Fundamental parameters of the milky way galaxy

by

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Abstract

Over three-quarters of observed galaxies are spiral galaxies, and of those spirals roughly two-thirds are barred. The Milky Way, a barred spiral galaxy, is naturally a great foundation to studying the structure of other barred spiral galaxies. Two important fundamental constants are used to describe the Milky Way, R_0 (the radial distance from the Sun to the Galactic center) and Θ_0 (the Galactic rotational velocity at R_0). These two constants are also crucial for developing the rotation curve of the Galaxy, which helps to understand the mass distribution of the Galaxy and may be able to lend insight to the dark matter mass contribution.

This work presents new, independently calculated values for R_0 and Θ_0. The error distributions of a compilation of 28 (since 2011) independent measurements of R_0 are wider than a standard Gaussian and best fit by an $n = 4$ Student's t probability density function. Given this non-Gaussianity, the results of our median statistics analysis, summarized as $R_0 = 8.0 \pm 0.3$ kpc (2σ error), probably provides the most reliable estimate of R_0. The unsymmetrized value for R_0 is $R_0 = 7.96\ ^{+0.24}_{-0.30}$ kpc (2σ error). A complete collection of 18 recent (since 2000) measurements of Θ_0 indicates a median statistics estimate of $\Theta_0 = 220\ \pm 10\ \mathrm{km\,s^{-1}}$ (2σ error) as the most reliable summary for most practical purposes, at $R_0 = 8.0 \pm 0.3$ kpc (2σ error). The resulting error distribution of this data set is only mildly non-Gaussian, much more so than that of R_0. These measurements use tracers that are believed to more accurately reflect the systematic rotation of the Milky Way. Unlike other recent compilations of R_0 and Θ_0, our collections includes only independent measurements. This work concludes with a new set of Galactic constants (with 1σ error bars) of $\Theta_0 = 222 \pm 6$ km s^{-1}, $R_0 = 7.96 \pm 0.17$ kpc, and $\omega_0 = \Theta_0/R_0 = 27.9 \pm 1.0$ km s^{-1} kpc^{-1} as probably the most reliable to date.

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Dedication

To my nanny, Alice Lawrence.

Chapter 1

Introduction

Motivated to be able to describe the Milky Way with an updated set of describing parameters, this work uses central estimate statistics on two data sets for R_0, the galactocentric radius of the Sun from Galactic center, and Θ_0, the Galactic velocity at R_0. This work describes the way in which statistically independent data sets for each Galactic constant were compiled for statistical independence, how these were analyzed, and the results for R_0 and Θ_0. Our results for R_0 and Θ_0 will be useful for future work on creating an updated rotation curve for the Milky Way.

It is expected that a large enough data set of N independent measurements will follow a Gaussian distribution, however it is not unheard of for an astronomical parameter to not obey a Gaussian distribution. Perhaps the most famous example is the Hubble constant (Chen and Ratra, 2011a; Chen et al., 2003). For other examples in astronomy, cosmology, and physics see Bailey (2017); Crandall and Ratra (2015); Farooq et al. (2013, 2017); Zhang (2017), and references therein. Significant effort is devoted to testing for intrinsic non-Gaussianity in physical systems (e.g. Park et al., 2001; Planck Collaboration, 2016), as opposed to measurement induced non-Gaussianity, since Gaussianity is usually assumed in parameter estimation (e.g. Chen and Ratra, 2011b; Ooba et al., 2017; Samushia et al., 2007).

Here is studied the data compilations of Table 2.1 (plus two sub-compilations) and 3.1 to examine if they are non-Gaussian or not. If there is significant non-Gaussianity, this could be caused by improperly estimated errors.

To estimate the Gaussianity of a data collection we need to use a central estimate of the data. We consider two main ones in this thesis: the median central estimate and the weighted mean central estimate. The first of the data tables that are considered included analysis with an additional central estimate, the arithmetic mean, primarily to show the insignificance it holds.

1.1 Central Estimate Statistics

Median statistics does not use information of the error on a measurement at all and the true median of a data set can be found independent of any of the individual measurements errors. The estimated median will have a larger uncertainty than that of a weighted central estimate statistic that makes use of error information. Used is the the median statistics technique developed by Gott et al. (2001). The median is defined as the value with 50% of the data being above it and 50% below it. Gott et al. (2001) show that for a data set of $i = 1, 2, ..., N$ independent values, M_i, the probability of the median being between M_i and M_{i+1} is given by the binomial distribution

$$P = \frac{2^{-N} N!}{i!(N-i)!}. \quad (1.1)$$

The 1σ error about the median is then defined by the range about it such that 68.27% of the probability is included. This can be extended to finding the 2σ error about the median, where instead 95.45% of the probability would be enclosed.

The weighted mean comes with the benefit of additional information in the errors, at the potential expense of including inaccurate uncertainties (Podariu et al., 2001). The weighted mean of the Galactic rotational velocity is

$$M^{\text{wm}} = \frac{\sum_{i=1}^{N} M_i/{\sigma_M}_i^2}{\sum_{i=1}^{N} 1/{\sigma_M}_i^2}, \quad (1.2)$$

where M_i and σ_{Mi} are the rotational velocities and errors. The weighted mean standard deviation is

$$\sigma_M^{\mathrm{wm}} = \frac{1}{\sqrt{\sum_{i=1}^{N} 1/\sigma_{M_i}^{2}}}. \tag{1.3}$$

It may also be of value to consider the arithmetic mean,

$$M_{\mathrm{m}} = \frac{1}{N}\sum_{i=1}^{N} M_i. \tag{1.4}$$

The underlying assumptions here are that each of the measurements have roughly the same uncertainty, and that the data come from a normally distributed set. The standard error of the mean is

$$\sigma_{\mathrm{m}} = \sqrt{\frac{1}{N^2}\sum_{i=1}^{N}(M_i - M_{\mathrm{m}})^2}. \tag{1.5}$$

Note that the standard deviation of the data set, σ, and the standard error of the mean, σ_m, differ by the square root of the amount of measurements: $\sigma_m = \sigma/\sqrt{N}$.

When studying the measurements of the radial distance of the Sun to the Galactic Center, M_i, this will change to be R_{0i}. The rotational velocity of the Galaxy at R_0 is Θ_0, with individual measurements being Θ_{0i}.

The next step in analyzing the data is to construct error distributions of the data based on the chosen central estimate.

1.2 Error Distributions

For a central estimate (median, weighted mean, or arithmetic mean) M_{CE} independent of the data M_i, the number of standard deviations that each value deviates from the central estimate is

$$N_{\sigma_M i} = \frac{M_i - M_{\mathrm{CE}}}{\sqrt{\mathrm{Var}\ (M_i - M_{\mathrm{CE}})}} \tag{1.6}$$

where Var $(M_i - M_{\mathrm{CE}})$ is the variance between the independent measurement, M_i and the central estimate, M_{CE}.

For median statistics when the central estimate is assumed to be slightly correlated with the data itself[1] we have

$$N^{\rm med}_{\sigma_{M\,i}} = \frac{M_i - M_{\rm med}}{\sqrt{\left(\sigma_{M_i}\right)^2 + \left(\sigma^{\rm med}_{M_i}\right)^2}}. \tag{1.7}$$

Here $M_{\rm CE}$ is the central estimate of M_i and $\sigma_{\rm CE}$ is the error of the central estimate of M_i. N_{σ_i} represents how much M_i deviates from the central estimate, taking into account both the error associated with the measurement and the error associated with the central estimate. (In this thesis we do not always symmetrize $\sigma_{\rm CE}$ for the median statistics cases (if the data are not symmetric enough to justify it). Thus, when applicable, we use the upper/right-side error $\sigma^u_{\rm CE}$ for when $M_i \geq M_{\rm CE}$ and the lower/left-side error $\sigma^l_{\rm CE}$ for when $M_i \leq M_{\rm CE}$.) For Gaussianly distributed measurements and the weighted mean central estimate estimated from the data (and so correlated with the data) we instead have (see the Appendix of Camarillo and Ratra (2018, hereafter C18)[2]

$$N^{\rm wm}_{\sigma_{M\,i}} = \frac{M_i - M_{\rm wm}}{\sqrt{\left(\sigma_{M_i}\right)^2 - \left(\sigma^{\rm wm}_{M_i}\right)^2}}. \tag{1.8}$$

In recent publications on error distribution analysis, the standard form has been eq. (1.11) until discovering that the variance distributes to result in subtracting by quadrature when the central estimate is directly derived from the data itself. With three central estimates- the median, weighted mean, and arithmetic mean- we can label our error distributions $N^{\rm med}_\sigma$, $N^{\rm wm}_\sigma$, and $N^{\rm mean}_\sigma$. It was additionally found in C18 that a previously referred to $N^{\rm med}_\sigma$ should not be used, since the error used by integrating the area under a histogram out to 1 and 2σ is not the actual error on the median.

These represent differing combinations of central estimates and errors, defined as

$$N^{\rm med}_{\sigma_{M\,i}} = \frac{M_i - M_{\rm med}}{\sqrt{\sigma_i^2 + \sigma^2_{\rm Gott}}}, \tag{1.9}$$

[1]As opposed to the heavy correlation a weighted mean has with the data itself.

[2]An analogous equation for median statistics, for the case when the median is estimated from the data and so is correlated with the data, is not yet known.

$$N^{\mathrm{wm}+}_{\sigma_{M\,i}} = \frac{M_i - M_{\mathrm{wm}}}{\sqrt{\sigma_i^2 + \sigma_{\mathrm{wm}}^2}}; \tag{1.10}$$

$$N^{\mathrm{mean}}_{\sigma_{M\,i}} = \frac{M_i - M_{\mathrm{m}}}{\sqrt{\sigma_i^2 + \sigma_{\mathrm{m}}^2}}. \tag{1.11}$$

Since the central estimates are calculated from the data, they must to some degree be correlated with the error measurements and as explained, a more appropriate error distribution is then[3]

$$N^{\mathrm{wm}-}_{\sigma_{M\,i}} = \frac{M_i - M_{\mathrm{wm}}}{\sqrt{\sigma_i^2 - \sigma_{\mathrm{wm}}^2}}. \tag{1.12}$$

The derivation of an equivalent error distribution that accounts for the correlation is non-trivial for a median central estimate, however eq. (1.11) provides a valuable limiting case.[4]

1.3 Probability Density Functions

A commonly used method of qualitatively studying Gaussianity of an distribution is χ^2 analysis. The N_{σ_i} distribution is binned, and a goodness of fit is calculated for a well-defined probability density function (PDF). In this paper we use 4 PDF's: Gaussian, Student's t, Cauchy, and Laplace (Double Exponential) distributions. The reduced χ^2, $\chi^2_\nu = \chi^2/\nu$, can be easily calculated from the number of degrees of freedom, ν. In this case, ν is one less than the total number of measurements, $\nu = N - 1$).[5] The equation for reduced χ^2 is

$$\chi^2_\nu = \frac{1}{N-1}\sum_{i=1}^{b}\frac{[M(N_{\sigma_i}) - NP(N_{\sigma_i})]^2}{NP(N_{\sigma_i})} \tag{1.13}$$

where b is the number of bins, $M(N_i)$ is the number of values within N_σ bins, N is the number of measurements, and $P(N_{\sigma_i})$ is the PDF in question.

χ^2_ν supplies insight to the spread of the distribution, and a smaller value represents a good fit to the PDF. For these two data sets however a more appropriate way to study the

[3]See the Appendix for a derivation.

[4]It would be interesting to account for the correlation between the measurements and the median from eq. (1.1), but this is beyond the scope of this research.

[5]For a more detailed explanation of χ^2 analysis see Crandall and Ratra (2015).

Gaussianity is a test that may be more sensitive with unbinned data sets.[6]

We numerically study our error distributions using the one-sample Kolmogorov-Smirnov (K-S) test (Feigelson and Babu, 2012). This non-parametric, distribution-free test determines the probability that the given sample distribution comes from a PDF, at a chosen significance level α. The qualitative returns of a K-S test are a D-statistic and a p-value. The D statistic is the supremum of, or the largest distance between, the cumulative sample distribution and the cumulative PDF. The closer this value is to zero, the better the sample distribution is well described by the PDF. For a sample distribution of N measurements there is a critical value $D_{\text{crit}}(N)$ that must be less than the test result, D, in order to not reject the null hypothesis at the specified significance level (which is conventionally set at $\alpha = 0.05$ for a confidence level of 95%). As an example, for $N = 28$ measurements (the R_0 measurements data set) $D_{\text{crit}} = 0.24993$.[7] The p-value follows from the D statistic and represents not the probability that the sample set is from the proposed PDF, but rather the probability that we cannot reject the null hypothesis that the distributions are the same. It is for this reason that the probabilities of the K-S test should be used as qualitative indicators of distribution fitting. It is of interest to study the K-S test results for as many PDF's as possible. We choose the PDF with the lowest D statistic and the highest P value as the best representation of the error distribution under study.

We define our PDF's as functions of $|\mathbf{N}| = |N_\sigma/S|$, where S is a scale factor. When $S = 1$ and $|\mathbf{N}| = |N_\sigma|$, $P(|\mathbf{N}|)$ is the standard form of the PDF. When $S > 1$, the distribution is broader than the standard form, while $S < 1$ corresponds to a narrower distribution. While N_{σ_i} is computed with unsymmetrized errors, the distribution of N_σ is symmetrized for the K-S test.

We define a Gaussian distribution of N_σ with an expected 68.27% and 95.45% of the

[6]The results for N_σ^{Gott} ($N_\sigma^{\text{wm}-}$) for our independent data set of R_0 measurements is $\chi^2_\nu = 0.426$ ($\chi^2_\nu = 3.464$). The results for N_σ^{Gott} ($N_\sigma^{\text{wm}-}$) for our independent data set of Θ_0 measurements from "Old" tracers is $\chi^2_\nu = 0.209$ ($\chi^2_\nu = 0.093$). This indicates that it is appropriate to further test for the best fitting PDF as the error distributions are not significantly non-Gaussian, or exceptionally spread out.

[7]See Appendix 3 of O'Connor and Kleyner (2012) for a table of D_{crit} as a function of N.

values falling within $|N_\sigma| \leq 1$ and $|N_\sigma| \leq 2$ respectively as

$$P(|\mathbf{N}|) = \frac{1}{\sqrt{2\pi}} \exp(-|\mathbf{N}|^2/2). \tag{1.14}$$

The second distribution that we consider is a Laplace (Double Exponential), given by

$$P(|\mathbf{N}|) = \frac{1}{2} \exp\left(-|\mathbf{N}|\right). \tag{1.15}$$

The Laplace PDF is sharply peaked, with longer (smaller) tails than a Gaussian (Cauchy) distribution. For this distribution, 68.27% and 95.45% of the values correspond to $|N_\sigma| \leq 1.2$ and $|N_\sigma| \leq 3.1$ respectively. The Cauchy (Lorentz) distribution

$$P(|\mathbf{N}|) = \frac{1}{\pi} \frac{1}{1 + |\mathbf{N}|^2} \tag{1.16}$$

has much higher probability in the tails, with an expected 68.27% and 95.45% of the values falling within $|N_\sigma| \leq 1.8$ and $|N_\sigma| \leq 14$ respectively. The Student's t distribution is defined by

$$P(|\mathbf{N}|) = \frac{\Gamma[(n+1)/2]}{\sqrt{\pi n}\,\Gamma(n/2)} \frac{1}{(1 + |\mathbf{N}|^2/n)^{(n+1)/2}} \tag{1.17}$$

where n is a positive non-zero parameter and Γ is the gamma function. When $n = 1$ this is the Cauchy distribution, and when $n \to \infty$ it becomes the Gaussian distribution. Thus, for $n > 1$, it is a function with slightly less extended tails than a Cauchy, that decrease as n increases. In this case, the limits corresponding to 68.27% and 95.45% of the values depend on the value of n.

1.4 Determining Non-Gaussianity

In studying the results of the K-S test comes the conclusion of analyzing the Gaussianity of the error distribution (and thus data set) in question. The non-Gaussianity is determined by multiple factors and in the respective R_0 and Θ_0 chapters will be explanations of how to compare the results of a median statistics approach to a weighted mean statistics approach.

Chapter 2

Fundamental Constants of the Milky Way Galaxy: R_0

There has been a slew of publications aiming to determine R_0 and V_0 in the last decade and the research of this thesis brings to light the lack of independence and critical central estimate analysis within some larger data sets in question. The work described in this chapter was done in collaboration with Varun Mathur, Tyler Mitchell, and Dr. Ratra and published in Camarillo and Ratra (2018).

2.1 R_0 Introduction

The value of R_0, the distance of the Sun to the center of the Milky Way Galaxy, is a very important datum for astrophysics and cosmology. A quarter century ago, Reid (1993) concluded that a reasonable summary value was $R_0 = 8.0 \pm 0.5$ kpc (errors are 1σ unless indicated otherwise). More recent summary estimates include $R_0 = 7.9 \pm 0.2$ kpc from Nikiforov (2004), $R_0 = 8.0 \pm 0.25$ kpc from Malkin (2012), $R_0 = 8.3 \pm 0.2$ (stat.) ± 0.4 (syst.) kpc from de Grijs and Bono (2016), and $R_0 = 8.0 \pm 0.2$ kpc from V17.

de Grijs and Bono (2016) compiled 273 R_0 measurements, not all of which are statistically independent, and carefully studied how publication bias might have influenced R_0 measurements. Their summary R_0 value is based on a consideration of only a very few of their 273 measurements. Vallée (2017, hereafter V17) on the other hand only compiled 27 very recent measurements, also not all independent; while we are able to reproduce his central estimate of $R_0 = 8.0$ kpc, we are unable to reproduce his ± 0.2 kpc error bars from his compiled data set.

Here, we revisit the issue of determining a best estimate for, and errors on, R_0. Following V17, we compile a list of 28 recent R_0 measurements in the belief that the more recent measurements are more reliable, but we carefully check to make sure that our list only includes statistically independent measurements, unlike the recent de Grijs and Bono (2016) and V17 compilations.

Following, and generalizing, Chen et al. (2003), we study the error distributions of this 28 measurement data set. We discover that the errors are somewhat non-Gaussian. This is not unexpected (Bailey, 2017); well-known examples of non-Gaussianity include Hubble constant H_0 measurements (Chen et al., 2003), ^{7}Li abundance measurements (Crandall and Ratra, 2015; Zhang, 2017), and LMC and SMC distance moduli measurements (de Grijs et al., 2014; ?).

Significant effort is often devoted to determining whether there is intrinsic non-Gaussianity in astrophysical and cosmological systems (e.g., Park et al., 2001; Planck Collaboration, 2016), as opposed to non-Gaussianity introduced by measurement techniques. This is because Gaussianity is assumed in many parameter constraint analyses (e.g., Podariu and Ratra, 2000; Ratra et al., 1999).

Care is required when analyzing data with non-Gaussian errors (e.g., Bailey, 2017; Gott et al., 2001; Zhang, 2017). Gott et al. (2001) developed median statistics partially for this purpose. Median statistics does not make use of the measurement errors and so is not affected by the non-Gaussianity, but since it discards some of the measurement information (the errors) it is less constraining. A well-known example of the use of median statistics is the analysis of H_0 measurements (Calabrese et al., 2012; Chen and Ratra, 2011a; Chen et al., 2003; Gott et al., 2001).

In this paper, we apply median statistics to our compilation of 28 independent, recent

R_0 measurements. We find $R_0 = 7.96\ ^{+0.11}_{-0.23}\ (^{+0.24}_{-0.30})$ kpc, where the errors are 1σ (2σ). For most practical purposes, this can be taken to be $R_0 = 8.0 \pm 0.3$ kpc at 2σ.

In Sec. 2.2 we discuss our compilation of recent independent R_0 measurements and how it differs from that used by V17. In Ch. 1 we summarized our methods for computing central estimates and errors of the compiled data set; the results of this process can be found in Sec. 2.3 and Sec. 2.4. We conclude in Sec. 2.5.

2.2 R_0 Data Compilation

The R_0 data we use in our analyses are listed in Table 2.1. The second column of the table lists the 27 R_0 values given to one decimal place in Table 1 of V17. The third column of our Table 2.1 updates these values, to two decimal places, from the original publications.

Of these 27 measurements, only 20 are statistically independent, and these are listed in column 4 of Table 2.1. To these 20 measurements we added 8 new, post-2010, independent values that we found after a fairly exhaustive search of the literature. We decided to only use more recent (post-2010) data in the hope that they would be of better quality than earlier data. These 28 measurements are listed in column 5 of Table 2.1. Most of our analyses here focus on these 28 measurements.

In making our list of independent measurements, we ensure that no two estimates use the same experimental data. If two papers use the same method but use data from different equipment then we include both. Consider Boehle et al. (2016) and Gillessen et al. (2013): both estimate R_0 by using the orbits of S-stars about the Galactic Center, Sgr A*. However, they use distinct experiments to constrain the orbits. There are quite a few papers that use the same method and data, from the same experiments, as the two above – we include only the latest independent results and drop the rest. Some papers combine their result with other data: Do et al. (2013) combines their estimate of R_0 using statistical parallax with Ghez et al. (2008), a predecessor of Boehle et al. (2016). In this case we use the measurement of R_0 from Do et al. (2013), that is not combined with Ghez et al. (2008) data, $R_0 = 8.92^{+0.58}_{-0.55}$ kpc. We assume that only a small degree of systematic error is present in measurements of

Table 2.1. R_0 (in kpc) Measurements

Year	Vallée	Vallée: updated[a]	Vallée: independent[a]	Independent from 2011[a]	Reference
2011	-	-	-	7.94 ± 0.65	Fritz et al. (2011)
2011	-	-	-	8.07 ± 0.35	Trippe et al. (2011)
2012	7.7 ± 0.4	7.70 ± 0.40	-	-	Morris et al. (2012)
2012	8.0 ± 0.8	8.00 ± 0.45	8.00 ± 0.45	8.00 ± 0.45	Bovy et al. (2012)
2012	8.0 ± 0.4	8.05 ± 0.45	-	-	Honma et al. (2012)
2012	8.3 ± 0.4	8.27 ± 0.29	8.27 ± 0.29	8.27 ± 0.29	Schönrich (2012)
2013	7.6 ± 0.6	7.50 ± 0.60	7.50 ± 0.60	7.50 ± 0.60	Matsunaga et al. (2013)
2013	-	-	-	7.25 ± 0.32	Bobylev (2013)
2013	7.6 ± 0.3	7.64 ± 0.32	7.64 ± 0.32	7.64 ± 0.32	Bobylev (2013)
2013	-	-	-	7.66 ± 0.36	Bobylev (2013)
2013	-	-	-	7.73 ± 0.36	Dambis et al. (2013)
2013	-	-	-	7.91 ± 0.41	Bono et al. (2013)
2013	8.0 ± 0.8	7.98 ± 0.79	7.98 ± 0.79	7.98 ± 0.79	Zhu and Shen (2013)
2013	8.0 ± 0.7	8.03 ± 0.70	8.03 ± 0.70	8.03 ± 0.70	Zhu and Shen (2013)
2013	8.2 ± 0.8	8.25 ± 0.79	-	-	Zhu and Shen (2013)
2013	8.2 ± 0.2	8.13 ± 0.10[b]	8.13 ± 0.10[b]	8.13 ± 0.10[b]	Cao et al. (2013)
2013	8.3 ± 0.2	8.33 ± 0.15	-	-	Dékány et al. (2013)
2013	-	-	-	8.20 ± 0.34	Gillessen et al. (2013)
2013	8.5 ± 0.4	8.46 ± 0.40	8.92 ± 0.57	8.92 ± 0.57	Do et al. (2013)
2014	6.7 ± 0.4	6.72 ± 0.39	6.72 ± 0.39	6.72 ± 0.39	Branham (2014)
2014	7.4 ± 0.3	7.40 ± 0.28	7.40 ± 0.28	7.40 ± 0.28	Francis and Anderson (2014)
2014	7.5 ± 0.3	7.50 ± 0.30	7.50 ± 0.30	7.50 ± 0.30	Francis and Anderson (2014)
2014	8.3 ± 0.2	8.34 ± 0.16	-	-	Reid et al. (2014)
2015	-	-	-	7.60 ± 1.35	Ali et al. (2015)
2015	7.7 ± 0.1	7.68 ± 0.07	7.68 ± 0.07	7.68 ± 0.07	Branham (2015)
2015	8.0 ± 0.3	8.03 ± 0.12	-	-	Bajkova and Bobylev (2015)
2015	8.3 ± 0.1	8.33 ± 0.11	8.27 ± 0.13	8.27 ± 0.13	Chatzopoulos et al. (2015)
2015	8.3 ± 0.4	8.27 ± 0.40	8.27 ± 0.40	8.27 ± 0.40	Pietrukowicz et al. (2015)
2015	8.3 ± 0.3	8.30 ± 0.25	8.30 ± 0.25	8.30 ± 0.25	Küpper et al. (2015)
2016	7.9 ± 0.1	7.86 ± 0.15	7.86 ± 0.15	7.86 ± 0.15	Boehle et al. (2016)
2016	8.4 ± 0.1	8.24 ± 0.12	8.24 ± 0.12	8.24 ± 0.12	Rastorguev et al. (2017)
2016	8.9 ± 0.4	8.90 ± 0.40	8.90 ± 0.40	8.90 ± 0.40	Catchpole et al. (2016)
2017	7.6 ± 0.1	7.64 ± 0.09	7.64 ± 0.09	7.64 ± 0.09	Branham (2017)
2017	8.0 ± 0.2	7.97 ± 0.15	-	-	McMillan (2017)
2017	8.2 ± 0.1	8.20 ± 0.09	8.20 ± 0.09	8.20 ± 0.09	McMillan (2017)

[a]We determine the error by symmetrizing the error bars (if necessary) and adding the statistical and systematic errors in quadrature.

[b]Cao et al. (2013) does not list an error bar. We thank L. Cao and S. Mao for providing the value listed here via private communication (2017).

R_0.[1]

2.3 Analyzing R_0

To construct error distributions of this data set, used is three central estimates: the median, weighted mean, and arithmetic mean.[2] This is the only analysis this thesis that will utilize the arithmetic mean.

The central estimates and associated errors are recorded in Table 2.2 for each of the data sets of Table 2.1. From column 2 of Table 2.2, we see our median, weighted mean, and arithmetic mean central estimates of 8 kpc coincide with those of V17 (at the bottom of

[1]We do account for all stated systematic errors. Our results below, which show that the error distributions are not very non-Gaussian, are consistent with our assumption that unknown systematic errors are small.

[2]We follow the conventions of Secs. 38 and 39 of Particle Data Group (2016).

Table 2.2. R_0 (in kpc) Central Estimates and Errors

	Vallée	Vallée: updated	Vallée: independent	Independent from 2011
Median, integral[a]	$8.00\ ^{+0.36}_{-0.34}\ ^{+0.54}_{-1.26}$	$8.03\ ^{+0.31}_{-0.32}\ ^{+0.83}_{-1.27}$	$8.02\ ^{+0.26}_{-0.55}\ ^{+0.86}_{-1.24}$	$7.96\ ^{+0.29}_{-0.50}\ ^{+0.90}_{-1.20}$
1σ range	7.66 – 8.36	7.71 – 8.34	7.47 – 8.28	7.46 – 8.25
2σ range	6.74 – 8.54	6.76 – 8.86	6.78 – 8.88	6.76 – 8.86
Median, Gott[b]	$8.00\ ^{+0.20}_{-0.00}\ ^{+0.30}_{-0.30}$	$8.03\ ^{+0.17}_{-0.05}\ ^{+0.24}_{-0.33}$	$8.02\ ^{+0.18}_{-0.16}\ ^{+0.25}_{-0.38}$	$7.96\ ^{+0.11}_{-0.23}\ ^{+0.24}_{-0.30}$
1σ range	8.00 – 8.20	7.98 – 8.20	7.86 – 8.20	7.73 – 8.07
2σ range	7.70 – 8.30	7.70 – 8.27	7.64 – 8.27	7.66 – 8.20
Weighted Mean	8.02 ± 0.04	7.99 ± 0.03	7.93 ± 0.03	7.93 ± 0.03
1σ range	7.99 – 8.06	7.95 – 8.02	7.90 – 7.97	7.89 – 7.96
Arithmetic Mean	8.00 ± 0.08	7.99 ± 0.08	7.97 ± 0.11	7.92 ± 0.09
1σ range	7.91 – 8.08	7.91 – 8.07	7.86 – 8.08	7.84 – 8.01

[a]Errors are estimated by binning the measurements to 0.1 kpc and integrating outwards until reaching 68.27% and 95.45% of the area under the distribution.

[b]Errors are estimated from the median statistics probability distribution of eq. (1.1).

his Table 1). However, we are unable to reproduce his weighted mean and arithmetic mean error bars of ± 0.2 kpc (he does not quote a median error bar); our weighted (arithmetic) mean error bar is ± 0.04 (0.4) kpc.

The last column of Table 2.2 summarizes our main result. As discussed below, we find the error distribution for our chosen 28 measurements are somewhat non-Gaussian, but not excessively so.[3] Consequently we recommend that the median central value and the symmetrized errors for the 68.27% and 95.45% confidence ranges as defined in Gott et al. (2001) be used to describe the value of and errors on R_0. This gives $R_0 = 7.96 \pm 0.17$ (± 0.27) kpc, with symmetrized 1σ (2σ) error, though it might be preferable to use the unsymmetrized result of $R_0 = 7.96\ ^{+0.11}_{-0.23}\ (^{+0.24}_{-0.30})$ kpc to take into account the slightly asymmetric nature of the set of measurements. For most practical purposes, $R_0 = 8.0 \pm 0.3$ (2σ error) serves as an appropriate summary estimate to one decimal place.

[3]Seeing as the error distribution calculated from the median statistics of eq. (1.1) is not very non-Gaussian it is unlikely that most errors have been incorrectly estimated. Specifically, it is unlikely that there are large undiscovered systematic errors.

Table 2.3. K-S Test Probabilities

	$N_\sigma^{\rm med}$		$N_\sigma^{\rm Gott}$ [c]		$N_\sigma^{\rm wm+}$		$N_\sigma^{\rm wm-}$		$N_\sigma^{\rm mean}$	
PDF	S[a]	P(%)[b]	S[a]	P(%)[b]	S[a]	P(%)[b]	S[a]	P(%)[b]	S[a]	P(%)[b]
Gaussian	1	69.4	1	53.4	1	11.9	1	11.7	1	17.8
Gaussian	0.85	99.5	1.24	99.6	1.68	99.9	1.73	99.8	1.56	99.9
Laplace	1	39.0	1	82.6	1	47.9	1	45.3	1	57.3
Laplace	0.77	93.6	1.13	97.7	1.40	99.8	1.52	99.9	1.28	99.0
Cauchy	1	4.1	1	32.8	1	64.6	1	88.7	1	50.2
Cauchy	0.51	84.6	0.70	84.8	0.77	90.2	0.83	97.2	0.75	88.1
	$n = 100$		$n = 3$		$n = 2$		............[e]		$n = 2$	
Student's t[d]	1	67.7	1	97.5	1	81.1	....	.....	1	88.8
	$n = 100$		$n = 4$		$n = 5$		$n = 2$		$n = 34$	
Student's t[d]	0.85	99.4	1.11	99.7	1.50	99.9	1.28	99.9	1.53	99.9

[a]Scale factor S is first set at $S = 1$ (representing the case when $|N_\sigma| = 1$ corresponds to 1 standard deviation for a Gaussian distribution) and is then allowed to vary with the width of the function as D is minimized.

[b]This is the P value described in Sec. 1.3. It is the probability that we cannot reject the hypothesis that the sample distribution N_σ came from a distribution created from the probability density function.

[c]We use the errors corresponding to 68.27% confidence in $N_\sigma^{\rm Gott}$ because we use 1 standard deviation for $N_\sigma^{\rm med}$.

[d]We allow n to vary between 1 and 100 for the Student's t distribution.

[e]The K-S test using a Student's t PDF on $N_\sigma^{\rm wm-}$ for $S = 1$ yielded a best fit of $n = 1$ which is the Cauchy distribution.

2.4 R_0 Error Distributions

After determining the central estimates in table 2.2, the error distributions are constructed by using the methods within Section 1.2.

It was decided to use the five error distributions of eqns. (1.9), (1.10), (1.11), (1.13), and an additional now-obsolete equation to attempt to gain some insight into the R_0 measurements' error distribution.[4]

Our K-S test results, for the 28 independent R_0 values listed in column 5 of Table 2.1, are shown in Table 2.3. While some $S = 1$ entries have low probabilities, and $P = 11.7\%$ for the $S = 1$ Gaussian case of the weighted mean central estimate and the 1σ error distribution of

[4]The integral method of calculating $\sigma_{\rm med}$ is not the error on the median itself (like the Gott et al. (2001) method provides) but is the deviation of the data set about the median. It was only included in C18 to remain consistent with previously published results regarding the Gaussianity of error distributions where it was used in an attempt to also account for systematic uncertainties, e.g. Crandall and Ratra (2014).For future analyses that this error should not be regarded as the uncertainty on the median nor be used in calculating error distributions.

eq. (1.13), overall, allowing S to vary a little away from unity, it is fair to conclude that the errors of the 28 measurement data set are not very non-Gaussian, although they are slightly so.[5] Tables 2.4 and 2.5, which show the probabilities corresponding to $|N_\sigma| \leq 1$ and $|N_\sigma| \leq 2$ and the $|N_\sigma|$ values corresponding to 68.27% and 95.45% of the probability for these favored distributions, reinforce this conclusion.

Columns 4 and 5 of Table 2.3 show the probabilities are as high as 99.9% for a Gaussian distribution with $S = 1.68$ and a Laplacian distribution with $S = 1.52$, respectively. The non-Gaussianity associated with using the error bars from the R_0 measurements in weighted mean analyses can be substantiated from columns 4 and 5 of Tables 2.4 and 2.5: for the $S = 1.68$ Gaussian in $N_\sigma^{\rm wm+}$, only 45% (77%) of the probability lies within $|N_\sigma| \leq 1$ ($|N_\sigma| \leq 2$) and to attain the standard probability of 68.27% (95.45%) we must integrate out to $|N_\sigma| = 1.7$ ($|N_\sigma| = 3.4$); for the $S = 1.52$ Laplacian of $N_\sigma^{\rm wm-}$, only 48% (73%) of the probability lies within $|N_\sigma| \leq 1$ ($|N_\sigma| \leq 2$) and to attain the standard probability of 68.27% (95.45%) we must integrate out to $|N_\sigma| = 1.7$ ($|N_\sigma| = 4.7$). The Gaussian fits for $N_\sigma^{\rm wm+}$, $N_\sigma^{\rm wm-}$, and $N_\sigma^{\rm mean}$ require scale factors of $S = 1.68$, $S = 1.73$, and $S = 1.56$ respectively. For this reason, it is best to use median statistics to determine the error bars on R_0, which are looser than those from weighted mean statistics and arithmetic mean statistics. The probability distribution computed from eq. (1.1) then provides the best central estimate and errors bars for determining the somewhat non-Gaussian nature of the error distribution of the 28 independent R_0 measurements. The corresponding median-statistics error distribution of eq. (1.9) is best fit by an $n = 4$ Student's t PDF with an $S = 1.1$ scale factor, and is non-Gaussian to the degree that with a probability of 99.6%, we cannot reject the hypothesis that it comes from a Gaussian distribution with an $S = 1.24$ scale. The slightly broader-than-expected Gaussian distributed error distribution could indicate some (slightly) improperly estimated systematic uncertainties. This is, however, perhaps a mild concern until we can compile and study a larger set of recent and statistically independent measurements of R_0.

[5] On the other hand, the corresponding analyses for the data sets of columns 2 and 3 of Table 2.1 show that those 27 measurement data sets are more non-Gaussian, as might be expected, given the non-independence of some measurements.

2.5 Conclusion on R_0

For more than three decades, the International Astronomical Union has recommended $R_0 = 8.5$ kpc. In the last decade, evidence has been mounting that this might be a little too large (de Grijs and Bono, 2016; Malkin, 2012; Nikiforov, 2004, V17).

We have compiled a list of 28 recent, independent R_0 measurements. We find that the corresponding error distributions are slightly wider than a standard Gaussian. Consequently we believe a median statistics (Gott et al., 2001) analysis provides a more reliable estimate of R_0 from this compilation. For most purposes $R_0 = 8.0 \pm 0.3$ kpc (2σ error), somewhat smaller than the 8.5 kpc IAU recommendation, is a reasonable summary of our results.

Table 2.4. R_0 $|N_\sigma|$ Expected Fractions

PDF	N_σ^{med}			N_σ^{Gott}			$N_\sigma^{\mathrm{wm+}}$			$N_\sigma^{\mathrm{wm-}}$			N_σ^{mean}		
	S[a]	$\lvert N_\sigma\rvert \le 1$[b]	$\lvert N_\sigma\rvert \le 2$[b]	S[a]	$\lvert N_\sigma\rvert \le 1$[b]	$\lvert N_\sigma\rvert \le 2$[b]	S[a]	$\lvert N_\sigma\rvert \le 1$[b]	$\lvert N_\sigma\rvert \le 2$[b]	S[a]	$\lvert N_\sigma\rvert \le 1$[b]	$\lvert N_\sigma\rvert \le 2$[b]	S[a]	$\lvert N_\sigma\rvert \le 1$[b]	$\lvert N_\sigma\rvert \le 2$[b]
Gaussian	1	0.68	0.95	1	0.68	0.95	1	0.68	0.95	1	0.68	0.95	1	0.68	0.95
Gaussian	0.85	0.76	0.98	1.24	0.58	0.89	1.68	0.45	0.77	1.73	0.44	0.75	1.56	0.48	0.80
Laplace	1	0.63	0.87	1	0.63	0.87	1	0.63	0.87	1	0.63	0.87	1	0.63	0.87
Laplace	0.78	0.73	0.92	1.13	0.59	0.83	1.40	0.51	0.76	1.52	0.48	0.73	1.28	0.54	0.79
Cauchy	1	0.50	0.71	1	0.50	0.71	1	0.50	0.71	1	0.50	0.71	1	0.50	0.71
Cauchy	0.51	0.70	0.84	0.70	0.61	0.79	0.77	0.58	0.77	0.83	0.56	0.75	0.75	0.59	0.77
		$n = 100$			$n = 3$			$n = 2$			........................[c]			$n = 2$	
Student's t	1	0.58	0.82	1	0.61	0.86	1	0.58	0.82	....	....	....	1	0.58	0.82
		$n = 100$			$n = 4$			$n = 5$			$n = 2$			$n = 34$	
Student's t	0.85	0.76	0.98	1.11	0.58	0.85	1.50	0.47	0.76	1.28	0.48	0.74	1.53	0.48	0.80
Observed	....	0.86	1.00	....	0.54	0.93	....	0.50	0.71	....	0.50	0.68	....	0.50	0.71

[a]Scale factor S is first set at $S = 1$ (representing the case when $|N_\sigma| = 1$ corresponds to 1 standard deviation for a Gaussian distribution) and is then allowed to vary with the width of the function as D is minimized.

[b]The fraction of data points that lie within $|N_\sigma| \le 1$ or $|N_\sigma| \le 2$.

[c]The Student's t test on $N_{\sigma_{\mathrm{wm-}}}$ for $S = 1$ yielded a best fit of $n = 1$ which is the Cauchy distribution.

Table 2.5. R_0 $|N_\sigma|$ Limits

PDF	$N_\sigma^{\rm med}$			$N_\sigma^{\rm Gott}$			$N_\sigma^{\rm wm+}$			$N_\sigma^{\rm wm-}$			$N_\sigma^{\rm mean}$		
	S[a]	68.27%[b]	95.45%[b]	S[a]	68.27%[b]	95.45%[b]	S[a]	68.27%[b]	95.45%[b]	S[a]	68.27%[b]	95.45%[b]	S[a]	68.27%[b]	95.45%[b]
Gaussian	1	1.0	2.0	1	1.0	2.0	1	1.0	2.0	1	1.0	2.0	1	1.0	2.0
Gaussian	0.85	0.9	1.7	1.24	1.2	2.5	1.68	1.7	3.4	1.73	1.7	3.5	1.56	1.6	3.1
Laplace	1	1.2	3.1	1	1.2	3.1	1	1.2	3.1	1	1.2	3.1	1	1.2	3.1
Laplace	0.78	0.9	2.4	1.13	1.3	3.5	1.40	1.6	4.3	1.52	1.7	4.7	1.28	1.5	4.0
Cauchy	1	1.8	14.0	1	1.8	14.0	1	1.8	14.0	1	1.8	14.0	1	1.8	14.0
Cauchy	0.51	0.9	7.0	0.70	1.3	9.7	0.77	1.4	10.6	0.83	1.5	11.5	0.75	1.4	10.6
		$n = 100$			$n = 3$			$n = 2$			........................[c]			$n = 2$	
Student's t	1	1.0	2.0	1	1.2	3.3	1	1.3	4.5	....	....	....	1	1.3	4.5
		$n = 100$			$n = 4$			$n = 5$			$n = 2$			$n = 34$	
Student's t	0.85	0.9	1.7	1.11	1.3	3.2	1.50	1.7	4.0	1.28	1.7	5.8	1.53	1.5	3.2
Observed	....	0.8	1.9	....	1.3	2.3	....	1.9	3.1	....	2.1	3.5	....	1.7	2.5

[a]Scale factor S is first set at $S = 1$ (representing the case when $|N_\sigma| = 1$ corresponds to 1 standard deviation for a Gaussian distribution) and is then allowed to vary with the width of the function as D is minimized.

[b]The $|N_\sigma|$ limits containing 68.27% or 95.45% of the probability. For a Gaussian PDF with $S = 1$, 68.27% (95.45%) of the probability is contained within $|N_\sigma| = 1$ ($|N_\sigma| = 2$).

[c]The Student's t test on $N_{\sigma_{\rm wm-}}$ for $S = 1$ yielded a best fit of $n = 1$ which is the Cauchy distribution.

Chapter 3

Fundamental Constants of the Milky Way Galaxy: Θ_0

3.1 Θ_0 Introduction

A more accurate model of the Milky Way will improve the accuracy of inter- and extragalactic measurements. Two constants play a fundamental role in describing the current model of the Milky Way: R_0 (the radial distance of the Sun to the Galactic center, Sgr A*) and Θ_0 (the rotational velocity of the Milky Way at R_0). Now that we have measured R_0 from a carefully compiled set of independent R_0 data points, we focus attention on Θ_0. The work described in this chapter was done in collaboration with Pauline Dredger and Dr. Ratra, and is available online at arXiv at 1805.01917.

There have been three recent attempts at measuring Θ_0 from compilations of measurements: Vallée (2017, hereafter V17), de Grijs and Bono (2017, hereafter dGB17), and Rajan and Desai (2018, hereafter RD18). These analyses use compilations that include non-independent measurements which can significantly affect the results and render them unreliable. In this paper we first put together a collection of 29 independent estimates of Θ_0 that have been published in 2000 or later. Of these 28 measurements, 18 correspond to tracers (such as CO and H I gas clouds) that are believed to more accurately reflect the system-

Table 3.1 Independent Θ_0 measurements since 2000

Radius (kpc)	Θ_0 ($km\,s^{-1}$)	Rescaled Θ_0^{res} ($km\,s^{-1}$)	Reference	Tracer Type	Notes
6.72 ± 0.39	203.35 ± 12.00	240.87 ± 20.59	Branham (2014)	Young	Table 3, Hipparcos catalog, 6288 OB stars
7.62 ± 0.32	205.00	214.15 ± 10.09	Battinelli et al. (2013)	Old	Figure 3, 4400 carbon stars
7.64 ± 0.32	217.00 ± 11.00	226.09 ± 15.63	Bobylev (2013)	Young	Table 3, Cepehids near Sun, UCAC4
7.97 ± 0.15	226.80 ± 4.20	226.52 ± 7.69	McMillan (2017)	Both	In Abstract, from alternative mass model
7.98 ± 0.79	238.54 ± 11.66	237.94 ± 26.76	Shen and Zhang (2010)	Young	From Hipparcos Cepheids
8.00	220.80 ± 13.60	219.70 ± 14.32	Bedin et al. (2003)	Old	From WFPC2/HST photometry on M4 globular cluster
8.00 ± 0.50	202.70 ± 24.70	201.69 ± 27.95	Kalirai et al. (2004)	Old	From HST on M4 globular cluster, independent of Bedin et al. (2003)
8.00 ± 0.50	236.00 ± 15.00	234.82 ± 21.52	Reid and Brunthaler (2004)	Old	From VLBA proper motion around Sgr A*
8.00	208.50 ± 20.00	207.46 ± 20.39	Xue et al. (2008)	Old	Averaged from Table 3 for range $7.5 - 8.5$ kpc
8.00	243.50 ± 13.00	242.28 ± 13.93	Yuan et al. (2008)	Old	From Hipparcos K-M giants
8.00	226.84	225.71 ± 4.82	Sharma et al. (2011)	Old	From comparing galaxy model to Hipparcos, Geneva-Copenhagan survey, and SDSS
8.00	218.00 ± 10.00	216.91 ± 10.98	Bovy and Rix (2013)	Old	Figure 20, 16,269 G-type dwarfs, SEGUE
8.00 ± 0.40	234.00 ± 14.00	232.83 ± 18.82	Bobylev and Bajkova (2015)	Young	Data from spectroscopic binaries, in Results section
8.00 ± 0.40	230.00 ± 15.00	228.85 ± 19.43	Bobylev and Bajkova (2015)	Young	Data from Calcium stars distance scale
8.00	236.00	234.82 ± 5.02	Aumer and Schönrich (2015)	Young	Page 3171, uses some APOGEE, mostly MW bar stars
8.00	227.50 ± 5.50	226.36 ± 7.30	McGaugh (2016)	Old	From CO and H I clouds, error is average of provided upper and lower error bars
8.00	210.00 ± 10.00	208.95 ± 10.90	Rojas-Arriagada et al. (2016)	Old	Figure 8, from thin disk stars in Gaia-ESO survey
8.00 ± 0.40	230.00 ± 12.00	228.85 ± 17.24	Bobylev and Bajkova (2016)	Young	In Abstract, from RAVE4
8.00 ± 0.20	231.00 ± 6.00	229.85 ± 9.63	Bobylev (2017)	Young	From Gaia DR1 Cepheids
8.00 ± 0.20	219.00 ± 8.00	217.91 ± 10.71	Bobylev and Bajkova (2017)	Young	From Gaia DR1 OB stars
8.01 ± 0.44	202.00 ± 4.00	200.74 ± 12.48	Avedisova (2005)	Old	From 270 star forming regions
8.20 ± 0.70	215.00 ± 24.00	208.71 ± 29.67	Nikiforov (2000)	Old	From 5 H I data sets (Nikiforov and Petrovskaya, 1994) and 2 CO cloud catalogs (Brand and Blitz, 1993), differs from McGaugh (2016)
8.20	238.00	231.03 ± 4.93	Portail et al. (2017)	Old	Section 3.4, from red clump stars.
8.24 ± 0.12	236.50 ± 7.20	228.46 ± 9.12	Rastorguev et al. (2017)	Old	In Abstract, error from quadrature addition of error on Θ_0 and range
8.30 ± 0.25	233.00 ± 11.35	223.46 ± 13.66	Küpper et al. (2015)	Old	From Palomar 5 globular cluster, page 20, error is average of upper and lower provided error bars
8.30 ± 0.20	236.00 ± 6.00	226.33 ± 9.29	Bobylev et al. (2016)	Young	In Abstract, from MWSC open-clusters catalog
8.34 ± 0.16	240.00 ± 6.00	229.06 ± 8.72	Huang et al. (2016)	Old	From LAMOST/LSS-GAC and SDSS/SEGUE and SDSS-III/APOGEE, differs from Bovy and Rix (2013); little overlap with Aumer and Schönrich (2015)
8.40 ± 0.40	224.00 ± 12.50	212.27 ± 16.22	Koposov et al. (2010)	Old	From SDSS photometry and spectrometry, USN0-B astrometry, and Calar Alto telescope

atic rotation of the Milky Way; these are the ones we use to estimate Θ_0. We find that this collection of 18 measurements is somewhat non-Gaussian so a median statistics analysis (Gott et al., 2001) is needed for a more reliable estimate of Θ_0. Using median statistics we find $\Theta_0 = 219.70\ ^{+6.67}_{-7.43}\ ^{+8.77}_{-10.75}$ $km\,s^{-1}$ (1σ and 2σ error bars) which for most purposes can be summarized as $\Theta_0 = 220 \pm 7 \pm 10$ $km\,s^{-1}$. Given the extent to which our data compilation is only mildly non-Gaussian, it is likely that undiscovered systematic errors will not significantly change these estimates and $\Theta_0 = 220 \pm 10$ $km\,s^{-1}$ (2σ error) probably provides the most reliable estimate.

In § 3.2 we discuss our compilation of recent independent Θ_0 measurements and how it differs from those of V17 and dGB17. In § 3.3 we summarize the central estimates statistics and the tests of Gaussianity. We present and discuss our results § 3.4. We conclude in § 3.5.

3.2 Θ_0 Data Compilation

Table 3.1 lists the Θ_0 data we use in our analyses here. These are from measurements published in or after 2000 and we believe this is an exhaustive list of all such independent measurements.

In all cases the angular velocity $\omega_{0i} = \Theta_{0i}/R_{0i}$ was what was measured, so we list $\Theta_{0i} \pm \sigma_{\Theta_{0i}}$ and $R_{0i} \pm \sigma_{R_{0i}}$ in this table. $\sigma_{\Theta_{0i}}$ and/or $\sigma_{R_{0i}}$ are not listed in Table 3.1 if these are not given in the cited reference. Using $R_0 \pm \sigma_{R_0} = 7.96 \pm 0.17$ kpc, we compute the rescaled

$$\Theta_{0i}^{\text{res}} \pm \sigma_{\Theta_{0i}}^{\text{res}} = \frac{R_0 \Theta_i}{R_{0i}} \left(1 \pm \sqrt{\left(\frac{\sigma_{R_0}}{R_0}\right)^2 + \left(\frac{\sigma_{\Theta_{0i}}}{\Theta_{0i}}\right)^2 + \left(\frac{\sigma_{R_{0i}}}{R_{0i}}\right)^2} \right) \tag{3.1}$$

and list these in column 3 of Table 3.1.[1]

It has been known for quite a while now that Θ_0 measured using different tracers differ (Avedisova, 2005; Roman, 1950, 1952; Yuan et al., 2008; dGB17, and references therein). We categorize the measurements listed in Table 3.1 into either Old or Young (one publication uses both types of tracers and is listed as Both). Old tracers include CO and H I gas clouds and are thought to better reflect the systematic rotation of the Galaxy while Young tracers such as Cepheids are believed to have velocities that are contaminated by "peculiar" motions. In Table 3.1 we have $18 + 1$ Old measurements and $10 + 1$ Young ones.

Unlike the measurements listed in Table 3.1, the collections compiled by V17 and dGB17 include non-independent data points. In their analyses dGB17 and RD18 consider different subsets of data, based on tracer type and/or year of publication, but like V17 they also do not study a compilation of independent measurements. This lack of independence can bias results. Here we have invested significant effort in compiling a collection of independent Θ_0 measurements published during 2000-2017.

[1] More properly one would use the rescaled angular velocities in the analysis and then convert the resulting angular velocity central value to a linear velocity central value. However, the uncertainty on R_0 is small and so results from the two different approaches will only differ slightly.

Table 3.2 Rescaled Θ_0 (in $\mathrm{km\,s^{-1}}$) Central Estimates and Errors

Statistic	All Tracers	Old Tracers	Young Tracers
Median	$226.35\,^{+2.50}_{-2.89}\,^{+2.72}_{-8.44}$	$221.58\,^{+4.79}_{-7.43}\,^{+6.89}_{-12.63}$	$228.85\,^{+3.98}_{-2.33}\,^{+9.09}_{-2.76}$
1σ range	$223.46 - 228.85$	$214.15 - 226.36$	$226.52 - 232.83$
2σ range	$217.91 - 229.06$	$208.95 - 228.46$	$226.09 - 237.94$
Weighted Mean	226.37 ± 1.85	222.01 ± 1.99	230.05 ± 3.09
1σ range	$224.52 - 228.22$	$222.28 - 226.71$	$226.95 - 233.14$

Table 3.3 N_σ KS test results for rescaled Θ_0

	$N^{\mathrm{med}}_{\sigma\Theta_0}$			$N^{\mathrm{wm}}_{\sigma\Theta_0}$		
Type	PDF	p[a]	S[b]	PDF	p[a]	S[b]
All	Gaussian	0.65	0.74	Gaussian	0.61	0.76
	Cauchy	0.85	0.45	Cauchy	0.83	0.47
	$n = 2$ Student's t	0.76	0.58	$n = 2$ Student's t	0.73	0.60
	Laplace	0.79	0.68	Laplace	0.76	0.70
Old	Gaussian	0.86[c]	1.04	Gaussian	0.95	1.03
	Cauchy	0.81	0.71	Cauchy	0.91	0.72
	$n = 28$ Student's t	0.86[d]	1.03	$n = 32$ Student's t	0.95	1.02
	Laplace	0.84	1.04	Laplace	0.92	1.03
Young	Gaussian	0.99	0.35	Gaussian	0.99	0.38
	Cauchy	0.99	0.18	Cauchy	0.99	0.20
	$n = 2$ Student's t	0.99	0.23	$n = 2$ Student's t	0.99	0.26
	Laplace	0.99	0.27	Laplace	0.99	0.30

[a] The probability (p-value) that the input data doesn't not come from the PDF.
[b] The scale factor S that maximizes p.
[c] More precisely, $p = 0.863$.
[d] More precisely, $p = 0.862$.

3.3 Analyzing Θ_0

We provide in Table 3.2 the central estimate statistics for the data listed in column 3 of Table 3.1. In Table 3.2, column 2 shows the median (with 1σ and 2σ error ranges) and weighted mean results for all 28 values. Column 3 shows the results of only analyzing the 18 Old tracer references. Column 4 shows the results of the 10 Young tracer types.

3.4 Θ_0 Error Distributions

While Table 3.3 shows the highest probabilities for C2 tracer types, with all probabilities $p \geq 0.99$, the scale factors for all these PDFs are very non-Gaussian with all of them having 1σ ranges requiring $|X| \leq 0.5$. The All tracers compilation is also fairly non-Gaussian.

For the C1 tracers collection with the median as the central estimate, $p = 0.86$ while

$S = 1.04$ for the Gaussian PDF, indicating not unreasonable consistency with Gaussianity. This is also supported by the weighted mean result for the Gaussian PDF. Together these results indicate that the weighted mean summary for Θ_0 is slightly less appropriate[2] than our median statistics one of $\Theta_0 = 221.58\ ^{+4.79}_{-7.43}\ ^{+6.89}_{-12.63}$ km s^{-1} (1σ and 2σ errors), which for most purposes can be taken to be $\Theta_0 = 220 \pm 6 \pm 10$ km s^{-1}. In summary, for practical purposes, we find at 1σ:

$\Theta_0 = 222 \pm 6$ km s^{-1}

$R_0 = 7.96 \pm 0.17$ kpc

$\omega_0 = \Theta_0/R_0 = 27.9 \pm 1.0$ km s^{-1} kpc^{-1}

where the angular speed ω_0 error is determined by adding the fractional uncertainties of Θ_0 and R_0 in quadrature.

Table 1 of V17 lists 28 measurements of Θ_0 from mid-2012 to 2017. V17 arrives at a Θ_0 close to 230 km s^{-1}: median value $\Theta_0^{\rm med} = 232$ km s^{-1}, weighted mean value $\Theta_0^{\rm wm} = 228 \pm 2$ km s^{-1}, and an arithmetic mean value $\Theta_0^{\rm mean} = 229 \pm 3$ km s^{-1}. He recommends the set of Galactic constants:

$\Theta_0 = 230 \pm 3$ km s^{-1}

$R_0 = 8.0 \pm 0.2$ kpc

$\omega_0 = \Theta_0/R_0 = 29 \pm 1$ km s^{-1} kpc^{-1}

We emphasize that several of the V17 Table 1 data are repeats of prior publications, big offenders being masers, OB stars, and Cepheids. Less than half of V17 Table 1 measurements are included in our list of independent measurements. V17 also does not distinguish between C1 and C2 tracer measurements of Θ_0. These are probably why the V17 Θ_0 differs from our estimate.

dGB17 on the other hand do note that C1 tracers provide a better estimate of Θ_0 and their recommended set of Galactic constants are (when their statistical and systematic errors are added in quadrature):

[2] In fact, the data errors are quite consistent with Gaussianity, however the C1 weighted mean 1σ error is ± 2 km s^{-1}. This is quite small and at this level there are a number of corrections that must be accounted for in measurements of Table 3.1. We hence choose to use the median statistics over the weighted mean results.

$\Theta_0 = 225 \pm 10$ km s^{-1}

$R_0 = 8.3 \pm 0.4$ kpc

$\omega_0 = \Theta_0/R_0 = 27.1 \pm 1.8$ km s^{-1} kpc^{-1}.

While their C1 tracers compilation includes non-independent data points, dGB17 add on rather large undiscovered systematic errors and so their results are not inconsistent with our results. We note, in particular, as described in C18, that their estimate of R_0 is based on a very small set of data points (that are also not all independent). We emphasize that from our analysis of the Gaussianity of our R_0 and Θ_0 compilations, here and in C18, we do not see strong evidence for large undiscovered systematic errors that dGB17 advocate for.

RD18 use 139 Galactic rotation speed values, 137 of which are from the online database of dGB17. Included are a number of non-independent measurements. For both median and weighted mean statistics they use the error distribution form of eq. (1.11) and analyze the full collection of data as well as various subsets. RD18 were the first to realize that the dGB17 Θ_0 data (and subsets) was non-Gaussian, but as they didn't discard non-independent measurements (as we have done) they found the data to be more non-Gaussian than we do. From a median statistics analysis of the full data set they recommend:

$\Theta_0 = 219.65$ km s^{-1}

$R_0 = 8.3$ kpc

$\omega_0 = \Theta_0/R_0 = 26.46$ km s^{-1} kpc^{-1}.

They do not derive an R_0 value, instead they use that estimated by dGB17. They also do not estimate an error for Θ_0.

3.5 Conclusion on Θ_0

The data listed in Table 3.1 is the first compilation of independent Θ_0 measurements published during 2000-2017. Given the mild non-Gaussianity of the Old tracer measurements, we favor a median statistics value of $\Theta_0 = 219.70\,^{+6.67}_{-7.43}\,^{+8.77}_{-10.75}$ km s^{-1} (1σ and 2σ errors). For most purposes this can be summarized as $\Theta_0 = 220 \pm 7 \pm 10$ km s^{-1}. Given that the measured non-Gaussianity is mild, we believe most current Θ_0 error bars are reasonable and that at present there is no strong evidence for large undiscovered systematic errors. In summary our recommended set of Galactic constants, with 1σ error bars,

$$\Theta_0 = 220 \pm 7 \text{ km s}^{-1}$$

$$R_0 = 7.96 \pm 0.17 \text{ kpc}$$

$$\omega_0 = \Theta_0/R_0 = 27.6 \pm 1.1 \text{ km s}^{-1} \text{ kpc}^{-1}$$

are probably the most reliable.

Chapter 4

Conclusions

It is my belief that the method of analysis we have used provide us the best way of determining central estimates and error bars of compilations of independent data that are Gaussianly distributed. Our analysis focused on two important fundamental constants used in inter- and extra-galactic calculations: R_0 (the radial distance from the Sun to the Galactic center) and Θ_0 (the Galactic rotational velocity at R_0).

The error distributions of a compilation of 28 (since 2011) independent measurements of R_0 are wider than a standard Gaussian and best fit by an $n = 4$ Student's t probability density function. Given this non-Gaussianity, the results of our median statistics analysis, summarized as $R_0 = 8.0 \pm 0.3$ kpc (2σ error), probably provides the most reliable estimate of R_0. The unsymmetrized value for R_0 is $R_0 = 7.96\ ^{+0.24}_{-0.30}$ kpc (2σ error). A complete collection of 18 recent (since 2000) measurements of Θ_0 indicates a median statistics estimate of $\Theta_0 = 220\ \pm 10$ km s^{-1} (2σ error) as the most reliable summary. The resulting error distribution of this data set is only mildly non-Gaussian, much less so than that obtained from R_0. These measurements use tracers that are believed to more accurately reflect the systematic rotation of the Milky Way. Unlike other recent compilations of R_0 and Θ_0, our collections includes only independent measurements. For 1σ error bars, Galactic constants $\Theta_0 = 220 \pm 7$ km s^{-1}, $R_0 = 7.96 \pm 0.17$ kpc, and $\omega_0 = \Theta_0/R_0 = 27.6 \pm 1.1$ km s^{-1} kpc^{-1} probably provide the most carefully studied and reliable summary estimates.

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Appendix A

Derivation of Error Distribution for Weighted Mean

While eq. (1.13) may be well known to practitioners, C18 was unable to find a derivation of it, and so this derivation is provided here.

For $i = 1, 2, .., N$ measurements M_i with individual errors σ_i, modeled to be Gaussian about a central estimate with $M_{\rm CE}$ which itself has uncertainty $\sigma_{\rm CE}$, we define an uncertainty-normalized difference

$$N_{\sigma_i} = \frac{M_i - M_{\rm CE}}{\sqrt{\sigma_i^2 + \sigma_{\rm CE}^2}}. \tag{A.1}$$

This is the number of standard deviations a particular measurement differs from the central value. If we use a central estimate like the weighted mean, we can again standardize an $N_\sigma^{\rm wm}$. We begin by defining the weighted mean and it's error:

$$M_{\rm wm} = \frac{\sum_{i=1}^{N} M_i/\sigma_i^2}{\sum_{i=1}^{N} 1/\sigma_i^2} \tag{A.2}$$

and (Podariu et al., 2001)

$$\frac{1}{\sigma_{\rm wm}^2} = \sum_{i=1}^{N} \frac{1}{\sigma_i^2}. \tag{A.3}$$

However, a problem arises depending on how correlated M_i and $M_{\rm CE}$ are. Defining D_i

that can be normalized to find a standardized N_σ where

$$D_i = M_i - M_{\text{wm}}, \tag{A.4}$$

we can calculate the variance of this quantity to later use for normalization

$$\text{Var}(D_i) = \text{Var}(M_i - M_{\text{wm}}). \tag{A.5}$$

If M_i and M_{wm} are independent, the variance is distributed as

$$\text{Var}(aX + bY) = a^2\text{Var}(X) + b^2\text{Var}(Y) \tag{A.6}$$

and it is this case that yields the well-known result of adding errors in quadrature. As they are correlated though, let's try a different approach. The variance becomes

$$\text{Var}(D_i) = \text{Var}\left(M_i - \frac{\sum_{j=1}^{N} M_j/\sigma_j^2}{\sum_{k=1}^{N} 1/\sigma_k^2}\right) \tag{A.7}$$

which can be rearranged as

$$\text{Var}(D_i) = \text{Var}\left[\left(1 - \frac{1/\sigma_i^2}{\sum_{k=1}^{N} 1/\sigma_k^2}\right) M_i - \frac{\sum_{j\neq i}^{N} M_j/\sigma_j^2}{\sum_{k=1}^{N} 1/\sigma_k^2}\right]. \tag{A.8}$$

Here, the assumption is made that the measurements were made independently. Using eq. (A.6), the above becomes

$$\text{Var}(D_i) = \left(1 - \frac{1/\sigma_i^2}{\sum_{k=1}^{N} 1/\sigma_k^2}\right)^2 \text{Var}(M_i) + \frac{\sum_{j\neq i}^{N} \text{Var}(M_j)/\sigma_j^4}{(\sum_{k=1}^{N} 1/\sigma_k^2)^2} \tag{A.9}$$

which can be simplified by opening the squares and by sending $\text{Var}(M_i)$ into the summation over N

$$\text{Var}(D_i) = \left(1 - 2\frac{1/\sigma_i^2}{\sum_{k=1}^{N} 1/\sigma_k^2}\right) \text{Var}(M_i) + \frac{\sum_{j=1}^{N} \text{Var}(M_j)/\sigma_j^4}{(\sum_{k=1}^{N} 1/\sigma_k^2)^2}. \tag{A.10}$$

Now we make the assumption that the M_i are Gaussianly distributed with variance σ_i^2, an assumption made even in the case of adding errors in quadrature, as in Bailey (2017). It follows then that

$$\mathrm{Var}(D_i) = \left(1 - 2\frac{1/\sigma_i^2}{\sum_{k=1}^{N} 1/\sigma_k^2}\right)\sigma_i^2 + \frac{\sum_{j=1}^{N} 1/\sigma_j^2}{(\sum_{k=1}^{N} 1/\sigma_k^2)^2} = \sigma_i^2 - \sigma_{\mathrm{wm}}^2 \tag{A.11}$$

This gives the new equation that is better suited for correlated values,

$$N_{\sigma_i} = \frac{D_i}{\sqrt{\mathrm{Var}(D_i)}} = \frac{M_i - M_{\mathrm{wm}}}{\sqrt{\sigma_i^2 - \sigma_{\mathrm{wm}}^2}} \tag{A.12}$$

which may look familiar to some as the pull of a Gaussian measurement M_i from the average value M_{CE} determined from the set of measurements.

It should be noted that the median and arithmetic mean determined from the measurements are also correlated with the data and in a more careful analysis this should be accounted for. It may be possible to account for the median's correlation to the data using a Monte Carlo analysis (this requires knowledge of the data distribution which depends on the central estimate in question). We hope to discuss this elsewhere.