Improving humanitarian supply chain operations through multi-agency collaborations using cooperative game theory

by

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B.S., Ershad University of Damavand, 2007
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AN ABSTRACT OF A DISSERTATION

submitted in partial fulfillment of the requirements for the degree

DOCTOR OF PHILOSOPHY

Department of Industrial and Manufacturing Systems Engineering
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Abstract

Humanitarian supply chains support agency goals regarding disaster response, recovery, and development operations. However, increased efficiency and effectiveness of humanitarian operations are essential as disaster occurrences multiply (CRED, n.d.). Although collaboration between agencies involved in humanitarian response, recovery, and development can help improve operations, identifying the collaborative arrangements that optimize societal goals and remain attractive to autonomous agencies is challenging. Two aspects must be addressed: coalition formation, or which agencies should collaborate with one another, and cost allocation, or how to share the costs and benefits of collaboration among partners in a way that is acceptable to them.

This research advances knowledge of coalition formation and cost allocation in a non-subadditive context, which encompasses settings in which large collaborative groups are not guaranteed to be optimal from the societal perspective. Few prior studies have considered this, and doing so is necessary to accurately model systems where collaboration can achieve cost savings among some partners and lead to increased costs among others. Humanitarian operations are one example, because scale economies can be achieved by collaborating, but large partnerships require additional costs to manage. This thesis presents new game theory and optimization models, solution methodologies, problem characterization, and computational analysis to address collaboration in non-subadditive settings.

First, the author presents a literature review of cooperation among supply chain entities, focusing on studies that utilize cooperative game theory. Findings from the review reveal that cooperative game theory has primarily been studied on systems with subadditive cost functions.

Next, the author introduces an optimization model focused on coalition formation to determine which humanitarian agencies should collaborate to minimize total societal cost. The problem is shown to be *NP*-complete, and a computational study is presented to compare the results from the optimization model with two proposed heuristics. Finally, the author examines cost allocations by proving analytical properties of the associated problems and introducing computational approaches to find stable and near-stable allocations. Notably, this thesis provides the first-known application of relaxation concepts to the non-subadditive game setting, which are generalized from the ε-core in subadditive games. Taken together, the coalition formation and cost allocation methods described in this research offer a foundation for improved collaboration in humanitarian supply chains and others characterized by non-subadditive costs.

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Table of Contents

Table of Contents viii
List of Figures x
List of Tables xi
Acknowledgements xii
Dedication xiv
Chapter 1 - Introduction 1
1.1 Research Motivation 2
1.2 Research Contribution and Organization of the Dissertation 3
1.2.1 State of Current Literature 5
1.2.2 Coalition Formation 6
1.2.3 Cost Allocation 7
1.3 Summary 7
Chapter 2 - Literature Review 9
2.1 Preliminaries on Cooperative Game Theory 9
2.1.1 Game Elements 9
2.1.2 Stability 10
2.1.3 Allocation Rules 12
2.1.4 Relaxing Stability Requirements 14
2.2 Current Literature on Coalition Formation and Cost Allocation 15
2.3 Research Focus 18
Chapter 3 - Coalition Formation 20
3.1 Optimal Coalition Structure 21
3.1.1 Optimal Coalition Structure Model 22
3.1.2 *OCS* Problem Complexity 23
3.2 Heuristic Approaches for Games with a Non-subadditive Cost Function 25
3.2.1 Cost Function 25
3.2.2 Heuristics 27
3.3 Computational Study and Results 38
3.3.1 Computational Study Design 38

3.3.2 Computational Study Results for Heuristics and *OCS* Model 39
3.4 Conclusion 44
Chapter 4 - Cost Allocation and Stability 45
4.1 Coalition Structure Core (*CSC*) Model 47
4.2 Analytical Characteristics of the Coalition Structure Core and Optimal Coalition Structure Problems 51
4.2.1 Fundamental Characteristics of *OCS* and *CSC* 51
4.2.2 Relationships that Enable More Efficient Computation of Stable Allocations 60
4.2.3 Summary of Analytical Results 67
4.3 Navigating the *CSC* Feasibility Challenge 68
4.3.1 Strong Least-Core Adapted to Non-subadditive Games 68
4.3.2 Solution Framework: Stability and Complexity Enhanced Algorithm 73
4.4 Computational Study 75
4.4.1 Examining *CSC* Feasibility 76
4.4.2 Comparing Allocation Rules 79
4.5 Discussion and Conclusions 84
Chapter 5 - Conclusion and Future Work 86
Bibliography 90
Appendix 94

List of Figures

Figure 3.1 Cost Function Illustration 26

Figure 3.2 Groups Order in Heuristics 29

Figure 3.3 Heuristics Cost Increase versus Optimal Solution for 16-Player Instances 40

Figure 3.4 Heuristics Cost Increase versus Optimal Solution for 18-Player Instances 40

Figure 3.5 Heuristics Cost Increase Standard Deviation for 16-Player Instances 41

Figure 3.6 Heuristics Cost Increase Standard Deviation for 18-Player Instances 41

Figure 4.1 Shapley/Proportional vs *LCSC*, Average Allocation Absolute Disparities per Player 82

Figure 4.2 Shapley vs Proportional, Average Allocation Absolute Disparities per Player 82

List of Tables

Table 3.1 Combined Coalitions and Resulting Savings 28
Table 3.2 Example to Illustrate Heuristics Steps and Comparison Between Optimal and Heuristic Solutions 36
Table 3.3 Player Distribution for Computational Studies 39
Table 3.4 Running Time Comparisons between Optimization Model and Heuristics 43
Table 4.1 Coalitions and Coalition Costs for the Instance in the Proof of Theorem 2 54
Table 4.2 Possible Coalition Structures for the Instance in the Proof of Theorem 2 55
Table 4.3 Coalitions and Coalition Costs for the Instance in the Proof of Theorem 3 59
Table 4.4 Possible Coalition Structures and Their Costs for the Instance in the Proof of Theorem 3 60
Table 4.5 Coalitions and Coalition Costs for the Example with Multiple Stable Allocations 66
Table 4.6 Coalitions and Coalition Costs in the Example Investigating the Tax and Subsidy Approaches for Determining the Value of $\varepsilon *$ 69
Table 4.7 Possible Coalition Structures and Their Costs in the Example Investigating the Tax and Subsidy Approaches for Determining the Value of $\varepsilon *$ 70
Table 4.8 Number of Instances with Non-empty Coalition Structure Core in Each Set of 10 Instances 77
Table 4.9 Average Ratio of Least-Core to *OCS* Model Objective Value and Count of Formed Coalitions in Each Set 78
Table 4.10 Number of Players Closer to *LCSC* Allocation in Each Set 80
Table 4.11 Average Allocation Absolute Disparities per Player 81

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Dedication

This dissertation is dedicated to Vahid and Golara, my loving husband and daughter, who are my motivation and inspiration throughout every stage of my life’s journey.

Chapter 1 - Introduction

Humanitarian supply chains are systems designed to provide goods and services in response to natural or human-induced disasters and continuous public health issues. Like commercial supply chains, humanitarian supply chains are concerned with efficiency and effectiveness. Efficiency involves optimizing the use of resources, while effectiveness focuses on the successful achievement of humanitarian goals in addressing crises. Increased efficiency and effectiveness of humanitarian operations are essential as disaster occurrences multiply (CRED, 2021).

The humanitarian supply chain comprises multiple independent, autonomous organizations and often operates in a decentralized context where decisions are made by multiple decision-makers. In contrast, centralized systems with a single decision-maker, who has a global view of the entire system, typically exhibit superior efficiency and effectiveness. Collaboration between agencies can contribute to enhancing overall operations. This collaboration can take the form of horizontal collaboration, where entities at the same level of the supply chain or with similar functions work together, or vertical collaboration, where organizations at different levels of the supply chain, such as manufacturers and distributors, form partnerships.

Collaboration involves various costs and challenges, such as coordination cost or communication difficulties. On the other hand, the benefits of collaboration include cost savings through economies of scale, improved service delivery, and enhanced negotiating power with suppliers. To make collaboration attractive to autonomous organizations, the associated costs and benefits must be distributed in a way that makes them better off, ensuring enduring collaboration.

Identifying the best collaborative partners in humanitarian supply chains and ways to distribute the associated costs and benefits have not been addressed in the literature. This dissertation addresses the gap. The remainder of this chapter further describes the research motivation, as well as the contributions and organization of the dissertation.

1.1 Research Motivation

Effective collaboration among entities may lead to enhanced operations. Considering humanitarian supply chains, where collaboration between agencies is within the context of humanitarian response, recovery, and development, this enhancement becomes imperative. Effective collaboration in supply chain is generally beneficial; however, in the humanitarian sector, it is especially crucial as it relates to facilitating the ease, quality, and cost of service delivery, impacting public well-being. While collaboration could yield benefits, question persist regarding how to form collaborative partnerships in ways that optimize society-level outcomes and how to allocate the cost among partners to ensure the stability of these collaborations. Stability refers to the condition that partners desire to participate in the collaboration because they find it beneficial.

Cooperative game theory provides tools for system analysis in which independent agencies work together for mutual benefit. Most studies in the cooperative game theory literature investigated subadditive games in which all players collaborate as a single entity to provide the least total cost to society. In this setting, the primary focus of the studies is on determining how to allocate the total cost among the players (Ben Jouida et al., 2017; Ergun et al., 2014). Work to date in this domain has not fully addressed the non-subadditive cost characteristics inherent in many humanitarian contexts, where scale economies may exist for some collaborative partnerships and diseconomies for others.

The primary motivation for this research is to advance the knowledge of cooperation in decentralized humanitarian supply chains within a non-subadditive setting by developing new models, heuristics, analysis, and computational studies. The ultimate goal is to provide a framework for humanitarian supply chain decision-makers to select cooperative partners and determine cost allocation methods, aiming to enhance supply chain operations by establishing stable collaborations that minimize the overall societal cost.

While the primary aim of this research is to improve humanitarian supply chain, this study intentionally avoids dependence on specific problem characteristics. The concepts presented in this study are designed to be versatile and adaptable, allowing for modification and application not only across various humanitarian scenarios but also other sectors.

1.2 Research Contribution and Organization of the Dissertation

Effective collaboration in humanitarian supply chains could align individual interests with the societal goal and improve the overall performance of the system. Cooperative game theory provides a tool to model the collaboration and address the two main questions of: 1) who is cooperating with whom? and 2) how should the gained savings of cooperation should be divided among cooperative entities (players)?

While cost allocation in subadditive games has been widely studied, there is a notable gap in the exploration of coalition formation and cost allocation in non-subadditive games. This research is dedicated to filling this gap by delving into coalition formation and cost allocation within a general non-subadditive cost structure, providing insights and solutions to address this previously neglected aspect of cooperative game theory.

First, this research considers the coalition formation problem by introducing an optimization model to identify optimal collaborative partners. As the author proves that this problem is *NP*-complete, the subsequent step involves the introduction of two efficient heuristics. In the final phase in this step, a computational study is conducted to examine the proposed approaches and determine the impact of player characteristics on both the running time and total cost.

Moving forward, this research examines the cost allocation problem. (Guajardo & Rönnqvist, 2015) introduced an optimization model that finds stable allocations. While examining this model, the author faces the complexity and feasibility issues inherent in this problem.

To navigate complexity challenge, the author chooses the path of studying the characteristics associated with both coalition formation and cost allocation problems. These explorations not only help navigate the complexity of the cost allocation problem in this study but also provide valuable insights for problem-solving strategies, laying the groundwork for future research in this domain.

The author tackles the problem feasibility challenge by applying a relaxation method. While the relaxation method has been extensively explored in subadditive games, this work introduces necessary modifications to the concepts, marking the first known application of relaxation to non-subadditive games.

The practical applicability of the proposed solutions within organizational settings is one of the concerns of this study. Consequently, in the subsequent step, the author selects and tests the proposed allocation approaches through two widely accepted and easily comprehensible allocation rules.

It is essential to note that, given that one of the primary goals of this research is to ensure its applicability across various problem scenarios, this study deliberately avoids relying on any specific characteristic of the problem. The concepts presented in this study are designed to be versatile and adaptable, allowing for modification and application in various scenarios.

In summary, this study utilizes game theory and operations research to make the mentioned contributions, ultimately benefiting society by enhancing collaboration within humanitarian public health organizations.

1.2.1 State of Current Literature

Chapter 2 conducts a comprehensive literature review on cooperative game theory. The existing literature in cooperative game theory can be categorized into two main groups. The first group, comprising the majority of works, investigates subadditive games, while the second group consists of limited number of studies exploring non-subadditive games. This review delves into coalition formation and cost allocations, computational complexity, and stability of collaboration, addressing both subadditive and non-subadditive scenarios. The primary contributions of the review encompass:

- Identifying the commonly used approaches to manage the complexity of the cost allocation problem, namely decreasing the number of players/coalitions andimplementing restrictions on the problem.
- Identifying the three most widely acknowledged cost allocation rules: ad hoc, Shapley, and proportional.
- Highlighting the fact that the relaxation method has not been used in non-subadditive games for determining near-stable solutions.

1.2.2 Coalition Formation

Chapter 3 applies cooperative game theory methods to reveal which humanitarian agencies should collaborate with each other to minimize total social cost. The focus is on supply chains with non-subadditive costs in which large collaborative groups are not necessarily optimal. Chapter 3 provides the following contributions:

- Introduction of an optimization model to identify optimal collaborative partners.
- Proof that the proposed problem is *NP*-complete.
- Introduction of two efficient heuristics to solve the problem, addressing the complexity challenge.
- Conducting a computational study to examine the proposed approaches and assess the impact of player characteristics on running time and total cost.

1.2.3 Cost Allocation

Chapter 4 utilizes game theory and operations research concepts to determine a stable allocation of costs within a non-subadditive setting. The focus of Chapter 4 is on various aspects of cost allocations, encompassing key concepts such as optimal cost allocation, computational complexity, stability, feasibility, and the applicability of the proposed approaches. Chapter 4 provides the following contributions:

- Proved certain characteristics of the coalition formation and cost allocation problems, uncovering properties that are useful in overcoming the complexity challenge posed by the cost allocation optimization model.
- Introduced a relaxation method to address the feasibility issue in the cost allocation problem.
- Compared the results generated by the proposed method with two allocation rules prevalent in the literature.
- Conducted a computational study to provide insights into how allocation methods perform under different player characteristics.

1.3 Summary

The combined contributions of this thesis further the understanding of finding collaborative partners and allocating cost within non-subadditive games. Although the primary research emphasis is humanitarian supply chains, the findings in this research could be applied to improve cooperation in any supply chain systems and not limited to humanitarian sector. In addition to filling important theoretical gaps in the literature, this research provides practical applications for the methods and concepts presented, as well as avenues for future research (Chapter 5).

Chapter 2 - Literature Review

This chapter synthesizes closely related literature and identifies important gaps. Comprehensive understanding of the academic literature requires recognition of the fundamental definitions of game theory concepts, so these are presented in the first section. This is followed by a summary of prior work in coalition formation and cost allocation. The focus of this dissertation is situated in context of this literature.

2.1 Preliminaries on Cooperative Game Theory

Cooperative game theory studies the class of games in which self-interested players form coalitions to obtain greater benefits. This section provides a brief introduction to the fundamental cooperative game theory concepts. For additional information, the reader is referred to (Aumann & Dreze, 1974; Peleg & Sudholter, 2007).

2.1.1 Game Elements

Let $N = \{1, \dots, |N|\}$ be a finite set of players. Subsets of N are called coalitions. For any coalition $\emptyset \subset S_i \subseteq N$, $|S_i|$ is the cardinality of coalition S_i. The coalition comprised of all players, N, is called the grand coalition. The set of all possible non-empty coalitions is denoted by Ω, and the cardinality of Ω is $|\Omega| = 2^{|N|} - 1$. A coalition structure is a partition of all players in N into disjoint coalitions, where each coalition's members cooperate with one another but not with members of other coalitions. A coalition structure $\varphi = \{S_1, \dots, S_m\}$ is such that $S_i \cap S_j = \emptyset$ for $i \neq j$ and $\bigcup_{i=1}^{i=m} S_i = N$.

A cooperative game can be either a cost game or a savings game, depending on whether the focus is on the costs incurred or the benefits derived from collaboration. Cost game is denoted by (N, c) and savings game is denoted by (N, v), where c (v in savings game) is a characteristic function that associates a real number c_i (v_i in savings game) with each subset $S_i \subseteq N$. These perspectives are interchangeable, where $v_i = \sum_{z \in S_i} c(\{z\}) - c_i$ for all $S_i \subseteq N$ (Peleg & Sudholter, 2007). This research utilizes the cost perspective.

A game is subadditive if $c_i + c_j \geq c_k$ for all disjoint coalitions $S_i, S_j \subseteq N$, where $S_k = S_i \cup S_j$. In other words, in subadditive games, large coalitions incur at most as much cost as their members would by operating alone. This implies that the collaboration between S_i and S_j creates synergy, and it is always profitable (or at least not unprofitable) to form a larger coalition. A game is monotonic if $S_i \subseteq S_j \subseteq N \Rightarrow c_i \geq c_j$. Monotonocity implies that large coalitions perform better. If a game is subadditive, then that game is monotonic; however, not all monotonic games are subadditive. The current research focus is on non-subadditive games, or games in which collaboration between coalitions does not always create synergy.

2.1.2 Stability

Cooperative game theory analyzes situations where a group of players act collectively to achieve a certain benefit. When a common transferable resource such as money is present, the players may distribute the gains among themselves in any possible way, but proper allocation of these gained benefits is crucial to maintain a stable collaboration. In a stable allocation, the formed collaboration is not threatened because no coalition of members can improve their own outcome by operating independently or by forming an alternative collaboration; players have no incentive to deviate from the stable allocation.

Stability is the key concept that maintains a collaboration and one of the most compelling properties in cooperative game theory. The core refers to the set of stable cost allocations in subadditive games, defined as:

$$\sum_{n \in S_j} x_n \leq c_j \qquad \forall \; S_j \in \Omega \tag{2.1}$$

$$\sum_{n \in N} x_n = c_N \tag{2.2}$$

Here $x = \{x_1, \dots, x_{|N|}\}$ refers to the set of allocations, x_i is the cost allocated to player i in that set, c_j is the cost of coalition S_j, and c_N is the grand coalition cost. Equation 2.1 represents the rationality condition, stating that no subset of players can deviate from the grand coalition, form their own coalition, and benefit from that deviation. This implies that the total cost allocated to the deviating coalition must be more than the sum of costs allocated to deviating players. If the deviating coalition has only one player, this constraint corresponds to the individual rationality condition, ensuring that the allocated cost to each player must not exceed the player's individual cost. Equation 2.2 corresponds to the efficiency/budget balanced condition, where the sum of costs allocated to all players must equal the grand coalition cost. An allocation that meets these two requirements is in the core and stable.

A coalition structure game refers to a non-subadditive game in which the cooperating players are organized in disjoint coalitions to form a coalition structure φ. This game is denoted by triplet (N, c, φ), where N is the set of players, c is a characteristic function that associates a real number c_i with each subset $S_i \subseteq N$, and φ is the coalition structure. A coalition structure φ is stable in the coalition structure core (Aumann & Dreze, 1974) if the cost allocated to players is budget balanced (efficiency constraint) and no group of players (within the same or different coalitions) is incentivized to leave the partition. The coalition structure core is formally defined as:

$$\sum_{n \in S_j} x_n \leq c_j \qquad \forall S_j \in \Omega \tag{2.3}$$

$$\sum_{n \in S_j} x_n = c_j \qquad \forall S_j \in \varphi^* \tag{2.4}$$

Similar to the core, the coalition structure core requires rationality constraints, as represented in equation 2.3. Here, φ^* refers to the chosen coalition structure. Equation 2.4 guarantees the budget balanced condition in each formed coalition in φ^*. When these two conditions, 2.3 and 2.4, are satisfied simultaneously, it is said that the allocation is strongly stable or in the coalition structure core.

2.1.3 Allocation Rules

While the core focuses on the stability of a cooperative arrangement, an allocation rule is a method or formula that dictates how to distribute the overall benefit or cost among the players in a cooperative game. A specific allocation may belong to the core, or the core may be empty, which means no allocation ensures the stability. This study employs a specific set of criteria for selecting allocation rules to examine, namely, that allocation rules should be commonly used to ensure relevance and easy to understand to broaden acceptability. Therefore, this study's focus is on two well-established allocation rules (Shapley value and proportional allocations) to examine via computational analysis.

The Shapley value, named after the mathematician and economist who introduced the value in 1953, is a fundamental method for allocating costs or benefits in cooperative settings (L. Shapley, 1953). The value allows allocation based on players' marginal contributions; each player *n* is allocated the average of the marginal cost implied when the player is added to a coalition. While the Shapley value is designed as an allocation rule for subadditive games, this research focuses on non-subadditive games. To apply Shapley value in a non-subaddive setting, each coalition is considered as a grand coalition. The classic Shapley value is defined as:

$$x_n = \sum_{S_j \in \Omega : n \in S_j} \left[\frac{(|N|-|S_j|)!-(|S_j|-1)!}{|S_j|!}\right]. \ [c_{S_j} - c_{(S_j \setminus n)}] \qquad \forall\, n \in N \tag{2.5}$$

The proportional allocation rule is a commonly used allocation method in cooperative game theory, where each player is assigned the same fraction of the formed coalition's costs as their fraction of the formed coalition's value of another parameter. Various methods exist for calculating these proportions based on the parameter selected, such as based on the proportion of total demand or order quantity. The proportional allocation is formally defined as:

$$x_n = \frac{q_n}{\sum_{i \in S_j} q_i}\, c_j \qquad \forall\, n \in S_j \tag{2.6}$$

where q_i is the selected parameter from which to calculate the proportion.

2.1.4 Relaxing Stability Requirements

Although stability is a highly desirable property, the core or coalition structure core is often empty. The strong least core (also called ε^*-core) is a relaxation method that has been widely utilized in subadditive games (Bachrach et al., 2013; Persien et al., 2016; L. S. Shapley & Shubik, 1963; Zick et al., 2013a), yet to the author's knowledge , the relaxation concept has not been applied in non-subadditive games. In subadditive games, the strong least core/ε^*-core (hereafter referred to as least core) corresponds to the smallest value ε^* by which the rationality condition for core allocations, equation 2.3, must be relaxed to obtain a solution that satisfies all core conditions. Formally, ε^* is given by the objective value of the following optimization model:

$$Minimize\ \varepsilon \tag{2.7}$$

subject to:

$$\varepsilon \geq \varepsilon_j \qquad \forall\, S_j \in \Omega \tag{2.8}$$

$$\sum_{n \in S_j} x_n \leq c_j + \varepsilon_j \qquad \forall\, S_j \in \Omega \tag{2.9}$$

$$\sum_{n \in N} x_n = c_N \tag{2.10}$$

$$\varepsilon_j \geq 0 \qquad \forall\, S_j \in \Omega \tag{2.11}$$

The ε^* could correspond to a tax that is added to rationality constraints to discourage deviations from the grand coalition, thereby making the grand coalition stable.

2.2 Current Literature on Coalition Formation and Cost Allocation

In their book *Theory of Games and Economic Behavior*, Von Neumann and Morgenstern (1944) established game theory, encompassing both cooperative and non-cooperative games. While both approaches study how individuals make strategic decisions, players in non-cooperative games act independently. Non-cooperative game theory examines how individuals make decisions based on their own self-interests, taking into account their expectations of other players' actions. Conversely, cooperative game theory analyzes how players work together to achieve mutually beneficial outcomes. It explores the potential for collaboration, communication, and formation of coalitions among players, with an emphasis on understanding player behavior when establishing binding agreements. The models are based on the premise that, even in cooperative settings, each player seeks to optimize their personal objectives.

Cooperative game theory addresses two key collaboration questions: who should collaborate with whom? and how should the cost or profit be shared among the collaborative partners? Cooperative games can be categorized as either subadditive or non-subadditive games. Most studies in the cooperative game theory literature investigate subadditive games in which the grand coalition provides the least total cost to society (Agarwal et al., 2009; Ergun et al., 2014; Kimms & Kozeletskyi, 2016; Lozano et al., 2013; Özener & Ergun, 2008; Xu et al., 2012). In subadditive games, the emphasis is typically on the allocation of cost/profit within the grand coalition, as it offers the minimum total social cost. However, because the focus of this study is non-subadditive games, both research questions are emphasized.

Allocation in cooperative games attempts to satisfy notions of stability. For example, subadditive games establish allocation to avoid incentives for players to operate apart from the grand coalition (Anily & Haviv, 2007). Young (1985) thoroughly reviewed basic cost allocation methods, while several other studies describe cost allocation among cooperative entities within specific application contexts, such as humanitarian operations (Ergun et al., 2014), supply networks (Xu et al., 2012), forestry (Frisk et al., 2010), cooperative replenishment (Ben Jouida et al., 2017), and transportation (Kimms & Kozeletskyi, 2016; Lozano et al., 2013). (Guajardo & Rönnqvist, 2016a) reviewed allocation approaches for costs and profits in 55 cooperative game theory articles, encompassing a wide scope of allocation methods in cooperative game theory.

While cost allocation in subadditive games has been widely studied, coalition formation and cost allocation in non-subadditive games has received less attention in the literature. Guajardo and Rönnqvist (2016a) reference only three studies (Audy et al., 2012; Guajardo & Rönnqvist, 2015; Guajardo & Rönnqvist, 2016b) on non-subadditive games, all of which focus on collaboration among eight number of forestry companies. Audy et al. (2012) investigates cost allocation and coalition formation, particularly when some players lead coalition members. Guajardo and Rönnqvist (2015) examine coalition formation by applying a cardinality constraint, where each coalition must contain at most *m* players. In Guajardo and Rönnqvist (2016b), multiple allocation rules were considered to determine the preference of players for each allocation rule.

Given the computational complexity associated with the coalition formation and cost allocation problems, some previous studies aim reduce the computational time required to solve these problems. Fatima & Wooldridge (2019)introduce a model that identifies optimal coalition structures in polynomial time, leveraging a monotonic value function and an algorithm to order coalitions in partitions. Elomri et al. (2012) employ a heuristic algorithm to enhance computation time by applying a filter that retains only cost-saving coalitions. In their investigation of coalition formation in a storage-sharing context with predetermined allocation rules, Ben Jouida et al. (2021) apply individual rationality principles to identify and discard non-profitable coalitions, thereby reducing problem complexity. Basso et al. (2020) propose a heuristic to address a scheduling problem in the wine bottling process, focusing on subsets of wineries that agree to cooperate as coalitions. Lastly, Schulte et al. (2017) evaluate collaboration among small groups of truckers to optimize truck scheduling.

Stability is a desirable but rare characteristic in cost/profit allocation. The most widely accepted notions of stable/fair allocation are core in subadditive and coalition structure core in non-subadditive games. Von Neumann & Morgenstern (1944) first introduced the concept of core, while Young (1985) first introduced the concept of *CSC* by considering the possibility of the formation of multiple coalitions or groupings of players. Since achieving a completely stable solution is challenging, a somewhat stable payoff is often attained by relaxing the core or coalition structure core constraints. For instance, a reasonable assumption could be that a subset of players would not deviate if the additional payment gained by deviating does not exceed $\varepsilon > 0$, and a coalition would choose to deviate only if a substantial gain was anticipated.

Relaxation is a commonly employed approach in solving real-world game theory problems (Ozener & Ergun, 2008). Shapley and Shubik (1963) first introduce ε^*-core as a method to relax core requirements in subadditive games. Zick et al. (2013) stabilize subadditive games by leveraging the least core to diminish the value of certain coalitions and reduce their bargaining power. In a similar vein, Bachrach et al. (2013) establish bounds on deviations from the stable allocation for various classes of cooperative games. Persien et al. (2016) explore the concept of stability and least core in path-disruption games. Notably, to the best of the author’s knowledge, the relaxation method has not been applied in non-subadditive games. Given the current research emphasis on non-subadditive games, the least core concepts are adapted to be applicable to the coalition structure core.

2.3 Research Focus

This study employs cooperative game theory methods to determine which humanitarian agencies should collaborate to minimize total social cost, and what allocation method should be applied so that the collaboration remains stable. The research gap in the literature is addressed by focusing on supply chains with non-subadditive costs in which large collaborative groups are not guaranteed to be optimal.

To address the first question, an optimization model is introduced to identify the coalition structure with minimum societal cost. Additionally, two efficient heuristics are proposed to address the same question and a computational study is conducted to analyze the effectiveness of these approaches.

To tackle the second question, this study employs an optimization model to determine the stable allocation. Due to the complexity and feasibility challenges inherent with this model, the author explores the characteristics of both coalition formation and cost allocation problems. These explorations yield insight to tackle the complexity of the problem. Furthermore, the author addresses the feasibility issue by introducing modifications to the ε^*-core concept, making it applicable in non-subadditive games —a novel method not previously explored. Two commonly used and easily comprehensible allocation methods are chosen, and all the proposed approaches and allocation rules undergo computational examination through randomly generated instances.

These computational studies provide valuable insights into how allocation methods perform under different player characteristics.

Chapter 3 - Coalition Formation

Humanitarian supply chains support agency goals related to disaster response, recovery periods, and development operations. The supply chain activities encompass the entire product and service delivery process, which includes procurement, storage, and transportation. The health and well-being supported by humanitarian operations are considered social goods.

Collaboration between humanitarian agencies could involve joint procurement, common warehouse space, or shared transportation, which all could save costs and/or improve service. For instance, joint procurement arrangements empower small, local agencies with negotiating power when collaborating, leading to possible advantages such as lower prices or preferred delivery, ultimately enhancing agency services. However, not all collaborative partnerships are inherently cost-saving or universally beneficial. Determining which agencies should collaborate becomes essential to ensure effective and efficient outcomes.

Cooperative game theory is a tool to analyze systems in which independent players may work together for mutual benefit. However, work to date in this domain has not fully addressed the inherent cost characteristics of many humanitarian contexts, particularly the notion that large collaborative groups may not always be the most cost-effective. Previous research has focused on whether collaboration is desirable for individual players, overlooking the significance of adopting a society-wide perspective in the context of humanitarian efforts.

This chapter employs cooperative game theory to determine which humanitarian agencies should collaborate to minimize total societal cost. The gap in the literature is addressed by focusing on supply chains with non-subadditive costs, where large collaborative groups are not necessarily optimal.

3.1 Optimal Coalition Structure

Collaboration in the humanitarian supply chain has the potential to significantly benefit both the public and humanitarian organizations through achieving synergies. For example, multiple organizations can reduce their fixed costs by sharing common inventory locations. In addition, information sharing of transportation orders could facilitate backhauling, or order consolidation could utilize quantity discounts. It's important to note that sharing common inventory locations primarily addresses storage efficiencies and cost reductions, whereas order consolidation focuses on streamlining transportation logistics to minimize shipping expenses.

Identifying optimal coalition structures, which involves grouping entities into collaborative partnerships, is a shared objective across various collaborations. However, optimal collaborative partners may vary when considering the perspectives of humanitarian organizations compared to societal perspectives. What might be deemed an optimal coalition structure for an organization may not necessarily lead to the lowest societal cost overall. This study specifically aimed to pinpoint a cooperative structure that minimizes the total cost for society, thereby improving operational efficiency and maximizing overall benefits.

For humanitarian supply chains, costs frequently exhibit non-subadditive characteristics, meaning that minimum societal cost does not necessarily result from large collaborations. For example, while certain collaborative arrangements may benefit from economies of scale, diseconomies can occur beyond a certain point, when coalitions become larger. In addition, even if a collaboration seems profitable for organizations in geographic proximity or offering similar services, barriers such as legal constraints or communication challenges can prevent cooperation, making a grand coalition neither optimal nor feasible. Therefore, this research delves into humanitarian supply chain collaborations within a non-subadditive context to pinpoint optimal partners for collaboration. The following sections introduce a general optimization model for determining the minimum-cost coalition structure, followed by an analysis of the model's complexity.

3.1.1 Optimal Coalition Structure Model

The author utilizes a mixed integer programming model to find a coalition structure that minimizes total societal cost. A solution to the following optimal coalition structure *(OCS)* model is an optimal coalition structure on set N. The notation for sets, parameters, and decision variables follows:

Sets

N: set of players

Ω: set of coalitions

Parameters

c_j: cost of coalition S_j

$$\alpha_{n,j} = \begin{cases} 1 & \text{if player } n \text{ belongs to coalition } S_j \\ 0 & \text{otherwise} \end{cases}$$

Decision variables

$$\beta_j = \begin{cases} 1 & \text{if coalition } S_j \text{ is formed} \\ 0 & \text{otherwise} \end{cases}$$

The optimization model below seeks to find the coalition structure that generates the minimum total social cost.

$$Minimize \sum_{S_j \in \Omega} c_j \, \beta_j \qquad 3.1$$

subject to:

$$\sum_{S_j \in \Omega} \alpha_{n,j} \beta_j = 1 \qquad \forall\, n \in N \qquad 3.2$$

$$\beta_j \in \{0,1\} \qquad \forall\, S_j \in \Omega \qquad 3.3$$

The objective function 3.1 minimizes the total societal cost across all selected coalitions. Constraint 3.2 requires that each player belongs to one and only one coalition in the coalition structure, derived from the definition of the coalition structure. Expression 3.3 represents the binary decision for the formation of a coalition.

3.1.2 *OCS* Problem Complexity

The input to the proposed optimization model is the set of players, N, and the costs of all $2^{|N|} - 1$ coalitions. The number of coalition structures is given by:

$\sum_{k=0}^{|N|-1} \binom{|N|-1}{k} B_k$, with $B_0 = 1$ (Rota, 1964). 3.4

Equation 3.4 shows that it is impractical to enumerate coalition structures for even modest values of $|N|$. The decision version of the coalition structure problem is *NP*-complete, as evidenced by its relationship to the exact cover problem.

Lemma 1. The optimal coalition structure problem is equivalent to the weighted exact cover problem; thus, the coalition structure problem is *NP*-complete.

Proof. To understand the relationship between weighted exact cover and the coalition structure problem, the exact cover problem must be considered. Given a matrix of 0s and 1s, an exact cover is a set of rows containing exactly one 1 in each column (Knuth, 2000). Exact cover is an *NP*-complete problem (Garey & Johnson, 1979; Jongen et al, 2004). In weighted exact cover, a weight (cost) is associated with each row. If the columns are players in set N and rows are subsets (coalitions) of N, then the coalition structure problem is to cover players in set N with disjoint subsets. With this representation, solving the weighted exact cover problem is equivalent to solving the coalition structure problem and vice versa, proving that the coalition structure problem is *NP*-complete. ☐

Knuth (2000) described an exponential algorithm to find all possible solutions to a weighted exact cover problem with an efficient data structure, while Vaissier et al. (2020) proposed a polynomial-time heuristic for a weighted exact cover problem in file compression for additive manufacturing applications in which the weights depend on problem-specific characteristics. This current research proposes heuristics for the weighted exact cover problem in which weights are given by a general non-subadditive function.

3.2 Heuristic Approaches for Games with a Non-subadditive Cost Function

This section considers the coalition structure problem for costs that follow a general functional form in which cost is a function of player or coalition order quantity. This approach is chosen since quantity influences many logistics costs, including order processing, handling, transportation, and storage costs. Organizations often collaborate by ordering jointly, but after a certain point, increased order quantities may result in increased marginal costs, thereby implying a non-subadditive cost function. This research introduces the proposed function and then characterizes the savings that result from collaboration between players. Based on savings observations, two heuristics are introduced that provide good solutions with reasonable computation time.

3.2.1 Cost Function

The notation used in the cost model and the heuristics is as follows:

A: fixed ordering cost

k: variable cost

d: variable cost discount

p: variable cost penalty

z_1, z_2: cutoff points in cost function

q_i: order quantity of coalition S_i

c_i: ordering cost of coalition S_i

φ_r: coalition structure r

Players may perform alone or collaborate to combine their orders into one order. If a player performs alone, the coalition order quantity is the same as the player's order quantity. If players collaborate, the coalition order quantity (q_i) is the sum of participating players' order quantities. Total ordering cost c_i, a function of coalition order quantity, is illustrated in Figure 3.1 and given by:

$$c_i = \begin{cases} A + kq_i & \text{if } 0 < q_i < z_1 \\ A + dz_1 + (k-d)q_i & \text{if } z_1 \leq q_i < z_2 \\ A + (k+p)q_i & \text{if } z_2 \leq q_i \end{cases} \quad 3.5$$

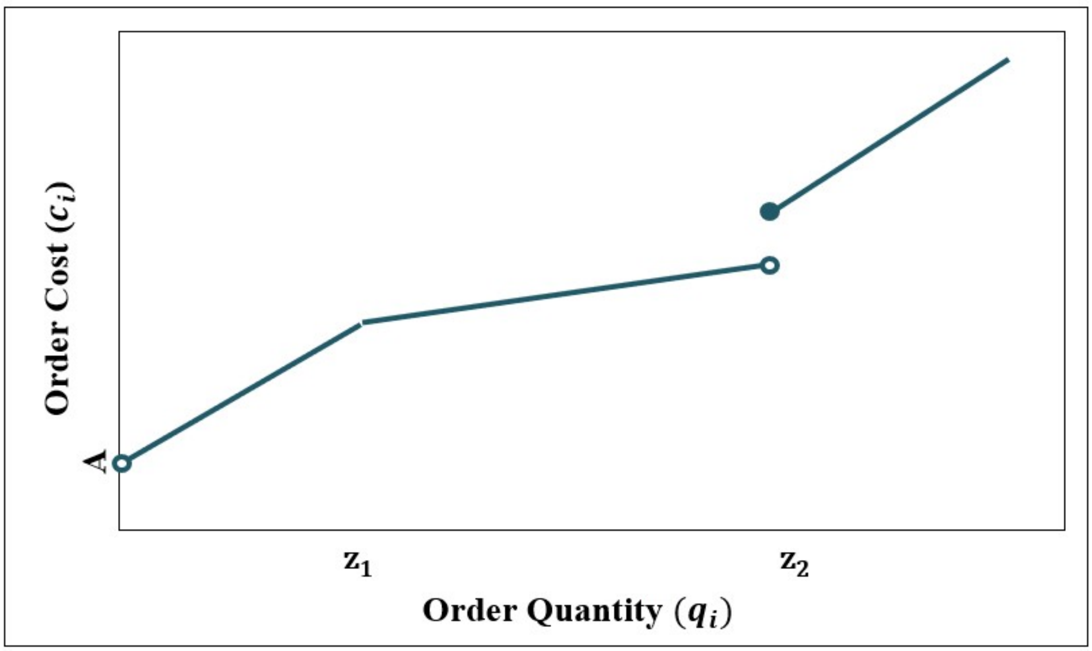

Figure 3.1 Cost Function Illustration

In the 3.5 cost function, while $q_i < z_1$ the variable cost is k, and when $z_1 \leq q_i < z_2$ a discount d on variable cost occurs. Eventually, if $q_i \geq z_2$ a penalty p occurs on the variable cost, implying that for coalitions with order quantities larger than z_2, adding more players to the coalition may not reduce the total societal cost. For any coalition structure, the total societal cost is given by the sum of the costs of the coalitions that form the structure.

3.2.2 Heuristics

This study presents two heuristics designed to leverage the characteristics of the introduced non-subadditive cost function, addressing the complexity of the problem and rapidly generating good solutions.

3.2.2.1 Savings from Coalition Formation

The proposed heuristics aim to identify low-cost coalition structures by merging coalitions to achieve savings. When two coalitions combine, they become one coalition. Table 3.1 depicts all possible scenarios for combined coalitions along with the resulting savings.

Group	Savings	q_i	q_j	$q_i + q_j$
1	A	$q_i < z_1$	$q_j < z_1$	$q_i + q_j < z_1$
2	$A + d(q_i + q_j - z_1)$	$q_i < z_1$	$q_j < z_1$	$z_1 \le q_i + q_j < z_2$
3	$A - p(q_i + q_j)$	$q_i < z_1$	$q_j < z_1$	$z_2 \le q_i + q_j$
4	$A + dq_j$	$z_1 \le q_i < z_2$	$q_j < z_1$	$z_1 \le q_i + q_j < z_2$
5	$A + d(z_1 - q_i) - p(q_i + q_j)$	$z_1 \le q_i < z_2$	$q_j < z_1$	$z_2 \le q_i + q_j$
6	$A - pq_i$	$z_2 \le q_i$	$q_j < z_1$	$z_2 \le q_i + q_j$
7	$A + dz_1$	$z_1 \le q_i < z_2$	$z_1 \le q_j < z_2$	$z_1 \le q_i + q_j < z_2$
8	$A + 2dz_1 - (d + p)(q_i + q_j)$	$z_1 \le q_i < z_2$	$z_1 \le q_j < z_2$	$z_2 \le q_i + q_j$
9	$A + dz_1 - (d + p)\, q_j$	$z_2 \le q_i$	$z_1 \le q_j < z_2$	$z_2 \le q_i + q_j$
10	A	$z_2 \le q_i$	$z_2 \le q_j$	$z_2 \le q_i + q_j$

Table 3.1 Combined Coalitions and Resulting Savings

As shown in Table 3.1, forming coalitions in groups 7, 4, 2, 1, and 10 results in positive savings for any parameter values, with a respective decreasing magnitude of savings. However, savings from groups 6, 3, 5, 9, and 8 may be positive or negative depending on the values of A, P, d, z_1, z_2, and q_i. Thus, increased costs (negative savings) may result by forming coalitions in these groups.

The objective of the proposed heuristics is to solve several smaller problems optimally and combine the solutions to produce a coalition structure for the original problem. Therefore, key decisions consist of which players/coalitions to include in each sub-problem and in which order to solve the sub-problems. The two heuristics, identified as Heuristic A and Heuristic B, solve sub-problems in which the players/coalitions are determined based on order quantity. The difference between these heuristics is the order of the sub-problems, as illustrated in Figure 3.2.

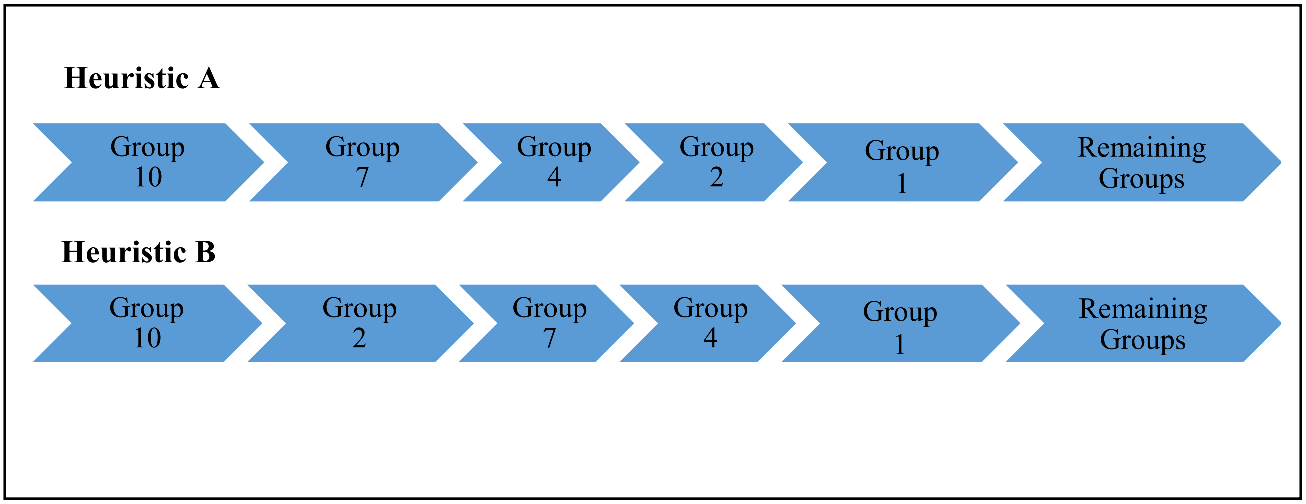

Figure 3.2 Groups Order in Heuristics

Both heuristics begin by considering only players whose order quantity is greater than or equal to z_2; this subset is denoted as group 10. The cost for coalitions comprised only of group 10 players is subadditive, and these players/coalitions cannot participate in any other groups in which positive savings are guaranteed (groups 7, 4, 2, or 1). Thus, to reduce the number of players and running time in both heuristics, group 10 is processed at the beginning of the heuristics and all group 10 players are combined into a single coalition. However, Heuristics A and B differ in the order they consider players with order quantity less than z_2. In Heuristic A, groups are considered in decreasing order of savings, while in Heuristic B, group 2 is considered before group 7, potentially increasing the number of coalitions that are considered in group 7 and decreasing those considered in group 4.

3.2.2.2 Heuristic Algorithms

This section provides the pseudocode for Heuristic A; complete pseudocode for Heuristic B is provided in the appendix. The following definitions and cost functions are used in the algorithm:

Definitions:

N: set of players $1, \ldots, |N|$

S_j: coalition j, $S_j \subseteq \Omega$; assume without loss of generality that coalitions $S_1, \ldots, S_{|N|}$ are the singleton coalitions consisting, respectively, of players $1, \ldots, |N|$

q_j: order quantity of coalition S_j

P_r: subset *r* of formed coalitions (including singleton coalitions) to be considered in phase *r* of the heuristic; $P_r \subseteq S$ for $r = 1, \ldots, 6$

$\Omega(P_r)$: set of non-empty coalitions on P_r; $\Omega(P_r) \subseteq \Omega$

$N(P_r)$: players that belong to at least one coalition in $S(P_r)$; $N(P_r) \subseteq N$

A: fixed ordering cost

k: variable cost

d: variable cost discount

p: variable cost penalty

z_1, z_2: cutoff points in cost function

Cost functions:

$$c_j^0 = \begin{cases} A + kq_j & \text{if } 0 < q_j < z_1 \\ A + dz_1 + (k-d)q_j & z_1 \leq q_j < z_2 \\ A + (k+p)q_j & \text{if } z_2 \leq q_j \end{cases} \qquad 3.6$$

$$c_j^1 = \begin{cases} A + dz_1 + (k-d)q_j & \text{if } z_1 \leq q_j < z_2 \\ \infty & \text{otherwise} \end{cases} \qquad 3.7$$

$$c_j^2 = \begin{cases} A + kq_j & \text{if } 0 < q_j < z_1 \\ \infty & \text{otherwise} \end{cases} \tag{3.8}$$

$$c_j^3 = \begin{cases} A + kq_j & \text{if } 0 < q_j < z_1 \\ A + dz_1 + (k-d)q_j & \text{if } z_1 \leq q_j < z_2 \text{ } and \text{ } \exists\, i \in S_j : z_1 \leq q_i < z_2 \\ \infty & \text{otherwise} \end{cases} \tag{3.9}$$

$$c_j^4 = \begin{cases} A + kq_j & \text{if } 0 < q_j < z_1 \\ A + dz_1 + (k-d)q_j & \text{if } z_1 \leq q_j < z_2 \\ \infty & \text{otherwise} \end{cases} \tag{3.10}$$

Algorithm 1. Heuristic A Pseudocode

Input: A player set N, a coalition set Ω, cost functions c_j^0, c_j^1, c_j^2, c_j^3, c_j^4

// *initialization*

for $r = 1, \dots, 6$

    $P_r \leftarrow \emptyset$

for $j = 1, \dots, |N|$

    if $q_j \geq z_2$

        $P_1 \leftarrow P_1 \cup S_j$

    else if $q_j < z_1$

        $P_3 \leftarrow P_3 \cup S_j$

    else $P_2 \leftarrow P_2 \cup S_j$

// *Phase 1; group 10 players form a single coalition*

$P_1 \leftarrow \bigcup_{S_j \in P_1} S_j$

// *Phase 2; group 7 optimization*

$P_2 \leftarrow MIP(P_2, c_j^1)$

// *Phase 3; group 4 optimization*

$P_3 \leftarrow P_3 \cup P_2$ // *coalitions resulting from Phase 2 are also included in this phase*

$P_3 \leftarrow MIP(P_3, c_j^3)$

// *Phase 4; group 2 optimization*

for all $S_j \in P_3$ // *coalitions resulting from Phase 3 are considered separately based on* q_j

    if $q_j < z_1$

        $P_4 \leftarrow P_4 \cup S_j$

    else $P_6 \leftarrow P_6 \cup S_j$

$P_4 \leftarrow MIP(P_4, c_j^4)$

// *Phase 5; group 1 optimization*

for all $S_j \in P_4$ // *coalitions resulting from Phase 4 are considered separately based on* q_j

    if $q_j < z_1$

        $P_5 \leftarrow P_5 \cup S_j$

    else $P_6 \leftarrow P_6 \cup S_j$

$P_5 \leftarrow MIP(P_5, c_j^2)$

// *Phase 6; combined optimization*

$\varphi_{final} \leftarrow MIP(P_5 \cup P_6 \cup P_1, c_j^0)$

return: φ_{final}

function: $MIP(P_r, c_j^i)$

    $\varphi_{temp} \leftarrow \emptyset$

    solve:

        $Min \sum_{S_j \in S(P_r)} c_j^i \beta_j$

        $s.t. \sum_{S_j \in S(P_r)} \alpha_{u,j} \beta_j = 1 \quad \forall\, u \in N(P_r)$

            $\beta_j \in \{0,1\} \qquad \forall\, S_j \in \Omega(P_r)$

    for all $S_j \in P_r$

        if $\beta_j == 1$

            $\varphi_{temp} \leftarrow \varphi_{temp} \cup S_j$

    return: φ_{temp}

Because Heuristics A and B differ in the order they solve the groups, the initialization steps that sort players based on order quantity (lines 7–9) also differ. In line 8, Heuristic B initially assigns players with order quantity less than z_1 to P_2, while in line 9, players with order quantity $z_1 \leq q_j < z_2$ are assigned to P_3. In addition, the phase 2 sub-problem (lines 12–13) considers group 2 coalitions, phase 3 (lines 14–16) considers group 7 coalitions, and phase 4 (lines 17–19 of Heuristic B) considers group 4 coalitions.

The following example presents the solution from the optimization model and compares the steps of each heuristic.

Example:

Consider a game with 18 players with order quantities of {60, 171, 28, 70, 130, 234, 165, 82, 450, 255, 606, 482, 731, 787, 580, 760, 1440, 1750}. Let $z_1 = 250$, $z_2 = 1000$, $A = 300$, $d = 4$, $p = 3,\ k = 10$.

Optimal Solution

Coalitions	Order quantity	Cost
(16)	760	5,860
(4, 14)	857	6,442
(1, 9, 12)	992	7,252
(3, 6, 13)	993	7,258
(5, 10, 11)	991	7,246
(2, 7, 8, 15)	998	7,288
(17,18)	3,190	41,770

Optimal total societal cost: 83,116

Heuristic A

Initialization:

$P_1 = \{(17), (18)\}$

$P_2 = \{(9), (10), (11), (12), (13), (14), (15), (16)\}$

$P_3 = \{(1), (2), (3), (4), (5), (6), (7), (8)\}$

Phase 1:

$P_1 = \{(17), (18)\}$

Phase 1 Solution

Coalitions	Order quantity	Cost
(17,18)	3,190	41,770

Phase 1 total cost: 41,770

Phase 2:

$P_2 = \{(10), (11), (12), (13), (14), (15), (16)\}$

Phase 2 Solution

Coalitions	Order quantity	Cost

Heuristic B

Initialization:

$P_1 = \{(17), (18)\}$

$P_2 = \{(1), (2), (3), (4), (5), (6), (7), (8)\}$

$P_3 = \{(9), (10), (11), (12), (13), (14), (15), (16)\}$

Phase 1:

$P_1 = \{(17), (18)\}$

Phase 1 Solution

Coalitions	Order quantity	Cost
(17,18)	3,190	41,770

Phase 1 total cost: 41,770

Phase 2:

$P_2 = \{(1), (2), (3), (4), (5), (6), (7), (8)\}$

Phase 2 Solution

Coalitions	Order quantity	Cost

(13)	731	5,686
(14)	787	6,022
(15)	580	4,780
(16)	760	5,860
(9,12)	932	6,892
(10,11)	861	6,466

Phase 2 total cost: 35,706

Phase 3:

$P_3 = \{(1), (2), (3), (4), (5), (6), (7), (8), (9{,}12), (10{,}11), (13), (14), (15), (16)\}$

Phase 3 Solution

Coalitions	Order quantity	Cost
(9,12)	932	6,892
(10,11)	861	6,466
(8,14)	869	6,514
(2,4,13)	972	7,132
(6,7,15)	979	7,174
(1,3,5,16)	978	7,168

Phase 3 total cost: 41,346

Phase 4:

$P_4 = \emptyset$

$P_6 = \{(1{,}3{,}5{,}16), (2{,}4{,}13), (6{,}7{,}15), (8{,}14), (9{,}12), (10{,}11)\}$

(1,2,3,4,5,6,7,8)	940	6,940

Phase 2 total cost: 6,940

Phase 3:

$P_3 = \{(9), (10), (11), (12), (13), (14), (15), (16), (1{,}2{,}3{,}4{,}5{,}6{,}7{,}8)\}$

Phase 3 Solution

Coalitions	Order quantity	Cost
(13)	731	5,686
(14)	787	6,022
(15)	580	4,780
(16)	760	5,860
(9,12)	932	6,892
(10,11)	861	6,466
(1,2,3,4,5,6,7,8)	940	6,940

Phase 3 total cost: 42,646

Phase 4:

$P_4 = \{(1{,}2{,}3{,}4{,}5{,}6{,}7{,}8), (9{,}12), (10{,}11), (13), (14), (15), (16)\}$

Phase 4 Solution

Coalitions	Order quantity	Cost
(13)	731	5,686
(14)	787	6,022
(15)	580	4,780
(16)	760	5,860

Phase 5:

$P_5 = \emptyset$

$P_6 = \{(1,3,5,16), (2,4,13), (6,7,15), (8,14), (9,12), (10,11)\}$

Phase 6:

$P_5 = \emptyset$

$P_6 = \{(1,3,5,16), (2,4,13), (6,7,15), (8,14), (9,12), (10,11)\}$

$P_1 = \{(17,18)\}$

Phase 6 Solution

Coalitions	Order quantity	Cost
(9,12)	932	6,892
(10,11)	861	6,466
(8,14)	869	6,514
(2,4,13)	972	7,132
(6,7,15)	979	7,174
(1,3,5,16)	978	7,168
(17,18)	3190	41,770

Final cost: 83,116

(9,12)	932	6,892
(10,11)	861	6,466
(1,2,3,4,5,6,7,8)	940	6,940

Phase 4 total cost: 42,646

Phase 5:

$P_5 = \emptyset$

$P_6 = \{(1,2,3,4,5,6,7,8), (9,12), (10,11), (13), (14), (15), (16)\}$

Phase 6:

$P_5 = \emptyset$

$P_6 = \{(1,2,3,4,5,6,7,8), (9,12), (10,11), (13), (14)(15), (16)\}$

$P_1 = \{(17,18)\}$

Phase 6 Solution

Coalitions	Order quantity	Cost
(13)	731	3,190
(14)	787	5,686
(15)	580	6,022
(16)	760	4,780
(9,12)	932	6,892
(10,11)	861	6,466
(1,2,3,4,5,6,7,8)	940	6,940
(17,18)	3190	41,770

Final cost: 84,416

Table 3.2 Example to Illustrate Heuristics Steps and Comparison Between Optimal and Heuristic Solutions

This example highlights similarities and notable differences between the three solutions. For example, all three solutions result in a coalition between players 17 and 18. This coalition is a group 10 case, so a single coalition of all players whose order quantity is larger than z_2 provides the greatest savings of fixed costs. The major difference between the heuristic solutions and the optimal solution is that coalitions in the optimal solution often form between players with very different order quantities because the optimization model solves the entire problem simultaneously without limiting which players are collaborative partners. In contrast, both heuristics solve sub-problems in which players are exposed to fewer collaborative partners. Although the heuristics are designed to identify promising potential partners, they may miss optimal combinations and result in higher costs.

The two heuristic solutions also differ in how players with order quantities less than z_1 appear in coalitions. Heuristic B considers only these players in Phase 2, and the resulting a coalition comprises all such players with order quantities between z_1 and z_2. Consequently, in Heuristic B, no remaining players or coalitions with order quantities less than z_1 reach Phase 4, so the associated cost savings opportunity is missed. In contrast, Heuristic A considers players with order quantities less than z_1 and players with order quantities between z_1 and z_2 in Phase 3. Coalitions formed between these players result in increased cost savings, leading to a lower total cost for the Heuristic A solution. Therefore, greater total societal cost savings are achieved when players with smaller order quantities collaborate with players with larger order quantities than when players with small order quantities collaborate exclusively with other players with small order quantities.

3.3 Computational Study and Results

This section presents a computational study for the *OCS* model and proposed heuristics. The optimal and heuristic solutions are compared based on running time and total societal cost. Additionally, the impact of player characteristics on running time and total cost is examined.

3.3.1 Computational Study Design

All the models and heuristics for the computational study were coded in Python 3.7 language. The optimization problems were solved using CPLEX 12.10.0.0 through the CPLEX Python API. This computational study utilized 100 randomly generated instances that differed in the total number of players in the instance, as well as in the fraction of players with order quantities less than z_1 (small players) and equal to or greater than z_1 and less than z_2 (medium players). Since players with order quantities greater than z_2 belong to subadditive games and form a single coalition, the computational study did not focus on those players. Each instance belonged to a set with a specified total number of players and distribution of player sizes. For example, an instance in set 16(8,8) had 16 total players (8 small and 8 medium). Each set had a total of 10 instances. Table 3.33 represents the number of players and order quantity for each instance.

# Instances	# Players	# Small players	# Medium players
10	16	8	8
10	18	9	9
10	16	4	12
10	18	4	18
10	16	12	4
10	18	14	4
10	16	2	14
10	18	2	16
10	16	14	2
10	18	16	2

Table 3.3 Player Distribution for Computational Studies

3.3.2 Computational Study Results for Heuristics and *OCS* Model

During the computational study, first, the total societal cost of solutions produced by Heuristics A and B were compared to the minimum total societal cost found by the optimization model, and then differences between the optimal cost and the heuristic cost were calculated for each instance. These differences were divided by the optimal cost to find the percent cost increase for each instance. Finally, the average percent cost increase for the 10 instances was computed in each set. The results are plotted for 16-players and 18-players instances, respectfully, in Figure 3.3 Hand Figure 3.4.

Similarly, the standard deviation of the percentage increase in cost was computed for each set of 10 instances, including computing the differences between cost generated by the heuristic and the optimal cost for all instances within a set. The resulting values were divided by the optimal cost, and the standard deviation was determined for that set of instances. The results are plotted for 16-players and 18-players instances, respectfully, in in Figure 3.5 igure 3.5 and Figure 3.6 where in each graph, the horizontal axis indicates the instance sets, and, moving from left to right on the graphs, the number of small players increases while the number of medium players decreases.

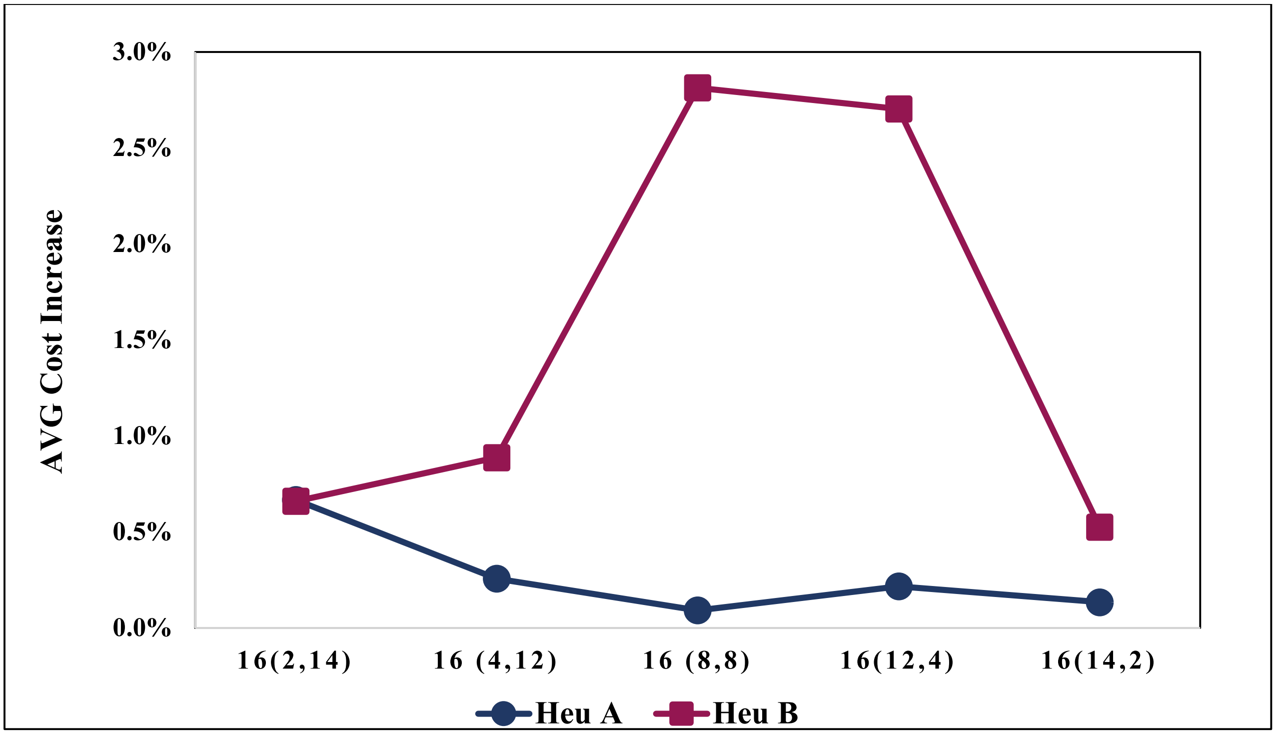

Figure 3.3 Heuristics Cost Increase versus Optimal Solution for 16-Player Instances

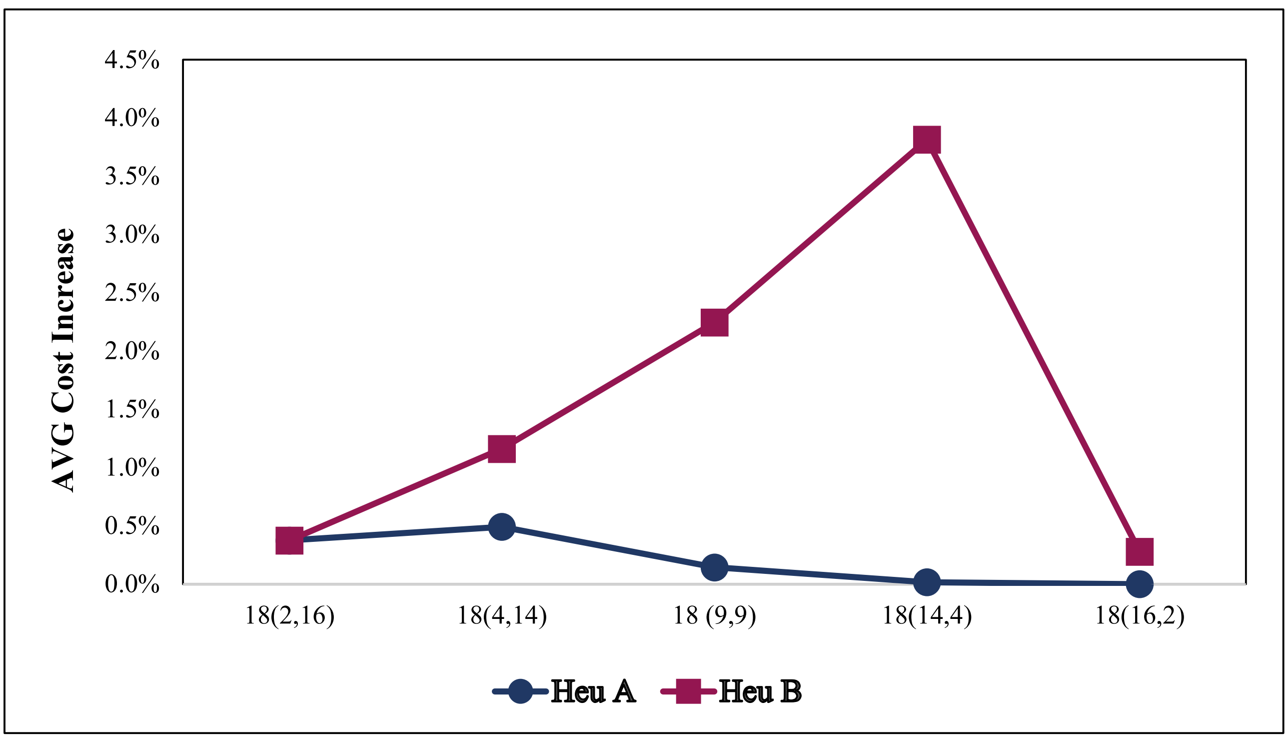

Figure 3.4 Heuristics Cost Increase versus Optimal Solution for 18-Player Instances

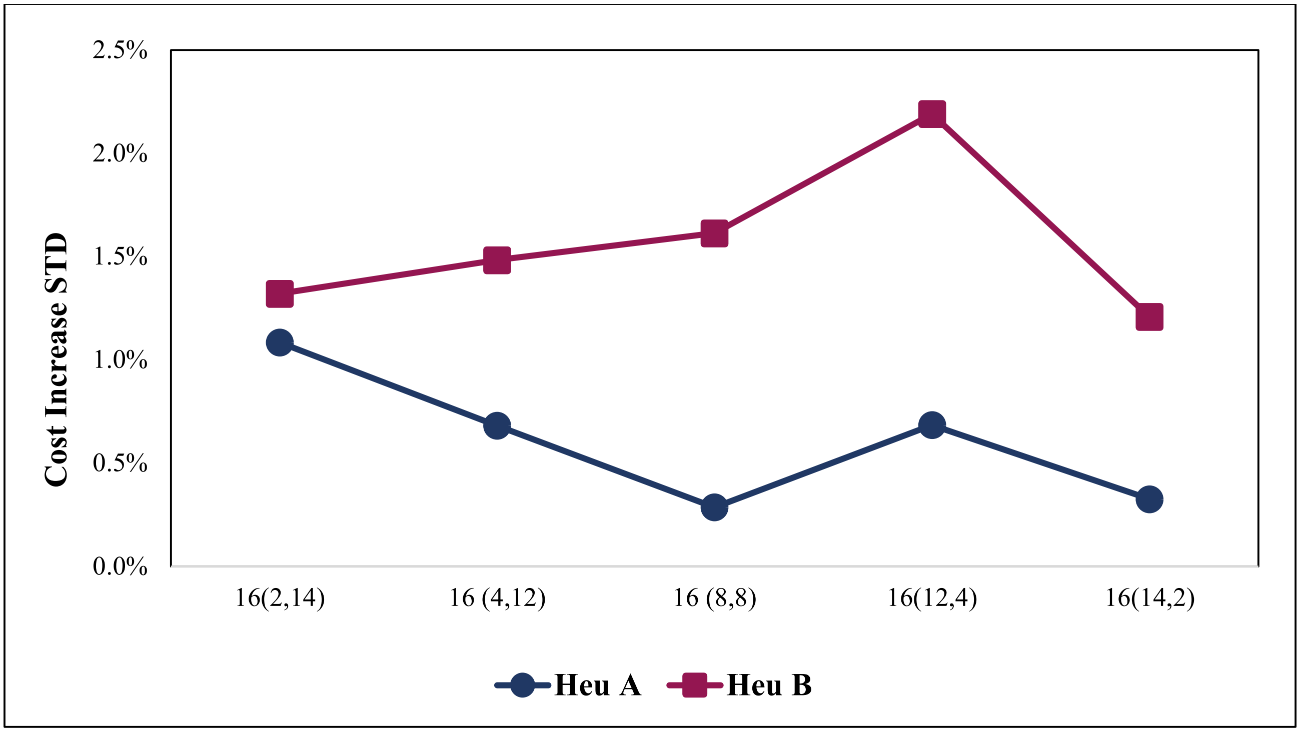

Figure 3.5 Heuristics Cost Increase Standard Deviation for 16-Player Instances

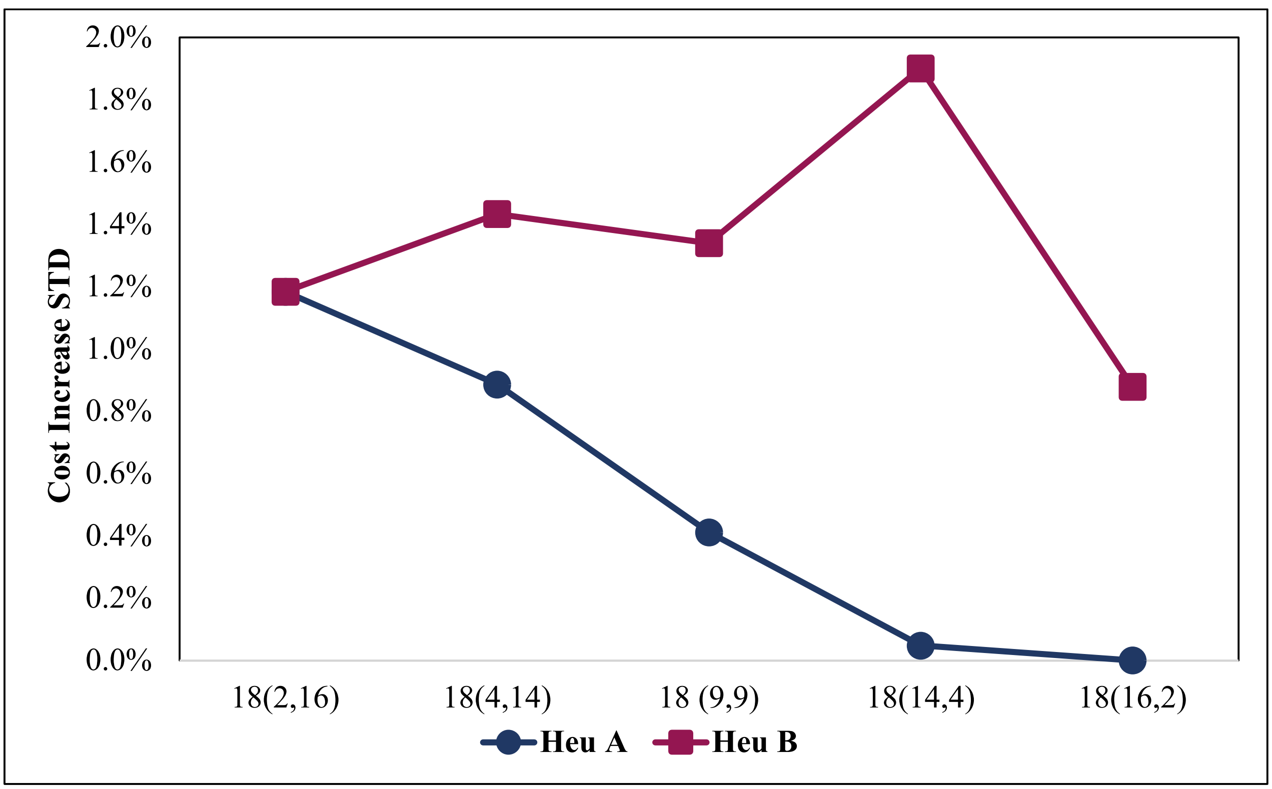

Figure 3.6 Heuristics Cost Increase Standard Deviation for 18-Player Instances

As shown in Figure 3.3 Hand Figure 3.4, on average, both heuristics find solutions with total societal cost within 5% of the optimal cost for each instance set. On average, Heuristic A finds solutions with total societal cost the same or less than costs found by Heuristic B for all instance sets. The average cost difference between Heuristic A and Heuristic B solutions is small when the number of small players is much less than or much greater than the number of medium players (i.e., sets 16(2,14), 18(2,16), 16(14,2), and 18(16,2)). In the remaining instance sets, the average cost difference between the two heuristics is larger when the number of small players is the same as or greater than the number of medium players in an instance set. Heuristic A also exhibits less variability in cost savings than Heuristic B for all instance sets as depicted in Figure 3.5 and Figure 3.6.

The solutions produced by the optimization model and heuristics differ in their composition of coalitions. For example, the number of coalitions in an optimal solution is less than or equal to the number in a heuristic solution for all instances and for both heuristics. Optimal coalition structures also include coalitions with more diverse player order quantity than coalitions in the heuristic solutions. Small players in Heuristic B often collaborate with other small players, and fewer coalitions involve both small and medium players than in the optimal or Heuristic A solutions. In all instances, the Heuristic B solution has a cost greater than or equal to the cost of Heuristic A. Therefore, the computational study results reveal increased societal benefit when coalitions are formed by players with diverse order sizes.

Table 3.4 compares the computation times for the optimal model and each heuristic and shows the difference between the time required to run the optimization model and the heuristic for each instance set. This difference was then divided by the time required to run the optimization model to find the percentage of time savings for each instance. The average percentage of time savings for the 10 instances in each set and the standard deviation of time savings for each set are shown in the table.

	AVG Time (Min)	AVG Time Diff.		STD Time Diff.	
Sets	Optimal solution	Heuristic A	Heuristic B	Heuristic A	Heuristic B
16(2,14)	2.35	-89.7%	-87.0%	12.1%	11.3%
16(4,12)	1.76	-90.7%	-97.6%	11.5%	0.7%
16(8,8)	1.76	-59.7%	-99.7%	56.8%	0.1%
16(12,4)	1.88	-53.2%	-98.1%	50.9%	1.6%
16(14,2)	17.54	-9.0%	-68.6%	84.7%	37.7%
18(2,16)	33.80	-91.1%	-84.2%	5.0%	9.0%
18(4,14)	34.22	-96.8%	-98.8%	4.2%	0.3%
18 (9,9)	34.20	-34.4%	-99.9%	88.4%	0.0%
18(14,4)	59.63	-19.5%	-99.1%	51.9%	1.1%
18(16,2)	73.53	-15.2%	-70.9%	86.3%	40.1%

Table 3.4 Running Time Comparisons between Optimization Model and Heuristics

The results showed that Heuristic A and Heuristic B offer measurable computational savings compared to the time required to find optimal solutions for the instances tested. Heuristic B was generally faster than Heuristic A with less variability in computation times across instances. The exceptions were instance sets 16(2,14) and 18(2,16), in which the average computation times for Heuristic A were slightly better than the times for Heuristic B. On average, computation times for the optimization model were longer for instance sets with 18 players than for sets with 16 players, and the computation times tended to increase as the number of small players increased. The time savings for Heuristic A, compared to the optimization model, decreased markedly as the number of small players increased. However, Heuristic B continued to provide substantial time savings (>97%) for most instance sets, with declining speed observed for instance sets 16(2,14) and 18(2,16), with time savings of 84%–87%, and for 16(14,2) and 18(16,2), with time savings of 68%–70% relative to the optimization model.

3.4 Conclusion

The primary research objective of this chapter was to help humanitarian organizations determining which players should collaborate to minimize total societal cost. In humanitarian contexts, this understanding could efficiently and effectively benefit supply chain systems, thereby increasing organizational response. The study built on existing literature by introducing a coalition structure optimization model and two heuristics for problems with a general non-subadditive cost function. The heuristics divided players into groups based on potential cost savings and utilized observations about cost-saving opportunities, while the computational study demonstrated the potential for the heuristics to quickly generate near-optimal solutions for the coalition structure problem. Evaluative computational analyses revealed promising results for both heuristics. Notably, although one of the heuristics produced slightly lower costs, the other heuristic consistently exhibited greater speed.

This multifaceted approach provides practical methods for optimizing collaboration in settings with non-subadditive costs. The work lays the foundation for answering other questions related to coalition structures in non-subadditive games. Further investigation is needed regarding cost allocation among participants to ensure coalition stability, and this aspect is the focus of Chapter 4.

Chapter 4 - Cost Allocation and Stability

In the preceding chapter, the focus was on finding the coalition structure with minimum societal cost. Insight about these optimal collaborations aids in improving supply chain operations, yet one remaining key question is how to share the costs incurred by the collaborative partners. To maintain collaboration, players should agree on a method for dividing costs among themselves, and the main concern is finding an allocation method that supports the stability of the collaboration. There is a growing body of literature in operations research that has applied principles of cooperative game theory to solve this sharing question in subadditive games. However, the question of cost sharing in non-subadditive games has received considerably less attention, despite its relevance and potential impact.

The core in subadditive games and the coalition structure core in non-subadditive games, respectively, are payoff divisions that have been most appealing and accepted as stable allocations. The core represents a set of outcomes where no subgroup of players can improve their situation by breaking away from the grand coalition to form a new coalition. The coalition structure core is an expansion on the core concept wherein the desired outcome may consist of multiple coalitions in contrast to the single grand coalition. Elomri et al. (2012) define coalition structure core as many disjoint budget balanced coalitions (with no side-payment between coalitions) where no group of players can be better off by deviating and forming another coalition (when constraints 2.3 and 2.4 are satisfied simultaneously).

The primary objective of this chapter is to establish an allocation framework that ensures the stability of collaboration while minimizing the total social cost. The methods introduced in Chapter 3 identify the coalition structure with minimum societal cost. However, it is not known whether there exist allocations that ensure that an optimal coalition structure is stable. Thus, the first question to be addressed in this chapter is how to identify coalition structures for which there exists a stable allocation while minimizing the societal cost among such structures. Using this framework, several additional research questions are investigated: How often do feasible (stable) allocations exist? When they do, what is the societal cost associated with stable coalition structures, and how does this compare to the minimum societal cost among all coalition structures? When stable allocations do not exist, can nearly-stable allocations be found? How well do common allocation rules perform?

This chapter introduces a framework aimed at providing coalition structure core allocations. To address these questions, the author employs not only an optimization model but also analytical proofs to develop algorithms that tackle the inherent complexity and feasibility challenges associated with this problem. The primary contributions of this chapter are as follows:

- Analytical proofs to uncover the characteristics of coalition structure core and optimal coalition structure problems. Some of these proofs serve as the foundation for a proposed algorithm, addressing feasibility and complexity challenges, while others offer insights for future research.

- A relaxation method to address the feasibility issue inherent in the cost allocation problem.

- An algorithm to navigate the feasibility and complexity of the coalition structure core problem.
- Computational studies to analyze the behavior of the optimization model and algorithm.

This chapter first describes the coalition structure core optimization model. To navigate the model's computational complexity and to gain a deeper understanding of the coalition structure core and optimal coalition structure problems, the author conducts analytical analyses. These analyses combined with the employment of the relaxation method serve as the foundation for this chapter's subsequent sections, in which an algorithm is introduced to address the feasibility and complexity of the coalition structure core problem. Finally, the author assesses the optimization model and proposed algorithm using the same instances employed in Chapter 3 for testing the *OCS* model and heuristics. The insights derived from this evaluation hold potential applications in real-world problem-solving.

4.1 Coalition Structure Core (*CSC*) Model

Coalition structure core in non-subadditive games is a distribution of costs that is widely recognized for its stability. It constitutes a set of outcomes where no subset of players can enhance their position by creating a new coalition. According to this definition, costs are allocated to players in a manner that ensures budget balance and strong stability within each coalition, preventing any group of players, whether within the same or different coalitions, from having an incentive to leave the structure (Aumann & Dreze, 1974).

The following mixed integer programming model finds an allocation in the coalition structure core, if at least one exists, with minimum total social cost. The following notation is used for sets, parameters, and decision variables.

Sets

N: set of players

Ω: set of coalitions

Parameters

c_j: cost of coalition S_j

$$\alpha_{n,j} = \begin{cases} 1 & \text{if player } n \text{ belongs to coalition } S_j \\ 0 & \text{otherwise} \end{cases}$$

Decision variables

$$\beta_j = \begin{cases} 1 & \text{if coalition } S_j \text{ is formed} \\ 0 & \text{otherwise} \end{cases}$$

$x_{n,j}$: cost allocated to player n in coalition S_j

***CSC* model:**

$$Minimize \sum_{S_j \in \Omega} c_j \, \beta_j \tag{4.1}$$

subject to:

$$\sum_{S_j \in \Omega} \alpha_{n,j}\, \beta_j = 1 \qquad \forall\, n \in N \qquad 4.2$$

$$\sum_{n \in N} (\alpha_{n,j} \sum_{S_{\bar{j}} \in \Omega} x_{n,\bar{j}}) \le c_j \qquad \forall\, S_j \in \Omega \qquad 4.3$$

$$\sum_{n \in N} \alpha_{n,j}\, x_{n,j} = \quad c_j \beta_j \qquad \forall\, S_j \in \Omega \qquad 4.4$$

$$\beta_j \in \{0,1\} \qquad \forall\, S_j \in \Omega \qquad 4.5$$

$$x_{n,j} \ge 0 \qquad \forall\, n \in N,\ \forall\, S_j \in \Omega \qquad 4.6$$

The objective function 4.1 finds a coalition structure that minimizes the total cost over all established coalitions. Constraint 4.2 indicates that each player must be assigned to one and only one coalition in the chosen coalition structure. This constraint is derived from the definition of coalition structure and ensures that the outcome solution is a partition on players in N. Constraint 4.3 indicates the strong stability condition and ensures that no player, either from the same or different coalitions in the structure, has an incentive to leave the structure. In other words, there is no set of players such that they can form a separate coalition where the cost of the newly created coalition is less than the sum of the allocated cost to those players in the initial coalitions. The left-hand side (LHS) is the sum of costs that are proposed to be allocated to players who are members of coalition S_j. The right-hand side (RHS) is the total cost of coalition S_j. If the LHS sum is less than or equal to the total cost of coalition S_j (RHS), then the players have no incentive to break away from the proposed coalition structure to form coalition S_j. Constraint 4.4 indicates the budget balanced condition and shows that the sum of costs allocated to all players in a formed coalition equals the total cost of such coalition. It also guarantees that the cost allocated to players in coalitions that are not formed is zero. Constraint 4.5 indicates that β_j is a binary variable and Constraint 4.6 ensures that $x_{n,j}$ is a non-negative value. The above coalition structure core optimization model was first introduced by Guajardo & Rönnqvist (2015).

In the *CSC* model, each possible vector β_j defines a unique coalition structure. For ease of exposition in the remainder of the chapter, x^k denotes the vector comprised only of non-zero $x_{n,j}$ values for a particular solution k; note that there is exactly one non-zero allocation for each player $n \in N$ in any solution. Thus, a solution to the *CSC* model includes the selected coalition structure and the associated allocations; it is denoted by (φ_k, x^k) and its cost is $c(\varphi_k)$.

In initial investigations with instances from the *OCS* computational study, described in Chapter 3, the proposed *CSC* model encountered complexity and feasibility challenges. This fact placed significant limitations on the ability to compare solutions to the *CSC* and *OCS* models on instances with even moderate size. In turn, this served as motivation to delve deeper into the nature of the *CSC* model and its connection to the *OCS* model. Doing so also led to the development of an algorithm to overcome the computational challenges.

The subsequent sections of this chapter present an extensive exploration into the *CSC* and *OCS* problems, highlighting the discoveries and insights gained. These findings serve as a foundation for simplifying the problem of the *CSC* model, offering a promising way forward.

4.2 Analytical Characteristics of the Coalition Structure Core and Optimal Coalition Structure Problems

This section presents insights regarding the nature of *OCS* and *CSC* models. These insights are divided into two main categories: fundamental characteristics of *OCS* and *CSC* models, and characteristics that support *CSC* problem computational approaches.

4.2.1 Fundamental Characteristics of *OCS* and *CSC*

The investigation in this phase has revealed crucial insights into the nature of *OCS* and *CSC* models. In the following theorem the author shows that any feasible solution within the *CSC* model is inherently optimal. In other words, any solution that adheres to the constraints of the *CSC* model is regarded as an optimal outcome.

Theorem 1 Consider a feasible instance of the *CSC* problem consisting of the player set $N = \{1, \dots, |N|\}$, the set of all nonempty coalitions on N , $\Omega = \{S_1, \dots, S_{2^{|N|}-1}\}$, and feasible solution (φ_f, x^f) where x_n^f represents the cost allocated to player n in the feasible solution. Then $c(\varphi_f) \leq c(\varphi_k)$ for all (φ_k, x^k) in the set of feasible solutions to the *CSC* instance; that is, (φ_f, x^f) is an optimal solution.

Proof of Theorem 1 Let (φ_o, x^o) be an optimal solution to the *CSC* instance. By definition,

$$\sum_{S_i \in \varphi_f} c_i = c(\varphi_f) \geq c(\varphi_o) = \sum_{S_j \in \varphi_o} c_j \,. \qquad 4.7$$

This proof will demonstrate that the expression is met at equality.

According to the budget balanced condition specified in the *CSC* model, Equation 4.4, the constraints 4.8 and 4.9 are established respectively, for feasible and optimal coalition structures.

Feasible coalition structure:

$$\sum_{n \in S_i} x_n^f = c_i \qquad \forall\, S_i \in \varphi_f \qquad 4.8$$

Optimal coalition structure:

$$\sum_{n \in S_j} x_n^o = c_j \qquad \forall\, S_j \in \varphi_o \qquad 4.9$$

According to the strong stability condition 4.3, the following constraints for both feasible and optimal coalition structures are established:

$$\sum_{n \in S_i} x_n^f \leq c_i \qquad \forall\, S_i \in \varphi_f \qquad 4.10$$

$$\sum_{n \in S_j} x_n^o \leq c_j \qquad \forall\, S_j \in \varphi_o. \qquad 4.11$$

Summing equation 4.8 over all coalitions in the feasible solution yields:

$$\sum_{S_i \in \varphi_f} \sum_{n \in S_i} x_n^f = \sum_{n \in N} x_n^f = \sum_{S_i \in \varphi_f} c_i = c(\varphi_f). \qquad 4.12$$

Similarly, summing equation 4.9 over all coalitions in the optimal solution yields:

$$\sum_{S_j \in \varphi_o} \sum_{n \in S_j} x_n^o = \sum_{n \in N} x_n^o = \sum_{S_j \in \varphi_o} c_j = c(\varphi_o). \qquad 4.13$$

Since the cost associated with a feasible solution must be equal to or greater than the optimal cost, combined from 4.12 and 4.13, it is concluded that:

$$\sum_{n \in N} x_n^f \geq c(\varphi_o). \qquad 4.14$$

Summing constraints 4.10 for those $S_j \in \varphi_o$ yields:

$$\sum_{S_j \in \varphi_o} \sum_{n \in S_j} x_n^f \leq \sum_{S_j \in \varphi_o} c_j \, , \qquad 4.15$$

which is equivalent to:

$$\sum_{n \in N} x_n^f \leq c(\varphi_o). \qquad 4.16$$

Combining equations 4.14 and 4.16 results in:

$$\sum_{n \in N} x_n^f = c(\varphi_o). \qquad 4.17$$

Combining equations 4.12 and 4.17 results in:

$$c(\varphi_o) = c(\varphi_f). \tag{4.18}$$

Expression 4.18 indicates that the feasible coalition structure cost is equal to the optimal coalition structure cost. Thus, a *CSC* feasible solution is also optimal. ☐

Theorem 1 demonstrates that any feasible solution to the *CSC* model is also optimal. However, Theorem 2 reveals that an instance of the *CSC* model may have no feasible solution.

Theorem 2 There exist *CSC* instances that are infeasible.

Proof of Theorem 2 Consider the following instance with $N = \{1, 2, 3\}$ and the associated coalition costs:

Coalition (S_i)	Coalition cost (c_i)
(1)	1,400
(2)	2,510
(3)	1,080
(1,2)	3,110
(1,3)	1,680
(2,3)	2,790
(1,2,3)	7,854

Table 4.1 Coalitions and Coalition Costs for the Instance in the Proof of Theorem 2

Table 4.2 depicts the possible coalition structures. The proof will demonstrate that no coalition structure admits a feasible cost allocation.

ID	Coalition structure (φ_j)
φ_1	{(1,2,3)}
φ_2	{(1), (2), (3)}
φ_3	{(1,3), (2)}
φ_4	{(1,2), (3)}
φ_5	{(2,3), (1)}

Table 4.2 Possible Coalition Structures for the Instance in the Proof of Theorem 2

In the following it is shown how each coalition structure is not stable.

Coalition structure $\boldsymbol{\varphi_1} = \{(\mathbf{1,2,3})\}$

The strong stability constraints require:

$$x_1 \leq 1{,}400 \qquad 4.19$$

$$x_2 \leq 2{,}510 \qquad 4.20$$

$$x_3 \leq 1{,}080 \qquad 4.21$$

No feasible solution to these constraints can satisfy the budget balanced constraint since:

$$1{,}400 + 2{,}510 + 1{,}080 < 7{,}854 \qquad 4.22$$

Coalition structure $\boldsymbol{\varphi_2} = \{(\mathbf{1}), (\mathbf{2}), (\mathbf{3})\}$

The budget balanced constraints require:

$$x_1 = 1{,}400 \qquad 4.23$$

$$x_2 = 2{,}510 \qquad 4.24$$

$$x_3 = 1{,}080 \qquad 4.25$$

However, this allocation violates the strong stability conditions since:

$x_1 + x_2 > 3{,}110$ 4.26

Coalition structure $\varphi_3 = \{(1, 3), (2)\}$

The budget balanced constraints require:

$x_1 + x_3 = 1{,}680$ 4.27

$x_2 = 2{,}510$ 4.28

The strong stability constraints require:

$x_1 + x_2 \leq 3{,}110$ 4.29

$x_3 \leq 1{,}080$ 4.30

$x_2 + x_3 \leq 2{,}790$ 4.31

Combining 4.28 and 4.29 results in:

$x_1 \leq 600$ 4.32

Combining 4.27, 4.30 and 4.32, results in:

$x_1 = 600$ 4.33

$x_3 = 1{,}080$ 4.34

Combining 4.28 and 4.34 results in:

$x_2 + x_3 > 2{,}790$ 4.35

Equation 4.35 violates strong stability constraint.

Coalition structure $\varphi_4 = \{(1, 2), (3)\}$

The budget balanced constraints require:

$$x_1 + x_2 = 3{,}110 \tag{4.36}$$

$$x_3 = 1{,}080 \tag{4.37}$$

And strong stability constraints require:

$$x_1 + x_3 \leq 1{,}680 \tag{4.38}$$

$$x_2 \leq 2{,}510 \tag{4.39}$$

$$x_2 + x_3 \leq 2{,}790 \tag{4.40}$$

Combining 4.37 and 4.38 results in:

$$x_1 \leq 600 \tag{4.41}$$

Combining 4.36 and 4.41 results in:

$$x_2 \geq 2{,}510 \tag{4.42}$$

Combining 4.39 and 4.42 results in:

$$x_2 = 2{,}510 \tag{4.43}$$

Combining 4.37 and 4.43 results in:

$$x_2 + x_3 = 3{,}590 \tag{4.44}$$

Equation 4.44 violates strong stability condition.

Coalition structure $\varphi_5 = \{(\mathbf{2}, \mathbf{3}), (\mathbf{1})\}$

The budget balanced constraints require:

$$x_2 + x_3 = 2{,}790 \quad 4.45$$

$$x_1 = 1{,}400 \quad 4.46$$

The strong stability constraints require:

$$x_2 \leq 2{,}510 \quad 4.47$$

$$x_3 \leq 1{,}080 \quad 4.48$$

$$x_1 + x_2 \leq 3{,}110 \quad 4.49$$

$$x_1 + x_3 \leq 1{,}680 \quad 4.50$$

Combining equations 4.45, 4.47, and 4.48 results in:

$$280 \leq x_3 \leq 1{,}080 \quad 4.51$$

$$1{,}710 \leq x_2 \leq 2{,}510 \quad 4.52$$

Substituting x_1= 1,400, combined with equation 4.50 results in:

$$x_2 \leq 1{,}710 \quad 4.53$$

Combining 4.52 and 4.53 results in:

$$x_2 = 1{,}710 \quad 4.54$$

Combining 4.45 and 4.54 results in:

$$x_3 = 1{,}080 \quad 4.55$$

Combining 4.46 and 4.55 results in:

$$x_1 + x_3 = 2{,}480 \quad 4.56$$

Equation 4.56 violates the 4.50 strong stability condition.

Thus, the above example shows that there exist *CSC* instances that are infeasible. ☐

While the first two analytical results contribute to a more comprehensive understanding of the *CSC* model, Theorem 3 demonstrates the potential existence of multiple optimal solutions to both the *CSC* and *OCS* models.

Theorem 3 There exist *OCS* and *CSC* instances with multiple optimal coalition structures.

Proof of Theorem 3 Consider the instance with players $N = \{1, 2, 3\}$ and the following parameters:

Coalition (S_i)	Coalition cost (c_i)
(1)	4
(2)	6
(3)	8
(1,2)	10
(1,3)	13
(2,3)	14
(1,2,3)	18

Table 4.3 Coalitions and Coalition Costs for the Instance in the Proof of Theorem 3

The coalition structures and their costs are:

ID	Coalition structure(φ_j)	Cost of coalition structure ($c(\varphi_j)$)
φ_1	{(1), (2), (3)}	18
φ_2	{(1,2), (3)}	18
φ_3	{(1,3), (2)}	19
φ_4	{(2,3), (1)}	18
φ_5	{(1,2,3)}	18

Table 4.4 Possible Coalition Structures and Their Costs for the Instance in the Proof of Theorem 3

In this instance, all the coalition structures with $c(\varphi) = 18$ are optimal solutions to the *OCS* model. Similarly, for the *CSC* model, coalition structures $\varphi_1, \varphi_2, \varphi_4, \varphi_5$ with allocation values $x_{1,1} = x_{1,2} = x_{1,4} = x_{1,5} = 4$; $x_{2,1} = x_{2,2} = x_{2,4} = x_{2,5} = 6$; and $x_{3,1} = x_{3,2} = x_{3,4} = x_{3,5} = 8$ are feasible and optimal. ☐

4.2.2 Relationships that Enable More Efficient Computation of Stable Allocations

The in-depth analysis of fundamental characteristics of *OCS* and *CSC* models contributes to a more comprehensive understanding of these models and sets the stage for further research and problem-solving strategies in cooperative game theory. The proofs that are covered in the current section provide valuable clarity and direction, ultimately leading to an approach to address the complexity challenge that previously hindered progress.

As mentioned earlier, an initial goal of this dissertation was to identify the gap between a societal optimal solution and a stable solution to evaluate the societal cost incurred due to stability considerations. Theorem 4 establishes that an optimal coalition structure in the *CSC* model is also an optimal solution to the *OCS* model for the same instance. This insight indicates that when a feasible coalition structure core allocation exists, there is no gap between a societal optimal solution and the best stable solution. Furthermore, this understanding serves as the foundation for the algorithm developed by the author to address the complexity challenge of the *CSC* model, which will be introduced later in this chapter.

Theorem 4 If there exists an optimal solution φ^*_{CSC} to the *CSC* model for a given instance, then the corresponding coalition structure yields an optimal solution φ^*_{OCS} to the *OCS* model for that instance.

Proof of Theorem 4 Let φ^*_{OCS} and φ^*_{CSC} denote optimal coalition structures in the *OCS* and *CSC* models, respectively, for the same instance, and let x^o_n denote the cost allocated to player n in φ^*_{CSC}. *CSC* and *OCS* models share identical objective functions, with *CSC* having constraints 4.3, 4.4, and 4.6 in addition to those common to both models. Consequently, every feasible solution in the *CSC* model is also feasible for the *OCS* model, and the following equation holds true:

$$c(\varphi^*_{OCS}) = \sum_{S_j \in \varphi^*_{OCS}} c_j \leq \sum_{S_k \in \varphi^*_{CSC}} c_k = c(\varphi^*_{CSC}). \tag{4.57}$$

Since every coalition structure is a partition on N, the following equations hold true.

$$\sum_{n \in S_j} \sum_{S_j \in \varphi^*_{OCS}} x^o_n = \sum_{n \in N} x^o_n \tag{4.58}$$

$$\sum_{n \in S_k} \sum_{S_k \in \varphi^*_{CSC}} x^o_n = \sum_{n \in N} x^o_n \tag{4.59}$$

Although strong stability belongs to the *CSC* model, summing the allocations for any formed coalition in the *OCS* solution must still satisfy this constraint (4.3), thus:

$$\sum_{n \in S_j} x_n^o \leq c_j \qquad \forall \; S_j \in \varphi_{OCS}^*. \qquad 4.60$$

Summing 4.60 over coalition coalitions S_j in φ_{OCS}^* results in:

$$\sum_{S_j \in \varphi_{OCS}^*} \sum_{n \in S_j} x_n^o \leq \sum_{S_j \in \varphi_{OCS}^*} c_j. \qquad 4.61$$

From the budget balanced condition (4.4):

$$\sum\nolimits_{n \in S_k} x_n^o = c_k \qquad \forall \; S_k \in \varphi_{CSC}^*. \qquad 4.62$$

Summing 4.62 over coalition coalitions S_k in φ_{CSC}^* results in:

$$\sum_{S_k \in \varphi_{CSC}^*} \sum_{n \in S_k} x_n^o = \sum_{S_k \in \varphi_{CSC}^*} c_k \,. \qquad 4.63$$

The coalition structure cost is equal to the sum of the costs of formed coalitions in each solution.

$$\sum_{S_j \in \varphi_{OCS}^*} c_j = \mathrm{c}(\varphi_{OCS}^*) \qquad 4.64$$

$$\sum_{S_k \in \varphi_{CSC}^*} c_k = \mathrm{c}(\varphi_{CSC}^*) \qquad 4.65$$

Combining 4.59, 4.61, and 4.64 results in:

$$\sum_{S_j \in \varphi^*_{OCS}} \sum_{n \in S_j} x^o_n = \sum_{n \in N} x^o_n \leq \sum_{S_j \in \varphi^*_{OCS}} c_j = \mathrm{c}(\varphi^*_{OCS}). \tag{4.66}$$

Combining 4.59, 4.63, and 4.65 results in:

$$\sum_{S_k \in \varphi^*_{OCS}} \sum_{n \in S_k} x^o_n = \sum_{n \in N} x^o_n = \sum_{S_j \in \varphi^*_{CSC}} c_k = \mathrm{c}(\varphi^*_{CSC}). \tag{4.67}$$

Combing 4.66 and 4.67 results in:

$$\mathrm{c}(\varphi^*_{CSC}) \leq \mathrm{c}(\varphi^*_{OCS}). \tag{4.68}$$

Combining 4.57 and 4.68 results in:

$$\mathrm{c}(\varphi^*_{CSC}) = \mathrm{c}(\varphi^*_{OCS}). \tag{4.69}$$

Thus, if there exists an optimal solution to the *CSC* model for a given instance, then the corresponding coalition structure yields an optimal solution to the *OCS* model for that instance.

□

As shown in Theorem 3, the *OCS* model may have multiple optimal solutions. An interesting question is whether or not, for a given instance, there may exist allocations in the coalition structure core corresponding to some of the optimal *OCS* solutions and not others. The following theorem rules out the possibility that two distinct optimal solutions to the *OCS* model may have different feasibility statuses in the *CSC* model.

Theorem 5 Consider an instance with players $N = \{1, \ldots, |N|\}$, the set of all non-empty coalitions $\Omega = \{S_1, \ldots, S_{2^{|N|}-1}\}$, and the set of coalition costs $C = \{c_1, \ldots, c_{2^{|N|}-1}\}$ for which at least one feasible *CSC* solution exists and for which two distinct optimal solutions, $\varphi^*_{OCS\_a}$ and $\varphi^*_{OCS\_b}$, to the *OCS* model exist. Given $(\varphi^*_{OCS\_a}, x^a)$, a solution to the *CSC* model where $x^a = \{x^a_1, \ldots, x^a_{|N|}\}$ is a set of players' cost allocations for which $\varphi^*_{OCS\_a}$ is stable, then $(\varphi^*_{OCS\_b}, x^a)$ is also a solution to *CSC* model. Furthermore, if there does not exist a stable allocation for $\varphi^*_{OCS\_a}$, then there also does not exist a stable allocation for $\varphi^*_{OCS\_b}$.

Proof of Theorem 5 First consider the case when $(\varphi^*_{OCS\_a}, x^a)$ is a feasible solution to the *CSC* model. To prove that $(\varphi^*_{OCS\_b}, x^a)$ also is feasible for the *CSC* model, it is necessary to show that the allocation meets both the budget balanced and strong stability conditions for all coalitions in $\varphi^*_{OCS\_b}$.

To examine the budget balanced condition, assume there exists $S_i \in \varphi^*_{OCS\_b}$ such that

$$\sum_{l \in S_i} x^a_l > c_i. \qquad 4.70$$

Since $S_i \in \Omega$, if this inequality is true, then x^a was not a strongly stable allocation for $\varphi^*_{OCS\_a}$.

Now, assume, there exists $S_i \in \varphi^*_{OCS\_b}$ such that

$$\sum_{l \in S_i} x^a_l < c_i. \qquad 4.71$$

Since $\varphi^*_{OCS\_a}$ and $\varphi^*_{OCS\_b}$ have the same total cost, there exists $S_j \in \varphi^*_{OCS\_b}$ such that

$$\sum_{l \in S_j} x_l^a > c_j. \tag{4.72}$$

This means that the strong stability condition is violated for coalition S_j and this conflicts with the original assumption. Thus

$$\sum_{l \in S_i} x_l^a = c_i \qquad \forall\, S_i \in \varphi^*_{OCS_b}, \tag{4.73}$$

and the allocations x^a satisfy the budget balanced condition for all coalitions in $\varphi^*_{OCS\_b}$.

Turning to the strong stability conditions, since $(\varphi^*_{OCS\_a}, x^a)$ is strongly stable, the below equation holds true:

$$\sum_{l \in S_i} x_l^a \leq c_i \qquad \forall\, S_i \in \Omega. \tag{4.74}$$

While the strong stability condition is satisfied in $\varphi^*_{OCS\_a}$, this condition is likewise satisfied in $\varphi^*_{OCS\_b}$ since both $\varphi^*_{OCS\_a}$ and $\varphi^*_{OCS\_b}$ share the same C and Ω. Therefore, if there exist multiple optimal solutions in the *OCS* model for a given instance, then a stable allocation in one coalition structure is also stable in other optimal solutions.

Second, consider the case in which there does not exist a coalition structure core allocation for $\varphi^*_{OCS\_a}$. Then there also does not exist a coalition structure core allocation for $\varphi^*_{OCS\_b}$. If there were a feasible solution $(\varphi^*_{OCS\_b}, x^b)$ to the *CSC* model, then $(\varphi^*_{OCS\_a}, x^b)$ likewise would be feasible as was shown in the first case in the proof. Having considered both cases, it is shown that in the instances where multiple optimal coalition structures exist in the *OCS* model, these solutions exhibit consistent stability statuses within *CSC* model. ☐

The next example shows that when the coalition structure core is non-empty, multiple stable allocations may exist.

Example:

Consider the a game with 3 players and the coalitions and associated costs shown in Table 4.5. The minimum total social cost belongs to coalition structure {(1,2), (3)} with coalition structure cost of 290. Then both $x_1^1 = 50, x_2^1 = 90, x_3^1 = 150$ and $x_1^2 = 55, x_2^2 = 85, x_3^2 = 150$ belong to the coalition structure core.

ID	Coalition (S_j)	Coalition cost (c_j)
S_1	(1)	55
S_2	(2)	100
S_3	(3)	150
S_4	(1,2)	140
S_5	(1,3)	210
S_6	(2,3)	250
S_7	(1,2,3)	310

Table 4.5 Coalitions and Coalition Costs for the Example with Multiple Stable Allocations

The presence of multiple core allocations gives decision-makers flexibility in dividing costs, thereby increasing the likelihood that allocations will not only be stable but also understandable. The ability to choose from multiple stable allocations empowers decision-makers by providing them flexibility, thereby enhancing the overall robustness and effectiveness of collaborative efforts.

4.2.3 Summary of Analytical Results

The results proven in this section encompass fundamental aspects of both *OCS* and *CSC* models. They also provide a foundation for computational approaches that do not require that the *CSC* model be solved directly to find a stable allocation. This is an important step, because doing so proved computationally expensive when the author attempted to run the *CSC* model on the same instances used for the *OCS* model in the computational study described in Chapter 3.

The analytical results demonstrated that while the *CSC* model may lack a solution in certain cases (Theorem 2), when a feasible solution exists, it is proven to be optimal in both the *CSC* and *OCS* models (Theorem 4). Additionally, it is proven that when the *OCS* model has multiple optimal solutions, all share the same feasibility status within the *CSC* model (Theorem 5). Thus, it is a straightforward conclusion that if an optimal solution for the *OCS* model meets the strong stability and budget balanced conditions in the *CSC* model, then that solution is also optimal for the *CSC* model.

These revelations demonstrate that, when stability is feasible, achieving it does not result in additional societal costs. Consequently, in response to the initial research question, the author concludes that the gap between the stable coalition structure with least societal cost and the minimum societal cost of any coalition structure is zero, whenever any stable coalition structure exists.

Given the above findings, an alternative to directly solving the *CSC* problem is to solve the *OCS* problem and then check whether the identified optimal coalition structure admits a cost allocation that satisfies the strong stability and budget balanced conditions. This approach has the potential to reduce the computational time necessary to identify allocations in the coalition structure core. However, this approach does not address the feasibility challenge, since the *CSC* model may not admit a feasible solution in all instances. This is found to be a significant issue; in the instances chosen for testing with the outlined steps, the majority did not yield any feasible solution.

While a relaxation approach is frequently employed to address the feasibility challenge in subadditive games, finding near-stable allocations when the core is empty, studies in the cooperative game theory literature have not addressed the corresponding question for non-subadditive games when the coalition structure core is empty. Consequently, the author introduces this approach in the subsequent section.

4.3 Navigating the *CSC* Feasibility Challenge

The studies in the literature utilize the concepts of ε-core and strong least-core, described in detail in what follows. The author then adapts these concepts to establish the foundation for tackling the feasibility challenge for non-subadditive games.

4.3.1 Strong Least-Core Adapted to Non-subadditive Games

Unfortunately, although stability is a highly attractive property, the core is often empty (L. S. Shapley & Shubik, 1963). In subadditive games, the ε-core is the set of cost allocations that satisfy the core conditions when the rationality condition is relaxed by ε. The smallest value ε^* for which the ε-core is non-empty determines the strong least-core/ε^*-core (referred to 'least-core' hereafter) (Bachrach et al., 2014; Zick et al., 2013b).

Intuitively a game is unstable when the cost of the grand coalition is not sufficiently low to meet the demands of each player. In such scenarios, ε^* can be thought of as a subsidy that an external party is interested to provide to reduce the grand coalition cost, thereby achieving stability. Alternatively, if ε^* is a tax added uniformly to the cost of each coalition in the strong stability constraints, the costs of these coalitions each would appreciate by ε^* (Zick et al., 2013b).

The following example facilitates a deeper understanding of the least-core concept using both subsidy and tax approaches to determine the value of ε^*.

Example:

Consider an instance with players $N = \{1, 2, 3\}$ and the following costs associated with each coalition:

Coalition (S_i)	Coalition cost (c_i)
(1)	10
(2)	10
(3)	10
(1,2)	15
(1,3)	15
(2,3)	15
(1,2,3)	24

Table 4.6 Coalitions and Coalition Costs in the Example Investigating the Tax and Subsidy Approaches for Determining the Value of ε^*

Given the above, the possible coalition structures and their associated costs are as follows:

ID	Coalition structure (φ_j)	Cost of coalition structure ($c(\varphi_j)$)
φ_1	{(1), (2), (3)}	30
φ_2	{(1,2), (3)}	25
φ_3	{(1,3), (2)}	25
φ_4	{(2,3), (1)}	25
φ_5	{(1,2,3)}	24

Table 4.7 Possible Coalition Structures and Their Costs in the Example Investigating the Tax and Subsidy Approaches for Determining the Value of ε^*

In this example, the grand coalition {1, 2, 3} with cost 24 has the minimum total cost. However, the core is empty because there is no way to allocate the cost among the players that satisfies the budget-balance and strong stability constraints, as demonstrated below.

The following conditions must be fulfilled for *k* to be a stable solution.

$$x_1^k \leq 10 \tag{4.75}$$

$$x_2^k \leq 10 \tag{4.76}$$

$$x_3^k \leq 10 \tag{4.77}$$

$$x_1^k + x_2^k \leq 15 \tag{4.78}$$

$$x_1^k + x_3^k \leq 15 \tag{4.79}$$

$$x_2^k + x_3^k \leq 15 \tag{4.80}$$

$$x_1^k + x_2^k + x_3^k = 24 \tag{4.81}$$

From equations 4.75 , 4.76, and 4.77, the maximum value any x_i^k can take is 10.

Now, adding up equations 4.78, 4.79, and 4.80 results in:

$x_1^k + x_2^k) + (x_1^k + x_3^k) + (x_2^k + x_3^k) \leq 15 + 15 + 15$

$2 * (x_1^k + x_2^k + x_3^k) \leq 45$

$x_1^k + x_2^k + x_3^k \leq 22.5.$

However, the budget balanced constraint (4.81) requires:

$x_1^k + x_2^k + x_3^k = 24.$

This and the preceding inequality cannot be satisfied simultaneously, therefore, the given instance does not have a feasible *CSC* solution.

Consider the two methods to determine the least-core for the above example: the subsidy approach and the tax approach.

Least-core, subsidy approach. The game described above can yield a feasible solution by reducing the cost of the grand coalition by 1.5 units (from 24 to 22.5). In this adjusted scenario, the budget balanced condition becomes $x_1^k + x_2^k + x_3^k = 22.5$, and one of the solutions for this game is: $x_1^k = x_2^k = x_3^k = 7.5$.

Least-core, tax approach. The game described above can achieve a feasible solution by increasing the right-hand side of each strong stability constraint by 1.5 units. The updated constraints are as follows.

$x_1^k \leq 11.5$

$x_2^k \leq 11.5$

$x_3^k \leq 11.5$

$x_1^k + x_2^k \leq 16.5$

$x_1^k + x_3^k \leq 16.5$

$x_2^k + x_3^k \leq 16.5$

In this scenario, the game has feasible solutions, and one such solution is $x_1^k = x_2^k = x_3^k = 8$.

Unlike the subsidy approach, in the tax approach, the budget balanced constraints remain unchanged, and the total cost c_N is still divided as intended. Stability is achieved by limiting the power of the coalitions that are not meant to form. The additional tax incorporated into strong stability constraints are not intended to be collected. However, in the subsidy approach the allocated cost to players is less than c_N, and there is no manipulation of coalitions' power.

In the context of this research, the players represent public health or humanitarian organizations, and the goal is to minimize the overall cost, thereby improving public services. While these organizations may prefer the subsidy approach, the preference from the public's perspective may vary depending on the funding source for the subsidy.

In this research the author adopts the tax approach and set E is defined as $E = \{\varepsilon_1, \dots, \varepsilon_{2^{|N|}-1}\}$, where ε_j represents the minimum tax amount that needs to be added to the coalition cost c_j in order for coalition S_j to become strongly stable. In this context ε^* is determined as the maximum value among all ε_j for the $S_j \in \Omega$:

$$\varepsilon^* = \max_{S_j \in \Omega} \varepsilon_j. \qquad 4.82$$

The author extends the least-core concept using tax approach to meet coalition structure core requirements, hereafter called the 'least-coalition structure core (*LCSC*)'. The formal definition of *LCSC* is outlined below.

$$Minimize\ \varepsilon \qquad 4.83$$

subject to:

$$\varepsilon \geq \varepsilon_j \qquad \forall S_j \in \Omega \qquad 4.84$$

$$\sum_{n \in S_j} x_n \leq c_i + \varepsilon_j \qquad \forall S_j \in \Omega \qquad 4.85$$

$$\sum_{n \in S_j} x_n = c_j \qquad \forall S_j \in \varphi^* \qquad 4.86$$

$$\varepsilon_j \geq 0 \qquad \forall S_j \in \Omega \qquad 4.87$$

$$x_n \geq 0 \qquad \forall n \in N \qquad 4.88$$

It is important to highlight that, for sufficiently large ε_j, there will always be a solution within the least-coalition structure core. Additionally, when the objective value is zero, it signifies that the allocation lies within the coalition structure core. Conversely, a positive objective value indicates the minimum tax that must be added to the strong stability constraints.

4.3.2 Solution Framework: Stability and Complexity Enhanced Algorithm

As discussed during the previous sections, the likelihood of discovering a stable allocation within the coalition structure core is low. Considering the attractiveness of finding a stable allocation, and in response to this challenge, the author develops *Stability and Complexity Enhanced (SCE)* algorithm. *SCE* identifies the least-coalition structure core as an alternative to the coalition structure core and addresses both the complexity and feasibility challenges inherent with the *CSC* problem. The algorithm's main premise is first to solve the *OCS* model to find a coalition structure that minimizes the total social cost, and then to solve the *LCSC* model to generate a stable or near-stable allocation.

SCE successfully tackles the complexity and feasibility challenges of the *CSC* model by finding the optimal coalition structure by solving *OCS* model and then replacing *CSC* model with *LCSC* model. If the *LCSC* objective function value is zero, then the solution produced by *SCE* is optimal for the *CSC* model. A positive objective function value indicates ε^*, the minimum deviation from stability.

The *SCE* algorithm is presented formally in Algorithm 3. The following definitions are used in the pseudocode:

Definitions:

N: set of players $1, \dots, |N|$

S_j: coalition S_j; $S_j \subseteq \Omega$

φ^*_{OCS}: Optimal coalition structure for *OCS* model

x_n: Cost allocated to player *n*

c_j: cost for coalition S_j; $C = \{c_1, \dots, c_{2^{|N|}-1}\}$

Algorithm 3. *SCE* Pseudocode

Input: A player set N, a coalition set Ω, a cost set C

// *Step 1*

$\varphi^*_{OCS} \leftarrow OCS(\Omega, C)$

// *Step 2*

$(\varphi_{LCSC}, x) \leftarrow \emptyset$

solve:

$Min\ \varepsilon$

s.t. $\varepsilon \geq \varepsilon_j$ $\qquad \forall\, S_j \in \Omega$

$\sum_{n \in S_j} x_n \leq c_j + \varepsilon_j$ $\qquad \forall\, S_j \in \Omega$

$\sum_{n \in S_j} x_n = c_j$ $\qquad \forall\, S_j \in \varphi^*_{OCS}$

$\varepsilon_j \geq 0 \qquad \forall S_j \in \Omega$

$(\varphi_{LCSC}, x) \leftarrow (\varphi^{*}_{OCS}, x)$

return: (φ_{LCSC}, x)

function: $OCS(\Omega, C)$

$\varphi \leftarrow \emptyset$

solve:

$Min \sum_{S_j \in \Omega} c_j \, \beta_j$

$s.t. \sum_{S_j \in \Omega} \alpha_{n,j} \beta_j = 1 \qquad \forall n \in N$

$\beta_j \in \{0,1\} \qquad \forall S_j \in \Omega$

for all $S_j \in \Omega$

if $\beta_j == 1$

$\varphi \leftarrow \varphi \cup S_j$

return: φ

Given the feasibility challenge inherent with *CSC* problem, it becomes crucial to explore solutions in proximity to stability. Addressing this objective, *SCE* takes on the dual challenge of reducing running time and seeking solutions that closely approximate stable solutions. This approach aligns with the initial direction of the research, while shifting the focus from optimal solutions to examining the characteristics of near-stable solutions. The author now investigates factors influencing stability with a computational study employing *SCE*.

4.4 Computational Study

This section demonstrates the applicability of *SCE* to find stable or near-stable allocations and evaluates the stability properties of candidate allocations. The study contains two main components. First, *SCE* is used to identify a coalition structure core allocation (if one exists) or a least coalition structure core allocation for each instance. Second, common allocation rules are examined to determine whether these produce allocations in the coalition structure core or least coalition structure core.

These experiments are conducted using the same instances employed in the computational study of *OCS* model presented in Chapter 3. Ten sets with 10 instances each are considered; instance sets differ in the total number of players in each instance, as well as in the proportion of small and medium players. For example, an instance in set 16(8,8) has 16 total players, 8 small and 8 medium. The computations are implemented on a system with 32.0 GB RAM in Python 3.7 language, and CPLEX 12.10.0.0 via the CPLEX Python API is used to solve all optimization problems.

4.4.1 Examining *CSC* Feasibility

The objective in Chapter 4 is to establish a framework for stable cost allocation that minimizes the total societal cost. While the *LCSC* model can address the proposed question, it faces challenges related to problem complexity and practicality. To overcome these issues, the author introduces the *SCE* algorithm.

The first goal is to understand the characteristics of the coalition structure core. As the initial phase of the computational study, the author identifies the number of instances in each set with a non-empty coalition structure core. The results are summarized in Table 4.8.

Sets	Number of Instances with Non-empty Coalition Structure Core
16(2,14)	5
16(4,12)	3
16(8,8)	6
16(12,4)	0
16(14,2)	1
18(2,16)	6
18(4,14)	5
18(9,9)	5
18(14,4)	0
18(16,2)	0

Table 4.8 Number of Instances with Non-empty Coalition Structure Core in Each Set of 10 Instances

Overall, only 31% of the instances have non-empty coalition structure core. The number of the players does not have considerable impact on these results. Specifically, within the 16-player category, 30% of the instances have stable solutions, and this value is 32% among the 18-player instances.

However, when the instance sets are categorized into 3 groups based on the player size distribution, a trend is observed. It is evident that 16(8,8) and 18(9,9) sets, where the number of small and medium players is equal, exhibit a higher likelihood of possessing stable solutions, with 55% of such instances meeting this criterion. In contrast, in the 16(12,4), 16(14,2), 18(14,4), and 18(16,2) sets, where the majority of players are small, only 2.5% of instances have non-empty coalition structure core. The 16(2,14), 16(4,12), 18(2,16), and 18(4,14) sets, where the majority of players are medium, perform better, with 47.5% of instances possessing stable allocations.

These findings indicate a correlation between the players' order quantities and the coalition structure stability. This motivates an exploration into factors that contribute to the variability in the likelihood of a non-empty coalition structure core.

Table 4.9 provides information on the average ratio between the smallest relaxation (ε^*) required to obtain a stable solution and the *OCS* model objective value ($c(\varphi^*_{OCS})$) (middle column), as well as the average number of the formed coalitions (right column) for each instance set.

Sets	Avg $\left(\varepsilon^* / c(\varphi^*_{OCS})\right)$	Avg # Coalitions in φ^*_{OCS}
16(2,14)	0.46%	10.4
16(4,12)	0.67%	9.3
16(8,8)	0.40%	6.7
16(12,4)	1.30%	4.3
16(14,2)	1.63%	4.0
18(2,16)	0.36%	12.3
18(4,14)	0.55%	10.1
18(9,9)	0.44%	8.5
18(14,4)	1.28%	4.9
18(16,2)	1.54%	3.8

Table 4.9 Average Ratio of Least-Core to *OCS* Model Objective Value and Count of Formed Coalitions in Each Set

The ratio in the middle column include solutions for which the objective function value is 0, meaning that the coalition structure core is non-empty, in addition to those with non-zero values. The results indicate that as the number of small players increases, the average magnitude of ε^* relative to the *OCS* objective function value generally increases as well. Further investigation reveals that the increase in small players coincides with a reduction in the number of coalitions within the optimal coalition structure. For a fixed number of players, fewer formed coalitions means that coalitions have more players, on average. As the number of coalitions decreases and number of players in coalitions increases, the likelihood of an allocation that ensures stability decreases.

4.4.2 Comparing Allocation Rules

The ultimate goal of this research is to apply these findings in humanitarian sectors to enhance their operations through collaborations in logistics. While the least coalition structure core produces allocations that are closest to stable solutions, they might not be easily understandable and thus, may be unacceptable for organizations. Given that, the author tests two commonly used and easy-to-understand allocation rules, evaluating the resulting stability. This approach ensures that the chosen rules are both relevant and practical for further examination. Gujardo and Rönnqvist (2016) examined 57 studies and published that ad hoc, Shapley, and proportional are among the most frequently investigated allocation rules in the academic literature. In this dissertation, Shapley and proportional allocations have been selected for examination, with the coalition structure core conditions considered as the requirement for stability. Shapley value allocations are calculated by applying the classic formula to each formed coalition, and proportional allocations are computed based on each player's proportion of its coalition's total order quantity.

The analysis in this section unfolds in two phases. First instances where ε^*equals zero are examined; this indicates the presence of a stable allocation. Here, the question is whether specific allocation rules can generate feasible solutions under coalition structure core conditions. Second, the chosen allocation rules are examined in all instances whether ε^* equals or exceeds zero, with the goal of assessing how close the chosen allocations are to those generated by the *LCSC* model.

As previously presented, out of the 100 instances, 31 had a zero-objective value in *LCSC* model. In first examination, the Shapley and proportional allocation rules are examined within this subset of 31 instances. The findings from this test reveal a consistent outcome: neither allocation rule produced a stable solution within this subset of instances.

Consequently, the analysis is extended by adding the remaining 69 instances. Table 4.10 provides insights into the application of Shapley and proportional allocations. The values in the columns indicate the number of players in each set whose allocations under the given rule are, regardless of their specific instance within that set, closer to the *LCSC* allocation.

Sets	Shapley	Proportional
16(2,14)	73	87
16(4,12)	62	98
16(8,8)	22	138
16(12,4)	30	130
16(14,2)	27	133
18(2,16)	94	86
18(4,14)	63	117
18(9,9)	43	137
18(14,4)	31	149
18(16,2)	63	117

Table 4.10 Number of Players Closer to *LCSC* Allocation in Each Set

In each instance, there are either 16 or 18 players, and each set is comprised of 10 instances. For example, in the case of set 16(2,14), the total number of players is 160. Among these 160 players, for 73 players the Shapley allocation is closer to the *LCSC* allocation, and for 87 players the proportional allocation is closer.

As is evident from Table 4.3, the proportional allocation produces allocations that are closer to the *LCSC* allocations in nearly all sets (except 18(2,16)). As the number of small players increases, the proportional allocations are more likely to be closer to the *LCSC* allocations. However, it was shown in the prior analysis that *LCSC* itself generates larger ε^* as the number of small players increases.

These findings serve as the motivation for the next stage of the analysis, which involves quantifying the difference between Shapley and proportional allocations in comparison to the *LCSC* allocation. Table 4.11 provides a summary of the average allocation absolute disparities per player between the Shapley and proportional allocations compared to the *LCSC* allocations (second and third columns), and a similar comparison between the Shapley and proportional allocations directly.

	Shapley vs *LCSC*	Proportional vs *LCSC*	Shapley vs Proportional
16(2,14)	280	197	93
16(4,12)	312	211	121
16(8,8)	292	155	142
16(12,4)	141	51	101
16(14,2)	131	70	70
18(2,16)	377	293	104
18(4,14)	287	186	121
18(9,9)	299	162	151
18(14,4)	137	35	112
18(16,2)	160	118	70

Table 4.11 Average Allocation Absolute Disparities per Player

For instance, in the comparison between Shapley and *LCSC* in set 16(2,14), the absolute difference between the Shapley allocation and the *LCSC* allocation is measured for all players, these absolute differences are summed, and this sum is divided by the total number of players (160 in this instance set). The result equals 280, as shown in Table 4.11.

These results also are depicted graphically to illustrate patterns. Figure 4.1 and Figure 4.2 depict cost allocation absolute difference per player. Figure 4.1 compares Shapley and proportional allocations with *LCSC* results, and Figure 4.2 compares Shapley with proportional allocations. Each figure is divided into two segments, with the first five points on the horizontal axis representing 16-player instances, and the following five points representing 18-player instances.

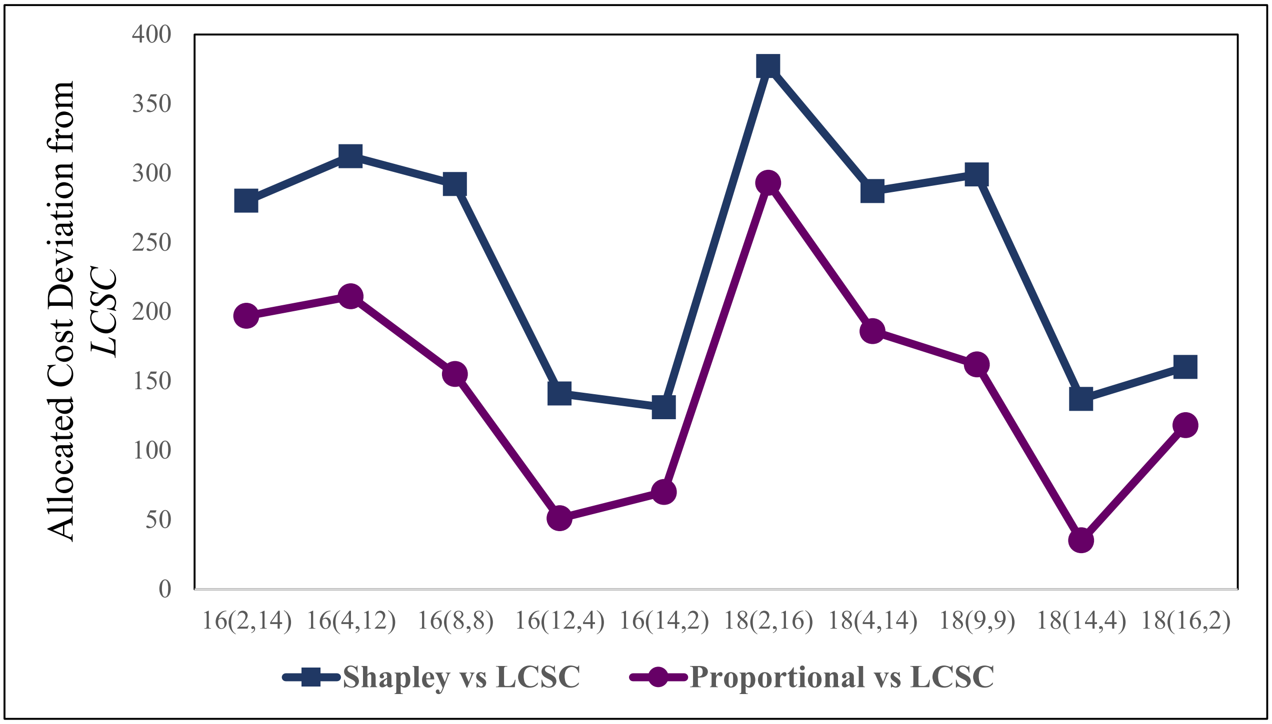

Figure 4.1 Shapley/Proportional vs *LCSC*, Average Allocation Absolute Disparities per Player

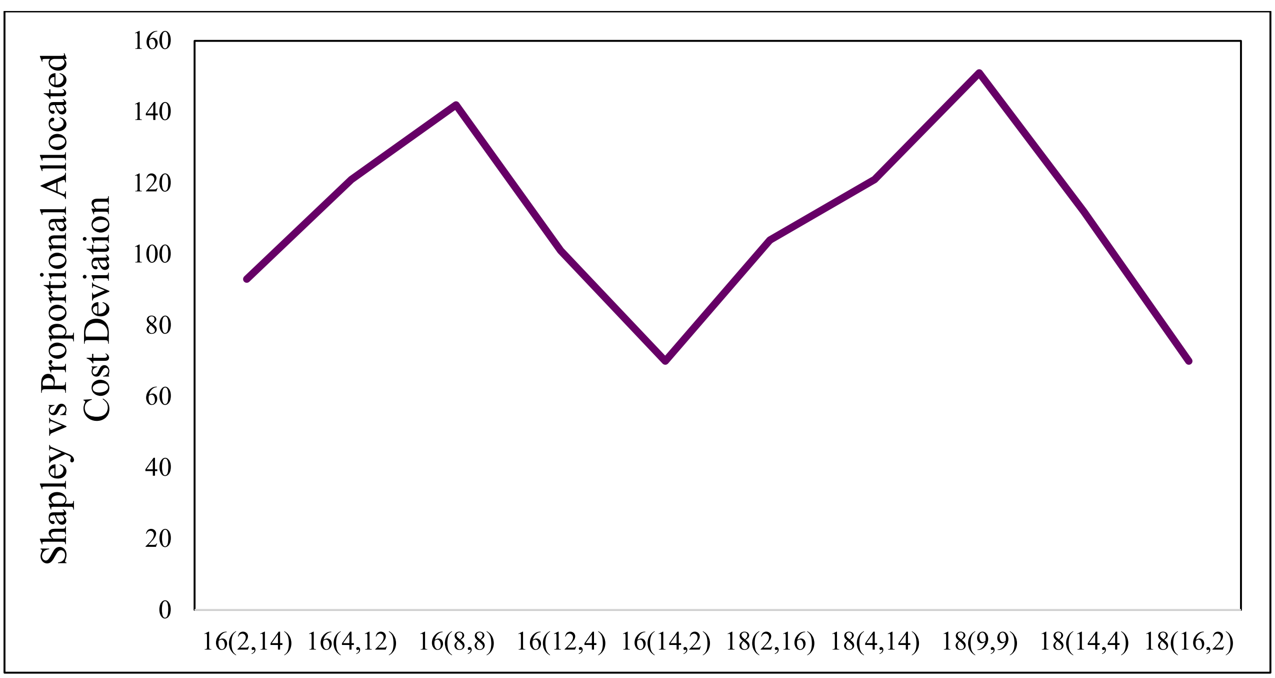

Figure 4.2 Shapley vs Proportional, Average Allocation Absolute Disparities per Player

As observed in Figure 4.1 similar pattern is depicted in both the first and second halves of the graphs. With an increase in the number of small players, the average per player absolute allocation difference between Shapley and proportional allocations with the $LCSC$ allocation diminishes. Figure 4.2 shows that the Shapley and proportional disparity is largest when the numbers of small and medium players are equal. As either the number of small or medium players increases, the proportional and Shapley allocations become closer to each other.

Recall from previous analysis that as the number of small players increases, more coalitions are formed, and finding a stable solution becomes increasingly challenging. In other words, the ratio between the required relaxation to the stability requirement, ε^*, and the total social cost in the optimal solution grows larger. Combining these observations, note that as the number of small players increases, the difference between $LCSC$ and $c(\varphi^*_{OCS})$ increases, while the absolute difference between Shapley and proportional allocation with $LCSC$ allocation decreases. This suggests that as the number of players and their collaborative efforts increase, and the $LCSC$ model's stability diminishes, both Shapley and proportional allocations, while never reaching the $LCSC$ allocation, move closer to the $LCSC$ allocations. This means that as collaboration increases, the probability of attaining stable solutions diminishes, but the performance of allocations becomes increasingly aligned with $LCSC$ allocations. Conversely, when the number of small players is limited, leaving little room for collaboration, there is a higher likelihood of identifying stable solutions. However, Shapley and proportional allocations tend to deviate further from these stable allocation points.

These findings demonstrate the varied performance of allocation methods under different player compositions. Proportional allocation consistently outperforms the Shapley value in two aspects: the number of players with allocations closer to the $LCSC$ allocation and the disparity between their allocations and what they would receive in the *LCSC* model. Additionally, it is observed that while proportional allocation consistently outperforms the Shapley value in all examined instances, this difference increases when there are more diverse groups in terms of small and medium players. When players are less diverse, the Shapley and proportional allocations exhibit more similar performance.

In conclusion, when the majority of players are medium-sized, the *LCSC* allocation significantly outperforms Shapley and proportional allocations, with proportional being slightly better. When the number of small and medium players is equal, the performance gap between *LCSC* and allocation rules diminishes, and in this scenario, proportional outperforms Shapley by a large margin. Lastly, when the majority of players are small, proportional continues to outperform Shapley, although all allocations (*LCSC*, Shapley, and proportional) are relatively close.

4.5 Discussion and Conclusions

In summary, Chapter 4 utilizes the *CSC* model to establish stable cost allocations among players. The discoveries on the nature of *OCS* and *CSC* models combined with the relaxation method lead to developing the *SCE* algorithm to address the complexity and feasibility issues inherent with the *CSC* problem. To evaluate the proposed algorithm's applicability, Shapley and proportional allocation rules are chosen. These allocation rules, along with the proposed *SCE* algorithm, undergo a computational study using previously employed instances. The results offer insights into how allocation methods perform under different conditions and player compositions, providing implications for real-world applications and decision-making.

This research introduces models, solution procedures, and analytical proofs for cooperative games with non-subadditive costs. These are demonstrated using computational analyses with randomly generated instances. The approach serves as a proof-of-concept that may be applied to collaborative opportunities in humanitarian and other supply chain contexts.

To apply the proposed methods in practice, data are required to instantiate the models. Specifically, for a given set of potential collaborating agencies, it is necessary to know or be able to estimate the cost associated with each possible coalition. With these data, agencies or a third party facilitator can determine the coalition structure that minimizes the total social cost by solving the *OCS* problem.

Given that players operate independently, it is crucial for them to comprehend and accept the allocation method. If the *LCSC* solution is not easily understandable to the players, a proportional method is recommended. Proportional allocation aligns closely with *LCSC* allocations for a significant portion of players across various instances, as indicated by research findings from randomly generated scenarios.

However, as the number of smaller players increases, the performance differentials between *LCSC*, proportional, and Shapley methods diminish. Therefore, while proportional allocation appears more critical when there are fewer small players, its importance decreases as the number of small players rises.

Chapter 5 - Conclusion and Future Work

The primary objective of this study was to help humanitarian organizations realize supply chain synergies via collaboration by determining which players should collaborate to minimize total societal cost. Humanitarian supply chains often operate as decentralized systems in which decision makers strive to maximize individual objectives. However, conflicts in supply chain objectives negatively impact the system, reducing the quality of humanitarian services and preventing optimal systemwide outcomes. This study sought to narrow the gap between optimality and feasibility in a decentralized system by forming stable coalitions. Cooperative game theory is applied in a non-subadditive context to determine collaboration partners for minimizing total cost and then to allocate costs among them to ensure the stability of collaboration.

This work begins with a literature review on cooperation among supply chain entities with a focus on studies that utilize cooperative game theory. As collaboration in humanitarian applications is still emerging, the review draws from all supply chain sources. The review indicates that cooperative game theory has been extensively explored in systems with subadditive cost functions in which the grand coalition yields the lowest societal cost. Consequently, this study addresses a literature gap by investigating systems with non-subadditive cost functions. While the original motivation is humanitarian, the study's applicability extends to all supply chain collaborations in non-subadditive settings.

The second component of this work involves determining optimal collaborative pairing by introducing the Optimal Coalition Structure (*OCS*) model. Next, it is proved that solving the

OCS problem is mathematically complex, so two heuristics are developed tailored for problems with a general non-subadditive cost function. The heuristics and the optimization model are demonstrated on 100 randomly generated instances. Both heuristics demonstrate strong performance in the computational analyses, with one achieving a slightly lower cost and the other consistently displaying faster processing times. The computational study not only considers running time and total cost, but also examines correlation with the problem size and players' order quantities. The comprehensive analysis provides a holistic assessment of the proposed solutions, including their practical applicability and real-world performance.

Next, this research employs the Coalition Structure Core (*CSC*) model to determine stable cost allocation among players. The characteristics of the mathematically complex *CSC* problem and its relationship to the *OCS* problem are identified. The discoveries provide the foundation for developing the Stability and Complexity Enhanced (*SCE*) algorithm to simultaneously tackle the complexity and feasibility issues in the *CSC* problem. Furthermore, the research introduces the *LCSC* model, which identifies a coalition structure with minimal deviation from the optimal *CSC* solution and marks the first application the relaxation approach to non-subadditive games.

A computational study demonstrates the applicability of *SCE* algorithm and compares its output to Shapley and proportional allocation rules are selected. The computational results shed light on how the allocation methods perform under various conditions and player compositions, providing insights into the implications of these findings for real-world applications and decision-making.

The research outcomes have potential applications in domains where collaborative decision-making, resource allocation, and cost-sharing are critical components. The *OCS* model can be applied to optimize collaborative pairing and *CSC* model can be utilized to allocate costs among players in supply chain logistics, where various entities need to work together efficiently. This could result in cost savings and better resource allocation. The computational study on Shapley and proportional allocation rules can be applied to real-world scenarios, such as budget allocation in organizations. Understanding how these rules perform under different conditions helps in making informed decisions on resource distribution. The *LCSC* model, as a relaxation approach can be employed in business scenarios involving partnerships. Identifying a coalition structure with minimal deviation from the optimal solution can enhance collaboration and resource-sharing agreements between organizations.

This study unveils numerous opportunities for future research that leverages game theory to optimize humanitarian operations. Within the humanitarian context, the models could be adapted to account for conditions not yet captured. One potential area for extending the *OCS* model would be integrating geographical constraints on allowable partnerships. By incorporating geographical considerations into collaborative decision-making, this enhancement could elevate the model's relevance to real-world scenarios. It also may be relevant to explore cost functions explicitly considering distances within the supply chain network. While this study primarily focuses on collaboration in the context of order quantity optimization, the approach could be generalized to other supply chain functions. For instance, humanitarian agencies could decrease transportation costs by consolidating shipments, securing more favorable rates with carriers, as studied in the private sector by Özener and Ergun (2008).

Extensions of this study could explore alternative allocations not considered in the current research. Additionally, a valuable direction for future research is the exploration of the closest cost allocation to the proposed heuristics, providing nuanced insights into their effectiveness compared to the *OCS* solution.

The study's applicability is not limited to humanitarian supply chains; the optimization model, heuristic approaches, and algorithms could be employed in any industry. The heuristic approach, involving solving subproblems by dividing players into groups based on potential cost savings, could also be adapted to other cost functions.

In conclusion, this work establishes a solid foundation for addressing critical questions related to coalition structure and cost allocation within non-subadditive games. It offers valuable insights into the complexities of optimizing collaboration among players. Promising avenues for expansion upon this research and the exploration of other dimensions of game theory and cost allocation in non-subadditive settings are also available. These future research directions collectively contribute to advancing the field of game theory both in humanitarian logistics and in broader settings, addressing critical challenges and optimizing decision-making processes for enhanced operational outcomes.

Bibliography

Agarwal, R., Ergun, O., Houghtalen, L., & Ozener, O. O. (2009). Collaboration in cargo transportation. In *Optimization and logistics challenges in the enterprise* (Vol. 30, pp. 373–409). http://www.econis.eu/PPNSET?PPN=605477698

Aumann, R., & Dreze, J. (1974). Cooperative games with coalition structures. *International Journal of Game Theory*, *3*(4), 217–237. https://doi.org/10.1007/BF01766876

Bachrach, Y., Lav, O., Lovett, S., Rosenschein, J., & Zadimoghaddam, M. (2014). Cooperative weakest link games. *Proceedings of the 13th International Conference on Autonomous Agents and Multiagent Systems (AAMAS-14)*, 589–596.

Bachrach, Y., Porat, E., & Rosenschein, J. S. (2013). Sharing rewards in cooperative connectivity games. *Journal of Artificial Intelligence Research*, *47*, 281–311. https://doi.org/10.1613/jair.3841

Basso, F., Guajardo, M., & Varas, M. (2020). Collaborative job scheduling in the wine bottling process. *Omega*, *91*, 102021. https://doi.org/10.1016/j.omega.2018.12.010

Ben Jouida, S., Guajardo, M., Klibi, W., & Krichen, S. (2021). Profit maximizing coalitions with shared capacities in distribution networks. *European Journal of Operational Research*, *288*(2), 480–495. https://doi.org/10.1016/j.ejor.2020.06.005

CRED, Centre for Research on the Epidemiology of Disasters (2021). (2021). https://cred.be/sites/default/files/2021_EMDAT_report.pdf

Elomri, A., Ghaffari, A., Jemai, Z., & Dallery, Y. (2012). Coalition formation and cost allocation for joint replenishment systems. *Production and Operations Management*, *21*(6), 1015–1027. https://doi.org/10.1111/j.1937-5956.2012.01333.x

Ergun, Ö., Gui, L., Heier Stamm, J. L., Keskinocak, P., & Swann, J. (2014). Improving humanitarian operations through technology-enabled collaboration. *Production and Operations Management*, *23*(6), 1002–1014. https://doi.org/10.1111/poms.12107

Fatima, S., & Wooldridge, M. (2019). Computing optimal coalition structures in polynomial time. *Autonomous Agents and Multi-Agent Systems*, 33, 35-83.

Garey, M. R., & Johnson, D. S. (1979). *Computers and intractability: A guide to the theory of NP-completeness*. San Francisco : W. H. Freeman.

Guajardo, M., & Rönnqvist, M. (2015). Operations research models for coalition structure in collaborative logistics. *European Journal of Operational Research*, *240*(1), 147–159. https://doi.org/10.1016/j.ejor.2014.06.015

Guajardo, M., & Rönnqvist, M. (2016). A review on cost allocation methods in collaborative transportation. *International Transactions in Operational Research*, *23*(3), 371–392. https://doi.org/10.1111/itor.12205

Jongen, H. Th., Meer, K., & Triesch, E. (Eds.). (2004). Some NP-completeness results. In *Optimization Theory* (pp. 313–328). Springer US. https://doi.org/10.1007/1-4020-8099-9_21

Kimms, A., & Kozeletskyi, I. (2016). Core-based cost allocation in the cooperative traveling salesman problem. *European Journal of Operational Research*, *248*(3), 910–916. https://doi.org/10.1016/j.ejor.2015.08.002

Knuth, D. E. (2000). Dancing links. *arXiv:Cs/0011047*. http://arxiv.org/abs/cs/0011047 (Accessed: 03-29-2021)

Lozano, S., Moreno, P., Adenso-Díaz, B., & Algaba, E. (2013). Cooperative game theory approach to allocating benefits of horizontal cooperation. *European Journal of Operational Research*, *229*(2), 444–452. https://doi.org/10.1016/j.ejor.2013.02.034

Özener, O. O., & Ergun, O. (2008). Allocating costs in a collaborative transportation procurement network. *Transportation Science*, *42*(2), 146–165. https://doi.org/10.1287/trsc.1070.0219

Peleg, B., & Sudholter, P. (2007). *Introduction to the theory of cooperative games* (Second, Vol. 34). Berlin ; New York : Springer.

Persien, V., Rey, A., & Rothe, J. (2016). Cost of stability and least core in path-disruption games. *Frontiers in Artificial Intelligence and Applications*, 99–110.

Rota, G.-C. (1964). The number of partitions of a set. *The American Mathematical Monthly*, *71*(5), 498. https://doi.org/10.2307/2312585

Schulte, F., Lalla-Ruiz, E., González-Ramírez, R. G., & Voß, S. (2017). Reducing port-related empty truck emissions: A mathematical approach for truck appointments with collaboration. *Transportation Research Part E: Logistics and Transportation Review*, *105*, 195–212. https://doi.org/10.1016/j.tre.2017.03.008

Shapley, L. (1953). A value for n-person games*, Annals of Mathematics Studies*. *28*, 307–318.

Shapley, L. S., & Shubik, M. (1963). *The core of an economy with nonconvex preferences* (p. 0051). Rand Corporation.

Vaissier, B., Pernot, J.-P., Chougrani, L., & Véron, P. (2020). Lightweight mesh file format using repetition pattern encoding for additive manufacturing. *Computer-Aided Design*, *129*, 102914. https://doi.org/10.1016/j.cad.2020.102914

Von Neumann, J., & Morgenstern, O. (1944). *Theory of Games and Economic Behavior*.

Xu, X., Pan, S., & Ballot, E. (2012). Game theoretic contribution to horizontal cooperation in logistics. *Proceedings of the 4th International Conference on Information Systems, Logistics and Supply Chain- ILS 2012*. https://hal-mines-paristech.archives-ouvertes.fr/hal-00733499

Young, P. (1985). *Cost allocation: Methods, principles, applications*. Amsterdam: North-Holland ; New York: North-Holland ; New York, N.Y.: North-Holland.

Zick, Y., Plukarov, M., & Jennings, N. (2013a). *Taxation and stability in cooperative games*. 523–530.

Appendix

Algorithm 2. Heuristic B Pseudocode

input: A player set N, a coalition set Ω, cost functions c_j^0, c_j^1, c_j^2, c_j^3, c_j^4

// *initialization*
for $r = 1, \dots, 6$
$\quad P_r \leftarrow \emptyset$
for $j = 1, \dots, |N|$
$\quad$ **if** $q_j \geq z_2$
$\qquad P_1 \leftarrow P_1 \cup S_j$
$\quad$ **else if** $q_j < z_1$
$\qquad P_2 \leftarrow P_2 \cup S_j$
$\quad$ **else** $P_3 \leftarrow P_3 \cup S_j$
// *Phase 1; group 10 players form a single coalition*
$P_1 \leftarrow \bigcup_{S_j \in P_1} S_j$
// *Phase 2; group 2 optimization*
$P_2 \leftarrow MIP(P_2, c_j^4)$
// *Phase 3; group 7 optimization*
$P_3 \leftarrow P_3 \cup P_2$ // *coalitions resulting from Phase 2 are also included in this phase*
$P_3 \leftarrow MIP(P_3, c_j^1)$
// *Phase 4; group 4 optimization*
$P_4 \leftarrow P_4 \cup P_3$ // *coalitions resulting from Phase 3 are also included in this phase*
$P_4 \leftarrow MIP(P_4, c_j^3)$
// *Phase 5; group 1 optimization*
for all $S_j \in P_4$ // *coalitions resulting from Phase 4 are considered separately based on* q_j
$\quad$ **if** $q_j < z_1$
$\qquad P_5 \leftarrow P_5 \cup S_j$
$\quad$ **else** $P_6 \leftarrow P_6 \cup S_j$
$P_5 \leftarrow MIP(P_5, c_j^2)$

// *Phase 6; combined optimization*

$\varphi_{final} \leftarrow MIP(P_5 \cup P_6 \cup P_1, c_j^0)$

return: φ_{final}

function: $MIP(P_r, c_j^i)$

$\quad \varphi_{temp} \leftarrow \emptyset$

$\quad$ **solve:**

$\qquad Min \ \sum_{S_j \in S(P_r)} c_j^i \beta_j$

$\qquad s.t. \ \sum_{S_j \in S(P_r)} \alpha_{u,j} \beta_j = 1 \quad \forall \, u \in N(P_r)$

$\qquad\qquad \beta_j \in \{0,1\} \qquad\qquad \forall \, S_j \in \Omega(P_r)$

$\quad$ **for all** $S_j \in P_r$

$\qquad$ **if** $\beta_j == 1$

$\qquad\quad \varphi_{temp} \leftarrow \varphi_{temp} \cup S_j$

$\quad$ **return:** φ_{temp}