a student truly did not know/remember, the student still has some level of flexibility in terms of brushing up and reviewing these specific skills on their own time before seeing it applied to new content in calculus. Had they seen these review topics in a pure "just-in-time" setting, there would not have been a long enough break in time to sit down and learn/re-learn/re-visit the review topic on their own. Overall, the review assignments created here serve as a catalyst for triggering connections between prior knowledge and new calculus topics, even when the "just-in-time" aspect is somewhat relaxed.

5.3 Research Question 3

Do students with the "just-in-time" review intervention access optional additional feedback differently than students without the intervention?

Ideally, we would have liked to seen a significantly fewer number of help views, on average, on the target calculus assignments during the spring 2012 semester than the spring 2011 term. While students on average, viewed help clips somewhat less frequently on the limits, chain rule and quotient rule assignments (outlined in Table 5.1), the difference between semesters is quite minimal.

Table 5.1: *Mean Number of Help Views*

Assignment	Spring 2011	Spring 2012
Limits	1.64	1.50
Chain Rule	1.31	1.00
Quotient Rule	2.75	1.96
Differentiation	1.48	1.23

Upon analyzing individual student help data, i.e., the number of times each student viewed a help clip for each assignment during the spring 2011 and spring 2012 semesters, the Wilcoxon rank-sum Test did not reveal anything statistically significant (See Appendix C for individualized help data). When looking at the percentage of students viewing help three or fewer times during each semester, results did not reveal any further insight regarding patterns in spring 2012 students accessing help less often than spring 2011 students.

It is worth noting that some of the review topics resulted in more average help views than others. As Table 4.4 illustrates, students accessed help clips the most on the rational expression review assignment, viewing help 2.31 times on average; students accessed help fewer than one time per assignment on the other three reviews. Moreover, students took a longer time to achieve a perfect score on the rational expression review; on average, it took 2.67 attempts to earn a perfect score; students also attempted this assignment more often, averaging 2.57 overall attempts. These results affirm that the topic of rational expressions is certainly a problematic area for students coming into the course; having access to such a review appears to be useful.

Patterns in how students approach the help links are worth exploring further, as it is possible that some students purposely miss questions on the first attempt of the online homework in order to view the help explanations (which are very detailed, outlining every step of a student's specific problem). In order to gain additional clarity as to why students access optional help links even after having completed a relevant review module ahead of time, it may be worthwhile to conduct a qualitative study that investigates these ideas.

5.4 Student Feedback

Student feedback regarding the modified "just-in-time" reviews indicate success. As shown by Table 4.23, our results align well with student reactions to such teaching tools that have been reported in the literature. With regard to our teaching experiment, approximately 76% of the students who responded to the survey either agreed or strongly agreed that the review assignments were helpful to them, while roughly 50% of the students enjoyed completing the review assignments. About 85% of the students reported seeing value in the assignments. That students find the reviews a valuable resource is very welcoming and rewarding. Put together, positive student feedback and gains in learning on the topics of limits and the chain rule reveal that the modules are useful in various ways; not only do they improve student achievement, but they also convey value and added worth as a study tool in the eyes of students.

5.5 Summary of Research Findings

The modified JiTT tools created for this teaching experiment affected student achievement in Calculus-I on the topics of limits and chain rule by significantly increasing the number of perfect scores on these online calculus assignments during the spring 2012 semester. The tools also helped with improving performance on the quotient rule online homework assignment, but this improvement was not statistically significant. Moreover, this online review tool can be easily incorporated into any homework schedule given its flexibility with respect to timing of the review assignments. Although the reviews do not affect how quickly students get to a perfect score, the reviews help with bringing more students to perfection on the topics listed above. Along with resulting in gains in learning, this tool is also viewed positively by students.

5.6 Implications and Future Work

Based on results and analysis of a modified “just-in-time” teaching approach, the following areas might be worth investigating in relation to student achievement in calculus, and more generally, in mathematics courses as a whole. In relation to this study, investigating student patterns in accessing help links through a qualitative study might help better determine if students who completed review modules are truly accessing help on a specific calculus assignment less often than students who did not complete the reviews. Clinical interviews might also help us better understand what difficulties students encountered in the quotient rule assignment, which was the target calculus assignment whose chi-square tests had the lowest level of statistical significance. Spring 2012 students still outperformed spring 2011 students on this assignment, but there is perhaps room for improvement in the design of its associated review module. In fact, one might experiment with expanding the order of operations review module by including additional exercises on properties of exponents and manipulation of radical expressions.

The fourth review module, over trigonometry, was created based on students encountering trigonometric relationships in many calculus applications. In particular, the module was intended to prepare students for related rates problems. The effectiveness of the trigonometry module was not directly analyzed in this study, as it was not possible to collect data on performance of related rates problems from the control group of spring 2011 students. A future study might investigate whether or not the trigonometry module helps with such applications in calculus.

While the spring 2012 semester only adapted four modified JiTT review assignments, it might always be worth exploring other areas of review. In particular, an assignment that tests students’ knowledge on graphing common functions might be useful. The motivation for such a review lies behind the concept of graphing the derivative function; being able to quickly recall graphs of known functions is a necessary skill. Finding equations of lines in point slope form might also be a relevant review module when students embark on computing tangent line equations; likewise, reviewing concepts from physics, such as displacement, velocity, and acceleration might also be another area to look into when students begin the differentiation unit in Caclulus-I.

Assessing the effectiveness of a modified "just-in-time" teaching approach in subsequent calculus courses might also yield useful information pertaining to retention of students at the university level. In particular, since we have demonstrated that such tools do lead to gains in student achievement, adapting them might lead to more success in mathematics courses that traditionally struggle with producing higher pass rates. Perhaps a longitudinal study across the whole calculus sequence and differential equations in which modified JiTT tools are utilized in each course along the way might better address questions surrounding retention of students from one course to the next.

While this study was conducted during a spring semester, fall semester data might lead to different results. One might investigate how effective these review modules are on an incoming class of students who are predominantly freshmen having just graduated from high school and worked with mathematics very recently. In particular, whether they perceive the review modules with as much value as the spring 2012 students did would be an interesting question to answer. One of the other limitations mentioned in Chapter 1 is that this study does not identify student characteristics that may have contributed to the findings. Once again, future qualitative research might help to better address this area.

5.7 Conclusion

The research study presented here investigated the effectiveness of a modified "just-in-time" set of review modules covering algebra topics in context of relevant calculus material. By administering these review modules via the online homework system traditionally used in this calculus course, we have discovered a flexible, time-independent method for addressing gaps in background knowledge that also leads to a statistically significant increase in student achievement on the topics of limits and the chain rule. With limits being an inherently difficult topic for first semester calculus students to master, the success of this tool in context of this topic is encouraging. With added flexibility of dropping the "just-in-time" feature, easy implementation is another positive aspect of these review modules. We hope that future research might address the limitations encountered in this study, offering new perspectives and insight regarding connections that students make between background mathematical knowledge and new content.

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Appendix A: Review Assignments

Math 220 Composition of Functions
Problem Set 1

You may print out this problem set using your browser's print feature, work on the problems at your leisure, and submit your answers later if you want (when printed the problems may include extra space for you to do your work). You may also save your work without it being graded using the "Save" button at the bottom of the page (this requires your password). **Once you submit your answers for grading, you will need to go through the online homework home page to obtain new problems.** If you go back and correct the answers to the old problems and try to resubmit them, the grading software will catch you. You may attempt this problem set as many times as you want. You will receive whatever your best score is over all your attempts.

Most problems in this course only ask that you write the correct answer and don't worry about the form. However, since this assignment is reviewing some algebraic skills, on the first problem you are required to simplify your answer completely. **Note that the answers to the *first problem* are polynomials and if you have parentheses in your answer, the parser will treat it as not completely simplified.** Answers to the first problem that are correct but not completely simplified will receive partial credit. Use ^ for powers and you may enter multiplication either explicitly (e.g. 2*x) or implicitly (e.g. 2x). **Problem 2 will involve more general functions and you will need to use parentheses.** Click this link to see full details on how to enter formulas.

1. Given the functions

- $f(x) = 4x^2 - 5x + 3$
- $g(x) = 5x + 5$

Compute the following (you must simplify each polynomial as far as possible to receive full credit)

$f(g(x)) =$ ______

$g(f(x)) =$ ______

Math 220 Composition of Functions
Problem Set 1 , continued

---

2. Find functions $f(x)$ and $g(x)$ such that
$f(g(x)) = \ln(2\sin(x))$

$f(x) =$ [ ]

$g(x) =$ [ ]

---

3. Using the table at the right, evaluate the following:

$f(g(-1)) =$ [ ]

$g(f(0)) =$ [ ]

x	$f(x)$	$g(x)$
-2	1	1
-1	-1	-1
0	0	0
1	6	-3
2	-3	6

Save Grade

Math 220 Rational Function Review
Problem Set 1

You may print out this problem set using your browser's print feature, work on the problems at your leisure, and submit your answers later if you want (when printed the problems may include extra space for you to do your work). You may also save your work without it being graded using the "Save" button at the bottom of the page (this requires your password). **Once you submit your answers for grading, you will need to go through the online homework home page to obtain new problems.** If you go back and correct the answers to the old problems and try to resubmit them, the grading software will catch you. You may attempt this problem set as many times as you want. You will receive whatever your best score is over all your attempts.

Simplify the following rational expressions as completely as possible. When entering an answer into the box for the numerator (or denominator), you may leave it either in factored form, such as (x-3)(x+2), or, you may multiply it out completely (for instance x^2-x-6). Each problem below has specific directions that you should read carefully. You may save your work at any time and come back to it. Once you submit your answers for grading the first time, you will have one chance to correct your answers (unless you earn a perfect score the first time)

1. Simplify the rational expression

$$\frac{\dfrac{1}{x} - \dfrac{1}{8}}{x - 1}$$

Enter the numerator and denominator of your answer below. You must simplify your answer to receive full credit. Hint: when simplified the numerator will have no parentheses. The denominator can be written in several reasonable forms which may or may not include parentheses.

2. Simplify the rational expression. You may leave in factored form.

$$\frac{-4}{x^2 - 4x + 3} - \frac{7}{x - 1}$$

Enter the numerator and denominator of your answer below:

$$\frac{\square}{\square}$$

3. Compute $f(t+h) - f(t)$ for the function below. Your answer will be a rational function of t and h. You must simplify your answer to receive full credit. Hint: when simplified the numerator will have no parentheses. The denominator can be written in several reasonable forms which may or may not include parentheses.

$$f(t) = \frac{-12t + 10}{t - 12}$$

Enter the numerator and denominator of your answer below:

$$f(t+h) - f(t) = \frac{\square}{\square}$$

Save Grade

Math 220 Order of Operations
Problem Set 1

1. Use your calculator to compute:
ln(39)/28

- 0.331
- 1.099
- 0.131
- 0.144

---

2. Consider the expression $(T + R)/S^2$
Which of the following are equivalent to this expression? Please select all that apply.

a.) $T + R/S^2$

b.) $T/S^2 + R/S^2$

c.) $T + (R)/S^2$

d.) $(T/S^2) + (R/S^2)$

---

3. Consider the expression $(6x^5e^x - x^6e^x)/e^{2x}$
Which of the following are equivalent to the above expression? Please select all that apply.

a.) $6x^5/e^x - x^6/e^x$

b.) $(6x^5 - x^6)/e^x$

c.) $6x^5e^x - (x^6e^x)/e^{2x}$

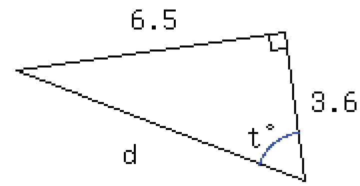
d.) $6x^5e^x - x^6/e^{3x}$

e.) $6x^5/e^x - x^6e^{-x}$

Math 220 Trigonometry Review
Problem Set 1

1.

Find the marked angle in degrees and the length of the unknown side of the triangle shown at the right. Your answers must be written as decimals and accurate to within 0.1.

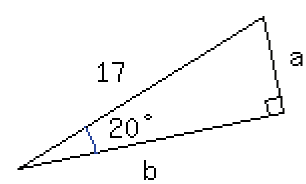


t = [ ] °

d = [ ]

2.

Find the lengths of the sides of the triangle shown at the right, with the marked side of length 17 and the marked angle of 20°. Your answers must be written as decimals and accurate to within 0.1.



a = [ ]

b = [ ]

---

3. Find the lengths of the sides of the triangle shown at the right, with the marked side of length 13 and the marked angle of 64°. Your answers must be written as decimals and accurate to within 0.1.

a = [ ]

b = [ ]

Save | Grade

Appendix B: Raw Data

Spring 11- RAW DATA

quiz	name
1	Limits
2	Differentiation
3	The Product Rule
4	The Quotient Rule
5	The Chain Rule
6	Finding Extreme Points
7	Natural Logarithms
8	Exponential Functions
9	Definite Integrals
10	Integration By Substitution
11	Integration Using ln AND exp

KEY: started: number of students who started the assignment
scored: number of students who actually earned a score
perfect: number of students who earned a perfect score
mean: average score on the assignment
m_atts: average number of attempts on assignment
s_atts: standard deviation of m_atts
atp: average time to getting a perfect score on assignment
help: average number of help views

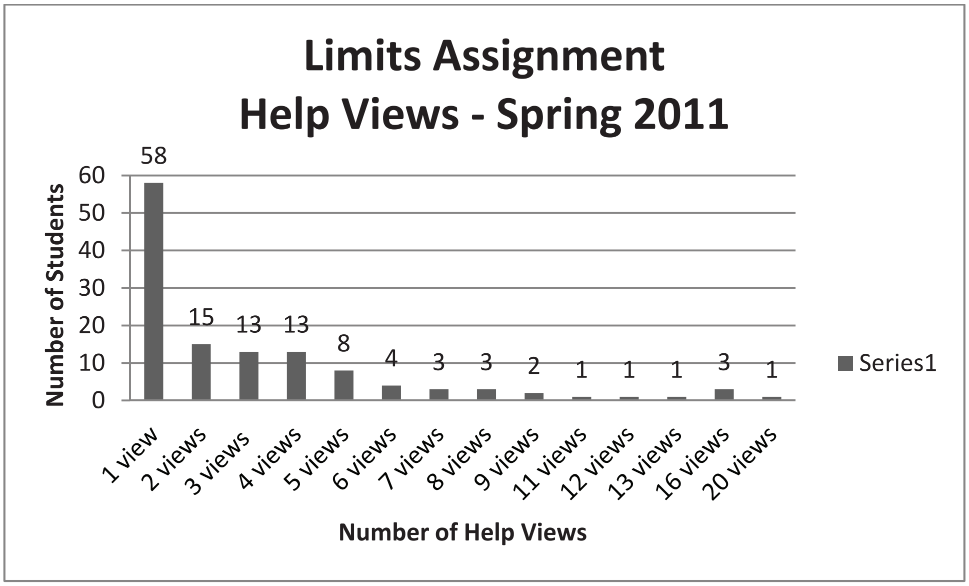
quiz	started	scored	perfect	mean	m_atts	s_atts	atp	help
1	285	256	172	9.08	2.38	2.21	2.28	1.64
2	278	260	235	9.55	2.44	2.14	2.39	1.48
3	240	208	185	9.44	2.38	2.53	2.23	1.64
4	248	223	126	7.52	5.02	6.00	4.25	2.75
5	245	230	200	9.39	2.40	2.40	2.20	1.31
6	237	212	146	8.47	2.59	2.13	2.56	1.78
7	216	182	121	8.21	3.34	2.59	3.82	3.43
8	223	197	134	8.72	3.42	3.94	3.60	3.74
9	206	192	158	8.93	2.60	2.03	2.33	2.05
10	192	177	136	8.50	2.94	3.09	2.80	2.68
11	169	155	119	8.58	3.57	2.53	3.86	2.78

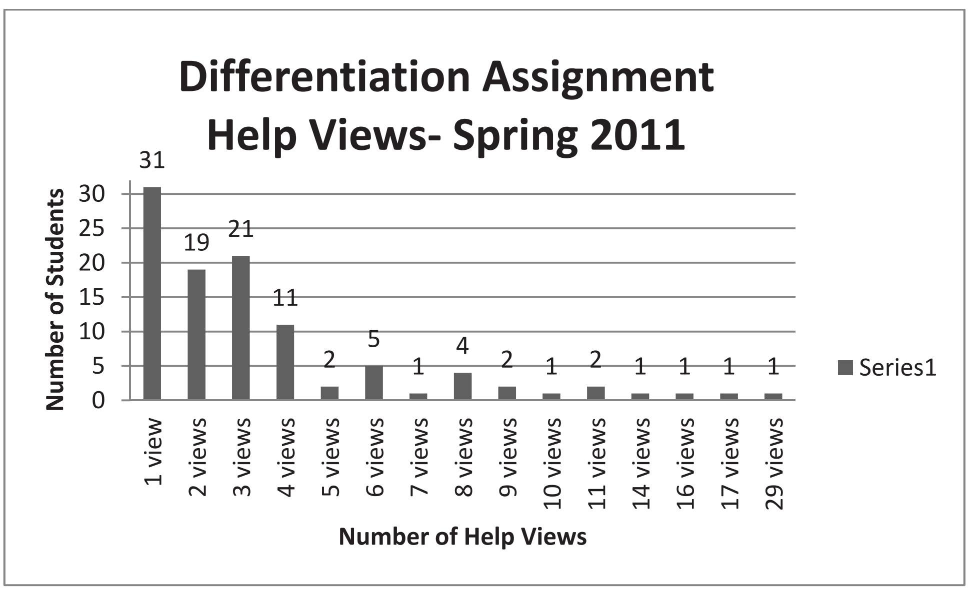
Spring 2012- RAW DATA

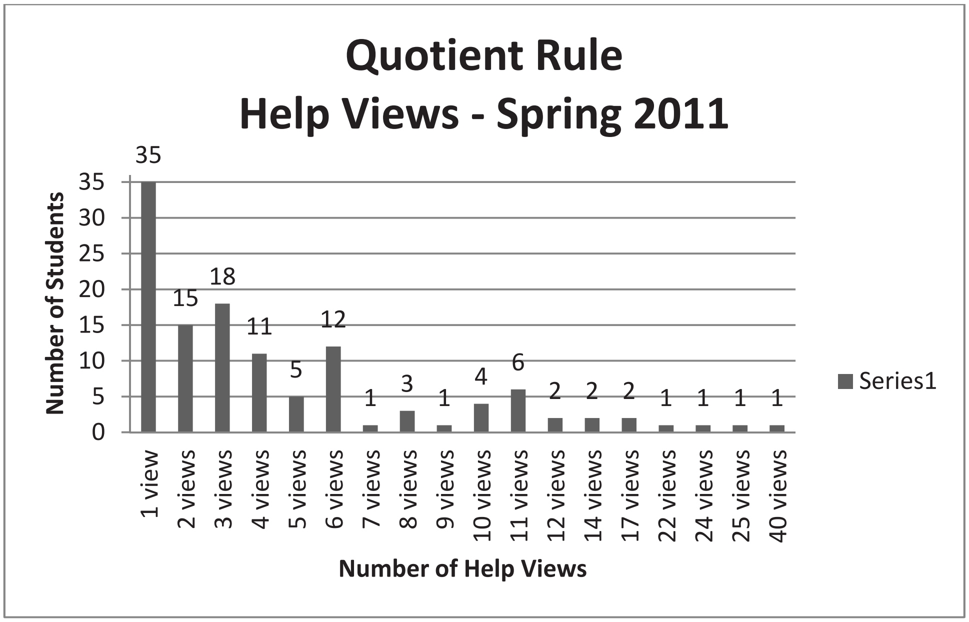
quiz	name
1	Rational Function Review
2	Limits
3	Trigonometry Review
4	Composition of Functions
5	Differentiation
6	Order of Operations
7	The Product Rule
8	The Quotient Rule
9	The Chain Rule
10	Natural Logarithms
11	Exponential Functions
12	Finding Extreme Points
13	Definite Integrals
14	Integration By Substitution
15	Integration Using ln and exp

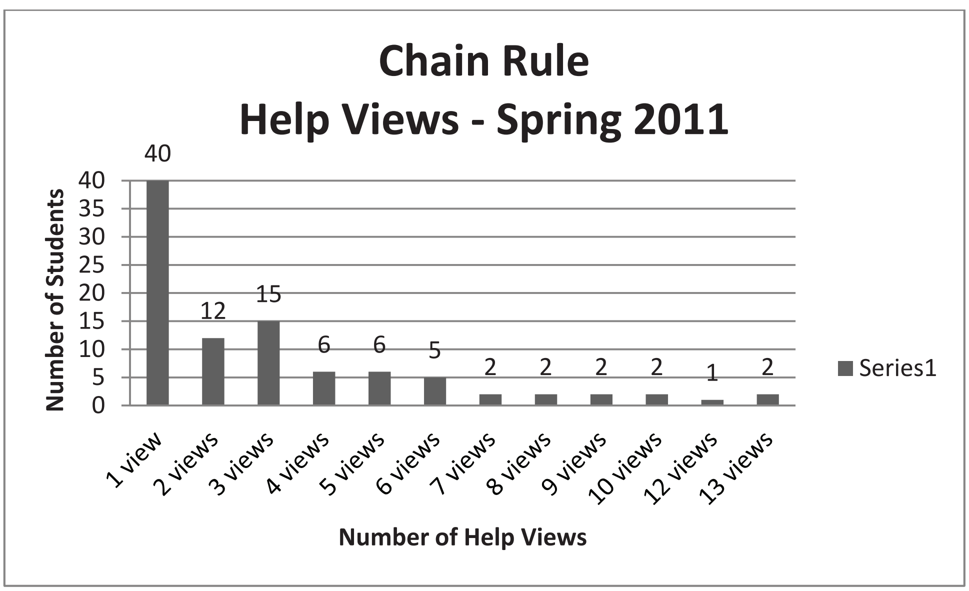
KEY: started: number of students who started the assignment
scored: number of students who actually earned a score
perfect: number of students who earned a perfect score
mean: average score on the assignment
m_atts: average number of attempts on assignment
s_atts: standard deviation of m_atts
atp: average time to getting a perfect score on assignment
help: average number of help views

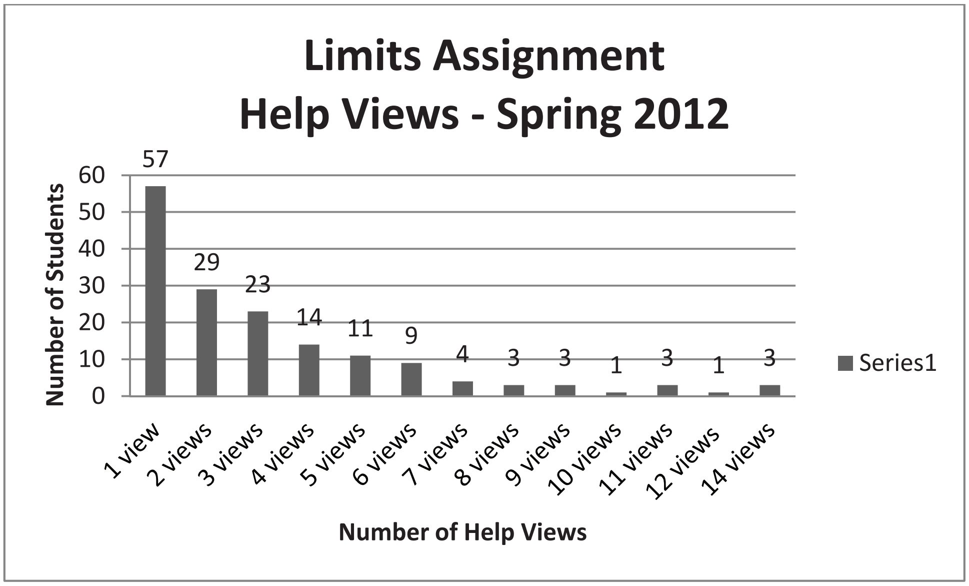
quiz	started	scored	perfect	mean	m_atts	s_atts	atp	help
1	345	321	236	8.71	2.57	1.76	2.67	2.31
2	341	330	248	9.32	2.18	2.18	2.18	1.50
3	337	322	285	9.51	1.52	0.89	1.47	0.81
4	326	313	259	9.42	2.06	1.48	2.04	0.83
5	326	314	284	9.55	2.56	2.11	2.50	1.23
6	306	299	269	9.82	1.79	0.93	1.83	0.29
7	306	288	261	9.53	1.97	1.54	1.77	1.06
8	289	275	176	7.72	4.11	4.59	3.78	1.96
9	279	263	245	9.66	2.14	1.93	2.08	1.00
10	275	261	186	8.56	3.29	2.64	3.46	2.71
11	55	35	24	8.07	2.56	2.18	2.63	2.59
12	16	8	3	5.41	1.38	0.52	1.67	1.88

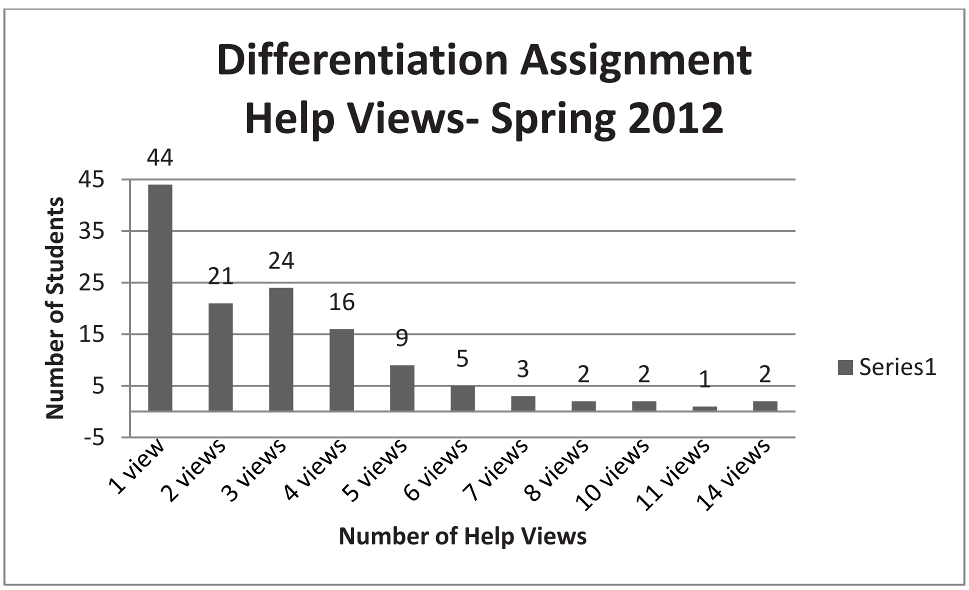

Limits Assignment
Help Views - Spring 2011
Number of Students
60
50
40
30
20
10
0
58
15
13
13
8
4
3
3
2
1
1
1
3
1
1 view
2 views
3 views
4 views
5 views
6 views
7 views
8 views
9 views
11 views
12 views
13 views
16 views
20 views
Number of Help Views
Series1

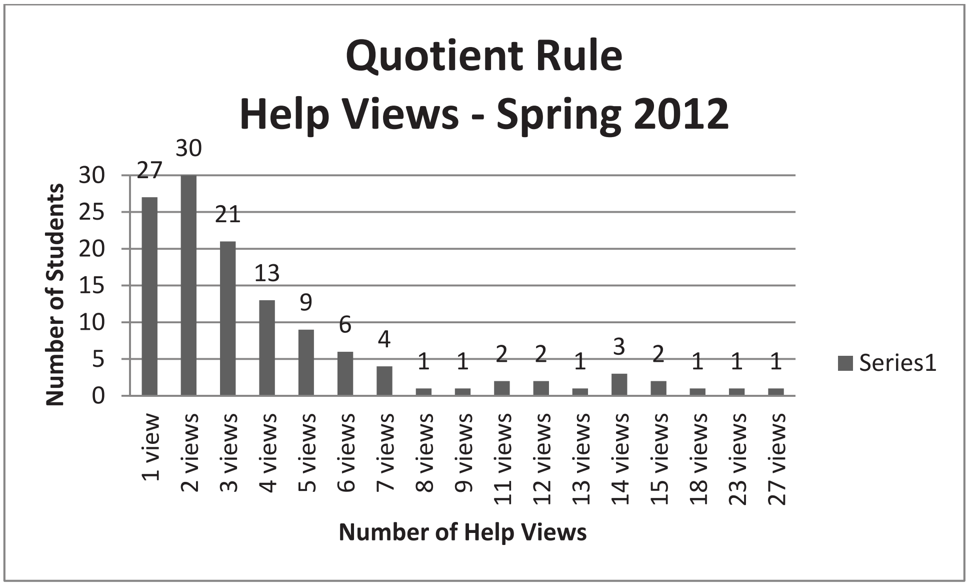

Differentiation Assignment
Help Views- Spring 2011
Number of Students
30
25
20
15
10
5
0
31
19
21
11
2
5
1
4
2
1
2
1
1
1
1
1 view
2 views
3 views
4 views
5 views
6 views
7 views
8 views
9 views
10 views
11 views
14 views
16 views
17 views
29 views
Number of Help Views
Series1


Quotient Rule
Help Views - Spring 2011
Number of Students
Number of Help Views
Series1
1 view
2 views
3 views
4 views
5 views
6 views
7 views
8 views
9 views
10 views
11 views
12 views
14 views
17 views
22 views
24 views
25 views
40 views
35
15
18
11
5
12
1
3
1
4
6
2
2
2
1
1
1
1

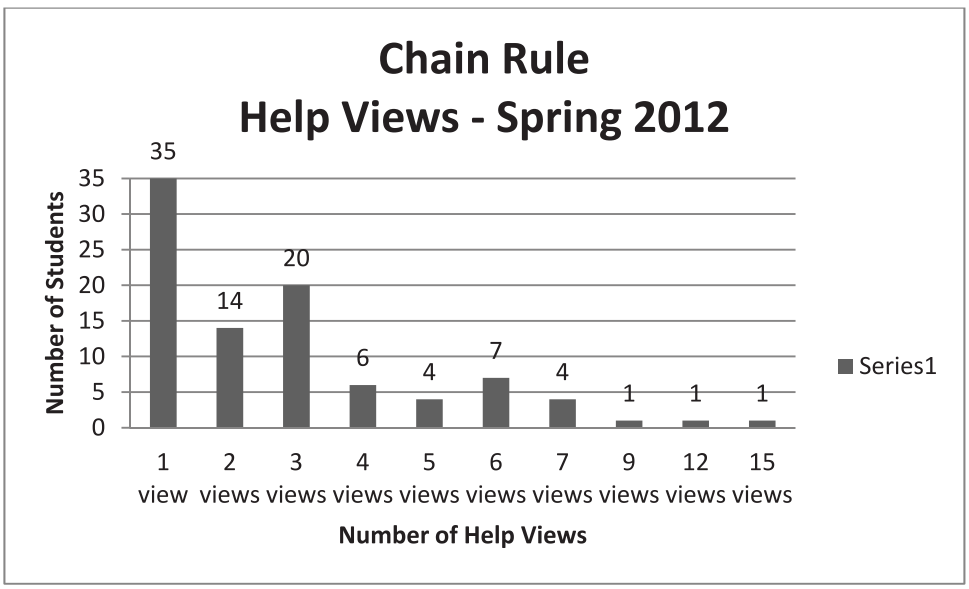

Chain Rule
Help Views - Spring 2011
Number of Students
Number of Help Views
Series1
1 view
2 views
3 views
4 views
5 views
6 views
7 views
8 views
9 views
10 views
12 views
13 views
40
12
15
6
6
5
2
2
2
2
1
2


Limits Assignment
Help Views - Spring 2012
Number of Students
60
50
40
30
20
10
0
57
29
23
14
11
9
4
3
3
1
3
1
3
Series1
1 view
2 views
3 views
4 views
5 views
6 views
7 views
8 views
9 views
10 views
11 views
12 views
14 views
Number of Help Views


Differentiation Assignment
Help Views- Spring 2012
Number of Students
45
35
25
15
5
-5
44
21
24
16
9
5
3
2
2
1
2
Series1
1 view
2 views
3 views
4 views
5 views
6 views
7 views
8 views
10 views
11 views
14 views
Number of Help Views



Quotient Rule
Help Views - Spring 2012
Number of Students
0
5
10
15
20
25
30
27
30
21
13
9
6
4
1
1
2
2
1
3
2
1
1
1
1 view
2 views
3 views
4 views
5 views
6 views
7 views
8 views
9 views
11 views
12 views
13 views
14 views
15 views
18 views
23 views
27 views
Number of Help Views
Series1



Chain Rule
Help Views - Spring 2012
Number of Students
0
5
10
15
20
25
30
35
35
14
20
6
4
7
4
1
1
1
1 view
2 views
3 views
4 views
5 views
6 views
7 views
9 views
12 views
15 views
Number of Help Views
Series1

Appendix C: Inverse Time To Perfect Data

Quotient Rule

SP11	SP12		SP11	SP12		SP11	SP12		SP11	SP12		SP11	SP12
0.03	0.03		0.25	0.2		0.5	0.33		1	0.5			1
0.05	0.04		0.25	0.2		0.5	0.33		1	0.5			1
0.05	0.07		0.25	0.2		0.5	0.33		1	0.5			1
0.06	0.07		0.25	0.2		0.5	0.33		1	0.5			1
0.06	0.07		0.25	0.2		0.5	0.33		1	0.5			1
0.07	0.07		0.25	0.2		0.5	0.33		1	0.5			1
0.07	0.09		0.25	0.2		0.5	0.33		1	0.5			1
0.08	0.11		0.25	0.2		0.5	0.33		1	0.5			1
0.08	0.11		0.25	0.2		0.5	0.33		1	0.5			1
0.09	0.13		0.25	0.25		0.5	0.33			0.5			1
0.1	0.13		0.33	0.25		0.5	0.33			0.5			1
0.11	0.13		0.33	0.25		0.5	0.33			0.5			1
0.11	0.13		0.33	0.25		1	0.33			0.5			1
0.13	0.13		0.33	0.25		1	0.33			0.5			1
0.13	0.13		0.33	0.25		1	0.33			0.5			1
0.13	0.13		0.33	0.25		1	0.33			0.5			1
0.13	0.13		0.33	0.25		1	0.33			0.5			1
0.13	0.14		0.33	0.25		1	0.33			0.5			1
0.14	0.14		0.33	0.25		1	0.33			0.5			1
0.14	0.14		0.33	0.25		1	0.33			0.5			1
0.14	0.14		0.33	0.25		1	0.33			0.5			1
0.14	0.14		0.33	0.25		1	0.33			0.5			1
0.14	0.14		0.33	0.25		1	0.33			0.5			1
0.17	0.14		0.33	0.25		1	0.5			0.5			1
0.17	0.14		0.33	0.25		1	0.5			0.5			1
0.17	0.14		0.33	0.25		1	0.5			0.5			1
0.17	0.17		0.33	0.25		1	0.5			0.5			1
0.2	0.17		0.33	0.25		1	0.5			0.5			1
0.2	0.17		0.33	0.25		1	0.5			1			1
0.2	0.17		0.33	0.25		1	0.5			1			1
0.2	0.17		0.33	0.25		1	0.5			1			1
0.2	0.17		0.33	0.25		1	0.5			1			1
0.2	0.2		0.5	0.25		1	0.5			1			
0.2	0.2		0.5	0.33		1	0.5			1			
0.25	0.2		0.5	0.33		1	0.5			1			
0.25	0.2		0.5	0.33		1	0.5			1			

Limits

SP11	SP12		SP11	SP12		SP11	SP12		SP11	SP12		SP11	SP12		SP11	SP12
0.09	0.06		0.5	0.33		1	0.5		1	1			1			1
0.09	0.07		0.5	0.33		1	0.5		1	1			1			1
0.1	0.08		0.5	0.33		1	0.5		1	1			1			1
0.1	0.08		0.5	0.33		1	0.5		1	1			1			1
0.11	0.08		0.5	0.33		1	0.5		1	1			1			1
0.13	0.1		0.5	0.33		1	0.5		1	1			1			1
0.13	0.11		0.5	0.33		1	0.5		1	1			1			1
0.13	0.13		0.5	0.33		1	0.5		1	1			1			1
0.14	0.14		0.5	0.33		1	0.5		1	1			1			1
0.14	0.14		0.5	0.33		1	0.5		1	1			1			1
0.17	0.17		0.5	0.33		1	0.5		1	1			1			1
0.17	0.17		0.5	0.33		1	0.5		1	1			1			1
0.17	0.17		0.5	0.33		1	0.5		1	1			1			1
0.17	0.17		0.5	0.33		1	0.5		1	1			1			1
0.2	0.17		0.5	0.5		1	0.5		1	1			1			1
0.2	0.17		0.5	0.5		1	0.5		1	1			1			
0.2	0.2		0.5	0.5		1	0.5		1	1			1			
0.2	0.2		0.5	0.5		1	0.5		1	1			1			
0.2	0.2		0.5	0.5		1	0.5			1			1			
0.2	0.2		0.5	0.5		1	0.5			1			1			
0.2	0.2		0.5	0.5		1	0.5			1			1			
0.25	0.25		0.5	0.5		1	0.5			1			1			
0.25	0.25		0.5	0.5		1	0.5			1			1			
0.25	0.25		0.5	0.5		1	0.5			1			1			
0.25	0.25		0.5	0.5		1	0.5			1			1			
0.25	0.25		0.5	0.5		1	1			1			1			
0.25	0.25		0.5	0.5		1	1			1			1			
0.25	0.25		0.5	0.5		1	1			1			1			
0.25	0.25		0.5	0.5		1	1			1			1			
0.25	0.25		0.5	0.5		1	1			1			1			
0.33	0.25		0.5	0.5		1	1			1			1			
0.33	0.25		0.5	0.5		1	1			1			1			
0.33	0.25		0.5	0.5		1	1			1			1			
0.33	0.25		0.5	0.5		1	1			1			1			
0.33	0.25		1	0.5		1	1			1			1			
0.33	0.33		1	0.5		1	1			1			1			
0.33	0.33		1	0.5		1	1			1			1			
0.33	0.33		1	0.5		1	1			1			1			
0.5	0.33		1	0.5		1	1			1			1			
0.5	0.33		1	0.5		1	1			1			1			
0.5	0.33		1	0.5		1	1			1			1			
0.5	0.33		1	0.5		1	1			1			1			
0.5	0.33		1	0.5		1	1			1			1			
0.5	0.33		1	0.5		1	1			1			1			
0.5	0.33		1	0.5		1	1			1			1			
			1	0.5		1	1			1			1			

Chain Rule

SP11	SP12		SP11	SP12		SP11	SP12		SP11	SP12		SP11	SP12		SP11	SP12
0.08	0.08		0.33	0.33		1	0.5		1	1			1			1
0.09	0.08		0.33	0.33		1	0.5		1	1			1			1
0.1	0.09		0.33	0.33		1	0.5		1	1			1			1
0.11	0.1		0.33	0.33		1	0.5		1	1			1			1
0.13	0.11		0.33	0.33		1	0.5		1	1			1			1
0.13	0.11		0.33	0.33		1	0.5		1	1			1			1
0.14	0.13		0.5	0.33		1	0.5		1	1			1			1
0.14	0.13		0.5	0.33		1	0.5		1	1			1			1
0.17	0.14		0.5	0.33		1	0.5		1	1			1			1
0.17	0.14		0.5	0.5		1	0.5		1	1			1			1
0.17	0.17		0.5	0.5		1	0.5		1	1			1			1
0.17	0.17		0.5	0.5		1	0.5		1	1			1			1
0.17	0.17		0.5	0.5		1	0.5		1	1			1			1
0.17	0.17		0.5	0.5		1	0.5		1	1			1			1
0.17	0.17		0.5	0.5		1	0.5		1	1			1			1
0.2	0.17		0.5	0.5		1	0.5		1	1			1			1
0.2	0.2		0.5	0.5		1	0.5		1	1			1			1
0.2	0.2		0.5	0.5		1	0.5		1	1			1			1
0.2	0.2		0.5	0.5		1	1		1	1			1			
0.25	0.2		0.5	0.5		1	1		1	1			1			
0.25	0.2		0.5	0.5		1	1		1	1			1			
0.25	0.2		0.5	0.5		1	1		1	1			1			
0.25	0.2		0.5	0.5		1	1		1	1			1			
0.25	0.2		0.5	0.5		1	1		1	1			1			
0.25	0.25		0.5	0.5		1	1		1	1			1			
0.25	0.25		0.5	0.5		1	1		1	1			1			
0.25	0.25		0.5	0.5		1	1		1	1			1			
0.25	0.25		0.5	0.5		1	1		1	1			1			
0.25	0.25		0.5	0.5		1	1		1	1			1			
0.25	0.25		0.5	0.5		1	1		1	1			1			
0.25	0.25		0.5	0.5		1	1		1	1			1			
0.33	0.25		0.5	0.5		1	1		1	1			1			
0.33	0.33		0.5	0.5		1	1		1	1			1			
0.33	0.33		0.5	0.5		1	1		1	1			1			
0.33	0.33		0.5	0.5		1	1		1	1			1			
0.33	0.33		0.5	0.5		1	1		1	1			1			
0.33	0.33		0.5	0.5		1	1		1	1			1			
0.33	0.33		0.5	0.5		1	1		1	1			1			
0.33	0.33		0.5	0.5		1	1		1	1			1			
0.33	0.33		0.5	0.5		1	1		1	1			1			
0.33	0.33		0.5	0.5		1	1		1	1			1			
0.33	0.33		1	0.5		1	1		1	1			1			
0.33	0.33		1	0.5		1	1		1	1			1			
0.33	0.33		1	0.5		1	1			1			1			
			1	0.5		1	1			1			1			
						1	1			1			1			