

THE DYNAMIC RESPONSE OF COLD ROOMS

by

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INTRODUCTION

For many years the problem of automatic control has been on the forefront of interest in refrigeration engineering. It is well known that automatic control is economically profitable and provides maximum technical security.

In many fields of engineering the study of system dynamics has become an integral part of the basic theoretical concepts related to that field. In refrigeration, however, the study of automatic control systems has not yet outgrown the early stage of a certain empiricism. The reason is twofold.

First, no simple relations are available to relate the temperature and the humidity control. In the second place, a lack of knowledge exists as to the dynamic response of the different parts which essentially constitute a cold room. In the present thesis the former difficulty has been removed by the introduction an "equivalent evaporator surface", the latter by a series of simplifications, resulting in some approximate responses.

The purpose of this thesis is to provide an analytical basis for the study of the dynamic response of cold rooms. The simplest assumptions are made to approach this problem. A deeper analysis would automatically require the use of computer simulation techniques. Due to the complexity and nonlinearity of the governing differential equations, this computer would be of extraordinary size. The subject of this thesis is in line with that of a current ASHRAE research project, conducted at the University of Wisconsin (1).

(1) Numbers in parenthesis refer to similarly numbered references in bibliography at end of thesis.

THE FUNDAMENTAL EQUATIONS OF ENVIRONMENTAL CONTROL

1. The variables to be controlled

Basically a system of environmental control in a cold room must maintain:

- a. a low storage temperature: everybody is acquainted with the fact that foods in general preserve better the lower the storage temperature.
- b. a constant storage temperature: this is particularly true for frozen foods. The 9th International Congress of Refrigeration (Paris, September 1955) was devoted to this subject (2). Temperature variations shift the thermodynamic equilibrium conditions between the ice-crystals and the remainder of cell-liquid in frozen foods. They hereby force higher kinetic rates of transition to occur between both phases. This results in a recrystallization and an excessive growth of large crystals due to their smaller surface energy (per unit volume). These large crystals finally destroy the cell membrane, the main cause of the so-called drip of deep-freeze products. Variations in the storage temperature thus reduce or annihilate the advantages obtained by quick-freezing procedures. More on this subject can be found in the extensive literature of food technology, for instance references (3,4,5).
- c. an optimum value of the room humidity: low humidities result in product desiccation, weight losses, in shriveling (fruits, vegetables) or in a loss of firmness (meat); high room humidities on the other hand may cause the deposition of moisture and possibly of mold on

the goods, followed by their decay. Since both phenomena (desiccation and mold formation) can more or less be remedied by the use of diffusion tight packages, the practice of packing the foodstuffs becomes quite general.

In practice it would be very hard to measure a representative value for the temperature within the whole storage load. Therefore the emphasis in the environmental control of cold rooms is shifted towards keeping the temperature of the room air constant. By doing so possible variations of the storage temperature are intercepted before they can occur except, of course, during loading periods.

For optimum preservation of foods the highest permissible variation of the room air conditions is rather small. The temperature should not vary more than ± 0.5 C and the relative humidity no more than ± 0.025 (2.5%). As will be shown later, a better criterion for the actual storage temperature of the foods would be the thermodynamic wet-bulb temperature of the room air.

2. Cold Storage Room Model

It is advantageous to assume a reasonable analytical model in an effort to simplify the analysis of the dynamic characteristics of a cold storage room. The model assumed further will present the following basic features:

- a. It is a storage room in the literal sense, with no other purpose than to keep at a predetermined temperature, goods, which before entering the room were precooled just to said storage temperature.
- b. During the total storage period the moisture content of the goods never decreases to the so-called "equilibrium moisture content" with respect to the surrounding air. In other words, the goods act as an

inexhaustible source of moisture to the room air. It is moreover assumed that the food surfaces act as wet or frosted, i.e. as saturated, surfaces. This is true, with almost no exceptions, in the case of unpacked food products (fruits, vegetables, meat, etc.) and has been confirmed by numerous experiments. As an example reference is made to (6).

The assumption of perfect mixing is assumed to hold. In this assumption the cold air coming from the evaporator is considered to mix directly and completely with the total air content of the storage room. The cold air stream is looked upon as a cold source as distinct from the various heat sources in the room. According to the perfect mixing assumption the temperature and the humidity ratio of the air are uniformly distributed throughout the whole room and air in this state is ducted to the evaporator. Although such assumption without doubt reveals a poor image of the actual situation in cold rooms, in practice it turns out to be a more than satisfactory approximation.

A last assumption is made concerning the influx of heat and moisture from external sources into the room, more particularly the influx due to infiltration, heat conduction and moisture diffusion through the walls. Actually these quantities are a rather complex function of many variables, two of these variables being the temperature and the humidity ratio of the room air itself. The dependence on external heat and moisture sources will not be expressed explicitly in the analysis to follow as these gains from outside sources will be represented by arbitrary time-dependent functions. Although a weak point of the proposed method, the fact just mentioned should not be considered as

a major disadvantage for following reasons. Unlike similar analyses of the same topic (for instance by means of rheo-electric simulation techniques) the present study is more concerned with the dynamic response of the room humidity to arbitrary heat input functions. The same is true for the automatic control activities discussed further: they aim at intercepting all possible perturbances of the heat and mass influx apart from the fact whether and how they occur. However it must be noted too that even the largest deviation of the room air conditions from a predetermined value remains insignificant, so that the effect of such deviations accounts for only very few percent of the influx quantities.

3. The Unsteady-state Balances of Cold Rooms

The perfect mixing assumption permits writing two differential equations that govern the dynamic behavior of a cold room. These equations are found by making an enthalpy balance on the air and a mass balance on the water-vapor within the air in the cold room. They are:

$$Q_{in}(\theta) + Q_F - \delta Q_E = R \frac{dH}{d\theta} \quad \dots (1)$$

$$M_{in}(\theta) + M_F - \delta M_E = R \frac{dW}{d\theta} \quad \dots (2)$$

where $Q_{in}(\theta)$: the influx of heat from external sources (kcal/hr)

$M_{in}(\theta)$: the influx of water-vapor from external sources (kg/hr)

Q_F : heat flowing from the cooling load to the room air
because of temperature fluctuations (kcal/hr)

M_F : moisture loss of the cooled goods (kg/hr)

Q_E : the heat removed by the evaporator (kcal/hr)

M_E	: the moisture removal by the evaporator	(kg/hr)
R	: the total air content of the cold room	(kga)
H	: the room air enthalpy	(kcal/kga)
W	: the humidity ratio of the room air	(kg/kga)
θ	: the time coordinate	(hr)
δ	: the so-called delta-function or Dirac-function;	

$\delta = 1$ when the compressor is running,

$\delta = 0$ during stand-still periods of the compressor.

The use of the delta-function implies neglecting the start-up time when the compressor is working at a reduced rate of heat removal (the evaporator pressure first has to drop off to its working point). This assumption is justified because of its simplicity.

The equations (1) and (2) completely describe the dynamic behavior of a cold room. The different terms they contain must therefore be worked out in terms of appropriate variables. This will be done in the subsequent chapters.

BASIC HEAT AND MASS TRANSFER EQUATIONS

The total heat and the mass transferred from a wet or frosted surface element into a passing air stream can be expressed by the following equations

$$dQ = G dq = h_c (T_s - T) dA + h_d (W_s - W) (H_g - H_{fw}) dA \quad \dots (3)$$

$$dM = G dW = h_d (W_s - W) dA \quad \dots (4)$$

where

Q	: the heat flow; q when related per kg air;	(kcal/hr)(kcal/kghr)
M	: the mass flow (transferred water-vapor)	(kg/hr)
G	: the air flow rate	(kg/hr)
A	: surface area	(m ²)
T	: temperature	(C)
H	: enthalpy	(kcal/kg)
W	: humidity ratio	(kg/kg)
h_c	: convection heat transfer coefficient	(kcal/m ² hr C)
h_d	: convection mass transfer coefficient	(kg/m ² hr kg kga ⁻¹)
-	: no subscript: bulk flow properties of the air	
g	: properties of water-vapor	
f	: properties of liquid water or ice	
s	: properties evaluated within the saturated interface layer of air, immediately next to the surface	
w	: denotes properties to evaluate at the liquid water or ice temperature.	

An accurate value of h_c should account for the fact that the molecular transfer of heat takes place in a kind of downstream motion, caused by the presence of mass transfer (7). A short cut approximation was given as early as in 1937 (8). In psychrometric applications the correction factor is neglectible (9).

One may introduce the perfect gas relations for moist air (10), a fair approximation of more accurate values (11), and the dimensionless group $h_c/h_d C_{ps}$, designated as the "Lewis-number" (Le) (see ref. (12) and (13)). This results by straight-forward substitutions in the following equation:

$$\frac{dq}{dW} = Le \frac{(H_s - H)}{(W_s - W)} + (1 - Le) H_g - H_{fw}$$

The factor C_{ps} appearing in Le is the specific heat of moist air, which in foregoing derivation has been evaluated at the saturation conditions existing next to the wet or frosted surface.

Various methods have been used to determine Le. Arnold's film theory leads to a complicated expression (14)(15), and boundary layer theory arrives at a result

$$h_c/h_d = (Pr/Sc)^m (k/D)$$

where Pr represents the Prandtl number, Sc the Schmidt number, k the thermal conductivity of air, D the diffusion coefficient of water-vapor into air, and m an exponent depending on the geometry of the problem and the type of flow (for turbulent flow m equals 0.33). An average value for Le on the basis of the last equation would be: $Le = 0.91$.

For air-vapor mixtures the Lewis number may be taken equal to one, which was shown again in 1937 (16). In psychrometric calculations the equality

$$Le = 1 \quad \dots (5)$$

is generally encountered. Hence the equation above can be written:

$$\frac{dq}{dW} = \frac{(H_s - H)}{(W_s - W)} - H_{fw} \quad \dots (6)$$

As such equation (6) excludes all heat transfer other than that following from a mixing process of bulk air with saturated air in the boundary layer. On the

basis of this explanation the equation has often been used to describe cooling and dehumidification processes, for instance in the famous paper (17).

Besides the heat and mass transfer, described by the Eqs. (3) and (4), the transport of enthalpy from the surface to the air can be expressed in a similar way. It will consist of two parts: the heat transferred plus an additional term accounting for the transport of moisture itself, carrying the enthalpy it possesses as liquid water (ice). The transport of enthalpy is found by dropping the term H_{fW} from Eq. (3), which results in:

$$G dH = h_c (T_s - T) dA + h_d (W_s - W) H_g dA \quad \dots (7)$$

Following the same procedure used in developing the heat equation yields enthalpy transferred per unit mass of water-vapor, an expression analogous to Eq. (6) concerning the heat transferred. Thus, it is derived:

$$\frac{dH}{dW} = \frac{(H_s - H)}{(W_s - W)} = \frac{1}{L} \quad \dots (8)$$

Eq. (8) represents a straight line element on the psychrometric chart. The slope $W_s - W/H_s - H$ of this element will further be called the "humidity slope" and designated by the symbol L .

For practical reasons Eqs. (3), (4) and (7) are simplified by substitution of the perfect gas relations for moist air, by taking the Lewis relation into account and by introducing the humidity slope. The following equations are easily obtained:

$$G dH = \frac{h_c}{C_{ps}} (H_s - H) dA \quad \dots (9)$$

$$G dW = \frac{h_c}{C_{ps}} (W_s - W) dA \quad \dots (10)$$

$$G dq = \frac{h_c}{C_{ps}} (H_s - H)(1 - LH_{fw}) dA \quad \dots (11)$$

In the last equation the term LH_{fw} is usually neglected: in the normal range of refrigeration applications it accounts for no more than a few percent of the total heat transfer (depending upon the relative importance of the latent with respect to the sensible heat transfer, the former causing the effect of moisture deposition on the surface). Neglecting the term just discussed reduces Eq. (11) into the extremely simple form:

$$G dq = \frac{h_c}{C_{ps}} (H_s - H) dA \quad \dots (12)$$

Neglecting the term H_{fw} however does not result in a simplification when a graphical procedure on the psychrometric chart is used. As shown in fig. 1 by drawing a line of constant enthalpy e and an adiabatic saturation line p , both going through the representation point S of the saturated air conditions next to the surface, the following rule may be derived:

"The actual driving force in wet-surface heat-transfer is an enthalpy difference, given directly by a vertical line segment on the psychrometric chart; lying on a line of constant humidity ratio through the entering air state, it is measured by the vertical distance from this air state to the intersection with the isotherm of the surface temperature T_s in the fog region; moreover, if the length of this segment is measured in temperature units, corresponding with the line of constant humidity W_s , an equivalent temperature difference is obtained by means of which the performance of a wet heat exchanger can be calculated directly as that of a dry surface."

This rule is based on the analytical expression of the adiabatic saturation line p (fig. 1), which is given by

$$H - H_s = H_{fw} (W - W_s)$$

and, moreover, on the fact that in the fog region of the psychrometric chart, the lines of constant thermodynamic wet-bulb temperature or adiabatic saturation lines exactly coincide with isothermal lines of constant temperature T_s .

THE COOLING LOAD AS A HEAT AND MASS EXCHANGER

1. Mass Exchange

The cooling load in cold storage rooms acts as a heat and mass exchanger. The air next to its surface is saturated, an assumption made before and strictly speaking only valid for unpacked food products.

Eq. (10) may directly be applied to this case and becomes:

$$dM_F = G dW = \frac{h_c}{C_{ps}} (W_{sF} - W) dF$$

where M_F moisture loss of the foods (kg/hr)

W_{sF} humidity ratio of the saturated interface layer
of air (subscript s denotes saturation) (kg/kg a)

F total surface area of the foods (m²)

The foregoing equation may be integrated over the total surface area of the cooled products. According to the perfect mixing assumption, the humidity ratio of the room air is constant throughout the room. Likewise, W_{sF} may be taken as a constant for all products, a uniform storage temperature throughout the room being a reasonable assumption. Thereby the integration simply results in:

$$M_F = \frac{h_{cF}}{C_{psF}} (W_{sF} - W) F \quad \dots (13)$$

Both h_{cF} and C_{psF} must be evaluated at the surface conditions of the goods.

The quantity M_F , as found in Eq. (13), is the total moisture loss of the goods. It is to be substituted into the balance equation (2).

2. Heat Exchange

For the cooling load a thermal balance can be set up, which takes the form

$$Q_{\text{rad}} + Q_{\text{bio}} - Q_{\text{air}} = m_F C_F \frac{d}{d\theta} (\bar{T}_F)$$

where	Q_{rad}	thermal radiation absorbed by the goods	(kcal/hr)
	Q_{bio}	biologic heat generation within living storage products as fruits, vegetables, etc., when kept above 0 C	(kcal/hr)
	Q_{air}	sensible and latent heat transferred from the goods to the surrounding room air	(kcal/hr)
	m_F	total mass of the storage products	(kg)
	C_F	the specific heat of the storage products	(kcal/kg C)
	\bar{T}_F	mean product temperature from surface to core	(C)

Fortuitously in almost all refrigeration applications the quantities Q_{rad} and Q_{bio} are negligibly small. Values of Q_{bio} as listed in (18,19) are available to prove this. Q_{rad} however depends slightly on the type of evaporator used. Therefore in deep-freeze storage rooms large bare coil evaporators may be preferred to finned units as far as product desiccation goes. Both terms Q_{rad} and Q_{bio} tend to shift the representation point for the surface conditions of the products towards higher humidity values on the saturation curve of moist air. If both terms are zero, steady-state will be reached at the thermodynamic wet-bulb temperature of the room air. With the approximation that Q_{rad} and Q_{bio} both are zero, the transient equation above becomes, according to Eq. (12),

$$Q_{\text{air}} = \frac{h_c}{C_{ps}} (H_{SF} - H) F = -m_F C_F \frac{d}{d\theta} (\bar{T}_F)$$

This equation has been integrated on the same basis as Eq. (13).

For simplicity it is assumed now that the thermal resistance at the food

surface constitutes the major part of the total resistance against heat flow of the goods. The type of heat flow allowed for is thus Newtonian heating or cooling and temperature gradients within the foods are considered negligible. A more complete analysis would require methods out of the scope of this work. If only small temperature fluctuations, as for example in a storage chamber for precooled goods, are taken into consideration the heat exchange between air and products is small. Moreover the air velocities in cold rooms are small and therefore the surface resistance high. For these reasons the assumption of Newtonian heating and cooling although roughly approximate, seems appropriate to be used here.

Hence the mean temperature of the foods may be replaced by their surface temperature. Linearizing the saturation curve and introducing the slope

$$\beta_F = \left. \frac{dH_s}{dT_s} \right|_{T_{sF}} \quad \dots (14)$$

permits one to write for the heat exchange between goods and air:

$$Q_F = \frac{h_{cF}}{C_{psF}} (H_{sF} - H) F = - m_F C_F \beta_F^{-1} \frac{dH_{sF}}{d\theta} \quad \dots (15)$$

where Q_F is the heat flow from goods to air, to substitute into Eq. (1)

h_{cF} and C_{psF} are to be evaluated at the food surface, which is indicated by the subscript F.

Equation (14) apparently justifies a former statement, made on page 3, that the best criterion for the actual storage temperature of the foods is the thermodynamic wet-bulb temperature of the room air. This follows from the fact that changes in the temperature of the food surface are caused by differences in the wet-bulb temperature, approximately the enthalpy, between the room air and that in the saturated layer next to the food surface.

THE EVAPORATOR AS A HEAT AND MASS EXCHANGER

1. Heat and Mass Transfer from/to a Surface at Uniform Temperature

Eqs. (9), (10) and (11) - or, as an approximation (12) - describe the heat and mass transfer from or to a surface element. In order to be complete, one more equation can be added to this group. This equation contains the first term of the right-hand member of Eq. (3) and describes the sensible heat transferred from the surface element to the air:

$$G C_p dT = h_c (T_s - T) dA \quad \dots (16)$$

No difficulties arise in the integration of the group of equations, just mentioned, over a surface at a constant temperature T_s , whence W_s and H_s are constant too. An arbitrary state (no subscript) of the air between the inlet state 1 and the outlet state 2, where the letter a denotes a part of the total surface area A, is characterized by the following equations:

$$H - H_s = (H_1 - H_s) e^{-h_c a / G C_{ps}} \quad \dots (17)$$

$$W - W_s = (W_1 - W_s) e^{-h_c a / G C_{ps}} \quad \dots (18)$$

$$T - T_s = (T_1 - T_s) e^{-h_c a / G C_p} \quad \dots (19)$$

Applying these equations to the particular case of the final air state and rearranging, results in:

$$H_1 - H_2 = (H_1 - H_s)(1 - e^{-h_c A / G C_{ps}}) \quad \dots (20)$$

$$W_1 - W_2 = (W_1 - W_s)(1 - e^{-h_c A / G C_{ps}}) \quad \dots (21)$$

$$T_1 - T_2 = (T_1 - T_s)(1 - e^{-h_c A / G C_p}) \quad \dots (22)$$

Dividing Eq. (18) by Eq. (17) shows that the process curve 1-2 is a

straight line on the psychrometric chart, for the humidity slope, L , defined in Eq. (8), remains constant during the whole process. This is the so-called straight-line law, analysed in (20) and (21), investigated further in (22) and (23). Since L remains constant, the heat transfer for the whole process is given by:

$$Q = G (1 - LH_{fw})(H_s - H_1)(1 - e^{-h_c A/G C_{ps}}) \quad \dots (23)$$

Eq. (23) is obtained by integrating Eq. (11). If however the approximate form (12) is used (with omission of the water deposited on or withdrawn from the surface) the total heat transfer reduces to the simple expression:

$$Q = G (H_s - H_1)(1 - e^{-h_c A/G C_{ps}}) \quad \dots (24)$$

Since the specific heat C_p of the passing air stream does not notably differ from C_{ps} , the former may be replaced by the latter in Eqs. (19) and (22). All equations from (20) up to (25) then contain the common factor

$$J = (1 - e^{-h_c A/G C_{ps}}) \quad \dots (24b)$$

This factor is usually referred to as the "surface contact factor". Its use is based entirely on the assumption of Le being one. In fig. 1 it is shown how the surface contact factor may be used directly in the graphical determination of the process curve on the psychrometric chart.

In cases where extreme accuracy is required or where the Lewis relation does not apply, the basis for the computation of process curves is Eq. (19) and the following, more original form of Eq. (18):

$$W - W_s = (W_1 - W_s) e^{-h_d a/G}$$

For this particular case reference is made to (24). In all cases the mass transfer is given by:

$$M = G (W_s - W_1)(1 - e^{-h_d A/G}) \quad \dots (25)$$

2. Heat and Mass Transfer from/to a Surface at Non-uniform Temperature

In the foregoing section the point of view was restricted to the presence of some surface, exchanging heat and mass with the surrounding air, and being at a uniform temperature. Consideration was actually given to the most outer surface of an exchanger, in this case the liquid film or frost layer.

In practice however the assumption of a uniform surface temperature turns out to be highly unrealistic. Three possible reasons may be proposed to explain why direct expansion evaporators display non-uniform surface temperatures.

A. As a first reason there is the presence of a pressure drop and, in flooded evaporators, of liquid columns. Ingenious systems have been designed to counter these drawbacks, definitely disadvantageous in low-temperature systems. A fine example thereof, with many others, can be found in (25).

B. Variations of the heat flow resistance between surface and refrigerant, always occurring in extended surfaces, so widely used, also cause non-uniform temperatures. In finned evaporators the temperature of the film surface is a function of the temperature along the fin itself and of the film thickness, which varies along the fin. This variation in turn depends on geometric factors, on the action of gravitational and hydrodynamic forces exerted by the passing air stream, and last but not least on the rate of latent heat transfer or condensation itself. This last effect in particular is of basic importance since it forms an essential mechanism of wetted surface heat transfer.

An exact solution of the problem, even when the fin surface would be at a uniform temperature, seems almost impossible. Ref. (26) presents a most profound analysis, based on Nusselt's condensation theory.

Although somehow unrealistic, the assumption of a constant film thickness

is very common. Thoroughly investigated in ref. (27) under the additional assumption of a complete covariance of sensible and latent heat transfer (in practice this comes down to neglecting the curvature of the saturation curve of moist air) it provides some simple results and seems to be a reliable approximation, for the proper influence of the (varying) film thickness on the total thermal resistance in most cases turns out to be small. From this study, the following results (in a slightly adapted form) will be retained.

Per definition, an overall heat transfer coefficient U is introduced by the equation:

$$dQ = G \, dq = U (H_{sR} - H) \, dA = G \, dH \quad \dots (26)$$

an equation, which, according to the convention of signs adopted before, describes the heat flow from the evaporator to the air. More particularly, Eq. (26) describes the amount of heat dQ delivered to the air by a small but finite part dA of the evaporator surface; dA should be large enough to consist of both pipe and fins, in proportion to their total surface areas, but limited, so that the passing air can be described by a single value H .

H_{sR} finally refers to a fictitious enthalpy, that saturated moist air would have at the refrigerant's evaporating temperature T_R .

The overall heat transfer coefficient U can be expressed in terms of two thermal resistances: an outside surface resistance R_s and an internal one, R_i :

$$U = \frac{1}{R_i + R_s} \quad \dots (27)$$

These resistances are defined as follows:

$$R_s = \frac{C_{ps}}{h_c} \quad \dots (28)$$

$$R_i = \frac{\beta_R A}{h_i A_i} + \frac{A}{A_p + \phi A_\phi} \frac{\beta_\phi y}{k} + \frac{(1 - \phi) A}{A_p + \phi A_\phi} \frac{C_{ps}}{h_c} \quad \dots (29)$$

where:

- A_1, A_p, A_ϕ : the inside tube surface area and the outside surface areas respectively of bare pipe and fins
 h_1, h_c : both. the in and outside heat transfer coefficients
 k, y : thermal conductivity and average thickness of moisture film

Furthermore (cf. Eq. (14)):

$$\beta_R = \left. \frac{dH_s}{dT_s} \right\}_{T_R} \quad \text{and} \quad \beta_\phi = \left. \frac{dH_s}{dT_s} \right\}_{T_{fin \text{ av.}}} \quad \dots (30)(31)$$

The quantity ϕ finally denotes a wet fin efficiency, defined as

$$\phi = \frac{Q_{(\text{actual fin})}}{Q_{(\text{for fin at base temperature})}}$$

According to the author of the original treatment (27), this wet fin efficiency can be derived directly from the theoretical expression for the dry fin efficiency merely by replacing the outside heat transfer coefficient h_c in the expression for the latter by an analogous coefficient h_c^w , taking on the role of h_c in the wet case, and defined as

$$\frac{1}{h_c^w} = \frac{C_{ps}}{\beta_\phi h_c} + \frac{y}{k}$$

If heavy frost conditions are avoided, h_c^w is approximately

$$h_c^w = \beta_\phi h_c / C_{ps}$$

Despite all the work undertaken to determine boiling and outside heat transfer coefficients, the most reliable method to evaluate the performance of a particular evaporator remains the experimental determination of the overall heat transfer coefficient. This method however should be based on and make use of the possibilities offered by the Eqs. (27), (28) and (29). It is easy to see that for a given type of evaporator, within a given range of evaporating temperatures, the expression for U contains three essentially variable quantities, namely h_1, h_c and y . For under these circumstances the wet fin

efficiency (see ref. (28)) and the thermal conductivity of water films or frost layers (strictly spoken time-dependent) may be looked upon as fairly constant. As for the variable parameters, just mentioned, some theoretical correlations were set up, recommended for practical use in the "ASHRAE Guide and Data Book" or in the "Handbuch der Kältetechnik". For a complete description of the use of these formulas, consult the original references.

These correlations are:

a. the refrigerant side heat transfer coefficient h_1 :

- flooded evaporators:

$$h_1 = c (Q/A)^n \quad \dots (32)$$

where c and n are constants, depending on each particular case.

The following values have been proposed for the exponent n :

for F12: $n = 0.60$ (29); for NH_3 : $n = 0.565$ at $T_e = -10$ C

for F22: $n = 0.70$ (29); (31)(32) $n = 0.602$ $T_e = -20$ C

for SO_2 : $n = 0.54$ (30); $n = 0.643$ $T_e = -30$ C

- dry expansion evaporators: the most reliable correlation for the forced convection boiling coefficient in this case, together with its justification, is given in (33) to be:

$$h_1 = c \frac{(gQ/A)^{0.5}}{d^{1.5}} \quad \dots (33)$$

It is based on ref. (34) and is in excellent agreement with the results of other investigators (35)(36)(37)(38)(39)(40).

In Eq. (33) g represents the amount of refrigerant flowing through the dry expansion coil of inside diameter d . As in Eq. (32) Q/A denotes the heat removed per unit surface area and c is a constant.

In this type of evaporator the amount of refrigerant g is directly proportional to the heat removal, at least when steady-state exists, the degree of superheating being regulated by the feeding valve.

Eq. (33) for a given evaporator therefore may be written:

$$h_i = K Q \quad \dots (34)$$

This linear law corresponds closely to another relationship, often used in design study and experimentally verified in (41), namely

$$h_i = K_1 + K_2 Q/A \quad \dots (35)$$

b. the outside heat transfer coefficient h_c :

- free convection systems: will not be considered here.
- forced convection systems: the critical Reynolds number, 100, is always exceeded; the correlation recommended in (42), from more specified data (43)(44)(45)(46), is that of turbulent flow heat transfer

$$\overline{Nu} = c Re_{max}^{0.6} Pr^{0.33}$$

in the present analysis simply rewritten in the following form:

$$h_c = c G^{0.6} \quad \dots (36)$$

c. the thickness of the moisture deposit: $y = f \quad \dots (37)$

- for water films: treated as a constant.
- for frost layers: a function f , depending on the frost conditions.

Combining the Eqs. (27)(28)(29)(36)(37) and either (32) or (34), depending on the case, results in the following "semi-theoretical" relationships for the overall heat transfer coefficient U , expressed in terms of the total heat removal Q , the air flow rate G , and the thickness f of the moisture layer:

- flooded evaporators:

$$U = \frac{Q^n}{A + B (f + CG^{-0.6}) Q^n} \quad \dots (38)$$

where A , B and C are constants.

- dry expansion evaporators (formulas based on Eq. (34) and not on Eq. (35) are given):

$$U = \frac{Q}{A + B (f + CG^{-0.6}) Q} \quad \dots (39)$$

For a fixed air flow rate and given average frost conditions (or water film) Eq. (39) may even further be reduced to:

$$U = \frac{Q}{A + B Q} \quad \dots (40)$$

If the overall enthalpy difference $\Delta H_m = H_{sR} - H_m$ (cf. Eq. (43)) is preferred as a variable, Eq. (39) takes the form:

$$U = \frac{\Delta H_m - A/F}{B (f + C G^{-0.6})} \quad \dots (41)$$

where F stands for the total surface area of the evaporator. It should be remarked here that for flooded evaporators there exists no simple relation, analogous to Eq. (41), in terms of the overall enthalpy difference (i.e. the so-called log mean enthalpy difference).

The practical advantages of the foregoing equations are obvious:

1. They permit extrapolations from a minimum of experimental data.
2. The number of experimental measurements can be reduced to a reasonable number, necessary to evaluate the constants A, B and C.

Some other aspects are worth being noted:

1. If the heat flux becomes large, U tends to an asymptotic value:

$$U = \frac{1}{B (f + C G^{-0.6})}$$

2. If the heat flux tends to zero, the slope of the function U is given by:

$$\left. \frac{dU}{dQ} \right\}_{Q=0} = \frac{1}{A}$$

and the value of U itself is found from Eqs. (38) and (39):

$$U_{Q=0} = 0$$

(In this particular case Eq. (41) cannot be derived from Eq. (39) because of mathematical considerations).

The foregoing analysis of the overall heat transfer coefficient concludes the study of the heat flow resistance from surface to refrigerant in evaporators.

C. The third reason why non-uniform surface temperatures occur in evaporators are the enthalpy variations of the air itself along the evaporator surface. For at each point the surface temperature takes an equilibrium value between the air and refrigerant temperatures. This effect may well be compared with the kind of situation occurring in a counter-flow heat exchanger.

In theoretical studies the effect of pressure drops and liquid columns in evaporators is usually neglected. Hence the refrigerant's evaporating temperature may be considered as a constant throughout the evaporator. As far as the overall heat transfer coefficient U is concerned, the simplest method is to take for U a constant value, evaluated on the basis of the average surface conditions. With these assumptions Eq. (26) may be integrated, which results in

$$-Q_E = Q = G (H_{sR} - H_1) (1 - e^{-UA/G}) = G (H_2 - H_1) \dots (42)$$

Eq. (42) is often written in terms of an overall or logarithmic mean enthalpy difference ΔH_m as follows

$$-Q_E = Q = U A \Delta H_m \dots (43)$$

where

$$\Delta H_m = \frac{H_2 - H_1}{\lg \frac{H_{sR} - H_1}{H_{sR} - H_2}} = \frac{H_2 - H_1}{\frac{UA}{G}} \dots (44)$$

It should be remarked that according to the convention of signs, adopted thus far, the positive direction of heat flow is taken from the surface to the air, whence in the foregoing equations the quantity of heat Q_E removed by the evaporator, is given the opposite sign. .

Instead of being integrated over the whole evaporator surface, Eq. (26) can also be integrated over a part A of this surface. Such integration would

result in the following expression for the enthalpy of the air during its passage

$$H - H_{sR} = (H_1 - H_{sR}) e^{-UA/G} \quad \dots (45)$$

Eq. (45) however implies that the air flow displays following characteristics:

- a. the air is completely cross-mixed: it is at uniform thermodynamic conditions across its flow passage which contains the entire flow.
- b. the air is unmixed as a whole but completely mixed in separate air flow channels: at the exit of each channel the air is in the same state; (of course, area A denotes the contact surface of each channel apart)

On the basis of Eq. (27), where the total thermal resistance of the system was subdivided into an internal and a surface resistance, and on the basis of Eq. (26), a representative mean surface enthalpy H_s can be associated with each little part dA of the evaporator surface. At a particular cross-section the surface enthalpy H_s is given by

$$H_s - H_{sR} = UR_1 (H_1 - H_{sR}) e^{-UA/G} \quad \dots (46)$$

obtained from Eq. (45) and from the equality

$$\frac{H_s - H_{sR}}{H - H_{sR}} = U R_1 \quad \dots (47)$$

derived from Eq. (26) by equalizing:

$$dQ = U (H_{sR} - H) dA = \frac{1}{R_1} (H_{sR} - H_s) dA$$

Hitherto, the Eqs. (42) and (45) have always been considered as the final results obtainable by analytical procedures in the theory of cooling and dehumidification of moist air. An analogous derivation for the air humidity was

taken for impracticable (47)(48). Since the problem is of basic importance in the design of cooling towers and evaporators, various graphical procedures have been worked out (49)(50) but the first and by far the simplest one remains the method of Merkel and Mickley, conceived by the first as early as in 1927 (51)(52)(53). In this method one makes use of the temperature-enthalpy diagram for moist air, which becomes more and more popular in Europe. An excellent survey of the method is presented in (54) and (55). Of theoretical interest is another procedure proposed in (56).

The author however, strongly impressed by the simplicity of Merkel's procedure, which led him to introducing Eq. (46), was able to set up a completely mathematical solution of the problem under consideration. The derivation thereof is given now.

Basic assumptions are: 1. The effect of both pressure drop and liquid columns within the evaporator are neglected. The refrigerant's evaporating temperature is constant everywhere in the evaporator (cf. Merkel's method).

2. The type of air flow can be described by one of both theoretical models, described on page 24 (cf. the analogous solution for the enthalpy).

3. The overall heat transfer coefficient is fairly constant throughout the evaporator and may locally be replaced by its mean value for the whole evaporator (cf. all previous methods).

4. The saturation curve of moist air can be approximated by a polynomial (algebraic function) up to any degree of accuracy.

5. Mathematically it is assumed that the air enthalpy H , the air humidity W , as well as the enthalpy H_s , the temperature T_s and the humidity W_s of the air next to the evaporator surface are continuous

and integrable functions in terms of the contact surface area. These assumptions are fulfilled on the basis of Eq. (45), the conservation of mass, Eq. (46) and the saturation curve of moist air.

The equations to start the discussion with are:

Eq. (45) for the enthalpy of the air stream:

$$H - H_{sR} = (H_1 - H_{sR}) e^{-UA/G} \quad \dots (45)$$

Eq. (46) for the enthalpy of the surface air layer:

$$H_s - H_{sR} = U R_1 (H_1 - H_{sR}) e^{-UA/G} \quad \dots (46)$$

Eq. (10) for the transfer of water-vapor from the surface to the air:

$$G dW = \frac{h_c}{C_{ps}} (W_s - W) dA \quad \dots (10)$$

The mathematical expression of the saturation curve given by the humidity of saturated air as a function of its enthalpy. A high degree of accuracy can be obtained with a quadratic expression:

$$W_s - W_{sR} = m (H_s - H_{sR}) + n (H_s - H_{sR})^2 \quad \dots (48)$$

In the foregoing equations following symbols are used:

H, W	enthalpy and humidity ratio of the air flow
H_s, W_s, C_{ps}	enthalpy, humidity ratio and specific heat of saturated air at the temperature of the evaporator surface
H_{sR}, W_{sR}	enthalpy and humidity ratio of saturated air at the refrigerant's evaporating temperature
U, R_1, R_s	respectively the overall heat transfer coefficient, the internal and the surface thermal resistances, the last being equal to $R_s = C_{ps} / h_c$ (cf. Eq. (28))
h_c	the outside heat transfer coefficient
m, n	two coefficients, obtainable from the methods of finite differences or least squares.

A the evaporator surface area the air at a given cross-section has already been in contact with

dA a small part of the evaporator surface as specified on page 18, see Eq. (26).

On the basis of the continuity assumptions (sub 5, page 25) symbol d however may be treated as a differential.

Combining the Eqs. (10), (46) and (48) results in the differential equation:

$$\frac{dW}{dA} + \frac{W}{GR_s} = \frac{W_{sR}}{GR_s} + \frac{mUR_1}{GR_s} (H_1 - H_{sR}) e^{-UA/G} + \frac{nU^2R_1^2}{GR_s} (H_1 - H_{sR})^2 e^{-2UA/G} \dots (49)$$

The solution of the reduced equation equals

$$W = C e^{-A/GR_s}$$

and that of the complete equation will take the form:

$$W = C e^{-A/GR_s} + K_1 + K_2 e^{-UA/G} + K_3 e^{-2UA/G}$$

By substitution of this expression into the differential equation the coefficients K_1 , K_2 and K_3 can be evaluated. The general solution of the differential equation then becomes

$$W = C e^{-A/GR_s} + W_{sR} + m(H_1 - H_{sR}) e^{-UA/G} + \frac{n(H_1 - H_{sR})^2}{1 - R_s^2/R_1^2} e^{-2UA/G} \dots (50)$$

The constant of integration C is to be found from the boundary condition:

$$\text{for } A = 0, W = W_1$$

It is directly found to be

$$C = (W_1 - W_{sR}) - m(H_1 - H_{sR}) - \frac{n(H_1 - H_{sR})^2}{1 - R_s^2/R_1^2} \dots (51)$$

The equations (50) and (51) may be used instead of any of the graphical procedures which are known. The humidity ratio of the air at the evaporator outlet can directly be calculated by substituting for A the total surface area of the evaporator. The method is easy and almost completely correct, for the saturation curve of moist air is very closely approximated by a quadratic curve within the limited range of evaporator temperatures. Nothing prevents the replacement of Eq. (48) by an algebraic expression of the third degree (or even higher).

From the practical point of view however, a more important remark is to be made. In almost all practical cases the assumption of a constant evaporating temperature of the refrigerant is highly unrealistic. One therefore even may content himself with a linear approximation of the saturation curve:

$$W_s = W_{s,\bar{T}_s} + \gamma (H_s - H_{s,\bar{T}_s}) \quad \dots (52)$$

where \bar{T}_s stands for the (expected) average surface temperature and γ represents the slope

$$\gamma = \left. \frac{dW_s}{dH_s} \right|_{\bar{T}_s} \quad \dots (53)$$

clearly distinct from another slope β of the saturation curve, some particular values of the latter being shown in Eqs. (14), (30) and (31).

From Eq. (52) can easily be derived that

$$W_s - W_{sR} = \gamma (H_s - H_{sR}) \quad \dots (54)$$

Replacing Eq. (48) by Eq. (54) and solving by an analogous procedure the system of Eqs. (10), (45), (46) and (54) leads to a simple solution for the humidity ratio of the air at an arbitrary cross-section of the evaporator:

$$W - W_{sR} = (W_1 - W_{sR}) e^{-A/GR_s} + \gamma (H_1 - H_{sR}) \left\{ e^{-UA/G} - e^{-A/GR_s} \right\} \quad (55)$$

or:

$$W - W_{sR} = (W_1 - W_{sR}) e^{-h_c A / GC_{ps}} + \gamma (H_1 - H_{sR}) \left\{ e^{-UA/G} - e^{-h_c A / GC_{ps}} \right\}$$

The simplified solution, given by Eq. (55), will be taken as a basis for the further development of this study. From it an expression for M , the total transfer of water-vapor from the evaporator to the air (M is a negative quantity) is easily found to be:

$$M = G (W_2 - W_1) = G (W_{SR} - W_1) \left\{ 1 - e^{-h_c A / GC_{ps}} \right\} + \gamma G (H_1 - H_{SR}) \left\{ e^{-UA/G} - e^{-h_c A / GC_{ps}} \right\} \dots (56)$$

Eq. (56) as to its form and significance is completely analogous to Eq. (42) for the total heat transfer.

One may safely say that the assumption of a linearized saturation curve does not give rise to considerable errors (certainly lesser than those resulting from neglecting the pressure drop and liquid columns inside the evaporator). Therefore the following statement may be derived from Eqs. (42) and (56):

THEOREM I

"An evaporator, in general, may be thought of as a simple heat and mass exchanging surface with uniform surface conditions; this substitution has no effect, neither upon the heat nor upon the mass transfer, provided that said surface conditions are represented by a point on the saturation curve, which has a corresponding enthalpy value:"

$$H_{SE} = H_{SR} + \frac{Q_E}{G} \left\{ \frac{1}{1 - e^{-UA/G}} - \frac{1}{1 - e^{-h_c A / GC_{ps}}} \right\} \dots (57)$$

where E is a subscript which will be used throughout this text to denote the "equivalent evaporator surface" introduced and defined in foregoing statement. This equivalent evaporator surface will often be referred to as the "evaporator surface," without any further specification, unless misunderstanding would be possible. Q_E is the heat removed by the evaporator, taken positive.

PROOF. The same amount of heat must be removed respectively by the actual evaporator and by the equivalent surface. Hence Eqs. (42) and (24) must be equal to each other. Eq. (57) is directly obtained from

$$Q_E = G(H_1 - H_{sR})(1 - e^{-UA/G}) = G(H_1 - H_{sE})(1 - e^{-h_c A/GC_{ps}}) \quad \dots (57b)$$

Similarly the amount of moisture removed from the air must be the same in both cases. Denoting this quantity by the symbol M_E (taken positive), one may write on the basis of Eqs. (56) and (25)

$$\begin{aligned} M_E &= G(W_1 - W_{sR})(1 - e^{-h_c A/GC_{ps}}) - \gamma G(H_1 - H_{sR})(e^{-UA/G} - e^{-h_c A/GC_{ps}}) \\ &= G(W_1 - W_{sE})(1 - e^{-h_c A/GC_{ps}}) \end{aligned}$$

Hence the equivalent surface must be represented by a point on the saturation curve which has a corresponding humidity value:

$$W_{sE} = W_{sR} + \gamma(H_1 - H_{sR}) - \gamma(H_1 - H_{sR}) \frac{1 - e^{-UA/G}}{1 - e^{-h_c A/GC_{ps}}}$$

On the basis of Eq. (42) this equality can be written

$$W_{sE} = W_{sR} + \gamma(H_1 - H_{sR}) - \frac{\gamma Q_E/G}{1 - e^{-h_c A/GC_{ps}}}$$

which in comparison with Eq. (58) simply equals

$$W_{sE} = W_{sR} + \gamma(H_{sE} - H_{sR})$$

The conclusion is obvious: according to Eq. (54) both H_{sE} and W_{sE} are properties of one and the same point on the saturation curve. This point may thus be taken to represent the total evaporator surface. In this case both the heat and mass transfer will remain unchanged.

THEOREM II

"For all evaporators, where a linear approximation of the saturation curve may be used as a satisfactory simplification, the heat and moisture removal

from the air may be expressed in terms of the equivalent surface conditions as follows:"

$$Q_E = G \mathcal{V} (H - H_{SE}) \quad \dots (59)$$

$$M_E = G \mathcal{V} (W - W_{SE}) \quad \dots (60)$$

This theorem directly follows from the definition of the equivalent surface conditions. The variables H and W refer to the room air properties. \mathcal{V} is the evaporator surface contact factor, defined in Eq. (24b).

It need not be said that the introduction of an equivalent evaporator surface forms an original contribution to the study of heat and mass exchangers.

Moreover, it is thanks to this procedure only, that the problem of the dynamic response of cooled rooms becomes solvable. But the steady-state solution for the heat and mass exchange of cooling coils, will in the future be considerably simplified too, by the use of the new method.

THE DYNAMIC RESPONSE OF THE EVAPORATOR-COMPRESSOR LOOP

1. The Problem

Comparing with each other Eq. (15) and one of the expressions (59) or (42) directly reveals a striking difference. In both cases some quantity of heat (Q_F ; Q_E) is expressed in terms of two variables (H , H_{SF} ; H , H_{SE} or H_{SR}). In the first case Q_F , the heat transfer from the storage goods to the air, is determined by two equations, permitting the elimination of one of the variables H or H_{SF} . The quantity Q_F is then completely defined in terms of the second variable and of the time coordinate, appearing in one of said equations. In system dynamics the air-foods system would therefore be designated as a closed-loop system. But in the second case things seem to be different: apparently the heat removal is a function of two input variables viz. H and H_{SE} or H_{SR} . A close investigation of a cooling system however reveals an additional relationship exists between the heat removal and the variables under consideration, to wit the capacity output of the refrigeration system as a whole, i.e. the compressor-condensor unit and the evaporator performances synthesised together.

The study of this additional relationship will be the subject of this chapter. Its final purpose consists in finding the feedback relation, which would result in bringing the air-evaporator interaction into the form of a closed-loop. The problem, as it is posed here, is new and, due to its complexity, yet incomplete. Nevertheless the results, already obtained, seem to be far-reaching.

2. Performance Characteristics of the refrigeration Unit

The capacity of a refrigeration unit, as one often calls the compressor and condenser as a whole, is given by the following expressions, the last of which applies if the suction line to the compressor is thermally insulated:

$$Q_{\text{ex}} = M_o(I_o - I_i) = \frac{\lambda D}{v^*} (I_o - I_i) = \frac{\lambda D}{v^*} (I^* - I_i) \quad \dots (61)$$

where	Q_{ex}	the heat evacuated by the refrigeration unit	(kcal/hr)
	M_o	the mass flow rate to the compressor	(kg/hr)
	D	the rate of displacement of the compressor	(m ³ /hr)
	λ	the compressor's actual volumetric efficiency	
	I	the refrigerant enthalpy in various states	(kcal/kg)
	i	subscript, denoting the refrigerant inlet state	
	o	id. for the outlet state of the evaporator	
	$*$	superscript, denoting the refrigerant's properties just before the entrance of the compressor	
	v	specific volume	(m ³ /kg)

If the refrigerant gas is assumed to behave as a perfect gas, Eq. (61) may be rewritten as

$$Q_{\text{ex}} = \frac{\lambda D}{v''} (I''_{p^*} - I_i v''_{p^*}/v^*) \quad \dots (62)$$

where	$''$	superscript, denoting saturated vapor properties
	p^*	the compressor's suction pressure

For all practical calculations, ref. (57) recommends the following approximation, resulting in a small error of about one percent:

$$Q_{\text{ex}} = \frac{\lambda D}{v''} (I''_{p^*} - I_i) \quad \dots (63)$$

To make the analysis simpler and more meaningful, this approximation

will not be used in the sequel. It will be replaced by another expression, based on the following three assumptions:

- a. the displacement rate of the compressor is constant (the rotating speed and number of operating cylinders are fixed).
- b. the subcooling temperature in the condenser remains substantially constant, whence I_1 may be treated as a constant value.
- c. for each evaporating temperature there is one and only one value for the compressor capacity, resulting in a steady-state operation of the installation. (The foregoing assumptions are assumed to be fulfilled here).

From these assumptions it follows directly that the capacity of a refrigeration unit, operating at steady-state conditions, may be represented by a function

$$Q_{\text{ex}} = f(T_R) \quad \dots (64)$$

This function does not differ too much from a straight line at least for a limited range of evaporating temperatures. Hence it may be linearized as

$$Q_{\text{ex}} = a + b T_R \quad \dots (65)$$

In the further treatment this function will be assumed to be known or computable. Most manufacturers provide sufficient data for this purpose.

An important remark however must be made here. Up to now it has been assumed that the suction line does not contain an evaporator pressure-regulating valve. If such valve is present in the suction line to the compressor, the refrigerant gas undergoes a throttling process along a line of constant enthalpy, while passing through it. Thereby the gas pressure drops from the evaporating pressure down to that in the main suction line, the difference Δp between these pressures depending on the opening of the valve. If one assumes the refrigerant gas to behave as a perfect gas, the

refrigerant density before and behind the valve will be proportional to the pressure it has on both places. The volumetric efficiency of the compressor moreover depends on the suction pressure. These factors result in reducing the capacity. This reduced capacity equals the capacity the system would have without the pressure regulating valve, multiplied by a correction factor in order to take the above phenomena into account. The reduced capacity can thus be written

$$Q_{ex} = f(P^*/P_R) \cdot f(T_R) \quad \dots (66)$$

where P^* is the suction pressure of the compressor.

Once again, it should be emphasized that all the expressions, hitherto given, are related to steady-state operations of the compressor-evaporator unit.

The capacity curves of the unit, Eqs. (65) and (66), are expressed in terms of the evaporating temperature. It turns out however, that in many cases an expression in terms of the variable H_{SR} be more convenient, H_{SR} being the enthalpy of saturated moist air at the evaporating temperature T_R . The transition from the latter to the former variable presents no difficulties and may graphically be performed by means of the saturation curve of moist air. Linearization of this curve directly results in a new expression for Eq.

$$(65): \quad Q_{ex} = a^R + b^R H_{SR} \quad \dots (67)$$

$$\text{where} \quad b^R = b \beta_R^{-1} \quad (\text{cf. Eq. (30)})$$

and where a^R is a constant which can be determined from a single value of the function Q_{ex} .

3. Steady-state of the Evaporator

The evaporator reaches dynamic equilibrium (a so-called balance point) when the heat it receives from the air equals the instantaneous capacity of the refrigeration unit. An excellent treatment of this subject can be found in ref. (58). Mathematically, steady-state thus implies:

$$Q_E = Q_{ex} \quad \dots (69)$$

Replacing these quantities by their respective expressions, as given by the Eqs (42) and (67), gives the value of the steady-state evaporating temperature, which the refrigerant has after a sufficient time and which will depend on the room air enthalpy. Expressed by the function H_{sR} in terms of the prevailing room air enthalpy H , this temperature is characterized by

$$H_{sR} = \frac{G H \left(1 - e^{-\frac{UA}{G}} \right) - a^R}{G \left(1 - e^{-\frac{UA}{G}} \right) + b^R} \quad \dots (70)$$

The numerical value of U , which is to be used in the calculation of each particular value of H_{sR} , depends on the corresponding heat flow (Q_E or Q_{ex}) and is found from Eq. (38) or (39) according to the type of evaporator used.

One may even go further and relate to each particular value of the room air enthalpy the corresponding surface conditions (i.e. the mean or equivalent surface conditions as described on page 29) the evaporator will take after a sufficient time. Combining the Eqs. (70) and (57) and expressing Q_E in terms of H_{sE} and H is the next step. For the locus of surface conditions H_{sE} in terms of the room air enthalpy H the following expression is found:

$$H_{sE} = H - \frac{b^R \odot H}{G \updownarrow \odot + \updownarrow b^R} - \frac{a^R \odot}{G \updownarrow \odot + \updownarrow b^R} \quad \dots (71)$$

where in an effort to simplify the notation the "evaporator contact factor" \odot

has been introduced. This coefficient will be used throughout this text to designate the expression

$$\Theta = (1 - e^{-UA/G}) \quad \dots (72)$$

As such it is clearly distinct from the surface contact factor \mathcal{V} , defined by Eq. (24b).as

$$\mathcal{V} = (1 - e^{-h_c A/GC_{ps}})$$

Eq. (71) seems somewhat complicated. Therefore another method will be presented now which at the same time is simpler and more meaningful. Instead of expressing the evaporator surface conditions H_{SE} as a function of the room air enthalpy, the same surface conditions can be expressed in terms of the heat flux to the evaporator Q_E . This flux is, at least during steady-state operation, the heat evacuated by the refrigeration unit Q_{ex} . Therefore one simply eliminates the variable H_{SR} from the Eqs. (57) and (67). The resulting expression

$$H_{SE} = Q_E \left\{ \frac{1}{G\Theta} - \frac{1}{G\mathcal{V}} + \frac{1}{b^*} \right\} - \frac{a^*}{b^*} \quad \dots (73)$$

would be a straight line if the overall heat transfer coefficient U , appearing in the evaporator contact factor Θ , were a constant. Since this is not the case, one profits by a graphical procedure of solving the system of Eqs. (57), (67) and (38) or (39). As shown in fig. 3, this method is extremely simple. The result is a curve which delivers the steady-state values of the evaporator surface conditions as a function of the heat removed by the evaporator (or the heat evacuated by the refrigeration unit). This curve, $H_{SE}(Q_E)$, may then be approximated by a straight line. Within the range of expected surface temperatures this consequently provides an expression of the form:

$$Q_E = Q_{ex} = a^* + b^* H_{SE} \quad \dots (74)$$

This expression, together with the familiar expression of Q_E , given by Eq. (59)

immediately delivers an expression for H_{SE} , which is in form very similar to Eq. (70). This expression is

$$H_{SE} = \frac{G H (1 - e^{-h_c A / GC_{ps}}) - a^e}{G (1 - e^{-h_c A / GC_{ps}}) + b^e} \quad \dots (75)$$

The coefficients a^e , b^e evidently are to be determined graphically and will depend not only upon the evaporator used, but also upon the refrigeration unit and the specific range of evaporator temperatures for which the system is fit.

The Eqs. (74) and (75) permit a very simple analysis when they are applied to the influence of the evaporator upon the humidity in cold rooms. Their application however remains restricted to the steady-state or final value the evaporator surface conditions will reach, when a certain heat load is imposed upon the refrigeration system. The principal features of the method proposed here are:

1. the similarity of the equations, governing the heat and mass transfer to the evaporator, when expressed in terms of the equivalent evaporator surface conditions (one should compare the Eqs. (59) and (60) with the analogous pair of Eqs. (42) and (56)). By this fact, the straight-line law becomes fully applicable.
2. the effect of variations of the overall heat transfer coefficient with changing heat loads is already incorporated into the form of the modified capacity curve of the unit. (Of this curve Eq. (74) is the linear approximation).

The conclusion is obvious: if there would be no transient response of the evaporator system (the system would go from one steady-state value to another), then Eq.(75) would be the feedback relation, discussed on page 32.

4. The Dynamic Characteristics of Dry Expansion Evaporators

A. Evaporator Model

Based on a considerable number of simplifications, the analysis to follow is presented as an example of how the problem of the dynamic response of dry expansion coils can be approached. The model of evaporator investigated consists of multiple circuit coils supplied with refrigerant through a distributor. The individual circuit outlets join into a common suction header.

According to the types of design most commonly used, a cross-flow tube arrangement will be considered in which the direction of the air flow and the overall refrigerant motion through the evaporator are opposite to each other. In other words, the refrigerant enters the coils at that extremity of row depth where the air is leaving the evaporator. The warm entering air is brought in contact with the row of tubes which discharges into the suction header.

The individual circuits are identical in form and located in planes, parallel to each other and to the direction of air flow.

B. Heat Transfer Assumptions

In an effort to simplify the analysis and to make it more meaningful, the following assumptions are made as to the heat transfer part of the problem:

1. In order to account for the pressure drop and for the superheating in part of the coils, an average refrigerant temperature will be taken as representative for the total evaporator. It will be denoted by T_R , the corresponding enthalpy value of moist air being H_{SR} .

2. The thermal parameters of the system remain substantially constant at each moment and at all places within the evaporator.

3. The evaporator surface area and the mass of the material of pipe and fins is constant per unit depth of the evaporator in the direction of the air flow. This material, moreover, is considered to be homogeneous.

4. The material of an individual refrigerant pipe and of the fins attached to it, if any, is assumed to have a uniform temperature in any cross-section perpendicular to the pipe axis. This uniform temperature is assumed to be the average surface temperature of the element under consideration.

5. At the evaporator entrance section the air has uniform properties and a uniform and constant mass flow rate. The flow rate remains constant and uniform throughout the evaporator.

The foregoing assumptions, combined together, may be said to describe a special type of heat exchanger, in which the thermodynamic properties of the air are uniform at each cross-section of the evaporator. This is true because all possible variations of the system's characteristics in the direction of the refrigerant tubes have systematically been neglected. This procedure is often called lumping of the parameters.

As a conclusion, the total analysis of the coils is simplified by considering the air as a perfectly mixed fluid, i.e. as having the same properties across its flow passage, which contains the entire flow, at each cross-section of the evaporator.

C. Heat Transfer Equations

The following equations entirely describe the various quantities of heat which are involved in the heat transfer through a single row of tubes, across which the air flows in a perpendicular direction (cross-flow arrangement) (59).

Air side heat transfer:

$$- G \frac{dH}{dx} + \frac{A_x}{R_s} (H_s - H) = S_m \frac{\partial(\rho E)}{\partial \theta} = S_m \frac{\partial(C_v P/R)}{\partial \theta} = 0 \quad \dots (76)$$

where following symbols are used:

H	enthalpy of moist air, dry basis	(kcal/kg a)
E	internal energy of moist air, wet basis	(kcal/kg
H _s	enthalpy of saturated air, corresponding with the mean surface temperature of the tubes (including fins, if any)	(kcal/kg a)
R _s	the outside surface resistance of extended surfaces (cf. Eq. (28)), the driving force being an enthalpy difference	(m ² hr/kg a)
ρ	the density of moist air, wet basis	(kg/m ³)
C _v	the specific heat of moist air at constant volume, expressed on the basis of the kg as the unit of mass	(kcal/kg C)
R	gas constant of moist air, same basis as C _v	(m ² /sec ² C)
P	absolute (atmospheric) pressure	(kg m/sec ² m ²)
G	mass rate of air flow	(kg a/hr)
x	the direction of the air flow	(m)
θ	time	(hr)
A _x	outside surface area of the evaporator related to the unit of length in the x direction; A _x is an idealized parameter, introduced to overcome discontinuities in the contact surface; A _x is equal to	

the outside surface area of a single row of tubes, divided by
the distance between two consecutive rows; the dimensions of

A_x are (m^2/m)

S_m the average flow passage of the air (m^2)

In Eq. (76) use is made of the perfect gas relationships, for moist air may be considered as an ideal mixture of two perfect gases, to wit dry air and water-vapor. For the sake of completeness it should however be remarked that Eq. (76) may result in a small error. In fact it is possible that at a given point in the evaporator the composition of the air mixture varies with time. This would result in a time dependent variation of the coefficient C_v and Eq. (76) would not equal zero. The possible changes of C_v however are very small and any departure of this sort may therefore be neglected.

Thermal balance of tube material: the following energy balance may be set up:

$$\frac{A_x}{R_s} (H - H_s) - \frac{A_x}{R_i} (H_s - H_{sR}) = Z_x C \beta_A^{-1} \frac{\partial H_s}{\partial \theta} \dots (77)$$

where H_{sR} enthalpy of saturated air at the refrigerant's evaporating temperature $(kcal/kg_a)$

R_i the internal resistance of extended surfaces (cf. Eq. (29)), related to the total external surface area, which includes both pipe and fins; the driving force being an enthalpy difference, R_i is expressed in $(m^2 hr/kg_a)$

Z_x mass of the pipe- and fin material used per unit depth of the evaporator in the x direction (kg/m)

C specific heat of the pipe- and fin material $(kcal/kg \text{ } ^\circ C)$

$Z_x C$ is the thermal capacity of the evaporator material per unit depth of the evaporator.

β_A is defined as the following slope of the saturation curve of moist air:

$$\beta_A = \left. \frac{dH_s}{dT_s} \right\}_{T_{\text{surf. av.}}} \quad \dots (78)$$

This slope is evaluated at the average temperature of the total evaporator surface. The value of this temperature need not be known with great accuracy, for it is not very critical

Heat Transfer to the refrigerant:

Introducing the symbol J to designate the total heat flow, which reaches the refrigerant between the air entrance section of the evaporator, where $x = 0$, and between the row of tubes to which the present analysis applies, one may write the following equation

$$\frac{\delta J}{\delta x} = \frac{A_x}{R_1} (H_s - H_{sR}) \quad \dots (79)$$

The quantity $J(x, \theta)$, introduced by Eq. (79), is a function of the time and of the distance from the air entrance section of a particular tube row. It has a mathematical rather than a physical significance.

The dimensions of the function J evidently are those of a heat flow: kcal/hr.

Transform Operations.

It is advantageous, in order to apply the Laplace transform method, to express the above equations in terms of deviation variables from some steady-state initial values, found as outlined in section 3 of this chapter.

It should be pointed out that the steady-state initial values, just mentioned, are functions of the space coordinate x . In each evaporator section these functions usually take a different value (except for H_{SR}).

The following notation will be used throughout the further treatment:

(a) deviation variables are denoted by small characters, whereas capitals designate the absolute value of the corresponding variable; (b) the superscript o denotes initial values; (c) unless misunderstanding is to be avoided, no special notation is used to indicate Laplace-transformed functions.

The deviation variables, which will be introduced now, are:

$$\begin{aligned} h(x, \theta) &= H(x, \theta) - \overset{o}{H} & \text{where } \overset{o}{H} &= H(x, 0) \\ h_S(x, \theta) &= H_S(x, \theta) - \overset{o}{H}_S & \text{where } \overset{o}{H}_S &= H_S(x, 0) \\ h_{SR}(\theta) &= H_{SR}(\theta) - \overset{o}{H}_{SR} & \text{where } \overset{o}{H}_{SR} &= H_{SR}(0) \\ j(x, \theta) &= J(x, \theta) - \overset{o}{J} & \text{where } \overset{o}{J} &= J(x, 0) \end{aligned}$$

Writing the Eqs. (76), (77) and (79) in terms of these variables, and Laplace transforming with respect to θ , while noting s as the subsidiary domain of θ , results in

$$G \frac{dh(x, s)}{dx} = \frac{A_X}{R_S} (h_S(x, s) - h(x, s)) \quad \dots (80)$$

$$\begin{aligned} Z_X C \beta_A^{-1} s h_S(x, s) &= \frac{A_X}{R_S} (h(x, s) - h_S(x, s)) \quad \dots (81) \\ &\quad - \frac{A_X}{R_1} (h_S(x, s) - h_{SR}(s)) \end{aligned}$$

$$\frac{dj(x, s)}{dx} = \frac{A_X}{R_1} (h_S(x, s) - h_{SR}(s)) \quad \dots (82)$$

As far as the left-hand member of the second equation is concerned, it may be recalled that the initial conditions for the deviation variable h_S are (in the original time domain): $h_S(x, 0) = 0$

The Eqs. (80), (81) and (82) may be integrated in the space domain, i.e. over the total depth X of the evaporator. Integrating from $x = 0$ up to $x = X$ results in

$$G (h (X,s) - h (0,s)) = \frac{A_X}{R_s} \int_0^X [h_s(x,s) - h(x,s)] dx \quad \dots (83)$$

$$Z_X C \beta_A^{-1} s \int_0^X h_s(x,s) dx = \frac{A_X}{R_s} \int_0^X [h(x,s) - h_s(x,s)] dx \quad \dots (84)$$

$$- \frac{A_X}{R_i} \int_0^X [h_s(x,s) - h_{sR}(s)] dx$$

$$j(X,s) - j(0,s) = \frac{A_X}{R_i} \int_0^X [h_s(x,s) - h_{sR}(s)] dx \quad \dots (85)$$

These equations contain quantities, which must be explained further.

$h(0,s)$ and $h(X,s)$ are the Laplace transforms of possible deviations or changes of the air enthalpy at the inlet and outlet of the evaporator. As such they may be represented by the more familiar symbols $h_1(s)$ and $h_2(s)$, but one should not forget to consider them as deviation variables.

From the definition of the heat flux J , given on page 43, it follows directly that $J(0,\theta)$ and consequently also $j(0,\theta)$ and $j(0,s)$ are identically zero at all times.

The quantity $J(X,\theta)$ and its transform $J(X,s)$ represent the total heat flow delivered to the boiling refrigerant inside the evaporator; the quantities $j(X,\theta)$ and its transform $j(X,s)$ on the other hand, represent the deviations of this heat flow from a steady-state initial value. All these quantities are functions of time only for X is a constant. In an effort to make a directly meaningful symbol, the total depth X of the evaporator will be dropped as a variable and replaced by the subscript q .

Taking the above remarks into account, one can rewrite Eq. (85) as

$$\int_0^X h_s(x,s) dx = \frac{R_1}{A_x} j_q(s) + X h_{sR}(s) \quad \dots (86)$$

Substituting Eqs. (83), (85) and (86) into Eq. (84) results in

$$G (h_1(s) - h_2(s)) = (1 + \tau_2 s) j_q(s) + K'_w s h_{sR}(s) \quad \dots (87)$$

$$\text{where} \quad \tau_2 = \frac{Z_x C R_1}{\beta_A A_x} \quad \dots (88)$$

$$\text{and} \quad K'_w = \frac{Z_x C X}{\beta_A} \quad \dots (89)$$

τ_2 has the dimension of a time (hr); K'_w has the dimension of a mass (kg).

Recalling that, on the basis of Eqs. (76) and (83), the left side of Eq. (87) represents the Laplace transform of the deviation, q_E , of the quantity Q_E from its initial and steady-state value, one may draw the following conclusion. Eq. (87) is the mathematical expression of the forward loop in the closed-loop system, formed by the evaporator and the refrigeration unit. However, an additional backward or return-loop relationship between the variables j_q and h_{sR} must be found. To find this relationship constitutes the purpose of the following pages.

D. Assumptions concerning the Internal Vaporization

The vaporization of a refrigerant inside a dry expansion evaporator is a very complex phenomenon, which is not yet completely understood.

Since both the liquid and the vapor phase of a pure substance exist simultaneously the type of flow encountered is usually classified under the general name of two-phase flow. This type of flow evidently obeys the usual mass-, momentum- and energy equations, which are all partial differential equations. But the major problem is more that of predicting the flow pattern which occurs when phases of different densities flow together in the same direction. The general problem, however, in the present analysis may be considerably simplified by making the following assumptions, which in most cases may be looked upon as reasonable approximations:

1. The evaporator pressure will be taken as a constant throughout the coils. Variations of this pressure are considered to be directly transmitted or, in other words, to be the same throughout the evaporator. This eliminates the possibility of compression waves inside the coils. As a matter of fact, the foregoing assumption comes down to completely disregarding the momentum equation of the flow, and to lumping the space parameters of the flow in the system.

2. The part of the heat transfer to the boiling refrigerant which, in case of variations of the evaporating temperature, is absorbed by the liquid phase will be neglected. This is permitted because of the small liquid content of the coils. Moreover, the reference state (zero point) for the refrigerant's internal energy and enthalpy will be taken at the saturated liquid state of the refrigerant corresponding with the (initial) evaporating temperature. These

assumptions permit the writing of a mass and an energy balance both related to the vapor phase only, i.e. dropping in the formulation of balance equations those terms which deal with the presence or the state of the refrigerant liquid in the coils.

3. In assumption 1 on page 39 an average refrigerant temperature was introduced in order to simplify the heat transfer part of the problem. Little error will be made by doing the same for the density of the refrigerant. Since the superheat region is small, the superheat will have only a slight effect on the weighted average temperature T_R . Therefore T_R will be considered as the actual temperature in the saturated region of the coils. As to the mean density, just mentioned, the density of saturated refrigerant vapor at the mean temperature T_R will be taken as representative of the entire vapor content of the coils. Exactly the same method will be applied to the internal energy of the vapor. The error resulting from this procedure is negligible. This becomes clear if one considers the fact that the mean quantities just defined appear in the balance equations of the coils under the form of derivatives only. Therefore their absolute magnitude has little importance.

4. Finally it will be assumed also that the vapor velocity inside the coils is high enough to neglect the delay, caused by the finite gas velocity, as far as the heat transfer to the system is concerned. More specifically, it will be assumed that at each moment, the state of the gas leaving the coils describes the relative proportions of the total heat taken up by the refrigerant during its vaporization, and of the heat transfer absorbed during the refrigerant's superheating. Both quantities of heat are taken at the same moment along the entire length of the coils (lumping of space parameters).

E. The Refrigerant's Mass and Energy Balances

With the foregoing assumptions, the two following mass and energy balances can be set up. These balances describe the average state of the refrigerant vapor inside the evaporator. They are related to the vapor phase only.

$$M_i'' - M_o + \frac{J_q}{I_{fg}} \frac{I'' - I_i}{I_o - I_i} = V \frac{\partial \rho''}{\partial \theta} \quad \dots (90)$$

$$M_i'' I_{fg} - M_o (I_o - I') + J_q = V \frac{\partial}{\partial \theta} e'' (E'' - E') \quad \dots (91)$$

Symbols:	M_i''	flow rate of entering flash gas	(kg/hr)
	M_o	flow rate of gas leaving the coils	(kg/hr)
	I	refrigerant enthalpy	(kcal/kg)
	E	refrigerant internal energy	(kcal/kg)
	I_{fg}	latent heat of vaporization	(kcal/kg)
	ρ	refrigerant density	(kg/m ³)
	J_q	total heat flow rate	(kcal/hr)
	V	total internal volume of the coils	(m ³)
	"	superscript, denoting saturated vapor	
	'	superscript: saturated liquid	
	i	subscript, referring to the refrigerant inlet conditions	
	o	subscript: refrigerant outlet conditions	

The procedure to follow in order to solve the Eqs. (90) and (91) is to express all the terms, appearing in the system, in terms of a few basic variables. These variables will be the following deviation variables, measured with respect to the initial steady-state values (as before, these values are denoted with the superscript o):

$$\begin{aligned}
 m_o &= M_o - \overset{o}{M}_o \\
 m_i'' &= M_i'' - \overset{o}{M}_i'' \\
 t_R &= T_R - \overset{o}{T}_R \\
 j_q &= J_q - \overset{o}{J}_q \\
 t^* &= T^* - \overset{o}{T}^*
 \end{aligned}$$

The only new variable, introduced by these Eqs., is the superheat of the leaving gas, at each moment defined as

$$T^* = T_o - T_R$$

The balance equations (90) and (91) in terms of these deviation variables may now be rewritten as approximatively equal to

$$(\overset{o}{M}_i'' + m_i'') - (\overset{o}{M}_o + m_o) + \frac{(\overset{o}{J}_q + j_q)}{\overset{o}{I}_{fg} + c_I t_R} \left(1 - \frac{c_p (\overset{o}{T}^* + t^*)}{I'' - I_i} \right) = v c_e \frac{dt_R}{d\theta}$$

$$\begin{aligned}
 (\overset{o}{M}_i'' + m_i'')(\overset{o}{I}_{fg} + c_I t_R) - (\overset{o}{M}_o + m_o)(\overset{o}{I}_{fg} + c_I t_R + c_p \overset{o}{T}^* + c_p t^*) + (\overset{o}{J}_q + j_q) \\
 = v \overset{o}{E}_{fg} c_e \frac{dt_R}{d\theta} + v \overset{o}{c}_E c_E \frac{dt_R}{d\theta}
 \end{aligned}$$

In order to arrive at these forms some coefficients have been introduced which follow from a linearization of the refrigerant's thermodynamic properties in the neighbourhood of the initial steady-state conditions. They are

$$c_p = \frac{\partial I_o}{\partial T^*}; \quad c_e = \frac{d\overset{o}{c}_e}{dT_R}; \quad c_I = \frac{dI_{fg}}{dT_R}; \quad c_E = \frac{dE_{fg}}{dT_R};$$

Subtracting from the above equations the solutions of the initial steady-state operations and dropping second-order differences result in the following expressions (after some mathematical rearrangements):

$$\begin{aligned}
 m_i'' - m_o + j_q \frac{1}{\overset{o}{I}_{fg}} \left(1 - \frac{c_p \overset{o}{T}^*}{I'' - I_i} \right) = v c_e \frac{dt_R}{d\theta} + t^* \frac{\overset{o}{J}_q c_p}{\overset{o}{I}_{fg} (I'' - I_i)} \\
 + t_R \frac{c_I \overset{o}{J}_q}{\overset{o}{I}_{fg}^2} \left(1 - \frac{c_p \overset{o}{T}^*}{I'' - I_i} \right) \dots (92)
 \end{aligned}$$

$$\begin{aligned}
m_i'' \dot{I}_{fG} - m_o (\dot{I}_{fG} + C_p \dot{T}^*) + j_q = & V \dot{E}_{fG} c_e \frac{dt_R}{d\theta} + V \dot{c}_E \frac{dt_R}{d\theta} \dots (93) \\
& + c_I (\dot{M}_o - \dot{M}_i'') t_R + C_p \dot{M}_o t^*
\end{aligned}$$

It may be noted here that in an effort to simplify the problem the enthalpy difference $I'' - I_i$ has been considered as a constant, resulting in a very small error. The error introduced by neglecting the second term on the right side of Eq. (93) also is negligible. This term is very small compared with the first term and is smaller than a corresponding energy term for the liquid phase, which has been left out of the analysis on the basis of one of the foregoing assumptions (page 47 and 48).

The approach described here for obtaining a linear form for the balance equations is always possible in practice. It must be emphasized that the linear forms, given by Eqs. (92) and (93), strictly speaking apply only to small deviations from the initial operating point. As such they may be used as a means to find the response of a refrigeration unit which is operating continuously under varying heat loads. In practice abrupt changes of the heat load do not occur. The Eqs. (92) and (93) cannot be considered for describing the system's behaviour during starting periods of the compressor like they occur in on-off control systems for refrigeration plants.

The Eqs. (92) and (93) contain five variables, namely t_R , t^* , j_q , m_i'' and m_o . The ultimate purpose is to express the evaporating temperature t_R in terms of the heat load j_q . Therefore it will be necessary to eliminate two more dependent variables, namely the entering and the leaving mass flow rates of refrigerant vapor.

In order to eliminate these variables, additional relationships between the five variables, just mentioned, will be found in the next sections.

F. Analysis of the Thermostatic Expansion Valve

The time constant of a typical pneumatic valve is on the order of 10 sec. That of a thermometer or a sensing bulb clamped to a coil usually does not differ too much from a minute. The reason why these numerical values are given here, is to show that the combination of valve and sensing bulb may be treated as a first-order system.

Assuming that the valve itself contributes negligible dynamic lag, the transfer function relating the valve stem position to the superheat may be written in terms of deviation variables as

$$\frac{y(s)}{t^*(s)} = \frac{K_1}{1 + \tau_v s} \quad \dots (94)$$

where K_1 is a constant, having the dimensions of a length divided by a temperature. K_1 often called the steady-state gain, is the proportionality factor between the steady-state stem position and the superheat reached after an infinite period of time. y is the stem position, τ_v is the time constant of the entire bulb and valve system.

The area of flow passage through the valve orifice is a function of the stem position:

$$S_v = f(Y)$$

Linearizing this relationship in the neighbourhood of the steady-state condition at time zero, results in

$$S_v = K_2 y + f(Y)^0 \quad \dots (95)$$

The flow of refrigerant through the valve orifice may be expressed by the well-known hydraulic formula

$$M_1 = K_3 S_v (P_C - P_R)^{\frac{1}{2}} \quad \dots (96)$$

where P_C is the condenser pressure and P_R the evaporator pressure. K_3 is a

coefficient which may be looked upon as substantially constant for a particular valve in a particular application.

In Eq. (96) the condenser pressure is considered to be constant. From the saturated refrigerant tables the functional relation between P_R and T_R , given as $P_{R_i}(T_R)$, is obtained. Hence Eq. (96) can easily be linearized, as shown for instance in ref. (60). Therefore one expands M_i about the initial operating point in a Taylor series of which only the linear terms are retained. The result will be given in deviation variables. When combined with the Eqs. (94) and (95), one obtains the following relationship for the Laplace transformed valve characteristics in terms of two constants k^* and k_R :

$$m_i(s) = \frac{k^* t^*(s)}{1 + \tau_v s} + k_R t_R(s) \quad \dots (97)$$

This expression is not directly usable in the present analysis. or the variable which occurs in the previous balance equations is the amount of flash gas entering the coils. This quantity is given by

$$M_i'' = K_3 S_v (P_C - P_R)^{\frac{1}{2}} X_i \quad \dots (98)$$

where X_i represents the quality of the refrigerant entering the coils. An analogous procedure makes it possible to derive, in exactly the same way as was done for m_i , the following relationship for the deviation variable m_i''

$$m_i''(s) = \frac{K^* t^*(s)}{1 + \tau_v s} + K_R t_R(s) \quad \dots (99)$$

The relation between the coefficients introduced here and the coefficients introduced before is easily found to be

$$K^* = \overset{0}{X_i} k^* \quad (\text{kg/hr } ^\circ\text{C})$$

$$K_R = \overset{0}{X_i} k_R + \overset{0}{M_i} c_x \quad (\text{kg/hr } ^\circ\text{C})$$

where the coefficient c_x is given by

$$c_x = \frac{\partial X_i}{\partial T_R} = \frac{\partial}{\partial T_R} \frac{I_i - I'}{I'' - I'}$$

G. The Mass Rate of Flow to the Compressor

The mass rate of flow to the compressor at each moment is given by (cf. Eq. (61)):

$$M_o = \frac{\lambda D}{v^*} \quad \dots (100)$$

where λ the compressor's actual volumetric efficiency

D the rate of displacement of the compressor (m³/hr)

v^* the specific volume of the refrigerant gas before it enters the
compressor (m³/kg)

A distinction must now be made between two possible cases:

A. If there is no appreciable pressure drop in the suction line, except the inevitable friction losses, then the suction pressure nearly equals the evaporator pressure. Moreover, the suction line to the compressor is usually well insulated, whence the temperature of the refrigerant gas remains unchanged from the outlet of the coils to the compressor. Using the perfect gas relationships for superheated refrigerant gas, one may write Eq. (100) as follows

$$M_o = \frac{\lambda D P_R}{R(T_R + T^*)}$$

where R is the refrigerant's gas constant

T_R is given in absolute degrees

Many experiments have shown that the volumetric efficiency of a reciprocating compressor obeys a linear law when expressed in terms of the compression ratio (ref. (61)).

Putting $\lambda = c_\lambda - c_\lambda (P_C/P_R)$

and substituting this formula into the expression above results in

$$M_o = \frac{C_A D P_R - c_A D P_C}{R (T_R + T^*)} \quad \dots (101)$$

where C_A and c_A are constants. P_R can directly be expressed in terms of T_R . Then, by means of the same procedure as applied before, Eq. (101) is ready to be linearized. Expanding in a Taylor series and retaining the linear terms only, yields for the deviation variable m_o a relation of the form

$$m_o = B_R t_R + B^* t^* \quad \dots (102)$$

where B_R and B^* are two coefficients whose value must be calculated in each particular case.

B. If the suction line contains an evaporator pressure-regulating valve, the problem becomes a little more complicated, for now the suction pressure, i.e. the pressure which prevails in the main suction line, intervenes as to determine the mass flow, ducted to the compressor. Due to the high velocity of the refrigerant gas in the suction line, one may safely assume that at each moment steady-state flow prevails in the suction line. As a consequence, the mass flow rate at each moment is the same along the entire length of the suction line. The following method may then be used in order to linearize the outgoing mass function in terms of the appropriate deviation variables introduced before. It should, however, directly be pointed out that this method merely constitutes an example. It is a guide as to how one may proceed in order to arrive at an acceptable result, and can be applied to almost any case. Due to the great variety in valve characteristics, as given in the manufacturer's data, the method is outlined in general terms only and must be adapted as to each particular application. Although not strictly necessary, it will also be assumed for the sake of simplicity that the refrigerant gas, when super-

heated, behaves as a perfect gas.

The method proceeds as follows: The flow rate of gas through the evaporator pressure-regulating valve can always be expressed by a function

$$M_o = f(Y, P_R, P^*)$$

where Y is the valve stem position, P_R the evaporating pressure and P^* the back pressure in the suction line. Often one finds that this function has a form analogous to Eq. (96). A more extended treatment is found in ref. (62).

On the other hand, the flow of refrigerant to the compressor is given by a function

$$M_o = f(T_R, T^*, P^*)$$

Eq. (101) after some obvious modifications gives a general example of this function.

Eliminating the variable P^* between the foregoing expressions, and making use of the relationship between P_R and T_R which is given directly by the refrigerant's vapor pressure curve, result in

$$M_o = f(Y, T_R, T^*) \quad \dots (103)$$

Function (103) has a total derivative which in the neighbourhood of the initial operating point can be evaluated to give:

$$dM_o = \frac{\partial M_o}{\partial Y} dY + \frac{\partial M_o}{\partial T_R} dT_R + \frac{\partial M_o}{\partial T^*} dT^* \quad \dots (104)$$

which may be written in terms of deviation variables and of constant coefficients as

$$m_o = B_Y y + B_R t_R + B^* t^* \quad \dots (105)$$

Equation (105) is a generalization of Eq. (102) for the case where an evaporator pressure-regulating valve is used. It contains one more independent variable, to wit the deviation stem position y , which further will be treated as a quantity acted upon by parameters external to the system. Because of its generality Eq. (105) will be used in the sequel with the understanding that,

if no pressure regulating valve is installed in the suction line, the coefficient B_y may be put equal to zero.

H. Dynamic Response. Transfer Functions

The dynamic response of the internal part of the evaporator, connected to a given type of compressor, is found by combining the Eqs. (92), (93), (99) and (105). Laplace-transforming those equations which have not yet been transformed, and substituting the Eqs. (99) and (105) into the balance equations (92) and (93), yields the following equations

$$G_q j_q(s) = (B_R - K_R - G_R + \mu s) t_R(s) + (B^* - G^* - \frac{K^*}{1 + \tau_v s}) t^*(s) + B_y y(s) \quad \dots (106)$$

$$j_q(s) = (b_R - k_R - g_R + \varepsilon s) t_R(s) + (b^* - g^* - \frac{k^*}{1 + \tau_v s}) t^*(s) + b_y y(s) \quad \dots (107)$$

where the various coefficients, which appear in these equations, are defined as follows:

K_R, K^* coefficients introduced by Eq. (99)

B_R, B^*, B_y coefficients introduced by Eq. (105)

k_R, k^* coefficients, by definition equal to:

b_R, b^*, b_y coefficients, by definition equal to:

$$\begin{aligned} K_R &= K_R \overset{o}{I}_{fg} \\ K^* &= K^* \overset{o}{I}_{fg} \\ B_R &= B_R (\overset{o}{I}_{fg} + C_P \overset{o}{T}^*) \\ B^* &= B^* (\overset{o}{I}_{fg} + C_P \overset{o}{T}^*) \\ b_y &= B_y (\overset{o}{I}_{fg} + C_P \overset{o}{T}^*) \end{aligned}$$

Furthermore

$$G_q = \frac{1}{\overset{\circ}{I}_{fg}} \left(1 - \frac{C_p \overset{\circ}{T}^*}{I'' - I_i} \right)$$

$$G_R = - \frac{c_I \overset{\circ}{J}_q}{\overset{\circ}{I}_{fg} 2} \left(1 - \frac{C_p \overset{\circ}{T}^*}{I'' - I_i} \right)$$

$$G^* = - \frac{\overset{\circ}{J}_q C_p}{\overset{\circ}{I}_{fg} (I'' - I_i)}$$

$$z_R = - c_I (\overset{\circ}{M}_O - \overset{\circ}{M}_I^*)$$

$$g^* = - C_p \overset{\circ}{M}_O$$

$$\mu = v c_e$$

$$\epsilon = v c_e \overset{\circ}{e}_{fg} + v \overset{\circ}{e}'' c_E$$

In order to make the analysis more meaningful, the following physical significance may be attached to the foregoing coefficients.

The coefficients K_R and K^* represent the partial derivatives of the entering vapor flow with respect to the evaporating temperature and the superheat. The coefficients k_R and k^* represent the corresponding enthalpy changes per degree of temperature change.

The coefficients B_R , B^* and B_y represent the partial derivatives of the outgoing mass flow rate with respect to changes in the evaporating temperature, the superheat and the stem position of the evaporator pressure-regulating valve in the suction line, if any. Again the corresponding derivatives of the outgoing enthalpy flow are represented by b_R , b^* and b_y .

The coefficients G_q , G_R and G^* represent the change of the amount of vapor, which is formed inside the coils per unit variation respectively of the heat flow, the evaporating temperature and of the superheat. As such these coefficients may be considered as partial derivatives too). The liquid refrigerant flowing through the coils needs, in order to vaporize, an amount of energy which will vary at each moment. This amount will change

according to the evaporating temperature. The variation of this amount of energy per unit change in the evaporating temperature, is evaluated for the total liquid flow, passing through the coils per unit time, and represented by the coefficient- g_R . Coefficient- g^* on the other hand represents the change in the amount of heat, absorbed by the total refrigerant flow, when the superheat increases by one degree.

The coefficients μ and ϵ respectively are the changes in the total mass and energy content of the vapor inside the coils when the evaporating temperature (the mean refrigerant temperature) increases by one degree.

The variable t^* must now be eliminated from the Eqs. (106) and (107). By doing so, one gets the following expression for the evaporating temperature as a function of the heat inflow and of the stem position of the evaporator pressure-regulating valve, if any,

$$t_R(s) = \frac{j_Q(s) \{ (B^* - G^* - K^*) - G_Q(b^* - g^* - k^*) + s \tau_V [(B^* - G^*) - (b^* - g^*) G_Q] \} + s^2 \tau_V [\epsilon(B^* - G^*) - \mu(b^* - g^*)] + s [\epsilon(B^* - G^* - K^*) - \tau_V(b^* - g^*)(B_R - y(s) \{ B_Y G_Q(b^* - g^* - k^*) - b_Y(B^* - G^* - K^*) + s \tau_V [B_Y G_Q(b^* - g^*) - K_R - G_R) + \tau_V(B^* - G^*)(b_R - k_R - g_R) - \mu(b^* - g^* - k^*)] - (B_R - K_R - b_Y(B^* - G^*)) \}]}{G_R(b^* - g^* - k^*) + (B^* - G^* - K^*)(b_R - k_R - g_R)} \quad \dots (108)$$

This equation in terms of Laplace transformed deviation variables shows clearly that the response of the internal system, formed by the evaporator's inner part and by the compressor, is essentially second-order. This result is in excellent agreement with some physical phenomena, often encountered in practice, as for example valve hunting or 'sluggish' evaporator response. Eq.

(108) can be brought into the standard form

$$t_R(s) = \frac{t_R(0) (1 + \tau_3 s) j_q(s)}{j_q(0) \tau^2 s^2 + 2\zeta \tau s + 1} + \frac{t_R(0) (1 + \tau_4 s) y(s)}{y(0) \tau^2 s^2 + 2\zeta \tau s + 1} \quad (109)$$

In this equation τ , τ_3 , τ_4 are time constants; ζ is the so-called damping factor.

Eq. (109) also contains the following so-called steady-state gains, reached after an infinite period of time, for $s = 0$, and easily computed from Eq. (108)

$$\frac{t_R(0)}{j_q(0)} = \frac{(B^* - G^* - K^*) - G_q(b^* - g^* - k^*)}{-(B_R - K_R - G_R)(b^* - g^* - k^*) + (B^* - G^* - K^*)(b_R - k_R - g_R)} \quad (110)$$

$$\frac{t_R(0)}{y(0)} = \frac{B_y G_q(b^* - g^* - k^*) - b_y(B^* - G^* - K^*)}{-(B_R - K_R - G_R)(b^* - g^* - k^*) + (B^* - G^* - K^*)(b_R - k_R - g_R)} \quad (111)$$

The overall transfer function, which directly relates the evaporating temperature of the refrigerant to the inlet and outlet conditions of the air stream through the evaporator, is found by combining the Eqs. (87) and (109). Eq. (87) may first be transformed and expressed in terms of the evaporating temperature. By means of Eq. (30) one may write

$$G(h_1(s) - h_2(s)) = (1 + \tau_2 s) j_q(s) + K_w s t_R(s) \quad \dots (112)$$

where in accordance with Eq. (89) a new coefficient K_w is introduced, equal to

$$K_w = \frac{Z_x C X \beta_R}{\beta_A} = \beta_R K_w^I \quad \dots (112b)$$

Elimination of $j_q(s)$, the instantaneous heat flow from the walls of the coils to the refrigerant, allows the combination of (109) and (112) into the following overall relation:

$$G(h_1(s) - h_2(s)) = t_R(s) \frac{j_q(0)}{t_R(0)} \frac{(1 + \tau_2 s)(\tau_2^2 s^2 + 2\zeta \tau_2 s + 1) + K_w s(1 + \tau_3 s)}{(1 + \tau_3 s)} - y(s) \frac{j_q(0)}{y(0)} \frac{(1 + \tau_2 s)(1 + \tau_4 s)}{(1 + \tau_3 s)} \quad \dots (113)$$

Eq. (113) completely describes the dynamic behaviour of dry expansion evaporators, connected to a particular type of compressor and whether or not provided with an evaporator pressure-regulating valve. In this equation three Laplace transformed variables are related to each other, namely the heat transfer from the air to the coils, the evaporating temperature of the refrigerant and the stem position of the evaporator pressure-regulating valve. Any of these three variables may be considered as the output variable in terms of the two others (input variables). In order to avoid confusion, it should be repeated that the constant values $j_q(0)$, $t_R(0)$ and $y(0)$ in Eq. (113) are Laplace transforms, so that the zero coordinate refers to the value of $s = 0$ after an infinite period of time. The constant values, just mentioned, are thus the final steady-state values those respective functions will take.

As to the quotients $\frac{j_q(0)}{t_R(0)}$ and $\frac{j_q(0)}{y(0)}$ appearing in Eq. (113) and also in Eqs. (109) and (110), some practical considerations must be made. These quotients are coefficients which can be found from a complete graph of the capacity curve of the refrigeration unit. A distinction, however, must be made between two possible cases.

A. There is no evaporator pressure-regulating valve in the system. Then, by definition of the coefficient b in Eq. (65), the following equality exists

$$b = \frac{j_q(0)}{t_R(0)} \quad \dots (114)$$

The coefficient β is, by definition, equal to the expression given by the inverse of Eq. (110). This expression will be the slope of the unit's capacity curve at the evaporating temperature used.

B. The refrigeration system contains an evaporator pressure-regulating valve. In this case one may go back to Eq. (103) and write the capacity of the unit as a product:

$$M_0(I_0 - I_1) = f(Y, T_R, T^*) \cdot f'(T_R, T^*) = Q_{ex}$$

Now one can use the approximate assumption of Eq. (63), which actually puts forward that the capacity of the system changes but negligibly with the superheat of the leaving refrigerant gas. This permits one to write the foregoing functional relationship as

$$Q_{ex} = F(Y, T_R)$$

This function is graphically represented by a group of capacity curves, each of them applying to a particular opening of the evaporator pressure-regulating valve. The partial derivatives of this function with respect to its variables, and evaluated in the neighbourhood of the operating point of the system at time zero, are precisely the quotients needed here. In order to keep the notation coherent these partial derivatives of the capacity function in this thesis will be represented by

$$\frac{\partial Q_{ex}}{\partial T_R} = \frac{j_q(0)}{t_R(0)} = b' \quad \dots (115)$$

$$\frac{\partial Q_{ex}}{\partial Y} = \frac{j_q(0)}{y(0)} = b'' \quad \dots (116)$$

I. Response of the Evaporator Surface

Not the evaporating temperature of the refrigerant but the evaporator surface temperature governs the heat and mass transfer from the room air to the cooling coils. For the study of the overall response of cold rooms it will therefore be of direct importance to find the function which governs the instantaneous state of the evaporator surface.

The analysis which will follow now has much in common with similar studies, found in the references (62), (59), (63) et al.

The starting point will be the Eqs. (80) and (81), which may directly be combined into a single expression

$$\frac{dh(x,s)}{dx} + \frac{h(x,s)}{G} \frac{A_x}{R_s} \left(1 - \frac{A_x/R_s}{s \frac{Z_x C}{\beta_A} + \frac{A_x}{R_s} + \frac{A_x}{R_i}} \right) = \frac{A_x h_{sR}(s) / G R_i}{s \frac{Z_x C}{\beta_A} + \frac{A_x}{R_s} + \frac{A_x}{R_i}}$$

This equation is a simple differential equation which can be solved directly with respect to the variable x .

The boundary condition, imposed on the equation, is given by

$$h(x,s) = h_1(s) \quad \text{at} \quad x = 0$$

Solving the equation yields

$$h(x,s) = h_1(s) - \left\{ 1 - \exp \left[- \frac{\frac{A_x}{G R_s} \left\{ s \frac{Z_x C}{\beta_A} + \frac{A_x}{R_i} \right\}}{s \frac{Z_x C}{\beta_A} + \frac{A_x}{R_i} + \frac{A_x}{R_s}} x \right] \right\} \left\{ h_1(s) - \frac{R_s}{R_i} \left\{ s \frac{Z_x C}{\beta_A} + \frac{A_x}{R_i} \right\} h_{sR}(s) \right\} \dots (117)$$

This solution represents the enthalpy of the airstream at each moment and at each cross-section of the evaporator model described before.

For practical use the expression just derived is too complicated. It may therefore be simplified on the basis of the following considerations.

In actual evaporators the evaporating temperature inside the coils varies very slowly. A kilogram of air, passing through the evaporator, is not able to detect any variation of this temperature. Therefore when the state of the air at the entrance of the evaporator changes, the refrigerant temperature will follow, but very slowly and gradually. Therefore one may consider the state of the leaving air as a function of the state of the entering air only.

As a consequence, one may neglect the influence of the deviation variable $h_{sR}(s)$ in the above solution for $h(x,s)$. Hence a transfer function can be derived directly from this solution. This transfer function relates the air enthalpy at the outlet to the same property at the inlet of the evaporator and is given by

$$\frac{h_2(s)}{h_1(s)} = - \exp \left[- \frac{X A_x}{G R_s} \left\{ s \frac{Z_x C}{\beta_A} + \frac{A_x}{R_i} \right\} \right] \dots (118)$$

$$s \frac{Z_x C}{\beta_A} + \frac{A_x}{R_i} + \frac{A_x}{R_s}$$

This transfer function presents a few very interesting characteristics, that will follow now. Applying the initial and the final value theorem, one may calculate that at time zero, i.e. for $s = \infty$,

$$\frac{h_2(\infty)}{h_1(\infty)} = - \exp \left(- \frac{X A_x}{G R_s} \right) = - \exp \left(- \frac{h_c A}{G C_{ps}} \right)$$

where use is made of Eq. (28) and of the definition of A_x on page 41.

On the other hand, after an infinite period of time, i.e. for $s = 0$, one easily finds:

$$\frac{h_2(0)}{h_1(0)} = - \exp \left(- \frac{A}{G} \frac{1}{R_i + R_s} \right) = - \exp (-UA/G)$$

One may conclude:

As soon as the enthalpy of the air, entering an evaporator, is subjected to a variation, the first reaction of the evaporator is that of a surface which is at constant temperature. The longer the period this variation lasts the more the evaporator will react as a composite wall. The transition between these responses depends on the quantity of heat absorbed by the walls themselves.

After an infinite period of time, the system again reaches steady-state and the only effect of the entire event will be a very small change of the refrigerant's evaporating temperature.

In well designed evaporators the external surface resistance constitutes by far the major part of the total resistance. Therefore only a small error will be made if one takes for the exponent in the above transfer functions the value it has after an infinite time, namely

$$\frac{h_2(s)}{h_1(s)} = 1 - e^{-UA/G}$$

Considering the fact that these enthalpy values are deviation variables from an initial enthalpy distribution, which obeys the law

$$\frac{H_2^0 - H_{sR}^0}{H_1^0 - H_{sR}^0} = 1 - e^{-UA/G}$$

and neglecting the small changes in the evaporating temperature, one obtains by adding the foregoing relations

$$\frac{H_2(s) - H_{sR}(s)}{H_1(s) - H_{sR}(s)} \approx 1 - e^{-UA/G}$$

The meaning of this equation is that the relationships, which govern the heat transfer to evaporators during steady-state operation, may be used for unsteady-state conditions too. This evidently will result in a certain error but, for lack of a simple and better approximation, there is no alternative.

The concept of an equivalent evaporator surface, introduced on page 29, will thus be generalized as to apply also during changes in the heat load. Eq. (57) keeps its validity but, of course, will be given an enlarged significance in order to allow for possible variations of the evaporating temperature. Mathematically it will be rewritten as (cf. Eq. (72)):

$$H_{sE}(s) = H_{sR}(s) + \frac{G(H_1(s) - H_2(s))}{G} \left(\frac{1}{\Theta} - \frac{1}{\vartheta} \right) \quad \dots (119)$$

or in terms of deviation variables

$$h_{sE}(s) = h_{sR}(s) + (h_1(s) - h_2(s)) \left(\frac{1}{\Theta} - \frac{1}{\vartheta} \right) \quad \dots (120)$$

In these expressions, however, one can make use of the basic property of the equivalent surface conditions themselves. More particularly, since on the basis of Eq. (59),

$$Q_E = G(H_1 - H_2) = G\vartheta(H_1 - H_{sE})$$

where ϑ is the evaporator surface contact factor, one may rewrite the equalities above as

$$H_{sE}(s) = H_{sR}(s) + \vartheta(H_1(s) - H_{sE}(s)) \left(\frac{1}{\Theta} - \frac{1}{\vartheta} \right) \quad \dots (121)$$

$$h_{sE}(s) = h_{sR}(s) + \vartheta(h_1(s) - h_{sE}(s)) \left(\frac{1}{\Theta} - \frac{1}{\vartheta} \right) \quad \dots (122)$$

Similarly, one may rewrite Eq. (113) in terms of h_{sE} and of h_{sR} (on the basis of Eq. (30)), which will result in an expression

$$h_{sR}(s) = \frac{G \hat{V}(h_1(s) - h_{sE}(s)) \beta_R}{b'} \frac{(1 + \tau_3 s)}{(1 + \tau_2 s)(\tau^2 s^2 + 2\zeta \tau s + 1) + K_W s(1 + \tau_3 s)} + \frac{y(s) \beta_R b''}{b'} \frac{(1 + \tau_2 s)(1 + \tau_4 s)}{(1 + \tau_2 s)(\tau^2 s^2 + 2\zeta \tau s + 1) + K_W s(1 + \tau_3 s)} \dots (123)$$

In these equations use is made also of the equalities (115) and (116).

By elimination of the variable h_{sR} it now becomes possible to find the response of the evaporator surface. Eliminating h_{sR} between the Eqs. (122) and (123) results in

$$h_{sE}(s) = h_1(s) \frac{\frac{G \hat{V} \beta_R}{b'} (1 + \tau_3 s) + \hat{V} \left(\frac{1}{\Theta} - \frac{1}{\hat{V}} \right) P(s)}{\frac{G \hat{V} \beta_R}{b'} (1 + \tau_3 s) + \left(1 + \frac{\hat{V}}{\Theta} - \frac{\hat{V}}{\hat{V}} \right) P(s)} + y(s) \frac{\frac{\beta_R b''}{b'} (1 + \tau_2 s)(1 + \tau_4 s)}{\frac{G \hat{V} \beta_R}{b'} (1 + \tau_3 s) + \left(1 + \frac{\hat{V}}{\Theta} - \frac{\hat{V}}{\hat{V}} \right) P(s)} \dots (124)$$

$$\text{where } P(s) = (1 + \tau_2 s)(\tau^2 s^2 + 2\zeta \tau s + 1) + K_W s(1 + \tau_3 s)$$

Eq. (124) at each moment relates the equivalent evaporator surface enthalpy to the enthalpy h_1 of the air stream, entering the coils, and to the instantaneous stem position y of the evaporator pressure-regulating valve in the suction line to the compressor.

Because of its importance in the sequel Eq. (124) will be represented by the following notation, the functions Φ_h and Φ_y being directly found from Eq. (124),

$$h_{sE}(s) = \Phi_h(s) h_1(s) + \Phi_y(s) y(s) \dots (125)$$

The functions Φ_h and Φ_y are the transfer functions which relate the evapo-

rator surface conditions respectively to the enthalpy of the entering air and to the stem position of the evaporator pressure-regulating valve, provided that the other variable in each case is kept constant.

It is easy to see, by completely writing down these transfer functions, that one may consider the response of the evaporator surface conditions as an essentially second-order function. This result agrees with how one intuitively expects the system to behave: the evaporator surface may be expected to follow the response of the internal part of the evaporator, which in turn is a second-order function in terms of the entering air stream properties. In their complete form the above transfer functions are

$$\begin{aligned} \Phi_h(s) &= \frac{G\beta_R}{b'} (1 + \tau_3 s) + \mathcal{J}\left(\frac{1}{\Theta} - \frac{1}{\mathcal{J}}\right) \left[(1 + \tau_2 s)(\tau^2 s^2 + 2\zeta\tau s + 1) + K_w s(1 + \tau_3 s) \right] \\ &\quad \frac{G\beta_R}{b'} (1 + \tau_3 s) + \left(1 + \frac{\mathcal{J}}{\Theta} - \frac{\mathcal{J}}{\mathcal{J}}\right) \left[(1 + \tau_2 s)(\tau^2 s^2 + 2\zeta\tau s + 1) + K_w s(1 + \tau_3 s) \right] \\ &\quad \dots (126) \\ \Phi_y(s) &= \frac{\beta_R b''}{b'} (1 + \tau_2 s)(1 + \tau_4 s) \\ &\quad \frac{G\beta_R}{b'} (1 + \tau_3 s) + \left(1 + \frac{\mathcal{J}}{\Theta} - \frac{\mathcal{J}}{\mathcal{J}}\right) \left[(1 + \tau_2 s)(\tau^2 s^2 + 2\zeta\tau s + 1) + K_w s(1 + \tau_3 s) \right] \\ &\quad \dots (127) \end{aligned}$$

THE TOTAL RESPONSE OF COLD STORAGE ROOMS

Now one may go back to the original balances, given by Eqs. (1) and (2), and one may express these equations in terms of deviation variables from the initial steady-state operating point. Laplace-transforming further results in

$$q_{in}(s) + q_F(s) - q_E(s) = R s h(s) \quad \dots (128)$$

$$m_{in}(s) + m_F(s) - m_E(s) = R s w(s) \quad \dots (129)$$

In these equations $h(s)$ and $w(s)$ are the deviation enthalpy and humidity ratio of the air in the room; $q_{in}(s)$ and $m_{in}(s)$ represent sudden perturbations of the influx of heat and moisture from external sources into the room; on the other hand, $q_E(s)$ and $m_E(s)$ are the increases of the heat and moisture which will be removed by the evaporator, because of the load perturbations just mentioned.

The Action of the Food Surfaces.

As to the influence of the surface of the storage goods upon the air conditions inside the cooled room, one goes back to Eq. (15). This equation may be written also in terms of deviation variables and Laplace-transformed. By doing so one easily obtains:

$$q_F(s) = \frac{h_{cF}}{C_{psF}} F (h_{sF}(s) - h(s)) = -m_F C_F \beta_F^{-1} s h_{sF}(s) \dots (130)$$

Eq. (130) may be put also in the form of a transfer function, relating the surface enthalpy (more accurately, the enthalpy of the air in the saturated layer at the food surface) of the food, to the enthalpy of the air in the room. This transfer function takes the form:

$$\Phi_F(s) = \frac{h_{sF}(s)}{h(s)} = \frac{1}{1 + \frac{m_F C_F \beta_F^{-1} C_{psF}}{h_{cF} \cdot F} s} \quad \dots (131)$$

With the approximate assumption of Newtonian heating or cooling of the goods this transfer function, of course, is first-order.

The quantity $q_F(s)$ may be written directly by means of Eq. (131) as

$$q_F(s) = \frac{h_{cF}}{C_{psF}} F (\Phi_F - 1) h(s) \quad \dots (132)$$

In order to find the term m_F in Eq. (129) one may go back to Eq. (13). The term m_F stands for the increase of the product desiccation, due to the change of the room air conditions because of variations in the heat and vapor influxes into the cold room. Eq. (13), expressed in terms of deviation variables and Laplace transformed, simply becomes

$$m_F(s) = \frac{h_{cF}}{C_{psF}} F (w_{sF}(s) - w(s)) \quad \dots (133)$$

The quantities which appear in this equation are not all independent variables.

The humidity ratio w_{sF} of the saturated air in the surface layer of the foods is related directly to the enthalpy of the same air, enthalpy, which is given by h_{sF} (thought is of all these properties as of deviations from the initial steady-state conditions). The relationship between w_{sF} and h_{sF} follows from the saturation curve of moist air. Linearizing this curve in the neighbourhood of the wet-bulb temperature of the room air permits the introduction of

$$y_F = \left. \frac{dH_S}{dH_S} \right\} \bigg|_{T_{SF}^0} \quad \dots (134)$$

(the initial wet-bulb temperature of the room air in the previous discussion

on page 14, has been shown to be the temperature that initially also the surface of the goods must have, for the initial conditions are assumed to be steady-state conditions).

The introduction of the slope γ_F directly gives Eq. (133) the following form

$$m_F(s) = \frac{h_{cF}}{C_{psF}} F (\gamma_F h_{sF}(s) - w(s))$$

and further, because of the dependence of h_{sF} on h given by the transfer function (131),

$$m_F(s) = \frac{h_{cF}}{C_{psF}} F (\gamma_F \Phi_F h(s) - w(s)) \quad \dots (135)$$

Equation (135) gives a perfect example of how, in the response of cooled spaces, the phenomena of heat and mass transfer are inherently interwoven.

The Action of the Evaporator.

The presence of the evaporator in the cold room constitutes the biggest factor in determining the conditions, taken by the room air, for the evaporator is responsible for the removal of heat and moisture from the cold room.

For this heat removal by the evaporator, one has on the basis of Eq. (59)

$$Q_E = G \mathcal{J} (H - H_{sE})$$

which in terms of deviation variables becomes

$$q_E = G \mathcal{J} (h - h_{sE})$$

In this equation, however, the variables h and h_{sE} are related to each other by the equality (121). Introducing this expression into the above equation results in

$$q_E(s) = G \mathcal{J} \left\{ (1 - \Phi_h) h(s) - \Phi_y y(s) \right\} \quad \dots (136)$$

The moisture removal by the evaporator in terms of the equivalent surface

conditions is given by Eq. (60) as

$$M_E = G \int (w - w_{sE})$$

and, in terms of deviations from the initial steady-state operation of the system, becomes

$$m_E = G \int (w - w_{sE})$$

The variable w_{sE} in this equation is directly related to the enthalpy value h_{sE} by the saturation curve of moist air. Using the same linear approximation of this curve as done on page 28, Eqs. (52) and (53), the deviation variable w_{sE} may be expressed as

$$w_{sE} = \gamma h_{sE}$$

Here again Eq. (125) may be used to relate the Laplace transforms of the functions $h(s)$, $h_{sE}(s)$ and consequently also $w_{sE}(s)$. All this permits one to write for $m_E(s)$:

$$m_E(s) = G \int (w(s) - \gamma \Phi_h h(s) - \gamma \Phi_y y(s)) \dots (137)$$

In this result, the mutual interaction of heat and mass removal from the room by the evaporator surface is clearly illustrated by the form of the right-hand member.

Solution of the Balance Equations

The Eqs. (132), (135), (136) and (137) all represent quantities of heat or vapor transfer that appear in the balance equations (128) and (129). Substitution of these equations into the balances yields the following expressions

$$q_{in}(s) = h(s) \left\{ R_s + G \int (1 - \Phi_h(s)) + \frac{h_{cF}}{c_{psF}} F (1 - \Phi_F(s)) \right\} - y(s) \left\{ G \int \Phi_y(s) \right\} \dots (138)$$

and

$$\begin{aligned}
 \dot{m}_{in}(s) = & \dot{w}(s) \left\{ R_s + G\mathcal{J} + \frac{h_{cF}}{C_{psF}} F \right\} - h(s) \left\{ G\mathcal{J}_Y \Phi_h(s) + \frac{h_{cF}}{C_{psF}} F Y_F \Phi_F(s) \right\} \\
 & - y(s) \left\{ G\mathcal{J}_Y \Phi_Y(s) \right\} \quad \dots (139)
 \end{aligned}$$

These equations completely express the behaviour of the cold room under sudden changes of the heat load and of the moisture infiltration. They relate directly to each other the changes of the room air conditions and the stem position of the evaporator pressure-regulating valve, considered as an independent variable. As can be seen from Eq. (138), the solution of the heat transfer problem in a cooled space does not depend on the quantities of vapor present in the air entering or leaving it. The vapor content of the room air, however, depends on the enthalpy content. In other words, the heat transfer problem affects the mass transfer problem. It should be remarked that the foregoing discussion is set up with the assumption that the enthalpy and the humidity ratio of the room air are the two independent variables of the system.

Steady-state Solution of the Balance Equations

A particular solution of the Eqs. (138) and (139), which is of importance, is the final steady-state solution for the conditions, the room air will take after an infinitely long period of time, i.e. a certain time after the perturbances in the heat or moisture load of the room did occur.

In order to find this solution, a small digression is to be made as to the final values of the various "transfer functions Φ " appearing in the Eqs. (138) and (139).

From Eq. (131) one directly finds, by putting $s=0$, that

$$\Phi_F(0) = 1 \quad \dots(140)$$

From Eq. (126) one finds that after an infinite period of time, i.e. for $s = 0$, that

$$\Phi_h(0) = 1 - \frac{1}{\frac{G\beta_R}{b'} + \frac{\gamma}{\theta} - \frac{\gamma}{\gamma} + 1} \quad \dots(141)$$

From Eq. (127) one finds by an analogous procedure

$$\Phi_y(0) = \frac{\beta_R b''}{G\beta_R + b' \left[1 + \frac{\gamma}{\theta} - \frac{\gamma}{\gamma} \right]} \quad \dots(142)$$

By substituting these function values into the balance equations, one finds that for $s=0$, the following final conditions will be fulfilled by the state of the room air

$$\begin{aligned} q_{in}(0) &= h(0) \frac{G\gamma}{\frac{G\beta_R}{b'} + \frac{\gamma}{\theta} - \frac{\gamma}{\gamma} + 1} \\ &\quad - y(0) \frac{G\beta_R b''}{G\beta_R + b' \left[1 + \frac{\gamma}{\theta} - \frac{\gamma}{\gamma} \right]} \end{aligned} \quad \dots(143)$$

ILLUSTRATIVE PROBLEMS

In this chapter a few problems are solved, which illustrate the theory, set forth in the foregoing chapters.

PROBLEM I.

Air at -10°C , with a humidity degree of 80%, enters an evaporator. The refrigerant evaporating temperature in the coils is -15°C . The air flow rate is 2200 kg/hr. The total external surface area of the coils is 42 m^2 .

Experiments have shown that the overall heat transfer coefficient is given by

$$U = 50 \text{ kcal/hr m}^2 (\text{kcal/kg})$$

The outside heat transfer coefficient equals

$$h_c = 14.6 \text{ kcal/hr } ^{\circ}\text{C m}^2$$

Define directly, without using any graphical method, the state of the air leaving the coils.

SOLUTION: From the psychrometric chart one is able to obtain the following values:

$$\text{Enthalpy of the air } H = -1.68 \text{ kcal/kg}$$

$$\text{Humidity ratio } W = 0.00128 \text{ kgw/kg}$$

$$\text{Saturated air enthalpy at } -15^{\circ}\text{C} \quad H_{SR} = -3.01 \text{ kcal/kg}$$

$$\text{Specific heat of saturated moist air at } -15^{\circ}\text{C}$$

$$c_{ps} = 0.24 + 0.46(-15)(0.00101) = 0.233 \text{ kcal/kg } ^{\circ}\text{C}$$

Furthermore, one calculates the evaporator contact factor

$$\Theta = \frac{1 - e^{-UA/G}}{1 - e^{-(50)(42)/(2200)}} = 1 - 0.385 = 0.615$$

and the evaporator surface contact factor

$$\begin{aligned}
 \mathfrak{J} &= 1 - e^{-h_c A / G C_{ps}} = 1 - e^{-(14.6)(42) / (2200)(0.233)} = 1 - e^{-(42) / (2200) \times (0.016)} \\
 &= 1 - 0.303 = 0.697
 \end{aligned}$$

The total heat transfer from the air to the coils is given by

$$Q_E = G \Theta (H - H_{SR}) = (2200)(0.615)(1.33) = 1800 \text{ kcal/hr}$$

Now, by means of Eq. (57), one calculates the equivalent surface enthalpy

$$\begin{aligned}
 H_{SE} &= H_{SR} + \frac{Q_E}{G} \left\{ \frac{1}{\Theta} - \frac{1}{\mathfrak{J}} \right\} = -3.01 + \frac{1800}{2200} \left\{ \frac{1}{0.615} - \frac{1}{0.697} \right\} \\
 &= -3.01 + (0.82)(1.622 - 1.432) = -3.01 + 0.154 = -2.864
 \end{aligned}$$

From the psychrometric chart, the corresponding equivalent evaporator surface temperature equals -14.51°C . The corresponding value for the humidity ratio of saturated air is

$$W_{SE} = 1.061 \times 10^{-3} \text{ kgw/kg}$$

The mass transfer from the air to the coils is given by

$$\begin{aligned}
 M_E &= G \mathfrak{J} (W - W_{SE}) = (2200)(0.697)(1.280 - 1.061)(10^{-3}) \\
 &= 0.325 \text{ kgw/hr}
 \end{aligned}$$

Since

$$Q_E = G (H - H_2)$$

$$M_E = G (W - W_2)$$

one directly finds for the state of the leaving air

$$H_2 = -1.68 - (1800)/(2200) = -2.50 \text{ kcal/kg}$$

$$W_2 = 0.00128 - (0.325)/(2200) = 0.001122 \text{ kgw/kg}$$

The transfer of sensible heat to the coils can be found

$$sQ_E = G (T - T_2)(C_{pa}) = G \mathfrak{J} (0.24)(T - T_{SE})$$

However, it is not necessary to determine sQ_E , for the temperature of the leaving air, according to Eq. (22), is given by the expression

$$\begin{aligned}
 T_2 &= T - \mathfrak{J}(T - T_{SE}) = -10 - (0.697)(-10 + 14.51) \\
 &= -13.14^\circ\text{C}
 \end{aligned}$$

DISCUSSION

The foregoing example shows clearly how the introduction of an equivalent evaporator surface permits the analytical solution of the problem of heat and mass transfer in cooling coils. A few simple calculations provide the same result as a long and inaccurate method, in which the process curve of the air, piece by piece, is determined on the psychrometric chart. The method is completely revolutionary. In the future it will supersede all the methods, at present commonly used in the field of refrigeration, air conditioning and chemical engineering.

The foregoing example made use of Eq. (57). This means that a linear approximation of the saturation curve was considered to be good enough. If, however, a greater accuracy is required, as for instance in the field of chemical engineering, one should take preferably a quadratic approximation of the saturation curve and use the Eqs. (50) and (51). These equations permit the finding of the total mass transfer. From there on, the procedure to find the state of the leaving air flow, remains the same as in the foregoing example.

PROBLEM II

Find the parameters which govern the dynamic behaviour of the internal evaporator-compressor system for the evaporator encountered in problem I.

Additional information:

Evaporator: the total internal volume, occupied by the refrigerant is

$$V = 0.008 \text{ m}^3$$

Compressor: the volumetric efficiency is a linear function of the compression ratio (ref. 61)

$$\lambda = 0.9756 - 0.456 (P_C/P_R)$$

Thermostatic Expansion Valve: the thermostatic expansion valve has the following characteristics:

Mass flow rate of refrigerant (kg/hr):

$$M_i = 46,900 S_v (P_C - P_R)^{\frac{1}{2}}$$

Orifice (m^2) (linear valve):

$$S_v = 5.8 \times 10^{-6} + 0.925 \times 10^{-6} \times t^*$$

where t^* is the deviation of the superheat from its setting value $\frac{0}{T^*} = 5 \text{ C}$. Thus.

$$t^* = T^* - \frac{0}{T^*}$$

Time constant: the time constant of the valve and of its feeling bulb is given to be

$$\tau_v = 0.01 \text{ hr} \quad (36 \text{ seconds})$$

Remark: The valve characteristics, just described, are completely fictitious values. A setting of 5 C superheat would require the presence of a solenoid valve in order to protect the coils against "flood back" in this case.

Condenser temperature: 20 C

Subcooling temperature: 15 C

The initial steady-state operation is the same as described in problem I

Evaporating temperature: - 15 C (T_R^0)

Heat load: 1800 kcal/hr (J_q^0)

Refrigerant used: F12

SOLUTION

From the refrigerant chart and tables the following values are determined

$$P_R = 18,600 \text{ kg/m}^2$$

$$P_C = 57,800 \text{ kg/m}^2$$

$$I' = 96.68 \text{ kcal/kg}$$

$$I'' = 135.30 \text{ kcal/kg}$$

$$I_i = 103.50 \text{ kcal/kg}$$

$$I_o = 136.03 \text{ kcal/kg}$$

$$X_i = 0.175 \text{ (17.5\%)}$$

$$v_o = 0.096 \text{ m}^3/\text{kg}$$

Introducing the variables

$$\begin{aligned} t_R &= T_R - T_R^0 = T_R + 15 \\ t^* &= T^* - T^0 = T^* - 5 \end{aligned}$$

the following linear approximations can be found from the refrigerant chart.

$$P_C/P_R = 3.11 - 0.13 t_R$$

$$(P_C - P_R)^{\frac{1}{2}} = 198 - 6.2 t_R$$

$$I_{fg} = I_{fg}^0 + c_I t_R = 38.62 - 0.1 t_R$$

$$\rho'' = \rho''^0 + c_\rho t_R = 10.8 + 0.35 t_R$$

$$v_o = 0.096 - 0.0036 t_R + 0.00031 t^*$$

$$X_i = X_i^0 + c_X t_R = 0.175 - 0.005 t_R$$

The specific heat of the refrigerant gas at constant pressure is

nearly

$$C_p = 0.145 \text{ kcal/kg C}$$

After the foregoing data have been gathered, determining the system's dynamic parameters will be a matter of calculations.

The outgoing mass flow rate

$$\text{At each moment} \quad M_o = \frac{\lambda D}{v_o}$$

In this expression λ is given by (cf. the original data) :

$$\lambda = 0.9756 - 0.456 (3.11 - 0.13 t_R) = 0.85 + 0.006 t_R$$

The rate of displacement D is given indirectly by the initial steady-state operation of the system:

$$D = \frac{\frac{Q}{J_q}}{I_o - I_i} \frac{\frac{Q}{v_o}}{\lambda} = \frac{(1800)(0.096)}{(33.53)(0.85)} = 6.075 \text{ m}^3/\text{hr}$$

The expression for M_o then becomes

$$M_o = \frac{(0.85 + 0.006 t_R)(6.075)}{(0.096 - 0.0036 t_R + 0.00031 t^*)}$$

Partially differentiating this expression with respect to the variables t_R and t^* , results in the following expression for M_o

$$M_o = 53.8 + 2.46 t_R - 0.1730 t^* \quad \dots (a)$$

The entering mass flow rate (in the form of flash gas)

One may combine into a single expression the functions, which were given in the original data to describe the valve characteristics. Further introducing the linearized expression for the pressure difference between the condenser and the evaporator results in the following equation:

$$M_i = 46,900 (5.8 + 0.925 t^*)(10^{-6})(198 - 6.2 t_R)$$

The amount of flash gas entering the coils is obtained as follows:

$$M_i^f = M_i X_i = 46,900 (5.8 + 0.925 t^*)(10^{-6})(198 - 6.2 t_R)(0.175 - 0.005 t_R)$$

Linearization of the last expression results in

$$M_1^H = 9.42 - 0.565 t_R + 1.50 t^* \quad \dots (b)$$

(It should be pointed out here that this expression, as well as the foregoing, holds during steady-state operations of the thermostatic valve only).

The vaporization coefficients

On the basis of the relations, given on page 58, one may calculate the following coefficients:

$$G_q = \frac{1}{135.3 - 96.68} \left(1 - \frac{(0.145)(5)}{135.3 - 103.5} \right) = \frac{1}{38.62} (0.977) = 0.0252$$

$$G_R = - \frac{(1800)(-0.1)}{(38.62)^2} (0.977) = 0.1178$$

$$G^* = - \frac{(1800)(0.145)}{(38.62)(135.3 - 103.5)} = - 0.2127$$

$$g_R = - (-0.1)(53.8 - 9.42) = 4.42$$

$$g^* = - (0.145)(53.8) = - 7.82$$

The changes in the mass and energy content of the coils

As defined on page 58, these coefficients are

$$\mu = (0.008)(0.35) = 0.0028$$

$$\varepsilon = (0.008)(34.62)(0.35) + (0.008)(10.8)(0.1) = 0.105$$

In the last computation the value of $\dot{E}_{fg} = \dot{I}_{fg} - \frac{P_R (v'' - v')}{427} = 34.62$ has been introduced. The coefficient c_E approximately equals c_I .

The inlet and outlet coefficients

From the equations (a) and (b) one finds directly that

$$B_R = 2.46 \text{ kg/hrC}$$

$$B^* = -0.1730 \text{ kg/hrC}$$

$$K_R = -0.565 \text{ kg/hrC}$$

$$K^* = 1.50 \text{ kg/hrC}$$

By means of the formulas, given on page 57, one further calculates

$$b_R = (2.46)(136.03 - 96.68) = 96.9 \text{ kcal/hrC}$$

$$b^* = (-0.1730)(136.03 - 96.68) = -6.80 \text{ kcal/hrC}$$

$$k_R = (-0.565)(135.3 - 96.68) = -21.81 \text{ kcal/hrC}$$

$$k^* = (1.50)(135.3 - 96.68) = 57.9 \text{ kcal/hrC}$$

PROBLEM III

Find the function which, in terms of the heat influx, describes the dynamic response of the system, consisting of the internal evaporator, the compressor and the valve.

SOLUTION

The solution of this problem is found by substituting into Eq. (108) the time constant τ_v and the dynamic parameters found in problem II.

By doing so, one obtains the following transfer function

$$\frac{t_R(s)}{j_q(s)} = \frac{-0.092676 + 0.00014 s}{0.00134775 s^2 + 0.002295 s - 7.73913}$$

According to this result, the capacity curve of the unit has a slope

$$\frac{7.73913}{0.09267} = 83.5 \text{ kcal/hr } ^\circ\text{C}$$

This value is normal for a unit of 1800 kcal/hr, working between -15 and 20 $^\circ\text{C}$.

DISCUSSION

Eq. (108), derived in this thesis, arrives at the result that the response of an evaporator-compressor-valve system, in terms of the heat influx is essentially second order.

As shown in the above problem, however, Eq. (108) is very difficult to apply. Because of the small numbers it involves, the slightest error in the refrigerant properties leads to erroneous results. In almost all cases, however, the refrigerant properties cannot be found with the necessary accuracy.

As a conclusion, one may say that for all practical applications, it is better to start from experimental response curves of cooling coils, and to cal-

culate by means of these data the complete response of cold rooms. How this can be done, will be shown in the following problem.

PROBLEM IV

Experiments have shown that the response of a certain refrigeration unit, working under the same circumstances as in problem I, may well be approximated by the following transfer function

$$\frac{t_R(s)}{q_E(s)} = \frac{0.01}{(1 + 0.02 s)^2}$$

In other words, the unit would have a critically damped second-order response. The steady-state gain would be 100 kcal/hr C (slope of the capacity function)

This unit will be used to cool a cold storage room, containing 2000 kg of frozen fish (specific heat approximatively equal to 0.4 kcal/kg C). The convection heat transfer coefficient at the food surface is 7 kcal/hr m² C. The total surface area of the foods is 12 m².

The total air content of the room is approximately 200 kg.

The problem is to determine the transfer functions, which relate the enthalpy of the room air to a sudden change in the heat load on the room, and which relate the humidity ratio of the room air to sudden variations either of the vapor or of the heat influx (or both) into the storage room.

The initial conditions of the room air are to be taken as given in problem I.

SOLUTION

The slope of the saturation curve in the neighbourhood of -15°C is equal to

$$\beta_R = \frac{dh_s}{dT} = 0.3 \text{ kcal/}^{\circ}\text{C}$$

Hence, the above transfer function may be written

$$\frac{h_{sR}}{q_E} = \frac{0.0033}{(1 + 0.02 s)^2}$$

This function is completely equivalent to Eq. (123) on page 67.

The variable q_E can be expressed in terms of the equivalent surface enthalpy h_{sE} , as follows

$$q_E = G \int (h_l - h_{sE})$$

On the other hand one has the equality

$$\int (h_l - h_{sE}) = \Theta (h_l - h_{sR})$$

Elimination of the variables q_E and h_{sR} from the above equations results in the following transfer function

$$\frac{h_{sE}(s)}{h_l(s)} = 1 - \frac{1}{\frac{\Theta}{G} + \frac{0.0033 G \Theta}{(1 + 0.02 s)^2}} = \Phi_h(s)$$

Using the data and the values $C_{psF} = 0.235$ and $\beta_F = 0.32$, obtained from the psychrometric chart, one directly finds on the basis of Eq. (131) the transfer function for the food surface enthalpy in terms of the enthalpy of the room air:

$$\frac{h_{sF}(s)}{h_l(s)} = \frac{1}{1 + \frac{(2000)(0.4)(0.235)}{(0.32)(7)(12)} s} = \Phi_F(s)$$

The transfer functions $\Phi_h(s)$ and $\Phi_F(s)$ thus become (cf. the data in problem I)

$$\Phi_h(s) = 1 - \frac{1}{1.132 + \frac{5.1}{(1 + 0.02s)^2}}$$

$$\Phi_F(s) = \frac{1}{1 + 7s}$$

The complete response of the room air, given by the Eqs. (138) and (139), can now be written as follows. On the basis of Eq. (138) one has

$$\frac{h(s)}{q_{in}(s)} = \frac{1}{200 s + \frac{(2200)(0.697)}{1.132 + \frac{5.1}{(1 + 0.02s)^2}} + \frac{(7)(12)/(0.235)}{1 + 7s}(7s)}$$

A similar equation can be set up to describe the response of the humidity ratio of the room air. (cf. Eq. (139)). By means of the values $\gamma = 0.0004$ and $\gamma_F = 0.00045$ one has

$$w(s) = \frac{m_{in}}{200 s + (2200)(0.697) + \frac{(7)(12)}{0.235}} + \frac{q_{in} \left[(0.4599) \left\{ 1 - \frac{1}{1.132 + \frac{5.1}{(1 + 0.02 s)^2}} \right\} + \frac{(0.160)}{1 + 7 s} \right]}{\left(200 s + 2200(0.697) + \frac{7(12)}{0.235} \right) \left(200s + \frac{2200(0.697)}{1.132 + \frac{5.1}{(1 + 0.02s)^2}} + \frac{2597s}{1 + 7s} \right)}$$

Thus, the humidity of the air depends not only upon the influx of vapor into the room, but also upon the heat influx to the room.

After an infinite period of time the room air enthalpy will increase by an amount

$$\frac{h(0)}{q_{in}(0)} = 0.000405 \frac{\text{kcal/kg}}{\text{kcal/hr}}$$

for each additional kcal/hr of heat influx into the room.

Similarly, after an infinite period of time, the humidity ratio of the room air will increase by an amount

$$\frac{w(0)}{m_{in}(0)} = 0.00053 \frac{\text{kgw/kg}}{\text{kgw/hr}}$$

for each additional kgw/hr of vapor influx into the room.

But, the humidity ratio of the room air will also increase

$$\frac{w(0)}{q_{in}(0)} = 0.000,00133 \frac{\text{kgw/kg}}{\text{kcal/hr}}$$

for each additional kcal/hr of heat influx into the room, whether this be sensible or latent heat. This conclusion is rather surprising, for it is common knowledge that the humidity in cold rooms decreases under heavy heat loads. Therefore it must be pointed out that the last statement is perfectly true for cold rooms with on-off control systems. It does not apply to systems where the compressor would be adjusted to the instantaneous heat load and thus would run 24 hrs a day. For then the evaporating temperature does increase slightly when the heat load undergoes a small and upward variation.

As to the form of the response curves, the dynamic response of the room air enthalpy to a step input function for the heat load is fourth-order. That of the humidity ratio of the room air is first-order with respect to a step perturbation of the vapor influx, but at the same time, it is fifth-order with respect to the perturbances occurring in the heat influx, accompanying this vapor in the form of latent heat. By means of the results, calculated above, the response of cold rooms can be found for any forcing function $q_{in}(s)$ and $m_{in}(s)$.

CONCLUSIONS

The problems just solved show clearly that the concepts, set forth in this thesis, are completely revolutionary. By means of two equations, given as Eq. (138) and Eq. (139) in the text, it becomes possible to calculate directly how the enthalpy and the humidity ratio of the room air will respond to sudden or gradual perturbances in the heat and moisture load imposed upon the room.

Wilfully these final equations have been formulated in general terms. This is done to allow for a further development of the theory and to facilitate computer simulations. In each particular case the transfer functions which describe the dynamic response of the various parts of the system must be substituted into the above equations. A complete example has been given in problem IV.

A special theory was set up to find the dynamic response of dry expansion evaporators. From a theoretical viewpoint the results are completely satisfactory. Moreover, they are in close accordance with phenomena as valve hunting and sluggish action of the evaporator, often encountered in practice. However, it was found in problem III that the direct application of these results encounters practical difficulties of a mathematical character.

The most important feature of the present thesis is that it constitutes the first attempt to solve the problem of the dynamic response of cold rooms in terms of two variables of the room air, for these variables, the enthalpy and humidity ratio of the room air, are inherently related to each other. A function which would describe the response of the room air in terms of its temperature only would be either incomplete or false. The special approach,

used in this thesis was made possible by the introduction of an "equivalent heat and mass exchanging surface" (Eq. (57)). Since actual heat and mass exchangers have non-uniform temperature distributions along their surface, complicated graphical procedures were necessary to derive from the heat transfer the quantity of mass transferred to the surface.

The procedure introduced in this thesis replaces these graphical methods by a very simple analytical calculation. An example was given in problem I. It may be expected that the new method will find general acceptance in the fields of chemical and environmental engineering for use in the design of cooling devices.

The analytical approach to dynamic response problems is necessarily restricted to the simplest systems and applications. However, once the basic concepts have been fixed, many specialized techniques are available to the engineer. It may therefore be expected that the analytical approach, grounded in this thesis, will be elaborated further and become a basic tool in the design of refrigeration systems.

EXPLANATION OF PLATE I

Fig. 1. The driving potential difference in wetted surface heat exchange. In Fig. 1. the driving potential difference is given by segment 1-Z, the distance Z-Z' being equal to $H_{fW}(W_1 - W_s)$. The equivalent temperature difference is measured by the segment X-S. The surface contact factor is equal to the ratios $(1-3)/(1-Z')$ and $(1-2)/(1-S)$.

Fig. 2. The equivalent surface enthalpy is given by H_{SE} . By definition, the indicated surfaces are equal.

PLATE I

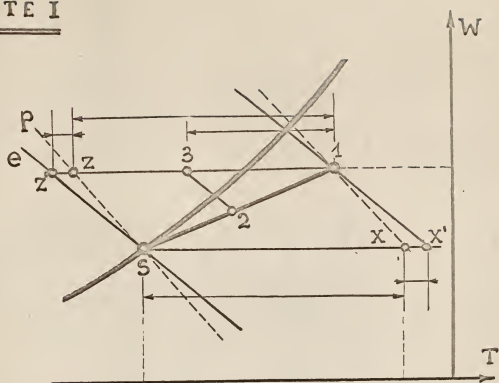


Fig. 1.

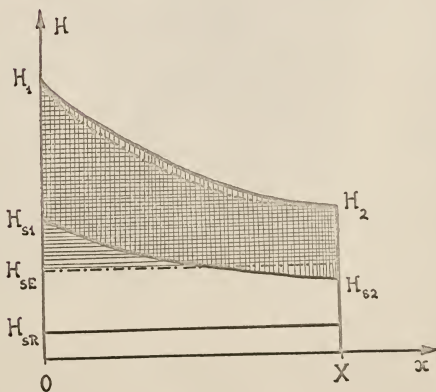


Fig. 2.

PLATE II

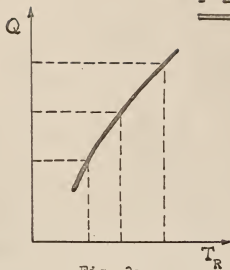


Fig. 3a.

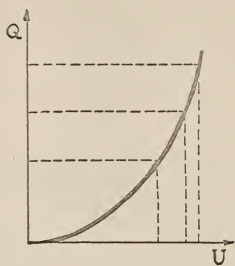


Fig. 3b.

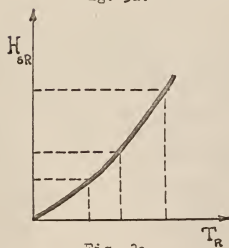


Fig. 3c.

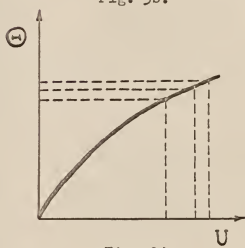


Fig. 3d.

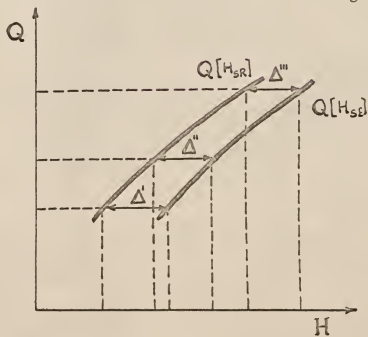


Fig. 3e.

EXPLANATION OF PLATE II

Fig. 3. The capacity curve in terms of the equivalent surface enthalpy. The various steps in the procedure can be followed. The segments Δ are equal to $(Q/G)[\frac{1}{\theta} - \frac{1}{\theta_1}]$.

ACKNOWLEDGMENT

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NOMENCLATURE

A	surface area
A	constant
a	part of the total evaporator surface area
a, a^R, a^E	coefficients in the capacity curve of the refrigeration unit
B	constant
b, b^R, b^E	coefficients in the capacity curve of the refrigeration unit
B_R, B^*, B_y	partial derivatives of the mass flow rate to the compressor
b_R, b^*, b_y	partial derivatives of the enthalpy flow to the compressor
C	constant
C	specific heat for liquids and solids
C_p	specific heat at constant pressure for gases
C_v	specific heat at constant volume for gases
c	constant
c	first degree coefficient in the linear approximation of thermodynamic properties
D	diffusion coefficient
D	displacement rate of the compressor
d	diameter
d	differential
E	internal energy
F	surface area
F, f	function
f	the thickness of a frost layer as a function of other variables
G	air flow rate
G_q, G_R, G^*	partial derivatives of the amount of liquid evaporating in coils

$\varepsilon_R, \varepsilon^*$	partial derivatives of two enthalpy quantities
\dot{G}	refrigerant flow rate
H, h	enthalpy of moist air (absolute value and deviation variable)
h_c	air side heat transfer coefficient
h_d	air side mass transfer coefficient
h_c^w	special coefficient in wetted surface heat transfer
h_i	refrigerant side heat transfer coefficient
I	refrigerant enthalpy
J, j	heat flux quantity (absolute and deviation variable)
j_q	heat flux from the walls of an evaporator to the refrigerant
K	constant
K_w, K_w^i	special coefficients for the heat transfer through evaporator walls
K_R, K^*	partial derivatives of the amount of flash gas, entering an evaporator
k_R, k^*	id. for the corresponding enthalpy flow
k	thermal conductivity
L	humidity slope of a process line on the psychrometric chart
M	mass flow rate of water vapor
m	constant coefficient of approximative saturation curve of moist air
m	quantity of mass
N	constant coefficient
n	coefficient of approximation of saturation curve of moist air
P	pressure
Q	heat flow
q	heat flow related to the unit mass of (passing) air
R	the total air content of a cold room

R	gas constant
R	thermal resistance
S_v	the area of a valve orifice
S_m	the average flow passage of air through an evaporator
s	subsidiary domain of the time
T, t	temperature (actual and deviation value)
U	overall heat transfer coefficient
V	total internal volume, occupied by the refrigerant in cooling coils
v	specific volume
W, w	humidity ratio of moist air (actual and deviation value)
X	quality of refrigerant
X	total length of an evaporator in the direction of the air flow
x	distance
y	layer thickness
Y, y	stem position of valve
Z_x	the total mass of the evaporator material per unit depth in the direction of the air flow

GREEK LETTERS

β	slope of the enthalpy of saturated air with respect to the temperature
γ	slope of the humidity ratio with respect to the enthalpy
$\partial; \Delta$	partial derivative; finite difference
ε	partial derivative of the energy content of the vapor phase inside an evaporator, with respect to the evaporating temperature
Θ	evaporator contact factor
\mathfrak{J}	evaporator surface contact factor
θ	time

ζ	damping factor
λ	actual volumetric efficiency of compressor
μ	partial derivative of the vapor content of cooling coils with respect to the evaporating temperature
ρ	density
τ	time constant
Φ	fin efficiency
Φ	transfer function

SUBSCRIPTS

f	liquid water
g	water vapor
fg	vaporization
i	refrigerant entrance
i	thermal coefficients related to the inner part
m	mean value
o	refrigerant exit
s	saturated air conditions
x	quantities, measured per unit depth of the evaporator
y	values related to the evaporator pressure-regulating valve
in	influx
ex	efflux, evacuation
A	evaporator surface
E	equivalent evaporator surface conditions
F	product surface, products
R	refrigerant evaporating temperature
P	bare pipe
Φ	fins

- 1 air inlet
- 2 air outlet

SUPERSCRIPTS

- ° initial steady-state value
- average value
- ' liquid refrigerant
- " saturated refrigerant vapor
- * properties of the refrigerant as sucked to the compressor, in particular
superheat

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THE DYNAMIC RESPONSE OF COLD ROOMS

by

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AN ABSTRACT OF A MASTER'S THESIS

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The environmental control of cold rooms involves the control of two thermodynamic variables of the room air, to wit its enthalpy and humidity ratio. During steady-state these variables take an equilibrium value, for which the influx of heat and moisture from external sources and from the storage goods is equal to the removal of heat and moisture by the evaporator.

Variations of the heat and moisture load, imposed upon the room, shift the equilibrium conditions of the room air. The purpose of the present thesis is to determine the response of the room air to these load variations.

The complexity of the problem is due to the fact that the heat and mass balances are not independent. For the heat and mass exchangers inside the room are saturated surfaces. Moreover, they have non-uniform temperature distributions. These difficulties are removed by the introduction of a new concept: the equivalent surface conditions. The introduction of equivalent surface conditions permits an easy and analytical solution for a problem which before could only be solved by a lengthy graphical procedure.

In order to find the dynamic response of the total room, one must know the response of the various parts, which constitute a cold room. The response of the storage goods is obtained by assuming that they follow the type of transient heat flow, which is known as Newtonian heating or cooling. For the dynamic response of the evaporator a new and approximative theory is set up. In this theory, only the refrigerant vapor phase is taken into consideration as to determine the response of the internal part of the evaporator in dry expansion coils. A great deal of the discussion deals with the linearization of the governing differential equations. The theory brings to light some very interesting aspects, concerning the dynamic response of dry expansion coils. A direct application of the results, however, turns out to be unreliable

because of mathematical difficulties. This does not constitute a major inconvenience. For, once the general pattern of the evaporator response is determined, it is not difficult to find experimentally the response curve which applies in a particular case.

In terms of the response curves of the various parts, and by combining these responses together, the total response of the cold room is found.

Practical calculations show a close agreement between the calculated and the expected values.

Since the final results of the thesis are kept in general terms, further developments of the theory can directly build on the analytical results obtained here.

