

THE DISTINCTION OF SIMULATED
FAILURE DATA BY THE
LIKELIHOOD RATIO TEST

by

DARRYL D. DRAYER

B.S., Kansas State University, 1980

A MASTER'S THESIS

submitted in partial fulfillment of the
requirements for the degree

MASTER OF SCIENCE

Department of Nuclear Engineering

KANSAS STATE UNIVERSITY

Manhattan, Kansas

1981

Approved by:


Major Professor

SPEC
COLL.
LD
2668
T4
1981
D72
C.2

TABLE OF CONTENTS

	<u>Page</u>
LIST OF FIGURES	ii
LIST OF TABLES	iii
1.0 INTRODUCTION	1
2.0 THEORY	4
2.1 Failure Models	4
2.1.1 Homogeneous Model	5
2.1.2 Compound Model	6
2.2 Notes on the Gamma Distribution	7
2.3 Estimation of Gamma Parameters	14
2.3.1 Matching Moments to the Prior Method	14
2.3.2 Marginal Matching Moments Method	15
2.3.3 Marginal Maximum Likelihood Method	17
2.4 Hypothesis Testing Concepts	18
2.5 Likelihood Ratio Test	24
2.6 Data Simulation	32
2.7 Power	34
3.0 CODE OPERATION	38
3.1 GENERATE	38
3.1.1 Description	38
3.1.2 Input	39
3.1.3 Sample Output	40
3.2 GAMMA MODIFIED	40
3.2.1 Description	40
3.2.2 Input	47
3.2.3 Sample Output	47
3.3 GAMMA GENERATE	47
3.3.1 Description	47
3.3.2 Input	49
3.3.3 Sample Output	50
3.4 GAMMAP GENERATE	50
3.4.1 Description	50
3.4.2 Input	52
3.4.3 Sample Output	52
4.0 DATA SIMULATION AND ANALYSIS	55
4.1 Generation of Power Curves	55
4.1.1 General Procedure	55
4.1.2 Parameters Set in Program	56
4.2 Results	57
4.3 Analysis	82
5.0 ACKNOWLEDGEMENTS	89
6.0 REFERENCES	89
7.0 APPENDIX	90

LIST OF FIGURES

<u>Figure</u>		<u>Page</u>
2.1	Gamma distribution with $\alpha = 0.5$, $\beta = 10$	9
2.2	Gamma distribution with $\alpha = 1.0$, $\beta = 10$	10
2.3	Gamma distribution with $\alpha = 1.5$, $\beta = 10$	11
2.4	Gamma distribution with $\alpha = 2.0$, $\beta = 10$	12
2.5	Gamma distribution with $\alpha = 2.0$, $\beta = 10$	13
2.6	Density Function	22
2.7	Probability that an observation x lies between x_a and x_b	23
2.8	Cumulative distribution generated from distribution with $T = 10,000$, $\alpha = 1.1$. $\beta = 20,000$	33
2.9	Number of failures corresponding to a generated random number of 0.781	33
2.10	Power versus value of second parameter	36
2.11	Power versus value of difference between standard parameter and second parameter	36
2.12	Ideal power curve	36
4.1	Power curve for $\alpha = 1.5$, using MMPM	59
4.2	Power curve for $\alpha = 1.5$, using MMMM	61
4.3	Power curve for $\alpha = 1.5$, using MMLM	63
4.4	Power curve for $\alpha = 2.0$, using MMPM	65
4.5	Power curve for $\alpha = 2.0$, using MMMM	67
4.6	Power curve for $\alpha = 2.0$, using MMLM	69
4.7	Power curve for $\alpha = 1.0$, using MMPM	72
4.8	Power curve for $\alpha = 1.5$, using MMPM	75
4.9	Power curve for $\alpha = 2.0$, using MMPM	78
4.10	Power curve for $\alpha = 2.5$, using MMPM	81

LIST OF TABLES

<u>Table</u>		<u>Page</u>
3.1	Sample output from GENERATE	41
3.2	Sample output from GAMMA MODIFIED	48
3.3	Sample output from GAMMA GENERATE	51
3.4	Sample output from GAMMAP GENERATE	53
4.1	Power for $\alpha = 1.5$, using MMPM	58
4.2	Power for $\alpha = 1.5$, using MMMM	60
4.3	Power for $\alpha = 1.5$, using MMLM	62
4.4	Power for $\alpha = 2.0$, using MMPM	64
4.5	Power for $\alpha = 2.0$, using MMMM	66
4.6	Power for $\alpha = 2.0$, using MMLM	68
4.7	Power for $\alpha = 1.0$, using MMPM	70
4.8	Power for $\alpha = 1.5$, using MMPM	73
4.9	Power for $\alpha = 2.5$, using MMPM	76
4.10	Power for $\alpha = 2.5$, using MMPM	79
4.11	FWO.5 for generated power curves	86

1 INTRODUCTION

In some applications failure rates are viewed as random variables, following a known type of distribution, e.g., a gamma distribution. Often the parameters of the known type of failure rate distribution are unknown. The integration of the failure rate distribution times the probability of failure in a test time (e.g., a Poisson distribution) over all failure rate space (0 to ∞) yields a marginal probability distribution, i.e., the probability of failure during a given test time for a process with a known type of failure rate distribution. This marginal distribution leads to Bayesian analysis, or Bayesian estimation [1]. In Bayesian estimation sample evidence obtained through direct observation is combined with prior information about the failure rate distribution, to make estimates of the parameters of their distribution.

If a group of items or components are required to perform a certain task, the components either operate or fail to operate. For example, when switched on a light bulb will light or remain dark. These kinds of components may be viewed in two ways, classical or Bayesian.

When a group of similar components is viewed in a classical sense all of the components are assumed to be completely identical. Because they are assumed identical all of the components are assumed to have the same probability of failure when energized or the same probability of failure per unit time. For example, all the light bulbs produced in a certain batch may be assumed to have a 0.01% probability of failing to light when turned on (i.e., once per one hundred tries the light bulb will fail) and may have a 25% probability of failing during a year of operation. If the same group of components is viewed in a Bayesian sense, the components are not assumed to be identical but are viewed as having