TRACKING LOOP DESIGN

by

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Abstract

In this thesis, we investigate two carrier tracking loops. We provide a basic overview of phase-lock loops. We derive a two-state EKF tracking loop. The two-state EKF estimates phase error and frequency error. The estimate of frequency error is fed back to an NCO to complete the tracking loop.

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Chapter 1

PLL Designs

In this chapter, we introduce the reader to basic concepts in Phase-lock loops (PLLs). The concepts include a generic transfer function, steady-state performance analysis, tracking and acquisition metrics, and loop filter design. Then, we examine the tracking and acquisition metrics by simulating a basic second-order PLL.

1.1 Phase-Lock Loops

In traditional receiver design, the carrier tracking loop is a phase-lock loop. In this section, we investigate phase-lock loops (PLL). We develop a PLL's transfer function. We introduce a set of metrics that quantify the tracking and acquisition limitations of a particular design. We explain the basic concept of a Costas PLL.

1.1.1 PLL Transfer Function

A PLL is a highly non-linear feedback loop. Figure 1.1 shows the loop approximated as a linear feedback system, this approximation holds when it operates in it's phase tracking region. The PLL can be broken into three subparts, the discriminator, the loop filter, and the numerically controlled oscillator.

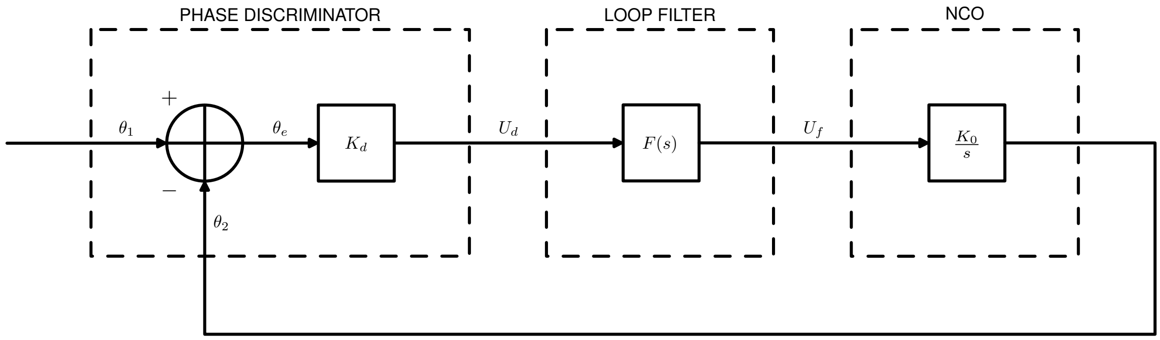


Figure 1.1: *Generic Tracking Loop*

Discriminator Model

The discriminator takes the incoming carrier wave and replica carrier wave and finds the phase difference between them. If the input carrier wave and the replica carrier wave are expressed as two different phases, θ_1 and θ_2, then the output of the phase discriminator, U_d, can be expressed as the phase difference, θ_e, and a gain, K_d. The discriminator output, U_d, can be expressed in the Laplace domain as

$$\theta_e(s) = \theta_2(s) - \theta_1(s) \tag{1.1}$$

$$U_d(s) = K_d \theta_e(s) \tag{1.2}$$

Generic Loop Filter

The output of the discriminator, U_d, contains the phase error, however it is corrupted by noise, so a loop filter is used to reduce the noise to a tolerable level. The output of the loop filter can be expressed as,

$$U_f(s) = F(s)U_d(s) \tag{1.3}$$

where $F(s)$ is the transfer function of the loop filter.

Numerically-Controlled Oscillator

The output of the loop filter is used by the NCO to adjust the frequency of replica carrier wave. The NCO is represented as an integrator with a gain, K_0. The NCO takes in frequency which integrates to phase. Hence, the output of the NCO is the phase of the replica carrier wave. The NCO is represented by

$$\theta_2(s) = \frac{K_0}{s} U_f(s) \tag{1.4}$$

Phase-Lock Loop Transfer Function

Using the above equations, a transfer function, $H(s)$, between the replica phase, θ_2, and the incoming phase, θ_1, can be formulated. Also, a transfer function, $H_e(s)$, between the phase error, θ_e and the input phase, θ_1, can be formulated. The transfer functions are,

$$H(s) = \frac{\theta_2(s)}{\theta_1(s)} = \frac{K_0 K_d F(s)}{s + K_0 K_d F(s)} \tag{1.5}$$

$$H_e(s) = \frac{\theta_e(s)}{\theta_1(s)} = \frac{s}{s + K_0 K_d F(s)} \tag{1.6}$$

Loop Filter Design

The loop filter, $F(s)$, attracts the most attention in the design of a PLL. The other two parameters, the discriminator gain, K_d, and the oscillator gain, K_0, are primarily functions of the particular discriminator and NCO. Therefore, when the PLL's characterestics need to change, the loop filter is modified. A common loop filter is a first order active filter. The filter transfer function is

$$F(s) = \frac{1 + \tau_2 s}{\tau_1 s} \tag{1.7}$$

A first-order loop filter leads to a second-order PLL. This is obvious when examining the simplified the error transfer function,

$$H_e(s) = \frac{\tau_1 s^2}{\tau_1 s^2 + K_d K_d \tau_2 s + K_d K_d} \tag{1.8}$$

Using transfer functions as a performance metric can be difficult.

1.1.2 PLL Metrics

To aid in design of PLL, multiple metrics have been devised that quantify the performance of a PLL. These metrics can be used to design the tracking loop. These metrics estimate the ability of a PLL to track a signal and the ability of a PLL to acheive phase lock on a signal.

Natural Frequency and Damping Coefficient

The transfer function for a second-order PLL with an active loop filter can be rewritten as

$$H(s) = \frac{2\zeta w_n s + w_n^2}{s^2 + 2\zeta w_n s + w_n^2} \tag{1.9}$$

where the natural frequency, w_n, and the damping coefficient, ζ are defined as

$$w_n = \sqrt{\frac{K_0 K_d}{\tau_1}} \tag{1.10}$$

$$\zeta = \frac{\tau_2}{2}\sqrt{\frac{K_0 K_d}{\tau_1}} \tag{1.11}$$

While these aren't metrics themselves, they are used to caculate the metrics presented in the following sections.

Hold Range

[1] defines the hold range of a PLL as the frequency range around the center frequency of the VCO/NCO which the PLL is guarenteed maintain phase tracking. The range is defined for active filters as

$$\Delta\omega_H = \infty \tag{1.12}$$

This is regarded as the maximum operating range of a PLL, which, for an active filters, is theoretically over an infinite operating range.

Pull-out Range

[1] defines the pull-out range of a PLL as the frequency range around the tracking frequency which a step in frequency can occur and the PLL maintain phase tracking. The pull-out range is approximated by,

$$\Delta\omega_{PO} \approx 1.8\omega_n(\zeta + 1) \tag{1.13}$$

This equations quantifies the ability of a tracking loop handle sudden change in the incoming carrier wave and still maintain phase tracking, i.e. not have to reaquire phase tracking.

Pull-in Range

[1] defines the pull-in range of a PLL as the frequency range around the center frequency ofthe VCO/NCO which an unlocked PLL will be able to eventually obtain phase tracking. The pull-in range is defined as.

$$\Delta\omega_P = (8/\pi)\sqrt{\zeta\omega_n K_0 K_d - \omega_n^2} \tag{1.14}$$

It can be approximated for high gain loops, i.e. $\omega_n \gg K_0 K_d$, as

$$\Delta\omega_P \approx (8/\pi)\sqrt{\zeta\omega_n K_0 K_d} \tag{1.15}$$

This metric sets the maximum deviation an acquisition algorithm must place the frequency, when attempting to gain phase tracking.

Lock Range

[1] defines the lock range of a PLL as the frequency range around the center frequency of the VCO/NCO which an unlocked PLL will obtain phase tracking a single beat note between the reference oscillator and the signal. The lock range is approximated by,

$$\Delta\omega_L \approx \pm\omega_n \tag{1.16}$$

Typically, the lock range is considered the nominal operating range of the PLL.

[1] also defines a lock time. This is the time it takes for the unlocked PLL to obtain phase tracking. The lock time is approximated by,

$$T_L \approx (1/\omega_n) \tag{1.17}$$

These metrics can be used by the designer to evaluate their designs. Again, the above metrics are approximatoins only for a second-order PLL with an active filter.

1.1.3 Costas PLL

An alternative PLL design, called a Costas PLL, uses two replica carrier waves. Figure 1.2 shows a generic model of a Costas PLL. The PLL multiplies the incoming carrier wave by a

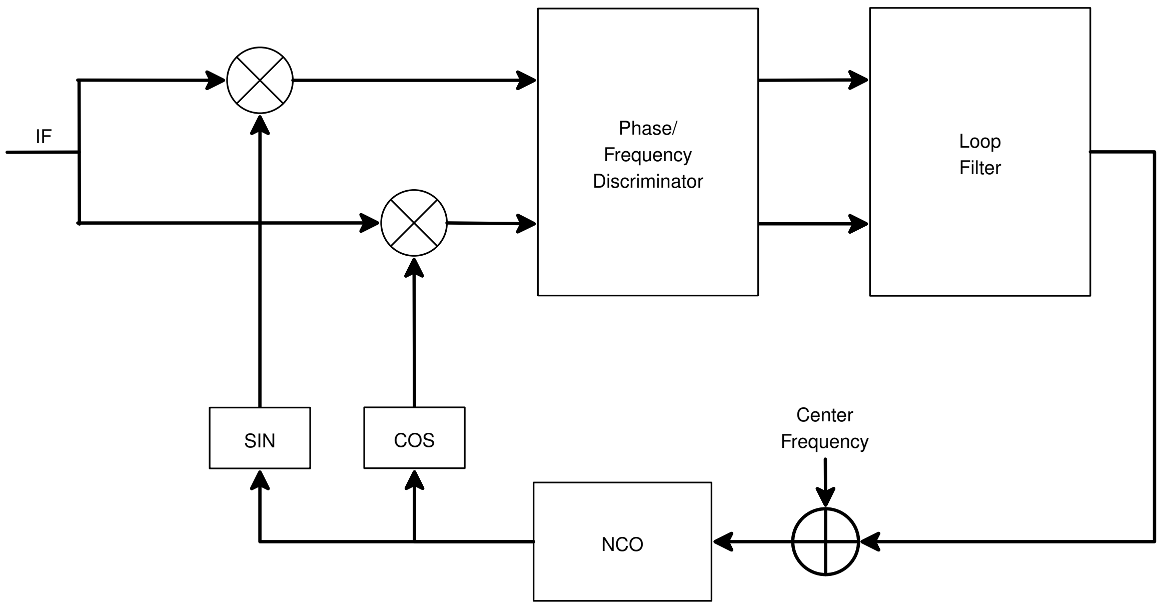


Figure 1.2: *Costas PLL*

replica carrier wave and a quadrature version (90° out of phase) of the replica carrier wave. The resulting elements are called the in-phase and quadrature elements. These elements are fed into a phase discriminator, typically an arctan operator to estimate the phase error. Alternatively, two consecutive in-phase and quadrature values can be used to find frequency error. The phase error and/or frequency error are fed into loop filters, similar to the one presented in the previous sections.

1.2 Performance Results

In this section, we will examine the design process for a second-order PLL.

1.2.1 Second-Order PLL

Let us investigate a design of a second-order PLL. The design process will be outlined, including an actual example. Then the design will be tested to verify the analytical results for pull-out range and pull-in range.

Design Process

Many approaches can be used to design a PLL. The following method lets the designer pick the lock range, the loop gain, and the damping coefficient. All other terms will fall out of these three parameters. The first step is to pick these parameters.

1. Specify the lock range, $\Delta f_L = 200\text{kHz}$
2. Specify the loop gain, $K_L = 1\text{MHz}$
3. Specify the damping coefficient, $\zeta = 0.707$

The next step is to calculate the rest of the loop parameters based on the lock range, loop gain, and damping coefficient.

1. Calculate Natural Frequency, $f_n = \frac{\Delta f_L}{2\zeta} = 141.4kHz$
2. Calculate Filter Coefficient, $\tau_1 = \frac{2\pi K}{(2\pi f_n)^2} = 7.955 * 10^{-6}$
3. Calculate Filter Coefficient, $\tau_2 = 2\zeta\sqrt{\frac{\tau_1}{2\pi K}} = 1.591 * 10^{-6}$

Finally, calculate the metrics based on the specified parameteres.

1. Lock range, $\Delta f_L = 200\text{kHz}$
2. Hold Range, $\Delta f_H = \infty$
3. Pull-Out Range, $\Delta f_{PO} = f_n(\zeta + 1) = 270kHz$
4. Pull-In Range, $\Delta f_P = \sqrt{\zeta f_n K - f_n^2} = 282.6kHz$

Simulink Model

The design was simulated in Simulink®. Figure 1.3 shows the model. The loop filter transfer function is $F(s) = \frac{1+\tau_2 s}{\tau_1 s}$. The model is a continuous-time model.

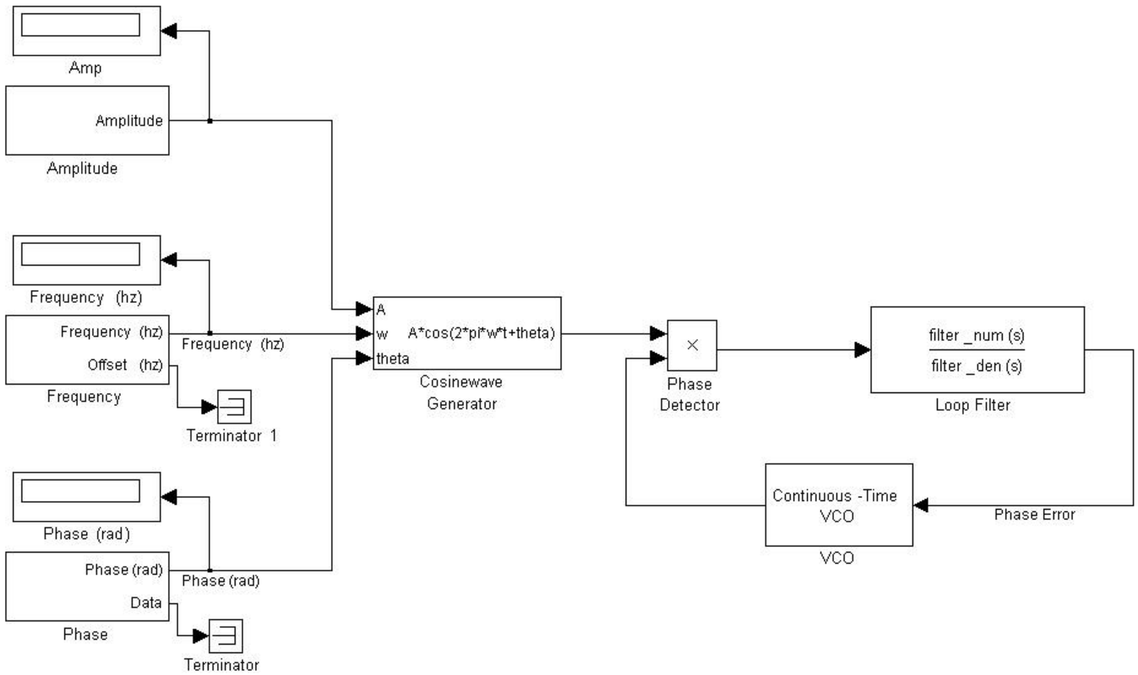


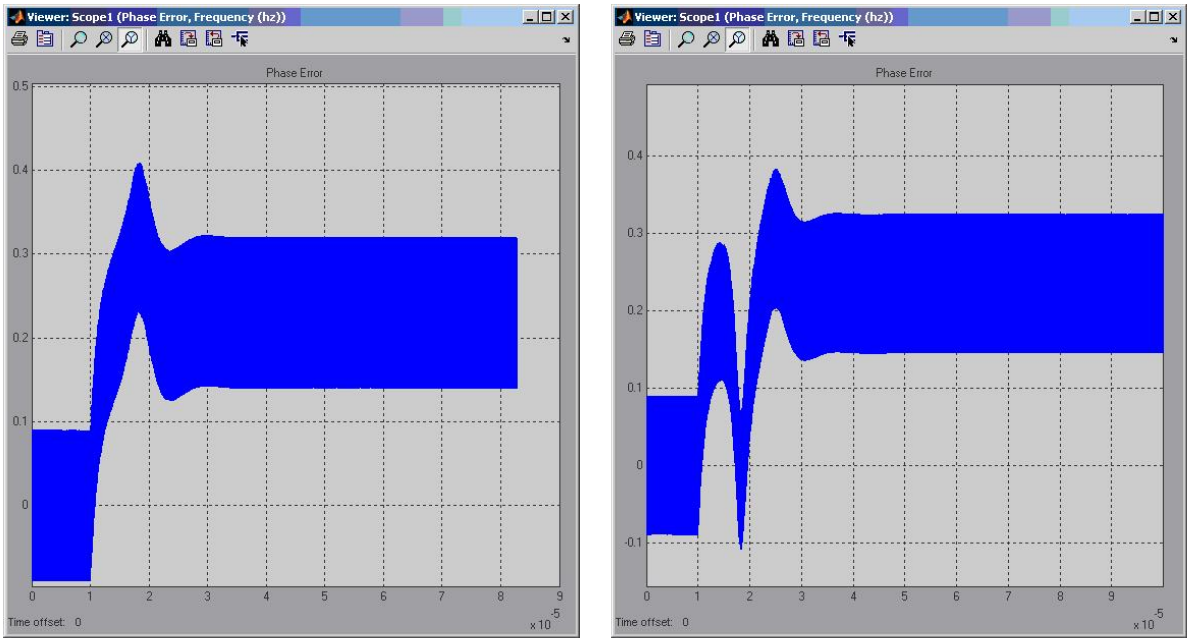
Figure 1.3: *Second-Order PLL Model*

Test Pull-Out Range

Figure 1.4 shows two test cases for pull-out range. The plots show the loop filter output of the PLL. In both test cases, the PLL is in phase-lock until the input signal undergoes a frequency step at 10 microseconds. Remember that pull-out range shows the maximum frequency step the PLL can undergo without a cycle slip. The PLL design is rated for a 270 kHZ pull-out range. Figure 1.4(a) undergoes a 230 kHZ frequency step without a cycle slip, while Figure 1.4(b) undergoes a 235 kHz frequency step and slips one cycle. The metric provides a ballpark estimate on the capabilities of the tracking loop. The actual pull-out range is within 15 percent of the metric estimate.

Test Pull-In Range

Figure 1.5 shows two test cases for pull-in range. The plots show the loop filter output of the PLL. In both test cases, the PLL is unlocked. Remeber that pull-in range is the maximum range around the nominal center frequency that an unlocked PLL will be able to maintain



(a) Pull-Out Range, 230 kHz

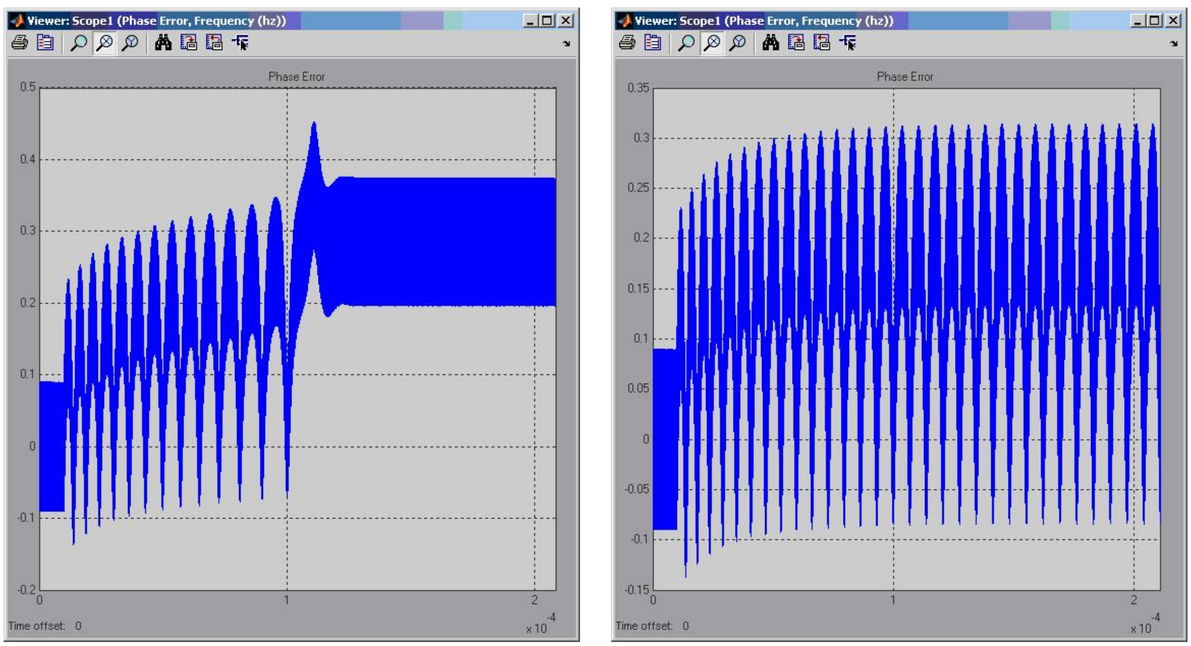
(b) Pull-Out Range, 235 kHz

Figure 1.4: *Second-Order PLL, Pull-out Range Example*

phase-lock. This design is rated for around 282 kHz. Figure 1.5(a) is 285 kHz away from the nominal frequency and is able to eventually acheive phase-lock. Figure 1.5(b) is 290 kHz away from the nominal frequency and is not able to acheive phase-lock.

1.3 Summary

In this chapter, we introduced the reader to a variety of related topics. We introduced basic concepts related to phase-lock loops. These included PLL transfer functions, metrics to evaluate tracking and acquisition limitations, and the costas PLL. Then we introduced how to design a basic second-order PLL.



(a) Pull-In Range, 285 kHz

(b) Pull-In Range, 290 kHz

Figure 1.5: *Second-Order PLL, Pull-in Range Example*

Chapter 2

EKF Tracking Loops

In this chapter, we introduce an alternative tracking loop design. We explore the use of an Extended Kalman Filter (EKF) to estimate error parameters, including frequency error, based on the correlator measurements. The estimate of frequency error can be used to correct the replica carrier wave. We develop an analytical expression for the in-phase and quadrature correlator measurements. We design a two-state EKF to estimate the errror parameters.

2.1 Measurement Model

The measurement equations are based on a discrete-time model of the correlator outputs (I and Q). The I and Q signals are based on the absolute phase of the incoming carrier wave and the absolute phase of the replica carrier wave. The correlator outputs are defined as

$$I = \sum_{k=1}^{N} AT \cos(\Theta(kT)) \sin(\hat{\Theta}(kT)) + v_I, \tag{2.1}$$

$$Q = \sum_{k=1}^{N} AT \cos(\Theta(kT)) \cos(\hat{\Theta}(kT)) + v_Q, \tag{2.2}$$

where, A is the amplitude of the incoming carrier wave, T is the summation timestep, and k is the accumulate and dump counter. The correlators are modeled with additive white gaussian noise, v_I and v_Q. Since I and Q are orthogonal, their noise is uncorrelated. The combined measurement noise covariance, R is a diagonal matrix, $R = diag\{\begin{bmatrix}\sigma_I^2 & \sigma_Q^2\end{bmatrix}\}$. Note that σ_I^2 and σ_Q^2 are directly related to the SNR of the incoming signal. Θ, the absolute phase of the incoming carrier wave, is defined as

$$\Theta(k) = \left[\sum_{i=1}^{k} f_{hot}(\Theta)T + \dot{\theta}_0\right] T + \theta_0. \tag{2.3}$$

which can be approximated by ignoring higer order terms. By ignoring the higher order terms, the model assumes that $\dot{\theta}$ is constant over the correlation accumulate and dump period. The approximation can be written as

$$\Theta(k) = \left[\sum_{i=1}^{k} \dot{\theta}_0\right] T + \theta_0 \tag{2.4}$$

The absolute phase of the replica carrier wave, $\hat{\Theta}$, is defined as

$$\hat{\Theta}(k) = \left[\sum_{i=1}^{k} \hat{\dot{\theta}}_0\right] T + \hat{\theta}_0 \tag{2.5}$$

Note that $\hat{\dot{\theta}}$ is not a simplification. This term does not change over the integration period, which means it is not an approximation.

Equations (2.4) and (2.5) can be rewritten as

$$\Theta(k) = k\dot{\theta}_0 T + \theta_0, \tag{2.6}$$

$$\hat{\Theta}(k) = k\hat{\dot{\theta}}_0 T + \hat{\theta}_0. \tag{2.7}$$

Our objective is to reduce the frequency and phase error to zero. Using basic trigonometric identities to include phase error θ_e and frequency error $\dot{\theta}_e$, Equations (2.1) and (2.2) can be rewritten as

$$I = \sum_{k=1}^{N} -\frac{AT}{2}\sin(\Theta_e), \tag{2.8}$$

$$Q = \sum_{k=1}^{N} -\frac{AT}{2}\cos(\Theta_e). \tag{2.9}$$

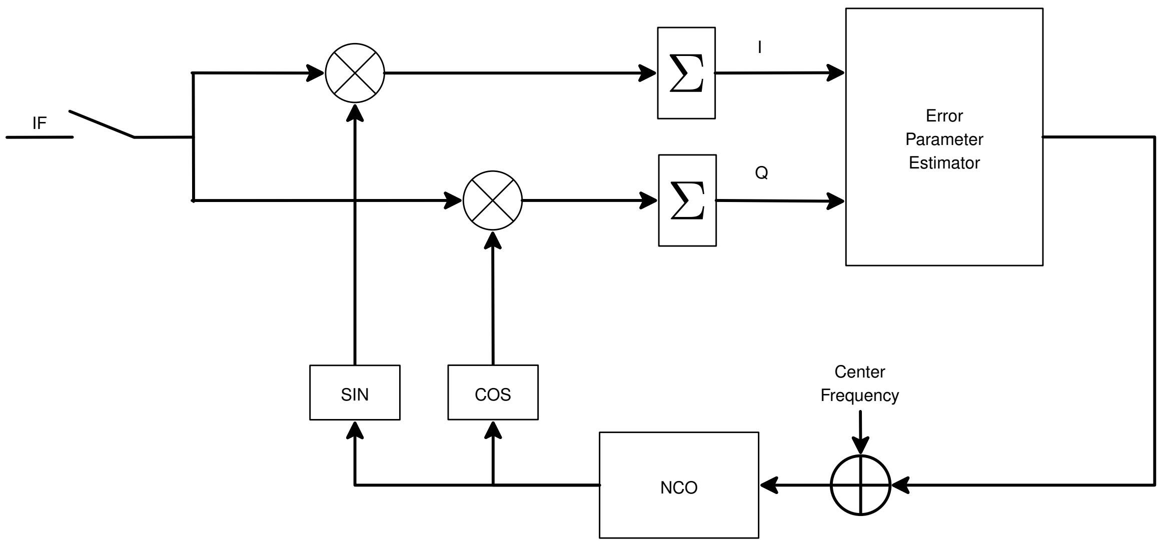


Figure 2.1: *Generic Tracking Loop*

Here, Θ_e corresponds to

$$\Theta_e = k\dot{\theta}_e T + \theta_e, \tag{2.10}$$

with θ_e (phase error) and $\dot{\theta}_e$ (frequency error) defined as

$$\theta_e = \theta_0 - \hat{\theta}_0, \tag{2.11}$$

$$\dot{\theta}_e = \dot{\theta}_0 - \hat{\dot{\theta}}_0 \tag{2.12}$$

Since Equations (2.8) and (2.9) are non-linear, a linear Kalman Filter cannot be used to estimate θ_e and $\dot{\theta}_e$. Instead, an extended Kalman Filter can be used, and is described next.

2.2 Two-state EKF Tracking Loop

Two-state EKF Tracking Loop

A two-state EKF can be used for error parameter estimation in Figure 2.1. The two-state EKF estimates the phase error and frequency error between the incoming signal and the replica signal based on measurements of the in-phase and quadrature correlator outputs. The states are defined in continuous time as $\mathbf{x}(t) = \begin{bmatrix} \theta_e(t) & \dot{\theta(t)}_e \end{bmatrix}^T$. The propagation model is based on the fact that the derivative of phase error is equal to frequency error. The frequency error state is used to correct the replica signal. Therefore, the correction applied to the replica carrier frequency is represented as an input, $u(t)$, into the state propagation model. Since phase error is pure integration of frequency error, there is no process noise in the phase error state. However, there is still process noise in the frequency error state. The state propagation model corresponds to

$$\dot{\mathbf{x}}(t) = F\mathbf{x}(t) + Gu(t) + Gw_{c2} \tag{2.13}$$

where, F and G are

$$F = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \tag{2.14}$$

$$G = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{2.15}$$

w_{c_2} represents the process noise as a Gaussian random variable, $N(0, \sigma^2_{\dot{\theta}_e})$. To find the process covariance of the entire state vector, we use the transform $G\sigma^2_{\dot{\theta}_e}G$. The process noise variance captures the accuracy of the error propagation model. The process noise covariance is

$$Q_{c_2} = G\sigma^2_{\dot{\theta}_e}G = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \sigma^2_{\dot{\theta}_e} \tag{2.16}$$

Using the approximations given in [2], the continuous-time model is approximated by a discrete-time model. The continuous-time model is transformed into a discrete-time model by the following approximation,

$$\Phi = I + FT + \frac{(FT)^2}{2!} \tag{2.17}$$

$$\Gamma = (IT + \frac{FT^2}{2!} + \frac{F^2T^3}{3!})G \tag{2.18}$$

$$Q_d = \int_0^T \exp^{F(t-\tau)} Q_c \exp^{F(t-\tau)} d\tau \tag{2.19}$$

where I is a 2x2 identity matrix, Q_c is assumed constant over the interval, and $\exp^{F(t-\tau)}$ is approximated using

$$\exp^{F(t-\tau)} \approx I + F(t-\tau) + \frac{1}{2}F^2(T-\tau). \tag{2.20}$$

The discrete-time states are $\mathbf{x}(\mathbf{k}) = \begin{bmatrix}\theta_e(k) & \theta(\dot{k})_e\end{bmatrix}^T$. The discrete-time state propagation equation is

$$\mathbf{x}(k) = \Phi\mathbf{x}(k-1) + \Gamma u(k) + \mathbf{w_2} \tag{2.21}$$

where, Φ and Γ are

$$\Phi = \begin{bmatrix}1 & T\\ 0 & 1\end{bmatrix}, \tag{2.22}$$

$$\Gamma = \begin{bmatrix}0\\ 1\end{bmatrix}. \tag{2.23}$$

Here, T is the correlator period. The discrete-time process noise $\mathbf{w_2}$ has a covariance corresponding to

$$Q_{d_2} \approx \begin{bmatrix}\frac{T^3}{3} & \frac{T^2}{2}\\ \frac{T^2}{2} & T\end{bmatrix}\sigma^2_{\dot{\theta}_e} \tag{2.24}$$

The in-phase and quadrature signals are used as measurements. Equations (2.8) and (2.9) represent the measurement equations.

An extended Kalman filter is used because the the in-phase and quadrature signals are non-linear functions of the states. Therefore, the measurement equations need to be linearized. The Jacobian of Equations (2.8) and (2.9) must be found with respect to the states, $\mathbf{x}$, and then evaluated at the current estimate, $\hat{\mathbf{x}}(k|k-1)$.

$$H = \left.\begin{bmatrix}\frac{\partial I}{\partial \theta_e} & \frac{\partial Q}{\partial \theta_e}\\ \frac{\partial I}{\partial \dot{\theta}_e} & \frac{\partial Q}{\partial \dot{\theta}_e}\end{bmatrix}\right|_{\hat{\mathbf{x}}(k|k-1)} \tag{2.25}$$

where $\frac{\partial I}{\partial \theta_e}$, $\frac{\partial Q}{\partial \theta_e}$, $\frac{\partial I}{\partial \dot{\theta}_e}$, and $\frac{\partial Q}{\partial \dot{\theta}_e}$ are defined as

$$\frac{\partial I}{\partial \theta_e} = \sum_{k=1}^{N} -\frac{AT}{2}\cos(\Theta_e) \tag{2.26}$$

$$\frac{\partial Q}{\partial \theta_e} = \sum_{k=1}^{N} -\frac{AT}{2}\sin(\Theta_e) \tag{2.27}$$

$$\frac{\partial I}{\partial \dot{\theta}_e} = \sum_{k=1}^{N} -\frac{AkT^2}{2}\cos(\Theta_e) \tag{2.28}$$

$$\frac{\partial Q}{\partial \dot{\theta}_e} = \sum_{k=1}^{N} -\frac{AkT^2}{2}\sin(\Theta_e) \tag{2.29}$$

The extended-Kalman filter equations are used to estimate the states. Since the measurement noise covariance matrix, R, is a diagonal matrix, the sequential extended Kalman filter is used [2]. The sequential extended Kalman filter equations are,

$$\hat{\mathbf{x}}_k^- = \Phi \mathbf{x}_{k-1}^+ + \Gamma U_{k-1} \tag{2.30}$$

$$P_k^- = \Phi P_{k-1}^+ \Phi^T + Q \tag{2.31}$$

$$\hat{\mathbf{x}}_{0k}^+ = \hat{\mathbf{x}}_k^- \tag{2.32}$$

$$P_{0k}^+ = P_k^- \tag{2.33}$$

$$\text{for } i = 1, 2 \tag{2.34}$$

$$K_{ik} = \frac{P_{(i-1),k}^- H_{ik}^T}{H_{ik} P_{(i-1),k}^- H_{ik}^T + R} \tag{2.35}$$

$$\hat{x}_{ik}^+ = \hat{x}_{(i-1),k}^+ + K_{ik}(y_{ik} - h_{ik}(\hat{x}_{(i-1),k}^+, 0)) \tag{2.36}$$

$$P_{ik}^+ = (I - K_{ik} H_{ik}) P_{(i-1),k}^- (I - K_{ik} H_{ik})^T + K_{ik} R K_{ik}^T \tag{2.37}$$

$$\text{end} \tag{2.38}$$

where Φ is defined in Equation (2.22), Γ is defined in Equation (2.23), $\hat{\mathbf{x}}_k^-$ is the propogated state estimate at timestep k before the measurement is applied, $\hat{\mathbf{x}}_k^+$ is the state estimate at timestep k after the measurement is applied. P_k^- is the variance of the state estimate at timestep k before the measurement is applied, P_k^+ is the variance of the state estimate at timestep k after the measurement is applied. Q is the covaraince of the state propagation noise at time step k, defined in Equation (2.24). R is the covariance of the measurement noise at time step k, defined in Section 2. H_k, defined in Equation (2.25), is the Jacobian of the measurement equations. K_k is the Kalman gain. y_k is the measurement vector, consisting of the in-phase and quadrature measurments. $h_k(\mathbf{x}, \mathbf{u})$ is the non-linear measurement function. I is a two-by-two identity matrix. Since a sequential EKF was used, each measurement is applied seperately. The notation with the i subscript denotes which measurement is being applied. This simplifies the Kalman gain calculation from a matrix inversion to a scalar inversion, which is mathematically simpler.

2.3 Summary

In this chapter, we introduced an alternative tracking loop to a PLL. The tracking loop used an extended Kalman filter to estimate the frequency error based on the in-phase and quadrature correlator values. First, a discrete-time model was developed for the in-phase and quadrature values, based on phase error, frequency error, and higher order terms. Then a two-state EKF was developed, with states estimating phase error and frequency error.

Chapter 3

Conclusion

We examined second-order phase lock loops. We derived the transfer function of a basic PLL. We introduced costas phase-lock loops. We explained design metrics for second-order PLLs. We designed a second-order PLL.

We also introduced the use of Kalman filters to replace PLL loop filters. A novel measurement model was developed. Then a two-state EKF based tracking loops was derived.

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[2] D. Simon, *Optimal State Estimation.* Hoboken, NJ: Wiley Interscience, 2006.