APPLICATION OF GENERAL AUTOREGRESSIVE MOVING-AVERAGE STOCHASTIC MODELS TO TIME SERIES AND SIMULATION, PROBLEM

by

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CHAPTER I

DESCRIPTION OF ARMA MODEL

Box and Jenkins define a time series as a set of observations generated sequentially in time [1]. If the set is continuous, the time series is said to be continuous. If the set is discrete, the time series is said to be discrete. In this thesis, we consider only the discrete time series where observations are made at some fixed interval $h$. However, the value of the time interval $h$ is often unimportant in the appropriate model for the given time series.

As indicated by Casimer Micheal Stralkowski [2], a desirable mathematical analysis of a time series should be general enough to accommodate all types of the time series and should be embody the following qualities:

1. Parsimony of the model parameter, i.e. the models should contain as few parameters as possible.
2. The model should be simple to interpret and apply.
3. The model should accommodate theoretical as well as empirical information, i.e. should be empirical-mechanistic in nature.

Box and Jenkins introduce the autoregressive model as a mathematical model which is extremely useful in the representation of certain practically occurring series. In this model, the current value of the process is expressed as a finite, linear aggregate of previous
values of the process and a random shock $a_t$. [3].

Another kind of model, of great practical importance in the representation of the observed time series, is the so-called finite moving average process. Box also introduces the moving average model as making $z_t$, i.e., is the time series observation $z_i$ minus its mean $\bar{z}$, linearly dependent on a finite number of previous random shocks. [4]

To achieve greater flexibility in the fitting of actual time series, it is sometimes advantageous to include both autoregressive and moving-average terms in the model. This model is the so-called autoregressive moving-average model.

When the general autoregressive moving-average model is mentioned later on in this thesis, it includes all possible models, either the autoregressive process, moving-average process or autoregressive moving-average process. However, if the autoregressive moving-average model is mentioned, it includes only an autoregressive moving-average process.

The general autoregressive moving-average model is capable of representing any type of time series problem and is empirical-mechanistic in nature. The parameters in the model are as parsimonious as possible and are simple to interpret. In practice, it is frequently true that an adequate representation of an actual time series can be obtained with a low order model. The order of the model is usually not greater than two and often less than two. Hence, it possesses the characteristics of being a good mathematical model.

Recently, the general autoregressive moving-average model has been developed to represent many practical time series occurring in nature.
Examples are: scientific phenomena, such as the movement of tide, the vibrations of violin strings, the motion of the pendulum, etc [5]; in a business situation, such as the common stock market, gasoline sales by all oil company, international airline passenger fluctuation [6]; in an industrial production process, such as temperature variation, gas furnace process; in a simulation process, such as an inventory control process [7]. Not only may this model be used to represent the ongoing process, but it may also be used to forecast future situations.

An iterative cycle of identification, fitting, diagnostic checking and its forecasting are developed in this thesis to arrive at the appropriate function-stochastic model for the time series. This technique is to be applied to three sets of data obtained from chemical process, international airline passenger situations and simulated inventory process respectively. Both the nature of the system of the process and the optimal forecasts of future values can be acquired from this methodology.

1.1. Linear Filter Model

The mathematical models we employ are based on the idea that a time series in which successive values are highly dependent can be generated from a series of independent "shocks" $a_t$. [8]. These shocks are random drawings from a fixed distribution, usually assumed normal with mean zero and variance $\sigma^2$, such a sequence of random variable $a_t$, $a_{t-1}$, $a_{t-2}$, is called a white noise process.

The white noise process $a_t$ is supposed to transform the process $x_t$ by what is called a "linear filter", as shown in Fig. 1.1. [8]. The
THIS BOOK CONTAINS NUMEROUS PAGES WITH DIAGRAMS THAT ARE CROOKED COMPARED TO THE REST OF THE INFORMATION ON THE PAGE. THIS IS AS RECEIVED FROM CUSTOMER.
Figure 1.1 Representation of a time series as the output from a linear filter
linear filtering operation simply takes a weighed sum of previous observations, so that

\[ z = \mu + \psi(B) a_t \]

\[ = \mu + a_t + \psi_1 a_{t-1} + \psi_2 a_{t-2} + \ldots \] (1.1.1)

where \( \mu \) is a parameter that determines the "level" of the process; \( B \) is the backward shift operator, i.e. \( B z_t = z_{t-1} \); and

\[ \psi(B) = 1 + \psi_1 B + \psi_2 B + \ldots \] (1.1.2)

is the linear function that transforms \( a_t \) to \( z_t \) and is called the transfer function of the filter. The sequence \( \psi_1, \psi_2, \ldots \) formed by the weights may be finite or infinite. If this sequence is finite, or infinite and convergent, the filter is said to be stable and the process \( z_1, \ldots, z_t \) to be stationary. [9]. The parameter \( \mu \) in (1.1.1) is then the mean about which the process varies. Otherwise, \( z_t \) is non-stationary and \( \mu \) has no specific meaning except as a reference point for the level of the process.

1.2. Stationary Process

Stationary processes play a very important role in the time series problem. Most of the time series phenomena occurring in nature, which are non-stationary or seasonal process, have to be transformed to a stationary process so that the appropriate model can be identified and the forecast values obtained.

The stationary process is based on the assumption that the process remains in equilibrium about a constant mean level. The time series is
said to be strictly stationary if it is independent of time differences; that is, if the joint distribution associated with \( m \) observations \( z_{t_1}, z_{t_2}, \ldots, z_{t_m} \) made at any set of times \( t_1, t_2, \ldots, t_m \), is the same as that associate with \( m \) observations \( z_{t_1+k}, z_{t_2+k}, \ldots, z_{t_m+k} \) made at times \( t_1+k, t_2+k, \ldots, t_m+k \); thus for a discrete process to be strictly stationary, the joint distribution of any set of observations must be unaffected by shifting all of the observation times forward or backward by any integer amount \( k \).

In Fig. 1.2, the observations of series C and D appear to fluctuate about a fixed mean with similar pattern of irregularity. Series of this type are said to be "stationary in mean and variance". Series E, appears to fluctuate about a fixed mean but with a changing pattern of irregularity. Series of this type are said to be "stationary in the mean but nonstationary in variance"; Series A and B appear to drift with time, but appear to exhibit constant patterns of irregularity if allowance is made for the changing level and direction about which the observations are fluctuating. Series of this type are said to be "non-stationary in the mean". A more complete discussion of non-stationary process is presented in Section 1.6.

The stationary process implies that the probability distribution \( P(z_t) \) is the same for all times \( t \) and may be written \( P(z) \). Hence its process has a constant mean where

\[
\mu = \mathbb{E}[z_t] = \int_{-\infty}^{\infty} z \ P(z) \ dz. \tag{1.2.1}
\]
Figure 1.2 Typical Time Series
Which defines the level about which the process fluctuate; and with a constant variance

$$\sigma^2_z = E [(z - \mu)^2] = \int_{-\infty}^{\infty} (z - \mu)^2 \, p(z) \, dz$$  \hspace{1cm} (1.2.2)

which measures its spread about the level \( \mu \).

The mean \( \mu \) of the time series process can be estimated by

$$\bar{z} = \frac{1}{N} \sum_{t=1}^{N} z_t$$

and the variance \( \sigma^2_z \) can be approximated by

$$\sigma^2_z = \frac{1}{N} \sum_{t=1}^{N} (z_t - \bar{z})^2$$  \hspace{1cm} (1.2.4)

1.3. Autoregressive Model

The autoregressive model of order \( p \), or abbreviated as AR(\( p \)), can represent the given time series \( z_t, z_{t-1}, z_{t-2}, \ldots \), observed at a constant time interval, as

$$\tilde{z}_t = \phi_1 \tilde{z}_{t-1} + \phi_2 \tilde{z}_{t-2} + \ldots + \phi_p \tilde{z}_{t-p} + a_t$$  \hspace{1cm} (1.3.1)

where \( \tilde{z}_t = z_t - \mu \), \( \mu \) is the mean of time series observation. \( a_t \) is assumed random normal and independent.

(1.3.1) can be related as a "dependent" variable \( \tilde{z}_t \) regressed on a set of "independent" variable \( \tilde{z}_{t-1}, \tilde{z}_{t-2}, \ldots, \tilde{z}_{t-p} \), plus an error term \( a_t \). The autoregressive model can also be written as
\[ z_t - \phi_1 z_{t-1} - \phi_2 z_{t-2} - \cdots - \phi_p z_{t-p} = a_t \]

or

\[ (1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p) \tilde{z}_t = a_t \]

where  \[ B^m \tilde{z}_t = \tilde{z}_{t-m} \]

or

\[ \phi(B) \tilde{z}_t = a_t \]

where  \[ \phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p \]

The model contains \( p+2 \) unknown parameters \( \mu, \phi_1, \phi_2, \ldots, \phi_p, \sigma_a^2 \), which are estimated from the time series data. The additional parameter \( \sigma_a^2 \) is the variance of the white noise process \( a_t \).

The finite parameter in the autoregressive model \( \phi(B) \tilde{z}_t = a_t \) can be inverted to an infinite number of random shock \( a_t \), as

\[ \tilde{z}_t = \phi^{-1}(B) a_t = \psi(B) a_t \]

where

\[ \phi^{-1}(B) = \psi(B) = 1 + \psi_1 B + \psi_2 B^2 + \cdots \quad (1.3.2) \]

Comparing (1.3.2) with (1.1.1) in section 1.1, for the autoregressive process to be stationary, the \( \phi \)'s must be so chosen that
the weights $\psi_1, \psi_2, \ldots$, in $\psi(B)$ form a convergent series. The auto-
regressive process can be thought of as the output $\tilde{z}_t$ from a linear
filter with transfer function $\phi^{-1}(B)$, where the input is white noise $a_t$.

1.4. Moving Average Model

The autoregressive model which expresses the deviation $\tilde{z}_t$ of the
process as a finite weighted sum of $p$ previous deviation $\tilde{z}_{t-1}, \tilde{z}_{t-2},$
\ldots, $\tilde{z}_{t-p}$, plus a random shock $a_t$, can be inverted as an infinite weighted
sum of $a$'s. The moving average model may be defined as a linear function
of a number of previous shocks $a$'s, which can be finite or infinite. [10].

For the moving average model of order $q$, or abbreviated as MA($q$),
the model form is

$$\tilde{z}_t = a_t - \theta_1 \tilde{a}_{t-1} - \theta_2 \tilde{a}_{t-2} - \ldots - \theta_q \tilde{a}_{t-q}$$ (1.4.1)

(1.4.1) is also called a moving average process of order $q$. It
may be written as,

$$\tilde{z}_t = \theta(B) a_t$$ (1.4.2)

where

$$\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \ldots - \theta_q B^q$$ (1.4.3)

The model contains $q+2$ unknown parameters $\mu, \theta, \ldots, \theta_q, \sigma^2$, which in
practice are determined from the data. The moving average process can
be thought of as the output $\tilde{z}_t$ from a linear filter with transfer function
$\theta(B)$, when the input is white noise $a_t$. 
1.5. Autoregressive Moving-Average Model

The first order moving-average model,

\[ \tilde{z}_t = a_t - \theta_1 a_{t-1} = (1 - \theta_1 B) a_t \]  \hspace{1cm} (1.5.1)

can be inverted to an infinite autoregressive process, as

\[ a_t = (1 - \theta_1 B)^{-1} \tilde{z}_t \]

or

\[ a_t = \tilde{z}_t + \theta_1 \tilde{z}_{t-1} + \theta_1^2 \tilde{z}_{t-2} + \ldots \]

or

\[ \tilde{z}_t = -\theta_1 \tilde{z}_{t-1} - \theta_1^2 \tilde{z}_{t-2} - \ldots + a_t \]  \hspace{1cm} (1.5.2)

The higher order moving average model can be inverted to an infinite autoregressive process by the same derivation. Hence, if the process can be represent by MA(1), it is impractical to obtain a non-parsimonious representation in terms of an autoregressive model; Conversely, an autoregressive model of first order can not be parsimoniously represented using a moving average process. To achieve greater flexibility in fitting actual time series, it is sometimes advantageous to include both autoregressive and moving-average terms in the model. This leads to the so-called autoregressive moving-average model, as,

\[ \tilde{z}_t = \phi_1 \tilde{z}_{t-1} + \ldots + \phi_p \tilde{z}_{t-p} + a_t - \theta_1 a_{t-1} - \ldots - \theta_q a_{t-q} \]  \hspace{1cm} (1.5.3)

or

\[ \phi(B) \tilde{z}_t = \theta(B) a_t \]  \hspace{1cm} (1.5.4)
which is also called the autoregressive moving-average process of order \((p,q)\), or abbreviated as ARMA\((p,q)\). The ARMA\((p,q)\) employs \(p+q+2\) unknown parameters \(\mu; \phi_1, \ldots, \phi_p; \theta_1, \ldots, \theta_q; \sigma_a^2\), that can be obtained from the data.

(1.5.4) may be inverted to

\[
\tilde{z}_t = \phi^{-1}(B) \theta(B) a_t = \frac{\theta(B)}{\phi(B)} a_t = \frac{1 - \theta_1 B - \ldots - \theta_q B^q}{1 - \phi_1 B - \ldots - \phi_p B^p} a_t
\]

The autoregressive moving-average process can then be thought of as the output \(\tilde{z}_t\) from a linear filter, whose transfer function is the ratio of two polynomials \(\theta(B)\) and \(\phi(B)\), when the input is white noise \(a_t\).

1.6. Non-stationary Process

Non-stationary time series have the property that their mean or variance or both may be changed with time. In other words, a non-stationary time series is depend on the time difference. When there is doubt about the choice of a nonstationary model or a stationary model to represent a time series, it is advantageous to employ the nonstationary model rather than the stationary alternative. [11]. Because the trend of a non-stationary time series can be transformed to a stationary process by differencing the data. Thus the nonstationary time series operator \(\psi(B)\) can be defined as

\[
\psi(B) = \phi(B) (1-B)^d
\]

(1.6.1)

where \(\phi(B)\) is a stationary operator. Thus a general model, which can represent nonstationary behavior, is of the form
\[ \psi(B) z_t = \phi(B) (1-B^d) z_t = \theta(B) a_t \]

or

\[ \phi(B) w_t = \theta(B) a_t \]  \hspace{1cm} (1.6.2)

where

\[ w_t = \nabla^d z_t; \quad \nabla z_t = (1-B) z_t \]

Nonstationary behavior can therefore be represented by a model which calls for the \( d \)'th difference of the process to be stationary. In practice, \( d \) is usually one or at most two.

The process defined by (1.6.2) provides a good way for describing non-stationary time series and is called an autoregressive moving-average process of order \((p,d,q)\), or abbreviated as ARMA\((p,d,q)\).

1.7. Seasonal Model

A seasonal time series is defined as a series which exhibits periodic behavior with period \( S \); i.e. when similarities in the series occur after \( S \) basic time intervals.

The monthly international airline passenger in Fig. 1.3, for example, is highly correlated twelve months apart. Sales of a particular product, like baseball equipment, will also be expected to have the same highly correlated situation. The highest sales occurring during the summer months and the months of December. Series of this type are called seasonal time series.

The fundamental part about seasonal time series with period \( S \) is that the observations which are \( S \) intervals apart are similar. Therefore,
Figure 1.3 Totals of international airline passengers in thousands
\( B^S z_t = z_{t-s} \) will be a powerful tool to analyze seasonal time series.

The linking of the observation \( z_t \) to an observation in the previous period with period \( S \) by a general ARMA model is defined as [12],

\[
\phi(B^S) \psi_s^D z_t = \Theta(B^S) a_t \quad (1.7.1)
\]

where

\[
\psi_s = 1 - B^S \quad \text{and} \quad D \text{ is the number of seasonal difference.}
\]

\( \phi(B^S), \Theta(B^S) \) are polynomials in \( B^S \) of degrees \( P \) and \( Q \), respectively.

(1.7.1) is the autoregressive moving-average process which represents the seasonal time series, or abbreviated symbolically as ARMA\((P,D,Q)_S\).

1.8. Multiplicative Model

Suppose that a time series has shown a tendency to increase over a particular period and also to follow a seasonal pattern. Then the time series may be represented by the form

\[
\phi(B) \phi(B^S) \psi_s^d \psi_s^D z_t = \theta(B) \Theta(Q(B^S)) a_t \quad (1.8.1)
\]

which is the multiplication of

\[
\phi(B^S) \psi_s^D z_t = \Theta(B^S) a_t \quad (1.8.2)
\]

and

\[
\phi(B) \psi_s^d a_t = \theta(B) a_t \quad (1.8.3)
\]

(1.8.2) and (1.8.3) are used to take care of seasonal fluctuations and non-stationary trend respectively. [13]. \( \alpha_t \) and \( a_t \) are defined as.
a white noise process; \( \phi(B) \) and \( \theta(B) \) are polynomials in \( B \) of degrees \( p \) and \( q \), respectively, and \( V = V_{-1} = 1-B \).

This most general autoregressive moving-average process is said to be of order \((p,d,q) \times (P,D,Q)_s\). It represents the time series process having a non-stationary trend and cyclic pattern and can also be denoted symbolically as \( \text{ARMA}(p,d,q) \times (P,D,Q)_s \).  

1.9. The Selection of An Appropriate Model

The purpose of this thesis is to find an appropriate model to represent a time series process and also forecast its future value. The method to select an appropriate model can be explained briefly in Fig. 1.4. The function at different stages can be illustrated as follows.

(1) The theory and practice are to be interacted to entertain the appropriate model. The autocorrelation and the partial autocorrelation function and the knowledge of the system are employed to suggest an appropriate parsimonious model. In addition, a rough estimate of the model parameters can be achieved in the process of model identification.

(2) The efficient estimate of parameters in the tentatively entertained model is the heart of this stage. The rough estimates of the parameters obtained during the identification stage can now be used as the starting points for the least square estimation of the parameters.

(3) The entertained model is subjected to a diagnostic check to test the goodness of fit. If no inadequacy of fit is indicated, the model is ready to use.
Figure 1.4 Stages in the iterative approach to model building
(4) If the model is adequate enough to represent the given time series process, the future situation can be forecasted and its confidence interval computed.

More details of model building and its application will be described in the following chapters.
CHAPTER II

MODEL IDENTIFICATION

Identification methods are the rough procedures applied to a set of time series data to indicate the kind of representational model which is worthy of further investigation. The specific aim in the identification stage is to obtain some idea about the number of parameters and the degrees of differences needed in the appropriate model and also to obtain initial estimates for the parameters. The tentative models so obtained provide a starting point for the application of the more formal and efficient estimation methods in the estimation stage.

Our approach to identify an appropriate model from the general autoregressive moving-average model family, which is

\[ \phi(B) \nabla^d z_t = \theta(B) a_t \]

are as follows.

(a) To identify the possibility of nonstationary and cycle trend, the original series \( z_t \) is to be differenced as many time as needed.

For the nonstationary time series,

\[ \phi(B) w_t = \theta(B) a_t \]

where \( w_t = (1-B)^d z_t = \nabla^d z_t \);

For the seasonal time series,

\[ \phi(B^s) w_t = \Theta (B^s) a_t \]
where \( w_t = v^D_s z_t = (1-B^s)^p z_t \).

(b) To identify the appropriate model form of the time series data. The distribution of the time series can be well defined by its theoretical autocorrelation function, its mean and variance. Every kind of model has its specific autocorrelations and partial autocorrelation function. In view of these facts, a powerful technique for identifying a candidate model form can be achieved by estimating the correlation and partial correlation pattern from the data and mentally comparing them with the theoretical patterns. Then select the model which has the estimated correlation and partials most similar with the theoretical correlation and partials. Many charts of lower order autoregressive moving-average models for this purpose are constructed by Box and Jenkins [14]. The unique pattern of the autocorrelation and partial autocorrelation of the general ARMA process can be used not only to identify the model, but to obtain the appropriate estimate of the parameters.

2.1. Autocorrelation Function

Each different type of autoregressive moving-average model has its own specific autocorrelation coefficient pattern. The autocorrelation coefficient process can be plotted out as a scatter diagram using pairs of values \((z_t, z_{t+k})\), of the time series, separated by \(k\) lags apart. It is easy to select the appropriate model for the given time series by the plotted form of its autocorrelation function. Theoretically,
the autocorrelation coefficient at lag $k$ is

$$\rho_k = \frac{E[(z_t - \mu)(z_{t+k} - \mu)]}{\sqrt{E[(z_t - \mu)^2] E[(z_{t+k} - \mu)^2]}} \tag{2.1.1}$$

And the covariance between $z_t$ and $z_{t+k}$, which is also called the autocovariance at lag $k$, is

$$\gamma_k = \text{Cov} [z_t, z_{t+k}] = E[(z_t - \mu)(z_{t+k} - \mu)] \tag{2.1.2}$$

To estimate the autocorrelation coefficient and autocovariance, Box and Jenkins recommends the following method [15], for autocovariance is estimated as

$$\hat{\gamma}_k = \frac{1}{N-k} \sum_{t=1}^{N-k} (z_t - \overline{z})(z_{t+k} - \overline{z}), \quad k = 0, 1, 2, \ldots, K \tag{2.1.3}$$

The estimated autocorrelation coefficient is

$$\hat{\rho}_k = \frac{\hat{\gamma}_k}{\hat{\gamma}_0}$$

with its variance

$$\text{Var} [\hat{\rho}_k] = \frac{1}{N} \left\{ 1 + 2 \sum_{j=1}^{k-1} \hat{\rho}_j^2 \right\} \tag{2.1.4}$$

The square root of (2.1.4) is called the large-lag standard error [16]. It is based on the assumption that the theoretical autocorrelation
\( \rho_k \) are all essentially zero beyond some hypothesized lag \( k = q \). The large lag standard error approximated the standard deviation of \( \rho_k \) for suitably large lags \((k>q)\). Hence, usually, \( \pm \hat{\sigma}_p \) or \( \pm 2\hat{\sigma}_p \) is plotted as "control" lines about zero. This is an rough indication of whether the autocorrelation coefficient is zero beyond some specific lag, or, in other words, the autocorrelation function is being cut off after a particular lag.

The theoretical autocorrelation matrix is usually symmetrical \([17]\), \( \rho_{-k} = \rho_k \), it is only necessary to plot the positive half of the autocorrelation matrix to analysis its process. When the autocorrelation function is mentioned later, it means only the positive half of its function.

2.2. Partial Autocorrelation Function

Every autoregressive, moving-average or autoregressive moving-average model has its own specific partial autocorrelation function. Hence the partial autocorrelation function is used as an auxiliary device to identify the appropriate model for a given time series among the general ARMA family. The correlation, \( \rho_k \), represents the dependence between \( z_t \) and \( z_{t-k} \). However, the partial correlation, \( \rho'_k \), represents the dependence between \( z_t \) and \( z_{t-k} \), given that observations \( z_{t-k+1}, \ldots, z_{t-1} \) are known. Hence, for the AR(p) model,

\[
\tilde{z}_t = \phi_1 \tilde{z}_{t-1} + \phi_2 \tilde{z}_{t-2} + \ldots + \phi_p \tilde{z}_{t-p} + a_t \tag{2.2.1}
\]

It can be observed that given the observations \( z_{t-1}, z_{t-2}, \ldots, \)
$z_{t-p}$, no dependence exist between $z_t$ and the observation occurring before time $t-p$. The partial correlation, $\rho_k'$, will therefore be zero when $k>p$. In other words, the partial autocorrelation process of AR(p) model will be cut off after lag $p$. For both the moving-average process MA(q)

$$\tilde{z}_t = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \ldots - \theta_q a_{t-q}$$ (2.2.2)

and the autoregressive moving-average process ARMA(p,q)

$$\tilde{z}_t = \phi_1 \tilde{z}_{t-1} + \ldots + \phi_p \tilde{z}_{t-p} + a_t - \theta_1 a_{t-1} - \ldots - \theta_q a_{t-q}$$ (2.2.3)

can be inverted to the infinite autoregressive process. This implies that the partial autocorrelation function of the moving-average process or ARMA process tails off rather cuts off.

The estimation of the partial autocorrelation is developed by Dubin as [18],

$$\rho_{k+1}' = \frac{\rho_{k+1} - \sum_{j=1}^{k} \rho_{k,j}' \rho_{k+1-j}'}{1 - \sum_{j=1}^{k} \rho_{k,j}' \rho_{j}'}$$ (2.2.4)

$$\rho_{k+1,j}' = \rho_{k,j}' - \rho_{k+1}' \rho_{k,k-j+1}' \quad (j = 1, 2, \ldots, K)$$

$$\rho_0' = 1$$

$$\rho_1' = \rho_1$$
with variance of the partial correlation coefficient as

$$\text{Var}(\hat{\rho}_k) = \frac{1}{n-k}$$  \hspace{1cm} (2.2.5)

The standard error, which is the square root of (2.2.5), can be used as a rough indication of the lag q where the partial autocorrelation is cut off.

2.3.1. The Autocorrelation Function of the Autoregressive Process

The specific autocorrelation pattern of the autoregressive process is a powerful tool to distinguish it from MA or ARMA process. On the autoregressive process

$$\tilde{z}_t = \phi_1 \tilde{z}_{t-1} + \phi_2 \tilde{z}_{t-2} + \ldots + \phi_p \tilde{z}_{t-p} + a_t$$  \hspace{1cm} (2.3.1)

Multiplying each term in (2.3.1) by $\tilde{z}_{t-k}$

$$\tilde{z}_{t-k} \tilde{z}_t = \phi_1 \tilde{z}_{t-k} \tilde{z}_{t-1} + \phi_2 \tilde{z}_{t-k} \tilde{z}_{t-2} + \ldots + \phi_p \tilde{z}_{t-k} \tilde{z}_{t-p} + \tilde{z}_{t-k} a_t$$  \hspace{1cm} (2.3.2)

Then take the expected value of (2.3.2), we obtain

$$E[\tilde{z}_{t-k} \tilde{z}_t] = \phi_1 E[\tilde{z}_{t-k} \tilde{z}_{t-1}] + \phi_2 E[\tilde{z}_{t-k} \tilde{z}_{t-2}] + \ldots$$

$$+ \phi_p E[\tilde{z}_{t-k} \tilde{z}_{t-p}] + E[\tilde{z}_{t-k} a_t]$$

or
\[ \gamma_k = \phi_1 \gamma_{k-1} + \phi_2 \gamma_{k-2} + \cdots + \phi_p \gamma_{k-p} \quad k > 0 \] (2.3.4)

\( z_{t-k} \) can only involve \( a_i \) up to time \( t-k \); for \( a_t \) is beyond \( t-k \), it is uncorrelated with \( z_{t-k} \); so the expected value of \( E[\tilde{z}_{t-k} a_t] \) vanishes. Dividing (2.3.4) by \( \gamma_0 \), the autocorrelation function of the AR process is

\[ \rho_k = \phi_1 \rho_{k-1} + \phi_2 \rho_{k-2} + \cdots + \phi_p \rho_{k-p} \quad k > 0 \] (2.3.5)

or

\[ (1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p) \rho_k = 0 \]

\[ \phi(B) \rho_k = 0 \] (2.3.6)

No matter how large a \( k \) we take in (2.3.5), \( \rho_k \) is still obtainable. This fact implies that the autocorrelation function of an AR process tails off rather than cuts off.

Box and Jenkins discuss the roots of \( \phi(B) \) in (2.3.6) and conclude that the autocorrelation function of an autoregressive process is either a damped exponential or damped sine wave or a mixture of damped exponential and damped sine wave [19].

2.3.2. The Partial Autocorrelation Function of AR Process and Yule-Walker Equations

To decide which order of autoregressive process to fit an observed time series is analogous to decide the number of independent variables to be included in a regression equation. For an AR process is finite itself and MA process can be inverted to an infinite AR process, any
The general ARMA model can be expressed in AR form, either finite or infinite. Although the proper order of an AR model to fit the time series is unknown, its parameter can be easily calculated.

For the autoregressive process

\[ \rho_k = \phi_1 \rho_{k-1} + \phi_2 \rho_{k-2} + \cdots + \phi_p \rho_{k-p} \quad k > 0 \quad (2.3.1) \]

By substituting \( k = 1, 2, \ldots, p \), in (2.3.1) one by one, and for \( \rho_{-k} = \rho_k \), it yields,

\[ \begin{align*}
\rho_1 &= \phi_1 + \phi_2 \rho_1 + \cdots + \phi_p \rho_{p-1} \\
\rho_2 &= \phi_1 \rho_1 + \phi_2 + \cdots + \phi_p \rho_{p-2} \\
&\vdots \quad \vdots \quad \vdots \\
\rho_p &= \phi_1 \rho_{p-1} + \phi_2 \rho_{p-2} + \cdots + \phi_p 
\end{align*} \quad (2.3.2) \]

(2.3.2) are the Yule-Walker equation [20]. The matrix form of Yule-Walker equations can be written as

\[
\begin{pmatrix}
\rho_1 \\
\rho_2 \\
\vdots \\
\rho_p
\end{pmatrix} = 
\begin{pmatrix}
\phi_1 \\
\phi_2 \\
\vdots \\
\phi_p
\end{pmatrix} 
\begin{pmatrix}
1 & \rho_1 & \rho_2 & \cdots & \rho_{p-1} \\
\rho_1 & 1 & \rho_2 & \cdots & \rho_{p-2} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\rho_{p-1} & \rho_{p-2} & \cdots & 1
\end{pmatrix}
\]

or

\[ \rho_p = P \phi \]
hence

\[ \phi = p^{-1} \rho_p \]

Initially, which order of an AR process to fit is unknown; suppose \( \phi_{kj} \) is the jth coefficient in an autoregressive process of order k, so that \( \phi_{kk} \) is the last coefficient. (2.3.1) can be written as

\[ \rho_j = \phi_{k1} \rho_{j-1} + \cdots + \phi_{k(k-1)} \rho_{j-k+1} + \phi_{kk} \rho_{j-k} \quad j = 1, 2, \ldots, k \]  

(2.3.5)

Hence (2.3.2) can be extended to

\[
\begin{bmatrix}
1 & \rho_1 & \rho_2 & \cdots & \rho_{k-1} \\
\rho_1 & 1 & \rho_1 & \cdots & \rho_{k-2} \\
\vdots & & & & \vdots \\
\rho_{k-1} & \rho_{k-2} & \rho_{k-3} & \cdots & 1 \\
\end{bmatrix}
\begin{bmatrix}
\phi_{k1} \\
\phi_{k2} \\
\vdots \\
\phi_{kk} \\
\end{bmatrix}
= 
\begin{bmatrix}
\rho_1 \\
\rho_2 \\
\vdots \\
\rho_p \\
\end{bmatrix}
\]

(2.3.6)

or the matrix form

\[ P_k \phi_k = \rho_k \]

\[ \phi_k = p^{-1} p_k \rho_k \]  

(2.3.7)

The quantity \( \phi_{kk} \) is regarded as a partial autocorrelation function [21]. To the autoregressive parameter \( \phi_{kk} \), the values between \( \rho_1, \ldots, \rho_k \) have to be known. In other words, \( \phi_{kk} \) is dependent on the observation \( z_1, \ldots, z_k \).
(2.3.7) can be used also as the rough estimate of the autoregressive model parameter.

2.4.1. The Autocorrelation Function of a Moving-Average Process

The moving-average process has its own specific autocorrelation function. For MA process,

\[ z_t = a_t - \theta_1 a_{t-1} - \cdots - \theta_q a_{t-q} \quad (2.4.1) \]

multiplying (2.4.1) by \( z_{t-k} \), which is

\[ z_{t-k} = a_{t-k} - \theta_1 a_{t-k-1} - \cdots - \theta_q a_{t-k-q}, \quad k = 1, 2, \ldots, q \quad (2.4.2) \]

then obtain

\[ z_t z_{t-k} = (a_t - \theta_1 a_{t-1} - \cdots - \theta_q a_{t-q})(a_{t-k} - \theta_1 a_{t-k-1} - \cdots - \theta_q a_{t-k-q}) \quad (2.4.3) \]

takes expectation value on (2.4.3),

\[ E[z_t z_{t-k}] = E[(a_t - \theta_1 a_{t-1} a_{t-1} - \cdots - \theta_q a_{t-q})(a_{t-k} - \theta_1 a_{t-k-1} - \cdots - \theta_q a_{t-k-q})] \quad (2.4.4) \]

Here, the random variables \( a_t \) are assumed uncorrelated [22],

\[ \gamma_k = E[a_t, a_{t-k}] = \begin{cases} \sigma^2, & k = 0 \\ \sigma_a^2, & k \neq 0 \end{cases} \quad (2.4.5) \]

Then the solution of (2.4.4) is shown as [23],
\[
\gamma_k = \begin{cases} 
\left(-\theta_k + \theta_1 \theta_{k+1} + \theta_2 \theta_{k+2} + \cdots + \theta_{q-k} \theta_q\right) \sigma_a^2, & k = 1, 2, \ldots, q \\
0 & , k > q
\end{cases} 
\] 

(2.4.6)

with

\[
\gamma_0 = 1 + \theta_1^2 + \theta_2^2 + \cdots + \theta_q^2
\]

Hence the autocorrelation coefficient, \(\rho_k = \frac{\gamma_k}{\gamma_0}\), will be

\[
\rho_k = \begin{cases} 
\frac{-\theta_k + \theta_1 \theta_{k+1} + \cdots + \theta_{q-k} \theta_q}{1 + \theta_1^2 + \cdots + \theta_q^2}, & K = 1, 2, \ldots, q \\
0 & , K > q
\end{cases} 
\] 

(2.4.7)

(2.4.7) reveals that the autocorrelation function of MA process is zero or cut off beyond a lag of q. This fact provides the means to identify the model of the observed time series. (2.4.7) can also be employed to obtain an approximate parameter estimate for the moving-average model.

2.4.2. The Partial Autocorrelation of MA Process

The finite moving-average model \(z_t = \phi(B) a_t\) can be inverted to an infinite autoregressive model \(\phi^{-1}(B) z_t = a_t\) with an infinite number of parameters. In other words, the finite moving-average model can be expressed in terms of an autoregressive model with its order to be decided. Here Box and Jenkins recommend the partial autocorrelation coefficient of the moving-average process to be expressed by its inverted autoregressive parameter as [24],
\[
\phi_{kk} = - \theta_1^k \left\{ 1 - \theta_2 \right\} / \left\{ 1 - \theta_1^{2(k+1)} \right\}
\]  \hspace{1cm} (2.3.2.1)

with

\[
\rho_1 = - \frac{\theta_1}{1 + \theta_1^2}
\]

and

\[
\rho_0 = 0
\]

to decide \( \theta_1 \) value.

From (2.4.2.1), if \( \rho_1 \) is positive, then \( \theta_1 \) is negative, so that \( \phi_{kk} \) is positive. Conversely, if \( \rho_1 \) is negative, \( \theta_1 \) is positive, so that \( \phi_{kk} \) is negative. This implies that the partial autocorrelation function of MA process is damped exponential. For \( k \) values in (2.4.2.1) can be substituted by any positive integer and \( \phi_{kk} \) is still obtainable, the process has a cut off after a lag of \( q \), whereas the autocorrelation function of the AR process tails off, whereas the partial autocorrelation function of an AR process cuts off.

2.5. Autocorrelation and Its Partial for the Autoregressive Moving-Average Process

\[
\tilde{z}_t = \phi_1 \tilde{z}_{t-1} + \cdots + \phi_p \tilde{z}_{t-p} + \alpha_t - \theta_1 \alpha_{t-1} - \cdots - \theta_q \alpha_{t-q}
\]
or

\[ \phi(B) z_t = \theta(B) a_t \quad \text{(2.5.1)} \]

has its autocovariance function

\[ \gamma_k = \phi_1 \gamma_{k-1} + \ldots + \phi_p \gamma_{k-p} + \gamma_{za}(k) - \theta_1 \gamma_{za}(k-1) - \ldots - \theta_q \gamma_{za}(k-q) \quad \text{(2.5.2)} \]

(2.5.2) is obtained by multiplying (2.5.1) with \( z_{t-k} \) and then taking the expected value. Suppose \( \gamma_{za}(k) = E[z_{t-k} a_t] \); since \( z_{t-k} \) depend only on random shock \( a_j \) which have occurred up to time \( t-k \), it follows that [25],

\[ \gamma_{za}(k) = 0 \quad k > 0 \]

\[ \gamma_{za}(k) \neq 0 \quad k \leq 0 \quad \text{(2.5.3)} \]
Hence the autocorrelation function (2.5.2) will become

\[ \gamma_k = \phi_1 \gamma_{k-1} + \phi_2 \gamma_{k-2} + \cdots + \phi_p \gamma_{k-p} \quad k \geq q + 1 \]  \hspace{1cm} (2.5.4)

by dividing (2.5.4) with \( \gamma_0 \), the autocorrelation function can be expressed as,

\[ \rho_k = \phi_1 \rho_{k-1} + \phi_2 \rho_{k-2} + \cdots + \phi_p \rho_{k-p} \quad k \geq q + 1 \]  \hspace{1cm} (2.5.5)

or

\[ \phi(B) \rho_k = 0 \]  \hspace{1cm} (2.5.6)

For any \( k \) value bigger than \( q+1 \) in (2.5.5), \( \rho_k \) is always obtainable. Thus the autocorrelation function of an ARMA process tails off rather than cuts off. Box and Jenkins also investigate the roots of \( \phi(B) \) in (2.5.6), and find the autocorrelation function of an ARMA process consists of a mixture of damped exponentials and/or damped sine wave. (2.5.5) can also be used to estimate the rough parameter \( \phi \) for the appropriate ARMA model. (2.5.1) can be inverted as

\[ \tilde{z}_t = \phi^{-1}(B) \theta(B) a_t \]

For \( \phi^{-1}(B) \) is an infinite series in \( B \). Hence the partial autocorrelation function of an ARMA process will be infinite in extent. Box and Jenkins conclude that the partial autocorrelation function will behave like a mixture of damped exponentials and/or damped sine waves, depending on the order of the ARMA process and the values of the parameters. [26].
We now can conclude that the appropriate model may be an ARMA process if the autocorrelation function and its partials tail off rather than cut off.

The first order ARMA model is the most common and practical to represent the appropriate ARMA process. An approximate value of the parameters of ARMA(1,d,1) process can be computed by [27],

$$
\rho_1 = \frac{(1 - \theta_1 \phi_1)(\phi_2 - \phi_1)}{1 + \theta_1^2 - 2 \phi_1 \theta_1} \quad -1 < \phi <
$$

$$
\rho_2 = \rho_1 \theta_1 \quad -1 < \theta_1 < 1
$$

A general method for obtaining initial estimate of the parameter for any ARMA process is derived by Box and Jenkins [28].

2.6. Identification of Appropriate Model

The autocorrelation function and the partial autocorrelation function have been used as a powerful tool to identify the appropriate model for a given time series. The low order model can represent the given time series quite well. The order is usually no more than 2. Many charts and tables have been constructed to describe the autocorrelation functions, partial autocorrelation functions and to provides an estimate of parameters for low order ARMA models, see Box and Jenkins [29]. The general process to identify the appropriate model for a given time series can be summarized as follows.

(a) For the non-stationary time series, its autocorrelation function will not die out quickly, or will fall off slowly, or is very nearly linearly [30]. Therefore, a tendency for the autocorrelation function
not to die out quickly is taken as an indication that a nonstationary time series may exist. Then we can treat the time series as nonstationary in $z_t$, but possibly as stationary in $\nabla z_t$, or in some higher difference.

It is assumed that the degree of difference $d$, which is required for stationarity, has been reached when the autocorrelation function of $w_t = \nabla^d z_t$ dies out fairly quickly. In practice, $d$ is normally either 0, 1 or 2. It is usually sufficient to inspect about the first 20 estimated autocorrelation of the time series.

(b) The AR process has a autocorrelation function which is infinite in extent, but has a partial autocorrelation function that is zero beyond a certain point. Conversely, the MA process has an autocorrelation function of zero beyond a certain point, but with a partial autocorrelation function which is infinite is extend.

Table 2.1. is the summary of the properties of AR, MA and ARMA process.
Table 2.1. Summary of Properties of Autoregressive, Moving-average and ARMA Process

<table>
<thead>
<tr>
<th></th>
<th>autoregressive process</th>
<th>moving-average process</th>
<th>ARMA process</th>
</tr>
</thead>
<tbody>
<tr>
<td>model in terms of previous $\tilde{z}$'s</td>
<td>$\phi(B) \tilde{z}_t = a_t$</td>
<td>$\theta^{-1}(B) \tilde{z}_t = a_t$</td>
<td>$\theta^{-1}(B) \phi(B) \tilde{z}_t = a_t$</td>
</tr>
<tr>
<td>model in terms of previous a's</td>
<td>$\tilde{z}_t = \phi^{-1}(B) a_t$</td>
<td>$\tilde{z}_t = \theta(B) a_t$</td>
<td>$\tilde{z}_t = \phi^{-1}(B) \theta(B) a_t$</td>
</tr>
<tr>
<td>autocorrelation function</td>
<td>infinite (damped exponentials and/or damped sine waves), tail off</td>
<td>finite, cut off</td>
<td>infinite (damped exponentials and/or damped sine waves after the first q-p lags), tail off</td>
</tr>
<tr>
<td>partial autocorrelation function</td>
<td>finite, cut off</td>
<td>infinite (dominated by damped exponentials and/or sine waves after the first p-q lags), tail off</td>
<td>infinite (dominated by damped exponentials and/or sine waves after the first p-q lags), tail off</td>
</tr>
</tbody>
</table>
CHAPTER III

MODEL ESTIMATION AND DIAGNOSTIC CHECK

The tentative formation of the model for the given time series is obtained in the identification stage. The more efficient estimate of the parameters in the appropriate model will be computed so as to construct a more perfect model. The rough estimated parameters obtained in the identification stage will be used as the starting points. After the model is built, it will be subject to diagnostic checks to test the fit of the model. If the model is inadequate, the time series process will be reviewed again and another modified model tried. If no lack of fit is indicated, the model is ready to use.

3.1. Maximum Likelihood Estimation of Parameters of ARMA Model

After a candidate model has been selected, it is necessary to estimate more accurate parameters to fit the time series data. The best estimate, from many points of view, is the maximum likelihood estimate [31].

For the ARMA process

\[ \tilde{z}_t = \phi_1 \tilde{z}_{t-1} + \cdots + \phi_p \tilde{z}_{t-p} - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \cdots - \]

\[ \theta_q a_{t-q} + a_t \]  \hspace{1cm} (3.1.1)

the random shocks \( a_1, a_2, \ldots, a_t, \ldots a_n \) are assumed normally independently distributed, so,
\[ f(a_1, a_2, \ldots, a_n \mid \hat{\theta}, \theta, \sigma^2) = \frac{1}{(2\pi)^{n/2} \sigma^n} e^{-\frac{1}{2\sigma^2} \sum a_t^2} \]

\[ = \frac{1}{(2\pi)^{n/2} \sigma^n} e^{-\frac{1}{2\sigma^2} S(\hat{\theta}, \theta)} \]  

(3.1.2)

where

\[ S(\hat{\theta}, \theta) = \sum a_t^2 \]  

(3.1.3)

and

\[ a_t = \tilde{z}_t - \phi_1 \tilde{z}_{t-1} - \cdots - \phi_p \tilde{z}_{t-p} + \theta_1 a_{t-1} + \theta_2 a_{t-2} + \cdots + \theta_q a_{t-q} \]  

(3.1.4)

The likelihood function of \( \hat{\theta}, \theta \) and \( \sigma^2 \), can be obtained by substituting the observed value of \( a \) into (3.1.2), as

\[ L(\hat{\phi}, \theta, \sigma^2 \mid a_1, a_2, \ldots, a_n) \sim \frac{1}{\sigma^n} e^{-\frac{1}{2\sigma^2} S(\hat{\phi}, \theta)} \]  

(3.1.5)

The likelihood function is maximized when \( S(\hat{\phi}, \theta) \) is minimized. The maximum likelihood estimates of \( \hat{\phi} \) and \( \hat{\theta} \), denoted by \( \hat{\phi} \) and \( \hat{\theta} \), corresponds to the minimum sum of squares, \( S(\hat{\phi}, \hat{\theta}) \).

Differentiating (3.1.5) with respect to \( \sigma^2 \) reveals that the maximum likelihood estimate of \( \sigma^2 \) is given by
\[ \hat{\sigma}^2 = \frac{1}{n} s(\hat{A}, \hat{B}) \]

The maximum likelihood estimation of \( \hat{A}, \hat{B} \) are equal to the least square estimate, which minimize the sum of squares of the residuals. The parameters in the model can be linear or non-linear after extention. For instance, \( a_t = \phi^{-1}(B) \theta(B) z_t \) or \( \hat{a}_t = \phi(B) \phi(B^\theta) \nabla^d D \nabla s z_t \). Hence the non-linear parameter least square estimation procedure is employed to meet every possible model, either linear or non-linear, [32]. Sub-routine UWHAUS in the Appendix is used to obtain the estimated parameter by a nonlinear least square method.

In order to insure that the estimation of parameters will converge to the least square point and also save machine time, it is necessary to obtain a good initial estimate of the parameters to start the computation. The initial estimate of parameters are obtained from autocorrelation and/or partial autocorrelation functions as discussed on Chapter Two.

The general ARMA model can be transformed to an equation like (3.1.4) in order to pursue the least square estimate of the parameters in the model [33]. For starting the problem, the \( p \) values \( z_0, z_{-1}, z_{-2}, \ldots, z_{-p+1} \) among the \( n=N-d \) \( w_1 \), which is \( w_1 = \nabla^d z_1 \), and the \( q \) values \( a_0, a_{-2}, \ldots, a_{-q+1} \) of \( a \) are unknown. For the practical purpose, if the sample size is moderately large, the unknown \( a \)'s can be assumed zero and also sacrifice the first \( p \) observations with an effective size of \( n-p \) [34].

3.2. Diagnostic Check

After the model has been identified and the parameters estimated for a time series, the model should be subject to investigation to test
the fit of the model. If there is evidence of serious inadequacy of fit, the model will be adjusted and the modified model tried again.

No model form can ever represent the true time series absolutely. However, the model should have no indicated lack of fit under different statistical tests. Box and Jenkins suggest many statistical tests for the general autoregressive moving-average model.

3.2.1. Diagnostic Checks Applied to Residuals—Autocorrelation Check

Theoretically, the random shock in the ARMA process is assumed to be white noise. Therefore, it is reasonable to expect that the study of the \( \hat{a}_t \) in the ARMA model can indicate the model inadequacy. The autocorrelation function of the residual \( \hat{a}_t \) is therefore a good device to test the fit of the model. Assume a general ARMA model,

\[
\phi(B) \hat{w}_t = \theta(B) a_t
\]

where

\[
\hat{w}_t = z_t - \mu
\]

being built from the interaction of the theory and practice discussed in Chapter Two and Three. Thus the residuals of the model

\[
\hat{a}_t = \theta^{-1}(B) \phi(B) \hat{w}_t
\]

are subject to test. It is possible to show that, if the model is adequate, then [35]

\[
\hat{a}_t = a_t + 0 \left( \frac{1}{\sqrt{n}} \right)
\]
As the series length increases, the \( \hat{a}_t \)'s become close to the white noise \( a_t \)'s. Hence the estimated autocorrelation coefficient \( \gamma_k(a) \), of the \( a \)'s, distributed approximately about zero with variance \( n^{-1} \), or, with a standard error of \( n^{-1/2} \) [36]. We can use these facts to assess the statistical significance of apparent departures of these autocorrelations from zero. If all the estimated autocorrelation coefficients of the residuals are inside the "control" line, then no inadequacy of the model is indicated. However, if the estimated autocorrelation coefficients are out of the "control" line, the suspicion of the lack of fit is hence aroused.

3.2.2. A Portmanteau Lack of Fit Test [37]

Box and Jenkins also suggest another statistical method to test the model fit. Rather than consider the \( \gamma_k(a) \)'s individually, the first few autocorrelation coefficients of the \( a \)'s, suppose about 20, are taken as a whole to test the fit of the model. Suppose we take the first \( k \) autocorrelation coefficients \( \gamma_k(\hat{a})(k=1,2,...,K) \) from general ARMA\((p,d,q)\) process, then if the model is appropriate, the value of

\[
Q = n \sum_{k=1}^{K} \gamma_k^2(a)
\]

will be distributed as \( \chi^2(k-p-q) \), where \( n=N-d \) is the number of transformed observations \( w_t \), where \( w_t = V^d z_t \), used to fit the model. On the other hand, if the model is inappropriate, the average values of \( Q \) will be inflated. Therefore, an appropriate, general, or "portmanteau" test of the fit of the model can be achieved by obtaining the value of \( Q \) and comparing it with the percentage points on the \( \chi^2 \) table. If \( Q \) is greater
than the percentage points on the $\chi^2$ table, then the inadequacy of the
model is indicated. Conversely, if $Q$ is no greater than the critical
$\chi^2$ value, then no inadequacy of the model is indicated.
CHAPTER IV

FORECASTING

The model is supposed to represent the time series data adequately as no lack of fit is indicated under the statistical investigation. Then the appropriate model can represent the stochastic process as well as be used to forecast the future situations. The approximation of the forecast value of the time series process will be presented in this Chapter. The confident limits of the forecast value will be developed.

4.1. The Forecast Function of the ARMA Model

The forecast function of ARMA model, as indicated by Box and Jenkins, has three model forms, either in terms of the difference equation, or in terms of an infinite weighted sum of previous random shock $a_j$, or in terms of an infinite weighted sum of previous observations plus a random shock. The simplest and the most practical form is the difference equation form, which will be discussed here [38]. For the general ARMA model

$$\psi(B) z_t = \phi(B) \varphi$$

where

$$\psi(B) = \phi(B) \psi^d$$

the forecast value is defined as $z_{t+\ell}$, $\ell \geq 1$, and its estimated value is $z_t(\hat{\varphi})$. In other words, the forecast $z_{t+\ell}$ is said to be made at origin $t$ for lead time $\ell$ when we are currently standing at time $t$. 
An observation $z_{t+\ell}$ generated by the process may be expressed directly in terms of the difference equation by

$$z_{t+\ell} = \varphi_1 z_{t+\ell-1} + \ldots + \varphi_{p+d} z_{t+\ell-p-d} - \theta_1 a_{t+\ell-1} - \ldots - \theta_q a_{t+\ell-q} + a_{t+\ell} \tag{4.1.1}$$

Now, suppose, standing at time $t$, then the forecast function \( \hat{z}_t(\ell) \) of $z_{t+\ell}$ will be a linear function of current and previous observations $z_t, z_{t-1}, z_{t-2}, \ldots$ and also a linear function of current and previous shocks $a_t, a_{t-1}, a_{t-2}, \ldots$; the forecast function may be written as,

$$\hat{z}_t(\ell) = \varphi_1 \hat{z}_{t+\ell-1} + \ldots + \varphi_{p+d} \hat{z}_{t+\ell-p-d} - \theta_1 \hat{a}_{t+\ell-1} - \ldots - \theta_q \hat{a}_{t+\ell-q} + \hat{a}_{t+\ell} \tag{4.1.2}$$

Box and Jenkins indicate (4.1.2) is the minimum mean square error forecast function [39]. To obtain the forecast value $\hat{z}_t(\ell)$, the right hand side of the forecast function in (4.1.2) should be treated as follows:

1) The $z_{t-j}$ ($j=0,1,2,\ldots$), which have already happened at time $t$, are left unchanged.

2) The $z_{t+j}$ ($j=0,1,2,\ldots$), which have not yet happened, are replaced by their forecasts $\hat{z}_t(j)$.

3) The $a_{t-j}$ ($j=0,1,2,\ldots$), which have happened, are available from $z_{t-j} - z_{t-j-1}$ (1).
4) The $a_{t+j}$ (j=1,2,...), which have not yet happened, are replaced by zero.

4.2. The Confidence Limits of the Forecast Value

Suppose the forecasts at lead time 1,2,..., L, are required. To obtain probability limits for these forecast value, it is necessary to calculated the weights $\psi_1$, $\psi_2$, ..., $\psi_{L-1}$, which are the parameters of the pure moving-average model; it may be written as,

$$z_t = \psi(B) a_t$$  \hspace{1cm} (4.2.1)

for the general ARMA model,

$$\psi(B) z_t = \theta(B) a_t$$  \hspace{1cm} (4.2.2)

Comparing (4.2.1) with (4.2.2), we can obtain

$$\psi(B) (1+\psi_1 B + \psi_2 B^2 + ...) = \theta(B)$$  \hspace{1cm} (4.2.3)

or

$$\psi(B) \psi(B) = \theta(B)$$  \hspace{1cm} (4.2.4)

$$(1 - \psi_1 B - \psi_2 B^2 - ... - \psi_{p+d} B^{p+d})(1 + \psi_1 B + \psi_2 B^2 + ...) =$$

$$(1 - \theta_1 B - \theta_2 B^2 - ... - \theta_q B^q)$$  \hspace{1cm} (4.2.5)

As we equate the coefficients of powers of B in (4.2.5), we can obtain the pure moving-average parameters in terms of general ARMA parameters $\psi$'s and $\theta$'s, which are known. Then we obtain,
\[ \psi_1 = \psi_1 - \theta_1 \]

\[ \psi_2 = \psi_1 + \psi_2 - \theta_2 \]

\[ \vdots \]

\[ \psi_j = \psi_{j-1} + \cdots + \psi_{p+d} \psi_{j-p-d} - \theta_j \]  \hspace{1cm} \text{(4.2.6)}

where

\[ \psi_0 = 1, \quad \psi_j = 0 \quad \text{for} \quad j < 0 \]

and

\[ \theta_j = 0 \quad \text{for} \quad j > q \]

If \( k \) is the greater of the integers \( p+d-1 \) and \( q \), then for \( j > k \), the \( \psi \)'s satisfy the difference equation [40];

\[ \psi_j = \psi_{j-1} + \psi_{j-2} + \cdots + \psi_{j-p-d} \]

Thus the \( \psi \)'s can be easily calculated recursively.

Box and Jenkins suggest the variance of the forecast error \( \ell \) steps ahead for any origin \( t \) is the expected value of \( \hat{e}_t^2(\ell) = (z_{t+\ell} - \hat{z}_t(\ell))^2 \), it can be estimated by [41]

\[ \nu(\ell) = \left( 1 + \sum_{j=1}^{\ell-1} \psi_j^2 \right) \sigma_a^2 \]

Then assuming that the \( a \)'s are normal, and given information up to time \( t \), the conditional probability distribution \( P(z_{t+\ell}/z_t, z_{t-1}, \ldots) \) of a future value \( z_{t+\ell} \) of the process will be normal with mean \( \hat{z}_t(\ell) \) and
standard deviation \( \left\{ 1 + \sum_{j=1}^{l-1} \psi_j^2 \right\}^{1/2} \sigma_a \).

The variance \( \sigma_a \) can be estimated by \( S_a \) if the numbers of observations on which such an estimate is based is at least fifty; \( S_a^2 \) is the minimum sum of square of residual and can be acquired by \( \frac{S(\hat{\phi}, \hat{\theta})}{n} \), [42].

Hence the approximate 1-\( \varepsilon \) probability limits \( z_{t+\ell}(-) \) and \( z_{t+\ell}(+) \) for \( z_{t+\ell} \) are given by

\[
z_{t+\ell} (\pm) = \hat{z}_t (\ell) \mp \frac{\varepsilon}{2} \left\{ 1 + \sum_{j=1}^{l-1} \psi_j^2 \right\}^{1/2} S_a
\]  \hspace{1cm} (4.2.7)

Where \( \frac{\varepsilon}{2} \) is the deviate exceeded by a proportion \( \varepsilon/2 \) of the unit normal distribution,

- for 50% limits, \( \frac{\varepsilon}{2} \) is 0.674
- for 95% limits, \( \frac{\varepsilon}{2} \) is 1.960

The \( z_{t+\ell} (-) \) and \( z_{t+\ell} (+) \) mean that, given the information available at origin \( t \), there is a probability of 1-\( \varepsilon \), that the actual value \( z_{t+\ell} \), when it occurs, will be within them; it can be expressed statistically as,

\[
P \{ z_{t+\ell} (-) < z_{t+\ell} < z_{t+\ell} (+) \} = 1 - \varepsilon
\]  \hspace{1cm} (4.2.8)

The confidence limits obtained here is applied to individual forecasts \( z_{t+\ell} \) only and not jointly to the forecast values at all the different lead times.

The Program FORCAT in the appendix will calculate the forecast values as well as its confidence limits.
CHAPTER FIVE

APPLICATION

The technique of model building for time series have been discussed in previous chapters. The computer programs in the Appendix provides the model calculations [43]. The model has to be constructed by computer calculation and human reasoning.

The process of model building is concerned with relating a class of statistical models to the data at hand and involves much more than model fitting. Thus, identification techniques, designed to suggest what particular kind of model might be worth considering, are developed first and make use of the autocorrelation and partial autocorrelation function. The fitting of the identified model to a time series using the likelihood function can then supply maximum likelihood estimate of the parameters. The initially fitted model will not, necessarily provide adequate representation. Hence diagnostic checks are developed to detect model inadequacy and thus, where necessary, to initiate a further iterative cycle of identification, estimation and diagnostic checking. When the forecast is the objective, the fitted statistical model with past data is used directly to generate optimal forecasts by simple recursive calculation.

The application of these techniques are presented by three examples of time series, which are obtained from industry process [44], business situation [45] and inventory simulation process [7] respectively.
5.1. Example One

A set of data shown on Table 5.1. about an industrial chemical process is to be analyzed here. [44]. This series represent "uncontrolled" outputs of concentration from the chemical process. And they were collected on full scale processes where it was necessary to maintain some output quality characteristics as close as possible to a fixed level. To achieve this control, another variable had been manipulated to approximately cancel out variation in the output. However, the effect of these manipulation on the output was in each case accurately known, so that it was possible to compensate numerically for the control action. That is to say, it was possible to calculate very nearly, the values of the series that would have been obtained if no corrective action been taken. It is these compensated value which are recorded here and referred to as "the uncontrolled" series [46].

The obtaining of the appropriate model will be explained step by step in the following sub-sections. Not only will we understand the system from the derived model, but we will acquire optimal forecast values for the series.

5.1.1. Identification of the Model

Program IDENT calculates the autocorrelations and partial autocorrelation of the time series. Since the series represent the "uncontrolled" behavior of the process output, we might expect it possess non-stationary characteristics. So differences of data are taken to see what kind of model can properly represent the series. The output of $z$, $\nabla z$ and $\nabla^2 z$ are shown on Table 5.2., 5.3 and 5.4. respectively. The plotting
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Table 5.2 Estimated Autocorrelations and its Partial of Chemical Process Concentration Readings about $z$

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<td>Partial Autocorrelations</td>
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<td>------------------</td>
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<td>-0.07</td>
</tr>
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<td>13</td>
<td>-0.01</td>
<td>-0.10</td>
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<tr>
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<td>0.10</td>
</tr>
<tr>
<td>15</td>
<td>-0.17</td>
<td>-0.08</td>
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<tr>
<td>16</td>
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<td>-0.13</td>
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<td>-0.09</td>
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<tr>
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<td>-0.07</td>
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<td>0.09</td>
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<tr>
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<td>-0.07</td>
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<tr>
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<td>0.02</td>
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Table 5.4 Estimated Autocorrelations and Its Partial of Chemical Process Concentration Readings about $\nu^2z$

<table>
<thead>
<tr>
<th>Lag</th>
<th>Autocorrelations</th>
<th>Partial Autocorrelations</th>
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</thead>
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<td>-0.65</td>
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<td>-0.31</td>
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<td>0.03</td>
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<tr>
<td>5</td>
<td>-0.04</td>
<td>-0.17</td>
</tr>
<tr>
<td>6</td>
<td>-0.04</td>
<td>-0.31</td>
</tr>
<tr>
<td>7</td>
<td>0.13</td>
<td>-0.17</td>
</tr>
<tr>
<td>8</td>
<td>-0.11</td>
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<tr>
<td>9</td>
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<td>-0.14</td>
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<tr>
<td>10</td>
<td>0.02</td>
<td>-0.05</td>
</tr>
<tr>
<td>11</td>
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<td>0.02</td>
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<td>0.02</td>
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<tr>
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<tr>
<td>14</td>
<td>0.17</td>
<td>0.05</td>
</tr>
<tr>
<td>15</td>
<td>-0.19</td>
<td>0.06</td>
</tr>
<tr>
<td>16</td>
<td>0.07</td>
<td>-0.00</td>
</tr>
<tr>
<td>17</td>
<td>-0.03</td>
<td>-0.12</td>
</tr>
<tr>
<td>18</td>
<td>0.09</td>
<td>0.01</td>
</tr>
<tr>
<td>19</td>
<td>-0.16</td>
<td>-0.12</td>
</tr>
<tr>
<td>20</td>
<td>0.19</td>
<td>0.07</td>
</tr>
<tr>
<td>21</td>
<td>-0.16</td>
<td>-0.02</td>
</tr>
<tr>
<td>22</td>
<td>0.10</td>
<td>0.06</td>
</tr>
<tr>
<td>23</td>
<td>-0.08</td>
<td>0.01</td>
</tr>
<tr>
<td>24</td>
<td>0.05</td>
<td>-0.01</td>
</tr>
<tr>
<td>25</td>
<td>-0.01</td>
<td>0.01</td>
</tr>
</tbody>
</table>
of the autocorrelation function and its partials for \( z \), \( Vz \) and \( V^2z \) are also shown on Fig. 5.1., 5.2., 5.3., respectively.

From Fig. 5.1, the autocorrelation function decreases fairly regularly after the first lag, and the partial autocorrelation has the tendency of tailing off; this is to suggest that the process might be ARMA \((1,0,1)\). However, the autocorrelation function of \( z \) does not fall quickly. This suggests that the series might be nonstationary. The appropriate estimate of the initial parameters can be calculated from

\[
\rho_1 = \frac{(1-\theta_1\phi_1)(\phi_1-\theta_1)}{1 + \phi_1^2 - 2\phi_1\theta_1} , \quad \rho_2 = \rho_1\phi_1
\]

and hence we obtain \( \phi_1 = 0.86 \), \( \theta_1 = 0.78 \); the model can thus be written as

\[(1-0.86 B) z_t = (1-0.78 B) a_t \quad (5.1.1.1)\]

From Fig. 5.2, the autocorrelation function are small after the first lag, and the partial autocorrelation tails off. This suggests an MA(1) process; the approximate estimate of initial parameters can be calculated from

\[
\rho_1 = \frac{-\theta_1}{1+\theta_1^2}
\]

and hence \( \theta_1 = 0.5 \); the model can thus be written as

\[Vz_t = (1-0.5 B) a_t\]

or
Figure 5.1 Estimated Autocorrelation and its Partial of Chemical Process Concentration Readings about z
Figure 5.2 Estimated Autocorrelation and its Partials of Chemical Process Concentration Readings about νz
Figure 5.3 Estimated Autocorrelation and its Partials of Chemical Process Concentration Readings about $\nu^2$
\[(1 - B) z_t = (1 - 0.5 B) a_t \quad (5.1.1.2)\]

Comparing (5.1.1.1) with (5.1.1.2), we see two possible result in the same form. Either form might represent the time series. However, in doubtful cases, it may be advantageous in employing the nonstationary model rather than the stationary alternative. Hence, the MA(1,1) is to be adapted to represent the given time series and will be subjected to a diagnostic check.

5.1.2. Efficient Estimation of Parameters

Program ESTIM performs the maximum likelihood estimate of the parameters in (5.1.1.2). The convergent situations is shown on Table 5.5. Hence, the appropriate model to represent the time series can now be written in more perfect form as

\[Vz_t = (1 - 0.7 B) a_t \quad (5.1.1.3)\]

(5.1.1.3) will be subject to further test for the goodness of fit. The statistical methods described in Section 3.2, are applied here to investigate the model. The sample correlation coefficients of residuals is also obtained from the output of program ESTIM and are shown in Table 5.6. By the autocorrelation check method, we compare the autocorrelation coefficients of residuals on Table 5.6, with the "control" line \(2n^{-1/2}\). It is revealed that all the correlation coefficients of the residuals are within the "control" lines. Thus there is no suspicion of inadequacy of the model.

To test the goodness of fit by the method of a portmanteau lack of fit, the value of
Table 5.5 Iterative estimation of $\theta_i$ for Chemical process concentration data

<table>
<thead>
<tr>
<th>Iteration</th>
<th>$\theta_i$</th>
</tr>
</thead>
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<tr>
<td>0</td>
<td>0.500</td>
</tr>
<tr>
<td>1</td>
<td>0.521</td>
</tr>
<tr>
<td>2</td>
<td>0.596</td>
</tr>
<tr>
<td>3</td>
<td>0.657</td>
</tr>
<tr>
<td>4</td>
<td>0.680</td>
</tr>
<tr>
<td>5</td>
<td>0.688</td>
</tr>
<tr>
<td>6</td>
<td>0.691</td>
</tr>
<tr>
<td>7</td>
<td>0.691</td>
</tr>
</tbody>
</table>
Table 5.6 The Sample Correlation Coefficients of Residuals for the Chemical Process Concentration Data

<table>
<thead>
<tr>
<th>Lag</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.091</td>
</tr>
<tr>
<td>2</td>
<td>0.010</td>
</tr>
<tr>
<td>3</td>
<td>-0.096</td>
</tr>
<tr>
<td>4</td>
<td>-0.112</td>
</tr>
<tr>
<td>5</td>
<td>-0.118</td>
</tr>
<tr>
<td>6</td>
<td>0.003</td>
</tr>
<tr>
<td>7</td>
<td>0.146</td>
</tr>
<tr>
<td>8</td>
<td>0.022</td>
</tr>
<tr>
<td>9</td>
<td>0.041</td>
</tr>
<tr>
<td>10</td>
<td>0.001</td>
</tr>
<tr>
<td>11</td>
<td>-0.099</td>
</tr>
<tr>
<td>12</td>
<td>-0.119</td>
</tr>
<tr>
<td>13</td>
<td>-0.036</td>
</tr>
<tr>
<td>14</td>
<td>0.062</td>
</tr>
<tr>
<td>15</td>
<td>-0.131</td>
</tr>
<tr>
<td>16</td>
<td>-0.010</td>
</tr>
<tr>
<td>17</td>
<td>0.045</td>
</tr>
<tr>
<td>18</td>
<td>0.073</td>
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<tr>
<td>19</td>
<td>-0.034</td>
</tr>
<tr>
<td>20</td>
<td>0.085</td>
</tr>
<tr>
<td>21</td>
<td>-0.091</td>
</tr>
<tr>
<td>22</td>
<td>-0.027</td>
</tr>
<tr>
<td>23</td>
<td>-0.058</td>
</tr>
<tr>
<td>24</td>
<td>0.037</td>
</tr>
<tr>
<td>25</td>
<td>0.041</td>
</tr>
</tbody>
</table>
\[ Q = n \sum_{k=1}^{k} \gamma^2_k(\hat{\alpha}) \]  

(5.1.1.4)

is assumed to distribute approximately as \( \chi^2(k-p-q) \) if the model is adequate, when \( m = N - d \) is the number of \( z \)'s to fit the model. By taking the first 20 autocorrelation coefficients on Table 5.6, to substitute on (5.1.1.4), we obtain

\[ Q = n \sum_{k=1}^{20} \gamma^2_k(\hat{\alpha}) = 23.58 \]

with 19 degrees of freedom. The 10% and 5% points for \( \chi^2 \) with 19 degrees of freedom, are 27.2 and 30.1 respectively. For 27.2 and 30.1 both far greater than 23.58, there is no significant inadequacy of the model.

5.1.3. Forecasting

Now MA(1,1) is supposed to represent the time series. The forecast values and its individual confidence limits are obtained by Program FORCAT. The forecast function can be written as

\[ z_{t+\ell} = (1-0.7 B) a_{t+\ell} \]

or

\[ z_{t+\ell} = z_{t+\ell-1} + a_{t+\ell} - 0.7 a_{t+\ell-1} \]

The \( a_{t+\ell} \) beyond the present time is assumed as zero. Hence, for all lead time, the forecasts at origin \( t \) will follow a straight line parallel to the time axis. Table 5.7 shows the forecast values and its confidence intervals. Fig. 5.4 shows parts of the chemical process and its forecast values.
Table 5.7 Forecast Value and its 95% Confidence Limits for the Chemical Process Concentration Data

<table>
<thead>
<tr>
<th>Time</th>
<th>Forecast Value</th>
<th>Upper Limit</th>
<th>Lower Limit</th>
</tr>
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<td>16.879</td>
<td>18.124</td>
</tr>
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<td>17.501</td>
<td>16.850</td>
<td>18.153</td>
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<tr>
<td>200</td>
<td>17.501</td>
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<td>203</td>
<td>17.501</td>
<td>16.745</td>
<td>18.258</td>
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<td>17.501</td>
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<td>18.282</td>
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<td>17.501</td>
<td>16.698</td>
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<tr>
<td>206</td>
<td>17.501</td>
<td>16.675</td>
<td>18.328</td>
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</table>
5.2. Example Two

The totals of international airline passengers for 1952, 53, and 54 shown on Fig. 1.3. is to be analyzed here. It is part of a longer series (twelve years of data) quoted by Brown. [47]. The series shows a marked seasonal pattern since travel is at its highest in the late summer months.

Many other series, particularly sales data, show similar seasonal characteristics. In general, we say that a series exhibits periodic behavior with period $S$, when similarities in the series occur after $S$ basic time intervals. In this example, we can see apparently from Fig. 1.3., the basic time interval is one month and the period is $S=12$ months.

When we have series exhibiting seasonal behavior with known periodicity $S$, it is of value to set down the data in the form of a table containing $S$ columns. The logarithms of the airline data taken by Box and Jenkins is shown on Table 5.8. As indicated by Box and Jenkins, logarithm are often taken before analyzing sales data and other series of this kind, because it is the percentage fluctuation which might be expected to be comparable at different sales volumes [48].

5.2.1. Identification of Model

Program IDENT provides the autocorrelation and its partials of the original time series and their differences. The outputs are shown on Table 5.9., 5.10., 5.11., 5.12. and Fig. 5.5., 5.6., 5.7., 5.8. respectively.
<table>
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<th>YEAR</th>
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<th>FEB.</th>
<th>MAR.</th>
<th>APR.</th>
<th>MAY</th>
<th>JUNE</th>
<th>JULY</th>
<th>AUG.</th>
<th>SEPT.</th>
<th>OCT.</th>
<th>NOV.</th>
<th>DEC.</th>
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<td>5.147</td>
<td>5.182</td>
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<td>5.293</td>
<td>5.215</td>
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<td>5.278</td>
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<td>5.460</td>
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<td>5.753</td>
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<td>5.872</td>
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<td>6.146</td>
<td>6.146</td>
<td>6.001</td>
<td>5.849</td>
<td>5.720</td>
<td>5.817</td>
</tr>
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</table>
Table 5.9 Estimated Autocorrelations and its Partials for the International Airline Passenger Data about $z_t$

<table>
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<th>Lag</th>
<th>Autocorrelation</th>
<th>Partial Autocorrelation</th>
</tr>
</thead>
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<td>1</td>
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Table 5.10  Estimated Autocorrelations and its Partial for the International Airline Passenger Data about $vz_t$

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<td>Partial Autocorrelation</td>
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<td>Partial Autocorrelation</td>
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<td>----------------</td>
<td>------------------------</td>
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<td>20</td>
<td>-0.106</td>
<td>-0.157</td>
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</table>
Figure 5.5 Estimated Autocorrelation and its Partials of the airline data about $z_t$.
Figure 5.6 Estimated Autocorrelation and its Partials of the airline data about $\nabla z_t$
Figure 5.7 Estimated Autocorrelation and its Partials of the airline data about $\nu_{iz}z_t$
Figure 5.8 Estimated Autocorrelation and its Partials of the airline data about $v'v_{ik}z_t$
In Fig. 5.5., the autocorrelations for z are large and fail to die out at higher lags. This implies the possibility of nonstationarity. The highly correlated periods at lags 12, 24 suggest the seasonal period of this time series 12.

In Fig. 5.6., while the first difference reduces the correlation in general, a very heavy periodic component remains. This is inducted by the large lag of 12.

In Fig. 5.7., simple differencing with respect to a period of twelve results in correlation which are first persistently positive and then persistently negative. This implies that the cyclic component of twelve periods and nonstationarity both exist in the time series.

In Fig. 5.8., the differencing \( \nabla_1 \nabla_{12} \) markedly reduces correlation throughout. The autocorrelation beyond the first lag are comparatively small. The partial autocorrelation has the tendency to tail off.

Besides, as indicated by Box and Jenkins, a simple and widely applicable stochastic model for the analysis of nonstationary time series is MA(1) [49]. Hence the model to represent the time series is suggested to be

\[
\nabla_1 \nabla_{12} z_t = (1 - \theta B) (1 - \Omega B^{12}) a_t
\]

which will be subject to further investigation.

As with the seasonal model, by equating the observed correlation to their expected values, approximate values can be obtained for the parameters \( \theta \) and \( \Omega \). On substituting the sample estimates \( \hat{\rho}_1 = -0.34 \) and \( \hat{\rho}_{12} = -0.39 \) in the expressions, which is obtained from (2.4.7.),
\[ \rho_1 = \frac{-\theta}{1+\theta^2}, \quad \gamma = \frac{-\Theta}{1+\Theta^2} \]

The rough estimate of parameters in (5.2.1.1) is \( \hat{\theta} = 0.39 \) and \( \hat{\Theta} = 0.48 \).

5.2.2. Efficient Estimate of Parameters.

Program ESTIM provides the maximum likelihood estimates of nonlinear parameters of (5.2.1.1). Table 5.13 shows the converge situations of parameters. The entertained model of the time series can be expressed as

\[ \psi_{12} z_t = (1 - 0.436 B) (1 - 0.486 B^{12}) a_t \quad (5.2.2.1) \]

The program ESTIM also provides the sample correlations of residuals. Table 5.14 shows the sample correlation coefficients of residuals of the time series. The goodness of fit can be tested as follows.

1. By autocorrelation check, comparing the autocorrelation coefficients of residuals on Table 5.14, with the "control" line \( n^{-1/2} \), few individual correlations appear little large. However, among 20 random deviates one would expect some large deviation. We will further investigate the model to check the goodness of fit.

2. By the method of a portmanteau lack of fit test, the value of

\[ Q = n \sum_{k=1}^{k} \gamma_k^2(\hat{a}) \]

is approximately distributed as \( \chi^2(k-p-q) \) if the model is appropriate. Hence, by taking the first 20 autocorrelation of the \( a \)'s as a whole from Table 5.14., we can obtain

\[ Q = n \sum_{k=1}^{20} \gamma_k^2(\hat{a}) = 20.44 \quad (5.2.2.2) \]
Table 5.13 Iterative Estimation of $\Theta$ and $\Psi$ for the logged airline data

<table>
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<tr>
<th>Iteration</th>
<th>$\Theta$</th>
<th>$\Psi$</th>
</tr>
</thead>
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<tr>
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<tr>
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<tr>
<td>2</td>
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</tr>
<tr>
<td>3</td>
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<tr>
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<tr>
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<td>0.486</td>
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</table>
Table 5.14 Correlation Coefficients of Residuals for the Logged Airline Data

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<td>0.014</td>
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<tr>
<td>4</td>
<td>-0.177</td>
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<tr>
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<td>0.035</td>
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<td>6</td>
<td>0.113</td>
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<td>7</td>
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<td>8</td>
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<tr>
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<tr>
<td>10</td>
<td>-0.162</td>
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<tr>
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<td>-0.033</td>
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<tr>
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<tr>
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<td>19</td>
<td>-0.084</td>
</tr>
<tr>
<td>20</td>
<td>-0.077</td>
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</tbody>
</table>
Comparing $Q$ with the value of $\chi^2(18)$ on $\chi^2$ table, the 10% and 5% points for $\chi^2$ value, with 18 degrees of freedom, are 27.2 and 30.1, respectively. The $Q$ value in (5.2.2.2) is smaller than 27.2, no indication of lack of fit is indicated. Hence (5.2.2.1) is proposed as the appropriate model to represent the international airline passenger situation.

5.2.3. Forecasting

Program FORCAT provides the forecast values and its confidence limits of the model with given time series. The results are shown on Table 5.15. and Fig. 5.9. We can predict the future business of the international airline passengers is to be increased with the cyclic period of twelve. Travel is at its highest in the summer months, while a secondary peak occurs in the spring.

5.3. Example Three

A set of observations about an inventory simulation process shown on Table 5.16. will be analyzed [7].

5.3.1. Identification of the Model

Program IDENT computes the autocorrelation and its partials for the original time series and its differences. Table 5.17, 5.18. and Fig. 5.10., 5.11. show the output of the autocorrelation and its partials.

In Fig. 5.10., the autocorrelation function is damped exponentially and tails off. While the partial autocorrelation is cut off after the first lag. This suggests that the process might possibly be a first
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<th>Lower Limit</th>
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Figure 5.9 Part of the airline passenger data and its forecast
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<td>296.056</td>
<td>285.056</td>
<td>269.960</td>
</tr>
<tr>
<td>265.960</td>
<td>263.600</td>
<td>253.961</td>
<td>243.961</td>
<td>235.961</td>
</tr>
<tr>
<td>229.961</td>
<td>223.601</td>
<td>263.056</td>
<td>248.056</td>
<td>235.056</td>
</tr>
<tr>
<td>217.056</td>
<td>201.056</td>
<td>189.056</td>
<td>170.056</td>
<td>260.056</td>
</tr>
<tr>
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<td>225.056</td>
<td>211.056</td>
<td>596.056</td>
<td>585.056</td>
</tr>
<tr>
<td>561.056</td>
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<td>518.056</td>
<td>495.056</td>
<td>487.056</td>
</tr>
<tr>
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<td>456.056</td>
<td>449.056</td>
<td>440.056</td>
</tr>
<tr>
<td>414.056</td>
<td>397.056</td>
<td>383.056</td>
<td>364.056</td>
<td>346.056</td>
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<tr>
<td>328.056</td>
<td>512.056</td>
<td>983.056</td>
<td>963.056</td>
<td>945.056</td>
</tr>
<tr>
<td>928.056</td>
<td>512.056</td>
<td>903.056</td>
<td>890.056</td>
<td>875.056</td>
</tr>
</tbody>
</table>
Table 5.17 Sample Correlation and its Partials of the Inventory Simulation Process about $z_t$

<table>
<thead>
<tr>
<th>Lag</th>
<th>Autocorrelation</th>
<th>Partial Autocorrelation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.96</td>
<td>0.96</td>
</tr>
<tr>
<td>2</td>
<td>0.92</td>
<td>-0.02</td>
</tr>
<tr>
<td>3</td>
<td>0.88</td>
<td>0.01</td>
</tr>
<tr>
<td>4</td>
<td>0.84</td>
<td>-0.07</td>
</tr>
<tr>
<td>5</td>
<td>0.80</td>
<td>-0.00</td>
</tr>
<tr>
<td>6</td>
<td>0.76</td>
<td>-0.01</td>
</tr>
<tr>
<td>7</td>
<td>0.72</td>
<td>-0.00</td>
</tr>
<tr>
<td>8</td>
<td>0.69</td>
<td>0.02</td>
</tr>
<tr>
<td>9</td>
<td>0.66</td>
<td>0.01</td>
</tr>
<tr>
<td>10</td>
<td>0.63</td>
<td>0.02</td>
</tr>
<tr>
<td>11</td>
<td>0.60</td>
<td>0.02</td>
</tr>
<tr>
<td>12</td>
<td>0.58</td>
<td>0.01</td>
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<tr>
<td>13</td>
<td>0.56</td>
<td>0.16</td>
</tr>
<tr>
<td>14</td>
<td>0.53</td>
<td>-0.00</td>
</tr>
<tr>
<td>15</td>
<td>0.52</td>
<td>0.03</td>
</tr>
<tr>
<td>16</td>
<td>0.51</td>
<td>0.02</td>
</tr>
<tr>
<td>17</td>
<td>0.49</td>
<td>0.01</td>
</tr>
<tr>
<td>18</td>
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<td>0.02</td>
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<td>19</td>
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<td>-0.04</td>
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<td>0.45</td>
<td>-0.00</td>
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<tr>
<td>21</td>
<td>0.43</td>
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</tr>
<tr>
<td>22</td>
<td>0.41</td>
<td>-0.04</td>
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<tr>
<td>23</td>
<td>0.39</td>
<td>-0.05</td>
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<tr>
<td>24</td>
<td>0.37</td>
<td>0.01</td>
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<tr>
<td>25</td>
<td>0.35</td>
<td>0.02</td>
</tr>
<tr>
<td>Lag</td>
<td>Autocorrelation</td>
<td>Partial Autocorrelation</td>
</tr>
<tr>
<td>-----</td>
<td>-----------------</td>
<td>------------------------</td>
</tr>
<tr>
<td>1</td>
<td>-0.000</td>
<td>-0.000</td>
</tr>
<tr>
<td>2</td>
<td>-0.031</td>
<td>-0.030</td>
</tr>
<tr>
<td>3</td>
<td>0.049</td>
<td>0.049</td>
</tr>
<tr>
<td>4</td>
<td>-0.021</td>
<td>-0.021</td>
</tr>
<tr>
<td>5</td>
<td>-0.014</td>
<td>-0.011</td>
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<tr>
<td>6</td>
<td>-0.013</td>
<td>-0.017</td>
</tr>
<tr>
<td>7</td>
<td>-0.041</td>
<td>-0.040</td>
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<tr>
<td>8</td>
<td>-0.029</td>
<td>-0.030</td>
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<tr>
<td>9</td>
<td>-0.041</td>
<td>-0.043</td>
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<tr>
<td>10</td>
<td>-0.040</td>
<td>-0.039</td>
</tr>
<tr>
<td>11</td>
<td>-0.020</td>
<td>-0.023</td>
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<tr>
<td>12</td>
<td>-0.032</td>
<td>-0.033</td>
</tr>
<tr>
<td>13</td>
<td>-0.010</td>
<td>-0.012</td>
</tr>
<tr>
<td>14</td>
<td>-0.035</td>
<td>-0.042</td>
</tr>
<tr>
<td>15</td>
<td>-0.036</td>
<td>-0.040</td>
</tr>
<tr>
<td>16</td>
<td>-0.017</td>
<td>-0.028</td>
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<tr>
<td>17</td>
<td>-0.022</td>
<td>-0.030</td>
</tr>
<tr>
<td>18</td>
<td>0.039</td>
<td>0.031</td>
</tr>
<tr>
<td>19</td>
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<tr>
<td>20</td>
<td>0.024</td>
<td>0.019</td>
</tr>
<tr>
<td>21</td>
<td>0.037</td>
<td>0.022</td>
</tr>
<tr>
<td>22</td>
<td>0.039</td>
<td>0.032</td>
</tr>
<tr>
<td>23</td>
<td>-0.015</td>
<td>-0.025</td>
</tr>
<tr>
<td>24</td>
<td>-0.028</td>
<td>-0.038</td>
</tr>
<tr>
<td>25</td>
<td>-0.022</td>
<td>-0.032</td>
</tr>
</tbody>
</table>
Figure 5.10 Estimated Autocorrelation and its Partials of the simulated Inventory Process about $z_t$
Figure 5.11  Estimated Autocorrelation and its Partials of the simulated inventory process about $v z_t$
order autoregressive process. The estimated parameter of AR(1) can be obtained by applying equation (2.3.7.)

$$\hat{\phi} = \rho_1 = 0.96$$

so, the recommended model might be

$$(1 - 0.96 B) z_t = a_t$$  \hspace{1cm} (5.3.1.1)$$

In Fig. 5.11, the correlation dies out completely after the first difference. This suggests a possible model might be

$$\nabla z_t = a_t$$

or

$$(1-B) z_t = a_t$$  \hspace{1cm} (5.3.1.2)$$

Comparing (5.3.1.1) with (5.3.1.2), the model form is similar except the parameter is a little different. The forecast value of (5.3.1.2) are all the same beyond the lead time \(\ell=1\). However, the inventory will be depleted little by little. Hence (5.3.1.1) is to be entertained to represent the simulated inventory process and will be subjected to diagnostic check.

5.3.2. Efficient Estimation of Parameter

Program ESTIM computes the maximum likelihood estimation of parameter of (5.3.1.1) with given time series. After the first iteration, the program is stopped execution with the efficient estimation parameter of \(\theta = 0.96\). Hence the tentative model is
\( (1 - 0.96 B) z_t = a_t \) \hspace{1cm} (5.3.2.1)

The process of testing the fit the model is explained as follows.

(1) An autocorrelation check is based on the assumption that the estimated autocorrelation of residuals \( \gamma_k(a) \) are uncorrelated and distributed approximately about zero with a standard error of \( n^{-1/2} \), if the model can represent the time series appropriately. Hence comparing the residual autocorrelation coefficients shown on Table 5.19, with the "control" line \( n^{-1/2} \), a few correlations are slightly larger than \( n^{-1/2} \). Hence, the model should be subjected to more investigation.

(2) To make a more formal assessment, the portmanteau lack of fit test which is based on the assumption that if the model is appropriate, the value of \( Q = n \sum_{k=1}^{k} \gamma_k^2(a) \) is approximately distributed as \( \chi^2(k-p-q) \). Hence, taking 20 autocorrelation coefficients of residual as a whole, we obtain,

\[
Q = n \sum_{k=1}^{k} \gamma_k^2(a) = 13.68
\]

with 19 degree of freedom. From \( \chi^2 \) tables, the 10% and 5% points for \( \chi^2 \), with 19 degrees of freedom are 27.2 and 30.1 respectively. For \( Q = 13.68 \) is smaller than 27.2, thus no lack of fit is indicated. Hence the model of (5.3.2.1) is recommended to represent the time series.
<table>
<thead>
<tr>
<th>Lag</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<tr>
<td>2</td>
<td>-0.0194</td>
</tr>
<tr>
<td>3</td>
<td>0.0595</td>
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<tr>
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<td>-0.0104</td>
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<tr>
<td>5</td>
<td>-0.0052</td>
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<tr>
<td>6</td>
<td>-0.0045</td>
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<tr>
<td>7</td>
<td>-0.0325</td>
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<td>-0.0213</td>
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<tr>
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</tr>
<tr>
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<td>17</td>
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<td>0.0451</td>
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<tr>
<td>20</td>
<td>0.0300</td>
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<tr>
<td>21</td>
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<td>22</td>
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<td>-0.0238</td>
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<tr>
<td>25</td>
<td>-0.0183</td>
</tr>
</tbody>
</table>
5.3.3. Forecasting

Program FORCAT provides the forecast values and its confidence intervals. The outputs are shown on Table 5.20., Fig. 5.12. respectively.

From Fig. 5.12. of the simulated inventory process, we see the inventory is being replenished when it is depleted to some extent. However, there is no definite replenishment cycle. Hence the model will not forecast replenishments. The AR(1) of $(1 - 0.96 B) z_t = a_t$ can represent this process satisfactorily. From the forecast values of Fig. 5.12., we can predict when the inventory stock will be dropped to what level, and hence, can prepare in advance to order the needed stock.
Table 5.20 Forecast Value and its 95% Confidence Limits for the Simulated Inventory Process Data

<table>
<thead>
<tr>
<th>Time</th>
<th>Forecast Value</th>
<th>Upper Limit</th>
<th>Lower Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1001</td>
<td>576.053</td>
<td>443.28</td>
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<td>1002</td>
<td>553.011</td>
<td>368.96</td>
<td>737.05</td>
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<tr>
<td>1003</td>
<td>530.890</td>
<td>309.88</td>
<td>751.89</td>
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<tr>
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<td>509.654</td>
<td>259.37</td>
<td>759.93</td>
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<tr>
<td>1005</td>
<td>489.268</td>
<td>214.75</td>
<td>763.78</td>
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<tr>
<td>1006</td>
<td>469.697</td>
<td>174.61</td>
<td>764.78</td>
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<tr>
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<td>450.909</td>
<td>138.05</td>
<td>763.76</td>
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<tr>
<td>1008</td>
<td>432.873</td>
<td>104.49</td>
<td>761.25</td>
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<td>757.61</td>
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<td>753.13</td>
</tr>
<tr>
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<td>748.01</td>
</tr>
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<td>742.39</td>
</tr>
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<td>-52.50</td>
<td>730.71</td>
</tr>
<tr>
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<td>325.280</td>
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<td>723.73</td>
</tr>
<tr>
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<td>717.17</td>
</tr>
<tr>
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<td>299.778</td>
<td>-110.97</td>
<td>710.53</td>
</tr>
<tr>
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<td>276.275</td>
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</tr>
<tr>
<td>1020</td>
<td>265.224</td>
<td>-160.11</td>
<td>690.56</td>
</tr>
</tbody>
</table>
Figure 5.12 Part of Simulated Inventory Process and its Forecast Values
BIBLIOGRAPHY


APPENDIX A

Program IDENT

A-1. Description of Program

Program IDENT is developed to provide the sample correlation and partial correlation functions so as to identify the appropriate model for the given time series. The calculations performed are based on equations (2.1.3), (2.1.4), (2.2.4) and (2.2.5) in Chapter Two. Program IDENT can accommodate stationary time series, non-stationary time series and seasonal time series. The program consists of a main program and three subroutines. Subroutine DIFFER performs data differences if non-stationary or seasonal time series is analyzed, Subroutine CORRCE calculates the correlations and partial correlations of the time series. Subroutine GRAPH is modified from IBM scientific subroutine PLOT to plot out the correlations and partial correlations within $\pm 1$ range. The flow chart of Program IDENT is constructed on next page.
START

Read in Data

Call Subroutine Differ to Take Differences if Nonstationary Or/And Seasonal Time Series is Analyzed

Take Seasonal Differences

Take Nonstationary Differences

Call Subroutine CORRCE to Calculate the Autocorrelations and its Partials

Continue

Call Subroutine GRAPH to Plot the Autocorrelation and its Partials

STOP
### A.2. Description of Input Data

<table>
<thead>
<tr>
<th>CARD</th>
<th>VARIABLE IN PROGRAM</th>
<th>FORMAT</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>KK</td>
<td>(I10)</td>
<td>Number of correlations to be calculated.</td>
</tr>
<tr>
<td>2</td>
<td>IDW</td>
<td>(I10)</td>
<td>Number of differences required. For stationary time series, a zero is entered.</td>
</tr>
<tr>
<td>3</td>
<td>IDB,IS</td>
<td>(2I10)</td>
<td>Number of seasonal differences followed by seasonal lag. For stationary time series, two zeros are entered.</td>
</tr>
<tr>
<td>4</td>
<td>N</td>
<td>(I10)</td>
<td>Number of observations in original series.</td>
</tr>
</tbody>
</table>

Last N Cards $Z(I,1,1)$ (2X,(F20.5)) Observations of original series.
A.3. Description of Output Data

The following output can be generated by Program IDENT.

1. Graphs of the sample correlations and partials of the original
time series and of the differenced time series.

2. Sample mean of observations; sample variance of observations;
Sample correlations, partials and standard errors.

3. Estimates of the autoregressive parameters for "candidate"
AR(p) models, with p taken from 1 to KK, based on the sample partial
correlation function.
APPENDIX A. PROGRAM IDENT

A.4 Computer Program
DIMENSION Z(1000,3,2),NOB(5,4),X(1000),P(1000)
COMMON/A1/Z
COMMON/A2/NOB
COMMON/A3/X,P
C READ IN DATA
200 FORMAT(4I10)
C KK TOTAL NO. OF CORRELATIONS AND PARTIALS TO BE CALCULATED
C IDW NO. OF DIFFERENCES REQUIRED.
C FOR STATIONARY TIME SERIES, A ZERO IS ENTERED.
C IDB,IS NO. OF SEASONAL DIFFERENCES FOLLOWED BY SEASONAL LAG.
C FOR STATIONARY TIME SERIES, TWO ZEROS ARE ENTERED.
C N NO. OF OBSERVATIONS IN ORIGINAL SERIES.
C Z OBSERVATIONS OF ORIGINAL SERIES.
READ (1,200) KK
READ (1,200) IDW
READ (1,200) IDB,IS
READ (1,200) N
201 FORMAT(2X,(F20.5))
READ(1,201) (Z(I,1,1),I=1,N)
CALL DIFFER(IDW,IDB,IS,N,WW,MB)
DO 202 J=1,MB
DO 202 K=1,WW
CALL CORRCE(KK,N,J,IS)
END
SUBROUTINE DIFFER(IDW,IDB,IS,MM,WH,PB)
C SUBROUTINE DIFFER PERFORMS DIFFERENCING OPERATIONS ON THE DATA
C IF NON-STATIONARY OR SEASONAL SERIES ARE ANALYZED.
C
DIMENSION Z(1000,3,2),NOB(5,4)
COMMON/A1/Z
COMMON/A2/NOB
C CALCULATION OF NOB(M,J)
MW=IDW+1
PB=IDB+1
DO 151 M=1,MW
CO 151 J=1,MB
151 NOB(M,J)=N+1-M+IS-J*IS
C DIFFERENCING WITH RESPECT TO DW, NO. OF DIFFERENCES
C NOBD=NOB(M,1) IS NO. OF OBSERVATIONS AFTER DIFFERENCE
C Z(I,M,1) IS THE OBSERVATIONS AFTER DIFFERENCE BY SUBTRACTING
C THE PRECEEDING OBS. FROM THE CURRENT OBS. M IS THE DIFFERENCE NO. INDE:
C
IF(MW-1)152,152,153
153 DO 154 M=2,MW
154 NOBD=NOBD(M,1)
CO 154 I=1,NOBD
C DIFFERENCING WITH RESPECT TO DB
C NOBD=NOB(M,J) IS THE NO. OF OBSERVATIONS AFTER DIFFERENCES M AND
C SEASONAL DIFFERENCES J.
C Z(I,M,J) IS THE OBSERVATIONS AFTER SEASONAL DIFFERENCE,
C J IS THE SEASONAL DIFFERENCE NO. INDEX HERE.
C
IF(PB-1)152,155,156
156 DO 157 P=1,MM
157 J=2,MB
158 NOBD=NOBD(M,J)
159 DO 157 I=1,NOBD
157 Z(I,M,J)=Z(I+IS,M,J-1)-Z(I,M,J-1)
155 CONTINUE
155 RETURN
END
SUBROUTINE CORRCE(KK,M,L,IS)
C SUBROUTINE CORRCE CALCULATES THE SAMPLE CORRELATIONS AND PARTIALS AND C THEIR STANDARD ERRORS.
C
DIMENSION ZDUM(1000,3,2),NDB(5,4),Z(1000),X(1000),P(1000),C(1000)
C DIMENSION R(1000),VAR(1000),S(1000),T(I1000),VART(1000),U(I1000)
C DIMENSION (Z250)
COMMON/A4/R
COMMON/A4/1,ZDUM
COMMON/A2/NCB
COMMON/A3/X,P
COMMON/A6/T
101 FORTRAN12X31HCORRELATION INFORMATION FOR DW=14,2X3HDB=14,2X,
  12HS=I4/11)
IDW=M-1
IDB=L-1
WRITE (I3,1011) IDW,IDB,IS
N=NCB(M,L)
CO 100 I=1,N
100 IF(I)=ZDUM(I,M,L)
ZBAR=C.*
XN=N
CO 102 I=1,N
102 ZBAR=ZBAR+Z(I)
ZBAR=ZBAR/XN
CO=C.*
CO 103 I=1,N
103 CO=CO+(Z(I)-ZBAR)**2
CO=CO/XN
C CALCULATION OF R, ESTIMATED CORRELATION FUNCTION
DO 104 K=1,KK
C(K)=C.*
NN=N-K
CO 105 J=1,NN
105 C(K)=C(K)+(Z(J)-ZBAR)*(Z(J+K)-ZBAR)
C(K)=C(K)/NN
104 R(K)=C(K)/CO
C CALCULATION OF T
IF(KK-101) 106,106,107
103 KKK=101-1
107 GO TO 109
106 KKK=KK-1
C RECURSIVE RELATIONS FOR FINDING TK(K,K), PARTIAL CORRELATIONS
C FUNCTION OF GIVEN R(K).
109 T(1,1)=R(1,1)
38 T(2,2)=(R(2,2)-R(1,1)**2)/(1-R(1,1)**2)
T(2,1)=T(1,1)-T(2,2)*T(1,1)
304 CO 203 K=2,KKK
304 B=0.*
304 A=G.*
304 CO 202 J=1,K
304 A=A+T(K,J)*R(K+1-J)
304 CO 202 B=B+T(K,J)*R(J)
304 A=R(K+1)-A
304 B=1.-B
304 T(K+1,K+1)=A/B
304 CO 203 J=1,K
304 203 T(K+1,J)=T(K,J)-T(K+1,K+1)*T(K,K-J+1)
C PLOT OUT AUTOCORRELATION AND PARTIAL AUTOCORRELATION FUNCTION
C E IS PLOTTED FUNCTION
0051 DO 112 K=1,KK
0052   K1=KK+K
0053   E(K)=K
0054 112 E(K1)=R(K)
0055 E(2*KK+1)=T(1,1)
0056 E(2*KK+2)=T(2,2)
0057 DO 113 K2=2,KKK
0058   K2=(2*KK+2)+K-1
0059 113 E(K2)=T(K+1,K+1)
C CALL GRAPH (I,E,KK,3,KK)
C CALCULATION OF VAR AND VART
C VAR(K) IS AN ESTIMATE OF THE VARIANCE OF THE ESTIMATE OF THE CORRELATIONS,
C WHICH CAN BE USED IN A ROUGH TEST FOR WHETHER CORRELATION R IS
C EFFECTIVELY ZERO.
C
0061 VAR(1)=1./XN
0062 S(1)=R(1)/SQRT(VAR(1))
0063 A=2./XN
0064 DO 204 K=2,KK
0065   VAR(K)=VAR(K-1)+A*(R(K-1)**2)
0066 204 S(K)=R(K)/SQRT(VAR(K))
C VART(K) IS AN APPROXIMATE ESTIMATE OF THE VARIANCE OF THE SAMPLE
C PARTIAL CORRELATIONS, GIVEN THAT THE MODEL IS AR(K-1)
C
0067 KK=KK+1
0068 DO 205 K=1,KKK
0069   A=1./N-K
0070 205 L(K)=T(K,K)/SQRT(VAR(K))
C WRITE OUT
C
0072 FORMAT(/2X,5HZBAR=,F20.5,5X,7HVAR(2)=,F20.5///)
0073 WRITE (3,601) ZBAR,A,
0074 SVAR=SQRT(VAR(KK))
0075 SVAT=SQRT(VAR(KK))
0076 WRITE(3,250) SVAR,SVAT
0077 250 FORMAT(/2X,12HS-D*(R(KK))=,F10.6,6X,14HS-D*(PR(KK))=,F10.6)
0078 300 FORMAT(/2X,3HSAMPLE CORRELATION COEFFICIENTS///)
0079 WRITE(3,300)
0080 301 FORMAT(2X,2HR((I3,2H)=,F10.5,6X,10HR/S-D*(R)=,F10.5)
0081 DO 302 I=1,KK
0082 302 WRITE (3,301) I,R(I),S(I)
0083 303 FORMAT(/2X,3HSAMPLE PARTIAL CORRELATION COEFFICIENTS///)
0084 WRITE(3,303)
0085 304 FORMAT(2X,3HPR((I3,2H)=,F10.5,6X,12HPR/S-D*(PR)=,F10.5)
0086 DO 305 I=1,KK
0087 305 WRITE(3,304) I,T(I),U(I)
0088 310 FORMAT(1H12X,25SHAUTOREGRESSIVE PARAMETERS///)
0089 WRITE(3,310)
0090 311 FORMAT(2X,2HP=,I3,5X,4HPHI((I3,2H)=,F10.5)
0091 DO 312 K=1,KK
0092 312 WRITE(3,311) K,I,T(K,I)
0093 RETURN
END
SUBROUTINE GRAPH (NO, A, N, M, NL)

DIMENSION OUT(101), YPR(11), ANG(9), A(1)

1 FORMAT (1H1, 60X, 7H CHART )
2 FORMAT (1H , F11.4, 5H+ ,101A1)
4 FORMAT (1OH *X015678+)
5 FORMAT (10A1)
7 FORMAT (1H , 16X, 101H-
1 -
8 FORMAT (1HO, 9X, 11F10.4/)
200 FORMAT (10X, ' PLOT OF AUTO-CORRELATION AND PARTIAL AUTO-CORRELATION FUNCTION')
201 FORMAT (10X, ' PLOT OF AUTO-CORRELATION FUNCTION')

NLL=NL

PRINT TITLE

20 WRITE(3,1)
91 WRITE(3,200)
92 WRITE(3,201)
19 CONTINUE

DEVELOP BLANKS AND DIGITS FOR PRINTING

REWIND 4
WRITE(4,4)
REWIND 4
READ(4,5) BLANK, (ANG(I), I=1,9)
REWIND 4

FIND SCALE FOR BASE VARIABLE

XSCAL=(A(N)-A(1))/(FLOAT(NLL-1))

FIND SCALE FOR CROSS VARIABLES

M1=N+1
M2=M*N
YMIN=-1.
YMAX=+1.
YSCAL=(YMAX-YMIN)/100+0

FIND BASE VARIABLE PRINT POSITION

XB=A(1)
MY=M-1
DO 108 I=1, NLL
45 F=I-1
XPR=XB+F*XSCAL
C FIND CROSS VARIABLES

C

51 DO 55 IX=1,101
55 CUT(IX)=BLANK
57 DO 60 J=1,MY
58 LL=I+J*NY
59 JP=((A(IX)-YMIN)/YSCALE)+1.0
60 CUT(JP)=ANG(J)
61 IF(JP.EQ.51) GO TO 60
62 CUT(51)=ANG(9)
63 IF(JP.EQ.101) GO TO 60
64 CUT(101)=ANG(9)
65 CONTINUE

C PRINT LINE AND CLEAR, OR SKIP

C

108 CONTINUE

C PRINT CROSS VARIABLES NUMBERS

C

86 WRITE(3,7)
7 YPR(1)=YMIN
90 DO 90 KN=1,9
91 YPR(KN+1)=YPR(KN)+YSCALE*10.0
92 YPR(11)=YMAX
93 WRITE(3,8)(YPR(IR),IR=1,11)
94 RETURN
95 END
APPENDIX B

PROGRAM ESTIM

B.1. Description of Program

Program ESTIM is developed to determine the least square estimates of the parameters of the models entertained as candidates for acceptance. The program consists of the main program, and five subroutines; they are subroutine MODEL, subroutine CALA, subroutine DIFFER, subroutine MULT and subroutine UWHAUS. The main program provides the data needed for subroutine UWHAUS and calculates the autocorrelation function of the residuals based on the least square estimates. Program UWHAUS is used in conjunction with subroutine MODEL, CALA and MULT, and perform the operation of locating the least square estimates in an iterative manner. The complete description of subroutine UWHAUS is presented on Section B.2. Subroutine DIFFER performs the required differences on the original observations if seasonal or nonstationary time series is analyzed. Subroutine MODEL, CALA and MULT calculate the residuals of original time series required by subroutine UWHAUS for each set of parameter tested. Its calculation is based on the following equation.

\[
a_t = \tilde{w}_t - \phi_1 \tilde{w}_{t-1} - \phi_2 \tilde{w}_{t-2} - \cdots - \phi_p \tilde{w}_{t-p} + \theta_1 a_{t-1} + \theta_2 a_{t-2} + \cdots + \theta_q a_{t-q}
\]

(B.1.1)

where \( \tilde{w}_t = \nabla^d \bar{z}_t \) and \( \tilde{w}_t = \bar{w}_t - \mu \) with \( E[\tilde{w}_t] = \mu \), \( a_t \) is the residual of general autoregressive model and is assumed as white noise process.
The flow chart of program ESTIM is as follows.

START

Read in Data:
No. of Observation,
Time Series Observation, No. of Models Analyzed.

Model Number

Read in Data:
No. of Differences,
The Model Form Being Analyzed.

Print Out the Model Form

Read in Data:
Parameters for the Analyzed Model

Is This Series Stationary

Yes

Is This The First Analyzed Model

No

C

A
C

A

Yes

Compute the Mean of the Time Series and Standardized Data

Print Out Mean and Standardized Data

Call Subroutine DIFFER to Take Differences if Non-stationary or/and seasonal Time Series is Analyzed

Define the data needed in the argument of subroutine UWHAUS

Call Subroutine UWHAUS to locate the least square estimate of non-linear parameter

Call subroutine CALA to compute the residual of time series using the obtained estimated parameters

B
B

Print Out the Residual of Time Series

Compute the Autocorrelation Function of the Residual of the Series

Print Out the Autocorrelation Function of the Residual

CONTINUE

STOP
### B.2. Description of Input Data of Program ESTIM

<table>
<thead>
<tr>
<th>CARD</th>
<th>FORMAT</th>
<th>VARIABLES IN PROGRAM</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(I10)</td>
<td>N</td>
<td>Sample size of original series.</td>
</tr>
<tr>
<td>Next</td>
<td>(2X,(F20.5))</td>
<td>ZD(I,1,1)</td>
<td>Observations of original series.</td>
</tr>
<tr>
<td>N cards</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Next</td>
<td>(I10)</td>
<td>INDS</td>
<td>Number of models to be fitted to original series.</td>
</tr>
<tr>
<td>card</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

[For each model, add the following parameter input data]

<table>
<thead>
<tr>
<th>Next card</th>
<th>(3I10)</th>
<th>IPW</th>
<th>Number of autoregressive parameters.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>IDW</td>
<td>Number of nonstationary differences.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>IQW</td>
<td>Number of moving-average parameters.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Next card</th>
<th>(4I10)</th>
<th>IPB</th>
<th>The largest power of the shift operator associated with the seasonal autoregressive parameters.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>IDB</td>
<td>The number of seasonal differences.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>IQB</td>
<td>The largest power of the shift operator, associated with the seasonal moving-average parameters.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>IS</td>
<td>The seasonal lag.</td>
</tr>
<tr>
<td>CARD</td>
<td>FORMAT</td>
<td>VARIABLES IN PROGRAM</td>
<td>DESCRIPTION</td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
<td>----------------------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Next</td>
<td>(4I10)</td>
<td>I1</td>
<td>The number of autoregressive parameters.</td>
</tr>
<tr>
<td>card</td>
<td></td>
<td>I2</td>
<td>The number of moving-average parameters.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>I3</td>
<td>The number of non-zero seasonal autoregressive parameters.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>I4</td>
<td>The number of non-zero seasonal moving-average parameters.</td>
</tr>
<tr>
<td>Next</td>
<td>(I10,F20.5)</td>
<td>J,PHIW(J)</td>
<td>The initial estimated value of the Jth autoregressive parameter. If I1=0, no card is entered.</td>
</tr>
<tr>
<td>card</td>
<td>(I10,F20.5)</td>
<td>J,THETW(J)</td>
<td>The initial value of the Jth moving-average parameter. If I2=0, no card is entered.</td>
</tr>
</tbody>
</table>

[Add the following cards, for seasonal models, only]

| Next   | (I10,F20.5)| J,PHIB(J)            | The Jth non-zero seasonal autoregressive parameters with index J, corresponding to the power of the adjacent shift operator in the model. |
| card   | (I10,F20.5)| J,THETB(J)           | The Jth non-zero seasonal moving-average parameter with index J corresponding to the power of the adjacent shift operator in the model. |
B.3. Description of Output Data

The following data can be generated by Program ESTIM.

1. List of the initial estimates of the parameters.

2. For stationary time series only; a print out of the standardized observations, in other words, the observations formed by subtracting the sample mean from the original observations.

3. UWHAUS prints out summary information at each iteration which can be analyzed to determine the path in the parameter space taken by the iterations to converge on the least square estimates.

4. Tabulation of the first 25 sample correlations of the residuals based on the least square estimates.

B.4.1. Description of Subroutine UWHAUS

Subroutine UWHAUS is developed to obtain least square estimates of parameters entering non-linearly into a mathematical model. An iterative technique is used, the estimate at each iteration is obtained by a method due to Marquardt which combines the Gauss (Taylor series) method and the method of steepest descent [51]. The main program must be provided by the user to supply the input for subroutine UWHAUS. Subroutine MODEL is to specify what mathematical model is to be used. As for the general Auto-regressive Moving-average model, its model form is equation (B.1.1).

The theory behind subroutine UWHAUS can be described as follows. Suppose the mathematical model which is tentatively entertaining is

\[ \eta = f(\theta, \xi) \]  

(8.2.1)
where $\theta$ is a pxl vector of unknown parameters and $\xi$ is a vector of independent variables.

Suppose $n$ actual observations $Y$ are made. When the $u$th observation $Y_u$ is made, the value of the independent variable is $\xi_u$. Because of experimental error, an observation $Y_u$, different from the model response, $\eta_u$. Hence,

$$Y_u = \eta_u + \epsilon_u, \quad u = 1, \ldots, n \quad (B.2.2)$$

Marquardt also assured the theoretical optimum properties of least square estimates. Some assumption on the errors, $\epsilon_u$, have to be made: [50]

1. The errors, $\epsilon$, are independent random variables with equal variance from the same probability observation (independence implies that knowledge of $\epsilon_i$ does not give any information about $\epsilon_j$, $i \neq j$).

2. The expected value of the errors is zero.

3. The probability distribution of the error is the normal (Gaussian) distribution with variance $\sigma^2$.

Under the assumption (1), (2), and (3), the least square estimate, $\hat{\theta}$, is a maximum likelihood estimates and thus had certain desirable properties [51].

Now, from (B.2.2), it is desired to use the observed data to obtain estimates of the unknown parameter, $\theta$. An estimate of $\theta$, say $\hat{\theta}$, obtained by minimizing

$$S(\theta) = \sum_{u=1}^{n} [Y_u - \eta_u]^2 = \sum_{u=1}^{n} [Y_u - f(\theta, \xi_u)]^2 \quad (B.2.3)$$
as a function of \( \theta \), is frequently referred to as a least square estimate. Subroutine UWHAUS is intended to provide a least square estimate of \( \theta \) when the model (B.2.1) is nonlinear in the parameters, \( \theta \).

When dealing with the autoregressive moving-average problem, \( u \) in (B.2.2) is the random error \( a_t \) in model (B.1.1). The expected value of the errors, which are \( Y_u \) in (B.2.2), are assumed zero; this conforms with the assumption of Box and Jenkins [52] and Marquardt [50]. Hence, in the case of an autoregressive moving-average model, (B.2.3) may be written as,

\[
S(\theta) = \sum_{u=1}^{n} [Y_u - \eta_u]^2 = \sum_{u=1}^{n} a_t^2
\]  

(B.2.4)

Now, suppose \( \hat{\theta}^{(0)} \) is an initial guess, the first order Taylor series expansion about \( \hat{\theta}^{(0)} \) is

\[
\eta_u(\hat{\theta}) = \eta_u(\hat{\theta}^{(0)}) + \sum_{i=1}^{p} (\hat{\theta}_i - \hat{\theta}_i^{(0)}) \left. \frac{\partial f(\theta, \xi_u)}{\partial \hat{\theta}_i} \right|_{\hat{\theta}^{(0)}}
\]  

(B.2.5)

\( u=1, \ldots, n \)

or more compactly,

\[
\eta(\hat{\theta}) = \eta^{(0)} + X \delta
\]

where \( X \) is the \( nxp \) matrix

\[
X_{nxp} = \left\{ \left. \frac{\partial f(\hat{\theta}, \xi_u)}{\partial \hat{\theta}_i} \right|_{\hat{\theta}^{(0)}} \right\}, \quad u=1, \ldots, n
\]

\( i=1, \ldots, p \)
where \( \delta = \theta - \theta^{(0)} \) is the p\times1 vector; \( \eta(\theta) \) is the n\times1 vector 
\([f(\theta, \xi_1), \ldots, f(\theta, \xi_n)]\), and \( \eta^{(0)} \) is the n\times1 vector \( \eta(\theta^{(0)}) \).

Now the approximation on the right hand side of (B.2.6) is linear in the parameters \( \delta \); by substituting (B.2.6) to (B.2.3), an approximation for \( S(\theta) \) is,

\[
S(\theta) \approx (y - \eta^{(0)} - X \delta)'^t (y - \eta^{(0)} - X \delta)
\]  

(B.2.7)

where

\[
\delta = D^{-1/2} (D^{-1/2} X' X D^{-1/2} + \lambda I)^{-1} D^{-1/2} X' y
\]  

(B.2.8)

is the correction vector, which is adapted from Marquardt's algorithm; [50]. \( D \) is a p\times p diagonal matrix whose i-th diagonal element is the same as that of \( X'X \); \( \lambda \) is a non-negative number.

\( \lambda \) should be decreased only if the progress is satisfactory, i.e., only if the sum of squares, \( S(\theta) \), at the new estimate is smaller than at the old. Thus, at i-th iteration, the basic strategy as indicated by Marquardt is as follows [50]:

Denote by \( S(\lambda) \) the value of \( S(\theta) \) obtained by using \( \lambda \) in (B.2.8) to get \( \theta^{(1)} \) from \( \theta^{(i-1)} \). Let \( \lambda^{(i-1)} \) be the value of \( \lambda \) from the previous iteration. Let \( \nu > 1 \).

Compute \( S(\lambda^{(i-1)}) \) and \( S(\lambda^{(i-1)} / \nu) \).

1. if \( S(\lambda^{(i-1)} / \nu) \leq S(\theta^{(i-1)}) \), let \( \lambda^{(i)} = \lambda^{(i-1)} / \nu \)

2. if \( S(\lambda^{(i-1)} / \nu) > S(\theta^{(i-1)}) \), and \( S(\lambda^{(i-1)}) \leq S(\theta^{(i-1)}) \), let \( \lambda^{(i)} = \lambda^{(i-1)} \)
otherwise, increase \( \lambda \) by successive multiplication by \( v \) until
for smallest \( w \), \( S(\lambda^{(i-1)} w) \leq S(\hat{\theta}^{(i-1)}) \). Let \( \lambda^{(i)} = \lambda^{(i-1)} w \)

Hence, by the definition of \( \hat{\bar{\theta}} \) in (B.2.8), the new guess is
\[ \hat{\bar{\theta}}^{(1)} = \hat{\bar{\theta}}^{(0)} + \hat{\bar{\theta}}^{(0)} \]
and the next iteration can be started by expanding about \( \hat{\theta}^{(1)} \).

B.4.2. Description of the variable in the Argument of Subroutine UWHAUS

UWHAUS is called from the main program with a FORTRAN statement of
the form:

CALL UWHAUS (NPROB, NOB, Y, NP, TH, DIFF, SIGNS, EPS1, EPS2, MIT, FLAM,
               PNU, SCRAT)

NPROB is the problem number.

NOB is the number of observations.

Y is a real one-dimensional array containing the vector of
observed function values; i.e., \( Y(I) \) is the \( I \)th observed
function value, \( I = 1, \ldots, NOB \).

NP is an integer indicating the number of unknown parameters.

TH is a real one-dimensional array of the parameter values.

i.e. \( TH(J) \) is the \( J \)th parameter value, \( J = 1, \ldots, NP \).

It is very important to obtain reasonable starting guess
for the parameters; not only will the computation time be
decreased by a good choice of starting values, but there
is also the possibility of converging to a more reasonable
estimate.

DIFF is a real one-dimensional array containing a vector of
proportions in \( \bar{\theta} \), for use in computing the difference
quotients of the model function values. The derivatives,
\[
\frac{\partial f(\theta, \xi_u)}{\partial \theta_i}
\] in (B.2.5) are approximated by difference quotients within the program.

\[
\frac{\partial f(\theta, \xi_u)}{\partial \theta_i} \approx \frac{f(\theta_1, \ldots, \theta_i + \Delta \theta_i, \ldots, \theta_p, \xi_u) - f(\theta, \xi_u)}{(\theta_i + \Delta \theta_i) - \theta_i}
\]

Thus at any point in the calculations, the denominator of the above difference quotient will be expressed as:

\[(TH(I) + DIFF(I) \times TH(I)) - TH(I) = DIFF(I) \times TH(I)\]

In any case, DIFF(I) must satisfy \(0 < |DIFF(I)| < 1\), \((I=1, \ldots, NP)\). Using a starting guess of zero for any parameter is prohibited for this method of calculation. SIGNs is a real one-dimensional array indicating the existence of a prior sign restrictions on each of the parameters.

If SIGNs(I) is set equal to any positive quantity, UWHAUS will not allow the Ith parameter to change its sign during the calculations, thus the Ith parameter, TH(I), retains the same sign as the starting guess for that parameter.

If SIGNs(I) = 0, this feature is disabled for the Ith parameter.

EPS1 is a real constant indicating the sum of squares convergence criterion and is used to terminate the calculation based on the relative change in the sum of squares from one iteration to the next iteration. More precisely, if at the completion of the Ith iteration, it is true that
\[ \left| \frac{S(\hat{\theta}^{(i)}) - S(\hat{\theta}^{(i-1)})}{S(\hat{\theta}^{(i-1)})} \right| \leq \text{EPS1} \]

then the calculations are terminated. Roughly, this means that if \( \text{EPS1} = 10^{-k} \), the calculations will be stopped if the sum of squares for the \((i-1)\)st and \(i\)th iteration agree to \(k\) decimal places. If \( \text{EPS1} \) is set equal to zero, this feature is disabled.

\( \text{EPS2} \) is a real constant which is the parameter convergence criterion and is used to terminate the calculations based on the relative change in the parameter values from one iteration to the next iteration. Suppose that after the \(i\)th iteration, the value of the \(j\)th parameter is \( \hat{\theta}_j^{(i)} \) \((j=1,\ldots,p)\). If, at the completion of the \(i\)th iteration, the following holds:

\[ \left| \frac{\hat{\theta}_j^{(i)} - \hat{\theta}_j^{(i-1)}}{\hat{\theta}_j^{(i-1)}} \right| < \text{EPS2} \]

for all \(j=1,\ldots,p\), then the calculations are terminated. Roughly, this means that if \( \text{EPS2} = 10^{-k} \), the calculations will be stopped if the value of each parameter after the \(i\)th iteration agree to \(k\) decimal place with the value of the same parameter after the \((i-1)\)st iteration. This feature is disabled if \( \text{EPS2} \) is set to zero.

\( \text{MIT} \) is an integer constant (where \(0 < \text{MIT} < 1000\)) which is the maximum number of iterations to be performed. If the
calculations have not been terminated for some other reasons, they will be terminated when the number of iteration equals MIT.

FLAM

starting value for \( \lambda \).

FNU

is the value of \( \nu \).

SCRAT

is an optimal parameters used to specify temporary storage for use by UWHAUS. When present in the calling sequence, SCRAT must be the name of an array containing at least the number of storage locations given by:

\[ 5 \times NP + 2 \times NP^2 + 2 \times NOB + NP \times NOB \]

The contents of these locations will be destroyed during execution of UWHAUS.

B.4.3. The restrictions of subroutine UWHAUS

At the beginning of each problem run, UWHAUS checks the input arguments to see that the following are obeyed:

1. \( 1 \leq NP \leq 50 \);

2. \( NOB \geq NP \)

3. \( TH(I) \neq 0, I=1, \ldots, NP \). Each starting parameter guess is non-zero.

4. \( 0 < |DLFF(I)| < 1, I=1, \ldots, NP \). Each difference proportion is between 0 and 1 in absolute value.

5. \( 0 < MIT < 1000 \), the maximum number of iteration is between 0 and 1000.

6. \( FNU \geq 1 \). The starting value of \( \nu \) is greater than or equal to 1.
If any of these restrictions are not obeyed, the message:

**PARAMETER ERROR**

will be printed on the printer output for the job, and control will be returned to the main program.

The flow chart of subroutine UWHANS is shown as follows. The notations $\varepsilon_1$, $\varepsilon_2$, $\Delta$ and $j_{\text{max}}$ are the input argument EPS1, EPS2, DIFF and MLT, respectively. Other notations have the same meanings as defined in Section B.4.
Compute Parameter Estimation
complete one iteration
Iterating Complete
APPENDIX B. PROGRAM ESTIM

B.5 Computer Program
PROGRAM ESTIM IS DEVELOPED TO DETERMINE THE LEAST SQUARES ESTIMATES
OF THE PARAMETERS OF MODELS TERMINATED AS CANDIDATEA FOR ACCEPTANCE.
THE PROGRAM ALSO DEVELOPS THE APPROXIMATE COVARIANCE MATRIX FOR THE
ESTIMATES, AND PROVIDES THE RESIDUALS BASED ON THE LEAST SQUARES ESTIMATES.

C DIMENSION SCRAT(4018)
0002 DIMENSION R(100),S(100),VAR(100)
0003 DIMENSION B(20)
0004 DIMENSION ZERO(1000)
0005 DIMENSION DIFF(20),SIGNS(20)
0006 COMMON Z(1000,4,3),WOB(4,3),Z1(1000),A(1000),PHI(100),THETA(100),
     1 PHI(100),THET1(100),PHIB(100),THETB(100),LOC1(10),LOC2(10),LOC3(10),LOC4(10),IND1(1),IND2(1),IND3(1),IND4(1),INDP(1),INDQ(1)
0007 KCREP=0

C N SAMPLE SIZE OF ORIGINAL SERIES
0008 REAC 2002, N
0009 2021 FORMAT(2X,F20.5)
0100 REAC 2021, (ZD111,1,1),INDS
C READ IN DATA AND PRINT OUT
C 2D111,1,1) OBSERVATIONS OF ORIGINAL SERIES
C IND1 NO. OF MODELS TO BE FITTED TO ORIGINAL SERIES
C IPW NO. OF AUTOREGRESSIVE PARAMETERS
C IDW NO. OF NON-STATIONARY DIFFERENCES.
C THIS IS ZERO FOR STATIONARY SERIES.
C IQW NO. OF MOVING AVERAGE PARAMETERS.
0011 REAC 2002, IND1
0012 DO 1010 ICQP=1,INDS
0013 2002 FORMAT(4110)
0114 REAC 2002, IPW,IDW,IQW
C IPB THE LARGEST POWER OF THE SHIFT OPERATOR ASSOCIATED WITH
C THE SEASONAL AUTOREGRESSIVE PARAMETERS.
C IC THE SEASONAL DIFFERENCES.
C ICW LARGEST POWER OF THE SHIFT OPERATOR ASSOCIATED WITH
C THE SEASONAL MOVING AVERAGE PARAMETERS.
C IS SEASONAL LAG.
C FOR STATIONARY SERIES, THE FOLLOWING CARD IS BLANK
0015 REAC 2002, IPB,IDW,IQW,IS
0016 2003 FORMAT(11,12X,6NHARMA(13,1H,,13,3H)),X(13,1H,,13,3H),X
     11,13,1H,))
0017 PRINT 2003, IPW,IDW,IQW,IPB,IDW,IQW,IS
0018 IP=IPB+IPB
0019 IC=ICW+ICW
0020 (1)=IP
0021 ICW(1)=IP
0022 ICW(1)+IP
0023 CO IDP=1,IP
0024 PHIB(1)=0*
0025 PHIB(1)=0*
0026 PHIB(1)=0*
0027 THET1(1)=0*
0028 THET1(1)=0*
0029 THET1(1)=0*
0030 2090 FORMAT///2X,20HINITIAL GUESS VALUES///
0031 PRINT 100
0032 204 FORMAT(11,10,F20.5)
0033 2005 FORMAT(2X,5PHIB1(13,2H)+,F11.5)
0034 2007 FORMAT(2X,5PHIB1(13,2H)+,F11.5)
0035 2006 FORMAT(2X,6HTHET1(13,2H)+,F11.5)
2C08 FORMAT(2X,6HTHETB(I3,2H)=,F11.5)
C I1 THE NO. OF AUTOREGRESSIVE PARAMETERS
C I2 THE NO. OF MOVING AVERAGE PARAMETERS
C I3 THE NO. OF NCN-ZERO SEASONAL AUTOREGRESSIVE PARAMETERS
C I4 THE NO. OF NCN-ZERO SEASONAL MOVING AVERAGE PARAMETERS.

READ 2002, I1,I2,I3,I4
K=0

IF(I1)2009,2009,2010
C PHIW(J) THE INITIAL GUESS VALUE OF THE JTH AUTOREGRESSIVE PARAMETERS.
C LOC1(I) ASSOCIATED THESE J.
C THETW(J) THE INITIAL GUESS VALUE OF THE JTH MOVING AVERAGE PARAMETERS.
C LOC2(I) ASSOCIATED THESE J.
C PHIB(J) THE JTH NON-ZERO SEASONAL AUTOREGRESSIVE PARAMETER,
C CORRESPONDING TO THE POWER OF THE ADJACENT SHIFT OPERATOR
C IN THE MODEL.
C LOC3(I) ASSOCIATED THOSE J.
C THETB(J) THE JTH NCN-ZERO SEASONAL MOVING AVERAGE PARAMETER.
C LOC4(I) ASSOCIATED THESE J.

2010 CC 2011 I=1,I1
READ 2004, J,PHIW(J)
LOC1(I)=J
K=K+1

2011 PRINT 2005, J,PHIW(J)

2013 CC 2014 I=1,I2
READ 2004, J,THETW(J)
LOC2(I)=J
K=K+1

2014 PRINT 2006, J,THETW(J)

2016 CC 2017 I=1,I3
READ 2004, J,PHIB(J)
LOC3(I)=J
K=K+1

2017 PRINT 2007, J,PHIB(J)
2015 IF(I4)2018,2018,2019

2019 CC 2020 I=1,I4
READ 2004, J,THETB(J)
LOC4(I)=J
K=K+1

2020 PRINT 2008, J,THETB(J)

2018 ISUM=I1+I2+I3+I4
I1(I1)=I1
I2(I1)=I2
I3(I1)=I3
I4(I1)=I4

IF(IDW)9000,9000,9001
CONTINUE
C IF THIS IS AFTER FIRST MODEL, JUST GO TO 9001, NOT NECESSARY
C TO COMPUTE AGAIN.
C
KCREP=KCREP+1

IF(KCREP)16721,6721,9001

6721 CONTINUE
XN=N
ZBAR=G.
CO 9002 I=1,N
ZBAR=ZBAR+Z(I,1,1)
ZBAR=ZBAR/XN
CO 9003 I=1,N
Z(I,1,1)=Z(I,1,1)-ZBAR
FORMAT(1H12X,17HSTANDARDIZED DATA///)
PRINT 9004
FORMAT(12X,5H57BAR=,F20.5///)
PRINT 9005,ZBAR
PRINT 205, (Z(I,1,1),I=1,N)
CONTINUE
CALL DIFFER(IOW,IDB,IS,N,NW,MB)
NCB8 IS NO. OF OBSERVATION AFTER DIFFERENCES AND SEASONAL DIFFERENCES.
NOBB=NCB8(PW,MB)
CO 2022 I=1,NOBB
ZERO(I)=0.
2C22 I(I)=ZD(I,1,1,MB)
CAUSSELAUS DATA
CC 11 I=1,ISUM
DIFF(I)=-01
11 SIGNS(I)=0.
EPS1=0.
EPS2=0.0001
MIT=18
FLM=50.
FN=10.
APRC=1DCP
CALL LSAGUS(NPROB,NCB8,ZERO,ISUM,B,DIFF,SIGNS,EEPS1,EEPS2,MIT,FLM,
1FN,SCRAT)
PUNCH OUT OF A FOR DIAGNOSTIC CHECKING
CALL CAL4(NCB8,IP,IQ)
PRINT 2CC2, NOBB
FORMAT(12X,6F20.5)
PRINT 205, (A(I),I=1,NOBB)
FORMAT(1H12X,19HDIAGNOSTIC CHECKING///)
PRINT 7001
CO 7000 I=1,NOBB
ZERO(I)=Z(I)-A(I)
CONTINUE
7CCQ CONTINUE
KK=25
N=NCB8
XN=N
ZBAR=0.
CO 102 I=1,N
Z(I)=A(I)
ZBAR=ZBAR+Z(I)
ZBAR=ZBAR/XN
CO=0.
CO 103 I=1,N
CO=CO+(Z(I,1,1)-ZBAR)**2
CO=CO/XN
CALCULATION OF R
DO 104 K=1,KK
C(K)=C*
DO 105 J=1,NN
0130 1C5 C(K)=C(K)+(Z(J)-ZBAR)*(Z(J+K)-ZBAR)
0131 C(K)=C(K)/XN
0132 1C4 R(K)=C(K)/XN
0133 VAR(L)=1./XN
0134 S(I)=R(I)/SCRT(VAR(I))
0135 AAA=2./XN
0136 DO 204 K=2,KK
0137 VAR(K)=VAR(K-1)+AAA*(R(K-1)**2)
0138 2C4 S(K)=R(K)/SCRT(VAR(K))
0139 300 FORMAT(1H12X,44HSAMPLE CORRELATION COEFFICIENTS OF RESIDUALS//)
0140 PRINT 300
0141 3C5 FORMAT(/2X,5HZBAR=,F10.5//)
0142 PRINT 3C5, ZBAR
0143 301 FORMAT(2X,2HR(I),13,2H)=,F10.5,6X,10HR/S.D*(R)=,F10.5
0144 DO 302 I=1,KK
0145 3C2 PRINT 301, I*R(I), S(I)
0146 1C CONTINUE
0147 STOP
0148 END
SUBROUTINE MODEL(NPROM, B, F, NOBB, ISUM)

C SUBROUTINE EST CHAGE B(K) TO DIFFERENT PARAMETERS, WHICH IS THE SAME AS
C READ IN DATA VECTOR PARAMETER. THEN, COMPUTE THE VALUE OF
C THE MODEL.
C
C DIMENSION B(1)
C DIMENSION F(1)
COMMON 2D11000, 4, 3, NOB(4, 3), I(1000), A(1000), PHI(100), THETA(100),
1 PHI(100), THETA(100), PHIB(100), THETB(100), LOC1(10), LOC2(20), LCC3(100), LCC4(10), III(1), I11(1), I12(1), I13(1), I14(1), IP(1), IQ(1)
C
IP=IP(1)
IC=IQ(1)
IPIC=IP+IQ
C
11=II(1)
12=I12(1)
13=I13(1)
14=I14(1)
K=0
C
II(1) 100, 100, 101
01
101 CO 102 I=1, 11
02 J=LCC1(I)
03 K=K+1
04 PHI(1J)=B(K)
05 IF(12) 103, 103, 104
06 CO 103 I=1, 12
07 J=LCC2(I)
08 K=K+1
09 THETA(J)=B(K)
10 IF(13) 106, 106, 107
11 CO 106 I=1, 13
12 J=LCC3(I)
13 K=K+1
14 PHIB(J)=B(K)
15 IF(14) 109, 109, 110
16 CO 110 I=1, 14
17 J=LCC4(I)
18 K=K+1
19 THETB(J)=B(K)
20 CALL MULTS (PHI, PHIB, PHI, IP)
C PHI(I) IS THE MINUS VALUE OF THE AUTOEGRESSIVE PARAMETERS
C BY MULTIPLYING NON-SEASONAL AND SEASONAL PARAMETERS TOGETHER.
C
CALL MULTS (THETA, THETB, THETA, IC)
C THETA(I) IS THE MINUS VALUE OF THE MOVING AVERAGE PARAMETERS AFTER
C MULTIPLYING NON-SEASONAL AND SEASONAL PARAMETER.
C
CALL CALA(NOBB, IP, IQ)
23 CO 50 I=1, NOBB
24 F(I)=-A(I)
25 RETURN
END
SUBROUTINE CALA(N,IPID,IQ)

C SUBROUTINE CALA IS TO CALCULATE THE VALUE OF MODEL BY TIME SERIES.

C

COMMON ZD(1000,4,3),NGB(4,3),Z(1000),A(1000),CPHI(100),THETA(100),
 PHW(100),THETM(100),PHB(100),THETB(100),LOC1(10),LOC2(210),LCC3(10),LOC4(10),III1(1),II2(1),II3(1),II4(1),IIP(1),IIQ(1)

C FIX INITIAL VALUES

IF(IQ-IPID)201,201,200

200 ISTART=IQ

GO TO 203

201 ISTART=IPID

203 CC 204 I=1,ISTART

204 A(I)=C-

ISTART=ISTART+1

CC 206 I=ISTART,N

A(I)=Z(I)

IF(I=IQ)207,207,208

208 CC 209 J=1,IPID

209 A(I)=A(I)-CPHI(J)*Z(I-J)

207 IF(IQ) 206,206,211

211 CC 212 J=1,IQ

212 A(I)=A(I)+THETA(J)*A(I-J)

211 CONTINUE

RETURN

END
SUBROUTINE DIFFER(IDX, IDB, IS, N, MW, MB)

CCMMCN Z(1000, 4, 3), NOB(4, 3), C(1000), A(1000), PHI(100), THETA(100),
1 PHIW(100), THETW(100), PHIB(100), THETB(100), LOC1(10), LOC2(10),
2 LCC3(10), LCC4(10), III(1), II2(1), II3(1), II4(1), III(1), IIQ(1)

C CALCULATION OF NOB(M, J)

MW = IDX + 1
MO = IDB + 1
CO 151 M = 1, MW
CO 156 M = 1, MB

151 NOB(M, J) = N + 1 - M + IS - J * IS

C DIFFERENCING WITH RESPECT TO DM

IF(MM - 1) 152, 152, 153
153 DO 154 M = 2, MW
154 NOBC = NCB(M, 1)
155 CO 154 I = 1, NOBC
156 II1(1, M, 1) = II1(I + 1, M - 1, 1) - II1(I, M - 1, 1)

C DIFFERENCING WITH RESPECT TO DB

IF(PP - 1) 155, 155, 156
155 DO 157 J = 2, MB
158 NOCD = NCB(M, J)
159 CO 157 I = 1, NOCD
160 II1(1, M, J) = II1(I + M, M, J - 1) - II1(I, M, J - 1)

155 CONTINUE
156 RETURN
157 END
SUBROUTINE MULS (PHIW, PHIB, PHI, IPWPB)
C SUBROUTINE MULS IS TO MULTIPLY THE PARAMETERS OF NON-SEASONAL
C ANG SEASONAL MODELS.
C
DIMENSION PHIW(100), PHIB(100), PHI(100)

IF(IPWPB)101,101,105

105 DO 100 I=1,IPWPB
100 PHI(I)=PHIW(I)+PHIB(I) IF(IPWPB-1)101,101,102

102 DO 104 I=2,IPWPB

JDUM=I-1

104 DO 104 J=1,JDUM

PHI(I)=PHI(I)-PHIB(J)*PHIW(I-J)

101 CONTINUE

RETURN

END
SUBROUTINE UWHAUS(NPROB,
NCB,Y,NP,TH,DIFF,SIGNS,EPS1,EPS2,
1 IMIT, FLAM, FNU, SCRAT)

DIMENSION SCRAT(1)

DIMENSION Y(1),TH(1),DIFF(1),SIGNS(1)

IA=1

IB=IA+NP

IC=IB+NP

ID=IC+NP

IE=ID+NP

IF=IE+KP

IG=IF+NPB

IH=IG+NCP

II = IH + NP * NCB

IJ = IH

CALL HAUS59(NPROB,
NCB,Y,NP,TH,DIFF,SIGNS,EPS1,EPS2,MITU)

1 ,FLAM,FNU,SCRAT(IA), SCRAT(IB), SCRAT(IC), SCRAT(ID),

2 SCRAT(IE), SCRAT(IF), SCRAT(IG), SCRAT(IH), SCRAT(IJ),

3 SCRAT(IJ) }

RETURN

END
SUBROUTINE HAUSS9(NPRB0, NBC, Y,NQ,TH,DIFZ,SIGNS,EP15,EP25,UTHA,20
1*IT,FLAP,FNU, C,P,E,PHI,TE,F,R,A,D,DELZ)
C PROGRAM UHAUS IS USED IN CONJUNCTION WITH SUBROUTINES EST, CALA
C AND PLTS, AND PERFORMS THE OPERATION OF LOCATING THE LEAST SQUARES
C ESTIMATES IN AN ITERATIVE MANNER.
C PROGRAM UHAUS IS BASED ON A METHOD DUE TO MARQUARDT WHICH COMBINES
C THE GAUSS-TAYLOR SERIES METHOD AND THE METHOD OF STEEPEST ASCENT.
C UHAUS PRINTS OUT SUMMARY INFORMATION AT EACH ITERATION WHICH CAN BE
C ANALYZED TO DETERMINE THE PATH IN THE PARAMETER SPACE
C TAKEN BY THE ITERATION TO CONVERGE ON THE LEAST SQUARE ESTIMATES.
C
FORTRAN II VERSION
H. J. WERTZ
ADAPTED FOR THE UNIVAC 1108 (HJW 12/68)
C
DIMENSION TH(NQ), DIFZ(NQ), SIGNS(NQ), Y(NBO)
C DIMENSION C(NQ), P(NQ), E(NQ), PHI(NQ), TB(NQ)
C DIMENSION F(NBO), R(NBO)
C DIMENSION A(NQ,NQ), C(NQ,NQ), DELZ(NBO,NQ)
C DIMENSION TH11,DIFZ1, SIGNS11,Y11,C11,P11,E11
C DIMENSION PHI11,TE11,F11,R11,A11,C11,DELZ1
C DIMENSION CO12
C
AGOSI=F-ATAN(SQRT(1-x2-1-0))
AP = A
APB = NPRB0
NCB = NBC
EPS1 = EP15
EPS2 = EP25
PRINT 1000, NPRB0,NBO,AP
PRINT 1001
CALL GASS60(1,APB,TH,00)
PRINT 1002
CALL GASS60(1,APB,DIFZ,00)
C TEST INPUT VALUE
IF(MIN(NP-1,0.5-NP,NCB-NP,NIT-1,999-NIT))999,15,15
CONTINUE
GO 10 1=1, NP
C TEMP = ABS(DIFZ1)
IF(EPMIN11-0.2-TEMP ABS(TH11))999,99,19
C CONTINUE
GO 15
C GA = FLAP
NIT = 1
MAY=1
MAY=1
LUCY=1
C EPS1 CANNOT BE NEGATIVE
C EPS2 LESS THAN OR EQUAL TO ZERO, EPS1 GREATER THAN ZERO, THEN MAY=3
C EPS2 GREATER THAN ZERO, EPS1 LESS THAN OR EQUAL TO ZERO, THEN MAY=2
C EPS2 GREATER THAN ZERO, EPS1 GREATER THAN ZERO
C EPS2 GREATER THAN ZERO, EPS1 LESS THAN OR EQUAL TO ZERO, THEN LUCY=2
C SQ=SLV*CF SQUARE CF RESIDUAL
IF(EPS11 5, 1, 10
EPS1 = 0
IF(EPS2) 40, 40, 30
IF(EPS1) 60, 60, 50
MAY=2
GO TO 70

(Continues on next page)
50 MAY=3          UHWA 63
GO TO 70         UHWA 64
30 IF (EPSL*100 .GE. 80, 80, 70)   UHWA 65
80 LUCY=2    UHWA 66
70 SSC = 0    UHWA 67
CALL PMDEL(INPBO, TH, F, NOB, NP)   UHWA 68
50040 CQ 90 I = 1, NOB     UHWA 69
R(I) = Y(I) - F(I)     UHWA 70
90 SSC = SSC + R(I) * R(I)     UHWA 71
CPRINT 1003, SQQ     UHWA 72
C
BEGIN ITERATION
C
100 GA = GA / FNU     UHWA 73
00444 INTCNT = 0     UHWA 74
PRINT 1004, NIT     UHWA 75
101 JS = 1 - NOB     UHWA 76
00446 130 J = 1, NP     UHWA 77
00478 TEMP = TH(J)     UHWA 78
00479 PH(J) = OFZ(J) * TH(J)     UHWA 79
00489 TH(J) = TH(J) + PH(J)     UHWA 80
00499 C(J) = 0     UHWA 81
00505 JS = JS + NOB     UHWA 82
00511 CALL PMCEL(INPBO, TH, DELZ(JS), NOB, NP)     UHWA 83
C
DELZ IS THE NEW PREDICTED FUNCTION VALUE THROUGH THE MODIFIED PARAMETERS
C
00521 IJ = JS - 1     UHWA 84
00529 CQ 120 I = 1, NOB     UHWA 85
00535 IJ = IJ + 1     UHWA 86
00541 DELZ(J) = DELZ(J) + F(I)     UHWA 87
00551 120 C(J) = C(J) + DELZ(J) * R(I)     UHWA 88
00559 C(J) = C(J) / P(J)     UHWA 89
C
Q = XT*R (STEEPEST DESCENT)     UHWA 90
00619 TH(J) = TEMP     UHWA 91
130 GO TO 131, 414, MARY     UHWA 92
00628 GC TC(131, 141), MARY     UHWA 93
00638 131 CO 150 I = 1, NP     UHWA 94
00649 CO 151 J = 1, I     UHWA 95
00659 SUM = 0     UHWA 96
01659 WJ = WCB(J + 1)     UHWA 97
00669 KI = NCB(I - 1)     UHWA 98
00679 CO 160 K = 1, NOB     UHWA 99
00689 KI = KI + 1     UHWA 100
00695 KJ = KJ + 1     UHWA 101
00701 SUM = SUM + DELZ(KJ) * DELZ(KJ)     UHWA 102
00719 TEMP = SUM / (P(I) * P(J))     UHWA 103
00729 JI = J + NP*(I - 1)     UHWA 104
00739 C(J) = TEMP     UHWA 105
00749 IJ = I + NP*(J - 1)     UHWA 106
00759 C(IJ) = TEMP     UHWA 107
00769 151 E(IJ) = SCRTD(D(IJ))     UHWA 108
00779 150 CONTINUE     UHWA 109
00789 666 CONTINUE     UHWA 110
00799 CO 153 I = 1, NP     UHWA 111
00809 IJ = I - NP     UHWA 112
00819 CO 153 J = 1, I     UHWA 113
00829 IJ = I + NP     UHWA 114
00839 A(IJ) = D(IJ) / (EI(J)**2)     UHWA 115
00849 EI = J + NP*(I - 1)     UHWA 116
00859 AIJ = A(IJ)     UHWA 117
C
A = SCALED MOMENT MATRIX     UHWA 118
258
ILLEGIBLE

THE FOLLOWING DOCUMENT (S) IS ILLEGIBLE DUE TO THE PRINTING ON THE ORIGINAL BEING CUT OFF

ILLEGIBLE
FORTRAN IV G LEVEL 18  

II = - NP
CO 155 I=1,NP
P(I)=C(I)/E(I)
PHI(I)=P(I)
II = NP + 1 + II
A(I) = A(I) + GA
C
I=1
CALL MATINI(A,NP,P,I,DET)
P/E = CORRECTION VECTOR
C
STEP=1.0
SUM1=0.
SUM2=0.
SUM3=0.
CC 231 I=1,NP
SUM1=P[I]*PHI[I]+SUM1
SUM2=P[I]*PHI[I]+SUM2
SUM3 = PHI[I] * PHI[I] + SUM3
231 PHI(I) = P(I)
TEMP = SUM1/QRAT(SUM2*SUM3)
TEMP = AMIN1(TEMP, 1.0)
TEMP = 57.295*ACOS(TEMP)
PRINT 1041, DET, TEMP
CO 220 I = 1, NP
PRINT 1042,TB(I), TEMP
CONTINUE
PRINT 7000
CC COPY FIXED POINT PARAMETER VALUES
PRINT 2006, (TB(I), I = 1, NP)
CO 221 I = 1, NP
IF(SIGN(S(I)) 221, 221, 222
222 IF(SIGN(1.0,TH(I)*SIGN(1.0,TB(I)))) 663, 221, 221
221 CONTINUE
CALL MODELINPROB, TB, F, NOP, NP)
CO 130 I=1,NOP
R(I)=Y(I)-F(I)
230 SIMB=SUMB+R(I)*R(I)
PRINT 1043, SUMB
IF(SUMB - (1.0+EPS1)*SSQ) 662, 662, 663
663 IF(A MIN1(TEMP-30.0, GA)) 665, 665, 664
STEP=STEP/2.0
INTCNT = INTCNT + 1
IF(INTCNT = 36) 170, 2700, 2700
GA=GA*FNU
INTCNT = INTCNT + 1
IF(INTCNT = 36) 666, 2700, 2700
PRINT 1007
CO 133 IC 669 I=1,NP
669 TH(I)=TB(I)
CALL GASS60(I, NP, TH, 00)
PRINT 1040, GA, SUMB
GC TC (225,270,265), MAY
CO 225 I = 1, NP
IF(ABS(P(I))>1.0-E-20+ABS(TH(I)))*EPS2) 240, 240, 241
241 GC TC (265,270), LUCY
240 CONTINUE
141
0142 PRINT 1009, EPS2
0143 GO TO 280
0144 265 IF (ABS(SUMB - SSQ) - EPS1*SSQ) 266, 266, 270
0145 266 PRINT 1010, EPS1
0146 GO TO 280
0147 270 SSQ = SUMB
0148 NIT = NIT + 1
0149 IF (NIT = M1T) 100, 100, 280
0150 270 GO PRINT 2710
0151 2710 FORMAT (/145H0** The sum of squares cannot be reduced to the sum
1CF squares at the end of the last iteration - iterating stops /*UHWA188

C

END ITERATION

C

280 PRINT 1011
0152 PRINT 2001, (F(I), I = 1, NOB)
0153 PRINT 1012
0154 PRINT 2001, (R(I), I = 1, NOB)
0155 SSC = SUMB
0156 IDF = NC8 - NP
0157 I = 0
0158 CALL MATRINT(D, NP, P, I, DET)
0159 DO 7692 I = 1, NP
0160 II = I + NP*(I-1)
0161 7692 E(I) = SQRT(D(II))
0162 DO 340 I = 1, NP
0163 JI = I + NP*(I-1) - 1
0164 340 DJ = I + NP*(I-2)
0165 DO 340 J = I, NP
0166 340 DJ = J + 1
0167 IF (J = I) 48(1)(1) = DI/JJ / (E(I)*E(I))
0168 IF (J = I) 48(1) = DI/JJ
0169 48(1)(1) = A(IJ) = A(IJ)
0170 340 A(IJ) = A(IJ)
0171 PRINT 1016
0172 CALL GASS60(1, NP, E, GO)
0173 IF (IDF) 341, 410, 341
0174 341 SDEV = SSQ / IDF
0175 PRINT 1014, SDEV, IDF
0176 SDEV = SQRT(SDEV)
0177 DO 391 I = 1, NP
0178 P(I) = TH(I)*Z20*E(I)*SDEV
0179 391 TB(I) = TH(I)*Z20*E(I)*SDEV
0180 PRINT 1039
0181 CALL GASS60(2, NP, TB, P)
0182 MARY = 2
0183 GO TO 101
0184 414 CC 415 K = 1, NOB
0185 TEMP = 0
0186 CC 420 I = 1, NP
0187 CC 420 J = 1, NP
0188 ISLB = K + NC8*(I-1)
0189 C CBUG1 = DELZ(ISUB)
0190 C CBUG1 = DELZ(K + NOB*(I-1))
0191 ISLB = K + NC8*(J-1)
0192 C CBUG2 = DELZ(ISUB)
0193 C CBUG2 = DELZ(K + NOB*(J-1))
0194 LI = I + NP*(J-1)
0195 C DEBUG3 = D(IJ) / (DIFZ(I)*TH(I)*CIFZ(J)*TH(J))
0194  420  TEMP = TEMP + DEBUG1 * DEBUG2 * DEBUG3
0195      TEMP = 2.0*SCRT(TEMP1)*SDEV
0196
0197  R(K)=F(K)+TEMP
0198
0199     PRINT 1008
0200     IE=0
0201     ID = 425, I=1,NGB,10
0202     IF(GB*IE) 430,435,435
0203  430  IE=NGB
0204
0205  435  PRINT 2001, (RIJ), J = 1, IE
0206
0207  425  PRINT 2006, (F(IJ), J = 1, IE
0208
0209  410  PRINT 1033, NPROB
0210  99  RETURN
0211
0212  GO TO 410
0213
0214  1000 FORMAT(33HINON-LINEAR ESTIMATION, PROBLEM NUMBER I3,// I5,
0215      1 14H OBSERVATIONS, I5, 11H PARAMETERS I14, 17H SCRATCH REQUIRED)
0216
0217  1CC1 FORMAT(/25H INITIAL PARAMETER VALUES )
0218
0219  1CC2 FORMAT(/54H PROPORTIONS USED IN CALCULATING DIFFERENCE QUOTIENTS
0220      1CC3 FORMAT(/15H INITIAL SUM OF SQUARES = E12.4)
0221  1CC4 FORMAT(/75HITERATION NO. I4)
0222  1CC7 FORMAT(/32H PARAMETER VALUES VIA REGRESSION )
0223
0224  1CD8 FORMAT(/54H APPROXIMATE CONFIDENCE LIMITS FOR EACH FUNCTION VALUE
0225      1LE )
0226
0227  1CC5 FORMAT(/72H ITERATION STOPS - RELATIVE CHANGE IN EACH PARAMETER LEUHA 263
0228      1SS THAN E12.4)
0229
0230  1CC4 FORMAT(/72H ITERATION STOPS - RELATIVE CHANGE IN SUM OF SQUARES LEUHA 265
0231      1SS THAN E12.4)
0232
0233  1CL1 FORMAT(/22H FINAL FUNCTION VALUES )
0234  1CL2 FORMAT(/10H RESIDUALS )
0235
0236  1CL4 FORMAT(/24H HOVARIANCE OF RESIDUALS = E12.4, I1, I4,
0237      1CL5 DEGREES OF FREEDOM )
0238  1CL6 FORMAT(/21H NORMALIZING ELEMENTS )
0239  1CL3 FORMAT(/19H END OF PROBLEM NO. I3)
0240  1CL4 FORMAT(/16H PARAMETER ERROR )
0241  1CL5 FORMAT(/71H INDIVIDUAL CONFIDENCE LIMITS FOR EACH PARAMETER ON LINE
0242      1INEAR HYPOTHESIS )
0243
0244  1OC9 FORMAT(/90H LAMBDA = E10.3, 40X, 33H SUM OF SQUARES AFTER REGRESSION =
0245      1CE15.7)
0246
0247  1OC4 FORMAT(/14H DETERMINANT = E12.4, 6X, 25H ANGLE IN SCALED COORDS =
0248      1F5.2, 8H DEGREES )
0249
0250  1OC3 FORMAT/60H TEST POINT SUM OF SQUARES = E12.4)
0251
0252  2CC1 FORMAT/10E12.4)
0253
0254  2CC6 FORMAT/10E12.4)
SUBROUTINE MATIN(A, NVAR, B, NB, DET)

DIMENSION A(NVAR,1), B(NVAR,1)

CCMPOK/GASPAR/DUMIES(7), PIVOTM

PIVOTM = A(1,1)

DET = 1.0

CC 550 ICOL = 1, NVAR

PIVOT = A(ICOL, ICOL)

PIVOTM = AMIN1(PIVCT, PIVOTM)

DET = PIVOT * DET

C

DIVIDE PIVOT ROW BY PIVOT ELEMENT

C

A(ICCL, ICCL) = 1.0

C

PIVCT = AMAX1(PIVCT, 1.E-20)

PIVCT = A(ICCL, ICOL)/PIVCT

CC 350 L=1, NVAR

ATICCL, L) = A(ICCL, L)*PIVCT

IF(NB .EQ. 0) GO TO 371

CC 370 L=1,NB

B(ICOL, L) = B(ICOL, L)*PIVCT

C

REDUCE NON-PIVOT ROWS

C

CC 550 L=1,NVAR

C F(L1 .EQ. ICOL) GC TO 550

T = A(L1, ICOL)

A(L1, ICOL) = 0.

CC 450 L=1,NVAR

45C A(L1, L) = A(L1, L) - A(ICOL, L)*T

IF(NB .EQ. 0) GO TO 550

CC 550 L=1,NB

50C B(L1, L) = B(L1, L) - B(ICOL, L)*T

CONTINUE

RETURN

END
SLERCUTINE GASS60 (IETYPE, NO, A, B)
DIMENSION A(NO), B(NO)
NP = NO
NR = NP/10
LCW = 1
LUP = 10
IF (NR .GE. LUP) RETURN
RETURN
LI RETURN
LUP = NP
IF (LCW .GT. LUP) RETURN
PRINT 500, (J, J = LOW, LUP)
GO TO (40, 60), ITYPE
PRINT 6CC, (A(J), J = LOW, LUP)
GO TO 100
PRINT 600, (B(J), J = LOW, LUP)
GO TO 40
LCW = LCW + 10
LUP = LUP + 10
NR = NR - 1
GO TO 10
FORMAT (/I8, 9I12)
FORMAT (10E12.4)
END
APPENDIX C

PROGRAM FORCAT

C.1. Description of program

Program FORCAT is developed to provide the forecast value and its confidence interval for the appropriate model of the time series, which may be stationary, nonstationary or/and seasonal. The program consists of a main program and seven subroutines. The functions of subroutines are as follows.

MULTS and EXPAND: convert the general seasonal nonstationary model into regular ARMA(p,q) forms. The form is used to calculate the \( \pi \) weights (pure AR weights), \( \psi \) weights (pure MA weights), confidence intervals of forecast values for the original series and the reduced stationary model.

CALPSI : calculate the \( \psi \) weights for the original model and for the reduced stationary model. The calculations are based on equation (4.2.6)

CALPIE : calculates the pure AR weights for the original model and for the reduced stationary model. The calculation is based on equation (C.1.4). For pure moving-average model, its form may be

\[
\tilde{z} = \psi(B) a_t
\]  \hspace{1cm} (C.1.1)

where

\[
\psi(B) = 1 + \psi_1 B + \psi_2 B^2 + \ldots
\]
For pure autoregressive model, its form may be

\[ \pi(B) \hat{z}_t = a_t \]  \hspace{1cm} (C.1.2)

where \( \pi(B) = 1 - \pi_1 B - \pi_2 B^2 - \ldots \)

comparing (C.1.1) with (C.1.2), we obtain

\[ \pi(B) \psi(B) = 1 \]  \hspace{1cm} (C.1.3)

equating the power of B in (C.1.3), getting

\[ \pi_1 = \psi_1 \]

\[ \pi_2 = \psi_2 - \psi_1 \pi_1 \]

\[ \pi_3 = \psi_3 - \psi_2 \pi_1 - \psi_1 \pi_2 \]

\[ \vdots \]

or in more general equation

\[ \pi_i = \psi_i - \sum_{j=1}^{i-1} \psi_j \pi_{i-j} \]  \hspace{1cm} (C.1.4)

hence, the \( \pi \) weights of pure AR model can be calculated recursively.

**FORCAT**
: calculate the forecasts and confidence interval

for the original time series. Its calculation is based on equations (4.1.2) and (4.2.7).

**CALA**
: calculate the residuals of the original time series.

**PLOT**
: plot the forecast values and its confidence intervals,
START

READ IN DATA
1. No. of Forecasts to be Calculated
2. Model Form to be Analyzed

Print out the Model Form

Read in the Parameters of the Appropriate Model

Call Subroutine Mults --
Multiples the Parameters of Seasonal Nonstationary Model of AR Process. The Operation is Skipped if the Model is Non-AR Process.

Subroutine Mults --
Multiple the Parameters of Seasonal Nonstationary Model of MA Process. The Operation is Skipped if the Model is Non MA Process.

Subroutine Expand --
Compute the Parameters Associated with Powers of Backward Shift Operator About AR Process

A
Subroutine Expand --
Compute the MA Parameters
Associated with the Powers
of The Backward Shift Operator if Seasonal Time Series is
Analyzed

Subroutine Mults --
Multiple the Parameters
Associated with the Backward
Shift Operator of AR Process
about the Nonstationary and/
or Seasonal Model

Subroutine Mults --
Multiple the AR Parameters
with Parameters Purely Associated with Backward Shift Operator
to Obtain General AR Parameters

Does the Model
Contain the AR
Process

Yes

Print AR Parameter PHI(I)

No

B
B

Does the Model Contain the MA Process

No

Yes

Print out MA Parameters

THETA(I)

Does the Model Contain the AR Parameters with Nonstationary and/or Seasonal Difference

No

Yes

Print General AR Parameters

Subroutine CALPSI -- Calculate $\psi$ Weights, PSI1(I)

Subroutine CALPIE -- Calculate TL Weights, PIE1(I)

C
C

Print \( \psi \) Weights and \( \pi \) Weights

Read in Data:
The no. of Observations,
The Time Series Observations

Subroutine CALA --
Calculate the Residuals of
the Time Series

Subroutine Plot --
Plot the time
Series Process,
Forecast Value,
Its Confidence
Interval

Subroutine FORCAT --
Calculate the Forecast
Values and its Confidence
Interval

STOP
C.2. Description of input data

<table>
<thead>
<tr>
<th>card</th>
<th>variable in program</th>
<th>FORMAT</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>first card</td>
<td>IFOR</td>
<td>(I10)</td>
<td>number of forecasts to be calculated</td>
</tr>
</tbody>
</table>

next block of cards

same as the parameter input block given in Appendix B for program ESTIM

<table>
<thead>
<tr>
<th>next card</th>
<th>MAX</th>
<th>(I10)</th>
<th>number of correlation to be calculated</th>
</tr>
</thead>
<tbody>
<tr>
<td>next card</td>
<td>N</td>
<td>(I10)</td>
<td>the number of observations to be read in.</td>
</tr>
<tr>
<td>next cards</td>
<td>Z(I,1)</td>
<td>(2X,(F20.5))</td>
<td>the observations of the time series.</td>
</tr>
</tbody>
</table>

C.3. Description of output data

The following data can be generated by program FORCAT;

1. Print out parameters and model form.
2. Pure autoregressive and pure moving-average weights of the original and reduced stationary form of the model.
3. Forecast values and confidence limits.
4. Plot out the forecast values and confidence limits.
APPENDIX C. PROGRAM FORCAT

C. 4 Computer Program
FORTRAN IV G LEVEL 18 MAIN CATE = 71113 03/42/28

0001 DIMENSION PHI(200),THETW(200),PHIB(200),THETB(200)
0002 DIMENSION PHI(200),THETA(200),C(200)
0003 DIMENSION 2(1500,1),A(1500)
0004 DIMENSION PI(200),PIE1(200),PSI(200),PSI1(200)
0005 DIMENSION CI(200),C2(200),C1(200)
0006 CCMP=CA/1/PHI/2/THETW/3/PHIB/4/THETB/5/PHI/6/THETA/7/C
0007 CCM=CA/8/4/PA/9/PIE/10/PIE1/11/PIE1/12/PSI/13/PSI1/14/RHO
0008 CCM=CA/15/RHO
0009 CCM=CA/16/TREND

C INSERT TO READ IN DATA AND PRINT OUT

0010 250 FORMAT(110)
C MAX NUMBER OF CORRELATIONS AND PARTIAL TO BE CALCULATED

0011 READ 250,MAX
C PROGRAM PARAMETERS
C END PROGRAM PARAMETERS
C READ IN MODEL FROM PARAMETERS

0012 2CC2 FORMAT(4110)
0013 READ 2002,IPW,IDW,IQW
0014 READ 2002,IPR,IDB,IQB,IS
C END READ IN MODEL FORM PARAMETERS
C READ IN PARAMETERS

0015 2CC3 FORMAT(12L2X,6H10RARMA(I3,1H,,I3,1H,,I3,1H,,I3,3H)X
1(I3,1H,,)X
0016 PRINT 2003,IPW,IDW,IQW,IPB,IDB,IQB,IS
0017 IP=IP+W+IPB
0018 IP=IP+W+IPB
0019 IDS=IDB+IS
0020 IFD=IP+IDW+IDS
0021 IG=IG+W+IDB
0022 CC 2000 I=1,100
0023 PHW(I)=0.
0024 PHIB(I)=0.
0025 PHI(I)=0.
0026 THETW(I)=0.
0027 THETB(I)=0.
0028 THETA(I)=0.
0029 CPH(I)=0.
0030 CI(I)=0.
0031 C2(I)=0.

2050 C(I)=0.

2CC4 FORMAT(10,F20.5)
0033 2CC5 FORMAT(2X,5PHW(I3,2H)=,F11.5)
0035 2CC7 FORMAT(2X,5PHIB(I3,2H)=,F11.5)
0036 2CC8 FORMAT(2X,6THETW(I3,2H)=,F11.5)
0037 2CC8 FORMAT(2X,6THETB(I3,2H)=,F11.5)
0038 2CC7 FORMAT(2X,6TREND(I3,2H)=,F11.5)
0039 READ 2002,23,24,25,16
0040 IF(10)2009,2009,2010
0041 2010 CC 2011 I=1,11
0042 READ 2004,J,PHIW(J)
0043 2011 PRINT 2005,J,PHIW(J)
0045 2013 CC 2014 I=1,12
0046 READ 2004,J,THETW(J)
0047 2014 PRINT 2CC5,J,THETW(J)
0049 2016 CC 2017 I=1,13
0050 READ 2004,J,PHIB(J)
FORTAN IV G LEVEL 18  MAIN  DATE = 71113  03/42/20

0051  2017 PRINT 2007, J, PHIB(J)
0052  2015 IF(IP(14), 2025, 2025, 2019)
0053  2019 EQ 2020 1, 1, 14
0054  REAC 2004, J, THETB(J)
0055  202C PRINT 2008, J, THETB(J)
0056  2025 CONTINUE
0057  C  ENC REACT IN PARAMETERS
0058  C  OPERATIONS TO REDUCE SEASONAL TC NCN SEASONAL MODEL
0059  C  CALL MULTS (PHIB, PHI1, PHI2, IP)
0060  C  CALL MULTS (THETB, THETB, THETA, IQ)
0061  C  CALL EXPANDC1, ICW, I1
0062  C  CALL EXPANDC2, ID9, I5
0063  C  IDC=IC5+IDW
0064  C  CALL MULTS (C1, C2, C, IDC)
0065  C  CALL MULTS (PHI, C, PHI, IQ)
0066  C  ENC OPERATIONS
0067  2021 FORMAT(//'2X, 2THPHI, THETA, AND CPHI VALUE//'
0068  2021 PRINT 2021
0069  107 FORMAT(2X, 6H PHII, I3, 2H)=, F10.5
0070  108 FORMAT(2X, 6HTHETA, I3, 2H)=, F10.5
0071  IF(IP(11), 109, 110)
0072  110 EQ 111 I=1, 1, IP
0073  111 PRINT 107, I, PHII(I)
0074  112  CONTINUE
0075  115 FORMAT(///'2X, 33H GENERAL AUTOREGRESSIVE PARAMETERS///
0076  115 PRINT 115
0077  116 FORMAT(2X, 6HCPHI, I3, 2H)=, F10.5
0078  IF(IP(10), 117, 110)
0079  117 IF(116, I, CPHII(I))
0080  118  CONTINUE
0081  C  CALCULATION OF PSI WEIGHTS
0082  116 IF(IP(10), 4, 7000, 7000, 7001)
0083  70CC IF(IP(9)-2, 7002, 7002, 7001)
0084  7002 KGZ=MAX
0085  7003 KGZ TO 7003
0086  70C1 KGZ=200
0087  70C2 CONTINUE
0088  70CC C  TC GET PURE MOVING AVERAGE PARAMETERS PSI FOR STATIONARY TIME SERIES
0089  70C CALL CALPSI(PHI, THETA, PSI, IP, IC, KGZ)
0090  C  TC GET PURE AUTOREGRESSIVE PARAMETERS PIE FOR STATIONARY TIME SERIES
0091  C  CALL CALPIE(PSI, PIE, MAX)
0092  300 FORMAT(///'2X, 4HPSI AND PIE WEIGHTS FOR STATIONARY SERIES///
0093  300 PRINT 300
0094  301 FORMAT(2X, 4HPSi, I3, 2H)=, F10.6, 7X, 4HPIE, I3, 2H)=, F10.6
0095  CC 303 I=1, MAX
0096  303 PRINT 302, I, PSI(I), PIE(I)
0097  C  IF IPIC=IP, MEANS NO ANY DIFFERENCES AND SEASONAL DIFFERENCE, SO WE CAN
0098  C  LCCK ORIGINAL SERIES AS STATIONARY TIME SERIES.
0099  C  TC GET PURE MOVING AVERAGE PARAMETERS PSI FOR ORIGINAL SERIES
0100  501 CALL CALPSI(PHI, THETA, PSI, IP, IQ, MAX)
0101  C  TC GET PURE AUTOREGRESSIVE PARAMETERS FOR ORIGINAL SERIES
CALL CALPIE(PSI, PIE, MAX)
5002 FORMAT(////2X,3(9HPSI AND PIE WEIGHTS FOR ORIGINAL SERIES///)
PRINT 5002
CC 5003 I=1,MAX
5003 PRINT 302, I, PSI(I), I, PIE(I)
GO TO 5006
5004 CC DO 5005 I=1,MAX
5005 PSI(I)=PSI1(I)
CONTINUE
251 FORMAT(2X,F20.5)
IFCR NC OF FORECASTS TO BE CALCULATED, MAXIMUM FORECAST LAG
READ 250, IFOR
READ 250, N
READ 251, (Z(I,1), I=1,N)
CALL TC GENERATE A(I)
CALL CALA(N, IPID, IG, SIGMA)
C FORECASTING
CALL TC GENERATE FORECASTS AND HD INTERVAL
CALL FORCAT(N, IPID, IG, IFOR, SIGMA)
C END FORECASTING
STOP
END
SUBRoutines CALPSI (CHPI, THETA, PSI, IPP, IG, MAX)

C SUBRoutines CALPSI IS TO CALCULATE THE PURE MOVING AVERAGE WEIGHTS
C FOR THE MODEL

DIMENSION CHPI(200), THETA(200), PSI(200)

C FIX INITIAL VALUES SO THAT IPP+IG=IPSI

IF(IPP+IG).GT.100,101,102
102 IDUM=IPP+1
103 DO 104 I=IDUM,1Q
104 CHPI(I)=0
105 DO 106 I= IPP+1, IPPL, 1
106 CHPI(I)=1
107 GO TO 110
108 IDUM=IDUM,1P
109 THETA(I)=0
110 CONTINUE
111 IPSI=IPP
112 CONTINUE

C ENC FIX
C CALCULATION OF PSI(I), PSI(I), PSI(I)

113 DO 114 I=1,IPSI
114 CHI(I)=CHPI(I)-THETA(I)
115 DO 116 I=2,IPSI
116 CHI(I)=CHI(I)-CHPI(I-1)
117 DO 118 I=1,IPSI
118 PSI(I)=PSI(I)+CHPI(I)*PSI(I-1)
119 CONTINUE

C ENC CALCULATION
C CALCULATION OF PSI(IPSI), PSI(IPSI), PSI(MAX)

120 DO 121 I=IPSI+1, MAX
121 PSI(I)=0
122 DO 123 I=1,IPSI
123 PSI(I)=PSI(I)+CHPI(I)*PSI(I-1)
124 CONTINUE

RETURN
END
SUBROUTINE CALPIE(PSI, PIE, MAX)

DIMENSION PSI(200), PIE(200)

CC 100 I=1, MAX

100 PIE(I)=PSI(I)

CC 101 I=2, MAX

JCUM=I-1

CC 101 J=1, JCUM

0008 PIE(I)=PIE(I)-PSI(J)*PIE(I-J)

101 CONTINUE

0010 RETURN

0C11 END
SUBROUTINE CALA(N, IPID, IQ, SIGMA)
DIMENSION THETA(200), CPHI(200), Z(I500,1), A(I500)
C COPHON/A8/Z, A/A6/THETA/A7/CPHI

C FIX INITIAL VALUES
IF(IQ-IPID)201,201,200
ISTART=IQ
GO TO 203
C1 ISTART=IPID
C0 C0 204 I=1, ISTART
C0 204 A(I)=0.
ISTART=ISTART+1
DO 206 I=ISTART, N
A(I)=Z(I,1)
IF(IPI=207,207,208
DO 206 J=1, IPID
A(I)=A(I)-CPHI(J)*Z(I-J,1)
207 IF(IQ) 206, 206, 211
208 DO 211 J=1, IQ
A(I)=A(I)+THETA(J)*A(I-J)
209 CONTINUE
210 FORMAT(1H12X,27HPRINT OUT OF CALA AND ERROR/)2
211 PRINT 210
212 FORMAT(2X,5HTIME=, I4, 2X, 2HZ(I, I4, 2H)=, F20.5, 5X, 2HA(I, I4, 2H)=, F20.5)
213 SIGMA=0.
DO 218 I=1, N
PRINT 217, I, I, Z(I,1), I, A(I)
SIGMA=SIGMA+A(I)*A(I)
CONTINUE
216 SIGMA=SQRT(SIGMA/N)
RETURN
END
SUBROUTINE FORCAT(IT,IPID,IPQ,IFCR,SIGMA)
DIMENSION H(1000)
DIMENSION G(1000)
DIMENSION CPHI(2000),THETA(2000),PSI(2000),Z(1500,1),A(1500)
COMMON/A7/CPHI/A6/THETA/A12/PSI/A27/Z,A/A9/X,P

100 FORMAT(1H12X,48HFORECASTS AND 95 PER CENT LIMITS FOR BASE TIME =,I
14///)

PRINT 100,IT
101 X = Z(1,1)
500 CONTINUE
ITI = IT + 1
ITFCR = IT + IFCR
102 X = Z(1,1)
K = 1
CO 102 I = ITI, IFCR
105 CO 102 J = 1/IPID
106 CO 102 K = 1/Q
107 CO 102 L = 1/THETA(K)*A(I-J+K)
108 CO 102 M = 1/ZHAT(I)
300 CONTINUE
103 CONTINUE
C CALCULATION OF UPPER AND LOWER 95 PER CENT POINTS
200 CO 200 I = 1, IFCR
201 CO 201 I = 2, IFCR
202 CO 202 J = 1, L
203 CO 203 K = 1, IFOR
204 CONTINUE
205 CO 205 I = 1, IFCR
206 CO 206 I = 2, IFCR
207 CO 207 J = 1, L
208 CO 208 K = 1, IFOR
209 CO 209 L = 1, I
210 CONTINUE
211 CO 211 I = 1, SIGMA
212 CO 212 I = 2, IFCR
213 CO 213 J = 1, IFOR
214 CONTINUE
215 CO 215 I = 1, IFCR
216 CO 216 I = 2, IFCR
217 CO 217 J = 1, L
218 CO 218 K = 1, IFOR
219 CONTINUE
220 CO 220 I = 1, IFCR
221 CO 221 I = 2, IFCR
222 CO 222 J = 1, L
223 CO 223 K = 1, IFOR
224 CONTINUE
225 CO 225 I = 1, IFCR
226 CO 226 I = 2, IFCR
227 CO 227 J = 1, L
228 CO 228 K = 1, IFOR
229 CONTINUE
230 CO 230 I = 1, IFCR
231 CO 231 I = 2, IFCR
232 CO 232 J = 1, L
233 CO 233 K = 1, IFOR
234 CO 234 I = 1, IFCR
235 CO 235 I = 2, IFCR
236 CO 236 J = 1, L
237 CO 237 K = 1, IFOR
238 CO 238 I = 1, IFCR
239 CO 239 I = 2, IFCR
240 CO 240 J = 1, L
241 CO 241 K = 1, IFOR
242 CO 242 I = 1, IFCR
C THIS LOOP IS TO SUPPLY DATA TO SUBROUTINE PLOT TO PRINT OUT
C THE FORECAST VALUE ZHAT(1), THEIR CORRESPONDING UPPER AND LOWER LIMITS
300 PRINT 300, IT
301 CONTINUE
CALL PLC(1,H,IFOR,4,IFOR,0)
CC 120 I=1,ITFOR
11=ITFOR+I
G(I)=I
12C C(I)=ZHAT(I)
CALL PLCT(1,C,ITFOR,2,ITFOR,0)
351 FORMAT(1H12X,18H1.96*5.0*(ZHAT(I))///)
PRINT 351
352 FORMAT(2X,2H1=,15,5X,5HU(L)=-,F20.5)
353 I=1,IFOR
C L(I) IS THE CONTROL LIMITS FOR CORRESPONDING FORECAST VALUE
PRINT 352,1,U(I)
RETURN
END
SUBROUTINE EXPAND(C, ID, IS)
C DIMENSION C(200)
IF(ID)250,250,251
251 CONTINUE
CO 120 I=1,IO
JA1=1
JA2=1
JA3=1
CO 140 J=1,ID
140 JA1=JA1*J
IF(ID-I)132,132,133
133 JDUM=ID-I
CO 141 J=1,JDUM
141 JA2=JA2*J
CO 142 J=1,LD
132 JDUM=JD-U
JA3=JA3*J
JJJ=(-1)**I
JJJ=JJJ*JA1/(JA2*JA3)
C(I*IS)=-JJJ
120 CONTINUE
250 CONTINUE
RETURN
END
SUBROUTINE MULTS(PHIW,PHIB,PHI,IPWPB)
DIMENSION PHIW(200),PHIB(200),PHI(200)

IF(IPWPB)101,101,105
105 CC 100 I=1,IPWPB
100 PHI(I)=PHIW(I)+PHIB(I)

IF(IPWPB-1)101,101,102
102 CC 104 I=2,IPWPB
104 PHI(I)=PHI(I)-PHIB(J)*PHIW(I-J)
101 CONTINUE
RETURN
END
SUBROUTINE PLOT(NO,A,N,M,NL,NS)

C DIMENSION OUT(101),YPR111,ANG(9),A(1)

1 FCMAT(1H1,60X,7H CHART,13,//)
2 FCMAT(1H,F11.4,5H+,101A1)
4 FCMAT(1OH,123456789)
5 FCMAT(1OA1)
7 FCMAT(1H,16X,101H+)
1 FCMAT(1HO,9X,11F10.4///)

2CC FCMAT(10X,*,PLOT OF FORCAST VALUE*)
2C1 FCMAT(10X,*,PLOT OF AUTO CCRR- FUNCTION*)
2C2 FCMAT(10X,*,PLOT OF SPECTRUM*)

C ALL=NL
C IF(NS)16,16,10
C $CRT BASE VARIABLE IN ASCENDING ORDER

10 DO 15 I=1,N
15 CC 14 J=1,N
IF(A(I)=A(J))14,14,11
11 L=I-N
16 LL=J-N
17 CC 12 K=1,M
18 L=L+N
19 LL=LL+N
20 F=A(L)
21 A(LL)=A(LL)
22 12 ALLL=F
23 14 CONTINUE
24 15 CC CONTINUE
C TEST NLL
C 16 IF(NLL)20,18,20
18 ALL=50
C PRINT TITLE
C 20 WRITE(3,1)NO
21 CC GO TO (91,92,93),NO
22 91 WRITE(3,200)
23 CC GO TO 21
24 92 WRITE(3,201)
25 CC GO TO 21
26 93 WRITE(3,202)
27 21 CONTINUE
C DEVELOP BLANKS AND DIGITS FOR PRINTING
C
28 CC REWIND 4
29 WRITE(4,4)
30 CC REWIND 4
READ(4,5)BLANK,(ANG(I),I=1,9)
        REWINC 4
        C
        FIND SCALE FOR BASE VARIABLE
        XSCAL=(A(I)-A(1))/FLOAT(NLL-1))
        C
        FIND SCALE FOR CROSSED VARIABLES
        #1=I+1
        YMIN=A(M1)
        YMAX=YMIN
        M2=IM
        DO 40 J=1,M2
        IF(A(J)-YMIN)28,28,26
        26 IF(A(J)-YMAX)40,40,30
        28 YMIN=A(J)
        30 YMAX=A(J)
        CONTINUE
        40
        YSCAL=(YMAX-YMIN)/100.0
        C
        FIND BASE VARIABLE PRINT POSITION
        XB=A(1)
        L=1
        MY=I-1
        I=1
        F=I-1
        XPR=XB+F*XSCAL
        IF(F(I)-XPR)51,51,70
        C
        FIND CROSSED VARIABLES
        51 CC 55 IX=1,101
        55 CUT(I)=BLANK
        57 CC 60 J=1,MY
        LL=L+J*CN
        JP=((ALL)-YMIN)/YSCAL)+1.0
        60 CONTINUE
        C
        PRINT LINE AND CLEAR, OR SKIP
        WRITE(3,2)XPR,(OUT(I),I=1,101)
        L=L+1
        GO TO 80
        20 I=I+1
        80 IF(I-ALL)45,84,86
        84 XPR=A(N)
        GO TO 51
        C
        PRINT CROSSED VARIABLES NUMBERS
        86 WRITE(3,7)
        YPR(I)=YMIN
        90 G0 KN=1,9
        75 YPR(KN+1)=YPR(KN)+YSCAL*10.0
APPLICATION OF GENERAL AUTOREGRESSIVE MOVING-AVERAGE STOCHASTIC MODELS TO TIME SERIES AND SIMULATION PROBLEM

by

Steven Shyan-Ming Ko

B.S., Taiwan Cheng-Kung University, 1966

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AN ABSTRACT OF A MASTER'S THESIS

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MASTER OF SCIENCE

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This thesis is concerned with the development of a stochastic model (general autoregressive moving-average model) to represent a time series and forecast its future values. An iterative model-building methodology, including model identification, model estimation, model diagnostic checking and employment of the model to forecast the time series are explored and illustrated in this thesis.

Application of the general autoregressive moving-average model is illustrated by identifying the appropriate model and forecasting for an industrial chemical process, a simulated inventory system and international airline passenger fluctuation. The computer programming and human judgement both contribute to these experiments.

From the computational results, it is found that the general autoregressive moving-average model not only represents the discrete time series in the time domain, but also possess the characteristics of maximum simplicity and minimum number of parameters with representational adequacy.

Finally, further research is suggested to put the entertained model under more strictly diagnostic checks in order that it can represent the time series process adequately.