The role of norms in facilitating productive struggle

by

Tegan William Nusser

B.S., Kansas State University, 2013
M.S., Kansas State University, 2018

AN ABSTRACT OF A DISSERTATION

submitted in partial fulfillment of the requirements for the degree

DOCTOR OF PHILOSOPHY

Department of Curriculum and Instruction
College of Education

KANSAS STATE UNIVERSITY
Manhattan, Kansas

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Abstract

This study links the teaching practice of productive struggle, a psychological perspective of classroom activities, and the classroom microculture, an interactionist perspective that combines sociocultural and psychological considerations (Cobb & Yackel, 1996). The research question for this study is: *In what ways does a teacher negotiate the establishment of classroom norms in order to facilitate productive struggle?* To answer this question, this study uses an analytic autoethnographic approach (Anderson, 2006) with analysis and findings preceding and following layered accounts (Ronai, 1995) describing instructional episodes. Reflexive journal entries following days of instruction in addition to lesson plans and curricular materials generated the data for this study. Coding of data revealed connections between the classroom microculture and productive struggle framework (Warshauer, 2015a).

This study suggests classroom norms are continually renegotiated over time as teachers and students intersubjectively determine what is acceptable in the classroom—whether norms are “taken as shared” (Yackel, 2001, p. 6). Regarding the classroom microculture, this study suggests that social norms provide supports for how students can collectively engage in struggle, that sociomathematical norms contribute to how students approach engaging in mathematics, and that classroom mathematical practices influence how students engage in struggling with novel mathematics. It further suggests that teachers’ responses to student struggle shape the reestablishment and renegotiation of classroom norms. Implications, limitations, and suggestions for future research are discussed.

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Acknowledgements

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Dedication

I dedicate this dissertation to my family. To my wife, Kirsten Maye: for giving patience and grace in allowing graduate school to dominate the first six years of our relationship. To Mom and Dad: for inculcating an inquisitive mind, for continual encouragement, and providing a sympathetic ear to my many passions and frustrations. To my sister, Anna: for sharing your passion for education with me—our conversations refresh and inspire me to continually improve my teaching. To Grandpa Warren and Grandma Phyllis: for the homecooked meals, conversation, and financial support. And to Teddy and Delia—our miniature schnauzers who unreservedly share their outsized personality and love.

Chapter 1 - Introduction

September 2012: Entering my Block 2 classwork was my first field experience inside a mathematics classroom. My cooperating teacher, Mr. Boomer, welcomed me into his classroom with his characteristic warmness and excitement for education. These moments not only laid the foundation for a meaningful mentor-mentee relationship but also laid the groundwork for an ongoing friendship. I recall being shocked in those moments by the differences in my experience of learning math in high school.

Rather than sitting in neat rows facing the front of the classroom, these students faced each other. Rather than the informal and hushed peer tutoring that sometimes occurs, these students were explicitly encouraged to work together, to collaborate. One of the classroom requirements for Mr. Boomer before helping any group of students requesting assistance was to "ask three (other students) before me (the teacher)." The physical orientation, explicit encouragement, and classroom expectation established the classroom social norm that mathematics is learned together, that collaboration allows them to make sense of the mathematics at hand.

This social norm of collaboration was aided by the nature of the curricula at hand. Students investigated a contextual problem with guided questions provided by the curriculum (Core-Plus Mathematics Project, 2008). Rather than students listening to a lecture, taking notes, and then finally becoming involved in the learning process with stale worksheets, these students were expected to begin exploring and constructing their initial understandings for themselves. The direct instruction in this classroom instead occurred as a summarization of the guided investigations, which was only then formalized into notes, or as the students knew it, a "toolkit" of index cards that also contained their notes from previous years. Not only was collaboration an *established social norm, but so too was using their resources when struggling with the mathematics at hand. Students frequently used resources such as their toolkit and their graphing calculators.*

If the norms of collaboration and using resources to help students build knowledge did not provide enough of a culture shock for me, I also saw an excellent example of a teacher acting as a facilitator of knowledge. Following the regular classroom procedures, of asking students if they had "asked three before me," ensured that students had read and understood the question and sought out help from others appropriately. Mr. Boomer also utilized, what he termed at the time, "Socratic questioning" to facilitate student understanding. He would guide or probe students' thinking in the right direction instead of removing their struggle and simply telling them what to do. Looking back, I clearly see an environment set up to support productive student struggle.

Historical Context

Mathematics educational theorists focused primarily on individual student's learning without consideration of the classroom environment. Thorndike (1927) introduced the behavioristic stimulus-response theory applied to learning. Shortly thereafter, Brownell shifted focus from memorization to students learning the conceptual basis of mathematics (Brownell & Sims, 1947; Skemp, 1978). How this conceptual understanding came about was described as traditional in nature until Bruner (1960) began describing discovery learning and types of questioning. Over time, mathematics educational theorists continued to focus on the individual's learning with various cognitive psychologists describing different ways as to how that individual's learning came about (Ausubel, 1963; Gagné, 1985; Piaget, 1952; Skemp, 1978; Vygotsky, 1930/1978). However, limiting focus to a single student's learning does not provide a picture of the whole learning environment. A learning environment, or learning community, is defined by the interactions between students and between students and their teacher. One perspective of these communities is that of Lave and Wenger's (1991) conception of situated learning in communities of practice. A perspective more focused on mathematics classroom learning is Cobb & Yackel's (1996) idea of the classroom microculture. This learning environment adds norms, a shared way of interacting, that completes the picture of both individual and group interactions and collective classroom learning.

Norms and Productive Struggle

Teachers establish and negotiate norms with their students, norms that collectively represent the classroom environment. This environment, a classroom microculture, is made up of the taken-as-shared agreements or norms that represent the accepted ways of behaving in the classroom (Cobb & Bowers, 1999; Cobb & Yackel, 1996; Cobb et al., 2001). When norms are taken as shared, students and teachers intersubjectively agree on appropriate behaviors and ways of engaging in learning in the classroom. Yackel and Cobb (1996) identify three types of norms that comprise the learning environment in math classrooms: social norms, sociomathematical norms, and classroom mathematical practices. Establishing norms in classrooms happens without explicit actions from the teacher. However, teachers that do take a role in explicitly negotiating norms with their students have more productive interactions in developing inquiry cultures in classrooms (Guven & Dede, 2015; Partanen & Kaasila, 2014), especially when considering that the norms that students interpret may not align with the norms that teachers aim to establish in their classrooms (Levenson et al., 2009).

Norms influence the way that students interact in classrooms and the way they engage, productively and unproductively, in learning mathematics. Struggling with rigorous tasks is an essential element of learning mathematics conceptually (Hiebert & Grouws, 2007). The National Council of Teachers of Mathematics (NCTM) identified supporting productive struggle as one of the eight research-based mathematics teaching practices that, when used with the other teaching practices, help establish students' conceptual understanding for procedural fluency (NCTM, 2014). They describe productive struggle as an aspect of "effective teaching of mathematics [that] consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships" (p. 48).

Rationale for the Study

Norms and productive struggle are linked loosely in the literature. Warshauer (2015a) investigated productive struggle in middle school classrooms and found that "the nature of student struggles seemed related to the sociomathematical norms that were in place in each class. In other words, struggles were not only cognitive in nature" (p. 393). However, the establishment and negotiation of these norms was not the focus of her research. Instead, she looked at the nature of struggle for students and the interactions between teachers and students that either facilitated that struggle or removed struggle on the task at hand. Baker and colleagues (2020) offer takeaways aimed at practitioners looking to establish productive struggle in their classrooms; one of which being that teachers should reflect on their efforts promoting productive struggle as "the idea of productive struggle may be unfamiliar and may take time to become a classroom norm" (p. 366).

Research points to aspects of the learning environments that support struggle: the importance of valuing persistence (Kapur, 2010, 2011), perseverance (Gresalfi et al., 2009; NCTM, 2014), allowing time for struggle, explicitly valuing struggle in the learning of mathematics (Stein et al., 2009; Warshauer, 2015a, 2015b), establishing a community of learners and valuing student thinking and highlighting their authorship of mathematics (Doerr, 2006; Franke et al., 2015; Gresalfi et al., 2009). However, these accounts do not provide clear views of how practitioners engage in the negotiation of this learning environment; nor is there discussion of the links to the microculture framework made up of social norms, sociomathematical norms, and classroom mathematical practices.

This study investigated the reestablishment of norms following nearly a year and a half of a learning environment impacted by safety concerns to prevent the spread of COVID-19. The one constant of the learning environment during this time was a blend of instructional modality where teachers often taught in person while at the same time broadcasting instruction for hybrid or remote students. With constant change and challenges of providing instruction during a pandemic, many teachers reported not covering the entirety of their curriculum (Hamilton et al., 2020; Kaufman et al., 2021). Lack of curriculum coverage and normalcy seem to have led to learning gaps, and in some cases, learned helplessness. In addition, students were physically separated within classrooms preventing the establishment of typical learning communities. In such an isolation-centered environment, sociomathematical norms of mathematical discourse, problem-solving, inquiry, or productive struggle were a challenge to establish.

Research Purpose

The purpose of this qualitative study was to investigate the influence of norms on the facilitation of productive struggle. Productive struggle, a psychological perspective of student knowledge construction, and norms, a social perspective, intersect and can be characterized as an interactionist perspective that captures individual and collective activities in a classroom (Cobb & Yackel, 1996). To investigate the intersection of this psychological and social perspective in my classroom, I implemented an analytic autoethnography as a self-as-subject researcher. I taught approximately 60 algebra 1 and 60 geometry students at a high school with an enrollment of 450 in a Midwestern town of approximately 4,800 people, according to 2017 estimates and according to the U.S. Census Bureau (2013). Adult ancillary informants included my fellow math teachers at this high school; my wife Kirsten, a middle school math teacher; and my major professor, Dr. Sherri Martinie. The data for this study was generated and documented in a reflexive journal. Data was coded by way of emergent analysis in an inductive, cyclical, and iterative fashion.

Research Question

The research question this study explored was: In what ways does a teacher negotiate the establishment of classroom norms in order to facilitate productive struggle? Supporting research questions help frame and inform my perspective as a researcher-participant-teacher perspective. These included:

1. How does a teacher perceive norms changing over time in a classroom?
2. How does a teacher utilize curriculum resources and ancillary informants to guide their negotiation of classroom norms?
3. How does a teacher's reflective practice influence their negotiation of norms and facilitation of student struggle?
4. How does a teacher perceive their responses to student struggle as influencing the renegotiation and reestablishment of norms?
5. What norms does a teacher perceive students utilizing when engaging in solving problems?
6. How do student behaviors and actions influence a teacher's negotiation of norms?

The first supporting research question helped to guide the construction of narratives to answer the main research question. The second and third dealt with core teaching practices: using curriculum resources and reflecting upon pedagogy—moreover, they align well with the autoethnographic method I used in researching my experiences as a teacher. The remaining research questions all dealt with what happens in the moments of teaching in the classroom, both from a memory standpoint as well as from the standpoint of an observer watching the classroom. In this way, student reactions contributed not only to my understanding but also to the more realistic construction of narratives that occurred in my classroom.

Operationalization of Constructs

Productive struggle describes a student's "effort to make sense of mathematics, to figure something out that is not immediately apparent" (Hiebert & Grouws, 2007, p. 287). A student's struggle is productive if cognitive demand was sustained throughout their engagement with the task.

Successful implementation of tasks requires that the intent and cognitive demand of the task remain unchanged by any facilitating actions a teacher might take so that a student can move forward in completing the task (Warshauer, 2015a).

Cognitive demand refers to the mental strain that students feel in response to a given task. To characterize the cognitive demand of the task, I will use the guide developed by Stein et al. (2009). Low-level cognitive demand tasks involve memorization or procedures without connections. High-level cognitive demand tasks involve procedures with connections or doing mathematics tasks.

Instructional strategies that promote productive struggle include four identified strategies promote productive struggle: 1) the use of focusing questions to aid students; 2) encouraging students to reflect on their work while recognizing and supporting student effort; 3) giving adequate time for tasks overall, and not stepping in to make the task easier for students too early in the attempt of the task; 4) establishment of a classroom norm that struggle represents an essential component of learning mathematics conceptually (Warshauer, 2015b).

Social norms represent norms that could exist in any classroom regardless of subject areas (Yackel & Cobb, 1996). Examples of "social norms include explaining interpretations and solutions, attempting to make sense of explanations given by others, indicating understanding or nonunderstanding, and questioning alternatives when a conflict in interpretations has become apparent" (Bowers et al., 1999, p. 27).

Sociomathematical norms are norms that have an inherent mathematical characteristic. For example, "the understanding that students are expected to explain their solutions and their ways of thinking is a social norm, whereas the understanding of what counts as an acceptable mathematical explanation is a sociomathematical norm" (Yackel & Cobb, 1996, p. 461).

Classroom mathematical practices represent the collective learning that has taken place in a math classroom. These "classroom mathematical practices evolve as the teacher and students discuss problems and solutions, and these practices involve means of symbolizing, arguing, and validating in specific task situations" (Bowers et al., 1999, p. 28)

Methodological Framework

This study is an analytic autoethnography. Analytic autoethnographies aim to do more than describe personal experiences by contributing to the body of knowledge for theoretical advancement (Anderson, 2006). To connect narratives or instructional episodes with themes and analysis, I utilized a layered account (Ronai, 1995). In this study, I described how I negotiated a microculture for the facilitation of productive struggle. Norms were explicitly negotiated with my students and documented from my perspective as a self-as-subject researcher. The data was generated through the use of a reflexive journal where I documented my experiences and thoughts. This reflexive journal contained more than just my experiences as a teacher and my interactions with my students, as other informants, including my fellow math teachers, who have been great sources for collaboration and inspiration for many of my years as a teacher. Lesson plans and curricular resources complimented the data in my reflexive journal.

Analysis of this data was, by necessity, an emergent process wherein data was cyclically and iteratively reflected upon as data was collected. This data was coded. The codes eventually came to collectively represent distinct categories, and categories comprised themes in the data. To organize and manage such a vast amount of data, I utilized a data matrix to document codes and memos with hyperlinks to raw data. The data matrix enabled me to see when I reached data saturation, and I began identifying themes and writing up findings. Essential to connecting my classroom's microculture to productive struggle was identifying lessons that were particularly relevant to how norms support the facilitation of productive struggle. These focus lessons became the instructional episodes documented in Chapters 4-7. The coding of the facilitation of productive struggle built from existing research done by Warshauer (2015a) and was informed by the framework for productive struggle (see Figure 3.2 in Chapter 3 in the Data Analysis section).

Autoethnographic rigor has five criteria: subjectivity, self-reflexivity, resonance, credibility, and contribution (Le Roux, 2016a). In short, subjectivity refers to the central nature of the researcher in the research findings. Self-reflexivity refers to a researcher's "self-awareness, self-exposure, and self-conscious introspection" (p. 205). Resonance implies the connection and meaningfulness of the author's writings with the reader. Credibility means clear methodological reporting. Finally, contribution implies the discovery of new knowledge in the field of research. Each of these five criteria come through the narratives written by autoethnographers—a process which contributes to both data generation and analysis. The storytelling in analytic autoethnography must not only achieve the above criteria, demonstrating subjectivity and self-reflexivity, but also reach resonance with the reader and contribute to the body of knowledge.

Theoretical Framework

As an analytic autoethnography, the knowledge discovered through the following narratives represents no final word. Instead, it represents my subjective experience in my classroom. Analysis in autoethnographies proceeds in a cyclical, iterative, and reflective process (Throne, 2019). Meaning is uncovered by constantly moving from raw data to groups of data and the themes that comprise the data. Reflection on this data, an important aspect of self-reflexivity, helps to build the meaning that autoethnographers discover in the creation of narratives (Chang, 2013). More specific to this study, analytic autoethnography borrows from grounded theory's central process (Corbin & Strauss, 1990), emergent analysis (Lofland, 1995; Anderson, 2006).

Emergent analysis is built from existing knowledge regarding both classroom norms and how teachers support productive struggle. Key to this study is the word borrow—I did not use purely grounded theory. Analytic autoethnographers borrow from grounded theory but make no claim of objectivity in their analysis. Further, I did not start my analysis from a point of view that there is no existing theories or knowledge explaining norms or productive struggle. I stress that autoethnography is a way of reflecting on one's experiences to discover more general knowledge. Analysis in this autoethnography was a creative and singularly unique method to this study which required a deep engagement of subjectivity and self-reflexivity.

The theoretical frameworks that contributed to this study include the interactionist perspective of classroom microcultures (Cobb, 1994; Cobb & Yackel, 1996). This perspective is built from symbolic interactionism (Blumer, 1969), a stance that reality consists of meaning discovered through people's interactions. Cobb and Yackel's (1996) framework that captures classroom microcultures consists of the interactions between the social perspective and the psychological perspective. In the social perspective, student's collective actions are represented by classroom social norms, sociomathematical norms, and classroom mathematical practices. Their individual beliefs and mathematical conceptions, however, are the individual correlates of the collective actions taken by students, seen by researchers. The interactionist perspective implies that these perspectives are complementary, informing one another, to give a better understanding of what is happening in the classroom environment (Cobb, 1994).

In addition, the productive struggle framework contributed to the theoretical basis of this study (Warshauer, 2015a). This framework is utilized within the tasks students are engaged in, as characterized by the task analysis guide: memorization, procedure with no concept, procedure with concept, or a doing math task (Stein et al., 2009). These tasks provoke types of struggles in students such as: getting started, carrying out a process or procedure, mathematical sense making, or expressing misconceptions or errors. Teachers respond to these struggles by either telling, providing directed guidance, probing guidance, or affordance (Warshauer, 2015b). Struggle is then resolved as productive if the cognitive demand is maintained, low level productive if the cognitive demand of the task was lowered, or unproductive if cognitive demand of the task was completely removed (Warshauer, 2011, 2015a).

Teacher response to struggle is informed by the use of purposeful questions (NCTM, 2014) as well as the classroom environment. Examples of purposeful questions include focusing questions (Wood, 1998), assessing questions and advancing questions (Smith et al., 2008), and judicious telling (Lobato et al., 2005). A specific teacher response to struggle includes affordance responses which refer to teacher moves such as encouraging and reassuring students that struggle is normal and important in learning mathematics, as well as giving students more time to struggle. Affordance responses are of relevance to this study as they influence the negotiation and establishment of norms by helping to create an environment in which students know expectations for engaging in math tasks (Kapur, 2011; Livy et al., 2018; Smith & Stein, 2018; Valentine & Bolyard, 2018; Zaslavsky, 2005). Affordance and probing guidance are responses to student struggle that reestablish norms. Examples of social norms that support struggle include collaboration, peer tutoring, student discourse, sociomathematical norms of justification, and classroom mathematical practices for students to structure their thinking.

Subjectivity

One of the primary characteristics of autoethnography is the use of subjective experiences and reflection to drive data collection, analysis, and the generation of narratives. As a result, my subjectivities and positionalities are not only revealed here, but they are also revealed throughout my writings, most explicitly in my reflective narratives and instructional episodes. Le Roux (2016a) identifies subjectivity as an aspect of autoethnographic rigor, characterizing it by the centrality of the researcher in the narratives. With the researcher at the center of narratives, describing different aspects of positionality helps to establish subjectivity. Throne (2019) identifies the following as describing positionality:

1. family/tribe status
2. occupation/profession/economics
3. religion/spirituality/beliefs

4. ethics/esthetics/creativity
5. age/gender/ability/sexuality
6. language/heritage/culture/geography (p. 30)

I am a happily married man with two vivacious miniature Schnauzers. I like to joke that the family business is education with my father, mother, sister, wife, and one of my first cousins and his wife all involved in education either as teachers, principals, counselors, or coaches. Among the seven of us we have a collective one-hundred and thirteen years in education. I grew up in a middle-class household and have thankfully always had that privilege to rely on—never has money been a true worry. Even with educators being perennially underpaid, and my decision to extend my current college, bachelors to doctorate, experience to eleven years running, the only complaint I have is that I have not been able to put down a payment to buy a house yet. I am incredibly privileged, lucky, and spoiled.

My privilege is only extended by the fact that I am a White male, cis-gendered, heterosexual Christian inculcated in the White-Anglo-Saxon-Protestant culture that dominates the United States. The only time I can recall any sort of lack of privilege was when I ruptured my patella a month before I married Kirsten. This short-lived experience of having a physical dis/ability, lacking the ability to move around and physically care for myself, was humbling and has given me a persistent recognition of buildings with outdated or little ADA compliance. I cannot imagine the blind spots I have regarding my White, male, sexual, religious, and cultural privilege. I try to—I am passionate about reading various literature from Critical Race Theory to Black Feminist thought as well as attempting to purposefully engage in an anti-racist life. But my perspective is always constrained and limited by the privilege I have.

Throne (2019) notes how important it is to disclose all relationships to others and informants. As I am doing research in my classroom, these relationships inform the very being of teaching. I have the relationships established with sophomore students I had in class last year, and the relationships I will establish with new freshmen and sophomores will be too numerous to address. Further, the need to protect the identity of students in my research is paramount to consider disclosing relationships with students. Somewhat easier to address, and even more critical to informing my perspective, is the relationships with the other ancillary informants in my study—my fellow math teachers.

The most important relationship to disclose is my relationship with my wife, Kirsten. We met pursuing our Master's degrees in Curriculum and Instruction in Mathematics Education—and I have since used her as a sounding board for my ideas, my teaching, as well as my writing. Two more relationships to disclose are both my friends as well as mentors for my teaching. I student-taught in both of their classrooms, and they have both grown into being my work family: Mrs. Garfield and Mr. Boomer. My initial experience of student teaching with them was so powerful that I always wanted to return to work in that school. I had that dream fulfilled five years ago, and I have had my dream teaching job ever since.

I have already spoken some about the culture shock I experienced in Mr. Boomer's classroom as a student-teacher. It is the perfect starting point to document my experiences as a math teacher because I have been passionate about reform-oriented mathematics education ever since. Beyond the culture and approach of the classrooms, this department truly had a collaborative approach to planning lessons, grading, and teaching. It is so very normal to walk into each other's classrooms and begin co-teaching with them. One of my favorite memories is Mr. Boomer and I walking into Mrs. Garfield's Math 4 class to help facilitate her students' *engagement of trigonometric identities. That semester Mr. Boomer and I both had student teachers, so we could float in and out of each other's classrooms. A group was stuck on one of the identities, and both John and I started helping different students in that group. We took different approaches to the problem and then got super excited comparing them. Looking back, we could have had the students compare the approaches to support their engagement with the task, but our excitement led us to simply "geek out" in front of our students and show how we are passionate about mathematics.*

I am right across the hallway from Mr. Boomer, and oftentimes he has a small lab class for students to receive extra help in his last block of our day. I can't count how many times he crosses the hallway and shares his excitement for what we are exploring that day. He always has something productive to add; and I am always excited to continue learning from my colleague, mentor, and friend. This collegial attitude is echoed in our professional learning community meetings. To an outside observer it probably seems like we argue about minutia, but we both get just so passionate about what we do that disagreements, raised voices and all, are just a byproduct. We always have Mrs. Garfield to bring us back into what we need to discuss and move forward— the peacekeeper of us all.

My collaboration with Mrs. Garfield began with her mentoring me through not only student teaching, but also teaching Integrated Math 2 in our previous curriculum (Core-Plus Mathematics Project, 2008). At first it began with warnings about lessons that were a little more teacher-led than purely exploratory, which questions were the best opportunities for formative assessment, discussions of classroom management, and types of purposeful questions. As the years continued, we began experimenting with ways to improve the explorations as the curriculum's dated nature started to show in less-relatable contexts for student engagement.

Further, we even started to create our own investigations. For all its strengths, Core-Plus (2008) was not written for the Common Core Standards (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010a), which influenced the writing of the current Kansas Math Standards (Kansas State Department of Education (KSDE), 2017). With a new curriculum to teach, collaboration with my fellow math teachers will undoubtedly look different. We will go back to the beginning of the cycle of curriculum adoption, back to a deep focus on discussing the mathematics and the intent of the writers of Illustrative Mathematics (2019) rather than modifying Core-Plus (2008) to fit current standards.

Limitations

To begin with, the most important limitation is central to the idea of autoethnography. This dissertation is *an* account of establishing norms for the purpose of supporting productive student struggle—not *the* account. There is no final word in autoethnography. As a result of the unique nature of autoethnographic methodology, each researcher has some variation in their implementation. The way that I established and negotiated norms is by no means the only way to establish and negotiate norms. Just as each teacher has their own unique quirks to implement the science of teaching as an art, any reader or researcher wishing to replicate my actions as a teacher will find diverse results. Never again will I have the same mix of students with the same chemistry in my classroom. Other school years might bring differences in who I work with in the math department, building leadership changes, and hopefully, an end to the pandemic. My negotiation of norms and facilitation of struggle was slightly different for each of my classes, and the microculture for each class evolved over time as the demands of students changed.

More specific to the context of my classroom is the usefulness of this research to different readers. I am in a department committed to progressive math teaching—where teachers are guides on the side rather than sages on the stage; where student discourse is prioritized; where students explore mathematics to construct their knowledge. As such, it might be more useful to readers looking to support existing reform efforts as opposed to an account of how to create change from a more traditional classroom. The context of this research site also limits the account of this dissertation in terms of equity in relation to diverse students. I teach a heavily White and largely middle-class population of students. Equitable and responsive teaching practices are of course what research suggests as best practice for all students (Gay, 2000; NCTM, 2014), but any particular focus on equity for diverse students will be best informed by other research.

Significance

Possibilities for this research, once again, begin with the unique advantage that autoethnography offers. This account offers an insider's view of a classroom microculture as opposed to an observer's view of that microculture. A researcher-observer must gauge the intent of, not only teacher participants, but also how students are interpreting and acting on that intent. As a self-as-subject researcher, I wholly know my intent as to the norms I negotiated and established, as well as the detailed knowledge of my students which informs my specific teaching actions. Rather than gauging the teacher's view of what they want to establish for norms in their environment and the student perception and engagement of those norms, it is my intent and my perception as to how students perceived these norms.

This detailed account of teaching offers contributions to both theory and practice. Knowledge in this study is discovered through reflections and narratives of my practice of teaching. I explicitly connected this knowledge to existing theory, and to further theorize in two frameworks. The establishment and negotiation of norms literature in math education consists of observer researcher accounts of norms in classroom microcultures (Bowers et al., 1999; Cobb et al., 2001; Levenson et al., 2009; Makar & Fielding-Wells, 2018; Mottier-Lopez & Allal, 2007; Partanen & Kaasila, 2015; Planas, & Gorgorió, 2004; Smith et al., 2007; Tatsis & Koleza, 2008; Yackel & Cobb, 1996; Yackel et al., 2000). While some of these accounts do include expert teacher-researchers as participants to engineer classroom research, they do not capture the essence of a singular focus on the establishment of norms from a self-as-subject perspective. Nor do they have purposeful ties to supporting productive struggle—instead, their aims usually included the establishment of norms that support mathematical inquiry or discourse.

This study is unique with its focus on connecting sociomathematical norms to the teaching practice of productive struggle and in its research location at a high school level for algebra 1 and geometry students. The majority of reviewed research regarding sociomathematical norms took place in primary or elementary schools (Bowers et al., 1999; Cobb et al., 2001; Franke et al., 2015; Kang & Kim, 2016; Leveson et al., 2009; Makar & Fielding-Wells, 2018; Morrison et al., 2021; Mottier Lopez & Allal, 2007; Roy et al., 2014; Sánchez & García, 2014; Yackel & Cobb, 1996; Van Zoest et al., 2012), or with calculus students (Partanen & Kaasila, 2015), gifted and talented classrooms (Çakır & Akkoç, 2020), undergraduate math students (Fukawa-Connelly, 2012; Yackel et al., 2000), preservice teachers (Güven & Dede, 2017; Kang & Kim, 2016; Roy et al., 2014; Sánchez & García, 2014; Van Zoest et al., 2012), and teachers (Zembat & Yasa, 2015).

This work further contributes to existing knowledge by providing more context to the productive struggle framework (Warshauer, 2015a), specifically with a focus on the classroom microculture (Cobb & Yackel, 1996). At this time, recommendations for classroom environment exist; however, these recommendations have not arisen from studies specifically focused on the environment for productive struggle. Rather, they come from research informed practitioner articles (Amidon et al., 2020; Baker et al., 2020; Barlow et al., 2018; Freeburn & Arbaugh, 2017; Livy et al., 2018; Lynch et al., 2018; Murawska, 2018; Townsend et al., 2018; Warshauer, 2015b). In sum, the aforementioned literature on productive struggle or environments that support struggle do not arise from a specific focus on establishing that environment.

Beyond contributions to the knowledge base of math microcultures and the productive struggle framework, this study was timely following one and a half years of disrupted learning environments from the COVID-19 pandemic. This disruption provides an opportunity to re-establish productive classroom cultures, assuming they were established at all. Also timely was the research site's adoption of a new curriculum: *Illustrative Mathematics* (2019). While not the focus of this dissertation, based on the analysis it did play a part in contributing to the research background for this curriculum and provided another use for the data collect for autoethnographic work. Most importantly, as a teacherm this research allowed me to systematically improve my practice as an educator.

Chapter Summary

Based on the literature, the context of this study loosely connects two sets of research - that of classroom microculture and that of supporting productive student struggle. It is known that the classroom environment is important to establishing a culture that values struggle as an essential part of learning mathematics. However, how teachers go about establishing that culture is less clear. At the onset of this study, many schools returned to in-person learning following an ever-present pandemic that has led to changing classroom environments and gaps in learning. To link the two frameworks of research and investigate a timely issue, this study investigated in what ways does a teacher negotiate the establishment of classroom norms in order to facilitate productive struggle?

Prominent constructs in this research include the idea of productive struggle, qualifying whether struggle is productive or not, and teacher moves that support struggle. Essential microculture constructs include the ideas of social norms, sociomathematical norms, and classroom mathematical practices. The literature review in Chapter 2 expands upon these ideas. These constructs helped to frame my thinking in implementing this analytic autoethnography, an autoethnography which endeavored to use subjective experiences as data, shape those experiences into narratives, and use those narratives to contribute to the body of knowledge. Chapter 3 will detail this methodology in detail. Shaping experiences into knowledge required self-reflexivity and ongoing emergent analysis of data. This emergent analysis was informed by the interactionist perspective, which combines the sociocultural framework of microculture and the individual correlates of student learning and beliefs, a psychological perspective.

A major aspect of credibility in analytic autoethnography is informed by clear self-reflexivity; reflective and cyclical iteration of shifting between data and themes of data; and subjectivity, the centrality of the self in research and in narratives. These narratives do not present the final word on establishing cultures, they will only represent an account of a teacher going about establishing a learning environment to support productive struggle. These narratives are spread throughout Chapters 4-7 to illustrate my findings, including a discussion of self-reflexivity in my thought process to illustrate credibility in the analysis of my data. The power of narratives originates from connecting the experiences of a classroom teacher-researcher to the body of knowledge. In this way, this study contributed not only to practitioners, but also to the base of literature for both mathematics classroom microcultures and the productive struggle framework. These contributions and connections to existing research are the basis for Chapter 8. Chapter 9 presents concluding thoughts by first discussing concise answers to the overarching research question and supporting research question. Answers to the research questions are followed by ruminations on autoethnography which presents a final dialogue detailing my path as an autoethnographer, as well as brief discussions of challenges, trustworthiness, limitations, and research implications.

Chapter 2 - Literature Review

This chapter consists of two parts, ‘Norms’ and ‘Productive Struggle.’ Norms are first discussed from a classroom standpoint without a specific focus on mathematics. Following that general discussion, I describe the emergent perspective that informs how the social perspective, and the psychological perspective are complimentary in informing what is happening in a classroom environment. Following this theoretical background is a discussion on how norms are established, and within are specific focuses on social norms, sociomathematical norms, and classroom mathematical practices.

Having established the general classroom environment, I begin discussing productive struggle with a specific focus on the tasks students engage in, as those tasks provide opportunities for struggle and opportunities for the continual establishment and renegotiation of social and sociomathematical norms. Following a discussion of those tasks are suggestions for task implementation with a specific focus on teacher questioning, as teacher questioning is an essential interaction with students that will support the establishment of norms. The chapter is concluded with a summary that blends important takeaways from both ‘Norms’ and ‘Productive Struggle.’

Norms

Norms in educational literature and in math education literature represent the shared meanings of the classroom and can provide positive benefits toward the learning environment. Yackel (2001) characterizes norms as “a sociological construct” that “refers to understandings or interpretations that become normative or taken-as-shared by the group” (p. 6). The group in the classroom environment comprises both the teacher and their students. Further, classroom norms “describe[s] the expectations and obligations that are constituted in the classroom” (p. 6). George

Homans (1951, as cited in Guven & Dede, 2017), one of the fathers of sociology, is credited with coining norms as a term to help describe social exchange theory.

Research indicates there are benefits of establishing positive classroom norms. Brophy (2000) found that the establishment and maintenance of norms allow teachers to spend more time engaged in learning activities rather than focusing on discipline or classroom management. Positive norms include explicit identification of ways of interacting respectfully with other students as well as productive ways of engaging in learning material (Brophy, 1998; Brophy, 2000; Good & Brophy, 2000; Sergiovanni, 1994; all as cited in Evertson et al., 2003). Norms play a critical role in social-emotional learning (SEL). Durlak and colleagues (2011) found four conditions in reviewing the literature that support SEL and better school performance:

> (a) peer and adult norms that convey high expectations and support for academic success, (b) caring teacher–student relationships that foster commitment and bonding to school, (c) engaging teaching approaches such as proactive classroom management and cooperative learning, and (d) safe and orderly environments that encourage and reinforce positive classroom behavior. (p. 418)

Bisson (2018) similarly links norms to fostering a sense of belonging and the establishment of a community of learners wherein students play a critical role in holding one another accountable to the rules and expectations of the classroom.

Early research on norms in education focused on the establishment of rules and procedures for teachers to use in classroom management (Emmer, 1984; Emmer et al., 1981). Notably, early research focused on teacher expectations imposed on students to foster a productive learning environment as opposed to a focus on how norms influence the construction of mathematical concepts by students seen in math education research (e.g., Bowers et al., 1999;

Cobb et al., 2001; Levenson et al., 2009; Makar & Fielding-Wells, 2018; Mottier-Lopez & Allal, 2007; Partanen & Kaasila, 2015; Planas, & Gorgorió, 2004; Smith et al., 2007; Tatsis & Koleza, 2008; Yackel & Cobb, 1996; Yackel et al., 2000;).

More general education on research on culture in classrooms is broadly construed. Redding (2014) provides a characteristic example:

> The culture of a teacher's classroom reflects values and is seen in its rituals, routines, expected behaviors, and relationships among teachers and students. How the teacher organizes the classroom and establishes and reinforces its rules and procedures constitute classroom management, and classroom management operationalizes much of what is broadly called classroom culture. (p.13)

Cobb and Yackel's (1996) use a classroom microculture framework to describe mathematics classroom environments. This microculture consists of shared meanings or norms between all individuals in the classroom environment. Cobb and Yackel (1996) further identify three types of norms creating this microculture: classroom social norms, sociomathematical norms, and classroom mathematical practices—each addressed in subsequent sections. This section begins with a discussion of the theoretical basis of social interaction, symbolic interactionism, and the emergent perspective created to interpret classroom environments (Cobb & Yackel, 1996). This is followed by research on how norms are established, with specific focuses on the constructs of social norms, sociomathematical norms, and classroom mathematical practices.

Interactionist Perspective

Discussing Cobb & Yackel's (1996) interactionist perspective must start with the sociological theory it built from symbolic interactionism. Symbolic interactionism originated with George Mead's (1934) work as a pragmatist and as a social behaviorist. Blumer (1969), a leading symbolic interactionist, published a collection of essays demonstrating symbolic interactionism's place in sociology. Yackel (2001) characterizes symbolic interactionism succinctly in saying that "meaning arises through interaction" (p. 7). More specifically, the first defining principle of symbolic interactionism is that "each person's actions are formed, in part, as she changes, abandons, retains, or revises her plans based on the actions of others" (p. 4), and the second defining principle is that "meaning is seen as a social product" (p. 5).

This study investigates norms, a sociocultural perspective (Lave & Wenger, 1991), and productive struggle, a teaching practice focused on what is happening in students' construction of knowledge, a constructivist perspective (von Glasersfeld, 1989). Cobb (1994) demonstrates that constructivist and sociocultural perspectives are complementary in informing what is going on in the classroom environment and how students are learning. In short, "the sociocultural perspective informs theories of the conditions for the possibility of learning, whereas theories deployed from the constructivist perspective focus on what students learn and the process by which they do so" (p. 13).

Cobb and Yackel (1996) created a framework to analyze students' individual and collective activities in a learning environment, see Figure 2.1. On the left side is the social perspective, and on the right is the psychological perspective. The social perspective informs collective meanings, whereas the psychological perspective implies the individual meanings of a given student. Cobb and Yackel (1996) describe the interaction between "each row. . . embodies a conjectured relation between an aspect of the classroom microculture and the activity of the individuals who participate and contribute to it" (p. 177). Relevant to this study is "the conjectured relation between classroom social norms and individual beliefs implies that a teacher who initiates and guides the renegotiation of classroom social norms is simultaneously supporting individual students' reorganization of the corresponding beliefs" (p. 177).

Figure 2.1

An interpretive framework for analyzing individual and collective activity at the classroom level.

SOCIAL PERSPECTIVE	PSYCHOLOGICAL PERSPECTIVE
Classroom social norms	Beliefs about own role, others' roles, and the general nature of mathematical activity in school
Sociomathematical norms	Mathematical beliefs and values
Classroom mathematical practices	Mathematical conceptions and activity

Note. Reproduced with permission from "Constructivist, Emergent, and Sociocultural Perspectives in the Context of Developmental Research," by P. Cobb, and E. Yackel, 1996, *Educational Psychologist, 31*(3/4), p. 177 (https://doi.org/10.1080/00461520.1996.9653265). Copyright 1996 by Taylor & Francis Group.

This study focused on the conditions for productive struggle in the learning environment with a specific emphasis on norms. These conditions were interpreted through my perspective as an autoethnographer. A perspective in the section entitled Epistemology in Chapter 3, neatly combines the sociocultural perspective and the constructivist perspective. More specifically, I constructed the meaning of the learning environment in my classroom through the interpretation and analysis of collected data.

Establishing Norms

Early research on establishing norms specifically focuses on how a teacher establishes norms by way of expectations, rules, and discipline that make up classroom management. Emmer (1984) reviews literature to suggest a three-phase model. In the first phase the teacher establishes rules and expectations prior to the school year. In the second phase, the teacher socializes “students into the classroom setting and establish[es] appropriate behavior” (p. 5). The third phase involves the maintenance of these norms throughout the school year. Glasser (1969, 1990) however, suggests that rules and procedures be negotiated and established between both teachers and their students, which is closer to the view that math education researchers take in viewing norms that support the learning of mathematics. Bisson (2018), in a similar vein, discusses the construction of norms between teachers and students as following a classroom discussion, the direct teaching of those norms, and the continual modeling and practice of the norms.

Güven and Dede (2015), in their work of examining social and sociomathematical norms in classroom microcultures, suggest explicit negotiation of norms as “important for students, as well as for the productivity of norms” (p. 287). From an ethnographic standpoint, this explicit negotiation of norms is termed “culture building” (Fine, 2003). Importantly, the research establishing the investigation of norms in mathematics classrooms identifies norms as “not predetermined criteria introduced into the classroom from the outside. Instead, these normative understanding are continually regenerated and modified by the students and the teacher through their ongoing reactions” (Yackel & Cobb, 1996, p. 474). While some might view only the teacher as influencing students in their establishment of norms, Yackel (2001) demonstrates the interaction between teacher and students as true negotiation in a discussion of a teacher establishing norms of mathematical discourse:

> It might seem that the teacher is the only one who contributes to the renegotiation of social norms. However, norms are interactively constituted as individuals participate in interaction. In this case, as the episode evolved, students contributed to the negotiation of the norms by increasingly acting in accordance with the expectations. (p. 11)

The students' interpretations of a teacher's modeling of norms are a critical element of the renegotiation of norms as well as characteristic of both the theoretical perspectives of symbolic interactionism and the sociocultural perspective of classroom learning. Student interpretations of teacher modeling and expectations are not necessarily one in the same. Levenson and colleagues (2009) investigated student perceptions of teacher endorsed norms and found that "even when the observed enacted norms are in agreement with the teachers' endorsed norms, the students may not perceive these same norms" (p. 171).

The establishment of norms, social and sociomathematical, allow teachers to support students' beliefs toward mathematics— their mathematical dispositions (Cobb et al., 2001; Yackel & Cobb, 1996). Student mathematical dispositions, beliefs, and actions in the engagement of mathematics, are connected to norms in several areas of research: inquiry (Makar & Fielding-Wells, 2017), collaboration (Webb, 2009), motivation (Megowan-Romanowicz et al., 2013), and problem-solving (Tatsis & Koleza, 2008). Webb (2009) identified teacher moves that promote collaboration and dialogue in the classroom. One important theme included the social norm where students explain and justify their thinking and reasoning and the corresponding teacher moves of using questions to promote student thinking and reasoning (e.g., posing purposeful questions, NCTM, 2014).

Megowan-Romanowicz and colleagues (2013) identified researcher perceived norms for participation in observing classrooms and their connection with student motivation, including:

> (1) the time eventually passes and if you wait quietly, little will be asked of you, (2) knowledge resides in the book—not usually in the teacher nor in peers (although answers may sometimes reside in their peers), (3) if the teacher asks you a question, you may respond by asking her for help, (4) during small group work one should not attract the teacher's attention, (5) there is no penalty for not finishing, (6) there is no penalty for guessing, (7) if the blanks are filled in the task is satisfactorily completed, (8) if everyone remains silent the teacher will eventually supply the right answer. (p. 59)

Based on the list above, it is clear that not all norms that exist in classrooms serve to create productive struggle or positive dispositions toward mathematics. The authors posit that these modes of participation "serve to reinforce the cultural norms that have been established in this mathematics class: they serve to free up time to engage in nonmathematical social chatter, providing the students with their goal fulfillment" (p. 59). One gap in the literature is intentionally harnessing the establishment of norms in a positive way (Megowan-Romanowicz et al., 2013). This study is well positioned to partially address that gap in the literature in investigating the establishment of norms for the facilitation of productive struggle.

According to Tatsis & Koleza (2008) there are connections between norms and problem solving. "We have found norms related to particular aspects of the problems posed. Our results show that most of these norms, once established, enhance the problem-solving process" (p. 89). Norms enhancing the problem-solving process are collaboration, justification, avoidance of threat, non-ambiguity, third person comprehension, mathematical justification, mathematical differentiation, validation, and relevance. I speculate that many of these norms, social and sociomathematical, will contribute to the facilitation of productive struggle. I now turn to specific classifications of aforementioned norms: social norms and sociomathematical norms.

Social Norms

Social norms are not specific to any single mathematics classroom, in fact, social norms exist in every classroom. What distinguishes social norms is the fact that they do not hold any inherent mathematical characteristic (Yackel & Cobb, 1996). For example, Tatsis & Koleza (2008) found norms enhancing problem solving, many of which are social in nature as these norms do not have a singularly mathematical nature. Students in all classrooms could have norms of collaboration, expectations of justification, the avoidance of threat (i.e., students feel safe expressing themselves), and that student explanations be understood by a third person (third person comprehension) and hold relevance to the problem situation. One important aspect of social norms related to mathematical learning is that students tend to comply with the norms and expectations of their classrooms (Megowan-Romanowicz et al., 2013; Webb, 2009). Thus, when promoting productive struggle, the establishment of social norms is every bit as important as developing sociomathematical norms.

Sociomathematical Norms

The importance of the teacher in establishing social norms is echoed in the establishment of sociomathematical norms: “The analysis of sociomathematical norms indicates that the teacher plays a central role in establishing the mathematical quality of the classroom environment and in establishing norms for mathematical aspects of students’ activity” (Yackle & Cobb, 1996, p. 475). Sociomathematical norms by definition have a mathematical nature. Yackel and Cobb (1996) offer a definition frequently used in subsequent sociomathematical norms literature: “the understanding that students are expected to explain their solutions and their ways of thinking is a social norm, whereas the understanding of what counts as an acceptable mathematical explanation is a sociomathematical norm” (p. 461). They distinguish sociomathematical norms as the beliefs that students collectively hold regarding the level of sophistication of mathematical explanations including different, acceptable, efficient, and sophisticated.

Bowers and colleagues (1999) offered a method of negotiating norms in such a way to teachers assist students in the development of sociomathematical norms.

> At the beginning of the teaching experiment, the teacher initiated the negotiation of social norms, which included expectations that students were to (a) explain and justify their solutions, (b) listen to (and make sense of) the explanations offered by others, (c) ask a clarifying question if an explanation is unclear, and (d) resolve disagreements by discussing the viability of various solution methods. (p. 39)

To explain and justify is social in nature and a general expectation that could occur in any content area classroom. However, more specific types of justification could be a sociomathematical norm. According to Yackel et al. (2000), “for first-order differential equations, we document the sociomathematical norm that explanations be grounded in an interpretation of the rates of change.” Tatsis and Koleza’s (2008) operationalized sociomathematical norms in a comparable way in observing interactions of pre-service math teachers. These interactions produced the sociomathematical norms of mathematical justification, mathematical differentiation, and validation. Further, sociomathematical norms, in particular, enhanced the problem-solving process.

Research highlights this subtle distinction between social and sociomathematical norms, expanding characterization of sociomathematical norms as compared to the original focus of acceptable mathematical explanations. Instead of a social norm of inquiry, Makar, and Fielding Wells (2017) characterize it as mathematical inquiry, a sociomathematical norm due to the centrality of mathematics in that norm. Specifically, "Norms of mathematical inquiry engage students in productive social interactions and improve their mathematical knowledge, as well as their interest, valuing and capacity to solve complex problems" (Makar & Fielding-Wells, 2017, p. 54). Another sociomathematical norm is that of creative investigating. Partanen and Kaasila (2015) characterize creative investigating as "when investigating mathematics, one should approach the topic in a creative way" (p. 927)— a sociomathematical norm that connects to what some mathematicians see as central to their discipline (Ellenberg, 2021).

Sociomathematical norms covered to this point have either been ways of assessing others' solutions or ways of approaching mathematics. However, researchers have also characterized collective ways of interacting in problem solving as sociomathematical. Mottier-Lopez and Allal (2007) identify norms of social interaction with mathematical elements as sociomathematical. Individual expectations are reframed as "sociomathematical norms regarding problem solving" (p. 254). Examples of individual expectations regarding problem solving were to "try out several procedures; distinguish the given elements from what is to be found; verify results" (p. 254). Further, they reframed collaborative expectations as "sociomathematical norms regarding problem solving in joint problem-solving activity" (p. 254). Examples of these collaborative expectations involved explaining to others and expressing opinions on other students' methods of solving. Similar to collaborative expectations are Classroom Mathematical Practices, which are collectively agreed-upon ways of doing and discussing mathematics.

Classroom Mathematical Practices

Classroom mathematical practices represent the “collective mathematical learning of the classroom community” (Bowers et al., 1999, p. 26). For example, once established as a valid method of solving systems of equations, referencing the coordinates of an intersection of functions on a graph would not require lengthy justification, discussion, or explanation in an algebra 1 classroom. Bowers and colleagues (1999) speculate that “in general, classroom mathematical practices evolve as the teacher and students discuss problems and solutions, and these practices involve means of symbolizing, arguing, and validating in specific task situations” (p. 28). I characterize classroom mathematical practices further as practices that hold utility throughout a student’s mathematical learning. For example, using the connections between tables, graphs, and function rules for high school mathematics. The relationship between sociomathematical norms and classroom mathematical practices is close, “If sociomathematical norms are specific to mathematical activity, then mathematical practices are specific to particular mathematical ideas” (Cobb et al., 2001).

Norms Summary

Theoretically speaking, in this study norms are constructed from an emergent perspective that blends both the social and psychological perspectives of student learning. Establishing norms is a continual negotiation between teachers and students. Initially, teachers should explicitly negotiate norms with students and formalize those norms to fit their classrooms best. Teachers establish expectations of these formalized norms by both modeling and repeatedly reestablishing expectations in their interactions with students. Formalized norms represent social norms, agreed upon ways of interacting that have no inherent mathematical nature. Social norms can emerge as sociomathematical when students implement these norms in inherently mathematical ways. In doing so, they can develop productive dispositions toward mathematics. Over time, established learning of the classroom is formalized into classroom mathematical practices. Students use classroom mathematical practices to anchor their reasoning as well as build future knowledge.

Productive Struggle

There is an extensive line of educational theorists that have emphasized the importance of struggle (Brownell & Sims, 1946; Bruner 1960; Dewey, 1910; 1926; 1929; Festinger, 1957; Hatano, 1988; Polya, 1957). Hiebert and Grouws (2007) identified student struggle as one of the critical features in teaching that promotes conceptual understanding. They also identify how student struggle, usually characterized as an inquiry-based instructional feature, utilizes aspects of direct instruction to make student's struggle strategic and, in that way, productive. Productive struggle refers to a student's "effort to make sense of mathematics, to figure something out that is not immediately apparent" (Hiebert & Grouws, 2007, p. 287). Bruner's (1960) description of discovery frames productive struggle as valuing the process of learning over the product of learning:

> Let me propose instead that discovery is better defined not as a product discovered but as a process of working, and that the so-called method of discovery has as its principal virtue the encouragement of such a process of working or, if I may use the term, such an attitude. (p. 612)

Unproductive struggle, by contrast, refers to a situation in which a student "make[s] no progress towards sense-making, explaining, or proceeding with a problem or task at hand" (Warshauer 2011, p. 21).

Warshauer (2015a) identified four types of student struggle: getting started, difficulty in carrying out a procedure or process, uncertainty in explaining and sense-making, and expressing misconceptions and errors. Types of struggles follow from the tasks students are engaged in—tasks of higher cognitive demand can be sustained through struggle, which is considered productive struggle in the literature. The next section will address the design and use of mathematical tasks. Following the discussion of tasks will be a review of how teachers should implement those tasks. Including how norms inform how a teacher implements tasks and supports student engagement of mathematics.

Tasks

Research on the use of tasks in math classrooms focuses on the characteristics of problems that students solve. Bennett and Desforges (1988) discuss two studies that address the connection between the design of a task and the concept of productive struggle, stating "children may make many errors but be provoked into thinking about how to solve a particular type of problem, that is, they may have learned a great deal" (p. 223). Hiebert and Wearne (1994) found classrooms that take a conceptual approach, using tasks with meaningful problem situations and engaging students in discourse, showed higher levels of performance on an end-of-year test and more growth as measured by a pre-and post-test. Multiple authors have described these meaningful problem situations (Bennet & Desforges, 1988; Stein et al., 1996; Stein & Lane, 1996; Stein & Smith, 1998).

Bennet and Desforges (1988) investigation of tasks led to the characterization of the diverse types of problems teachers use: incremental, restructuring, enrichment, and practice. Incremental tasks introduce new related ideas, while restructuring tasks provoke the discovery of new ideas. Enrichment tasks apply existing ideas to new context and practice makes use of concepts on familiar problems. The Task Analysis Guide builds upon these descriptions and includes the cognitive demand that tasks require (Stein et al., 2009). Tasks that only require a low level of cognitive demand involve memorization or non-contextual procedures, and tasks with high levels of cognitive demand require the use of procedures in context or with connections to other mathematical concepts as well as "doing mathematics" tasks. Kapur (2008) characterizes lower and higher demand tasks as well-structured or ill-structured. An ill-structured task would require higher cognitive demand and thus be more likely to prompt struggle for students. Tasks that prompt productive struggle best come from the categories requiring higher-level cognitive demand (Baker et al., 2020), but are best represented by the "doing mathematics" characterization. "Doing mathematics" tasks:

- require complex and non-algorithmic thinking. . .
- require students to explore and understand the nature of mathematical concepts, processes, or relationships.
- demand self-monitoring or self-regulation of one's own cognitive processes.
- require students to access relevant knowledge and experiences and make appropriate use of them in working through the task
- require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions.
- require considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required. (Stein et al., 2000, p. 16)

Other task research utilizes similar design features. Schwartz and Martin (2004) in investigating the effects of student invention of mathematics used tasks in which they gave students contrasting cases—similar to requiring students to analyze tasks and consider task constraints. Kapur's (2012, 2014) research on productive failure utilized tasks with complex problem contexts, ones that allow for multiple representations and solution methods, prompt prior knowledge retrieval, and challenge students to both solve and investigate their solutions—task features which are similar to and compliment doing mathematics tasks. Zaslavsky (2005) used three types of tasks to provoke uncertainty in students: tasks with "competing claims. . . unknown path or questionable conclusion. . . [and] non-readily verifiable outcomes" (p. 305). Task characteristics that, once again, align with doing mathematics tasks.

Rich math tasks provide opportunities to assist in the development of classroom norms. Smith and colleagues (2007) illustrate this when discussing how pattern-based math tasks assist in the development of multiple solution paths. Sharing solution paths supports "students [in] learn[ing] how to participate in the classroom community" (p. 41). Jilk and Erickson (2017) identify norms for participation meant to provide access and equity considerations in how students engage in high quality tasks. Examples of these social norms for participation include: "Students write on their own papers. . .All team papers are in the middle of the table. . .Think out loud. . .All papers match. . .(and) All voices are heard" (p. 21).

Task Summary

The literature on mathematical tasks that would prompt productive struggle reflect the doing mathematics task characteristics. Despite knowing the characteristics of these tasks, challenges arise in ensuring tasks have both a low-threshold and high ceiling (LTHC, McClure, Woodham, & Borthwick, n.d.). Bennett and Desforges (1988) relate to this challenge in finding that teachers tend to misjudge the abilities of both high and low attaining students in the use and design of tasks. Implementing those tasks with fidelity, maintaining the initial cognitive demand may be yet another challenge. The practical results of student interaction with tasks rest partially in the interaction between student and facilitating teacher, interactions in which literature has suggestions for successful, or productive, outcomes.These interactions also are where teachers can reestablish and continue to negotiate social norms that support sociomathematical norms.

Implementation

Literature has numerous suggestions as to how teachers can successfully implement tasks with high cognitive demand. Structuring lessons with a Launch-Explore-Summarize (LES) format is one way to successfully implement high demand tasks (Livy et al., 2018; Smith & Stein, 2018). In this lesson format, teachers launch a task providing appropriate task context, reestablish expectations for engagement in the task, and then afford students time to work and struggle on a given task in the explore phase. During this exploration phase, teachers are facilitating student learning and documenting student thinking to reference in the summarize portion of the lesson. The summarize portion of the lesson includes the teacher orchestrating discourse to develop student thinking, while sequencing student ideas from least sophisticated to most sophisticated.

Stein and colleagues (1996) identified ways in which teachers lowered cognitive demand. These include proceduralizing difficult task characteristics, shifting the focus from meaning to the correctness of an answer, providing insufficient or too much time, classroom management issues, inappropriate task design for student development, and not pressing students to explain their thinking fully. However, sustained cognitive demand involves several teacher actions. Teachers scaffold student's thinking and reasoning, provide students with opportunities to self-monitor, continually press students to fully explain their thinking, ensure appropriate task design that builds on prior knowledge, and provide sufficient time (Stein et al., 1996). Much of the following literature reflects or builds upon these recommendations for successful task implementation.

Zaslavsky (2005) notes that an appropriate classroom environment is essential in successfully implementing tasks intended to provoke uncertainty, specifically an environment that supports mathematical discourse between students. Kapur (2011) describes more specific sociomathematical norms for productive struggle. In productive failure classrooms, teachers establish the expectation that students value the process of engaging in the problem rather than finding the product, the right answer. Students should "more importantly, generate multiple representations and methods even if they do not lead to a successful solution" (p. 575). Further, teachers should resist the impulse to rescue students, only providing enough assistance for where the student's zone of proximal development is (Kapur, 2011; Vygotsky, 1930/1970). Assisting students also would look like encouraging students to try harder and to work together to find a solution (Kapur, 2011).

Warshauer (2011) examined teacher-student interactions and characterized them as either productive or unproductive. The outcome of the interaction was productive if the teacher did not lower the cognitive demand in their facilitation, and also accurately responded to the student struggle. However, if the teacher lowered the cognitive demand, or did not accurately respond to the student struggle, the interaction was unproductive. In essence, responses were either closed-ended and supplying information or open-ended and affording opportunity. Closed-ended responses involved teachers "telling" or providing "directed guidance," and open-ended responses providing "probing guidance: or "affordance" (Warshauer, 2015a, p. 387). Probing guidance involved "made students' thinking visible and served as the basis for addressing the students' struggles" (p. 389), whereas an affordance response requires teachers to prompt for further explanation from students—as well as affording enough time for students to work through their struggle. Warshauer (2015a) identified a complicating factor in these interactions—trying to sustain student engagement throughout the classroom environment with students with varying struggles and varying levels of engagement (e.g., Stein et al., 1996).

Warshauer's (2015b) provides four suggestions for practitioners focused on the classroom environment: (1) acknowledge and recognize the essential nature of struggle in the learning of mathematics. (2) provide enough time for the task itself and give enough waiting time for students to have to persist in their struggle. (3) Encourage students to reflect upon their problem-solving process, and to recognize effort—not just getting the correct answer. (4) Question students to assist them in focusing and clarifying their thinking. Studying the implementation of inventing activities, Schwartz, and Martin (2004) identified similar effective instructional moves: asking students to explain what they were doing, asking students whether their solution appeals to common sense, and pushing students towards more generalized solutions.

Questioning

Purposeful questions support productive teacher-student interactions. Question types that promote information gathering on the part of the teacher include probing thinking, making mathematics visible, and encouraging reflection and justification (NCTM, 2014). These question types are also characterized as advancing questions (Smith et al., 2008) or focusing questions (Herbel-Eisenmann & Breyfogle, 2005; Wood, 1998). Advancing questions are meant to move student thinking forward, and focusing questions are meant to encourage students to clarify their thoughts in sense-making. Whatever the label, purposeful questions help to sustain the cognitive demand of tasks that allow teachers to facilitate student struggle. When cognitive demand is sustained, an episode of struggle is more productive rather than unproductive. Unfortunately, the temptation to rescue students, or the discomfort in waiting for students to think during wait time, (Chapin et al., 2003) makes successfully using purposeful questioning more difficult.

Research indicates the style of questioning in classrooms impacts what is learned and how it is learned. Traditional teacher-student interactions follow the response patterns of Initiate-Response-Evaluate (IRE) wherein a teacher poses a question, the student responds, and the teacher evaluates that response (Mehan, 1979). This response pattern can be characterized as funneling, as teachers are funneling students down a single, teacher inspired, path of reasoning (Herbel-Eisenmann & Breyfogle, 2005; Wood, 1998). Another IRE questioning pattern is identified in the literature as assessing questions (Smith et al., 2008. Assessing questions allow teachers to elicit and use student thinking as an informal formative assessment (e.g., NCTM, 2014). Assessing questions are not inherently unproductive or productive. The productivity of assessing questions depends on how the teacher uses their student's ideas. If a teacher funnels students down their prescribed path of reasoning, they reduce the cognitive demand of the task. Alternatively, if teachers use the thinking they elicit from students to advance their thinking or to encourage students to collaborate in their thinking (Freeburn, 2015), they sustain the cognitive demand of the task, which increases the likelihood that the struggle is productive.

Judicious telling is one method of eliciting student thinking by way of assessing questions. A teacher uses student responses to frame student thinking so that students can build upon their ideas (Lobato et al., 2005). Framing student thinking, or revoicing student ideas, is an important aspect of judicious telling (Freeburn & Arbaugh, 2017); so is providing extra information on problem contexts for students to frame their thinking around and providing correct mathematical terminology when students are expressing misconceptions and errors (Chazan & Ball, 1999; Freeburn 2015).

Implementation Summary

To summarize, the instructional environment must be one in which students and teachers value struggle as an essential part of the learning process (Baker et al., 2020). Teachers should leverage purposeful questions to guide students exploring mathematics in such a way that maintains the cognitive demand, and thereby facilitates productive struggle—but only after giving students requisite time to persevere in their struggle, and only just enough guidance. Teachers must also encourage and value student effort, pushing for students to fully explain their thinking—prizing the development of understanding, and not necessarily getting the right answer. Several instructional models have prominent elements of productive struggle and provide empirical evidence of its efficacy in improving student knowledge outcomes, the LES instructional model is a prominent example.

Productive Struggle Summary

For students to productively struggle, the task they engage in must not only be in their ZPD, but also have sufficient challenges for requisite growth in learning to occur. Sufficient challenges typically implies that tasks are either procedures with connections or doing mathematics tasks. In implementing these tasks, teachers can turn to various questioning strategies (see ‘Questioning’) as well as establish and negotiate their classroom environment to be one in which students understand the value of struggle. Valentine and Bolyard (2018) in reviewing literature on classroom environment characteristics that support productive struggle find four essential elements:

1) all students can learn/growth mindset (teacher is responsible for supporting the learning of all students),

2) student agency (learners are sensemakers of mathematics),

3) nature of mathematics (authority lies in mathematics), and

4) struggle is part of learning (mistakes are natural occurrences in the learning process). (p. 9)

Such a classroom has norms that support struggle. For example, this classroom may have social norms such as collaboration in problem solving, sociomathematical norms of justification, and various classroom mathematical practices students can use in struggle to construct their learning.

Literature Review Summary

The intersection of the establishment of norms and the facilitation of productive student struggle is the focus of this study. The selection of tasks for students to engage in struggle day to day was determined by the curricula I piloted in teaching algebra 1 and geometry (Illustrative Mathematics, 2019). This curriculum fits the characteristics of being within student ZPD, as well as having a challenging enough nature to spark student growth. In implementing these tasks, I utilized a LES instructional sequence, and during the task implementation, explicitly negotiated norms (see Chapter 3 subsection ‘Establishment of Norms’). These tasks provoke productive struggle at various levels: getting started, carrying out a procedure, explaining mathematics, and expressing misconceptions or errors. Teachers respond to struggle in diverse ways that either sustain or remove the cognitive demand of the task by telling, using directed or probing guidance, or affording students time or resources to resolve their struggle. Struggle is resolved in one of three ways: productively, productive at a low level, or unproductively.

Explicitly negotiated norms represent social norms, norms that do not have a specific mathematical characteristic. The norms of particular interest to this study afforded student learning in the facilitation of productive struggle. Norms should be explicitly negotiated initially, and over time teachers and students intersubjectively decide what ends up as accepted in each classroom. Social norms are created from this explicit negotiation of what counts as acceptable. Sociomathematical norms develop from students first explaining and justifying their thinking, second, sharing their ideas, and third, critiquing and comparing ideas shared from their peers. Over time classroom mathematical practices develop, these practices represent the collectively agreed upon mathematics that requires no justification following its use in the classroom—both students and teachers accept these practices as valid.

Chapter 3 - Methodology

Fast forward to the 2020-2021 school year. We began in-person schooling after a 6-month hiatus from seeing students in person. The people who make teaching worth it. I remember feeling sluggish and uninspired at the beginning of the year. It felt like the February slump in August! Teachers in my district spent the first two and a half weeks of what is usually the beginning of our academic year preparing for the first units of study we would teach for each class. We also participated in professional development about utilizing technology to teach in a "hybrid" fashion— where half of our students were in person, and half of our students were at home receiving "remote" instruction. This sucked any life and excitement that I usually have for teaching out of me. I was afraid that I was reaching burnout in only my 8th year of teaching!

School was the same and yet different. My school had all students in-person to begin the school year, except everyone had masks on, and included time for cleaning the classroom after each block. Oh. Students now could not share any materials—so goodbye to manipulatives that help breathe life into learning mathematics. At the time we were also not supposed to collect student work on paper to prevent the transmission of COVID-19. So, my student teacher and I went about the exhaustive process of creating Google Docs for each investigation so that students could type in their answers and submit their work on Google Classroom. Students would be there one day, gone the next, quarantined. I even had one student pulled out of class who logged in before the end of the block. She began the class period in person and ended it at home. Bizzaro land.

My favorite part of teaching has always been facilitating collaboration among students and the investigation of mathematics. Hearing students discuss ideas is what helps me continue to love my job year in and year out. Except these students had just spent 6 months *communicating with each other through their phones. Dialogue in the classroom was stilted and unreliable. When students spoke with each other, I could hardly hear them through their masks. As soon as my students and I began getting used to speaking through masks and talking with one another to establish some of the environment I am used to— collaboration, discussion, and collective exploration of mathematics, it changed once again.*

The dividers that our district ordered finally came in, two months into school. If we had these dividers (sneeze guards) up, somehow students would not have to be quarantined if they were across from one another. The dialogue, collaboration, and discussion in my classroom was once again stopped. Each year I establish this dialogue and collaboration before I really start pushing for productive struggle as I have always believed it is a prerequisite for successful student struggle. And it was sidelined once again. I am supposedly an expert in mathematics instruction. And my facilitation of struggling students was poor most of the time during the fall, and nonexistent at other times. How are teachers supposed to establish a community of learners when random groups of students are effectively wiped out of that community for two weeks as the school year continues? No continuity meant the regular social norms in my classroom had to be continually renegotiated. It felt like the first week of school stretched the entire Fall semester!

Purpose of the Study

This study examined the re-establishment of classroom norms following nearly one and a half school years of ever-changing learning environments in classrooms around the world. In some places, students had not returned to in person learning, and in other places students spent much of the year transitioning from a fully in person learning environment, to hybrid instruction where some students were online and some were in person, to fully remote learning. Teachers were challenged to plan, change plans, and sometimes deliver multiple modalities of instruction at the same time (Hamilton, 2020; Hamilton et al., 2020; Kaufman et al., 2021). Students had little to no consistency in their learning environment. Some students would begin the school day in school and end it at home following being pulled out of the classroom to quarantine. Within the school environment itself, students had physical barriers if not space preventing normal communication with desk dividers and masks to prevent the spread of COVID-19.

This disruption of a normal school environment led to a lack of engagement and coverage of the curriculum (Hamilton et al., 2020; Kaufman et al., 2021), which leads to learning gaps and learned helplessness. This challenge, however, provided teachers with an opportunity to start anew in re-establishing what a classroom environment should look like. This study looked at how the establishment of classroom norms can assist in changing learned helplessness into student productive struggle. The research question this study explored was: In what ways does a teacher negotiate the establishment of classroom norms to facilitate productive struggle?

A focus on my teaching using autoethnography rather than a more traditional qualitative lens provided an opportunity in looking at the culture of a classroom that researching as an outside observer would not have. I purposefully looked to establish social classroom norms that support productive student struggle; whereas as a researcher-observer in a classroom, I would not fully be able to ascertain the intent of the actions of teacher-participants in their classrooms regarding the establishment of norms. Further, learning the culture of a classroom as an outside researcher or traditional ethnographer is not as advantageous as interrogating the culture in my classroom as a self-as-subject researcher. Navigating these challenges presents a more contextual view of the challenges that teachers face following COVID-19. Jones et al. (2013) identify the use of insider knowledge and personal experience as one of the purposes that "make autoethnography, as a method, unique and compelling" (p. 32).

Research Questions

The overarching research question for study was: In what ways does a teacher negotiate the establishment of classroom norms to facilitate productive struggle? Sub-questions below informed my perspective as a teacher and provided framing for thinking about data and findings for this study.

1. How does a teacher perceive norms changing over time in a classroom?
2. How does a teacher utilize curriculum resources and ancillary informants to guide their negotiation of classroom norms?
3. How does a teacher's reflective practice influence their negotiation of norms and facilitation of student struggle?
4. How does a teacher perceive their responses to student struggle as influencing the renegotiation and reestablishment of norms?
5. What norms does a teacher perceive students utilizing when engaging in solving problems?
6. How do student behaviors and actions influence a teacher's negotiation of norms?

Autoethnography

After implementing a case study investigating how a novice teacher engages in facilitating productive struggle (Nusser & Martinie, 2022), my interest in productive struggle only continued. One of the themes emerging from that study was how classroom norms assisted in the facilitation of productive struggle. I grappled with developing another case study with an experienced teacher facilitating struggle or developing a multiple case study in designing a study for my dissertation. However, Dr. Todd Goodson gave some advice regarding my future research, and that was "to research wherever you are"and to consider autoethnography as a *current classroom teacher (T. Goodson, personal communication, April 6th, 2021). This kernel of an idea stuck with me, and I kept coming back to autoethnography to further investigate productive struggle and to investigate norms in my classroom.*

Throne (2019) identifies this interest and opportunity as one of the common motivations for autoethnographers: "more than likely the researcher also has a personal or lived experience related to the study phenomenon and may desire to explore the phenomenon from a self-as-subject stance or dive more deeply into the experience" (p. 14). Ellis and Bochner (2000) break down autoethnography into its roots as *auto* meaning the self as a researcher-participant, *ethno* meaning culture being investigated, and *graphy* meaning the research process itself. Denzin (1997, as cited in Minge, 2013) identifies autoethnography as the "ethnographic gaze inward on the self (auto), while maintaining the outward gaze of ethnography, looking at the larger context where self-experiences occur" (p. 217). In this study, I was an autoethnographer investigating the culture of my classroom and interpreting my experiences to learn more about the intersection of norms and the facilitation of productive struggle. Douglas and Carless (2013) support the investigation of personal experiences to advance scholarship:

> We cannot know or tell anything without (in some way) being involved and implicated in the knowing and the telling. . . one of the unique opportunities that autoethnography provides: to learn about the general— the social, cultural, and political— through an exploration of the personal. (pp. 84-85)

Autoethnography may seem new and extremely different to some readers; however, historically speaking, the use of personal experience to learn about reality has deep roots that only momentarily fell out of use in the rise of positivistic methods of research.

History

Douglas and Carless (2013) identify the use and growth of autoethnography in various fields as a "recognition of a growing need for a way to address, consider, and include what is found to be missing from writings based solely on scientific research methods: the voice of personal experience" (p. 89). One of the earliest reflective writings meant to illuminate future action and personal growth is Marcus Aurelius' (2002) *Meditations.* At the onset of the 16th century, Michel de Montaigne turned reflective writings into meditations for others (de Montaigne, 1991, as cited in Douglas and Carless (2013). William James, one of the intellectual fathers of pragmatism, used his personal experiences to help establish the field of psychology (James, 1892 as cited in Douglas and Carless (2013)), and had a stance toward ideas particularly suited to autoethnography in that:

> ideas are not 'out there' waiting to be discovered, but are tools—like forks, knives, and microchips—that people devise to cope with the world in which they find themselves. . . that ideas do not develop according to some inner logic of their own, but are entirely dependent, like germs, on their human carriers and environment. (Menand, 2001, p. xi)

This connection between ideas developing between people to solve problems within an environment is a prescient demonstration of the need for autoethnography in a mathematics classroom. Examining the culture of a classroom and how it influences the development of mathematical thinking from the perspective of an individual inside that microculture, namely, the teacher.

The earliest description of autoethnography came from Karl Heider to describe the ethnographic research contributions given by members of a culture (Heider, 1975 as cited by Austin & Hickey, 2007). Douglas and Carless (2013) trace the development of autoethnography as a research method starting with Patricia and Peter Adler (1987) describing the connection between field researchers and the phenomena being investigated. Ellis and Bochner (2000) identified the crisis of confidence spurred by postmodernism in the 1980s as a reason for the speedy growth of autoethnography as a method— identifying over forty separate ways authors described self-as-subject research from the 1970s to 2000. Rather than a growth of a novel method, Douglas and Carless (2013) identify it as a rediscovery:

> Rather than appearing now for the first time, personal and subjective experience has *been systematically removed from* human and social science research over the course of the past century in response to calls for methods that more closely parallel research in the natural sciences. (p. 89)

Epistemology

Ethnography itself is rooted in a stance on reality that there is no final or complete truth that captures what occurs in any environment (Lofland, 1995). This is remarkably similar to the nuance in education in that each classroom will have different dynamics and unique needs. Autoethnography is like ethnography in that it is both a process and a product and relies on a premise that "knowledge as discoverable via narrative and the individual lived experience as well as narrative truth is existent among textual data that may not be readily apparent without deep and iterative introspection" (Throne, 2019, p. 15). In this way the art of teaching in one classroom can be contextualized and reflexively analyzed to contribute to existing knowledge in mathematics education. Autoethnographers embrace the social construction of research in that it:

> allows researchers to ponder and "write in" the nature of the encounter and the research process, thus illustrating how a research outcome is always *constructed*— made,

> negotiated, and renegotiated by both researchers and researched who are involved in the *process*. (Tomaselli et al., 2013, p. 583)

Mingé (2013) identifies six epistemological lessons learned in her approach and products of autoethnography, several of which are of relevance to this study. The first of which is that "realities and knowledges are messy, complex, and multiple" (p. 428) which rings true to the reality of the unique makeup of each classroom microculture. The second lesson is that "we construct these knowledges from a particular point of view within a particular context" (p. 428). Rather than claim some sort of objectivity, autoethnography is rooted in the personal and subjective experience. The job of autoethnographers is then to connect the subjective lessons learned to more general theoretical knowledge. The fifth lesson she identifies is that "we enact change and create knowledge through mindful action" (p. 428). This is the primary goal of this study— to purposefully enact change in my classroom through informed action to create knowledge.

Criticisms

My first exposure to autoethnography came from listening to Dr. Alex RedCorn describe his experiences as a member of the Osage Nation in the Fall of 2019. The research method seemed compelling but unrelated to anything I might do, I am a math teacher. Math teachers should research something that relates to their math classroom. And writing some fluffy dissertation with lots of emotions and descriptions is quite simply not in my repertoire of skills.

I also knew that one of my close friends, Shannon, was considering using autoethnography for her dissertation. Shannon is incredibly gifted with writing compelling and heartwarming stories. Opposite to my typical, succinct, academic writing. Our collaboration in the Spring of 2020 to create a new graduate class offering held that unique balance between her *warmth and creativity, and my hyper analytical approach to academics. Once again, it was a unique approach that seemed interesting, but just not for me.*

The growth of autoethnography has roused criticism from some researchers. Douglas and Carless (2013) provide two major examples of this criticism: (1) the relevance of the issues chosen by autoethnographers, and (2) the growth of evocative autoethnography in that it does not advance theoretical knowledge. Delamont (2007) provides six arguments against autoethnography, four of which are relevant to this study: (1) It cannot fight familiarity, (2) It cannot be published ethically, (3) It is experiential not analytic, and (4) It focuses on the wrong side of the power divide" (Conclusions section). Fighting familiarity is best done with contextualized writings which serve a clear theoretical purpose. This critique shows the differing epistemology between autoethnographers and their critics. Autoethnographers claim their work "represents understandings and insights captured at one point (or more) in temporal and sociocultural contexts" (Anderson & Glass-Coffin, 2013, p. 78) rather than any final word on any topic. The ethics of autoethnography will be addressed in the section entitled "Validity, Credibility, and Rigor." The third critique will be addressed in the section that follows entitled Analytic Autoethnography. The fourth critique is compelling when the research is done only in the context of the experiences of the powerful; however, an autoethnography done inside a classroom purposefully intended to develop more student agency is anything but on the wrong side of the power divide. Research done in classrooms for the purpose of improving the learning environment is far from the critique that autoethnographers only document their experiences as tenured professors in so-called ivory towers.

Analytic Autoethnography

The idea that autoethnography is not for me held, even after Dr. Goodson recommended it. Autoethnography is evocative, what I like to call fluffy. It is writing that bursts with creativity and unique approaches to highlight personal experiences that hold truths that need to be heard. And then I found Analytic Autoethnography. That seemed to fit.

Delamont (2007) critiques autoethnography for lack of theoretical purpose and analytical power. Leon Anderson (2006) did much the same in calling for a greater emphasis on *analytic* autoethnography, describing its features as "research in which the researcher is (1) a full member in the research group or setting, (2) visible as such a member in published texts, and (3) committed to developing theoretical understandings of broader social phenomena" (p. 373). Further, Anderson describes analytic autoethnography as aligning with traditional symbolic interactionist epistemology, in that it is aligned with the belief that individuals construct meaning of the world around them through the meanings they imbue to the objects and people they interact with (e.g., Benzies & Allen, 2001). The intellectual genealogy of analytic autoethnography comes from both analytic ethnography and autoethnography (Anderson, 2006).

Anderson (2006) identifies five key features of analytic autoethnography. The first, complete member researcher status has two possibilities, an opportunistic researcher or a convert researcher. Opportunistic researchers are researchers who investigate contexts they are already living in whereas convert researchers "begin with a purely data-oriented research interest in the setting but become converted to complete immersion and membership during the course of the research" (p. 379). In this study I was an opportunistic autoethnographer as I was in my sixth year of teaching at the research site. The second key feature, analytic reflexivity, "entails self-conscious introspection guided by a desire to better understand both self and others through examining one's actions and perceptions in reference to and dialogue with those of others" (p. 382). This reflexivity will be more detailed in the section entitled "Validity, Credibility, and Rigor." The third key feature, the researcher's narrative visibility, informs the storytelling process that makes the foundation of the narratives in chapter four. The fourth key feature, interaction with informants informs the data collection process. Informants in this study included any person I interacted with who provided any sort of insight into norms, productive struggle, or any other relevant idea. Interaction with informants is addressed in the section entitled "Data Collection." The fifth key feature, commitment to theoretical analysis, is the most important feature of analytic autoethnography for this study's contribution to theoretical knowledge. This means that not only do narratives have to provide thick descriptions, but they must also have theoretical explanatory power (Winkler, 2018). More specifically, these narratives must demonstrate "develop theoretical understandings of broader social phenomena" (Anderson, 2006, p. 373), whether that be regarding productive struggle or the distinct types of norms that support the facilitation of struggle. This key feature is the focus of my conclusions—how do my findings connect with what is already known about productive struggle and about social, sociomathematical norms, and classroom mathematical practices? Further, how do these findings contribute to further knowledge about the intersection of norms and the facilitation of productive struggle?

In line with traditional expectations for dissertation methodologies (Throne, 2019) and for analytic autoethnographies intended for publication (Anderson & Glass-Coffin, 2013), the following sections describe the implementation and explicit methodology of this study not only for repeatability but also describe "the ethnographer's path" (Lofland, 1995, p. 49).

Research Design and Methods

"Perhaps methodological innovation is a hallmark of the approach and a requirement in every autoethnographic study" (Douglas & Carless, 2013, p. 103).

There is no one set way to embark upon implementing an autoethnography. Chang (2013) notes it as an intensely personal and social process— a process influenced not only by personal preference but also by informants around the autoethnographer. Anderson and Glass-Coffin (2013) describe autoethnographers as "eclectic bricoleurs" in that "not only do various autoethnographic scholars collect and interpret their "data" in different ways, but even individually they often improvise and experiment, changing their methods and ways of interpreting their data as they go" (p. 64). With this methodological openness in mind, the following sections describe what informed my establishment norms in my classroom, the data collection process, data analysis, data trustworthiness and ethics, storytelling, and a summary to conclude the chapter.

Establishment of Norms

In previous years of teaching, norm establishment in my classroom began with establishing classroom expectations and procedures, and following that, occurring on the sly. Opportunistic moments in teaching to encourage certain behaviors or reestablish expectations that imply the norms I wish to establish—much as the narrative at the beginning of this chapter illustrated. Mr. Boomer's expectations and classroom procedures coming together to gradually establish the norms of collaboration and using resources to help create the belief that students could co-construct knowledge with each other with appropriate teacher facilitation.

However, in the process of reviewing literature on norms to write my literature review, it turns out that explicit negotiation of norms is by far the best practice for the initial establishment *of norms. The realization that norms should be explicitly negotiated coincided with recommendations from a curriculum training I attended. Even knowing what is suggested as best practice, I still was reluctant to try this in my classroom. It is something I have never done before. Does that mean that I am a bad teacher? What does it mean when a highly educated teacher is reluctant to change their practice when reviewing research? How am I supposed to convince other teachers in the future to adopt professional development?*

The fall semester of the 2021-2022 school year, I piloted the *Illustrative Mathematics* curriculum for both algebra 1 and geometry (Illustrative Mathematics, 2019). When preparing to implement this curriculum, the training facilitators provided access to a bevy of resources including a process to build a mathematical community inside a classroom by co-constructing norms with students (Gray et al., 2018). This process details the first days of class with an explicit focus on students thinking about mathematical thinking and about the actions that support mathematical thinking. I used these explicitly co-constructed norms as a guide to help shape student behavior and thinking in my classroom. See Appendix A for the suggested process. Polsgrove and colleagues (2019) documented this process with middle school students who were not acclimated to working together to learn mathematics.

As this process was intended for middle school students, I anticipated some reticence to engage in the construction of norms. However, when I have approached the co-construction of norms in the previous years to help support Math Lab students, a class that afforded extra time and support to assist student learning, both freshmen and sophomores have positively responded. If lower attaining students can positively respond to co-constructed norms, there's reason to believe that grade level and higher attainings students can also positively respond.

Ray-Riek (2019) described how the co-construction of norms fits within *Illustrative Mathematics'* (2019) philosophy of problem-based learning as well as student and teacher actions that help to build norms and change a classroom culture. These co-constructed norms were assisted by the instructional routines that *Illustrative Mathematics* (2019) includes throughout their curriculum that value student thinking and engagement rather than centering knowledge construction around the teacher. Two of the most common instructional routines in the *Illustrative Mathematics* (2019) curriculum are "Notice and Wonder" (Rumack and Huinker, 2019) and "Which one doesn't belong?" (Danielson, 2016). Both routines supported openness and creativity in mathematics that helped establish the sociomathematical norm that at the onset of the problem-solving process there is no one right answer, instead there is engaging with and discussing student thinking to understand the situation at hand.

Data Collection

Despite the methodological openness and bricoleur nature of autoethnography leading to new ways of collecting data, the types of data collected are standard in some cases and more unique to autoethnography in others. Throne (2019) describes data collection for doctoral studies as "presented in advance. . . as rigorous, systematic, and often meet institutional requirements for appropriate data collection for the research focus and the discipline" (p. 14). Autoethnographic data frequently generates from observations, interviews, recordings, as well as journals, diaries, and fieldnotes (Anderson & Glass-Coffin, 2013; Chang 2013; Throne, 2019). Implementing an autoethnography as a teacher studying his classroom is particularly valuable according to Thorne (2019) who notes it is "ideal to gather data from everyday experiences" in autoethnography (p. 14).

This study generated most of its data from creating a reflexive journal. The data collection for this reflexive journal will be recorded from self-interviews and musings using Zoom, an online video conferencing software, to record and transcribe the entry. This reflexive video journal not only had reflections on the day itself, but also included reflexive engagement of the data as “contemporaneously written fieldnotes may at times involve less detailed reflexivity in the moment, followed by more reflexive engagement at a later point in time” (Anderson & Glass-Coffin, 2013, p. 67). Lesson plans and curricular materials additionally informed this reflexive journal. Further data came from discussions from ancillary informants: my students, my teacher colleagues, and even from discussing elements of my study with my wife, Kirsten, a middle school math teacher. Reflections on these discussions were recorded via Zoom to be documented in my reflexive journal. Griffin (2018) described the utility and value of video recordings for autoethnographies and further, the use of video diaries much like the reflexive journal for this study.

Less traditional qualitative data also informs autoethnographies, such as memory work (Chang, 2013; Giorgio, 2013) and even the writing and storytelling process (Denshire, 2014; Throne, 2019). Memory work in autoethnography can involve self-interviews, where the researcher engages with their “past and present selves” to build new understandings about the topic of study, in this case, my growth as a teacher negotiating norms in my classroom, and my facilitation of struggle (Anderson & Glass-Coffin, 2013, p. 69; Chang 2013). The data generation from the writing and storytelling process is part of the messy nature of autoethnographic work: data collection and analysis often happen concurrently, and the process of writing up results and narratives often spurs memory of related experiences thus creating more data.

Data Analysis

Analytic autoethnographers borrow from analytic ethnography in their analysis process (Anderson, 2006; Lofland, 1995). Lofland (1995) describes

> The central process of grounded theory—of emergent analysis—is nevertheless an integral element of analytic ethnography. Its key features are the gradual accumulation of data through witnessings observation and the slow inductive analysis of these data. Even though much has been written on the mechanical aspects of doing this, the process has also rested on the sensitivities and intuition of the researcher. As such, emergent analysis has been—historically—also very much a creative act. (p. 47)

Autoethnography in a similar vein has a creative act of analysis whereby the steps of the research process are not so delineated as more traditional qualitative work (Chang, 2013). A prime example comes from the coding of data where coding and emergent analysis begins at the onset of the study (Chang, 2013; Ellis et al., 2011; Lofland, 1995; Throne, 2019). Coding takes place just as in a traditional qualitative study where a researcher memo or takes notes on common topics, elements emergent from the data. From these codes emerge categories, combinations of codes, and from categories come themes which demonstrate critical relationships within the data that often comprise the central findings of qualitative work (e.g., Saldana, 2009; Maxwell, 2010). However, autoethnographers are warned to not rush to create categories: "At earlier stages of coding, researchers are advised not to impose several external categories too soon so as to avoid losing sight of meanings emerging from raw data" (Chang, 2013, p. 116).

This connection with raw data helps to create meaning from the data "whereby layered meaning is uncovered and discoverable through the cyclical, iterative processes used for analyses" (Throne, 2019, p. 124). Chang (2013) describes this cyclical process in depth, describing how meaning making goes beyond the coding process:

> This analytical process of dissecting and grouping is a way to make meaning but is not meaning-making per se. Meaning-making is like holding chunks of data against a backdrop and understanding what the data mean in relation to other segmented data and within the broader context. Meaning making also requires determining how the data are connected to the realities of other people with similar experiences and to existing research. To make meaning of seemingly unconnected data, researchers need to transcend minute details and see a big picture, hear an overtone, or imagine a smell that is not buried in data. By reading others' work, reviewing data over and over, using intuition to grab something out of thin air, and imagining what they hope for researchers will reach "aha" moments— moments when they begin to see contours of data that were not there previously and connections among fragments that they had not noticed before. (p. 116)

Existing theory in the teaching practice of supporting productive student struggle became a critical element of emergent analysis (Hiebert & Grouws, 2007; NCTM, 2014; Warshauer, 2015a), in addition to the diverse types of norms in a mathematics classroom (e.g.,Yackel & Cobb, 1996). Existing theoretical understanding of supporting productive struggle as well as norms in math classroom microcultures was the emphasis in Chapter 2. Considering existing theoretical understanding in emergent analysis aligns with the commitment to theoretical analysis, a key feature of analytic autoethnography (e.g., Anderson 2006).

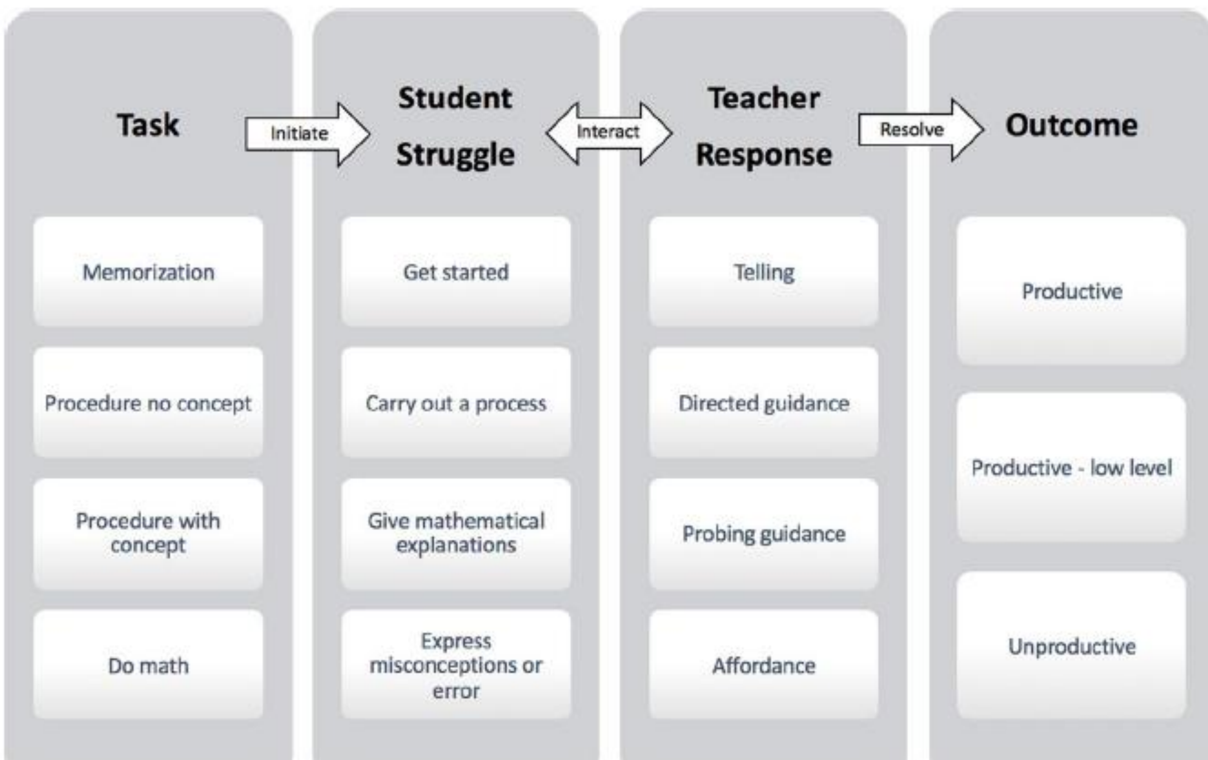
One of the critical frameworks for engaging in productive struggle research is Warshauer's (2015) productive struggle framework. This framework was essential in coding an episode of student struggle from video observations and pictures of student-task engagement. Warshauer and colleagues (2021) describe coding instances of student struggle:

> The resolution of the student struggle involves going back to the originating mathematical task, assessing whether the struggle was addressed and/or built on the student's thinking, and identifying if there was progress made toward the task goals without lowering the task's cognitive demand. (p. 94)

Figure 3.1 below illustrates the interactions between task, student struggle, teacher response, and outcome.

Figure 3.1

Productive Struggle Framework



Note. Reproduced with permission from "Developing Prospective Teachers' Noticings and Notions of Productive Struggle with Video Analysis in a Mathematics Content Course," by H. K.

Warshauer, C. Starkey, C. A. Herrara, and S. Smith, 2021, *Journal of Mathematics Teacher Education*, *24*, p. 94 (https://doi.org/10.1007/s10857-019-09451-2). Copyright 2021 by Springer Nature.

The challenge of engaging with this meaning making process was facilitated by a tool I created for data management. To address the challenge of organizing the vast amount of data generated from the reflexive journal, videoed lessons, and discussions with ancillary informants, I created a data matrix within Google Sheets that hyperlinked documents with raw data and memos and records the codes within that raw data in the cells beside the hyperlink. See Figure 3.2 below for a partial example. In this way, codes that recurred became apparent, and I had easy access to the raw data which assisted me in engaging in the cyclical and iterative process of meaning making. This data matrix ensured that I could see when I reached data saturation to begin the findings writing process (Chapters 4-7). More specifically, the data matrix illustrated when no new codes are emerging, when I observed strong categories that comprised themes in the metadata.

Figure 3.2

Data Matrix

	A	B	C	D	E	F
1		**Date**	Establishing norms	Meaning Making	Organizing data	Methodology
2	**Codes**	6/24/21	x	x	x	x

Trustworthiness

Qualitative rigor typically has adopted Lincoln and Guba's (1985) trustworthiness criteria of credibility, transferability, dependability, and confirmability. Credibility is of relevance to this study. Traditionally it is seen as the believability of results that follows from practices such as triangulation, member checking, negative case analysis, or use of a reflexive journal (J. Johnson, personal communication, September 30, 2020; Lincoln & Guba, 1985). In autoethnography, credibility "may be assured as data are checked and re-checked throughout the data collection and analysis process to assess repeatability and as one measure for overall data trustworthiness" (Throne, 2019, p. 105). Tracy (2010) built upon this general set of criteria to attempt to provide eight universal measures of qualitative trustworthiness: worthy topic, rich rigour, sincerity, credibility, resonance, significant contribution, ethical, and meaningful coherence.

Describing techniques to ensure rigor in an analytic autoethnography starts from epistemological view. Ethnographers disregard any form of final word or analysis— truth is never complete. Accordingly, analytic ethnography does not hold with more traditional notions of validity and reliability:

> Assessing analytic trueness has been accorded much more attention than assessing factual trueness. Tending to eschew the narrow, positivist conceptions of "reliability" and "validity," attention has turned, instead to criteria of a personal testimony character in the absence of any more direct and straightforward way to assess trueness (Lofland, 1995, p. 48).

In describing personal testimony criteria, Lofland turns to Sanjek's (1990) criteria of validity for ethnography: "theoretical candor," "the ethnographer's path," and "fieldnote evidence" (Lofland, 1995, p. 49). In ethnography, theoretical candor implies a clear description of how an analysis developed over time, a "candid exposition of when and why" (Sanjek, 1990, p. 396). The ethnographer's path in a manuscript provides chronological evidence of the interactions between the ethnographer and their informants. Fieldnote evidence refers to clear reports of data collection and presentation in the ethnographic report. An ethnographer then attempts to reach analytic consistency within their data and reporting procedures rather than any objective trueness.

Further, autoethnographers recognize that all analyses are socially constructed (Mingé, 2013; Tomaselli et al., 2013). From this epistemological standpoint, and from a literature review and results of a survey questionnaire of autoethnographers, Le Roux (2016a) identified five criteria for autoethnographic rigor and trustworthiness:

- *Subjectivity*: The self is primarily visible in the research. The researcher re-enacts or re-tells a noteworthy or critical personal relational or institutional experience – generally in search of self-understanding. The researcher is self-consciously involved in the construction of the narrative which constitutes the research. (p. 10)

Subjectivity in my study means that my voice, my opinions, thoughts, feelings, and experiences are central and clear in my narratives. My narratives are not just a story, they are my story.

- *Self-reflexivity*: There is evidence of the researcher's intense awareness of his or her role in and relationship to the research which is situated within a historical and cultural context. Reflexivity points to self-awareness, self-exposure, and self-conscious introspection. (p. 10)

Self-reflexivity is a challenge to communicate through narratives—describing my thought process and the connections I make to math education literature must be clear from narrative to theoretical analysis.

- *Resonance*: Resonance requires that the audience be able to enter, engage with, experience, or connect with the writer's story on an intellectual and emotional level. There is a sense of commonality between the researcher and the audience, an intertwining of lives. (p. 10)

Resonance is judged by each reader. Does my classroom come to life? Can you imagine what I experience in teaching? My frustrations, joys, and contentments?

- *Credibility*: There should be evidence of verisimilitude, plausibility, and trustworthiness in the research. The research process and reporting should be permeated by honesty. (p.10)

Credibility in part is addressed in the clear methodological descriptions provided in this chapter. Otherwise, it comes through in the clear reporting of my analysis and findings.

- *Contribution*: The study should extend knowledge, generate ongoing research, liberate, empower, improve practice, or contribute to social change. Autoethnography teaches, informs, and inspires. (p. 10)

Contribution is the entire goal of analytic autoethnography. In the case of this study, have I documented, described, and developed a clear view of the establishment of norms that assist the facilitation of productive student struggle? Have I connected these ideas to existing literature and contributed to the body of knowledge?

Reflexivity, Subjectivity, and Ethics

There are multiple aspects of reflexivity for autoethnographers. Ruby (1980) characterizes researcher reflexivity as when they: "systematically and rigorously reveal their methodology and themselves as the instrument of data generation" (p. 153, as cited in Tomaselli et al., 2013). This view of reflexivity then implies a clear description of methodology with the researcher's presence at the center much as the narratives of autoethnographers center around their experience. A second common view of reflexivity is the "strategy of reflexive inquiry involves describing and reflecting on oneself and experience at different points in time" (Anderson & Glass-Coffin, 2013, p. 73). This aspect of reflexivity contributes to data generation and analysis for autoethnographers. Further, it was implemented in this study by comparing reflections captured following an instructional episode, and later analyzing the instructional episode captured on video. This increased trustworthiness of findings, and crucially impacted my views and trust in my initial reflective analysis of teaching. Related but different to researcher reflexivity is researcher subjectivity.

Throne (2019) characterizes subjectivity in describing intersectional aspects of a researcher's background: family/tribe status, occupation/profession/economics, religion/spirituality/beliefs, ethics/esthetics/creativity, age/gender/ability/sexuality, language/heritage/culture/geography (p. 30 from Figure 1). However, Gannon (2013) notes an increased importance of disclosing researcher relationships as an aspect of subjectivity for autoethnography:

> We bring all the relationships we have in the world onto the pages and, with them, come unconscious thoughts and desires that are difficult or impossible to articulate. There is no neutral space from which we write, or from which we read. (p. 229)

This is an especially important part of subjectivity as I have established friendly and working relationships with the ancillary informants for this study. Protecting these relationships was not only morally right, but also ethically right as a researcher.

Tullis (2013) has established the following ethical guidelines for autoethnography:

1. Do no harm to self and others.
2. Consult your IRB (Institutional Review Board).
3. Get informed consent.
4. Practice process consent and explore the ethics of consequence.
5. Do a member check.

6. Do not present publicly or publish anything you would not show the people mentioned in the text.
7. Do not underestimate the afterlife of a published narrative. (pp. 256-257)

Of particular relevance to this study are guidelines one, three, four, and five. Doing no harm implies the protection of identity and putting in measures of confidentiality by keeping secure records, de-identifying data, and creating pseudonyms. Informed consent, process consent, and member checks involved keeping ancillary informants aware of the research process, beginning, during data collection, and during the writing process.

Positionality

The central complicating factor of an autoethnography is the researcher's simultaneous position as research subject, observer, and analyst. This is where subjectivity and self-reflexivity come to true importance in this study as supports for qualitative and autoethnographic trustworthiness. I must not only be central to the narrative in construction, but I must also further demonstrate clear consideration of the development of my thought process. That is the true challenge of lending credibility and trustworthiness to research centered in personal narrative. My reflexive journal plays a key role in being able to position myself as a teacher, observer, and researcher. Reflections or anticipations on a day of teaching is clear enough— I am positioned as a teacher. However, collecting and analyzing video evidence of my teaching moves my position to that of an observer-researcher. But how can I ensure that I am not simply seeing what I wish to see?

I am reminded of negative case analysis as a method for demonstrating credibility and trustworthiness (Lincoln & Guba, 1985). Implementing negative case analysis in this study would then involve searching for evidence that disproves any tentative codes, categories, or *themes. Reflections on larger periods of time demonstrate this negative case analysis and self-reflexivity as well— as I reflect on the week or month, have I seen these phenomena emerge in all my classes consistently over time? Is there a class that does not fit the mold? Or alternatively, is the emergence of themes across classes consistent enough to demonstrate triangulation within my data? These are the considerations I must take to move my position toward my research into that of an analyst.*

While I embrace the fact that I know my intent as a teacher, I must also gauge that intent with what appears to be happening in the classroom as an observer. Balancing the two, even going above the two in perspective is the challenge as an analyst embarking upon an analytic autoethnography. Accurately reporting the development in analysis demonstrates both self-reflexivity and credibility as characterized by the criteria for trustworthiness established by Le Roux (2016a). To balance these three perspectives, I need to keep a clear view as to which role I am acting as a researcher when collecting, coding, and analyzing data. It is this blended role that will assist me in clearly reporting the research process and the development of analysis in the construction of my teaching narratives.

Storytelling

Storytelling is more than just the process of autoethnography, it is a data generation and even analysis process. The challenge of writing autoethnography is significant in its aims: "Autoethnographers strive to write accessible prose that is read by a general audience, but they also try to construct the work so that it steps into the flow of discussion around a topic of interest to researchers" (Jones et al., 2013, p. 22). Further, the style of writing must itself make the argument *and* demonstrate theoretical understanding of the research phenomena to fulfill the aims of analytic autoethnography (Goodall, 2013).

Chang (2008) identified four types of autoethnographic writings: “imaginative-creative,” “confessional-emotive,” “descriptive-realist,” and “analytical-interpretive” (pp. 141-148). This study will follow the “analytical-interpretive” style:

> *Analytical-interpretive* writing tends to engage a more typical academic discourse common to social science research reports and to incorporate theoretical and conceptual literature sources. In this style, narration tends to support researchers’ socio-cultural analyses and interpretation. (Chang, 2013, p. 119)

To take on such a large task, I borrowed Sanjek’s (1990) concept of “the ethnographer’s path,” or in my case “the teacher’s path,” as a metaphor to create a sense of continuity and chronology in my narrative. I aimed to establish my path, as it was, to establish and negotiate norms in my classroom. Further, I had to make the connection between the norms in my classroom and the struggle, productive or not, that I observed my students engage in. If that were the only goal, perhaps analytic autoethnography would not be quite so challenging. However, these instructional episodes that I illustrate also serve a theoretical purpose, with clear connections between narrative, existing theory, and serve the purpose of building upon existing theory. To make the connection between theory and instructional episodes clear, I borrowed from Ronai’s (1995) in using layered accounts, which are in *italics,* accounts which are preceded and followed by more traditional qualitative analysis (Ellis et al., 2011).

Methodology Summary

This study is an analytic autoethnography setting out to explore the question of: In what ways does a teacher negotiate the establishment of classroom norms to facilitate productive struggle? I followed an established protocol to negotiate norms in my classroom (Gray et al., 2018). Collected data includes a reflexive journal, primarily consisting of reflections upon teaching and student learning, as well as discussions with ancillary informants recorded post hoc in the reflexive journal.

To organize this vast amount of data, a data matrix hyperlinked to transcripts of reflexive journal entires and held memos and codes which assisted me in observing code consistency or change overtime in the metadata. Analysis began as soon as data was collected and as I engaged in the recursive process of reflexively engaging in then current experiences, and in doing memory work and analysis of previous experiences. I began creating categories and themes as I started to see consistency in the codes and a lack of new or changing elements which indicated data saturation.

With preliminary themes identified, I began to write first person narratives that illustrate the theme. These narratives protected ancillary informants with appropriate pseudonyms generated for both students and fellow teachers. When directly using contributions from informants, I checked my interpretation of the events with them, and allowed them to read a preliminary draft of the narrative to practice process consent. Each narrative has a purpose by explicit connection to theory, whether that be regarding norms in a mathematics classroom, or productive student struggle, or emergent theory that connects the two.

Prelude to Chapters 4-7

The journey of the fall semester for the 2021-2022 school year is incredibly hard to capture. Students in and out of quarantine—somehow catching them back up when they return. Adopting a new curriculum for both of my preps, algebra 1 and geometry. Continuing the process of engaging in school redesign meant I felt pulled in multiple directions. Trying to square the circle of reimagining how math classes can respond to changing student needs while still fulfilling standards. Further, I found myself questioning whether a new curriculum that aligns with NCTM reform efforts even needs reimagining? On a more personal level, I found out that I am going to be a father in April, which makes the timeline of making ***sense*** *of all of this so much more urgent. Suddenly, I needed to deliver this intellectual baby before the real one arrives!*

The following four chapters depart stylistically from previous chapters. Telling the story of what happened and providing analysis of what happens occurs so fluidly that the storytelling flows with the analysis. Instructional episodes in italics demarcate the layered account (Ronai, 1995), episodes which were reconstructed from memory and reflexive journal entries. In addition to instructional episodes, other storytelling appears much like a journal entry of metacognition.

The continual renegotiation and reestablishment of norms following the “Building a Mathematics Learning Community Plan” (aka the Plan)—a process that is described in sections entitled “Initiating the Plan,” “Implementing the Plan,” and “Following Through with the Plan”—represents the primary finding of this study. The following themes contribute to this overall idea. Within “Implementing the Plan” and “Don’t let the media become the message,” I describe a theme that involves how I self-sabotaged teaching this new curriculum with several short instructional episodes. These supplementary themes are described in Chapter 4. Following “the Plan,” a second major theme emerged: the importance of several negotiated social norms to the facilitation of productive struggle—the focus of Chapter 5. I discussed this second theme with multiple instructional episodes in the section entitled “Social Norms.” The expectation that students explain and justify their work led to the development of sociomathematical norms for my students. The development of sociomathematical norms in my classroom led to the realization of sociomathematical norms’ influence on the facilitation of productive struggle, a third major theme and the topic of Chapter 6. The fourth major theme, the influence of classroom mathematical practices on productive struggle is investigated in Chapter 7.

To answer the main research question for this study, “*in what ways does a teacher negotiate the establishment of classroom norms in order to facilitate productive struggle?,”* I recorded reflexive journal entries and coded and analyzed them. Throughout this iterative process, I chose several episodes of instruction that exemplify emergent themes. Autoethnographies demonstrate findings by storytelling as opposed to traditional qualitative work which describes themes from discussion of coding and analysis. Organizing storytelling and thick descriptions by way of the microculture theoretical framework helps to fulfill the aims of analytic autoethnography, contributing to theoretical understanding (Anderson, 2006). I use Cobb and Yackel’s (1996) microculture framework of social norms, sociomathematical norms, and classroom mathematical practices as opposed to Warshauer’s (2015a) productive struggle framework as this study’s purpose is to investigate the influence of norms on productive struggle. Organizing findings by norms instead of by productive struggle better demonstrates the influence of norms on productive struggle as norms influence the phenomena of productive struggle, not vice versa.

Throughout each of these chapters, I indicate teaching actions and student behaviors that I observed to answer the main research question for this study: In what ways does a teacher negotiate the establishment of classroom norms to facilitate productive struggle? I delve deeper into Chapter 9 in answering the supporting research questions as a part of the discussion and conclusions for this study.

Chapter 4 -

Theme 1: Continual Renegotiation and Reestablishment of Norms

Building the Mathematical Learning Community

Algebra 1

My beginning lesson for algebra 1 dealt with statistics: introducing gathering data and investigating types of statistical questions. The positivity of my freshman classes pleasantly surprised me. It seemed like last year my freshman classes came in already disaffected with school— as if they had 'senioritis' to begin their freshman year. I observed a huge reluctance to engage or to even act like they cared about their learning. Teachers last year at my school hypothesized that students had such a bad experience with 'remote learning' during the 'continuous learning' period of the 2020 spring semester that their attitude toward school soured. Hopefully not permanently. However, this new group of freshmen seemed eager to begin their high school career and much more open to learning math. This generally positive attitude has continued throughout the semester. Further, the 'doing math' actions they identified as part of the "Building a Mathematical Classroom Community Plan" reflect this positivity (Gray et al., 2018). At the end of each lesson each day, I asked my students, "What does it look and sound like to do math together as a mathematical community?" First, students discussed their ideas as a small group. Following discussion, students reported their ideas to the whole class. I recorded student ideas on a Google Slide projected on the whiteboard and checked with students to make sure I represented what they thought accurately. They produced the ideas shown below (see Table 4.1). I revisited these ideas with each class of students at the end of the next few class *periods. My students' conceptions of math rooted in discussion, argumentation, and collaboration impressed me.*

Table 4.1

Algebra 1 Doing Math Actions

Block 1 Doing Math Actions
• Debating, arguing • Discussing strategies and ideas • Loud • Communication • Being engaged/engaged conversation
Block 4 Doing Math Actions
• Loud • Sounds like numbers (talking about math) • Getting along and putting in all your thoughts into it • Groups working together • Communication is key • A lot of talking • Working together as a group • People talking over each other • Arguing/debating • PENCILS ON PAPER
Block 8 Doing Math Actions
• Chaotic • Agreeing and disagreeing—making a solution out of that • Putting your heads together/working together • Each member has a different opinion • Discussing math • Frustrating/Stressful • Fun Fresh and Funky?

Note. These 'doing math' actions were developed by following the "Building a Mathematical Classroom Community Plan" (Gray et al., 2018).

Geometry

Geometry began with an in-depth set of investigations into constructions—a focus on congruent circles and reasons why we can construct regular polygons from circles. My sophomore math classes in the past have developed into positive learning environments. Geometry and advanced tend to be placed opposite of each other on the school line schedule—this meant that I typically have had a larger concentration of better readers in these sophomore-level math classes. In my experience, better reading skills have led to more successful reasoning in mathematics. This increased success usually means that these classes have positive dispositions toward mathematics. It came as a surprise this year to find one of my sophomore math classes with a seemingly negative disposition. My students in block 3 seemed to already be anxious towards learning math to begin the year! Their answers (Table 4.2) to "What does it look and sound like to do math together as a mathematical community?" compared to my other four classes seemed different.

Of the ten actions identified by block 3, three actions oriented positively toward mathematics. One action framed negatively by my students represents an essential part of learning mathematics: Getting frustrated. It surprised me that block 3 did not mention any sort of discussion in their actions, whereas each of my other classes all held it central to their ideas of the actions of "doing math". It puzzled me as to what makes this one group of students so different. A unique group of students made up block 3 in that semester. I would describe their behavior as both gregarious and ornery in equal measure. Last year, the successful implementation of many lessons hinged on using these sophomores (then freshmen) positively for their energy. Upon reflection it seemed that block 3, more than my other classes, might uniquely inform how I can negotiate an environment of productive struggle. Two questions came to mind at the start of the fall semester: How can I make this a positive learning experience for all my students? And how can I take these feelings of frustration, helplessness, and anxiety and help my students grow into having positive mathematical experiences?

Table 4.2

Geometry Doing Math Actions

Block 3 Doing Math Actions
• Groaning and asking the teacher for help—sometimes happy • Crying—copying off one person (asking others for help) • Parents frustrated that you do not do it the "right way" • Working with other people—can't do it alone • Mental health awareness • Collaborating—working off one another's ideas • Getting frustrated • Being off task • Problem solving • Everyone contributing
Block 7 Doing Math Actions
• Teaching/Helping each other with what we know • Collaboration: working together to solve problems • Disagreements • Sounding smart • Discussing your ideas • Being able to debate your ideas

Note. These 'doing math' actions were developed by following the "Building a Mathematical Classroom Community Plan" (Gray et al., 2018).

Looking at both block 3 and 7 for productive dispositions, I observed them collaborating and peer tutoring one another through mathematics—in line with the same actions identified by my freshmen algebra 1 students. In geometry and algebra 1 alike, students engaged in an environment rooted in discussion, argumentation, and collaboration. I found myself thinking, what else did my classroom need to create an environment to support productive struggle?

Implementing the Plan

At this point, I questioned myself in two key ways. First, I questioned my implementation of the community building plan. After writing down the student-generated ‘doing math’ actions, I revisited those actions in subsequent days with each block. I wanted to know if my students recognized any missing actions. I had students write down their thoughts individually and then debriefed with their groups. Each class had the same reaction to this concluding activity: general confusion. I found myself puzzled as to why my students looked at me with confusion as they revisited their respective lists of ‘doing math’ actions. Looking back, the first two lessons held similar characteristics in what students engaged in for both algebra 1 and geometry. It might make sense then that my students did not see any difference in the actions they identified from ‘doing math’ those days.

I also questioned myself in my implementation of the new curriculum. This question occurred because of my experience with the previous curriculum. The previous curriculum, due to its dated nature (Core-Plus Mathematics Project, 2008) required extensive teacher modification to fit the current math standards (KSDE, 2017) as its writing occurred prior to the Common Core State Standards (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010a) which informed the writing of the current Kansas Math Standards (2017). While schools normally revisit curriculum every five years, funding issues meant that the school district delayed curriculum adoption until the infusion of funds from the CAREs act (Coronavirus Aid, Relief, and Economic Security Act, 2020). Because of the age of the curriculum, and the seeming change in students’ engagement in class, the math department at my school became experienced at modifying the curriculum to fit students' needs. It seemed as if each year students had more difficulty following the guided investigations of *Core-Plus Mathematics* that led them to discover different mathematical ideas. Whether that difficulty came from students struggling more with reading, or a change in student ability to sustain prolonged periods of concentration through our 95-minute blocks is not clear.

The curriculum change helped in both aspects with how *Illustrative Mathematics* (2019) chunked their lessons. While the authors designed the existing curriculum for 45-minute time periods, they also included recommendations for how to modify lessons for schools following block schedules. The authors' recommended inviting students into the mathematics lesson with a warm-up activity, to sequence together two consecutive lessons of activities, and to end each lesson with a cool-down activity designed to formatively assess student understanding. Activity lengths range between five and twenty minutes each. I consistently found through coding reflexive journal entries that the shorter activity length kept the class period moving and better fit student attention spans as the launch-explore-summarize (LES) instructional sequence appeared multiple times per block. The previous curriculum had one LES instructional sequence that occurred over the entire block time period, 95 minutes. Over the course of the fall semester, these shorter instructional sequences resulted in increased student engagement and learning. Coincidentally, shorter instructional sequences align with how we modified our previous curriculum from its original design.

The habit of modifying curriculum, unfortunately, is a hard habit to break. It goes directly against one of the recommendations made by the curriculum training I attended to implement *Illustrative Mathematics* (2019) with fidelity (Pesce, personal communication, June 2021). Not trusting the curriculum resulted in several lessons that fell metaphorically on their face! Once, it occurred when I uncharacteristically made the lesson far too teacher centered. This occasionally happens even though I know and believe wholeheartedly that the most meaningful learning occurs when students are at the center of doing and discussing mathematics.

Geometry: Don't let the media become the message

The second time I unwisely failed to trust the curriculum occurred when I found the Geogebra version of Illustrative Mathematics (2019) online when it came time to introduce digital tools to create constructions in geometry. The problem here came in that "the media became the message" as I have heard from one of my colleagues and mentors, *Dr. Allen Sylvester (originates with Marshall, 1964). Technology exists to supplement and enhance the curriculum, not to become the vehicle for it. Instead of exploring constructions, students focused too much on where to record their work on the website, the applet on the website crashing, and trying to learn how to navigate the constructions themselves with the digital tools. This focus became the message instead of the learning goal: exploring the use of perpendicular bisectors*.

Striking the balance between technology enhancing the curriculum rather than students only learning the use of technology became key in the following few days of instruction. Students seemed to focus better when they followed instructions in their workbooks exploring what constructions and eventually transformations to create. For many students, this seemed like the perfect balance. I observed some students disappointed by not using compasses to create physical constructions, and a few students becoming overwhelmed in having to manipulate internet applets while trying to learn mathematics. It is the students that gave an interesting window into facilitating struggle.

For these students, the media became the message. Navigating the Geogebra applet became the learning goal instead of exploring the use of perpendicular bisectors, and a few lessons later, exploring rigid transformations. At first, I performed the construction for them so they could reflect on what the question asked. Some might consider this removing struggle as I did the construction for them; however, I would claim that it removes struggle that is irrelevant to the learning goal. Struggle is only productive when it is relevant to the learning goal for the lesson. A few lessons later I anticipated this struggle occurring once again, and I instead had technology-savvy students work with the less savvy ones. Instead of having to make teacher responses to struggle, I observed collaboration and discussion between those students. I believe that anticipating struggle and making moves to make struggle relevant to the learning goal is potentially an under-explored avenue of productive struggle literature as a guiding post to inform my negotiation of an environment that facilitates productive struggle.

Following Through with the IM Community Building Plan

Geometry: Constructions, Transformations, and Symmetry

I fell behind on implementing Illustrative Mathematics' (Gray et al., 2019) suggested community-building protocol due to various issues that arose such as running out of time in class, the school day canceled from a broken air conditioner, and for one class, a series of odd days where the environment of the classroom went from positive to negative without an obvious cause. I asked each class "What norms, or expectations, were we mindful of as we did math together in our mathematical community?" The class that at the beginning of the year seemed very unsure in their collective mathematical dispositions, my block 3 geometry class, produced positive and productive norms:

- Show your work
- Attempt something before asking for help
- Be on task

- Work as a team/group
- Everyone contributes
- Always justify and explain your thinking

I found myself pleasantly surprised by these norms, but not completely taken aback. While block 3 originally recorded fairly negative dispositions towards mathematics at the beginning of the year, they began to come around in their dispositions toward geometry and producing these norms supported this hypothesis. I helped students shape the wording of norm number two: "Attempt something before asking for help." I remember one of my students raised his/her hands and said "Well, this sounds dumb, but don't you actually have to try?" And so, I helped them reframe that as the wording "attempt something before asking for help" has more positive implications as opposed to a negative view that would imply that a student was lazy by "not trying."

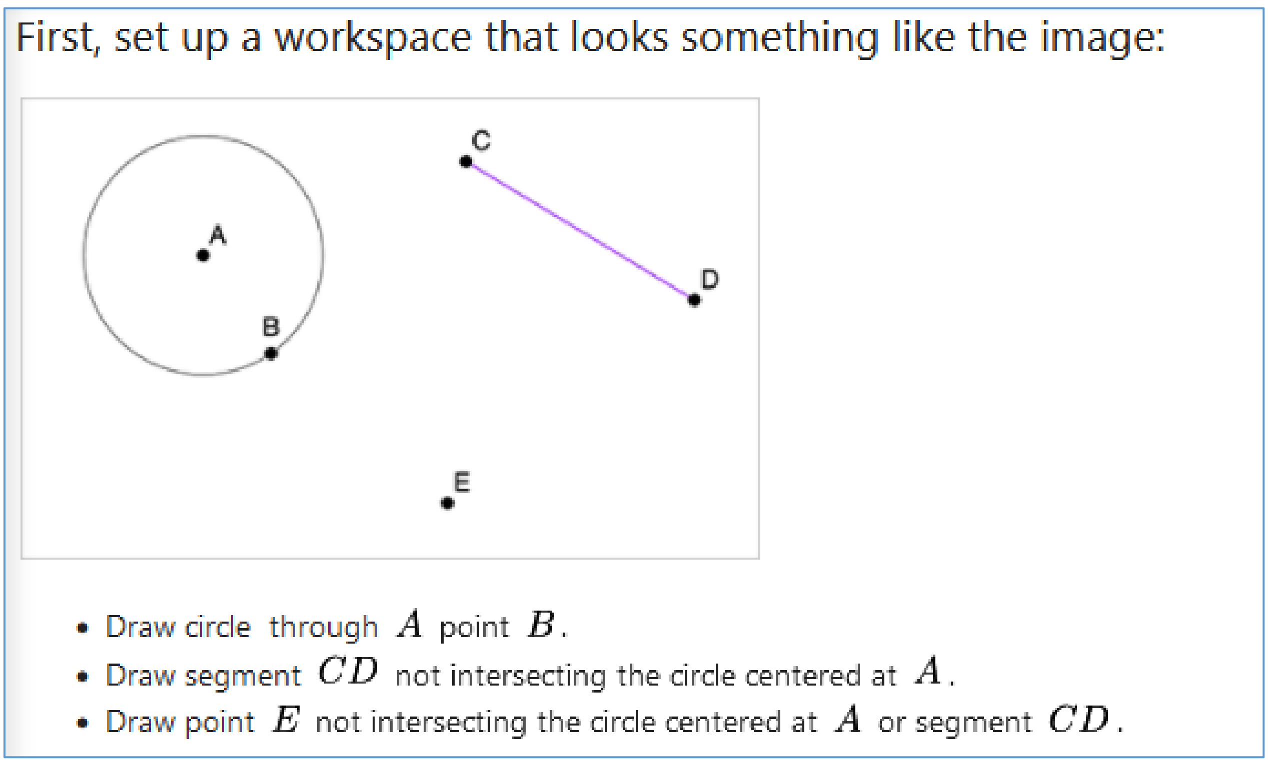
Digital Construction Tools

Following our first mid-unit assessment for geometry, I could tell students had started to begin thinking of constructions as a classroom mathematical practice and further developed reasoning based on those constructions. More specifically, students began to see the construction of congruent circles as the basis of reasoning for why we could construct regular polygons and even perpendicular and angle bisectors. An interaction with a student in block 3 pleasantly surprised me. According to Sophie, she consistently unproductively struggled in learning mathematics in previous years of school. I pushed students in her class when struggling to always describe the task situation, and further, to identify something that they know from that problem description to get them started in the problem-solving process. Sophie raised her hand and proceeded to describe to me the problem situation.

That day, I tried to introduce using digital construction tools so that students could see how they are alike and different from physical constructions. Digital construction tools allow students to create points, lines, and shapes for students to explore properties of shapes or parallel and perpendicular lines. In this task (Figure 4.1) Sophie struggled with creating a similar webpage on Geogebra based on a given picture, and then creating a circle with the exact same radius, alternatively centered at point D. I chose to follow the already created lesson on Geogebra, and unfortunately, one task asked students to use a tool that was not available on the linked applet. My failure in planning is not the point of this short story, however.

Figure 4.1

Geogebra: Introducing Digital Construction Tools



Note. This work was adapted by GeoGebra Classroom Activities from "Using Technology for Constructions" from IM Geometry © 2019 Illustrative Mathematics. Licensed under the Creative Commons Attribution 4.0 license.

When I asked her to do her best in making a circle the same size as A centered at E (without the compass tool available), she said, "But how am I supposed to know that it's the same size? I could with my compass (gesturing to her physical compass in hand), but there's no way to do that on the screen."

This interaction excited me for several reasons: one, she recognized the need to attend to precision (CCSS.MATH.PRACTICE.MP6); two, she wanted to base her reasoning and work off based on a classroom mathematical practice; and three, she had already sought assistance from her peers. I had evidence of her developing a more positive mathematical disposition, adherence to a social norm (asking her peers), and a classroom mathematical practice (of using compasses) to justify her reasoning (a sociomathematical norm). Further, this prime example of struggling to make sense of mathematics shows the interconnectivity between struggle and the math classroom microculture.

During these beginning weeks in block 3, I was intentional as I facilitated struggle throughout this time. I pushed for students to justify their reasoning and created opportunities to allow students to peer tutor one another through their struggles. I began seeing numerous instances of spontaneous peer tutoring—an advantage of having all my students on the same activity all at once in following the LES instructional sequence. Often this required some supervision as students did not initially know how to help guide their peers in reasoning as opposed to simply telling them what they should do. At the beginning when students asked, "What do I do now?" I repeatedly emphasized that they should "get up and go see if anyone needs help." Further descriptions of this peer tutoring and how I negotiated this norm are found in Chapter 5 in the "Peer Tutoring" section.

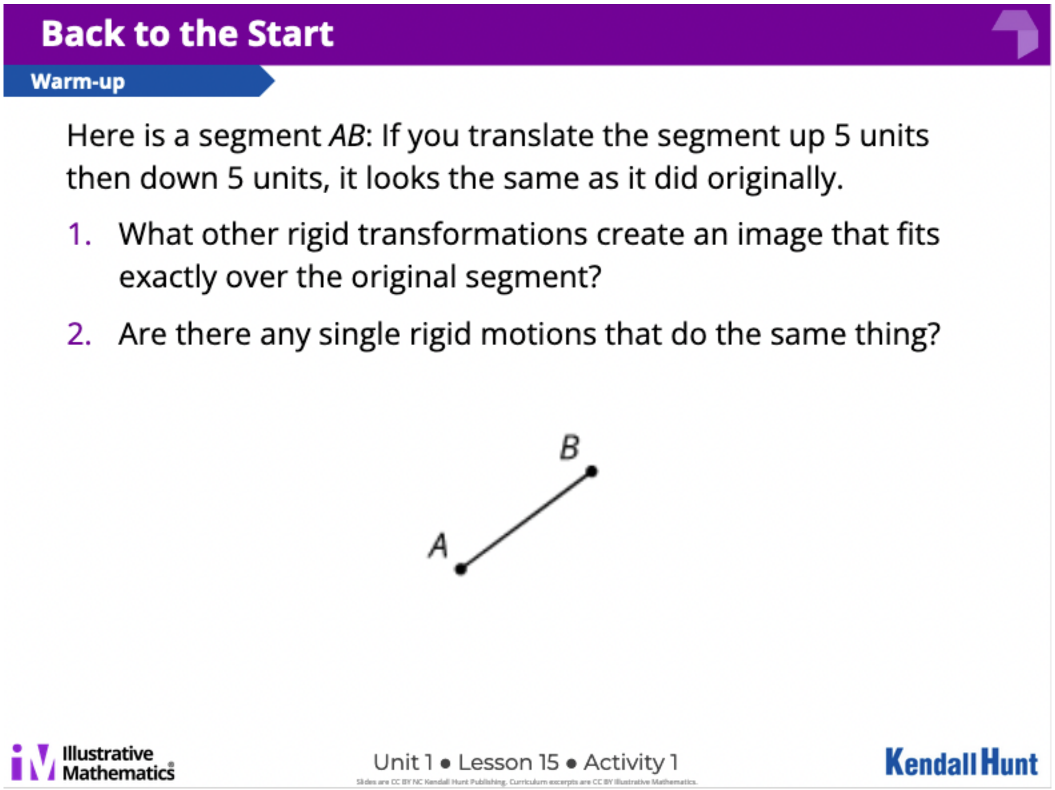
Lines of Symmetry and Rotational Symmetry

The fruits of these efforts came through in one block where we investigated lines of symmetry and rotational symmetry in given shapes. Up to this point, my geometry students had explored and formalized rigid transformations: translations, reflections, and rotations. In this lesson, students began making connections with how certain rigid transformations result in the exact same image, and more importantly, the emphasis that rigid transformations always result in congruent figures. To begin this line of thinking, I asked students to contemplate the prompt in Figure 4.2.

These questions prompted great discussions where students first wanted to rely on other translations (left 4 units, right 4 units), but instead, I challenged them to think of reflections and rotations that result in the same image. Predictably, students thought of two 180-degree rotations around the same point—which led to the realization that a 360-degree rotation would result in the same image. Students found reflections more challenging. One student, Tyson, described a reflection over a perpendicular line through point B, followed by a reflection back over that same line. Another student, Tony, wanted to do a reflection over a midpoint of AB. I asked if anyone had a problem with that—sadly, no student remembered that reflections are defined using a line. After providing this definition, I challenged my students,"How could we make the reflection over a line through that midpoint more specific?" My students were stuck. So, I instructed them: "Draw in that line of reflection through the midpoint—talk with your partner to see if you can come up with how to best describe it." After about twenty seconds, I finally heard it repeated throughout the room,"It's a perpendicular bisector." To which one of my students, Colin responded, "Ugh, I thought we were done with those things."

Figure 4.2

Geometry, Unit 1, Lesson 15, Activity 1



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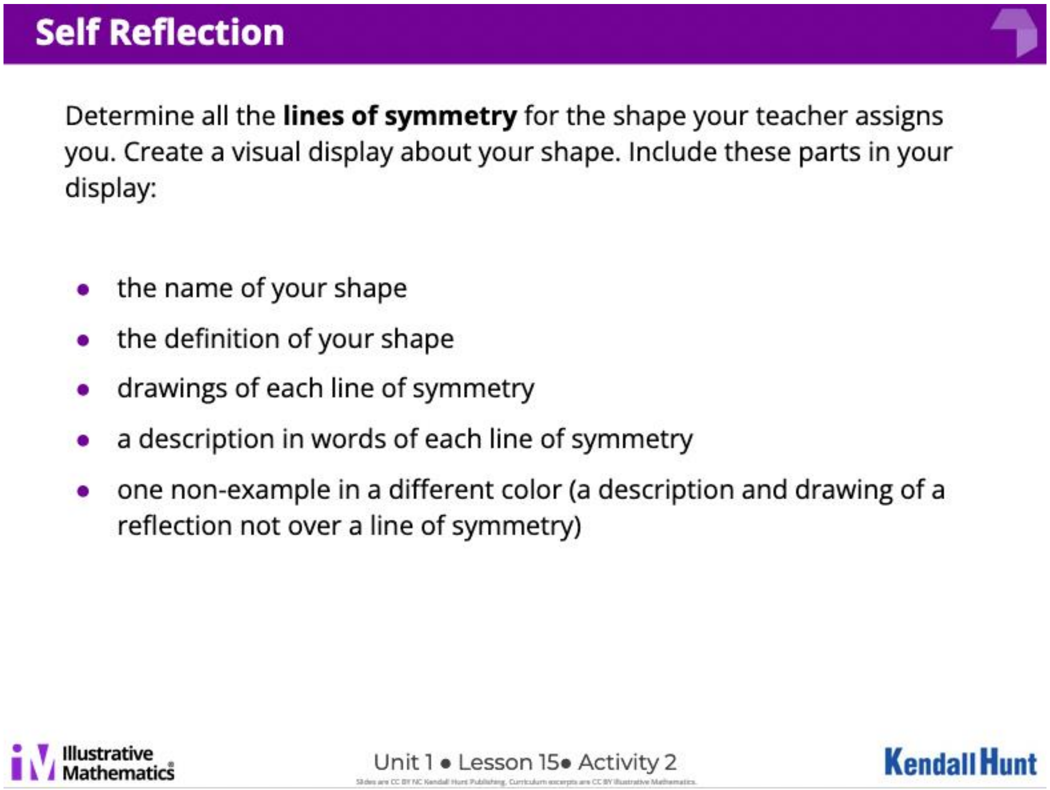
Following the warmup, I handed each student a shape (square, equilateral triangle, rectangle, regular pentagon, regular hexagon, circle, rhombus, and a parallelogram that is not a rhombus). I asked students to provide the name, definition, line of symmetries as sketches and descriptions, as well as a non-example for a line of symmetry for their assigned shape. (See Figure 4.3).

Figure 4.3

Geometry, Unit 1, Lesson 15, Activity 2



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Once finished, I asked for volunteers to demonstrate their shapes on the document camera that projected onto the whiteboard. At first, I had to press for more specific definitions. For example, students wanted to define a square as a shape with four congruent sides—ignoring that the description of a shape with four congruent sides also describes a rhombus. Universally, my students could see their lines of symmetry, but defining those proved a little more difficult. It was not until the second student presented their shape with missing lines of symmetry (the *regular pentagon) that I asked, "What do you notice about her line of symmetry?" Tony piped in "ohhh, it's just the perpendicular bisectors." This led the rest of the presentations to proceed very precisely in describing the lines of symmetry. Students recognized for the most part that lines of symmetry lie on perpendicular bisectors (minus the non-rhombus parallelogram, the rhombus with the line of symmetry through its diagonals, and the circle).*

Summary: Initial Negotiation of Norms

My findings suggest that the remainder of my classes did not have such a unique development of their learning communities. When asked the same prompt, "What norms, or expectations, were we mindful of as we did math together in our mathematical community," each class produced variants of norms that value productivity, effort, collaboration, and discussion of mathematics (see Table 4.3). The only norm I promoted in each of these classes was "Try SOMETHING before asking for help." I framed this norm and expectation as not only something that will help me to facilitate their learning but also as something that I am focusing on for my research. Table 4.3 represents the summary of the initial negotiation of norms for each class.

Table 4.3

Class Negotiated Norms

Geometry Block 7 Norms	Geometry Block 3 Norms
• Working together • Sharing strategies /opinions • TRIAL AND ERROR • Try SOMETHING before asking for help • Ask 3 before me • Be respectful of the person who has the floor to speak	• Show your work • Attempt something before asking for help • Be on task • Work as a team/group • Everyone contributes • Always justify and explain your thinking
Algebra 1 Block 1 Norms	**Algebra 1 Block 8 Norms**
• Showing your work • Talking to each other/listening to each other (do not talk over others) (engaged in math conversations) • Coming into class with an open mind/positive set • NOT GIVING UP • Use your resources • Try even if you are unsure • Question yourself to see if your work is right/check your work • Put your distractions away • Take corrective criticism	• Listen to others • Ask for help— ask 3 before me (the teacher) • Discuss the problems with each other • Collaborate with each other (work with your peers) • Try SOMETHING before you ask for help **Algebra 1 Block 4 Norms** • Work as a team/partner • Talk about the problem • Stay on task • Try SOMETHING before asking for help • Give effort • Don't goof around

Note. This figure shows class negotiated norms that were developed by following the "Building a Mathematical Classroom Community Plan" (Gray et al., 2018).

Chapter 4 - 5 Interlude

Algebra 1: Engagement and Pedagogy Changes

In the last few lessons of the first unit of study in algebra 1, one-variable statistics, an odd change came over my students in block 8. At the onset of the year, these students presented as gregarious and excited to learn. As the shine of the new year began to wear off so seemingly did their excitement in learning. Three straight class periods I observed reluctant engagement from the students I relied on as leaders in the classroom. Unfortunately, the rest of the students took that message of nonchalance to mean they too could be nonchalant in their engagement. I took this reluctance to engage as an indication that I needed to modify my pedagogy or interactions with students in that class. The only unique aspect of block 8 is that it is the first algebra 1 class that I teach before I reflect, modify, and improve the lessons for my remaining algebra 1 classes, blocks 1 and 4. At my school, 'White' days begin each "White-Red" combination of the alternating day schedule. White days have blocks 5, 6, 7, and 8, whereas 'Red' days have blocks 1, 2, 3, and 4.

Over the course of those three class periods of a lackluster classroom atmosphere, I brainstormed to try to figure out what adjustments I could make: should I create a seating chart? Do my students need more structure in the lesson? Or do they need less structure? Do they simply need more accountability toward their learning? I planned to create a seating chart to build more of a community atmosphere after the first unit assessment to try to balance higher and lower attaining students in groups in addition to expanding the students they felt comfortable interacting and collaborating with. As I knew algebra 1 had an upcoming Unit assessment, I did not see a point in collecting homework grades or doing a daily assignment grade that reflected their compliance in adhering to class expectations—especially because my *personal philosophy toward grades is that grades should reflect a student's understanding as opposed to reflecting a combination of understanding and behavioral compliance.*

I tried both more structure and less structure. In past years, I have launched activities with clearly defined time limits to give students a sense of urgency for engaging in the activity at hand. Sadly, that backfired as I misjudged the amount of time the activities of the next class period would take. This caused a few students to shut down as I failed to give them adequate time to process the task. The next lesson I initially judged as one where students could do some independent work where we could do an overall lesson style of LES instead of doing an LES for each individual activity with smaller discussions in between activities. This too backfired, as my more motivated students went through their work without fanfare while I pushed my students with less internal motivation to work with little success aside from when I directly worked with each group. Further, the lack of classwide mathematical discourse throughout that block led to a day of inconsistency where I did not re-establish norms promoting discourse.

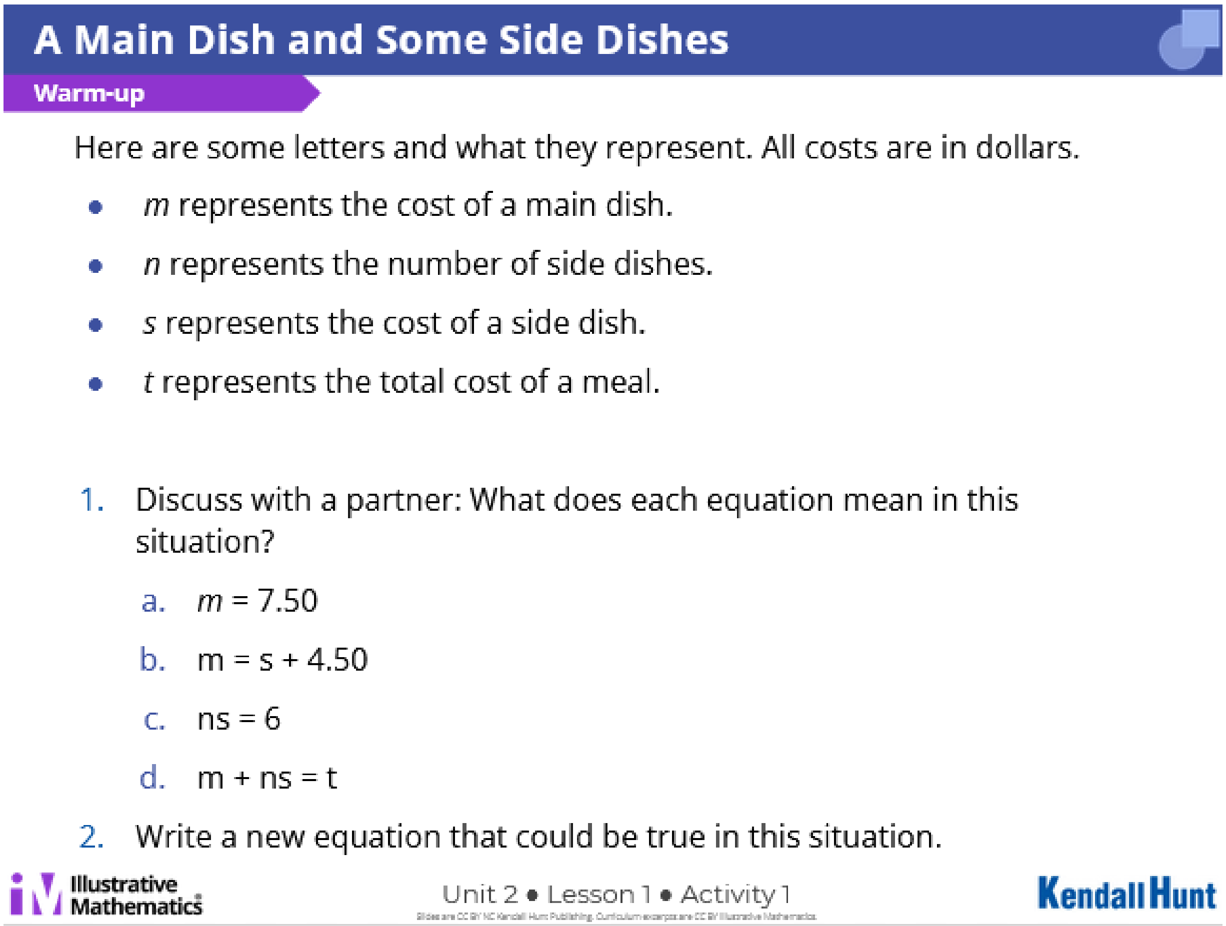
Block 8 did surprisingly well on their first Unit assessment, with the most frequently misunderstood concepts being standard deviation and how outliers are quantified by way of formulae and estimation on a box and whisker plot. This lackluster atmosphere did not truly represent their learning, and instead reflected their frustration with the abstract nature of learning one-variable statistics? I can clearly recall several of my students in each of my algebra 1 classes indicated their readiness to be done with "these stupid graphs."

A Main Dish and Some Side Dishes

What I do know is that in the first lesson we did for unit 2 ("Linear Equations, Inequalities, and Systems", Illustrative Mathematics, 2019), my students exhibited excitement *and engagement in their learning. Students were asked to first consider the situation shown in Figure 4.4.*

Figure 4.4

Algebra 1, Unit 2, Lesson 1, Activity 1



Note. Reproduced from Kendall Hunt Publishing slide deck © 2020, "Planning a Pizza Party" from IM Algebra I © 2019 Illustrative Mathematics. Licensed under the Creative Commons Attribution 4.0 license.

My students' skills pleasantly surprised me here—they could clearly recognize the situation and describe what each equation meant. While skill typically represents a baseline expectation for incoming algebra 1 students, these same students reluctantly engaged with the *last few lessons of the one-variable statistics unit. The equations that students produced by combining the equations surprised me even more (e.g., 7.50m+3n=t). In retrospect, students' zones of proximal development (ZPD, Vygostsky, 1930/1978) could explain the difference in engagement. In the last two COVID-19 dominated years of school, teachers in my experience have focused more on the essential skills that students need for successive math classes. Based on previous conversations, I know that the middle school in my district heavily prioritized a focus on linear functions, followed closely by quadratic and exponential functions. My students' comfort with this first activity and discomfort with the beginning algebra 1 unit over one-variable statistics reflects that priority.*

The next activity surpassed any higher expectations I had based on the engagement of my students during the warmup. I asked students to imagine that they had to plan a pizza party for the class (see Figure 4.5). After around ten minutes had passed, I had students voice out some of the variables they considered, and how they found the information to create their equations. I made sure to emphasize to students that each group should have a different set of equations because we all make different assumptions. Further, I emphasized that the only wrong answer in writing their equations is a non-attempt. In this time, I heard notable examples of collaboration, inquiry, disagreements, frustration, and working through that frustration. To most questions, I responded: "I am not sure, is there anywhere you could look that up?" or "Do you think you're forgetting to consider anything?" or "Have you tried asking another group for inspiration?"

Figure 4.5

Algebra 1, Unit 2, Lesson 1, Activity 2

How Much Will It Cost?

Imagine your class is having a pizza party.

Work with your group to plan what to order and to estimate what the party would cost.

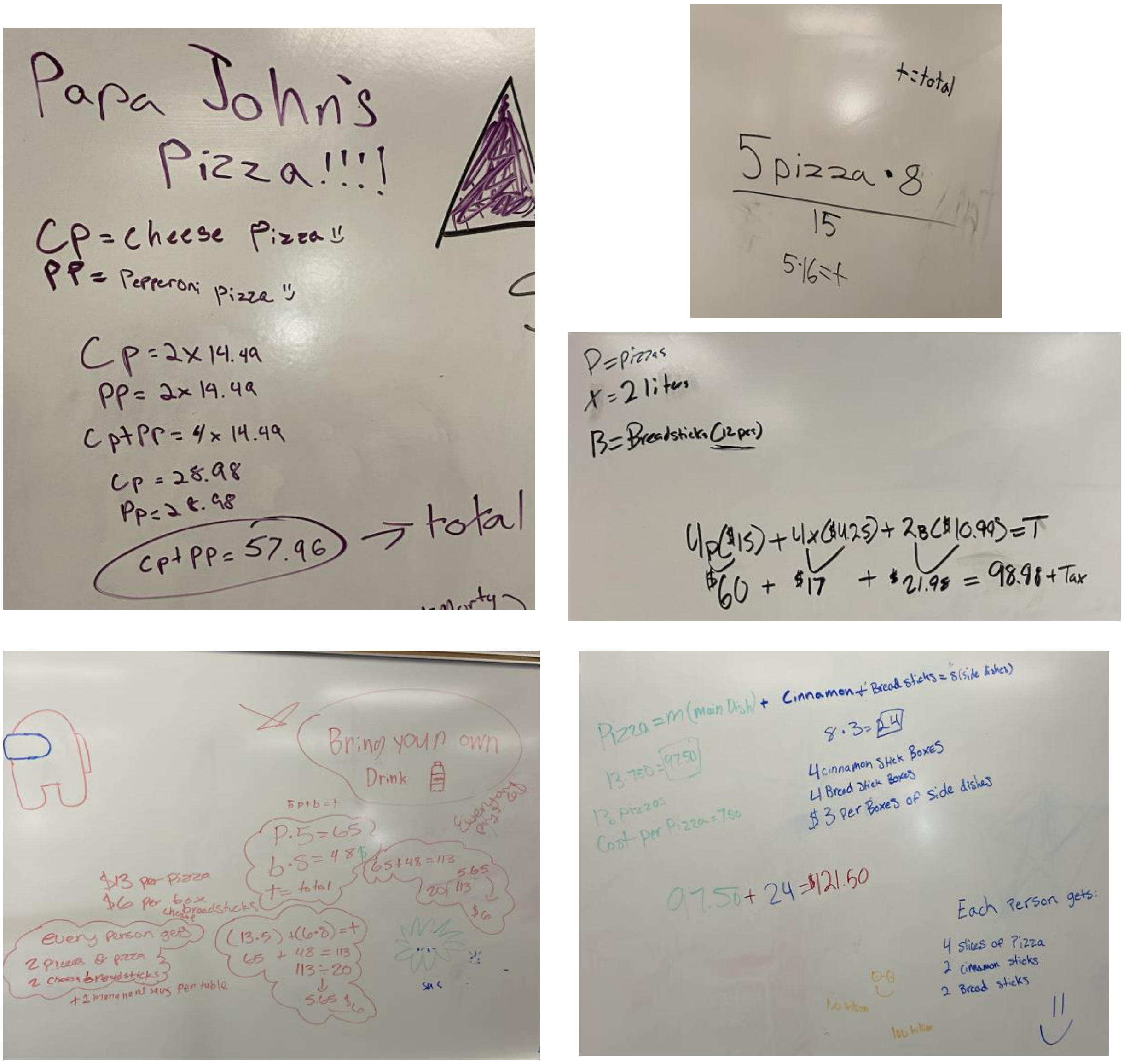
1. Record your group's plan and cost estimate. What would it take to convince the class to go with your group's plan? Be prepared to explain your reasoning.
2. Write down one or more expressions that show how your group's cost estimate was calculated.
3.
   a. In your expression(s), are there quantities that might change on the day of the party? Which ones?
   b. Rewrite your expression(s), replacing the quantities that might change with letters. Be sure to specify what the letters represent.

Illustrative Mathematics   Unit 2 • Lesson 1 • Activity 2   Kendall Hunt

Following another 5-10 minutes, I had students present to one other group. They were frustrated they could not present to the whole class! Sadly, the block only had a few minutes remaining. It is so incredibly rare that a class at the end of the day has that sort of energy and excitement—it was a true pleasure to experience. The five different models the groups produced are shown in Figure 4.6.

Figure 4.6

Student Pizza Party Mathematical Models



Another factor that might have supported student engagement is the lack of structure and "open" nature of the task, both open middle (no one single path), and open-ended (no one single answer). For one short class period, I observed sociomathematical norms of inquiry, collaboration, and even struggle. Instead of shutting down as some students do with certain teacher responses to struggle, students kept trying, kept searching for information, and used each other to move forward in problem-solving. It is lessons like this one that continue my love and drive for teaching. It is lessons like this that I will continue searching for in trying to negotiate a classroom environment that facilitates struggle.

Chapter 5 - Theme 2: Social Norms

Identifying social norms that contribute to student struggle was not difficult. Within the first few days of doing reflexive journal entries following a day of instruction, I purposefully recorded instances of collaboration, peer tutoring, and student discourse in my reflexive journal entries. Connecting those reflections to how I interpreted student struggle followed the first time I began coding the data. In this coding process, I realized I needed a more specific focus as to how those norms influenced students struggling. It is this focus that contributed to the specific discussion of how each social norm connects with both productive and unproductive student struggle.

My findings show that following the community-building process (Gray et al., 2018), the development of the learning communities in my classroom ebbed and flowed. There are times when collaboration seemed to spark and flourish on its own, and there were times when it just doesn't seem to be there. Peer tutoring is similar as well—whenever students have finished the launched activity, I encourage them to roam the classroom to see if any of their peers need assistance. There are times when students alternatively want to sit and zone out instead. And there are times when students are willing to go try to help others but the students needing help are not willing to receive it from a classmate. Social norms in a classroom microculture are in constant flux, and whether a student complied with a classroom norm on any given day is influenced by any number of hidden variables.

A major barrier toward these social norms that predictably occurred during a once-in-a-century pandemic was physical in nature. At the beginning of the year, my school began the year in masks as a precaution for the resurgent cases of COVID-19 due to the delta variant. Two weeks into the school year the table dividers the district ordered finally came in, and so did the expectation that classes would put up dividers as an extra precaution.

Despite the increased safety from the transmission of the virus within the classroom because of the dividers, I found myself incredibly frustrated at its dampening effect on the classroom culture. The natural collaboration and peer tutoring that occurs at a table where students could hear one another suddenly had two physical barriers: masks and dividers made from cardboard and plastic. Not only did this extra physical barrier cut down the progress of the developing learning communities, but it also hurt my teaching in other ways. For one, normally I can informally observe student work and hear student conversations walking around the classroom—this now proved much more difficult. To continue developing the learning communities in my classroom, I had to become much more intentional about prompting students to peer tutor, to provide intentional opportunities for collaboration, and to provoke more opportunities for discussion.

Coding and analysis of reflexive journal entries support the finding that teachers and students continually renegotiate social norms, and further, that the effect of social norms on any one given lesson is hard to predict. One lesson that seems to go smoothly in one block *because* of social norms and fails in another block *because* students do not adhere to those same norms for whatever reason. Something as small as a few students being gone for a marching band performance can so completely change the dynamic of classroom culture negatively for one lesson, but those same students in the same class being gone for a marching band performance can prompt even better collaboration and problem-solving a few weeks later. Ultimately, the importance of social norms in a classroom environment cannot be understated, but at the same time, it is difficult to predict how those social norms will be "taken-as-shared" day by day (Yackel, 2001, p. 6).

Collaboration

Student collaboration is an idea that has captivated me ever since I first witnessed it when student teaching with Mr. Boomer. I tried to capture it with a traditional curriculum and traditional classroom setup with desks set up in rows in my first few years of teaching. The essence of collaboration never seemed to develop quite as well as when the physical environment, the curriculum, and my teaching actions promoted the idea that students learn math together as a community—not as a set of individuals who happen to be in a classroom together. When I had the opportunity to return to where I student taught, I found myself thrilled with the opportunity. Coincidentally, I began the pursuit of my master's degree that year. I took "Research in Mathematics Education" with Dr. Sherri Martinie, my major professor for both my masters and doctoral degrees. Creating a brief review of research over a topic of my choosing for that class became one of my favorite projects of my master's degree—I chose cooperative learning. This review of research supported what I had always felt was right.

Every year I have taught I aim to establish collaboration as a strong social norm. Not only do I believe the best way to learn is through social interaction, which is backed by research (see Table 5.1), and it thankfully fits with what has always felt right in my classroom. To begin with, my students seemed receptive based on the 'doing math' actions generated from the "Building a Mathematical Classroom Community Plan" (Gray et al., 2018) that align with collaboration.

Table 5.1

Summary of Cooperative Learning Research

Summarized Findings	Reference
• When academic contents are not novel and simply more complex, the most effective group was collaboration while the least effective was peer tutoring. When academic contents were building upon concepts learned previously, no significant differences appeared. When previous knowledge proves insufficient for academic content requiring a strong conceptual change, the peer tutor relationship is most effective for fostering effective learning.	Pons, R. M., Prieto, M. D., Lomeli, C., Bermejo, M. R., & Bulut, S. (2014). Cooperative Learning in Mathematics: A study on the effects of the parameter of equality on academic performance. *Anales de Psicología/Annals of Psychology*, *30*(3), 832-840.
• Participant math teacher utilized roles in cooperative learning groups: discussion leader, computational role, and recorder. Participant's advanced classes had more time cooperative learning and more disagreements in discussion	Siegel, C. (2005). Implementing a research-based model of cooperative learning. *The Journal of Educational Research*, *98*(6), 339-349.
• Results suggest that students who participate in group work some or half the time show significantly higher achievement than those who did not participate in group work.	Smith, T. J., McKenna, C. M., & Hines, E. (2014). Association of group learning with mathematics achievement and mathematics attitude among eighth-grade students in the US. *Learning Environments Research*, *17*(2), 229-241.
• Experimental class that implemented cooperative learning saw greater achievement gains than the control group in addition to a more positive attitude toward cooperative learning as measured by pre-test, post-test survey.	Walmsley, A. L., & Muniz, J. (2003). Cooperative learning and its effects in a high school geometry classroom. *The Mathematics Teacher*, *96*(2), 112.
• Questionnaires and interviews for students revealed that students find student centered mathematics more applicable to real world scenarios and more oriented toward learning as a community.	Zain, S. F. H. S., Rasidi, F. E. M., & Abidin, I. I. Z. (2012). Student-centred learning in mathematics-constructivism in the classroom. *Journal of International Education Research*, *8*(4), 319.

My algebra 1 classes produced actions that closely aligned with collaboration: Putting your heads together/working together; each member having a different opinion; agreeing and disagreeing—making a solution out of that; getting along and putting all your thoughts into it; groups working together; working together as a group. And my geometry classes produced similar ‘doing math’ actions: Working with other people—can’t do it alone; collaborating—working off one another’s ideas; everyone contributing; collaboration: working together to solve problems. Based on these ‘doing math’ actions, my algebra 1 and geometry classes all produced norms that signal collaboration as a social norm, either explicitly or implicitly. The geometry collaboration related norms were Work as a team/group; everyone contributes; working together; and sharing strategies/opinions. The algebra 1 collaboration related norms were similar: Work as a team/partner; and collaborate with each other (work with your peers). The student developed ‘doing math’ actions and their negotiated norms related to collaboration suggest that they view collaboration as a social norm in my classroom.

My findings suggest that despite the many barriers to natural collaboration, my students still worked with one another and viewed it as part of how they engage in and learn mathematics. Engagement in rigorous mathematics hopefully leads to iterations of productive student struggle but neatly mapping how collaboration facilitates student struggle is not necessarily obvious. First, collaboration can quickly turn into peer tutoring where a student responds to another student’s struggle of various forms, ‘getting started’ or ‘carrying out a process’ most frequently in my classroom. Further, students in collaboration have opportunities for mathematical discourse; however, I define the social norm of discourse in my classroom as structured opportunities to talk about teacher-directed prompts as opposed to the organic discussions that occur between students collaborating. Collaboration can also, unfortunately, remove opportunities for struggle when students give up and choose to either copy another's work or the other student simply tells them what to do—an iteration of unproductive struggle. However, I have seen far more occasions of organic student discussions sparking ideas of how to 'get started' and 'giving mathematical explanations'—which aligns perfectly with what I aimed for in establishing an environment that helps to facilitate productive struggle.

The following lessons offered powerful opportunities for students to learn collaboratively, and both are more interesting than anecdotal accounts of students working together with their peers on a given activity. Both lessons are from geometry when my students were learning about triangle congruence and are presented in the chronological order they occurred—the first lesson is from the beginning of the Unit and the second occurs toward the end of the unit where students were beginning to *use* triangle congruence theorems in proofs.

Invisible Triangles

Unit 2, lesson 3 involved students beginning to understand one way to claim figures are congruent: the idea that if all the corresponding parts of figures are congruent, then the whole figures are congruent to one another. Their previous understanding of congruence built from the idea that rigid transformations preserve congruence, so this activity served as a bridge between that understanding, and their ability to recognize the corresponding parts of congruent figures. Further, this activity (Figures 5.1 and 5.2) marked a point where transformations became a classroom mathematical practice in my geometry classes.

Figure 5.1

Geometry, Unit 2, Lesson 3, Activity 2

Invisible Triangles

Player 1: You are the transformer. Take the transformer card.

Player 2: Select a triangle card. Do not show it to anyone. Study the diagram to figure out which sides and which angles correspond. Tell Player 1 what you have figured out.

Player 1: Take notes about what they tell you so that you know which parts of their triangles correspond. Think of a sequence of rigid motions you could tell your partner to get them to take one of their triangles onto the other. Be specific in your language. The notes on your card can help with this.

Player 2: Listen to the instructions from the transformer. Use tracing paper to follow their instructions. Draw the image after each step. Let them know when they have lined up 1, 2, or all 3 vertices on your triangles.

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Invisible Triangles

Invisible Triangles

Transformer

Listen to hear which parts of the triangles correspond. Then give instructions to take one triangle onto the other.

Possible instructions:

- Translate ______ from ______ to ______.
- Rotate ______ using ______ as the center by angle ______.
- Rotate using ______ as the center so that ______ coincides with ______.
- Reflect ______ across ______.
- Reflect ______ across the perpendicular bisector of ______.

$\overline{JK} \cong \overline{PQ}$, $\overline{JL} \cong \overline{PR}$, $\overline{KL} \cong \overline{QR}$, $\angle J \cong \angle P$, $\angle K \cong \angle Q$, $\angle L \cong \angle R$

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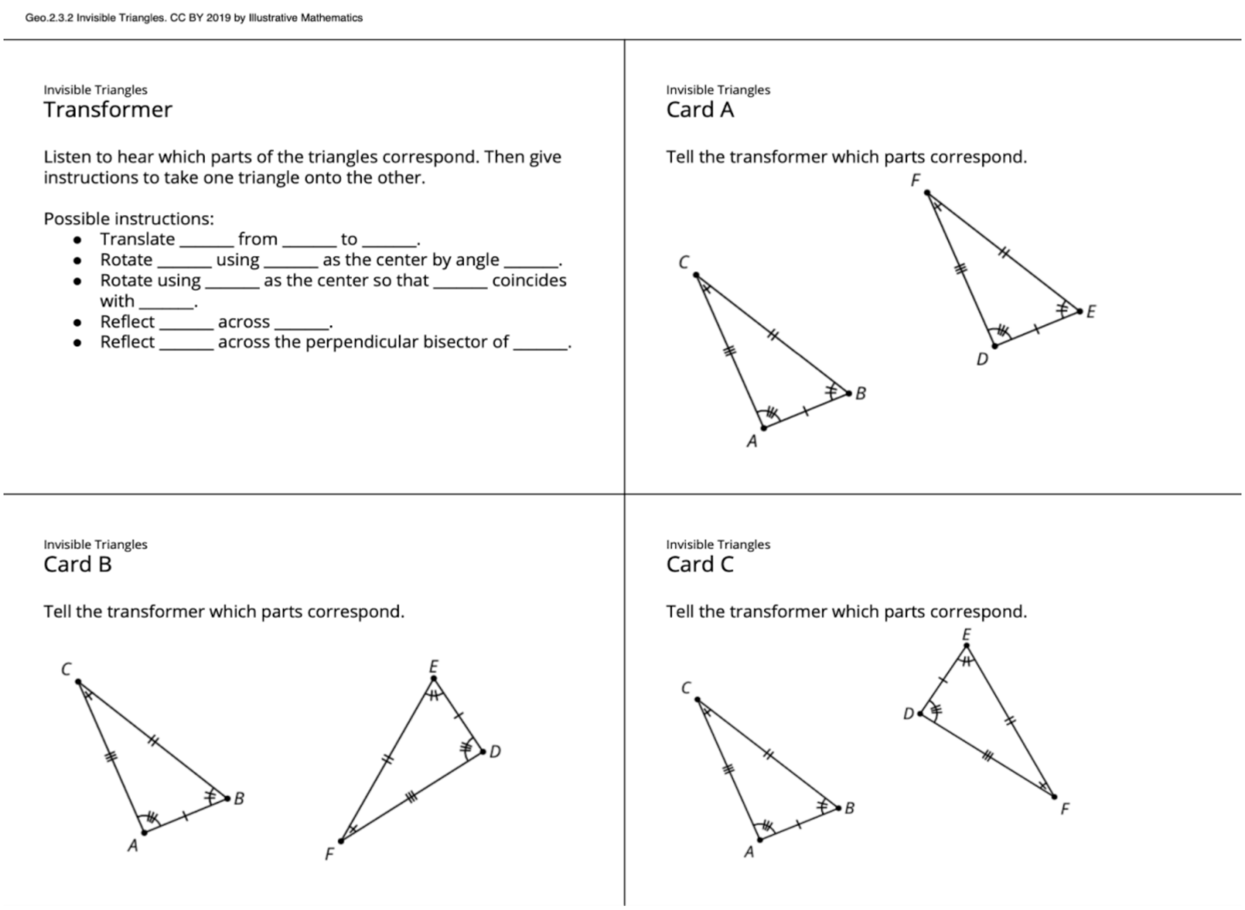
Figure 5.2

Geometry, Unit 2, Lesson 3, Activity 2 Cards



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This activity is a prime example of collaborative and cooperative learning as students worked together to accomplish the given task and provided students the opportunity to create meaning together in their shared learning. One key physical manipulative that scaffolded my student's learning is tracing paper. Students used tracing paper so they could see the triangles moving as they used translations, reflections, and rotations to move the triangles on top of one another.

Having never taught this activity, I found myself skeptical—in an aspect of mathematics like geometry, student ability to reason visually with shapes is incredibly important. When checking to see if there were any questions before we started the activity, one of my students asked what to do if they got stuck? I gave students a few sentence stems to provide each other assistance: (1) what have you tried so far, and (2) so if you have tried ____ and it doesn't work, what else could you try? To my surprise, after doing one of the invisible transformations as a class, students quickly proceeded to engage in the activity, eventually even being disappointed when there were no more invisible triangle cards. I observed my students being somewhat shocked that when they kept trying something else, they eventually had success.

Students who were the invisible transformers had the opportunity to focus on the learning goal itself as they developed an understanding of the corresponding parts of congruent figures. I provided these students with scratch paper to write down the corresponding parts as well as their transformations so I could check their work. Students had the opportunity to struggle and by necessity try out multiple attempts in engaging in the task. The students who held the data cards alternatively had the valuable opportunity to continue practicing their fluency of transformations using the tracing paper. Further, these students had the opportunity to think more like a teacher in thinking about how they could assist their struggling partner.

The struggles I observed fell into the categories of ‘getting started’ and ‘carrying out a process.’ A student asked what to do if they got stuck? This question prompted me to create a way for students to help my facilitation of their peers’ struggles. Students responding with those sentence stems used ‘probing guidance’ to help facilitate their peers’ struggles. However, I also witnessed some students taking a more hands-on approach and providing more ‘directed guidance’ by rephrasing my questions in a more direct way. For example, they asked questions more like “so you’ve translated to make one set of points line up (coincide), then you tried reflecting and no more points lined up (coincided), what’s the last thing you could do?”

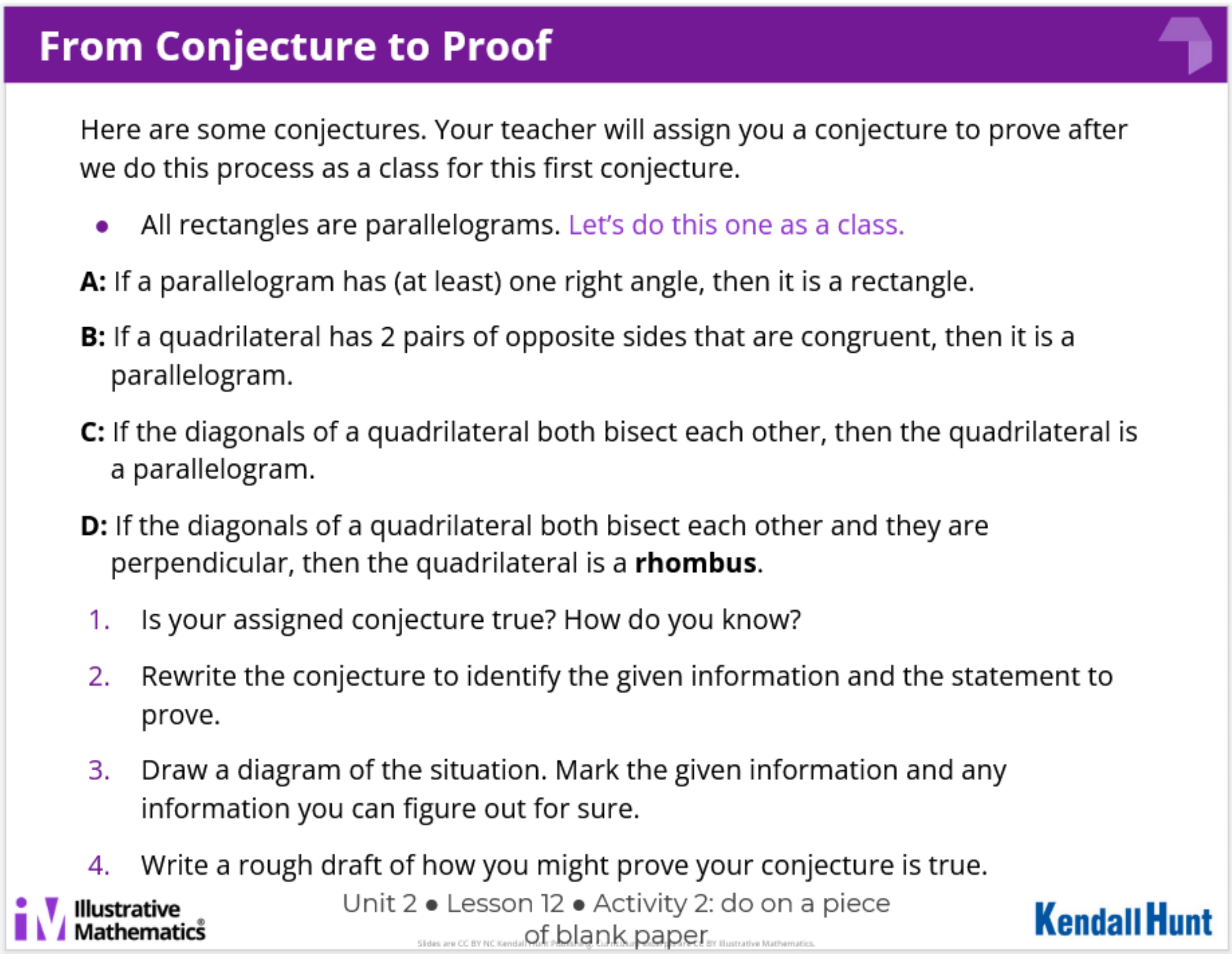
Based on the lack of struggle that I observed from the students holding the cards with the triangles in this activity, I felt confident in claiming transformations as a classroom mathematical practice at that time. Additionally, I observed students throughout the second unit of geometry basing their reasoning on how they could use rigid transformation to make points of triangles coincide to establish congruence of two triangles. This belief continued in the following Unit over similarity where students once again had an invisible triangle activity, but that time involving dilations—a non-rigid transformation.

Proofs About Quadrilaterals

For Unit 2, lesson 12, I modified from the original curriculum to make the activity (Figure 5.3) more collaborative and cooperative, as well as to provide students an opportunity to practice more triangle proofs with their peers than the original task, which had students choose only one conjecture to prove. I provided more structure and some direct instruction at the onset of this lesson because using triangle congruence theorems is not an aspect of mathematics that students can typically discover on their own. I continued to encourage students to base their *reasoning and thinking on the idea that they need to prove triangle congruence to prove their claims about the properties of the parts in parallelograms.*

Figure 5.3

Geometry, Unit 2, Lesson 12, Activity 2



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I provided some direct guidance to help students be able to put their thoughts onto paper by printing off the 'Proofs tips!' in Figure 5.4. We first went through the conjecture that "all rectangles are parallelograms" as a class. This conjecture came together as a class wide conversation where I helped them shape their thoughts into a coherent proof. When students were collectively stuck during this discussion, I modeled the need for students to use their reference sheets (see Appendix E), which served as their notes for both fall and spring semesters.

Figure 5.4

Proofs tips!

Proofs tips!

1. Draw in your auxiliary line to create triangles!
2. We've introduced triangle congruence theorems so that we can use them! Try to find SAS, ASA, or SSS in your shape.
3. Mark what you know on your picture (what sides or angles are congruent), and then write down why you could mark that in your proof.
4. If you're stuck, use your reference sheets!
5. Another way to help yourself out if you're stuck is to check out prior proofs we have done-- there might be similarities that you can use.
6. When we're trying to prove **parts** of figures are congruent, CPCTC might be useful.... Remember, it allows us to claim that corresponding **parts** of congruent triangles are congruent!

Following this classwide discussion, I assigned students proofs A, B, C, or D in a random fashion. Since most students are at tables of four, each student had a different proof in each group. Students worked through prompts 1-4 to get started on this activity. I observed various levels of struggle: 'getting started,' 'giving mathematical explanations,' and 'expressing *misconceptions and errors.' In anticipating these struggles, I tried to think of ways to facilitate different student struggles by creating the "Proofs Tips" (Figure 5.4).*

Tip 1, drawing in auxiliary lines, referred to an established class practice of creating multiple triangles from a larger figure to reason about specific parts of that figure. I created Tips 2 and 6 to focus student learning and thinking toward the learning goal at hand: using triangle congruence theorems in parallelograms. Tips 3 and 4 I intended to address students struggling with 'giving mathematical explanations.' I meant for tip 5 to assist students in 'getting started;' however, the students who struggled with 'getting started' did not have the organizational skills to keep track of where they kept the prior proofs covered previously as a class—even with their work kept in their student workbooks. Developing these 'Proofs tips!' is a prime example of the essential work of anticipating struggle—in thinking about how to set up parameters for engaging in math tasks that give students guideposts on where to turn when students get stuck.

I observed some cross-proof collaboration, mostly assisting one another getting started. This mostly came from students using a frequently referenced theorem about parallelograms in class to that point in time: if a quadrilateral is a parallelogram, then the opposite sides are congruent. And in reminding one another to use the auxiliary line as a "free" side to claim as congruent to itself—an amusing way students referred to the reflexive property. Collaboration really shone as a social norm when students moved to new groups where they could collaborate with one another on polishing up and finishing the same proof together. During this polishing time, I could hear evidence of traditionally conceptualized sociomathematical norms: students critiqued one another's reasoning and offered feedback on better ways to write their claims. In *this way, students demonstrated an awareness of different, acceptable, and to an extent, sophisticated explanations as sociomathematical norms.*

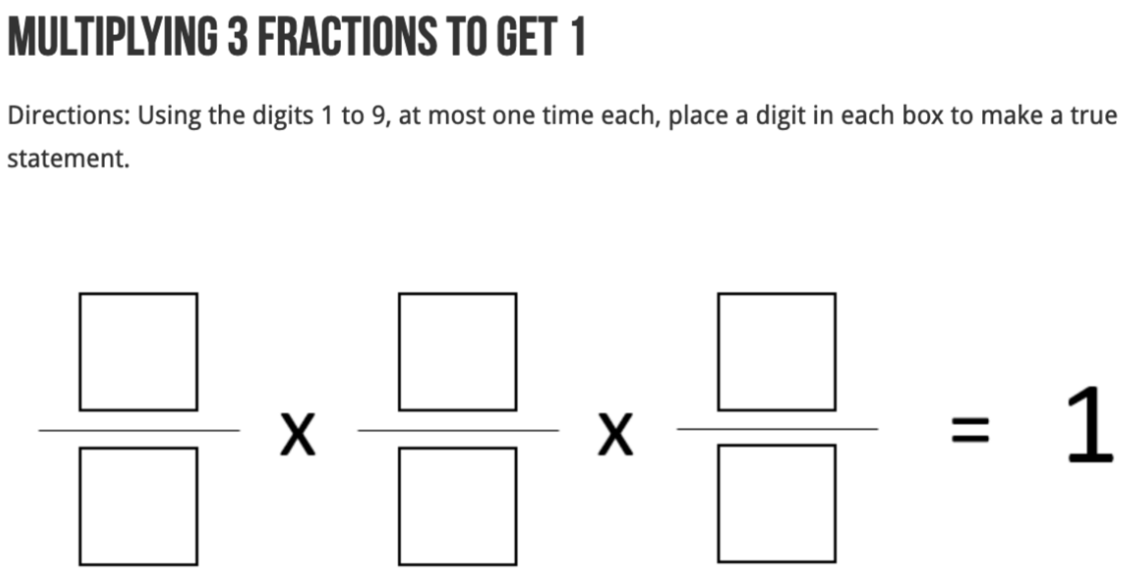
As time ran out to begin another activity, I shifted the typical classwide discourse that would normally have occurred to conclude an activity to instead include presentations for each group to demonstrate their thinking. Following each presentation, I asked the class "were there any statements that they claimed but didn't justify?" Unfortunately, my students did not seem quite ready or comfortable with critiquing one another's proofs in front of the whole class. Instead, pivoted and provided each group feedback to improve the drafts of their proofs.

Open Middle Proofs?

I learned about these types of problems several years ago. Sherri came into my classroom to observe my student teacher at the time, and she shared the problem shown in Figure 5.5. This prompt led quite a few high school students to struggle and enjoy playing around with numbers to solve this problem. While I imagine that Sherri's interest in open-middle problems came from her interest in developing conceptual understanding and procedural fluency, my interest came from the seemingly obvious connections to problems that would provoke occasions of productive struggle. What better to begin developing an environment where students are willing to engage in struggle than problems that are well within students' ZPDs and provide the opportunity to polish up underlying skills? My student teacher at the time, Keri, collaborated with me in using these types of problems and in modifying our curriculum at the time to have some of the characteristics of 'open' mathematics problems (Boaler, 1998), open beginnings, middles, and ends.

Figure 5.5

Open Middle Fractions



Note. Reproduced from *"Multiplying 3 Fractions to get 1"* from Open Middle © 2022 Open Middle. Licensed under the Creative Commons Attribution 4.0 license.

Out of all the popularized textbook makeover problem-solving strategies, 'open middle' problems captured my interest most easily. This is likely due to the connections I see in their use with trying to provoke productive struggle. In the past, I have used them as bell-ringers, as extension problems for students who completed tasks quickly, and even as opportunities to earn candy as a reward for my bored homeroom students. 'Open middle' problems are characterized by their open-middle, where students can take different approaches to try to solve a problem (Johnson & Kaplinsky, n.d.). There are also naturally spin-offs of 'open-middle' problems: problems with an open beginning—where students could use different information as assumptions for creating constraints and equations; or problems with an open end, where students are given information and could think about what questions we could ask and solve given that information (KSDE Regional Training, 2019). OpenMiddle hosts the most popularized version of open middle problems (https://www.openmiddle.com/). Typical problems have boxes where students try to create solutions for a given equation by using the numbers 1 through 9 at most one time each.

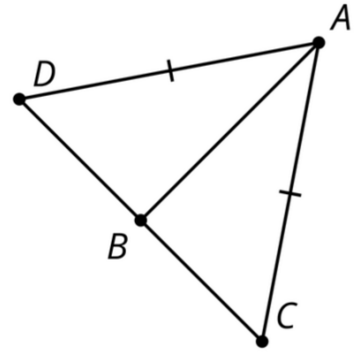
I did not anticipate noticing characteristics of 'open-middle' problems in observing students engaging in proofs. The characteristic that I saw come to light most frequently is that proofs have an open middle where students can take multiple pathways to prove the same idea—a closed end. At first, I observed students hesitant to engage in the parallelogram proofs because they began their proof in a different way than their classmates. As this happened several times in working with students across the classroom, I re-established the expectations that the most important part of a proof is the communication and justification of reasoning. And further, that there are many ways of being able to reason through these proofs. I saw students approaching their proofs using each of the triangle congruence theorems covered to that time in class (side-side-side, side-angle-side, and angle-side-angle).

Later, while we reviewed for the unit 2 exam, these efforts paid off unexpectedly. I created a unit review and included a challenging problem: being able to prove two triangles in a figure with each of the triangle congruence theorems (Figure 5.6). To my surprise, it was not only my higher attaining students attempting and succeeding at these problems but also students who had come for further assistance outside of class time in the week prior to the exam.

Figure 5.6

Open Middle Proofs

Prove that ΔDAB and ΔCAB are congruent if B is the midpoint of DC. For a challenge, try to find ways to prove these triangles congruent by all three congruence theorems: SSS, SAS, and ASA.



Not Latching onto the Learning Goal.

I had a group of around six students in my block 7 class feeling as if they had no understanding of geometry content through lessons 9 and 11 of unit 2. To that point in the unit, students proved the triangle congruence theorems in class using transformations. This group had not latched on to the big ideas of triangle congruence. They negatively judged their understanding of proofs because they did not follow the much more rigorous proofs that established the triangle congruence theorems. However, I assess students over whether they can use these theorems, not whether they can understand their basis. Luckily, these students listened to my advice that they should come into advisory so I could work with them as a small group of students.

Using small whiteboards so that students can feel safer in making mistakes is one of my favorite scaffolds. Whiteboards seem to make students feel as if their work is not as permanent, more modifiable, and thus less threatening if they make a mistake. I began with an overview of the triangle congruence theorems—quizzing students over recognizing triangles already marked *as side-side-side, side-angle-side, or angle-side-angle congruence. I wanted to align with the learning goals taught to that point and to lay a foundation for the future learning goals— using triangle congruence theorems in parallelograms.*

Having established their ability to recognize the end goal of a triangle congruence theorem, I wanted to leverage the social norms established in class: collaboration, peer-tutoring, and discourse. I gave these students a single figure to draw on their whiteboards, told them to get out their reference sheet, and asked them to try to mark their picture on their own with what they recognized from the given information. After around a minute of quiet work time, I had students share their work with the group. To try to develop some of the discourse typically facilitated as a whole class, I asked each student, one after the other, to 'tell me about your markings on your figure.' After a few iterations of marking figures, this group began to put proofs together through their collective ideas.

Around a week and a half later, part of this group of students came in to work on their unit reviews together. Unfortunately, at the time, I had several algebra 1 students come in to work with me on solving systems algebraically—namely by using substitution and elimination. In the hopes that their previous instances of collaboration bore fruit, I projected the review for them to work together on the whiteboard. To my pleasant surprise, this group only needed me to check their work—with none of the 'getting started,' 'expressing misconception and error, 'or 'giving mathematical explanation' struggles that plagued them weeks earlier.

Peer Tutoring

I knew from the onset of the school year that alongside the social norms of collaboration and student discourse, I wanted to promote and establish a norm of peer tutoring. Normally I tell my classes some version of:

> When you help your peers, you learn the content more deeply. You must take on the teacher's perspective because you must determine what they are thinking and try to lead them in the right direction. It is challenging, but I promise, it helps you learn the content more deeply, and it helps the person you are working with because the language you use might be more effective than the language I use.

The beginning of the school year is normally an especially advantageous time to work on this norm, as most curricula begin the sequence of content with review over the prior year's content. As I piloted a new curriculum that year, this advantage did not necessarily occur. For algebra 1, the curriculum began with one-variable statistics. Normally middle school curricula cover one-variable statistics in part in both sixth and seventh grade (KSDE, 2017). However, given that these freshmen had COVID-19 influence both their seventh and eighth-grade years, teachers may have chosen to emphasize other standards. A comparable situation existed for geometry where the curriculum began with learning about constructions and transformations. Middle school curricula in my experience cover a basis for constructions and transformations without the depth or rigor of high school standards.

Students identified fewer 'doing math' actions and official norms that related to peer tutoring. My block 7 geometry class recognized "Teaching/Helping each other with what we know" as a 'doing math' action and later on that recognition developed into a norm in the form of "Ask 3 before me" (the teacher). Block 8 also explicitly identified a peer tutoring type norm which took the form of “Ask classmates for help.” I ended up asking students if I could instead phrase “ask classmates for help” as “ask 3 before me” so I could remember that norm—they agreed.

Because of this lesser recognition from students that peer tutoring is a part of ‘doing math’ and an important norm in classrooms, I knew I had to take direct action to establish this norm in my classroom. To prompt this social norm, I encouraged students to find people to assist across the classroom when finished with a given task at their groups. At the beginning of the year, this move worked well as students had yet to miss class periods from illness, absence from sports, or COVID-19 quarantines. One finding of coding and analyzing reflexive journal entries suggests that the novelty of asking peers if they wanted help, as well as the willingness of students to accept help decreased over time. Most of the peer tutoring ended up occurring in their table groups. With the use of dividers to prevent the spread of COVID-19, this norm restricted even further to typically only occur with students on the same side of the table—who I call “shoulder partners.”

With these challenges to establishing a social norm of peer tutoring, I looked for specific opportunities to promote peer tutoring when students struggled. When students struggled with ‘getting started,’ I looked for a student nearby asked them to tell their peers “What helped them begin their work on the problem?” Students who struggled with ‘carrying out a process’ also found help with peer tutoring. In this case, I prompted students to tutor one another in carrying out a given process. I tried to stay nearby so that I could ensure students used ‘directed guidance’ with leading questions rather than telling a peer how to move forward in the problem. When establishing this sort of peer tutoring, I say “remember, don’t tell them, ask them questions to help move their thinking forward.” I observed students' abilities in using advancing questioning with another student with varying levels of efficacy. I vividly recall a student becoming frustrated trying to use advancing questions, proceeding to retrieve her notes from her bag so she could use them as an anchor for her thinking, and using questions like “well, what did we do next? So, what should we do next?”

Discourse

One of the challenges I have faced in my teaching career to this point is launching and facilitating purposeful student discussions. In general, when teachers put students at a table facing one another, conversations occur. Making sure that those student discussions aligned with the learning goal for the day is slightly more difficult. Especially with the innumerable distractions that adolescents deal with in the modern classroom.

With our prior curriculum, *Core-Plus Mathematics* (2008), students engaged in guided investigations of mathematics that did not require nearly as much teacher guidance for the class. We would launch an investigation, and most of the class period involved student exploration of the mathematics. During this exploration, we worked with small groups, using purposeful questions to aid student inquiry. At the end of the block, we summarized and formalized the mathematics with classwide mathematical discourse. Changing to *Illustrative Mathematics* (2019) shifted this mindset not necessarily in philosophy, but in practice. *Core-Plus* (2008) used a single LES instructional cycle over the course of a lesson. Whereas with *Illustrative Mathematics* (2019), teachers use LES cycles with each instructional activity. There might be five or six activities in a block! This increased teacher guidance, to say the least, and increased the frequency of classwide discourse. For example, classwide discourse occurred at the end of the lesson with *Core-Plus* (2008) whereas with *Illustrative Mathematics* (2019), discourse occurred at the end of each instructional activity. I knew that I had to change my approach to facilitating discourse; specifically, I had to become more purposeful in how I prompted student discussions.

Illustrative Mathematics' (2019) curriculum is designed to prompt and use discourse to summarize learning activities. As a result, I have found it no surprise that the learning activities tend to naturally promote student discussion. And I found that my skill in orchestrating mathematical discourse has grown given the intentional design of the learning activities. By necessity, the social norm of discourse connected with another social norm in my classroom, collaboration. When students learned together and worked toward the same learning goal, discussion of a problem occurred naturally. By that same logic, a social norm of discourse intersected with multiple sociomathematical norms in my classroom. Social norms framed students' discussions of mathematical ideas. Prior to these discussions, I established and re-established the expectation that they justify their reasoning over time. The resulting discussions where students evaluated whose ideas are acceptable and valid represented the sociomathematical norm.

Establishing this norm thankfully is not a new way of engaging in mathematics for my students based on both their ‘doing math’ actions and their identified norms. My algebra 1 and geometry students identified numerous ‘doing math’ actions related directly to discussion in the classroom. The norms they generated for both algebra 1 and geometry reflected this clear recognition of discourse as a part of doing mathematics. In Tables 5.2 and 5.3, I highlighted discussion related actions and norms.

Table 5.2

Doing Math Actions Table

Block 7 Doing Math Actions	Block 4 Doing Math Actions
• Teaching/Helping each other with what we know • Collaboration: working together to solve problems • Disagreements • Sounding smart • Discussing your ideas • Being able to debate your ideas	• Loud • Sounds like numbers (talking about math) • Getting along and putting in all your thoughts into it • Groups working together • Communication is key • A lot of talking • Working together as a group • People talking over each other • Arguing/debating • PENCILS ON PAPER
Block 3 Doing Math Actions	Block 8 Doing Math Actions
• Groaning and asking the teacher for help—sometimes happy • Crying—copying off one person (asking others for help) • Parents frustrated that you do not do it the “right way” • Working with other people—cannot do it alone • Mental health awareness • Collaborating—working off one another’s ideas • Getting frustrated • Being off task • Problem solving • Everyone contributing	• Chaotic • Agreeing and disagreeing—making a solution out of that • Putting your heads together/working together • Each member has a different opinion • Discussing math • Frustrating/Stressful • Fun Fresh and Funky?
	Block 1 Doing Math Actions
	• Debating, arguing • Discussing strategies and ideas • Loud • Communication • Being engaged/engaged conversation

Note. This figure shows class negotiated ‘doing math’ actions that were developed by following the “Building a Mathematical Classroom Community Plan” (Gray et al., 2018).

Table 5.3

Class Negotiated Norms

Geometry Block 7 Norms	Geometry Block 3 Norms
• Working together • Sharing strategies /opinions • TRIAL AND ERROR • Try SOMETHING before asking for help • Ask 3 before me • Be respectful of the person who has the floor to speak	• Show your work • Attempt something before asking for help • Be on task • Work as a team/group • Everyone contributes • Always justify and explain your thinking
Algebra 1 Block 1 Norms	**Algebra 1 Block 8 Norms**
• Showing your work • Talking to each other/listening to each other (do not talk over others) (engaged in math conversations) • Coming into class with an open mind/positive set • NOT GIVING UP • Use your resources • Try even if you are unsure • Question yourself to see if your work is right/check your work • Put your distractions away • Take corrective criticism	• Listen to others • Ask for help— ask 3 before me (the teacher) • Discuss the problems with each other • Collaborate with each other (work with your peers) • Try SOMETHING before you ask for help
	Algebra 1 Block 4 Norms
	• Work as a team/partner • Talk about the problem • Stay on task • Try SOMETHING before asking for help • Give effort • Don’t goof around

Note. This figure shows class negotiated norms that were developed by following the “Building a Mathematical Classroom Community Plan” (Gray et al., 2018).

My students' recorded views of 'doing math' supports the strength of discourse as a social norm. Further, the various codified norms in each class related to discourse also support that strength. Unfortunately, as my students had a natural tendency to engage in conversation, they also at times became distracted and disengaged, talking about anything but the assigned task.

To try to harness student discussion as a positive influence on the learning environment, I made various teaching moves to prompt on-task student discussion. Oftentimes this took the form of instructing students to 'turn and talk' with their shoulder partner based on various prompts such as: "tell your partner what do you notice and wonder about this situation or picture;" "tell your partner what are the most important constraints in this problem situation;" or more specific prompts that dealt with a recently introduced concept the curriculum included on the Google Slides for any given lesson. Another teaching action I have used to take advantage of discourse is providing students with quiet work time to begin their work on a task, followed by prompting students to share their strategies at their table groups. These teaching actions are complementary to the learning activities and structures in the curriculum itself which prompt discourse, but beyond that, also promote a culture of inquiry in learning mathematics—a culture discussed in the following section.

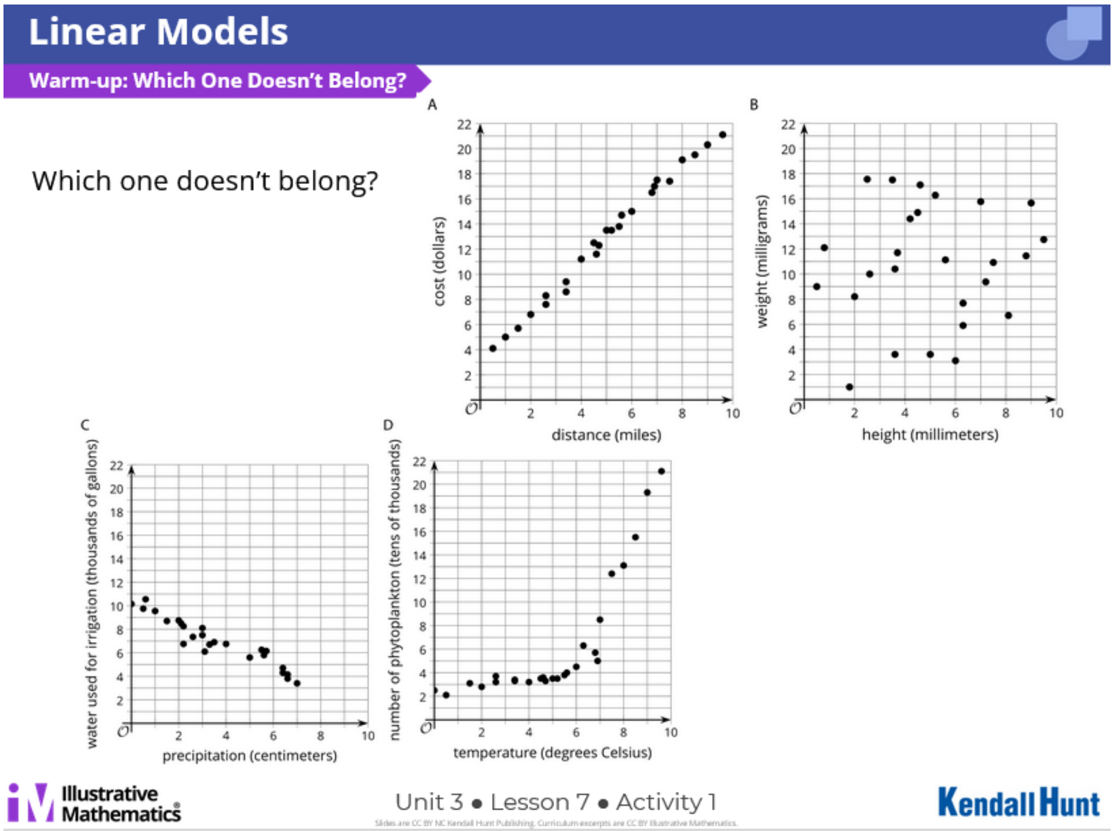
My findings indicate that discourse as a social norm had a less direct influence on facilitating student struggle than either collaboration or peer tutoring. It also had fairly obvious connections with the social norm of collaboration as it is through student discussion and sharing their thinking that collaboration occurred. Yet another intersection with the social norm of peer tutoring occurred when two shoulder partners had a clear understanding differential. However, it is the structured discussion prompts that gave discourse a more nuanced nature than either collaboration or peer tutoring. The reason I tried to purposefully direct student discussion on certain occasions is to address various levels of student struggle: ‘getting started,’ ‘giving mathematical explanations,’ and ‘expressing misconceptions or errors.’ Notice and wonder tasks prompted student discussion that assisted with any range of struggle. More specific prompts like the ones discussed in the following focus lesson typically addressed higher levels of student struggle like ‘giving mathematical explanations’ and ‘expressing misconceptions or errors.’

Correlation Coefficient

As one of the final blocks of the semester for algebra 1, a combination of lessons investigated the idea of correlation coefficient to see how to evaluate the fit for regression models and data. A “which one doesn’t belong” task invited students into the mathematics and began the structured discourse for the day (Figure 5.7). Most of my students identified scatterplot B as the one that does not belong. However, my students are familiar with this structure now and are well versed in my further challenge in rationalizing why the other graphs might not belong. Without prompting, students continued the task by volunteering why other graphs do not belong. For example, graph A could have a line of best fit that fits better than all the other data; graph C is the only data with the appearance of a negative slope; and graph D appears to be the only one that is ‘curved’ [students to this point have not learned about exponential models].

Figure 5.7

Algebra 1, Unit 3, Lesson 7, Activity 1



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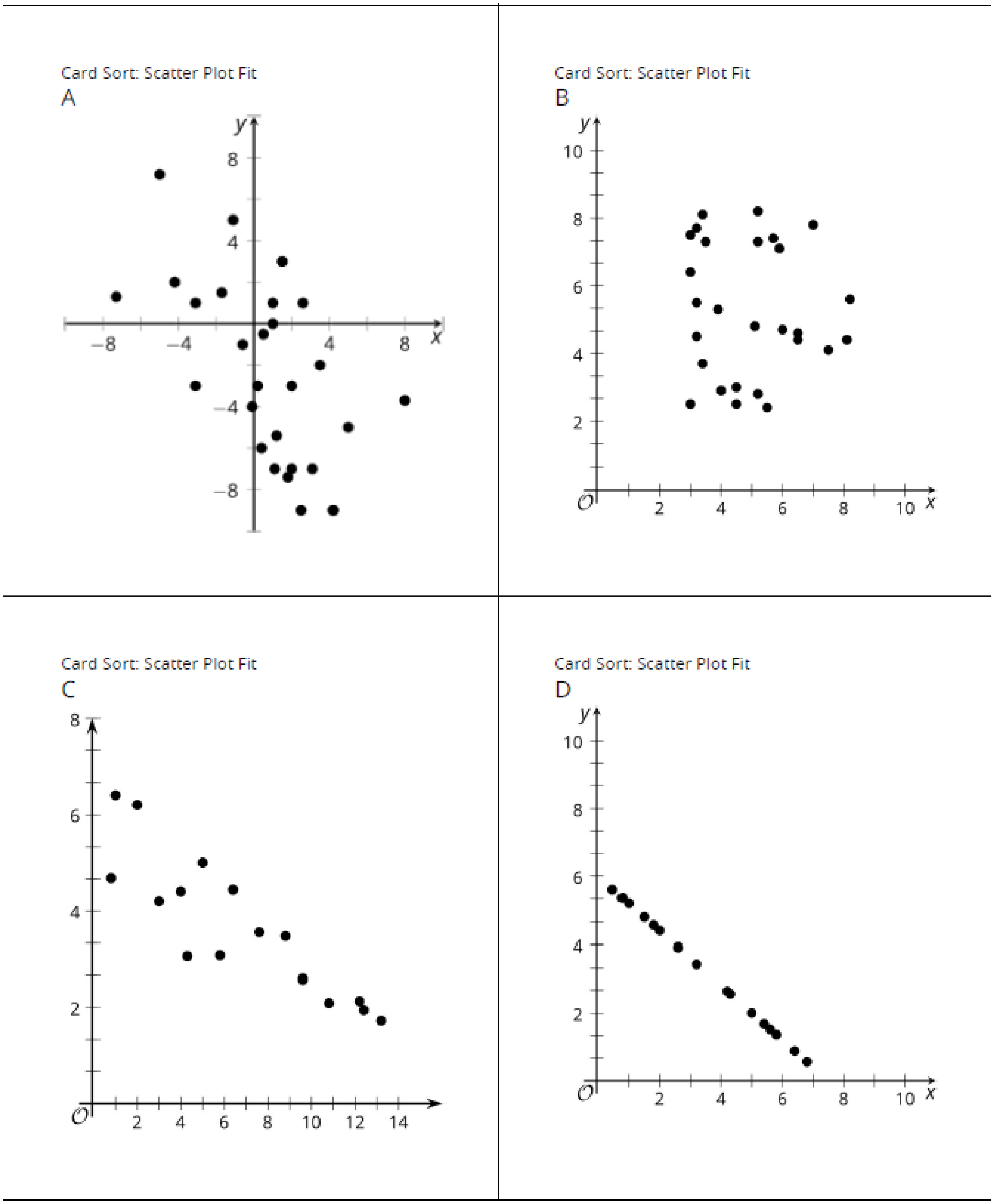
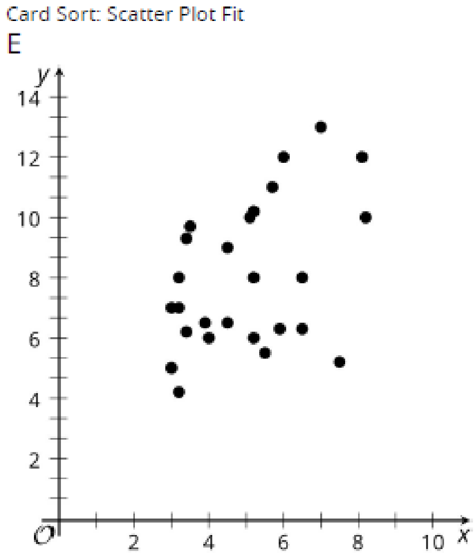
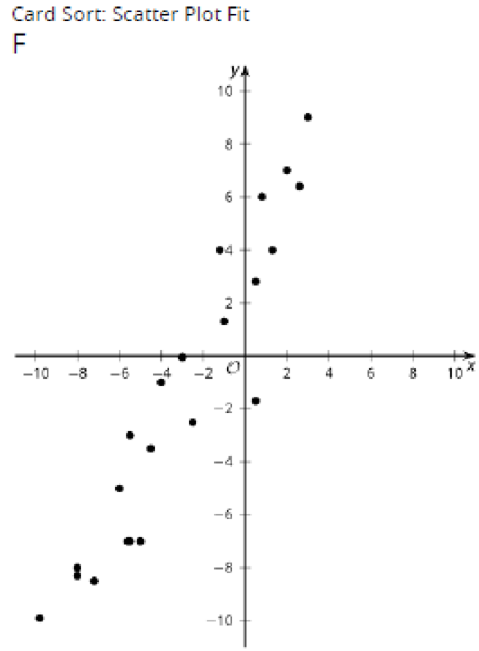
The discourse continued with a task also requiring collaboration: a card sort (see Figure 5.8). Card sorts require students to sort cards into categories of their own choosing and explaining those categories to either another group, the class, or the teacher. In this lesson, the students sorted the cards in the figure below. In this instance, the students had to produce two categories with an explanation, and then find two more categories still with an explanation. Students were then introduced to the definition of a correlation coefficient.

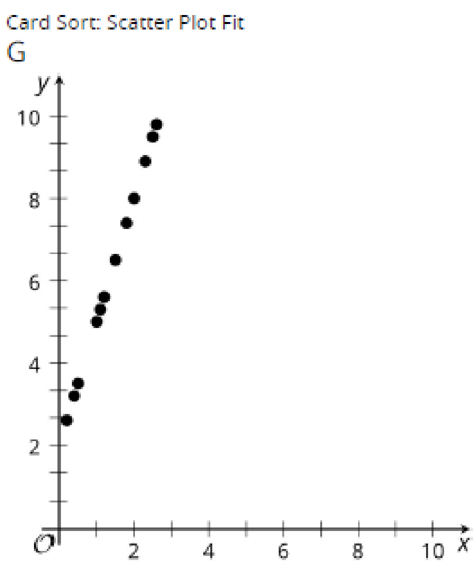
Figure 5.8

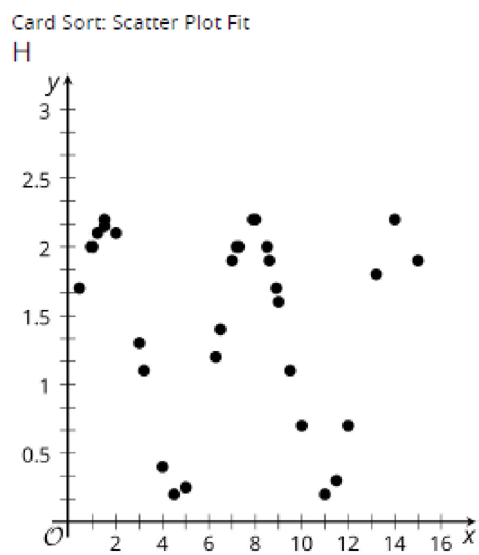
Algebra 1, Unit 3, Lesson 7, Activity 2 Card Sort




Card Sort: Scatter Plot Fit
E


Card Sort: Scatter Plot Fit
F


Card Sort: Scatter Plot Fit
G

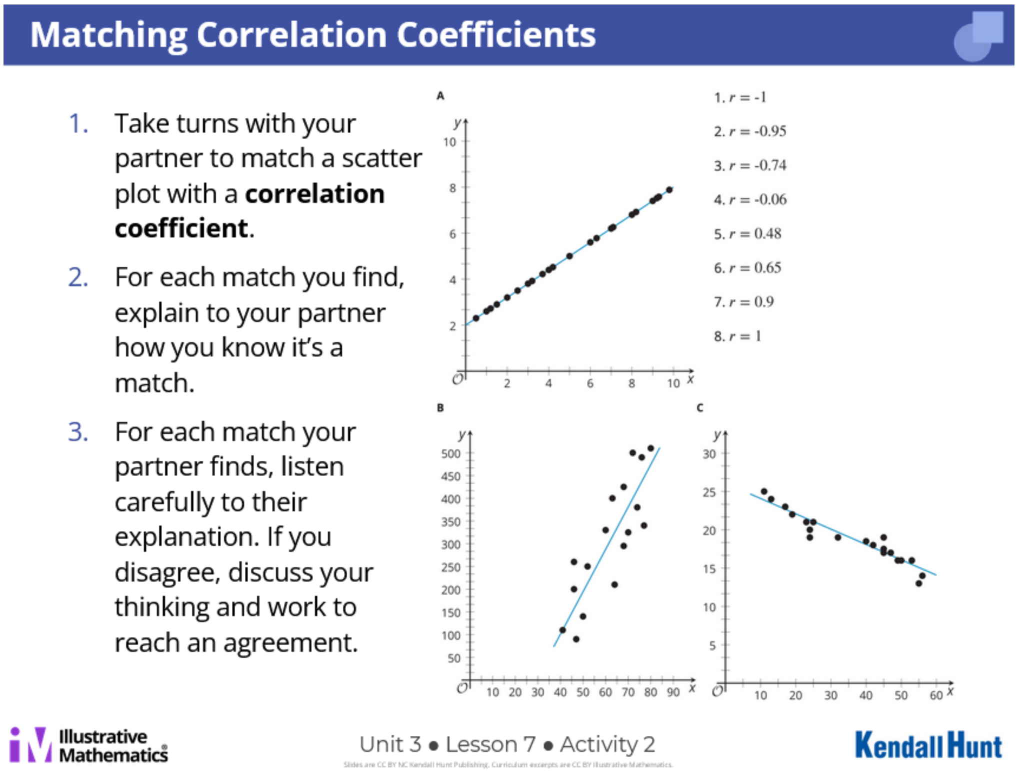

Card Sort: Scatter Plot Fit
H

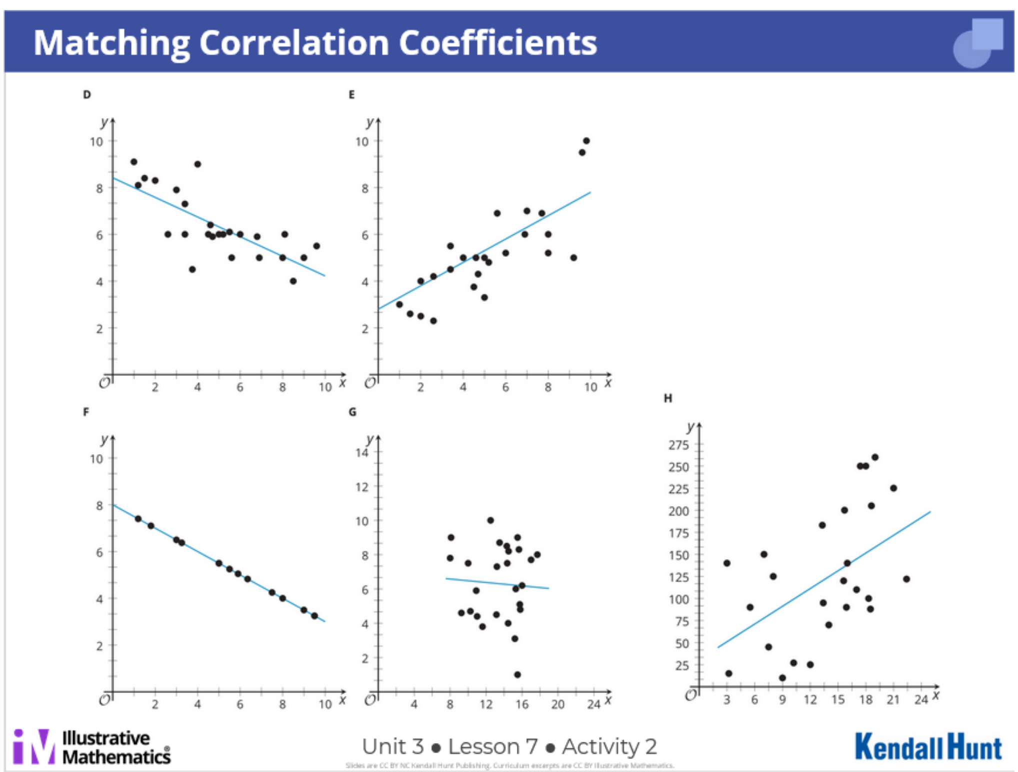
The following task (Figure 5.9) required students to use this definition collaboratively. Students took turns matching a correlation coefficient with a scatterplot, explaining their thinking to their partner. A prime example of the intersection of the social norms of collaboration and discourse.

During this activity, I observed numerous examples of students facilitating one another's struggle. I recall one student asking the question "well, which graph would be a perfect fit? So, which [r-value] one would that be?" to help facilitate another's 'getting started' struggle. 'Giving mathematical explanations' was partially addressed in the structure of the activity itself; however, I had to respond to this struggle several times. As the activity was directly using the definition of correlation coefficient just introduced, I used 'directed guidance' in redirecting students to use the definition in their thinking and explanations. When students were finished with the activity early, I encouraged them to get up to see if they could help (peer tutor) any struggling pairs. I witnessed several iterations of students facilitating other students 'expressing misconceptions and errors,' using 'directed guidance' by mimicking my actions—redirecting their peers to the definition.

Figure 5.9

Algebra 1, Unit 3, Lesson 7, Activity 2





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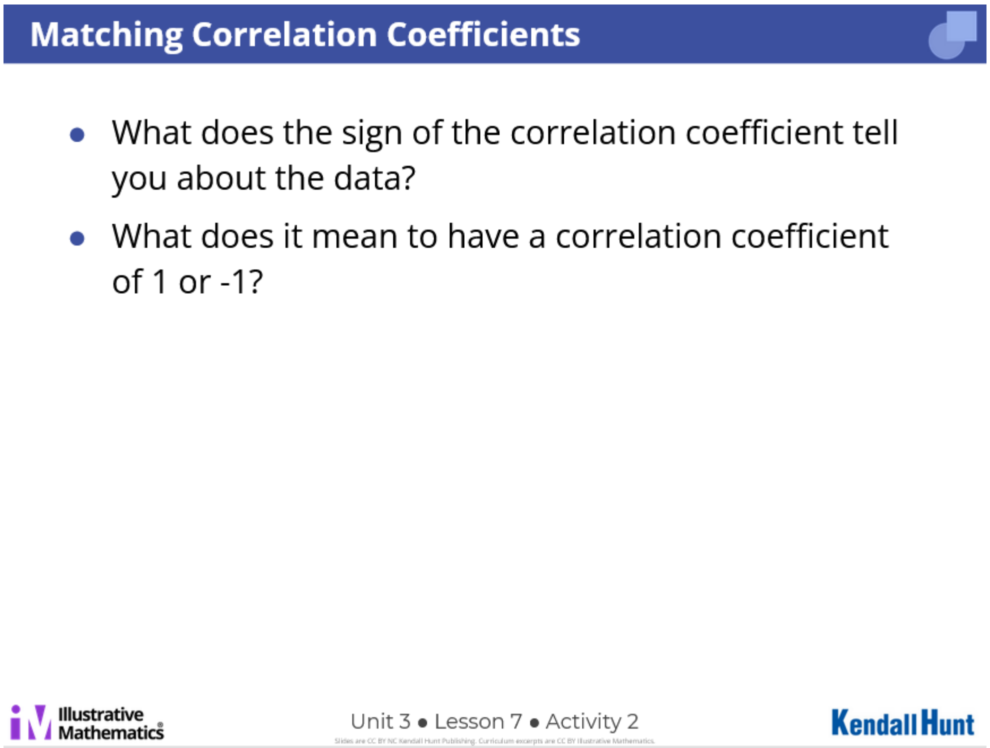
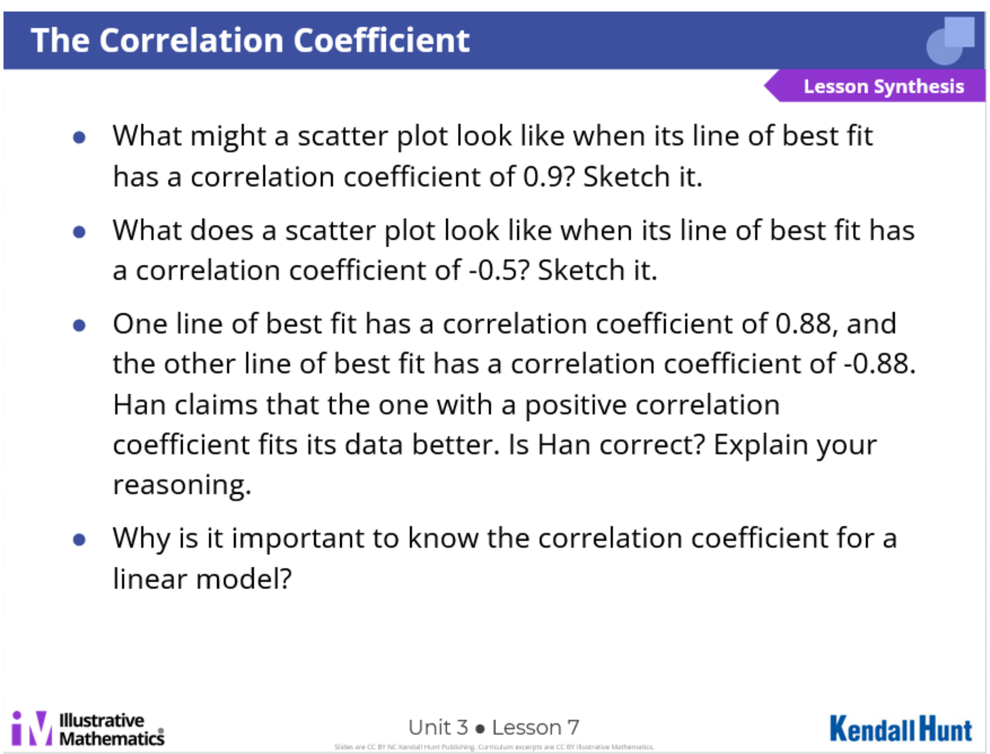
Student discourse helped to summarize the lesson with the prompts shown in Figure 5.10. I had students choose one of the prompts to tell their partner to conclude activity 7.2 (Figure 5.10 Google Slide on the lower left), whereas for the lesson summary I provided students some quiet work time to make their sketches and think about the two discussion prompts (the bottom two bullets on the lower Google Slide). Lessons like this, and tasks like the activities described in this lesson have helped to develop a culture of inquiry in my classroom.

Figure 5.10

Algebra 1, Unit 3, Lesson 7, Concluding Discussion Prompts





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Summary: Social Norms and Productive Struggle

The development of social norms occurred throughout the course of the semester. Further, the efficacy of social norms in contributing to an environment that facilitates productive struggle varies each day. Students viewed collaboration as part of doing mathematics and engaged in learning together as a community. Collaboration tended to remove 'getting started' struggles and with some teacher use of probing guidance to help students reflect on their work resolved 'giving mathematical explanations' struggles. Initially, students did not recognize peer tutoring as a part of doing mathematics and this social norm required teacher moves to establish. Safety concerns led to peer tutoring occurring in table groups and with students' shoulder partners. Peer tutoring mostly led to students assisting one another with struggles in getting started and carrying out procedures. Student collaboration sparked organic student discussions that helped students in 'getting started' and in 'giving mathematical explanations.' Students viewed discourse as an integral part of how they engage in mathematics, and the norms developed by students reflect that. I classified discourse in this chapter as discourse that is prompted directly from teacher moves or from the curriculum itself. The social norm of discourse had fewer connections to how the classroom environment assisted in the facilitation of struggle. Specific prompts at times helped to address struggle characterized as 'giving mathematical explanations' and 'expressing misconceptions or errors.'

Chapter 6 - Theme 3: Sociomathematical Norms

The emergence and recognition of sociomathematical norms in my classroom came through extensive reflection and analysis in coding reflexive journal entries. Noticing relevant social norms to productive struggle came quite quickly by comparison. I kept asking myself, “what are the key ways students are interacting that are inherently mathematical in nature?” Initially, I just was not sure. It took reaching data saturation and even being deep in the draft writing process for this chapter to understand what happened in my classroom regarding sociomathematical norms.

I knew for sure that I needed to identify the contribution of the curriculum to the environment. For one, it is an identified supporting research question for this study: How does a teacher utilize curriculum resources and ancillary informants to guide their negotiation of classroom norms? But beyond that, the piloting of this new curriculum meant not only is it shaping my students' engagement and interaction with mathematics, but it also shaped my evolving mathematical understanding in terms of pedagogical content knowledge. I began by identifying what I saw as key tasks and question types that are used repeatedly in the curriculum for both algebra 1 and geometry. The three discussed in the section entitled ‘Developing a Culture of Inquiry’ became recognized by my math teacher colleagues as important for not only students’ developing conceptual understandings but also their engagement and enjoyment of mathematics.

My findings suggest that sociomathematical norms are not as easily identified within math classrooms as parsing out norms with inherent mathematical characteristics is more difficult than identifying social norms that could apply in any classroom. One such sociomathematical norm is the norm of mathematical inquiry, which is described in more detail in the following section. Alongside the social norms which govern expectations for student behavior are ways in which students have engaged in learning mathematics, more specifically, in solving problems and math tasks. To frame this norm, I described essential aspects of the curriculum which contribute to this culture of inquiry. A brief discussion of traditional sociomathematical norms of acceptable, different, efficient, and sophisticated concludes this section.

Developing a Culture of Inquiry

Within a math classroom environment, there are numerous factors: teacher expectations, and students' interpretations and responses to those expectations are only a few. As important as those taken-as-shared norms are the math tasks and curricula in which students engage. These math tasks helped to shape students' current and future dispositions toward mathematics based on coding and analysis of reflexive journal entries. One of my favorite aspects of teaching with *Illustrative Mathematics* (2019) are the designed warm-ups for each lesson. These warm-ups are designed to spark student thinking in a similar direction to the learning goal for the day. My findings suggest that beyond sparking student thinking, warm-ups are also intricately connected with the classroom microculture. These warm-ups helped to build a culture of mathematical inquiry—a sociomathematical norm. In reflection, they also helped to develop more productive student dispositions toward mathematics. More productive dispositions led to better engagement in mathematics, which increased instances of productive student struggle.

My only classes that generated 'doing math' actions indicative of mathematical dispositions, positive or negative, were my block 8 algebra 1 class and my block 3 geometry class. I imagine this is because there were enough students who were confident enough to voice negative experiences rather than only telling me a version of what they thought I wanted to hear.

In Table 6.1, I highlighted 'doing math' actions that indicate either a positive or negative view of mathematics. Of the highlighted actions, unfortunately, only one is positively oriented: "Fun Fresh and Funky"—which may also be a silly answer that group of students wanted me to write down on the board.

Table 6.1

Doing Math Actions: Connections to Mathematical Dispositions

Block 8 Doing Math Actions	Block 3 Doing Math Actions
• Chaotic • Agreeing and disagreeing—making a solution out of that • Putting your heads together/working together • Each member has a different opinion • Discussing math • Frustrating/Stressful • Fun Fresh and Funky	• Groaning and asking the teacher for help--sometimes happy • Crying—copying off one person (asking others for help) • Parents frustrated that you do not do it the "right way" • Working with other people—can't do it alone • Mental health awareness • Collaborating—working off one another's ideas • Getting frustrated • Being off task • Problem-solving • Everyone contributing

Note. These 'doing math' actions were developed by students as I implemented the "Building a Mathematical Classroom Community Plan" (Gray et al., 2018). The highlighted actions indicate positive or negative dispositions that students in these classes held.

One essential norm or expectation to building this culture that contributed to both engaging in problem solving and provoking student struggle is an expectation that I intentionally pushed for in negotiating norms for each of my classes, i.e., the idea that students must try something in engaging in the problem prior to receiving help. The exact norms and phrasing each class produced that indicated productive dispositions in mathematics toward productive struggle or sociomathematical norms are below in Table 6.2.

Table 6.2

Student Negotiated Norms: Connections to Mathematical Dispositions

Block 1 Algebra 1	Block 3 Geometry
• Coming into class with an open mind/positive set • Not giving up • Use your resources • Try even if you are unsure • Question yourself to see if your work is right/check your work	• Show your work • *Attempt something before asking for help* • Be on task • Always justify and explain your thinking
Block 4 Algebra 1	Block 7 Geometry
• *Try something before asking for help* • Give effort	• [engage in] Trial and error • *Try something before asking for help*
Block 8 Algebra 1	
• *Try something before you ask for help*	

Note. These norms were developed by students as I implemented the "Building a Mathematical Classroom Community Plan" (Gray et al., 2018). These actions indicate positive or negative dispositions that students in these classes held.

Both block 1 and block 3 demonstrated some awareness of norms that promote struggle and the development of sociomathematical norms. Despite this recognition, I have continually re-established the above expectations for engaging in problem-solving each class period. A key finding suggests that an established environment does not stay established without direction from the teacher and adherence from the students. Renegotiation and re-established norms were easier to continue in comparison to the initial negotiation and establishment. But they are without a doubt continually negotiated. This re-establishment occurred in my classroom with the launch of each activity by setting out the expectations for how students should either work alone or collaboratively, what they should talk about, the work I should see before giving them help, in addition to typical expectations that students “ask three before me” or always explain or justify their thinking.

Warm-ups

The warm-ups for each lesson are all intended to begin lessons as invitations to learn mathematics. One consistent finding from reflective journal entries suggests these invitations allow students to experience a small success at the onset of a lesson with a non-threatening task as well as oftentimes activate relevant prior knowledge necessary for success in the following tasks. For example, both ‘Which One Doesn’t Belong?’ (See Figures 6.1 and 6.2) and ‘Notice and Wonder’ (See Figures 6.3 and 6.4) promote mathematics that has no set ‘middle’ or ‘end,’ meaning the process of doing the task and the solution of the task are up for interpretation and discussion. It is this key characteristic, the idea that there is more than one solution, that has helped to change some of the terms of engagement in my classroom. Students frequently came into my classroom with the perspective that doing mathematics is finding one answer or way of thinking quickly. The remaining category of warm-ups is ‘Math Talk’ activities (See Figures 6.5 and 6.6) which help to promote student discourse.

Figure 6.1

Geometry, Unit 1, Lesson 11, Activity 1 Which One Doesn't Belong?

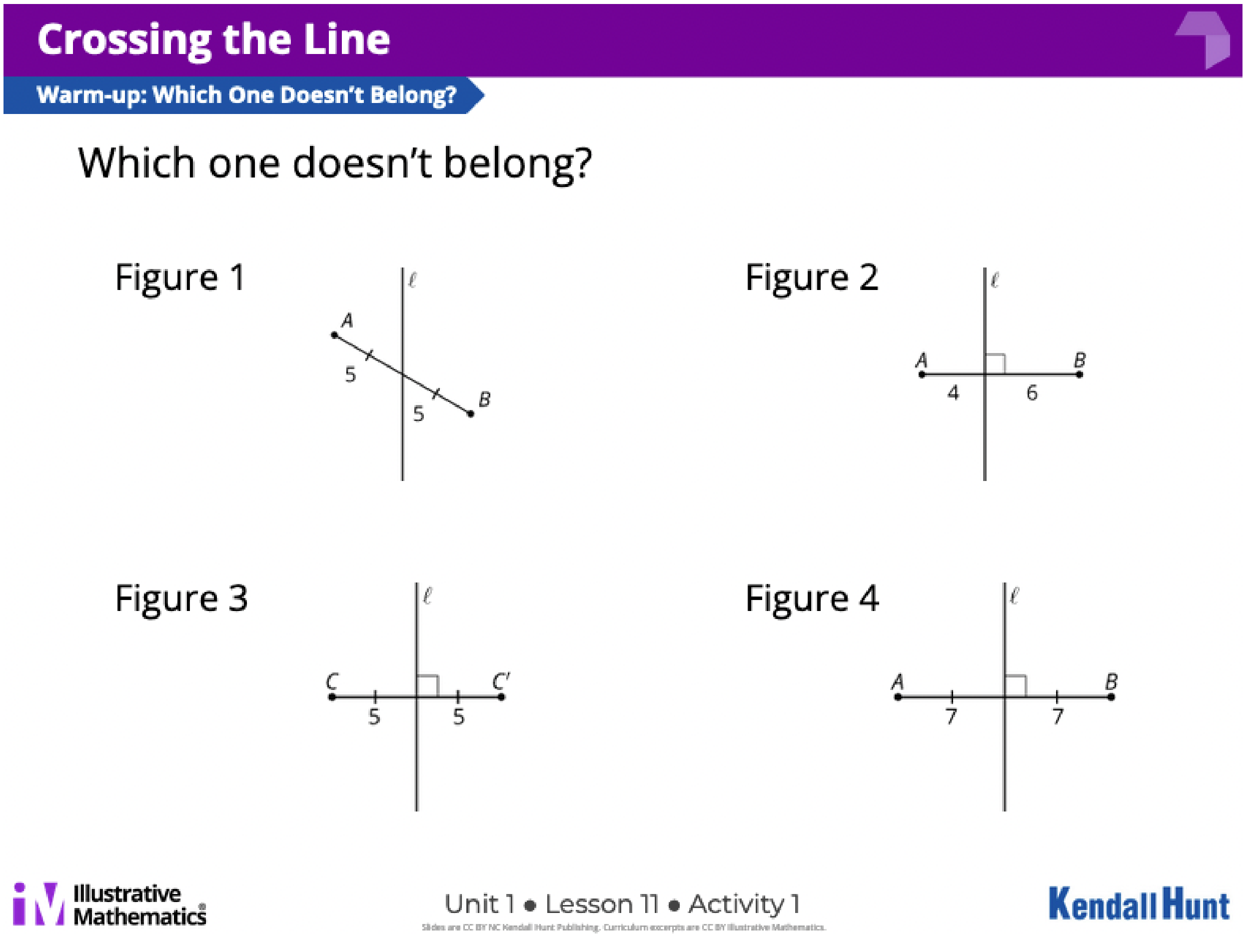

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Figure 6.2

Algebra 1, Unit 2, Lesson 5, Activity 1 Which One Doesn't Belong?

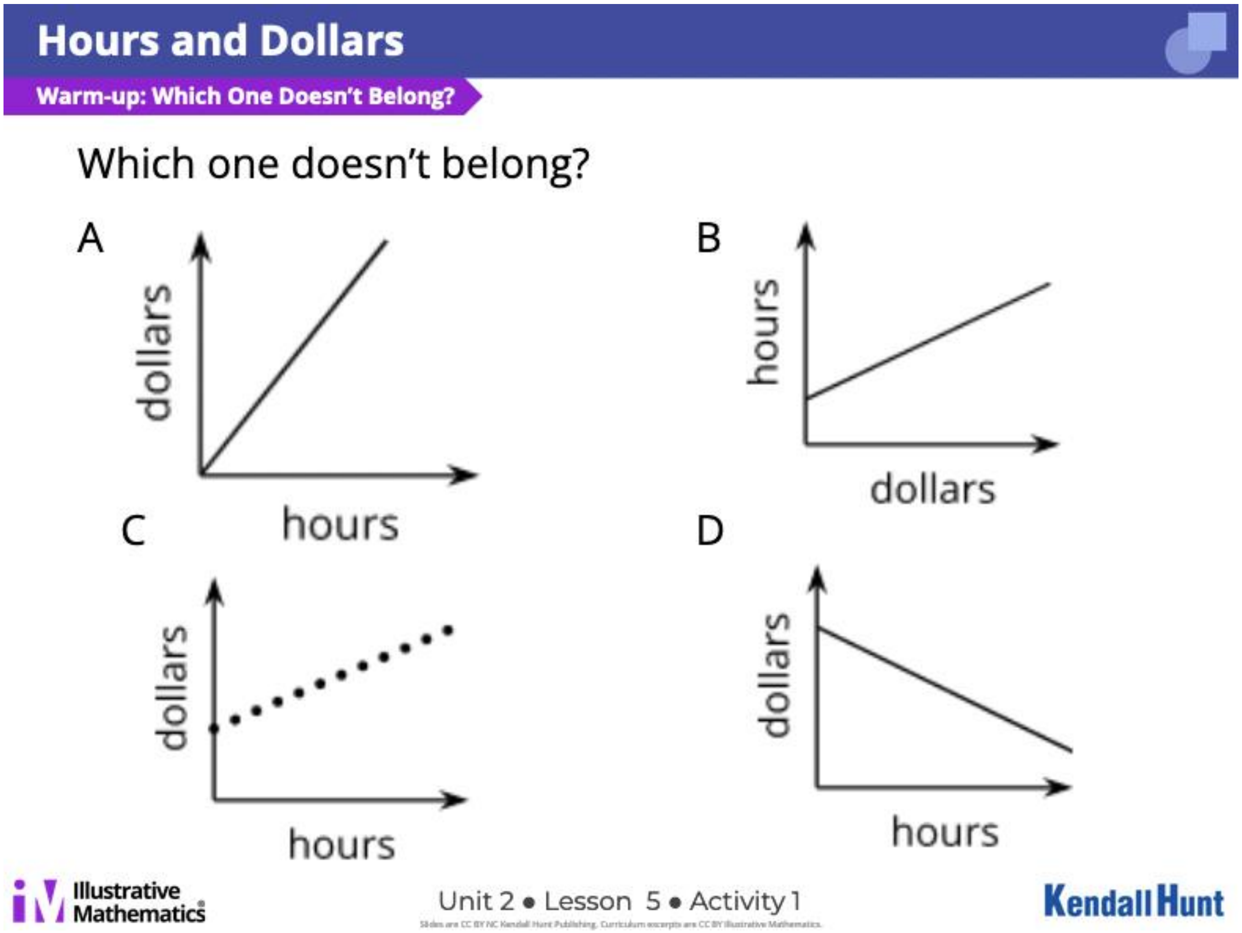

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Figure 6.3

Geometry, Unit 1, Lesson 4, Activity 1 Notice and Wonder

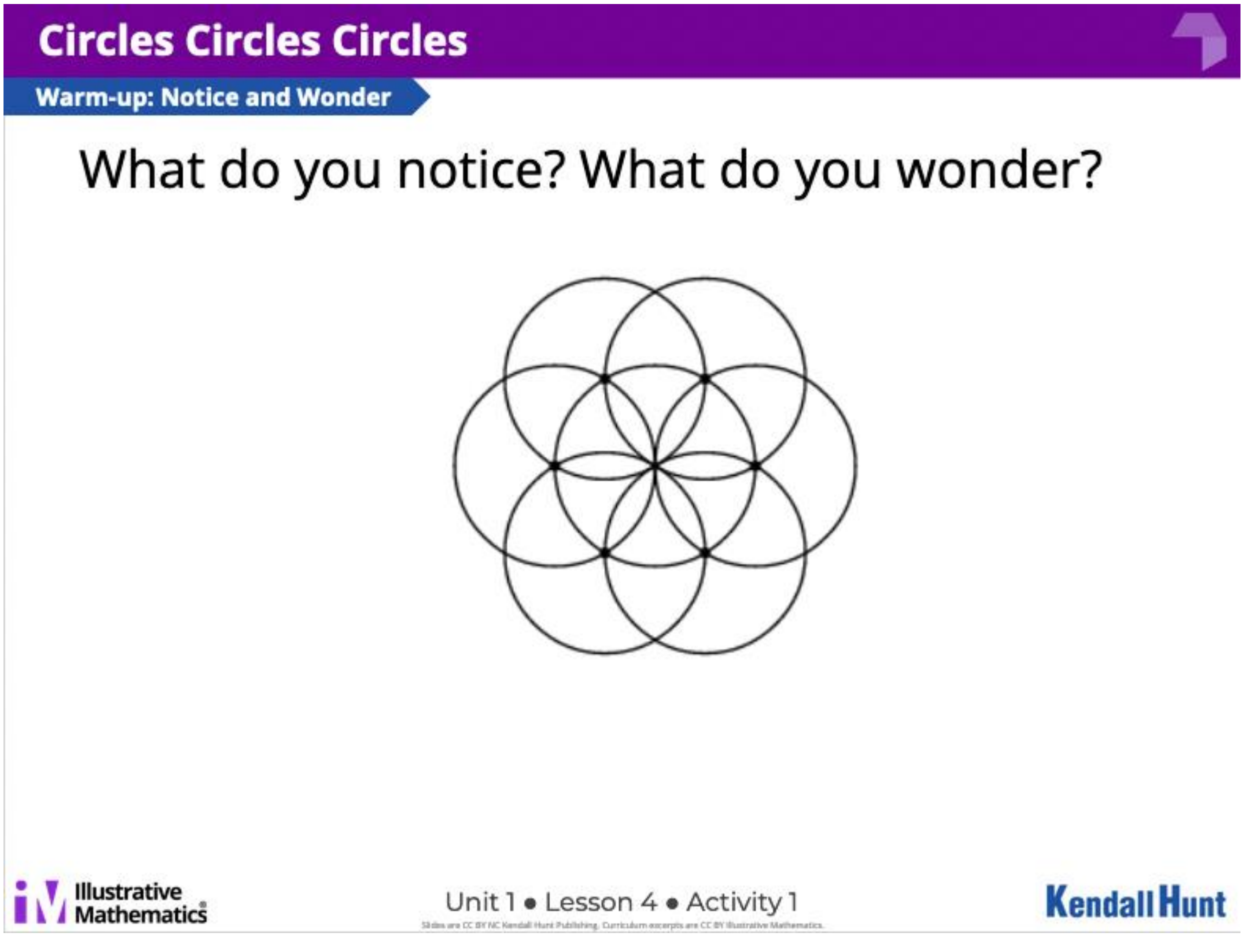

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Figure 6.4

Algebra 1, Unit 2, Lesson 14, Activity 1 Notice and Wonder

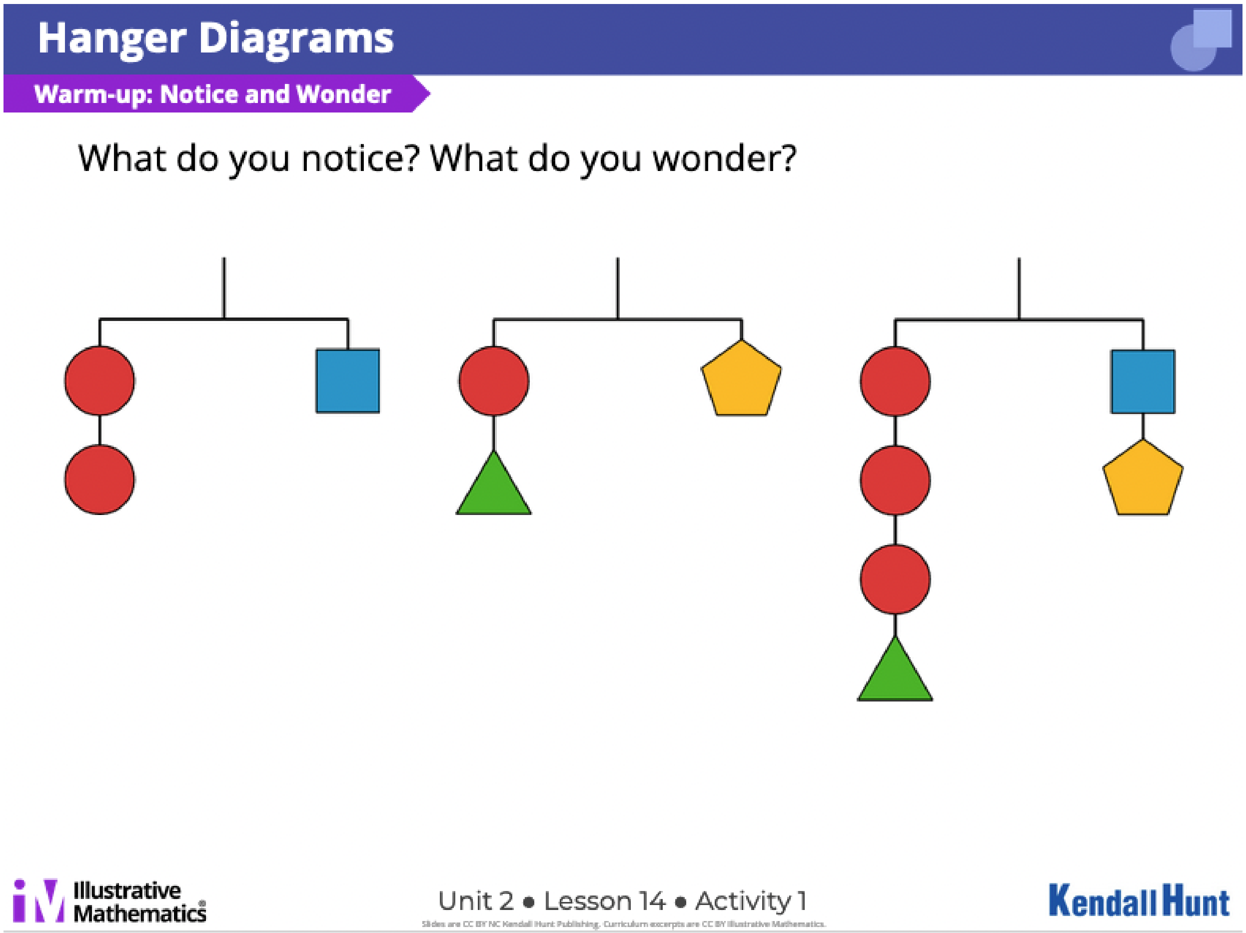

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Figure 6.5

Geometry, Unit 1, Lesson 17, Activity 1 Math Talk

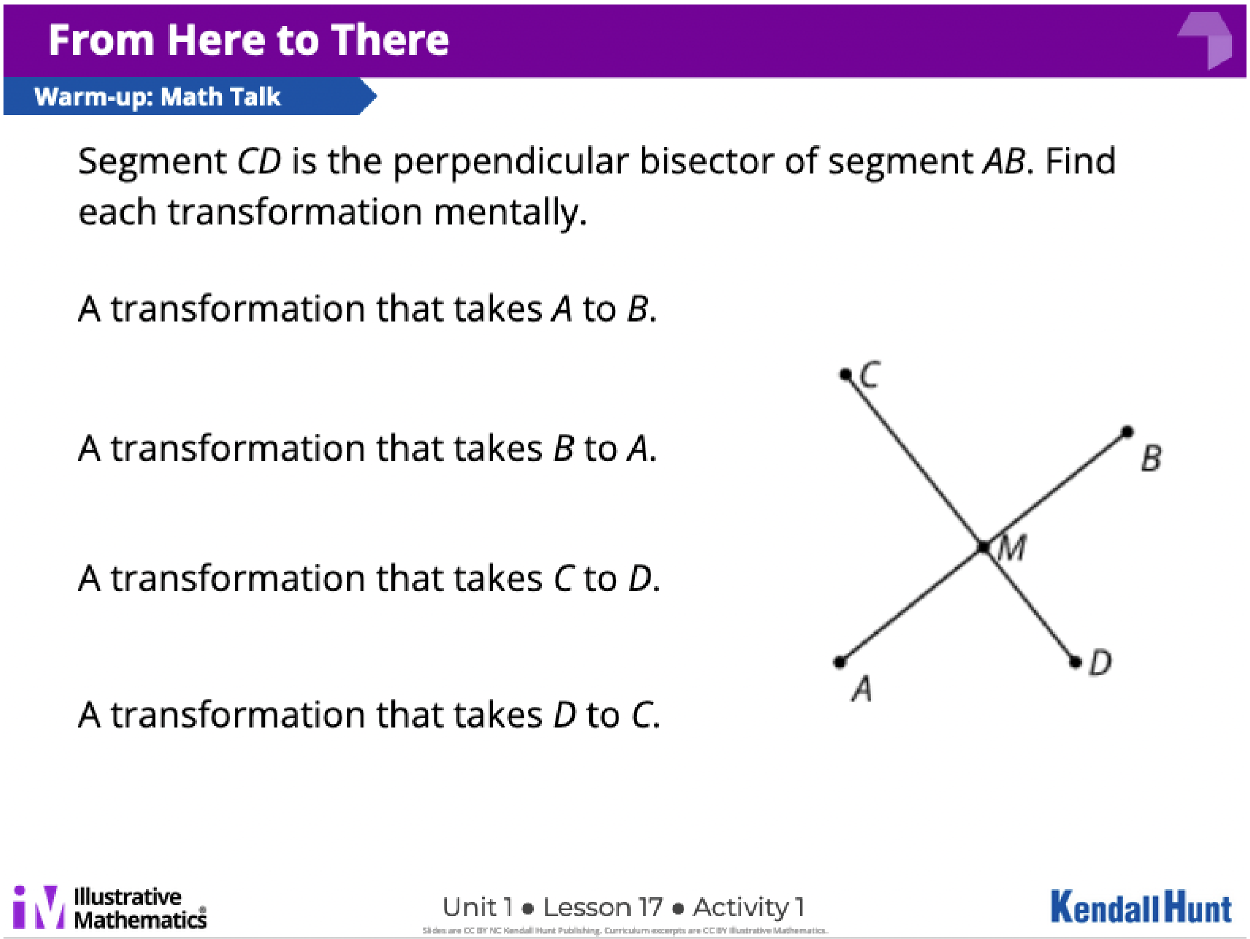

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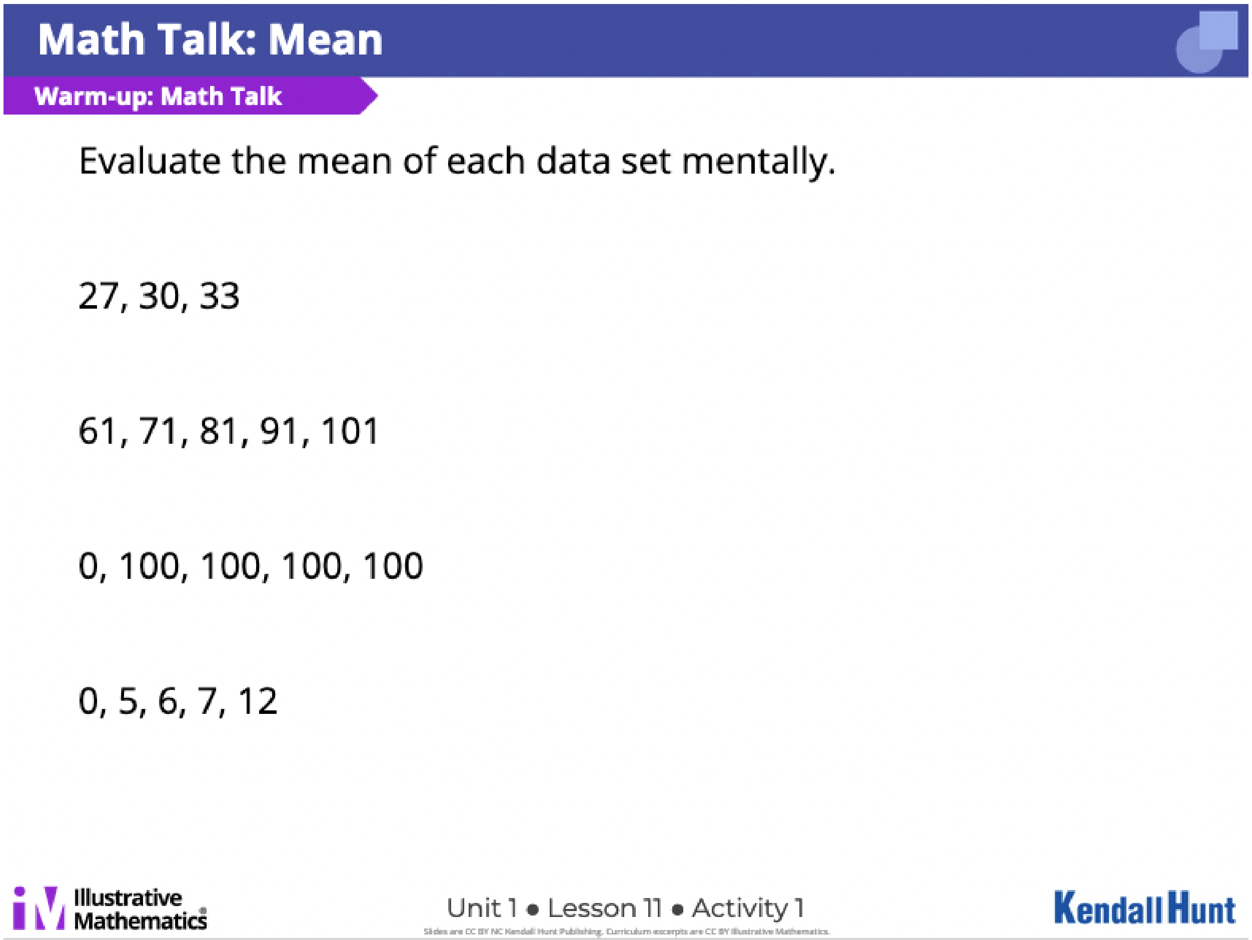
Figure 6.6

Algebra 1, Unit 1, Lesson 11, Activity 1 Math Talk



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Thankfully, there were no obvious connections to this attitude from the student responses to “What does it look and sound like to do math together as a mathematical community?” Regardless of the lack of indications of performance and speed-based mathematics, I still took purposeful action to launch warm-ups with a productive mindset for my students. For ‘Which One Doesn’t Belong’ warm-ups, I established the expectation that they find their first answer for which one does not belong, and to then challenge themselves to switch perspectives and find reasons for why *each figure* does not belong. Another expectation I had to stress is that I want to know how they justify their thinking. Similarly, I made the expectation clear for 'Notice and Wonder' tasks that "I'm not looking for one answer. I am looking to see what you remember and what you recognize. In these problems, no detail or pattern is too small to notice." Initially, my students struggled with wondering, so I had to frontload that part of the task with "what questions come to your mind when you look at this situation?" For 'Math Talk,' I had students turn and talk with their partners, helping to establish the previously described structured student discourse.

At first, my students did not exhibit enthusiasm for either 'Which one doesn't belong?' or 'Notice and Wonder.' The first time we did each of these prompts, it required waiting and pressing for more rationale for why a figure did not belong or more notices and wonders for the classwide discussion. After several iterations of these warm-ups, my reflexive journal entries suggest they became a positive influence on the lesson. On multi-lesson blocks, students are frequently disappointed when I skip the warm-up for the second lesson of the block, asking that we stop and do the second warm-up. These requests further suggest the positive influence of *Illustrative Mathematics* (2019) warmups on the classroom environment.

When students struggled, I tried to take advantage of the structure and culture that these warm-ups established. Because students became familiar with these structures and ways of thinking, they were more willing to engage in novel mathematical concepts. For example, 'Which one doesn't belong' activities activated relevant prior knowledge and started students on a line of pattern-seeking relevant to the learning goal. This pattern-seeking led to better engagement and productive dispositions toward mathematics based off student behavior. 'Math talk' warm-ups helped to spark structured student discussion emphasized earlier in my discussion over discourse as a social norm.

Most relevant to facilitating student struggle, however, is the prompt of "what do you notice and wonder?" I used this phrase repeatedly with multiple types of student struggle, namely 'getting started' and 'giving mathematical explanations.' Oftentimes this finding allowed students the opportunity to re-engage with the problem in a nonthreatening way for 'getting started' and helped them notice relevant problem or figure constraints necessary for answering a question.

Card Sorts

Card sorts were not a feature of every lesson. However, they tended to occur once or twice for each unit of study. For each card sort task, each student and their shoulder partner sorted cards into categories of their own choosing or of pre-identified ways that reflected ideas already learned in class. For example, the geometry card sort in Figures 6.7 and 6.8 had students engage with cards by their similarity. Then students practiced essential fluency skills with each similar pair: writing similarity statements by identifying corresponding parts of the similar triangles, identifying scale factors, and using scale factors to find the missing lengths. This geometry activity had a review-oriented nature, with the challenge for students came with putting each of these skills to use in a single task. The algebra 1 activity had students engage in exploring and classifying lines of best fit (Figures 6.9 and 6.10), but also practicing fluency in underlying skills with the remainder of task one in sorting by slope and y-intercept. Finally, students put these skills together in assessing if the line of best fit is a good fit or not in task two.

Card sorts helped to support the social norms of collaboration and discourse. Students collaborated in making meaning of their learning together during these activities. During their interactions, student discourse came as a natural consequence. At times, small iterations of students' peer tutoring occurred during these activities when one of the partners had a gap in learning that the other understood. However, I tried to take action to activate relevant prior knowledge and or reteach underlying concepts for students to engage in these tasks at their intended level rather than allowing for unproductive struggle when students either could not recall prior learning or did not initially learn that content in the first place.

Figure 6.7

Geometry, Unit 3, Lesson 12, Activity 2 Card Sort

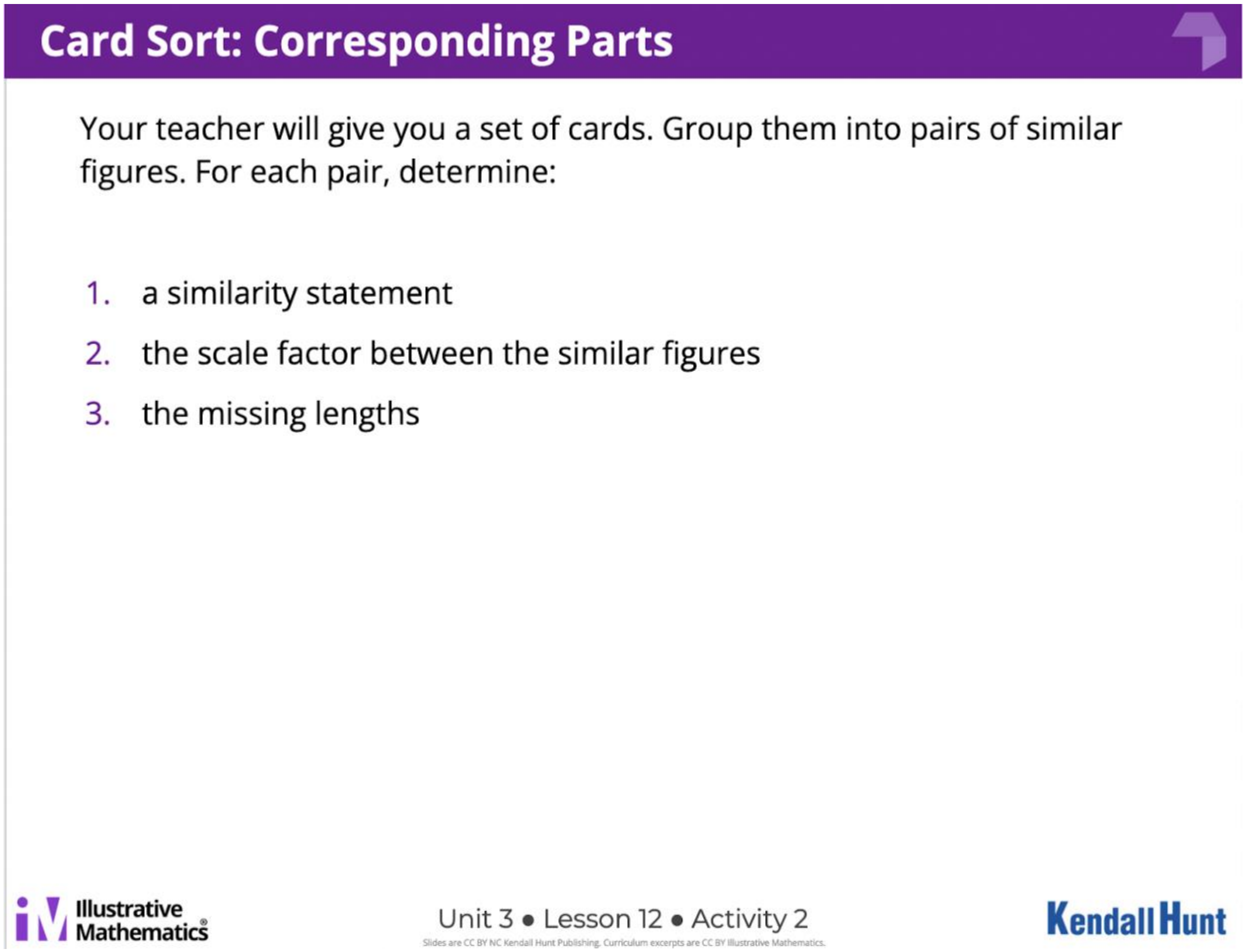

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Figure 6.8

Geometry, Unit 3, Lesson 12, Activity 2 Cards

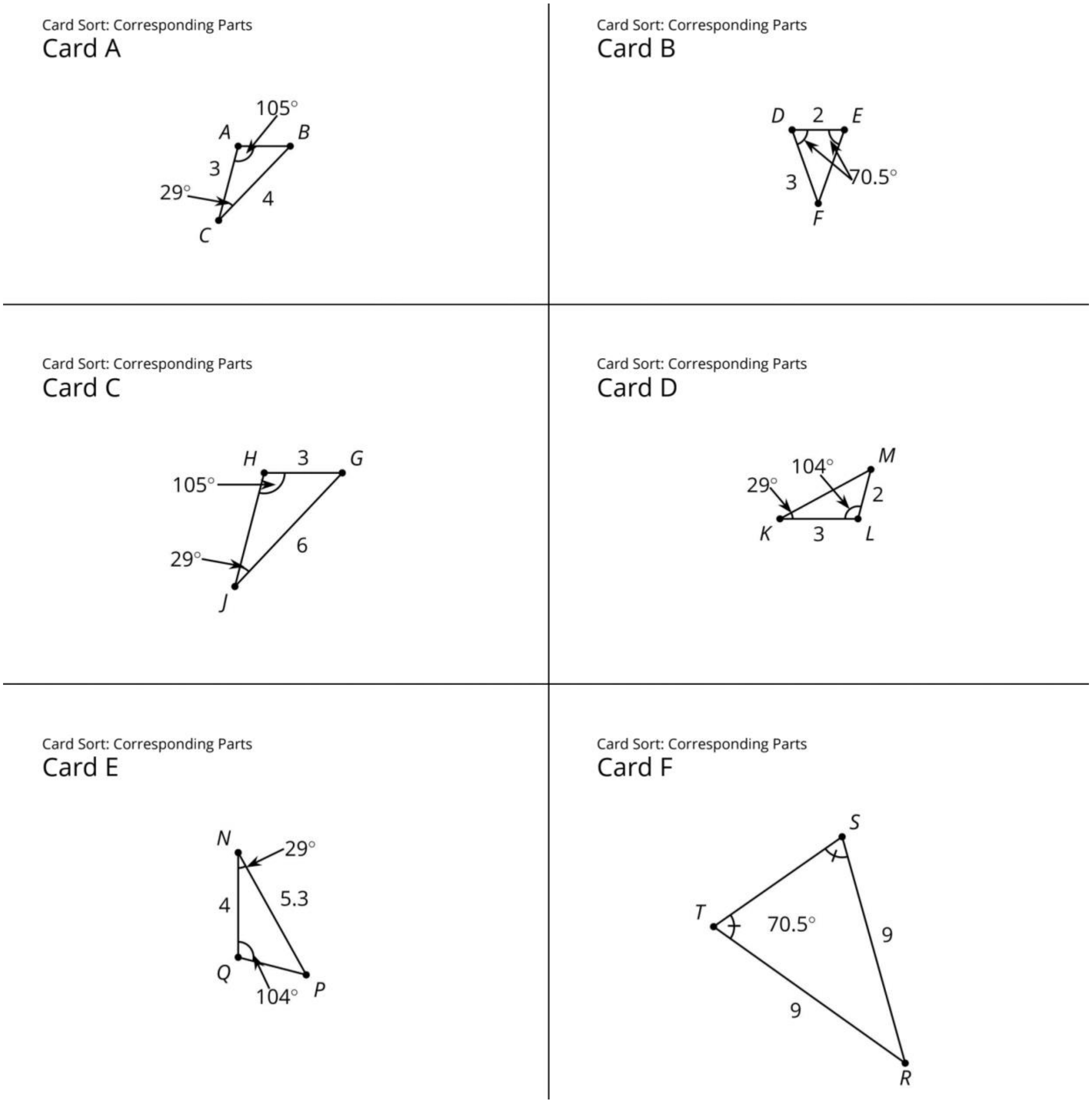

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Figure 6.9

Algebra 1, Unit 3, Lesson 5, Activity 2 Card Sort

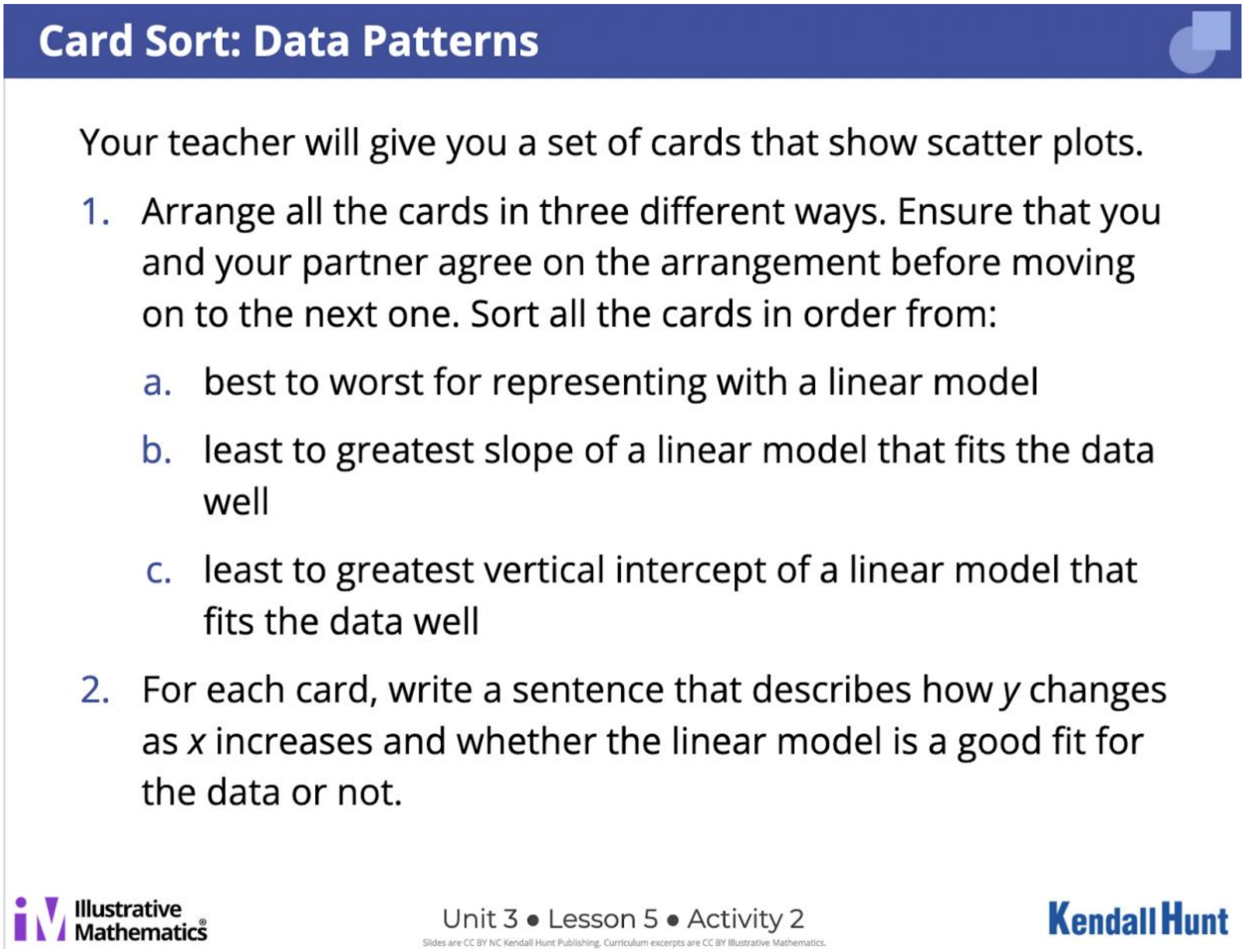
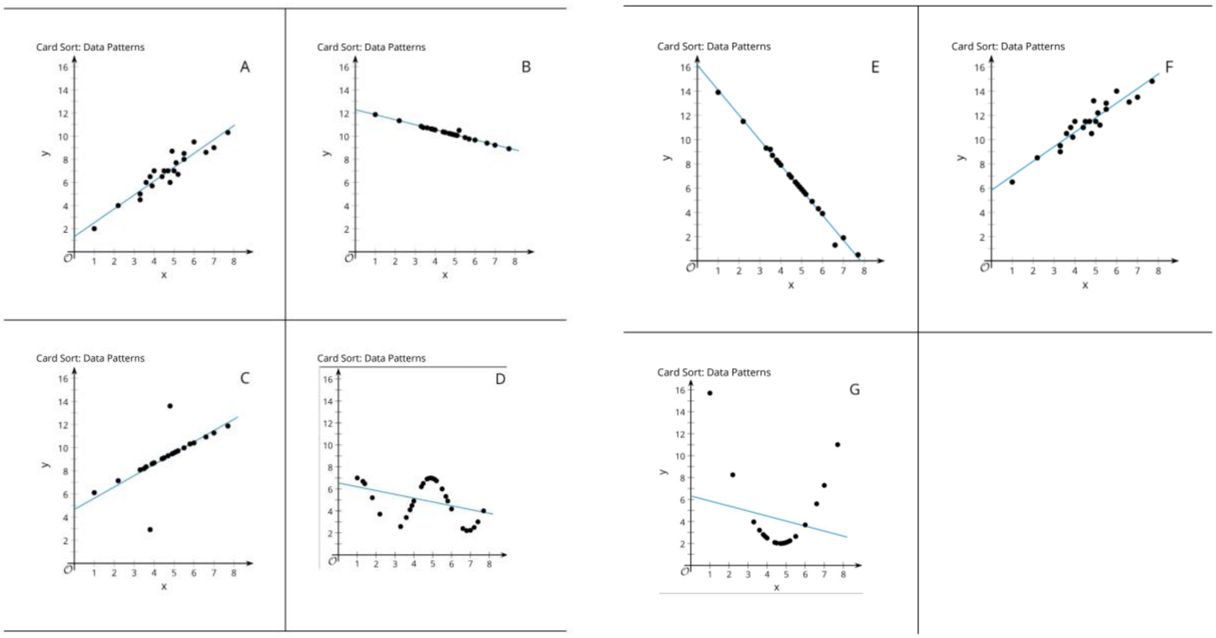


Figure 6.10

Algebra 1, Unit 3, Lesson 5, Activity 2 Cards



Note. Reproduced from "Fitting Lines" from IM Algebra I © 2019 Illustrative Mathematics.

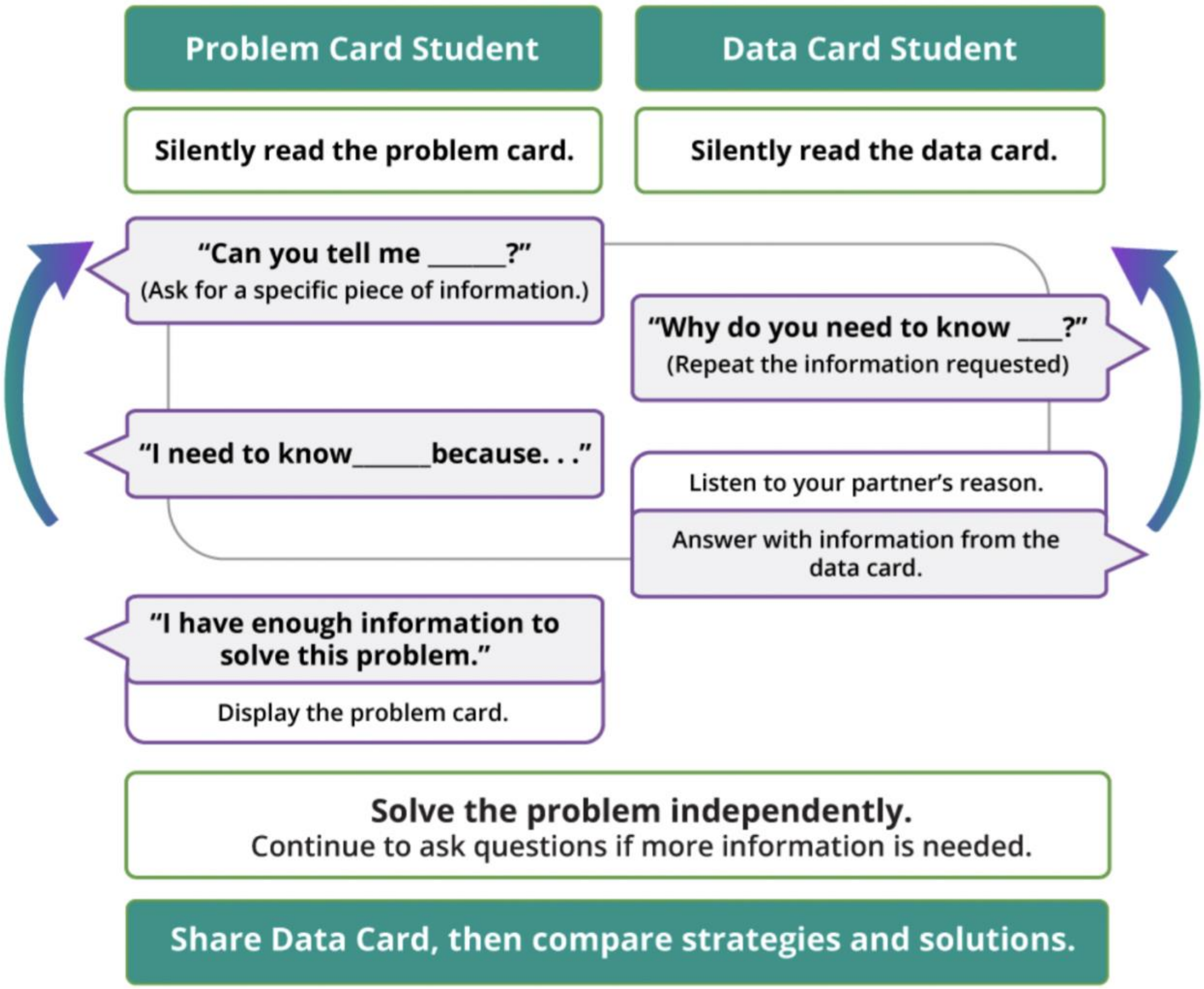
Info Gaps

Much like the card sorts, info gaps occurred once or twice each unit. For each info-gap (see Figure 6.11), students worked as partners. One student had the information card, and the other student had the problem card. The student with the problem card had to request relevant information from the student with the data card. The data card student required the problem card student to provide a rationale for why they needed that information. When the student with the problem card believed they had enough information, they shared the question with the data card partner, and both students solved the task.

For example, an algebra 1 student solving the problem cards in Figure 6.12 needed to inquire about the specific rules regarding team makeup and even write several equations and inequalities to represent the rules. This info-gap activity came as a culminating activity for a unit of study which required students to use their skills and fluency in writing systems of equations or inequalities, and partially in interpreting graphs. The geometry info gap in Figure 6.13 alternatively exists as an exploratory activity in a unit in which students discover triangle congruence theorems. To facilitate this activity a little more clearly given varying levels of precision for students, I made a GeoGebra applet so students used technology to modify their triangles to the combination of lengths and angle measures so congruence between triangles could be clearly seen or not between data card and applet.

Figure 6.11

Instructions for Info-Gap Activities



Note. Reproduced from "Instructional Routines" from IM © 2019 Illustrative Mathematics.

Figure 6.12

Algebra 1, Unit 2, Lesson 25, Activity 3 Info Gap

Info Gap: Terms of a Team

Problem Card 1

To participate in a competition, each team must follow the rules about the allowable number of adults and children on a team.

1. A team has 6 adults and 8 children. Does the team meet all the membership rules of the competition?
2. What is the maximum number of adults allowed on a team?

If helpful, use the coordinate planes in your workbook or graph paper.

Info Gap: Terms of a Team

Problem Card 2

Another competition has different membership rules.

1. What is the minimum number of members that a team could have? How many adults and children would be on that team?
2. What is the maximum number of adults allowed on a team?

If helpful, use the coordinate planes in your workbook or graph paper.

Info Gap: Terms of a Team

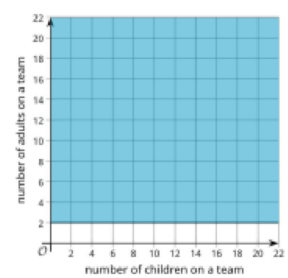
Data Card 1

- A team without adults is allowed.
- A team with only adults is not allowed.
- If 3 adults are on a team, there must be at least 6 children.
- Each team must have no more than 16 members in total.
- The number of adults must be at most half the number of children on the team.

Info Gap: Terms of a Team

Data Card 2

- A team with only adults is not allowed.
- A team with only children is not allowed.
- The number of children must be at least 3 times that of adults.
- The total number of people cannot exceed 20.
- This graph represents a rule about the number of adults on a team.



Note. Reproduced from "Solving Problems with Systems of Linear Inequalities in Two Variables" from IM Algebra I © 2019 Illustrative Mathematics. Licensed under the Creative Commons Attribution 4.0 license.

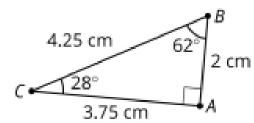
Figure 6.13

Geometry, Unit 2, Lesson 4, Activity 2 Info Gap

Info Gap: TMI
Problem Card 1

Construct a triangle that is congruent to your partner's. What is the least amount of information that you need?

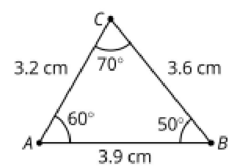
Info Gap: TMI
Data Card 1



Info Gap: TMI
Problem Card 2

Construct a triangle that is congruent to your partner's. What is the least amount of information that you need?

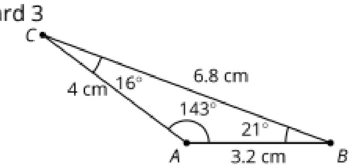
Info Gap: TMI
Data Card 2



Info Gap: TMI
Problem Card 3

Construct a triangle that is congruent to your partner's. What is the least amount of information that you need?

Info Gap: TMI
Data Card 3



Note. Reproduced from "Congruent Triangles, Part 2" from IM Geometry © 2019 Illustrative Mathematics. Licensed under the Creative Commons Attribution 4.0 license.

To make this activity work, I actively encouraged students to try to minimize and change up the information they requested from their partner, having students focus very clearly on the learning goal for the lesson: "Let's figure out if there are shortcuts for being sure two triangles are congruent." Students quickly learned they could ask for three sides and that provided the information they needed. However, with the end goal in mind of discovering not only side-side-

side triangle congruence, but also side-angle-side, and angle-side-angle, I suggested that students look for other combinations of angles and sides that allowed them to create congruent triangles. In summarizing the activity as a class, students volunteered their combinations of triangles that worked, which allowed the class to discover each triangle congruence theorem.

Student engagement in info-gap activities began on a rough footing, where students had little understanding of the purpose of the task. This is due to my less than stellar launch of these activities. But as the semester progressed, I improved my framing of the activity. Accordingly, student understanding of the terms of engagement increased, as did student ability to identify what types of information they needed in solving a problem. Over time, info-gap activities helped contribute to collaboration as a social norm and mathematical discourse framed from a teacher's standpoint. During these activities, students made meaning out of their experiences toward a learning goal. Most importantly, students had to question themselves and their partners to complete the task. This active questioning in pursuit of relevant knowledge contributed to creating an environment of shared inquiry.

Different, Acceptable, Efficient, and Sophisticated

Identifying where students engaged in traditionally characterized sociomathematical norms was difficult. For one, it was more difficult to hear student conversations with COVID-19 precautions. Additionally, most of my focus has centered on how I respond to struggle and how various aspects of the classroom environment contributed to students' engagement in productive or unproductive struggle. One finding is that the rich conversations where students debated mathematical ideas tended to occur when students engaged in struggle that I classified as 'giving mathematical explanations'—this struggle did not always require my intervention.

As a result, focusing on students shaping their explanations and understandings did not come to the forefront of my analysis until Sherri asked me to prepare a presentation for her Noyce scholars about norms in the mathematics classroom. Much of my thinking crystallized when I thought about my findings visually. Preparing for that presentation led me to create concept maps to visualize my results. This visualization ended up helping me understand the intersection of sociomathematical norms and productive struggle much more clearly. Going back to thinking about how I should communicate these ideas to preservice teachers seemed to take some of the fog away from my analysis and understanding of this research.

Traditionally characterized sociomathematical norms fall in the categories of different, acceptable, efficient, and sophisticated (Yackel & Cobb, 1996). Students in my classroom do not necessarily categorize shared solution methods or ideas in that way, but they had the opportunity to critique them, ask questions, or even add to them. Following the launch of an activity, where I described the expectations for engagement in a task, students proceeded to engage in the mathematics of the task. As students explored, I used purposeful questioning to provide ‘probing’ and ‘directed guidance’ to productive student struggle. I used a response of telling to scaffold when struggle became unproductive—mostly when that struggle did not represent the learning goal. There is one more important teaching action that occurred during this phase of the task: observing student work and asking their permission to share their ideas when I facilitated mathematical discourse to summarize as a class. Student ideas during mathematical discourse were most often shared in order of sophistication: acceptable or different, efficient, and sophisticated.

However, including strategies that had misconceptions or mistakes in addition to the correct solution strategies seemed to improve the quality of discourse in my classroom based on reflexive journal entries. Sharing some mistakes or misconceptions in my experience can be just as powerful as sharing only correct conceptualizations. To find these useful mistakes or misconceptions I asked students for their permission to use their work and offered to keep their work anonymous. My findings suggest that the classwide work of critiquing, offering feedback, and improving a common mistake or misconception is just as much a sociomathematical norm as any of the correct conceptualizations. A prime example of this occurring at a group level came in the discussion of the lesson entitled “Proofs About Quadrilaterals.” There, students critiqued one another’s proofs, offered feedback, and worked collaboratively toward a proof to be shared with the class.

Beyond the use of sociomathematical norms as a teaching strategy when orchestrating mathematical discourse, I classified students sharing mathematical ideas during warm-ups and even card-sorts as sociomathematical norms in action. Different, acceptable, and sophisticated noticings from ‘Notice and Wonder’ tasks came from students as they looked for and made use of structure in each figure (CCSS.MATH.PRACTICE.MP7). Further, these same sociomathematical norms can be seen and shared in tasks in which students select ‘Which One Doesn’t Belong?’ The non-threatening ways in which students shared diverse levels and types of thinking on tasks with an open end (no one correct answer) this semester led to better engagement and better development of student dispositions toward mathematics based on comments made in reflexive journal entries regarding student behaviors. Lower attaining students contributing to classwide discussions with their ideas and mistakes has helped to give them some role as authors of mathematics rather than sufferers of mathematics.

Different ways of engaging and interacting with card-sort activities, namely producing personal ways of sorting the cards and sharing those explanations, provided opportunities for correct but various levels of sophistication in the student categories. Beyond the curriculum's influence and contribution toward the development of social norms, the ways in which students interact while doing mathematics, the curriculum also clearly contributes to sociomathematical norms—the ways in which students interact in the classroom that are inherently mathematical.

Summary: Sociomathematical Norms and Productive Struggle

Sociomathematical norms in this study helped shape students' perceptions of mathematics. Aspects of *Illustrative Mathematics* (2019) curriculum helped to create a culture of inquiry: the warm-ups, card sort activities, and info-gap activities. The warm-ups in particular allowed students to feel a small amount of success at the onset of lessons, which may positively influence their engagement in the learning activities that follow. Further, the open nature of the curricular aspects may contribute to students seeing mathematics more productively and assist the development of their productive dispositions. Within tasks, students struggled, and when students had discussions where they compared their ideas, students critiqued and evaluated their conceptions of mathematics. As a result of this discussion of traditionally characterized sociomathematical norms, students resolved some iterations of struggle characterized as 'giving mathematical explanations.'

Chapter 7 - Theme 4: Classroom Mathematical Practices

Reflecting on the relevant collective learning of my classes required a look at the scope and sequence of *Illustrative Mathematics* (2019) algebra 1 and geometry classes. This collective learning in my experience is best characterized as mathematics that requires little or no justification or discussion in its implementation or rationale. Further, these practices repeatedly were the skills that my students used in engaging in math tasks and productively struggling with novel learning. Unfortunately, using classroom mathematical practices to engage in new content also meant that students transferred these practices to novel concepts and did not see the relevance of learning a new way of thinking about that concept. This balance will be discussed in the next two sections, “Constructions,” and “Transformations.”

Illustrative Mathematics’ (2019) geometry curriculum began with a sequence of lessons over constructions. Students explored constructions that had them create perpendicular bisectors, angle bisectors, and regular polygons such as squares, rhombuses, and hexagons. Accordingly, students began to see the usefulness and importance of being able to create congruent circles. The next sequence of lessons had students investigate rigid transformations– transformations that preserve congruence– specifically, translations, reflections, and rotations. Students used these constructions and transformations to engage in the next unit of study over congruence congruent triangles. The basis in this curriculum for establishing triangle congruence theorems is based on rigid transformations, and several lessons embedded throughout this unit over congruence allowed students to use the fluency with constructions they previously established. The third unit of study over similarity began once again with transformations, introducing a non-rigid transformation, dilations, to add to students’ conceptual understanding of transformations.

Illustrative Mathematics' (2019) algebra 1 curriculum alternatively began with a unit of study over one-variable statistics. In their study of one-variable statistics, students developed an understanding of measures of center: mean, median, mode; measures of variability: range, interquartile range, mean absolute deviation, and standard deviation; as well as a particular focus on interpreting and assessing outliers following the investigations of measures of variability. The second unit of study had students investigate linear equations, inequalities, and systems. Students began the second unit by writing equations that build models of contextual situations—briefly described in the section above entitled "A Main Dish and Some Side Dishes." Students continued investigating the structure of equations and manipulating equations in subsequent lessons. Following this investigation of structure, a series of lessons had students learn methods to solve systems including substitution, elimination, and graphing. The final portion of the unit had students write systems of inequalities, and graph inequalities. The semester concluded with a unit of study investigating two-variable statistics, in particular, investigations over two-way tables, correlation, lines of best fit, and interpreting linear regression models.

The algebra 1 first semester scope and sequence as compared to geometry does not have a series of consistent underlying skills that connect large ideas between units of study. As a result, I am reluctant to identify any one classroom mathematical practice that students relied on throughout the first three units that clearly connects with supporting productive student struggle. Within units there are consistent mathematical practices. For example, within the second unit of study students consistently write models about problem situations to support their contextual reasoning, interpret models in context, and evaluate solutions for reasonableness. The closest mathematical practice that encompasses the breadth of the first semester is interpreting contextual models and graphs. However, these interpretations do not occur frequently enough, nor consistently enough for its justification as a support for student reasoning while they struggle. Further, the difficulty students have in individually making these interpretations makes any claim I make about the existence of ‘interpreting mathematical models in context’ as a classroom mathematical practice dubious. As a result of the lack of a semester-long unifying mathematical practice or practices, the classroom mathematical practice that best unified the algebra 1 experiences is using resources, the concluding section that follows “Constructions” and “Transformations.”

Constructions

Initially looking at *Illustrative Mathematics’* (2019) geometry curriculum, I found myself skeptical. Beginning a year with a unit of study over constructions? When I taught geometry previously, constructions always felt like one of those add-ons that students had to do to fulfill the standards, i.e., something that felt disparate and disconnected from the actual meat of the curricula. My experience as a student likely did not help me with this attitude—I can scarcely recollect doing any constructions. So, my teacher either skipped them or I did not see the point and did not remember. Luckily, my department head, Mrs. Garfield, had experience in teaching geometry with a central focus on constructions in previous schools in which she taught. She assured me that it is not pointless, and it most definitely helps to make geometric thinking more concrete. But I doubt that she anticipated the centrality that constructions would take in some of our students’ thinking. The established reasoning with circles, and perpendicular bisectors became a touchpoint for all our students to engage in learning new concepts.

Constructions in *Illustrative Mathematics* (2019) geometry in unit 1 involved students using compasses, straight edges, and pencils to create: regular polygons, polygons with all sides congruent; parallel lines, lines that do not intersect; perpendicular lines, lines that intersect at a right angle; segment bisectors, lines that intersect other lines at their midpoint; and angle bisectors, lines that cut angles in half. Students in my classroom created each of these constructions by drawing congruent circles (circles of the same size) and straight lines.

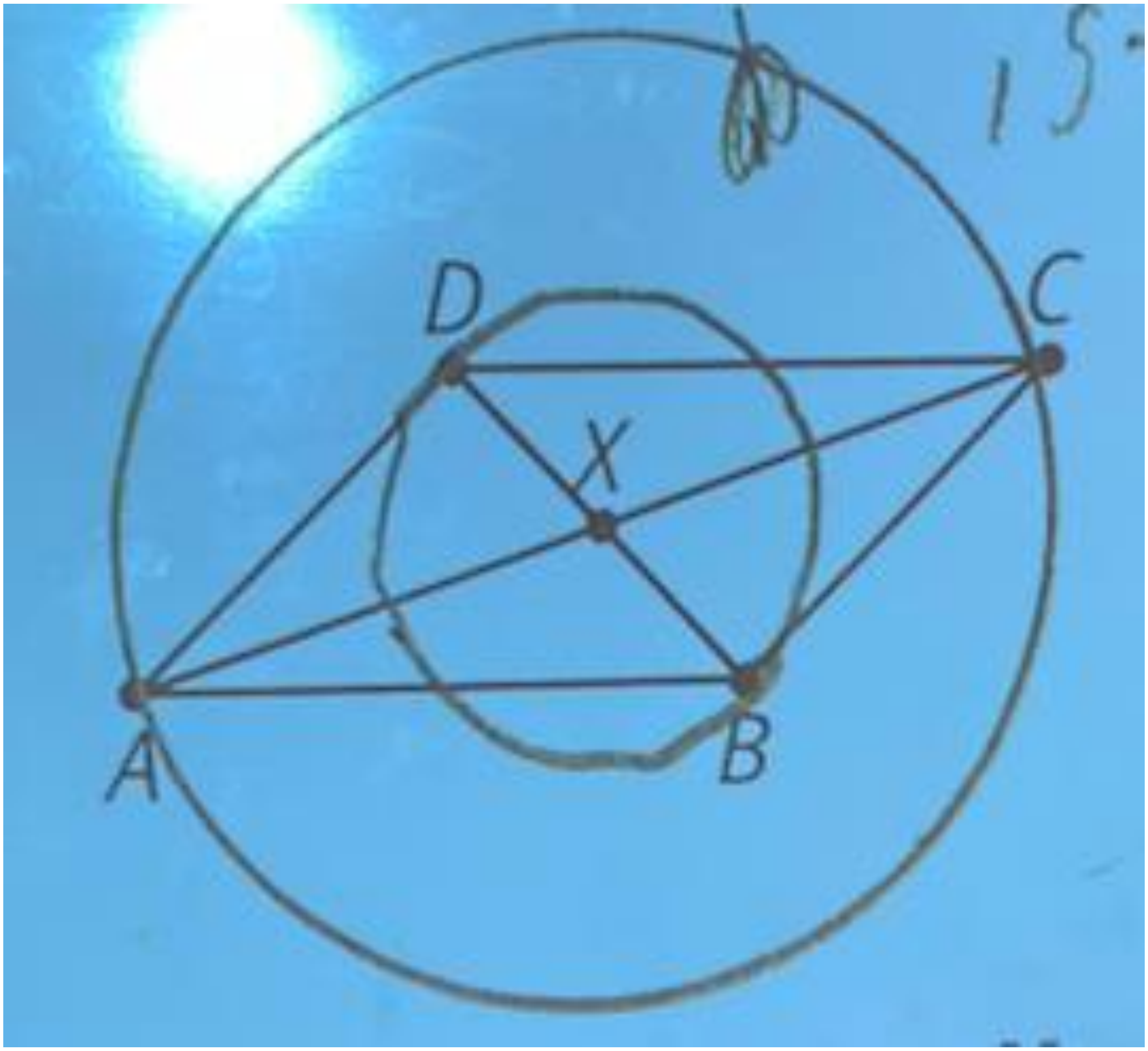
To my surprise and delight, my students applied constructions where I did not see the relevance—in reasoning with congruent triangles. Initially in unit 2, which involves reasoning about congruent triangles, students wanted to prove that triangles were congruent by demonstrating that each set of corresponding sides would create congruent circles as they had to use the same spot and center for their compasses to create those circles. This example demonstrates a finding in how classroom mathematical practices can allow students to engage in novel learning situations. Unfortunately, it also demonstrates how this mathematical practice can prevent new learning or new reasoning from occurring as the learning goal for that lesson involved how students could use transformations to engage in reasoning about congruent triangles. That shift in reasoning led to the establishment of rigid transformations becoming a new classroom mathematical practice.

Previously I described a lesson entitled "Proofs About Quadrilaterals." In that lesson, one of my students, Harry, selected the proof that diagonals bisect in a parallelogram. Harry asked if he could use a compass. Not knowing where he was going with this, I replied "sure you can." Harry smirked at me and said, "I can prove this using two circles" and proceeded to construct the circles in Figure 7.1. I was shocked to say the least. I had never considered an alternative approach to a proof normally requiring reasoning with triangle congruence. I told him "That's one of the most clever and elegant ways I've seen proving that diagonals bisect each other in a parallelogram." However, this provides yet another example of reasoning with classroom mathematical practices that are not aligned with the learning goal. To make sure

Harry could also reason along those lines, I pushed him to move forward in doing the proof using the triangle congruence theorems we had recently discovered and proven.

Figure 7.1

Student Work: Proof by Construction



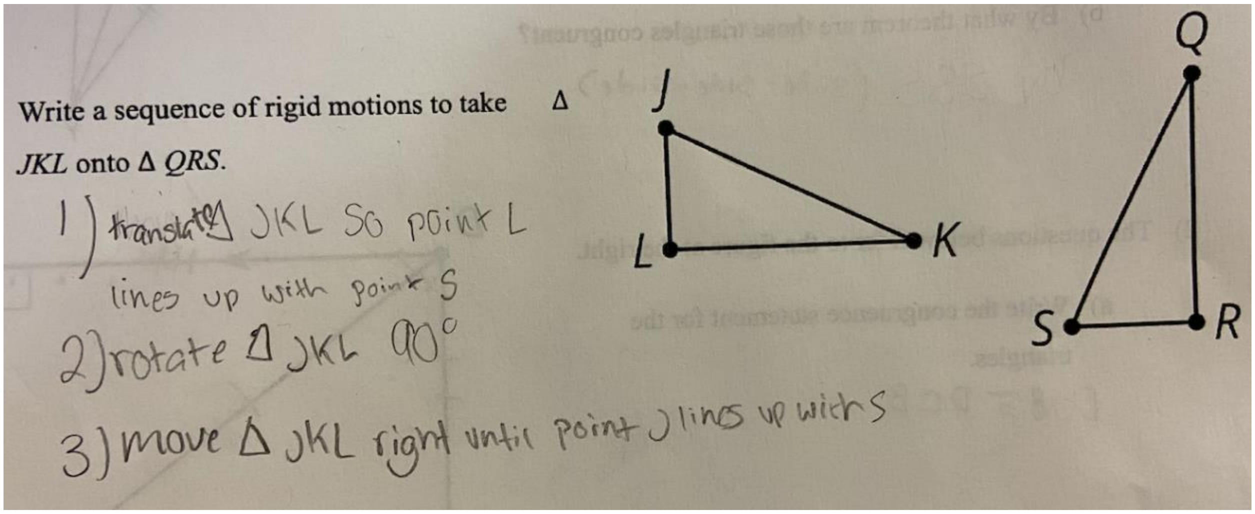
Transformations

In our previous curriculum, students only engaged in transformations using coordinate geometry. Points were translated by vectors, reflected over lines, and rotated around the origin in our Integrated Mathematics Course II—the sophomore year course (Core-Plus Mathematics Project, 2008). To explore these concepts, students used Geogebra, a free, exploratory geometry software. Beyond that initial engagement, around three or four class periods, I moved on to other topics and never touched on transformations again in Course II.

This year alternatively has had the opposite experience for my students. Figure 7.2 is a typical task in which students engage in demonstrating rigid transformations. As I previously described in the lesson episode entitled "Invisible Triangles," transformations shaped how my students viewed congruence and similarity with rigid (preserves congruence) and non-rigid (preserves angle measures) transformations, respectively. This worked out well when the students needed to prove triangle congruence theorems. Unfortunately, when the time came to engage in using triangle congruence theorems, many of our students wanted to continue using rigid motion to demonstrate that two triangles are congruent. Like working with Harry with constructions demonstrating that diagonals of a parallelogram bisect, I had to encourage these students to move forward in engaging with the learning goal: using the triangle congruence theorems to prove various conjectures about parallelograms and other figures.

Figure 7.2

Student Work: Rigid Motion Transformations



Use Resources

While my algebra 1 students had no algebra 1 specific classroom mathematical practice that spanned the scope of the semester *and* supported their struggle, they did use resources to support their struggle—resources that allowed them to engage in two standards for mathematical practice (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010b): Use appropriate tools strategically (CCSS.MATH.PRACTICE.MP5); and Model with mathematics (CCSS.MATH.PRACTICE.MP4). More accurately, they used appropriate tools to model with mathematics. For Unit 1, a study of one-variable statistics, students used a statistics calculator applet developed by Geogebra to calculate 5-number summary statistics (mean, min, Q_1, median, Q_3, and max) as well as a measure of standard deviation. Further, students used this applet to create histograms and box and whisker plots. For Unit 2, students wrote linear equations and inequalities to represent contextual situations. Reasoning with these contextual situations came through algebraic manipulations, solving by substitution and elimination, and graphing using Desmos—another online applet with powerful graphing features. For Unit 3, students used these features to find linear regression equations.

In each Unit, students made interpretations of mathematical models. For Unit 1, they applied ideas of measures of center and variability to make comparisons between models. In Unit 2, students interpreted solutions and solution areas on graphs of systems of equations and inequalities in terms of what those solutions mean in context. And for Unit 3, students interpreted the contextual meaning of the slope and y-intercept given by regression equations.

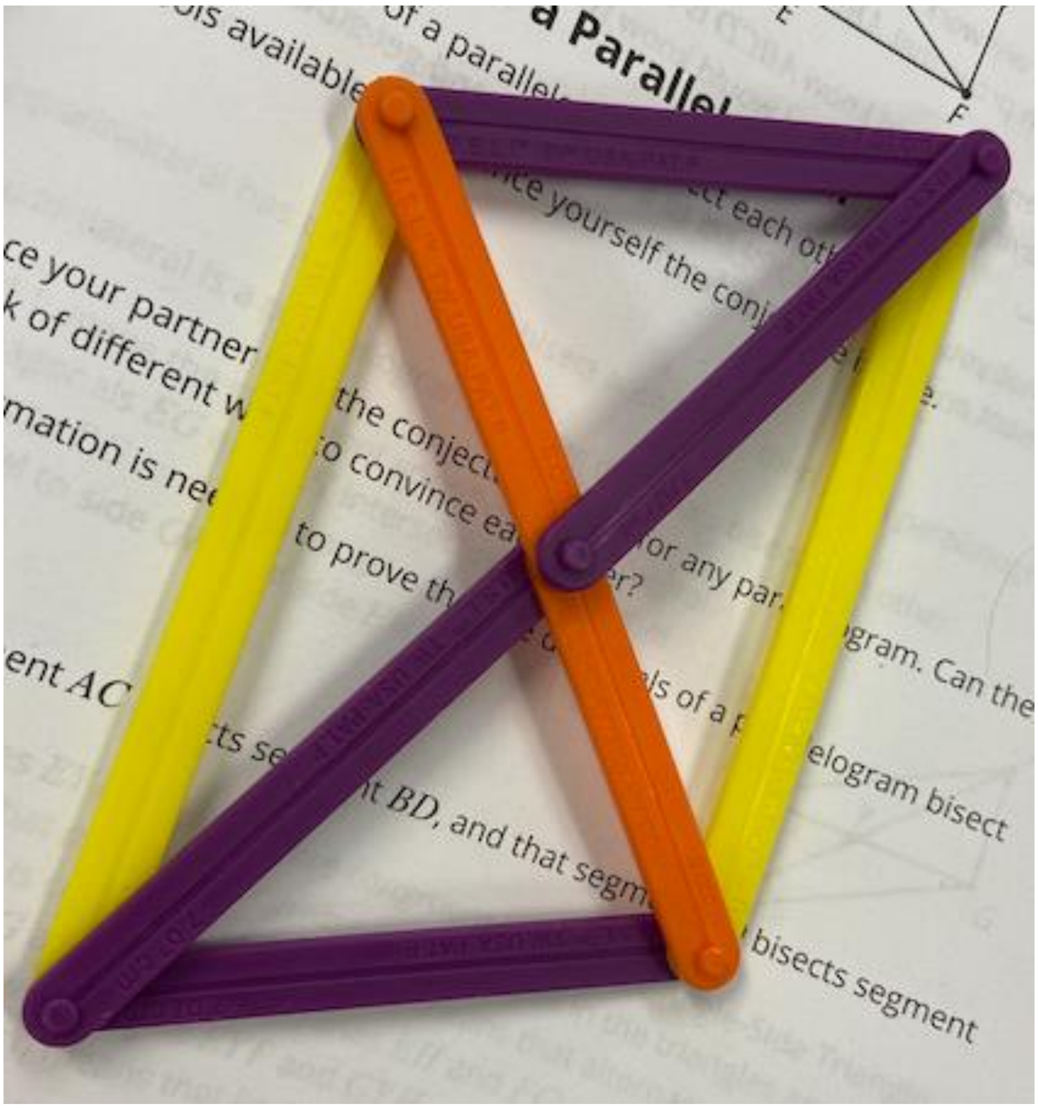
Students in Geometry engaged with mathematics requiring or relating to constructions and transformations often used a standard for mathematical practice (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010b): Use appropriate tools strategically (CCSS.MATH.PRACTICE.MP5). By no means is this the only standard occurring in my classroom, however, it is the standard that is most specific to the classroom mathematical practices of constructions and transformations. Introducing tracing paper to scaffold student thinking became one of the major turning points for some of my struggling students in understanding transformations conceptually. Students having the power of being able to trace the figure and then apply requisite transformations could finally physically do the transformations that they previously struggled visualizing. I described an instructional episode in which tracing paper played a pivotal role previously in the section above entitled "Invisible Triangles."

Beyond tracing paper, students engaging in reasoning about similar triangles and quadrilaterals repeatedly chose to use AngLegs (see Figure 7.3) to create both examples and counterexamples to support their reasoning with similarity theorems for triangles and congruence theorems for quadrilaterals. AngLegs allow students to create shapes with specific side length—

legs with the same color have the same length. Congruence theorems for quadrilaterals was a lesson that shortly followed the lesson described in the section entitled “Proofs About Quadrilaterals.”

Figure 7.3

Student Work: Investigating Parallelogram Diagonals with AngLegs ®

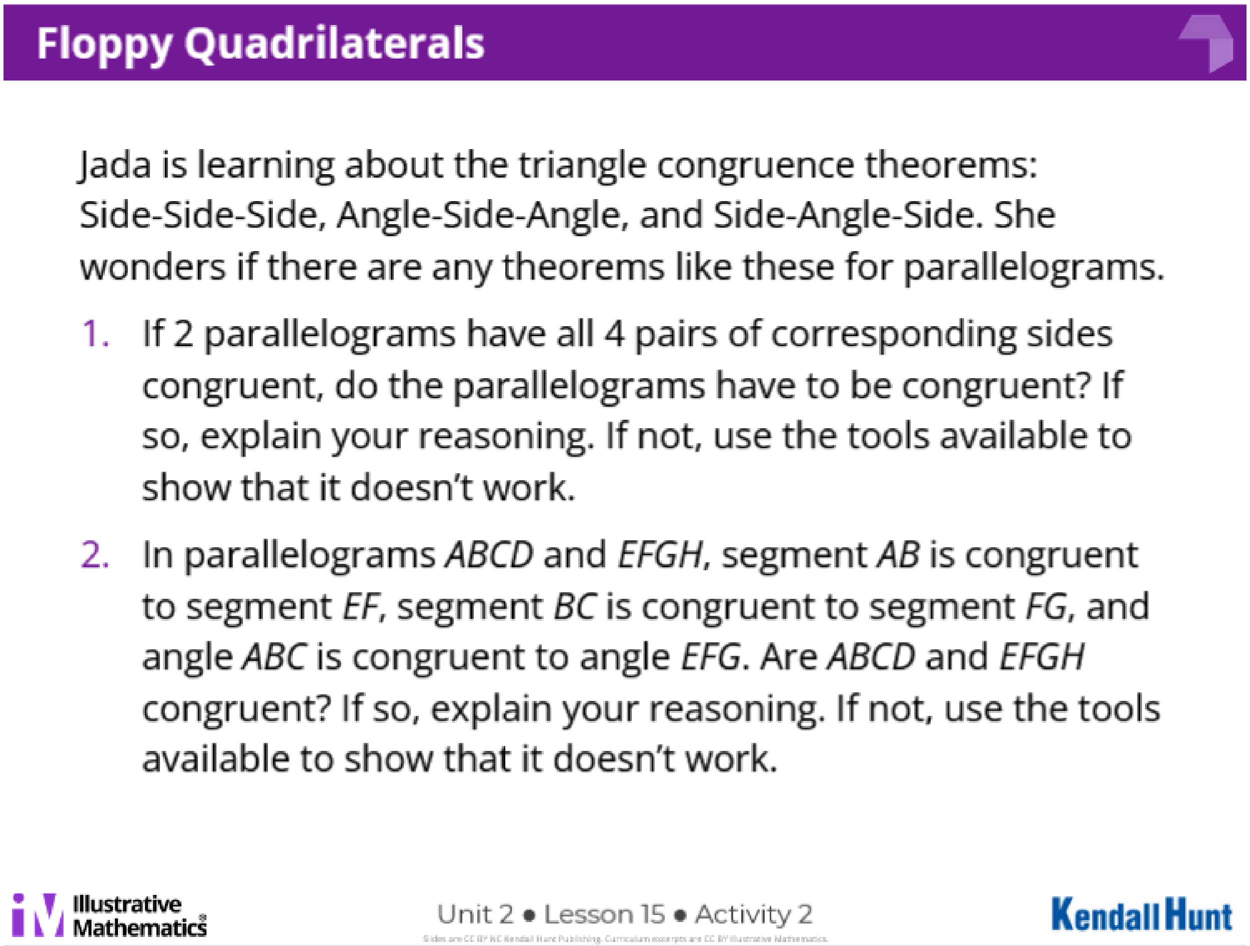


Congruence for Quadrilaterals

One of the concluding lessons for geometry unit 2 was in exploring whether congruence theorems for triangles are applicable to quadrilaterals. Students were given the following tasks to explore in Figure 7.4. The first prompt has students explore whether side-side-side-side would work as a quadrilateral congruence theorem while the second prompt had students explore whether side-angle-side would work as a quadrilateral congruence theorem. I encouraged students to focus on the part of the task that encourages them to use appropriate tools. Accordingly, students from each group went to retrieve the table sets of AngLegs. I only had one 'getting started' struggle with this task, to which I asked, "Have you tried making any models with the AngLegs?" Most of the struggle for this task occurred as a split between giving mathematical explanations and expressing misconceptions and errors. For those struggling with giving explanations, I asked them if they could write their explanation based on the figures they made with the AngLegs?

Figure 7.4

Geometry, Unit 2, Lesson 15, Activity 2



Alternatively, for students struggling with misconceptions and errors, it was a result of not being able to think through what parts of the figure were guaranteed to be congruent and which parts were left unspecified. For example, for task 1 (see Figure 7.4), if the sides are specified what does that imply about the angles? Being able to reason with AngLegs, students *could easily see how four congruent sides do not necessarily imply corresponding congruent angles—think of a rhombus versus a square. Both parallelograms have four corresponding congruent sides, but a rhombus does not have to have right angles for each corner.*

Students reasoning through task 2 (see Figure 7.4) had more frequent struggles than they had on task 1. Creating a model using AngLegs where students are given that they have a set of consecutive corresponding and congruent side-angle-sides does not obviously imply other properties about that parallelogram. Students across the classroom were stuck to say the least. More frustratingly, the purposeful question that I meant to provide as probing guidance, "what else do you know about this figure?" was met with mostly blank stares. Moving forward, I encouraged students to get their reference sheets out, hoping that they would individually find the key property of parallelograms that would allow students to move forward in reasoning— the fact that parallelograms have two pairs of opposite congruent sides. For a few groups, their recollection of that property allowed them to move past their struggle. Unfortunately, around half the class had hit their limit for their tolerance of struggle. I ended up going around to each of these groups using advancing questions and directed guidance to lead students through their reasoning.

Summary: Classroom Mathematical Practices and Productive Struggle

This study finds that students struggling with novel mathematical concepts tend to rely on classroom mathematical practices. For geometry, this involved students relying on constructions, transformations, and using resources. For algebra 1, students did not have consistent relevant skills that assisted their exploration of novel mathematics throughout the first three units of study. Instead, algebra 1 students turned to the use of resources—traditionally recognized as standards for mathematical practice (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010b) such as use appropriate tools strategically (CCSS.MATH.PRACTICE.MP5), and model with mathematics (CCSS.MATH.PRACTICE.MP4). Unfortunately for geometry students, relying on skill with constructions and transformations to reason through novel situations also meant that students did not always align their efforts with the learning goal for the class period.

Chapter 8 - Connections and Contributions

The purpose of this chapter is to fulfill the aims of this analytic autoethnography in contributing to the theoretical understanding of classroom microculture and productive struggle. This chapter is organized into two main parts. The first part, “Classroom Microculture: Elements of Struggle” makes connections between Cobb and Yackel’s (1996) microculture and elements of struggle that builds off descriptions and conceptualizations of student struggle as originally characterized by Warshauer (2011). Here, findings from Chapters 4-7 are summarized and discussed within the context of existing literature describing how they connect with and contribute to the current understanding of productive struggle. The second part, “Productive Struggle Framework in the Classroom Microculture,” theoretically discusses the classroom microculture in the context of Warshauer’s (2015a) overall productive struggle framework.

Classroom Microculture: Elements of Struggle

The sections that follow connect elements of the classroom microculture (Cobb & Yackel, 1996), social norms, sociomathematical norms, and classroom mathematical practices, to elements of student struggle. I begin by describing elements of how my analysis emerged and summarizing the main connection between that part of the classroom microculture and student struggle. I continue by synthesizing information from the stories and analysis of Chapters 4-7. I close each section by contextualizing these findings within existing literature, making note of this study’s contributions and how it connects with ongoing research, and identifying opportunities for future research. Following the discussion of each element of the microculture and its connection to struggle, I conclude this part of Chapter 8 with a summary and a concept map in Figures 8.1 and 8.2 to assist readers in visualizing key takeaways for teachers.

Social Norms

Social norms are norms that develop in all classrooms, with no inherent mathematical characteristics (Yackel & Cobb, 1996). Throughout the iterative process of coding, analyzing, and writing, several social norms emerged from their influence on student struggle. In Chapter 5, I discussed collaboration, peer tutoring, and student discourse in the context of instructional episodes and their impact on students' struggles. Each of these norms supported my students in collectively engaging in rigorous mathematics and in resolving struggle to varying degrees. In the following three paragraphs I discuss each social norm in relation to how they assist students in episodes of struggle.

Collaboration is distinct from peer tutoring in that both students are contributing equally toward learning as opposed to one student teaching the other. Collaboration works hand in hand with organic student discussions—distinct from the structured discourse I described as a social norm in Chapter 4. Collaboration among students enabled students to resolve struggles in two ways: 'giving mathematical explanations,' and to a lesser degree 'expressing misconceptions or errors.' Struggles that involve 'giving mathematical explanations' can resolve as students talk about the task and verbally process the mathematics. At times this required intervention on my part to provide 'probing' or 'directed guidance.' An example of 'probing guidance' used in my classroom was, "What can you both tell me about the units of this problem?" On the other hand, an example of directed guidance I used was in giving a directive, "Leia, you see if your notes will help, and Han can search through his workbook for similar questions we've done in the past." Students experiencing struggles that involved 'expressing misconceptions or errors' were much less likely to resolve their struggle without teacher intervention. Oftentimes students had competing conceptions of mathematical ideas—both partially right, one wrong and one right, or any permutation of partial correctness. To give students a chance to resolve this on their own, I asked students to explain their ideas to me—at times this resulted in the other student conceding that they were wrong. More often, I resorted to 'directed guidance' and led students down a set of guided reasoning.

The social norm of peer tutoring had a clearer interplay with struggle in my classroom. Peer tutoring, as opposed to collaboration, indicated that one student was at least partially teaching another student. At times this led to students telling their peers what to do. However, I attempted to establish the ideal that students should use questions to help their peers much like I use questions to help students process mathematics and frame their thinking. Peer tutoring assisted in resolving students' struggles with 'getting started.' This occurred organically and by teacher direction. With an established expectation turned norm of "ask three before me," students interact with one another to resolve their struggle. When that expectation was not taken as shared, one of my immediate actions was to direct students to ask a peer who had already moved past their initial point of struggle. Students struggling with 'carrying out a process' at times resolved their struggle through a peer tutoring interaction with varying levels of success. At times, these struggles did not resolve. This occurred when frustration bled into peer interaction and the tutor gave up, or the tutor confused themself while helping their peer, or the student being tutored shut down from reaching their tolerance for frustration. Other times when a process had already been formalized in class, I observed students leading their peers in how to use their notes to resolve their peers' struggles.

The norm of engaging in structured student discourse was prompted by curricular or teacher-driven questioning. Questions were provided to prompt students to engage in discourse with shoulder partners or with a small group of students with predefined roles. Structured discourse occurred at both the open and the close of tasks—the launch and summarize portions of the launch-explore-summarize instructional model. Structured discourse at the open of tasks tended to prevent ‘getting started’ struggles by allowing students to engage in the reasoning of the task more fully. During the ‘exploration’ portion of the task, I made a point of responding to struggle by pointing students back to our opening discourse when relevant as an affordance move. At times I witnessed students referring to the opening discourse in sense-making and resolving struggle through organic student discussions—a clear connection to the social norm of collaboration. Structured discourse was used when summarizing tasks to resolve struggle with ‘giving mathematical explanations’ that occurred while students were exploring the mathematics.

Throughout the process of negotiating these social norms, I kept in mind the idea that I wanted the student-identified social norms to align with my established expectations and teacher moves to help in the successful facilitation of productive struggle. This study partially addresses the gap in the literature regarding a math classroom that positively uses the negotiation of norms (Megowan-Romanowicz et al., 2013). Like Tatsis & Kolzea (2008), findings from my classroom suggest collaboration as a norm positively oriented toward problem-solving. Additionally, my findings indicate peer-tutoring and structured discourse as positive influences on the problem-solving process. The teaching actions described in Chapters 4-7 reflect some of Webb’s (2009) description of the teacher’s role in establishing collaborative dialogue:

> teachers have many roles to play when using small-group work in the classroom, including preparing students for collaborative work, making decisions about the group-work task and the composition of groups, making decisions about the structure and

requirements of group work, monitoring groups' functioning and intervening when necessary, and helping groups reflect on and evaluate their progress. (Webb, 2009, p. 21)

Webb's (2009) teaching actions also connect with teacher moves I made to support peer tutoring and student discourse. Productive failure research similarly suggests establishing participation structures that allow students to collaborate and discuss one another's solution methods (Kapur & Bielaczyc, 2011).

Sociomathematical Norms

Sociomathematical norms are norms with inherent mathematical characteristics (Yackel & Cobb, 1996). Sociomathematical norms in this study were more difficult to document, code, and analyze than social norms. Mottier Lopez and Allal (2007) also found sociomathematical norms more complex to understand in their work examining the microcultures of third-grade classrooms. When teaching and facilitating the learning of mathematics, there are numerous interactions between teacher and students, and between the students themselves. My task as a participant researcher was reflecting and processing these interactions. Through coding these reflections and concept mapping it became apparent sociomathematical norms were influencing classroom dynamics. Moreover, these sociomathematical norms had connections to how struggle was facilitated in my classroom. In Chapter 6, I characterized the culture of inquiry in my classroom as a sociomathematical norm—how the students in my classroom approached the learning of mathematics. Additionally, I discussed the traditionally characterized sociomathematical norms of different, acceptable, efficient, and sophisticated solution strategies in how they emerged in my classroom—in the context of facilitating mathematical discourse and learning activities. This culture of inquiry influenced how my students were oriented toward the learning of mathematics.

While the focus of this study is not student perceptions of mathematics, I observed the culture of inquiry positively influencing their perceptions. Makar and Fielding-Wells (2018) support this claim as “norms of mathematical inquiry engage students in productive social interactions and improve their mathematical knowledge, as well as their interest, valuing and capacity to solve complex problems” (p. 2). Implications regarding this culture of inquiry exist for curriculum developers and teacher educators. Curriculum developers should direct their efforts toward developing inquiry-oriented curricula. Teacher educators should expose preservice teachers toward inquiry-centered tasks like which one does not belong, notice and wonder, and info-gap activities.

Traditionally characterized norms of different, acceptable, efficient, and sophisticated solution strategies in this study were utilized in facilitating mathematical discourse while summarizing in my classroom. Mottier Lopez and Allal (2007) similarly found “sociomathematical norms [were] constructed during whole-class discussions” additionally they “provide[d] a reference for the elaboration of mathematical practices and for the interactive regulation of learning” (p. 252). While summarizing, I sequenced and connected the student reasoning that I anticipated and selected while planning and facilitating student exploration of the task (Smith & Stein, 2018). Different, acceptable, efficient, and sophisticated solution strategies played out as incorrect and partially correct solution strategies. I directed students to share with the class incorrect and partially correct strategies that helped advance all students’ learning by discussing useful misconceptions. By valuing different and incorrect ways of thinking, I promoted the message that all students’ thinking was valued and useful for the learning community. Cobb and Yackel (1996) also support the idea that the “teacher plays a central role in establishing the mathematical quality of the classroom environment and in establishing norms for mathematical aspects of students' activity" (p. 475). Sharing incorrect ways of thinking for the class to discuss may uncover opportunities for students to reengage in struggle. Granberg (2016) found when students reengage and reanalyze their work, they were more likely to be productive. Conversely, when students did not return to their work, new knowledge was not constructed, and struggle was not productive.

Sharing student solution strategies as described above supports the idea that even if a student is unsure, simply trying and engaging in the problem, not only begins the learning process for an individual but can also advance the learning process for the whole class. Believing their solution strategies are useful and valued leads to the development of productive dispositions toward mathematics (Cobb et al., 2001; Yackel & Cobb, 1996). Gresalfi and colleagues (2008) similarly suggest that students' competence is

> constructed as students and the teacher negotiate (1) the mathematical agency that the task and the participation structure afford, (2) what the students are supposed to be accountable for doing, and (3) whom they need to be accountable to to participate successfully in the classroom activity system. (p. 52)

In this study then, students engaging in a problem unsuccessfully would still represent a student earning competence by their participation in teacher's expectations and classroom norms. Megowan-Romanowicz and colleagues (2013) suggest similarly that the "use of strategies for encouraging participation can change the level of engagement of students who previously were observed to opt out in their mathematics class" (p. 68).

In addition, research aligns with this study's findings on teaching moves that support social and sociomathematical norms. Campbell and Yeo (2021) found teaching moves that

> support the development of productive social and sociomathematical norms in dialogic classrooms. These teaching moves include (1) inviting strategies and encouraging all members to actively listen; (2) exploring, clarifying, and questioning the mathematical details of all strategies as a group; (3) promoting understanding of differing strategies; (4) comparing and evaluating differing strategies as a group; and (5) connecting the group's thinking for making progress on the task/advancing current strategy to become viable. (p. 1)

Morrison and colleagues (2021) similarly noted that the negotiation of social norms was necessary to further develop sociomathematical norms in elementary classrooms. This study aligns with the finding that students' collaborative efforts allowed for the sharing and critiquing of one another's mathematical ideas.

Classroom Mathematical Practices

Classroom mathematical practices represent the collective learning of a classroom. The collective learning described in this study was focused on how it supported or prevented students from engaging in productive struggle. There were many concepts with which my students attained fluency; however, not all new concepts learned in algebra 1 or geometry were used throughout the semester. Skills that spanned the entirety of the semester became touchstones for how my students could interact with new content—skills that my students could and often turned to when stuck in struggle with novel mathematics.

Identifying these skills for geometry was quite simple in comparison to algebra 1. For geometry, my colleagues and I were pleasantly surprised by *Illustrative Mathematics'* (2019) use of constructions and transformations throughout the scope and sequence of the first semester. I was pleasantly surprised at the fluency with which students used these skills. The scope and sequence of algebra 1, however, did not have consistent skills that were used throughout the entirety of the first three units of study—this was described in more detail in Chapter 7. Both classes used resources in the classroom to support their struggle. One type of resource they used is better characterized as standards for mathematical practice (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010b). For algebra 1, students modeled with mathematics (CCSS.MATH.PRACTICE.MP4) and used appropriate tools strategically (CCSS.MATH.PRACTICE.MP5). These practices were combined when algebra 1 students used statistics calculator applets and graphing technology to create models—histograms, boxplots, and lines of best fit. Geometry students similarly used appropriate tools strategically (CCSS.MATH.PRACTICE.MP5) to scaffold their thinking. However, their use of classroom scaffolds also involved using physical manipulatives, like using compasses, to create constructions, wax paper to trace and visualize rigid transformations, and AngLegs (see Figure 7.3) to create visual representations of polygons.

The classroom mathematical practices described in this study fit these three characteristics: "normative purpose for engaging in mathematical activity; normative standards of mathematical argumentation; normative ways of reasoning with tools and symbols" (Cobb et al., 2011, p. 110). The use of digital classroom resources to support student reasoning and learning in this study is like the use of Geogebra to develop concepts of surface area (Dogruer & Akyuz, 2020). Hoyles (2018) describes Geogebra as a "dynamic… graphical tools that allow mathematics to be explored in diverse ways from different perspectives" (p. 3) that changes the way students think about mathematics.

The use of classroom mathematical practices in productive struggle connects with an extensive program of research on productive failure by Manu Kapur (2008, 2010, 2011, 2012,

2014) involves two design phases: (1) generating and exploring multiple representations and solution methods on ill structured/novel tasks, and (2) organization and knowledge assembly (Kapur & Bielaczyc, 2011). In the first phase, students use prior knowledge to collaborate in solving and discussing ill-structured tasks. During the second phase of instruction, teachers prompt comparison of student ideas by facilitating mathematical discourse (e.g., Smith & Stein, 2018). The productive failure instructional model as compared to direct instruction resulted in superior conceptual understanding and similar outcomes for procedural fluency (Kapur, 2008, 2011, 2012, 2014). It is the first phase of instruction that connects with classroom mathematical practices—when students use prior knowledge to engage in ill-structured tasks. While not all prior knowledge would represent classroom mathematical practices, it is likely that the use of these practices would be more relevant to the task students are engaged in than more general prior knowledge.

In my classroom environment, social norms influenced both sociomathematical norms and classroom mathematical practices. Selling (2016) studied the intersection of structured discourse as a social norm and the establishment of classroom mathematical practices by identifying teacher moves that make mathematical practices explicit following student exploration of mathematics. Selling (2016) characterizes teacher interactions with students as initiating, sustaining, or reprising reasoning. 'Initiating' reasoning might assist students in 'getting started' or 'carrying out a process' struggles and could be characterized as any of Warshauer's (2015a) teacher responses to struggle. 'Sustaining' student reasoning might represent 'directed guidance' or 'probing guidance' from a teacher and would likely be used with students struggling with 'carrying out a process' or 'giving mathematical explanations.' 'Reprising' connects with 'probing guidance' moves and classroom mathematical practices.

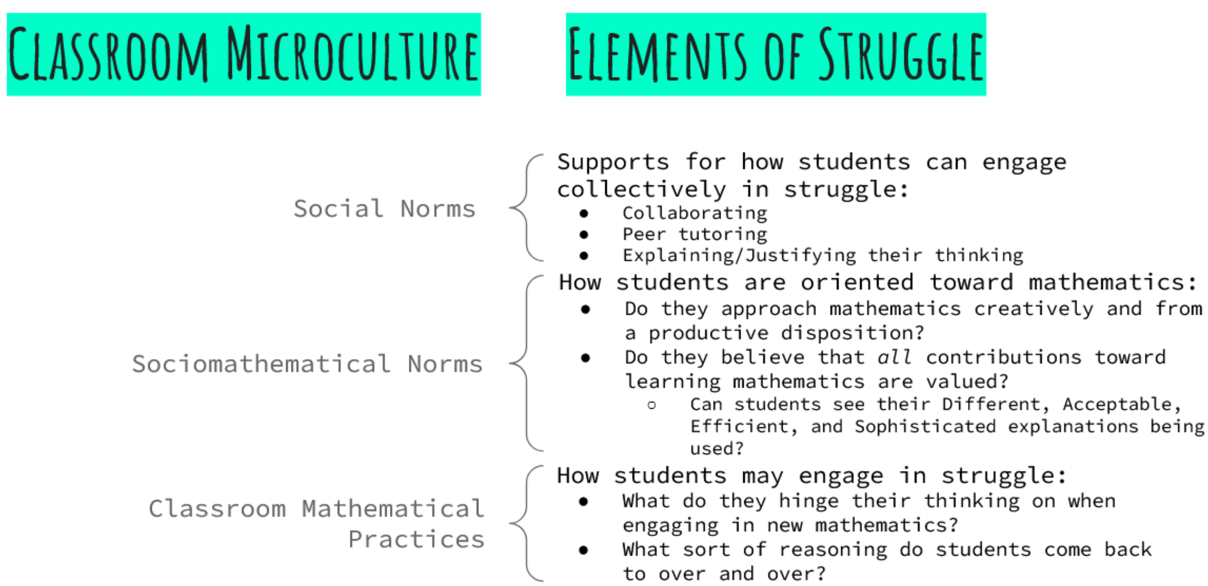
Reprising occurs “when the teacher explicitly reflects back on student participation in mathematical practices” (Selling, 2016, p. 518). Selling (2016) identified multiple types of reprising responses: naming, highlighting, evaluating, explaining the goal or rationale, connecting, framing, eliciting self-assessment, and referring to a teaching narrative.

Teacher Implications

There are several implications to be gleaned from examining how the classroom microculture framework (Cobb & Yackel, 1996) influences how students struggle with mathematics, and six are shared here (see Figure 8.1). A special focus is given to the connections between learning goals and both the negotiation of norms and facilitation of struggle.

Figure 8.1

Classroom Microculture: Elements of Struggle



First, the literature is clear that explicit negotiation of norms positively impacts how students engage in learning. Using an already established routine like “Building a Mathematical

Classroom Community" (Gray et al., 2018) could assist teachers who have not negotiated norms with students previously.

Second, the establishment of social norms can positively and negatively impact how students struggle. Figure 8.1 identifies positive supports for how students can collectively engage in struggle by collaborating, tutoring one another, and explaining and justifying their thinking. However, to imply that social norms only positively impact student engagement of mathematics would not be accurate. I observed negative social norms to negatively impact student engagement and thus prevent struggle or make it such that struggle is unproductive. Examples include students copying work; students remaining quiet and disengaged with the understanding that the teacher does not hold them accountable for their work; or an understanding that every time a student struggles the teacher will unproductively resolve that struggle by responding with a "telling" response. Copying work prevents engagement in mathematics entirely and thus prevents students from struggling with important mathematics. Students allowed to remain quietly disengaged or students who act out with behaviors that cause them to be removed from the classroom would likewise be removing themselves from the community of learners and any possibility of struggling with important mathematics.

Third, the social norm of students being expected to explain and justify their thinking is essential for teachers intending to establish sociomathematical norms. It is in the justification of thinking and the engagement of student discussions that sociomathematical norms can become established—where students can discuss and debate one another's ideas. Sociomathematical norms are students' beliefs about mathematics (Yackel & Cobb, 1996). These beliefs inform their dispositions toward mathematics, and influence how they engage in mathematics. The curricula teachers use helps to shape these beliefs. In this study, the problem-based and inquiry promoting nature of *Illustrative Mathematics* (2019) influenced how students engaged with mathematics. Teachers should contemplate how the nature of their curricula impacts how their students engage in learning mathematics. Teaching actions also influence how students approach their learning. Several questions teachers can consider include: Do their teaching actions press students to explain and share their thinking? Do their teaching actions communicate that all students' thinking is valued and useful for the classroom community?

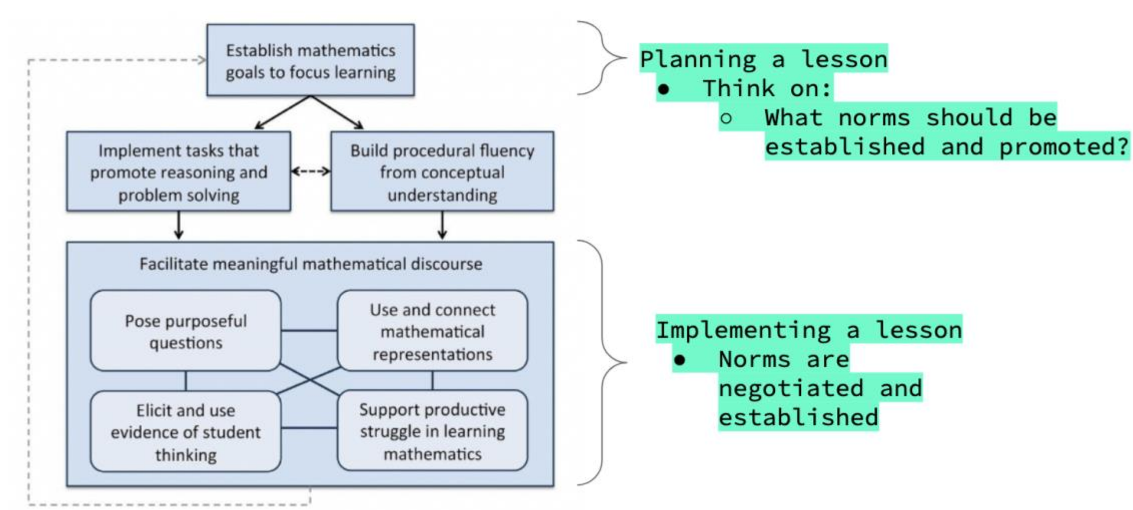
Fourth, classroom mathematical practices influence how students engage when struggling with mathematics. Teachers can reflect on the overarching skills that students use throughout the scope and sequence of their curricula. When anticipating how students will engage in novel learning, teachers should consider what classroom mathematical practices students will utilize. In addition, teachers can ask themselves "what sort of reasoning do students come back to over and over?" when reflecting on the overarching classroom mathematical practices that students use in struggle. An identified learning goal should guide teachers' implementation of lessons and help teachers consider what use of classroom mathematical practices will result in relevant productive struggle, or irrelevant unproductive struggle. Struggle is relevant if the student's implementation of the mathematical practice leads toward the understanding of new content and irrelevant if it prevents the learning of new content.

Fifth, the mathematics teaching framework (Boston et al., 2017) in Figure 8.2 can provide insight as to how teachers conceptualize negotiating and establishing norms in their classrooms. Teachers begin with appropriate learning goals that align with tasks that promote reasoning and problem-solving. When planning for lessons teachers can consider the following questions: What social norms might assist students engaging in these tasks? What are the different, acceptable, efficient, and sophisticated sociomathematical solutions that students might produce in pursuit of that learning goal? Based on the learning goal, teachers can determine what struggle is relevant to the learning goal and what struggle should be scaffolded so that students can access the learning goal.

Sixth, implementing the lesson requires the facilitation of mathematical discourse which involves posing purposeful questions, eliciting and using student thinking, connecting mathematical representations, and supporting productive struggle (NCTM, 2014). As teachers pose purposeful questions or interact with students, they should identify what norms they are renegotiating and re-establishing through their questioning. Teachers can examine how social norms give structure to how students interact, and further, how these interactions support productive struggle. This study finds that a teacher's moves and actions support the negotiation and renegotiation of various social norms and sociomathematical norms throughout the scope of a lesson. As teachers continuously establish expectations, students react positively or negatively to those expectations which determine whether norms are intersubjectively agreed upon (Yackel & Cobb, 1996).

Figure 8.2

Norms in the Mathematics Teaching Framework



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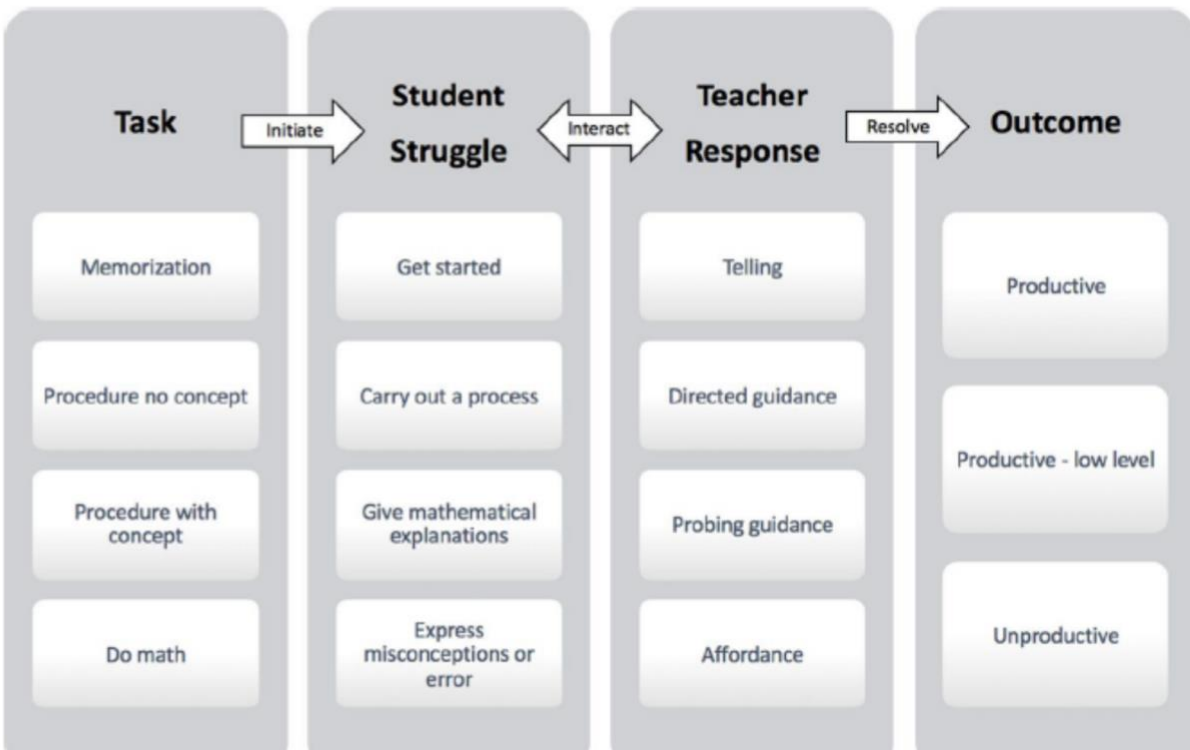
Seventh, when teachers facilitate mathematical discourse, they should follow Smith and Stein's (2018) five practices: (1) anticipate responses, (2) monitor students' discussions and collaboration, (3) select responses to highlight, (4) sequence responses from least to most sophisticated, and (5) connect responses in a coherent way. These five practices should be informed by the classroom microculture. When monitoring student discussions governed by the social norms established in the classroom, teachers should look for the different responses that they anticipated from students. These anticipated responses should be informed by their classroom mathematical practices. Teachers should select different responses that represent the various levels of sociomathematical norms so that when they are connecting them, students can evaluate and question these responses as the teacher sequences the responses from least to most sophisticated. Facilitating discourse in this way helps students to make connections between the sequenced responses.

Expanding the Productive Struggle Framework to the Classroom Microculture

Warshauer and colleagues' (2021) productive struggle framework in Figure 8.3 operationalizes aspects of student struggle and how teachers respond to that struggle. The task initiates the episode of student struggle with varying levels of cognitive demand: memorization, procedures with no concept, procedure with concept, or doing math (Stein et al., 2009). When engaged in the task, students' struggle can occur in four ways: getting started, carrying out a process, giving mathematical explanations, or expressing misconceptions or errors. Interaction between the teacher and student is prompted by students requesting help or the teacher noticing students are stuck. In this interaction, teachers respond with different responses: telling, directed guidance, probing guidance, or affordance. The interaction results in student engagement that is classified as productive, productive at a low level, or unproductive.

Figure 8.3

Productive Struggle Framework

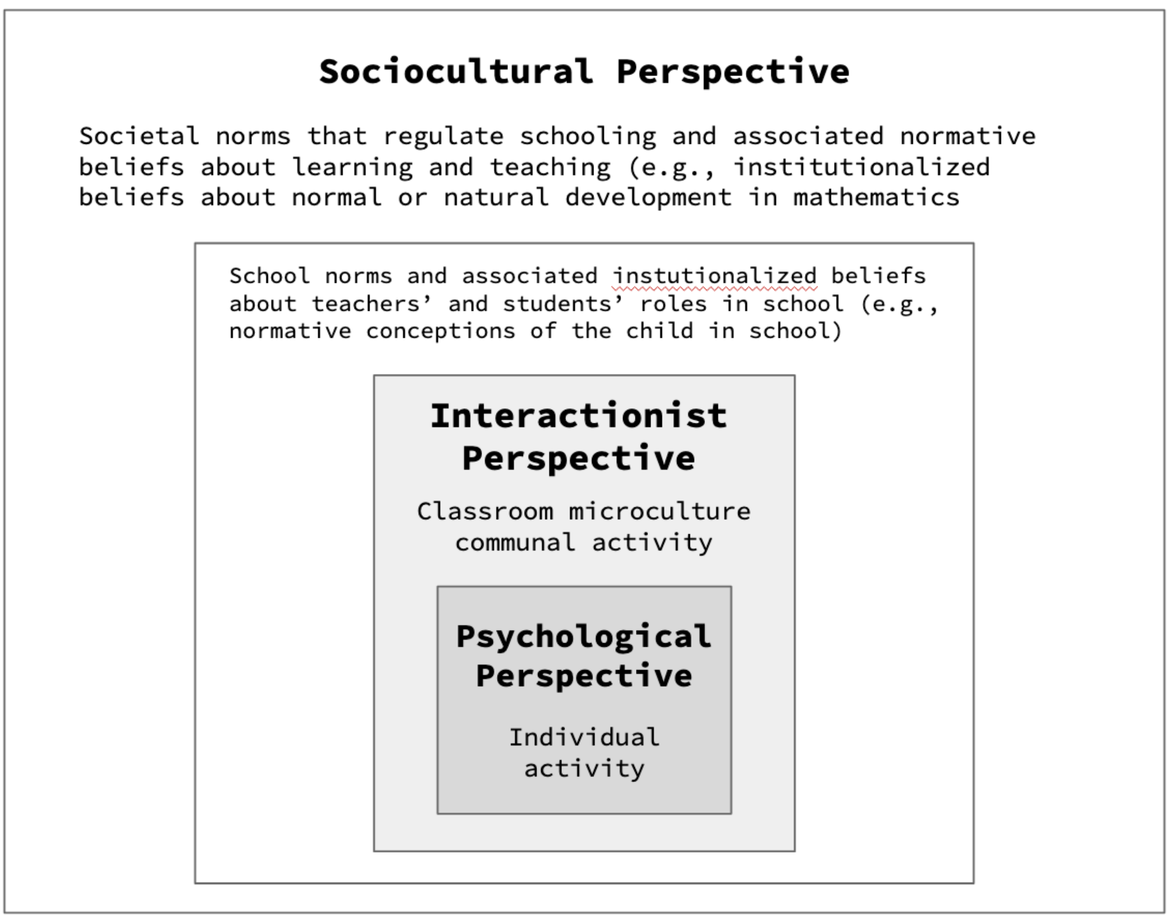


Note. Reproduced with permission from “Developing Prospective Teachers’ Noticings and Notions of Productive Struggle with Video Analysis in a Mathematics Content Course,” by H. K. Warshauer, C. Starkey, C. A. Herrara, and S. Smith, 2021, *Journal of Mathematics Teacher Education*, *24*, p. 94 (https://doi.org/10.1007/s10857-019-09451-2). Copyright 2021 by Springer Nature.

In the section that follows, I conceptualize findings connecting elements of struggle in the classroom microculture in context of the productive struggle framework. I add the classroom context to the existing framework. It is currently conceptualized as mostly a psychological perspective (See Figure 8.4) as action that happens within a student's mind and the interaction between the teacher and the student. The classroom microculture influences these interactions. Adding microculture to this framework conceptualizes the productive struggle framework as an interactionist perspective, a communal activity. Instead of struggle occurring in a student's mind and from their interaction with their teacher, the classroom microculture completes what happens as students engage in and resolve struggles.

Figure 8.4

An elaboration of the interpretive framework

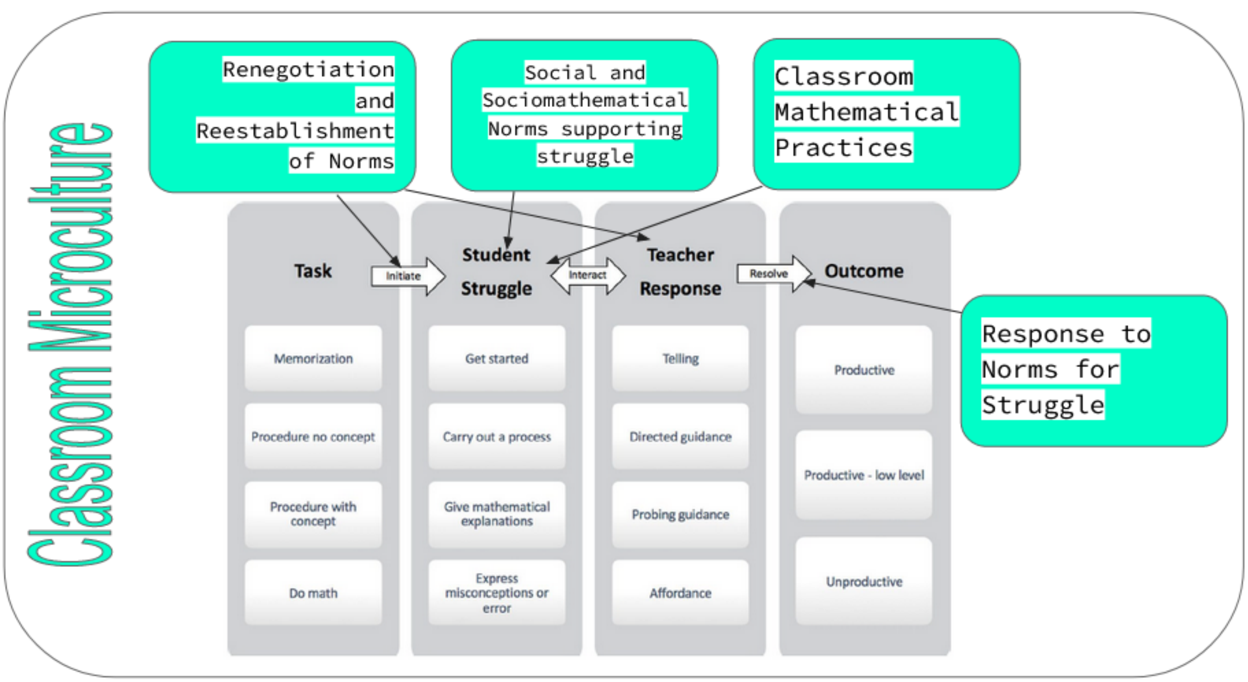


Note. Reproduced with permission from "Constructivist, Emergent, and Sociocultural Perspectives in the Context of Developmental Research," by P. Cobb, and E. Yackel, 1996, *Educational Psychologist, 31*(3/4), p. 181 (https://doi.org/10.1080/00461520.1996.9653265). Copyright 1996 by Taylor & Francis Group.

The interactions between Cobb and Yackel's (1996) microculture framework and Warshauer's (2015a) productive struggle framework encompasses an entire episode of struggle: From the initiation of a task provoking struggle, to the interaction between teacher and student resulting in a productive or unproductive resolution of struggle. Figure 8.5 conceptualizes this interaction in detail.

Figure 8.5

The Classroom Microculture and Productive Struggle Framework



Note. Adapted from “Developing Prospective Teachers’ Noticings and Notions of Productive Struggle with Video Analysis in a Mathematics Content Course,” by H. K. Warshauer, C. Starkey, C. A. Herrara, and S. Smith, 2021, *Journal of Mathematics Teacher Education, 24,* p. 94 (https://doi.org/10.1007/s10857-019-09451-2).

The renegotiation and reestablishment of norms within classrooms is driven by the teacher’s expectations when the task is initiated. Expectations might include whether students are expected to work by themselves or collaboratively, whether students are expected to discuss distinct aspects of the task or share strategies following individual work, or what sort of engagement from the students the teacher expects to see before they will help their students. Specific expectations for engagement include students demonstrating that they have engaged in routines like three-reads or other established reasoning routines within the classroom (e.g., Kelemanik et al., 2016) by reporting out what they have found and where they are stuck. Bowers and colleagues (1999) provide other examples of these expectations:

> (a) explain and justify their solutions, (b) listen to (and make sense of) the explanations offered by others, (c) ask a clarifying question if an explanation is unclear, and (d) resolve disagreements by discussing the viability of various solution methods. (p. 39)

Social norms and sociomathematical norms shape how students engage in tasks and struggle (see Figure 8.5). Examples of social norms include student collaboration, peer tutoring, and student discourse. Sociomathematical norms have a more complicated impact on productive struggle. First, they influence how students perceive mathematics. Second, sociomathematical norms contribute to how students engage in tasks, specifically, how they engage in discussing mathematics. When students share their ideas and compare and critique their peers’ ideas, they develop sociomathematical norms. These discussions assist students in resolving struggles characterized as giving mathematical explanations.

Classroom mathematical practices shape how students engage in struggle with mathematics. In this study, I observed students in geometry consistently turn to practices like constructions and transformations in learning new mathematics. These practices positively influenced their engagement as students were more willing to try out unfamiliar tasks. Unfortunately, it also meant their efforts towards exploring these tasks occurred in less productive directions when their efforts did not align with the learning goal. When classroom mathematical practices align with the learning goal, teachers should use directed or probing guidance to steer students into using these practices. However, when classroom mathematical practices do not align with the learning goal, teachers should launch the task with expectations that students avoid a given classroom mathematical practice or redirect students during the task so that their efforts better align with the learning goal.

Norms are renegotiated and re-established by the way teachers respond to struggle. Consider the following scenario. A teacher responds to a student’s struggle where that student has not met expectations laid out in the launch of a task. If the teacher does not reestablish those expectations, they send conflicting messages about the validity of the social norms or sociomathematical norms in the classroom. However, if a teacher responds to that struggle by reestablishing expectations, they respond to struggle with affordance, a response to struggle that does not remove the cognitive demand of the task. Warshauer (2011) characterizes probing affordance as “provid[ing] students' opportunity and time for further action and interaction” (p. 96). Alternatively, if teachers respond to struggle and students have engaged in the expectations for the task, students might require assistance in putting together or synthesizing their mathematical ideas—a struggle that requires a teacher to engage in probing guidance. Warshauer (2011) characterized probing guidance as

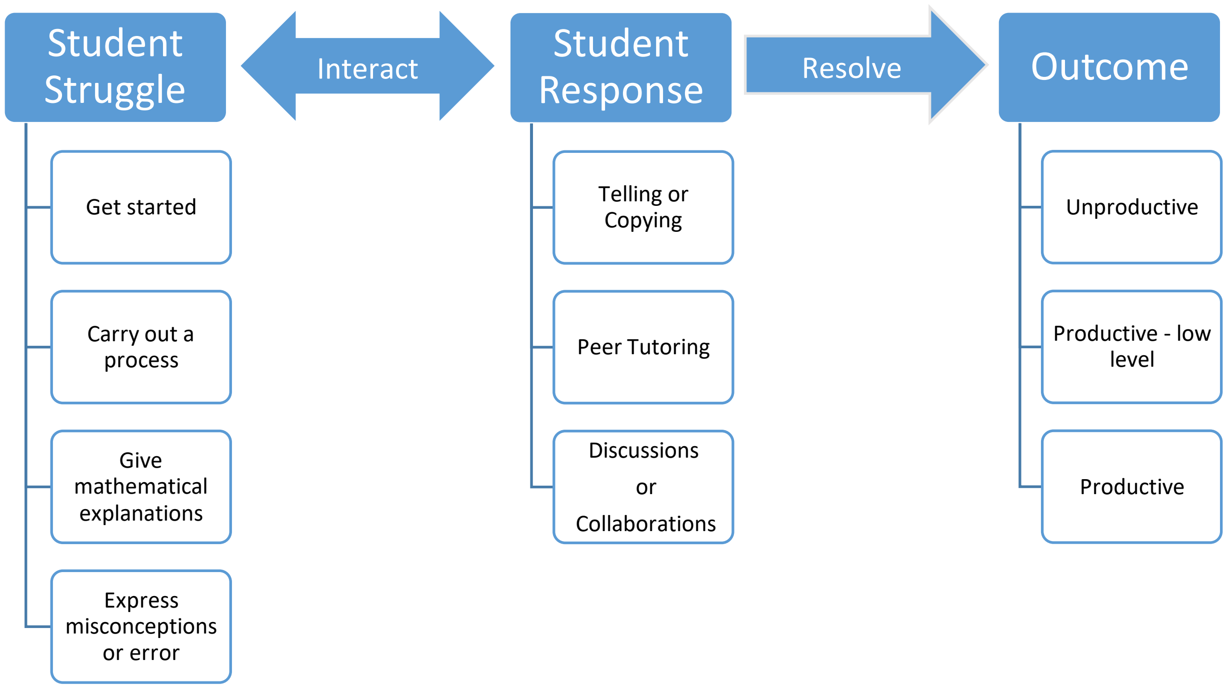
> responsive to the students' thinking, probing their ideas, suggesting mathematical concepts or procedures that related to and built on the students' thinking. The intellectual effort needed to tackle the problems rested with the students, but the responses served to clarify, connect, or confirm ideas the students presented, and were therefore made visible through the teacher responses. (p. 97)

The response to norms comes when students resolve their struggle, productively or unproductively. The interactions between themselves and their teachers or their interactions with their peers contributes back to the constitution of the classroom microculture. In this way, the classroom microculture sets the stage for how students engage in tasks and in struggle; however, the resolution of that struggle following interactions with peers or with teachers implies that the resolution of struggle then influences the evolution of the classroom microculture. Normative understandings are always "taken as shared" (Yackel, 2001, p. 6) or intersubjective. These intersubjective agreements are constructed by how students react to teacher's expectations and how teachers react to students meeting or failing to meet those expectations. The resolution of that interaction fleshes out whether both parties actually share interpretations of how to behave in class and how to learn mathematics.

The key difference between the framework illustrated in Figure 8.4 and the framework illustrated in Figure 8.6 is that 'Teacher Response' is now 'Student Response,' and the task column is removed. Social norms and sociomathematical norms shape the interactions students have when engaged in mathematics. Figure 8.6 conceptually demonstrates these interactions. These interactions involve different types of struggles, different responses to struggle, and different outcomes. For example, students struggling with ‘getting started’ can resolve their struggle unproductively with another student telling them the answer or allowing them to copy their work. That same struggle could also be resolved when another student interacts with them, such as a peer tutor, or by engaging in discussion or collaboration with any level of resolution (unproductive, productive - low level, or productive). And students struggling with ‘giving mathematical explanations’ most often resolved those struggles from engaging in discussions or in collaboration.

Figure 8.6

Social and Sociomathematical Norms Shaping Student Responses to Struggle



Note. Adapted from Warshauer et al. (2021)

Summary: Connections and Contributions

This study connected the classroom microculture (Cobb & Yackel, 1996), and the productive struggle framework (Warshauer et al., 2021). Within each part of the classroom microculture, elements of productive struggle were discussed in context of what occurred in my classroom and what is found in literature. Social norms provided support for how students engaged collectively in struggle. Sociomathematical norms influenced how my students approached mathematics as well as provided useful reference points for anticipating, sequencing, and connecting student ideas in facilitating discourse (e.g., Smith & Stein, 2018). Classroom mathematical practices influenced how my students engaged in struggle.

I concluded this chapter by theoretically discussing the classroom microculture and productive struggle framework (see Figure 8.5). In summary, the renegotiation and reestablishment of norms occurs within the initiation of a task and a teacher's response to student struggle. Social and sociomathematical norms support students in struggle. Classroom mathematical practices are implemented by students during episodes of struggle. The resolution of an episode of struggle also influences the response to norms for struggle. Findings discussed in this chapter are organized and addressed by specific research questions in Chapter 9.

Chapter 9 - Conclusions

This chapter is organized into three sections. The first section answers the research questions identified in Chapter 1. The second section has a more personal element: my musings on the process of autoethnography. The third section concludes with research implications and a summary of this study.

Research Questions

The research question this study explored is: In what ways does a teacher negotiate the establishment of classroom norms in order to facilitate productive struggle? Chapter 4 described the initial negotiation of these norms as I utilized the "Building a Mathematical Classroom Community Plan" (Gray et al., 2018). I began by having students reflect on what they did in "doing math" for that class period: "What does it look and sound like to do math together as a mathematical community?" In the days that followed, students modified or added to these actions if they felt some part of "doing math" was missing. Several days later, I asked students "What norms, or expectations, were we mindful of as we did math together in our mathematical community?"

Throughout Chapters 4-7, I described teaching actions where I continued to re-establish expectations, which in essence is re-negotiating these norms. In each chapter, I described my intentional teaching actions to promote norms that positively impacted how students engaged in struggle. I began by describing the initial negotiation and establishment of norms in Chapter 4. In Chapter 5, I covered social norms and their impact on struggle. In particular, I described the impact of collaboration, peer tutoring, and discourse. In Chapter 6, I laid out the impact of sociomathematical norms on the culture of the classroom and elements of student discourse. And Chapter 7 described students' use of classroom mathematical practices while engaged in and struggling with mathematics. The overarching theme of Chapters 4-7 is that classroom norms required continual re-establishment and re-negotiation.

Based on the literature and personal experience as a teacher, research questions were developed to explore various aspects of my teaching practice such as change over time, use of curricular resources, influence of reflective practice, and student behaviors and reactions. These supporting questions informed and framed the overarching research question for this study by informing various aspects of my teaching practice. The following paragraphs answer each supporting research question.

How do I perceive norms changing over time in my classroom?

Students generally grew in their abilities to engage in the norms established in my classroom. Social norms were in a state of constant change in my classroom, and they had varying effects on the outcome of the lesson. At times, lessons where one class of students engaged in productive collaboration or discourse, devolved into talking about anything *but* math with another class of students. Students' abilities to explain and justify their reasoning increased over time; however, without reminding students of that expectation, students did not by default explain and justify their reasoning. Justifying and explaining is a social norm that developed into a sociomathematical norm when students compared and critiqued one another's reasoning. It took much longer for students to become comfortable comparing and critiquing one another's reasoning. The more familiar students were with the math concept, the more they were able to compare and critique one another's reasoning. Classroom mathematical practices emerged over time and impacted how students engaged and struggled with novel mathematics.

How do I utilize curriculum resources and ancillary informants to guide my negotiation of classroom norms?

The nature of the curriculum itself as problem-based laid the foundation for the ways in which students engaged with mathematics. Three components of the curriculum contributed to this culture of inquiry: warm-ups, card sorts, and info-gap activities. Prior to the school year, I attended an *Illustrative Mathematics* (2019) curriculum implementation training that informed how I launched these activities (Pesce, personal communication, June 2021).

Regarding ancillary informants, I cannot overemphasize the impact that informal conversations with my wife and my colleagues played in understanding what occurred in my classroom. My wife had the curse of hearing and reading the initial drafts and analysis. She also provided critical first round feedback on my descriptions and analysis of instructional episodes. Conversations with math department colleagues provided insight into the impact of *Illustrative Mathematics* (2019) on their classroom communities. These conversations revealed what classroom mathematical practices students utilized when struggling in their classrooms. Conversations and feedback from Sherri, my major professor, assisted not only in the writing process but also in better understanding my analysis and ideas by asking questions and providing feedback when something did not make sense.

How does my reflective practice influence my negotiation of norms and facilitation of student struggle?

My reflective practice impacted my analysis and understanding of what occurred in my classroom. It had less impact on my negotiation of norms and facilitation of struggle. The initial negotiation of norms occurred by following the community-building process (Gray et al., 2018). This negotiation continued with re-establishing and renegotiating those norms as expectations laid out in the launch of learning activities. My facilitation of struggle was informed by prior research and writings about productive struggle (e.g., Hiebert & Grouws, 2007; NCTM, 2014; Warshauer, 2015a) and from collaborating with Sherri on a case study that sought to investigate a novice teacher's understanding and implementation of productive struggle (Nusser & Martinie, 2022).

How do I perceive my responses to student struggle as influencing the renegotiation and re-establishment of norms?

My response to struggle informed the renegotiation and re-establishment of norms when I ensured students were engaged with mathematics in line with the way I launched the task. At the launch of math tasks, I lay out terms for engagement in mathematics with what different student actions should occur:

- whether students are working alone or collaboratively;
- whether students engage in various established classroom routines (e.g., Kelemanik et al., 2016);
- what questions students should discuss or figures they should describe;
- and what resources they could refer to in solving a given problem.

When students met these expectations, I turned to probing guidance to assist students in giving mathematical explanations or expressing misconceptions or errors. I used directed guidance in assisting a student interpreting class notes for carrying out a process or trying out ideas they discussed in getting started. However, if a student did not meet my expectations or classroom norms and I failed to re-establish that expectation, I did not support the positive renegotiation and re-establishment of norms.

What norms do I perceive students utilizing when engaging in solving problems?

My students engaged in the social norms of collaboration, peer tutoring, and discourse; the sociomathematical norm of critiquing and assessing one another's solution strategies; and classroom mathematical practices of constructions, transformations, and using resources that at times did not align with the learning goal.

How do student behaviors and actions influence my negotiation of norms?

Student reactions to my expectations continually informed my negotiation of the classroom microculture. Their adherence to expectations implied that the norms in my classroom were "taken-as-shared" (Yackel, 2001, p. 6). However, students not meeting my expectations implied different possibilities and prompted different reflections (see Table 9.1). The first two possibilities influenced me to take action to modify how I negotiated the classroom environment.

Table 9.1

Possibilities and Reflections

Possibilities	Reflections
• Students did not understand my instructions • Students did not perceive or agree with the importance of the norm toward their learning • Students had unmet needs that influenced their engagement in class (e.g., Maslow, 1947)	• Reflection on how I launched and facilitated learning activities • Reflection on my interactions with that student and think about what I could do to better reach them • Limited actions—reach out to school administration to help students meet needs

Musings on Autoethnography

The Ethnographer's Path

I became a more purposeful teacher by recording reflexive journal entries focused on my teaching. Throughout my teaching career, I have consistently reflected during and after lessons to improve my pedagogy for the next implementation of that lesson. However, reflecting on teaching practices as well as student reactions over the course of many lessons, rather than looking at them individually, seemed different. The use of the data matrix to analyze codes allowed me to track changes to my teaching and to the classroom microculture much more intentionally. I could see how making changes in my instruction made long-term positive impacts on how students interacted in my classroom. It turns out that researching my teaching in my classroom does more than generate knowledge, it helped me improve as a practitioner.

Early on in this semester, it seemed like doing reflexive journal entries at the end of the school day and performing weekly preliminary coding and analysis felt remarkably similar to my previous experiences implementing qualitative research. The autoethnographic element did not seem unique. Yet. After the initial negotiation of norms, I knew I had the beginnings of a story to tell and Chapter 4 began to take shape. However, after recording the initial negotiation and the instructional episode entitled "A Main Dish and Some Side Dishes," my internal muse abandoned me. I continued to record reflexive journal entries and continued coding and analysis.

At this point, certain instructional episodes began to emerge as impactful. After writing about these episodes, I felt ready to apply some of the analysis I had done with coding to illustrate the social norm themes. My writings about social norms quickly began to take shape—write about instruction and analyze it by tying social norms to the iterations of student struggle.

After writing about social norms, I once again became stuck. I could describe how discourse, as a social norm, contributed to the classroom environment, and to how it assists in the facilitation of student's struggle. But the nuance between students discussing aspects of mathematics (a social norm) and students comparing and critiquing their ideas about mathematics (a sociomathematical norm) took more time to discover.

After a few more weeks, a few critical conversations with my math professional learning community (PLC) occurred. In these conversations, my colleagues shared important aspects of the curriculum that had made a positive impact on the cultures of the classroom. These conversations sparked my writings about curricular aspects of Illustrative Mathematics (2019). The mix of coding and analysis, writing instructional episodes, and discussing ideas with ancillary informants set the stage for my understanding of the big picture of what had and was occurring in my classroom. This big picture solidified from creating concept maps for a presentation shortly thereafter with how elements of struggle fit into the classroom microculture (see Figure 8.1), and how the classroom microculture fits into the productive struggle framework (see Figure 8.5). The messy nature of doing an autoethnography assisted me in fulfilling the aims of analytic autoethnography—contributing to theoretical understanding (Anderson, 2006). In this case, synthesizing the classroom microculture with the productive struggle framework.

The last piece to fall into place was understanding the influence of classroom mathematical practices on productive struggle. My math PLC spoke about how much students used constructions and transformations in their geometry classes. I began to see students engaged in these practices when struggling with novel mathematics. The difficulty arose in identifying the unifying practices for algebra 1. These classroom mathematical practices *seemingly developed in parallel with standards for mathematical practice rather than specific mathematical ideas.*

Challenges

I can echo other autoethnographers remarking on the messiness and the element of iterative reflection that lay at its heart (Chang, 2013; Ellis et al., 2011; Throne 2019). Analysis began early and continued throughout the scope of the research—through data saturation and the writing process. Coming back to codes and analysis during the writing process helped me to connect specific instructional episodes to emerging themes. Writing occurred at sporadic intervals and reconstructing the memories of instruction through reviewing lesson plans and reflexive journal entries played a critical role in how I discussed the ways in which students interacted with one another and how I launched and facilitated activities. Other autoethnographers also describe the element of memory as critical in autoethnography (Chang 2013; Giorgio, 2013).

Describing the ethnographer's path while also clearly communicating concise themes was more challenging. To confront this challenge, I aimed to supplement the existing description of emergent analysis in Chapters 4-7 with the chronological reflection that opened this section. Also supplementing Chapters 4-7 are answers to several of the research questions in the first section of this chapter. These ideas connected less theoretically with the classroom microculture. In Chapters 4-7, I intended to connect theory and narrative, the goal of analytic autoethnography. To connect theory and narrative, I described major themes, illustrated those themes using instructional episodes, and continued with further analysis by using the layered account (Ronai, 1995).

Trustworthiness

Trustworthiness considerations for this autoethnography include Le Roux's (2016a) five criteria for autoethnographic rigor and trustworthiness: subjectivity, self-reflexivity, resonance, credibility, and contribution. Readers can judge whether I established subjectivity, the centrality of myself in instructional episodes. Further, they can assess whether I demonstrate self-reflexivity, an awareness of the different roles I took as an autoethnographer in this study—as a researcher and analyst, and as a teacher participant. This awareness is best seen from my use of the layered account (Ronai, 1995) to demarcate where I took the role of a teacher in describing instructional episodes and researcher in describing themes and analyzing instructional episodes. Most importantly, readers can assess for resonance as to whether they could imagine myself interacting inside my classroom with students. For credibility, I intended to replicate the ethnographers' path previously described as existing in Chapters 4-7 and in the opening narrative to "Musings on Autoethnography." Finally, I aimed to establish contribution in Chapter 8: to establish connections with and contributions to existing research, as well as in describing limitations of this study and possibilities for future research.

Limitations

This study is limited in that it reflects my perceptions of what occurred in my classroom. While this describes the intent behind my teaching actions and moves to negotiate norms, it does not capture student perceptions of that intent. Rather, it captures my perceptions of how students reacted to my teaching moves. I make no claim of objectivity in my analysis—I attempted to separate myself internally to identify flaws as a teacher from the researcher perspective, but fully separating these roles is impossible. Further, the challenge of a teacher-researcher recording each iteration of struggle and its outcome as productive, low-level productive, or unproductive means that a researcher acting only as an observer or would better capture the specific outcomes of these iterations of struggle. Teachers who implement similar strategies to those described in this study will likely find similarities but also differences in that they negotiate and establish different norms. Another teacher may call a norm “collaboration” but conceptualize its implementation differently than I did. Furthermore, the unique groups of students in each of my blocks interpreted my expectations in unique ways that created their own unique microcultures. Each microculture will influence students’ struggles in different ways. This autoethnography only represents *an* account of establishing a microculture, but by no means does it represent *the* account. Other teachers should endeavor to respond to their students’ collective needs in establishing expectations and negotiating norms to create a microculture that responds to their students in its own unique way.

Implications for Research

Studies that are not entirely autoethnographic—from the teacher’s perspective solely—would better discover student perceptions of classrooms using inquiry or problem-based curriculums. Future research could shed further light on the negotiation of norms with multiple researchers. To better capture what occurs in a classroom, an autoethnographer could capture the teacher side of negotiation, while an observer could interpret student interpretations and compliance to teacher expectations. This research could look at both teacher and student perspectives for how sociomathematical norms connect with both productive struggle and the development of productive dispositions. Student perspectives toward social norms and sociomathematical norms are critical as Levenson and colleagues (2009) found that “even when the observed enacted norms are in agreement with the teachers’ endorsed norms, the students may not perceive these same norms” (p. 171). Capturing perspectives from both classroom teachers and students is necessary as “the negotiation of goals among students and teachers can be in competition, in cooperation/collaboration, or neutral (meaning both, but not favoring one over the other)” (Megowan-Romanowicz et al., 2013, p. 72). Sellers’ (2016) identified types of reprising responses could inform coding teacher-student interactions to categorize teacher responses to types of student struggle.   Reprising responses might provide better clarity between whether a response is directed guidance, probing guidance, affordance, or even fit as subcategories within those responses (e.g., Warshauer, 2015a).

Future research that involves observation of small groups of students can build upon the classroom microculture and productive struggle framework by better describing and clarifying how those interactions occur. This research should start by following the development and explicit negotiation of social norms in the classroom. As time goes on, a researcher can observe teacher moves that support and detract from these negotiated norms. Further, that researcher will then track the development of classroom mathematical practices and what practices students turn to while engaged in mathematics. While students engage in mathematics, the researcher will record student interactions to best understand how students interpret social norms and apply their adherence to those norms, and further, how students comply or ignore teacher expectations laid out in the launch of the task.

Future research is necessary to gauge the specific influence of classroom mathematical practices on specific instances of student struggle. For example, what classroom mathematical practices from prior grades are sustained in the following year and used in episodes of student struggle? What classroom mathematical practices come into use while students struggle over the course of an entire school year instead of a semester? This study focused on the influence of the classroom microculture on the facilitation of struggle, a much larger focus than tracking student use of classroom mathematical practices during instances of struggle. A study positioned to track student use of classroom mathematical practices might make extensive use of video recordings to focus on and capture student interactions rather than a focus on a teacher's reflective practice. Future research could also build from Hoyles (2018) research in observing both the development and student perspective of classroom mathematical practices through digital tools.

A mixed method approach to investigating the influence of classroom mathematical practices on student productive struggle could build from Kapur's (2008, 2010, 2011, 2012, 2014) productive failure research. Productive failure classrooms have students approach ill structured tasks with the understanding that they should use prior knowledge in their approaches. The approaches students use in a productive failure instructional model could be investigated to identify classroom mathematical practices qualitatively while their eventual conceptual understanding is measured by a pre-test post-test model. Post-hoc analysis could uncover whether current classroom mathematical practices used by students have a larger effect on conceptual understanding than basic prior knowledge that is not identified as a classroom mathematical practice.

Study Summary

This study was an analytic autoethnography that investigated: In what ways does a teacher negotiate the establishment of classroom norms in order to facilitate productive struggle? Classroom norms were described using Cobb and Yackel's (1996) microculture framework, whereas productive struggle was operationalized using Warshauer's productive struggle framework (2015a). My experiences were documented by reflexive journal entries and were supplemented with additional documents such as pictures of student work, lesson plans, and presentation materials. Ancillary informants included colleagues in the math department at the research site, Dr. Sherri Martinie, and my wife, Kirsten. Over time, four themes emerged in the iterative process of coding, analyzing, and writing of instructional narratives. The primary theme suggests that classroom norms are continually renegotiated over time as teachers and students intersubjectively determine what is acceptable in the classroom. The other themes connect each part of the classroom microculture to elements of struggle: social norms provide supports for how students can collectively engage in struggle, sociomathematical norms contribute to how students approach engaging in mathematics, and classroom mathematical practices influence how students engage in struggling with novel mathematics.

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Appendix A - IRB Approval

KANSAS STATE UNIVERSITY | University Research Compliance Office

TO: Sherri Martinie
Curriculum and Instruction
Manhattan, KS 66506

Proposal Number: IRB-10791

FROM: Rick Scheidt, Chair
Committee on Research Involving Human Subjects

DATE: 08/16/2021

RE: Proposal Entitled, " The Role of Norms in Facilitating Productive Struggle."

The Committee on Research Involving Human Subjects / Institutional Review Board (IRB) for Kansas State University has reviewed the proposal identified above and has determined that it is EXEMPT from further IRB review. This exemption applies only to the proposal - as written – and currently on file with the IRB. Any change potentially affecting human subjects must be approved by the IRB prior to implementation and may disqualify the proposal from exemption.

Based upon information provided to the IRB, this activity is exempt under the criteria set forth in the Federal Policy for the Protection of Human Subjects, **45 CFR §104(d), category:Exempt Category 1.**

Certain research is exempt from the requirements of HHS/OHRP regulations. A determination that research is exempt does not imply that investigators have no ethical responsibilities to subjects in such research; it means only that the regulatory requirements related to IRB review, informed consent, and assurance of compliance do not apply to the research.

Any unanticipated problems involving risk to subjects or to others must be reported immediately to the Chair of the Committee on Research Involving Human Subjects, the University Research Compliance Office, and if the subjects are KSU students, to the Director of the Student Health Center.

Electronically signed by Rick Scheidt on 08/16/2021 4:47 PM ET

203 Fairchild Hall, Manhattan, KS 66502 | (785) 532-3224 | fax: (785) 532-3278

Appendix B - Building a Mathematical Classroom Community Plan

Illustrative Mathematics®
LEARN MATH FOR LIFE Building a Mathematical Classroom Community Plan

Goal for Days 1–3 Understand what it means to learn math by doing math and identify the associated actions.
Day 1: Unit 1, Lesson 1

1. Prepare a space, such as a piece of poster paper, titled "Mathematical Community" and a two-column table with the headers "Doing Math" and "Norms."

Mathematical Community	
Doing Math	*Norms*

2. Do the warm-up as described in the lesson plan.

3. After the warm-up, ask students to reflect on both individual and group actions while considering the question "What does it look and sound like to do math together as a mathematical community?"

4. Record and display their responses under the "Doing Math" header. Students might mention things such as: we talked to each other and to the teacher, we had quiet time to think, we shared our ideas, or we thought about the math ideas and words we knew.

5. Do the rest of the lesson as described, monitoring for the recorded actions as students work.

6. After the cool-down, revisit the "Doing Math" list of actions. Ask students to discuss with a partner where they saw evidence of the actions during the rest of the day's lesson. As a whole group, add any missing actions and revise earlier ideas.

Day 2: Unit 1, Lesson 2
1. Tell students they will have an opportunity to revise their “Mathematical Community” ideas at the end of this lesson, so as they work today they should think about actions that may be missing from the current list. 2. Do the warm-up and Activities as described in the lesson plan. 3. After the cool-down, give students 2–3 minutes to discuss any revisions to the “Doing Math” actions in small groups. 4. Share ideas as a whole group and record any revisions.
Day 3: Unit 1, Lesson 3
1. Tell students that, at the end of the lesson, they will be asked to identify specific actions from their “Doing Math” list they personally experienced. 2. Do the warm-up and activities as described in the lesson plan. As students work, monitor for examples of the “Doing Math” actions. 3. After the cool-down, ask students to individually reflect on the question “Which ‘Doing Math’ action did you feel was most important in your work today, and why?” Have students write their responses on the bottom of their cool-down page, on a separate sheet of paper, or in a math journal. 4. Collect and read their responses after class. These responses will offer insight into how students feel about their own mathematical work and help you make personal connections to the norms they will be creating during Days 4–6.
Goal for Days 4–6

Relate the actions of doing math to norms that support that work.
Day 4: Unit 1, Lesson 4
1. Explain to students that norms are expectations that help everyone in the room feel safe, comfortable, and productive doing math together. Offer an example, such as “It may help us share our ideas as a whole class if we have the norm ‘Listen as others share their ideas.’” Tell students you will pause at two different points of the lesson to identify norms that help everyone do math. 2. Do the warm-up as described in the lesson plan. 3. After the warm-up, ask students to reflect on both individual and group actions while considering the question “What norms, or expectations, were we mindful of as we did math together in our mathematical community?” 4. Record and display their responses under the “Norms” header. 5. Complete the lesson plan as described. 6. After the cool-down, revisit the “Norms” list. Ask students to discuss with a partner when a norm was helpful as they did math, and add any missing ideas or revise earlier ones.
Day 5: Unit 1, Lesson 5
1. Tell students that, at the end of the lesson, they will be asked to identify specific examples of norms they experienced as they did math. 2. Do the warm-up and activities as described in the lesson. 3. After the cool-down, give students 2–3 minutes to discuss in small groups any revisions to the “Norms” section. 4. Collect and record any revisions.
Day 6: Unit 1, Lesson 6

1. Tell students they will reflect on their identified norms at the end of this lesson.

2. Do the warm-up and activities as described in the lesson.

3. After the cool-down, ask students to individually reflect on the following question: "Which one of the norms did you feel was most important in your work today, and why?" Students can write their responses on the bottom of their cool-down page, on a separate sheet of paper, or in a math journal.

4. Tell students that as their mathematical community works together over the course of the year, the group will continually add to and revise its "Doing Math" and "Norms" actions and expectations.

Appendix C - Informed Consent Forms

Student Informants

The Role of Norms in Facilitating Productive Struggle

This research concerns the researcher's experiences in teaching mathematics. In particular, this study will investigate the impact of classroom norms, or ways of behaving and engaging in learning, on a math teaching practice called supporting productive struggle. The purpose of this study is to use narratives from the researcher's personal experience to describe the influence of the classroom environment on how a teacher supports productive struggle. Data for this study includes the researcher's reflective journaling, and video recordings of the researcher's teaching for the 2021-2022 academic school year. Reflective journaling includes memory work of his teaching— this means that interactions with students provide essential insights to what is happening in the classroom. Video recordings of teaching will only ever be seen and analyzed by the researcher, your child's teacher. To protect your child's privacy, these recordings and reflective journaling will be stored on a password protected server. Names in any narrative will be made anonymous through the use of pseudonyms and or constructions of fictional students who collectively represent various individuals. Presentations and research articles may come from analysis of this data. Information from this study may benefit and inform others on understanding classroom norms and math teaching practices. There are no anticipated risks or benefits to participating other than those encountered in daily life. The researcher is conducting this study as part of his doctoral dissertation at the College of Education at Kansas State University. This research project was IRB approved on 8/16/21.

If you have any questions or concerns about this research, you may contact the principal investigator, Tegan Nusser, nussert@usd320.com, 620.680.0341; and my Major Professor Sherri Martinie, martinie@ksu.edu, 785.532.8414. *For the participant/parent or guardian should he/she have questions or wish to discuss on any aspect of the research with an official of the university or the IRB. These are:* Rick Scheidt, Chair, Committee on Research Involving Human Subjects, 203 Fairchild Hall, Kansas State University, Manhattan, KS 66506, (785) 532-3224; Cheryl Doerr, Associate Vice President for Research Compliance, 203 Fairchild Hall, Kansas State University, Manhattan, KS 66506, (785) 532-3224.

Terms of participation: I understand this project is research, and that my participation is voluntary. I also understand that if I decide to participate in this study, I may withdraw my consent at any time, and stop participating at any time without explanation, penalty, or loss of benefits, or academic standing to which I may otherwise be entitled.

I verify that my signature below indicates that I have read and understand this consent form, and willingly agree to participate in this study under the terms described, and that my signature acknowledges that I have received a signed and dated copy of this consent form.

Participant name:

Participant signature: Date:

Parent/Guardian name:

Parent/Guardian signature: Date:

Colleague/Adult Informants

The Role of Norms in Facilitating Productive Struggle

This research concerns the researcher's experiences in teaching mathematics. In particular, this study will investigate the impact of classroom norms, or ways of behaving and engaging in learning, on a math teaching practice called supporting productive struggle. The purpose of this study is to use narratives from the researcher's personal experience to describe the influence of the classroom environment on how a teacher supports productive struggle. Data for this study includes the researcher's reflective journaling, and video recordings of the researcher's teaching for the 2021-2022 academic school year. Reflective journaling includes memory work of his teaching— this means that interactions with colleagues and other informants provide essential insights to what is happening in the classroom. To protect your privacy, these recordings and reflective journaling will be stored on a password protected server. Names in any narrative will be made anonymous through the use of pseudonyms. Presentations and research articles may come from analysis of this data. Information from this study may benefit and inform others on understanding classroom norms and math teaching practices. There are no anticipated risks or benefits to participating other than those encountered in daily life. The researcher is conducting this study as part of his doctoral dissertation at the College of Education at Kansas State University. This research project was IRB approved on 8/16/21.

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Terms of participation: I understand this project is research, and that my participation is voluntary. I also understand that if I decide to participate in this study, I may withdraw my consent at any time, and stop participating at any time without explanation, penalty, or loss of benefits, or academic standing to which I may otherwise be entitled.

I verify that my signature below indicates that I have read and understand this consent form, and willingly agree to participate in this study under the terms described, and that my signature acknowledges that I have received a signed and dated copy of this consent form.

Participant name:

Participant signature: Date:

Witness to signature: Date:

Appendix D - Doing Math Actions and Norms

Geometry

Norms

What norms, or expectations, were we mindful of as we did math together in our mathematical community?

Mathematical Community	
Doing Math	*Norms*
• Groaning and asking the teacher for help--sometimes happy • Crying--copying off of one person (asking others for help) • Parents frustrated that you don't do it the "right way" • Working with other people--can't do it alone • Mental health awareness • Collaborating--working off of one another's ideas • Getting frustrated • Being off task • Problem solving • Everyone contributing	• Show your work • Attempt something before asking for help • Be on task • Work as a team/group • Everyone contributes • Always justify and explain your thinking

Illustrative Mathematics Block 3 Kendall Hunt

Norms

What norms, or expectations, were we mindful of as we did math together in our mathematical community?

Mathematical Community	
Doing Math	*Norms*
• Teaching/Helping each other with what we know • Collaboration: working together to solve problems • Disagreements • Sounding smart • Discussing your ideas • Being able to debate your ideas	• Working together • Sharing strategies /opinions • TRIAL AND ERROR • Try SOMETHING before asking for help • Ask 3 before me • Burger • Be respectful of the person who has the floor to speak

Illustrative Mathematics Block 7 Kendall Hunt

Norms!

What norms, or expectations, were we mindful of as we did math together in our mathematical community?

Mathematical Community	
Doing Math	*Norms*
• Debating, arguing • Discussing strategies and ideas • Loud • Communication • Being engaged/engaged conversation	• Showing your work • Talking to each other/listening each other (don't talk over others) (engaged in math conversations) • Coming into class with open mind/positive set • NOT GIVING UP • Use your resources • Try even if you're unsure • Question yourself to see if your work is right/check your work • Put your distractions away • Take corrective criticism

Illustrative Mathematics

Block 1

Kendall Hunt

Norms!

What norms, or expectations, were we mindful of as we did math together in our mathematical community?

Mathematical Community	
Doing Math	*Norms*
• Loud • Sounds like numbers (talking about math) • Getting along and putting in all your thoughts into it • Groups working together • Communication is key • A lot of talking • Working together as a group • People talking over each other • Arguing/debating • PENCILS ON PAPER	• Work as a team/partner • Talk about the problem • Stay on task • Try SOMETHING before asking for help • Give effort • Don't goof around

Illustrative Mathematics

Block 4

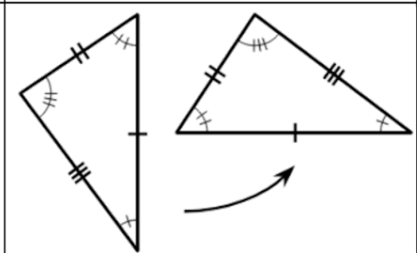
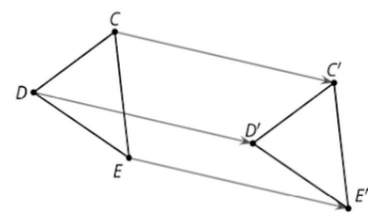
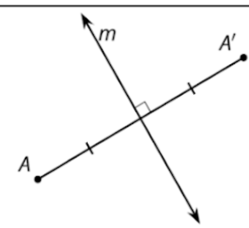
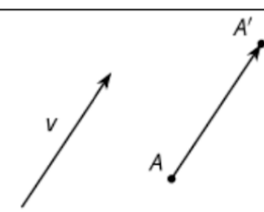
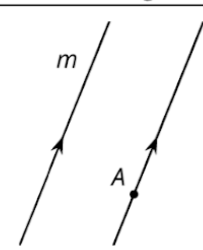
Kendall Hunt

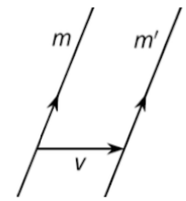
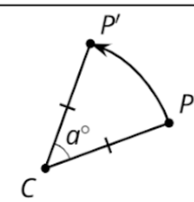
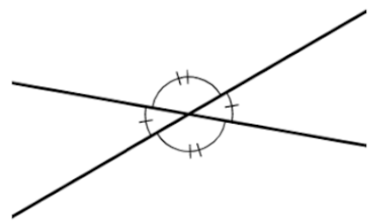
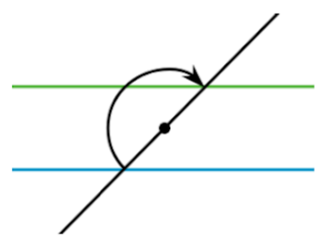
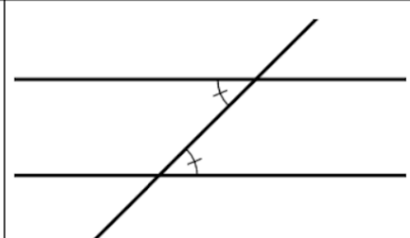
Norms!

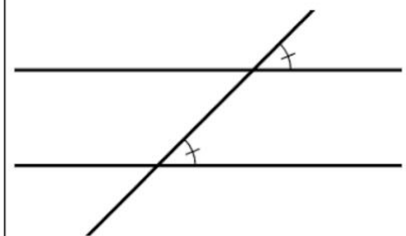
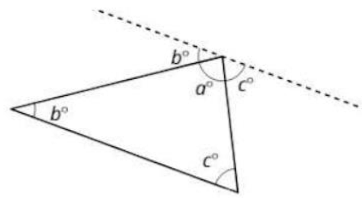
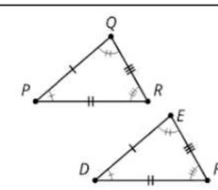
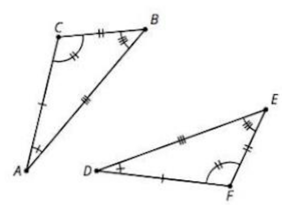
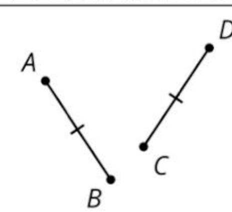
What norms, or expectations, were we mindful of as we did math together in our mathematical community?

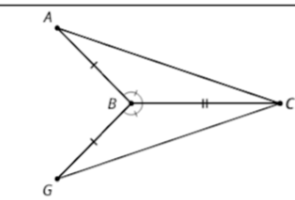
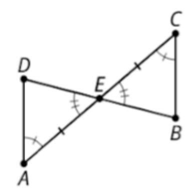
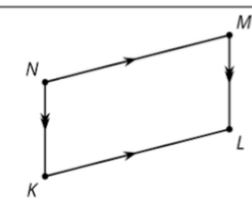
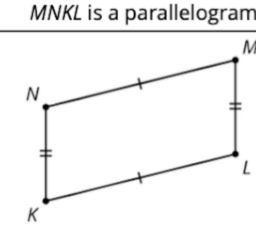
Mathematical Community	
Doing Math	*Norms*
• Chaotic • Agreeing and disagreeing--making a solution out of that • Putting your heads together/working together • Each member having a different opinion • Discussing math • Frustrating/Stressful • Fun Fresh and Funky?	• Listen to others • Ask for help-- ask 3 before me (the teacher) • Discuss the problems with each other • Collaborate with each other (work with your peers) • Try SOMETHING before you ask for help

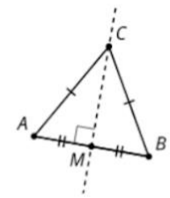
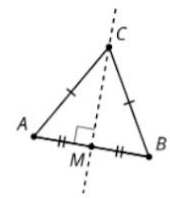
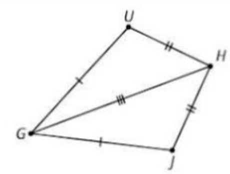
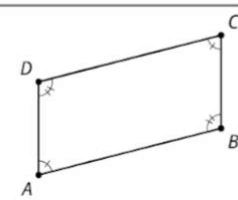
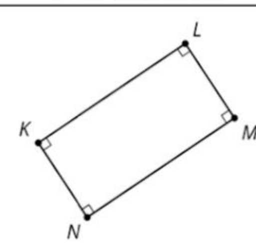
Appendix E - Geometry Reference Sheets

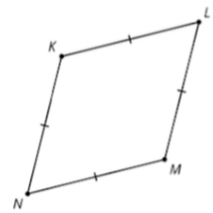
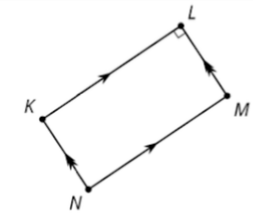
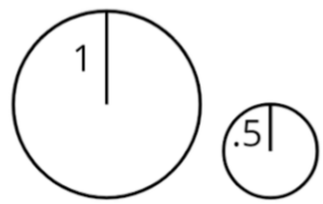
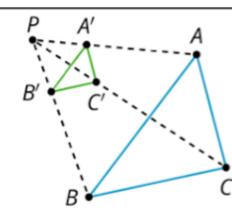
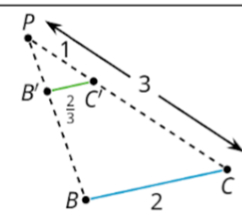
lesson, type	statement	diagram
U1, L10 (students write the date) assertion	A **rigid transformation** is a translation, reflection, rotation, or any sequence of the three. Rigid transformations take lines to lines, angles to angles of the same measure, and segments to segments of the same length.	
U1, L10 definition	One figure is **congruent** to another if there is a sequence of translations, rotations, and reflections that takes the first figure exactly onto the second figure. The second figure is called the image of the rigid transformation.	 $\triangle EDC \cong \triangle E'D'C'$
U1, L11 definition	**Reflection** is a rigid transformation that takes a point to another point that is the same distance from the given line, is on the other side of the given line, and so that the segment from the original point to the image is perpendicular to the given line. Reflect <u>(object)</u> across line <u>(name)</u>.	 Reflect *A* across line *m*.
U1, L12 definition	**Translation** is a rigid transformation that takes a point to another point so that the directed line segment from the original point to the image is parallel to the given line segment and has the same length and direction. Translate <u>(object)</u> by the directed line segment <u>(name or from [point] to [point])</u>.	 Translate *A* by the directed line segment *v*.
U1, L12 assertion	**Parallel Postulate:** Given a line *m* and a point *A* that is not on *m*, there is exactly one line that goes through *A* that is parallel to *m*.	

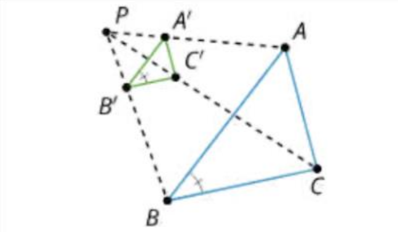
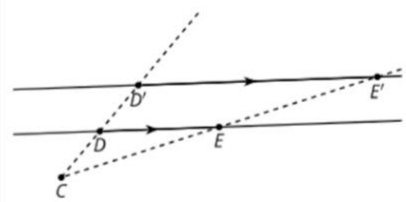
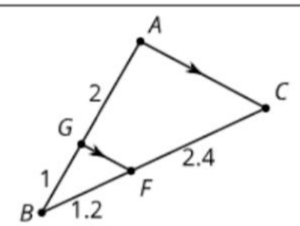
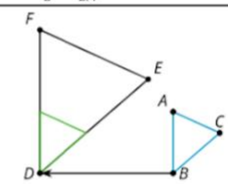
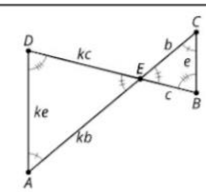
lesson, type	**statement**	**diagram**
U1, L12 theorem	Translations take lines to parallel lines or to themselves.	 $m \parallel m'$
U1, L14 definition	**Rotation** is a rigid transformation that takes a point to another point on the circle through the original point with the given center. The two radii to the original point and the image make the given angle. Rotate <u>(object)</u> (clockwise or counterclockwise) by <u>(angle or angle measure)</u> using center <u>(point)</u>.	 Rotate *P* counterclockwise by *a*° using center *C*.
U1, L19 theorem	Vertical angles are congruent.	
U1, L20 assertion	Rotation by 180 degrees takes lines to parallel lines or to themselves.	
U1, L20 theorem	**Alternate Interior Angle Theorem:** If two parallel lines are cut by a transversal, then alternate interior angles are congruent. Conversely, if two lines are cut by a transversal and alternate interior angles are congruent, then the lines have to be parallel.	

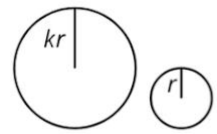
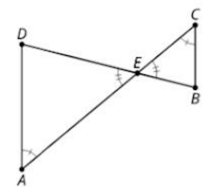
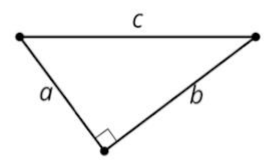
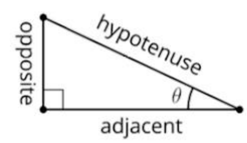
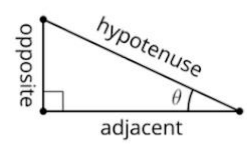
lesson, type	statement	diagram
U1, L20 theorem	**Corresponding Angle Theorem:** If two parallel lines are cut by a transversal, then corresponding angles are congruent. Conversely, if two lines are cut by a transversal and corresponding angles are congruent, then the lines have to be parallel.	
U1, L21 theorem	**Triangle Angle Sum Theorem:** The three angle measures of any triangle always sum to 180 degrees.	 $a + b + c = 180$
U2, L1 theorem	If two figures are congruent, then **corresponding** parts of those figures must be congruent	 $\triangle PQR \cong \triangle DEF$ so $PQ=DE$, $PR=DF$, $QR=EF$, $\angle P \cong \angle D$, $\angle Q \cong \angle E$, $\angle R \cong \angle F$
U2, L3 theorem	If all pairs of corresponding sides and all pairs of corresponding angles are congruent, then the triangles must be congruent.	 $AB=DE$, $BC=EF$, $CA=FD$, $\angle B \cong \angle E$, $\angle A \cong \angle D$, $\angle C \cong \angle F$ so $\triangle ABC \cong \triangle DEF$
U2, L5 theorem	If two segments have the same length, then they are congruent.	 $AB = CD$ so, $\overline{AB} \cong \overline{CD}$

lesson, type	statement	diagram
U2, L6 theorem	**Side-Angle-Side Triangle Congruence Theorem:** In two triangles, if two pairs of congruent corresponding sides and the pair of corresponding angles between the sides are congruent, then the two triangles are congruent.	 $AB=GB$, $BC=BC$, $\angle ABC \cong \angle GBC$ so $\triangle ABC \cong \triangle GBC$
U2, L6 theorem	**Isosceles Triangle Theorem:** In an isosceles triangle, the base angles are congruent.	 $AP=PB$ so $\angle A \cong \angle B$
U2, L7 theorem	**Angle-Side-Angle Triangle Congruence Theorem:** In two triangles, if two pairs of corresponding angles, and the pair of corresponding sides between the angles, are congruent, then the triangles must be congruent.	 $\angle A \cong \angle C$, $AE=EC$, $\angle DEA \cong \angle BEC$, so $\triangle DEA \cong \triangle BEC$
U2, L7 definition	A **parallelogram** is a quadrilateral with two pairs of opposite sides parallel.	 $NM \parallel KL$, $NK \parallel ML$, so $MNKL$ is a parallelogram
U2, L7 theorem	In a parallelogram, pairs of opposite sides are congruent.	 $MNKL$ is a parallelogram, so $NM=KL$, $NK=ML$

lesson, type	statement	diagram
U2, L8 theorem	If a point C is the same distance from A as it is from B, then C must be on the perpendicular bisector of AB.	 $AC=BC$, M is the midpoint, so $MC \perp AB$
U2, L8 theorem	If C is a point on the perpendicular bisector of segment AB, the distance from C to A is the same as the distance from C to B.	 $AB \perp CM$, $AM=BM$, so $AC=BC$
U2, L9 theorem	**Side-Side-Side Triangle Congruence Theorem:** In two triangles, if all three pairs of corresponding sides are congruent, then the triangles must be congruent.	 $HU=HJ$, $UG=JG$, $HG=HG$ so $\triangle HUG \cong \triangle HJG$
U2, L9 theorem	In a parallelogram, opposite angles are congruent.	 $ABCD$ is a parallelogram, so $\angle A \cong \angle C$, $\angle D \cong \angle B$
U2, L12 definition	A **rectangle** is a quadrilateral with four right angles.	

lesson, type	statement	diagram
U2, L12 definition	A **rhombus** is a quadrilateral with four congruent sides.	
U2, L12 theorem	If a parallelogram has (at least) one right angle, then it is a rectangle.	 *KLMN* has a right angle so it is a rectangle
U3, L1 definition	**Scale factor** is the factor by which every length in an original figure is multiplied when you make a scaled copy.	 Scale factor is 2 or ½
U3, L1 definition	A **dilation** with center P and positive scale factor k takes a point A along the ray PA to another point whose distance is k times further away from P than A is. Dilate _(object)_ using center _(point)_ and a scale factor of _(number)_.	 $PA' = k \cdot PA$
U3, L3 assertion	The dilation of a line segment is longer or shorter according to the same ratio given by the scale factor.	 $PC:PC' = 3:1$, $BC:B'C' = 2:\frac{2}{3}$

lesson, type	statement	diagram
U3, L4 assertion	If a figure is dilated, then corresponding angles are congruent.	 $\triangle A'B'C'$ is a dilation of $\triangle ABC$ so $\angle B \cong \angle B'$
U3, L4 theorem	A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.	 Dilate using center C. $DE \parallel D'E'$
U3, L5 theorem	If a line divides two sides of a triangle proportionally, the line must be parallel to the third side of the triangle.	 $\frac{1}{2} = \frac{1.2}{2.4}$ so $AC \parallel GF$
U3, L6 definition	One figure is **similar** to another if there is a sequence of rigid motions and dilations that takes the first figure so that it fits exactly over the second.	 Translation and dilation takes $\triangle ABC$ onto $\triangle DEF$ so $\triangle ABC \sim \triangle DEF$
U3, L7 theorem	If two triangles have all pairs of corresponding angles congruent and all pairs of corresponding side lengths in the same proportion, then the two triangles are similar.	 $\angle A \cong \angle C$, $\angle D \cong \angle B$, $\angle DEA \cong \angle BEC$, $\frac{AD}{CB} = \frac{DE}{BE} = \frac{EA}{EC}$ so $\triangle DEA \sim \triangle BEC$

lesson, type	statement	diagram
U3, L8 theorem	All circles are similar.	
U3, L9 theorem	**Angle-Angle Triangle Similarity Theorem:** In two triangles, if two pairs of corresponding angles are congruent, then the triangles must be similar.	 $\angle A \cong \angle C$, $\angle DEA \cong \angle BEC$, so $\triangle DEA \sim \triangle BEC$
U3, L14 theorem	**Pythagorean Theorem:** If a right triangle has legs with lengths a and b and hypotenuse with length c, then $a^2 + b^2 = c^2$.	 $a^2 + b^2 = c^2$
U4, L6 definition	The **cosine** of an acute angle in a right triangle is the ratio (quotient) of the length of the adjacent leg to the length of the hypotenuse.	 $\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$
U4, L6 definition	The **sine** of an acute angle in a right triangle is the ratio (quotient) of the length of the opposite leg to the length of the hypotenuse.	 $\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$

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