# Optimal Sizing and Placing of Distributed Generation in Distribution Networks

by

Fatehullah Nassery

B.S., Kabul University, 2012

A Report

submitted in partial fulfillment of the requirements for the degree

MASTER OF SCIENCE

Department of Electrical and Computer Engineering
College of Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

2017

Approved by:

Major Professor
Anil Pahwa

## Copyright

# Abstract

Due to the ongoing changes in the structure of the electricity markets, distribution networks have developed an appealing potential for housing distributed generation (DG). In order to make the most out of the present distribution network, this project report verifies the results and method developed in a paper (Optimal Allocation of Embedded Generation on Distribution Networks) by A. Kean and M. O'Malley, which discusses an efficient way of incorporating DG in the current power system. The methodology under consideration elaborates on how certain constraints should be adopted that will lead toward optimally sizing and placing DG in the network under examination. Along with that, the effect of voltage rise and short circuit current are observed which shows that a certain allocation to some buses will cause a sudden rise in voltage and short circuit levels throughout the network. Furthermore, the adopted methodology with its relative constraints is solved using linear programming. Linear programming provides a more accurate allocation than its heuristic counterparts do when it comes to embedding DG in smaller networks. The adopted methodology is then applied to a section of the Irish rural distribution network and the results pinpoint that appropriate placement of the DG will pave the way toward higher levels of penetration. The results obtained showed the same pattern as those recorded in the aforementioned source paper, there were only minor differences that are the result of using different software's than those that were used by the authors of the paper.

# Table of Contents

# List of Figures

# List of Tables

# Chapter 1 - Introduction

Small scale generating plants, which are placed in the distribution network or on the consumer side are known today by various names, distributed generation, decentralized generation, dispersed generation or embedded generation [1]. Today this method of power generation is favored for numerous factors and among those factors, global warming, and an alternative for fossil fuel and change in the power system topology are worth mentioning in here.

Nowadays the rise in demand for penetration of distributed generation has compelled the electricity network to move from its passive form to active form, a network that has the capability of accommodating distributed generation [2]. Apart from the above-mentioned factors, embedded or distributed generation offers advantages like lowering peak operating costs, better service to the end user, and reduction in further expenses for expansion of transmission and distribution network, as well as penetration of diverse energy resources into the system [1]. In the same way, placing optimal sizes of distributed generation units in optimal places can have significant impacts on the voltage level and loss levels in distribution networks. On the other hand, placing a non-optimal size of distributed generation in a non-optimal place may compromise system stability and reliability, eventually leading to higher levels of losses than the network had prior to installation of the distributed generation.

Distributed generation can be used both in an isolated and integrated manner, isolated means that each customer produces certain level of power to supply its own demand, but when it comes to integrated manner, apart from providing power for their own load, customers flood power into the distribution system as well [3]. Besides that, utilities have not reached into consensus on the size and voltage level of the distributed generation [4]. Therefore, the size and voltage level of the distributed generation differs from region to region and country to country. For instance France define a capacity of 40MW at 225kV, Portugal maintains a capacity limit of 10MW, but when comes to Australia, they have installed the largest capacity which is 130MW at 132kV in their grid.

Apart from the size and voltage level of the distributed generation, its type is also an eye-catching topic when it comes to electricity market. In today's electricity market, types of DG which are used for power generation encompass combined cycle turbines, hydro, combustion turbines, diesel and, last but definitely the most important are natural gas turbines, wind turbines and solar panels [5].

For placing an optimal size of distributed generation in an optimal location various models and methodologies are proposed and evaluated. These methods vary in approach, some have considered tackling the problem of sizing and placing using analytical methods, while others have gone for methods like numerical (Linear Programming, Sequential Quadratic programming, Nonlinear Programming and Ordinal Optimization), heuristic ( Genetic Algorithm, Particle Swarm Optimization, Ant Colony Optimization, Bee Colony Optimization, Differential Evolution) and evaluative [1]. In [3], a methodology for placement of distributed generation is developed in order to minimize losses and acquire acceptable levels of voltage, and is solved using genetic algorithms (GA). The methodology developed in [6], maximizes energy harvest from a certain area while considering the amount of energy resources available and is solved using linear programming (LP). In [7], a probabilistic methodology for optimally allocating wind-based DG units for reduction of annual energy losses has been developed and is solved using mixed integer non-linear programming (MINLP). [8] Has used a hybrid method of Tabu search and genetic algorithms (GA) to allocate DG and capacitor banks simultaneously in the network to take care of the reactive power. [9] has dealt with the problem of sizing and placing DG using an artificial bee colony algorithm; they used the ABC method to find the optimal place, size as well as power factor.

The approach used in this project report is based on the method developed in [2], and is aimed to reproduce the results obtained by the authors of [2], for optimally allocating and sizing distributed generation in distribution networks and taking into consideration the voltage and short circuit interdependency of buses, and is solved using linear programming (LP). LP is used to optimize a problem in which both the objective function and its relevant constraints are of the linear form.

# Chapter 2 - Methodology

The main target is to maximize distributed generation penetration subject to the prevailing constraints by using the method developed in [2]. Therefore, as per the developed method, the maximum size of DG must be placed in such a way that none of the technical constraints are undermined. Therefore, the objective function will look like,

$$f = \sum_{i=1}^{N} P_{DG\ i}\,, \qquad i \quad \forall\ N$$

In the objective function, $P_{DG\ i}$ stands for the capacity of distributed generation connected into the $i$th bus and $N$ is the total number of buses. Not to mention, it is assumed that only one generating unit can be connected at a time to a certain bus. Moreover, the objective function is maximized subject to the following constraints.

$$I_i < I_i^{rated}\,, \qquad i \quad \forall\ N \tag{1}$$

Constraint (1) is a stand-alone constraint in which $I_i$ is the current from generation unit $i$ flowing toward bus $i$. In the same way, $I_i^{rated}$ is the rated ampacity of line that connects DG unit to its respective bus.

$$\sum_{i=1}^{N} P_{DG\ i} - \sum_{i=1}^{N} P_{min\ i} \le P_{Tr\ cap}\,, \qquad i \quad \forall\ N \tag{2}$$

Constraint (2) will make sure that the flow of power from the utility to the transmission transformer does not rise beyond the capacity of the transformer $P_{Tr\ cap}$. Furthermore, it means that total connected generation minus the summer valley load ($P_{min\ i}$) must not exceed the capacity of the transformer.

$$\sum_{i=1}^{N} \delta_{jTx} P_{DG\ i} + \alpha_{Tx} \le SCC_{rated} , \qquad i \quad \forall\ N \tag{3}$$

In the above constraint, $\delta_{jTx}$ is the effect that power penetration on other buses reflects on the transmission bus's short circuit current. $\alpha_{Tx}$ is the short circuit current at the transmission bus prior to the installation of DG. On top of that, $SCC_{rated}$ is the rated maximum amount of current that the switchgear can interrupt when a fault rises.

$$\frac{P_{DG\ i}}{SC_{mva\ i} \bullet \cos\theta} \times 100 < 10\% , \qquad i \quad \forall\ N \tag{4}$$

Constraint (4) represents the short circuit ratio in each of the buses which is the ratio of injected power at that bus divided by the short circuit MVA at that bus, and it should not exceed beyond 10%. $SC_{mva\ i}$ Stands for short circuit MVA and the power factor is represented by $\cos\theta$.

$$SC_{mva\ i} = \alpha_i + \sum_{j=1}^{N} \delta_{ji} P_{DG\ j} , \qquad i \neq j , \qquad i \quad \forall\ N \tag{5}$$

Looking into constraint (5), $\alpha_i$ represents the initial short circuit MVA at each of the buses prior to installation of the DG units, and this is considered as the short circuit level at winter night. In the same manner $\delta_{ji}$ represents the effect of power injection at other buses ($j$th) on the short circuit level at the $i$th bus. By subbing equation (5) into equation (4) the relation become linear and we get the following equation (6).

$$P_{DG\ i} - 0.1\cos\theta \sum_{j=1}^{N} \delta_{ji} P_{DG\ j} \le 0.1\cos\theta\, \alpha_i , \qquad i \neq j , \qquad i \quad \forall\ N \tag{6}$$

For this constraint, $\alpha_i$ represents the short circuit level of the buses at a summer night valley level, which is provided by the utility company. In order to linearize $\cos\theta$, it will be replaced by power factor (PF) in the above equation (6) and it is considered constant.

$$V_{DG\ i} = V_i + \frac{R\ P_{DG\ i} + X\ Q_{DG\ i}}{V_i} + j\frac{X\ P_i + R\ Q_i}{V_i}\ , \qquad i \quad \forall\ N \tag{7}$$

In constraint (7), a relationship among the bus voltage $V_i$ and the voltage at the generator $V_{DG\ i}$ is represented. $R$ and $X$ are the parameters of the line through which DG is connected to its respective bus.

$$V_{min\ i} < V_i < V_{max\ i}\ , \qquad i \quad \forall\ N \tag{8}$$

Constraint (8) will make sure that voltage at each bus remains in an acceptable envelope, and that envelope at its maximum value reaches to 10% higher than the normal bus voltage.

$$\mu_i\ P_{DG\ i} + \ \beta_i + \sum_{j=1}^{N} \mu_{ji}\, P_{DG\ j}\ \leq V_{max\ i}, \qquad i \neq j\ , \qquad i \quad \forall\ N \tag{9}$$

In (9), $\mu_i$ stands for the effect that power injection at bus $i$ has on the voltage level at bus $i$. In the same way, $\mu_{ji}$ stands for the effects that power injection at other buses has on voltage level at bus $i$. While $\beta_i$ will represent the voltage at bus $i$, prior to the installation of distributed generation. So, this constraint makes sure that when DG is integrated into the system, voltages level must not rise beyond a predefined safe value at the bus.

$$P_{DG\min i} \leq P_{DG\ i} \leq P_{DG\ max\ i}, \qquad i \quad \forall\ N \tag{10}$$

As per constraint (10), the capacity of installed unit at each bus is limited to a certain amount. If there are resources pre-installed in a certain bus, then $P_{DG\min i}$ will start its initial value from there and rise to a maximum allowable capacity.

# Chapter 3 - Test System

The system under consideration is a rural network outside Dublin, Ireland. Due to the characteristics of rural networks, they are weak, that means they have higher levels of impedance, which leads to lower amounts of short circuit. In Ireland DG is mostly connected at a voltage of 38kV, which is counted as medium voltage. Furthermore, the dominant constraint in placing distributed generation in radial networks is voltage rise, because injection of each power unit will lead to rise in voltage levels system wide. Moreover, another factor is the impedance of long lines, which connects buses in the rural distribution network, this leads to lower voltage levels at distant buses from the distribution station. Therefore, the voltage at the substation bus is usually assigned higher than the normal (38kV) as (41kV), this is to make sure that customers at the far end buses have voltage at the standard range. In the meantime, this voltage rise will reduce the assignment of distributed generation system wide. On the other hand, if voltage at the substation bus is kept at nominal level, it will pave the way for a higher total penetration of the DG, but if DG is not present, the voltage at the distant buses of the system will decrease beyond standard levels and the customers will experience poor service.

DG is connected in various ways into the network and among them direct feed to 110kV/38 station and direct feed to 38kV medium voltage and as a T-connection to an existing line. However, among the mentioned installation methods, its connection to 38kV system is more common when the size of the overall project is somewhat smaller. In the system under evaluation, the connected units are induction generators with a fixed power factor of 0.95.

Figure 4.1 depicts the topology of the test system with its five buses, which is a typical rural distribution network in Ireland.

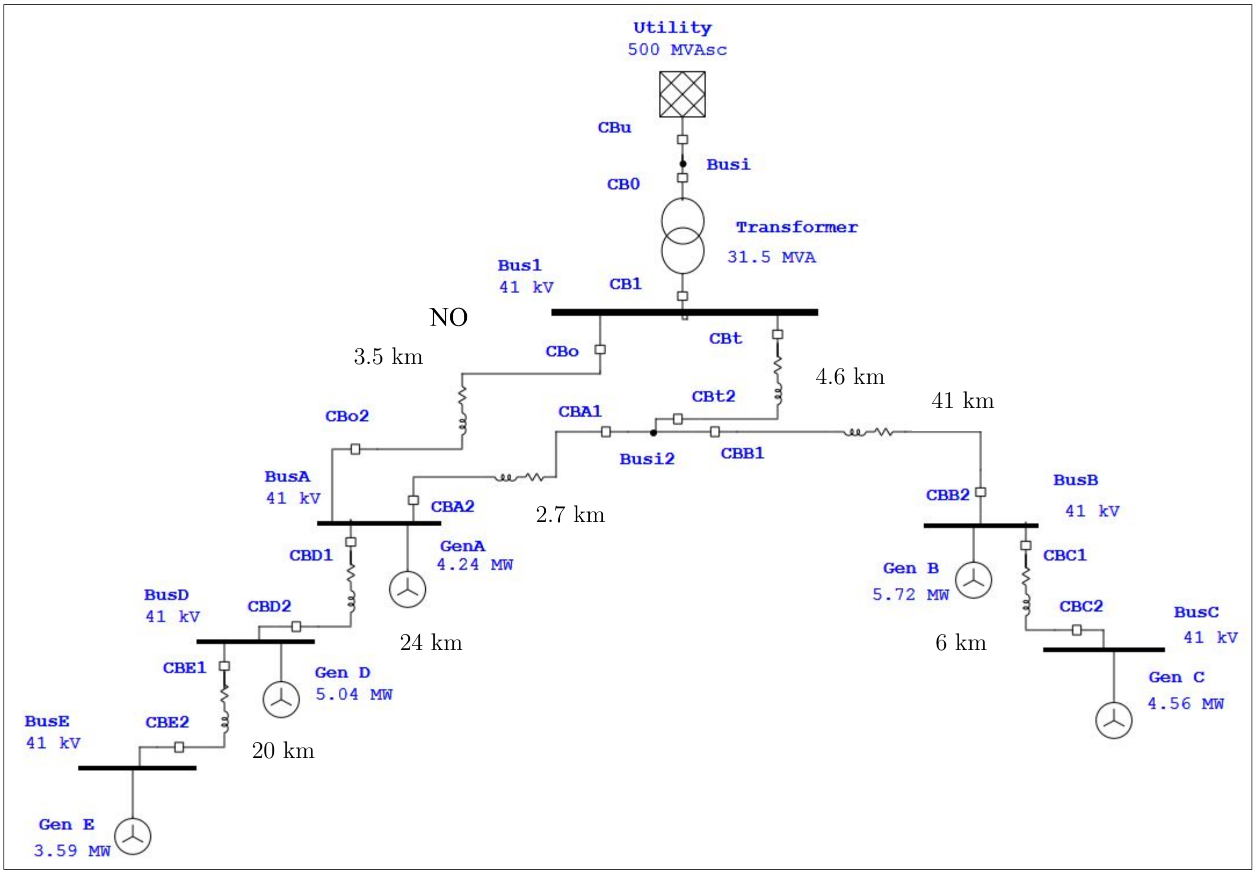

Figure 3.1: Irish Rural Distribution Network

In the depicted network above, all the buses are located at a certain geographical distance from one another. The 3.5km line, which binds bus A to the transmission bus, is considered to be normally open (NO) or out of service.

Throughout the system under consideration, voltages and short circuit levels are interdependent to each other. This means that if power is injected to a bus it will have its respective effects on voltage and short circuit levels of the other buses. Interdependence between the system buses is related to several factors, among those factors distance between the buses, conductor type and vicinity to transmission bus are mentionable in here.

Firstly, the impedance of a conductor connecting buses has a major effect on the voltage rise and fall of the interconnected buses. Meaning, the higher the impedance of a line connecting two buses the more significant will be the amount of voltage dip between them and the same will hold for the other way around.

Second factor is closeness to the transmission bus, the closer a bus is located to the transmission stations bus the higher will be its short circuit level and distance among other buses will also contribute toward lower and higher short circuit effects. Based on the impedance levels, power networks can be divided into two divisions: strong and weak. Strong networks have low impedance levels, which leads to higher short circuit levels. When it comes the weak networks, customers are located geographically away from the substation and that in turn means that they are connected via longer lines to the distribution systems, and long lines mean higher amount of impedance and lower short circuit level.

Third factor of interdependency is vicinity to transmission station and its effect on the voltage level of a bus. As per this factor, the closer a bus is located to the transmission bus the higher will be its voltage this is due to the radial characteristics of distribution networks.

# Chapter 4 - System Data

The first step in analyzing the system for DG placement and sizing is, load flow analysis. ETAP 16.0 [10] software is used to accomplish the task of load flow analysis. The main purpose was to find the voltages at each bus under minimum load conditions and prior to the installation of DG units, the results are shown in table 4.1.

Table 4.1: Bus Voltages

| Bus | Voltage (kV) |
|---|---|
| A | 40.9 |
| B | 39.51 |
| C | 39.40 |
| D | 40.30 |
| E | 39.99 |

Each of the individual buses in the system influence both the voltage and short circuit levels of each other, once a unit of power is injected to any of the buses its effects will be obvious on the system's balance. To find the voltage interdependency among the buses a single MW of power is injected to a bus and then by running load flow analysis we observe how much it effected voltage levels on the other buses. Additionally, power penetration to a certain bus will have more pronounced effect on voltage level of that bus than the rest of the system buses. Eventually, by doing this over for all the buses, the following voltage interdependency values will be produced. Table 4.2 shows voltage interdependency among system buses.

Table 4.2: **Voltage Interdependency among the Buses**[2]

| $\mu$ (kV/MW) | A | B | C | D | E |
|---|---|---|---|---|---|
| A | 0.0053 | 0.008 | 0.007 | 0.0021 | 0.016 |
| B | 0.012 | 0.218 | 0.18 | 0.009 | 0.007 |
| C | 0.012 | 0.191 | 0.238 | 0.009 | 0.007 |
| D | 0.026 | 0.008 | 0.007 | 0.162 | 0.11 |
| E | 0.026 | 0.008 | 0.007 | 0.135 | 0.234 |

In the same manner, power injection has effects of its own on the short circuit levels, system wide. In order to find the short circuit interdependency among the buses, a single unit of power is flooded to a single bus and its results are then observed on the short circuit levels of buses around the system. The results are depicted in Table 4.3.

Table 4.3: **Short Circuit Interdependency between Buses**[2]

| $\delta$ (MVA/MW) | A | B | C | D | E |
|---|---|---|---|---|---|
| A | − | 1.54 | 1.5 | 2.95 | 2.62 |
| B | 0 | − | 3.29 | 0 | 0 |
| C | 0 | 3.63 | − | 0 | 0 |
| D | 0 | 0 | 0 | − | 3.2 |
| E | 0 | 0 | 0 | 1.74 | − |

As shown in Table 5.3, power injection to a bus does not affect its own short circuit level, for that reason it is not calculated.

Power injection at the buses throughout the system has its own effects on the short circuit current level of the transmission bus. That means that a single unit of power injection at any of the buses whether close or far from the transmission bus will have an effect on the current levels at the transmission bus. Table 4.4 depicts the short circuit dependency at the transmission bus.

Table 4.4: **Short Circuit Interdependency of Transmission Bus**[2]

| | Bus A | Bus B | Bus C | Bus D | Bus E |
|---|---|---|---|---|---|
| $\boldsymbol{\delta_{Tx}}$ (kA/MW) | 0.18 | 0.11 | 0.10 | 0.14 | 0.11 |

Using linear programming, the system objective function is solved with respect to the prevailing constraints and the aforementioned data.

# Chapter 5 - Sensitivity Analysis

First of all sensitivity characteristics of the buses system wide are shown. Following that, DG placement and sizing is done using GAMS [11] software, by using the data previously extracted from ETAP software simulations. However, prior to that, we look in to how power injection changes voltage levels in each of the busses. The results are depicted in Figure 5.1 and are generated by multiplying voltage sensitivity of each bus from Table 4.2 by increasing levels of power at that bus.

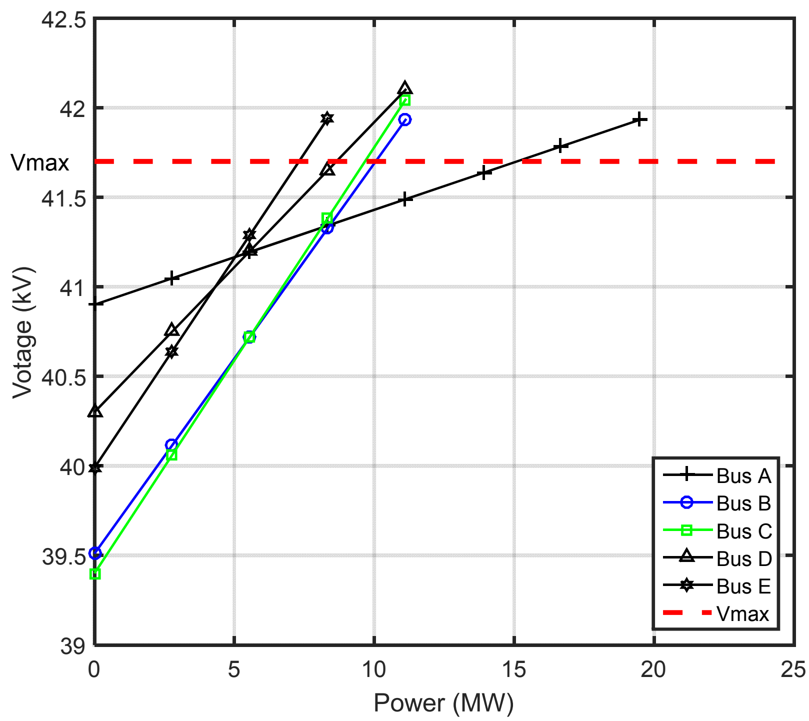


**Figure 5.1: Sensitivity of each individual bus to power injection at the same bus**

As per Figure 5.1, we can deduce that bus A can accommodate higher sizes of DG in comparison to the buses downstream. This is because bus A is located close to the transmission bus, which is also system slack bus, these results in least sensitivity of bus A to power injection.

In the same way, lower initial voltage levels in buses B and C are due to their distance of 41km from bus A and more than that number from the transmission station. To show how voltage levels in a certain bus behaves when the power is injected at bus C, we again use the sensitivity data of bus C from Table 4.2 and by multiplying to increased amount of power at bus C we eventually get the results of Figure 5.2.

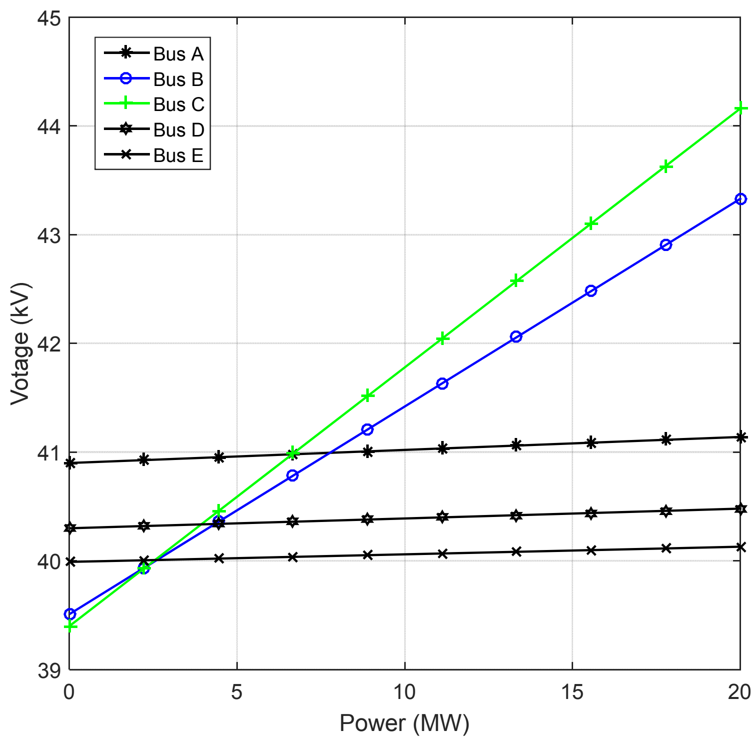


**Figure 5.2: Voltage Sensitivity to power injection at Bus C**

As is observable from Figure 5.2, buses A, D and E are less sensitive to power injection at bus C than the rest of the system buses because of the relatively longer distance among them and buses B and C. However, B and C can influence voltage at each other due to the small distance between them as mentioned in Chapter 3.

Bus D is the other bus which is worth examining for the same purposes. As can be seen from Figure 5.3, again generated using the data from Table 4.2, each of the buses A and D experience pronounced effects when power injection at E is increased. Especially bus

D, due to the relatively shorter distance among them. However, when observing the effects of power injection on buses B and C, we can see that due to the significant distance between them, voltages at bus B and C are almost non-sensitive to power injection at bus E.

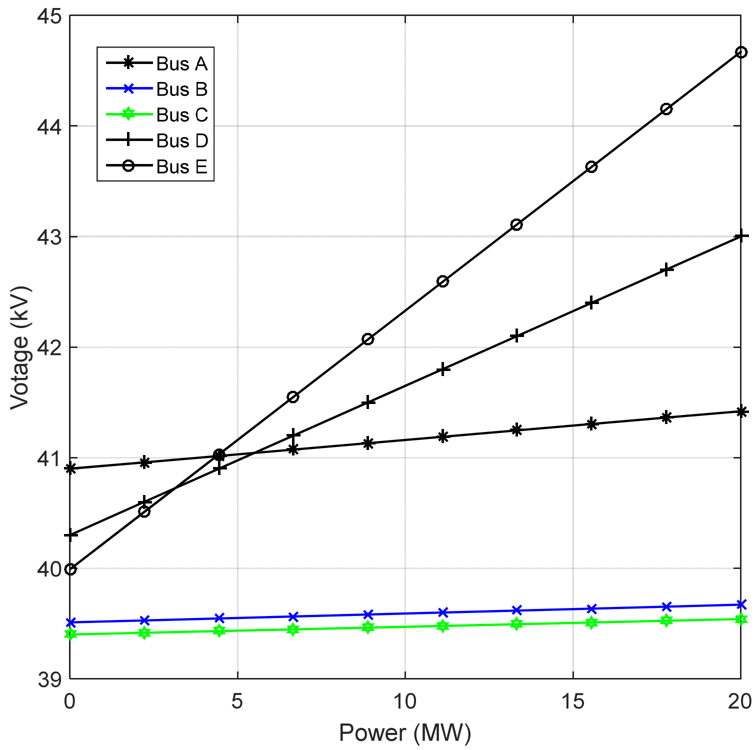


**Figure 5.3: Dependency of Voltage to Power injection at Bus E**

Power penetration at each of the buses also have a fair share of effects on the short circuit current levels in the system. One of most important short circuit current level is that of the transmission bus. The results of Figure 5.4 is obtained by multiplying dependency of the transmission bus to power injection at other buses in Table 4.4. It is observable from Figure 5.4 that the closest bus to the transmission station will have higher influence on the short circuit current and that bus is bus A. Similarly, bus D is relatively closer to the transmission bus, which has some observable effect as can be seen from Figure 5.4. On the contrary, buses B, C and E are slightly more distant from the transmission bus as is obvious from the figure. Figure 5.5 depicts the effects that power injection in bus C can have on the rest of the system. These results are obtained by multiplying short circuit dependency of other buses from Table 4.3 to power injection at bus C.

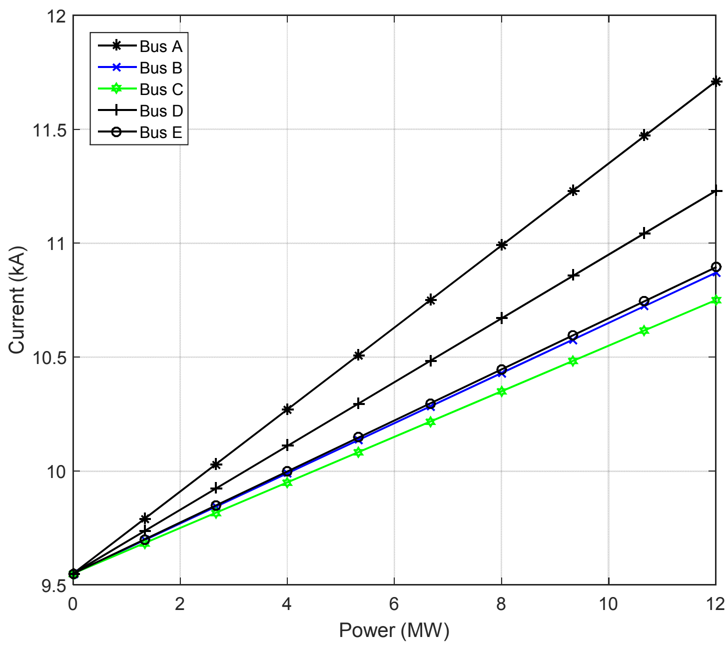


Figure 5.4: Short Circuit Dependency of T-Bus to power injection at other buses

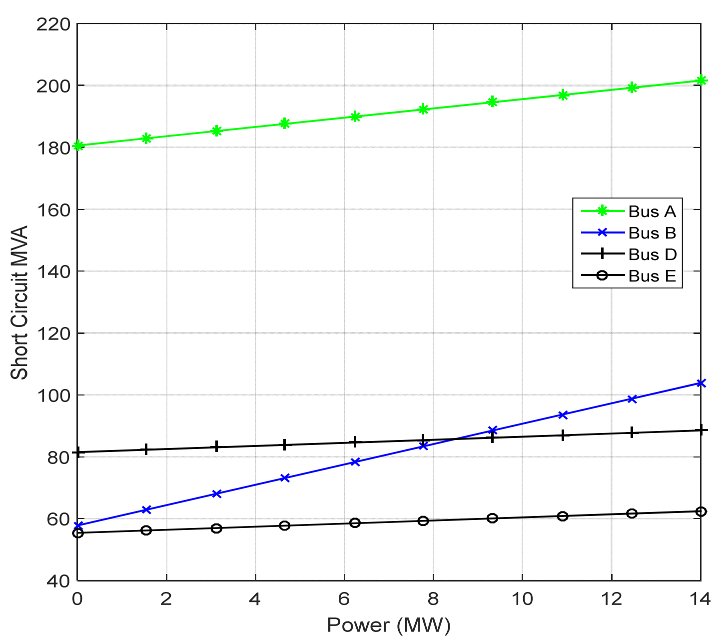


Figure 5.5: Short Circuit Contribution of power injection at Bus C

The sensitivity analysis is done by using the interdependency of buses listed in Tables 4.2, 4.3 and 4.4. Due to their linear characteristics with respect to each other, we are having linear graphs. However, if at each time that we inject power into a certain bus we run the load flow analysis again we will get non-linear graphs due to the fact that load flow model is nonlinear. For instance if we take a look at Figure 5.6, we can observe that by injecting power into bus C, not only voltage at the bus itself does not behave according to a linear model but it also does not have linear effects on the other buses of the system. This is more obvious from voltage rise at bus B.

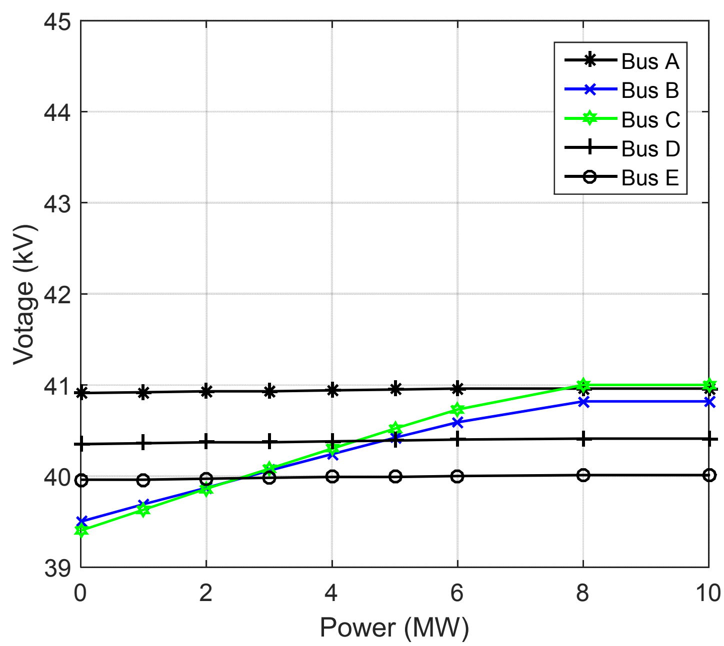


**Figure 5.6 : Voltage Sensitivity of other Buses to Power Injection at Bus C**

It should be noted that each of the points on the graph shows a separate load flow analysis. Similar behavior can be observed once we do load flow by injecting power into any other bus. Figure 5.7, shows how the voltages in the system behave when power is added to bus E. As we can see from this figure, power injection into bus E causes voltage rise at the bus close to it, which is bus D and we can observe that the effect is nonlinear as well.

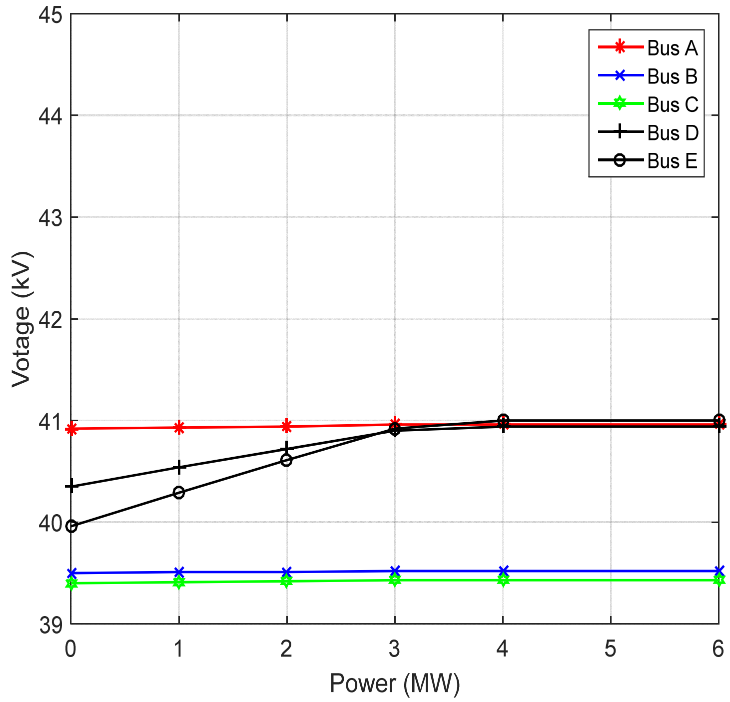


Figure 5.7: Voltage Behavior in the System due to Power Injection at Bus E

# Chapter 6 - Results

The methodology was solved using linear programming, because both our objective function and its prevailing constraints were defined in the form of linear equations. Three cases are examined for sizing and allocating DG in the system under test.

*Case 1: No Prior DG installed in the System:* For this Case, the developed optimization is solved by assigning $P_{DG\min i}$ as zero for all the buses of the network. When executed the results of Table 6.1 are obtained.

Table 6.1: Allocation of Case 1

| Bus | Power (MW) |
|---|---|
| A | 4.24 |
| B | 5.72 |
| C | 4.56 |
| D | 5.04 |
| E | 3.59 |
| Total | 23.15 |

The results show that when there is no prior generation installed at any of the buses, developed algorithm will tend to assign optimal sizes of DG at each of the locations. In order to make sure the results are optimal we look at the allocation of bus B. If we multiply 5.72MW of its integration by the voltage sensitivity of its own as well as the other buses, we get a voltage value of 41.699kV at bus B that is closest to the highest limit of voltage in the bus. The procedure can be applied to the rest of the buses. In the same way, to see its effects of power integration on the short circuit level of the transmission bus we multiply the result of each bus by its respective sensitivity of the transmission bus. They will all contribute 2.948kA to the initial short circuit current of the transmission bus, apart from that it will hit the limit in our constraint (3), which indicated the amount of fault current the switchgear can interrupt safely.

*Case 2: 5.7MW of prior DG is installed at bus E by customers:* In this case, there was a pre-integrated amount of power in bus E of the system, so $P_{DG\min E}$ will be assigned as 5.7MW. After execution, results of Table 6.2 were observed.

Table 6.2: Allocation of Case 2

| Bus | Power (MW) |
|---|---|
| A | 0 |
| B | 0 |
| C | 2.75 |
| D | 2.64 |
| E | 5.70 |
| Total | 11.09 |

From the results obtained, we can see that placing DG prior to planning by the distribution network operator (DNO) will significantly lower penetration levels in the network. Existence of 5.7MW power in bus E will lead to network sterilization which the authors of [2] defines as allocating DG capacity to bus or buses whose voltage and short-circuit levels are most sensitive to power injection, thus no more generation can be placed because the buses will be constrained. For instance, at its optimal value bus E (3.59MW) will increase short circuit MVA of bus D about 11.48MVA. However, when integration levels rise in E, it will contribute a higher amount of 18.24MVA, which is beyond acceptable sensitivity limit. In the same way, 5.7MW in bus E will cause a sudden rise in bus voltage as well, it can contribute (1.709kV) to its own bus E and this in turn will cause higher power flow upstream an cause limitation to DG allocation in other buses.

*Case 3: 6.35MW of Prior Capacity at bus C:* In this case, 6.35MW of prior generation is existent in bus C of the network. Therefore, in the formulation $P_{DG\min C}$ is assigned as 6.35MW. Table 6.3 depicts the results and total power.

The network sterilization effect is easily observable in the above allocations and placements. At its optimal value of 4.56MW, bus C will contribute only 15MVA to the short circuit levels at bus B and this is the maximum acceptable limit. On the contrary, placing 6.36MW in C will contribute 20.9MVA to the short circuit level at bus B and that is more than acceptable. Bus C will in this case will also cause sudden rise it is own voltage level, the 6.36MW will rise its voltage 1.51kV and beyond the effect that optimal allocation will produce. This means that, if there is no prior generation in the system, the developed method will assign the maximum amount network wide without breaching any of the constraints.

Table 6.3: Allocation of Case 3

| Bus | Power (MW) |
|---|---|
| A | 0 |
| B | 3.95 |
| C | 6.35 |
| D | 0 |
| E | 4.86 |
| Total | 15.16 |

However, if distributed generation is installed in the first come first served manner as happened in cases 2 and 3, the overall allocation will be reduced. This is due to the fact that a prior generation capacity in a certain bus will push the limits of voltage and short circuit current beyond the sustainable amount for the other buses in the network.

Figure 6.1 shows the effects of power integration in buses E and C and how it will change the total power allocation in the network. As we can see for bus E, when the allocation goes beyond its optimal value of 3.59MW until 5.5MW there is no significant reduction observable in total power integration. However, by crossing the max limit of 5.5MW any minute increase of power integration will reduce the total dedicated power significantly. The same behavior is observable in bus C, when DG size goes from the optimal value of 4.56MW to 6MW no decline can be felt in total power, but going beyond 6MW the total power declination can be observed quiet well.

However, if we look from a customer's point of view, we can see that a customer can place higher capacity DG than the optimally allocated values without hurting the total capacity of the network by much. For instance, customer at bus E can place up to 5.5MW instead of its optimal 3.59MW that was found by the optimization in Case 1. This can only be allowed in case customers connected to the other buses grant permission to customer at bus E or customers connected in other buses are not very fond of installing DG. Same can hold for customers connected at bus C, instead of its optimal capacity of 4.56MW of DG it can place 6.2MW without hurting the total network capacity a lot. Again, it is mentionable that these are allowed in case we run the optimization from a customer point of view not from the point of view of the utility or the DNO. Bus since the owner of the distribution network is the DNO, they will always try to make use of their existing network as much as possible.

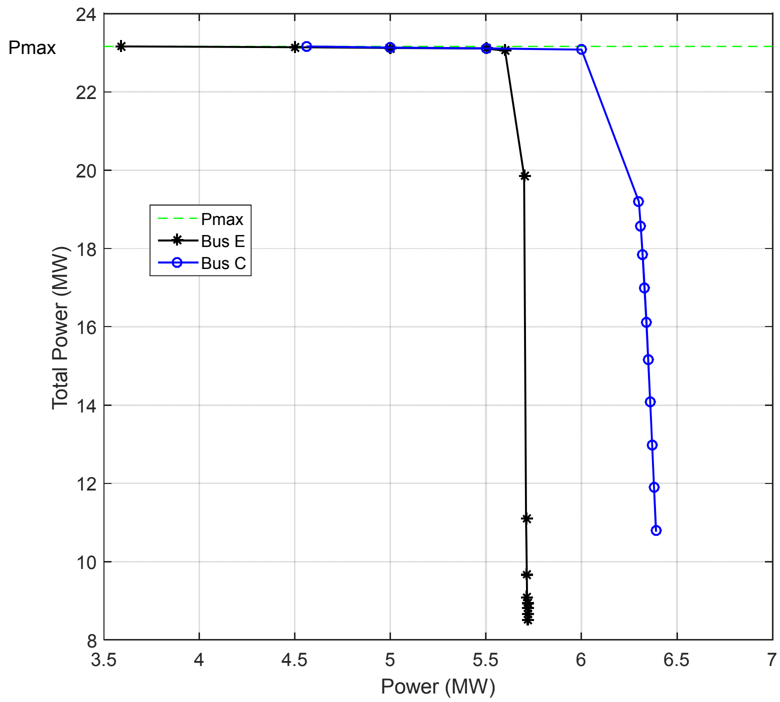


Figure 6.1: Total Power vs Power Injection at Buses E and C

# Chapter 7 - Conclusion

Since distribution networks around the globe have the same radial pattern, the tested methodology of this project report can be applied to all of them. The developed method is solved using linear programming by applying it on a section of the radial distribution network at the outskirts of Dublin, Ireland. The results obtained in this report were only slightly different (buses B and C) from the results of [2] but showed the same pattern, this was due to the fact that different platforms were used for power flow and optimization. In the same way, a customer-oriented optimization was also examined to give preference to DG friendly customers. Overall, this report comes to the same conclusion that the purpose of the methodology is distinction between the results when DG integration is planned by the DNO and when it is not planned. From the presented results, it can be seen that, if there is no prior DG in a section of the system, the method will assign the maximum possible amount throughout the network. However, if DG is dealt with on the basis of first come first served, the total penetration will reduce to a great extent. Additionally, if DG is installed on a first come first served basis it will lead to immense levels of network sterilization, which means, sudden rises of voltage and short circuit levels that will in turn lower the integration of DG in the system.

# References

[1] P. S. Georgilakis and N. D. Hatziargyriou, "Optimal distributed generation placement in power distribution networks: Models, methods, and future research," *IEEE Trans. Power Syst.*, vol. 28, no. 3, pp. 3420–3428, 2013.

[2] A. Keane and M. O'Malley, "Optimal allocation of embedded generation on distribution networks," *Power Syst. IEEE Trans.*, vol. 20, no. 3, pp. 1640–1646, 2005.

[3] C. L. T. Borges and D. M. Falcão, "Optimal distributed generation allocation for reliability, losses, and voltage improvement," *Int. J. Electr. Power Energy Syst.*, vol. 28, no. 6, pp. 413–420, 2006.

[4] M. F. Akorede, H. Hizam, I. Aris, and M. Z. a. Ab Kadir, "Effective method for optimal allocation of distributed generation units in meshed electric power systems," *IET Gener. Transm. Distrib.*, vol. 5, no. 2, p. 276, 2011.

[5] J. Teng, M. Ieee, T. Luor, and Y. Liu, "Strategic Distributed Generator Placements For Service Reliability Improvements," *IEEE Power Eng. Soc. Summer Meet.*, vol. 2, pp. 719–724 vol.2, 2002.

[6] A. Keane and M. O'Malley, "Optimal utilization of distribution networks for energy harvesting," *IEEE Trans. Power Syst.*, vol. 22, no. 1, pp. 467–475, 2007.

[7] Y. M. Atwa and E. F. El-Saadany, "Probabilistic approach for optimal allocation of wind-based distributed generation in distribution systems," *IET Renew. Power*

*Gener.*, vol. 5, no. 1, p. 79, 2011.

[8] B. R. Pereira, G. R. M. Martins Da Costa, J. Contreras, and J. R. S. Mantovani, "Optimal Distributed Generation and Reactive Power Allocation in Electrical Distribution Systems," *IEEE Trans. Sustain. Energy*, vol. 7, no. 3, pp. 975–984, 2016.

[9] F. S. Abu-Mouti and M. E. El-Hawary, "Optimal Distributed Generation Allocation and Sizing in Distribution Systems via Artificial Bee Colony Algorithm," *Power Deliv. IEEE Trans.*, vol. 26, no. 4, pp. 2090–2101, 2011.

[10] Operation Technology INC, "ETAP 16." https://etap.com/, Irvine, California.

[11] GAMS Development Corp., "GAMS." https://www.gams.com/.

# Appendix A - Methodology Code

The code written for the methodology was compiled using the GAMS software [11] .

```
set i /1*5/;
set j /1*5/;

scalar SCLrated /12.5/;

scalar R /0.815/;

scalar Irated /0.4/;

scalar pf /0.95/;

* scalar Vmin /38/;
* scalar Vmax /41.7/;

scalar alphaT /9.55/;

scalar ptrcap /31.5/;

* scalar pmin /6.5/;

parameter Vmax(i)
/1  41.7
2  41.7
3  41.7
4  41.7
5  41.7/;

parameter Vmin(i)
/1  38
2  38
3  38
4  38
5  41.7/;

parameter pmin(i)
/1  0.25
2  1.7
3  2.5
4  0.25
5  1.8/;
```

```
parameter pdgmin(i)
/1  0
2  0
3  0
4  0
5  0/;

parameter pdgmax(i)
/1  14
2  14
3  14
4  14
5  14/;

parameter sigmaT(i)
/1  0.18
2  0.11
3  0.1
4  0.14
5  0.11/;

parameter alpha(i)
/1  180.6
2  57.8
3  52.5
4  81.5
5  55.4/;

parameter beta(i)
/1  40.9
2  39.51
3  39.40
4  40.3
5  39.99/;

parameter Muu(i)
/1  0.053
2  0.218
3  0.238
4  0.162
5  0.234/;


table Mu(j,i)
1      2      3      4      5
1    0     0.008  0.007  0.021  0.016
2  0.012    0    0.18   0.009  0.007
```

```
3  0.012  0.191    0    0.009  0.007
4  0.026  0.008  0.007    0    0.11
5  0.026  0.008  0.007  0.135   0

$ontext
table Mu(j,i)
1      2      3      4      5
1    0    0.008  0.007  0.021  0.016
2  0.012    0    0.238  0.009  0.007
3  0.012  0.218    0    0.009  0.007
4  0.026  0.008  0.007    0    0.11
5  0.026  0.008  0.007  0.135   0
$offtext

table sigma(j,i)
1   2    3    4    5
1  0  1.54 1.5  2.95 2.62
2  0   0   3.29  0    0
3  0  3.63  0    0    0
4  0   0    0    0   3.2
5  0   0    0   1.74  0

variable Pdg(i);
variable F;
variable V(i);
variable Vdg(i);

Equations

obj
con1
con2
con31
con32
con33
con34
con35
* con41
* con42
* con43
* con44
* con45
con4
con5
con61
con62
con63
```

```
con64
con65
* con6
con7
con8
;

obj.. F =e= sum((i), Pdg(i));

con1.. sum(i, Pdg(i))- sum(i,pmin(i)) =l= ptrcap;

* con2.. sigmaT('1')*pdg('1')+ sigmaT('2')*pdg('2')+
sigmaT('3')*pdg('3')+ sigmaT('4')*pdg('4')+
sigmaT('5')*pdg('5')+ alphaT =l= SCLrated;

con2.. sum(i, sigmaT(i)*pdg(i))+ alphaT =l= SCLrated;

* con31.. pdg('1') - (0.095) *
(sigma('2','1')*pdg('2')+sigma('3','1')*pdg('3')+sigma('4','1')*
pdg('4')+sigma('5','1')*pdg('5')) =l= 0.095*alpha('1');
* con32.. pdg('2') - (0.095) *
(sigma('1','2')*pdg('1')+sigma('3','2')*pdg('3')+sigma('4','2')*
pdg('4')+sigma('5','2')*pdg('5')) =l= 0.095*alpha('2');
* con33.. pdg('3') - (0.095) *
(sigma('1','3')*pdg('1')+sigma('2','3')*pdg('2')+sigma('4','3')*
pdg('4')+sigma('5','3')*pdg('5')) =l= 0.095*alpha('3');
* con34.. pdg('4') - (0.095) *
(sigma('1','4')*pdg('1')+sigma('2','4')*pdg('2')+sigma('3','4')*
pdg('3')+sigma('5','4')*pdg('5')) =l= 0.095*alpha('4');
* con35.. pdg('5') - (0.095) *
(sigma('1','5')*pdg('1')+sigma('2','5')*pdg('2')+sigma('3','5')*
pdg('4')+sigma('4','5')*pdg('4')) =l= 0.095*alpha('5');



con31.. pdg('1') - (0.095) *
(sigma('1','2')*pdg('2')+sigma('1','3')*pdg('3')+sigma('1','4')*
pdg('4')+sigma('1','5')*pdg('5')) =l= 0.095*alpha('1');
con32.. pdg('2') - (0.095) *
(sigma('2','1')*pdg('1')+sigma('2','3')*pdg('3')+sigma('2','4')*
pdg('4')+sigma('2','5')*pdg('5')) =l= 0.095*alpha('2');
con33.. pdg('3') - (0.095) *
(sigma('3','1')*pdg('1')+sigma('3','2')*pdg('2')+sigma('3','4')*
pdg('4')+sigma('3','5')*pdg('5')) =l= 0.095*alpha('3');
con34.. pdg('4') - (0.095) *
(sigma('4','1')*pdg('1')+sigma('4','2')*pdg('2')+sigma('4','3')*
pdg('3')+sigma('4','5')*pdg('5')) =l= 0.095*alpha('4');
```

```
con35.. pdg('5') - (0.095) *
(sigma('5','1')*pdg('1')+sigma('5','2')*pdg('2')+sigma('5','3')*
pdg('3')+sigma('5','4')*pdg('4')) =l= 0.095*alpha('5');

* con3(i).. pdg(i) - (0.095) * sum(j, sigma(j,i)*peg(j)) =l=
0.095*alpha(i);

* con41(i).. Vdg(i) =e= V(i) + R * Irated;

* con41(i).. Vdg(i) =e= V(i) + R*pdg(i)/V(i);

* con41.. V('1')=e=
(mu('2','1')*pdg('2')+mu('3','1')*pdg('3')+mu('4','1')*pdg('4')+
mu('5','1')*pdg('5')) + beta('1');
* con42.. V('2')=e=
(mu('1','2')*pdg('1')+mu('3','2')*pdg('3')+mu('4','2')*pdg('4')+
mu('5','2')*pdg('5')) + beta('2');
* con43.. V('3')=e=
(mu('1','3')*pdg('1')+mu('2','3')*pdg('2')+mu('4','3')*pdg('4')+
mu('5','3')*pdg('5')) + beta('3');
* con44.. V('4')=e=
(mu('1','4')*pdg('1')+mu('2','4')*pdg('2')+mu('3','4')*pdg('3')+
mu('5','4')*pdg('5')) + beta('4');
* con45.. V('5')=e=
(mu('1','5')*pdg('1')+mu('2','5')*pdg('2')+mu('3','5')*pdg('3')+
mu('4','5')*pdg('4')) + beta('5');

con4(i).. V(i) =l= Vmax(i);

con5(i).. V(i) =g= vmin(i);

* con61..  Muu('1')*Pdg('1') +
(mu('2','1')*pdg('2')+mu('3','1')*pdg('3')+mu('4','1')*pdg('4')+
mu('5','1')*pdg('5')) + beta('1') =l= Vmax('1');
* con62..  Muu('2')*Pdg('2') +
(mu('1','2')*pdg('1')+mu('3','2')*pdg('3')+mu('4','2')*pdg('4')+
mu('5','2')*pdg('5')) + beta('2') =l= Vmax('2');
* con63..  Muu('3')*Pdg('3') +
(mu('1','3')*pdg('1')+mu('2','3')*pdg('2')+mu('4','3')*pdg('4')+
mu('5','3')*pdg('5')) + beta('3') =l= Vmax('3');
* con64..  Muu('4')*Pdg('4') +
(mu('1','4')*pdg('1')+mu('2','4')*pdg('2')+mu('3','4')*pdg('3')+
mu('5','4')*pdg('5')) + beta('4') =l= Vmax('4');
* con65..  Muu('5')*Pdg('5') +
(mu('1','5')*pdg('1')+mu('2','5')*pdg('2')+mu('3','5')*pdg('3')+
mu('4','5')*pdg('4')) + beta('5') =l= Vmax('5');
```

```
con61..  Muu('1')*Pdg('1')
+(mu('1','2')*pdg('2')+mu('1','3')*pdg('3')+mu('1','4')*pdg('4')
+mu('1','5')*pdg('5')) + beta('1') =l= Vmax('1');
con62..  Muu('2')*Pdg('2')
+(mu('2','1')*pdg('1')+mu('2','3')*pdg('3')+mu('2','4')*pdg('4')
+mu('2','5')*pdg('5')) + beta('2') =l= Vmax('2');
con63..  Muu('3')*Pdg('3')
+(mu('3','1')*pdg('1')+mu('3','2')*pdg('2')+mu('3','4')*pdg('4')
+mu('3','5')*pdg('5')) + beta('3') =l= Vmax('3');
con64..  Muu('4')*Pdg('4')
+(mu('4','1')*pdg('1')+mu('4','2')*pdg('2')+mu('4','3')*pdg('3')
+mu('4','5')*pdg('5')) + beta('4') =l= Vmax('4');
con65..  Muu('5')*Pdg('5')
+(mu('5','1')*pdg('1')+mu('5','2')*pdg('2')+mu('5','3')*pdg('3')
+mu('5','4')*pdg('4')) + beta('5') =l= Vmax('5');


* con6(i).. Muu(i)*Pdg(i) + sum(j, Mu(j,i)*pdg(i)) + beta(i) =l=
Vmax;

con7(i).. pdg(i)=l= pdgmax(i);

con8(i).. pdg(i)=g= pdgmin(i);

model DG /all/;

solve DG using lp maximizing F;

display
F.l, pdg.l, con31.m;
```