ECONOMY IN ULTIMATE STRENGTH DESIGN

by

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PURPOSE

The purpose of this report is to make a comparitive study of the two methods of design of reinforced concrete structure, 1. ultimate strength design, 2. working stress design, and show the economy of ultimate strength design over the working stress design, and the convenience in its design procedure.

INTRODUCTION

Ultimate load design of reinforced concrete members is not a new development. Its origin may be found far back to the concepts of elasticity and working stresses. The origin of Systematic thought regarding flexure of beams, Galilei's work of 1638, was exclusively devoted to ultimate strength design (1). Hook's law was formulated 40 years later, and over 180 years elapsed before the fundamental theorems of the elasticity were developed by Navier in 1821 (12).

However straight line theory was generally accepted in 1900 because it was mathematically simple and the resulting safety factors with respect to ultimate load observed in tests were sufficiently controlled to satisfy the requirements of that time (12).

In 1909 the point committee on standard specifications for concrete and reinforced concrete established the straight line theory (12). Until very recently most methods of structural design have been based on the assumption that stress and strain are proportional. This assumption is far from the truth especially for concrete. As shown in Plate I, stresses and strains in concrete are proportional only at a relatively low stress, but at the higher stresses the strain increases at a higher rate than the stress (21). As the short comings of straight line method became increasingly evident, arbitrary adjustments were added to the design codes to account for these recognized discrepancies. The result is the inconsistency of designing members by an ultimate strength method when the load is axial, by a straight line method when flexure is added, and by a mixture of methods when the member contains compression steel (5). It appears from recent research on members subjected to combined bending and flexual stress that the most important part of the straight line theory retained by codes is inaccurate (5). The only way to avoid the inconsistency and to achieve design of maximum economy is to base the design of all types of concrete members on the actual performance of concrete as depicted schematically in Plate 1. Recognizing this situation, the American Concrete Institute and American Society of Civil Engineers, in 1952 formed a point committee on ultimate strength design. This committee published its report in 1955, in which procedures were proposed for ultimate strength design of tension-reinforced

PLATE I

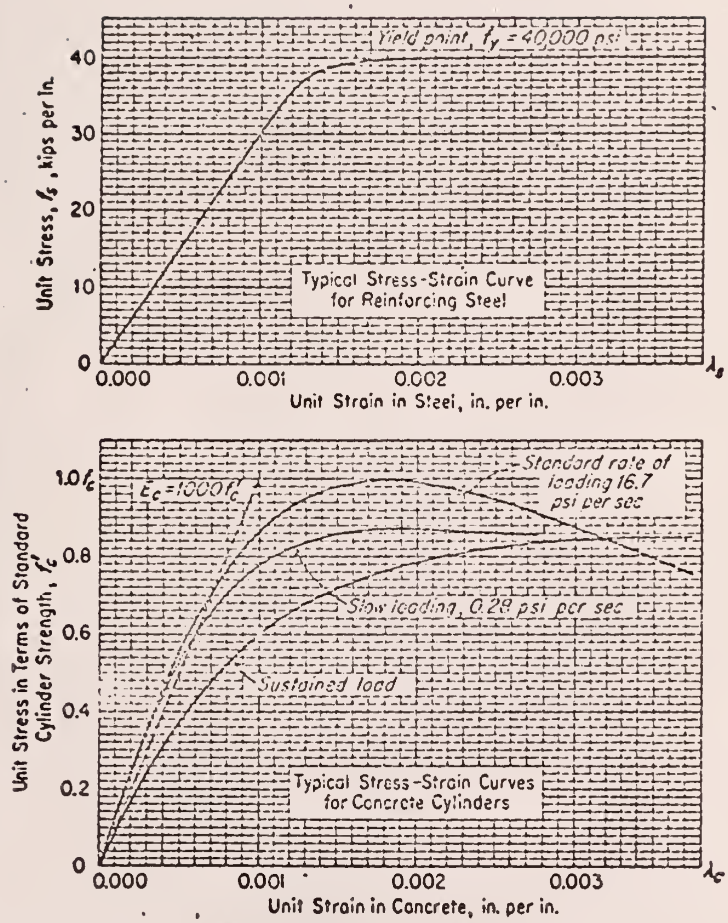


FIG. 4.2. The concrete curves of this figure are taken from C. S. Whitney, "Application of Plastic Theory of the Design of Concrete Structures," *J. Boston Soc. Civil Engrs.*, vol. 25, no. 1, 1948.

(a) Typical stress-strain curve for reinforcing steel.

(b) Typical stress-strain curves for concrete cylinders.

beams without and with compression reinforcement and of concentrically and eccentrically loaded columns, rectangular as well as circular.

HISTORICAL DEVELOPMENT

Ultimate load design is basically a return to forgotten fundamentals. Although reinforced concrete members are seldom subjected to a state of pure flexure in actual service, the ultimate strengthfor this type of loading has been the object of continued research since the development of reinforced concrete as a structural material. Several ultimate load theories have been proposed from time to time because the stress-strain curve of concrete is not very definite and is influenced by several factors e.g. quality of concrete, speed of loading, etc.

The first ultimate theory for beams in flexure was advanced by Koenen (26) in 1886 (11). This theory and others like it were used as the basis for design of beams prior to the introduction of the "straight line" theory about 1900. Thullies (27) flexural theory of 1897 and Rotter's (28) introduction of the parabolic distribution of concrete stresses in 1899, were ultimate load theories (1). (refer Fig. 1) The first theory of this type in America was developed by Talbot (29) in 1904. He recognized that even if straight line relations be accepted as sufficient for use with ordinary working stress the parabolic or other variable

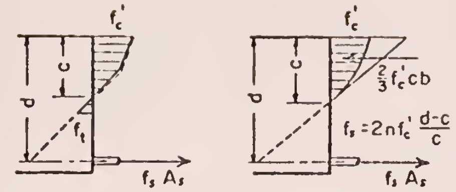


Fig. (1)

relation must be used in discussing experimental data, when any considerable deformation is developed in the concrete.

In 1921 McMillans study of column test data, showed that building columns under load may develop steel stress due to creep considerably higherthan those predicted by straight line theory (12).

In 1931 Emperger (30) presented a paper and shortly after this, in 1932 Stussi (31) in Switzerland presented an ultimate theory (11). These theories were further developed by Brandtozaeg (32) in 1935. Jensen (16) in 1943 and Chamband (33) in 1949 (11). In 1937 Whitney presented a paper in ultimate theory based on rectangular stress block and factors determined expirically from tests (20). In 1941 Cox (22) presented a similar theory, and in 1943 Jensen reported the development of a theory similar to that of Brandtzaeg, in which the character of the stress block was related to the strength of the concrete (16). In 1955 the report of ASCE-ACI joint committee on ultimate strength design was published, which recommended the use of ultimate strength design for simple structures (1). Jain in 1960 presented a paper in which he proved that elastic theory calculations can not estimate the ultimate strength of arches even approximately, and also stated that to get the true idea of factor of safety, ultimate load theory calculations are necessary (23). His tests proved that the elastic theory underestimated the ultimate strength by about forty per cent while the proposed theory gave results which agree closely with the test observation. In 1961 Mattick, Kriz and Hognestad in their paper proposed the use of an equivalent rectangular stress distribution and the applicability to the calculation of the ultimate strength of structural concrete (13). Wang in 1962 stated that with the aid of tables and curves, the ultimate strength design of concrete structure will be made more appealing to practical engineers as evidenced by his examples (8).

ULTIMATE STRENGTH DESIGN THEORY

The term ultimate strength design means the design of reinforced concrete sections by plastic theory to resist moment, thrust, and shear which have been determined from elastic analysis of the structure under the assumed design loads multiplied by specified load factors (10). It is a method of proportioning reinforced concrete members based on calculations of their ultimate strength. The theory is based on the idea of calculating the ultimate load a member will carry, then reducing that load by some factor of safety to determine the design load.

The basic three fundamental steps by which any structure is designed are: I. The determination of the service loads the structure is to carry; II. The determination of the forces, moments and deflections which these loads create; III. The determination of the dimensions of the members to resist economically the movement and forces produced by some multiple of the service loads. Ultimate load design is concerned solely with the last step.

The fundamental difference between the straight line and the ultimate strength theories is in the stress-strain relationship assumed for stresses near the ultimate load. The straight line theory assumes that stress and strain are proportional up to the ultimate capacity approached, and that the plastic range of the material reached when the stress and strain are no longer proportional. Therefore, at the ultimate load the stress distribution in the compressive zone is not trianglar but curvilinear. Various possible shapes ranging from simple rectangle to a parabola have been proposed as representative of the stress distribution at failure.

After various experiments it is concluded that rectangular stress distribution theory permits prediction with sufficient accuracy of the ultimate strength in bending in compression, and a combination of the two, of all types of structural concrete sections likely to be encountered in structural design practice (13). And according to ACI building code a rectangular concrete stress distribution is defined as follows:

> "At ultimate strength, a concrete stress intensity of 0.85 f'_c shall be assumed uniformly distributed over an equivalent compression zone bounded by the edges of the cross section and a straight line located parallel to the neutral axis at a distance $a=k_1c$ from the fiber of maximum compressive strain. The distance c from the fiber of maximum strain to neutral axis is measured in a direction perpendicular to that axis. The fraction k, shall be taken an 0.85 for strength f'_c, up to 4000 psi and shall be reduced continuously at a rate of 0.05 for each 1000 psi of strength in excess of 4000 psi."

Plate II shows a variety of stress diagrams which have been assumed during the period 1914 to 1949.

ASSUMPTIONS (10)

The assumption on which the ultimate strength design theory is based follows:

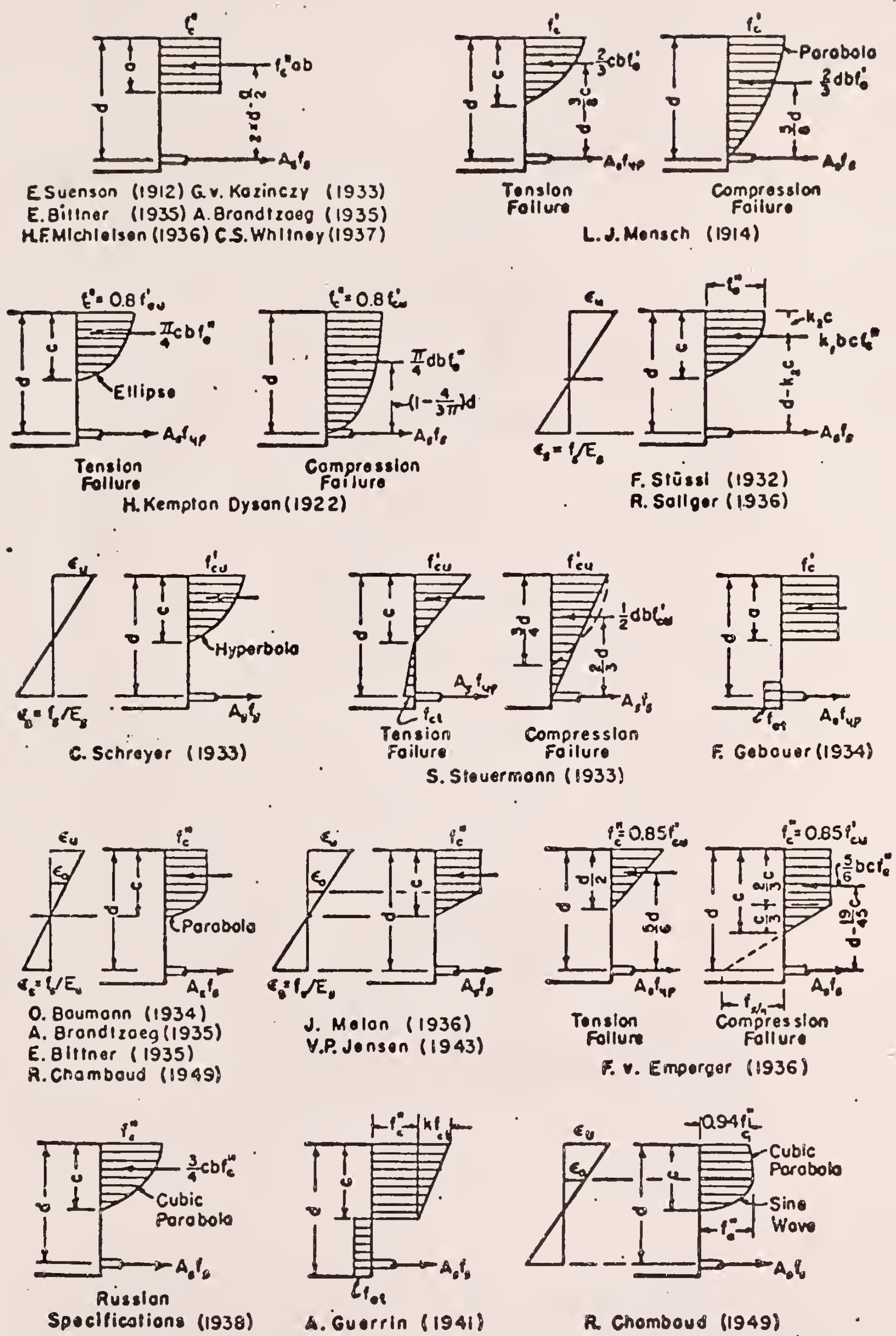
(1) Tensile strength in concrete is neglected in the designing of sections subject to bending.

(2) At ultimate load, stresses and strains are not proportional and the distribution of compression stresses in a section subject to bending is non linear.

(3) The compressive stress distribution used for design is assumed to be rectangular.

(4) The uniform compressive stress in the equivalent stress block is equal to 0.85 f'_c.

PLATE II



A variety of stress diagrams which have been assumed during the period 1914 to 1949.

(5) The total force and the location of the centroid in the rectangular stress blocks are the same as for the actual non linear stress distribution.

(6) Plane sections normal to the axis remain plane after bending.

LOAD FACTOR

The primary purpose of ultimate strength design is to obtain a more uniform over-load factor when the structure is loaded to near its ultimate strength. In ultimate design we deal with ultimate strength instead of allowable stresses. To obtain sufficient margin of safety, especially to take care of extra heavy loads, it is necessary that the service load be multiplied by suitable load factors, and the product of these two be used as design load.

The practical concept such as importance and service nature of structure, seriousness of failure, suddenness of failure have effect on margin of safety. So it is desirable to vary the margin of safety based on a sense of relative values and significance.

There are several different ways to provide margins of safety. It can be provided in stresses, by using suitable values of allowable stress. It can be provided in the loads by using suitable load factors, it can be provided in dimension of structure by making the size a little longer or it can be provided in the combination.

The method of providing safety margins should be logical and without involving unnecessary complications. The margin of safety should be provided where it belongs and wherever it is needed. The experiments revealed that providing different load factors is the logical and realistic approach, and this is in fact the most practical and simplest method, especially in connection with ultimate design of reinforced concrete (9).

The complexity of the problem must be first studied to choose the load factors correctly. Load factors will usually be different for different kind of loads such as dead loads, live loads, impact loads, snow loads, ice loads, strain loads, and lateral loads. Secondly, load factors may vary with different combinations and nature of loads, and also with the type of structure whether a bridge, a building, a tower or a tank. One must consider that members should be capable of carrying, without failure, the critical load combination. Thereby insuring an ample factor of safety against an increase in live load. And the strains under working loads should not be so large as to cause excessive cracking. The ACI code, suggest the following equations for the design load of the structures (24).

$$U = 1.5D + 1.8L \quad (1)$$

$$U = 1.25\,(D+L+W) \quad (2)$$

$$U = 0.9D + 1.1W \quad (3)$$

U = Ultimate load

D = Dead load

L = Live load

W = Wind load

E = Earthquake load

For those structures in which earthquake loading must be considered, E shall be substituted for W in equation (2).

ADVANTAGES OF ULTIMATE STRENGTH DESIGN (12)

As the ultimated load is approached the behavior of concrete is not elastic, and under some circumstances, the ultimate strength may be more than 50 per cent greater than that computed by the straight line theory. It shows that actual factor of saftey cannot be determined by straight line theory. This deficiency is eliminated by ultimate strength design.

(2) Dead load is the quantity that generally remains unchanged during the life of a structure, but actual live loads are a less predictable quantity. Therefore, it is unreasonable to apply the same load factors to dead and live loads. Ultimate strength design allows different factors of safety for live loads and dead loads.

(3) Conventional column design is a modified ultimate strength procedure, whereas the straight line theory is used for design for simple flexure. It is unavoidable, therefore, that various inconsistencies occur in design of

section subject to both axial load and bending. Designing all types of members on the basis of ultimate strength results in consistency in the design procedures.

(4) A better evaluation of the critical moment thrust ratio for members subject to combined bending and axial load is obtained by ultimate strength design procedure.

(5) For prestressed concrete it is necessary that design recommendations include investigation of ultimate strength to determine the factor of safety, since, at high loads, stresses do not vary linearly. The straight line theory is therefore not applicable so the ultimate strength theory must be used.

COMPARISON OF CONVENTIONAL AND ULTIMATE LOAD METHODS

A comparative study is made of designs by the working stress method and by the ultimate strength method for the beams and columns carrying different loads, using 3000 psi concrete and intermediate grade reinforcing beams with a yeild strength of 40,000 psi.

Problem -la- (By working stress method)

Design a round spiral column to carry an axial load 200,000 lb.

$$Ag = \frac{P}{0.25\ fc + f_s Ag} \qquad \text{(ACI Section 1402)}$$

$$= \frac{2,000,000}{0.25\ (3000) + 0.4(40,000)(0.04)}$$

$$= 141 \text{ sq. in.}$$

Assume outside diameter 14 in.

Ag = 154 sq. in.

Concrete will carry = (154)(0.75) = 116 kips

Remaining for steel = 200-116 = 84 kips

$$As = \frac{84}{(0.4)(40)}$$

$$= 5.25 \text{ sq. in.}$$

use 9 #7 bars As = 5.41 sq. in.

spiral reinforcement =

$$Ps = 0.45 \left(\frac{Ag}{Ac} - 1\right) \frac{f'_c}{f_y} \qquad \text{(ACI section 913)}$$

$$= 0.45 \left(\frac{14^2}{112} - 1\right) \frac{3000}{40{,}000}$$

$$= 0.026 = 2.6\%$$

i.e. $0.026\ (11)^2\ (\frac{\pi}{4}) = 1.96$ cu. in per in of column height.

Problem -1b- (By working stress method)

Design column for an axial load of 300,000 lb.

$$Ag = \frac{300{,}000}{0.25(3000) + 0.4\ (40{,}000)(0.04)}$$

$$= 216 \text{ sq. in.}$$

Assume outside diameter 17 in.

$$Ag = 227 \text{ sq. in.}$$

concrete will carry = (227)(0.75) = 170 Kips

Remaining for steel = 300-170 = 130 Kips

$$As = \frac{130}{(0.4)(40)}$$

$$= 8.15 \text{ sq. in.}$$

Use 11 #8 bars As = 8.64 sq. in.

spiral reinforcement

$$Ps = 0.45 \left(\frac{17^2}{14^2} - 1\right) \frac{3000}{40{,}000}$$

$$= 0.016 = 1.60\%$$

i.e. $0.016(14)^2 \left(\frac{\pi}{4}\right) = 2.46$ cu. in. per in. of column height.

Problem -1c- (By working stress method)

Design a column for an axial load of 400,000 lb.

$$Ag = \frac{400,000}{0.25(3000) + (0.4)(40,000)(0.04)}$$

$$= 288 \text{ sq. in.}$$

assume outside diameter 20 in.

$$Ag = 314 \text{ sq. in.}$$

concrete will carry = (314)(0.75) = 235 Kips

Remaining for steel = 400-235 = 165 Kips

$$As = \frac{165}{(0.4)(40)}$$

$$= 10.30 \text{ sq. in.}$$

Use 13 #8 bars As = 10.21 sq. in.

spiral reinforcement

$$Ps = 0.45 \left(\frac{20^2}{17^2} - 1\right)\frac{3000}{40,000}$$

$$= 0.013 = 1.3\%$$

i.e. $0.013 (17)^2 \left(\frac{\pi}{4}\right) = 2.95$ cu. in. per in. of column height.

Problem -1d- (By working stress method)

Design a column for an axial load of 500,000 lbs.

$$Ag = \frac{500,000}{0.25(3000) + 0.4(500,000)(0.04)}$$

$$= 360 \text{ sq. in.}$$

assume outside diameter 22 in.

$$Ag = 380 \text{ sq. in.}$$

concrete will carry = 380 (0.75) = 285 Kips

Remaining for steel = 500-285 = 205 Kips

$$As = \frac{205}{(0.4)(40)}$$

$$= 12.8 \text{ sq. in.}$$

Use 13 #9 bars As = 13.00 sq. in.

spiral reinforcement

$$Ps = 0.45 \left(\frac{22^2}{19^2} - 1\right) \frac{3000}{40,000}$$

$$= 0.013 = 1.3\%$$

i.e. $0.013 \, (17)^2 \, \frac{\pi}{4} = 3.40$ cu. in. per in. of column height.

Problem -1e- (By working stress method)

Design a column for an axial load of 600,000 lbs.

$$A_g = \frac{600,000}{0.25(3000) + (0.4)(40,000)(0.04)}$$

= 430 sq. in.

assume outside diameter 24 in.

A_g = 452 sq. in.

concrete will carry = (452)(0.75) = 340 Kips

Remaining for steel = 600-340 = 260 Kips

$$A_s = \frac{260}{(0.4)(40)}$$

= 16.2 sq. in.

Use 13 #10 bars A_s = 16.45 sq. in.

spiral reinforcement

$$P_s = 0.45 \left(\frac{24^2}{21^2} - 1\right) \frac{3000}{40,000}$$

= 0.0105 = 1.05%

i.e. $0.105 (21)^2 \frac{\pi}{4}$ = 3.61 cu. in. per in. column height.

Problem -1f- (By working stress method)

Design a column for an axial load of 700,000 lbs.

$$A_g = \frac{700,000}{(0.25)(3000) + (0.4)(40,000)(0.04)}$$

= 505 sq. in.

assume outside diameter 26 in.

$$Ag = 531 \text{ sq. in.}$$

concrete will carry = (531)(.75) = 417 Kips

Remaining for steel = 700-417 = 283 Kips

$$As = \frac{283}{(0.4)(40)}$$

$$= 17.70 \text{ sq. in.}$$

Use 14 #10 bars As = 17.72 sq. in.

spiral reinforcement

$$Ps = 0.45 \left(\frac{26^2}{23^2} - 1\right) \frac{3000}{40,000}$$

$$= 0.0094 = 0.94\%$$

i.e. $.0094 \ (23)^2 \ \frac{\pi}{4} = 3.92$ cu. in. per in. of column height.

Problem -1g- (By working stress method)

Design a column for an axial load of 800,000 lbs.

$$Ag = \frac{800,000}{(0.25)(3000) + (0.4)(40,000)(0.04)}$$

$$= 575 \text{ sq. in.}$$

assume outside diameter 18 in.

$$Ag = 616 \text{ sq. in.}$$

concrete will carry = (616)(0.75) = 464 Kips

Remaining for steel = (800-464) = 336 Kips

$$As = \frac{336}{(0.4)(40)}$$

$$= 21.00 \text{ sq. in.}$$

Use 14 #11 bars As = 17.72 sq. in.

spiral reinforcement

$$Ps = 0.45 \left(\frac{28^2}{25^2} - 1\right) \frac{3000}{40{,}000}$$

$$= 0.0085 = 0.85\%$$

i.e. $0.0085\ (25)^2 \frac{\pi}{4} = 4.18$ cu. in. per in. of column height.

Problem -1h- (By working stress method)

Design a column for an axial load of 900,000 lbs.

$$Ag = \frac{900{,}000}{(0.25)(3000) + (0.4)(40{,}000)(0.04)}$$

$$= 647 \text{ sq. in.}$$

assume outside diameter 29 in.

Ag = 661 sq. in.

concrete will carry = 661(0.75) = 496 Kips

Remaining for steel = (900-496) = 404 Kips

$$As = \frac{404}{(0.4)(40)}$$

$$= 26.00 \text{ sq. in.}$$

spiral reinforcement

$$Ps = 0.45 \left(\frac{29^2}{26^2} - 1\right) \frac{3000}{40,000}$$

$$= 0.0082 = 0.82\%$$

i.e. $0.0082\,(26)^2 \frac{\pi}{4} = 4.37$ cu. in. per in. of column height.

Problem -2a- (By ultimate strength design method)

Design an axial loaded spiral column to carry an ultimate load of 200,000 lbs. $f'_c = 3000$ PSI and $f_y = 40,000$ PSI.

ACI code states that round column must be designed for a minimum eccentricity of 0.05t.

Assuming a column of 10 inches in diameter

$$P_u = \phi \left[\frac{Astfy}{\frac{3e}{D_s} + 1} \quad \frac{Agf'_c}{\frac{12te}{(t+0.67D_s)^2} + 1.18} \right] \quad \text{(ACI section 1904)}$$

From ACI code section (1504) $\phi = .75$.

$$200,000 = .75 \left[\frac{40,000\ Ast}{\frac{3(.5)}{6} + 1} + \frac{(78.5)3000}{\frac{(12)(10)(.5)}{10 + (.67)(6)^2} + 1.18} \right]$$

$$200,000 = 24,000\ Ast + 119,000$$

Ast = 3.36 square inches

Use 6 #7 bars Ast = 3.61 square inches

Ag = 78 square inches

spiral reinforcement

$$Ps = 0.45 \left(\frac{Ag}{Ac} - 1\right) \frac{f'_c}{f_y} \qquad \text{(ACI section 913)}$$

$$= 0.45 \left(\frac{(10)^2}{(6)^2} - 1\right) \frac{3000}{40,000}$$

$$= 0.058 = 5.8\%$$

i.e. $= 0.05 \times 6^2 \times \frac{\pi}{4} = 1.41$ cubic inches per inch of column height.

Problem -2b- (By ultimate strength method)

Design for an axial load of 300,000 lbs.

Assume a column of 12 inches diameter.

$$300,000 = .75 \left[\frac{40,000 \text{ Ast}}{\frac{3(.6)}{8} + 1} + \frac{(113)\ 3000}{\frac{(12)(12)(.6)}{12 + (.67 \times 8)^2} + 1.18} \right]$$

$$300,000 = 24,200 \ As_t + 174,000$$

Ast = 5.2 square inches

Use 9 #7 bars Ast = 5.41 square inches

Ag = 113 square inches

spiral reinforcement

$$Ps = 0.45 \left(\frac{(12)^2}{(6)^2} - 1\right) \frac{3000}{40,000}$$

$$= .0422 = 4.22\%$$

i.e. $= 0.422\ (8)^2 \frac{\pi}{4} = 2.15$ cu. in. per in. of column height.

Problem -2c- (By ultimate strength method)

Design for an axial load of 400,000 lbs.

Assume a column of 13 inches diameter.

$$400,000 = .75 \left[\frac{40,000\ Ast}{\frac{3(.65)}{9} + 1} + \frac{(133)\ 3000}{\frac{12(13)(.65)}{13 + (.67)(9)}^{2} + 1.18} \right].$$

$$400,000 = 24,600\ As_t + 205,000$$

$$As_t = 7.95 \text{ square inches}$$

Use 8 #9 bars Ast = 8.00 square inches

$$Ag = 133 \text{ square inches}$$

spiral reinforcement

$$Ps = 0.45 \left(\frac{13^2}{9^2} - 1\right) \frac{3000}{40,000}$$

$$= 0.037 = 3.7\%$$

$$\text{i.e.} = 0.037\ (9)^2 \frac{\pi}{4}$$

= 2.36 cubic inches per inch of column height

Problem -2d- (By ultimate strength method)

Design for an axial load of 500,000 lbs.

Assume a column of 14 inches diameter.

$$500,000 = .75 \left[\frac{40,000\ Ast}{\frac{3(.7)}{10} + 1} + \frac{(154)\ 3000}{\frac{12(14)(.7)}{(14 + .67(10))^{2}} + 1.18} \right]$$

$$500,000 = 24,800\ As_t + 240,000$$

$$As_t = 10.5 \text{ square inches}$$

Use 11 #9 bars Ast = 11.00 square inches

Ag = 154 square inches

spiral reinforcement

$$Ps = 0.45 \left(\frac{14^2}{10^2} - 1\right) \frac{3000}{40,000}$$

$$= .0306 = 3.06\%$$

i.e. $= .0306 \, (10)^2 \, \frac{\pi}{4}$

= 2.42 cubic inches per inch of column height.

Problem -2e- (By ultimate strength method)

Design for an axial load of 600,000 lbs.

Assume a column of 15 inch diameter.

$$600,000 = 0.75 \left[\frac{40,000 \text{ Ast}}{\frac{3(.75)}{11} + 1} + \frac{(177)\,3000}{\frac{(12)(15)(.75)}{(15 + 0.67\,(11))^2} + 1.18} \right]$$

600,000 =253,000 Ast + 280,000

As_t = 12.6 cubic inches

Use 10 #10 bars Ast = 12.66 square inches

Ag = 177 square inches

spiral reinforcement

$$Ps = 0.45 \left(\frac{15^2}{11^2} - 1\right) \frac{3000}{40,000}$$

$$= 0.029 = 2.9\%$$

i.e. $0.029 \, (11)^2 \, \frac{\pi}{4}$

= 2.76 cubic inches per inch of column height.

Problem -2f- (By ultimate strength method)

Design for an axial load of 700,000 lbs.

Assume a column of 17 inches diameter.

$$700{,}000 = 0.75\left[\frac{40{,}000\ Ast}{\frac{3(.85)}{13}+1} + \frac{(227)\ 3000}{\frac{(12)(17)(.85)}{(17+(.67)(13))^2}+1.18}\right]$$

$$700{,}000 = 25{,}200\ Ast + 355{,}000$$

$Ast = 13.7$ square inches

Use 11 #10 bars $Ast = 13.92$ square inches

$Ag = 227$ square inches

spiral reinforcement

$$Ps = 0.45\left(\frac{17^2}{13^2} - 1\right)\frac{3000}{40{,}000}$$

$$= .024 = 2.4\%$$

$$\text{i.e.} = 0.024\ (13)^2\ \frac{\pi}{4}$$

= 3.20 cubic inches per inch of column height

Problem -2g- (By ultimate strength method)

Design for an axial load of 800,000 lbs.

Assume a column of 18 inch diameter.

$$800{,}000 = .75\left[\frac{40{,}000\ Ast}{\frac{3(.9)}{14}+1} + \frac{(255)\ 3000}{\frac{(12)(18)(.9)}{18+(.67)(14)}+1.18}\right]$$

$$800{,}000 = 25{,}200\ Ast + 398{,}000$$

Ast = 15.9 square inches

Use 13 #10 bars Ast = 16.45 square inches

Ag = 255 square inches

spiral reinforcement

$$Ps = 0.45 \left(\frac{18^2}{14^2} - 1\right) \frac{3000}{40,000}$$

$$= .0224 = 2.24\%$$

i.e. $0.0224\ (14)^2\ \frac{\pi}{4}$

= 3.44 cubic inches per inch of column height.

Problem -2h- (By ultimate strength method)

Design for an axial load of 900,000 lbs.

Assume a column of 19 inch diameter.

$$900,000 = .75 \left[\frac{40,000\ Ast}{\frac{3(.95)}{15} + 1} + \frac{(284)\ 3000}{\frac{(12)(19)(.95)}{(19 + 0.67\ (15))^2} + 1.18} \right]$$

900,000 = 25,300 Ast + 444,000

Ast = 18.00 square inches

Use 12 #11 bars Ast = 18.75 square inches

Ag = 284 square inches

spiral reinforcement

$$Ps = 0.45 \left(\frac{19^2}{15^2} - 1\right) \frac{3000}{40,000}$$

$$= .0205 = 2.05\%$$

i.e. $= 0.0205\ (15)^2\ \frac{\pi}{4}$

= 3.62 cubic inches per inch of column height.

COMPARISON OF REQUIRED CROSS SECTION
OF CONCRETE AND STEEL FOR COLUMNS
DESIGNED BY "ULTIMATE STRENGTH"
AND "WORKING STRESS DESIGN"

DESIGN METHOD	ITEM	AXIAL LOAD ON COLUMN IN KIPS							
		200	300	400	500	600	700	800	900
ULTIMATE STRENGTH DESIGN	Steel Sq. In.	3.36	5.20	7.95	10.50	12.60	13.70	15.70	18.00
	Concrete Sq. In.	78	113	133	154	177	227	255	284
WORKING STRESS DESIGN	Steel Sq. In.	5.25	8.15	10.30	12.80	16.20	17.70	21.00	26.00
	Concrete Sq. In.	154	227	314	380	452	531	616	661

TABLE 1

UNIT COST OF AXIAL LOADED ROUND
COLUMNS DESIGNED BY "WORKING STRESS METHOD"

LOAD in lb	Required Quantity per Linear Foot			COST* per lin ft
	CONCRETE @ 54¢ per cft	STEEL @ 10¢ per lb	FORM WORK @50¢ per sq ft	
200,000	1.07 cft	24.00 lb	3.66 sq ft	$ 4.77
300,000	1.57 "	36.00 "	4.45 "	6.45
400,000	2.18 "	44.00 "	5.22 "	8.19
500,000	2.64 "	55.00 "	5.74 "	9.80
600,000	3.14 "	68.00 "	6.30 "	12.15
700,000	3.70 "	73.00 "	6.80 "	12.70
800,000	4.27 "	85.00 "	7.42 "	14.51
900,000	4.70 "	103.00 "	7.60 "	16.62

TABLE 2

*Prices are taken from O. D. Milligan Construction Co., Inc., Manhattan, Kansas

UNIT COST OF AXIAL LOADED
COLUMNS DESIGNED BY "ULTIMATE STRENGTH METHOD"

LOAD IN LB	Required Quantity per Linear Foot			COST* per lin ft
	CONCRETE @ 54¢ per ft	STEEL @ 10¢ per lb	FORM WORK @50¢ per sq ft	
200,000	0.54 cft	16.00 lb	2.62 sq ft	$ 3.20
300,000	0.78 "	25.00 "	3.14 "	4.49
400,000	0.92 "	35.00 "	3.39 "	5.69
500,000	1.07 "	44.00 "	3.36 "	6.82
600,000	1.23 "	52.00 "	3.93 "	7.82
700,000	1.57 "	57.00 "	4.45 "	8.77
800,000	1.77 "	65.00 "	4.70 "	9.82
900,000	1.97 "	74.00 "	5.00 "	10.97

TABLE 3

*Prices are taken from O. D. Milligan Construction Co., Inc., Manhattan, Kansas

COMPARISON OF UNIT COST OF
COLUMN DESIGNED BY "ULTIMATE STRENGTH DESIGN"
AND "WORKING STRESS DESIGN" METHODS

AXIAL LOAD ON COLUMNS IN LB	ULTIMATE STRENGTH DESIGN METHOD	WORKING STRESS DESIGN METHOD
	Cost Per Lin Ft	Cost Per Lin Ft
200,000	$ 3.20	$ 4.47
300,000	4.49	6.45
400,000	5.69	8.19
500,000	6.82	9.80
600,000	7.82	12.15
700,000	8.77	12.70
800,000	9.82	14.51
900,000	10.97	16.62

TABLE 4

Problem -3a- (By working stress design)

Determine the cross section of concrete and area of steel required for simply supported rectangular beam with a span of 20 ft. which is to carry a uniform load of 700 lb. per lin ft. A 3000 psi concrete to be used the allowable stress in steel is 20,000 psi.

Assume the weight of beam 150 lb. per lin ft. The total load to be carried is 750 lb. per lin ft. and external bending moment.

$$M = 1/8 \times 860 \times 20^2 \times 12$$

$$= 510{,}000 \text{ in lb}$$

$$r = \frac{fs}{fc} = \frac{20{,}000}{1350}$$

$$k = \frac{n}{n+r}$$

$$= \frac{10}{10 + 14.8} = 0.403$$

$$j = 1 - \frac{k}{3}$$

$$= 1 - \frac{0.403}{3} = 0.866$$

$$Mc = K\ bd^2$$

$$Mc = \tfrac{1}{2}f'_c kjdb^2$$

$$510{,}000 = \tfrac{1}{2}\ (1350)(.403)(.866)\ bd^2$$

$$bd^2 = 2200 \text{ in}^3$$

let $d = 16.5$ in.

required concrete section = 8 by 18.5 in.

$$Ms = Asf_sjd$$

$$510{,}000 = As \times 20{,}000 \times 0.866 \times 16.5$$

$$As = 1.78 \text{ sq. in.}$$

use 2 #8 and 1 #4 bars As = 1.78 sq. in.

Problem -3b- (By working stress design)

Design the same beam to carry a uniform live load 800 lb per lin ft.

Assume weight of beam 185 lb. per lin. ft.

$$k = 0.403$$

$$j = 0.866$$

$$K = 235$$

$$M = 1/8 \times 985 \times 20^2 \times 12$$

$$= 590{,}000$$

$$bd^2 = \frac{590{,}000}{235}$$

$$= 2500 \text{ in}^3$$

use d = 18"

required concrete section = 9 by 20 in.

$$Ms = Asfsjd$$

$$590{,}000 = As \times 20{,}000 \times 0.866 \times 18$$

$$As = 1.87 \text{ sq. in.}$$

use 2 #8 and 1 #5 bar As = 1.88 sq. in.

Problem -3c- (By working stress design)

Design the same beam to carry a uniform live load 900 lb. per lin. ft.

Assume weight of beam 195 lb. per lin.ft.

$k = 0.403$

$j = 0.866$

$K = 235$

$M = 1/8 \times 1095 \times 20^2 \times 12$

$= 660,000$

$bd^2 = \frac{660,000}{235}$

$= 2800 \text{ in}^3$

let d = 17 in.

required concrete section = 10 by 19 in.

$M_s = A_s f_s j d$

$660,000 = A_s \times 20,000 \times 0.866 \times 17$

$A_s = 2.22$ sq. in.

use 2 #9 and 1 #5 bars As = 2.31 sq. in.

Problem -3d- (By working stress design)

Design the same beam to carry a uniform live load 1000 lb. per lin. ft.

Assume weight of beam 200 lb. per lin. ft.

$k = 0.403$

$j = 0.866$

$K = 235$

$M = 1/8 \times 1200 \times 20^2 \times 12$

$= 720{,}000$ in lb

$bd^2 = \frac{720{,}000}{235}$

$= 3060\ in^3$

let d = 18 in.

required concrete section = 10 by 20 in.

$Ms = Asfsjd$

$720{,}000 = As \times 20{,}000 \times 0.866 \times 18$

$As = 2.30$ sq. in.

use 2 #9 and 1 #5 bars As = 2.31 sq. in.

Problem -3e- (By working stress design)

Design the same beam to carry a uniform live load 1100 lb. per lin. ft.

Assume weight of beam 210 lb. per lin. ft.

$k = 0.403$

$j = 0.866$

$K = 235$

$M = 1/8 \times 1310 \times 20^2 \times 12$

$= 785{,}000$ in. lb.

$$bd^2 = \frac{785{,}000}{235}$$

$= 3340 \text{ in}^3$

let d = 18.5 in.

required cross section of concrete 10 by 20.5 in.

$$Ms = Asfsjd$$

$$785{,}000 = As \times 20{,}000 \times 0.866 \times 18.5$$

$$As = 2.45 \text{ sq. in.}$$

use 2 #9 and 1 #6 bars As = 2.60 sq. in.

Problem -3f- (By working stress design)

Design beam to carry a uniform live load 1200 lb. per lin. ft.

Assume weight of beam 230 lb. per lin. ft.

$$k = 0.403$$

$$j = 0.866$$

$$K = 235$$

$$M = 1/8\ (1430)\ 20^2 \times 12$$

$= 860{,}000$ in. lb.

$$bd^2 = \frac{860{,}000}{235}$$

$= 3660 \text{ in}^3$

let d = 19 in.

required cross section of concrete 10 by 21 in.

$$Ms = Asfsjd$$

$$860{,}000 = As \times 20{,}000 \times 0.866 \times 19$$

$$As = 2.62 \text{ sq. in.}$$

use 2 #10 and 1 #4 bars As = 2.70 sq. in.

Problem -4a- (By ultimate strength design)

Determine the cross section of concrete and area of steel required for simply supported rectangular beam with a span of 20 ft. which is to carry a uniform load of 700 lb. per lin. ft. A 3000 Psi concrete is used and yield strength of steel 40,000 Psi.

Assume weight of beam 130 lb. per lin. ft.

$$U = 1.5D + 1.8L \quad \text{(ACI section 1506)}$$

$$= 1.5\ (130) + 1.8\ (7.00)$$

$$= 1455 \text{ lb}$$

External bending moment

$$M = 1/8\ (1455)\ 20^2 \times 12$$

$$= 870{,}000 \text{ in. lb.}$$

$$Mu = \phi \left[Asf_y \left(d - \frac{a}{2}\right) \right] \quad \text{ACI Section 1601}$$

$$a = pmd$$

$$Mu = \phi \left[Asf_y d \left(1 - \frac{pm}{2}\right) \right]$$

$$K = \frac{Mu}{bd^2} = \phi \left[Pf_y \left(1 - \frac{pm}{2}\right) \right]$$

$$m = \frac{f_y}{0.85\ f'_c}$$

$$= 15.69$$

$$\phi = 0.9 \qquad \text{(ACI Section 1504)}$$

$$p = 0.015$$

$$K = 0.9 \left[0.015 \times 40{,}000 \ (1 - \frac{0.015(15.69)}{2}) \right]$$

$$= 530$$

$$bd^2 = \frac{870{,}000}{530}$$

$$= 1610 \text{ in}^3$$

use b = 8 in. d = 14 in.

required concrete cross section = 8 by 16 in.

$$As = pbd$$

$$As = 0.015 \times 8 \times 14$$

$$= 1.68 \text{ sq. in.}$$

use 2 #9 bars

$$As = 2.00 \text{ sq. in.}$$

Problem -4b- (By ultimate strength design)

Design the same beam to carry a uniform live load 800 lb. per lin. ft.

Assume weight of beam 150 lb. per lin. ft.

$$U = 1.5(150) + 1.8(800)$$

$$= 1665 \text{ lb.}$$

$$M = 1/8\ (1665)\ 20^2 \times 12$$

$$= 995{,}000 \text{ in. lb.}$$

$$bd^2 = \frac{995{,}000}{530}$$

$$= 1870 \text{ in}^3$$

use b = 8 in.   d = 15.5 in.

Required concrete cross section 8 by 17.5 in.

$$As = (8)(15.5)(.015)$$

$$= 1.86 \text{ sq. in.}$$

use 2 #9 bars

$$As = 2.00 \text{ sq. in.}$$

Problem -4c- (By ultimate strength design)

Design the same beam to carry a uniform live load 900 lb. lin. ft.

Assume weight of beam 160 lb. per lin. ft.

$$U = 1.5\ (160) + 1.8\ (900)$$

$$= 1860 \text{ lb.}$$

$$M = 1/8\ (1860)\ 20^2 \times 12$$

$$= 1{,}115{,}000 \text{ in. lb.}$$

$$bd^2 = \frac{1{,}115{,}000}{530}$$

$$= 2070 \text{ in}^3$$

use b = 9 in.   d = 15.5 in.

required concrete cross section = 9 by 17.5 in.

$$As = (9)(15.5)(0.015)$$

$$= 2.08 \text{ sq. in.}$$

use 2 #8 and 1 #7 bars

$$As = 2.18 \text{ sq. in.}$$

Problem -4d- (By ultimate strength design)

Design the same beam to carry a uniform live load 1000 lb. per lin. ft.

Assume weight of beam 180 lbs.

$$U = 1.5(180) + 1.8(1000)$$

$$= 2070 \text{ lb.}$$

$$M = 1/8\ (2070)\ 20^2 \times 12$$

$$= 1{,}1240{,}000 \text{ in. lb.}$$

$$bd^2 = \frac{1{,}240{,}000}{530}$$

$$= 2340 \text{ in}^3$$

use b = 9.5 in. d = 16 in.

required concrete cross section 9.5 by 18 in.

$$As = (9.5)(16)(0.015)$$

$$= 2.28 \text{ sq. in.}$$

Problem -4e- (By ultimate strength design)

Design the same beam to carry a uniform live load 1100 lb. per lin. ft.

Assume weight of beam 190 lb. per lin. ft.

$$U = 1.5\ (190) + 1.8\ (1100)$$
$$= 2265 \text{ lb.}$$

$$M = 1/8\ (2265)\ 20^2 \times 12$$
$$= 1{,}360{,}000 \text{ in. lb.}$$

$$bd^2 = \frac{1{,}360{,}000}{530}$$
$$= 2580 \text{ in}^3$$

use b = 10 in. d = 16 in.

required concrete cross section 10 by 18 in.

$$As = 10 \times 16 \times 0.015$$
$$= 2.4 \text{ sq. in.}$$

use 2 #9 and 1 #6 bars

$$As = 2.44 \text{ sq. in.}$$

Problem -4f- (By ultimate strength design)

Design the same beam to carry a uniform live load 1200 lb. per lin. ft.

Assume weight of beam 200 lb. per lin. ft.

$$U = 1.5(200) + 1.8(1200)$$
$$= 2460 \text{ lb.}$$

$$M = 1/8\ (2460)\ 20^2 \times 12$$
$$= 1{,}470{,}000 \text{ in. lb.}$$

$$bd^2 = \frac{1{,}470{,}000}{530}$$
$$= 2780 \text{ in}^3$$

use b = 10 in. d = 17 in.

required concrete cross section = 10 by 19 in.

As = 2.55 sq. in.

use 2 #9 and 1 #7 bars

As = 2.60 sq. in.

COMPARISON OF REQUIRED CROSS SECTIONS OF CONCRETE AND STEEL FOR 20 FT. LONG RECTANGULAR BEAM DESIGNED BY "ULTIMATE STRENGTH" AND "WORKING STRESS" METHODS

DESIGN METHOD	ITEM	LOAD IN LB. PER LIN. FT.					
		700	800	900	1000	1100	1200
ULTIMATE STRENGTH DESIGN	STEEL Sq. In.	1.68	1.86	2.08	2.28	2.40	2.60
	CONCRETE Sq. In.	128	140	157	171	180	190
WORKING STRESS DESIGN	STEEL Sq. In.	1.78	1.87	2.22	2.30	3.46	2.62
	CONCRETE Sq. In.	148	180	190	200	205	210

TABLE 5

UNIT COST OF 20 FT. LONG RECTANGULAR BEAM
DESIGNED BY "WORKING STRESS METHOD"

LOAD lb. per lin. ft.	Required Quantity per Linear Foot			COST* per lin.ft.
	CONCRETE @ 54¢ per cft	STEEL @ 10¢ per lb.	FORM WORK @ 50¢ per sq.ft	
700	1.03 cft.	5.59 lb.	3.74 sq. ft.	$3.03
800	1.25 "	6.36 "	4.07 "	3.30
900	1.32 "	7.55 "	4.00 "	3.47
1000	1.39 "	7.85 "	4.16 "	3.61
1100	1.42 "	8.35 "	4.25 "	3.73
1200	1.46 "	8.54 "	4.35 "	3.86

TABLE 6

*Prices are taken from O. D. Milligan Construction Co., Inc., Manhattan, Kansas.

UNIT COST OF 20 FT. LONG RECTANGULAR BEAM
DESIGN BY "ULTIMATE STRENGTH METHOD"

LOAD lb. per lin. ft.	Required Quantity per Linear Foot			COST* per lin.ft.
	CONCRETE @ 54¢ per cft.	STEEL @ 10¢ per lb.	FORM WORK @ 50¢ per sq.ft	
700	0.85 cft.	5.7 lb.	3.43 sq. ft.	$2.73
800	0.97 "	6.37 "	3.58 "	2.94
900	1.09 "	7.08 "	3.67 "	3.12
1000	1.18 "	7.75 "	3.75 "	3.27
1100	1.25 "	8.15 "	3.83 "	3.39
1200	1.32 "	8.85 "	4.00 "	3.59

TABLE 7

*Prices are taken from O. D. Milligan Construction Co., Inc., Manhattan, Kansas.

COMPARISON OF UNIT COST OF 20 FT. LONG RECTANGULAR BEAM DESIGNED BY "ULTIMATE STRENGTH DESIGN" AND "WORKING STRESS DESIGN" METHOD

LOAD lb. per lin. ft.	ULTIMATE STRENGTH DESIGN METHOD	WORKING STRESS DESIGN METHOD
	Cost per lin. ft.	Cost per lin. ft.
700	$2.73	$3.03
800	2.94	3.30
900	3.12	3.47
1000	3.27	3.61
1100	3.39	3.73
1200	3.59	3.83

TABLE 8

CONCLUSIONS

In this comparative study of "Ultimate Strength Design" and "Working Stress Design" methods, it was found out that the design of reinforced concrete structure by ultimate strength design is more rational and simpler.

Tables (1) and (4) show that the quantities of concrete and steel are less, when designed by ultimate strength method, contrasted with the quantities obtained from the working-stress design method. Although the saving in steel in the case of rectangular beams is small, it was found from experimental datas by Cowan (26) that for heavily reinforced concrete sections the use of ultimate theory results in a very considerable saving of steel. (refer to Plate III)

From Table (4) it can be calculated that the reduction in cost for the materials of columns for the ultimate strength design as opposed to the working stress design is approximately 35 per cent.

PLATE III

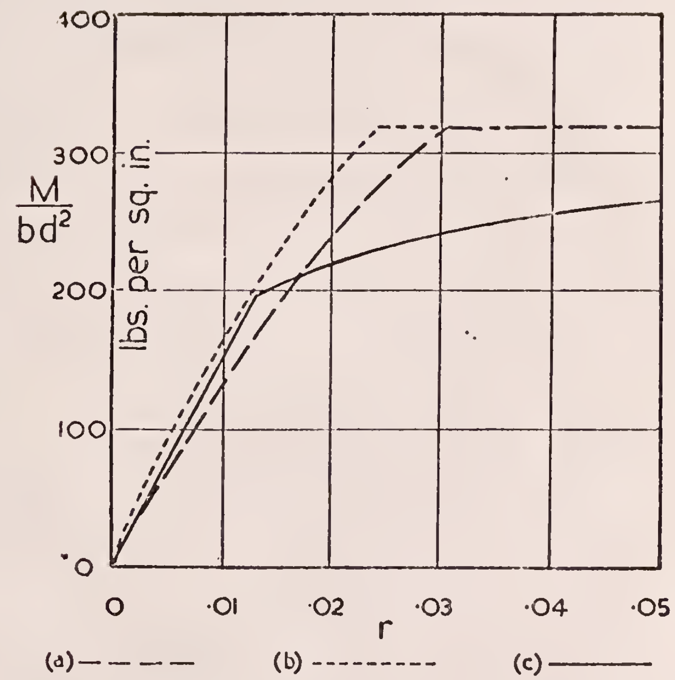


Fig. 14. Relation between maximum permissible moment and steel ratio for rectangular sections with tension reinforcement only, using a concrete of 1:2:4 nominal mix reinforced with mild steel: (a) according to the ultimate strength theory, taking the load factor as 2.5; (b) according to the ultimate strength theory, taking the load factor as 2.0 supplemented by a factor of safety of 1.25 for the concrete only; and (c) according to the working stress theory, taking the factor of safety for steel as 2.0, and the factor of safety for concrete as 2.5 (based on the cylinder crushing strength)

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NOTATION

A_g = Gross area of spirally reinforced column.

A_s = Area of tension reinforcement.

a = Depth of equivalent rectangular stress block.

b = Width of compression face of flexural member.

c = Distance from extreme compression fiber to neutral axis at ultimate strength.

D = Over all diameter of circular column.

D_s = Diameter of circle through center of reinforcement arranged in circular pattern.

e = Eccentricity of axial load at end of member measured from plastic centroid of the section.

e' = Eccentricity of axial load at end of member measured from plastic centroid of tension reinforcement.

f'_c = 28 day cylinder strength of concrete under standard loading.

f_c = Design strength of concrete.

f_s = Design strength of steel.

f_y = Yield strength of steel.

j = Ratio of lever arm of resisting couple to depth d.

k_d = Depth of neutral axis.

k_i = A fraction and shall be taken 0.85 for strength up to 4000 psi.

M = Bending moment.

Mu = Ultimate resisting moment.

n = $\frac{E_s}{E_c}$

p = As/bd

P_u = Ultimate load.

r = fs/fc

t = Total depth of rectangular section or diameter of circular section.

U = Required ultimate load capacity of section.

= Capacity reduction factor.

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ECONOMY IN ULTIMATE STRENGTH DESIGN

by

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B. S., Kansas State University, 1963

AN ABSTRACT OF A MASTER'S REPORT

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MASTER OF SCIENCE

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The assumption of working stress, theory that concrete is elastic with in the range of working stresses, has been the subject of controversy since the early days of reinforced concrete design, however, straight line theory was generally used for reinforced concrete design. As the short comings of straight line method became increasingly evident, the arbitrary adjustments were added to the design code. But there was complete inconsistant approach to the design of reinforced concrete structure.

It was the intent of this report to recognize the importance of ultimate strength design which avoids the inconsistancy of working stress design and provides a design of maximum economy and also to compare the ultimate strength design with working stress design in practical problems. Columns and beams with various loads were designed.

The calculations in this report demonstrate that the procedure of ultimate strength design for reinforced concrete member is simpler and provides saving in the amount of material used and economy in total cost and still maintains adequate factor of safety.