INFERENCE SYSTEM FOR SELECTION OF AN APPROPRIATE MULTIPLE ATTRIBUTE DECISION MAKING METHOD

by

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TABLE OF CONTENTS

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LIST OF FIGURES

LIST OF TABLES

1 INTRODUCTION

This report deals with the problem of choosing the best MADM (Multiple Attribute Decision Making) method from a number of solution techniques available to solve a MADM problem. A MADM problem is a decision making problem in which a decision maker selects the best alternative among a finite number of alternatives based on some attributes. Thus, the selection of the best MADM method itself is a MADM problem.

MADM problems are every day ocurrences (1). In personal or domestic decision making situations, examples are choosing a car, house, school, or job. In business decision making situations, examples are selecting a piece of equipment, manager, marketing strategy, plant site, etc. In public policy making situations, examples are choosing a means of transportation, energy storage system, or an area for R & D.

Although the study of multiple criteria has a long tradition, substantial advancement in MADM has been made only in the last two decades. As a result, a number of MADM methods have been developed (See Fig. 1 for a taxonomy of MADM methods). However, little research has been done on the problem of selection of an appropriate MADM method, given a specific MADM problem.

1.1 Literature Survey

Only two articles were found during the literature survey concerning the MADM method selection problem.

THIS BOOK CONTAINS NUMEROUS PAGES WITH DIAGRAMS THAT ARE CROOKED COMPARED TO THE REST OF THE INFORMATION ON THE PAGE.

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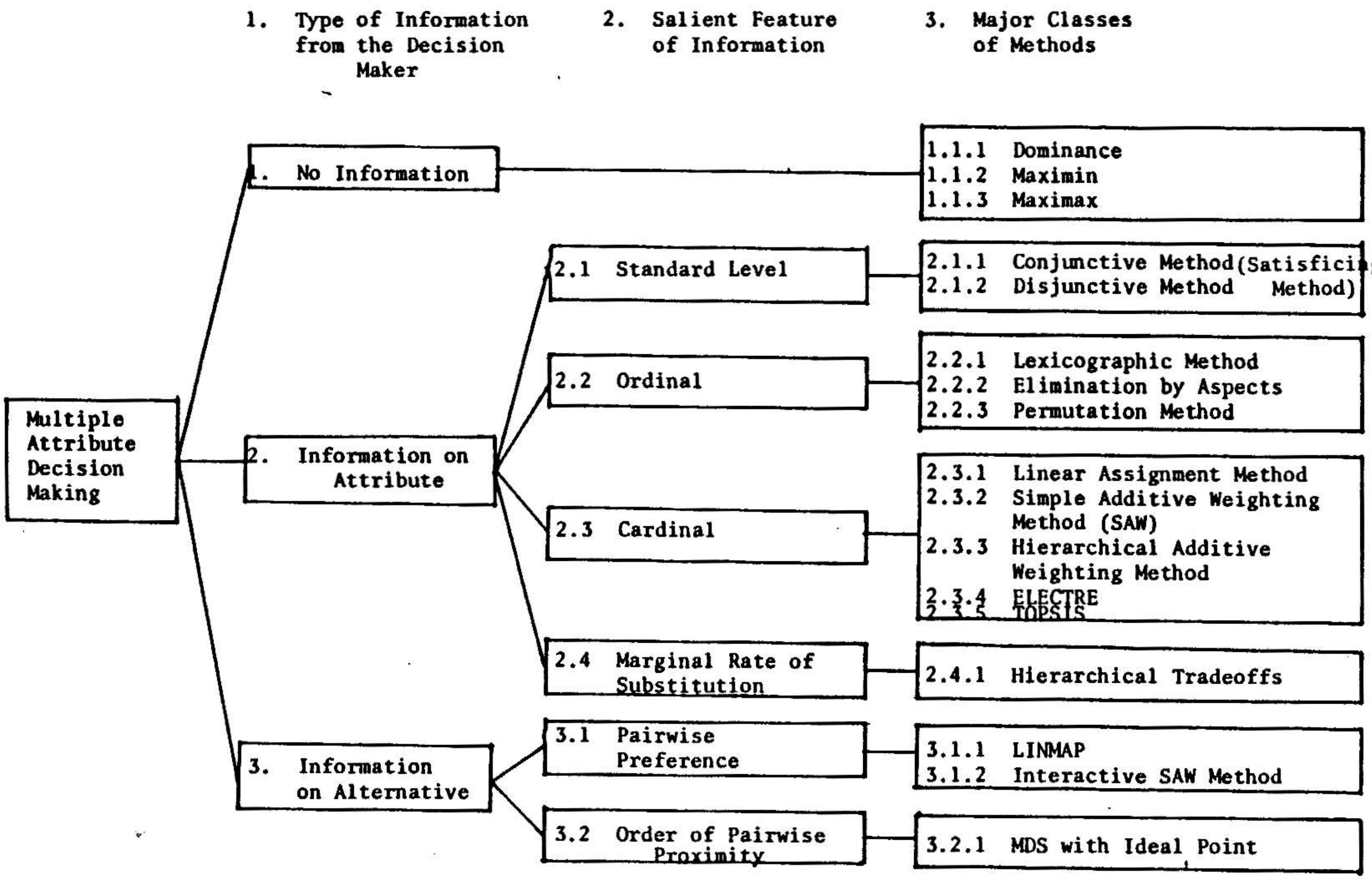
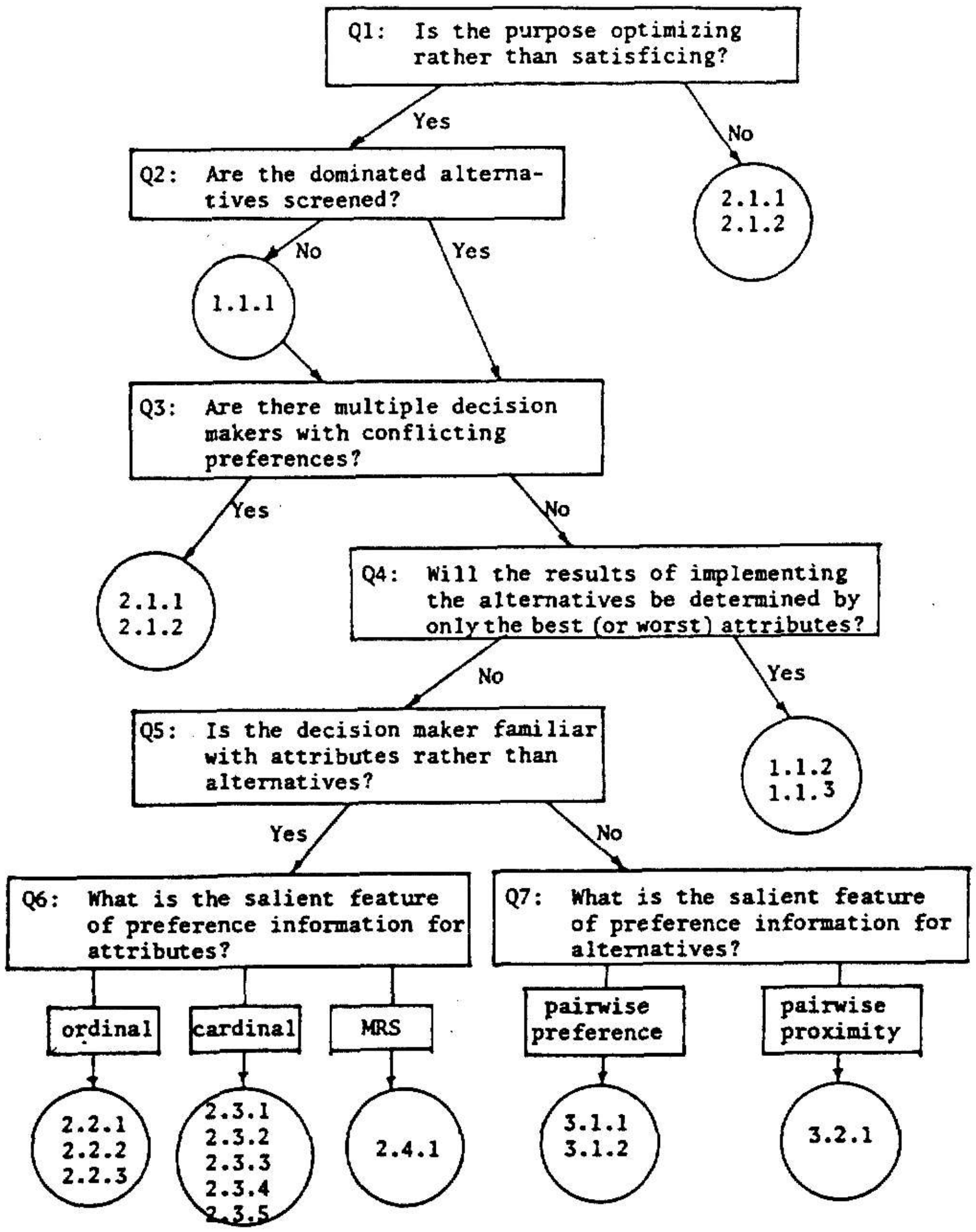


Fig. 1 A Taxonomy of Methods for MADM (2, p. 9)

Hwang and Yoon (2) developed a general choice rule using a tree diagram (Fig. 2). In Fig. 2, the proper method(s) emerge(s) by answering a sequence of questions. This choice rule is not specific enough to select the one best method, but rather leads to a group of methods. Some of the last nodes contain more than one MADM method, and a decision maker still has to choose one from among them to implement. For example, the LEXICOGRAPHIC method and ELIMINATION BY ASPECTS are grouped under the same ordinal preference information on attributes, but for application the two methods are used for quite different decision making situations.

Gershon and Duckstein (3) used the MADM approach to solve the MADM method selection problem. They evaluated the thirteen MCDM (Multiple Criteria Decision Making) techniques, including both MADM and MODM (Multiple Objective Decision Making) techniques, using twenty-eight attributes. The thirteen techniques are listed in Fig 3. They divided the 28 attributes into four groups, only one of which must be reevaluated for each decision problem encountered. The compromise programming was then used to select the appropriate technique for implementation. The weakest point of this approach is that there is no justification for the use of the compromise programming for selection. Should they have used a MADM method to justify the selection of the compromise programming? But which method should they use? Do they need another MADM method to select? This is a paradox.



Note; Refer to Fig. 1 for the methods in circles

Fig. 2 MADM Method Specification Chart (2, p. 211)

(1) Sequential optimization

(2) Weighting

(3) C-constraint

(4) Compromise programming

(5) Goal programming

(6) Cooperative game

(7) Multiattribute utility

(8) Surrogate worth tradeoff

(9) ELECTRE

(10) Q-analysis

(11) Dynamic compromise programming

(12) PROTRADE

(13) STEP

Fig. 3 Thirteen Selected Techniques (3)

1.2 Objective of Research

In this report, I attempted to develop an inference system which selects a MADM method based on information provided by a decision maker. The inference system is the decision rule (logic) representing a human MADM expert's intuitive approach to this problem.

The inference system can be computerized using any computer language. Expert system shells such as EXPERT-EASE which represent a decision table are more suitable for computerizing the inference system. However the inference system must be considered a part of a larger MADM Decision Support System (DSS). The organization of DSS is well explained using the architecture of an expert system. An expert system is a computer program which performs an intelligent task like a human expert in a certain specific problem domain. Expert systems are one of the principal Artificial Intelligence (AI) application areas. Fig. 4 shows the basic structure of an expert system (4).

The _knowlege base_ in an expert system contains facts and heuristics. Facts consist of the widely shared knowledge that is written in textbooks or that forms the basis of lectures in a classroom. Heuristics are rules of thumb or bases of good judgment that a human expert acquires over years of work in his field. The knowledge base in DSS is a collection of computer programs of MADM methods.

The _inference system_ in an expert system contains a problem-solving procedure or a reasoning procedure to act upon the combination of knowledge and problem data and make effective use of them to find a solution. The inference system in DSS determines which method in the knowledge base should be loaded to solve a problem, based on information acquired through the interaction with the decision maker.

The knowledge aquisition facility in an expert system eases the transfer of expertise from humans to the symbolic data structure by acquiring the knowledge automatically or semiautomatically. In the case of DSS, a human programmer programs MADM methods using some computer language.

The expert in an expert system is a person who has become extremely proficient at problem solving in a particular domain through years of training and experience. For DSS, the expert is a person proficient in the MADM area.

The knowledge engineer is a person who designs and builds the expert system or DSS.

The input/output system is a user-friendly interface to receive parameters and problem input data from the user and transmit the decision for the problem.

Chaper 2 explains the basic concepts of MADM. Chapter 3 describes the process of developing the inference system. Section 3.1 contains the classification of MADM problems with examples. Section 3.2 provides the best match between the problem types and the methods. Section 3.3 includes the decision rule (logic) to identify the problem type, and hence the best method(s) for a given MADM problem. Chapter 4 gives the conclusions.

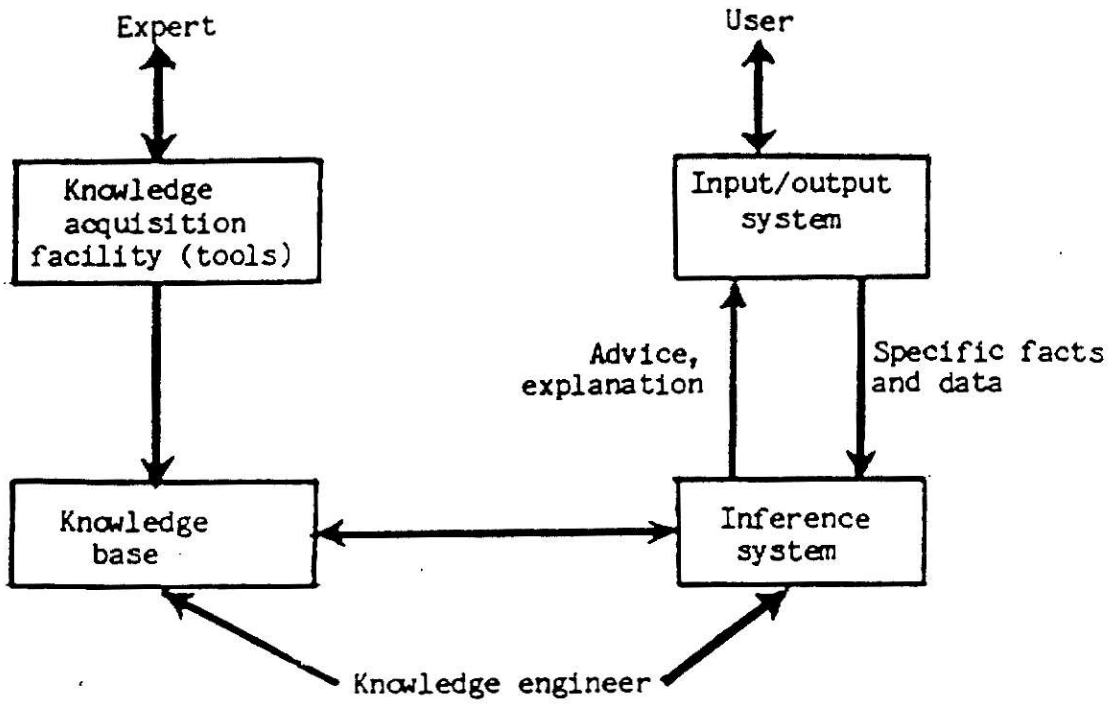


Fig. 4 Basic Structure of an Expert System (4, p. 76)

2 MULTIPLE ATTRIBUTE DECISION MAKING (MADM)

2.1 Definitions

MADM can be defined as decision aids to help a decision maker identify the best alternative among a finite number of alternatives that maximize his satisfaction with respect to more than one attribute.

MADM is one of the two categories of MCDM (Multiple Criteria Decision Making). The other category is MODM (Multiple Objective Decision Making).

To discuss the nature of MCDM (MADM) problems, the meanings of three terms must be clarified (2,5).

<u>Criteria</u> Criteria are measures, rules, and standards that guide decision making. Criteria form the basis for evaluation. Criteria emerge as a form of attributes and objectives in the actual problem setting.

<u>Attributes</u> Attributes are characteristics of objects in the world. Attributes should provide means of evaluating the levels of objectives. They can be measured in relative independence from the decision maker's needs or desires. Each alternative can be characterized by a number of attributes (chosen by decision maker's concept of criteria), e.g., gas mileage, purchasing cost, horsepower, etc. of a car.

<u>Objectives</u> After attributes are measured, a decision maker must decide which attributes, at what levels, to maximize or minimize, where his needs and desires come into play. An attribute becomes an objective when it is assigned a purpose, direction of desirability or improvement. For

example, horsepower is an attribute but "to maximize horsepower" is an objective. An objective may derive from an aggregate of attributes. For example, the objective of maximizing prestige may derive from the combined attributes of price, horsepower, scarcity, group affiliation, etc.

Multiple criteria decision making refers to making decisions in the presence of multiple, usually conflicting, criteria. Most of real world decision making deals with multiple criteria, and one can think of numerous examples easily.

2.2 Characterics

MCDM (MADM) problems usually have the following characteristics (2).

Multiple criteria (objectives/attributes) Each problem has multiple objectives/attributes. A decision maker must generate relevant objectives/attributes for each problem setting.

Conflict among criteria Multiple criteria usually conflict with each other. For example, in designing a car, the objective of higher gas mileage may reduce the objective of higher comfort as a result of the smaller passenger space.

Incommensurable units Each criterion (objective/attribute) has a different unit of measurement. In the car selection case, gas mileage is expressed by miles per gallon (MPG), comfort is by cu. ft if it is measured by passenger space.

The purpose of MADM is to <u>select</u> the best alternative among previously specified <u>finite</u> number of alternatives while the purpose of MODM is to <u>design</u> the best alternative (choose among <u>infinite</u> number of alternatives). For instance, a car a customer may purchase (select) is among the available finite models auto companies have produced, but a model which a company mass produced is among the infinite number of options which engineers may have designed. The MCDM process involves designing/searching for an alternative that is the most attractive over all criteria. Table 1 shows the contrast of the features between MADM and MODM.

2.3 An Example Problem

A typical example problem of MADM is shown in Table 2. Note this automobile (car) selection problem has the three characteristics of MCDM problems.

A potential car buyer is looking for a car. He has four models (alternatives - A_1, A_2, A_3, A_4) in his mind to select from. He has seven attributes to evaluate each alternative (<u>multiple criteria</u>); namely price, comfort (roominess), economy, safety, maintenance, depreciation, and appeal. Some of the attributes conflict with each other (<u>conflict among criteria</u>). For example, the alternative A_1 (presumably a small car) has better fuel economy, but is less comfortable and is not as safe as a big car in case of an accident (because the car is small and light, the driver is more likely to be injured severely). No car has both

Table 1 MADM vs MODM (2, p. 4)

	MADM	MODM
Criteria	Attributes	Objectives
Objective	Implicit	Explicit
Attribute	Explicit	Implicit
Constraint	Inactive	Active
Alternative	Finite number discrete	Infinite continuous
Interaction	Not much	Mostly
Usage	Selection	Design

Table 2 An Example Problem

(Automobile (Car) Selection)

Alternatives (A_i)	Price ($)	Comfort (roominess) (cu. ft)	Economy (gas mileage (MPG)	Safety (good-bad)	Maintenance ($)	Depreciation (%)	Appeal (grades (1-5)
	ATTRIBUTES (X_j)						
A_1	8,000	54	35	poor	200	25	3
A_2	15,000	85	28	very good	500	20	5
A_3	9,000	100	22	good	400	30	4
A_4	20,000	128	16	excellent	1000	40	5

o. Multiple attribute (criteria)
o. Conflicting criteria
o. Incommensurable units

good fuel economy and good comfort and safety at the same time. The price is measured in dollars, the comfort in cu. ft, economy in miles per gallon, and so forth (<u>incommensurable units</u>).

Also note that a typical MADM problem can be concisely expressed in a matrix form called <u>decision matrix</u>.

<u>Decision matrix</u> A decision matrix D is a (m x n) matrix whose element x_{ij}'s indicate evaluation or value of alternative i, A_i, with respect to attribute j, X_j. M is the number of alternatives, and n is the number of attributes. Hence, A_i, i = 1, 2, ..., m is denoted by

$$\underline{x}_i = (x_{i1}, x_{i2}, \ldots, x_{in})$$

and the column vector

$$\underline{x}_j = (x_{1j}, x_{2j}, \ldots, x_{mj})$$

shows the contrast of each alternative with respect to attribute j, X_j. The decision matrix for the automobile selection problem, then, is as follows.

$$D = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{ccccccc} X_1 & X_2 & X_3 & X_4 & X_5 & X_6 & X_7 \end{array} \\ \begin{bmatrix} 8000 & 54 & 35 & \text{poor} & 200 & 25 & 3 \\ 15000 & 85 & 28 & \text{very good} & 500 & 20 & 5 \\ 9000 & 100 & 22 & \text{good} & 400 & 30 & 4 \\ 20000 & 128 & 16 & \text{excellent} & 1000 & 40 & 5 \end{bmatrix} \end{array}$$

2.4 Assessing Importance of Attributes (Weights)

In a typical MADM problem, all attributes are not equally important for a decision maker. In the car selection problem, for example, for a decision maker with less money on hand, the price of a car is the primary concern for him, and other attributes are secondary. The information about the relative importance of each attribute is usually expressed by a set of weights which is usually normalized to sum to 1. In case of n attributes, a set of weights is expressed as follows.

$$\underline{w}^T = (w_1, w_2, \ldots, w_j, \ldots, w_n)$$

$$\sum_{j=1}^{n} w_j = 1$$

In some cases, particularly when a decision maker is experienced, he can directly assign a numerical value to each attribute. In the car selection case, for instance, for the decision maker above, a set of weights might be (Price: 0.4; Comfort: 0.1; Economy: 0.1; Safety: 0.1; Maintenance: 0.1; Depreciation: 0.1; Appeal: 0.1). But in most cases, a decision maker has difficulty to assign weights directly. In these cases the eigenvector method (6) is frequently used to assess weights. The eigenvector method is summarized in Appendix A. The eigenvector method utilizes pairwise comparisons of importance of attributes. It can equally be used to evaluate alternatives subjectively by using pairwise comparisons among alternatives.

In a complex decision making situation in the real world, attributes often form a hierarchical structure in a human mind. Such an example is shown in Fig. 5. In this case, the logical way to assess weights is to assign weights to the highest level attribute first (the highest level has only one attribute, so the weight is 1.0). For the car selection problem, the highest level attribute would be "best car." Then, the weight is distributed to the second level, and then, in turn, to the third level and so forth.

A model of a MADM problem with attributes with a hierarchical structure is shown in Fig. 6. In Fig. 6, the pairwise comparison of the final attributes in the decision matrix (X_{11}, ..., X_{33}) is not likely to assess weights correctly, because they are from different levels of the hierarchy. The more systematic way to assess weights is to have the decision maker first compare the relative importance of X_1, X_2, X_3 and assesse weights (X_1: 03; X_2: 0.2; X_3: 0.5). Next, he assesses the relative importance of X_{11} vs X_{12}, and the weight of X_1 (0.3) is distributed using the resultant weights. Then he evaluates X_{31}, X_{32}, X_{33} and so on.

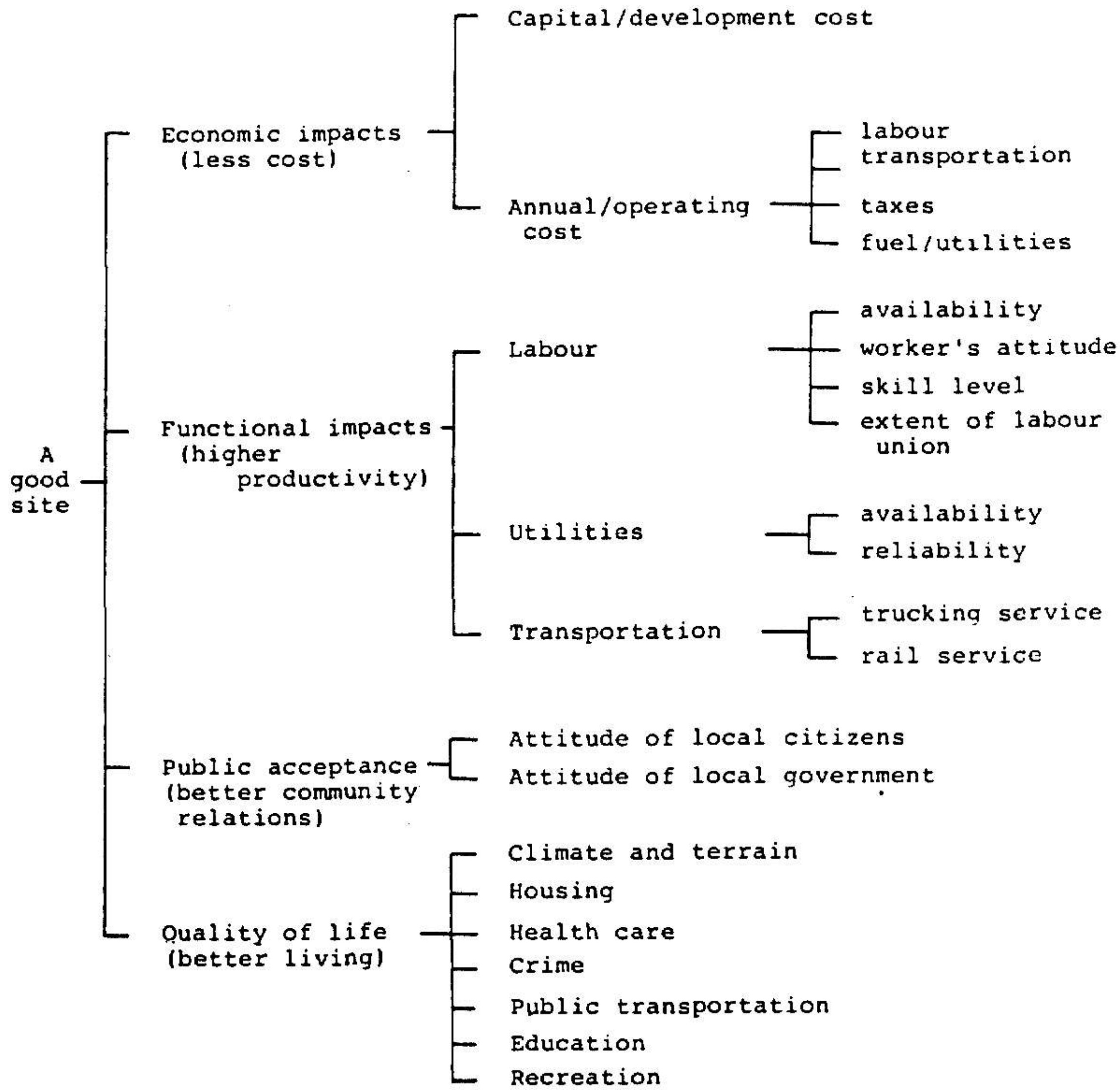


Fig. 5 A Hierarchy of Attributes for Evaluation Plant Sites
(7, p. 346)

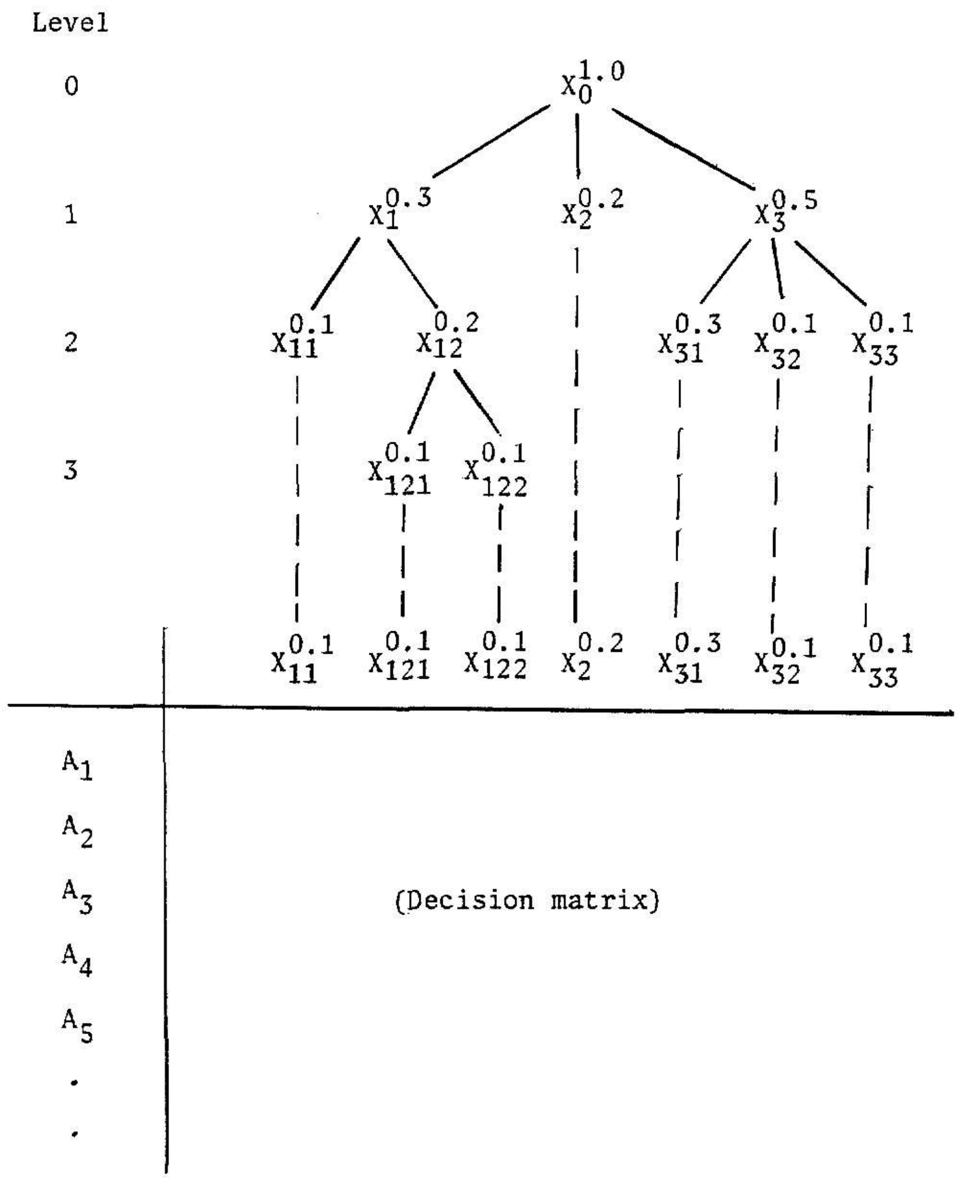


Fig. 6 A Hierarchical Structure of Attributes

3 INFERENCE SYSTEM

The inference system is a decision rule which selects the best MADM method to solve a problem. The decision rule represents a human MADM expert's intuitive approach by:

(1) classifying problems,

(2) matching the problem types with the methods,

(3) mining the decision maker's mind through a systematically ordered series of questions to identify the type of problem.

3.1 Classification of MADM Problems

3.1.1 Key concepts

MADM problems can be classified according to the characterictics they have. The characteristics of MADM problems can be described by the key concepts in a MADM problem (Fig 7).

Attributes A MADM problem is a problem of how attribute information is to be processed to arrive at a choice. So, the nature of attributes is an important aspect compared to the alternatives. Attributes refer to descriptor of objective reality. They may be actual objective traits, or they may be subjectively assigned traits. For example, when a man chooses a wife, candidates might be described in terms of height, weight, age, wealth. These are plainly objective. An age of 24 is an age of 24, and neither the decision maker nor the object of desire can do anything about it. Other attributes are more subjective: intellect, beauty, figure, companionship, social status, tastefulness, Attributes

objective vs subjective
(measurable vs poorly measureable)
incommensurable vs commensurable
weights (degree of importance)

Data

numerical evaluation
rank evauation
YES or NO evaluation

Solution aimed at

screen
select (maximize utility function)
minimize the distance from the target
prioritize

Trade-off

compensatory vs noncompensatory

Size

number of alternatives
number of attributes

Fig. 7 Key Concepts in a MADM Problem

and so on. Less well defined, these attributes are less precisely measurable than objective ones.

In most MADM problems attributes are incommensurable. In other words, you cannot compare apples with oranges. Again, in the car selection problem, you cannot say "very good" safety is better than 25 MPG fuel economy. Incommensurable attributes are different from incommensurable units. Test scores of both mathematics and English can be measured in percentage (%) ,but they are still apples and oranges.

Usually a decision maker wishes to consider all the attributes at the same time, although he may want to put different importance on each attribute (weights). But in some decision situations, a single attribute may predominate. For example, "buy the cheapest" rule is that in which the price is the most important attribute to the decision maker.

Data Typically the data in the decision matrix is numerical. Even qualitative data can be converted into numerical data utilizing an interval scale. An example of inverval scales is shown in Fig. 8. In some cases, the data could be the ranking order of the alternatives for each attribute. In some other cases, the data could be just "YES" or "NO." "YES" means that the alternative has that particular attribute (aspect), and "NO" means it does not.

Solution aimed at This decribes the way a decision maker wishes to select the best alternative. Generally a decision maker wishes to select the alternative which maximizes his utility functions, hence satisfies him most.

For cost attributes			For benefit attributes
	0	0	
very high	1.0	1.0	very low
high	3.0	3.0	low
average	5.0	5.0	average
low	7.0	7.0	high
very low	9.0	9.0	very high
	10.0	10.0	

Fig. 8 An Interval Scale

However, in some cases he may have a particular target alternative (which is not necessarily considered to be ideal in the general sense) and wishes to select the alternative which is closest to the target. In some cases, he needs the order of the alternatives (prioritize) to, say, allocate money. If a decision maker has many alternatives, he may want to screen undesirable alternatives first before he makes the final decision.

Trade-off In some decision situations, a decision maker allows a disadvantage or unfavorble value in one attribute to be offset by an advantage or favorable value in some other attribute (compensatory). In other situations, he does not allow a disadvantage to be offset by an advantage (noncompensatory).

Size The size of problem is described by the number of alternatives and the number of attributes.

3.1.2 Types of problems

Type 1

Solution	**select**
Attributes	**incommensurable weights**
Data	**numerical**
Trade-off	**compensatory**

Example 1-1 Fighter aircraft selection (2, p.18)

A country decides to purchase a fleet of jet fighters from the US.

	Maximum speed (mach)	Ferry range (NM)	Maximum payload (pound)	Cost ($ in million)	Relia-bility	Maneuver-ability
A_1	2.0	1500	20000	5.5	average	very high
A_2	2.5	2700	18000	6.5	low	average
A_3	1.8	2000	21000	4.5	high	high
A_4	2.2	1800	20000	5.0	average	average

Example 1-2 Facility layout selection (8, p.83, modified)

A production manager wishes to select the best layout for his new factory.

	Minimum investiment	Ease of supervision	Operating cost	Ease of expansion
A_1	A	C	C	C
A_2	B	C	B	C
A_3	B	B	C	C

(A = 4 pts, B = 3 pts, C = 2 pts)

Example 1-3 Determine the best thing to do about our marriage (9, p. 145, modified)

A man who has a problem with his wife tries to find the best course of action.

	Adverse impact on kids	Bitterness	Long-term resolution	Image in community
A_1	low	low	average	aveage
A_2	very low	very low	very good	bad
A_3	average	high	very bad	very good
A_4	very high	high	very bad	very bad

A_1 Separation

A_2 Divorce

A_3 Status quo

A_4 A_3 plus relationship with another woman

Type 2

Solution	**prioritize**
Attributes	**incommensurable**
	weights
Data	**numerical**
Trade-off	**compensatory**

Example 2-1 Military promotion

Selection of 1500 from 6500 Majors in US Army to promote to Lieutenant Colonel.

	Mel.	Cel.	Bear.	Qual.	Perf.	POT
A_1	2.00	3.00	4.750	5.000	6.500	7.000
A_2	2.00	1.00	5.000	3.000	1.500	2.000
A_3	2.00	3.00	5.000	5.000	2.000	3.000
A_4	3.00	3.00	5.000	7.000	9.375	9.000
A_5	2.00	6.00	5.000	5.000	4.000	7.000
A_6	2.00	3.00	5.000	3.000	5.125	3.000
A_7	2.00	3.00	5.000	7.000	6.000	5.250
.	.	.	.	.	.	.

Mel. : Military Education Level
Cel. : Civilian Education Level
Bear.: Physical Readiness and Military Bearing
Qual.: Officer Qualifications
Perf.: Duty Performance
POT : Officer Potential

Type 3

Solution	**select**
Attributes	***commensurable*** **(weights)**
Data	**numerical**
Trade-off	**noncompensatory (compensatory)**

Example 3-1 Grading

An instructor of intensive English language course acesses the students' command of English using TOEFL (Test Of English as Foreign Language) scores. The instructor may use the best score or the worst score (*noncompensatory*), or the average (compensatory) to grade the students.

TOEFL Score

	Jul.	Sep.	Nov.	Jan.
A_1	525	512	549	556
A_2	536	562	541	548
A_3	609	587	621	585
A_4	528	501	492	517

Type 4

Solution	**screen (conjunctive standards)**
Attributes	**incommensurable**
Data	**numerical**
Trade-off	**noncompensatory**

Example 4-1 Graduate school admission for international students

A graduate school official screens applicants for a new semester. To pass the screening, an applicant must meet all the standards.

	TOEFL score *	GRE score **	G.P.A.
A_1	532	1720	2.8
A_2	563	1650	3.5
A_3	620	1530	3.2
A_4	558	1680	3.0
Standards	**550**	**1600**	**3.0**

* Test of English as a Foreign Language
** Graduate Record Examination

Exapmple 4-2 Driver license examination

An officer at a Division of Vehicle does his routine screening of candidates for driver licence. To pass the examination, a candidate must score above 90% in all three tests.

	Vision test (%)	Written test (%)	Driving test (%)
A_1	85	95	70
A_2	90	85	95
A_3	95	90	95
A_4	95	100	75
Standards	**90**	**90**	**90**

Type 5

Solution	**screen (disjunctive standards)**
Attributes	**incommensurable**
Data	**numerical**
Trade-off	**noncompensatory**

Example 5-1 Best motion picture nomination

A motion picture can be nominated as best picture, best actor, best actress, best director, or some combination of them. A picture is nominated if at least one of its ratings is above 9.

	Best picture	Best actor	Best actress	Best director
A_1	8	7	10	9
A_2	6	9	5	8
A_3	10	8	7	9
A_4	9	8	8	7
Standards	**9**	**9**	**9**	**9**

Example 5-2 Field and track athletes selection

An Olympic committee selects athletes for the coming Olympic game. An athlete is qualified if he/she can meet at least one of the standards.

	100m (sec)	200m (sec)	Long jump (feet-inch)	high jump (feet-inch)
A_1	10.5	21.5	20-7	6-0
A_2	11.3	23.2	19-11	5-7
A_3	10.3	22.6	19-3	6-4
A_4	12.0	24.4	24-4	5-2
Standards	**10.5**	**22.0**	**20-0**	**6-0**

Type 6

Solution	**select**
Attributes	**incommensurable**
	weights
Data	**numerical**
Trade-off	**(implicitly) noncompensatory**

Example 6-1 Secretary selection (10, p. 193, modified)

A local company hires a small number of secretaries who are also required to do double duty as receptionist and sales clerk. The job requires both secretarial and sales ability.

	Secretarial ability score	Sales ability score
A_1	0	100
A_2	30	70
A_3	50	50
A_4	70	30
A_5	100	0

Example 6-2 Baseball outfielder selection

A professional baseball team considers scouting a few all-round players to enforce its outfielder lineup. An all-round player is a player who is fairly good in all three categories.

	Batting (hits/game)	Defense (errors/game)	Running (bases stolen/game)
A_1	.253	0.02	0.15
A_2	.342	0.08	0.02
A_3	.302	0.03	0.23
A_4	.285	0.04	0.11

Type 7

Solution	minimize distance from target
Attributes	incommensurable weights
Data	numerical
Trade-off	compensatory

Example 7-1 Roommate selection (10, p. 206, modified)

In "Three's Company," Jack and Janet wish to select a new roommate who has the closest profile to their former roommate Chris using I.P.A.T.16 PF Test Profile.

	A	B	C	E	F	G	H	I	L	M	N
A_1	5	10	8	8	2	5	5	4	6	7	5
A_2	9	7	9	9	6	5	6	5	6	5	5
A_3	6	8	9	8	5	3	5	5	5	5	5
Chris (Target)	7	9	9	7	4	6	7	3	5	6	3

	O	Q1	Q2	Q3	Q4
A_1	5	5	8	5	5
A_2	6	6	7	6	6
A_3	5	5	8	5	5
Chris (Target)	3	7	9	5	3

Explanation of the Sten Personality Factors (10)

Factor	A Person with a Low Score on this Factor Is Described as	A Person with a High Score on this Factor Is Described as
A	Reserved, detached, critical, cool	Outgoing, warmhearted, easy-going, participating.
B	Less intelligent, concrete thinking	More intelligent, abstract-thinking, bright.
C	Affected by feelings, emotionally less stable, easily upset	Emotionally stable, faces reality, calm
E	Humble, mild, obedient, conforming	Assertive, independent, aggressive, stubborn
F	Sober, prudent, serious, taciturn	Happy-go-lucky, heedless, gay, enthusiastic
G	Expedient, a law to himself, bypasses obligations	Conscientious, persevering, staid, rule-bound
H	Shy, restrained, diffident, timid	Venturesome, socially bold, uninhibited, spontaneous
I	Tough-minded, self-reliant, no-nonsense	Tender-minded, dependent, overprotected, sensitive
L	Trusting, adaptable, free of jealousy, easy to get along with	Suspicious, self-opinionated, hard to fool
M	Practical, careful, conventional, regulated by external realities, proper	Imaginative, wrapped up in inner urgencies, careless of practical matters, bohemian
N	Forthright, natural, artless, sentimental	Shrewd, calculating, worldly, penetrating
O	Placid, self-assured, confident, serene	Apprehensive, worrying, depressive, troubled
Q_1	Conservative, respecting established ideas, tolerant of traditional difficulties	Experimenting, critical, liberal, analytical, free-thinking
Q_2	Group-dependent, a joiner and a sound follower	Self-sufficient, prefers own decisions, resourceful
Q_3	Casual, careless of protocol, untidy, follows own urges	Controlled, socially-precise, self-disciplined, compulsive
Q_4	Relaxed, tranquil, torpid, unfrustrated	Tense, driven, overwrought, fretful

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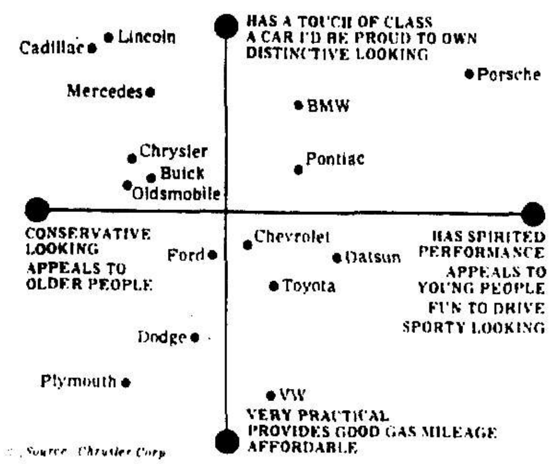
Example 7-2 Car brand image evaluation

A person wants to buy an American-made car which has the closest image to Toyata.

	Prestige (has a touch of class)	Appeal to young people (sporty looking)
BMW	7.3	6.4
Buick	5.8	3.4
Cadillac	8.5	2.1
Chevrolet	4.3	5.4
Chrysler	6.2	3.0
Datsun	4.1	7.3
Dodge	2.5	4.3
Ford	4.1	4.8
Lincoln	8.7	2.7
Mercedes	7.6	3.3
Oldsmobile	5.6	2.9
Plymouth	1.5	2.9
Pontiac	6.0	6.4
Porshe	8.0	10.0
VW	1.2	5.9
Toyata (Target)	3.6	6.0

PERCEPTUAL MAP-BRAND IMAGES (11)



Type 8

Solution	**select**
Attributes	**incommensurable**
	one predominant
Data	**numerical**
Trade-off	**noncompensatory**

Example 8-1 Airline selection

A coporate executive wishes to buy an airline ticket from New York to Los Angeles. The price of the ticket is most important for him.

	Price ($)	Punctuality (%)	Quality of service	Satety record
A_1	120	95	good	good
A_2	180	96	very good	very good
A_3	150	98	very good	very good
A_4	130	94	good	very good

Example 8-2 Fast food restaurant selection

A person selects one among the four restaurants in town. Because he does not have a car, the distance is crucial for him.

	Distance (min. on foot)	Quality of food	Quality of service	Price ($)
A_1	14	good	very good	2.00
A_2	7	good	average	2.15
A_3	21	good	good	2.05
A_4	15	very good	average	2.10

Type 9

Solution	**select**
Attributes	**incommensurable weights**
Data	**rank**
Trade-off	**compensatory**

Example 9-1 Martial art champion selection

A martial art champion is selected in a police academy from 8 contestants. Each contestant is ranked under each category using the result of each category's tournament.

	Judo	Fencing	Karate
A_1	2	7	4
A_2	5	3	8
A_3	7	4	6
A_4	1	5	3
A_5	3	2	7
A_6	6	6	1
A_7	8	1	2
A_8	4	8	5

Type 10

Solution	**select**
Attributes	**incommensurable**
Data	**YES or NO**
Trade-off	**noncompensatory**

Example 10-1 Gift selection (9, p. 120, modified)

A friend of Linda chooses a gift for her engagement.

	Practical	Attractive	Unusual	Memorable	$25 to $50
A_1	Y	Y	N	Y	N
A_2	Y	Y	N	Y	Y
A_3	Y	N	N	N	Y
A_4	Y	Y	N	N	Y
A_5	Y	Y	Y	Y	Y

A_1 Set of crystal

A_2 Wooden bowl

A_3 Towels

A_4 Bedding

A_5 Gourmet cookbook

Example 10-2 Long distance phone company selection

If you do not choose, someone is going to choose for you.

	Operator service	Imediate credit for a wrong number	World wide call	Clear connection	Daytime discount up to 70%
A_1	Y	Y	N	Y	Y
A_2	N	Y	N	Y	Y
A_3	Y	Y	Y	Y	Y
A_4	Y	N	N	Y	Y

Type 11

Solution	**screen**
Attributes	**incommensurable**
Data	**numerical**
Trade-off	**noncompensatory**
Size	**alternatives > 100**
	attributes < 5

Example 11-1 Plant site selection (12, modified)

A Japanese firm looks for a manufacturing plant site in the US. The company wishes to prescreen more than 100 candidate sites to 10 and invest some money to investigate the final ten for detail. The site selection team feels that transportation, raw materials, and operating cost are the three most important factors.

	Transportation	Raw material	Operating cost (million $)
A_1	good	available	6.0
A_2	fair	available	5.8
A_3	good	limited	5.7
A_4	very good	plentiful	5.6
A_5	excellent	available	5.6
A_6	fair	limited	6.0
A_7	fair	available	5.7
A_8	very good	available	5.4
.	.	.	.

3.2 Matching (Problem Types vs Methods)

The appropriate matching between the problem types in Section 3.1.2 and the methods is shown in Fig. 9. The fourteen MADM methods in Fig. 9 were selected from Hwang and Yoon (2) and Easton (10, 13) on the basis of reasonable demand of information required from and reasonable involvement of a decision maker. The methods are sumarized in Appendix B. For clarity, each problem type is identified with only the solution aimed at and major characteristics. For example, the attributes are incommensurable and the data is numerical unless otherwise stated.

3.3 Inference System

The flow diagram of the inference system is shown in Fig. 10. The inference system can be divided into three phases.

Phase I (Fig. 10 (a) and (b)) covers the input of information from a decision maker. The first input is the attribute input and the assessment of attribute weights (Fig. 10 (a)). The system assumes a hierarchical structure of attributes. The system asks for input of sub-attributes after each level 1 attribute and after each level 2 attribute, and so on (depth first search). After a group of attributes on the lowest level of hierarchy is input, the eigenvector method is used to assess the weights of the attributes (distribute 1.0 to the attributes).

Then, the system backtracks to the immediate higher level to complete attribute input, and so forth. After all

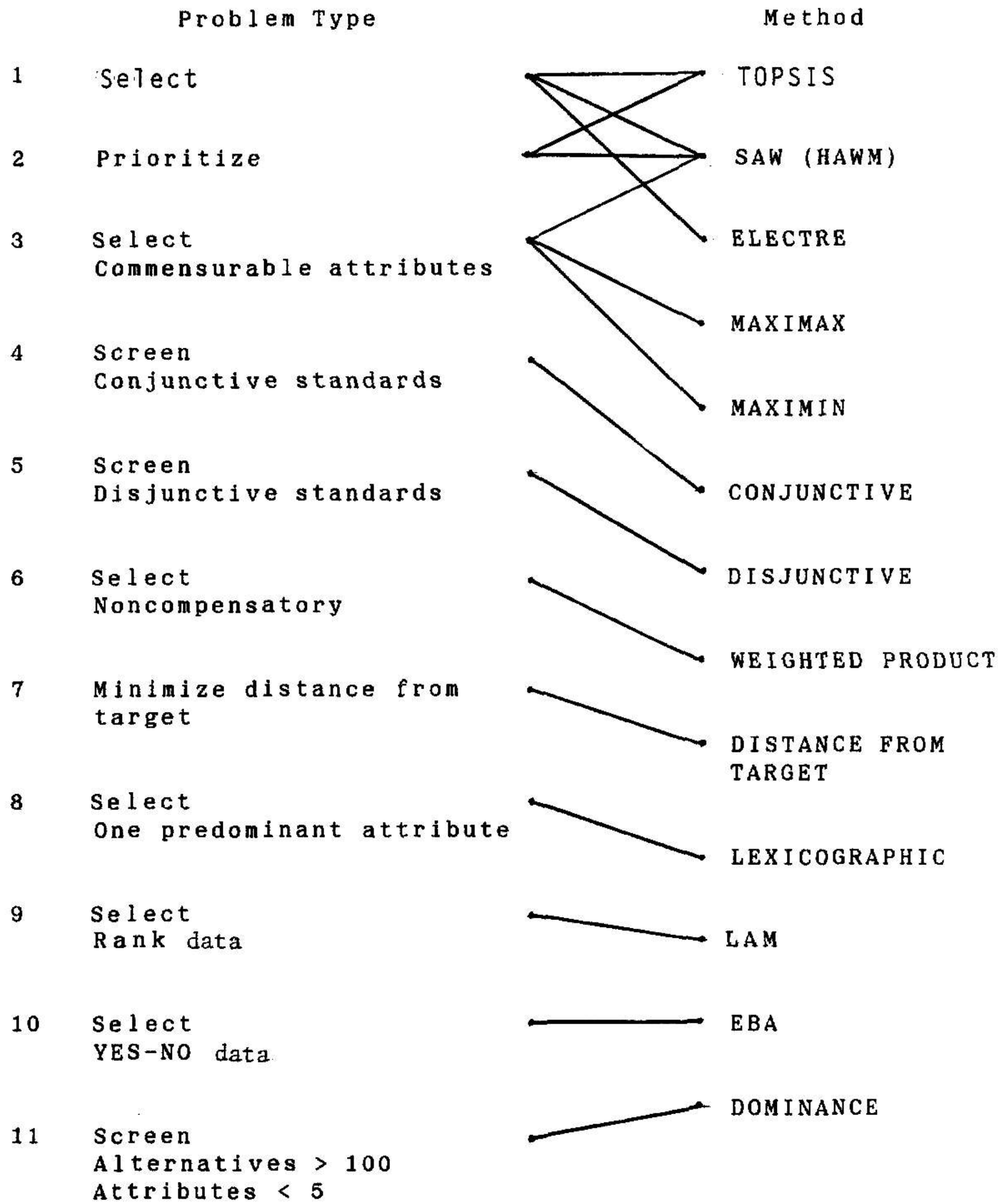


Fig. 9 MADM Problem Types vs Methods

the level 1 attributes are input and the weights are assessed using the eigenvector method, the overall weights are calculated by multiplication across the levels.

Second, the alternatives are input (Fig. 10 (b)). The system keeps tracks of the number of alternatives as well as the number of attributes.

Third, the evaluation data for each alternative is input. Based on the number of alternatives, the system uses two diffrent input modes. The difference between the two is for the input of subjective attributes. If the number of alternatives is greater than 15, the system uses direct input using an interval scale. If it is not greater than 15, the system uses the eigenvector method. The reason is the decision maker is not likely to be consistent in his judgment in the pairwise comparison of the alternatives if the number of alternatives is greater than 15. This completes the input of the decision matrix.

Finally, the number of solutions (alternatives to be selected) the decision maker needs is input (Fig. 10 (b)).

Phase II identifies problem types where prescreening of dominated alternatives by the DOMINANCE method should not be carried out (Fig. 10 (c)). An alternative A_i is dominated by A_j, if A_j has an equal or "better" value than A_i for all the attributes considered. There are a group of problem types which should not be prescreened by DOMINANCE. They are Type 2, 4, 5, 7, 9 and 10. In the case of Type 4 and 5 problems, for example, all the alternatives which satisfy the standards must be selected regardless whether some of the alternatives are dominated by others.

If the data is "rank" or "YES-NO," there is only one method for each, LAM and EBA respectively. The presence of minimal acceptable values (standards) indicates the CONJUNCTIVE or DISJUNCTIVE method as the solution method. The presence of a particular target alternative indicates the DISTANCE FROM TARGET method to use. To prioritize (cardinally rank) the alternatives, TOPSIS and SAW are the best methods.

Phase III covers problem types which can be used with DOMINANCE. DOMINANCE screens alternatives efficiently when the number of alternatives is greater than 100, and the number of attributes is less than 5 (Fig. 11). Also DOMINANCE is used for prescreening when the decision maker needs a "small" number of solutions out of "many." The appropriate ratio (solutions vs candidates) would be less than 10 %.

Most MADM problems have incommensurable attributes as stated in 2.2. In case they are commensurable, MAXIMAX or MAXIMIN can be used. The presence of one predominant attribute indicates the LEXICOGRAPHIC method. The final question the system asks is whether the problem is compensatory or not. If it is not, the method is WEIGHTED PRODUCT. Otherwise, the problem type is the most typical Type 1. The appropriate methods are TOPSIS, SAW, and ELECTRE. They are also fine methods because of their simple logic, full utilization of information contained in the decision matrix, and refined computational procedure. The choice of one from among these three methods is up to the decision maker based on his personal preference. Alternatively, the decision maker may use all of the three or two of the three and use group decision making methods such as Borda to aggregate the sets of preference order.

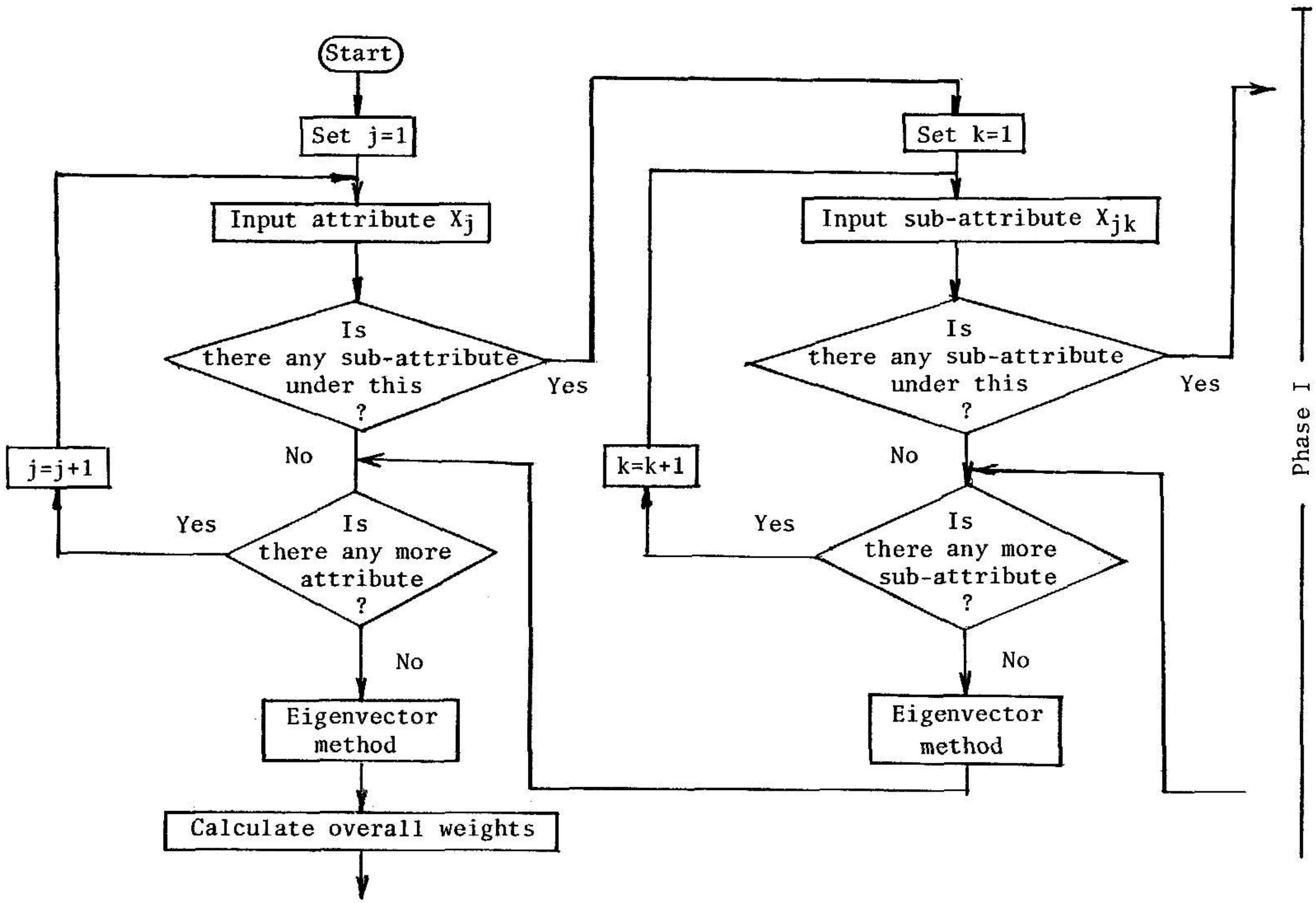


Fig. 10 (a) Inference System

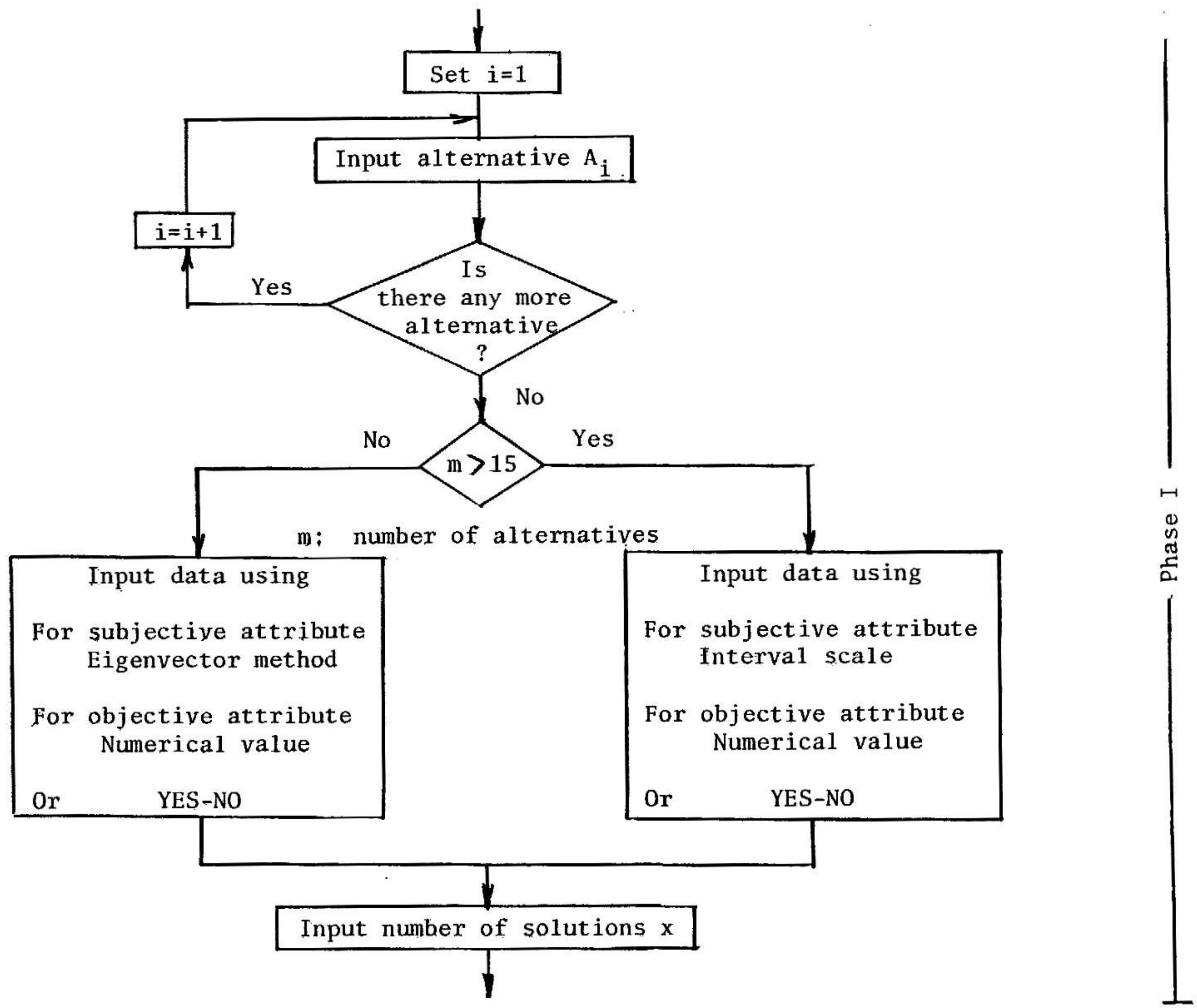


Fig. 10 (b) Inference System

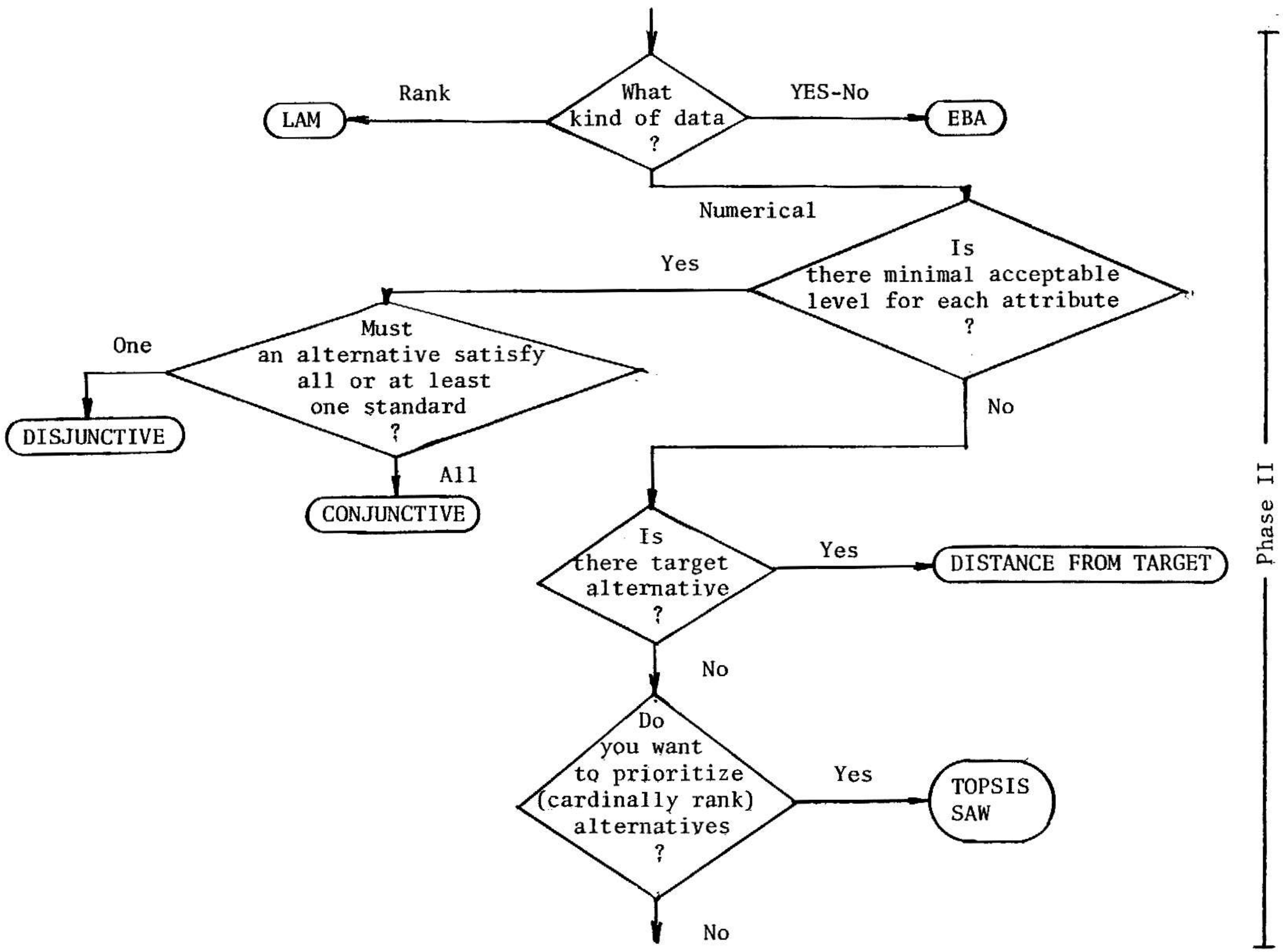


Fig. 10 (c) Inference System

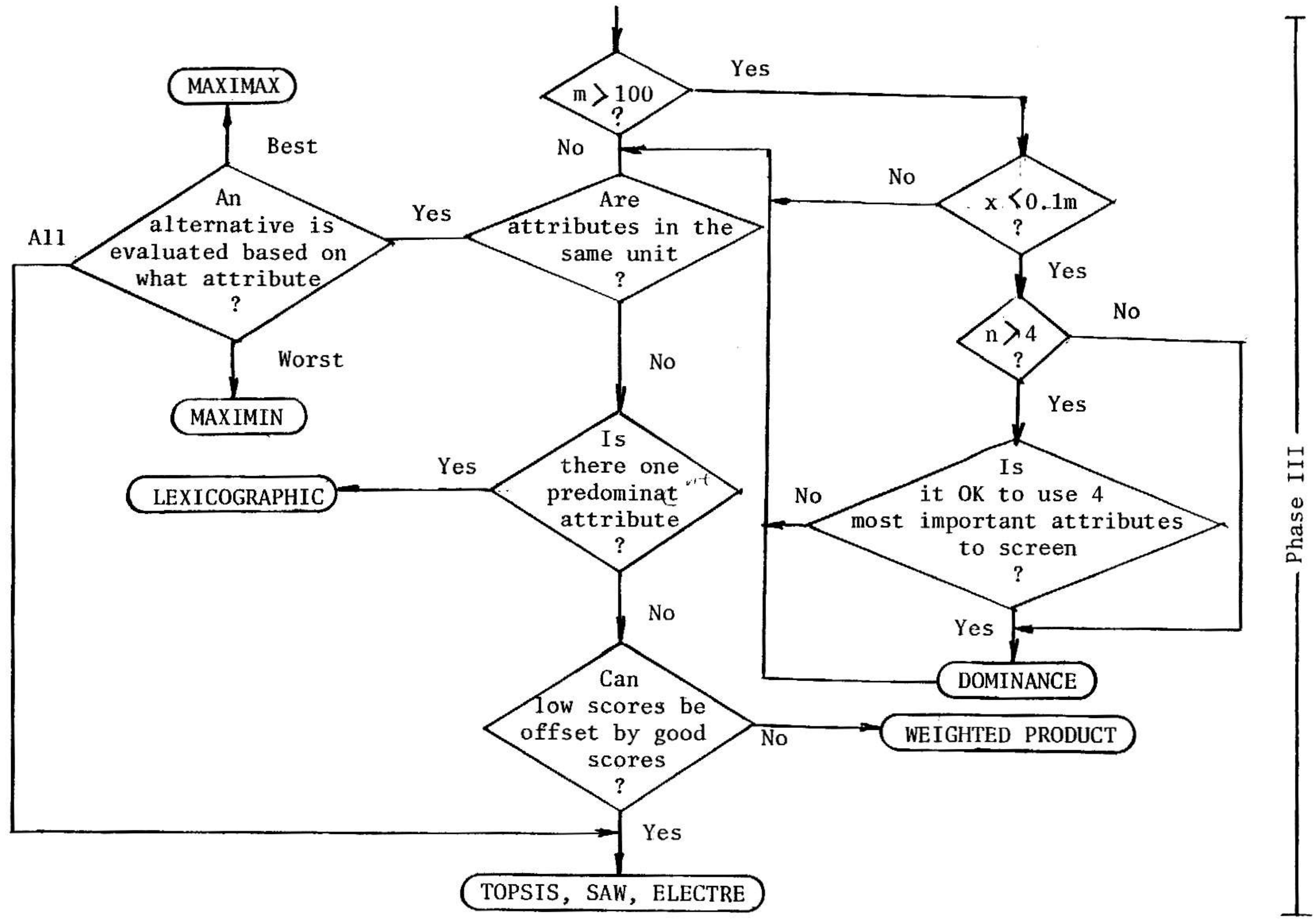


Fig. 10 (d) Inference System

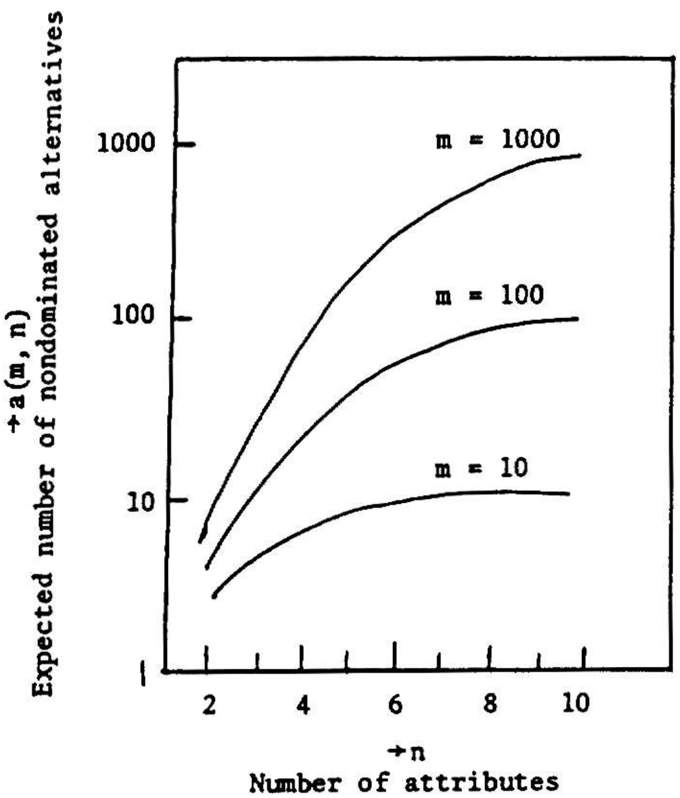


Fig. 11 Expected Number of Nondominated Alternatives (14)

4 CONCLUSIONS

A total of eleven MADM problem types were identified in this report. They seem to cover most of MADM problems encountered in the real world. I selected the examples for each problem type which are as realistic as possible. But a real world problem may be a combination of a couple of problem types. For instance, a decision maker may wish to screen alternatives using the CONJUNCTIVE method first, and the final decision may be made by TOPSIS.

Each problem type is solved by a particular method except Types 1, 2, and 3. By adding more criteria to characterize a problem, these problem types may be further divided so that a one to one correspondance may be established.

The inference system selects an appropriate method from among the fourteen MADM methods. The fourteen methods cover most of methods available at present. In the system, HAWM (Hierarchical Additive Weighting Method) or AHP (Analytical Hierarcy Process) is not shown. If the attributes in a problem have a hierarchical structure, and the data is input by the eigenvector method (pairwise comparison of the alternatives), and the problem is solved by the SAW (Simple Additive Weighting) method, the process is called HAWM or AHP.

The system may be further developed before and after implementation in a MADM DSS. However one of the important features of the MADM DSS realized by the inference system is that the user (decision maker) does not have to be concerned about what problem type he has and which solution technique he should use.

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APPENDIX A

EIGENVECTOR METHOD (2)

The decision maker is supposed to judge the relative importance of two criteria. The number of judgments is ${}_nC_2 = n(n-1)/2$. Some inconsistencies from these judgments are allowed. Saaty (6) introduced a method of scaling ratios using the principle eigenvector of a positive pairwise comparison matrix.

Let matrix A be

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{w_1}{w_1} & \frac{w_2}{w_2} & \cdots & \frac{w_2}{w_2} \\ \frac{w_2}{w_1} & \frac{w_2}{w_2} & \cdots & \frac{w_2}{w_n} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \frac{w_n}{w_1} & \frac{w_n}{w_2} & & \frac{w_n}{w_n} \end{bmatrix} \qquad (1)$$

This is a 'reciprocal matrix' which has all positive elements and has the reciprocal property

$$a_{ij} = 1/a_{ji} \tag{2}$$

and

$$a_{ij} = a_{ik}/a_{jk} \tag{3}$$

Multiplying A by $\underline{w} = (w_1, w_2, \ldots, w_n)^T$ yields

$$A\,\underline{w} = \begin{pmatrix} \frac{w_1}{w_1} & \frac{w_1}{w_2} & \cdots & \frac{w_1}{w_n} \\ \frac{w_2}{w_1} & \frac{w_2}{w_2} & \cdots & \frac{w_2}{w_n} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \frac{w_n}{w_1} & \frac{w_n}{w_2} & \cdots & \frac{w_n}{w_n} \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \\ \cdot \\ \cdot \\ \cdot \\ w_n \end{pmatrix} = n \begin{pmatrix} w_1 \\ w_2 \\ \cdot \\ \cdot \\ \cdot \\ w_n \end{pmatrix} = n\,\underline{w}$$

or

$$(A - nI)\,\underline{w} = 0 \tag{4}$$

Due to the consistency property of eq. (3), the system of homogeneous linear equations, eq. (4) has only trivial solutions.

In general, the precise values of w_i/w_j are unknown and must be estimated. In other words, human judgments can not be so accurate that eq. (3) be satisfied completely. We know that in any matrix, small perturbations in the coefficients imply small perturbations in the eigenvalues. If we define A' as the decision maker's estimate of A and $\underline{w}'$ is corresponding to A', then

$$A'\underline{w}' = \lambda_{max}\,\underline{w}' \tag{5}$$

where λ_{max} is the largest eigenvalue of A'. $\underline{w}'$ can be obtained by solving the system of linear equations, eq. (5)

Numerical Example

If the following positive pairwise comparison matrix is given

$$A = \begin{pmatrix} 1 & 1/3 & 1/2 \\ 3 & 1 & 3 \\ 2 & 1/3 & 1 \end{pmatrix}$$

then set the determinant of $(A - \lambda I)$ as zero. That is

$$\det(A - \lambda I) = \begin{vmatrix} 1-\lambda & 1/3 & 1/2 \\ 3 & 1-\lambda & 3 \\ 2 & 1/3 & 1-\lambda \end{vmatrix} = 0$$

The largest eigenvalue of A, λ_{max}, is 3.0536, and we have

$$\begin{pmatrix} -2.0536 & \frac{1}{3} & \frac{1}{2} \\ 3 & -2.0536 & 3 \\ 2 & \frac{1}{3} & -2.0536 \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = 0$$

The solution of the homogeneous system of linear equations gives (recall that $\sum_{i=1}^{3} w_i = 1$)

$$\underline{w}^T = (0.1571,\ 0.5936,\ 0.2493).$$

The Scale

For assessing the scale ratio w_i/w_j, Saaty (6) gives an intensity scale of importance for activities and has broken down the importance ranks as shown in the following table.

The scale and its description (6)

Intensity of importance	Definition	Explanation
1	Equal importance	Two criteria contribute equally to the objective.
3	Weak importance of one over another	Experience and judgment slightly favor one criterion over another.
5	Essential or strong importance	Experience and judgment strongly favor one criterion over another.
7	Demonstrated importance	A criterion is strongly favored and its dominance is demonstrated in practice.
9	Absolute importance	The evidence favoring one criterion over another is of the highest possible order of affirmation.
2,4,6,8	Intermediate values between the two adjacent judgments	When compromise is needed.

APPENDIX B

MULTIPLE ATTRIBUTE DECISION MAKING METHODS

Dominance (2)

Maximin (2)

Maximax (2)

Conjunctive method (2)

Disjunctive method (2)

Lexicographic method (2)

Elimination By Aspects (EBA) (2)

Linear Assignment Method (LAM) (2)

Simple Additive Weighting method (SAW) (2)

Hierarchical Additive Wieghting Method (HAWM)
(Analytical Hierarchy Process (AHP)) (2)

ELECTRE method (2)

TOPSIS

Weighted Product (10, 13)

Distance from Target (10, 13)

1. Dominance

An alternative is dominated if there is another alternative which excels it in one or more attributes and equals it in the remainder. The number of alternatives can be reduced by eliminating the dominated ones. In other words we screen the set of alternatives before the final choice is made. A set of nondominated solutions is one obtained through the sieve of dominance method.

This method does not require any assumption or any transformation of attributes. The sieve of dominance takes the following procedures; compare the first two alternatives and if one is dominated by the other, discard the dominated one. Next compare the undiscarded alternatives with the third alternative and discard any dominated alternative. Then introduce the fourth alternative and so on. After (m - 1) stages the nondominated set is determined. This nondominated set usually has multiple elements in it, hence the dominance method is mainly used for the initial filtering.

2. Maximin

An astronaut's life or death in the orbit may depend upon his worst vital organ, and a chain is only as strong as its weakest link. In this situation where the overall performance of an alternative is determined by the weakest or poorest attribute, a decision maker would examine the attribute values for each alternative, note the lowest value for each alternative, and then select the alternative with the most acceptable value in its lowest attribute. It is the selection of the maximum (across alternatives) of the minimum (across attributes) values, or the maximin.

Under this procedure only a single weakest attribute represents an alternative; all other (n-1) attributes for a particular alternative are ignored. If these lowest attribute values come from different attributes, as they often do, we may be basing our final choice on single values of attributes that differ from alternative to alternative. Therefore, the maximin method can be used only when interattribute values are comparable; that is, all attributes must be measured on a (common) scale; however, they need not be numerical. The alternative, A^+, is selected such that

$$A^+ = \{A_i \mid \max_i \min_j x_{ij}\}, \; j = 1, 2, \ldots, n; \quad i = 1, 2, \ldots, m$$

where all x_{ij}'s are in a common scale.

3. Maximax

In contrast to the maximin method, the maximax method selects an alternative by its best attribute value rather than its worst attribute value. In this case the highest attribute value for each alternative is identified, then these maximum values are compared in order to select the alternative with the largest such value, the maximax procedure.

Note that in this procedure, as with the maximin procedure only the single strongest attribute represents an alternative; all other (n-1) attributes for the particular alternative are ignored; and it may evaluate different attributes in a final choice among alternatives. Therefore, as with the maximin method, the maximax method can be used only when all attributes are measured *on* a *common* scale. The alternative, A^+, is selected such that

$$A^+ = \{A_i \mid = \max_i \max_j x_{ij}\}, \qquad j = 1, 2, \ldots, n; \qquad i = 1, 2, \ldots, m$$

where all x_{ij}'s are in a common scale.

4. Conjunctive method

Consider, for example, the position of a visiting American history teacher in a French school. An individual's effectiveness as a teacher will be limited by the lesser of his/her abilities in history and French; he/she cannot compensate for an insufficient knowledge of French by an excellent knowledge of history, or vice versa. The school wants to eliminate the candidates who do not possess the acceptable knowledge in both fields. In the conjunctive method (or satisficing method), all the standards must be passed in order for the alternatives to be acceptable.

To apply the method, the decision maker must supply the minimal attribute values (the cutoff values) acceptable for each of the attributes. The cutoff values given by the decision maker play the key role in eliminating the noncontender alternatives; if too high, none is left; if relatively low quite a few alternatives are left after filtering. Hence increasing the minimal standard levels in an iterative way, we can sometimes narrow down the alternatives to a single choice.

We classify A_i as an acceptable alternative only if

$$x_{ij} > x_j^0, \qquad j = 1, 2, \ldots, n$$

where x_j^0 is the standard level of x_j.

5. Disjunctive method

A disjunctive method is one in which an alternative (or an individual) is evaluated on its greatest value (or talent) of an attribute. For example, professional football players are selected according to the disjunctive method; a player is selected because he can either pass exceptionally, or run exceptionally, or kick exceptionally, etc. A player's passing ability is irrelevant if he is chosen for his kicking ability.

We classify A_i as an acceptable alternative only if

$$x_{ij} > x_j^0, \qquad j = 1 \text{ or } 2 \text{ or } \ldots \text{ or } n$$

where x_j^0 is a desirable level of x_j.

A disjunctive method guarantees selection of all individuals (candidates) with any extreme talent, while the conjunctive method guarantees rejection of all individuals with an extremely small talent.

6. Lexicographic method

In some decision situations a single attribute seems to predominate. For example, "buy the cheapest" rule is that in which the price is the most important attribute to the decision maker. One way of treating this situation is to compare the alternatives on the most important attribute. If one alternative has a higher attribute value than any of the other alternatives, the alternative is chosen and the decision process ends. However, if some alternatives are tied on the most important attribute, the subset of tied alternatives are then compared on the next most important attribute. The process continues sequentially until a single alternative is chosen or until all n attributes have been considered.

The method requires that the attributes be ranked in the order of importance by the decision maker. Let the subscripts of the attributes indicate not only the components of the attribute vector, but also the priorities of the attributes, i.e. X_1 be the most important attribute to the decision maker, X_2 the second most important one, and so on. Then alternative(s), A^1, is(are) selected such that

$$A^1 = \{A_i \mid \max_i x_{i1}\}, \qquad i = 1, 2, \ldots, m \tag{1}$$

If this set $\{A^1\}$ has a single element, then this element is the most preferred alternative. If there are multiple maximal alternatives, consider

$$A^2 = \{A^1 \mid \max_i x_{i2}\}, \qquad i \in \{A^1\} \tag{2}$$

If this set $\{A^2\}$ has a single element, then stop and select this alternative. If not, consider

$$A^3 = \{A^2 \mid \max_i x_{i3}\}, \qquad i \in \{A^2\} \tag{3}$$

Continue this process until either (a) some $\{A^k\}$ with a single element is found which is then the most preferred alternative, or (b) all n attributes have been considered, in which case, if the remaining set contains more than one element, they are considered to be equivalent.

7. Elimination By Aspects (EBA)

The decision maker as in conjunctive method, is assumed to have minimum cutoffs for each attribute. An attribute is selected, and all alternatives not passing the cutoff on that attribute are eliminated. Then another attribute is selected, and so forth. The process continues until all alternatives but one are eliminated. Like lexicographic method, it examines one attribute at a time, making comparisons among alternatives. However, it does differ slightly since it eliminates alternatives which do not satisfy some standard level, and it continues until all alternatives except one have been eliminated. Another difference is that the attributes are not ordered in terms of importance, but in terms of their discrimination power in a probabilistic mode.

Each alternative is viewed as a set of aspects. The aspects could represent values along some fixed quantitative or qualitative dimensions (attributes) (e.g., price, quality, comfort), or they could be arbitrary features of the alternatives that do not fit into any simple dimensional structure. Since the model describes choice as an elimination process governed by successive selection of aspects instead of cutoffs, it is called the Elimination By Aspects (EBA).

8. Linear Assignment Method (LAM)

The linear assignment method is based on a set of attributewise rankings and a set of attribute weights. The method features a linear compensatory process for attribute weights. The method features a linear compensatory process for attribute interaction and combination. In the process only ordinal data, rather than cardinal data, are used as the input. This information requirement is attractive in that we do not need to scale the qualitative attributes.

For instance, consider the following attributewise preferences with equal weight,

rank	X_1	X_2	X_3
1st	A_1	A_1	A_2
2nd	A_2	A_3	A_1
3rd	A_3	A_2	A_3

Let us define a product-attribute matrix π as a square (m x m) nonnegative matrix whose elements π_{ik} represent the frequency (or number) that A_i is ranked the k^{th} attributewise ranking. For example, the corresponding π matrix with the equal weight on the attributes is

$$\pi = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{ccc} \text{1st} & \text{2nd} & \text{3rd} \end{array} \\ \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix} \end{array}$$

For the different weight $\underline{w} = (w_1, w_2, w_3) = (.2, .3, .5)$, π matrix becomes

$$\pi = \begin{bmatrix} .2+.3 & .5 & 0 \\ .5 & .2 & .3 \\ 0 & .3 & .2+.5 \end{bmatrix} = \begin{bmatrix} .5 & .5 & 0 \\ .5 & .2 & .3 \\ 0 & .3 & .7 \end{bmatrix}$$

It is understood that π_{ik} measures the contribution of A_i to the overall ranking, if A_i is assigned to the k^{th} overall rank. The larger π_{ik} indicates the more concordance in assigning A_i to the k^{th} overall rank. Hence the problem is to find A_i for each k, k = 1, 2, ..., m which maximizes $\sum_{k=1}^{m} \pi_{ik}$. This is an m! comparison problem. An LP model is suggested for the case of large m.

Let us define permutation matrix P as (m x m) square matrix whose element $P_{ik} = 1$ if A_i is assigned to overall rank, k, and $P_{ik} = 0$ otherwise. The linear assignment method can be written by the following LP format,

$$\max \sum_{i=1}^{m} \sum_{k=1}^{m} \pi_{ik} P_{ik} \tag{1}$$

subject to

$$\sum_{k=1}^{m} P_{ik} = 1, \qquad i = 1, 2, \ldots, m \tag{2}$$

$$\sum_{i=1}^{m} P_{ik} = 1, \qquad k = 1, 2, \ldots, m \tag{3}$$

Recall that $P_{ik} = 1$ if alternative i is assigned rank k, and clearly alternative i can be assigned to only one rank, therefore, we have eq. (2). Likewise, a given rank k can only have one alternative assigned to it; therefore, we have the constraint of eq. (3).

Let the optimal permutation matrix, which is the solution of the above LP problem, be P*. Then, the optimal ordering can be obtained by multiplying A by P*.

9. Simple Additive Weighting method (SAW)

To each of the attributes in SAW, the decision maker assigns importance weights which become the coefficients of the variables. To reflect the decision maker's marginal worth assessments within attributes, the decision maker also makes a numerical scaling of intra-attribute values. The decision maker can then obtain a total score for each alternative simply by multiplying the scale rating for each attribute value by the importance weight assigned to the attribute and then summing these products over all attributes. After the total scores are computed for each alternative, the alternative with the highest score (the highest weighted average) is the one prescribed to the decision maker.

Mathematically, simple additive weighting method can be stated as follows: Suppose the decision maker assigns a set of importance weights to the attributes, $\underline{w} = \{w_1, w_2, \ldots, w_n\}$. Then the most preferred alternative, A^*, is selected such that

$$A^* = \{A_i \mid \max_i \sum_{j=1}^{n} w_j x_{ij} \sum_{j=1}^{n} / w_j\}$$

where x_{ij} is the outcome of the i^{th} alternative about the j^{th} attribute with a numerically comparable scale. Usually the weights are normalized so that $\sum_{j=1}^{n} w_j = 1$.

10. Hierarchical Additive Weighting Method (HAWM) (Analytical Hierarchy Process (AHP))

In simple additive weighting method (SAW), the weighted averages (or priority value) for alternative A_i are given by

$$\sum_{j=1}^{n} w_j x_{ij} \Big/ \sum_{j=1}^{n} w_j$$

where in general, $\sum_{j=1}^{n} w_j = 1$, and x_{ij} is in a ratio scale. The ratio x_{ij} can be interpreted as the subscore of the i^{th} alternative with regard to the j^{th} criterion. Then the vector $\underline{x}_j = (x_{1j}, x_{2j}, \ldots, x_{ij}, \ldots, x_{mj})$ may indicate the contribution, importance (another weight) of A_i's for the j^{th} criteria as the weight vector $\underline{w}$ represents the importance of different criteria for the decision problem. If we impose

$$\sum_{j=1}^{n} x_{ij} = 1, \qquad j = 1, 2, \ldots, n$$

the SAW is simple to compose weights from the different levels. This approach matches Saaty's (6) hierarchical structures.

11. ELECTRE method

This method consists of a pairwise comparison of alternatives based on the degree to which evaluations of the alternatives and the preference weights confirm or contradict the pairwise dominance relationships between alternatives. It examines both the degree to which the preference weights are in agreement with pairwise dominance relationships and the degree to which weighted evaluations differ from each other.

The ELECTRE method takes the following steps:

Step 1. Calculate the normalized decision matrix: This procedure transforms the various attribute scales into comparable scales. Each normalized value r_{ij} of the normalized decision matrix R can be calculated as:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^{2}}}, \quad R = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \cdot & & & \\ \cdot & & & \\ \cdot & & & \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix} \tag{1}$$

so that all attributes have the same unit length of vector.

Step 2. Calculate the weighted normalized decision matrix: This matrix can be calculated by multiplying each column of matrix R with its associated weight w_j. Therefore, the weighted normalized decision matrix V is equal to

$$V = RW$$

$$= \begin{bmatrix} v_{11} & \cdots & v_{1j} & \cdots & v_{1n} \\ \cdot & & & & \\ \cdot & & & & \\ \cdot & & & & \\ v_{m1} & \cdots & v_{mj} & \cdots & v_{mn} \end{bmatrix} = \begin{bmatrix} w_1 r_{11} & \cdots & w_j r_{1j} & \cdots & w_n r_{1n} \\ \cdot & & & & \\ \cdot & & & & \\ \cdot & & & & \\ w_1 r_{m1} & \cdots & w_j r_{mj} & \cdots & w_n r_{mn} \end{bmatrix} \tag{2}$$

Step 3. Determine the concordance and discordance set: For each pair of alternatives k and ℓ($k, \ell = 1, 2, \ldots, m$ and $k \neq \ell$), the set of decision criteria $J = \{j | j = 1, 2, \ldots, n\}$ is divided into two distinct subsets. The concordance set $C_{k\ell}$ of A_k and A_ℓ is composed of all criteria for which A_k is preferred to A_ℓ. In other words,

$$C_{k\ell} = \{j | x_{kj} \geq x_{\ell j}\} \tag{3}$$

The complementary subset is called the discordance set, which is

$$\begin{aligned} D_{k\ell} &= \{j | x_{kj} < x_{\ell j}\} \\ &= J - C_{k\ell} \end{aligned} \tag{4}$$

The successive values of the concordance indices $c_{k\ell}$($k, \ell = 1, 2, \ldots, m$ and $k \neq \ell$) form the concordance matrix C of ($m \times m$):

$$C = \begin{bmatrix} — & c_{12} & \cdots\cdot & c_{1m} \\ c_{21} & — & c_{23} & c_{21} \\ . & & & . \\ . & & & . \\ . & & & . \\ c_{m1} & c_{m2} & \cdots\cdot c_{m(m-1)} & — \end{bmatrix} \qquad (5)$$

It should be noted that matrix C is, in general, not symmetric.

Step 4. Calculate the concordance matrix: The concordance index $c_{k\ell}$ between A_k and A_ℓ is defined as:

$$c_{k\ell} = \sum_{j \in C_{k\ell}} w_j \Big/ \sum_{j=1}^{n} w_j$$

For the normalized weight set

$$c_{k\ell} = \sum_{j \in C_{k\ell}} w_j \qquad (6)$$

The concordance index reflects the relative importance of A_k with respect to A_ℓ. Obviously, $0 < c_{k\ell} < 1$. A higher value of $c_{k\ell}$ indicates that A_k is preferred to A_ℓ as far as the concordance criteria are concerned.

Step 5. Calculate the discordance matrix: The discordance index is defined as:

$$d_{k\ell} = \frac{\max_{j \in D_{k\ell}} | v_{kj} - v_{\ell j} |}{\max_{j \in J} | v_{kj} - v_{\ell j} |} \tag{7}$$

It is clear that $0 < d_{k\ell} < 1$. A higher value of $d_{k\ell}$ implies that, for the discordance criteria, A_k is less favorable than A_ℓ, and a lower value of $d_{k\ell}$, A_k is favorable to A_ℓ. The discordance indices form the discordance matrix D_x of (m x m):

$$D_x = \begin{bmatrix} — & d_{12} & \cdots & d_{1m} \\ d_{21} & — & d_{23} & d_{2m} \\ \vdots & & & \vdots \\ d_{m1} & \cdots & d_{m(m-1)} & — \end{bmatrix} \tag{8}$$

Obviously, matrix D_x is, in general, asymmetric.

Step 6. Determine the concordance dominance matrix: This matrix can be calculated with the aid of a threshold value for the concordance index. A_k will only have a chance of dominating A_ℓ, if its corresponding concordance index $c_{k\ell}$ exceeds at least a certain threshold value $\bar{c}$, i.e.,

$$c_{k\ell} > \bar{c}$$

This threshold value can be determined, for example, as the average concordance index, i.e.,

$$\bar{c} = \sum_{\substack{k=1 \\ k\neq \ell}}^{m} \sum_{\substack{\ell=1 \\ \ell\neq k}}^{m} c_{k\ell}/m(m - 1) \tag{9}$$

On the basis of the threshold value, a Boolean matrix F can be constructed, the elements of which are defined as

$$f_{k\ell} = 1, \text{ if } c_{k\ell} \geq \bar{c} \tag{10}$$

$$f_{k\ell} = 0, \text{ if } c_{k\ell} < \bar{c}$$

Then each element of 1 on the matrix F represents a dominance of one alternative with respect to another one.

<u>Step 7. Determine the discordance dominance matrix</u>: This matrix is constructed in a way analogous to the F matrix on the basis of a threshold value $\bar{d}$ to the discordance indices. The elements of $g_{k\ell}$ of the discordance dominance matrix G are calculated as

$$\bar{d} = \sum_{\substack{k=1 \\ k\neq \ell}}^{m} \sum_{\substack{\ell=1 \\ \ell\neq k}}^{m} d_{k\ell}/m(m - 1) \tag{11}$$

$$g_{k\ell} = 1, \text{ if } d_{k\ell} \leq \bar{d}$$

$$g_{k\ell} = 0, \text{ if } d_{k\ell} > \bar{d}$$

Also the unit elements in the G matrix represent the dominance relationships between any two alternatives.

Step 8. Determine the aggregate dominance matrix: The next step is to calculate the intersection of the concordance dominance matrix F and discordance dominance matrix G. The resulting matrix, called the aggregate dominance matrix E, is defined by means of its typical elements $e_{k\ell}$ as:

$$e_{k\ell} = f_{k\ell} \cdot g_{k\ell} \tag{12}$$

Step 9. Eliminate the less favorable alternatives: The aggregate dominance matrix E gives the partial-preference ordering of the alternatives. If $e_{k\ell} = 1$, then A_k is preferred to A_ℓ for both the concordance and discordance criteria, but A_k still has the chance of being dominated by the other alternatives. Hence the condition that A_k is not dominated by ELECTRE procedure is,

$$e_{k\ell} = 1, \text{ for at least one } \ell,\ \ell = 1, 2, \ldots, m,\ k \neq \ell \tag{13}$$

$$e_{ik} = 0, \text{ for all } i,\ i = 1, 2, \ldots, m,\ i \neq k,\ i \neq \ell$$

12. Technique for Order Preference by Similarity to Ideal Solution (TOPSIS)

TOPSIS is based upon the concept that the chosen alternative should have the shortest distance from the ideal solution and the farthest from the negative-ideal solution.

Assume that each attribute takes the monotonically increasing (or decreasing) utility; then it is easy to locate the "ideal" solution which is composed of all best attribute values attainable, and the "negative-ideal" solution composed of all worst attribute values attainable. TOPSIS considers the distances to both the ideal and the negative-ideal solutions simultaneously by taking the relative closeness to the ideal solution.

Step 1. Calculate the normalized decision matrix; this procedure transforms the various attribute scales into comparable scales. Each normalized value r_{ij} of the normalized decision matrix R can be calculated as:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}, \quad R = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \cdot & & & \\ \cdot & & & \\ \cdot & & & \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix} \qquad (1)$$

so that all attributes have the same unit length of vector.

Step 2. Calculate the weighted normalized decision matrix: This matrix can be calculated by multiplying each column of matrix R with its associated weight w_j. Therefore, the weighted normalized decision matrix V is equal to

$$V = RW$$

$$= \begin{bmatrix} v_{11} & \cdots & v_{1j} & \cdots & v_{1n} \\ \cdot & & & & \\ \cdot & & & & \\ \cdot & & & & \\ v_{m1} & \cdots & v_{mj} & \cdots & v_{mn} \end{bmatrix} = \begin{bmatrix} w_1 r_{11} & \cdots & w_j r_{1j} & \cdots & w_n r_{1n} \\ \cdot & & & & \\ \cdot & & & & \\ \cdot & & & & \\ w_1 r_{m1} & \cdots & w_j r_{mj} & \cdots & w_n r_{mn} \end{bmatrix} \tag{2}$$

Step 3. Determine ideal and negative-ideal solutions: Let the two artificial alternatives A^* and A^- be defined as

$$A^* = \{(\max_i v_{ij} \mid j \in J), (\min_i v_{ij} \mid j \in J') \mid i = 1, 2, \ldots, m\}$$

$$= \{v_1^*, v_2^*, \ldots, v_j^*, \ldots, v_n^*\} \tag{3}$$

$$A^- = \{(\min_i v_{ij} \mid j \in J), (\max_i v_{ij} \mid j \in J') \mid i = 1, 2, \ldots, m\}$$

$$= \{v_1^-, v_2^-, \ldots, v_j^-, \ldots, v_n^-\} \tag{4}$$

where $J = \{j = 1, 2, \ldots, n \mid j$ associated with benefit criteria)

$J' = (j = 1, 2, \ldots, n \mid j$ associated with cost criteria$\}$

Step 4. Calculate the separation measure: The separation between each alternative can be measured by the n-dimensional Euclidean distance. The separation of each alternative from the ideal one is then given by

$$S_{i*} = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^*)^2}, \qquad i = 1, 2, \ldots, m \qquad (5)$$

Similarly, the separation from the negative-ideal one is given by

$$S_{i-} = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^-)^2}, \qquad i = 1, 2, \ldots, m \qquad (6)$$

Step 5. Calculate the relative closeness to the ideal solution: The relative closeness of A_i with respect to A^* is defined as

$$C_{i*} = S_{i-}/(S_{i*} + S_{i-}), \quad 0 < C_{i*} < 1, \qquad i = 1, 2, \ldots, m \qquad (7)$$

It is clear that $C_{i*} = 1$ if $a_i = A^*$ and $C_{i*} = 0$ if $A_i = A^-$. An alternative A_i is closer to A^* as C_{i*} approaches to 1.

6. Rank the preference order: A set of alternatives can now be preference ranked according to the descending order of C_{i*}.

13. Weighted Product

The decision maker assigns importance weights which become the coefficients of the variables. To reflect the decision maker's marginal worth assessments within attributes, the decision maker also makes a numerical scaling of intra-attribute values. The decision maker can then obtain a total score for each alternative simply by multiplying the scale rating for each attribute value by the importance weight assigned to the attribute and then multiplying these products over all attributes. After the total scores are computed for each alternative, the alternative with the highest score is the most preferred alternative to the decision maker.

Mathematically,

$$A^{*} = \{A_i \mid \max_i \left(\prod_{J=1}^{n} (x_{ij})^{w_j} \right)^{\frac{1}{\Sigma w_j}}$$

where x_{ij} is the outcome of the i^{th} alternative about the j^{th} attribute with a numerically comparable scale. Usually the weights are normalized so that $\sum_{j=1}^{n} w_j = 1$.

14. Distance from Target

The decision maker establishes a target alternative. The deviation of all alternatives from the target is computed. The alternative that has the shortest distance from the target is the best.

The coordinates of the target can be expressed as

$$T = (t_1, t_2, \ldots, t_j, \ldots, t_n) \tag{1}$$

and those for the i^{th} alternative as

$$A_i = (X_{i1}, X_{i2}, \ldots, X_{ij}, \ldots, X_{in}) \tag{2}$$

Then the distance between the i^{th} alternative and the target is

$$d_i = \sqrt{\sum_{j=1}^{n} (x_{ij} - t_j)^2}, \quad i = 1, 2, \ldots, m \tag{3}$$

INFERENCE SYSTEM FOR SELECTION OF AN APPROPRIATE
MULTIPLE ATTRIBUTE DECISION MAKING METHOD

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ABSTRACT

An inference system was developed for a future development of Multiple Attribute Decision Making Decision Support System (MADM DSS). A MADM DSS will contain a number of MADM solution methods prgrammed so that it is applicable to any given MADM problem such as selection of a car, school, or job. The inference system is the part of the DSS which is used to select an appropriate MADM method for a specific problem.

The system was developed as a decision rule through the stages which represent a human MADM expert's intuitive approach when he selects an appropriate method. First, MADM problems were classified into eleven types based on the characteristics they have. The charateristics were identified using the five concepts on MADM problems. They are attributes, data, solution aimed at, trade-off, and size of a problem. Next, fourteen frequently used MADM methods were chosen, and the matching between the problem types and the methods was constructed. The fourteen methods selected were TOPSIS, SAW, HAWM, ELECTRE, MAXIMAX, MAXIMIN, CONJUNCTIVE method, DISJUNCTIVE method, WEIGHTED PRODUCT, DISTANCE FROM TARGET, LEXICOGRAPHIC method, LAM, EBA, and DOMINANCE. Finally, the flow diagram of the inference system was developed in such a way that as a user (decision maker) inputs necessary information, the system identifies the best method(s) by a systematically ordered series of questions to him.

The system consists of three phases. Phase I covers problem information input from the user; attributes, alternatives, data, the number of solutions needed. Phase II identifies MADM methods which should not be used with DOMINANCE (a prescreening method). Phase III identifies MADM methods which can be used with DOMINANCE. The most common type of problem is soloved by TOPSIS, SAW, or ELECTRE.