A NON-EQUILIBRIUM LWR FUEL CYCLE MODEL

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NOMENCLATURE

$I^{\ell}_{i,j,k}$	the amount (Kg) of nuclides of species i, in the reactor of region j, during the irradiation period k, at the beginning of that irradiation period for $\ell = 1$, and the end of that year for $\ell = 2$.
θ_j	average neutron fluence in region j
f	thermal utilization factor
ε	fast fission factor
P	thermal neutron nonleakage probability
p	resonance escape probability
Σ_s	macroscopic cross section of reactor structure material
η	ratio of the number of neutrons produced by fission to the number of neutrons absorbed in the fuel.
POUR	percentage of uranium recycled
POPR	percentage of plutonium recycled
ν_m	average number of fast neutrons produced per fission event for nuclides of species m
σ_{am}	microscopic absorption cross section of species m
σ_{fm}	microscopic fission cross section of species m
σ_{cm}	microscopic capture cross section of species m

Subscript (m) for symbols

0	^{240}Pu
1	^{241}Pu
5	^{235}U
6	^{236}U
8	^{238}U
9	^{239}Pu
7	Fission Products

1.0 INTRODUCTION

Nuclear fuel management can be defined as the collection of principles and practices required for the planning, refueling, and safe operations of nuclear power plants while minimizing total plant and system energy costs to the extent possible through the timely procurement of nuclear fuel and related services (14).

Each utility must develop its own unique fuel management program because of the characteristics, limitations and variations involved in each individual design. However, parts of these programs can be studied generally and independently of the total program.

Typical areas of fuel management studies can be summarized by the following categories:

(1) fuel shuffling techniques

(2) "cash flow" mechanisms

(3) optimization theories

Fuel shuffling deals with various kinds of fuel loading methods such as "batch" (15), "out-in" (15,20) and "rondelay" (15).

In "batch" irradiation, the reactor is loaded uniformly with a complete core at one time and this is irradiated without moving the fuel elements. When the system ceases to be critical the whole core is discharged and reloaded. This method leads to a smaller degree of burnup than other schemes.

In the "Out-In" method, the fresh fuel is charged near the outher edge and moved progressively toward the center from which it is discharged. In this case considerably better burnup is attained than in batch irradiation. Another advantage is that it leads to a fairly uniform power density distribution in the radial direction. However, the "out-in" zone refueling patterns can lead to undesirable flux shapes in large reactors, because the higher average burnups cause large reactivity difference between the new and old fuel in their respective zones. The situation can be mitigated by placing the fresh fuel elements adjacent to older elements in a scatter pattern. Such refueling methods are usually called "scatter-loading" or "rondelay" schemes. They are similar to the "out-in" zone refueling scheme but they are different because the "zones" are distributed throughout the core and are intimately mixed. Although "rondelay" can eliminate the problem of undesirable flux shapes, there is no basis to assure that the optimum reloading pattern is obtained. Additionally, it makes computer calculations much more difficult and complicated.

Cash flow studies take into consideration every activity involved in the fuel cycle, estimate the overall cost of the electric power generation, and determine the sensitivity of each individual step of the fuel cycle to the total cost (16).

Optimization theories include, dynamic programming (17), direct search method (18), linear and quadratic programming (19,20).

Studies have been made, either individually or for combinations of these areas, since numerous variables and constraints are involved in fuel management problems. General studies, considering the compatibility with computing machines, are always simplified by making assumptions and neglecting minor factors.

In this model, a relatively simple yet accurate method was developed to calculate the minimum amount of uranium required for six years of continuous operation. The first six years were considered to be the time period when the reactor core composition is in non-equilibrium stage. After this period the loading requirement and the composition of the spent fuel are relatively the same for each year. Studies were concentrated on the effect of plutonium and spent uranium recycling. The reason for this emphasis is that the material characteristics and the technology involved in plutonium reprocessing has been the subject of much effort recently. Yet, the relative worth of plutonium and the effect of recycling from the nuclear point of view have not been firmly fixed. Furthermore, the cost of re-enriching the discharged uranium is not well established, i.e., the incremental cost for re-enriching this uranium which will contaminate the cascades with neutron poison uranium such as ^{236}U has not been established. The results of this model can serve as a basis for the cost analysis between different recycling methods.

2.0 BASIC CONCEPTS OF LINEAR PROGRAMMING

2.1. Introduction

Linear Programming (LP) deals with those problems in which all relations among the variables are linear. This includes the objective function (OF), that function which is to be maximized or minimized, and all model constraints.

The general LP problem can be described as follows (4):

> Given a set of m linear inequalities or equations in n variables, find the non-negative values of these variables which will satisfy every constraint and maximize (or minimize) some linear function of the variables.

Mathematically, this can be written as:

optimize (minimize or maximize)

$$OF = C_1X_1 + C_2X_2 + \ldots\ldots + C_nX_n \tag{2.1.1}$$

subject to:

$$\begin{aligned} a_{11}X_1 + a_{12}X_2 + \ldots\ldots + a_{1n}X_n &\gtreqless b_1 \\ a_{21}X_1 + a_{22}X_2 + \ldots\ldots + a_{2n}X_n &\gtreqless b_2 \\ &\vdots \\ a_{m1}X_1 + a_{m2}X_2 + \ldots\ldots + a_{mn}X_n &\gtreqless b_m \end{aligned} \tag{2.1.2}$$

where $X_j \geq 0$, $j = 1,\ldots,n$ are state variables,

b_i, c_j, a_{ij}, $i = 1,\ldots,m$, $j = 1,\ldots,n$ are constant coefficients.

2.2. LP Example

As an illustration of a simple LP problem consider a nuclear fuel fabrication company who wants to begin fabrication of nuclear fuel. It is decided not to hire more than 20 workmen for this new plant. Assume there are only two kinds of fabrication machines, namely model A and model B, available for producing fuel elements. Model A costs \$15,000 each and model B costs \$8,000. Both machines need 2 workmen during operation.

It is estimated each model A machine is capable of making \$4,000 annual net profit, and \$3,000 for model B. The total budget for purchasing is \$122,000. The problem is how many model A machines as well as model B machines that this nuclear fuel fabrication company should buy to maximize their profit.

To solve this problem, let X_A and X_B be the number of machines of model A and B, respectively.

The objective function is

$$OF = 4X_A + 3X_B \quad . \qquad (2.2.1)$$

The constraint on purchasing is

$$15\ X_A + 8\ X_B \leq 122 \quad . \qquad (2.2.2)$$

The constraint on workmen is

$$2\ X_A + 2\ X_B \leq 20 \quad . \qquad (2.2.3)$$

This simple problem can be solved by a graphical method:

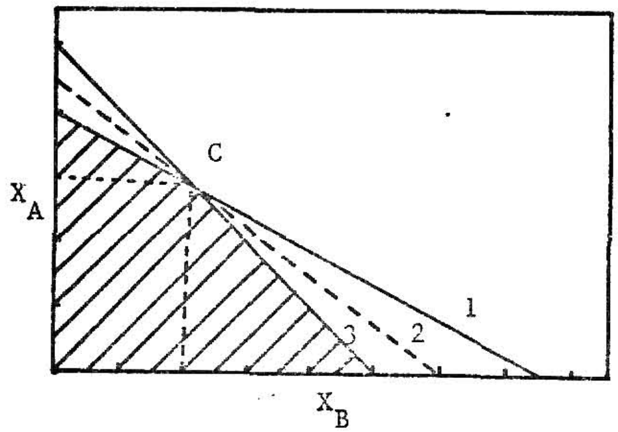

Figure 2.2.a Graphical solution for sample problem.

Equation (2.2.2) constrains the solution to lie under line 1.

Equation (2.2.3) constrains the solution to lie under line 3.

Also, both X_A and X_B should be positive.

These constraints enclose the shaded area in Figure 2.2.a. This area is called the feasible region (4). All allowable solutions should be in the feasible region.

Equation (2.2.1), the objective function, defines the slope of an isoprofit line (4). When the isoprofit line moves away from the origin the profit increases. Point C in Figure 2.2.a gives the optimal solution, and the final position for the isoprofit line is line 2 in Figure 2.2.a. Hence, the solution is

$$X_A = 6 \quad ,$$

$$X_B = 4 \quad ,$$

and the objective function

$$OF = 36 \quad .$$

For a problem which consists of more than 3 variables, the simplex and revised simplex methods (4) were developed both of which require the set up of an initial tableau and a standard iteration process to obtain the optimal solution (4).

2.3. LP Computer Code Setup

The IBM-MPS-360 (7) which uses the revised simplex method (4) of solving LP problems requires that users setup the input data and enter the constant coefficients of each linear equation in a matrix form. The control program statements take only the following few cards with each statement being a builtin subprogram:

```
PROGRAM
INITIALZ
MOVE (XDATA 'NAME')
MOVE (XPBNAME 'PBFILE')
CONVERT ('SUMMARY')
SETUP
MOVE (XOBJ 'ROW1000')
MOVE (XRHS 'LIMITS')
PRIMAL
SOLUTION
EXIT
PEND
```

A detailed description of these statements and input data formats can be found in IBM-MPS-360 manual, H-20-0476-02 (7).

3.0 MODEL DEVELOPMENT

3.1. The Problem

Given a typical design of a base load power reactor, assume the reactor has to be refueled once a year. During the refueling, one third of the core will be discharged, and the irradiation scheme is assumed to be "out-in" batch irradiation. The problem to be solved is to find a simple model that can be used to calculate the minimum uranium requirement in each irradiation period starting from the first clean core with different percentages of plutonium and spent uranium recycled. The first step is to establish a reference reactor.

3.2. The Reference Reactor

It is very difficult to develop a model which can apply to all types of reactors. The proposed Liquid Metal Fast Breeder Reactor (LMFBR) demonstration plant will not be in operation until mid 1980's. The High Temperature Gas-Cooled Reactor (HTGR) is only now beginning a strong entry to the market. Hence, most nuclear power plants in operation before 1985 will be Light Water Reactors (LWR), either the Boiling Water Reactor (BWR) or the Pressurized Water Reactor (PWR). In this model the PWR was chosen as the reference reactor. The general characteristics of a large scale power PWR can be summarized as follows (8,9):

(1) Cylindrical core, 11 ft in diameter and 12 ft in height,

(2) Three equivolume regions, concentrically distributed,

(3) Average flux relation between the three regions (outer to inner) is 1:1.225:1.264,

(4) Initial UO_2 inventory is 110 short tons,

(5) Initial fuel enrichment is 3.2% in the outer region, 2.7% in the intermediate region, and 2.2% in the inner region,

(6) Power output is 3250 Mw(th) (1100 Mwe),

(7) Operating temperature is 575°F,

(8) Thermal neutron nonleakage probability (P) is 0.9912,

(9) Resonance escape probability (p) is 0.832,

(10) Thermal utilization factor (f) is 0.91,

(11) Fast fission factor (ε) is 1.0534,

(12) $K_{eff} > 1.114$ at Beginning of Life (BOL) for each burn period, and $K_{eff} > 1.003$ at End of Life (EOL) for each burn period.

3.3. Refueling

Generally, for a PWR, at each refueling time one-third of the core will be replaced. To reduce the fuel cost, low enriched uranium is recommended for fabricating new fuel elements. For a base load reactor each refueling will be expected to last about one year to eighteen months. Since the reactor has to be shut down to do the refueling, it is most realistic and economical to do the refueling during those few weeks when the power demand is the lowest. Therefore, in this model, the refueling is assumed to take place once a year during the predetermined refueling window selected by consideration of the seasonal load curve. Furthermore, during refueling, the fuel elements in the inner region are assumed to be discharged to the fuel reprocessing plant. Those of the intermediate region will be moved into the inner region and the outer region fuel elements are moved into the intermediate region. Finally, new fuel elements will be loaded into the outer region.

In the fuel reprocessing plant, uranium and plutonium will be separated from the fission products. In this model the uranium from the spent fuel contains approximately 0.8% fissionable material ^{235}U, 0.4% neutron poison ^{236}U, and 98.8% fertile material ^{238}U. The plutonium from the spent fuel contains approximately 60% ^{239}Pu, 30% ^{240}Pu, and 10% ^{241}Pu. It is conceivable that this uranium and plutonium has the potential for recycling. Of course, additional "clean" uranium has to be added to assure the burnup and reactivity requirements. The model allows for different percentages of recycling for both uranium and plutonium, while minimizing the total amount of additional ^{235}U required for six years. The enrichment for this additional uranium was fixed at 3.2%. Figure 3.3.a is the general schematic drawing which illustrates the material flow scheme from the first year to the sixth year.

3.4. Model Constraints

Linear Programming requires that all the constraint equations be written as a linear combination of the state variables. In this model the state variables, $I^{\ell}_{i,j,k}$, are defined to be the amount of material of species i in the reactor of region j during the irradiation period k at the beginning of life (BOL) for $\ell = 1$ and at the end of life (EOL) for $\ell = 2$. Table 3.4.1 describes the indices i, j, k and ℓ.

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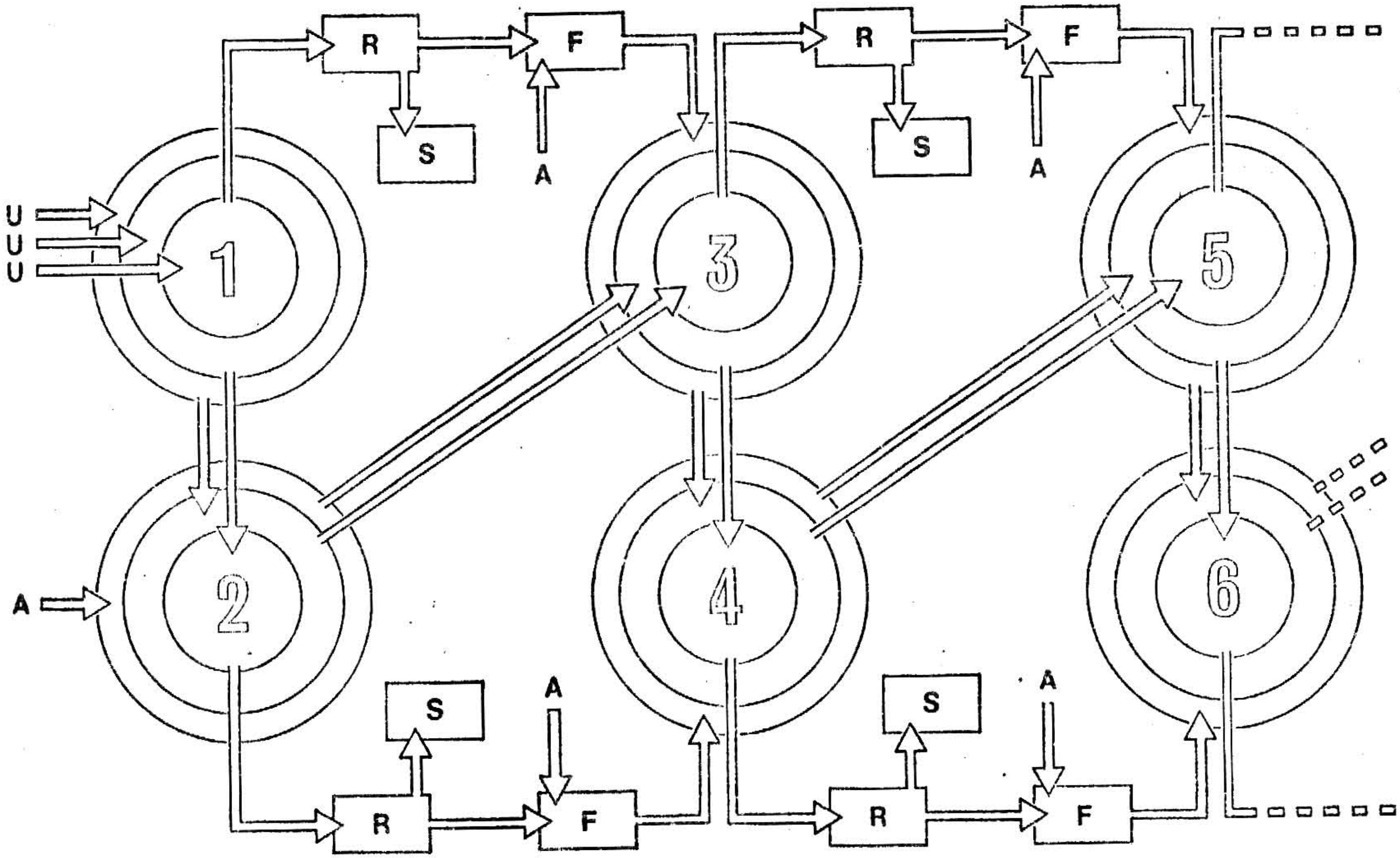

Figure 3.3.a. The material flow scheme from the first year to the sixth year.

U = Initial Loading S = Storage R = Fuel Reprocessing Plant

A = Additional "Clean" Uranium 1 = First Core F = Fuel Fabrication Plant

Table 3.4.1 The descriptions of indices i, j, k and ℓ for defining state variables.

i	Corresponding Material Name	j	Corresponding Region	k	Corresponding Irradiation Period
1	^{235}U	1	Outer	1	1st year
2	^{236}U	2	Intermediate	2	2nd year
3	^{238}U	3	Inner	3	3rd year
4	^{239}Pu	ℓ	Corresponding Status	4	4th year
5	^{240}Pu	1	Beginning of Life	5	5th year
6	^{241}Pu	2	End of Life	6	6th year
7	Fission Products				

There are 185 constraint equations formulated in this model. These equations are grouped into the following seven categories:

(1) initial load constraint,

(2) burnup constraint,

(3) enrichment constraint,

(4) fabrication constraint,

(5) energy release constraint,

(6) volume constraint,

(7) reactivity constraint.

3.4.1. Initial Load Constraint

It was estimated (8) that for a typical large scaled PWR power station the initial UO_2 inventory is 110 short tons, and the initial fuel enrichment is 3.2% for outer regions, 2.7% for intermediate region, and 2.2% for inner region. Based on these data, the initial ^{235}U and ^{238}U content in each region can be calculated and formulated into six equations:

For outer region ^{235}U content

$$I^{1}_{1.1.1} = \frac{110}{3}\ (\text{short tons}) \times \frac{1}{0.001102}\ \left(\frac{\text{Kg}}{\text{short ton}}\right) \times 0.032 \times \frac{238}{(238 + 32)}$$

$$= 932.27\ \text{Kg} \qquad (3.4.1)$$

Similar equations for ^{238}U and for other regions can be formulated. The calculated values are presented in Table 3.4.2.

Table 3.4.2 Initial load in each region.

	Outer	Intermediate	Inner
^{235}U	932.27 (Kg)	786.60 (Kg)	640.94 (Kg)
^{238}U	28201.0 (Kg)	28347.0 (Kg)	28492.0 (Kg)

3.4.2. Burnup Constraint

During irradiation, production or consumption of the various nuclides present in the reactor are modeled by a series of differential equations which are commonly called burnup equations. To simplify the model only seven of the most important nuclides were chosen. The derivation of the burnup equations is based on the following assumptions:

(1) one homogeneous region,

(2) a known neutron fluence for each irradiation period,

(3) one neutron energy group (thermal),

(4) negligible decay of fission product nuclides,

(5) batch irradiation.

The only independent variable in the differential equation is the fluence, θ_j, which is defined as

$$\theta_j = \int_0^t \bar{\phi}_j(t')\, dt' \quad , \tag{3.4.2}$$

where $\bar{\phi}_j(t')$ is the averaged neutron flux in region j.

The derivation of the burnup equations based upon these assumptions can be found in reference (6). For convenience the equations derived are listed as follows:

For ^{235}U:

$$I^2_{1jk} = a_{11j}\, I^1_{1jk} \tag{3.4.3}$$

where $a_{11j} = \exp(-\sigma_{a5} \times \theta_j)$

For ^{236}U:

$$I^2_{2jk} = a_{21j}\, I^1_{1jk} + a_{22j}\, I^1_{2jk} \tag{3.4.4}$$

where $a_{22j} = \exp(-\sigma_{a6} \times \theta_j)$

$a_{21j} = \sigma_{c5}(a_{11j} - a_{22j})/(\sigma_{a6} - \sigma_{a5})$

For ^{238}U:

$$I^2_{3jk} = a_{33j}\, I^1_{3jk} \tag{3.4.5}$$

where $a_{33j} = \exp(-\sigma_{a8} \times \theta_j)$

For ^{239}Pu:

$$I^2_{4jk} = a_{41j}\, I^1_{1jk} + a_{43j}\, I^1_{3jk} + a_{44j}\, I^1_{4jk} \tag{3.4.6}$$

define $C_1 = \varepsilon\, P(1 - p)\, \eta_9 \sigma_{a9} - \sigma_{a9}$

$C_2 = \varepsilon\, P(1 - p)\, \eta_5 \sigma_{a5}$

$C_3 = -\sigma_{a8}/C_1$

$C_4 = \sigma_{a5} + C_1$

and $a_{44j} = \exp(C_1 \times \theta_j)$

$$a_{43j} = -C_3(a_{44j} - 1)$$

$$a_{41j} = C_2(a_{44j} - a_{11j})/C_4$$

For ^{240}Pu:

$$I^2_{5jk} = a_{51j}\, I^1_{1jk} + a_{53j}\, I^1_{3jk} + a_{54j}\, I^1_{4jk} + a_{55j}\, I^1_{5jk} \qquad (3.4.7)$$

define $C_5 = \sigma_{a5} - \sigma_{a0}$

$$C_6 = \sigma_{a0} + C_1$$

and $a_{55j} = \exp(-\sigma_{a0} \times \theta_j)$

$$a_{54j} = \sigma_{c9}(a_{44j} - a_{55j})/C_6$$

$$a_{53j} = -C_3\left[\frac{\sigma_{c9}(a_{55j} - 1)}{\sigma_{a0}} + a_{54j}\right]$$

$$a_{51j} = \frac{C_2}{C_4}\left[a_{54j} + \frac{\sigma_{c9}}{C_5}(a_{11j} - a_{55j})\right]$$

For ^{241}Pu:

$$I^2_{6jk} = a_{61j}\, I^1_{1jk} + a_{63j}\, I^1_{3jk} + a_{64j}\, I^1_{4jk} + a_{65j}\, I^1_{5jk} + a_{66j}\, I^1_{6jk} \qquad (3.4.8)$$

define $C_7 = \sigma_{a1} + C_1$

$$C_8 = \sigma_{a1} - \sigma_{a5}$$

$$C_9 = \sigma_{a1} - \sigma_{a0}$$

$$C_{10} = C_6 \times C_7$$

$$C_{11} = C_5 \times C_8$$

$$C_{12} = \frac{1}{C_5} + \frac{1}{C_6}$$

and $$a_{66j} = \exp(-\sigma_{a1} \times \theta_j)$$

$$a_{65j} = \frac{\sigma_{a0}}{C_9}(a_{55j} - a_{66j})$$

$$a_{64j} = \frac{\sigma_{a0}\ \sigma_{c9}}{C_{10}}(a_{44j} - a_{66j}) - \frac{\sigma_{c9}}{C_4} \times a_{65j}$$

$$a_{63j} = -C_3[\sigma_{c9}(\frac{(a_{66j} - 1)}{\sigma_{a1}} + \frac{a_{65j}}{\sigma_{a0}}) + a_{64j}]$$

$$a_{61j} = \frac{\sigma_{a0}\ \sigma_{c9}\ C_2}{C_4}[\frac{(a_{44j} - a_{66j})}{C_{10}} + \frac{(a_{11j} - a_{66j})}{C_{11}}] - \frac{\sigma_{c9}\ C_2\ C_{12}\ a_{65j}}{C_4}$$

For fission products:

$$I^2_{7jk} = a_{71j}\ I^1_{1jk} + a_{73j}\ I^1_{3jk} + a_{74j}\ I^1_{4jk} + a_{75j}\ I^1_{5jk} + a_{76j}\ I^1_{6jk} + a_{77j}\ I^1_{7jk} \quad (3.4.9)$$

define $C_{13} = 2.0 \times C_3$

$$C_{14} = C_{13} \times \sigma_{f1} \times \sigma_{c9}$$

$$C_{15} = C_{14} \times \sigma_{a0}$$

$$C_{16} = 2.0 \times C_2$$

$$C_{17} = C_{16} \times \sigma_{f1} \times \sigma_{c9}$$

$$C_{18} = C_{17} \times \sigma_{a0}$$

and $$a_{77j} = 1.0$$

$$a_{76j} = \frac{2.0\ \sigma_{f1}}{\sigma_{a1}}(1 - a_{66j})$$

$$a_{75j} = \frac{2.0\ \sigma_{f1}\ \sigma_{a0}}{C_q}[\frac{(a_{66j} - 1)}{\sigma_{a1}} - \frac{(a_{55j} - 1)}{\sigma_{a0}}]$$

$$a_{74j} = \frac{2\ \sigma_{f9}}{C_1}(a_{44j} - 1)(1 + \frac{\sigma_{a0}\ \sigma_{c9}}{\sigma_{a1}}) + \frac{2\ \sigma_{f1}\ \sigma_{c9}}{C_6\ C_9}(a_{55j} - 1)$$

$$- \frac{\sigma_{a0}\ \sigma_{c9}}{C_6}(\frac{1}{C_7} + \frac{1}{C_9}) \times a_{76j}$$

$$a_{73j} = \theta_j(C_{13}\ \sigma_{f9} + \frac{C_{14}}{\sigma_{a1}}) - \frac{(a_{44j} - 1)}{C_1}(C_{13}\ \sigma_{f9} + \frac{C_{15}}{C_{10}})$$

$$+ \frac{C_{14}}{C_9}(\frac{1}{\sigma_{a0}} - \frac{1}{C_6})[(a_{55j} - 1) - \frac{\sigma_{a0}}{\sigma_{a1}}(a_{66j} - 1)]$$

$$+ \frac{C_{15}}{\sigma_{a1}}(\frac{1}{\sigma_{a0}\ \sigma_{a1}} - \frac{1}{C_{10}})(a_{66j} - 1)$$

$$a_{71j} = \frac{(a_{44j} - 1)}{C_1\ C_4}[C_{16}\ \sigma_{f9} + \frac{C_{18}}{C_{10}}]$$

$$+ \frac{(a_{11j} - 1)}{\sigma_{a5}}[\frac{C_{16}\ \sigma_{f9}}{C_6} - 2\ \sigma_{f5} - \frac{C_{18}}{C_{11}\ C_4}]$$

$$+ \frac{C_{12}\ C_{18}}{C_4\ C_9}[\frac{(a_{55j} - 1)}{\sigma_{a0}} - \frac{(a_{66j} - 1)}{\sigma_{a1}}] + \frac{C_{18}(a_{66j} - 1)}{C_4\ \sigma_{a1}}[\frac{1}{C_{10}} + \frac{\ }{C}$$

The following temperature corrections, using Westcott factors (3), on all cross sections were made in all calculations.

Table 3.4.3 Cross section temperature corrections.

Cross Section	20°C (Kb)	575°F (Kb)	Cross Section	20°C (Kb)	575°F (Kb)
σ_{a5}	0.694	0.412	σ_{f5}	0.582	0.3414
σ_{a6}	0.0067	0.0048	σ_{f9}	0.738	0.6772
σ_{a8}	0.00237	0.00173	σ_{f1}	0.950	0.773
σ_{a9}	1.025	1.028	σ_{c5}	0.102	0.0706

Table 3.4.3 (Cont'd)

Cross Section	20°C (Kb)	575°F (Kb)	Cross Section	20°C (Kb)	575°F (Kb)
σ_{a0}	0.250	0.1772	σ_{c9}	0.287	0.3508
σ_{a1}	1.222	1.121	σ_{c1}	0.372	0.348
σ_{aFP}	0.025	0.025			

3.4.3. Enrichment Constraint

The enrichments of the first clean core were defined by the initial load constraints. Additional "clean" uranium is required to fabricate the fuel elements for the subsequent years. For practical purposes without losing any significance, the enrichment of this additional uranium was fixed at 3.2%. Thus, the following equations were formulated

$$0.968\ I^{1}_{112} - 0.032\ I^{1}_{312} = 0 \ ,$$

$$0.968\ U_{1k} - 0.032\ U_{3k} = 0 \ , \quad k = 3,4,5,6 \qquad (3.4.10)$$

where U_{1k} and U_{3k} represent the amount of ^{235}U and ^{238}U added to the fabrication plant before kth year, respectively.

3.4.4. Fabrication Constraint

Fabrication constraint equations are used to describe the composition of the nuclides in the new fuel elements from the fuel fabriaction plant. POUR and POPR are defined to be the percentage of uranium and plutonium recycled, respectively:

$$I^{1}_{i1k+2} - POUR \times I^{2}_{i3k} - U_{ik+2} = 0 \quad \text{for} \quad i = 1,2,3 \quad k = 1,2,3,4 \qquad (3.4.11)$$

$$I^{1}_{i1k+2} - POPR \times I^{2}_{i3k} = 0 \quad \text{for} \quad i = 4,5,6 \quad k = 1,2,3,4 \qquad (3.4.12)$$

In this model, POUR and POPR were varied from 0% to 100% to determine their effect on the minimization of the total additional uranium required for the six irradiation periods.

3.4.5. Energy Release Constraint

The decay of fission product nuclides is neglected in the derivation of the burnup equations for fission products. Therefore, the equation derived will be used to calculate the amount of fission product nuclides accumulated in each region at the end of each irradiation period. Every fission event will deliver about 200 MeV of recoverable energy and produce two fission product nuclides; therefore, there exists a proportional relationship between the energy release and the amount of fission products produced in each irradiation period.

The reactor is assumed to operate at full power (0.9 load factor) for 24 hours a day and 350 days a year. The production of fission product nuclides corresponding to the total energy released was calculated to be 1056.3 Kg. Therefore

$$\sum_{j=1}^{3} (I^{2}_{7jk} - I^{1}_{7jk}) \geq 1056.3 \qquad \text{for each } k \quad . \qquad (3.4.13)$$

3.4.6. Volume Constraint

The total volume that can be occupied by the fuel pellets is estimated (8,9) to be 480 ft^3. The maximum density of PuO_2 and UO_2 is 11.46 g/cc and 10.96 g/cc, respectively (5). Thus, the volume constraints were formulated:

$$\frac{(238 + 32)}{238} \times \frac{10^3}{10.96}[\sum_{i=1}^{3} I^1_{i1k}] + \frac{(239 + 32)}{239} \times \frac{10^3}{11.46}[\sum_{i=4}^{6} I^1_{i1k}] \leq \frac{480}{3} \times 2.832 \times 10^4 \quad . \tag{3.4.14}$$

which can be simplified as:

$$0.9124[\sum_{i=1}^{3} I^1_{i1k}] + 0.8726[\sum_{i=4}^{6} I^1_{i1k}] \leq 40,000 \quad . \tag{3.4.15}$$

3.4.7. Reactivity Constraint

Two reactivity or multiplication factor constraints for each year were formulated in this model. One for the beginning of the year and the other for the end of the year. The later one has to be satisfied to assure criticality throughout the irradiation period.

The multiplication factor, k_{eff}, is (5)

$$k_{eff} = P\ k_\infty \quad ,$$

$$= P\ p\varepsilon f\eta \quad ,$$

where P = thermal neutron nonleakage probability,

k_∞ = infinite multiplication factor,

p = resonance escape probability,

ε = fast fission factor,

f = thermal utilization factor,

η = the number of neutrons produced per neutron absorbed in the fuel.

To simplify the model P, p, ε, f were assumed to be constant (5) throughout the irradiation processes. The only factor assumed to change was η.

$$\eta = \frac{k_{eff}}{Pp\varepsilon f} = \frac{\sum_{j=1}^{3} \sum_{i=1}^{7} \nu_i\ \sigma_{fi}\ I^\ell_{ijk}}{\sum_{j=1}^{3} \sum_{i=1}^{7} \sigma_{ai}\ I^\ell_{ijk} + \Sigma_s} \quad \text{for every } k, \tag{3.4.16}$$

where Σ_s is the total macroscopic cross section of all the materials present in the reactor except those mentioned in Section 3.3.2.

In this model, the minimum value for k_{eff} at the end of each year was assumed to be 1.003. Therefore, if Σ_s is assumed to be constant, equation (3.4.16) can be used to calculate Σ_s. The minimum value of k_{eff} at the beginning of the year can be calculated using the initial load data and equation (3.4.16). The value found was 1.114.

3.5. The Objective Function

Fuel management consists of the methods and procedures to optimize the utilization of the available fuel (10). It can be either "in-core" fuel optimization or minimization of the total amount of fuel required throughout plant life. This model deals with the latter one. Therefore, the objective function of this model was selected to be the total amount of ^{235}U, excluding the initial load, required for six years.

$$\text{Objective Function} = I^{1}_{112} + U_{13} + U_{14} + U_{15} + U_{16} \qquad (3.5.1)$$

Since the enrichment of the uranium for this reference reactor was fixed at 3.2% as indicated in Section 3.4.3, this objective function, if divided by the enrichment, is essentially equivalent to the total amount of additional uranium required for six years.

3.6. Model Formulation and Analysis

There are 173 variables considered in this model which include 159 as the nuclide concentrations, $I^{\ell}_{i,j,k}$, and 6 fluence values (one for each year). The other 8 variables are U_{13}, U_{14}, U_{15}, U_{16}, U_{33}, U_{34}, U_{35} and U_{36} as defined in Section 3.4.3. The number of equations is also 173 and that includes 126 burnup equations (7 for each region, 21 for each year), 6 for the initial load, 24 for refueling (fabrication constraint), 5 for enrichment, 6 for energy release, and another 6 for the end of life (EOL) reactivity constraint. The number of inequalities considered are 12 that includes 6 volume constraints and 6 beginning of life (BOL) reactivity constraints. Theoretically, this problem can be solved exactly because the number of variables is equal to the number of equations. However, the burnup constraints contain cross product terms and other complicated expressions of fluence and nuclide concentrations which make it very difficult to solve. If those six fluence values are known quantities, the problem can be greatly simplified, since all the expressions become a set of first order linear equations. Based on this, the following method was developed.

The first step is to use the initial load conditions and the energy release requirement to estimate the fluence for the first year, which is around 1.20 n/Kb, i.e., 1.2×10^{21} n/cm^2. Next, arbitrarily chose 5 numbers in the neighborhood of 1.20 to represent the fluence for the subsequent years, e.g., 1.0628, 0.9832, 0.9465, 0.9803, 0.9796. The number of variables now become 166. Exclude the 6 EOL reactivity equations from the system to make the number of variables equal to the number of equations, then, use the IBM mathematical programming system (MPS-360) (7) which is the linear programming code developed by IBM for solving first order linear equations to solve the problem. The solutions obtained are then used to calculate the EOL reactivities for each year using the equations described in Section 3.4.7. In the case of 0% plutonium recycle and 10% uranium recycle, the EOL reactivity values obtained were 1.003, 1.0013, 0.9972, 0.9933, and 1.0065. Up to this stage, it is understandable that the set of fluence values 1.20, 1.0628, 0.9832, 0.9465, 0.9803, 0.9796, will generate a set of reactivity values like

1.003, 1.003, 1.0013, 0.9972, 0.9933, 1.0065. In matrix representations;

$$\begin{Bmatrix} 1.003 \\ 1.003 \\ 1.0013 \\ 0.9972 \\ 0.9933 \\ 1.0065 \end{Bmatrix} = [A] \begin{Bmatrix} 1.20 \\ 1.0628 \\ 0.9832 \\ 0.8465 \\ 0.9803 \\ 0.9796 \end{Bmatrix} \tag{3.6.1}$$

[A] represent the unknown six by six matrix that, in principle, describes the behavior of the system. The desired situation is to find the set of fluence values that will make the EOL reactivity values always at the minimum value, 1.003. Theoretically, the minimum K_{eff} should be exactly equal to one. However, some excess reactivity is always required to compensate for the effect caused by structure defects, the variation of moderator properties, boron shim concentration, as well as the reactor control mechanism. Therefore, the minimum K_{eff} in this model was chosen to be 1.003. If matrix [A] is known then the desired fluence values, vector {d}, should be

$$\{d\} = [A]^{-1} \begin{Bmatrix} 1.003 \\ 1.003 \\ 1.003 \\ 1.003 \\ 1.003 \\ 1.003 \end{Bmatrix} \tag{3.6.2}$$

This is called the least squares solution of the system. The mathematical proof of this can be found in reference (11).

$$\text{Define} \quad \begin{Bmatrix} 1.20 \\ 1.0628 \\ 0.9832 \\ 0.9465 \\ 0.9803 \\ 0.9796 \end{Bmatrix} = \{I_1\} \tag{3.6.3}$$

$$\text{and} \quad \begin{Bmatrix} 1.003 \\ 1.003 \\ 1.0013 \\ 0.9972 \\ 0.9933 \\ 1.0065 \end{Bmatrix} = \{O_1\} \tag{3.6.4}$$

$$\text{therefore,} \quad \{O_1\} = [A]\,\{I_1\} \tag{3.6.5}$$

Repeat the procedure described above six times, each time with a different set of fluence values to obtain six sets of corresponding K_{eff} values. Hence;

$$[A]\ [\{I_1\}\{I_2\}\{I_3\}\{I_4\}\{I_5\}\{I_6\}] = [\{O_1\}\{O_2\}\{O_3\}\{O_4\}\{O_5\}\{O_6\}] \qquad (3.6.6)$$

Let

$$[I] \equiv [\{I_1\}\{I_2\}\{I_3\}\{I_4\}\{I_5\}\{I_6\}] \qquad (3.6.7)$$

and

$$[O] \equiv [\{O_1\}\{O_2\}\{O_3\}\{O_4\}\{O_5\}\{O_6\}] \qquad (3.6.8)$$

then

$$[A][I] = [O] \qquad (3.6.9)$$

and

$$[A] = [O][I]^{-1} \qquad (3.6.10)$$

$$[A]^{-1} = [I][O]^{-1} \qquad (3.6.11)$$

Substitute this expression into equation (3.6.2)

$$\{d\} = [I][O]^{-1} \begin{Bmatrix} 1.003 \\ 1.003 \\ 1.003 \\ 1.003 \\ 1.003 \\ 1.003 \end{Bmatrix} \qquad (3.6.12)$$

Thus, a satisfactory set of fluence values can be obtained, and the optimum solution was obtained by using these fluence values as the input to MPS-360 program.

A computer program was developed, based on given values of fluences, POUR, and POPR, to calculate all the coefficients and prepare a data set on disc for MPS-360. The control cards was written so that once the data set is prepared the MPS-360 will begin execution and print out the results. The program is called the LP code. This code is listed in appendix B.1. The equation and inequalities defined in this LP program are described in Appendix A.1 and A.2.

Another program was developed to perform the matrix inversion and calculate the vector {d}. This program is listed in Appendix B.2.

4.0 RESULTS AND CONCLUSIONS

4.1. Fluence Variations

The fluence values obtained for each scheme using the method indicated in Section 3.5 are tabulated in Table 4.1.1 and plotted in Fig. 4.1.a.

Table 4.1.1 The fluence values obtained for each scheme.

POUR	POPR	1st year	2nd year	3rd year	4th year	5th year	6th year
0%	0%	1.20	1.062846	0.986174	0.952301	0.990521	0.996727
0%	25%	1.20	1.062846	0.983271	0.946476	0.980342	0.979628
0%	50%	1.20	1.062846	0.980423	0.940834	0.970320	0.963262
0%	75%	1.20	1.062846	0.977629	0.935375	0.960457	0.947625
0%	100%	1.20	1.062846	0.974891	0.930100	0.950751	0.932715
10%	0%	1.20	1.062846	0.982496	0.943897	0.975233	0.980072
10%	100%	1.20	1.062846	0.971213	0.921690	0.935463	0.916060

For each scheme the fluence decreased sharply in the first four years and then gradually oscillated to an equilibrium value. The reason for this fluence decrease is as follows: The reactor started with a clean core. During the first irradiation period there will be some neutron poison buildup (fission products). Part of these poisons will be carried over to the second year. In order to keep the reactor in a critical condition until the end of the second year, more fissionable material must be loaded into the reactor at the beginning of the second year than was loaded in the first year to counter-balance the effect of the fission products. The energy release in each irradiation period is proportional to the total macroscopic fission cross section, calculated from the material compositions, as well as the fluence of that year. Therefore, if the relative content of fissionable material is increased, going from first year to the second year, the fluence should be decreased to maintain a constant energy release in both years. Eventually, when the core composition reaches an equilibrium condition, the fluence should also reach an equilibrium value.

The effect of recycle plutonium is equivalent to an increase of the enrichment of the additional uranium which is added to the fabrication plant in the prior year. However, the quality of plutonium, as will be indicated in Section 4.2, became poorer when more plutonium was recycled. As a result, compared with 0% Pu recycle, there will be a further increase of fissionable materials in the reactor as more Pu is recycled. Therefore, the fluence decreased further.

Figure 4.1.b describes the averaged fluence decrease versus the percentage of Pu recycled. Since the fluence for first two years are the same for each scheme, the average value was calculated using the fluence data of the last four years.

The effect of recycle uranium from the spent fuel will result in a decrease of the enrichment of the additional uranium added to the fabrication plant. Therefore, more uranium will be required to satisfy the reactivity constraint. The solution to the model becomes infeasible if the percentage of spent uranium recycled is more than 10% because the volume constraint was violated. Higher percentage of uranium recycling can be achieved by raising the enrichment of the additional "clean" uranium which is to be blended together with the spent uranium in the fabrication plant. This will provide more fissionable material to satisfy the reactivity constraint without violating the volume constraint.

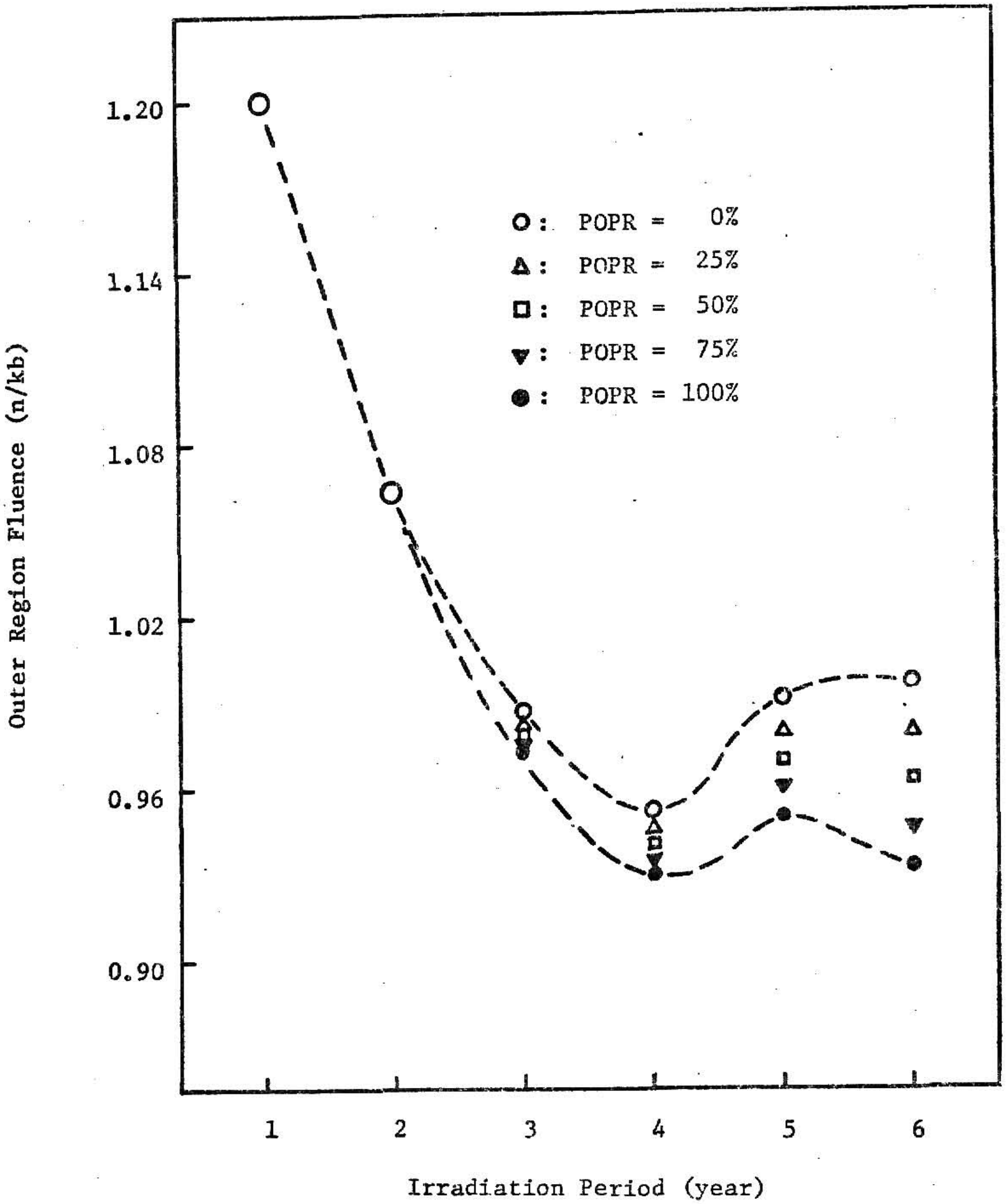

Figure 4.1.a The outer region fluence vs. irradiation period.

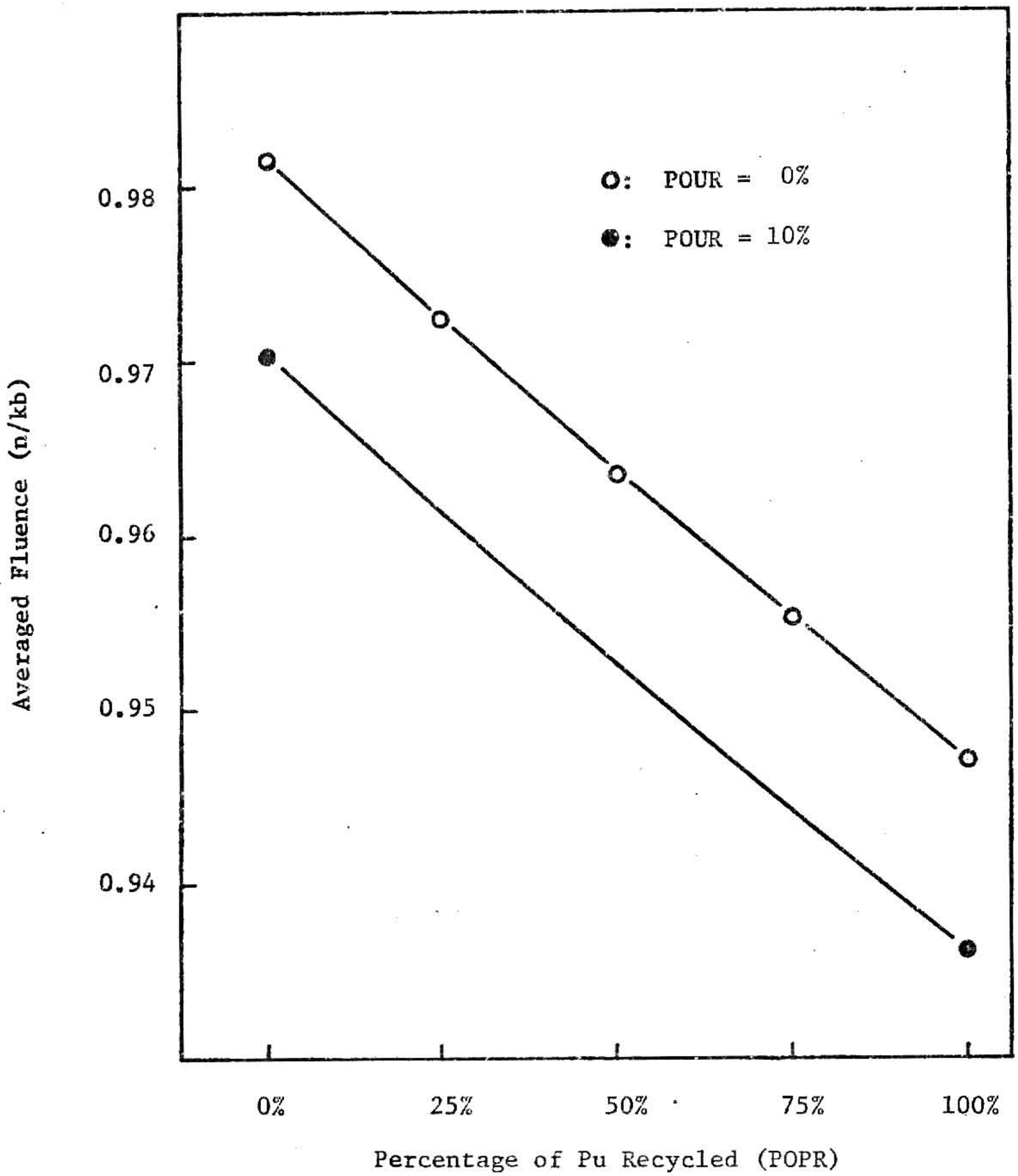

Figure 4.1.b The averaged fluence for the last four years vs. the percentage of Pu recycled.

4.2. Variations of Nuclei Concentrations

The optimal solution of the state variables (nuclei concentrations) was obtained using the fluence values indicated in Section 4.1. The results are plotted in Figs. 4.2.a through 4.2.f.

Figure 4.2.g is a plot of the ratio of the amount of fissionable plutonium to the total amount of plutonium in the reactor versus time. The quality of plutonium, which can be represented by this ratio, decreased when POPR increased. This is due to the accumulation of the non-fissionable ^{240}Pu caused by repeated irradiation of the plutonium atoms. However, it will be mentioned in Section 4.3 that no matter what the plutonium quality is, the worth of ^{239}Pu atom is going to be primarily independent of the percentage of plutonium recycled. These results indicate that, if the price of plutonium is to be related to the price of uranium in the future, the price of ^{239}Pu atoms is going to be essentially independent of the plutonium quality. This is not the case for uranium where the worth of ^{235}U atoms increases with the enrichment of uranium.

4.3. Solutions for the Objective Function

The objective function of this model is defined to be the total amount of ^{235}U required for six years, excluding the initial load. Table 4.3.1 lists the minimized values for the objective function for each scheme. Also included in the table is the total amount of ^{239}Pu in the storage after six years operation. ΔU represents the amount of ^{235}U saved going from 0% Pu recycle to that particular scheme. That is, ΔPu is replacing ΔU in each scheme. Hence, $\Delta U/\Delta Pu$ was calculated for each scheme which gives the relative worth of the ^{235}U atom corresponding to one ^{239}Pu atom. Figure 4.3.b is a plot of $\Delta U/\Delta Pu$ versus POPR. $\Delta U/\Delta Pu$ is essentially independent of the percentage of Pu recycled. This ratio is important in relating the value of plutonium to the cost of uranium. The value of ^{235}U is a fixed and known quantity for 3.2% enriched uranium, while the value of plutonium has not been firmly established. Since the result of this model shows that this ratio, $\Delta U/\Delta Pu$, is essentially constant no matter what the percentage of Pu recycle is, the price of ^{239}Pu can be determined by this ratio. However, the ratio may have a different value for different reactors, but the fact, that this ratio will be independent of the percentage of Pu recycle should still be true for different reactors.

Table 4.3.1 Results of objective function.

POUR	POPR	Total additional ^{235}U required (Kg)	ΔU (Kg)	Total ^{239}Pu in the storage (Kg)	ΔPu (Kg)	$\frac{\Delta U}{\Delta Pu}$
0%	0%	5766.204	--	883.998	---	---
0%	25%	5619.817	146.387	660.664	223.334	0.6555
0%	50%	5471.513	294.691	443.420	440.578	0.6689
0%	75%	5321.207	444.997	218.714	665.284	0.6689
0%	100%	5168.866	597.338	0.0	883.998	0.6757
10%	0%	5680.488	--	896.371	--	---
10%	100%	5087.193	593.195	0.0	896.371	0.6618

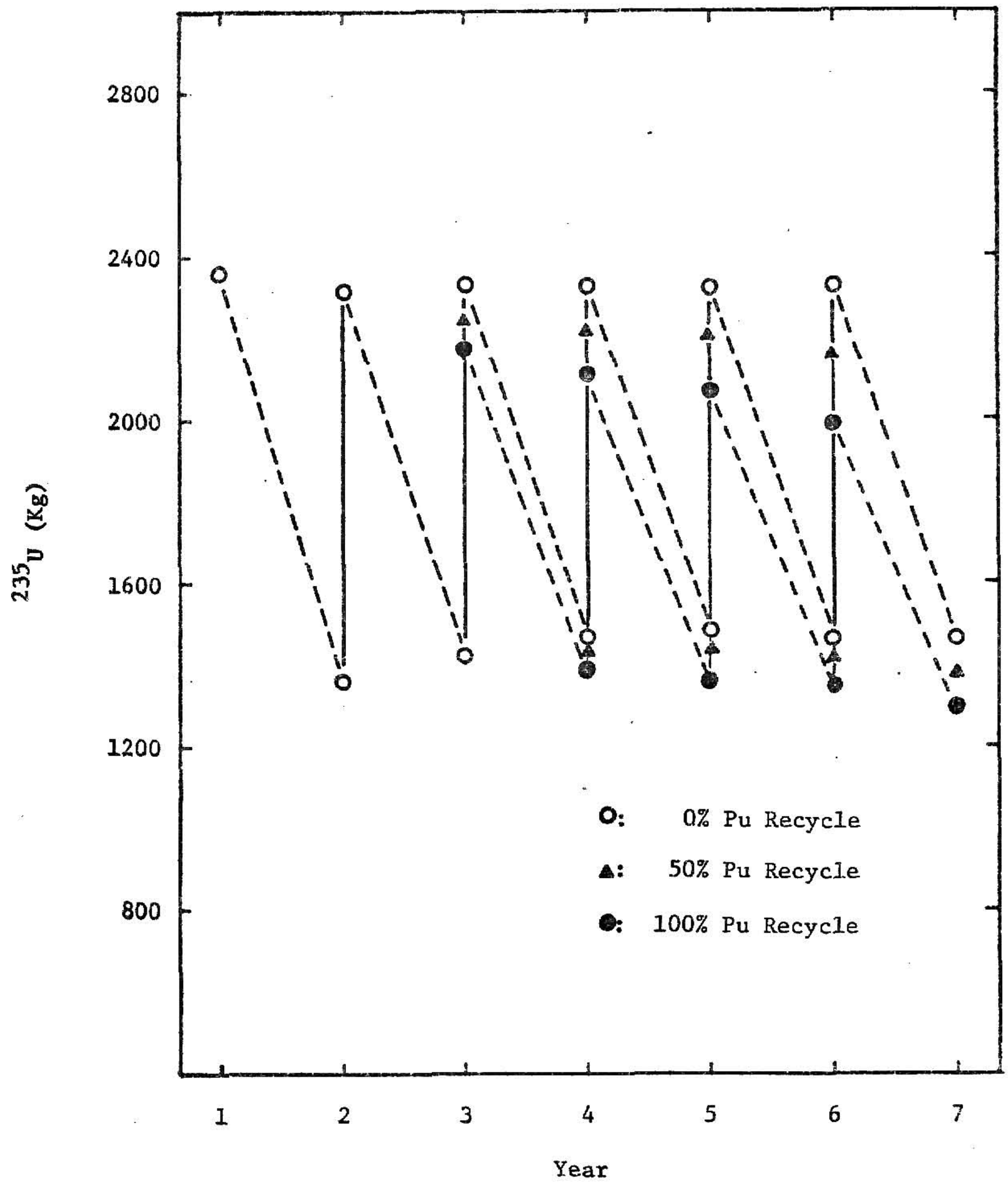

Figure 4.2.a Total amount of ^{235}U in the reactor vs. time for 0% Uranium recycle.

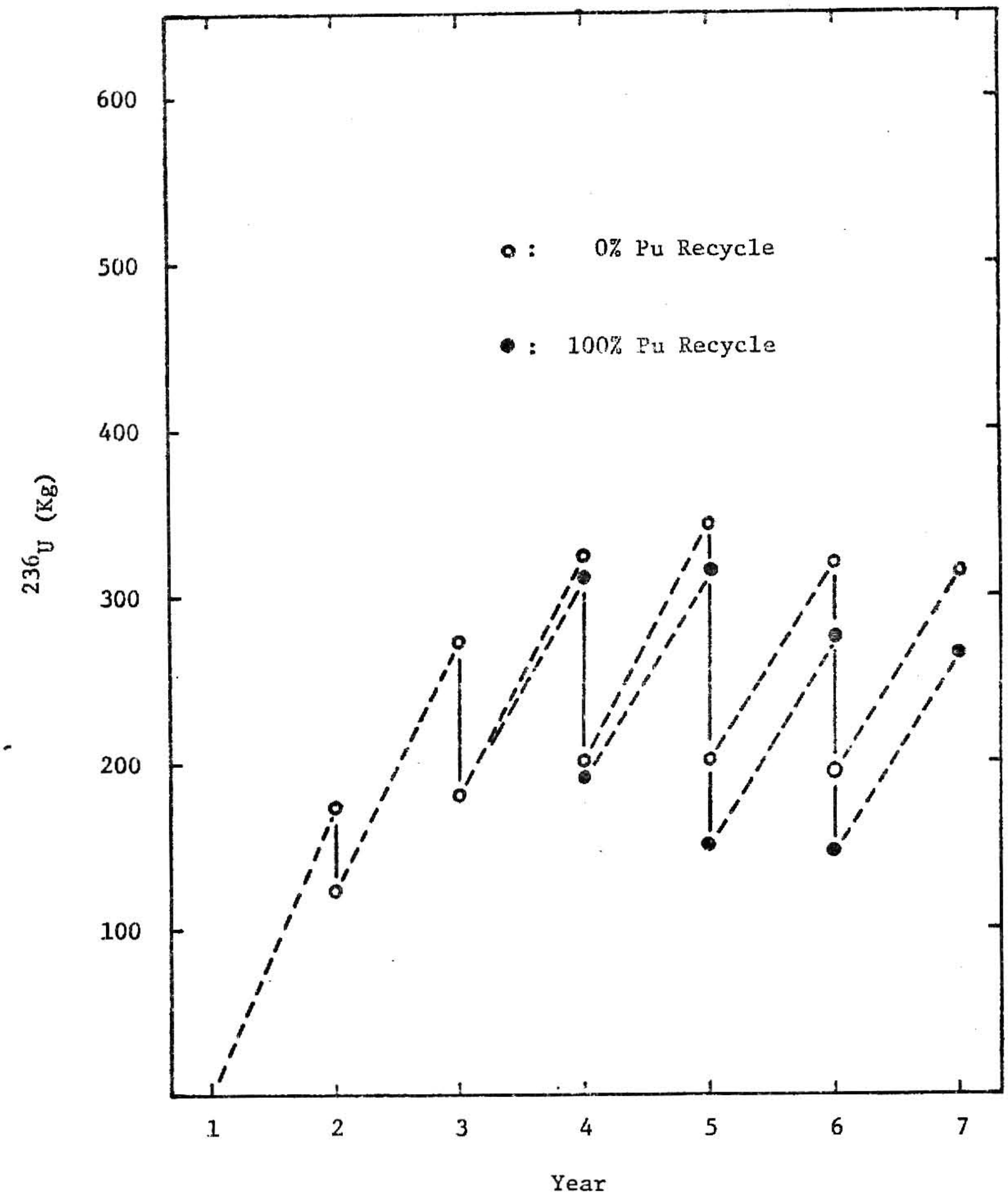

Figure 4.2.b Total amount of ^{236}U in the reactor vs. time for 0% uranium recycle.

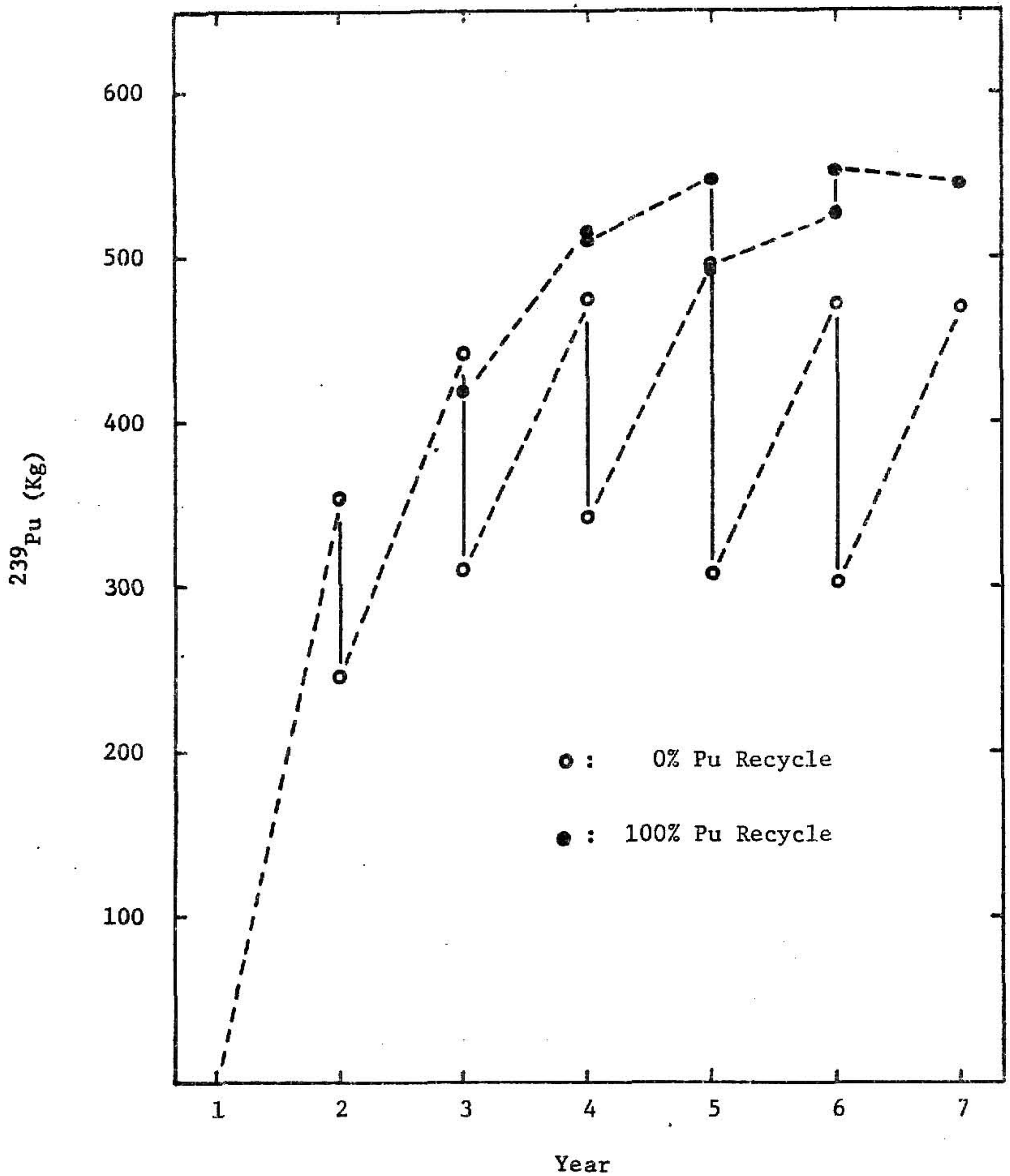

Figure 4.2.c Total amount of ^{239}Pu in the reactor vs. time for 0% uranium recycle.

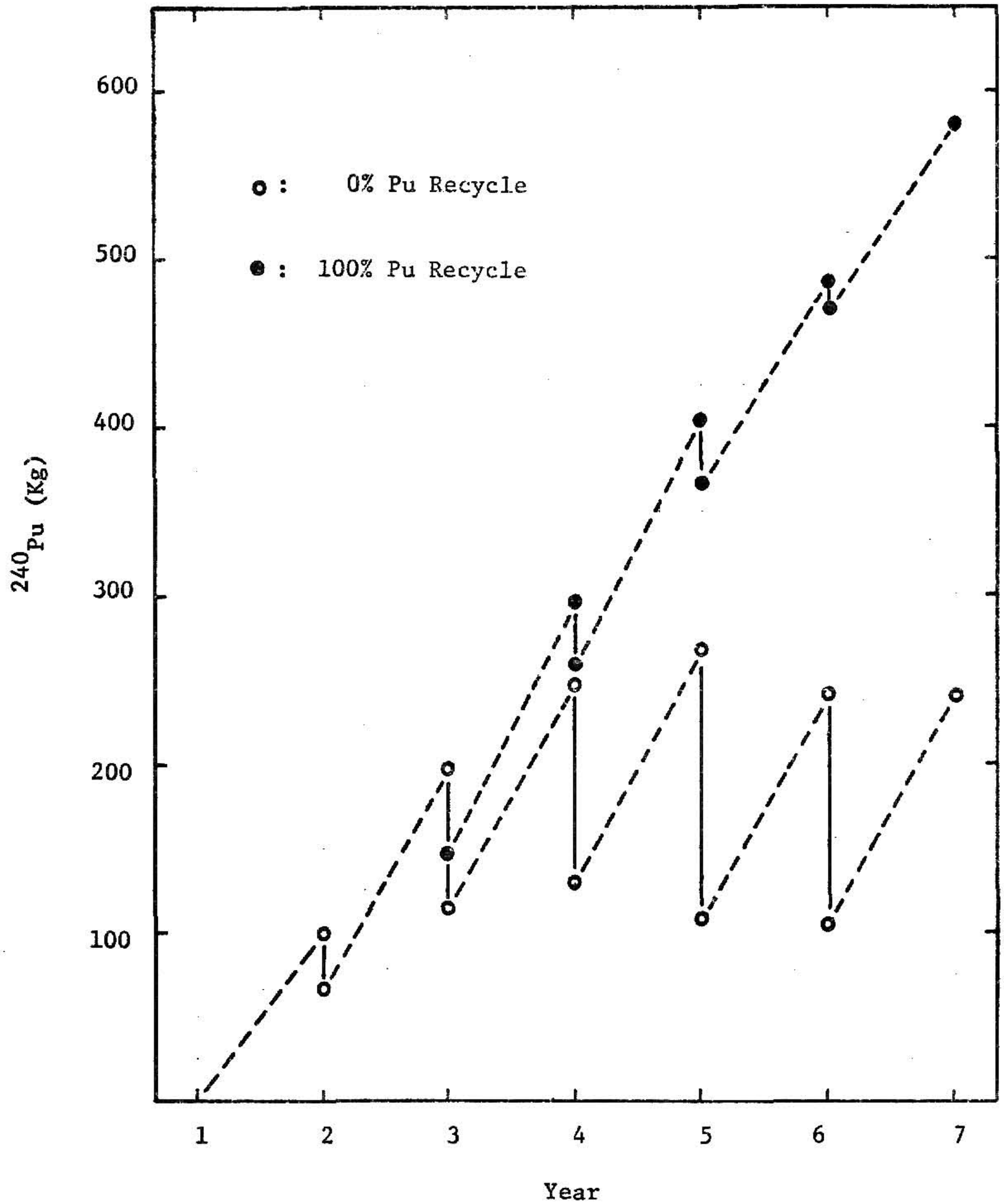

Figure 4.2.d Total amount of ^{240}Pu in the reactor vs. time for 0% uranium recycle.

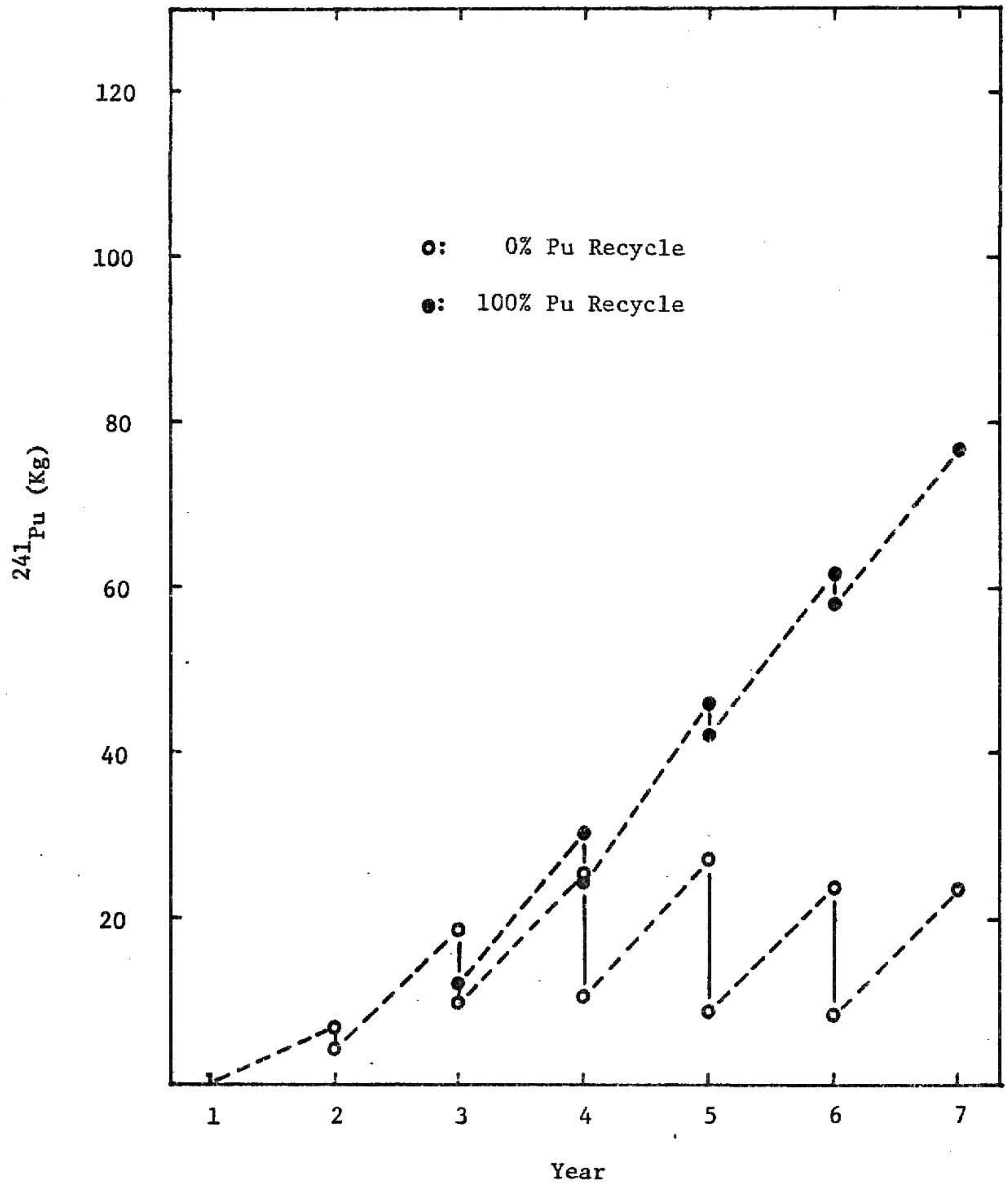

Figure 4.2.e Total amount of ^{241}Pu in the reactor vs. time for 0% uranium recycle.

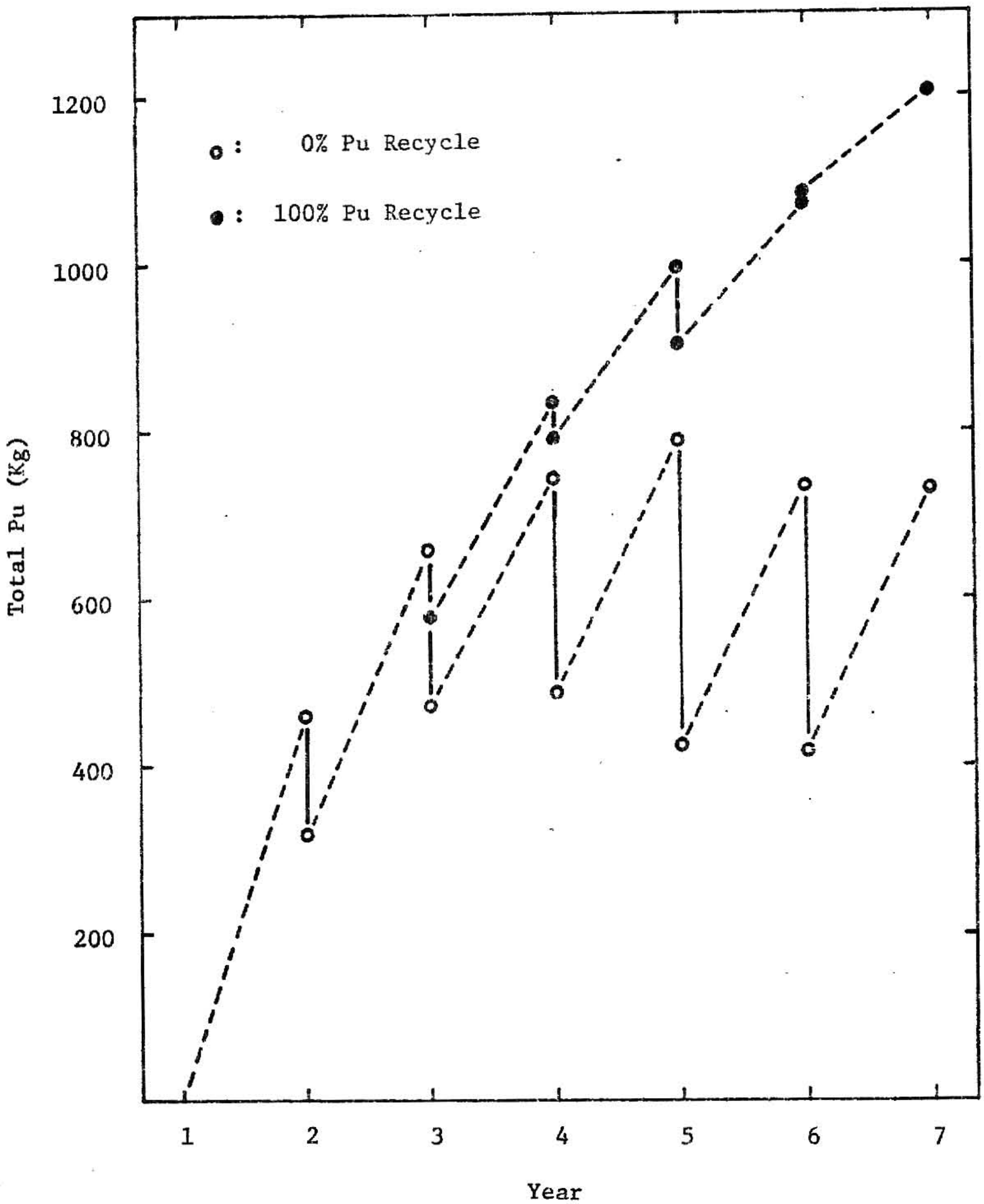

Figure 4.2.f Total amount of plutonium in the reactor vs. time for 0% uranium recycle.

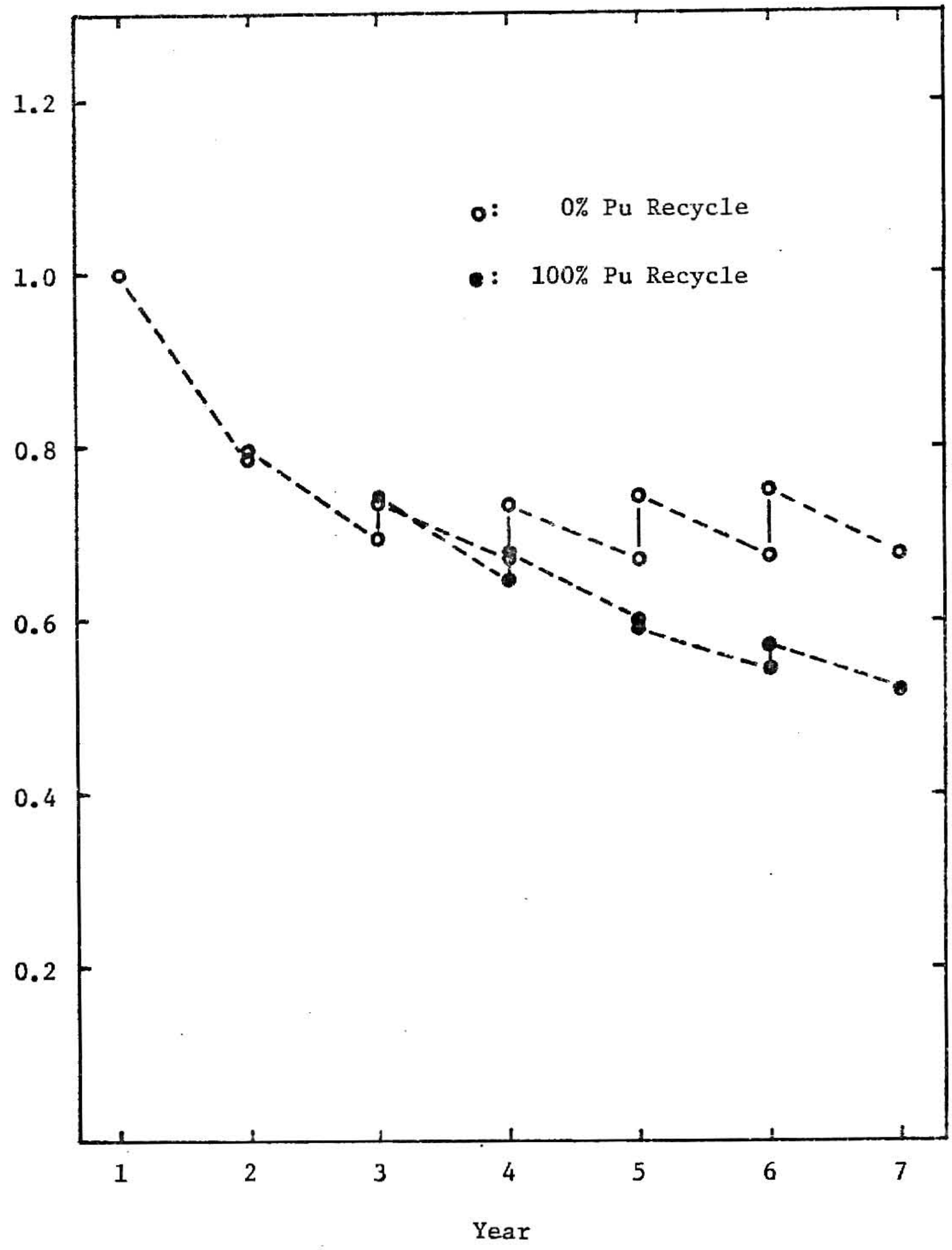

Figure 4.2.g Ratio of the total amount of fissionable Pu to the total plutonium vs. time for 0% uranium recycle.

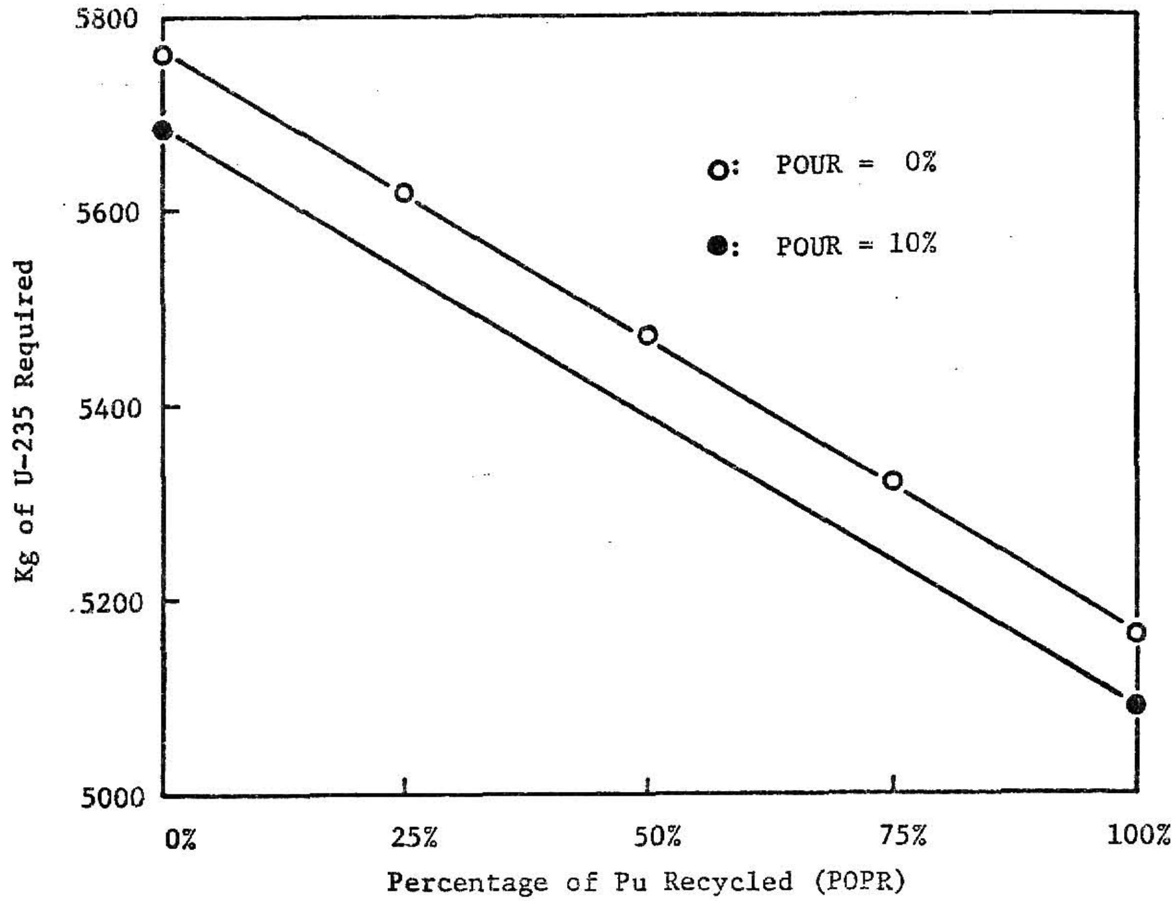

Figure 4.3.a The amount of U-235 required for six irradiation periods vs. percentage of Pu recycled.

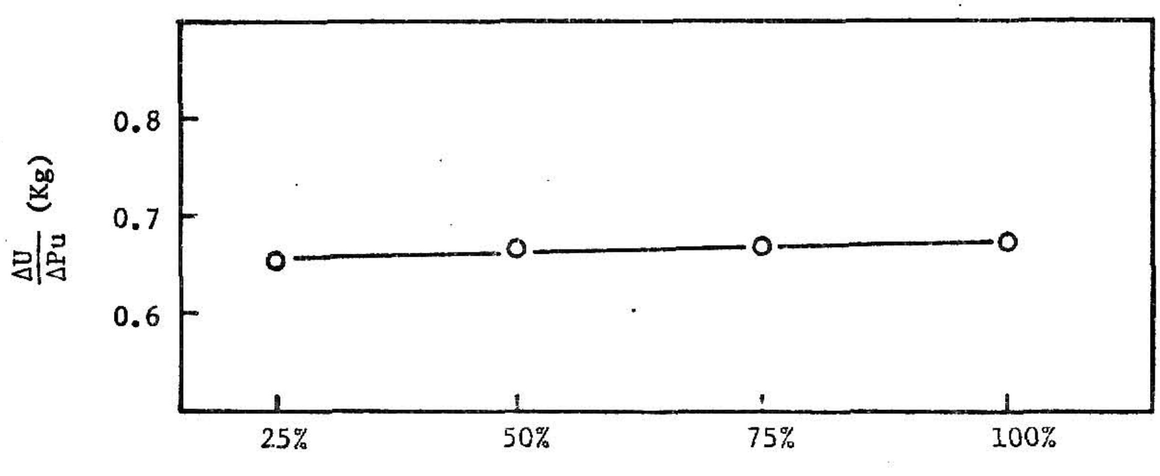

Figure 4.3.b The ratio of the amount of U-235 saved to the amount of Pu-239 lost vs. the percentage of Pu recycled.

5.0 DISCUSSION

This model provides a good starting point for the cost analysis of various options that the utilities may have dealing with fuel management. Theoretically, mixing more highly enriched uranium with lower enrichments means devaluation of ^{235}U atoms (12). Therefore, it might be disadvantageous to recycle spent uranium without recycling plutonium as well (13), not only because it will devaluate the "clean" additional uranium, but also because the spent uranium contains neutron poisons, such as, ^{236}U which tend to further increase the uranium requirement. The solution then exceeds the volume constraint. However, the value of ^{235}U in the spent fuel may be worth less than the "clean" uranium with the same enrichment because of the presence of neutron poisons (^{236}U). Therefore, it might still be feasible to mix more highly enriched (greater than 3.2%) uranium or plutonium with the spent uranium to produce an equivalent quality to 3.2% enriched uranium and recycle the mixture. One should perform the cost analysis on both options to determine which is economically more feasible.

The ratio $\Delta U/\Delta Pu$ provides a reasonably good criterion for a utility to estimate the worth of their own plutonium production. By comparing the estimated worth with the current Pu market price, they could make the decision whether they should save the plutonium for their own purpose or sell it to the open market.

In summary, the results obtained indicate that this model is a realistic representation of a large scale power reactor refueling scheme. The whole methodology is applicable to other types of reactors. The model can be made more complex and accurate by using a better burnup code.

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APPENDICES

APPENDIX A.1

Name of the Constraint Equations and the Corresponding Row Number in the Computer Program

Abbreviations:

*	= Fixed Bounded Constraint	O.R.	= Outer Region
+	= Loosely Bounded Constraint	M.R.	= Intermediate Region
Bn.	= Burnup Constraint	I.R.	= Inner Region
Re.	= Reactivity Constraint	U-5	= ^{235}U
En.	= Enrichment Constraint	U-6	= ^{236}U
I.L.	= Initial Load Constraint	U-8	= ^{238}U
E.R.	= Energy Release Constraint	P-9	= ^{239}Pu
Vol.	= Volume Constraint	P-0	= ^{240}Pu
Fab.	= Fabrication Constraint	P-1	= ^{241}Pu
BOL	= Beginning of Life	F.P.	= Fission Product
EOL	= End of Life	OF	= Objective Function

CU-2 = "Clean Uranium" added to the outer region of the 2nd core.

1-3 = From 1st year to 3rd year.

Row Number	Name of Constraint	Status	Row Number	Name of Constraint	Status
1000	OF		1020	Bn. (1st yr. M.R. P-1)	*
1001	En. (CU-2)	*	1021	Bn. (1st yr. M.R. F.P.)	*
1002	I.L. (O.R. U-5)	*	1022	Bn. (1st yr. I.R. U-5)	*
1003	I.L. (O.R. U-8)	*	1023	Bn. (1st yr. I.R. U-6)	*
1004	I.L. (M.R. U-5)	*	1024	Bn. (1st yr. I.R. U-8)	*
1005	I.L. (M.R. U-8)	*	1025	Bn. (1st yr. I.R. P-9)	*
1006	I.L. (I.R. U-5)	*	1026	Bn. (1st yr. I.R. P-0)	*
1007	I.L. (I.R. U-8)	*	1027	Bn. (1st yr. I.R. P-1)	*
1008	Bn. (1st yr. O.R. U-5)	*	1028	Bn. (1st yr. I.R. F.P.)	*
1009	Bn. (1st yr. O.R. U-6)	*	1029	E.R. (1st yr.)	+
1010	Bn. (1st yr. O.R. U-8)	*	1030	Vol. (1st yr.)	+
1011	Bn. (1st yr. O.R. P-9)	*	1031	BOL Re. (1st yr.)	+
1012	Bn. (1st yr. O.R. P-0)	*	1032	EOL Re. (1st yr.)	+
1013	Bn. (1st yr. O.R. P-1)	*	1033	Bn. (2nd yr. O.R. U-5)	*
1014	Bn. (1st yr. O.R. F.P.)	*	1034	Bn. (2nd yr. O.R. U-6)	*
1015	Bn. (1st yr. M.R. U-5)	*	1035	Bn. (2nd yr. O.R. U-8)	*
1016	Bn. (1st yr. M.R. U-6)	*	1036	Bn. (2nd yr. O.R. P-9)	*
1017	Bn. (1st yr. M.R. U-8)	*	1037	Bn. (2nd yr. O.R. P-0)	*
1018	Bn. (1st yr. M.R. P-9)	*	1038	Bn. (2nd yr. O.R. P-1)	*
1019	Bn. (1st yr. M.R. P-0)	*	1039	Bn. (2nd yr. O.R. F.P.)	*

Row Number	Name of Constraint	Status	Row Number	Name of Constraint	Status
1040	Bn. (2nd yr. M.R. U-5)	*	1060	En. (CU-3)	*
1041	Bn. (2nd yr. M.R. U-6)	*	1061	E.R. (2nd yr.)	*
1042	Bn. (2nd yr. M.R. U-8)	*	1062	Vol. (2nd yr.)	+
1043	Bn. (2nd yr. M.R. P-9)	*	1063	BOL Re. (2nd yr.)	+
1044	Bn. (2nd yr. M.R. P-0)	*	1064	EOL Re. (2nd yr.)	+
1045	Bn. (2nd yr. M.R. P-1)	*	1065	Bn. (3rd yr. O.R. U-5)	*
1046	Bn. (2nd yr. M.R. F.P.)	*	1066	Bn. (3rd yr. O.R. U-6)	*
1047	Bn. (2nd yr. I.R. U-5)	*	1067	Bn. (3rd yr. O.R. U-8)	*
1048	Bn. (2nd yr. I.R. U-6)	*	1068	Bn. (3rd yr. O.R. P-9)	*
1049	Bn. (2nd yr. I.R. U-8)	*	1069	Bn. (3rd yr. O.R. P-0)	*
1050	Bn. (2nd yr. I.R. P-9)	*	1070	Bn. (3rd yr. O.R. P-1)	*
1051	Bn. (2nd yr. I.R. P-0)	*	1071	Bn. (3rd yr. O.R. F.P.)	*
1052	Bn. (2nd yr. I.R. P-1)	*	1072	Bn. (3rd yr. M.R. U-5)	*
1053	Bn. (2nd yr. I.R. F.P.)	*	1073	Bn. (3rd yr. M.R. U-6)	*
1054	Fab. (1-3, U-5)	*	1074	Bn. (3rd yr. M.R. U-8)	*
1055	Fab. (1-3, U-6)	*	1075	Bn. (3rd yr. M.R. P-9)	*
1056	Fab. (1-3, U-8)	*	1076	Bn. (3rd yr. M.R. P-0)	*
1057	Fab. (1-3, P-9)	*	1077	Bn. (3rd yr. M.R. P-1)	*
1058	Fab. (1-3, P-0)	*	1078	Bn. (3rd yr. M.R. F.P.)	*
1059	Fab. (1-3, P-1)	*	1079	Bn. (3rd yr. I.R. U-5)	*

Row Number	Name of Constraint	Status	Row Number	Name of Constraint	Status
1080	Bn. (3rd yr. I.R. U-6)	*	1100	Bn. (4th yr. O.R. P-9)	*
1081	Bn. (3rd yr. I.R. U-8)	*	1101	Bn. (4th yr. O.R. P-0)	*
1082	Bn. (3rd yr. I.R. P-9)	*	1102	Bn. (4th yr. O.R. P-1)	*
1083	Bn. (3rd yr. I.R. P-0)	*	1103	Bn. (4th yr. O.R. F.P.)	*
1084	Bn. (3rd yr. I.R. P-1)	*	1104	Bn. (4th yr. M.R. U-5)	*
1085	Bn. (3rd yr. I.R. F.P.)	*	1105	Bn. (4th yr. M.R. U-6)	*
1086	Fab. (2-4, U-5)	*	1106	Bn. (4th yr. M.R. U-8)	*
1087	Fab. (2-4, U-6)	*	1107	Bn. (4th yr. M.R. P-9)	*
1088	Fab. (2-4, U-8)	*	1108	Bn. (4th yr. M.R. P-0)	*
1089	Fab. (2-4, P-9)	*	1109	Bn. (4th yr. M.R. P-1)	*
1090	Fab. (2-4, P-0)	*	1110	Bn. (4th yr. M.R. F.P.)	*
1091	Fab. (2-4, P-1)	*	1111	Bn. (4th yr. I.R. U-5)	*
1092	En. (CU-4)	*	1112	Bn. (4th yr. I.R. U-6)	*
1093	E.R. (3rd yr.)	*	1113	Bn. (4th yr. I.R. U-8)	*
1094	Vol. (3rd yr.)	+	1114	Bn. (4th yr. I.R. P-9)	*
1095	BOL Re. (3rd yr.)	+	1115	Bn. (4th yr. I.R. P-0)	*
1096	EOL Re. (3rd yr.)	+	1116	Bn. (4th yr. I.R. P-1)	*
1097	Bn. (4th yr. O.R. U-5)	*	1117	Bn. (4th yr. I.R. F.P.)	*
1098	Bn. (4th yr. O.R. U-6)	*	1118	Fab. (3-5, U-5)	*
1099	Bn. (4th yr. O.R. U-8)	*	1119	Fab. (3-5, U-6)	*

Row Number	Name of Constraint	Status
1120	Fab. (3-5, U-8)	*
1121	Fab. (3-5, P-9)	*
1122	Fab. (3-5, P-0)	*
1123	Fab. (3-5, P-1)	*
1124	En. (CU-5)	*
1125	E.R. (4th yr.)	*
1126	Vol. (4th yr.)	+
1127	BOL Re. (4th yr.)	+
1128	EOL Re. (4th yr.)	+
1129	Bn. (5th yr. O.R. U-5)	*
1130	Bn. (5th yr. O.R. U-6)	*
1131	Bn. (5th yr. O.R. U-8)	*
1132	Bn. (5th yr. O.R. P-9)	*
1133	Bn. (5th yr. O.R. P-0)	*
1134	Bn. (5th yr. O.R. P-1)	*
1135	Bn. (5th yr. O.R. F.P.)	*
1136	Bn. (5th yr. M.R. U-5)	*
1137	Bn. (5th yr. M.R. U-6)	*
1138	Bn. (5th yr. M.R. U-8)	*
1139	Bn. (5th yr. M.R. P-9)	*
1140	Bn. (5th yr. M.R. P-0)	*
1141	Bn. (5th yr. M.R. P-1)	*
1142	Bn. (5th yr. M.R. F.P.)	*
1143	Bn. (5th yr. I.R. U-5)	*
1144	Bn. (5th yr. I.R. U-6)	*
1145	Bn. (5th yr. I.R. U-8)	*
1146	Bn. (5th yr. I.R. P-9)	*
1147	Bn. (5th yr. I.R. P-0)	*
1148	Bn. (5th yr. I.R. P-1)	*
1149	Bn. (5th yr. I.R. F.P.)	*
1150	Fab. (4-6, U-5)	*
1151	Fab. (4-6, U-6)	*
1152	Fab. (4-6), U-8)	*
1153	Fab. (4-6, P-9)	*
1154	Fab. (4-6, P-0)	*
1155	Fab. (4-6, P-1)	*
1156	En. (CU-6)	*
1157	E.R. (5th yr.)	*
1158	Vol. (5th yr.)	+
1159	BOL Re. (5th yr.)	+

Row Number	Name of Constraint	Status	Row Number	Name of Constraint	Status
1160	EOL Re. (5th yr.)	+	1173	Bn. (6th yr. M.R. P-1)	*
1161	Bn. (6th yr. O.R. U-5)	*	1174	Bn. (6th yr. M.R. F.P.)	*
1162	Bn. (6th yr. O.R. U-6)	*	1175	Bn. (6th yr. I.R. U-5)	*
1163	Bn. (6th yr. O.R. U-8)	*	1176	Bn. (6th yr. I.R. U-6)	*
1164	Bn. (6th yr. O.R. P-9)	*	1177	Bn. (6th yr. I.R. U-8)	*
1165	Bn. (6th yr. O.R. P-0)	*	1178	Bn. (6th yr. I.R. P-9)	*
1166	Bn. (6th yr. O.R. P-1)	*	1179	Bn. (6th yr. I.R. P-0)	*
1167	Bn. (6th yr. O.R. F.P.)	*	1180	Bn. (6th yr. I.R. P-1)	*
1168	Bn. (6th yr. M.R. U-5)	*	1181	Bn. (6th yr. I.R. F.P.)	*
1169	Bn. (6th yr. M.R. U-6)	*	1182	E.R. (6th yr.)	*
1170	Bn. (6th yr. M.R. U-8)	*	1183	Vol. (6th yr.)	+
1171	Bn. (6th yr. M.R. P-9)	*	1184	BOL Re. (6th yr.)	+
1172	Bn. (6th yr. M.R. P-0)	*	1185	EOL Re. (6th yr.)	+

APPENDIX A.2

Name of the State Variable and the Corresponding Column Number in the Computer Code

It is indicated in Section 3.3 that the state variables, $I^{\ell}_{i,j,k}$, are defined to be the amount of nuclides of species i in the reactor of region j during the irradiation period k at the beginning of that irradiation period for $\ell = 1$ and at the end of that year for $\ell = 2$. The description of the indices, i, j, k is given in Table 3.3.1.

By the way it is defined, $I^{2}_{1,1,1}$ would represent the amount of ^{235}U in the outer region at the end of the first year, and $I^{1}_{1,2,2}$ would be the amount of ^{235}U in the intermediate region at the beginning of the second year. However, the refueling scheme for this model was chosen to be the "Out-In" batch irradiation. Therefore, $I^{2}_{1,1,1}$ and $I^{1}_{1,2,2}$ refer to the same quantity. In the computer code, only one name is assigned to this quantity, both names are listed in the following table.

Abbreviations:

A = Column number defined in the computer code

B = Corresponding name defined in Section 3.4

C = Other name corresponding to the same quantity

A	B	C
1001	I^1_{111}	--
1002	I^1_{311}	--
1003	I^1_{121}	--
1004	I^1_{321}	--
1005	I^1_{131}	--
1006	I^1_{331}	--
1007	I^1_{112}	--
1008	I^1_{312}	--
1009	I^2_{111}	I^1_{122}
1010	I^2_{211}	I^1_{222}
1011	I^2_{311}	I^1_{322}
1012	I^2_{411}	I^1_{422}
1013	I^2_{511}	I^1_{522}
1014	I^2_{611}	I^1_{622}
1015	I^2_{711}	I^1_{722}
1016	I^2_{121}	I^1_{132}
1017	I^2_{221}	I^1_{232}
1018	I^2_{321}	I^1_{332}
1019	I^2_{421}	I^1_{432}
1020	I^2_{521}	I^1_{532}

A	B	C
1021	I^2_{621}	I^1_{632}
1022	I^2_{721}	I^1_{732}
1023	I^2_{131}	--
1024	I^2_{231}	--
1025	I^2_{331}	--
1026	I^2_{431}	--
1027	I^2_{531}	--
1028	I^2_{631}	--
1029	I^2_{731}	--
1030	U_{13}	--
1031	I^1_{113}	--
1032	I^1_{213}	--
1033	I^1_{313}	--
1034	I^1_{413}	--
1035	I^1_{513}	--
1036	I^1_{613}	--
1037	U_{33}	--
1038	I^2_{112}	I^1_{123}
1039	I^2_{212}	I^1_{223}
1040	I^2_{312}	I^1_{323}

A	B	C
1041	I^2_{412}	I^1_{423}
1042	I^2_{512}	I^1_{523}
1043	I^2_{612}	I^1_{623}
1044	I^2_{712}	I^1_{723}
1045	I^2_{122}	I^1_{133}
1046	I^2_{222}	I^1_{233}
1047	I^2_{322}	I^1_{333}
1048	I^2_{422}	I^1_{433}
1049	I^2_{522}	I^1_{533}
1050	I^2_{622}	I^1_{633}
1051	I^2_{722}	I^1_{733}
1052	I^2_{132}	--
1053	I^2_{232}	--
1054	I^2_{332}	--
1055	I^2_{432}	--
1056	I^2_{532}	--
1057	I^2_{632}	--
1058	I^2_{732}	--
1059	U_{14}	--
1060	I^1_{114}	--

A	B	C	A	B	C	A	B	C
1061	I^1_{214}	—	1081	I^2_{133}	--	1101	I^2_{614}	I^1_{625}
1062	I^1_{314}	—	1082	I^2_{233}	--	1102	I^2_{714}	I^1_{725}
1063	I^1_{414}	—	1083	I^2_{333}	--	1103	I^2_{124}	I^1_{135}
1064	I^1_{514}	—	1084	I^2_{433}	--	1104	I^2_{224}	I^1_{235}
1065	I^1_{614}	—	1085	I^2_{533}	--	1105	I^2_{324}	I^1_{335}
1066	U_{34}	—	1086	I^2_{633}	--	1106	I^2_{424}	I^1_{435}
1067	I^2_{113}	I^1_{124}	1087	I^2_{733}	--	1107	I^2_{524}	I^1_{535}
1068	I^2_{213}	I^1_{224}	1088	U_{15}	--	1108	I^2_{624}	I^1_{635}
1069	I^2_{313}	I^1_{324}	1089	I^1_{115}	--	1109	I^2_{724}	I^1_{735}
1070	I^2_{413}	I^1_{424}	1090	I^1_{215}	--	1110	I^2_{134}	--
1071	I^2_{513}	I^1_{524}	1091	I^1_{315}	--	1111	I^2_{234}	--
1072	I^2_{613}	I^1_{624}	1092	I^1_{415}	--	1112	I^2_{334}	--
1073	I^2_{713}	I^1_{724}	1093	I^1_{515}	--	1113	I^2_{434}	--
1074	I^2_{123}	I^1_{134}	1094	I^1_{615}	--	1114	I^2_{534}	--
1075	I^2_{223}	I^1_{234}	1095	U_{35}	--	1115	I^2_{634}	--
1076	I^2_{323}	I^1_{334}	1096	I^2_{114}	I^1_{125}	1116	I^2_{734}	--
1077	I^2_{423}	I^1_{434}	1097	I^2_{214}	I^1_{225}	1117	U_{16}	--
1078	I^2_{523}	I^1_{534}	1098	I^2_{314}	I^1_{325}	1118	I^1_{116}	--
1079	I^2_{623}	I^1_{634}	1099	I^2_{414}	I^1_{425}	1119	I^1_{216}	--
1080	I^2_{723}	I^1_{734}	1100	I^2_{514}	I^1_{525}	1120	I^1_{316}	--

A	B	C	A	B	C	A	B	C
1121	I^{1}_{416}	—	1136	I^{2}_{525}	I^{1}_{536}	1151	I^{2}_{616}	—
1122	I^{1}_{516}	—	1137	I^{2}_{625}	I^{1}_{636}	1152	I^{2}_{716}	—
1123	I^{1}_{616}	—	1138	I^{2}_{725}	I^{1}_{736}	1153	I^{2}_{126}	—
1124	U_{36}	—	1139	I^{2}_{135}	—	1154	I^{2}_{226}	—
1125	I^{2}_{115}	I^{1}_{126}	1140	I^{2}_{235}	—	1155	I^{2}_{326}	—
1126	I^{2}_{215}	I^{1}_{226}	1141	I^{2}_{335}	—	1156	I^{2}_{426}	—
1127	I^{2}_{315}	I^{1}_{326}	1142	I^{2}_{435}	—	1157	I^{2}_{526}	—
1128	I^{2}_{415}	I^{1}_{426}	1143	I^{2}_{535}	—	1158	I^{2}_{626}	—
1129	I^{2}_{515}	I^{1}_{526}	1144	I^{2}_{635}	—	1159	I^{2}_{726}	—
1130	I^{2}_{615}	I^{1}_{626}	1145	I^{2}_{735}	—	1160	I^{2}_{136}	—
1131	I^{2}_{715}	I^{1}_{726}	1146	I^{2}_{116}	—	1161	I^{2}_{236}	—
1132	I^{2}_{125}	I^{1}_{136}	1147	I^{2}_{216}	—	1162	I^{2}_{336}	—
1133	I^{2}_{225}	I^{1}_{236}	1148	I^{2}_{316}	—	1163	I^{2}_{436}	—
1134	I^{2}_{325}	I^{1}_{336}	1149	I^{2}_{416}	—	1164	I^{2}_{536}	—
1135	I^{2}_{425}	I^{1}_{436}	1150	I^{2}_{516}	—	1165	I^{2}_{636}	—
						1166	I^{2}_{736}	—

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APPENDIX B.1

The LP Code Listing

```
      DIMENSION RE(8),RB(8),V(7),U(6)
      COMMON /Z1/C1,C2,C3,C4,C5,C6,C7,C8,C9,C10
      COMMON /Z2/SA5,SA6,SA8,SA9,SA0,SA1
      COMMON /Z3/MM,A(7,7,3),B(63,29)
      COMMON /Z5/SAF,SF5,SF9,SF1,SC5,SC9,SC1
      COMMON /Z4/C11,C12,C13,C14,C15,C16,C17,C18
      DATA W1,W2,POUR,POPR/1.20,1.062846,0.1,1.0/
      DATA W3,W4,W5,W6/0.971213,0.92169,0.935463,0.916060/
      DATA P,P1,EPS,ET5/0.832,0.9912,1.0584,2.0423/
      DATA P,P1,EPS,ET5/0.832,0.9912,1.0584,2.0423/
      DATA AN5,AN9,AN1,EF/2.47,2.905,3.06,0.91/
      DATA ENA,ENB,ET9,SAS/0.968,-0.032,1.8107,260.0/
      DATA V1,V2,V3,V4,V5,V6,V7/3*0.9124,3*0.8726,43000.0/
   10 FORMAT(T5,'COL',I4,T15,'ROW1000',T26,
     1'0.10000E 01',T40,'ROW',I4,T51,'0.10000E 01')
   11 FORMAT(T5,'COL',I4,T15,'ROW',I4,T25,E12.5)
   12 FORMAT('RHS')
   13 FORMAT(T5,'LIMITS',T15,'ROW',I4,T25,
     1E12.5,T40,'ROW',I4,T50,E12.5)
   14 FORMAT(T5,'COL1007',T15,'ROW1000',T26,'0.10000E 01')
   20 FORMAT('NAME',T15,'YFC')
   21 FORMAT('ROWS')
   22 FORMAT('  N ROW1000')
   23 FORMAT('  E ROW',I4)
   24 FORMAT('  G ROW',I4)
   25 FORMAT('ENDATA')
   26 FORMAT('COLUMNS')
   27 FORMAT('  L ROW',I4)
      U(1)=W1
      U(2)=W2
      U(3)=W3
      U(4)=W4
      U(5)=W5
      U(6)=W6
      V(1)=V1
      V(2)=V2
      V(3)=V3
      V(4)=V4
      V(5)=V5
      V(6)=V6
      V(7)=V7
      SA5=0.412
      SA6=0.0048
      SA8=0.00173
      SA9=1.028
      SA0=0.1772
      SA1=1.121
      SAF=0.025
      SF5=0.3414
      SF9=0.6772
      SF1=0.773
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```
      SC5=0.0706
      SC9=0.3508
      SC1=0.348
      ETB=1.114/(P*EPS*P1*EF)
      ETE=1.003/(P*EPS*P1*EF)
      RE(1)=AN5*SF5-ETE*SA5
      RB(1)=AN5*SF5-ETB*SA5
      RE(2)=-ETE*SA6
      RB(2)=-ETB*SA6
      RE(3)=-ETE*SA8
      RB(3)=-ETB*SA8
      RE(4)=AN9*SF9-ETE*SA9
      RB(4)=AN9*SF9-ETB*SA9
      RE(5)=-ETE*SA0
      RB(5)=-ETB*SA0
      RE(6)=AN1*SF1-ETE*SA1
      RB(6)=AN1*SF1-ETB*SA1
      RE(7)=-ETE*SAF
      RB(7)=-ETB*SAF
      MM=9
      RB(8)=ETB*SAS
      RE(8)=ETE*SAS
      RE(8)=370.0
      C1=EPS*P1*(1.0-P)*ET9*SA9-SA9
      C2=EPS*P1*(1.0-P)*ET5*SA5
      C3=-SA8/C1
      C4=SA0+C1
      C5=SA5-SA0
      C6=SA5+C1
      C7=SA1+C1
      C8=SA1-SA5
      C9=SA1-SA0
      C10=C4*C7
      C11=C5*C8
      C12=2.0*C3
      C13=C12*SF1*SC9
      C14=C13*SA0
      C15=2.0*C2
      C16=C15*SF1*SC9
      C17=C16*SA0
      C18=(1.0/C5+1.0/C4)
      WRITE(MM,20)
      WRITE(MM,21)
      WRITE(MM,22)
      DO 30 I=1001,1028
   30 WRITE(MM,23)I
      I=1029
      WRITE(MM,24)I
      I=1030
      WRITE(MM,27)I
```

```
      DO 31 I=1031,1032
   31 WRITE(MM,24)I
      I1=1033
      I2=1061
      DO 32 K=1,5
      DO 33 I=I1,I2
   33 WRITE(MM,23)I
      I=I2+1
      WRITE(MM,27)I
      I=I2+2
      WRITE(MM,24)I
      I=I2+3
      WRITE(MM,24)I
      I1=I2+4
      I2=I1+28
      IF(K.GE.4)I2=I1+21
   32 CONTINUE
      WRITE(MM,26)
      DO 99 I=1,7
      DO 99 J=1,7
      DO 99 K=1,3
   99 A(I,J,K)=0.0
      DO 101 I=1,63
      DO 101 J=1,29
  101 B(I,J)=0.0
      B(1,7)=ENA
      B(1,8)=ENB
      DO 102 I=1,6
  102 B(I+1,I)=0.01
      DO 103 I=8,28
  103 B(I,I+1)=-1.0
      DO 104 K=1,3
      B(31,2*K-1)=RB(1)
      B(31,2*K)=RB(3)
      B(K+53,K+22)=POUR
      B(K+56,K+25)=POPR
  104 B(29,K*7+8)=1.0
      IY=1
      TU=U(IY)
      CALL BURNUP(IY,TU)
      DO 106 K=1,3
      DO 106 I=1,7
      B(K*7+I,K*2-1)=A(I,1,K)
  106 B(K*7+I,K*2)=A(I,3,K)
      IY=2
      TU=U(IY)
      CALL BURNUP(IY,TU)
      DO 107 I=1,7
      B(32+I,7)=A(I,1,1)
  107 B(32+I,8)=A(I,3,1)
```

```
      DO 108 I=1,7
      B(32,I+8)=RE(I)
      B(32,I+15)=RE(I)
      B(32,I+22)=RE(I)
      B(63,I+8)=RB(I)
      B(63,I+15)=RB(I)
      DO 110 K=2,3
      DO 110 J=1,I
  110 B(K*7+25+I,K*7-6+J)=A(I,J,K)
  108 CONTINUE
      B(30,1)=V(1)
      B(30,2)=V(3)
      B(62,7)=V(1)
      B(62,8)=V(3)
      B(63,7)=RB(1)
      B(63,8)=RB(3)
      B(61,15)=-1.0
      B(61,22)=-1.0
      I1=1000
      J1=1000
      I2=31
      J2=6
      J3=1
      CALL WTOUT(I1,J1,I2,J2,J3)
      WRITE(MM,14)
      I2=63
      J2=29
      J3=7
      CALL WTOUT(I1,J1,I2,J2,J3)
      IR3=1054
      JCOL=1030
      WRITE(MM,10)JCOL,IR3
      IR4=IR3+6
      WRITE(MM,11)JCOL,IR4,ENA
      DO 201 I=1,63
      DO 201 J=1,29
  201 B(I,J)=0.0
      DO 202 I=1,21
  202 B(I,I+7)=-1.0
      DO 203 I=1,6
      B(I+21,I)=-1.0
      B(62,I)=V(I)
  203 B(63,I)=RB(I)
      DO 204 I=1,7
      B(32,I+7)=RE(I)
      B(32,I+14)=RE(I)
      B(32,I+21)=RE(I)
      B(63,I+7)=RB(I)
  204 B(63,I+14)=RB(I)
      B(24,7)=1.0
```

```
      B(61,14)=-1.0
      B(61,21)=-1.0
      B(28,7)=ENB
      DO 205 I=1,3
      B(I+53,I+21)=POUR
      B(I+56,I+24)=POPR
  205 B(29,I*7+7)=1.0
      I1=1032
      J1=1030
      I2=63
      J2=28
      J3=1
      DO 210 IY=3,6
      TU=U(IY)
      CALL BURNUP(IY,TU)
      DO 212 I=1,7
      DO 212 J=1,I
      B(I+32,J)=A(I,J,1)
      B(I+39,J+7)=A(I,J,2)
  212 B(I+46,J+14)=A(I,J,3)
      IF(IY.GE.6)GO TO 210
      CALL WTOUT(I1,J1,I2,J2,J3)
      IR3=I1+54
      IR4=IR3+6
      I1=I1+32
      J1=J1+29
      JCOL=J1
      WRITE(MM,10)JCOL,IR3
      WRITE(MM,11)JCOL,IR4,ENA
  210 CONTINUE
      DO 213 I=1,3
      B(I+53,I+21)=0.0
      DO 214 J=1,21
  214 B(I+53,J)=B(I+60,J)
  213 CONTINUE
      I1=1128
      J1=1117
      I2=56
      J2=28
      J3=1
      CALL WTOUT(I1,J1,I2,J2,J3)
      DO 215 I=2,4
  215 B(22,7*I)=1.0
      DO 216 I=8,28
  216 B(25,I)=B(32,I)
      I1=1160
      J1=1138
      I2=25
      J2=28
      J3=8
```

```
      CALL WTOUT(I1,J1,I2,J2,J3)
      WRITE(MM,12)
      XEN=0.032
      DO 218 IR1=1002,1006,2
      IR2=IR1+1
      RV1=(XEN*109.265/(0.1102*3.0))*(238.0/270.0)
      RV2=((1.0-XEN)*109.265/(0.1102*3.0))*(238.0/270.0)
      WRITE(MM,13)IR1,RV1,IR2,RV2
  218 XEN=XEN-0.005
      IR1=1029
      RV1=2112.50
      IR2=1030
      RV2=V(7)
      RV3=RB(8)
      RV4=RE(8)
      DO 300 I=1,6
      WRITE(MM,13)IR1,RV1,IR2,RV2
      IR1=IR1+2
      IR2=IR2+2
      WRITE(MM,13)IR1,RV3,IR2,RV4
      IA=30
      IF(I.GE.5)IA=23
      IR1=IR1+IA
  300 IR2=IR2+IA
      WRITE(MM,25)
      STOP
      END
```

```
      SUBROUTINE BURNUP(IY,TU)
      DIMENSION T(3)
      COMMON /Z1/C1,C2,C3,C4,C5,C6,C7,C8,C9,C10
      COMMON /Z2/SA5,SA6,SA8,SA9,SA0,SA1
      COMMON /Z3/MM,A(7,7,3),B(63,29)
      COMMON /Z4/C11,C12,C13,C14,C15,C16,C17,C18
      COMMON /Z5/SAF,SF5,SF9,SF1,SC5,SC9,SC1
      T(1)=TU
      T(2)=1.225*T(1)
      T(3)=1.264*T(1)
      DO 40 K=1,3
      A(1,1,K)=EXP(-SA5*T(K))
      A(2,2,K)=EXP(-SA6*T(K))
      A(3,3,K)=EXP(-SA8*T(K))
      A(4,4,K)=EXP( C1*T(K))
      A(5,5,K)=EXP(-SA0*T(K))
      A(6,6,K)=EXP(-SA1*T(K))
      A(7,7,K)=1.0
      D1=A(5,5,K)-1.0
      D2=A(6,6,K)-1.0
      D3=A(4,4,K)-1.0
      A(2,1,K)=SC5*(A(1,1,K)-A(2,2,K))/(SA6-SA5)
      A(4,3,K)=-C3*D3
      A(4,1,K)=C2*(A(4,4,K)-A(1,1,K))/C6
      A(5,4,K)=SC9*(A(4,4,K)-A(5,5,K))/C4
      A(5,3,K)=-C3*(SC9*D1/SA0+A(5,4,K))
      A(5,1,K)=C2*(A(5,4,K)+SC9*(A(1,1,K)-A(5,5,K))/C5)/C6
      A(6,5,K)=SA0*(A(5,5,K)-A(6,6,K))/C9
      A(6,4,K)=SA0*SC9*(A(4,4,K)-A(6,6,K))/
     1C10-SC9*A(6,5,K)/C4
      A(6,3,K)=-C3*(SC9*(D2/SA1+A(6,5,K)/SA0)+A(6,4,K))
      A(6,1,K)=SA0*SC9*C2*((A(4,4,K)-A(6,6,K))/C10+
     1(A(1,1,K)-A(6,6,K))/C11)/C6-SC9*C2*A(6,5,K)*C18/C6
      A(7,6,K)=-2.0*SF1*D2/SA1
      A(7,5,K)=2.0*SF1*SA0*(D2/SA1-D1/SA0)/C9
      A(7,4,K)=2.0*SF9*D3*(1.0+SA0*SC9/C10)/C1+2.0*SF1*
     1SC9*D1/(C9*C4)-A(7,6,K)*SA0*SC9*(1.0/C7-1.0/C9)/C4
      A(7,3,K)=T(K)*(C12*SF9+C13/SA1)-D3*(C12*SF9+
     1C14/C10)/C1+C13*(1.0/SA0-1.0/C4)*(D1-D2*SA0/
     2SA1)/C9+D2*C14*(1.0/(SA1*SA0)-1.0/C10)/SA1
      A(7,1,K)=D3*(C15*SF9+C17/C10)/(C1*C6)+
     1(A(1,1,K)-1.0)*(C15*SF9/C6-SF5*2.0-
     2C17/(C11*C6))/SA5+C17*C18*(D1/SA0-
     3D2/SA1)/(C6*C9)+C17*D2*(1.0/C10+1.0/
     4C11)/(SA1*C6)
   40 CONTINUE
      A(7,7,1)=0.0
      RETURN
      END
```

```
      SUBROUTINE WTOUT(I1,J1,I2,J2,J3)
      COMMON /Z3/MM,A(7,7,3),B(63,29)
   15 FORMAT(T5,'COL',I4,T15,'ROW',I4,T25,
     1E12.5,T40,'ROW',I4,T50,E12.5)
   16 FORMAT(T5,'COL',I4,T15,'ROW',I4,T25,E12.5)
      DO 50 J=J3,J2
      JC=J+J1
      I=1
   51 IF(B(I,J).EQ.0.0)GO TO 53
      IR1=I+I1
      RV1=B(I,J)
   52 IF(I.GE.I2)GO TO 54
      I=I+1
      IF (B(I,J).EQ.0.0)GO TO 52
      IR2=I+I1
      RV2=B(I,J)
      WRITE(MM,15)JC,IR1,RV1,IR2,RV2
   53 IF(I.GE.I2)GO TO 50
      I=I+1
      GO TO 51
   54 WRITE(MM,16)JC,IR1,RV1
   50 CONTINUE
      RETURN
      END
```

```
MPS-PTF4        CONTROL PROGRAM COMPILER - MPS/360 V2-M10

        0001                     PROGRAM
        0002                     INITIALZ
        0065                     MOVE(XDATA,'YFC')
        0066                     MOVE(XPBNAME,'PBFILE')
        0067                     CONVERT('SUMMARY')
        0068                     SETUP
        0069                     MOVE(XOBJ,'RCW1000')
        0070                     MOVE(XRHS,'LIMITS')
        0071                     PRIMAL
        0072                     SOLUTION
        0073                     EXIT
        0074          SV1         DC(0.0)
        0075          SV2         DC(0.0)
        0076                     PEND
```

```
MPS-PTF4        EXECUTOR.    MPS/360 V2-M10

SOLUTION   (OPTIMAL)

TIME =   3.47 MINS. ITERATION NUMBER =   174

         ...NAME...     ...ACTIVITY...       DEFINED AS

         FUNCTIONAL       5087.19277         ROW1000
         RESTRAINTS                          LIMITS
```

SECTION 1 - ROWS

	NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
	1	ROW1000	BS	5087.19277	5087.19277-	NONE	NONE	1.00000
	2	ROW1001	EQ	.	.	.	.	.26520-
	3	ROW1002	EQ	9.32270	.	9.32270	9.32270	34.37134
	4	ROW1003	EQ	282.01000	.	282.01000	282.01000	.67534
	5	ROW1004	EQ	7.86600	.	7.86600	7.86600	16.27750-
	6	ROW1005	EQ	283.47000	.	283.47000	283.47000	.08572
	7	ROW1006	EQ	6.40940	.	6.40940	6.40940	20.42233
	8	ROW1007	EQ	284.92000	.	284.92000	284.92000	.26840
	9	ROW1008	EQ	.	.	.	.	.49073-
A	10	ROW1009	EQ	.	.	.	.	.
	11	ROW1010	EQ	.	.	.	.	.06610-
	12	ROW1011	EQ	.	.	.	.	.37482-
	13	ROW1012	EQ	.	.	.	.	.45348-
	14	ROW1013	EQ	.	.	.	.	.17365
A	15	ROW1014	EQ	.	.	.	.	.
	16	ROW1015	EQ	.	.	.	.	.27153
A	17	ROW1016	EQ	.	.	.	.	.
	18	ROW1017	EQ	.	.	.	.	.00111-
	19	ROW1018	EQ	.	.	.	.	.21109
	20	ROW1019	EQ	.	.	.	.	.22454-
	21	ROW1020	EQ	.	.	.	.	.44057
A	22	ROW1021	EQ	.	.	.	.	.
	23	ROW1022	EQ	.	.	.	.	.09511-
A	24	ROW1023	EQ	.	.	.	.	.
	25	ROW1024	EQ	.	.	.	.	.00016-
	26	ROW1025	EQ	.	.	.	.	1.49164-
	27	ROW1026	EQ	.	.	.	.	.15424-
	28	ROW1027	EQ	.	.	.	.	1.31683-
A	29	ROW1028	EQ	.	.	.	.	.
	30	ROW1029	BS	2112.57061	.07061-	2112.50000	NONE	.
	31	ROW1030	BS	26581.19555	16418.80445	NONE	43000.00000	.
	32	ROW1031	BS	419.99971	55.33971-	364.66000	NONE	.
	33	ROW1032	BS	401.55217	31.55217-	370.00000	NONE	.
	34	ROW1033	EQ	.	.	.	.	1.35724-
A	35	ROW1034	EQ	.	.	.	.	.
	36	ROW1035	EQ	.	.	.	.	.00614-
	37	ROW1036	EQ	.	.	.	.	2.11384-
	38	ROW1037	EQ	.	.	.	.	.46958-
	39	ROW1038	EQ	.	.	.	.	1.54490-
	40	ROW1039	EQ	.	.	.	.	.49318
	41	ROW1040	EQ	.	.	.	.	1.15666-
A	42	ROW1041	EQ	.	.	.	.	.
	43	ROW1042	EQ	.	.	.	.	.00363-
	44	ROW1043	EQ	.	.	.	.	2.04704-
	45	ROW1044	EQ	.	.	.	.	.45813-
	46	ROW1045	EQ	.	.	.	.	1.49982-
	47	ROW1046	EQ	.	.	.	.	.49318
	48	ROW1047	EQ	.	.	.	.	.08254-
A	49	ROW1048	EQ	.	.	.	.	.

	NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
	50	ROW1049	EQ	.	.	.	.	.00058-
	51	ROW1050	EQ	.	.	.	.	.79013-
	52	ROW1051	EQ	.	.	.	.	.32198-
	53	ROW1052	EQ	.	.	.	.	.39996-
	54	ROW1053	EQ	.	.	.	.	.49318
	55	ROW1054	EQ	.	.	.	.	.95113-
A	56	ROW1055	EQ	.	.	.	.	.
	57	ROW1056	EQ	.	.	.	.	.00162-
	58	ROW1057	EQ	.	.	.	.	1.49164-
	59	ROW1058	EQ	.	.	.	.	.15424-
	60	ROW1059	EQ	.	.	.	.	1.31683-
	61	ROW1060	EQ	.	.	.	.	.05048-
	62	ROW1061	EQ	2112.50000	.	2112.50000	2112.50000	.49318
	63	ROW1062	BS	36921.32321	6078.67679	NONE	43000.00000	.
	64	ROW1063	BS	440.46990	75.80990-	364.66000	NONE	.
	65	ROW1064	BS	401.55597	31.55597-	370.00000	NONE	.
	66	ROW1065	EQ	.	.	.	.	.15786-
A	67	ROW1066	EQ	.	.	.	.	.
	68	ROW1067	EQ	.	.	.	.	.00006-
	69	ROW1068	EQ	.	.	.	.	.28402-
	70	ROW1069	EQ	.	.	.	.	.01354-
	71	ROW1070	EQ	.	.	.	.	.27566-
	72	ROW1071	EQ	.	.	.	.	1.33796-
	73	ROW1072	EQ	.	.	.	.	.43055-
A	74	ROW1073	EQ	.	.	.	.	.
	75	ROW1074	EQ	.	.	.	.	.00276-
	76	ROW1075	EQ	.	.	.	.	1.11302-
	77	ROW1076	EQ	.	.	.	.	.29108-
	78	ROW1077	EQ	.	.	.	.	.70562-
	79	ROW1078	EQ	.	.	.	.	1.33796-
	80	ROW1079	EQ	.	.	.	.	.08775-
A	81	ROW1080	EQ	.	.	.	.	.
	82	ROW1081	EQ	.	.	.	.	.00040-
	83	ROW1082	EQ	.	.	.	.	.91268-
	84	ROW1083	EQ	.	.	.	.	.29721-
	85	ROW1084	EQ	.	.	.	.	.47771-
	86	ROW1085	EQ	.	.	.	.	1.33796-
	87	ROW1086	EQ	.	.	.	.	.82537-
A	88	ROW1087	EQ	.	.	.	.	.
	89	ROW1088	EQ	.	.	.	.	.00577-
	90	ROW1089	EQ	.	.	.	.	.79013-
	91	ROW1090	EQ	.	.	.	.	.32198-
	92	ROW1091	EQ	.	.	.	.	.39996-
	93	ROW1092	EQ	.	.	.	.	.18040-
	94	ROW1093	EQ	2112.50000	.	2112.50000	2112.50000	1.33796-
	95	ROW1094	BS	31025.85955	11974.14045	NONE	43000.00000	.
	96	ROW1095	BS	450.69127	86.03127-	364.66000	NONE	.
	97	ROW1096	BS	401.60426	31.60426-	370.00000	NONE	.
	98	ROW1097	EQ	.	.	.	.	.83970-
A	99	ROW1098	EQ	.	.	.	.	.
	100	ROW1099	EQ	.	.	.	.	.00452-

	NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
	101	ROW1100	EQ	.	.	.	.	.80002-
	102	ROW1101	EQ	.	.	.	.	.31141-
	103	ROW1102	EQ	.	.	.	.	.35427-
	104	ROW1103	EQ	.	.	.	.	.30831-
	105	ROW1104	EQ	.	.	.	.	.07587
A	106	ROW1105	EQ	.	.	.	.	.
	107	ROW1106	EQ	.	.	.	.	.00013
	108	ROW1107	EQ	.	.	.	.	.11902
	109	ROW1108	EQ	.	.	.	.	.01239
	110	ROW1109	EQ	.	.	.	.	.10501
	111	ROW1110	EQ	.	.	.	.	.30831-
	112	ROW1111	EQ	.	.	.	.	.09598-
A	113	ROW1112	EQ	.	.	.	.	.
	114	ROW1113	EQ	.	.	.	.	.00013-
	115	ROW1114	EQ	.	.	.	.	1.58433-
	116	ROW1115	EQ	.	.	.	.	.13071-
	117	ROW1116	EQ	.	.	.	.	1.45031-
	118	ROW1117	EQ	.	.	.	.	.30831-
	119	ROW1118	EQ	.	.	.	.	.97752-
A	120	ROW1119	EQ	.	.	.	.	.
	121	ROW1120	EQ	.	.	.	.	.00405-
	122	ROW1121	EQ	.	.	.	.	.91268-
	123	ROW1122	EQ	.	.	.	.	.29721-
	124	ROW1123	EQ	.	.	.	.	.47771-
	125	ROW1124	EQ	.	.	.	.	.12653-
	126	ROW1125	EQ	2112.50000	.	2112.50000	2112.50000	.30831-
	127	ROW1126	BS	28410.41282	14589.58718	NONE	43000.00000	.
	128	ROW1127	BS	451.56824	86.90824-	364.66000	NONE	.
	129	ROW1128	BS	401.55017	31.55017-	370.00000	NONE	.
	130	ROW1129	EQ	.	.	.	.	1.15243-
A	131	ROW1130	EQ	.	.	.	.	.
	132	ROW1131	EQ	.	.	.	.	.00192-
	133	ROW1132	EQ	.	.	.	.	1.92795-
	134	ROW1133	EQ	.	.	.	.	.18154-
	135	ROW1134	EQ	.	.	.	.	1.62839-
	136	ROW1135	EQ	.	.	.	.	.10369
	137	ROW1136	EQ	.	.	.	.	1.18488-
A	138	ROW1137	EQ	.	.	.	.	.
	139	ROW1138	EQ	.	.	.	.	.00203-
	140	ROW1139	EQ	.	.	.	.	1.86721-
	141	ROW1140	EQ	.	.	.	.	.19075-
	142	ROW1141	EQ	.	.	.	.	1.65375-
	143	ROW1142	EQ	.	.	.	.	.10369
A	144	ROW1143	EQ	.	.	.	.	.
A	145	ROW1144	EQ	.	.	.	.	.
A	146	ROW1145	EQ	.	.	.	.	.
A	147	ROW1146	EQ	.	.	.	.	.
A	148	ROW1147	EQ	.	.	.	.	.
A	149	ROW1148	EQ	.	.	.	.	.
	150	ROW1149	EQ	.	.	.	.	.10369
	151	ROW1150	EQ	.	.	.	.	.95977-

	NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
A	152	ROW1151	EQ	.	.	.	.	.
	153	ROW1152	EQ	.	.	.	.	.00133-
	154	ROW1153	EQ	.	.	.	.	1.58433-
	155	ROW1154	EQ	.	.	.	.	.13071-
	156	ROW1155	EQ	.	.	.	.	1.46031-
	157	ROW1156	EQ	.	.	.	.	.04156-
	158	ROW1157	EQ	2112.50000	.	2112.50000	2112.50000	.10369
	159	ROW1158	BS	31782.55510	11217.44490	NONE	43000.00000	.
	160	ROW1159	BS	449.79546	85.13546-	364.66000	NONE	.
	161	ROW1160	BS	402.87031	32.87031-	370.00000	NONE	.
A	162	ROW1161	EQ	.	.	.	.	.
A	163	ROW1162	EQ	.	.	.	.	.
A	164	ROW1163	EQ	.	.	.	.	.
A	165	ROW1164	EQ	.	.	.	.	.
A	166	ROW1165	EQ	.	.	.	.	.
A	167	ROW1166	EQ	.	.	.	.	.
	168	ROW1167	EQ	.	.	.	.	1.64963-
A	169	ROW1168	EQ	.	.	.	.	.
A	170	ROW1169	EQ	.	.	.	.	.
A	171	ROW1170	EQ	.	.	.	.	.
A	172	ROW1171	EQ	.	.	.	.	.
A	173	ROW1172	EQ	.	.	.	.	.
A	174	ROW1173	EQ	.	.	.	.	.
	175	ROW1174	EQ	.	.	.	.	1.64963-
A	176	ROW1175	EQ	.	.	.	.	.
A	177	ROW1176	EQ	.	.	.	.	.
A	178	ROW1177	EQ	.	.	.	.	.
A	179	ROW1178	EQ	.	.	.	.	.
A	180	ROW1179	EQ	.	.	.	.	.
A	181	ROW1180	EQ	.	.	.	.	.
	182	ROW1181	EQ	.	.	.	.	1.64963-
	183	ROW1182	EQ	2112.50000	.	2112.50000	2112.50000	1.64963-
	184	ROW1183	BS	29192.99165	13807.00835	NONE	43000.00000	.
	185	ROW1184	BS	452.55546	87.89546-	364.66000	NONE	.
	186	ROW1185	BS	401.87935	31.87935-	370.00000	NONE	.

SECTION 2 - COLUMNS

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
187	COL1001	BS	932.27000	.	.	NONE	.
188	COL1002	BS	28201.00000	.	.	NONE	.
189	COL1003	BS	786.60000	.	.	NONE	.
190	COL1004	BS	28347.00000	.	.	NONE	.
191	COL1005	BS	640.94000	.	.	NONE	.
192	COL1006	BS	28492.00000	.	.	NONE	.
193	COL1007	BS	1294.91708	1.00000	.	NONE	.
194	COL1008	BS	39171.24164	.	.	NONE	.
195	COL1009	BS	568.62976	.	.	NONE	.
196	COL1010	BS	62.11995	.	.	NONE	.
197	COL1011	BS	28142.62393	.	.	NONE	.
198	COL1012	BS	125.16925	.	.	NONE	.
199	COL1013	BS	29.83409	.	.	NONE	.
200	COL1014	BS	1.71534	.	.	NONE	.
201	COL1015	BS	727.93400	.	.	NONE	.
202	COL1016	BS	429.27122	.	.	NONE	.
203	COL1017	BS	60.99532	.	.	NONE	.
204	COL1018	BS	28274.99862	.	.	NONE	.
205	COL1019	BS	121.32059	.	.	NONE	.
206	COL1020	BS	36.17460	.	.	NONE	.
207	COL1021	BS	2.43226	.	.	NONE	.
208	COL1022	BS	747.45324	.	.	NONE	.
209	COL1023	BS	343.09518	.	.	NONE	.
210	COL1024	BS	50.63359	.	.	NONE	.
211	COL1025	BS	28417.35096	.	.	NONE	.
212	COL1026	BS	108.59597	.	.	NONE	.
213	COL1027	BS	33.33410	.	.	NONE	.
214	COL1028	BS	2.28963	.	.	NONE	.
215	COL1029	BS	637.18337	.	.	NONE	.
216	COL1030	BS	991.53958	1.00000	.	NONE	.
217	COL1031	BS	1025.84910	.	.	NONE	.
218	COL1032	BS	5.08336	.	.	NONE	.
219	COL1033	BS	32835.80740	.	.	NONE	.
220	COL1034	BS	108.59597	.	.	NONE	.
221	COL1035	BS	33.33410	.	.	NONE	.
222	COL1036	BS	2.28963	.	.	NONE	.
223	COL1037	BS	29994.07230	.	.	NONE	.
224	COL1038	BS	835.73948	.	.	NONE	.
225	COL1039	BS	78.47068	.	.	NONE	.
226	COL1040	BS	39099.16655	.	.	NONE	.
227	COL1041	BS	164.28386	.	.	NONE	.
228	COL1042	BS	34.21504	.	.	NONE	.
229	COL1043	BS	1.78305	.	.	NONE	.
230	COL1044	BS	903.12247	.	.	NONE	.
231	COL1045	BS	332.55685	.	.	NONE	.
232	COL1046	BS	102.04872	.	.	NONE	.
233	COL1047	BS	28079.30303	.	.	NONE	.
234	COL1048	BS	145.46342	.	.	NONE	.
235	COL1049	BS	81.15458	.	.	NONE	.

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
236	COL1050	BS	7.90306	.	.	NONE	.
237	COL1051	BS	1381.98507	.	.	NONE	.
238	COL1052	BS	246.80519	.	.	NONE	.
239	COL1053	BS	91.76092	.	.	NONE	.
240	COL1054	BS	28209.40062	.	.	NONE	.
241	COL1055	BS	130.74888	.	.	NONE	.
242	COL1056	BS	82.97673	.	.	NONE	.
243	COL1057	BS	8.64100	.	.	NONE	.
244	COL1058	BS	1302.77970	.	.	NONE	.
245	COL1059	BS	898.26074	1.00000	.	NONE	.
246	COL1060	BS	922.94126	.	.	NONE	.
247	COL1061	BS	9.17609	.	.	NONE	.
248	COL1062	BS	29993.32732	.	.	NONE	.
249	COL1063	BS	130.74888	.	.	NONE	.
250	COL1064	BS	82.97673	.	.	NONE	.
251	COL1065	BS	8.64100	.	.	NONE	.
252	COL1066	BS	27172.33726	.	.	NONE	.
253	COL1067	BS	687.55484	.	.	NONE	.
254	COL1068	BS	62.98581	.	.	NONE	.
255	COL1069	BS	32780.64324	.	.	NONE	.
256	COL1070	BS	181.44901	.	.	NONE	.
257	COL1071	BS	76.30091	.	.	NONE	.
258	COL1072	BS	6.78137	.	.	NONE	.
259	COL1073	BS	772.68456	.	.	NONE	.
260	COL1074	BS	511.90715	.	.	NONE	.
261	COL1075	BS	133.34429	.	.	NONE	.
262	COL1076	BS	39018.62227	.	.	NONE	.
263	COL1077	BS	202.52854	.	.	NONE	.
264	COL1078	BS	99.70663	.	.	NONE	.
265	COL1079	BS	9.08361	.	.	NONE	.
266	COL1080	BS	1763.62269	.	.	NONE	.
267	COL1081	BS	200.54508	.	.	NONE	.
268	COL1082	BS	123.99802	.	.	NONE	.
269	COL1083	BS	28019.77490	.	.	NONE	.
270	COL1084	BS	132.36838	.	.	NONE	.
271	COL1085	BS	119.35695	.	.	NONE	.
272	COL1086	BS	14.47851	.	.	NONE	.
273	COL1087	BS	1861.30029	.	.	NONE	.
274	COL1088	BS	1015.83972	1.00000	.	NONE	.
275	COL1089	BS	1035.89423	.	.	NONE	.
276	COL1090	BS	12.39980	.	.	NONE	.
277	COL1091	BS	33531.12906	.	.	NONE	.
278	COL1092	BS	132.36838	.	.	NONE	.
279	COL1093	BS	119.35695	.	.	NONE	.
280	COL1094	BS	14.47851	.	.	NONE	.
281	COL1095	BS	30729.15157	.	.	NONE	.
282	COL1096	BS	631.32874	.	.	NONE	.
283	COL1097	BS	58.98830	.	.	NONE	.
284	COL1098	BS	29945.63793	.	.	NONE	.
285	COL1099	BS	179.62496	.	.	NONE	.
286	COL1100	BS	118.71647	.	.	NONE	.

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
287	COL1101	BS	13.57478	.	.	NONE	.
288	COL1102	BS	704.89045	.	.	NONE	.
289	COL1103	BS	431.79819	.	.	NONE	.
290	COL1104	BS	106.24446	.	.	NONE	.
291	COL1105	BS	32716.72098	.	.	NONE	.
292	COL1106	BS	188.31202	.	.	NONE	.
293	COL1107	BS	130.28850	.	.	NONE	.
294	COL1108	BS	14.34711	.	.	NONE	.
295	COL1109	BS	1511.21660	.	.	NONE	.
296	COL1110	BS	316.76302	.	.	NONE	.
297	COL1111	BS	165.93920	.	.	NONE	.
298	COL1112	BS	38940.19484	.	.	NONE	.
299	COL1113	BS	189.87428	.	.	NONE	.
300	COL1114	BS	154.17479	.	.	NONE	.
301	COL1115	BS	17.91239	.	.	NONE	.
302	COL1116	BS	2432.70020	.	.	NONE	.
303	COL1117	BS	886.63565	1.00000	.	NONE	.
304	COL1118	BS	918.31195	.	.	NONE	.
305	COL1119	BS	16.59392	.	.	NONE	.
306	COL1120	BS	30714.74788	.	.	NONE	.
307	COL1121	BS	189.87428	.	.	NONE	.
308	COL1122	BS	154.17479	.	.	NONE	.
309	COL1123	BS	17.91239	.	.	NONE	.
310	COL1124	BS	26820.72940	.	.	NONE	.
311	COL1125	BS	704.58418	.	.	NONE	.
312	COL1126	BS	68.98177	.	.	NONE	.
313	COL1127	BS	33476.80863	.	.	NONE	.
314	COL1128	BS	194.27967	.	.	NONE	.
315	COL1129	BS	152.92397	.	.	NONE	.
316	COL1130	BS	19.23291	.	.	NONE	.
317	COL1131	BS	794.24878	.	.	NONE	.
318	COL1132	BS	393.74079	.	.	NONE	.
319	COL1133	BS	99.25636	.	.	NONE	.
320	COL1134	BS	29886.34557	.	.	NONE	.
321	COL1135	BS	178.36438	.	.	NONE	.
322	COL1136	BS	163.17545	.	.	NONE	.
323	COL1137	BS	20.49511	.	.	NONE	.
324	COL1138	BS	1419.25624	.	.	NONE	.
325	COL1139	BS	265.28385	.	.	NONE	.
326	COL1140	BS	134.08955	.	.	NONE	.
327	COL1141	BS	32649.97887	.	.	NONE	.
328	COL1142	BS	167.25427	.	.	NONE	.
329	COL1143	BS	172.40480	.	.	NONE	.
330	COL1144	BS	22.07106	.	.	NONE	.
331	COL1145	BS	2115.10203	.	.	NONE	.
332	COL1146	BS	629.62222	.	.	NONE	.
333	COL1147	BS	65.87483	.	.	NONE	.
334	COL1148	BS	30665.21858	.	.	NONE	.
335	COL1149	BS	211.11831	.	.	NONE	.
336	COL1150	BS	191.91829	.	.	NONE	.
337	COL1151	BS	24.29389	.	.	NONE	.

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
338	COL1152	BS	769.47763	.	.	NONE	.
339	COL1153	BS	443.75416	.	.	NONE	.
340	COL1154	BS	113.17769	.	.	NONE	.
341	COL1155	BS	33411.86362	.	.	NONE	.
342	COL1156	BS	196.70098	.	.	NONE	.
343	COL1157	BS	196.46417	.	.	NONE	.
344	COL1158	BS	25.81293	.	.	NONE	.
345	COL1159	BS	1576.45057	.	.	NONE	.
346	COL1160	BS	244.35947	.	.	NONE	.
347	COL1161	BS	124.22797	.	.	NONE	.
348	COL1162	BS	29826.57238	.	.	NONE	.
349	COL1163	BS	156.01548	.	.	NONE	.
350	COL1164	BS	194.38198	.	.	NONE	.
351	COL1165	BS	26.66189	.	.	NONE	.
352	COL1166	BS	1980.07683	.	.	NONE	.

APPENDIX B.2

The Fluence Optimization Code Listing

```
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION X(5,5),Y(5,5),R(5,5),YY(5,5),P(5),XX(5,5),Q(5)
  100 FORMAT(T10,'CHECK MATRIX INVERSION')
  101 FORMAT(T10,5F15.9)
  102 FORMAT(T10,'VECTOR NEED TO FEED IN')
  103 FORMAT(13X,5F13.8)
  104 FORMAT(T10,'INPUT MATRIX')
  105 FORMAT(T10,'OUTPUT MATRIX')
  106 FORMAT(T10,'VECTOR DESIRED AT OUTPUT')
  107 FORMAT(T10,'INVERTED MATRIX')
  108 FORMAT(T10,'VECTOR OBTAINED AT OUTPUT')
  109 FORMAT(1H1,//////////)
      KM=1
      CO=401.55217
      WRITE(6,109)
      DO 9 I=1,5
    9 READ(5,103)X(1,I),X(2,I),X(3,I),X(4,I),X(5,I)
      CALL MATINV(X,XX)
      DO 11 IM=1,KM
      DO 10 I=1,5
   10 READ(5,103)Y(1,I),Y(2,I),Y(3,I),Y(4,I),Y(5,I)
      WRITE(6,104)
      DO 12 I=1,5
   12 WRITE(6,101)X(I,1),X(I,2),X(I,3),X(I,4),X(I,5)
      WRITE(6,105)
      DO 13 I=1,5
   13 WRITE(6,101)Y(I,1),Y(I,2),Y(I,3),Y(I,4),Y(I,5)
      CALL MATINV(Y,YY)
      WRITE(6,107)
      DO 14 I=1,5
   14 WRITE(6,101)YY(I,1),YY(I,2),YY(I,3),YY(I,4),YY(I,5)
      DO 20 I=1,5
      DO 20 J=1,5
      R(I,J)=0.D0
      DO 21 K=1,5
   21 R(I,J)=R(I,J)+Y(I,K)*YY(K,J)
   20 CONTINUE
      WRITE(6,100)
      DO 22 I=1,5
   22 WRITE(6,101)R(I,1),R(I,2),R(I,3),R(I,4),R(I,5)
```

```
      DO 24 I=1,5
      DO 24 J=1,5
      R(I,J)=0.D0
      DO 25 K=1,5
   25 R(I,J)=R(I,J)+X(I,K)*YY(K,J)
   24 CONTINUE
      DO 26 I=1,5
      P(I)=0.D0
      DO 27 J=1,5
   27 P(I)=P(I)+R(I,J)
   26 P(I)=P(I)*CO
      WRITE(6,102)
      WRITE(6,101)P(1),P(2),P(3),P(4),P(5)
      DO 29 I=1,5
      DO 29 J=1,5
      R(I,J)=0.D0
      DO 30 K=1,5
   30 R(I,J)=R(I,J)+Y(I,K)*XX(K,J)
   29 CONTINUE
      DO 31 I=1,5
      Q(I)=0.D0
      DO 32 J=1,5
   32 Q(I)=Q(I)+R(I,J)*P(J)
   31 CONTINUE
      WRITE(6,106)
      WRITE(6,101)CO,CO,CO,CO,CO
      WRITE(6,108)
      WRITE(6,101)Q(1),Q(2),Q(3),Q(4),Q(5)
   11 CONTINUE
      WRITE(6,109)
      STOP
      END

      SUBROUTINE MATINV(A,AINV)
      IMPLICIT REAL*8(A-H,O-Z),INTEGER(I-N)
      DIMENSION A(5,5),AINV(5,5),D(5,10)
      N=5
      N2=N*2
      DO 10 I=1,N
      DO 10 J=1,N
   10 D(I,J)=A(I,J)
```

```
      DO 20 I=1,N
      DO 21 J=1,N
   21 D(I,N+J)=0.0
   20 D(I,N+I)=1.0
   54 DO 30 J=1,N
      DIV=D(J,J)
      IF(DIV.EQ.0.0)GO TO 50
      S=I.0/DIV
      DO 31 K=J,N2
   31 D(J,K)=D(J,K)*S
      DO 30 I=1,N
      IF(I.EQ.J)GO TO 30
      DIJ=-D(I,J)
      DO 32 K=J,N2
   32 D(I,K)=D(I,K)+DIJ*D(J,K)
   30 CONTINUE
      DO 61 I=1,N
      DO 61 J=1,N
      J1=N+J
   61 AINV(I,J)=D(I,J1)
      GO TO 53
   50 M=J
   51 M=M+1
      IF(M.GT.N)GO TO 53
      IF(D(M,J).NE.0.0)GO TO 52
      GO TO 51
   52 DO 55 L=J,N2
      DUM=D(J,L)
      D(J,L)=D(M,L)
   55 D(M,L)=DUM
      GO TO 54
   53 RETURN
      END
```

$ENTRY

INPUT MATRIX

1.062846000	1.062846000	1.062846000	1.062846000	1.062846000
0.983271000	0.970271000	0.983271000	0.983271000	0.983271000
0.946476000	0.946476000	0.925000000	0.946476000	0.946476000
0.980342000	0.980342000	0.980342000	0.960342000	0.980342000
0.979628000	0.979628000	0.979628000	0.979628000	0.995028000

OUTPUT MATRIX

401.555970000	401.555970000	401.555970000	401.555970000	401.555970000
400.879450000	412.370970000	400.879450000	400.879450000	400.879450000
399.254460000	396.251140000	419.908690000	399.254460000	399.254460000
397.682740000	397.280080000	392.145790000	415.571320000	397.682740000
402.964030000	408.232570000	402.102380000	398.167720000	389.647010000

INVERTED MATRIX

0.153927140	-0.137042396	-0.054872145	-0.035767852	0.075091875
-0.086874082	0.087020690	-0.000000000	-0.000000000	0.000000000
-0.060771047	0.012653630	0.048416232	-0.000000000	0.000000000
-0.076127977	0.005875383	0.014986000	0.055901586	0.000000000
0.072336280	0.031492693	-0.008530088	-0.020133734	-0.075091875

CHECK MATRIX INVERSION

1.000000000	0.000000000	0.000000000	0.000000000	-0.000000000
0.000000000	1.000000000	-0.000000000	-0.000000000	-0.000000000
0.000000000	-0.000000000	1.000000000	-0.000000000	-0.000000000
0.000000000	-0.000000000	0.000000000	1.000000000	-0.000000000
0.000000000	-0.000000000	-0.000000000	-0.000000000	1.000000000

VECTOR NEED TO FEED IN

1.062835942	0.982496376	0.943890144	0.975233072	0.980071865

VECTOR DESIRED AT OUTPUT

401.552170000	401.552170000	401.552170000	401.552170000	401.552170000

VECTOR OBTAINED AT OUTPUT

401.552170000	401.552170000	401.552170000	401.552170000	401.552170000

A NON-EQUILIBRIUM LWR FUEL CYCLE MODEL

by

YUH-FUU CHEN

B.S., National Tsing Hua University, 1969

AN ABSTRACT OF A MASTER'S THESIS

submitted in partial fulfillment of the

requirement for the degree

MASTER OF SCIENCE

Department of Nuclear Engineering

KANSAS STATE UNIVERSITY

Manhattan, Kansas

1973

ABSTRACT

A model was developed to simulate a large scale power PWR fuel cycle of six, consecutive, one year irradiation periods.

The reference reactor was assumed to have a three concentrical equivolume region core with the refueling scheme being an "Out-In" batch irradiation. Linear programming was used to minimize the total amount of additional "clean uranium" required for the six irradiation periods. A technique was developed to obtain the fluence for each year so that the optimal solution would satisfy both the energy release and the reactivity constraints.

A fuel burnup code was used which, by use of a given known fluence value, would correlate the beginning of life and end of life nuclide concentrations for the seven species: ^{235}U, ^{236}U, ^{238}U, ^{239}Pu, ^{240}Pu, ^{241}Pu, and fission products.

The effect of recycling plutonium and spent uranium was studied which revealed that the value of ^{239}Pu is essentially independent of the percentage of Pu recycled. However, the quality of plutonium decreases when the percentage of Pu recycled increases.