ANALYSIS OF ENERGY EFFICIENCY OF COOPERATIVE MIMO SCHEMES

by

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Abstract

Recently, user-cooperative MIMO (multi-input multi-output) systems have been generating significant interest due to their capacity/performance gains over SISO (single-input single-output) systems. In cooperative MIMO architectures, individual nodes with single antennas collaborate with each other to act as a MIMO unit. As a result, the individual node complexity associated with traditional MIMO implementations is alleviated. This feature is especially beneficial in sensor networks and cellular systems where individual node energy, size and cost are important constraints. Additionally, cooperative MIMO schemes provide all the benefits of traditional MIMO systems. In this work, we classify the cooperative MIMO systems into three different categories equivalent to classical MIMO, MISO and SIMO systems. For the three protocols, we quantify and compare the energy efficiency of Amplify-and-Forward (AF) and Decode-and-Forward (DF) schemes on a basic three node cooperative network. Total energy is calculated considering circuit energy as well as transmission energy. For AF and DF schemes, we set a target Symbol Error Probability (SEP) and evaluate the minimum transmission energy for achieving the target SEP. In this process, we first derive an approximation for SEP at high SNR. Then, we formulate the transmission energy calculation as an optimization problem subject to the target SEP and present the theoretical solution. The result is used to compare the total energy consumption of AF and DF for the three protocols. This is unlike most of the prior efforts that primarily focus on optimum allocation of limited total power to maximize the some peformance criterion. Since any wireless systems in order to operate have a set performance criterion, we intend to minimize the resources that is capable of achieving that.

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Chapter 1

Introduction

We encounter many applications of wireless systems our daily lives. While cellphones and wireless LANs have already become an indespensable commodity, wireless sensor networks and embedded wireless systems have the potential to radically affect the way we interact with the physical world. Apart from a host of applications in battle fields, nuclear reactors and in other inaccessibe terrains, wireless networks also find applications in habitat monitoring, health monitoring, home networking and practically in any hand held or wearable computing devices. Research efforts have been focussing on understanding the fundamental limits of wireless networks and at the same time improving the performance of present systems to achieve these limits. Space time wireless communications is one of the major breakthroughs [4] in the recent times where the new dimension of "space" was utlized to improve the performance in terms of capacity and bit error rate of wireless systems. Additionally, the concept of space time wireless communications is utilized in wireless sensor networks through sharing of resources by the nodes of the network. This is termed as cooperative communications which has many potential applications in wireless networks. In this chapter, we first introduce to the reader some basics in space time wireless communication and cooperative wireless communication. We then elucidate on the prior work, the uncharted research areas, our motivation and finally the contribution of this thesis.

1.1 Space Time Wireless communications

Conventional wireless devices uses single antenna to transmit and single antenna to receive. It is commonly refered to as Single Input Single Output (SISO) communication system. It is well known that SISO systems are yet to achieve the performance limits in capacity. With increasing demand for wireless services, wireless system designers are finding it difficult to meet the growing demands for higher data rates and better quality of service (QoS). In this context, the use of multiple antennas at the transmitter as well as receiver opens new dimension to exploit - space. This form of communication is called Space Time (ST) wireless communication. Figure 1.1 illustrates the typical ST wireless communication system where there are multiple antennas attached to a node either at transmitter or at receiver or both. The different antenna configuration are SIMO (Single Input Multiple Output) which has single transmitter antenna and multiple receive antennas, MISO (Multiple Input Single Output) has multiple transmit antennas and a single receive antenna and MIMO (Multiple input Multiple Output) has multiple antennas in both transmitter and receiver. The performance enhancement in ST wireless communcation system is illustrated and will be clear through a passage extracted from [4],

> ... Assuming a target SNR of 20dB, current single antenna transmit and receive technology can offer a data rate of 0.5 Mbps. A two-transmit and one-receive antenna system would acheive 0.8 Mbps. A four-transmit and four-receive antenna system can reach 3.75 Mbps. It is worth noting that 3.75 Mbps is also achievable in a single antenna transmit and receive technology, but needs 10^5 times higher SNR or transmit power compared with four-transmit and four-receive antenna configuration. ...

ST wireless communcation system, usually termed as MIMO systems, offer the following benefits over SISO systems [4]:

Array Gain: Array gain refers to the increase in SNR at the receiver that arises from coherent combining of signals from different antennas of the receiver. This gain is easily

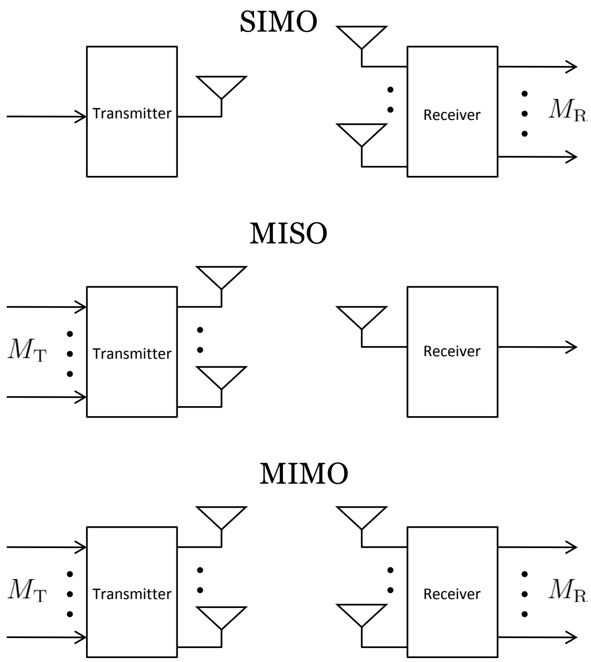

Figure 1.1: Diagram of Space Time Wireless Channel

exploited in SIMO configuration. For MISO and MIMO cases, the channel state information is required to exploit array gain.

Diversity Gain The radio signal from a single antenna transmitter arrives through multiple paths ("multipath effect") to the receiver having multiple antennas (SIMO system). Each of these multipaths provides different channel of communication. Signal power at the receiver fluctuates differently in each of these channel and the fluctuation is random. This is called fading. The multiple antennas at the receiver receives various versions of the same signal but which are faded independently and combines the signal coherently. Since the fading is independent, the probability of two or more channels, experiencing deep fade is very low. This reduces the probability of error in detection for the coherently combined received signal. The reduction in probability of error is due to the increase in SNR, which in turn is due to combining of the signals received at different antennas. This is termed as diversity gain. Channel knowledge is required for MISO and MIMO systems to exploit diversity.

Spatial Multiplexing Spatial Multiplexing offers increase in capacity for the same bandwidth and with no increase in power expenditure. The independent paths created by the multipath can be exploited and treated as different channels and two different signals can be sent simultaneously. The signals can then be seperated at the receiver by exploiting some properties of the channel. Spatial multplexing is possible only for MIMO systems and the capacity increase is proportional to the increase in the number of antennas.

It is to be noted here that all these advantages of MIMO systems cannot be exploited simultaneously. There is a tradeoff between obtaining diversity gain and spatial multiplexing gain. However, the gains of MIMO systems are phenomenal and find many applications like in wireless LANs, base stations in cellular systems, satellite communications etc. MIMO technology it is already incorporated in IEEE 802.11n standard for wireless LANs. The transmit diversity is already incorporated into 2.5G and 3G wireless standards.

There are, however, certain applications where multiple antennas cannot be installed in wireless nodes. Nodes in wireless sensor networks, cellular mobiles, ad-hoc networks and Micro Electro Mechanical Systems (MEMS) are constrained by their size, cost or expendable battery power that multiple antennas and the consequent complex circuitry may not be supported. However, the compelling advantages of MIMO systems gained by exploiting space and multipath has fueled research efforts focussed on exploiting the space dimension in single antenna nodes. The outcome of this effort is broadly classified as cooperative communications [5–7]. In cooperative communications, the wireless agents, instead of competing for resources, share it in order to enhance system performance. This is particularly relevant in sensor networks where a set of nodes are employed in order to achieve a particular task and in which the performance of the system is more important than that of individual nodes. The next section briefly describes cooperative communication.

1.2 Cooperative Communications

Cooperative communication is employed in systems where multiple antennas cannot be supported due to resource constraints. However, through cooperation between nodes the advantages of a MIMO system, like spatial diversity and multipath effects can be exploited. Cooperative communications also alleviates the individual node complexity associated with having multiple antennas at the cost of some overhead required for cooperation. The potential broadcast nature of a wireless network is exploited here. Consider a source node transmitting with some power to its destination. Because of the broadcast nature of the source, many nodes in the vicinity of the source listen to this message. However, since the transmission is not intended for them, they do not perform any decoding. Note that the nodes in the vicinity of the source receive stronger signal than destination. Consider a situation in which severe fading along the source to destination link results in SNR at destination falling below a threshold. This may result in the receiver decoding incorrectly and consequently retransmission of the message is required. In this context, if one of the node near the source is able to forward the message to the destination it may incur less transmission energy than due to retransmission from source. This node is called a "relay" as it forwards the signal received from the source. The advantage is not only in energy savings, but also in the form of diversity. The probability that both the source to destination and relay to destination channels will be in deep fade is quite low if they are uncorrelated. While it is assumed that the relay is in the vicinity of source, usually it is far enough so that the channel fades are independent (for correlated fades, the source-relay distance should be close to a fraction of a wavelength). Both signals, one from source even though it is very weak and other from relay can be coherently combined to provide significant SNR at the destination. The relay may either just act as a repeater where it amplifies the signal from source and forwards it or it may decode the signal from source, re-encode and forward it the destination. The former strategy is called non-regenerative relaying or amplify-forward (AF) relaying, while the latter technique is called regenerative relaying or decode-forward (DF) strategy. Also, it not necessary that only one relay forwards the message, but there can be multiple relays. This in essence is cooperative communication. In a more complex form, a set of sources collaborate with each other and transmits messages to the set of receiver cluster such that it mimicks a ST wireless system. In the next section we explain the recent results in cooperative communication and the motivation for our research.

1.3 Prior Work

The concept of cooperative diversity was first introduced in [5], [6]. Here, the authors establish that cooperation leads to increase in capacity even when the interuser channel is noisy. Cooperation also makes the system robust to channel variations that cause rate fluctuations. In [7], the performance analysis for several strategies like amplify-forward, decode-forward, selection relaying and incremental relaying are done. The performance criterion in [7] is outage probability and it is found that except for decode-forward, all protocols achieve full diversity without need for multiple transmit antennas and hence provide significant energy savings. Also, it is shown that cooperative diversity can be implemented in various wireless systems like ad-hoc, cellular and sensor networks. For example, the applications of cooperation in wireless sensor networks is explored in [8]. The performance analysis of cooperative diversity networks is analyzed widely in literature. Various performance critera such as symbol error probability, outage probability, capacity are explored. Authors in [9] derive closed form expressions and tight upperbound for the SEP of decode-forward relaying strategy. The modulation schemes considered in [9] are M-ary PSK and QAM. In [10], the performance in fading channels is found to depend significantly on the probability density function (pdf) of the SNR near origin. Based on this knowledge, the SEP and outage behaviour is analyzed. The result also provides a unified approach to evaluate the peformance of coded and uncoded transmissions. It is also applicable to almost all digital modulation schemes (e.g., M-ary PSK, QAM). Further, a high SNR approximation for SEP for general cooperative diversity links employing amplify-forward strategy is found in [11]. [11] also establishes that diversity gain does not depend on the fading distribution (e.g., Rayleigh, Ricean, Nakagami).

In a cooperative communication system, a three node network consisting of the source, relay and the destination constitutes the fundamental unit. This three node network was analyzed by Nabar et al. [2], where the authors put forth three different time division based protocols equivalent to the classical SIMO, MISO and MIMO communication systems. All the three protocols employ either amplify-forward or decode-forward mode. The ergodic capacity and the outage behaviour of these three protocols are found. Further they also went on to design space time codes for the amplify-forward schemes. The research in [5], [6], [7] uses this SIMO equivalent of the protocols proposed in [2].

With various applications of cooperative communication being closely related to energy and power constrainted systems like sensor networks and mobile cellular networks, researchers have focussed on optimal power allocation and energy efficiency. In general, the research is directed towards maximizing a predefined performance criterion with limited resources. Previous efforts ([5], [6], [7]) assume that both source and relay transmit with equal power, however, it is found that the knowledge of channel state information (CSI) can be exploited to optimally allocate power. Intuitively, the channel with higher gain will be allocated more power and lower gain will be allocated less power. In [9], the optimal power allocation for minimizing the SEP is found to be dependent on the instantaneous CSI. The instantaneous CSI, is therefore required at source, relay to optimally allocate power. A similar work was extended to the case of amplify-forward strategy in [12]. In certain applications, the overhead required to estimate CSI is prohibitive and hence [13] has looked into the case when only mean channel gains are available. A near optimal solution for power allocation is found in [13] that minimizes the outage probability. More than one relay could potentialy take part in forwarding the information through decode-forward strategy when the mean channel gain is greater than a threshold. In [14],[15], assuming knowledge mean channel gains, optimum cooperative ratio is found for allocating power for source and relay for amplify-forward based relaying strategy. Another area of research in cooperative communications is related to capacity maximization problems. [16], [17] investigates maximizing the capacity subject to power total power constraint, with full knowledge of CSI. Two cases, one in which there is no source-destination direct link while the other has a direct link is considered, for decode-forward and amplify-forward, respectively.

Broadly the power allocation problems in cooperative communication investigates how to efficiently allocate the limited total power among the source and the relays so that some performance criterion is met. However, only a few papers have looked into the energy efficency of a cooperative communication system. In energy efficiency we try to find the minimum energy required by the network to transmit one bit from a source to a destination while maintaining a performance criterion. This is unlike power allocation where we divide available power between source and relay(s) to maxmize/minimize some criterion(capacity/BER). A major work in this direction was initiated by [1], in their seminal paper. Energy efficiency of MIMO and Virtual MIMO schemes have been presented in [1]. It has been found in [1] that MIMO and Virtual MIMO with optimized modulation schemes [3] are more energy efficient relative to SISO at long range distances even when circuit energy consumptions of multi-antenna nodes is taken into consideration. However, these efforts do not consider any underlying AF or DF based cooperative communication model in their analysis. Authors in [18] determine the minimum transmit power employed by source and relay to maintain error free communication. In their paper, however, they consider that both source and relay transmit with the same power, which may not be optimal. Additionally [18] did not consider circuit energy consumption. [18] introduces the dependence of energy on path loss and employs knowledge of mean CSI in their cooperation. Finally, the comparison is made between conventional and cooperative relaying. The results indicate that cooperative schemes benefit from diversity. In [19], the criterion for minimizing energy was a target outage probability and they compare maximal ratio and equal gain combining strategies. In [20], the authors evaluate the energy benefits that can be obtained in cooperative communication system with a target SNR constraint. The cooperation protocol considered is akin to relaying and beyond a threshold distance, direct transmission is shown to consume more energy. The transmission energy is minimized while maintaining the required rate in [21] with option of choosing the best relay among multiple relay choices. In [22], outage constraint is used as the target performance criterion and the authors propose a simple relay selection criteria. The important contribution of [22] is that the cost of acquiring CSI is explicitly modeled, and relay cooperation is shown to be beneficial even after incorporating the cost. [23] examines the power allocation strategy to maximize the network life time of cooperative MIMO system with multiple relays for amplify-forward. It is shown that the strategy that consumes minimum energy subject to an SNR requirement is selective relaying, i.e., selecting the relay with the best channel towards destination. However, this does not necessarily seem to maximize the network lifetime. In order to maximize life time, it is proposed to exploit the residual energy infomation (REI) of the nodes.

1.4 Motivation

Surveying the literature, it is found that most of the prior research efforts concentrate on illustrating the benefits of cooperation over direct transmission. However, within the cooperation schemes there still remains an open question on which of the two strategies to employ, amplify-forward or decode-forward. Also, we need to investigate the reason on why we prefer one over the other. In view of the applications of cooperative communication in wireless sensor networks and cellular networks [8], energy efficiency is an important criterion. Applications like sensor networks are resource constrained and often employed in hostile environments where battery replacement is impossible. And in cellular systems the mobile should be able to endure long hours without being recharged frequently. Analyzing the energy efficiency of amplify-forward(AF) and decode-forward(DF) based cooperative MIMO scheme will provide insights into which of the two modes can be used to meet a specific performance criterion when nodes are in power starved situation. This may in turn help in increasing the longevity of the node. To the best of authors knowledge there has been no prior work that compares energy efficiency of AF/DF based cooperative communication schemes. The probable factors that may favor one against other are the relay(s) positions, number of relays, type of fading (rayleigh/ricean/nakagami), the seperation distances between the source and destination etc.

In this thesis, we consider a fundamental unit of cooperative communication system, a three node network operating in SIMO, MISO, and MIMO equivalent protocols [2] employing strategies of AF or DF for our analysis. We evaluate the energy consumed in transmitting one bit from the source to destination with the aid of a relay in order to meet a target SEP at the destination. The total energy consumption is found as the sum of transmission and circuit energy. The transmission energy is minimized subject to the target SEP. Finally, the energy consumption is compared between AF and DF for each of the different protocols.

1.5 Key Contributions

This section describes in detail, the key contributions of our thesis.

- *We model a circuit which implements the AF relaying strategy and calculate its circuit power consumption.* Further, in chapter 2, we calculate the circuit power consumption of the SIMO, MISO and MIMO protocols for both AF and DF relaying strategies.

- *We derive a new simple approximation to the SEP of SIMO-DF with imperfect relay.* In order to accomplish that we show that the instantaneous SNR at destination for SIMO-DF as a sum of exponential random variables. Previous works [9] although have exact SEP of DF are intractable to be used as a constraint in optimization problems. The simulation of SIMO-DF BPSK system confirm that the SEP expression is a good approximation.

- *We find the minimum transmssion energy for SIMO-DF and SIMO-AF subject to a target SEP. This formulation has not been explored so far in literature.* We first formulate an optimization problem to minimize transmission energy for SIMO-DF subject to the SEP expression found above. Although the problem is non-convex we approximate the objective to a linear function assuming some minimum SEP requirement at the relay. Analytical results thus obtained for transmission energy are matching with the numerical results. Further, we formulate a convex optimization problem to minimize the transmission energy for SIMO-AF subject to SEP (SEP was already available from literature). The analytical results are obtained for the optimum energy consumption at source and relay. The solution is confirmed by numerical methods.

- *For the first time we compare the energy efficiency of SIMO-AF and SIMO-DF based on three relay positions.* We find the total energy consumption as sum of transmission energy and circuit energy for both SIMO-DF and SIMO-AF. The energy efficiency comparison between the two relaying strategies is done for different relay positions such as,

    1. relay near source,
    2. relay near destination,
    3. relay is neither close to source or destination.

  In each of the above cases the analytic expression transmission energy consumption at source and relay is approximated to more tractable form. Arguments are then presented to support and understand the results intuitively.

- *We derive a upperbound for SIMO-DF SEP, which is tight.* We first prove that instantaneous SNR at destination for MISO-DF when there is no error is exponentially distributed. The SEP for SIMO-DF is then upperbounded using Jensen's inequality. The upperbound is found to be tight if the relay error is small enough and it is verified using simulation.

- *We discover that the optimal strategy for the MISO-DF protocol seems to be relaying of information rather than pure MISO in which source takes part in the second time slot. We show that implementation of MISO-DF is restricted to certain relay positions relative to source and destination. Also, it is shown that the transmission energy consumption for an imperfect relay is always greater than that incurred assuming a perfect relay.* We formulate a problem to minimize transmission energy subject to the SEP at the relay. The optimization problem is convex and is solved analytically as two trivially parallelizable peoblems. Numerical results confirm the accuracy of obtained results. Based on the obtained results for transmission energy we extend it to the case for imperfect relays.

- *Similar to MISO-DF, for MISO-AF we prove that pure relaying seems to be the (sub)optimal but better strategy and also that MISO-AF implementation is restricted to certain relay positions relative to source and destination.* In order for that, we derive an expression for the average received SNR at the destination for MISO-AF. It is found that in some cases AF relaying strategy does not aid in increasing the SNR at the destination. For MISO-AF, the problem of finding minimum transmission energy is formulated subject to satisfying a minimum SNR at the destination which in turn is required to satisfy the critical SEP. The optimization problem is found to be non-convex. It is further reformulated as a non-convex Quadratically Constrained Quadratic Program (QCQP). This result is supported by numerical methods employed on the QCQP problem.

- *We compare the energy efficiency for MISO-AF and MISO-DF.* We calculate the energy consumption of MISO-AF and MISO-DF as the sum of transmission energy and circuit energy. The optimal transmission strategy is already found to be relaying. Similar to SIMO protocol, the energy efficiency comparison is done for different possible relay positions.

- *We prove that the optimal strategy for MIMO-DF is in fact implementing it as SIMO-DF. The transmission energy assuming an imperfect relay is found and is proved to greater than that incurred using a perfect relay.* In order to accomplish that, the instantaneous SNR at the destination is derived and the SEP of a perfect relay is found. The problem of minimizing the transmission energy is posed as an optimization problem subject to the target SEP. The problem is found to be convex and numerical results readily gives the solution to the problem. In order to gain more insight we use primal decomposition to solve two sub problems seperately. Further, we analyze the problem as a combination of MISO and SIMO protocols.

- *We derive a SEP expression for MIMO-AF. We show that MIMO-AF system can be decomposed into a MISO and SIMO system and for certain relay positions it acts as MISO and in other cases it acts as a SIMO implementation.* To investigate the working of MIMO-AF we first derive the SEP and perform to simulations to validate the theoretical expression found. Then, we pose an optimization to minimize the transmission energy. The optimization problem is decomposed into two subproblems as earlier, each having characteristics of MISO and SIMO protocols.

- *Finally, the most important contribution to this research is to formulate a framework to compare the energy efficiency AF and DF strategies for cooperative MIMO systems in a three node network.* The comparison is applicable to a broad category of coded and uncoded cooperation schemes and for different digital modulation techniques like M-ary PSK and QAM.

1.6 Organization

This thesis is organized into seven chapters. Chapter 2 introduces the basic concepts of cooperative communication. Here, we explain different implementations of user-cooperation like virtual MIMO system and cooperative MIMO system. The fundamental differences between the two are clarified. Further, the three different protocols implementations in a three node network are explained. After briefly describing the AF and DF strategies, the signal model for the the three protocols in both AF and DF mode is derived.

In chapter 3, we first define energy efficiency and elaborate on the circuit energy consumption and transmission energy consumption model. A typical transmitter and receiver circuit is shown with the power consumption values for its components. We also describe a simple amplify-forward circuit in the process. The circuit energy consumption for the the three protocols are then found out. The transmission energy model that we use througout this work is then explained.

Chapter 4, 5 and 6 deal with the core contribution of the thesis. In chapter 4, we use a SIMO protocol for cooperative communication where we compare the energy efficiency of AF and DF modes. We first find the SEP of both SIMO-AF and SIMO-DF and consequently minimize the transmission energy subject to the SEP. This minimized transmission energy is used to find the total energy consumption. The comparision between AF and DF is done for different relay postions. In a similar manner, chapter 5 and 6 presents energy efficiency analysis of MISO and MIMO protocols, respectively. Finally, we conclude the thesis in chapter 7, with suggestions for possible future research and extensions to the present work.

Chapter 2

Cooperative Wireless Communications

In this chapter, we explain cooperative wireless communication and the different methods by which they are implemented in detail. This chapter also describes the protocols that are used in this research. In cooperative wireless communications individual nodes with single antennas collaborate with each other to act as a MIMO unit. Typically, a source destination pair takes the help of one or more intermediate nodes in transmitting information in order to combat the effects of fading. As a result, the individual node complexity associated with a MIMO unit is alleviated and at the same time providing the benefits of a classical MIMO system [4]. User cooperation is especially useful in certain mobile wireless systems where multiple antennas cannot be supported in individual nodes due to battery energy, cost and/or size constraints. Examples of such system includes cell phones, wireless sensor etc.

2.1 Types of Cooperative Communications

Based on the implementation of the cooperation protocol, user cooperation can be divided into virtual MIMO and cooperative MIMO techniques. Both techniques exploit the spatial independence, multipath charateristics and the broadcast nature of wireless channels.

2.1.1 Virtual MIMO

Figure (2.1) shows the diagram of a typical Virtual MIMO system for a source destination pair. The source transmits the data to a predetermined set of nodes which are physically close to itself. These nodes form the transmit cluster. Because of the physical nearness, the links between the source and the transmit cluster are considered to be AWGN and is unaffected by fading. Similarly, the destination also has a set of nodes which are physically close to iself which it considers as the receive cluster. The distance between the transmit and receive cluster is large and is subject to fading. The interaction between the transmit cluster and receiver cluster mimicks a MIMO system. However, there is an overhead of transmitting the required information from source to the transmit cluster and similarly from receive cluster to the destination. The overhead may be depend upon the eventual objective of Virtual MIMO implementation on whether to provide diversity or to improve capacity. One of the challenges involved is to reduce the overhead so that the maximum benefits of a MIMO system can be gained through cooperation.

2.1.2 Cooperative MIMO

Figure (2.2) shows the diagram of a Cooperative MIMO system. In this scenario, a source node may not have any other nodes which are close to each other. However, due to broadcast nature of the wireless system multiple nodes may be able to listen to the source and aid in forwarding the information to the destination. These intermediates nodes are called relays. One or more intermediate nodes can act as relays. At the destination, in addition to direct link from source, the transmitted information is received from different spatially independent path through relays and hence provides diversity advantage. A system consisting of a source destination pair aided by one relay is considered to be the fundamental unit in cooperative MIMO system. Figure (2.3) represents the model of a three node network with r_{SR}, r_{SD} and r_{RD} representing the link distances between the source to relay, relay to destination and source to destination, respectively. h_{SR}, h_{SD} and h_{RD} represents channel gain which may

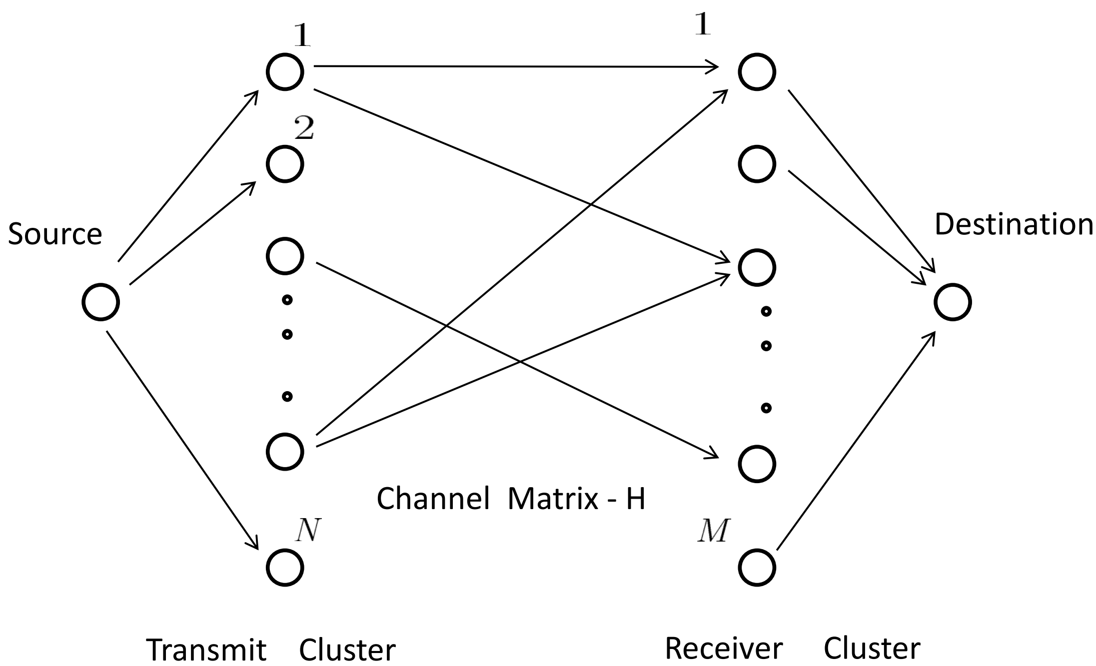

Figure 2.1: Diagram of a Virtual MIMO system

be due to rayleigh fading in dense urban areas [24] and ricean in case there is line of sight communication. In this thesis, the wireless channel is considered to be slow flat rayleigh fading. The relay is considered to be half duplex. At a higher protocol level, the rules of cooperative transmission can be further choreographed to implement a MIMO, MISO and SIMO schemes [2]. The protocols are time division based and are described in detail below.

SIMO Protocol

In this protocol, the source terminal broadcasts information. Both the relay and destination terminal receives the information in the first time slot. In the second time slot, relay communicates with the destination terminal. At the destination, the same information is received over two time slots providing receiver diversity. Hence the protocol is termed as SIMO. It was first proposed in [7].

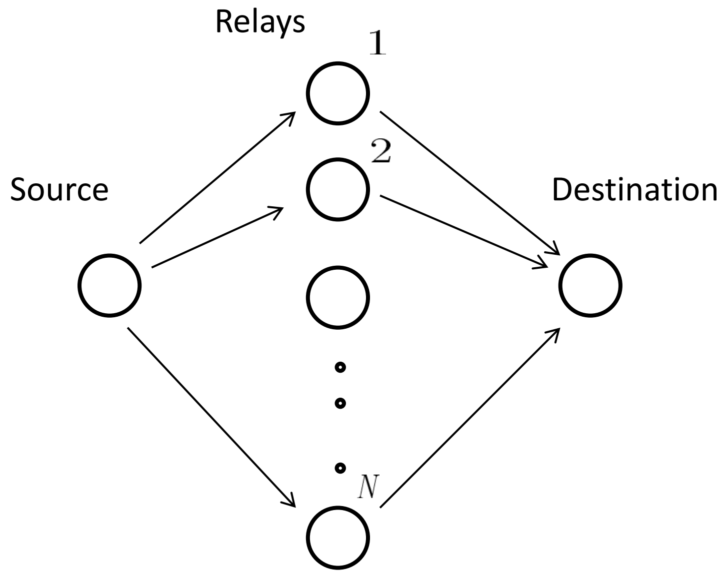

Figure 2.2: Diagram of Cooperative MIMO system

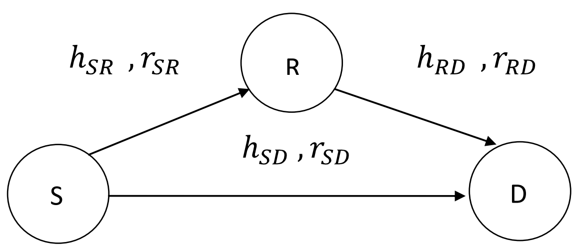

Figure 2.3: A three node Cooperative MIMO system

MISO Protocol

In this case, the source transmits only to the relay in the first time slot. In the second time slot, both source and relay transmit together to the destination. Since, at the destination the signals are received at the same time transmit precoding is required to extract diversity. It is named MISO because, the source and relay acts as a single unit with two transmit antennas in the second time slot and destination being the single antenna receiver.

MIMO Protocol

Here, the source terminal broadcasts and both relay and destination terminal receives in the first time slot. In the second time slot, both source and relay transmits simultaneously to the destination. This protocol can be considered as combination of MISO and SIMO protocols.

The three modes of cooperative MIMO communication in a three node network is summarized in the following table. S, R and D stands for source relay and destination terminals respectively. The indicator $A \rightarrow B$ signifies the communication between terminals A and B.

Time Slot	SIMO	MISO	MIMO
1	$S \rightarrow R, D$	$S \rightarrow R$	$S \rightarrow R, D$
2	$R \rightarrow D$	$S \rightarrow D, R \rightarrow D$	$S \rightarrow D, R \rightarrow D$

Table 2.1: Protocol table for Cooperative MIMO [2]

2.2 Modes of Relaying

Depending on the way in which the relay processes the information it obtains from the source and forwards it, the communication can be divided into the following categories.

2.2.1 Amplify and Forward

In Amplify and Forward (AF) strategy, the relay receives the signal from the source, then amplifies and forwards it to the destination in the next time slot. The implementation of AF mode is simple as no signal processing is invovled at the relay. AF strategy is also called non-regenerative relaying.

2.2.2 Decode and Forward

The signal received by the relay in the first time slot is decoded. In the second time slot, it is re-encoded and forwarded to the destination. It is assumed that after decoding at the relay it is possible to detect errors. Once the relay detects an error it will no longer cooperate in the transmission. DF strategy is also called regenerative relaying.

2.3 Signal Models

In this section, we find the received signal model at the destination for all the protocols. The following assumptions are made in order to derive the expressions for the received signal model. As mentioned earlier the channel is assumed to slow flat rayleigh and independently fading with Additive White Gaussian Noise (AWGN). The variance of the AWGN is assumed to N_0 on all the links. However, the results derived in the following chapter can be easily extended to the case when the variance is different. All the wireless nodes are considered half duplex, which means that they either listen or transmit at an instant but not listen and transmit together. Channel state information is not available at any of the transmitters, however they are available at the receivers. The multipath signals received is combined using Maximal Ratio Combining (MRC) technique to maximize the signal to noise ratio (SNR) received at the destination. In MRC combining the received signals at two time instants are multiplied by certain weights proportional to the instantaneous channel and added up so that their resultant SNR is maximized. The maximum SNR will be the sum of the instantaneous SNR received at the two instances.

2.3.1 SIMO with Amplify Forward

The protocol is denoted as SIMO-AF in this work. In this protocol at both instances the same information is transmitted. If x_1 is the complex transmitted symbol in the first time slot, and if $y_{R,1}$ and $y_{D,1}$ is the received signal at relay and destination, then

$$y_{R,1} = \sqrt{E_{SR}}h_{SR}x_1 + n_{R,1} \tag{2.1}$$

$$y_{D,1} = \sqrt{E_{SD}}h_{SD}x_1 + n_{D,1}. \tag{2.2}$$

where, E_{SR} is received signal energy at the relay, E_{SD} is the signal energy at the destination and $n_{R,1}$, $n_{D,1}$ is the AWGN noise. Since the relay just amplifies the signal that it receives from the source in the second time instant, the received signal $y_{D,2}$ is given by,

$$y_{D,2} = \frac{\sqrt{E_{RD}}h_{RD}y_{R,1}}{\sqrt{E\{\mid y_{R,1} \mid^2\}}} + n_{D,2}, \tag{2.3}$$

where, E_{RD} is the signal energy received at destination due to transmission from relay, $n_{D,2}$ is the noise at the destination, $E\{.\}$ denotes the expectation operator over the noise. The expectation is taken to normalize the signal power to unity before amplifying. The resultant signal received at the destination is given by

$$y_{D,2} = \frac{\sqrt{E_{RD}E_{SR}}h_{RD}h_{SR}}{\sqrt{E_{SR} \mid h_{SR} \mid^2 + N_0}}x_1 + \frac{\sqrt{E_{RD}}h_{RD}}{\sqrt{E_{SR} \mid h_{SR} \mid^2 + N_0}}n_{R,1} + n_{D,2}. \tag{2.4}$$

The effective noise variance in the received signal is different from N_0 and is equal to $N_0\left(1 + \frac{E_{RD}|h_{RD}|^2}{E_{SR}|h_{SR}|^2+N_0}\right)$. Later we will use these known equations to find the SNR at the destination and to calculate the symbol error probability.

2.3.2 SIMO with Decode Forward

The protocol is denoted as SIMO-DF in this work. The received signal at the relay and destination in the first time slot is given by equations (2.1) and (2.2). Assuming that the relay correctly decodes the symbol and forwards it, the received signal at the destination $y_{D,2}$, is given by,

$$y_{D,2} = \sqrt{E_{RD}}h_{RD}x_1 + n_{D,2}. \tag{2.5}$$

2.3.3 MISO with Amplify Forward

For MISO-AF, the received signal at relay in the first time slot is given by equation (2.1). The destination does not receive any signal at this time. In the second instant both source and relay transmits. Similar to SIMO-AF, $y_{R,1}$ is normalized to unit energy and amplified. For analytical tractability the expectation is taken over both channel and noise in this particular case. Hence, the received signal $y_{D,2}$ is given by,

$$y_{D,2} = \left(\frac{\sqrt{E_{RD}E_{SR}} h_{RD} h_{SR}}{\sqrt{E_{SR} + N_0}} + \sqrt{E_{SD}} h_{SD} \right) x_1 + \frac{\sqrt{E_{RD}} h_{RD}}{\sqrt{E_{SR} + N_0}} n_{R,1} + n_{D,2}. \tag{2.6}$$

2.3.4 MISO with Decode Forward

This protocol is denoted as MISO-DF in this work and the equation for the received signal at relay in the first time slot is given by (2.1). In both the time instants only one symbol is sent. Assuming that the decoding is perfect the received signal at the destination in the second time instant is given by,

$$y_{D,2} = \left(\sqrt{E_{SD}} h_{SD} + \sqrt{E_{RD}} h_{RD} \right) x_1 + n_{D,2}. \tag{2.7}$$

2.3.5 MIMO with Amplify Forward

MIMO can be treated as a combination of SIMO and MISO protocols described earlier and only one symbol is sent in two time slots. The relay is considered half duplex in this case. The source broadcasts and the received signal at the relay and destination in the first time slot is given by,

$$y_{R,1} = \sqrt{E_{SR,1}} h_{SR,1} x_1 + n_{R,1} \tag{2.8}$$

$$y_{D,1} = \sqrt{E_{SD,1}} h_{SD,1} x_1 + n_{D,1}, \tag{2.9}$$

where, $E_{SR,1}$, $E_{SD,1}$ is the energy of the received signal at source and relay. $h_{SR,1}$ and $h_{SD,1}$ are the channel realizations at the first time instant. In the second time instant both the source and relay transmits simultaneously to the destination. The signal at the destination is hence given by,

$$y_{D,2} = \left(\frac{\sqrt{E_{RD,2}E_{SR,1}}h_{RD,2}h_{SR,1}}{\sqrt{E_{SR,1}+N_0}} + \sqrt{E_{SD,2}}h_{SD,2} \right) x_1 + \frac{\sqrt{E_{RD,2}}h_{RD,2}}{\sqrt{E_{SR,1}+N_0}} n_{R,1} + n_{D,2}, \quad (2.10)$$

where, $E_{SD,2}$ and $E_{RD,2}$ is the energy received at the destination at the second instant from source and relay.

2.3.6 MIMO with Decode Forward

Similar to half duplex relay in MIMO-AF the equations for $y_{R,1}$ and $y_{D,1}$ is given by (2.8) and (2.9). At the second time instant the signal at the destination assuming perfect decoding at relay is given by,

$$y_{D,2} = \left(\sqrt{E_{SD,2}}h_{SD,2} + \sqrt{E_{RD,2}}h_{RD,2} \right) x_1 + n_{D,2}. \quad (2.11)$$

2.4 Summary

This chapter introduces the concept of cooperative communication and describes Virtual MIMO and Cooperative MIMO systems. Two common methods of relaying information is introduced. The signal model is found for the three node network for all the protocols. This will be further used to find the Symbol Error Probability (SEP) or SNR for each of the protocols.

Chapter 3

Energy Efficiency

In the previous chapter, we presented the basics of cooperative wireless communications. In this chapter, we define energy efficiency and explain the techniques used to calculate circuit energy consumption and transmission energy consumption.

3.1 Definition

In this thesis, we address the following question "What is the minimum (optimal) energy required by a three node network to transmit one bit/symbol from a source to a destination, subject to a target performance criterion?". We define this as energy efficiency. The three node network operates with protocols as mentioned in the previous chapter. The target performance criterion assumed is Symbol Error Probability (SEP). We intend to minimize the total energy consumed by the network while transmitting one bit/symbol. This is unlike power allocation where we divide available power between source and relay(s) to maxmize/minimize some criterion(capacity/BER). The power allocation has been analyzed widely in literature [9, 12–17]; however, energy efficiency has not been considered. Analyzing the energy efficiency of AF and DF based cooperative MIMO schemes provides insights into which of the two modes can be used to meet a specific performance criterion.

3.2 Circuit Energy Consumption

For a three node coperative communication system, the total energy consumption is defined as sum of circuit energy and transmission energy. The total circuit energy consumption is calculated as the sum of transmitter circuit energy and the receiver circuit energy of all transmitter and receiver circuits used in the three node to facilitate transmission of a bit. The transmitter and receiver circuits we consider are available from literature [1], is as shown in figure 3.1 and 3.2. We assume that all the nodes are equipped with

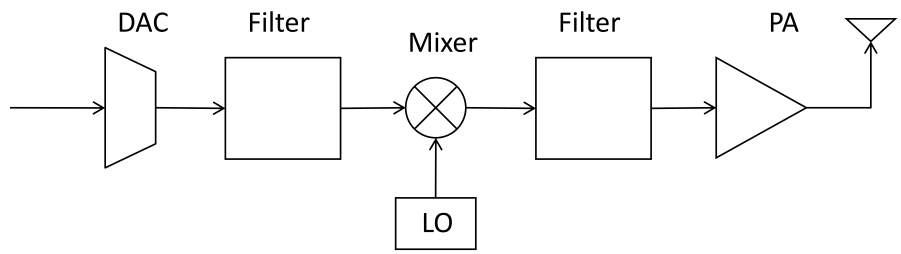

Figure 3.1: Diagram of a transmitter circuit, [1]

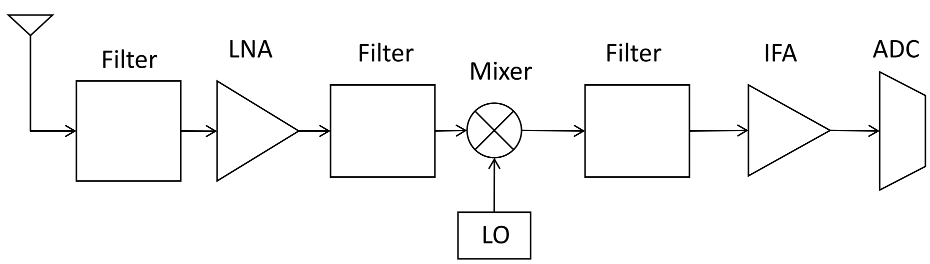

Figure 3.2: Diagram of a receiver circuit, [1]

similar transmitter and receiver circuit blocks. Also, the power consumed by the filter, in transmitter and receiver are assumed the same. The energy consumed is given by the sum of energy consumed by the individual components of the circuit blocks. The transmitter power consumption is given by,

$$P_c^t = P_{DAC} + 2P_{filter} + P_{mixer} + P_{LO} \tag{3.1}$$

and the receiver power consumption is given by,

$$P_c^r = P_{ADC} + 3P_{filter} + P_{mixer} + P_{LO} + P_{IFA} + P_{LNA} \tag{3.2}$$

where, P_{DAC}, P_{filter}, P_{mixer}, P_{ADC}, P_{IFA}, P_{LNA} and P_{LO} are the power consumption values for the Digital to Analog Converter (DAC), the filter, the mixer, the Analog to Digital Converter (ADC), the Intermediate Frequency Amplifier (IFA), the Low Noise Amplifier (LNA) and, the syntesizer respectively. The energy consumed from Power Amplifier (PA) in transmitter is considered separately when we calculate the transmission energy. The typical values are obtained from [1] and [3] and is reproduced here for convienence in table 3.1 shown below. In contrast, at the relay, we have a modified circuit block for AF circuit

Components	Power Consumed
P_{filter}	2.5 mW
P_{mixer}	30.3 mW
P_{IFA}	3 mW
P_{LNA}	20 mW
P_{LO}	50 mW
P_{ADC}	6.7 mW
P_{DAC}	15.4 mW

Table 3.1: Typical values of Power Consumption for Circuit Components, [1], [3]

which does not involve the entire transceiver chain. The AF circuit block is shown in the figure 3.3. It consists of a filter to filter out of band noise and then a low noise amplifier. The amplified signal is filtered again and transmitted using the power amplifier circuit. This is circuit is used by the AF scheme mentioned in section 2.2.1. The power consumed by the AF circuitry is given by,

$$P_c^{af} = 2P_{filter} + P_{LNA} \tag{3.3}$$

The energy consumed per bit can be calculated as the power divided by the constant rate of transmission R_b. Based on the transmitter, receiver and AF circuit model we can find the circuit energy consumed by different protocols of section 2.3.

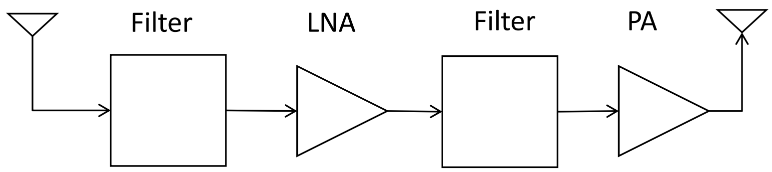

Figure 3.3: Amplify and Forward Circuit

Circuit Energy for SIMO-AF

The circuit energy consumption is given by,

$$P_c^{\mathrm{SIMO-AF}} = P_c^t + 2P_c^r + P_c^{af}. \tag{3.4}$$

where, $P_c^{\mathrm{SIMO-AF}}$ is the circuit energy consumption for SIMO-AF. In this case there is one complete transmission at source which involves all transmitter circuit blocks and similarly two complete receptions of the signal both at destination. In contrast at the relay there is a modified circuit block for amplify and forward circuit which consumes P_c^{af} amount of power.

Circuit Energy for SIMO-DF

Similarly, in SIMO-DF transmitting one bit involves two instances of transmission and three instances of reception.

$$P_c^{\mathrm{SIMO-DF}} = 2P_c^t + 3P_c^r \tag{3.5}$$

Circuit Energy for MISO-AF

The MISO protocols consumes more circuit energy than the SIMO protocols because it has an extra transmission initially to the relay. Similar to SIMO-AF we can find the energy consumption $P_c^{\mathrm{MISO-AF}}$ and is given by,

$$P_c^{\mathrm{MISO-AF}} = 2P_c^t + P_c^r + P_c^{af} \tag{3.6}$$

Circuit Energy for MISO-DF

The circuit energy consumption for MISO-DF is given by,

$$P_c^{\mathrm{MISO-DF}} = 3P_c^t + 2P_c^r \tag{3.7}$$

where, $P_c^{\mathrm{MISO-DF}}$ denotes the energy consumed.

Circuit Energy for MIMO-AF

Among the circuit energy consumption the MIMO protocol consumes the most energy as it involves more transmission and reception than other two protocols. The energy consumption $P_c^{\mathrm{MIMO-AF}}$ is given by,

$$P_c^{\mathrm{MIMO-AF}} = 2P_c^t + 2P_c^r + P_c^{af} \tag{3.8}$$

Circuit Energy for MIMO-DF

The energy consumption for MIMO-DF denoted by $P_c^{\mathrm{MIMO-DF}}$ is given by,

$$P_c^{\mathrm{MIMO-DF}} = 3P_c^t + 3P_c^r \tag{3.9}$$

3.3 Transmission Energy Consumption

The transmission power is a variable and depends on the distance between the node pairs. We use square law path loss model to find the energy consumed by the power amplifier for transmission [24]. We know that the average signal energy received from the source is E_{SD}. Therefore, assuming that the destination gets on an average E_{SD} amount of energy, we evaluate the power that is spent by the transmit power amplifier of the source, accounting for path loss, as in [1], [24]. Hence the signal power at the source by the power amplifier according to link budget relationship is,

$$P_{PA} = (1+\beta)E_{SD}R_b\frac{(4\pi r_{SD})^2}{G_t G_r \lambda^2}M_l N_f \tag{3.10}$$

where β the constant dependent on drain efficiency of RF power amplifier and Peak-to-Average ratio(PAR), R_b is the bit rate, G_t is the transmitter antenna gain, G_r is the receiver antenna gain, λ is the carrier wavelength, M_l is the link margin compensating the hardware process variations and other additive background noise or interference, N_f is the receiver noise figure defined as $N_f = \frac{N_r}{N_0}$ with $N_0 = -171 dBm/Hz$ the single-sided thermal noise Power Spectral density(PSD) at the room temperature and N_r is the PSD of the total effective noise at the receiver input. Consequently, the transmission energy per bit at source can be evaluated as $\frac{P_{PA}}{R_b}$. The values of variables used in this thesis are given in the table 3.2. Here, f_c refers to the frequency of operation and B is the bandwidth. Since the system

Parameters	Typical Values
f_c	2.5 GHz
$G_t G_r$	5 dbi
B	10 KHz
N_f	10 dB
M_L	40 dB
R_b	10^4 bps

Table 3.2: Typical values of Parameters of the System, courtsey [1],[3]

parameters remain unchanged through out the analysis we define c_0, a system constant as,

$$c_0 \triangleq (1+\beta)\frac{(4\pi)^2}{G_t G_r \lambda^2} M_l N_f. \tag{3.11}$$

Even though we use the square law path loss model in our research it is easily extendable to exponents with higher powers. However, it is necessary to adjust the system parameters accordingly.

3.4 Summary

In this chapter, we first define energy efficiency, the metric of interest in our analysis. We also provided details of circuit energy consumption assuming typical transmitter and receiver circuits available from literature. Based on that we modeled a simple AF circuit which will be used in our work. Further more, the transmission energy is calculated using a square law path loss model. In the coming chapters we use the framework described here to find the transmission energy consumed by using SIMO, MISO and MIMO protocols in order to compare the AF and DF modes of transmission in each case.

Chapter 4

Energy Efficiency of Cooperative SIMO scheme

This chapter compares the energy efficiency of SIMO-AF and SIMO-DF. Recall the SIMO protocol that was described in table 2.1. In the first instant, source broadcasts to both relay and destination. In the second instant, relay forwards the bit to destination. In this chapter, we first find the minimum overall transmission energy for both SIMO-AF and SIMO-DF which required to maintain a target SEP. The minimization is done by formulating it as an optimization problem. The SEP of SIMO-AF is already available in literature, however we derive an approximation for SEP of SIMO-DF in the process. Finally, the energy consumed per bit is found as the sum of transmission energy and circuit energy. The results are then compared for different relay positions.

4.1 Transmission Energy for SIMO-DF

4.1.1 Symbol Error Probability of SIMO-DF

The SEP for DF mode taking into consideration the error probability at the relay is derived in this section. The following assumptions are made for DF protocol. After decoding at the relay it will be able to detect error if any. In case an error is detected during the decoding it will not further cooperate in the transmission. As a result of this, the communication system is equivalent to the case of direct transmission (SISO). However, the relay receiver circuit spends some energy for decoding the received bit and hence overall more energy is consumed for this case than SISO. Define, η to be a discrete binary random variable taking values 1 for an error at the relay and 0 for succesful decoding at the relay. Therefore

$$\eta = \begin{cases} 1 & \text{with probability } E\{Q(\sqrt{k\gamma_{SR}})\} \\ 0 & \text{with probability } 1 - E\{Q(\sqrt{k\gamma_{SR}})\} \end{cases} \tag{4.1}$$

where k is a fixed positive constant dependent on modulation parameters, $Q(.)$ denotes the Marcum Q function, $E\{.\}$ denotes the expectation operator and γ_{SR} is the instantaneous SNR at the $S-R$ link given by,

$$\gamma_{SR} = \frac{E_{SR}|h_{SR}|^2}{N_0} \tag{4.2}$$

The instantaneous SNR γ_D at the destination D as per the assumptions is given by

$$\gamma_D = \begin{cases} \gamma_{SD} & \text{when } \eta = 1 \\ \gamma_{SD} + \gamma_{RD} & \text{when } \eta = 0 \end{cases} \tag{4.3}$$

where, γ_{SD} and γ_{RD} are defined similar to equation (4.2). From equation (4.3) the expected value of instantaneous SNR at the destination is given by $\gamma_{SD} + (1 - P(\eta = 1))\gamma_{RD}$. The value of $P(\eta = 1)$ is found out using the high SNR approximation for $E\{Q(\sqrt{k\gamma_{SR}})\}$ given in [10] and is equal to $\frac{1}{2k\overline{\gamma}_{SR}}$. Therefore the instantaneous SNR at the destination averaged over the error at relay can be written as,

$$\overline{\gamma}_D = \gamma_{SD} + \hat{\alpha}\gamma_{RD} \tag{4.4}$$

where, $\hat{\alpha} \triangleq \left(1 - \frac{1}{2k\overline{\gamma}_{SR}}\right)$. It is evident that $0 < \hat{\alpha} < 1$ as it is the probability of relay decoding correctly. An intuitive explanation for equation (4.4) would be, that a perfect relay would have resulted in an instantaneous SNR of $\gamma_{SD} + \gamma_{RD}$ at the destination, but due to the finite error probability of relay the actual instantaneous SNR at the destination is marginally lower than that. We now have the instantaneous SNR at destination as the sum of two exponential random variables (as a consequence of Rayleigh fading assumption) whose pdfs are given by,

$$f_{\gamma_{SD}}(x) = \frac{1}{\overline{\gamma}_{SD}} \exp\left\{\frac{-x}{\overline{\gamma}_{SD}}\right\} \tag{4.5}$$

$$f_{\hat{\alpha}\gamma_{RD}}(x) = \frac{1}{\hat{\alpha}\overline{\gamma}_{RD}} \exp\left\{\frac{-x}{\hat{\alpha}\overline{\gamma}_{RD}}\right\} \tag{4.6}$$

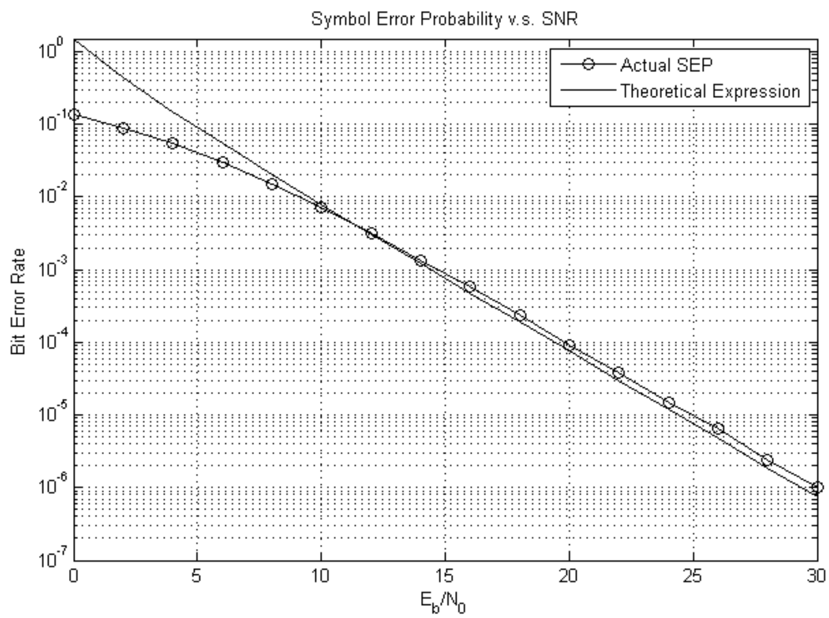

Figure 4.1: Symbol Error Probability v.s. SNR for the SIMO-DF

where, $\overline{\gamma}_{SD} = E\{\gamma_{SD}\}$, averaged over the channel h_{SD}. Similarly, $\overline{\gamma}_{RD}$ is also defined. Following the similar lines of proof as outlined in [10, 11] the asymptotic average SEP is given by,

$$\overline{P}_e \longrightarrow \frac{3}{4k^2}.\frac{1}{\gamma_{SD}\hat{\alpha}\gamma_{RD}} \tag{4.7}$$

Analyzing the expression we can see that the SEP of SIMO-DF is dependent on the product of SNR from two branches $S-D$ and $R-D$. Also from the result the SEP for an error prone relay is higher than that of perfect relay due to the $\hat{\alpha}$ term in the expression which depends on the relay error. We can also conclude the same from equation (4.4) that since the SNR at destination is lower than that for perfect relaying the SEP consequently is higher. The advantage of using this high SNR approximation is that, the expression thus obtained for SEP is applicable to different modulation schemes in an AWGN channel.

4.1.2 Simulation for SIMO-DF SEP

Figure 4.1 shows the Symbol Error Probability versus SNR for SIMO-DF when modulation is BPSK. Both theoretically found SEP and the SEP using Monte Carlo simulations are plotted. It is seen that the simulated SEP matches with the theoretical SEP. The theoretical expression of SEP derived here, at high SNR is marginally lower than the actual one due to approximation errors. In the next section we move on to formulate an optimization problem to minimize the transmission energy for SIMO-DF subject to a target SEP.

4.1.3 Problem Formulation SIMO-DF

The average transmission energy for SIMO-DF can be written as $c_0 r_{SD}^2 E_{SD} + \hat{\alpha} c_0 r_{RD}^2 E_{RD}$. Note that $\hat{\alpha}$ which is dependent on E_{SR} is probability of relay decoding correctly. We pose an optimization problem to minimize the average transmission energy of SIMO-DF subject to the target SEP as in equation (4.7). The optimization problem corresponds to:

$$\begin{aligned}
\phi_{\text{SIMO-DF}} = \underset{E_{SD}, E_{RD}}{\text{minimize}} \quad & c_0 r_{SD}^2 E_{SD} + \left(1 - \frac{N_0 c_2}{2k E_{SD}}\right) c_0 r_{RD}^2 E_{RD}, \\
\text{subject to} \quad & c_1 \left(\frac{1}{E_{SD} - \frac{N_0 c_2}{2k}}\right) \frac{1}{E_{RD}} \leq p_c, \\
& -E_{SD} \leq 0, \\
& -E_{RD} \leq 0.
\end{aligned} \tag{4.8}$$

Here, we substitute the value of $\hat{\alpha}$ in both the objective function and constraint and use the relation $c_2 = \frac{E_{SD}}{E_{SR}} = \frac{r_{SR}^2}{r_{SD}^2}$. p_c is the critical SEP that we want to maintain at the destination. It is important to remember that in SIMO-DF, the relay transmitter circuit block will not be used when there is an error in relay as it does take part in forwarding.

On analyzing the optimization problem, although the constraint is convex, the Hessian of the objective is found to be negative definite and hence concave. Therefore no global minimum is guaranteed on applying KKT conditions. Also the application of KKT conditions gives an intractable cubic equation to solve for E_{SD} and E_{RD}. Following a different approach for SIMO-DF, we define p^* to be the critical probability of error for a prospective node to act as a relay. If probability of error exceed p^*, the node will not be considered for relaying. It implicitly means an added constraint in the optimization problem. Accordingly the objective function is modified as $c_0 r_{SD}^2 E_{SD} + (1-p^*)\, c_0 r_{RD}^2 E_{RD}$. The new objective indicates the minimum energy that is consumed in SIMO-DF. The actual energy consumption is slightly greater than this value. The optimization problem is then reformulated as,

$$\begin{aligned}
\phi_{\text{SIMO-DF}} = \underset{E_{SD},E_{RD}}{\text{minimize}} \quad & c_0 r_{SD}^2 E_{SD} + (1-p^*)\, c_0 r_{RD}^2 E_{RD}, \\
\text{subject to} \quad & c_1 \left(\frac{1}{E_{SD} - \frac{N_0 c_2}{2k}} \right) \frac{1}{E_{RD}} \le p_c, \\
& \frac{N_0 c_2}{2k E_{SD}} \le p^*, \\
& -E_{SD} \le 0, \\
& -E_{RD} \le 0.
\end{aligned} \tag{4.9}$$

The resulting optimization problem is convex and hence KKT conditions will give a global minimum. Setting up the lagrangian and applying the KKT conditions we get the equations needed to solve for E_{SD} and E_{RD} for different cases of $\mu_1 \ge 0$, $\mu_2 \ge 0$, (the Kuhn-Tucker coefficients). The equations are,

$$\begin{aligned}
c_0 r_{SD}^2 - \frac{\mu_1 c_1}{(E_{SD} - \frac{N_0 c_2}{2k})^2 E_{RD}} - \frac{\mu_2 N_0 c_2}{2k E_{SD}^2} &= 0, \\
(1-p^*) c_0 r_{RD}^2 - \frac{\mu_1 c_1}{(E_{SD} - \frac{N_0 c_2}{2k}) E_{RD}^2} &= 0, \\
\mu_1 \left[\frac{c_1}{(E_{SD} - \frac{N_0 c_2}{2k}) E_{RD}} - p_c \right] + \mu_2 \left[\frac{N_0 c_2}{2k E_{SD}} - p^* \right] &= 0, \\
\mu_1 &\ge 0, \\
\mu_2 &\ge 0.
\end{aligned} \tag{4.10}$$

There are four possible cases associated with different values μ_1 and μ_2 can take and immediately we can discard $(\mu_1 = 0, \mu_2 = 0)$ and $(\mu_1 = 0, \mu_2 > 0)$ as it contradicts the KKT conditions. On solving the equations for the KKT conditions for case A defined as when

$(\mu_1 > 0, \mu_2 = 0)$ the candidate solutions are,

$$\overline{E}_{SD}^{A} = \frac{N_0}{2k}\left\{\frac{r_{SR}^2}{r_{SD}^2} + \frac{r_{RD}}{r_{SD}}\left[\frac{3(1-p^*)}{p_c}\right]^{\frac{1}{2}}\right\}$$
$$\overline{E}_{RD}^{A} = \frac{N_0}{2k}\frac{r_{SD}}{r_{RD}}\left[\frac{3}{p_c(1-p^*)}\right]^{\frac{1}{2}} \tag{4.11}$$

and for the case B defined as when $(\mu_1 > 0, \mu_2 > 0)$ the candidate solutions are,

$$\overline{E}_{SD}^{B} = \frac{N_0}{2k}.\frac{r_{SR}^2}{r_{SD}^2}.\frac{1}{p^*}$$
$$\overline{E}_{RD}^{B} = \frac{N_0}{2k}.\frac{r_{SD}^2}{r_{SR}^2}.\frac{3p^*}{p_c(1-p^*)} \tag{4.12}$$

On careful observation of case $(\mu_1 > 0, \mu_2 > 0)$, we can see that it is equivalent to the original problem mentioned in equation (4.8) with an additional constraint $\frac{N_0 c_2}{2kE_{SD}} = p^*$. Case $(\mu_1 > 0, \mu_2 = 0)$, is equivalent to the original problem itself if we relax the second constraint. The objective evaluated from solution (4.12) is larger than that evaluated out of (4.11). That is, if we try to maintain a fixed level performance at the relay at case $(\mu_1 > 0, \mu_2 > 0)$ there is undue strain on the source to transmit more energy. Practically, if we have an upper bound on the transmission power of source, the case $(\mu_1 > 0, \mu_2 > 0)$ becomes infeasible. Therefore the minimized transmission energy is given by $\phi_{\text{SIMO}-\text{DF}} = c_0 r_{SD}^2 E_{SD}^A + (1-p^*)\, c_0 r_{RD}^2 E_{RD}^A$.

4.2 Transmission Energy for SIMO-AF

4.2.1 Symbol Error Probability of SIMO-AF

The average SEP for SIMO-AF is evaluted for high SNR based on [10]. The expression for the case of a rayleigh fading channel is given by,

$$\overline{P}_e \longrightarrow \frac{3}{4k^2}\left[\frac{1}{\overline{\gamma}_{SR}} + \frac{1}{\overline{\gamma}_{RD}}\right]\frac{1}{\overline{\gamma}_{SD}}. \tag{4.13}$$

4.2.2 Problem Formulation for SIMO-AF

The total transmission energy per bit for SIMO-AF, is the sum of energy per bit spent at the source S and relay R. and is given by $c_0 r_{SD}^2 E_{SD} + c_0 r_{RD}^2 E_{RD}$. Similar to SIMO-DF we formulate a problem to minimize the transmission energy per bit subject to the target SEP as in equation (4.13). Formally, the optimization problem is defined as:

$$
\begin{aligned}
\phi_{\text{SIMO-AF}} = \underset{E_{SD},E_{RD}}{\text{minimize}} \quad & c_0 r_{SD}^2 E_{SD} + c_0 r_{RD}^2 E_{RD}, \\
\text{subject to} \quad & c_1 \left[\frac{c_2}{E_{SD}} + \frac{1}{E_{RD}} \right] \frac{1}{E_{SD}} \leq p_c, \\
& -E_{SD} \leq 0, \\
& -E_{RD} \leq 0. \qquad (4.14)
\end{aligned}
$$

Here, equation (4.14) is obtained by modifying equation (4.13) with $\overline{\gamma_b} = \frac{E_b}{N_0}$, $c_2 = \frac{E_{SD}}{E_{SR}} = \frac{r_{SR}^2}{r_{SD}^2}$, $c_1 = \frac{3N_0^2}{4k^2}$ and p_c the critical SEP. The optimization problem is to find the E_{SD} and E_{RD} that guarantees a SEP less than p_c and uses the minimium possible transmission energy. The problem can be shown to be convex and KKT conditions gives a global minimum. Applying KKT conditions and solving the optimization problem described in (4.14) analytically, we get the optimum values of E_{SD} and E_{RD} as

$$
\begin{aligned}
E_{SD}^{\star} &= 2 \left(\frac{c_1}{p_c} \right)^{\frac{1}{2}} \frac{r_{SR}}{r_{SD}} \left(\frac{1}{4} + \frac{1}{\sqrt{1 + \frac{8r_{SR}^2}{r_{RD}^2}} - 1} \right)^{\frac{1}{2}} \quad \text{and} \\
E_{RD}^{\star} &= \frac{1}{2} \left(\frac{c_1}{p_c} \right)^{\frac{1}{2}} \frac{r_{SD}}{r_{SR}} \left(\frac{2r_{SR}^2}{r_{RD}^2} + \frac{1}{2}\sqrt{1 + \frac{8r_{SR}^2}{r_{RD}^2}} - \frac{1}{2} \right)^{\frac{1}{2}} \qquad (4.15)
\end{aligned}
$$

The optimum energy expended per bit can be obtained from substituting the values of $E_{SD}^{\star}$ and $E_{RD}^{\star}$ in the objective of the optimization problem 4.14. Although the analytical expression appears to be complex, approximations based on various relay positions relative to source and destination can be used to better understand the results.

4.3 Results and Discussion

In this section, we focus on the comparision of the SIMO-AF and SIMO-DF scheme for three cases involving different relay positions. For each case the comparisions are done when absolute value of distance between source and destination is short (around $100-200m$) where circuit energy is dominant and distances are large ($> 700m$) when transmission energy is dominant.

4.3.1 Energy Efficiency Comparision: Relay near Source

We assume that the relay near the source i.e. $r_{SR} \ll r_{RD}, r_{SD}$ and is typically $\frac{r_{SR}}{r_{SD}} < \frac{1}{10}$. Substituting equation (4.15) in the objective in problem 4.14, we get the total transmission energy for SIMO-AF. Consequently, we can derive an simple result for the total transmission energy by using binomial approximation of $(1 + \frac{8r_{SR}^2}{r_{RD}^2})^{\frac{1}{2}} \approx 1 + \frac{4r_{SR}^2}{r_{RD}^2}$. The resulting expression for the transmission energy consumption for SIMO-AF from both source and relay is $c_0\left(\frac{c_1}{p_c}\right)^{\frac{1}{2}} r_{RD}r_{SD}$. In this case the transmission power is split equally among source and relay for the best performance.

For SIMO-DF after minor modification of the result and using the fact that the value of $\frac{r_{SR}^2}{r_{RD}^2} < \frac{1}{100}$ is insignificant, we can find that the transmission energy at the source and relay is both $c_0(1-p^*)^{\frac{1}{2}}\left(\frac{c_1}{p_c}\right)^{\frac{1}{2}} r_{RD}r_{SD}$. Once again the transmission energy is split equally among source and relay for optimum performance. Notice that the total transmission energy for $r_{SR} \ll r_{RD}$ is same for both SIMO-AF and SIMO-DF if not for a factor of $(1-p^*)^{\frac{1}{2}}$ which is very near to ≈ 1. However when it comes to total energy consumption, SIMO-DF consumes more energy per bit as the circuit energy is more for it. The circuit energy is of the order of 10^{-5} and since the transmission distances are less in the range of $100m$ the total transmission energy is of the order of 10^{-6}. Clearly circuit energy prevails over the transmission energy and so SIMO-AF seems to be a better option to consider for a relay near source. Figure 4.2 shows the comparision total energy consumption per bit of the two schemes for a BPSK transmission system for maintaining a target SEP of 10^{-3}. Since $r_{SD} = 100$ and $r_{SR} = 10$ the value of r_{RD} is restricted to $90 \leq r_{RD} \leq 110$. Although in the absolute sense there is not much difference between the total energy consumption per bit between the two schemes, the value of this multiplied by the fixed rate of transmission R_b, which is the average power is significantly higher for SIMO-DF. Figure 4.3 shows that the transmission energy is almost

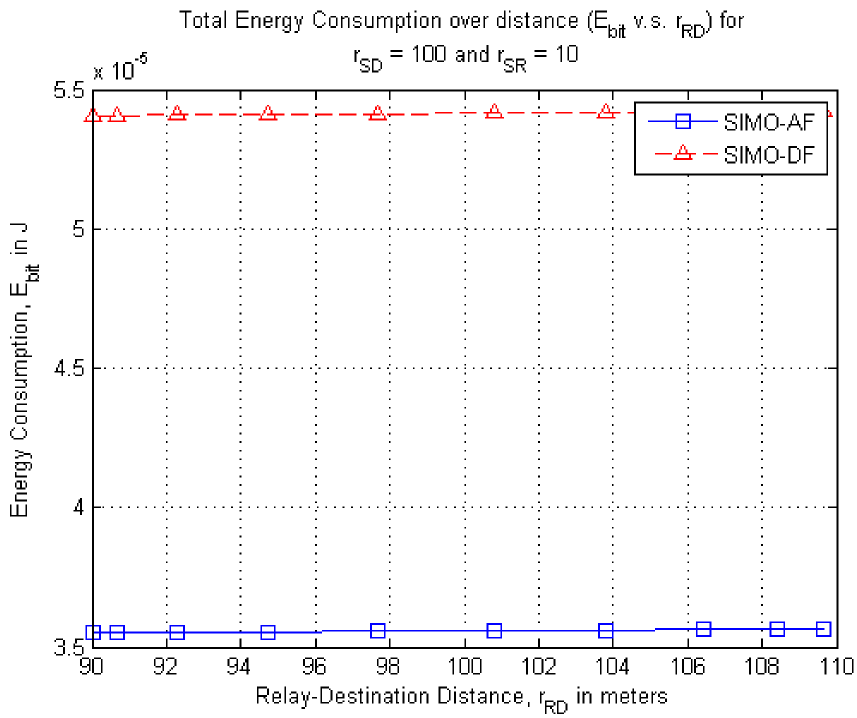

Figure 4.2: Total Energy consumption v.s. Relay-Destination Distance, for a fixed $r_{SD} = 100$ and $r_{SR} = 10$

Table 4.1: Comparision of Numerical results: Original SIMO-DF optimization problem and the approximated problem where $r_{SD} = 100$ and $r_{SR} = 10$

$r_{RD}(m)$	90.00	94.75	100.80	106.45	109.65
Approx. Problem $E_T * 10^{-6}$ J	0.6803	0.7161	0.7620	0.8044	0.8287
Original Problem $E_T * 10^{-6}$ J	0.6806	0.7164	0.7623	0.8047	0.8290

equal in both the cases. The numerical results for transmission energy in both SIMO-AF and SIMO-DF match exactly with the theoretically obtained results of the optimization problems. The numerical methods are employed using 'fmincon' function in MATLAB. Table 4.1 shows the transmission energy per bit required for the problem in 4.9 and of that in 4.8 for different values of r_{RD}. It is seen that numerical methods of both the optimizaion problems yield the same results. Therefore, we can conclude that the optimization problem 4.9 is good approximation and closed form expression for an intractable problem as defined in 4.8. Another case worth checking is when the distances are increased maintaning $r_{SR} \ll r_{RD}$

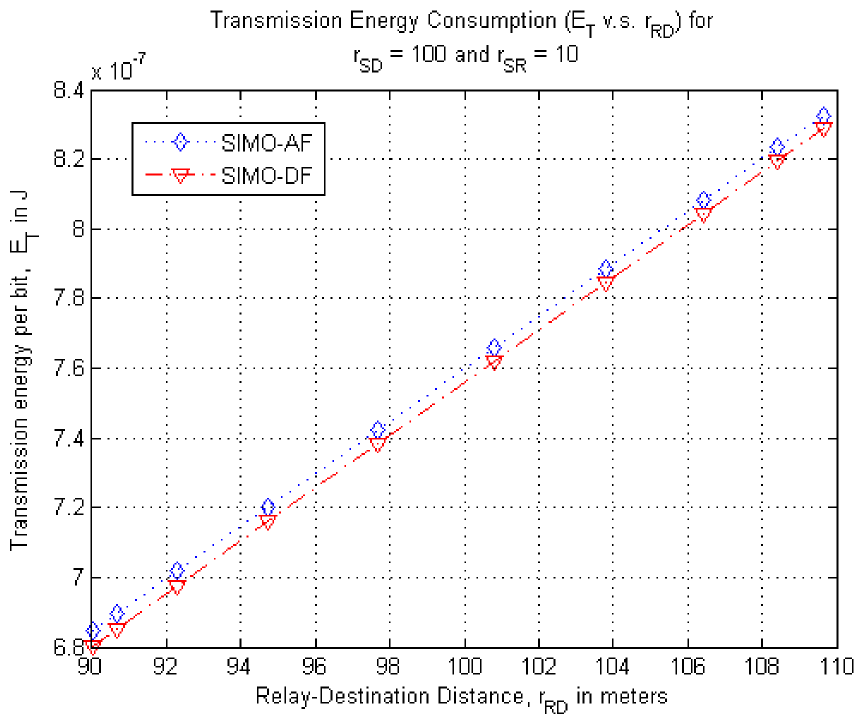

Figure 4.3: Transmission Energy consumption v.s. Relay-Destination Distance, for a fixed $r_{SD} = 100$ and $r_{SR} = 10$

so that transmission energy is comparable with the circuit energy. However, since the transmission energy is found to equal in both the cases, SIMO-DF consumes more energy per bit.

4.3.2 Energy Efficiency Comparision: Relay near Destination

Here we assume that $r_{RD} \ll r_{SD}$, $r_{RD} \ll r_{SR}$ and typically when $\frac{r_{RD}}{r_{SD}} < \frac{1}{10}$ the relay is close to the destination. Approximations are derived for transmission energies for both SIMO-AF and SIMO-DF. The approximation is based on the fact that $\frac{r_{SR}^2}{r_{RD}^2} \gg 1$ and using binomial approximation $(1+x)^n \approx 1 + nx$ for $x < 1$. As a result the transmission energy of SIMO-AF, used up by relay is calculated as $c_0 \left(\frac{c_1}{2p_c}\right)^{\frac{1}{2}} r_{SD} r_{RD}$, and that used up by the source is approximated as $c_0 \left(\frac{c_1}{p_c}\right)^{\frac{1}{2}} r_{SR} r_{SD} \left(1 + \frac{r_{RD}}{\sqrt{2} r_{SR}}\right)$. The source transmission energy for SIMO-AF is dependent on sum of two product terms, the $r_{SR}.r_{SD}$ term and $r_{SD}.r_{RD}$ term. It is clear that the source transmission energy is dominated by $r_{SR}.r_{SD}$ product and is significantly greater than the transmission energy of relay. This is intuitively correct as more energy is required to transmit a bit across larger distances (r_{SR} and r_{SD}) and relatively low energy to transmit a bit across the $R - D$ path when relay is close to destination.

For SIMO-DF, the transmission energy at the relay is given as $\left(\frac{2}{1-p^*}\right)^{\frac{1}{2}} c_0 \left(\frac{c_1}{2p_c}\right)^{\frac{1}{2}} r_{SD} r_{RD}$ which is greater than $\sqrt{2}$ times that used by SIMO-AF relay. This is expected because if the relay decodes the symbol correctly, a higher transmission energy at $R - D$ path will ensure a error free reception at the destination. The source transmission energy for SIMO-DF is given as $c_0 \left(\frac{c_1}{p_c}\right)^{\frac{1}{2}} \left[\left(\frac{p_c}{3}\right)^{\frac{1}{2}} r_{SR}^2 + (1 - p^*) r_{RD} r_{SD}\right]$. Notice that similar to SIMO-AF source transmission energy there are two product terms involved here, which are r_{SR}^2 and $r_{RD}.r_{SD}$ terms. However, the transmission energy required by the source in SIMO-DF is lower than that of SIMO-AF because of the the $\left(\frac{p_c}{3}\right)^{\frac{1}{2}}$ product in the first term. Therefore, for this case since source transmission energy dominates, and AF consumes more transmission energy than DF.

The following figure show plots for a particular case of BPSK transmission when the distances between nodes are less such that circuit energy is dominating. The critical probability of error is maintained at 10^{-3}. It is clear that when circuit energy is dominating then DF performs worse (Fig. 4.4) even though in the case of transmission energy AF is far worse than DF (Fig. 4.5). Since $r_{SD} = 200$ and $r_{RD} = 20$ the value of r_{SR} is restricted to $180 \leq r_{RD} \leq 220$. Maintaining the relay near the destination and if the distances are increased then it is seen that at SIMO-AF is less energy efficient than SIMO-DF because of it high transmission energy requirement. The Fig. 4.6 shows that given r_{RD} and r_{SD} to be 75 and 750 respectively if the r_{SR} is greater than around $710m$ the SIMO-AF starts to consume more energy to transmit a bit. Also the fact whether AF or DF is more efficient is topology dependent i.e. dependent on the two distances r_{SD} and r_{RD} that we fix initially. For example, higher the r_{SD} value maintaining the same r_{RD} as before will make SIMO-AF worse while reducing the r_{RD} and maintaining same r_{SD} will make SIMO-DF worse. Figure

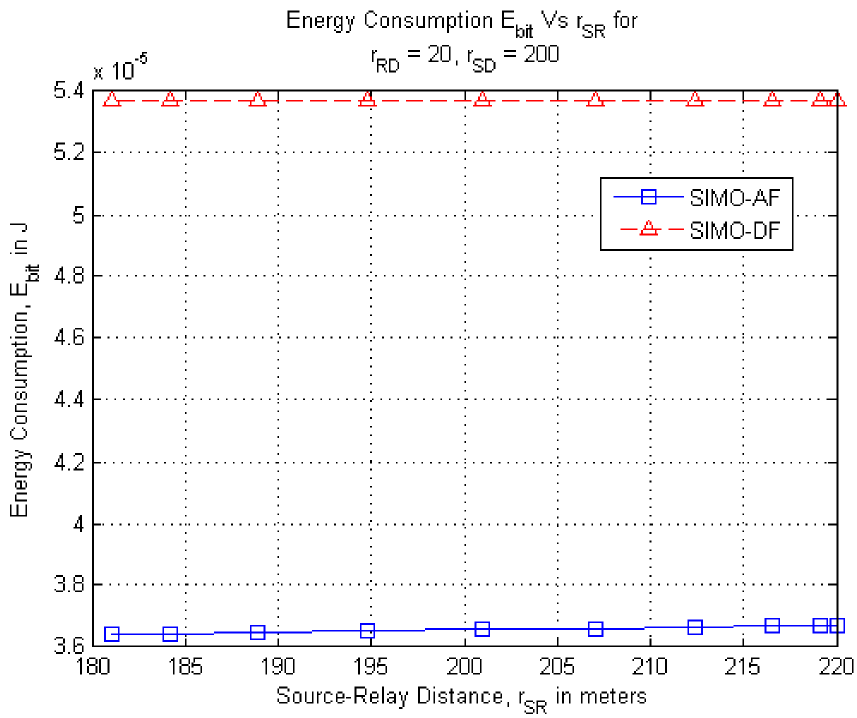

Figure 4.4: Total Energy consumption v.s. Source-Relay Distance, for a fixed $r_{RD} = 20$ and $r_{SD} = 200$

4.7 shows the transmission energy comparision. It is clear that at larger distances there will be transistion point distance where SIMO-DF will be favourable as compared to SIMO-AF.

4.3.3 r_{SR}, r_{SD} and r_{RD} all comparable distances

When all the distances are comparable the general trend is that for short distances between nodes SIMO-AF is more energy efficient and as the distances are increased SIMO-DF is more energy efficient. Figure 4.8 and 4.9 shows total energy consumption of a BPSK system for maintaining a target SEP of 10^{-3}, one for short distances and the other for larger distances, respectively. As regarding the transmission energy SIMO-AF always consumes a lot of energy than SIMO-DF. Numerical methods match with the analytical expressions for both SIMO-AF and SIMO-DF. In addition the approximated objective of SIMO-DF gives results

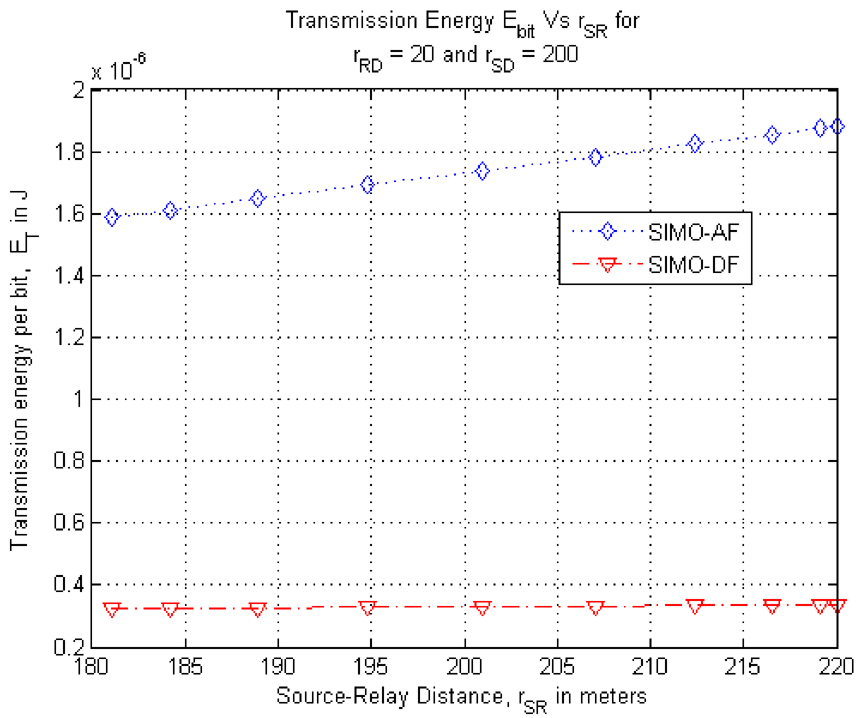

Figure 4.5: Transmission Energy consumption v.s. Source-Relay Distance, for a fixed $r_{RD} = 20$ and $r_{SD} = 200$

Table 4.2: Comparision of Numerical results: Original SIMO-DF optimization problem and the approximated problem where $r_{RD} = 900$ and $r_{SR} = 900$

$r_{RD}(m)$	280	817	1272	1603	1778
Approx. Problem $E_T * 10^{-3}$ J	0.0197	0.0561	0.0871	0.1097	0.1215
Original Problem $E_T * 10^{-3}$ J	0.0192	0.0556	0.0866	0.1091	0.1210

which are very close to that obtained using application of numerical methods on the original objective. The table 4.2 which compares the case corresponding to Fig. 4.9 shows that the approxiamted results has less than 2% deviation with actual results.

4.4 Summary

We analyzed the energy efficiency of SIMO-AF and SIMO-DF cooperative schemes on a three node cooperative network under the performance constraint of a target SEP. The total energy is calculated as the sum of circuit energy and transmission energy. We modeled

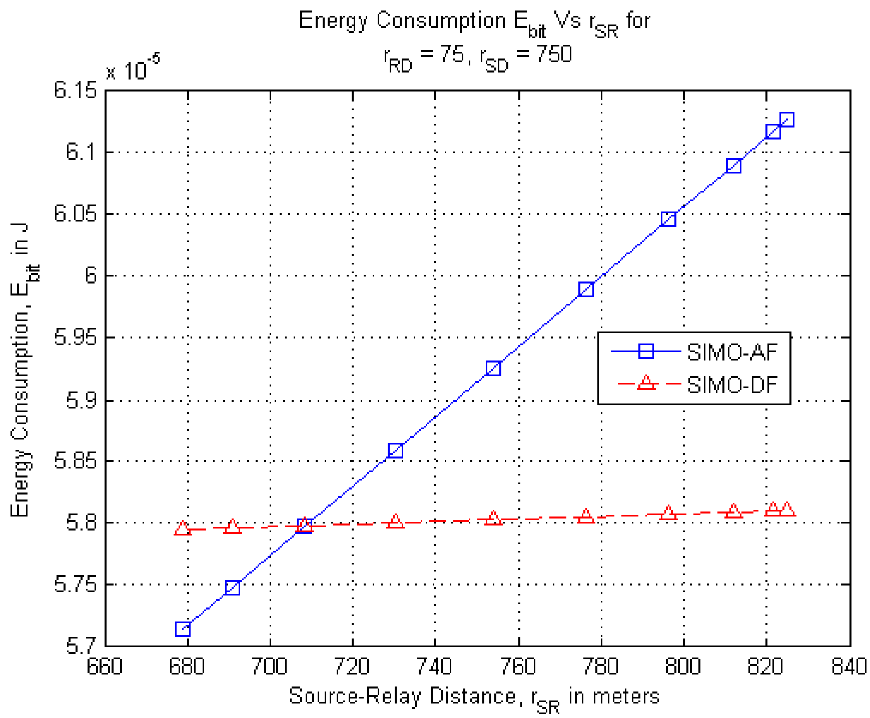

Figure 4.6: Total Energy consumption v.s. Source-Relay, for a fixed $r_{RD} = 75$ and $r_{SD} = 750$

a simple circuit for the AF at the relay and calculated its circuit energy consumption. The circuit energy is constant irrespective of the distance and the transmission energy formulated as a convex optimization problem subject to a target SEP for both SIMO-AF and SIMO-DF. A general approximation for SEP at high SNRs for DF with possible error in relays is formulated for the target SEP which is found to close to the Monte Carlo simulated SEP. The SEP obtained is also valid for a wide range of modulation schemes in AWGN channel. The expression for transmission energy of the SIMO-DF is found to be non-convex and is approximated by a linear expression which on minimization gives very close results to the original objective. This is verified using numerical simulations. The result is used to compare the total energy consumption of SIMO-AF and SIMO-DF for various relay positions: relay near source, relay near destination and all comparable distances. Approximations are derived for the analytic expressions for the first two cases which gives an intutive understanding of the transmission energy spent between source and relay. The

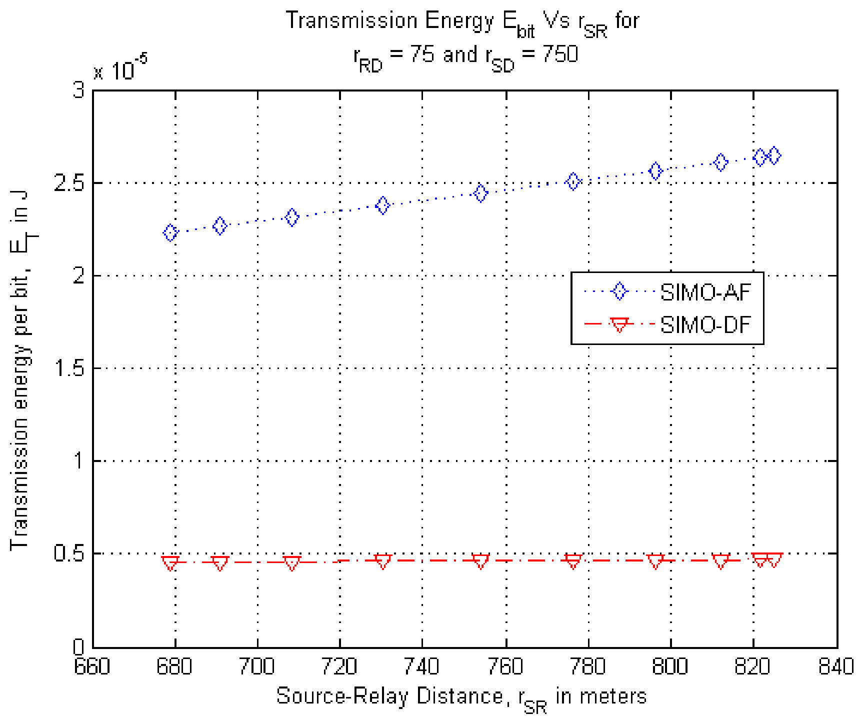

Figure 4.7: Transmission Energy consumption v.s. Source-Relay Distance, for a fixed $r_{RD} = 75$ and $r_{SD} = 750$

results generally shows a significant dependence on circuit energy for short distances where DF is found to perform worse and transmission energy at larger distances where AF is found to perform badly.

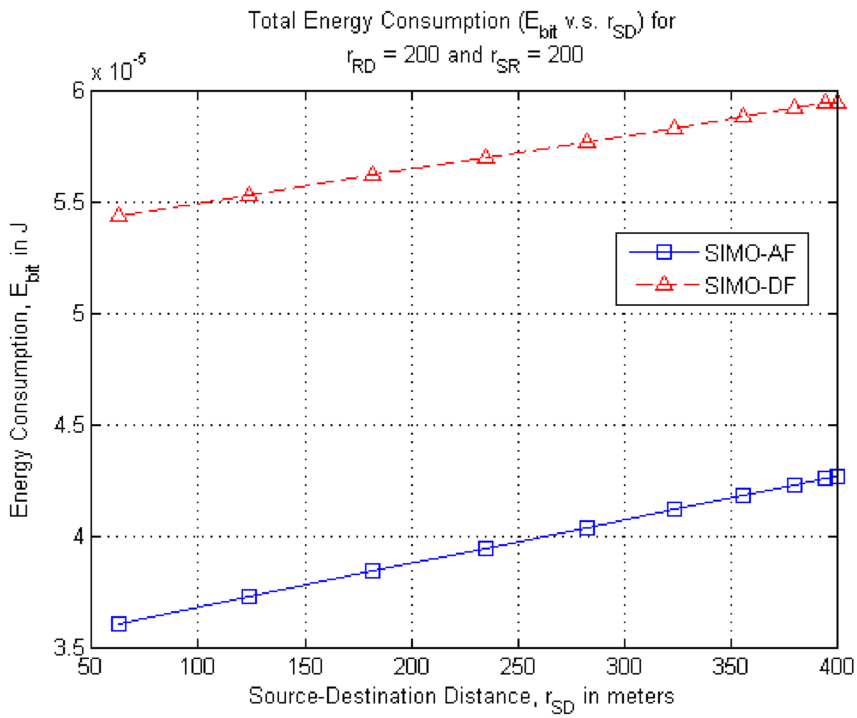

Figure 4.8: Total Energy consumption v.s. Source-Destination Distance, for a fixed $r_{RD} = 200$ and $r_{SR} = 200$

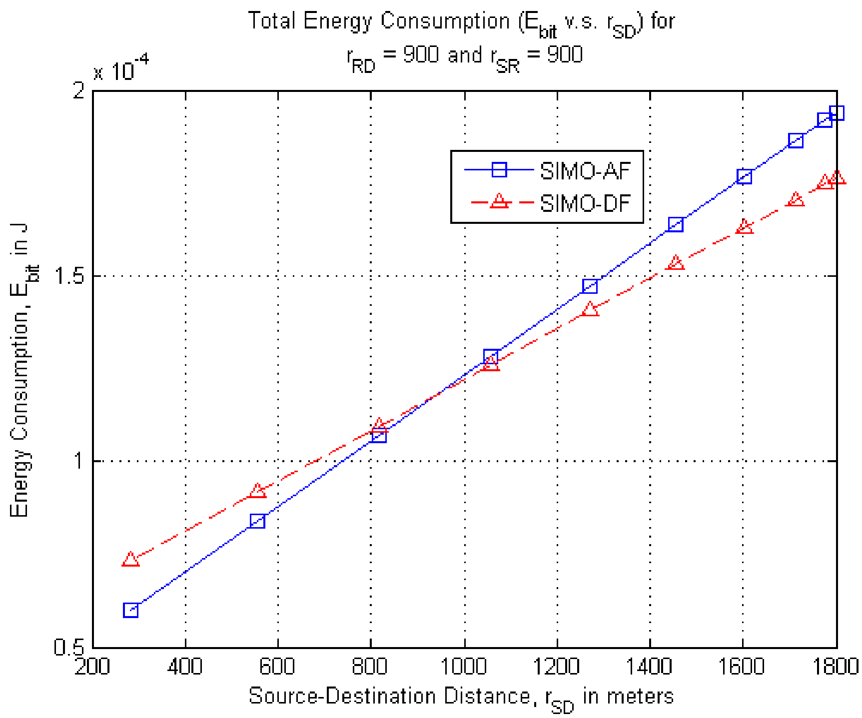

Figure 4.9: Total Energy consumption v.s. Source-Destination Distance, for a fixed $r_{RD} = 900$ and $r_{SR} = 900$

Chapter 5

Energy Efficiency of Cooperative MISO scheme

Similar to the SIMO analysis, in this chapter we compare the energy efficiency of MISO-DF and MISO-AF. Recall the MISO protocol described in table 2.1. In the first time instant the source transmits to relay and in the second instant both source and relay transmit to the destination. In order to compare, we first find the minimum overall transmission energy for both MISO-DF and MISO-AF which required to maintain a target SEP. The minimization is done by formulating it as an optimization problem. We derive an approximation for SEP of MISO-DF in the process. Finally, the energy consumed per bit is found as the sum of transmission energy and circuit energy.

5.1 Transmission Energy for MISO-DF

5.1.1 Symbol Error Probability of MISO-DF

In this section we derive the SEP of a MISO-DF scheme. The received signal at the destination at the second time slot is given by equation (2.11). When there is an error in the relay, it does not forward the symbol to the destination. The instantaneous SNR at the destination in this case is purely due to that from source alone and is equal to $\gamma_{D,e} = \frac{E_{SD}|h_{SD}|^2}{N_0}$. We need to find the statistics of instantaneous SNR when the relay forwards. The proof is shown in Appendix A.1 and it is found that the instantaneous SNR $\gamma_{D,ne} = \frac{(E_{SD}+E_{RD})|h|^2}{N_0}$.

$| h |^2$ is a chi square random variable with two degrees of freedom. This result was also confirmed using simulations. Therefore instantaneous SNR at the destination,

$$\gamma_D = \begin{cases} \gamma_{D,e} & \text{when } \eta = 1 \\ \gamma_{D,ne} & \text{when } \eta = 0 \end{cases} \tag{5.1}$$

where, η was defined in equation (4.1). The instantaneous SNR at the destination averaged over the relay is given by $\overline{\gamma}_D = \gamma_{D,e}\alpha + \gamma_{D,ne}(1-\alpha)$ and $\alpha = P(\eta = 1)$, the SEP at the relay. The instantaneous SEP P_e is given by $Q\{\sqrt{k(\gamma_{D,e}\alpha + \gamma_{D,ne}(1-\alpha))}\}$. The argument of Q function has the sum of random variables and it appears to be analytically intractable to find the average SEP. Applying Jensen's inequality as $Q\{\sqrt{\cdot}\}$ is a convex function we get,

$$\begin{aligned} P_e &= Q\{\sqrt{k(\gamma_{D,e}\alpha + \gamma_{D,ne}(1-\alpha))}\} \\ &\leq \alpha Q\{\sqrt{k\gamma_{D,e}}\} + (1-\alpha)Q\{\sqrt{k\gamma_{D,ne}}\} \end{aligned} \tag{5.2}$$

Here, we have upperbounded the instantaneous SEP by a more analytically tractable function. Therefore the average SEP $\overline{P}_e$ is given by,

$$\begin{aligned} \overline{P}_e &= E\{Q\{\sqrt{k(\gamma_{D,e}\alpha + \gamma_{D,ne}(1-\alpha))}\}\} \\ &\leq \alpha E\{Q\{\sqrt{k\gamma_{D,e}}\}\} + (1-\alpha)E\{Q\{\sqrt{k\gamma_{D,ne}}\}\} \end{aligned} \tag{5.3}$$

The SEP expression can be further modified as [11],

$$\overline{P}_e \leq \left(1 - \frac{1}{2k\overline{\gamma}_{SR}}\right) \frac{1}{2k(\overline{\gamma}_{SD} + \overline{\gamma}_{RD})} + \frac{1}{4k^2\overline{\gamma}_{SR}\overline{\gamma}_{SD}} \tag{5.4}$$

where, $E\{Q\{\sqrt{k\gamma_{D,ne}}\}\} = \frac{1}{2k(\overline{\gamma}_{SD}+\overline{\gamma}_{RD})}$ and $E\{Q\{\sqrt{k\gamma_{D,e}}\}\} = \frac{1}{2k\overline{\gamma}_{SD}}$. Figure 5.1 shows that this upperbound (5.4) is tight when α is small and can be considered as an approximation to the SEP of MISO-DF when the arguments of $Q\{\sqrt{\cdot}\}$ function are large. Intuitively, this is due to the combined effect of the fact that the $Q\{\sqrt{\cdot}\}$ is convex and the slope tends to zero as its argument increases.

5.1.2 Problem Formulation for MISO-DF

In this section we formulate an optimization problem to minimize the transmitted energy. Consider the case when there is no error in the relay. The problem of finding the optimum

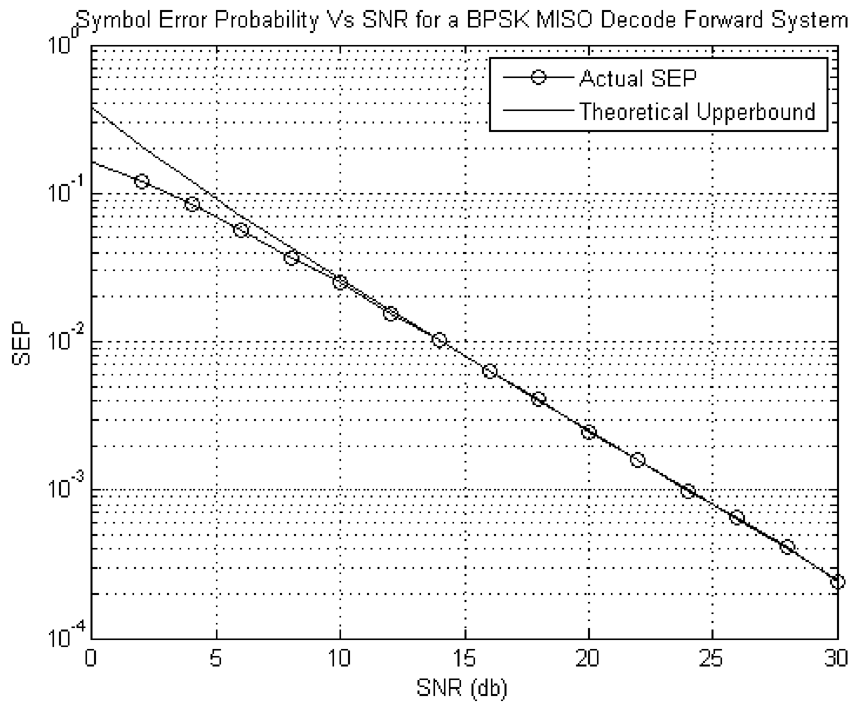

Figure 5.1: Symbol Error Probability v.s. SNR for the MISO-DF

transmission energy may be posed as,

$$
\begin{aligned}
\phi_{\text{MISO-DF}} = \underset{\overline{\gamma}_{SR},\overline{\gamma}_{SD},\overline{\gamma}_{RD}}{\text{minimize}} \quad & f_0\overline{\gamma}_{SR} + f_1\overline{\gamma}_{SD} + f_2\overline{\gamma}_{RD} \\
\text{subject to} \quad & \frac{1}{2k(\overline{\gamma}_{SD}+\overline{\gamma}_{RD})} \le p_c, \\
& \frac{1}{2k\overline{\gamma}_{SR}} \le \alpha, \\
& -\overline{\gamma}_{SR} \le 0, \\
& -\overline{\gamma}_{SD} \le 0, \\
& -\overline{\gamma}_{RD} \le 0. \qquad (5.5)
\end{aligned}
$$

where, $f_0 = c_0 r_{SR}^2$, $f_1 = c_0 r_{SD}^2$, $f_2 = c_0 r_{RD}^2$; c_0 is a constant for the system as defined in equation (3.11) and which corresponds to the system parameters like frequency, antenna gains etc.; the transmission energy consumed for one bit $E_b = N_0 * \phi_{\text{MISO-DF}}$ and p_c is the target SEP at the destination. Because of the charateristic of DF protocol we have to introduce a performance criterion at the relay given by the second constraint. The solution to the optimization problem 5.5 is the sum of the optimal values of two seperate problems given by,

$$\begin{aligned} \phi^{(1)}_{\mathrm{MISO-DF}} = \underset{\overline{\gamma}_{SR}}{\text{minimize}} \quad & f_0 \overline{\gamma}_{SR} \\ \text{subject to} \quad & \frac{1}{2k\overline{\gamma}_{SR}} \leq \alpha, \\ & -\overline{\gamma}_{SR} \leq 0. \end{aligned} \tag{5.6}$$

$$\begin{aligned} \phi^{(2)}_{\mathrm{MISO-DF}} = \underset{\overline{\gamma}_{SD}, \overline{\gamma}_{RD}}{\text{minimize}} \quad & f_1 \overline{\gamma}_{SD} + f_2 \overline{\gamma}_{RD} \\ \text{subject to} \quad & \frac{1}{2k(\overline{\gamma}_{SD} + \overline{\gamma}_{RD})} \leq p_c, \\ & -\overline{\gamma}_{SD} \leq 0, \\ & -\overline{\gamma}_{RD} \leq 0. \end{aligned} \tag{5.7}$$

Both the problems are convex and can be solved analytically. The solution to problem 5.6 is direct and the optimal $\overline{\gamma}^{\star}_{SR} = \frac{1}{2k\alpha}$. The second problem has an interesting solution. The optimal points are given by,

$$\begin{aligned} \overline{\gamma}^{\star}_{SD} = 0, \quad \overline{\gamma}^{\star}_{RD} = \frac{1}{2kp_c} \quad & \text{when} \quad f_1 > f_2, \\ \overline{\gamma}^{\star}_{SD} = \frac{1}{2kp_c}, \quad \overline{\gamma}^{\star}_{RD} = 0 \quad & \text{when} \quad f_1 < f_2. \end{aligned} \tag{5.8}$$

The results seems to suggest that it is optimal to use MISO protocol only when $f_1 > f_2 \rightarrow r_{SD} > r_{RD}$. Also, the optimal strategy is relaying of information. The proof of the result is given in Appendix A.2. The minimum energy consumed is given by $N_0 * (\phi^{(1)}_{\mathrm{MISO-DF}} + \phi^{(2)}_{\mathrm{MISO-DF}})$ assuming no error at the relay. This analytical result obtained is confirmed using numerical methods. This is true when the relay decodes the bit correctly. In order to accomodate for relay errors we assume that if an error occurs at the relay it is known to source through some acknowledgement. The source will then transmit to destination directly with energy $\phi_{\mathrm{SD}} = \frac{f_1}{2kp_c}$. Hence, the effective energy consumed will be $N_0 * \phi_{\mathrm{MISO-DF}}$ where,

$$\phi_{\mathrm{MISO-DF}} = \{(1-\alpha)(\phi^{(1)}_{\mathrm{MISO-DF}} + \phi^{(2)}_{\mathrm{MISO-DF}}) + \alpha(\phi^{(1)}_{\mathrm{MISO-DF}} + \phi_{\mathrm{SD}})\} \tag{5.9}$$

5.1.3 Conclusion for MISO-DF

It is found that the best strategy of transmission for the MISO protocol described in table 2.1, is relaying for information rather than involving both source and relay to transmit simultaneously in the second time slot. The implementation of MISO-DF is restricted to the region where $r_{SD} > r_{RD}$.

5.2 Transmission Energy for MISO-AF

5.2.1 Symbol Error Probability of MISO-AF

In this section we find the SEP of the MISO-AF scheme. The SEP can be calculated as $E\{Q\{\sqrt{k\gamma_D}\}\}$ where γ_D is the instantaneous SNR at the destination. However, since the expression for instantaneous SNR at the destination is analytically intractable we resort to find a lower bound given by $Q\{\sqrt{kE\{\gamma_D\}}\}$ where $E\{\gamma_D\} = \overline{\gamma}_D$ is the average SNR. This is a lower bound because $Q\sqrt{\{.\}}$ is a convex function and Jensen's inequality gives the result $E\{Q\{\sqrt{k\gamma_D}\}\} \geq Q\{\sqrt{kE\{\gamma_D\}}\}$. Accordingly, we first find the average SNR received at the destination. The noise power P_N in equation (2.6) is given by

$$P_N = \quad E\left\{\left(\frac{\sqrt{E_{RD}}h_{RD}n_{SR}}{\sqrt{E_{SR}+N_0}} + n_{D,2}\right)\left(\frac{\sqrt{E_{RD}}h_{RD}n_{SR}}{\sqrt{E_{SR}+N_0}} + n_{D,2}\right)^H\right\}, \tag{5.10}$$

where $E\{.\}$ denotes the expectation operator on all random variables. Since h_{RD}, n_{SR}, $n_{D,2}$ are all random variables which are statistically independent we can easily arrive at the result that,

$$P_N = N_0\left(\frac{E_{RD}}{E_{SR}+N_0} + 1\right). \tag{5.11}$$

Dividing the equation (2.6) by $\left(\frac{E_{RD}}{E_{SR}+N_0} + 1\right)^{\frac{1}{2}}$ and rearranging the terms so that we make the effective noise power in the equation (2.6) N_0 we get,

$$y_{D,2} = Ax_1 + n, \tag{5.12}$$

where,

$$A = \left\{ \frac{\sqrt{E_{RD}E_{SR}} h_{RD} h_{SR}}{\sqrt{E_{SR} + E_{RD} + N_0}} + \frac{\sqrt{E_{SD}} h_{SD} (E_{SR} + N_0)^{\frac{1}{2}}}{\sqrt{E_{SR} + E_{RD} + N_0}} \right\} \tag{5.13}$$

and n is $ZMCSCG(0, \frac{N_0}{2})$. The Average SNR of the recieved signal $\overline{\gamma}_D$ is given by $\frac{E\{AA^H\}}{N_0}$ and is calculated as,

$$\overline{\gamma}_D = \frac{\overline{\gamma}_{SR}\overline{\gamma}_{RD} + \overline{\gamma}_{SD}(1 + \overline{\gamma}_{SR})}{1 + \overline{\gamma}_{SR} + \overline{\gamma}_{RD}}, \tag{5.14}$$

where, any $\overline{\gamma}_b = \frac{E_b}{N_0}$ $\forall b \in \{SR,\, SD,\, RD\}$. Therefore the average SNR found as the ratio of signal power to noise power in $y_{D,2}$ is given by,

$$\overline{\gamma}_D \;\; = \;\; \overline{\gamma}_{SD} + \frac{\overline{\gamma}_{RD}(\overline{\gamma}_{SR} - \overline{\gamma}_{SD})}{1 + \overline{\gamma}_{SR} + \overline{\gamma}_{RD}}. \tag{5.15}$$

The result implies that as long as $\overline{\gamma}_{SR} > \overline{\gamma}_{SD}$ the relay contributes positively to the SNR, otherwise it degrades the SNR at destination. Therefore if the available $\overline{\gamma}_{SR}$ is less than $\overline{\gamma}_{SD}$, set $\overline{\gamma}_{RD} = 0$ (direct transmission) to prevent degradation in performance.

5.2.2 Problem Formulation for MISO-AF

Our objective is to determine the minimum energy required to guarantee a certain SEP at the destination. The SEP at destination corresponds to $E\{Q\{\sqrt{k\gamma_D}\}\}$, where $Q\{.\}$ denotes the Q function and k is some constant dependent on the modulation schemes used. To minimize the transmission energy we need to solve the optimization problem,

$$\begin{aligned} \phi_{\text{MISO}-\text{AF}} = \underset{\overline{\gamma}_{SR},\overline{\gamma}_{SD},\overline{\gamma}_{RD}}{\text{minimize}} \quad & f_0\overline{\gamma}_{SR} + f_1\overline{\gamma}_{SD} + f_2\overline{\gamma}_{RD}, \\ \text{subject to} \quad & E\{Q\{\sqrt{k\gamma_D}\}\} \le p_c, \end{aligned} \tag{5.16}$$

where, $f_0 = c_0 r_{SR}^2$, $f_1 = c_0 r_{SD}^2$, $f_2 = c_0 r_{RD}^2$; c_0 is a constant for the system which corresponds to the system parameters like frequency, antenna gains etc.; the transmission energy consumed for one bit $E_b = N_0 * \phi_{\text{MISO}-\text{AF}}$ and p_c is the target SEP at the destination. The constraint in problem (5.16) is not convex with respect to the variables $\overline{\gamma}_{SR}, \overline{\gamma}_{SD}$ and

$\overline{\gamma}_{RD}$. It is clear that since the problem is not convex finding a global minimum is almost impossible. The problem can reformulated as a non-convex QCQP (Proof in Appendix A.3) and a semidefinite or lagrangian relaxation does not seem to give any informative lower bounds on the objective [25]. Alternately, the convex-concave procedure [26] can be employed to analyze the problem. The obtained numerical results indicate that inorder to minimize long term energy consumption pure relaying seems to be more feasible option than implementing a MISO-AF scheme. For the QCQP formulation we transform the constraint $E\{Q\{\sqrt{k\gamma_D}\}\} \leq p_c$ to $Q\{\sqrt{kE\{\gamma_D\}}\} \leq p_c$ using jensen's inequality. In effect we are replacing the constraint with a lower bound for get analytical results.

To further confirm the observation and derive useful analytical insights, we attempt to reformulate (5.16). Specifically we consider a case where the relay maintains at least an SEP of α and hence at least an SNR of $\overline{\gamma}_{SR}$. This assumption is practical and is based on the reasoning that a signal at the relay is of acceptable quality. This implies that the source transmits at a constant power in the first time slot to guarantee this received signal quality at the relay [23]. Specifically, our approach is to find the optimal $\overline{\gamma}_{SD}$ and $\overline{\gamma}_{RD}$ assuming constant $\overline{\gamma}_{SR}$; and in the next stage find the best $\overline{\gamma}_{SR}$ among all possible values it can take which will minimize the objective. Therefore we can rewrite the optimization problem as,

$$\begin{aligned} \phi_{\text{MISO-AF}} = \underset{\overline{\gamma}_{SD},\overline{\gamma}_{RD}}{\text{minimize}} \quad & f_0\overline{\gamma}_{SR} + f_1\overline{\gamma}_{SD} + f_2\overline{\gamma}_{RD}, \\ \text{subject to} \quad & E\{Q\{\sqrt{k\gamma_D}\}\} \leq p_c, \\ & \overline{\gamma}_{SD} \leq \overline{\gamma}_{SR}. \end{aligned} \tag{5.17}$$

Here we add another constraint such that $\overline{\gamma}_{SR}$ is greater than $\overline{\gamma}_{SD}$ for cooperation to be effective as found in equation (5.15) and the problem is optimized for variables $\overline{\gamma}_{SD}$ and $\overline{\gamma}_{RD}$. Introducing a new variable γ and rewriting the SEP expression as $E\{Q\{\sqrt{k\gamma_D}\}\} \leq p_c \Rightarrow \overline{\gamma}_D \geq \overline{\gamma}_{SD,d}$ where, $\overline{\gamma}_{SD,d}$ is the SNR required at destination to maintain a SEP of p_c. This change of constraints is valid because the γ_D is found to be exponential random variable using simulations. The optmization problem can be rewritten as,

$$
\begin{aligned}
\phi_{\text{MISO-AF}} = \underset{\overline{\gamma}_{SD},\overline{\gamma}_{RD},\gamma}{\text{minimize}} \quad & f_0\overline{\gamma}_{SR} + f_1\overline{\gamma}_{SD} + f_2\overline{\gamma}_{RD}, \\
\text{subject to} \quad & S_1\overline{\gamma}_{SD} + S_2\overline{\gamma}_{RD} + S_3 \leq 0, \\
& -\gamma \leq 0, \\
& \overline{\gamma}_{SD} + \gamma = \overline{\gamma}_{SR}, \qquad (5.18)
\end{aligned}
$$

where, $S_1 = \overline{\gamma}_{SD,d} - \overline{\gamma}_{SR}$, $S_2 = -(1+\overline{\gamma}_{SR})$ and $S_3 = (1+\overline{\gamma}_{SR})\overline{\gamma}_{SD,d}$. The problem is convex (linear) and hence KKT conditions will give the global minima. Setting up the Lagrangian and solving for KKT conditions we get the solutions for the problem as,

$$
\begin{aligned}
\overline{\gamma}_{SD} &= \overline{\gamma}_{SR} - \gamma, \\
\overline{\gamma}_{RD} &= (1+\overline{\gamma}_{SR}) \left\{ \frac{\gamma - (\overline{\gamma}_{SR} - \overline{\gamma}_{SD,d})}{\overline{\gamma}_{SR} - \overline{\gamma}_{SD,d}} \right\}. \qquad (5.19)
\end{aligned}
$$

An implicit condition coming out of equation (5.19) is that $\overline{\gamma}_{SR} > \overline{\gamma}_{SD,d}$ and $\overline{\gamma}_{SR} - \overline{\gamma}_{SD,d} \leq \gamma \leq \overline{\gamma}_{SR}$ for both $\overline{\gamma}_{SD}$ and $\overline{\gamma}_{RD}$ to take a positive value. At this point we can consider γ as some parameter which can adjust the power used by both source and relay.

Substituting $\overline{\gamma}_{SD}$ and $\overline{\gamma}_{RD}$ for objective $\phi_{\text{MISO-AF}}$ in problem (5.18) we can reformulate the optimizaton problem to minimize transmission energy in terms of $\overline{\gamma}_{SR}$ and γ. The problem is stated thus,

$$
\begin{aligned}
\phi_{\text{MISO-AF}} = \underset{\overline{\gamma}_{SR},\gamma}{\text{minimize}} \quad & (f_0 + f_1 - f_2)\overline{\gamma}_{SR} + \frac{f_2\,(1+\overline{\gamma}_{SR})\,\gamma}{\overline{\gamma}_{SR} - \overline{\gamma}_{SD,d}} - f_1\gamma - f_2, \\
\text{subject to} \quad & -\overline{\gamma}_{SR} + \overline{\gamma}_{SD,d} < 0, \\
& \gamma - \overline{\gamma}_{SR} \leq 0, \\
& -\gamma + \overline{\gamma}_{SR} - \overline{\gamma}_{SD,d} \leq 0. \qquad (5.20)
\end{aligned}
$$

Although the problem is non-convex because of the non-convex objective function, we set out to find one possible suboptimal but local minimum point. We set up the Lagrangian and get the KKT conditions as follows,

$$(f_0 + f_1 - f_2) + \frac{f_2 \gamma}{\overline{\gamma}_{SR} - \overline{\gamma}_{SD,d}} - \frac{f_2(1+\overline{\gamma}_{SR})\gamma}{(\overline{\gamma}_{SR} - \overline{\gamma}_{SD,d})^2} - \lambda_1 - \lambda_2 + \lambda_3 = 0, \tag{5.21}$$

$$-f_1 + \frac{f_2(1+\overline{\gamma}_{SR})}{\overline{\gamma}_{SR} - \overline{\gamma}_{SD,d}} + \lambda_2 - \lambda_3 = 0, \tag{5.22}$$

$$\lambda_1 \left(-\overline{\gamma}_{SR} + \overline{\gamma}_{SD,d}\right) = 0, \tag{5.23}$$

$$\lambda_2 \left(\gamma - \overline{\gamma}_{SR}\right) = 0, \tag{5.24}$$

$$\lambda_3 \left(-\gamma + \overline{\gamma}_{SR} - \overline{\gamma}_{SD,d}\right) = 0, \tag{5.25}$$

where, $\lambda_1 > 0$, $\lambda_2 \geq 0$ and $\lambda_3 \geq 0$ are the lagrangian multipliers. It is clear that the solution out these is not a global minimum. Out of the possible cases only two are valid and they are, CASE A: $(\lambda_1, \lambda_2, \lambda_3 = 0)$ which corresponds to pure MISO transmission in which both source and relay are involved in the second time slot and CASE B: $(\lambda_1, \lambda_3 = 0, \lambda_2 > 0)$ which corresponds to pure relaying. We do not get direct transmission as a case in KKT conditions as MISO form of transmission entails that in the first time slot a signal to be transmitted to relay even though relay may not help in forwarding in the second time slot.

The candidate points corresponding to CASE A is obtained from solving equation (5.22) for $\overline{\gamma}_{SR}$ and substituting $\overline{\gamma}_{SR}$ in (5.21) to get γ. We get $\overline{\gamma}_{SR,A}$ and γ_A corresponding to CASE A as

$$\overline{\gamma}_{SR,A} = \frac{f_1 \overline{\gamma}_{SD,d} + f_2}{f_1 - f_2} \tag{5.26}$$

$$\gamma_A = \frac{f_2(f_0 + f_1 - f_2)(1 + \overline{\gamma}_{SD,d})}{(f_1 - f_2)^2} \tag{5.27}$$

For $\overline{\gamma}_{SR,A}$ to be valid, $f_1 > f_2$ or $r_{SD} > r_{RD}$ for constraint (**??**) be satisfied. γ_A is found to be always greater than $\overline{\gamma}_{SR} - \overline{\gamma}_{SD,d}$ (lower bound) but in order for it be lesser than upper bound, the condition $f_1 \geq f_2 + \sqrt{f_0 f_2}$ has to hold (Proof in Appendix A.4). The solution corresponding to Case B are

$$\gamma = \overline{\gamma}_{SR,B} = \overline{\gamma}_{SD,d} + \left(\frac{f_2 \overline{\gamma}_{SD,d}(\overline{\gamma}_{SD,d} + 1)}{f_0}\right)^{\frac{1}{2}} \tag{5.28}$$

Assume $\overline{\gamma}_{SD,d} \gg 1$, and we get,

$$\gamma = \overline{\gamma}_{SR,B} = \overline{\gamma}_{SD,d}\left[1 + \left(\frac{f_2}{f_0}\right)^{\frac{1}{2}}\right] \tag{5.29}$$

We also find that the lagrangian multiplier $\lambda_2 > 0$ if $f_1 > f_2 + \sqrt{f_0 f_2}$.

5.2.3 Conclusion for MISO-AF

We already have shown that the use of relay is limited to regions where the condition $f_1 > f_2 + \sqrt{f_0 f_2}$. The condition can be rewritten as $r_{SD}^2 > r_{RD}(r_{SR} + r_{RD})$ which gives some intuitive idea that the relay should be close to the source-destination pair. Assuming MISO-AF is applicable i.e. $f_1 > f_2 + \sqrt{f_0 f_2}$, we need to check the energy incurred in transmitting one bit from source to destination according to the formulation. This is done by substituting $\overline{\gamma}_{SR,A}$ and γ_A in the objective $\phi_{\mathrm{MISO-AF}}$ as in equation (5.20).

The value of $\phi_{\mathrm{MISO-AF}}$ corresponding to Case A is given as,

$$\phi^{A}_{\mathrm{MISO-AF}} = \frac{f_0(f_1\overline{\gamma}_{SD,d} + f_2)}{f_1 - f_2} + f_1\overline{\gamma}_{SD,d} \tag{5.30}$$

Similarly, the energy incurred while using the pure relaying is given by,

$$\phi^{B}_{\mathrm{MISO-AF}} = c_0\left\{(r_{SR} + r_{RD})^2\overline{\gamma}_{SD,d} + r_{SR}r_{RD}\right\} \tag{5.31}$$

Comparing $\phi^{A}_{\mathrm{MISO-AF}}$ and $\phi^{B}_{\mathrm{MISO-AF}}$ it is found that $\phi^{A}_{\mathrm{MISO-AF}}$ is always greater than $\phi^{B}_{\mathrm{MISO-AF}}$ (Proof in Appendix A.5). This implies that MISO form of transmission using a single fixed relay over a long period of time (multiple instances of channel) is not the most energy efficient transmission. Intuitively, the reason for MISO mode's energy inefficiency is the result of the condition $\overline{\gamma}_{SR} > \overline{\gamma}_{SD}$. This condition strains the transmission at the first instant and results in a higher transmission energy.

5.3 Results and Discussion

The following are the conclusions made on the analysis of MISO-DF and MISO-AF.

- If $r_{SD} < r_{RD}$, it implies that both MISO-DF and MISO-AF cannot be implemented.

- In the region $r_{SD} > r_{RD}$ but $r_{SD}^2 < r_{RD}(r_{SR} + r_{RD})$ our only option is to implement MISO-DF as it violates the required condition for MISO-AF.

- If $r_{SD}^2 > r_{RD}(r_{SR} + r_{RD})$ is true, it implies that $r_{SD} > r_{RD}$, the condition for MISO-DF. Therefore, we set out to compare the two schemes when $r_{SD}^2 > r_{RD}(r_{SR} + r_{RD})$. Assuming same performance at the relay, it can be seen that when relay is very near the source i.e., when $r_{SR} \ll r_{RD}, r_{SD}$ and $r_{RD} \approx r_{SD}$ the DF is found to consume less transmission energy. (Appendix A.6)

- When $r_{RD} \ll r_{SR}, r_{SD}$ and $r_{SR} \approx r_{SD}$ it is seen that AF consumes less transmission energy than DF, theoretically (Appendix A.6). However, for BPSK system with target SEP 10^{-3}, it is seen that this happens only when $r_{RD} \to 0$, which is generally not practical.

Further, we compare the energy consumption of MISO-AF and MISO-DF. We have already found the minimized transmission energy for both. In order to make the comparision we make sure that the relay performance is same in both the modes. Since the performance at the relay for MISO-DF can be decided arbitrarily, the SEP at relay is fixed to a value determined by the SNR as in equation 5.26. The $\overline{\gamma}_{SD,d}$ used to calculate $\overline{\gamma}_{SR,A}$ is found from the high SNR approximation for SEP given by $\frac{1}{2kp_c}$. As in SIMO we consider BPSK transmission example with target SEP to be 10^{-3} for various relay positions.

In the first case we set r_{SD} and r_{RD} constant and vary r_{SR}. Here, we consider a case when $r_{RD} \ll r_{SD}$. When $r_{RD} \approx r_{SD}$ the AF mode may become impossible for some r_{SR} and hence effective comparision is not possible. We consider a particular case when $r_{SD} = 550m$ and $r_{RD} = 50m$. The relay position can hence vary between $500m < r_{RD} < 600m$. The plot for the transmission energy and total energy consumption is as shown in figures 5.2 and 5.3 respectively. It is seen that AF consumes more transmission energy and eventually as the distance increases the effect of transmission energy is more than that of circuit

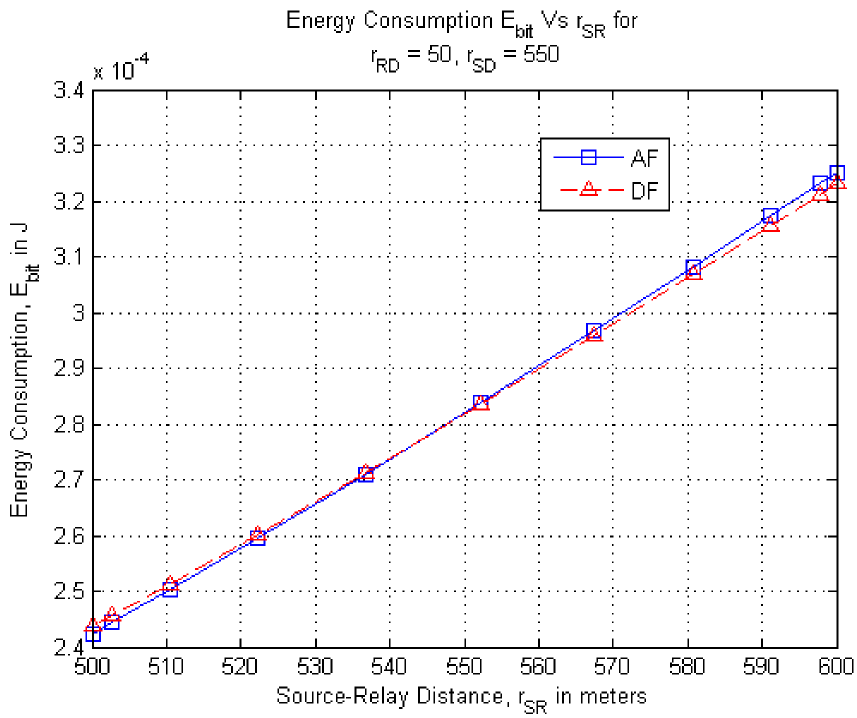

Figure 5.2: Total Energy consumption for MISO v.s. source-relay distance, for a fixed $r_{SD} = 550m$ and $r_{RD} = 50m$

energy consumption. Therefore, at large source-destination separation ($r_{SD} > 500m$) it is advisable to use DF if the relay is near destination. The reason being that DF consumes less trasmission energy. At small source-distance seperation AF is energy efficient as circuit energy dominates and is more for DF. As seen in figure 5.3 when $r_{SD} > 550m$ the transition occurs.

In the second case, we set r_{SD} and r_{SR} constant and vary r_{RD}. The case when $r_{SR} \ll r_{SD}$ is not considered as it violates the MISO-AF condition in most cases of corresponding r_{RD} values. Figure 5.4 and 5.5 shows the transmission energy and total energy required when $r_{SD} = r_{SR} = 400$. Figure 5.5 shows that DF is efficient after when $r_{RD} > 60m$. Another case when $r_{SD} = r_{SR} = 100$ gives the result that AF is energy efficient (Figure 5.6) but limited to a few relay positions. Therefore, when MISO-AF is not applicable there is no other option other than using DF.

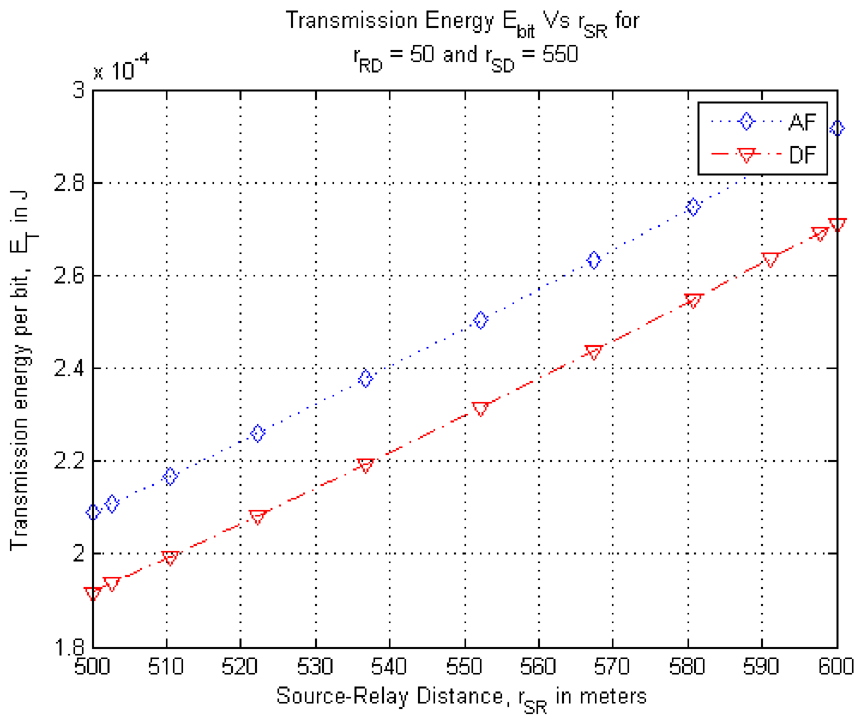

Figure 5.3: Transmission Energy consumption for MISO v.s. source-relay distance, for a fixed $r_{SD} = 550m$ and $r_{RD} = 50m$

5.4 Summary

In this chapter we found a good approximation for SEP for the MISO-DF case. We found the minimum transmission energy for both MISO-DF and MISO-AF. The optimal strategy for MISO-DF seems to be relaying. For MISO-AF although suboptimal the better strategy seems to be again relaying. Further, we compare the energy efficiency of the two protocols assuming the same performance at the relay and the results are analyzed.

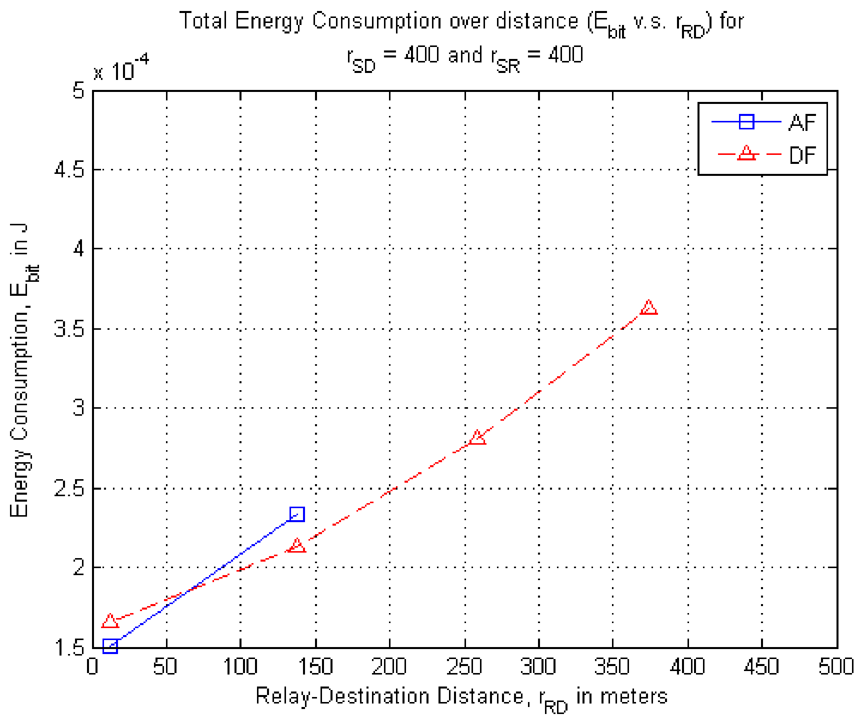

Figure 5.4: Total Energy consumption for MISO v.s. relay-destination distance, for a fixed $r_{SD} = 400m$ and $r_{SR} = 400m$

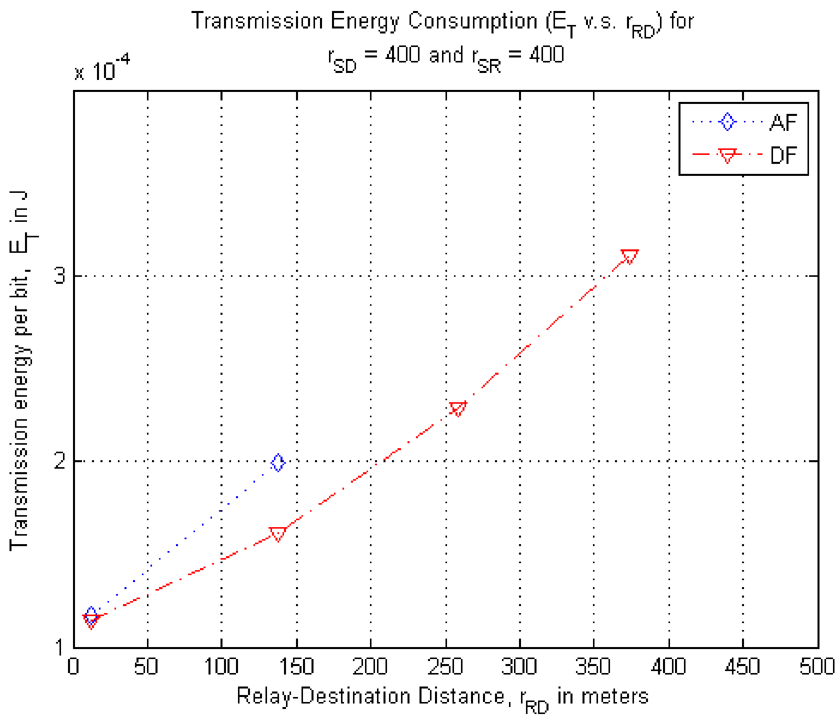

Figure 5.5: Transmission Energy consumption for MISO v.s. relay-destination distance, for a fixed $r_{SD} = 400m$ and $r_{SR} = 400m$

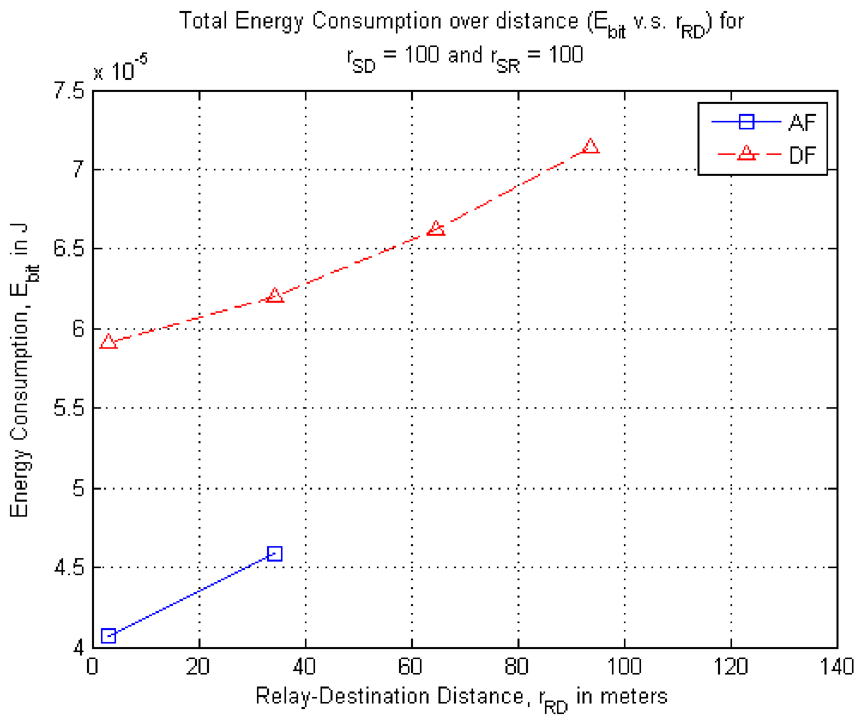

Figure 5.6: Total Energy consumption for MISO v.s. relay-destination distance, for a fixed $r_{SD} = 100m$ and $r_{SR} = 100m$

Chapter 6

Energy Efficiency of Cooperative MIMO scheme

In the previous two chapters, we analyze the energy efficiency of SIMO and MISO schemes. In this chapter we deal with the MIMO, which can be considered as a combination of SIMO and MISO protocols. We have already introduced the MIMO protocol and it is described in 2.1. Briefly describing it once again, in the first time instant source broadcasts to both relay and destination. In the second time instant both source and relay transmit again to the destination.

6.1 Transmission Energy for MIMO-DF

6.1.1 Symbol Error Probability of MIMO-DF

In this section, we find the SEP of a MIMO-DF scheme. The signal models are described by equations (2.8), (2.11). The instantaneous SNR at the destination in the second instant can be found out from equation (2.11) and is given by $\frac{(E_{SD,2}+E_{RD,2})|h|^2}{N_0}$ when there is no error in the relay, where $\mid h \mid^2$ is chi-square random variable with two degrees of freedom. The proof of this is similar to that presented in Appendix A.1. If there is an error in the relay, the SNR is given by $\frac{E_{SD,2}|h_{SD,2}|^2}{N_0}$ as the relay does not transmit. Also, from equation (2.8) the SNR received at the first instant is given by $\frac{E_{SD,1}|h_{SD,1}|^2}{N_0}$. Since we combine the signals at two instances using maximal ratio combining the resultant SNR at the destination (when there is no error at relay) is given by,

$$\gamma_D = \frac{(E_{SD,2} + E_{RD,2}) \mid h \mid^2}{No} + \frac{E_{SD,1} \mid h_{SD,1} \mid^2}{No}. \tag{6.1}$$

Since it is the sum of two independent random variables we can find the SEP as [11],

$$P_e = \frac{3}{4k^2 \overline{\gamma}_{SD,1}(\overline{\gamma}_{SD,2} + \overline{\gamma}_{RD,2})} \tag{6.2}$$

where, $\overline{\gamma}_b = E\{\frac{E_b |h_b|^2}{N_0}\} = \frac{E_b}{N_0}$ for $b \ \epsilon \ \{SD,1 \quad SD,2 \quad RD,2\}$. It is important to note that the result in 6.2 employs the high SNR approximation.

6.1.2 Problem Formulation MIMO-DF

In this section, we formulate an optimization problem to minimize the transmitted energy. Consider the case when there is no error in the relay. Similar to MISO-DF the problem of finding the minimum energy can be posed as,

$$\begin{aligned} \phi_{\text{MIMO-DF}} = \underset{\overline{\gamma}_{SR,1}, \overline{\gamma}_{SD,2}, \overline{\gamma}_{RD,2}}{\text{minimize}} \quad & f_0 \overline{\gamma}_{SR,1} + f_1 \overline{\gamma}_{SD,2} + f_2 \overline{\gamma}_{RD,2} \\ \text{subject to} \quad & \frac{3}{4k^2 \overline{\gamma}_{SD,1}(\overline{\gamma}_{SD,2} + \overline{\gamma}_{RD,2})} \le p_c, \\ & -\overline{\gamma}_{SR,1} \le 0, \\ & -\overline{\gamma}_{SD,2} \le 0, \\ & -\overline{\gamma}_{RD,2} \le 0, \\ & f_0 \overline{\gamma}_{SR,1} - f_1 \overline{\gamma}_{SD,1} = 0 \end{aligned} \tag{6.3}$$

where, the energy consumed can be found as equal to $N_0 * \phi_{\text{MIMO-DF}}$. The problem is convex and can be solved easily using numerical methods. However, in an attempt to get an insight into the MIMO protocol and its relation to the previous two protocols we reformulate the problem into two seperable sub problems using primal decomposition [25]. The optimization problem can be rewritten as,

$$\begin{aligned}
\phi_{\mathrm{MIMO-DF}} = \underset{\overline{\gamma}_{SR,1},\overline{\gamma}_{SD,2},\overline{\gamma}_{RD,2}}{\text{minimize}} \quad & f_0\overline{\gamma}_{SR,1} + f_1\overline{\gamma}_{SD,2} + f_2\overline{\gamma}_{RD,2} \\
\text{subject to} \quad & \frac{3}{4k^2(\overline{\gamma}_{SD,2} + \overline{\gamma}_{RD,2})} - \frac{p_c f_0}{f_1}\overline{\gamma}_{SR,1} \leq 0, \\
& -\overline{\gamma}_{SR,1} \leq 0, \\
& -\overline{\gamma}_{SD,2} \leq 0, \\
& -\overline{\gamma}_{RD,2} \leq 0.
\end{aligned} \tag{6.4}$$

The problem 6.4 can be decomposed in two subproblems given by,

$$\begin{aligned}
\phi^{(1)}_{\mathrm{MIMO-DF}}(t) = \underset{\overline{\gamma}_{SR,1}}{\text{minimize}} \quad & f_0\overline{\gamma}_{SR,1} \\
\text{subject to} \quad & -\frac{p_c f_0}{f_1}\overline{\gamma}_{SR,1} \leq -t, \\
& -\overline{\gamma}_{SR,1} \leq 0,
\end{aligned} \tag{6.5}$$

$$\begin{aligned}
\phi^{(2)}_{\mathrm{MIMO-DF}}(t) = \underset{\overline{\gamma}_{SD,2},\overline{\gamma}_{RD,2}}{\text{minimize}} \quad & f_1\overline{\gamma}_{SD,2} + f_2\overline{\gamma}_{RD,2} \\
\text{subject to} \quad & \frac{3}{4k^2(\overline{\gamma}_{SD,2} + \overline{\gamma}_{RD,2})} \leq t, \\
& -\overline{\gamma}_{SD,2} \geq 0, \\
& -\overline{\gamma}_{RD,2} \leq 0,
\end{aligned} \tag{6.6}$$

where, $t \in \mathbb{R}^+$ is any arbitrary variable. The transmission energy is given by minimizing $\phi^{(1)}_{\mathrm{MIMO-DF}}(t) + \phi^{(2)}_{\mathrm{MIMO-DF}}(t)$ over t. The problem 6.6 is exactly similar to the optimization problem described for MISO-DF and has a similar result. Hence, the MIMO scheme is possible only when $f_1 > f_2$. The problem 6.5 describes the effect of SIMO part. The variable t can be interpreted as that which depends on the relay SEP given by α. The relation can be made explicit by rewriting the constraint in problem 6.5 as $\frac{1}{2k\gamma_{SR,1}} \leq \frac{p_c f_0}{2kf_1 t}$, where, $\frac{1}{2k\gamma_{SR,1}}$ is the high SNR approximation for the SEP at relay. Therefore, $\frac{p_c f_0}{2kf_1 t} = \alpha$ can be treated as some critical relay SEP that atleast needs to be maintained at the relay.

Hence, in terms of α the optimization problem can be reformulated as,

$$\begin{aligned}
\phi^{(1)}_{\text{MIMO-DF}}(\alpha) = &\underset{\overline{\gamma}_{SR,1}}{\text{minimize}} && f_0\overline{\gamma}_{SR,1} \\
&\text{subject to} && \frac{1}{2k\gamma_{SR,1}} \leq \alpha, \\
& && -\overline{\gamma}_{SR,1} \leq 0, \qquad (6.7)\\
\phi^{(2)}_{\text{MIMO-DF}}(\alpha) = &\underset{\overline{\gamma}_{SD,2},\overline{\gamma}_{RD,2}}{\text{minimize}} && f_1\overline{\gamma}_{SD,2} + f_2\overline{\gamma}_{RD,2} \\
&\text{subject to} && \frac{1}{2k(\overline{\gamma}_{SD,2}+\overline{\gamma}_{RD,2})} \leq \frac{p_c f_0}{3f_1\alpha}, \\
& && -\overline{\gamma}_{SD,2} \leq 0, \\
& && -\overline{\gamma}_{RD,2} \leq 0. \qquad (6.8)
\end{aligned}$$

This problem is similar to the MISO-DF optimization problems 5.6 and 5.7 but not the same. The difference being that α couples the two subproblems in the MIMO case. This is due to the initial broadcast similar to SIMO from the source to relay and destination instead for just transmitting to relay as in MISO protocol. Assuming MIMO protocol can be implemented i.e., $f_1 > f_2$, the solution to the optimization problem is,

$$\begin{aligned}
\phi_{\text{MIMO-DF}} = &\underset{\alpha}{\text{minimize}} && \phi^{(1)}_{\text{MIMO-DF}}(\alpha) + \phi^{(2)}_{\text{MIMO-DF}}(\alpha), \\
\Rightarrow &\underset{\alpha}{\text{minimize}} && \frac{f_0}{2k\alpha} + \frac{3f_1f_2}{2kp_cf_0}\alpha. \qquad (6.9)
\end{aligned}$$

The function is convex and differentiating with respect to α we get the minimum and is given by $f_0\sqrt{\frac{p_c}{3f_1f_2}}$. The minimum objective is given by,

$$\phi_{\text{MIMO-DF}} = \frac{1}{k}\sqrt{\frac{3f_1f_2}{p_c}}. \qquad (6.10)$$

Similar to the approach in MISO-DF we can find the energy consumed considering the probablity of the error in the relay as,

$$\phi_{\text{MIMO-DF}} = (1-\alpha^\star)\{\phi^{(1)}_{\text{MIMO-DF}}(\alpha^\star) + \phi^{(2)}_{\text{MIMO-DF}}(\alpha^\star)\} + \alpha^\star\{\phi^{(1)}_{\text{MIMO-DF}}(\alpha^\star) + \phi_{\text{SD}}(\alpha^\star)\}. \qquad (6.11)$$

where, $\alpha^\star$ is the optimal α found from solving 6.9 and $\phi_{\text{SD}}(.)$ is the energy consumed by direct transmission from source when the relay is in error. Substituting the values of $\alpha^\star$ the objective is found to be,

$$\phi_{\text{MIMO-DF}} = \frac{1}{k}\sqrt{\frac{3f_1f_2}{p_c}} + \frac{f_0}{2kf_2}(f_1 - f_2). \tag{6.12}$$

6.1.3 Conclusion for MIMO-DF

The following conclusion can be drawn from our analysis of MIMO-DF protocol,

- It is shown that the optimal strategy of MIMO-DF protocol to minimize transmission energy is to operate as SIMO-DF protocol. The MIMO-DF protocol can be implemented only when $f_1 > f_2$. The transmission energy consumption for MIMO-DF protocol is exactly the same as that of SIMO-DF protocol. The circuit energy consumption is also found to be the same as that of SIMO-DF protocol.

- Observing equation (6.12) it is seen that the transmission energy required when the relay is imperfect is greater than that when we assume a perfect relay. For a perfect relay the transmission energy consumption does not depend on the source destination distance.

- Because we assume imperfect relay it is seen that the energy consumption depends on the relay destination distance. We can also conclude from equation (6.12) that when the relay is near source the average energy consumption is is lower than when relay is near destination. This is because when relay is near source $f_0 \ll f_1$,f_2 and $f_1 \approx f_2$. On the contrary, when relay is near destination $f_2 \ll f_0$,f_1 and $f_0 \approx f_1$.

6.2 Transmission Energy for MIMO-AF

6.2.1 Symbol Error Probability of MIMO-AF

In this section we find the SEP of MIMO-AF scheme. The signal models for MIMO-AF are described by equations (2.8), (2.9) and (2.10). From equation (2.9) the instantaneous SNR received at the destination at first time instant is given by $\gamma_{D,1} = \frac{E_{SD,1}|h_{SD,1}|^2}{N_0}$ and the average SNR is given by $\overline{\gamma}_{D,1} = \overline{\gamma}_{SD,1} = \frac{E_{SD,1}}{N_0}$. Similar to section 5.1.1 we find the average SNR at destination at the second time instant to be given by,

$$\overline{\gamma}_{D,2} = \frac{\overline{\gamma}_{SR,1}\overline{\gamma}_{RD,2} + \overline{\gamma}_{SD,2}(1+\overline{\gamma}_{SR,1})}{1+\overline{\gamma}_{SR,1}+\overline{\gamma}_{RD,2}}, \tag{6.13}$$

where, $\overline{\gamma}_b = \frac{E_b}{N_0}\ \forall b \in \{(SR,1),\ (SD,2),\ (RD,2)\}$. Assume that the instantaneous SNR at the second time instant is given by $\gamma_{D,2}$. It is found to be an intractable expression and we do not know its pdf. However, it not required in analysis. The total instantaneous SNR at the destination using MRC combining of the two signals at the destination is given by,

$$\gamma_D = \gamma_{D,1} + \gamma_{D,2}. \tag{6.14}$$

Inorder to invoke the results from [11] and [10], we investigate the properties of the pdf of γ_D and $\gamma_{D,2}$ around origin.

- It is found that the pdf of γ_D evaluated at origin is zero. This is because the source always transmit with a finite power ($E_{SD,1} > 0$, SNR is given by $\gamma_{D,1}$) at the first instant and therefore γ_D can never take a random value of zero.
- We observe on plotting the normalized histogram of $\gamma_{D,2}$ that it “appears” to belong to exponential distribution. Therefore we assume the pdf can be represented as infinite power series expansion.
- Most importantly, we observe that the value of pdf at zero in the normalized histogram given is equal to $\frac{1}{\overline{\gamma}_{D,2}}$. This is observation is verified for wide ranges of $\gamma_{D,2}$ as applicable in practical scenarios.

Hence, we can approximate the SEP at the destination as [10],

$$P_e \rightarrow \frac{3}{4k^2\overline{\gamma}_{D,1}\overline{\gamma}_{D,2}} \tag{6.15}$$

This result is further verified using simulations. The figure 6.1 shows the comparison between simulated plot and the theoretical equation found above. It is found to be a good match for high SNRs. Also plotted is the SEP for direct transmission for comparision, the theoretical expression of which is obtained from [11]. The diversity advantage gained in MIMO-AF is also observable from the plot.

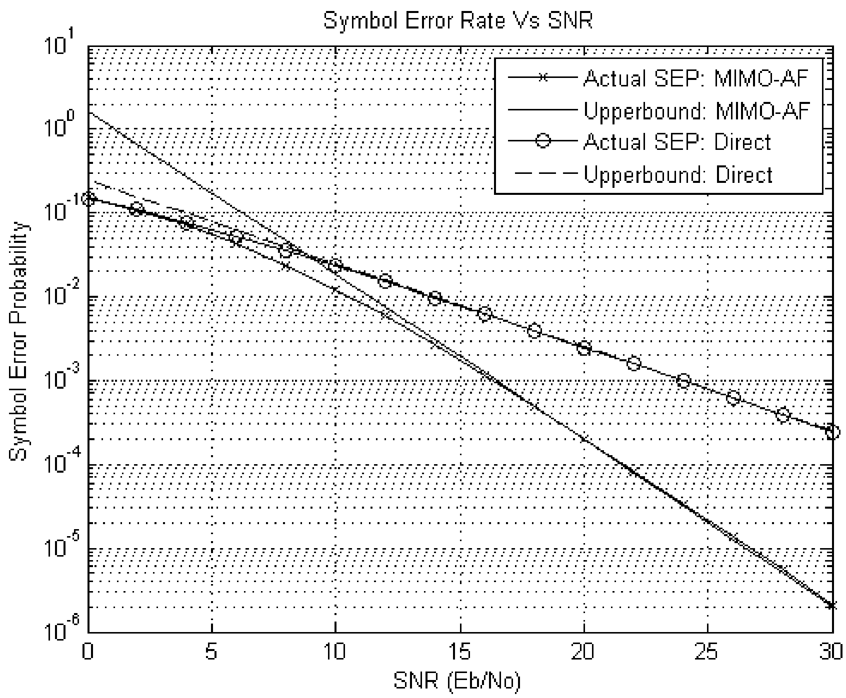

Figure 6.1: Symbol Error Probability v.s. SNR for the MIMO-AF

6.2.2 Problem Formulation for MIMO-AF

In this section, we formulate an optimization problem to minimize the transmission energy subject to target SEP found above. We pose the optimization problem as,

$$\begin{aligned}
\phi_{\text{MIMO-AF}} = \underset{\overline{\gamma}_{SR,1},\overline{\gamma}_{SD,2},\overline{\gamma}_{RD,2}}{\text{minimize}} \quad & f_0\overline{\gamma}_{SR,1} + f_1\overline{\gamma}_{SD,2} + f_2\overline{\gamma}_{RD,2} \\
\text{subject to} \quad & \frac{3}{4k^2\overline{\gamma}_{D,1}\overline{\gamma}_{D,2}} \leq p_c, \\
& \overline{\gamma}_{SD,2} \leq \overline{\gamma}_{SR,1}, \\
& -\overline{\gamma}_{SR,1} \leq 0, \\
& -\overline{\gamma}_{SD,1} \leq 0, \\
& -\overline{\gamma}_{SD,2} \leq 0, \\
& -\overline{\gamma}_{RD,2} \leq 0, \\
& f_0\overline{\gamma}_{SR,1} - f_1\overline{\gamma}_{SD,1} = 0.
\end{aligned} \tag{6.16}$$

where, the condition $\overline{\gamma}_{SD,2} \leq \overline{\gamma}_{SR,1}$ is as a result of the observation made, similar to equation (5.15) for MISO-AF protocol. The constraint $f_0\overline{\gamma}_{SR,1} - f_1\overline{\gamma}_{SD,1} = 0$ ensures that the transmission energy at source is consistent and the value of energy consumed is given by $N_0 * \phi_{\text{MIMO-AF}}$. Substituting for $\overline{\gamma}_{D,1}$ and $\overline{\gamma}_{D,2}$ we can reformulate the problem as,

$$\begin{aligned}
\phi_{\text{MIMO-AF}} = \underset{\overline{\gamma}_{SR,1},\overline{\gamma}_{SD,2},\overline{\gamma}_{RD,2}}{\text{minimize}} \quad & f_0\overline{\gamma}_{SR,1} + f_1\overline{\gamma}_{SD,2} + f_2\overline{\gamma}_{RD,2} \\
\text{subject to} \quad & \frac{\overline{\gamma}_{SR,1}\overline{\gamma}_{RD,2} + \overline{\gamma}_{SD,2}(1+\overline{\gamma}_{SR,1})}{1+\overline{\gamma}_{SR,1}+\overline{\gamma}_{RD,2}} \geq \frac{1}{\delta\overline{\gamma}_{SD,1}} \\
& \overline{\gamma}_{SD,2} \leq \overline{\gamma}_{SR,1}, \\
& -\overline{\gamma}_{SR,1} \leq 0, \\
& -\overline{\gamma}_{SD,1} \leq 0, \\
& -\overline{\gamma}_{SD,2} \leq 0, \\
& -\overline{\gamma}_{RD,2} \leq 0, \\
& f_0\overline{\gamma}_{SR,1} - f_1\overline{\gamma}_{SD,1} = 0.
\end{aligned} \tag{6.17}$$

where, $\delta = \frac{4k^2 p_c}{3}$. Similar to MIMO-DF we use primal decomposition technique to analyze the system as a combination of SIMO and MISO protocols. We decompose the problem into two subproblems as,

$$\begin{aligned}
\phi^{(1)}_{\mathrm{MIMO-AF}}(t) = \underset{\overline{\gamma}_{SR,1},\overline{\gamma}_{SD,2},\overline{\gamma}_{RD,2}}{\text{minimize}} \quad & f_0\overline{\gamma}_{SR,1} + f_1\overline{\gamma}_{SD,2} + f_2\overline{\gamma}_{RD,2} \\
\text{subject to} \quad & \frac{\overline{\gamma}_{SR,1}\overline{\gamma}_{RD,2} + \overline{\gamma}_{SD,2}(1+\overline{\gamma}_{SR,1})}{1+\overline{\gamma}_{SR,1}+\overline{\gamma}_{RD,2}} \geq t \\
& \overline{\gamma}_{SD,2} \leq \overline{\gamma}_{SR,1}, \\
& -\overline{\gamma}_{SR,1} \leq 0, \\
& -\overline{\gamma}_{SD,2} \leq 0, \\
& -\overline{\gamma}_{RD,2} \leq 0, \qquad (6.18)
\end{aligned}$$

$$\begin{aligned}
\phi^{(2)}_{\mathrm{MIMO-AF}}(t) = \underset{\overline{\gamma}_{SD,1}}{\text{minimize}} \quad & 0 \\
\text{subject to} \quad & -\frac{1}{\delta\overline{\gamma}_{SD,1}} \leq -t \\
& -\overline{\gamma}_{SD,1} \leq 0,
\end{aligned} \qquad (6.19)$$

where, $t \in \mathbb{R}^+$ is any variable and further, the minimum transmission energy is given by,

$$\begin{aligned}
\phi_{\mathrm{MIMO-AF}} = \underset{t}{\text{minimize}} \quad & \phi^{(1)}_{\mathrm{MIMO-AF}}(t) + \phi^{(2)}_{\mathrm{MIMO-AF}}(t), \\
\text{subject to} \quad & f_0\overline{\gamma}_{SR,1}(t) = f_1\overline{\gamma}_{SD,1}(t). \qquad (6.20)
\end{aligned}$$

In the problem 6.18 the variables $\overline{\gamma}_{SD,2}$ and $\overline{\gamma}_{RD,2}$ are called the private or local variables and $\overline{\gamma}_{SR,1}$ is called the public variable as it is connected to the variable $\overline{\gamma}_{SD,1}$ in problem 6.19. The two optimization problems can be considered as two interdependent systems constrained by the linear combination of public variables. The problem 6.19 is a feasibility problem and its solution is the entire range of possible $\overline{\gamma}_{SD,1}$ and is given by $[\ \frac{1}{\delta t}\ ,\ \infty\ )$.

The optimization problem given by 6.18 is similar to problem 5.18 for MISO-AF and it is interpreted as the MISO half of the MIMO-AF protocol. The variable t here is equivalent to $\overline{\gamma}_{SD,d}$ which is some SNR criterion that needs to maintained at the destination through

MISO scheme. An intuitive understanding can be obtained by seeing that if we try to increase t, i.e., when we try to satisfy a high SNR through the MISO part of the system, it tends to reduce the lower limit of the range of values that $\overline{\gamma}_{SD,1}$. It indicates that the initial broadcast from the source which is the SIMO part can be of lesser power.

We already know that the optimization problem 6.18 is non-convex and optimal solution does not exist. The analysis follows exactly as in section 5.2.2. However, we do have a solution to suboptimal strategy of "relaying" which gives,

$$\phi^{(1)}_{\mathrm{MIMO-AF}} = c\left\{(r_{SR}+r_{RD})^2 t + r_{SR}r_{RD}\right\} \text{ and} \tag{6.21}$$

$$\overline{\gamma}_{SR,1} = t\left[1+\left(\frac{f_2}{f_0}\right)^{\frac{1}{2}}\right]. \tag{6.22}$$

Recall that in relaying the source does not transmit in the second time slot. Further, solving for 6.20 we get the (sub)optimal solution as,

$$t^{\star} \geq \sqrt{\frac{r_{SD}^2}{\delta r_{SR}(r_{SR}+r_{RD})}}, \tag{6.23}$$

and $\phi_{\mathrm{MIMO-AF}} = \phi^{(1)}_{\mathrm{MIMO-AF}}(t^{\star})$ for minimum energy consumption.

6.2.3 Conclusion for MIMO-AF

It is clear that the MIMO protocol is inherently a SIMO protocol as there is no transmission at the second time slot. Further, these conclusions can be drawn from our analysis of MIMO-AF,

- The SEP expression for MIMO-AF SEP breaks down into the exact expression for SIMO-AF SEP given by equation (4.13) when $\overline{\gamma}_{SD,2} = 0$.
- The value of objective of MIMO-AF is fairly accurate with the value calculated from expression for SIMO for the cases (1) relay near destination and (2) for all comparable distances. The expression for circuit energy consumption also is equal to that of SIMO-AF.

- However, the objectives for MIMO-AF and SIMO-AF do not match when relay is near source. When relay is near source this implies that $r_{SR} \ll r_{RD}, r_{SD}$. Hence, observing equation (6.23), we can find that t will be quite large relative to other two cases of relay positions. The value of t being large implies that the lower limit of $\overline{\gamma}_{SD,1}$ tends to zero, i.e., the contribution to the SNR at destination due to source in the first transmission is very less. Recall that in MISO protocol, at the first time slot the source transmits to relay exclusively. Hence, the MIMO-AF protocol although inherently SIMO, tends to mimic MISO-AF (pure relaying) when relay is near source. This explains why the energy consumption is greater than that due to SIMO protocol for this particular case.

- Another conclusion that can be deduced from the analysis of MIMO-AF is the reason why MISO-AF is a suboptimal protocol to implement unless relay is near source. This assumes the broadcast nature of wireless networks. The source, at the first time transmits to relay, but both relay and destination listens to the signal. However, in MISO the destination does not decode the message in the first time instant. Clearly, the SNR at destination in the first time slot could be used for decoding and MRC combining with the signal received from relay in the second time slot. Therefore, in this particular case we are not using the energy available to us and hence implementing MISO is not desirable.

- MISO-AF can be implemented and suitable when (1) the relay is near source and (2) the source does not have enough power to transmit so as to realize a significant SNR through $S - D$ link, but the relay does.

6.3 Summary

In this chapter, we analyzed both MIMO-DF and MIMO-AF protocols by decomposing into two subproblems involving MISO and the SIMO parts. It is found that both MIMO-DF and MIMO-AF work as SIMO protocols. This is because, the source does not transmit at the second time instant when the protocol is operating in optimally. As a consequence of the result all the comparisons done for SIMO protocols are applicable to MIMO protocol. This concludes our work on cooperative communications and in the next chapter we present our conclusions of our thesis with directions for future work.

Chapter 7

Conclusions

So far, we have analyzed and compared the energy efficiency of three different cooperative communication schemes in chapters 4, 5 and 6, respectively. These chapters form the core contribution to our research. In this chapter we provide a concise summary of our contributions once again and comment on the possible future areas of reseach.

7.1 Summary of Key Contributions

Cooperative communications uses the spatial diversity inherent in wireless systems to provide diversity and combat multipath fading by sharing of resources. They are particularly useful in applications involving sensor networks and mobile nodes, and therefore energy efficiency is a critical factor. We presented three different protocols, namely SIMO, MISO and MIMO each operating in either AF or DF mode. We defined the metric of comparison as the energy consumed by a bit to get transmitted from source to destination with the aid of relay. Energy consumed is calculated as the sum of circuit and transmission energy. We modeled an AF circuit and calculated its energy consumption. We minimized the transmission energy for the SIMO-AF and SIMO-DF. We compared the energy consumed by each AF and DF mode and results seem to give useful insights into the working of the protocols. The result seems to show a significant dependency on the circuit energy at small distances between source and destination. At larger distances transmission energy seems to dominate and hence AF is found to be generally the worse peformer. We analyzed the energy consumed by source, relay under three different relay positions and gave intuitive explanations for the behaviour.

Next, we analyzed the MISO protocol. For the MISO protocol we found that the best strategy for communication is relaying of information rather that pure MISO form of transmission. This is true for both AF and DF mode in order to minimize the transmission energy. For the AF mode the relaying is actually a suboptimal solution as the optimization problem for minimizing the transmission enerygy is non-convex. It is however the optimal strategy for DF. Similar to SIMO we found the energy consumed by both the protocols and compare to for different positions of relay.

In the next chapter we investigated into the implementation of MIMO protocol. We decomposed the implementation of MIMO protocol as a combination of SIMO and MISO protocol. We found that the MIMO protocol is inherently a SIMO. This is because, as a part of the optimal strategy for both AF and DF, the source does not transmit in the second time slot. We showed that the transmission energy consumption for MIMO-AF and MIMO-DF is found to be the same as SIMO protocol.

We have tried to investigate the implementation of cooperative communication protocols on a fundamental unit consisting of a three node network. However, there still many questions unanswered which are left as future work.

7.2 Future Work

The most obvious extension of this work is to compare the AF and DF strategy across protocols. This will give us insight about which of the protocols is best suited for a three node network. Further, this work deliberatly ignored the power constraints of each nodes as well as the total available power to transmit one bit in a three node network. Incorporating the constraints in optimizing the transmission energy alters the problem to more of relay selection criteria. That is left as future work.

Further, this work presupposes a single relay cooperating to provide diversity. It will be an interesting challenge to consider multiple relays assisting the source in the transmission. However, the problem is complicated by the transmisson energy measure which depend on distances. The issue dealing with the actual distances also crops up in all cases when multiple relays or multiple hops are considered. In this context another problem worth mentioning is to analyze the energy efficiency of a relaying network when optimal choice of relay(s) among many choice of relays is performed at every instant. The optimal choice may be selecting the relay(s) with the best relay to destination channel. In these cases the relay positions might be modeled as randomly distributed in space. Another significant factor affecting performance is interference from other nodes of networks. We have not considered in our analysis and is to be considered for more practical scenarios. Further, we assumed rayleigh fading in our work and the optimal strategy MISO-AF and MIMO-AF is pure relaying in such channels. A interesting question is to find how ricean fading affect MIMO-DF and MISO-DF. Since there is a strong LOS for ricean fading channels in source to destination link as well as relay to destination links, a different strategy might be optimal.

More abstractly, it will be worthwhile to consider the energy efficiency performance of the networks as the number of nodes scale up. All the above factors such as relay selection, power constraints, interference becomes relevant as the number of nodes increases. It can be both in the form of a multibranch or multihop three node/single node networks.

Bibliography

[1] S. Cui, A. J. Goldsmith, and A. Bahai, "Energy Efficiency of MIMO and Cooperative MIMO Techniques in Sensor Networks," *IEEE Journal on Selected Areas in Communication*, vol. 22, pp. 1089–1098, 2004.

[2] R. U. Nabar, H. Bolcskei, and F. W. Kneubuhler, "Fading Relay Channels: Performance Limits and Space-Time Signal Design," *IEEE Transactions on Selected Areas in Communication*, vol. 22, pp. 1099–1109, 2004.

[3] S. Cui, A. J. Goldsmith, and A. Bahai, "Energy-Constrained Modulation Optimization," *IEEE transactions on Wireless Communications*, vol. 4, pp. 2349– 2360, 2005.

[4] A. Paulraj, R. Nabar, and D. Gore, *Introduction to Space-Time Wireless Communications.* Cambridge University Press, 2003, vol. 1st Ed.

[5] A. Sendonaris, E. Erkip, and B. Aazhang, "User Cooperation Diversity. Part I. System Description," *IEEE Transactions on Communications*, vol. 51, pp. 1927–1938, 2003.

[6] ——, "User Cooperation Diversity. Part II. Implementation Aspects and Performance Analysis," *IEEE Transactions on Communications*, vol. 51, pp. 1939–1948, 2003.

[7] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative Diversity in Wireless Networks: Efficient Protocols and Outage Behaviour," *IEEE Transactions on Information Theory*, vol. 50, pp. 3062–3080, 2004.

[8] Y. W. Hong, W. J. Huang, and C. C. J. K. F. H. Chiu, "Cooperative Communication in Resource-Constrained Wireless Networks," *IEEE Signal Processing Magazine*, May 2007.

[9] S. Weifeng, A. K. Sadek, and K. J. R. Liu, "SER Performance Analysis and Optimum Power Allocation for Decode-and-Forward Cooperation Protocol in Wireless Networks," *IEEE Wireless Communications and Networking Conference*, vol. 2, pp. 984–989, 2005.

[10] G. B. Giannakis and Z. Wang, "A Simple and General Parameterization Quantifying Performance in Fading Channels," *IEEE Transactions on Communications*, vol. 51, pp. 1389–1398, 2003.

[11] G. B. Giannakis, A. Ribeiro, and X. Cai, "Symbol Error Probabilities for General Cooperative Links," *IEEE Transactions on Wireless Communications*, vol. 4, pp. 1264–1273, 2005.

[12] Y. Wang and F. Lin, "SER Performance Analysis and Power Allocation for Amplify-and-Forward Relay Networks," *International Conference on Wireless Communications, Networking and Mobile Computing*, pp. 1274 – 1277, 2007.

[13] J. Luo, R. S. Blum, L. J. Cimini, L. J. Greenstein, and A. M. Haimovich, "Decode-and-Forward Cooperative Diversity with Power Allocation in Wireless Networks," *IEEE Global Telecommunications Conference*, vol. 5, pp. 5 pp.–3052, 2005.

[14] L. Zhao and Z. Liao, "On the Power Allocation for Cooperative Amplify-and-Forward Transmission," *IEEE 66th Vehicular Technology Conference*, pp. 1162 – 1166, 2007.

[15] ——, "On the Power Allocation for Cooperative Amplify-and-Forward Transmission over Rayleigh Fading Channels," *IEEE Journal of Communications*, vol. 3, 2008.

[16] Q. Zhang, J. Zhang, C. Shao, Y. Wang, P. Zhang, and R. Hu, "Power Allocation for Regenerative Relay Channel with Rayleigh Fading," *IEEE 59th Vehicular Technology Conference*, pp. 1167– 1171, 2004.

[17] ——, "Adaptive Optimal Transmit Power Allocation for Two Hop Non-regenerative Wireless Relay System," *IEEE 59th Vehicular Technology Conference*, vol. 2, pp. 1213–1217, 2004.

[18] K. Schwieger and G. Fettweis, "Power and Energy Consumption for Multi-hop Protocols: A Sensor Network Point of View," *Proceedings of the International Workshop on Wireless Ad-hoc Networks (IWWAN05)*, 2005.

[19] J. Yang and D. R. Brown, "The Effect of Receiver Diversity Combining on Optimum Energy Allocation and Energy Efficiency of Cooperative Wireless Transmission Systems," *IEEE International Conference on Acoustics, Speech and Signal Processing*, vol. 16, pp. III–493–III–496, 2007.

[20] A. K. Sadek, W. Yu, and K. J. R. Liu, "When Does Cooperation have Better Performance in Sensor Networks?" *3rd Annual IEEE Communications Society on Sensor and Ad Hoc Communications and Networks*, vol. 1, pp. 1157 – 1161, 2006.

[21] Z. Zhou, S. Zhou, J. H. Cui, and S. Cui, "Energy Efficient Cooperative Communication based on Power Control and Selective Relay in Wireless Sensor Networks," *IEEE Military Communications Conference*, pp. 1–7, 2007.

[22] R. Madan, N. B. Mehta, A. F. Molisch, and J. Zhang, "Energy-Efficient Cooperative Relaying over Fading Channels with Simple Relay Selection," *IEEE Global Telecommunications Conference*, pp. 1–6, 2006.

[23] W. J. Huang, Y. W. P. Hong, and C. J. J. Kuo, "Lifetime Maximization for Amplify-and-Forward Cooperative Networks," *IEEE Transactions on Wireles Communications*, vol. 7.

[24] T. S. Rappaport, *Wireless Communications: Principles and Practice.* Pretence Hall, 2002.

[25] S. P. Boyd and L. Vanderberghe, *Convex Optimization.* Cambridge University Press, 2004.

[26] S. Boyd and A. d'Aspremont, "Relaxations and Randomized Methods for Nonconvex QCQPs," EE392o, Stanford University, 2003.

[27] R. C. Palat, "Performance Analysis of Cooperative Communication for Wireless Networks," Ph.D. dissertation, Virginia Polytechnic Institute and State University, 2006.

[28] I. Mariac, "Cooperative Strategies for Wireless Relay Networks," Ph.D. dissertation, Rutgers State University of New Jersey, 2006.

[29] A. Papoulis and U. Pillai, *Probability Random Variables and Stochastic Processes.* McGraw-Hill Higher Education, 2002, vol. 4th Ed.

[30] E. K. P. Chong and S. H. Zak, *An Introduction to Optimization.* Wiley-Interscience, 2008.

[31] M. Grant and S. Boyd, "CVX: Matlab Software for Disciplined Convex Programming." Web Page and Software at http://stanford.edu/ boyd/cvx, February 2009.

Appendix A

A.1 Statistics of SNR at the destination for MISO-DF

In this section we find the statistics of the SNR received at the destination for the MISO-DF case when the relay forwards the symbol. The signal at the destination is given by equation 2.11 from that the instantaneous SNR at the destination is given by,

$$\gamma_{D,ne} = \frac{E_{SD} \mid h_{SD} \mid^2}{N_0} + \frac{E_{RD} \mid h_{RD} \mid^2}{N_0} + \frac{\sqrt{E_{SD}E_{RD}} h_{SD} h_{RD}^{\star}}{N_0} + \frac{\sqrt{E_{SD}E_{RD}} h_{SD}^{\star} h_{RD}}{N_0} \quad \text{(A.1)}$$

where, the superscript * stands for conjugate transposition. Let $h_{SD} = x + iy$ and $h_{RD} = u + iv$ where $x, y, u, v \sim N(0, \frac{1}{2})$ and are independent. Substituting this in equation (A.1) and rearranging we get,

$$\gamma_{D,ne} = \left(\sqrt{\frac{E_{SD}}{N_0}} x + \sqrt{\frac{E_{RD}}{N_0}} u\right)^2 + \left(\sqrt{\frac{E_{SD}}{N_0}} y + \sqrt{\frac{E_{RD}}{N_0}} v\right)^2 \quad \text{(A.2)}$$

Equation (A.2) can be interpreted as the sum of squares of two gaussian random variables with mean zero and variance $\frac{E_{SD}+E_{RD}}{2N_0}$ and hence a chi square distribution with two degrees of freedom. The probability density function is then found as,

$$p_{\gamma_{D,ne}}(x) = \frac{N_0}{E_{SD} + E_{RD}} e^{\frac{-xN_0}{E_{SD}+E_{RD}}} \quad \text{(A.3)}$$

The figure A.1 shows that the normalized histogram obtained from realizations of equation (A.1) matches with the theoretical pdf in equation (A.3). The value of $\frac{E_{SD}}{N_0}$ and $\frac{E_{RD}}{N_0}$ were taken arbitrarily and 10^7 simulations were done.

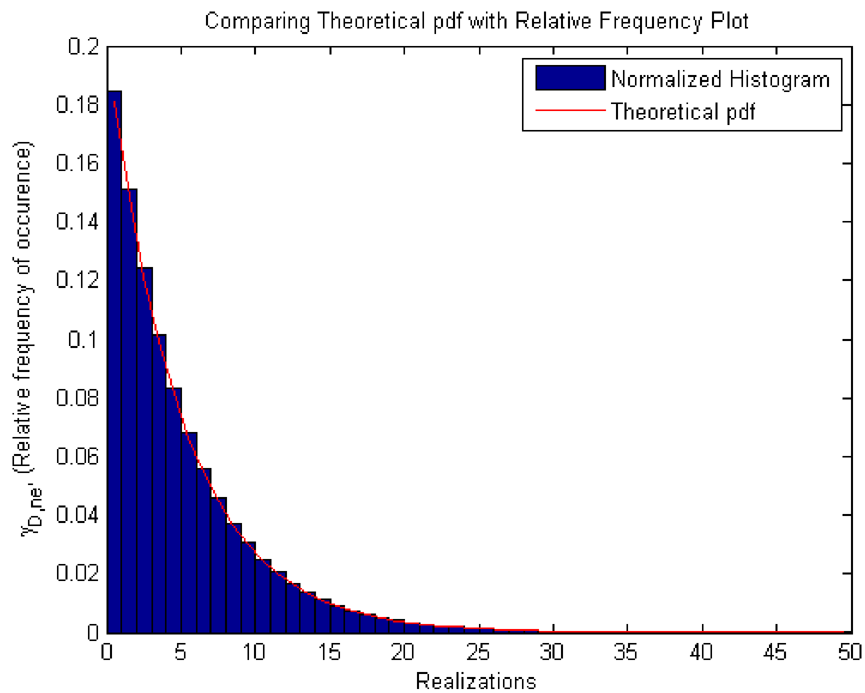

Figure A.1: Validating the theoretically found pdf of SNR for MISO-DF

A.2 Solution to Problem 5.7

The problem is given by,

$$\begin{aligned} \phi^{(2)}_{\mathrm{MISO-DF}} = \quad & \text{minimize} && f_1\overline{\gamma}_{SD} + f_2\overline{\gamma}_{RD} \\ & \text{subject to} && \frac{1}{2k(\overline{\gamma}_{SD}+\overline{\gamma}_{RD})} \leq p_c, \\ &&& -\overline{\gamma}_{SD} \leq 0, \\ &&& -\overline{\gamma}_{RD} \leq 0. \end{aligned} \tag{A.4}$$

The problem is convex and KKT conditions will given the optimal points. Setting up the lagrangian we get,

$$L(\overline{\gamma}_{SD}, \overline{\gamma}_{RD}, \lambda_0, \lambda_1, \lambda_2) = f_1\overline{\gamma}_{SD} + f_2\overline{\gamma}_{RD} + \lambda_0 \left\{ \frac{1}{(\overline{\gamma}_{SD}+\overline{\gamma}_{RD})} - 2kp_c \right\} - \lambda_1\overline{\gamma}_{SD} - \lambda_2\overline{\gamma}_{RD}, \tag{A.5}$$

where λ_0, λ_1, λ_2 are the langrangian variables and the KKT conditions are,

$$f_1 - \frac{\lambda_0}{(\overline{\gamma}_{SD} + \overline{\gamma}_{RD})^2} - \lambda_1 = 0, \tag{A.6}$$

$$f_2 - \frac{\lambda_0}{(\overline{\gamma}_{SD} + \overline{\gamma}_{RD})^2} - \lambda_2 = 0, \tag{A.7}$$

$$\lambda_0 \left\{ \frac{1}{(\overline{\gamma}_{SD} + \overline{\gamma}_{RD})} - 2kp_c \right\} - \lambda_1 \overline{\gamma}_{SD} - \lambda_2 \overline{\gamma}_{RD} = 0, \tag{A.8}$$

$$\lambda_0 \geq 0,$$

$$\lambda_1 \geq 0,$$

$$\lambda_2 \geq 0.$$

The candidate points exists for only two specific combinations, and they are when $(\lambda_0 > 0, \lambda_1 = 0, \lambda_2 > 0)$ and when $(\lambda_0 > 0, \lambda_1 > 0, \lambda_2 = 0)$.

Case $(\lambda_0 > 0, \lambda_1 = 0, \lambda_2 > 0)$

Since $\lambda_2 > 0$ it is clear that $\overline{\gamma}_{RD} = 0$ and as $\lambda_0 > 0$ we can get the value of $\overline{\gamma}_{SD} = \frac{1}{2kp_c}$. The value of λ_0 is always found always positive from equation (A.6) and λ_2 is positive when $f_2 > f_1$ from equation (A.7).

Case $(\lambda_0 > 0, \lambda_1 > 0, \lambda_2 = 0)$

Using similar arguments as the case above we can come to a conclusion that $\overline{\gamma}_{RD} = \frac{1}{2kp_c}$, $\overline{\gamma}_{SD} = 0$, $\lambda_0 > 0$ always and $\lambda_2 > 0$ when $f_1 > f_2$.

A.3 Formulation of MISO-AF as non-convex QCQP

From equation (5.15) we know that for effective communication $\overline{\gamma}_{SR} > \overline{\gamma}_{SD}$. Therefore we substitute $\overline{\gamma}_{SR}$ in equation (5.15) as $\overline{\gamma}_{SD} + \gamma$ where, γ is some dummy variable. We know that $Q(\sqrt{k\overline{\gamma}_D}) \leq p_c \Rightarrow \overline{\gamma}_D \geq \overline{\gamma}_{SD,d}$ where, $\overline{\gamma}_{SD,d}$ is the SNR required for direct transmission. On rewriting the constraint as $\overline{\gamma}_D \geq \overline{\gamma}_{SD,d}$, and substituing $\overline{\gamma}_{SD} + \gamma$ for $\overline{\gamma}_{SR}$, we can write the constraint as

$$\overline{\gamma}_{SD} + \overline{\gamma}_{SD}\overline{\gamma}_{SR} + \overline{\gamma}_{SD}\overline{\gamma}_{RD} + \overline{\gamma}_{RD}\gamma - \overline{\gamma}_{SD,d}\left(1 + \overline{\gamma}_{RD} + \overline{\gamma}_{SD} + \gamma\right) \geq 0 \tag{A.9}$$

Equation (A.9) can further is written as

$$x^T Q x + q^T x + r \geq 0, \tag{A.10}$$

where, $x = [\overline{\gamma}_{SD} \;\; \overline{\gamma}_{RD} \;\; \gamma]^T$,

$$\mathbf{Q} = \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix},$$

$q = -\overline{\gamma}_{SD,d}[1 \;\; 1 \;\; 1]^T$ and $r = -\overline{\gamma}_{SD,d}$.

The constraint (A.10) is found to be a concave constraint [**?**]. Therefore the non-convex QCQP optmization problem looks like,

$$\begin{aligned} &\text{minimize} && f^T x \\ &\text{subject to} && x^T Q x + q^T x + r \geq 0 \end{aligned} \tag{A.11}$$

where, $f = [f_0 + f_1 \;\; f_2 \;\; f_0]^T$.

A.4 Conditions when Case A is applicable

We need to check whether $\underline{\gamma} \leq \gamma \leq \overline{\gamma}$ where $\underline{\gamma} = \overline{\gamma}_{SR,A} - \overline{\gamma}_{SD,d}$ and $\overline{\gamma} = \overline{\gamma}_{SR,A}$. After trivial manipulations we get $\underline{\gamma} = \frac{f_2(1+\overline{\gamma}_{SD,d})}{f_1 - f_2}$. Comparing γ and $\underline{\gamma}$ it is clear that it the greater always. Similarly, we get $\gamma < \overline{\gamma}$ only if $(f_1 - f_2)^2 \overline{\gamma}_{SD,d} > f_0 f_2 (1 + \overline{\gamma}_{SD,d})$. Here we make an assumption that $\overline{\gamma}_{SD,d} \gg 1$ and hence arrive at the required condition.

A.5 Comparing $\phi^A_{\text{MISO-AF}}$ and $\phi^B_{\text{MISO-AF}}$

Calculating the value of $E \triangleq \phi^A_{\text{MISO-AF}} - \phi^B_{\text{MISO-AF}}$ we get,

$$E = \left\{ \frac{\left[r_{SD}^2 - r_{RD}(r_{SR} + r_{RD})\right]^2 \overline{\gamma}_{SD,d}}{r_{SD}^2 - r_{RD}^2} \right\} - \frac{r_{SR} r_{RD} \left[r_{SD}^2 - r_{RD}(r_{SR} + r_{RD})\right]}{r_{SD}^2 - r_{RD}^2} \tag{A.12}$$

We want to verify whether the expression $E > 0$ or not. Taking the common terms out we need to verify whether

$$\left[r_{SD}^2 - r_{RD}(r_{SR} + r_{RD})\right] \overline{\gamma}_{SD,d} - r_{SR} r_{RD} > 0; \tag{A.13}$$

which is true from our earlier assumptions that $\overline{\gamma}_{SD,d} \gg 1$ and $r_{SD}^2 > r_{RD}(r_{SR} + r_{RD})$.

A.6 Comparing $\phi_{\text{MISO}-\text{DF}}$ and $\phi_{\text{MISO}-\text{AF}}$

The energy consumed by MISO-AF $\phi_{\text{MISO}-\text{AF}}$ is given by equation (5.31) and that by MISO-DF is given by equation (5.9). Substituting for values for $\phi^{(1)}_{\text{MISO}-\text{DF}}$, $\phi^{(2)}_{\text{MISO}-\text{DF}}$ and ϕ_{SD}, we get,

$$\phi_{\text{MISO}-\text{DF}} = c_0 \left(\frac{r_{SR}^2}{2k\alpha} + \frac{r_{RD}^2}{2kp_c} + \alpha \frac{r_{SD}^2 - r_{RD}^2}{2kp_c} \right). \tag{A.14}$$

In order to compare MISO-DF and MISO-AF transmission energy we assume that both relays perform similarly and therefore the relay SEP α is given by,

$$\alpha = \frac{1}{2k\overline{\gamma}_{SR,B}} = \frac{p_c r_{SR}}{r_{SR} + r_{RD}}, \tag{A.15}$$

where, $\overline{\gamma}_{SR,B}$ is as defined in equation (5.29) and $\gamma_{SD,d} = \frac{1}{2kp_c}$. Accordingly, $\phi_{\text{MISO}-\text{DF}}$ can be rewritten as,

$$\phi_{\text{MISO}-\text{DF}} = c_0 \left\{ \frac{(r_{SR} + r_{RD})^2}{2kp_c} + \alpha \frac{r_{SD}^2 - r_{RD}^2}{2kp_c} - \frac{r_{SR} r_{RD}}{2kp_c} \right\}. \tag{A.16}$$

We define, $E_\phi \triangleq \phi_{\text{MISO}-\text{AF}} - \phi_{\text{MISO}-\text{DF}}$ and intend to verify whether $E_\phi > 0$ or not. Therefore,

$$E_\phi = r_{SR} r_{RD} + \frac{r_{SR} r_{RD}}{2kp_c} - \alpha \frac{r_{SD}^2 - r_{RD}^2}{2kp_c}. \tag{A.17}$$

We can see the following behaviour for E_ϕ,

- It can be seen that when relay is very near the source i.e., when $r_{SR} \ll r_{RD}, r_{SD}$ and $r_{RD} \approx r_{SD}$ $E_\phi > 0$ and hence, DF is found to consume less transmission energy.

- When $r_{RD} \ll r_{SR}, r_{SD}$ and $r_{SR} \approx r_{SD}$ it is seen that AF consumes less transmission energy than DF, theoretically.