

STATISTICAL ANALYSIS OF CUTTING FORCES
WHILE MACHINING NODULAR CAST IRON

by

NARAIN G. KEMALRAMANI

D. M. E., M. S. University of Baroda, India, 1955
B. S. (I. E.), Kansas State University, 1963

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INTRODUCTION

An Industrial Engineer is very sensitive to the cost of production. When dealing with production, he is concerned with every stage of a product in process. Tooling cost is of major concern in many instances. Costly machine equipment may be tied up, adding machine and operator cost and inventory would be built up while waiting to be machined. A frequent failure of tools may upset a production schedule. Allowing a dull tool to operate, until the next loading of the machine, may result in a sacrifice of tolerances of a part, or forcing a slow speed operation of the machine. The cost of the tool itself may necessitate a cost evaluation for quality and quantity output.

Some of the several factors influencing the machining process are:

1. Structure and mechanical properties of the workpiece; its prior deformation of strain, due to heat treatment or surface treatment;
2. Composition of workpiece, including effect of alloying additions; impurities in workpiece;
3. Cutting speed, feed, depth of cut and general machinery set-up;
4. Cutting fluids and lubricants, including method of application;
5. Tool geometry and tool composition;
6. Cutting pressure, or externally controlled cutting force.

The number of possible variables would become limitless. Even the number of variables ordinarily considered to be controllable is quite large. There are many other variables over which control is sometimes impractical even though they have significant effect on the result.

The principal aim of this thesis was to determine the association of certain important machining factors with the variations in the cutting forces developed at the tool tip, while machining Nodular Iron (Grade 60) with the grades (K4H and K6 of Kennametal) of cemented carbide tools, on a Reed-Prentice engine lathe.

Statistical methods have not been widely used in the experiments of this nature. Imperial Chemical Industries Ltd. took the initiative for the publication of the book, "Statistical Methods in Research and Production" edited by Owen L. Davies in 1947. Their next book, "The Design and Analysis of Industrial Experiments" edited by Owen L. Davies and published in 1954 deals with the design of experiments in industrial research. "Quality Control and Industrial Statistics" by Acheson J. Duncan and "Engineering Statistics" by Albert Bowker and Gerald J. Lieberman are a few of the many recent books of the last decade which have given recognition to the use of statistical methods in industrial experimentation on a wider scale. "Tool-Life Testing by Response Surface Methodology", A.S.M.E. Papers No. 63 - Prod. - 1 and No. 63 - Prod. - 7 by S. M. Wu of the University of Wisconsin, suggests a more profitable means of conducting experiments of that nature. Moore and Kibbey in their "Ceramic Tool Geometry and Preparation" used the analysis of variance technique to analyze the results of the multivariable testing procedure (1). Ram Varmha in his "Statistical Analysis of Metal Cutting Data" of stainless steel (2) used Randomized Complete Block design of Analysis of Variance.

The statistical technique of analysis of variance was chosen as the statistical method best suited to fulfill the aims of this paper. Analysis of variance is a technique for estimating how much of the total variation in a set of data can be attributed to one or more assignable causes of variation, and not attributable to any assignable cause (residual). It also provides for tests of significance, by which we can decide whether the assignable causes have probably

resulted in real variation or effects or whether the apparent variation ascribed to them is the result only of the chance causes which produce the error variation.

The statistical analysis was employed to investigate the effects of grade of tool material, tool angle, cutting speed and the rate of feed on the magnitude of cutting forces as follows:

1. Four-factor factorial design and preliminary analysis of variance based on this design, and the application of correlation techniques to compare the mutual effects between the cutting forces,
2. Breakdown of the factorial effects and determination of the linear, quadratic and cubic effects of the factors by means of regression analysis using orthogonal comparisons of the treatment effects.

The experiment was performed using a Randomized Complete Block Design with three replicates and four-factor factorial arrangement of two grades of tool material, with three different side rake angles in each, three levels of rate of feed and four different cutting speeds. Other relevant factors, that may influence the magnitude of cutting forces, were either held constant or minimized as far as technically feasible. All machining operations were performed dry, without the presence of any cutting fluid or gas.

The measurement of the cutting forces was accomplished by a lathe tool dynamometer and a strain gage amplifying and recording unit.

All calculations pertaining to the statistical analysis of the data were carried out on a Monroe desk calculating machine.

THE DEPENDENT VARIABLE
CUTTING FORCES AND THEIR IMPORTANCE

The cutting force developed at the tool during machining can be vectorially split into three components, the tangential, the feed and the radial forces as characterized in Fig. 1. Under the optimum conditions that the tool be perfectly sharp and the total resultant force acts at a point on the tool tip, the resultant total force may be resolved into components and represented in a three dimensional array as suggested by Merchant (3) and Shaw, Cook and Smith (4). It is necessary to recognize the nature of these forces, in order to determine tool wear, tool bit temperatures and the power requirements.

The Tangential force is the major force component which contributes to the total power consumption in machining. It determines the stiffness requirements of the tool to withstand the chip pressure on the tool face and is responsible for the heat generated at the tool-chip interface for land wear, tool cratering and tool-edge chipping (5).

The feed force is required to drive the tool into the machined shoulder of the workpiece. It is the second major factor in the heat generation at the tool-chip interface after the tangential force. However, feed (axial) force represents a minor part of the energy required in shearing the chip from the workpiece (Fig. 2).

The Radial force is present in oblique cutting and in the presence of a nose radius on the tool. It is the thrust imparted on the tool by the workpiece during machining. It, however, does not constitute an important source of interest.

Researchers and investigators have developed and established theories regarding the behavior and general characteristics of the cutting forces (6, 7, 8, 9) by using dynamometers measuring two or all the three components of the

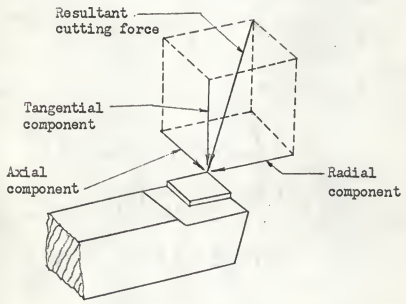
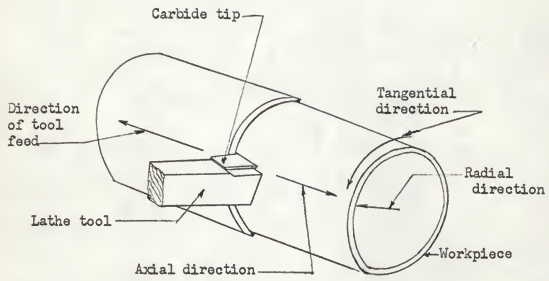


Fig. 1. Designation of cutting force components acting on lathe tool.

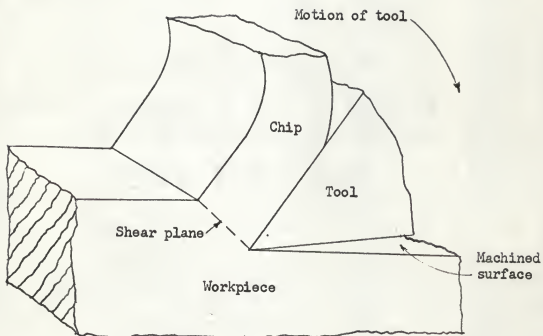


Fig. 2. Shearing action of tool penetrating workpiece.

forces (5, 10, 11). Metal produced by different techniques requires the analysis of tool forces generated with different tool geometry. F. W. Taylor's one-variable-at-a-time method is time consuming. Response surface methodology proposed by G. E. P. Box (12) in 1951 in the study of optimization problems in chemical engineering has been successfully applied to Tool-Life Testing by Prof. S. M. Wu (13). Multivariable testing is an attempt to gain information about more than one variable at a time. The number of tests necessary to provide the proper combinations of the variables furnishes reliable information about chance variation. By this technique it is possible to determine the effects of the main factors as well as the combinatorial effects of the main factors on the variation of cutting forces. With a multivariate factorial method, detailed information is revealed by choosing an appropriate design. The information so desired can be charted on graph paper as a guide to depict the magnitude of cutting forces under varied conditions of machining. A further regression analysis by "orthogonal comparison" leads to the derivation of empirical formulae to predict the cutting forces, and hence the power requirements and other factors depending on cutting forces.

The mutual relationship between two components of cutting forces is investigated with the aid of a correlation analysis together with Analysis of Variance (AOV).

This powerful technique of Analysis of Variance has been successfully employed to estimate the effects of machining (independent variable or dependent on each other) conditions on dependent variables by some researchers in the past. Mennel and Jeffery (14) studied accelerated tool-life testing methods under various machining conditions. Wu (13) has suggested the Tool-Life Testing by Response Surface Methodology. Drachmann and Jorgon (15) in France have made statistical analysis of correlation between the composition of cast iron and its

mechanical properties. Kibbey and Morris (1) used AOV for ceramic tool cutting demonstrating the involvement of several machining factors.

DESIGN OF THE EXPERIMENT

Analysis of Variance

In every experiment there is an experimental error that arises from two sources: lack of uniformity of the material and the inherent variability in the experimental technique. One purpose of the analysis of variance to be run on the data of an experiment is to separate the random variation, the so called sampling error from non-sampling error, such as might be caused by an interaction between main factors such as tool and angle or feed and speed.

The statistical technique of Analysis of Variance was chosen as the statistical method best suited to analyze the data of this experiment. A complete experiment usually consists of a series or combination of elementary experiments, and the results take the form of a mean value or a set of mean values. The purpose of the complete experiment is to test the hypothesis pertaining to the universe mean values, and to estimate the means or the various components of variance.

An experiment that calls for combining the levels of each factor with the levels of all the other factors is technically known as a Factorial Experiment. The general advantage of a Factorial Experiment lies in its efficiency in extracting certain kinds of information at a required cost. One of the features of the analysis of variance is that the variability of the components in an experiment is measured in terms of sums of squares of deviations about the general mean or average. The variation of data may come from several variables. The AOV informs the investigator as to which of the factors and/or their combination significantly effect the results. Snedecor's Variance Ratio or F-test

will make it evident as to how non-equal variances differ: (i) that they are significantly different as a result of cause factors, (ii) that they are different only because of random sampling error.

Hypothesis. Assuming the null hypothesis that there is no significant difference in the means, the calculated variance ratio if found to be greater than the tabulated F, the hypothesis is rejected indicating that there is significant difference in the means of the factors and their combination.

Selection of Factors Independent Variables

Considering the tremendous number of factors which will have some effect on the machining, especially since the optimum value for most of the variables will depend on the particular values of all the other variables, strict control over many of the variables becomes practically impossible. It would be highly desirable if all the known factors could be kept at a perfectly constant level. Quite probably, many of the more or less uncontrollable variables do have some effect on the results. However, it was considered appropriate to limit the controlling factors to (i) two grades of cutting tool (ii) with three different rake angles, (iii) four cutting speeds and (iv) three rates of feed. Various other factors, other tool angles, varying depth of cut, hot machining, and many more if included in a statistical analytical design would complicate the analysis. Instead it is always advisable to perform smaller experiments where it is possible to keep control over the independent variables.

The experimental data measured were:

- (i) The tangential (cutting) force
- (ii) The feed (axial) force.

It may be of interest to study other dependent variables viz., temperature at the tool bit, the chip thickness, tool wear, shearing force, total work done,

etc. For the purpose of this paper, the variables of interest were the two cutting forces.

Since the strain gage "Sanborn recorder" available had only two channels, as such the two more important components of cutting force, the tangential and the feed forces could be measured simultaneously. Since variation in depth of cut is more responsible for variations in radial force; by holding depth of cut at a constant level, the radial force study was isolated as non-significant.

Selection of the Levels of Factors

Tool material grade. Nodular cast iron has been produced in this country for the last 14 years. With a production in this country in 1963 of around 300,000 product tons, the industry has not seen any special tool material developed so far. In 1952, Trigger, Zylstra and Chao (16) investigated by using various grades of carbide tools. A recent experiment by Hitomi and Thuring (17) on machinability of Nodular cast iron reports three grades of carbide and another grade of ceramic tool. Hitomi and Thuring have used Kennametal grades K4H (steel cutting grade) and K6 (cast-iron cutting grade) when studying flank adhesion and grades K3H (steel cutting grade) and K6 and ceramic tools for tool-life testing on different grades of N.I. (Nodular Iron). They used throw-away type tips.

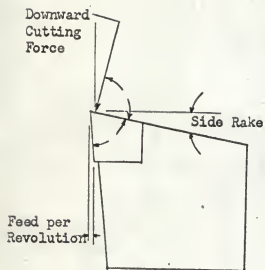
For the purpose of this experiment, two grades K4H and K6 (of Kennametal) of carbide were chosen, since both of these grades are Abrasion-Resistant and they have the same hardness (92 Rockwell A). Grade K3H is recommended as Crater-Resistant (18).

Since the tool shank size in the dynamometer available was $5/8$ inch square, throw-away type tool could not be used since the tips of the grades for this size tool holder were not readily available. The experiment was, therefore, performed with the brazed carbide-tip tools, No. AR10 and C10 (Kennametal

Catalog 64). The chemical composition of the carbide tips used is given in a table.

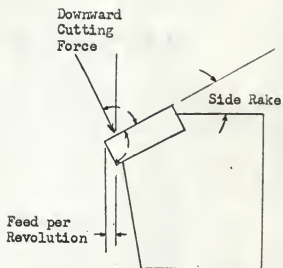
Tool Angles. The rake angles are the important features of a single point tool. Both the side and back rake angles are vital parts of carbide tool geometry. The side rake is defined as the angle between the top face of the carbide tip and the top plane of the tool shank, and is measured perpendicular to the side cutting edge angle. This inclination of the tool face from the horizontal (top) plane could be positive, neutral or negative. The back rake angle is the angle between the top face of the carbide tip and the top plane of the tool shank, and is measured in a plane parallel to the longitudinal axis and at right angles to the base of the tool. Like side rake, this angle can also be positive, neutral or negative (Fig. 3).

It is known that cemented carbides are very strong in compression, but comparatively weak in shear (19). Since downward cutting force is perpendicular to the top face of the tool, it can be seen in Fig. 4 that carbide in the positive rake is placed in shear, while in the negative rake tool is in compression. The included angle under the cutting edge of the positive rake tool is much smaller than the included angle of the negative rake tool. For these reasons, the cutting edge of the positive rake tool is inherently weaker. Because the negative rake strengthens the tool, it promotes longer tool life when taking interrupted cuts. It may be of interest to some investigators to use a combination of negative back rake and positive side rake for roughing cuts on steel. Here the negative back rake will tend to protect the weak nose of the tool, while the positive side rake would provide a freer cutting action. It was considered of importance, therefore, to include three different rake angles in each of the two carbide tool grades, in the basic model to investigate their



Positive side rake angle.

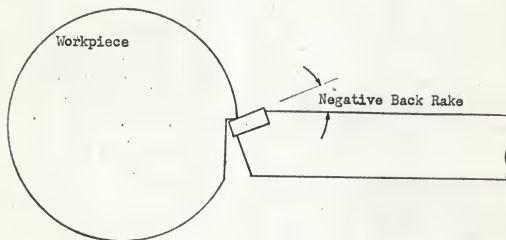
Tool in shear



Negative side rake angle.

Tool in compression

Fig. 3.



With a negative back rake, the initial shock of the cut is taken back of the weak point on the stronger part of the tool.

Fig. 4.

effect on the cutting forces under study. The levels of rake angles so chosen were classified as follows:

$T_1 A_1, T_2 A_1$ = Tool possessing rake angle of $+5^\circ$

$T_1 A_2, T_2 A_2$ = Tool possessing rake angle of 0°

$T_1 A_3, T_2 A_3$ = Tool possessing rake angle of -5°

Table 4 gives the complete tool geometry and the chemical composition of the tool material. A larger clearance angle was ground on all tools in order to avoid flank adhesion as recommended by Hitomi and Thuring (17).

Rate of Feed. After a few trial tests at different levels of feeds and speeds, and due to the limitations of the cross-sectional strength, it was decided to hold the depth of cut constant at 0.10 inch for the entire experiment to make the radial force insignificant. With 0.10 inch depth of cut, it was found that the tool suffered damage when the feed exceeded 0.015 inch/rev. Therefore, three levels of feed including 0.015 inch/rev. were chosen as given below.

F_1 = feed rate of 0.005 inch/rev.

F_2 = feed rate of 0.010 inch/rev.

F_3 = feed rate of 0.015 inch/rev.

Cutting Speeds. During the trial tests it was observed that with the levels of depth of cut and feeds chosen, the machine would not take up the load satisfactorily without generating undue heat when cutting at a speed over 547 s.f.p.m. (342 r.p.m.). Taking this speed as the limiting factor, four different speed levels including 547 s.f.p.m. were chosen for the purpose of this experiment, as given below.

S_1 = 73 r.p.m. 117 s.f.p.m.

S_2 = 136 r.p.m. 218 s.f.p.m.

$S_3 = 212$ r.p.m. 339 s.f.p.m.

$S_4 = 342$ r.p.m. 547 s.f.p.m.

Speeds of 171 r.p.m. (274 f.p.m.) and 276 r.p.m. (442 f.p.m.) did not indicate any significant variation, therefore they were not included in the experiment.

It will be of interest to note that as far as possible, the various levels of angles, feeds and speeds have been approximately equally spaced so as to be helpful when considering orthogonal comparisons by polynomials (20).

Advantages in Choosing Factorial Model

Many times an experimenter does not know which factors are important or whether each factor exerts its effect independent of the other factors. If we were to vary one factor at a time and study the effect, the experiment would give no information at all on the dependence of the effect of the factor on the levels at which other factors were held constant. It is rare in experimentation that interactions do not exist. By interaction is meant the failure of the levels of one factor to retain the same order and magnitude of performance (within random sampling errors) throughout all levels of another factor. Thus, if the effect of one factor is dependent on the level of another factor, then the two factors are said to interact. "In addition to giving equal information on main effects, the factorial experiment gives information on the interactions of the factors. If there are interactions between the factors, the factorial experiment will bring them to the attention of the experimenter, whereas with the other type of experiments the experimenter can obtain no knowledge of them." (21)

Some of the interactions involving several factors may be regarded as trivial and the interactions involving several factors may be used to estimate the experimental error.

The effect of including some non-zero high order interactions in the sum of squares, by which the error is estimated, will be to inflate this estimate of error variance. The actual effect may be expressed in terms of the true interactions. This inflation will be negligible for most types of factorial experiments. "Thus, (i) when there are no interactions the factorial design gives the maximum efficiency in the estimation of the effects and (ii) when interactions exist, their nature being unknown, a factorial design is necessary to avoid misleading conclusions. (iii) In the factorial design the effect of a factor is estimated at several levels of the other factors, and the conclusions held over wide range of conditions." (22)

Hence, the results obtained by changing two or more factors (using factorial design) will give the required information with the required degree of precision and with the minimum expenditure of effort.

The designs are compared on

1. Relative efficiency
2. Basis of Confidence Intervals derived from the residual error
3. Sensitivity in detecting a given difference.

Replications

In every experiment there is an experimental error that arises from two sources:

- (i) Lack of uniformity of the material and
- (ii) The inherent variability in the experimental technique.

Fisher (23) in his Design of Experiments states that the two conditions necessary to obtain a valid estimate of the experimental error are replication and randomization.

"If a treatment is applied to absolutely homogeneous material, there is no variation from experimental unit to experimental unit, in effect only one replicate from the population of possible replicates is obtained even though several observations are made." (24) Since variability is almost universal, the repetition of the set of treatments in the experiment is necessary. By dividing the experiment in several blocks (replicates), the sets of treatments could be tested in each block. A block is a part of the experimental material which is likely to be more homogeneous than the whole. Observations within blocks can therefore be compared with greater precision than observations distributed over the whole experimental material. Randomization insures that any comparison of treatments is estimated without bias by the same comparison of the observed mean yields (25).

In any randomized block experiment it is necessary to have at least two replications in order that an estimate of the experimental error variance may be obtained. With increasing replication the error variance will be estimated with increasing accuracy. But it is intuitively obvious that increasing replication results in increasing sensitivity of the experiment.

Three workpieces of 60 grade Nodular Iron served as three replicates. Although two workpieces supplied by one foundry were reported to have the same chemical composition, samples from both of the pieces showed a slightly different microstructure. A third piece supplied by another foundry showed almost similar microstructure as one of the pieces (MT-5) supplied by the first foundry but its chemical composition was different. The microstructure of the sample from each replicate is given in Fig. 5 while their chemical composition is given in Table 4. (see page 34).



Fig. 5(a).
Replicate 1
MT-4
X250

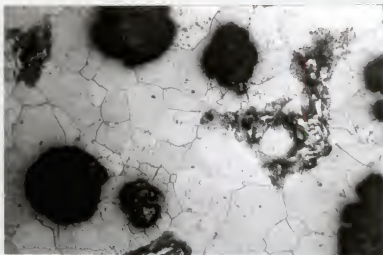


Fig. 5(b).
Replicate 2
MT-5
X250

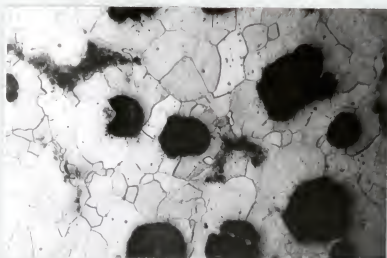


Fig. 5(c).
Replicate 3
Int. Harv.
X250

Fig. 5. Photomicrographs of typical test specimen Nodular cast iron grade 60.

With this information about the workpieces, it was hoped that completely randomized block design would help to signify such a lack of homogeneity.

Randomization within Blocks

Having decided upon two grades of tool material with three levels of tool angles, three levels of feed and four different speeds as factor levels, the total number of different treatments in a block would be $2 \times 3 \times 3 \times 4 = 72$. In order to effect complete randomization of the different combinations of levels of factors within each block, 72 small paper chips were made and each was marked with a number from one to seventy-two. These seventy-two chips correspond to the possible combinations of all the levels of factors as given in Table 1. These chips were placed in a bowl and thoroughly mixed. Using W. A. Shewart's method of randomization, chips were drawn from the bowl without replacement. Chip No. 1 designated a machining combination of T_1, A_1, F_1, S_1 and so on and chip No. 72 meant a machining combination of T_2, A_3, F_3, S_4 . The order of drawing chips from the bowl was noted serially. For each replicate the chips were thoroughly mixed in the bowl and drawn. Table 2 gives the designated machining combinations in sequence as drawn by randomization. Machining was performed in strict accordance of the pattern as indicated in Table 2.

TEST EQUIPMENT

The tests were conducted on a Reed-Prentice engine lathe with a 5 H.P. capacity motor. Tool forces were measured with a strain-gage-type lathe tool dynamometer and a two-channel Sanborn amplifier and recorder, model #60-13003. Due to the limitation of only two channels on the amplifier unit, it was decided to use only two main components of the cutting force. These were the tangential

Table 1. Randomized complete block design.

		S ₁	S ₂	S ₃	S ₄	
T ₁	A ₁	F ₁	1111	1112	1113	1114
		F ₂	1121	1122	1123	1124
		F ₃	1131	1132	1133	1134
	A ₂	F ₁	1211	1212	1213	1214
		F ₂	1221	1222	1223	1224
		F ₃	1231	1232	1233	1234
	A ₃	F ₁	1311	1312	1313	1314
		F ₂	1321	1322	1323	1324
		F ₃	1331	1332	1333	1334
T ₂	A ₁	F ₁	2111	2112	2113	2114
		F ₂	2121	2122	2123	2124
		F ₃	2131	2132	2133	2134
	A ₂	F ₁	2211	2212	2213	2214
		F ₂	2221	2222	2223	2224
		F ₃	2231	2232	2233	2234
	A ₃	F ₁	2311	2312	2313	2314
		F ₂	2321	2322	2323	2324
		F ₃	2331	2332	2333	2334

Tools T = 1, 2
 Rake Angles A = 1, 2, 3
 Feeds F = 1, 2, 3
 Speeds S = 1, 2, 3, 4

Table 2. Randomized design separate for each replicate.

	Replicate 1			Replicate 2			Replicate 3		
	TAFS			TAFS			TAFS		
1	1131	1322	2321	1222	2223	1221	2233	1332	1221
2	1334	2223	1132	1234	1132	2212	2131	2231	2321
3	2332	2212	1123	1113	1333	1214	2332	1124	1314
4	1133	1231	2121	2122	2313	2333	1134	1322	1213
5	2324	1232	2233	2222	2121	2213	2111	2313	2311
6	2221	2314	1233	1134	2124	1331	2214	1133	1132
7	1214	2113	2131	2133	1211	1231	1321	2121	2113
8	2214	2234	1223	1131	1224	1212	2211	1331	2324
9	1222	1122	2232	1124	2111	2134	1333	2133	1122
10	1314	1113	2331	2233	1133	1324	2222	2333	1222
11	2313	2222	1134	1213	2311	2113	1233	2122	2234
12	1111	2224	1221	1112	1123	2314	1223	1114	1211
13	2213	2231	2123	1321	2232	2322	2221	1311	1112
14	2311	1332	1213	2324	1114	2214	2312	2331	2334
15	1212	1324	2112	2332	1313	1311	1313	1214	1212
16	2111	1312	1234	1121	2323	1232	1113	2213	2134
17	2334	1321	1112	2221	1111	2224	1121	2314	1231
18	2114	1121	2323	2312	2234	1122	2212	1323	2223
19	1211	2211	2134	2132	1314	2112	1123	2224	2323
20	1323	2322	2132	1312	1334	2334	1131	1224	1234
21	1124	1114	2312	2331	1233	2131	1312	2114	2232
22	2133	1311	1313	1322	2231	1332	2123	1324	2124
23	2124	1331	2122	1223	1323	2321	1334	1232	2322
24	1333	1224	2333	2114	2211	2123	1111	2112	2132

T = 1, 2

A = 1, 2, 3

F = 1, 2, 3

S = 1, 2, 3, 4

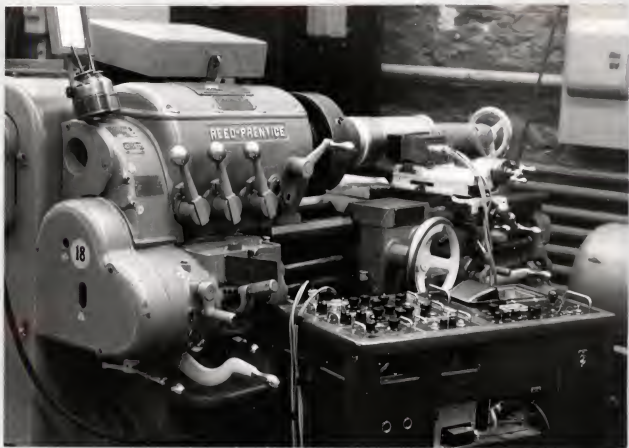


Fig. 6(a). Photograph of the experimental set-up, showing the workpiece and dynamometer with the Sanborn strain gage amplifier and recorder in the foreground.



Fig. 6(b). View of the dynamometer exposing the strain gages.

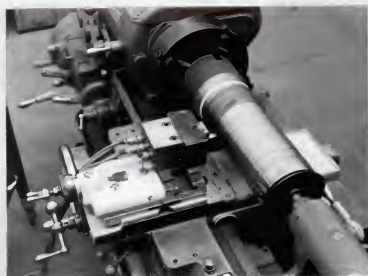


Fig. 6(c). Photograph showing the tool-post lathe dynamometer and the Nodular Iron workpiece mounted on the lathe.

and the feed forces. Before proceeding with the experiment, the dynamometer and the Sanborn recording unit were balanced, calibrated, and tested for accuracy. Figure 6 shows the set-up of the experiment.

Lathe Dynamometer

A three dimensional lathe dynamometer was used to measure the force components acting on the lathe tool. It consisted of a unit machined from one-piece of steel in order to provide the maximum possible stiffness for the required sensitivity and to achieve the high degree of linearity and freedom from hysteresis effects. Thus all the deflection taking place under load would be purely elastic and free of friction. The deflection was measured with suitably placed resistance strain gages to provide the means for converting forces into a conveniently measured electrical quantity. Figure 7 gives diagramatic picture of the dynamometer with the locations of the strain gages.

The strain gages were made up of thin tantalum strain gage wires, bonded and cemented on to the dynamometer block with epoxy resins, at positions indicated. Strain gages C_1 , C_2 , C_3 and C_4 were intended to measure the radial force component, C_5 , C_6 , C_7 and C_8 to measure the tangential force and C_9 , C_{10} , and C_{11} and C_{12} measured the feed force. Each of the three measuring gage units contained four 120 ohm strain gages and connections brought to three separate Wheatstone bridge connections representing the three directional forces as indicated in sketches in Fig. 8. The different strain gages affixed on the dynamometer are connected in such a way as to form three independent Wheatstone bridge circuits. The four arms of each of the Wheatstone bridges are formed by those appropriate strain gages intended to measure the three cutting forces. Four elements wired as a Wheatstone bridge circuit gave the full-bridge. Each strain gage element

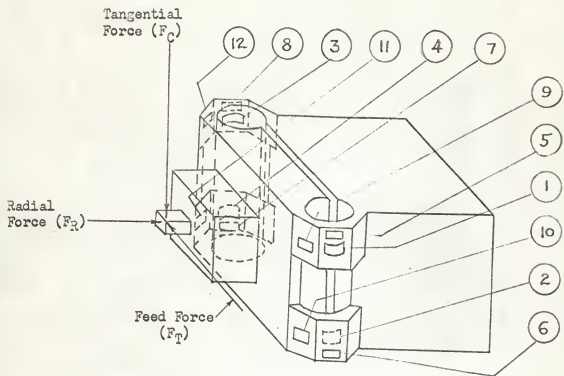


Fig. 7. Diagrammatic sketch of lathe dynamometer.
 Gages 1 to 4 are used to measure radial force.
 Gages 5 to 8 are used to measure tangential force.
 Gages 9 to 12 are used to measure feed force.

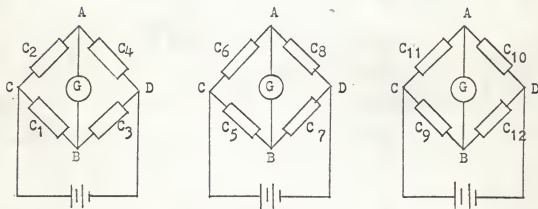


Fig. 8. Wheatstone Bridge.
 Three circuits formed by the appropriate strain gages to measure respective cutting forces.

is a length of wire fastened to a permanent backing (bonded strain gage) or held between supports (unbonded strain gage). With no strain on the wire, the wire has a certain length and resistance.

The dynamometer works under the principle that when the tool is under the effect of a particular force, the strain is applied to the wires, and the corresponding strain gages (wires) become longer and thinner or shorter and thicker. When wire becomes thinner, the resistance is increased and decreases when wire becomes thicker, with a result that the Wheatstone bridge circuit is unbalanced causing a voltage difference at the bridge terminals. This signal voltage from the bridge is measured by the Strain Gage Amplifier, which gives an estimate of the force responsible in bringing the change in resistance of the strain gages. The voltage input is between C and D and the voltage output between A and B.

It is evident from Fig. 7 that if a vertical force similar to the tangential force were to act on the tool tip, then due to the bending of the octagonal ring of the dynamometer, strain gages C_5 and C_8 would be stretched out since they would be in tension, and the strain gages C_6 and C_7 would be in compression allowing the wires to shorten in length. Thus the resistance to the electric current would be reduced through gages C_6 and C_7 and increased through gages C_5 and C_8 . The effect of Radial and Feed forces acting on the tool bit could be explained similarly.

Initially when there were no forces acting on the tool, all the strain gages $C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8$ and $C_9, C_{10}, C_{11}, C_{12}$ possessing equal resistances in their respective groups would balance the three Wheatstone bridges one by each group. As a result of this, the Wheatstone bridge ACEC would be in balance and would show no voltage difference between A and B and therefore no current would pass through G. When, however, a Feed force becomes

active at the tool tip, the changes in strain gage resistances would unbalance the bridge circuit. If C_9 is stretched out so that its resistance is increased by a small unit ($+\Delta R$), then the voltage at B would be increased from zero to a unit ($+\Delta V$), allowing a small amount of current to flow through G. While C_9 is under tension, C_{12} would be under compression, with reduction in its resistance by a small amount ($-\Delta R$) resulting in further change in voltage at B from $+V$ to $+2\Delta V$. [$+\Delta V - (-\Delta V) = +2\Delta V$]. With similar action at C_{10} (under tension) and at C_{11} (under compression), the total effect due to feed force results in an increase in voltage by $+4\Delta R$. It is possible to measure the strain by using single gage, but by using two gages in tension and other two in compression, the sensitivity is fourfold. This helps in measurement of a smaller magnitude of change in resistance, which is usually less than 0.5 percent in the strain gage.

The individual forces (feed, tangential, radial) acting on the tip of the tool may be considered, as each force developing a bending moment, putting a member in strain. The strain will be at a section with maximum +ve at one end to minimum -ve at the other end.

Connections A and B of the dynamometer are taken to the Sanborn strain gage amplifier and the potential difference of the Wheatstone bridge is measured. This is recorded by the movement of stylus on the running graph paper.

Thus two forces, tangential and feed or radial force could be recorded simultaneously on the two channel recorder, with appropriate calibration of recorder and the channels of the dynamometer.

Balancing of the Sanborn Amplifier

Before proceeding with the calibration of the amplifier, the unit was balanced in order to compensate for the residual unbalance of the bridge circuit and its cabling. Balancing procedure is as indicated below:

1. Plug in the amplifier unit and connect everything.
2. Turn on the power switch and the channel switches. Warm up for 30 minutes.
3. Set the panel controls:
R/T . . . T ATTENUATOR . . . OFF GAIN . . . FULL RIGHT
4. Set the FINE/COARSE switch to FINE. Remove all strain from the bridge. Center the stylus with the zero control then set the FINE/COARSE switch to COARSE.
5. Advance the Attenuator to the X100 position. Bring the stylus to its null position with the RES BAL and CAP BAL controls. Continue advancing the ATTENUATOR and bringing the stylus back to its null position until the ATTENUATOR is at X1.
6. Set the FINE/COARSE switch to FINE. Now make a fine adjustment of the RES BAL control, so that the stylus does not move when turning the ATTENUATOR between X1 and OFF. Then return the ATTENUATOR to OFF.
7. Check the electrical sensitivity of the system by pressing the calibration button and adjust the GAIN CONTROL for the required sensitivity for the dynamometer.
8. The full bridge is now balanced, and the system is ready for calibration.

Calibration

After balancing the Sanborn recorder under no load condition, the unit was calibrated. The method employed for calibration was to provide a known (on testing machine), varying force at the tool-tip affixed in the dynamometer and

measure the deflection of the stylus at different force intervals. By plotting the values of force applied on one axis and the deflection of the stylus on the other axis of the graph, "calibration curves" were drawn as guide lines. With no load, the stylus returned at null point. In order to obtain accurate "calibration curves", regression coefficients of forces and stylus deflection were calculated and the "lines of best fit" originating from 0-0 point were drawn. These "calibration curves" were used to determine forces during experiment.

The following steps were observed while calibrating the equipment:

1. The dynamometer was firmly affixed to the base plate of a Southwark Emery (Hydraulic) Universal Testing machine, of 120,000 lbs. capacity.
2. A tool of $5/8$ inch square section (of the size of dynamometer tool insertion) and $4\frac{1}{2}$ inches long was inserted into the dynamometer and secured by means of two Allen screws.
3. A point on the tool tip was carefully determined as to be in close approximation with the tool point wherein the cutting forces may be expected to act. A small hole of $1/16$ inch diameter was drilled at this location (Fig. 9).
4. A tool overhang of $3/4$ inch was adjusted. This overhang was fixed for further experiments. By using mild steel tool shank, it was felt, that larger overhang would not allow the rigidity of the tool and produce undesirable deflection of the tool during calibration instead of the desired absolute dynamometer deflection.
5. In order to apply the concentrated load vertically (to conform with the tangential force of cutting), a ball bearing of $1/8$ inch diameter was seated on the drilled hole on the tool tip face. A slender cylindrical steel rod of $1/2$ inch diameter and 3 inches in length, with a slight hole punched at one end, was placed vertically above the steel ball and perpendicular to the overhead ram.
6. The overhead ram was slowly lowered to barely touch the slender steel rod, with indicator showing zero load on the dial of press machine.

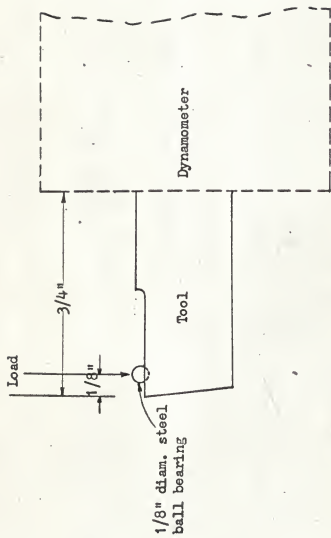


FIG. 9. Tool set-up for dynamometer calibration.

7. The Sanborn recorder that was already warmed up and balanced showed no deflection of stylus. The power switch that was turned on previously was then turned to RUN position to allow the graph paper (Permapaper) to flow out at the speed determined by the change-gears. The stylus gave zero line on the paper. This is the base line.

8. The Attenuator switch was then set at X10 position and the ram slowly depressed to obtain a total sweep of the stylus representing the tangential force.

9. The maximum load required a total sweep of 50 mm's on the Permapaper. This sweep converted gave 600 lbs. force value, a satisfactory upper limit of the experimental force.

10. With Attenuator still on X10, hydraulic ram was returned to zero load and stylus following the zero base line. The dynamometer was again continuously (vertically) and uniformly loaded. As the test machine dial indicator touched 0, 50, 100, 150 . . . pounds of force, the corresponding stylus deflections were noted by depressing the marker button on recorder. Thus, the stylus deflections pertaining to loads at 50 pound increment was noted.

11. After loading up to 600 lbs., the load was gradually decreased back to zero and the marker button was similarly pressed at the loads of 550, 500, 450, . . . 100, 50, 0 pounds.

12. The stylus deflection readings are given in Table 3(a) and 3(b) and the average values were then used to get the regression line plotted on the graph in Fig. 10

13. In order to measure the Feed or horizontal force, the dynamometer was tilted 90 degrees. The tool was turned to receive load through the slender rod on the ball bearing as before. Steps (6) through (11) were repeated. Table 3(b) shows the recordings and Fig. 10 represents the regression line representing the Feed force curve on the graph.

The unit so calibrated was then ready for experiment.

Table 3(a). The deflection in millimeters of the Sanborn amplifier recorder stylus needle during the process of calibration. Attenuator setting-X10.

Type of force	Force lbs.	Increasing force	Decreasing force	Average stylus deflection
VERTICAL	50	0.75	0.25	0.5
	100	1.0	1.0	1.0
	150	2.0	1.25	1.63
	200	2.25	2.00	2.13
	250	3.0	2.5	2.75
	300	3.75	3.1	3.43
	350	4.25	4.0	4.13
	400	5.0	4.8	4.9
	450	5.75	5.5	5.63
	500	6.5	6.0	6.25
	550	7.1	7.0	7.05
	600	8.0	7.75	7.87

Table 3(b). The deflection in millimeters of the Sanborn amplifier recorder stylus needle during the process of calibration. Attenuator setting-X10.

Type of force	Force lbs.	Increasing force	Decreasing force	Average stylus deflection
HORIZONTAL	50	2.0	1.8	1.9
	100	3.1	3.0	3.05
	150	4.6	4.1	4.35
	200	6.0	5.8	5.9
	250	7.1	7.0	7.05
	300	8.9	8.5	8.7
	350	10.1	10.1	10.1
	400	11.6	11.6	11.6
	450	13.1	13.0	13.1
	500	15.0	14.5	14.75

Calibration Curves

Attenuator setting: X10
 Recorder setting: Ave.-fine
 Calibration setting: 2 mm.
 Tool overhang: $3/4$ "

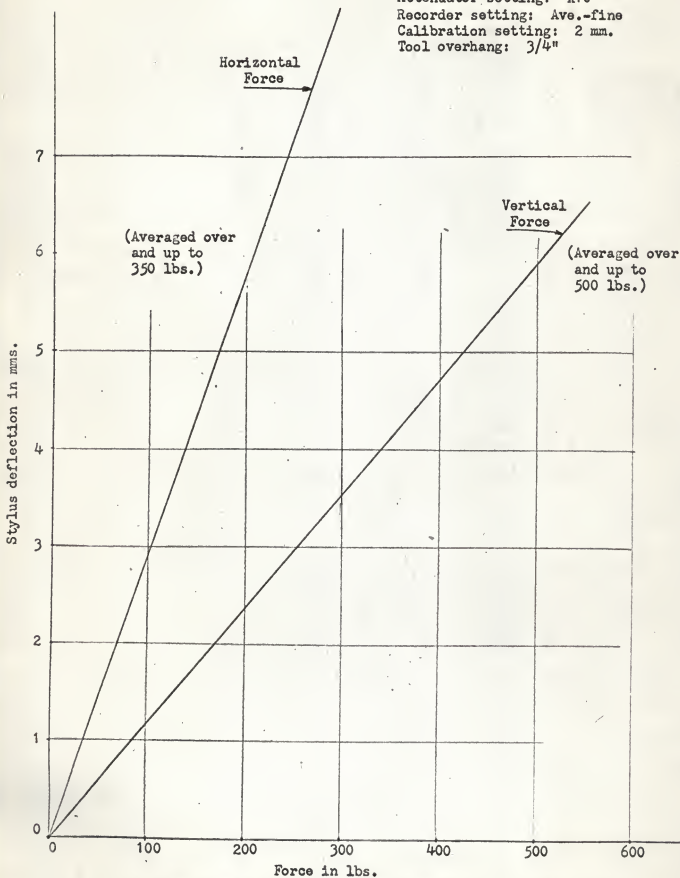


Fig. 8. Calibration graph showing the Sanborne recorder stylus deflection vs. the vertical and horizontal forces at attenuator setting X10..

Workpiece. The tests were run on Nodular cast iron of grade 60. The tubes supplied by two foundries were turned to obtain a uniform diameter and wall thickness. The outside diameter was turned down to 6.1 inches and the inside diameter was increased to 5.6 inches, thus obtaining a wall thickness of 0.25 inch. The chemical composition and the physical properties of the work material employed is given in Table 4. Figure 5 shows the microstructure.

Tools. All tests were run with brazed carbide tip tools. Grades employed were K6, a so-called cast-iron cutting grade; and K4H, a steel cutting grade (Kennametal Catalog 64). Tool shape of AR10 was easily converted to the required positive rake, and C10 provided the neutral rake and conversion to negative rake angle. These tools had shank of $5/8$ inch square section and $4\frac{1}{2}$ inch length. The physical properties and the chemical composition is given in Table 4. No chip breakers were necessary for this experiment with Nodular Cast Iron.

EXPERIMENTAL PROCEDURE

Preliminary Operations

The workpiece tubes supplied were turned to remove outer skin and provide a concentric hollow cylindrical workpieces. A light finish cut was then taken before any data were recorded to insure removal of any surface material, which may have been work-hardened by the previous rough machining. The three workpieces served the purpose of three replicates (blocks).

Tools. Three tools of each grade were accurately ground to have $+5^{\circ}$, 0° and -5° rake angles. Tools AR10 were ground for positive rake while C10 tools were ground for neutral and negative (-ve) rake angles. All other angles and the nose radius were kept constant for all the tools. Table 5 gives the tool geometry of three different forms of tools employed. The accuracy of the tools ground was checked by a tool protractor.

Table 4. Nodular cast iron grade 60.

Brinell hardness	MT-4 174	MT-5 158	INT. HARV. 162
Chemical composition:			
Replicate	1	2	3
Total carbon	3.60	3.60	3.81/3.73
Si	2.33	2.33	1.67/2.51
S	0.015	0.015	0.20/0.015
P	0.033	0.033	0.038
Mn	0.027	0.027	0.48
Ni	1.00	1.00	0.58
Cu	0.06	0.06	0.06
Cr	0.02	0.02	0.05
Mo	0.03	0.03	0.05
Mg	0.036	0.036	0.047
Tensile, yield, elongation	65-45-10	60-45-20	65-45-15

Table 5. Nomenclature of the three different forms of tools employed.

Grades K6, K4H	Tool A ₁	Tool A ₂	Tool A ₃
Side rake angle	+5°	0°	-5°
Back rake angle	+5°	0°	-5°
Side cutting edge angle	10°	10°	10°
End cutting edge angle	5°	5°	5°
End relief angle	10°	10°	10°
Side relief angle	15°	15°	15°
Nose radius	1/16"	1/16"	1/16"

Properties and chemical composition of carbide cutting tools.

Grade	Density g/cm ³	Hardness (Ra)	Composition (percent by weight)					
			W	C _o	T ₁	T _a	N _b	C
K6	14.9	92.0	86.4	5.75	-	2.0	-	5.8
K4H	12.4	92.0	72.6	6.75	4.7	3.4	5.8	6.9

Running the Test

The dynamometer was mounted on the tool-post of the lathe and the two outlets of F_C and F_T were connected to the two channels A and B respectively of the Sanborn amplifier. The unit was warmed up for 30 minutes and balanced under no load conditions and the styli were positioned at the required base lines.

Tests were run in the randomized order as indicated in Table 2. The alterations in feeds and speed and tools were manipulated as per the combination indicated in the table. Tool overhang was checked and maintained at $3/4$ inch after every tool change before running every test. The Sanborn recorder power switch was turned to RUN position for every test of machining. Graph paper travel was kept slow for slower speed and at fast for faster speeds of cutting.

Stylus deflection marked the magnitude of cutting forces on the graph permapaper of the Sanborn recorder.

The tool was closely inspected after every cut. Any flank adhesion found on the cutting edge was carefully removed by using an oil stone and then honing with a diamond hone. If there was doubt about wear, the tool was examined under a microscope, and then ground back to required dimensions. However, during the entire experiment, the tools did not need regrinding.

Due to the heat developed on the tool tip and in the workpiece during cutting, sufficient time was allowed between cuts to allow the workpiece and the tool to attain normal temperature.

After the tests, the readings from the graph-sheet of the recorder were analyzed and converted to force readings with the aid of calibration curves plotted in Fig. 10. The data so derived from the three replicates are listed in Tables 6 and 7 separately for the two components of the forces.

EXPECTATIONS OF MEAN SQUARES IN FACTORIAL DESIGN

Mathematical Model

The mathematical model postulated for the data obtained under four-factor factorial with one qualitative factor and three quantitative factors is as follows:

$$\begin{aligned}
 Y_{ijklm} = & M + R_i + G_j + A_k + F_l + C_m \\
 & + (GA)_{jk} + (GF)_{jl} + (GC)_{jm} + (AF)_{kl} + (AC)_{km} + (FC)_{lm} \\
 & + (GAF)_{jkl} + (GAC)_{jkm} + (GFC)_{jlm} + (AFC)_{klm} \\
 & + (G AFC)_{jklm} + E_{ijklm}
 \end{aligned}$$

where i = Replicates = $r = 3$ nos.

j = Tool grade = $t = 2$ grades

k = Rake angle = $a = 3$ levels

l = Rate of feed = $b = 3$ levels

m = Cutting speed = $c = 4$ levels,

and M = Mean effect

R_i = Effect of the i^{th} replicate

G_j = Effect of the j^{th} grade of tool

A_k = Effect of the k^{th} level of rake angle

Table 6. Tangential forces obtained during the machining of Nodular Iron (60 grade) in a four-factor factorial randomized complete block design.

		Replicate 1				Replicate 2				Replicate 3				
		S ₁	S ₂	S ₃	S ₄	S ₁	S ₂	S ₃	S ₄	S ₁	S ₂	S ₃	S ₄	
T ₁ -K6	F ₁	100	310	265	240	80	90	70	110	90	90	125	115	
	A ₁	F ₂	310	325	285	265	135	135	160	175	175	175	165	
		F ₃	205	310	210	290	200	210	150	135	220	240	220	220
T ₁ A ₂	F ₁	100	170	195	195	70	140	170	150	105	100	150	100	
	F ₂	250	230	225	250	120	165	170	170	175	185	185	170	
	F ₃	280	255	220	260	155	240	230	250	250	250	240	200	
A ₃	F ₁	165	240	255	230	90	80	160	195	105	100	135	130	
	F ₂	310	240	240	255	160	210	220	175	175	220	205	185	
	F ₃	340	255	240	230	210	280	300	200	280	235	265	240	
T ₂ -K4H	F ₁	105	130	100	125	70	70	105	130	90	100	105	125	
	A ₁	F ₂	160	195	165	140	90	125	150	105	140	185	165	160
		F ₃	175	205	115	175	160	150	205	135	230	230	230	210
T ₂ A ₂	F ₁	175	175	155	195	70	90	135	160	100	105	120	120	
	F ₂	230	250	250	285	135	175	160	165	170	195	175	150	
	F ₃	320	255	275	255	230	210	195	170	250	230	235	200	
A ₃	F ₁	215	250	230	310	100	100	175	170	95	175	125	115	
	F ₂	325	310	290	240	195	250	240	180	185	205	185	185	
	F ₃	390	310	335	310	325	275	255	240	265	265	265	225	

Table 7. Feed forces obtained during the machining of Modular Iron (60 grade) in a four-factor factorial randomized block design.

		Replicate 1				Replicate 2				Replicate 3			
		S ₁	S ₂	S ₃	S ₄	S ₁	S ₂	S ₃	S ₄	S ₁	S ₂	S ₃	S ₄
T ₁ -K6	F ₁	40	55	50	45	30	30	25	65	30	40	105	65
	A ₁ F ₂	80	80	75	65	40	40	80	65	45	125	100	65
	F ₃	100	105	95	80	50	90	95	100	65	135	100	65
T ₁ A ₂	F ₁	40	45	50	45	35	40	40	40	45	40	115	65
	F ₂	80	80	75	65	60	60	60	60	95	135	100	70
	F ₃	105	105	95	95	80	90	85	75	120	145	95	70
A ₃	F ₁	55	65	65	50	35	35	40	40	35	45	85	75
	F ₂	100	90	90	70	40	65	70	65	65	110	100	75
	F ₃	125	110	100	60	95	125	110	90	105	120	110	85
T ₂ -K4H	F ₁	30	40	40	45	35	35	35	40	25	45	60	45
	A ₁ F ₂	70	75	65	40	55	65	65	60	65	65	65	60
	F ₃	90	90	80	75	85	80	95	75	80	80	75	60
T ₂ A ₂	F ₁	55	55	50	45	40	35	35	40	40	65	85	75
	F ₂	85	85	75	70	60	75	60	65	75	110	110	80
	F ₃	115	100	105	95	95	90	90	90	110	125	100	85
A ₃	F ₁	60	60	60	55	35	35	40	40	35	50	75	70
	F ₂	100	95	90	75	75	75	70	65	70	105	90	80
	F ₃	135	120	110	100	105	100	100	100	125	125	95	85

- F_1 = Effect of the 1th level of rate of feed
 C_m = Effect of the mth level of speed
 (GA)_{jk} = Effect of the interaction of the jth grade of tool material and with kth level of rake angle
 (GF)_{jl} = Effect of the interaction of the jth grade of tool material and with lth level of rate of feed
 (GC)_{jm} = Effect of the interaction of the jth grade of tool material and with mth level of cutting speed
 (AF)_{kl} = Effect of the interaction of the kth level of rake angle and with lth level of rate of feed
 (AC)_{km} = Effect of the interaction of the kth level of rake angle and with mth level of cutting speed
 (FC)_{lm} = Effect of the interaction of the lth level of rate of feed and with mth level of cutting speed
 (GAF)_{jk_l} = Second order interaction effect of the jth grade of tool material, with kth level of rake angle and lth level of rate of feed
 (GAC)_{j_{kl}} = Second order interaction effect of the jth grade of tool material, with kth level of rake angle and mth level of cutting speed
 (GFC)_{j_{lm}} = Second order interaction effect of the jth grade of tool material, with lth level of rate of feed and mth level of cutting speed
 (AFC)_{k_{lm}} = Second order interaction effect of the kth level of rake angle, with lth level of rate of feed and mth level of cutting speed
 (GAFC)_{jk_{lm}} = Third order interaction effect of the jth grade of tool material, with kth level of rake angle, with lth level of rate of feed and mth level of cutting speed
 $E_{i,jklm}$ = Effect of the experimental unit in the ith replicate to which the (jklm)th treatment had been randomly assigned.

Derivation of Sum of Squares and Correlation of Forces

Denoting F_g as X and F_t as Y

Source	D.F.	X	Y	XY
Blocks	2	$(X_1 \dots -X \dots)^2$	$(Y_1 \dots -Y \dots)^2$	$(X_1 \dots -X \dots)(Y_1 \dots -Y \dots)$
Grades of tool	1	$(X_j \dots -X \dots)^2$	$(Y_j \dots -Y \dots)^2$	
Angles	2			
Feeds	2			
Speeds	3			
Tool x angle	2	$(X_{jk} \dots -X_j \dots - X_k \dots + X \dots)^2$	$(Y_{jk} \dots -Y_j \dots - Y_k \dots + Y \dots)^2$	
Tool x feed	2			
Tool x speed	3			
Angle x feed	4			
Angle x speed	6			
Feed x speed	6			
Tool x angle x feed	4	$(X_{jkl} \dots - X_j \dots - X_k \dots - X_l \dots + X \dots)^2$		

Assumptions and Test of Hypothesis

The assumptions needed for a fixed effect statistical factorial design done as randomized complete block design, of finite model, are:

1. The cumulative effects of the replicates, tool materials, angles, feed and speed and their first order, second order and third order interactions are all zero, i.e.:

$$\sum_{i=1}^r R_i = \sum_{j=1}^l G_j = \sum_{k=1}^a A_k = \sum_{l=1}^b F_l = \sum_{m=1}^c C_m = 0$$

$$\begin{aligned} \sum_{j=1}^t (GA)_{jk} &= \sum_{k=1}^a (GA)_{jk} = \sum_{j=1}^t (GF)_{jl} = \sum_{l=1}^b (GF)_{jl} \\ &= \dots \dots \dots = \sum_{j=1}^t (GAF)_{jkl} \\ &= \dots \dots \dots = \sum_{m=1}^c (GAFC)_{jklm} = 0 \end{aligned}$$

2. ϵ_{ijklm} , the random error of observations due to natural or unassignable causes are $NID(0, \sigma^2)$.

In conjunction with these assumptions, the null hypothesis to be tested, would be:

The cutting forces are not significantly affected by the different levels of the factors such as grade of tool material, rake angle, feed and speed; and there exists no interaction effects between any or all of the factors.

Each of them may be tested for significance by the F-test by dividing their respective mean squares, by the experimental error mean square, and comparing them with the ratios in the F-table at their appropriate degrees of freedom.

The level of significance was maintained at 0.05 for all the tests. (Higher significance is indicated at 0.01 level and very high significance at 0.001 level.)

Calculations

The computations of sum of squares and the mean squares were performed on a calculating machine. For ease of calculations, data was coded by dividing by 5.

The calculations involved in obtaining the sums of squares and mean squares for the analysis of variance of a four-factor factorial in a fixed effect randomized complete block design, are summarized below:

Source	Sum of squares F_C and F_T	Computations for $(F_C)(F_T)$
Blocks	$\sum_i \left(\sum_{jklm} X_{ijklm} \right)^2$	$\sum_i \left(\sum_{jklm} X_{ijklm} \right) \left(\sum_{jklm} Y_{ijklm} \right)$
Grade of tool	$\sum_j \left(\sum_{iklm} X_{ijklm} \right)^2$	$\sum_j \left(\sum_{iklm} X_{ijklm} \right) \left(\sum_{iklm} Y_{ijklm} \right)$
Tool x angle	$\sum_{jk} \left(\sum_{ilm} X_{ijklm} \right)^2$	$\sum_{jk} \left(\sum_{ilm} X_{ijklm} \right) \left(\sum_{ilm} Y_{ijklm} \right)$

The terms used in the tabular form of analysis of variance are expressed as:

R_{yy} = corrected replicate sum of squares

G_{yy} = corrected tool grade sum of squares

A_{yy} = corrected angle sum of squares

F_{yy} = corrected feed sum of squares

C_{yy} = corrected speed sum of squares

E_{yy} = corrected error sum of squares

$(GA)_{yy}$ = corrected (grade x angle) sum of squares

$(GAFC)_{yy}$ = corrected (grade x angle x feed x speed) sum of squares.

Table 8 gives the general analysis of variance for a four-factor factorial in the randomized complete block design.

While adding the sum of squares directly on calculating machine, accuracy of the work can be assured by the total value of the cell values.

Table 9 gives the computed calculations of Analysis of Variance for the tangential force (F_C), feed force (F_T) and the correlation between these two components of the cutting forces when the radial force is kept constant (by keeping the depth of cut as constant).

The test of significance on the variables is performed by dividing each of the variable mean squares by the experimental error mean square, and comparing the ratios for significance with the theoretical variance ratios tabulated in Snedecor's table 105.3 of variance ratio of appropriate degrees of freedom. If F exceeds the 5 percent level, it is called "significant"; when it exceeds the 1 percent level, it is called "highly significant" and its value exceeding 0.1 percent is called "very highly significant".

Table 3. Estimations of mean square for the 2 x 3 x 3 x 4 factorial, randomized complete block design.

Source of variation	Degrees of freedom	Mean square	Expected mean square
Replicates	(r-1)	$R_{yy}/(r-1)$	$\sigma^2 + tabc \sum_{j=1}^c (R_j)^2/(r-1)$
MAIN EFFECTS			
Grade of tool	(t-1)	$G_{yy}/(t-1)$	$\sigma^2 + rabc \sum_{j=1}^t (G_j)^2/(t-1)$
Angle of tool	(a-1)	$A_{yy}/(a-1)$	$\sigma^2 + rtbc \sum_{k=1}^a (A_k)^2/(a-1)$
Feed	(b-1)	$F_{yy}/(b-1)$	$\sigma^2 + rtac \sum_{l=1}^b (F_l)^2/(b-1)$
Speed	(c-1)	$C_{yy}/(c-1)$	$\sigma^2 + rtab \sum_{m=1}^c (C_m)^2/(c-1)$
INTERACTIONS			
Grade x angle	(t-1)(a-1)	$(GA)_{yy}/(t-1)(a-1)$	$\sigma^2 + rbc \sum_{j=1}^t \sum_{k=1}^a (GA_{jk})^2/(t-1)(a-1)$
Grade x feed	(t-1)(b-1)	$(GF)_{yy}/(t-1)(b-1)$	$\sigma^2 + rac \sum_{j=1}^t \sum_{l=1}^b (GF_{jl})^2/(t-1)(b-1)$
Grade x speed	(t-1)(c-1)	$(GC)_{yy}/(t-1)(c-1)$	$\sigma^2 + rab \sum_{j=1}^t \sum_{m=1}^c (GC_{jm})^2/(t-1)(c-1)$
Angle x feed	(a-1)(b-1)	$(AF)_{yy}/(a-1)(b-1)$	$\sigma^2 + rtc \sum_{k=1}^a \sum_{l=1}^b (AF_{kl})^2/(a-1)(b-1)$

Table 8 (cont.)

Source of variation	Degrees of freedom	Mean square	Expected mean square
Angle x speed	(a-1)(c-1)	$(AC)_{yy}/(a-1)(c-1)$	$\sigma^2 + tb \sum_{k=1}^a \sum_{l=1}^c (AC_{klm})^2 / (a-1)(c-1)$
Feed x speed	(b-1)(c-1)	$(FC)_{yy}/(b-1)(c-1)$	$\sigma^2 + ta \sum_{l=1}^b \sum_{m=1}^c (FC_{lkm})^2 / (b-1)(c-1)$
Grade x angle x feed	(t-1)(a-1)(b-1)	$(GAF)_{yy}/(t-1)(a-1)(b-1)$	$\sigma^2 + ro \sum_{j=1}^t \sum_{k=1}^a \sum_{l=1}^b (GAF_{jklm})^2 / (t-1)(a-1)(b-1)$
Grade x angle x speed	(t-1)(a-1)(c-1)	$(GAC)_{yy}/(t-1)(a-1)(c-1)$	$\sigma^2 + rb \sum_{j=1}^t \sum_{k=1}^a \sum_{m=1}^c (GAC_{jklm})^2 / (t-1)(a-1)(c-1)$
Grade x feed x speed	(t-1)(b-1)(c-1)	$(GFC)_{yy}/(t-1)(b-1)(c-1)$	$\sigma^2 + ra \sum_{j=1}^t \sum_{l=1}^b \sum_{m=1}^c (GFC_{jlm})^2 / (t-1)(b-1)(c-1)$
Angle x feed x speed	(a-1)(b-1)(c-1)	$(AFC)_{yy}/(a-1)(b-1)(c-1)$	$\sigma^2 + rt \sum_{j=1}^a \sum_{l=1}^b \sum_{m=1}^c (AFC_{jklm})^2 / (a-1)(b-1)(c-1)$
Grade x angle x feed x speed	(t-1)(a-1)(b-1)(c-1)	$(GAFC)_{yy}/(t-1)(a-1)(b-1)(c-1)$	$\sigma^2 + r \frac{\sum_{j=1}^t \sum_{k=1}^a \sum_{l=1}^b \sum_{m=1}^c (GAFC_{jklm})^2}{(t-1)(a-1)(b-1)(c-1)}$
RESIDUAL			
Experimental error	(r-1)(tabc-1)	$E_{yy}/(r-1)(tabc-1)$	σ^2
TOTAL	(rtabc-1)		

DISCUSSION

Results of Analysis of Variance
on Tangential Force

Tests of the analysis of variance reveal (Table 9) that the first order interaction of tool grade and tool angle is very highly significant and the first order interaction of cutting speed and feed rate is highly significant. This means that the interactions of tool grade x angle and feed x speed at various levels create significant variation in the tangential force component. Table 11 gives the regression analysis of the interactions found significant.

Tool-angle and feed rate have very highly significant effect on the tangential force. Variation of tool angles and feed rate separately is responsible in creating change in the magnitude of tangential force.

It is indicated by the Orthogonal-Comparison of the interaction factors that there is:

1. Significant linear effect of rake angle.
2. Significant linear effect of feed rate.
3. Significant quadratic effect of cutting speed.
4. Significant interaction between linear effect of tool grade and linear effect of rake angle.
5. Significant interaction between linear effect of feed rate and linear effect of cutting speed on the tangential force. Effects 2, 4 and 5 are very highly significant and 1 is highly significant.

Figures 11, 13 and 15 give a further analysis of the interaction source of variation in tangential force. The F-test does not give any clue as to how many differences there are in mean (M). In order to find as to whether each mean differs from all the rest, or some are differentiated, LSD (Least Significant Difference) is performed to locate the significant differences.

Table 9. Analysis of variance and covariance of factors: grade of tool, angle, feed and speed on the tangential (F_G) and feed (F_T) forces in machining of nodular cast iron of 60 grade, randomized complete block design for $2 \times 3 \times 3 \times 4$ factorial.

Source of variation	Degrees of freedom	Mean F_G	Square F_T	Covariance $F_G \times F_T$	Correlations coefficient
Replicates	2	100,046.18***	5,726.38***	9,107.98	.381
Tool grades	1	2,744.90	111.22	552.55	1.000
Angles	2	51,854.51***	3,756.60***	12,748.09	.913
Feeds	2	167,134.72***	40,921.18***	82,599.30	.999
Speeds	3	2,708.18	2,076.97***	1,923.38	.811
Grade x angle	2	18,630.68***	811.23*	3,755.15	.966
Grade x feed	2	722.69	25.12	82.06	.122
Grade x speed	3	923.61	430.67	525.54	.834
Angle x feed	4	1,586.11	254.34	551.56	.868
Angle x speed	6	958.99	170.49	99.94	.247
Feed x speed	6	6,404.48**	1,282.76***	2,523.61	.880
G x A x F	4	553.24	25.64	20.08	.169
G x A x C	6	421.87	136.22	-64.45	-.269
G x F x C	6	1,243.98	163.54	97.41	.216
A x F x C	12	591.74	69.16	109.55	.541
G x A x F x C	12	333.57	57.12	11.94	.086
Error	142	1,393.95	44.72		
Total	215				

N = 216

F Test

0.01 < P ≤ 0.05 * Significant
 0.001 < P ≤ 0.01 ** Highly significant
 P ≤ 0.001 *** Very highly significant

Table 10. Coefficients and divisors for orthogonal comparisons.

Comparison	Tool--0, 1		Angle--0, 1, 2		$\sum \lambda^2$	$(\sum \lambda^2)n$						
	00	01	11	12								
Linear tool	-1	-1	+1	+1	6	216						
Linear angle	-1	0	-1	0	4	144						
Quad. angle	-1	+2	-1	+2	12	432						
L.T. x L.A.	+1	0	-1	0	4	144						
L.T. x Q.A.	+1	-2	-1	+2	12	432						
S _{1j}	6,775	6,770	7,555	6,765	8,310							
F ₀	2,520	2,700	2,800	2,770	2,905							
F _T												
n = 36	$SS = \frac{(\sum \lambda_{1j} S_{1j})^2}{n \sum \lambda_{1j}^2}$											
Comparison	Feed--0, 1, 2			Speed--0, 1, 2, 3								
	00	01	02	03	10	11	12	13	20	21	22	23
Linear feed	-1	-1	-1	-1	0	0	0	0	+1	+1	+1	+1
Quad. feed	+1	+1	+1	+1	-2	-2	-2	-2	+1	+1	+1	+1
Linear speed	-3	-1	+1	+3	-3	-1	+1	+3	-3	-1	+1	+3
Quad. speed	+1	-1	+1	+1	+1	-1	-1	+1	+1	-1	-1	+1
Cubic speed	-1	+3	-3	+1	-1	+3	-3	+1	-1	+3	-3	+1
L.F. x L.S.	+3	+1	-1	-3	0	0	0	0	-3	-1	+1	+3
L.F. x Q.S.	-1	+1	+1	-1	0	0	0	0	+1	-1	-1	+1
L.F. x C.S.	-1	+1	+3	-1	0	0	0	0	-1	+3	-3	+1
Q.F. x L.S.	-3	-1	+1	+3	+6	+2	-2	-6	-3	-1	+1	+3
Q.F. x Q.S.	+1	-1	-1	-2	-2	+2	-2	-2	+1	-1	-1	+1
Q.F. x C.S.	-1	+3	-3	+1	+2	-6	+6	-2	-1	+3	-3	+1
S _{1j}	1,925	2,525	2,775	2,915	3,440	3,775	3,645	3,420	4,485	4,405	4,185	3,945
F ₀	700	815	1,055	945	1,260	1,535	1,440	1,195	1,785	1,935	1,735	1,485
F _T												
n = 18												

144
432
1,080
216
1,080
720
144
720
2,160
432
2,160

8
24
60
12
60
40
8
40
120
24
120

Table 11. Regression comparison.

Comparison	Tool angle		Feed speed		Comparison	Feed speed	
	F _C	F _T	F _C	F _T		F _C	F _T
Linear tool	2,744.90	11.227	329,667.36***	81,462.67***	Linear feed	329,667.36***	81,462.67***
Linear angle	102,133.51**	6,875.17***	4,602.08	379.69	Quad. feed	4,602.08	379.69
Quad. angle	1,575.52	637.15	301,333.33	159.47	Linear speed	301,333.33	159.47
L.T. x L.A.	35,941.84***	1,314.06*	6,337.50*	6,069.56***	Quad. speed	6,337.50*	6,069.56***
L.T. x Q.A.	1,319.50	308.39	453.70	1.88	Cubic speed	453.70	1.88
			35,701.25***	5,980.035***	L.F. x L.S.	35,701.25***	5,980.035***
			584.03	212.67	L.F. x Q.S.	584.03	212.67
			11.25	834.20*	L.F. x C.S.	11.25	834.20*
			1,450.42	95.85	Q.F. x L.S.	1,450.42	95.85
			602.08	398.67	Q.F. x Q.S.	602.08	398.67
			77.82	175.10	Q.F. x C.S.	77.82	175.10

0.01 < P ≤ 0.05

*Significant

0.001 < P ≤ 0.01

**Highly significant

P ≤ 0.001

***Very highly significant

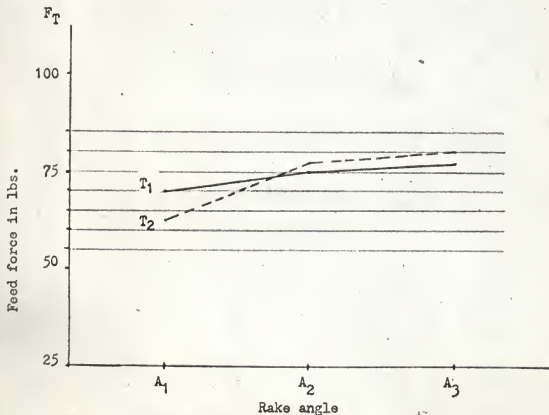
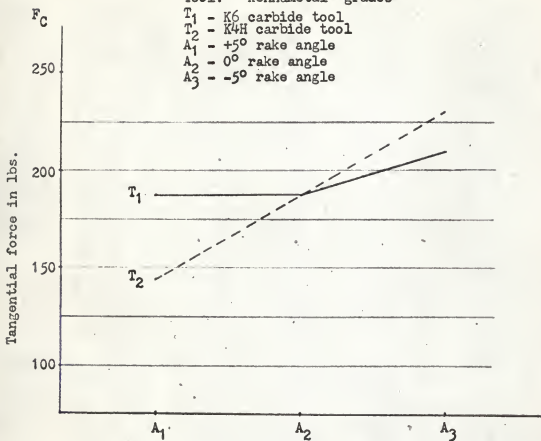
Tangential Force = $M_1 + b_1A + c_1F + d_1S^2 + e_1(GA) + f_1(FS)$ Feed Force = $M_2 + b_2A + c_2F + d_2S^2 + e_2(GA) + f_2(FS) + g_2(FS^3)$

Material: Nodular Iron (Grade 60)

Tool: "Kennametal" grades

T₁ - K6 carbide toolT₂ - K4H carbide toolA₁ - +5° rake angleA₂ - 0° rake angleA₃ - -5° rake angle

LSD = 17.25



LSD = 6.74

Fig. 12. Rake angle vs. feed force.

LSD = 24.39

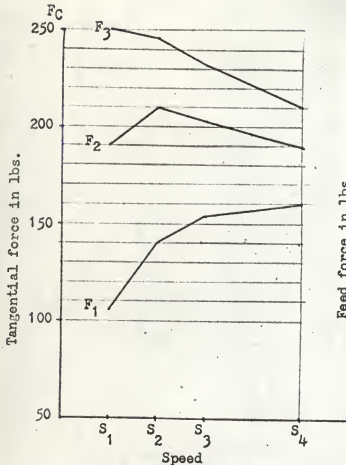


FIG. 13. Cutting speed vs. tang. force.

Workpiece material:

Nodular Iron
(grade 60)

Carbide tip tools:

Feed: $F_1 = 0.005''/\text{rev.}$
 $F_2 = 0.010''/\text{rev.}$
 $F_3 = 0.015''/\text{rev.}$
 Speed: $S_1 = 117 \text{ f.p.m.}$
 $S_2 = 218 \text{ f.p.m.}$
 $S_3 = 340 \text{ f.p.m.}$
 $S_4 = 550 \text{ f.p.m.}$

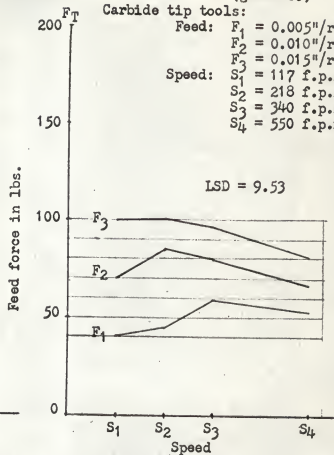


FIG. 14. Cutting speed vs. feed force.

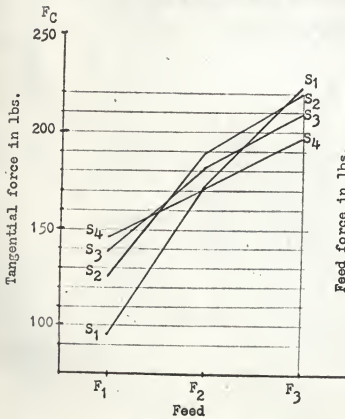


FIG. 15. Feed rate vs. tang. force.

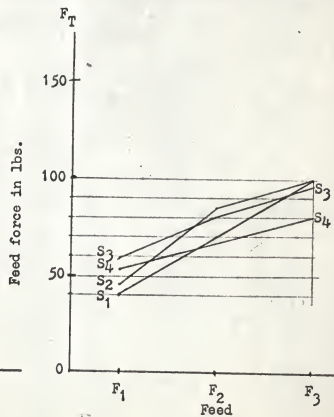


FIG. 16. Feed rate vs. feed force.

The values of LSD calculated for each of interaction are given in the respective figures. It is evident from Fig. 11 that:

1. Tool grade K4H (Kennametal) with positive rake angle develops significantly less tangential force than grade K6 with same rake angle.
2. With the neutral rake angle, both the grades of tool generate almost equal tangential cutting force. There is significant increase in the magnitude of the force when changing the rake angle of grade K4H of tool from positive ($+5^{\circ}$) to neutral.
3. With negative rake (-5°) both the grades of tool show significant increase in the tangential force. Increase in the magnitude of force (tangential) is very prominent with grade K4H tool, which develops significantly higher force than K6 when operated with negative rake angle.
4. The effect of changing of rake angle from positive to neutral to negative is almost linear in increasing the tangential force.

The analysis (Figs. 13, 15) of feed and speed shows that:

1. Higher feed rates develop very high amounts of tangential force at cutting speed of about 117 s.f.p.m.
2. An increase in cutting speed to about 218 s.f.p.m. brings in a significant increase in tangential force at feed rate of $0.005''/\text{rev}$. Increase in the magnitude of tangential force at $0.01''/\text{rev}$. and 218 s.f.p.m. is not significant. The decrease in the magnitude of tangential force at $0.015''/\text{rev}$. and 218 s.f.p.m. is not significant, however, it is encouraging to operate at higher cutting speed at this feed.
3. At cutting speed of about 340 s.f.p.m., at $0.005''/\text{rev}$. there is an increase in tangential force but it is not significant as compared with 218 s.f.p.m. speed. At feeds $0.01''/\text{rev}$. and $0.015''/\text{rev}$. there is an insignificant decrease in the magnitude of tangential force at this cutting speed as compared to 117 and 218 s.f.p.m.

4. A further increase in cutting speed to about 550 s.f.p.m. does not appreciably increase the force magnitude with 0.005"/rev. feed rate. The reduction in force magnitude with feed rates of 0.01"/rev. and 0.015"/rev. is not significant as compared to operation at cutting speed of 340 s.f.p.m. However, it is quite apparent that (i) with feed rate of 0.01"/rev. the magnitude of tangential force is almost same when operated at either speed 117 s.f.p.m. or 550 s.f.p.m., and (ii) the magnitude of force at 550 s.f.p.m. with 0.015"/rev. is almost same when operating at 218 s.f.p.m. with 0.01"/rev., and (iii) it is always advantageous to operate at higher speed of 550 s.f.p.m. when using feed rate of 0.015"/rev.

The effect of feed 0.015"/rev. shows almost linear trend with changes of cutting speed and that the regression line has a negative slope.

At all the speeds of operation used in this experiment, there is a significant increase in the magnitude of the tangential force when changing feed rate from 0.005"/rev. to 0.01"/rev. to 0.015"/rev. Speeds 117 s.f.p.m. and 550 s.f.p.m. very closely show linear regression in the amount of tangential force with positive slope when the feed rate increases from 0.005"/rev. to 0.01"/rev. to 0.015"/rev.

The significance of replicates effect will be discussed later along with their effect on feed force. The following equation gives the relationship of the tangential force with the influencing main factors and the interactions:

Tangential Force = $M_1 + b_1A + c_1F + d_1C^2 + e_1(GA) + f_1(FC)$ where b_1, c_1, d_1, e_1 and f_1 are all constants associated with the coefficients of regression and A, F, G, C, C^2 , etc. depict the linear and higher orders of angle, feed, grade of tool and cutting speed respectively.

Results of Analysis of Variance on Feed Force

The analysis of variance of the feed force is shown in Table 9. The regression analysis of the orthogonal comparisons is given in Table 11 along with the analysis of the tangential force. A further analysis by LSD test is shown in Figs. 12, 14 and 16.

There is first order interaction significant effect of the grade of tool and angle; and very highly significant effect of the interaction of feed rate and cutting speed.

The main factors, angle, feed and cutting speed, have very highly significant effect on feed force.

The orthogonal comparison indicates that there is:

1. Very highly significant linear effect of angle.
2. Very highly significant linear effect of feed.
3. Very highly significant quadratic effect of cutting speed.
4. Significant interaction of linear effect of grade of tool and linear effect of angle.
5. Very highly significant interaction of linear effect of feed rate and linear effect of cutting speed.
6. Significant interaction of linear effect of feed rate and cubic effect of cutting speed on the magnitude of feed force. The relationship of these effects with the feed force is given by:

Feed Force = $M_2 + b_2A + c_2F + d_2C^2 + e_2(GA) + f_2(FC) + g_2(FC^3)$ where b_2, c_2, d_2, e_2, f_2 and g_2 are all constants associated with the coefficients of regression and A, C, F, G, C^2, C^3 , etc. depict the linear and higher orders of angle, cutting speed, feed rate and grade of tool respectively.

The LSD test shows that:

1. The two grades of tool with positive ($+5^{\circ}$) rake angle generate significantly different amount of feed force and that the grade K4H is more advantageous to use than K6 grade tool.
2. With neutral rake angle, there is no significant difference in the feed force generated by both grades of tool. Increase in the magnitude of feed force with operation with K4H is highly significant while that with K6 is not significant when changing from positive rake to neutral rake.
3. When operating with negative rake angle, there is no appreciable difference in the magnitude of feed forces between the two grades K4H and K6. The K4H grade, however, develops higher feed force with negative rake angle. The increase in the feed force is not significant when changing rake angle from neutral to negative with any of the K4H or K6 grade.

Grade K6 shows a somewhat linear regression trend with change of rake angle from positive to neutral to negative.

The analysis of feed and speed shows:

1. Higher feed rate develops significantly higher amount of feed force at cutting speed of about 117 s.f.p.m. This speed shows linear regression between the feed rate and the feed force.
2. At about 218 s.f.p.m. cutting speed, there is significant increase in feed force at $0.01''/\text{rev.}$ feed rate. The increase is not significant with $0.005''/\text{rev.}$ and there hardly any change in the feed force at $0.015''/\text{rev.}$
3. At about 340 s.f.p.m. cutting speed, there is significant increase in feed force. The reduction in the magnitude of feed force at $0.01''/\text{rev.}$ and $0.015''/\text{rev.}$ feed rate is not significant. However at this cutting speed, there seems to be linear regression between the feed force and the feed rate.

4. At the cutting speed of about 550 s.f.p.m. the reduction in the magnitude of feed force at the feed rates of $0.01''/\text{rev.}$ and $0.015''/\text{rev.}$ is quite significant. At the feed rate of $0.005''/\text{rev.}$ and this speed the decrease in feed force is not significant. However, once again, there seems to be linear regression between the feed force and the feed rate at the speed of about 550 s.f.p.m.

It is interesting to note that (i) the cutting speed of 550 s.f.p.m. is more favorable at the feed rate of $0.015''/\text{rev.}$ and that the feed force developed is of the magnitude of operating at 218 s.f.p.m. cutting speed and $0.01''/\text{rev.}$ feed rate, (ii) when operating at $0.01''/\text{rev.}$ feed rate, the feed force developed at cutting speed of 550 s.f.p.m. is of the same magnitude when operating at 117 s.f.p.m., (iii) feed force at 550 s.f.p.m. and $0.01''/\text{rev.}$ is not significantly higher at 340 s.f.p.m. and $0.005''/\text{rev.}$

Replicates. Highly significant effect of the replicates on the magnitude of the tangential and the feed forces indicates the non-homogeneous nature of the workpiece material. This means that the three pieces of tubes used as three replicates differ in their physical and/or chemical properties of each other.

Workpieces used as replicates 1 and 2 were supplied by one foundry with the code of MT-4 and MT-5 respectively. The chemical composition of these two workpieces was reported to be the same (Table 4). Although, both of these tubes were cast as twins, together in the same mold, from the same heat of ladle, MT-5 piece was stress-relieved. This seems to have changed the microstructure (shown in Fig. 5) of this thin-walled tube. Replicate 3 used was a workpiece supplied by another foundry, meeting the specifications of grade 60 (minimum value, as marketed). This though had a different chemical composition, the microstructure closely relates to that of MT-5 (Fig. 5) due to the reason that it was annealed by the foundry. The ferritic microstructure of replicates 2 and 3 indicates their effect of better machinability than replicate 1 which shows about

ten percent of pearlite in microstructure. The graphite nodules are surrounded by pearlite in replicate 1 (MT-4).

Correlation between feed forces and tangential forces. Since, the tangential and feed forces were recorded simultaneously for all the tests run, a correlation analysis and a multivariate test (using Wilks's Λ) were attempted. The analysis and the test were necessary to judge as to whether any interdependency existed between the tangential and the feed forces. Correlation coefficients are given along with AOV of the tangential and feed forces in Table 9. The multivariate test by Wilks's Λ distribution confirmed the result of Table 9, as such it is not included in this report.

A high correlation exists due to:

1. Grade of tool
2. Angle
3. Feed rate
4. Cutting speed
5. Interaction effect of grade of tool and angle
6. Interaction effect of grade of tool and cutting speed
7. Interaction effect of angle and feed rate
8. Interaction effect of feed and cutting speed

However, the interaction effect of the factors angle, feed and cutting speed creates a lesser correlation as compared to 7 and 8 above. The interaction effect of grade of tool, angle and cutting speed show a negative correlation against 5 and 6 above. Figures 17 and 18 show the correlation trend between the tangential and the feed forces due to the varying effects of feed, cutting speed, angle and feed x speed.

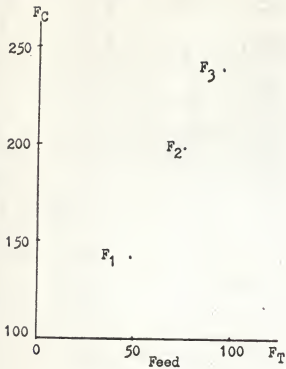


Fig. 17(a).

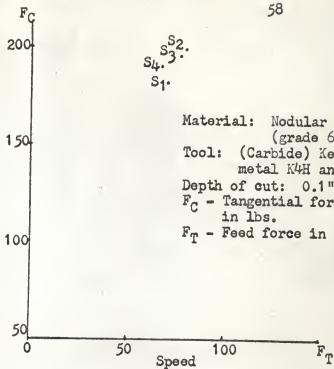


Fig. 17 (b).

Material: Nodular Iron (grade 60)
 Tool: (Carbide) Kennametal K4H and K6
 Depth of cut: 0.1"
 F_C - Tangential force in lbs.
 F_T - Feed force in lbs.

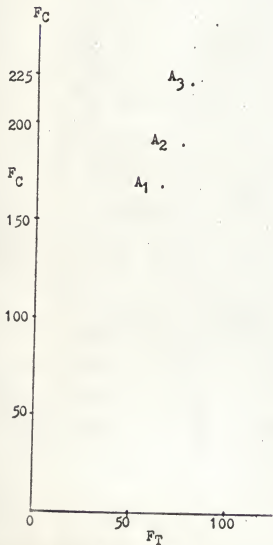


Fig. 18(a). Angle

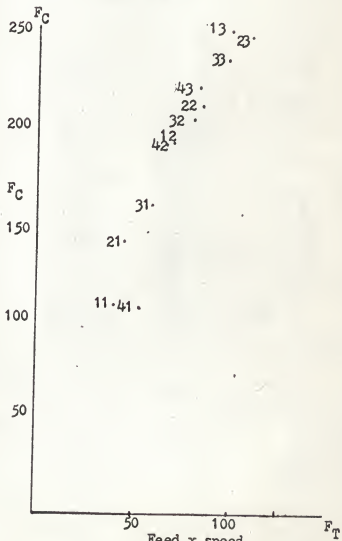


Fig. 18(b).

Correlation between tangential and feed force.

Figure 17(a) indicates that by increasing feed rate from $0.005''/\text{rev.}$ to $0.01''/\text{rev.}$ to $0.015''/\text{rev.}$ both the components of cutting force increase considerably. Figure 18(a) shows that with speed increase from 117 f.p.m. to 218 f.p.m. both the forces increase and then decrease at speed of 340 f.p.m. At 550 f.p.m. there is further drop in the magnitude of forces and that the feed force decreases at almost twice the rate the tangential force decreases. It is also indicated that high speed operation generates lesser amount of forces, a further experimentation at higher speed will further reduce the forces. This shows an agreement to the research work of Hitomi and Thuring (17).

By changing the rake angle from positive to neutral to negative, increases the tangential force while there is hardly any increase in the feed force. The interaction effect of feed x speed shows that the tangential and the feed forces increase and decrease together except when increasing speed from 117 s.f.p.m. to 218 s.f.p.m. at $0.015''/\text{rev.}$ where the feed force increases considerably while there is hardly any change in tangential force. Operating at higher feed and higher speed generates lesser forces than higher feed and lower speeds. This is quite apparent from Fig. 18b.

CONCLUSION

The technique of statistical analysis helped to formulate the following inferences:

Tool grades K6 and K4H (of Kennametal) can both be employed when machining Modular Iron (grade 60).

Machining at higher speeds (550 f.p.m.) at higher feed rate ($0.015''/\text{rev.}$) is advantageous. The cutting forces (hence the shearing force) have lower magnitude, as such an increase in the amount of chip removal at lower cost.

Tool grade K4H is preferable to K6 when machining with positive ($+5^\circ$) rake angle. Any of the two grades may be employed when operating with neutral rake angle. However, when it becomes necessary to use negative (-5°) rake angle, tool grade of K6 (so-called cast-iron cutting grade) should be employed. Due to the higher density and lower cobalt content in K6, there will be added advantage of lesser B.U.E. (built-up-edge) and take up the initial shock of the cut.

The trend of regression curves in Fig. 13 indicate that if the machine capacity permits, speed higher than 550 s.f.p.m. can be applied which will develop lesser cutting forces. Increase in the feed rate (more than $0.015"/\text{rev.}$) will increase the cutting forces. When it becomes necessary to machine at 117 s.f.p.m. the compromising feed of $0.01"/\text{rev.}$ will keep the cutting forces at lower level.

Since the feed and the speed do not interact with tool grade or angle, they can be changed as per the machining requirements without fearing any increase in cutting forces. However, the machining conditions will change if factors other than grades of tool, angle, feed and cutting speed are brought in to play any role.

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STATISTICAL ANALYSIS OF CUTTING FORCES
WHILE MACHINING NODULAR CAST IRON

by

NARAIN G. KEWALRAMANI

D. M. E., M. S., University of Baroda, India, 1955
B. S. (I. E.), Kansas State University, 1963

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The object of this thesis was to analyze statistically data comprising of two major components of cutting forces, so as to determine the manner in which independent known variables in metal cutting process, and their possible interactions affect the magnitude of the tangential and the feed forces (dependent variables).

The statistical model formulated was one of four-factor factorial, completely randomized block design.

The independent variable factors under control were two tool material grades (Kennametal K6 and K4H), three levels of rake angles ($+5^\circ$, 0° , -5°), three levels of feed rate (.005 ipr, .010 ipr, and .015 ipr), and four levels of cutting speed (117 s.f.p.m., 218 s.f.p.m., 340 s.f.p.m., and 550 s.f.p.m.).

The experiment was performed on Nodular Iron (60 grade), using brazed carbide tools of grades K6 (cast-iron cutting grade) and K4H (so-called steel cutting grade). Conventional form of machining was performed, without the use of any cutting fluid.

A three dimensional lathe tool dynamometer and "Sanborn amplifier" unit with two channels were employed to measure the tangential and the feed forces simultaneously.

The investigations revealed that the effects of first order interactions of feed x speed and tool grade x angle and the main factors, feed and tool angle were very highly significant on the magnitude of tangential force. The effect of first order interaction of tool grade x angle was found to be significant and the effects of first order interaction of feed x speed and the main factors, angle, feed and speed were very highly significant, on the feed force.

It also showed that the two components of the cutting forces were correlated for the grades of tool, feed rates; first order interactions of tool grade x

angle, angle x feed, feed x speed and the second order interaction effect of angle x feed x speed.

A further analysis showed that it was advantageous to operate with high feed rate and high cutting speed. It indicated that (if machine capacity could permit) higher cutting speed could be employed resulting in still lower cutting forces.

It also showed that tool grade K4H with positive ($+5^{\circ}$) rake angle developed appreciably less cutting forces than the grade K6 with same tool geometry. However, with negative (-5°) rake angle, tool grade K6 showed superiority over grade K4H. For neutral rake angle, both the grades performed equally.

The replicates showed a very highly significant effect. This means that the material supplied by the foundries (Nodular Iron, grade 60) was not homogeneous. This was also revealed by the microstructure. It showed that the workpieces used as replicates 1 and 2 had different microstructure though reported to have same chemical composition. Workpiece used as replicate 3, matched the microstructure of replicate 2, but was reported to have different chemical composition than replicates 1 and 2 material. Replicate 1 showed about 10 percent pearlite structure and 90 percent ferritic. Workpiece material of replicates 2 and 3 showed an entire ferritic structure.