Flexural performance enhancement of existing reinforced concrete beams using CFRP

by

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Abstract

Reinforced concrete structures often require strengthening or retrofitting due to aging structural elements, revisions to design codes, or increases in service loads. This report evaluates the use of externally bonded carbon fiber reinforced polymer (CFRP) systems for strengthening reinforced concrete beams with inadequate flexural capacity resulting from increased live loading. The design theory, step-by-step strengthening procedure, and installation methodology are presented in accordance with current design provisions. In addition, a comparative evaluation of structural performance and cost between externally bonded CFRP systems and externally bonded steel plate strengthening is conducted.

Results indicate that the addition of externally bonded CFRP significantly increases the flexural capacity of the reinforced concrete section, enabling the strengthened member to satisfy updated load demands. Externally bonded steel plates also provide an effective increase in flexural strength and offer greater fabrication flexibility, as steel plates can be manufactured to specified dimensions prior to installation, whereas CFRP materials are supplied in standardized roll widths and lengths that require field cutting. Although the steel plate method demonstrated lower initial cost, CFRP strengthening offers several performance advantages, including corrosion resistance, minimal increase in dead load, high strength-to-weight ratio, and ease of installation. However, CFRP systems also present certain limitations, such as higher initial material cost, brittle failure behavior, and reduced familiarity within segments of the construction industry.

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Chapter 1 – Introduction and Background

Reinforced concrete structures need to be strengthened or retrofitted for several reasons, including insufficient structural elements (beams, slabs, columns, etc.), changes in design codes and standards, and/or increased service loads, and deterioration of structural elements exposed to an aggressive environment. Traditional strengthening or retrofitting methods consist of externally bonded steel plates, steel jacketing, concrete section enlargement, or external post-tensioning. Adding steel or reinforced concrete to an existing structure increases the dead load and typically requires labor-intensive installation methods and heavy equipment. Concrete and steel are both susceptible to environmental factors which can cause corrosion and long-term durability issues. Since the mid-1980s, externally bonded fiber-reinforced polymers (FRP) have emerged as a promising method for the strengthening and retrofitting of concrete structures. Some of the earliest experimental work for retrofitting concrete structures as an alternative to steel was reported as early as 1978 in Germany (ACI Committee 440, 2023). What began as a few projects has evolved into a globally recognized practice, utilized in thousands of structural projects today. The implementation of FRP systems continues to expand as they become a standardized solution within the global construction industry.

Construction accounts for 26% of industrial applications of FRP where the automobile industry is leading with 31%. The past twenty years have seen a growing research focus on the use of FRP systems for flexural strengthening of beams. This increased attention is likely driven by an improved understanding of the advantages FRP composites offer for strengthening concrete structures (Jahami & Issa, 2024). While retrofitting and strengthening reinforced concrete beams is one of the focuses for this paper, FRP systems have been used on slabs, columns, walls, and connections, along with various other structural elements. Research was started on FRP due to its lightweight, high strength to weight ratios, and its resistance to corrosion (ACI Committee 440, 2023).

1.1 What is Carbon Fiber Reinforced Polymers (CFRP)

The carbon fiber in CFRP is responsible for the materials strength and stiffness whereas the polymer is used as a protective matrix that acts as a load transfer mechanism. Other benefits include CFRP's high durability, efficient fatigue endurance, and simple installation on reinforced concrete beams (Solahuddin & Yahaya, 2023). The polymeric resins which include primers, adhesives, putty fillers, and saturants are used with FRP systems. Epoxy, vinyl esters, polyesters, and polyurethane are commonly used resin types. Resins that are used by manufacturers are compatible with adhesion to the concrete substrate, adhesion to the FRP composite, and adhesion to the reinforcing fiber. Other factors that are considered are filling ability, resistance to environmental effects, workability, and others (ACI Committee 440, 2023).

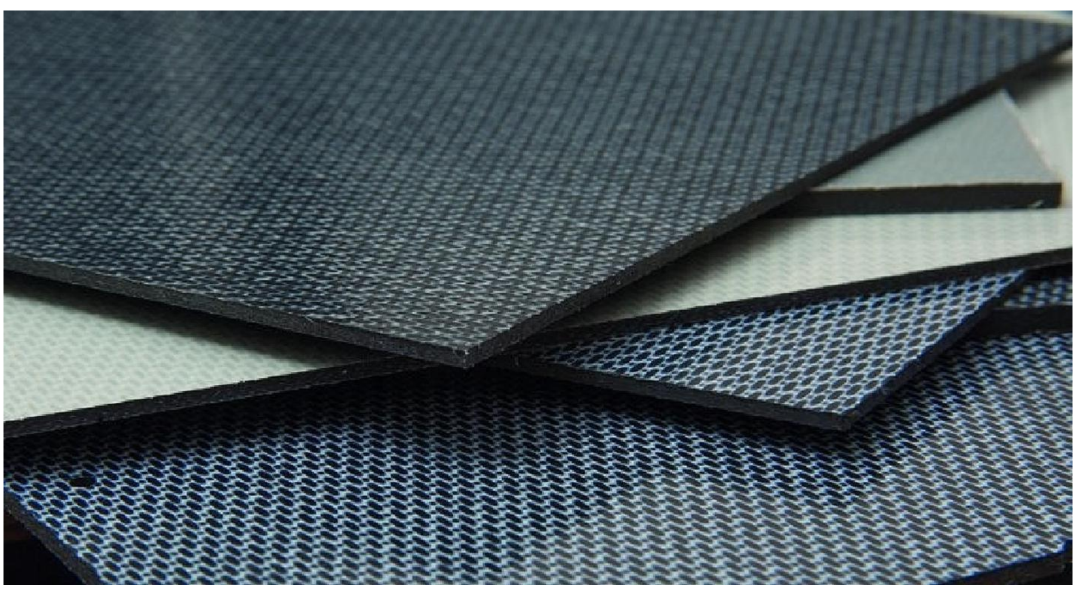

Figure 1. Carbon Fiber Reinforced Polymer (CFRP) (Solahuddin & Yahaya, 2023)

1.2 Tensile Behavior of CFRP

A linear elastic stress-strain relationship up until failure is used to characterize the tensile behavior of FRP systems that are unidirectional and consist of one fiber type. This failure is sudden and brittle. Factors that affect the tensile strength of the FRP is the type of fiber being used, orientation of the fibers, number of fibers being used (layers), and the method and conditions in which the composite system is being manufactured. Tensile properties obtained from laminate testing reflect the true composite behavior of an FRP system by accounting for fiber content, fiber–matrix interaction, fabric architecture, and manufacturing effects, rather than relying solely on idealized fiber material properties. Due to insufficient testing for externally bonded FRP in compression, it should not be used for compression reinforcement (ACI Committee 440, 2023).

1.3 Reinforced Concrete Structures Retrofitted or Strengthened with CFRP

Due to CFRP's many advantageous properties, it has been utilized in many structural engineering projects like reinforced concrete and masonry buildings and bridges to strengthen or retrofit the structures. Many buildings in the United States and other countries have used CFRP to strengthen existing reinforced concrete structures. A notable project was the 333 South Beaudry Ave building in Los Angeles that utilized CFRP to strengthen the floor slabs. A total of 20 floors in the 28-story building were strengthened. One floor was upgraded to accommodate a new data storage facility, while the remaining 19 floors were strengthened to meet the current live load requirements specified in the updated building code (Chambers, n.d.).

Bridge G-270 in Iron County, Missouri, constructed in 1922, is a simple-span reinforced concrete slab bridge that was strengthened using externally bonded CFRP sheets. The retrofit was completed in just three days with minimal impact on traffic, and surface preparation required only light sandblasting. The bridge had been load-posted, meaning a maximum vehicle weight was enforced to prevent overstressing the structure, as it is the only load-posted bridge on a heavy truck route serving a lead mining operation. The strengthening project aimed to increase the bridge's load-carrying capacity and allow the removal of the load posting (Mayo, 2000).

Another project that utilized CFRP for strengthening beams due to increased loading was Marie-Paule-Lanthier Birthing Center in Montreal, Canada, which installed water filled tubs that added substantial loads on the existing concrete structure. CFRP strips were used to ensure long term performance and sufficient load bearing capacity ("All About Structural Strengthening with CFRP," n.d.).

These represent only a small portion of the thousands of projects worldwide that have utilized CFRP systems for strengthening structural elements. While this report focuses on flexural strengthening of reinforced concrete beams, CFRP applications extend far beyond this single use. CFRP systems are commonly employed for shear strengthening, confinement of columns to enhance axial capacity and ductility, and seismic retrofitting.

Chapter 2 – Flexural Strengthening Using CFRP

2.1 Design Theory

The flexural design of reinforced concrete members strengthened with carbon fiber reinforced polymer (CFRP) is based on the principles of strain compatibility, force equilibrium, and material stress-strain relationships. The addition of externally bonded CFRP reinforcement enhances the flexural capacity of a beam by contributing additional tensile resistance and lowering the neutral axis.

Similar to conventional reinforced concrete design, it is assumed that plane sections remain plane, resulting in a linear strain distribution across the depth of the section. Concrete is assumed to carry no tensile stress after cracking, and its compressive behavior is typically represented using an equivalent rectangular stress block. Steel behaves elastically up to its yield point, after which it deforms at approximately constant stress, while CFRP exhibits linear elastic behavior up to failure without yielding.

The strain in the CFRP is governed by its location relative to the neutral axis and is typically limited by either rupture of the CFRP or debonding from the concrete substrate. Because CFRP does not yield, failure is generally brittle. Therefore, design provisions aim to ensure a ductile response by promoting yielding of the internal steel reinforcement before CFRP failure or concrete crushing occurs.

Force equilibrium is established by equating the compressive force in the concrete to the sum of the tensile forces in the steel and CFRP. From this, the neutral axis depth is determined iteratively. Once equilibrium and compatibility are satisfied, the nominal moment capacity is calculated by summing the moments of internal forces about a reference point, typically the location of the concrete compressive force. Positioning the CFRP at the extreme tension fiber (the soffit) maximizes its contribution to the moment capacity by providing the greatest possible lever arm for the tensile force. Below represents the nominal moment capacity equation for a reinforced concrete beam with tension steel and CFRP reinforcement.

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) + A_f f_{fe} \left(d_f - \frac{a}{2}\right) \tag{1}$$

2.2 Design Steps

Step 1: CFRP Design Material Properties

The first step in the flexural strengthening design of a reinforced concrete beam using CFRP is to establish the material properties of the selected system. The manufacturer provides the cured laminate properties, including the ultimate tensile strength and ultimate rupture strain. These values must be modified using the appropriate environmental reduction factor to account for exposure conditions.

For CFRP systems, the environmental reduction factor is 0.95 for interior exposure and 0.85 for exterior or aggressive environments. The reduced design tensile strength and corresponding rupture strain are obtained by multiplying the manufacturer-provided ultimate values by the applicable environmental reduction factor. These adjusted properties are then used in the flexural design calculations. Other fiber types, such as glass, basalt, and aramid, have different environmental reduction factors depending on material and exposure conditions.

$$f_{fu} = C_E f_{fu}^* \tag{2}$$

$$\varepsilon_{fu} = C_E \varepsilon_{fu}^* \tag{3}$$

$$E_f = {f_{fu}}/{\varepsilon_{fu}} = {f_{fu}^*}/{\varepsilon_{fu}^*} \tag{4}$$

Where:

f_{fu} = design ultimate tensile strength of FRP, psi

ε_{fu} = design rupture strain of FRP reinforcement, in./in.

E_f = chord tensile modulus of elasticity of FRP, psi

C_E = environmental reduction factor

* indicates the ultimate tensile strength of the FRP material as reported by the manufacturer and the ultimate rupture strain of the FRP reinforcement

Step 2: Preliminary Calculations

The next step in the design is to determine the properties of the concrete, existing reinforcing steel, and area of FRP reinforcement. It is important to determine the β_1 factor, which is the ratio of depth of equivalent rectangular stress block to depth of the neutral axis. This factor is based on the specified compressive strength of the concrete in accordance with the ACI 318-19(22). The factor adjusts the stress block depth based on the concrete compressive strength. Refer to the table below on how to determine the factor value.

Table 1: ß1 Factor ACI 318-19(22) Table 22.2.2.4.3

f'_c, psi	β_1
$2500 \leq f'_c \leq 4000$	0.85
$4000 < f'_c < 8000$	$0.85 - \frac{0.05(f'_c - 4000)}{1000}$
$f'_c \geq 8000$	0.65

Another property that needs to be calculated is the modulus of elasticity of the concrete. For normal weight concrete, the modulus of elasticity is calculated by the equation below from equation 19.2.2.1.b in the ACI 318-19(22).

$$E_c = 57{,}000\sqrt{f'_c} \tag{5}$$

Where:

E_c = modulus of elasticity of concrete, psi

f'_c = specified compressive strength of concrete, psi

It is necessary to determine the cross-sectional area of both the existing reinforcing steel and the FRP reinforcement. The area of the reinforcing steel is obtained by multiplying the number of bars by the cross-sectional area of an individual bar. Similarly, the area of the FRP reinforcement is calculated by multiplying the number of plies by the thickness and width of the FRP laminate.

Step 3: Existing Strain on Beam Soffit

The third step in the flexural design process is determining the existing strain state at the soffit of the reinforced concrete beam. The soffit refers to the bottom of the beam, which is also the tension zone. It is assumed that the beam is cracked and that only dead load is acting at the time of FRP installation.

Prior to strengthening, the reinforced concrete beam is subjected to sustained service loads, resulting in existing stresses and strains in both the concrete and reinforcing steel. Consequently, the concrete substrate possesses an initial strain state at the time of FRP installation. The externally bonded FRP does not begin at zero strain; rather, it engages from the strain level present in the substrate at the time of application. This initial strain must therefore be subtracted from the total FRP strain when determining the effective strain used in design. The FRP contributes additional tensile capacity only as further loading is applied. Therefore, the strain in the soffit is calculated by the equation below.

$$\varepsilon_{bi} = \frac{M_{DL}(d_f - kd)}{I_{cr}E_c} \tag{6}$$

Where:

ε_{bi} = strain in concrete substrate at time of FRP installation (tension is positive), in./in.

M_{DL} = moment due to dead load, in.-lb

d_f = effective depth of FRP flexural reinforcement, in.

d = distance from extreme compression fiber to centroid of tension reinforcement, in.

k = ratio of depth of neutral axis to reinforcement depth measured from extreme compression fiber

I_{cr} = moment of inertia of cracked section transformed to concrete, in.^{4}

A cracked section analysis is needed to determine the ratio of depth of neutral axis to reinforcement depth measured from the extreme compression fiber *k* and cracked moment of inertia I_{cr}.

Step 4: Design Strain of CFRP

The fourth step in the design process is determining the design strain of the FRP system. The design strain associated with the debonding failure mode is determined using a prescribed relationship, as shown below in Equation (7). The ultimate rupture strain is limited to 0.9 times the value determined in Step One. If the design strain is less than the ultimate rupture strain, debonding controls the design. Debonding may occur at locations other than the section where the externally bonded FRP terminates. This equation is used to prevent an intermediate crack-induced debonding failure mode (ACI Committee 440, 2023).

$$\varepsilon_{fd} = 0.083\sqrt{\frac{f'_c}{NE_f t_f}} \leq 0.9\varepsilon_{fu} \tag{7}$$

Where:

ε_{fd} = debonding strain of externally bonded FRP reinforcement, in./in.

ε_{fu} = design rupture strain of FRP reinforcement, in./in.

N = number of plies of FRP reinforcement

t_f = nominal thickness of one ply of FRP reinforcement

When a single flexural crack governs the behavior, the mechanism of intermediate crack (IC) debonding can be described as follows. Once a flexural crack forms in the beam, the tensile forces released by the cracked concrete are transferred to the FRP plate, generating high local interfacial stresses between the FRP plate and the surrounding concrete near the crack. As the applied loading increases, both the tensile stress in the FRP plate and the interfacial stresses between the FRP and the cracked concrete continue to increase. When these stresses reach critical levels, debonding initiates, or peels away, at the crack and propagates toward the plate ends, typically the nearer end of the externally bonded FRP reinforcement (Teng et al., 2003). Figure 2 below shows the intermediate crack induced debonding failure mode.

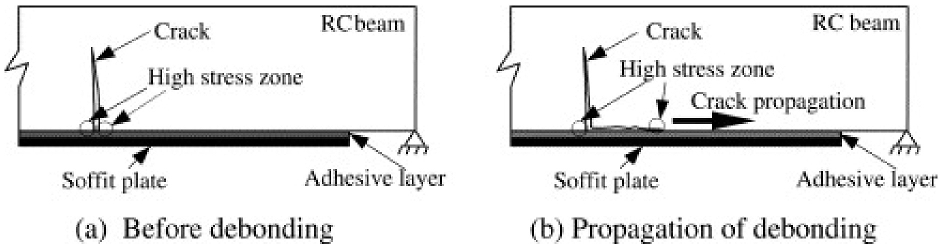

Figure 2. Intermediate crack-induced debonding in a simply-supported FRP-plated RC beam/slab (Teng et al., 2003)

The IC debonding strain Equation (7) above was developed by comparing average FRP strain measurements from flexural tests where intermediate crack-induced debonding was experienced. From this comparison, the coefficient of 0.083 was identified as the best fit (ACI Committee 440, 2023). Intermediate crack induced debonding is a prevalent failure mode in moderately reinforced and moderately strengthened RC beams that have FRP sheets or plates that extend close to the supports of the beam. Preventing this failure can be achieved by the addition of the anchorage systems or keeping the strain in the FRP below the debonding strain (Rasheed, 2014).

U-wraps and fiber anchors have been shown to help control the debonding failure mode of longitudinal FRP and can also improve the flexural strength and deformability of strengthened RC members. However, these anchorage systems do not necessarily prevent local FRP debonding or eliminate the brittle nature of the failure mode. Instead, they may mitigate sudden, complete debonding of the FRP, allowing the FRP to continue resisting force. (ACI Committee 440, 2023).

Step 5: Estimate Neutral Axis Depth

To continue the design process, an estimate of the neutral axis depth, c, is required. For singly reinforced sections, an initial approximation is obtained by taking 20% of the effective depth, d. The effective depth is measured from the extreme compression fiber (top of the RC beam) to the centroid of the combined tensile reinforcement, regardless of whether the reinforcement is arranged in one or multiple layers. It should be noted that this is only an approximate value for the neutral axis depth and may need to be recalculated. The final value will be verified in later design steps.

Step 6: Effective Level of Strain in CFRP

The sixth step is determining the effective level of strain in the FRP reinforcement. This is calculated by using similar triangles with the depth to the neutral axis and the distance to the FRP reinforcement with the subtraction of the strain in the substrate.

$$\varepsilon_{fe} = \varepsilon_{cu}\left(\frac{d_f - c}{c}\right) - \varepsilon_{bi} \leq \varepsilon_{fd} \tag{8}$$

Where:

ε_{fe} = effective strain in FRP reinforcement attained at failure, in./in.

ε_{cu} = ultimate axial strain of unconfined concrete corresponding to $0.85f'_c$ or maximum usable strain of unconfined concrete, in./in., which can occur at $f_c = 0.85f'_c$ or $\varepsilon_c = 0.003$

c = distance from extreme compression fiber to the neutral axis, in.

FRP systems behave in a linear elastic manner until failure; therefore, it is important to determine the strain in the FRP, as this directly dictates the stress developed in the material. The maximum strain that can be achieved in the FRP reinforcement is governed by the controlling failure mode. The strain developed in the FRP may be limited by concrete crushing, FRP rupture, or FRP debonding from the substrate. If the right-hand term controls the inequality, either debonding or FRP rupture will govern the failure mode, depending on what controlled in (6). If the left-hand term controls, concrete crushing governs the failure mode (ACI Committee 440, 2023). Concrete crushing and FRP rupture occurring after yielding of the tension reinforcement are considered desirable failure modes due to the warning signs exhibited by the RC beam prior to failure (Rasheed, 2014). If the FRP controls the inequality, the maximum compressive strain in the concrete cannot be assumed to be 0.003 and must be recalculated. This is necessary because if the maximum compressive strain in the concrete differs from 0.003, the strength and ductility may be overestimated (ACI Committee 440, 2023). The revised concrete strain can be calculated using the equation below.

$$\varepsilon_c = \left(\varepsilon_{fe} + \varepsilon_{bi}\right)\left(\frac{c}{d_f - c}\right) \tag{9}$$

Where:

ε_c = strain in concrete, in./in.

Step 7: Strain in Existing Reinforcing Steel

Calculating the strain in the existing reinforcing steel is the seventh step in the design process. Like the previous step used to determine the updated concrete strain, the strain in the reinforcing steel can also be calculated using strain compatibility and similar triangles. This may be accomplished either by relating the strain in the concrete substrate to the strain in the FRP, or `by using the newly calculated concrete strain directly. When the updated concrete strain is used, the strain in the reinforcing steel can be determined from the equation provided below.

$$\varepsilon_s = \varepsilon_c\left(\frac{d - c}{c}\right) \tag{10}$$

Alternatively, the strain in reinforcing steel can be computed by the equation below.

$$\varepsilon_s = (\varepsilon_{fe} + \varepsilon_{bi})\left(\frac{d - c}{d_f - c}\right) \tag{11}$$

Where:

ε_s = strain in nonprestressed steel reinforcement, in./in.

Similar triangles are used to determine the strains in the concrete, steel and CFRP as shown in Figure 3.

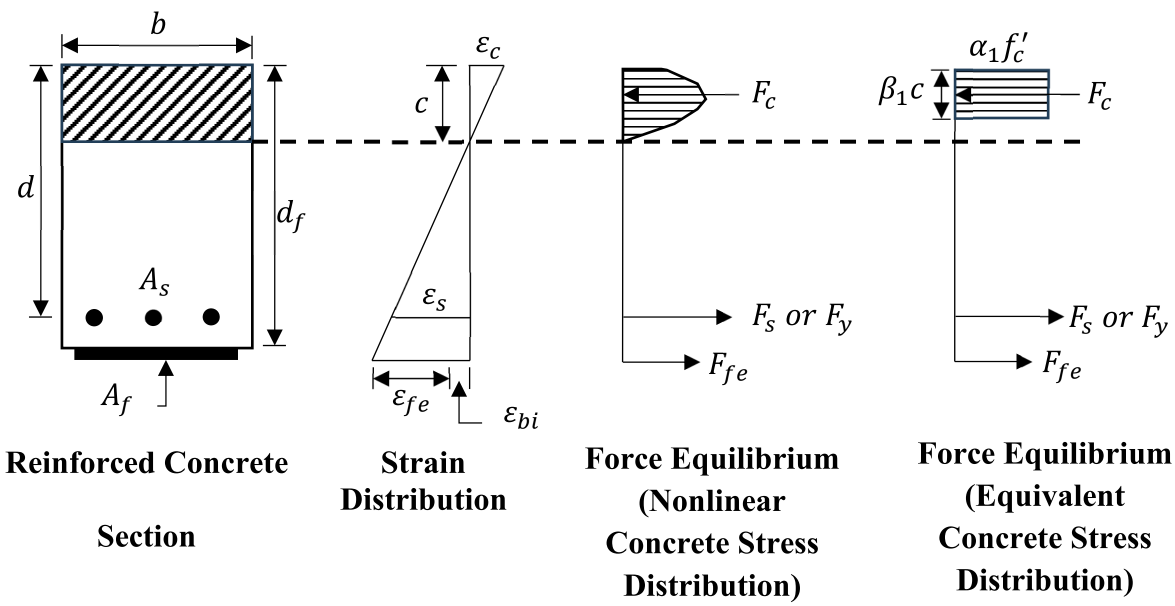

Figure 3. Strain and Stress Distribution for a Rectangular Section Under Flexure at Ultimate Limit State

Step 8: Stress in Steel and CFRP

Now that the strain has been determined in the reinforcing steel and the FRP, the stress level can be calculated. With the assumption of an elastic-perfect plastic stress-strain curve, the stress in the steel is determined from the strain in the steel. This equation is stating that the steel will not be able to take any more stress after its yielding point.

$$f_s = E_s \varepsilon_s \leq f_y \tag{12}$$

Where:

f_s = stress in nonprestressed steel reinforcement, psi

E_s = modulus of elasticity of steel, psi

f_y = specified yield strength of nonprestressed steel reinforcement, psi

To determine the stress in FRP, the assumption is that the CFRP is linearly elastic until failure, so Hooke's law is used.

$$f_{fe} = E_f \varepsilon_{fe} \tag{13}$$

Where:

f_{fe} = effective stress in the FRP; stress attained at section failure, psi

The effective stress in the FRP reinforcement is the maximum stress that can develop in the FRP prior to the section reaching flexural failure, based on its effective strain and modulus of elasticity. For design purposes, CFRP is applied unidirectionally, meaning the fibers are oriented to act in a single direction. Illustrations of CFRP application on flexural members and its elastic behavior are shown in the figures below for visualization.

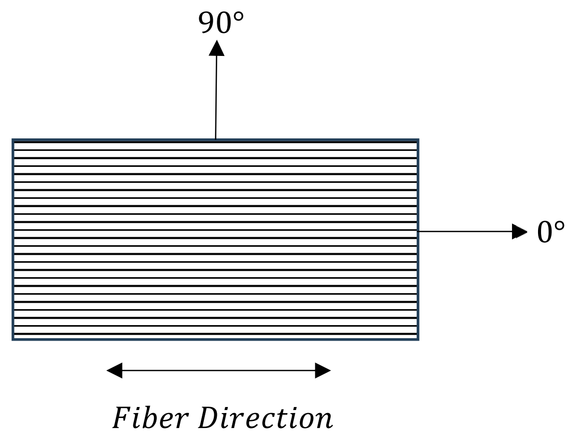

Figure 4. Fiber Angles

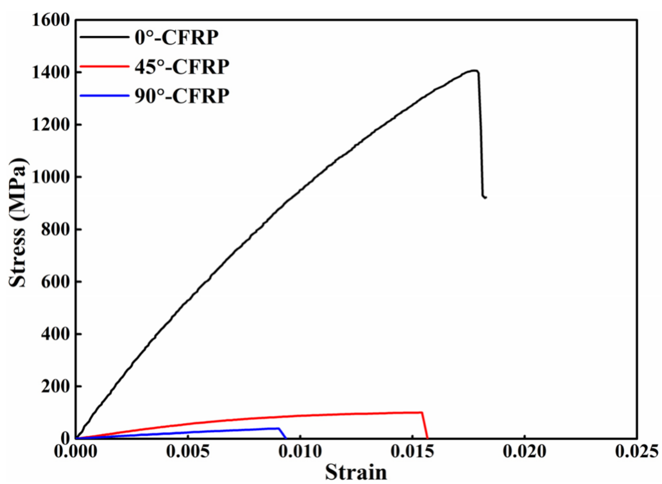

Figure 5. Stress Strain Behavior Based on Fiber Angle

Step 9: Internal Force Resultants and Equilibrium Check

To complete step nine, the assumed depth to the neutral axis is verified now that the stresses in both the steel and the FRP have been determined from Equations (12) and (13). By enforcing equilibrium between the internal compression and tension forces, an equation can be formulated to solve for the neutral axis depth, c.

$$\alpha_1 f_c' \beta_1 bc = A_s f_s + A_f f_{fe} \tag{14}$$

Rearranging the equation to solve for *c* yields

$$c = \frac{A_s f_s + A_f f_{fe}}{\alpha_1 f_c' \beta_1 b} \tag{15}$$

Where:

A_s = area of reinforcement steel, in.^{2}

A_f = area of fiber reinforcement, in.^{2}

b = width of compression face of member, in.

α_1 = multiplier on f'_c to determine intensity of an equivalent rectangular stress distribution for concrete

β_1 = ratio of depth of equivalent rectangular stress block depth of the neutral axis

Associated with the Whitney stress block from the ACI 318, α_1 may be taken as 0.85 and β_1 will vary depending on the concrete compressive strength if concrete crushing is the governing failure mode. This mode of failure can occur before or after the yielding of steel. If concrete crushing is not the controlling mode of failure, and FRP debonding or FRP rupture is the governing mode of failure, using the Whitney stress block will give reasonably accurate results. A nonlinear stress distribution in the concrete, or a more refined stress block corresponding to the strain level attained in the concrete at the ultimate limit state, may also be adopted. Parabolic Stress-Strain relationship for concrete has approximate stress block factors, α_1 and β_1, that also may be calculated.

$$\beta_1 = \frac{4\varepsilon_c' - \varepsilon_c}{6\varepsilon_c' - 2\varepsilon_c} \tag{16}$$

$$\alpha_1 = \frac{3\varepsilon_c'\varepsilon_c - \varepsilon_c^2}{3\beta_1 {\varepsilon_c'}^2} \quad (17)$$

Where:

ε'_c = the strain corresponding to the specified compressive strength of concrete, f'_c

$$\varepsilon_c' = \frac{1.7 f_c'}{E_c} \quad (18)$$

Step 10: Adjust Neutral Axis Depth until Equilibrium is Achieved

Using the stress block factors, the previously assumed value of c can now be verified. If the assumed neutral axis depth (0.20d) does not match the value of c calculated from Equation (15), Equations (8) – (17) must be repeated until equilibrium is satisfied. Once the assumed neutral axis depth agrees with the value obtained from Equation (15), the design process can proceed to the next step.

Step 11: Nominal Flexural Strength

Once the correct depth to the neutral axis has been established, the flexural strength can be calculated by considering both the steel contribution and the CFRP contribution. The nominal flexural strength provided by the steel reinforcement is determined using the equation provided below.

$$M_{ns} = A_s f_s \left(d - \frac{\beta_1 c}{2}\right) \quad (19)$$

Where:

M_{ns} = contribution of steel reinforcement to nominal flexural strength at a section, in.-lb

The contribution of the CFRP to the flexural strength can be determined using a similar approach to that used for the internal tensile reinforcement. In this case, however, the moment arm is produced by the tensile force developed in the externally bonded CFRP. The flexural strength of the member increases as the distance between the CFRP plate or fabric and the extreme compression fiber increases, since a larger distance results in a greater lever arm and, consequently, a higher moment capacity.

$$M_{nf} = A_f f_{fe}\left(d_f - \frac{\beta_1 c}{2}\right) \tag{20}$$

Where:

M_{nf} = contribution of FRP reinforcement to nominal flexural strength at a section, in.-lb

Reduction factors will be applied to the design flexural strength. The overall reduction factors ϕ, will be determined based on the strain in the steel reinforcement.

$$\phi = 0.90\ for\ \varepsilon_s \geq 0.005$$

$$\phi = 0.65 + \frac{0.25(\varepsilon_s - \varepsilon_{sy})}{0.005 - \varepsilon_{sy}}\ for\ \varepsilon_{sy} < \varepsilon_s < 0.005$$

$$\phi = 0.65\ for\ \varepsilon_s < \varepsilon_{sy}$$

Where:

ε_s = strain in nonprestressed steel reinforcement, in./in.

ε_{sy} = strain corresponding to yield strength of nonprestressed steel reinforcement, in./in.

An additional reduction factor $\Psi_f = 0.85$, will be applied to the CFRP flexural strength. From “An Updated Review on the Effect of CFRP on Flexural Performance of Reinforced Concrete Beams” *“Zhang et al. (2022) suggested a method based on experimental data to* *predict flexural capacity using the ensemble learning (EL) algorithm. Other machine learning (ML) methods were surpassed by their models, and a reduction factor of 0.85 is proposed for developing FRP strengthened RC beams."*(Jahami & Issa, 2024).

It should be noted, however, that the 0.85 factor was already in use prior to this research. It was originally derived from a reliability analysis of experimentally calibrated statistical data and was primarily introduced to account for the relatively unpredictable failure mode associated with FRP delamination (Rasheed, 2014).

Step 12: Design Flexural Strength

The next step is to calculate the design flexural strength of the reinforced concrete beam. At this stage, the appropriate strength reduction factors are applied to obtain the design flexural capacity of the member. Since the strain in the tensile reinforcement was determined in the previous steps, it is necessary to apply the correct strength reduction factor based on the section classification (tension-controlled, transition, or compression-controlled). The flexural strengths contributed by the tensile reinforcement and the CFRP can then be combined to determine the total design flexural strength, as expressed in the equation below.

$$\phi M_n = \phi\left[M_{ns} + \Psi_f M_{nf}\right] \tag{21}$$

Where:

ϕM_n = design flexural strength at a section, in.-lb

The design flexural strength is then compared to the required factored moment strength, which reflects the additional loads the beam is being designed to resist or the demands necessary to meet current code requirements. It is essential that the design flexural strength be greater than or equal to the required (ultimate) moment strength to ensure adequate structural capacity.

$$\phi M_n \geq M_u \tag{22}$$

Where:

M_u = factored moment at a section, in.-lb

Step 13: Service Stresses in Steel and CFRP

It is important to ensure that the strengthened beam performs adequately under service loads; therefore, the stress limits in both the reinforcing steel and the CFRP must be checked. The stress in the steel under service-level loading is evaluated first. Using a cracked-section analysis of the strengthened reinforced concrete beam, the steel stress can be calculated using the equation provided below. In this analysis, the tensile strength of the concrete is neglected, and only the steel reinforcement and CFRP are assumed to resist the tensile forces.

$$f_{s,s} = \frac{\left[M_s + \varepsilon_{bi} A_f E_f \left(d_f - \frac{kd}{3}\right)\right](d - kd)E_s}{\left[A_s E_s \left(d - \frac{kd}{3}\right)(d - kd) + A_f E_f \left(d_f - \frac{kd}{3}\right)\left(d_f - kd\right)\right]} \tag{23}$$

Where:

$f_{s,s}$ = stress in nonprestressed steel reinforcement at stress levels, psi

M_s = service moment at a section, in.-lb

The distribution of strain and stress is similar to that of a conventional reinforced concrete section. However, at service load levels, the depth to the neutral axis, kd, is determined by taking the first moment of the areas of the transformed cracked section. In this approach, the initial strain in the FRP is neglected, as it does not significantly affect the neutral axis depth within the elastic response range of the strengthened member.

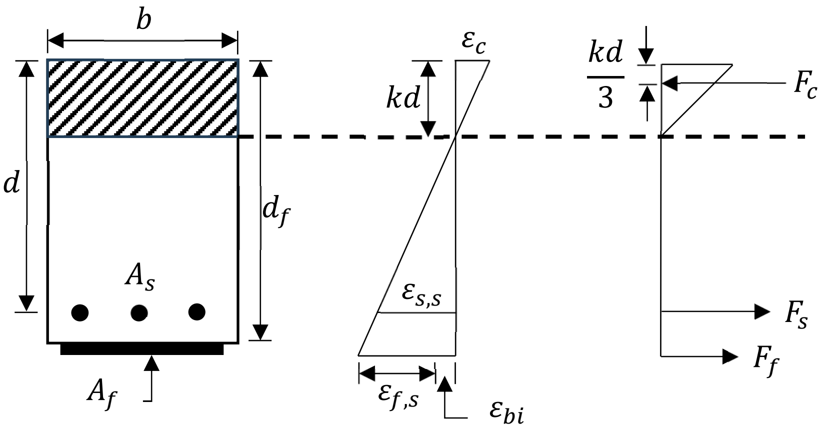

Figure 6. Elastic Strain and Stress Distribution

To calculate the elastic depth to the cracked neutral axis, *k*, is solved by the equation below for rectangular concrete beams without compression steel.

$$k = \sqrt{\left(\rho_s \frac{E_s}{E_c} + \rho_f \frac{E_f}{E_c}\right)^2 + 2\left(\rho_s \frac{E_s}{E_c} + \rho_f \frac{E_f}{E_c}\left(\frac{d_f}{d}\right)\right)} - \left(\rho_s \frac{E_s}{E_c} + \rho_f \frac{E_f}{E_c}\right) \tag{24}$$

$$\rho_s = \frac{A_s}{bd} \tag{25}$$

$$\rho_f = \frac{A_f}{bd_f} \tag{26}$$

Where:

ρ_s = steel reinforcement ratio

ρ_f = FRP reinforcement ratio

With the value of k established, the stress in the reinforcing steel can be calculated. The computed steel stress should not exceed the recommended limit, $f_{s,s} \leq 0.8f_y$. The moment M_s represents the moment produced by all sustained loads, including dead loads and the sustained portion of live loads, as well as the maximum moment experienced during a fatigue loading cycle.

Verifying the steel and CFRP stresses at service load levels ensures elastic behavior, controls cracking and deflection, guards against brittle or time-dependent failures, and promotes acceptable long-term performance of the strengthened member.

After evaluating the steel stress, the FRP stress must also be checked under service load conditions, including the sustained-plus-cyclic stress limit. The FRP stress is determined using an elastic analysis based on the bending moment from sustained loads (dead load and the long-term portion of live load) together with the maximum moment occurring during a fatigue cycle. The corresponding equation is formulated to prevent creep rupture under sustained stress and fatigue failure due to cyclic loading.

$$f_{f,s} = f_{s,s}\left(\frac{E_f}{E_s}\right)\left(\frac{d_f - kd}{d - kd}\right) - \varepsilon_{bi}E_f \tag{27}$$

$$f_{f,s} \leq 0.55f_{fu} \tag{28}$$

Where:

$f_{f,s}$ = stress in FRP caused by a moment within elastic range of member, psi

f_{fu} = design ultimate tensile strength of FRP, psi

Step 14: Creep Rupture Limit at Service of CFRP

Creep rupture refers to the sudden failure of an FRP system subjected to sustained stress over time. Among FRP systems, CFRP is the least susceptible to creep rupture; however, the sustained stress must be limited to $f_{f,s} \leq 0.55 f_{fu}$ to maintain long-term safety. When this limit is satisfied, the CFRP retains sufficient strength to resist additional short-term or transient (nonsustained) loads.

In addition to creep effects, tensile fatigue gradually reduces CFRP strength by approximately 5–8% per decade of logarithmic load cycles. By comparison, steel reinforcement typically degrades at a higher rate, on the order of 20% per decade (ACI Committee 440, 2023).

2.3 Installation of CFRP

The installation of CFRP systems can vary between manufacturers, so it is important to follow the manufacturer's installation procedures along with the relevant code procedures. The contractor's competence in the preparation of the surface and the installation of the CFRP can be verified by providing evidence of training and documentation of previously completed work, or through the installation of FRP systems and proper surface preparation on sections of the structure.

Performance of FRP systems can be affected by relative humidity, temperature, and surface moisture of the concrete at the time of installation. Regarding the conditions of the concrete before and during installation, the surface temperature and moisture conditions should be observed. Improper saturation of the fibers and curing of the resin constituent materials can occur when the concrete surface temperature falls below the minimum level specified by the CFRP manufacturer, which typically occurs on cold or frozen surfaces.

Wet or damp surfaces should be avoided when applying resins and adhesives unless those products are specifically designed for such conditions. CFRP systems should not be installed on concrete surfaces that exhibit moisture vapor transmission, as vapor passing through uncured resin can create surface bubbles and compromise the bond between the CFRP and the concrete substrate.

Surface preparation and substrate repair are the next step in the process of applying the CFRP system to the reinforced concrete beam. This is a vital step in the preparation process because if the surface is inadequate, debonding or delamination of the CFRP can occur before achieving the design load transfer. The repair of the substrate should be addressed before the surface preparation starts, and ACI 564R has detailed methods for the repair and surface preparation of the concrete. If there is any actively corroding reinforcing steel within the RC beam specimen, externally bonded CFRP should not be applied until the corrosion-related deterioration is repaired. This is due to the expansive forces that are linked to corrosion and are difficult to determine which can jeopardize the externally bonded CFRP structural integrity. Cracks that are larger than 0.010 in. should be pressure-injected with epoxy before any installation of CFRP in accordance with ACI 224.1R due to the cracks negatively affecting the performance of the externally bonded reinforcement.

Externally bonded CFRP on beams for flexure is a bond-critical application and not a contact critical application like it is for columns, so different surface preparation requirements are required. The concrete that is to be strengthened by the addition of CFRP sheets or plates should be freshly exposed and free of loose or unsound materials. Abrasive or water-blasting techniques can be used to remove all laitance, dust, dirt, oil, curing compound, existing coatings, or anything else that could interfere with the bond of the CFRP system to the concrete substrate. The ICRI 310.2R states that the concrete surface should be prepared to a concrete surface profile (CSP) not less than 3 which is a light to medium surface texture that will aid in proper bonding of the concrete substrate and CFRP composite. Figure 7 shows different surface preparation methods and their range of concrete surface profiles. If the CFRP manufacturer states different tolerances, that also may be followed.

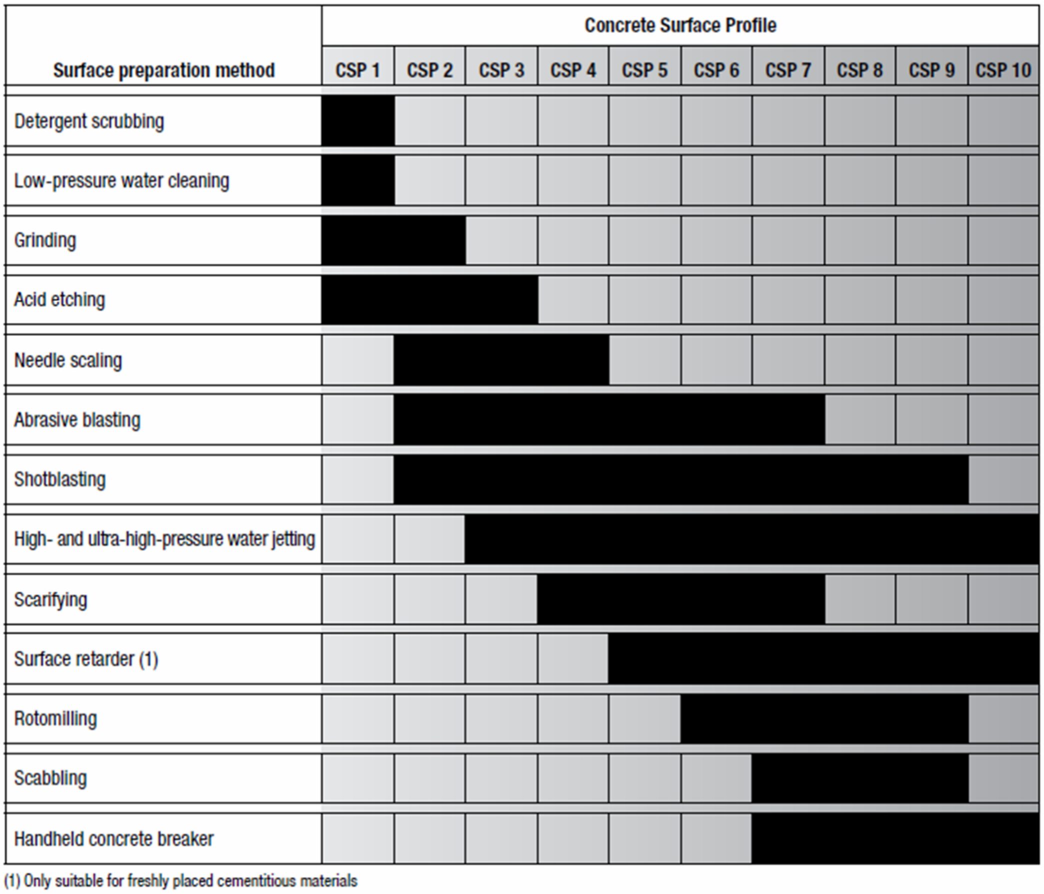

Figure 7. Common Surface Preparation Methods and CSP Comparison (ICRI, 2013)

Localized out-of-plane variations which include form lines, bumps, or irregularities in the concrete should not exceed 1/32 in. or the tolerances recommended by the CFRP manufacturer. These variations should be removed by grinding before the surface is prepared by abrasive or water-blasting techniques. If variations in the flatness or elevation of the beam specimen are very small, a resin-based putty may be used. All surfaces receiving the strengthening system should be dried to the level recommended by the FRP system manufacturer.

There are two common ways of applying the CFRP to the reinforced concrete beam which consist of wet layup systems and precured systems. The wet layup process uses dry carbon fiber reinforcement that is impregnated on site with a saturating resin and cured in place. Impregnation can be performed using a saturation machine, where the fabric is passed through resin-filled rollers set to achieve the manufacturer-specified fiber-to-resin ratio or rate, or it can be done manually using rollers. Once saturated, the fiber sheets are applied to the prepared surface. When multiple layers are required, each subsequent layer should be placed before the previous one has fully cured, ensuring that the uncured layer does not shift during application.

Precured systems for beam applications consist of CFRP plates. Instead of saturating resins that are used in the wet layup system, adhesives are applied uniformly to the prepared concrete beam surface. The precured CFRP plates should be placed on or into the adhesive in a manner that is recommended by the manufacturer. The adhesive should be applied at the specified rate and all entrapped air bubbles should be released or rolled out before the adhesive sets (ACI Committee 440, 2023).

Chapter 3 – Design Examples and Comparative Evaluation

3.1 Design Example

A reinforced concrete beam is to be strengthened in order to carry an increased live load resulting from a change in floor usage from a corridor (80 psf) to a light storage within a warehouse (125 psf). A partial plan of the floor is shown in Figure 8. The slab is designed as one-way slab, and the beam spans 30 ft. Externally bonded CFRP will be applied to the soffit of the beam to increase the flexural strength. The simply supported beam is adequate for shear strength and also meets the deflection and crack control serviceability requirements from the increase in live load. The beam is located inside and is not exposed to an exterior or an aggressive environment.

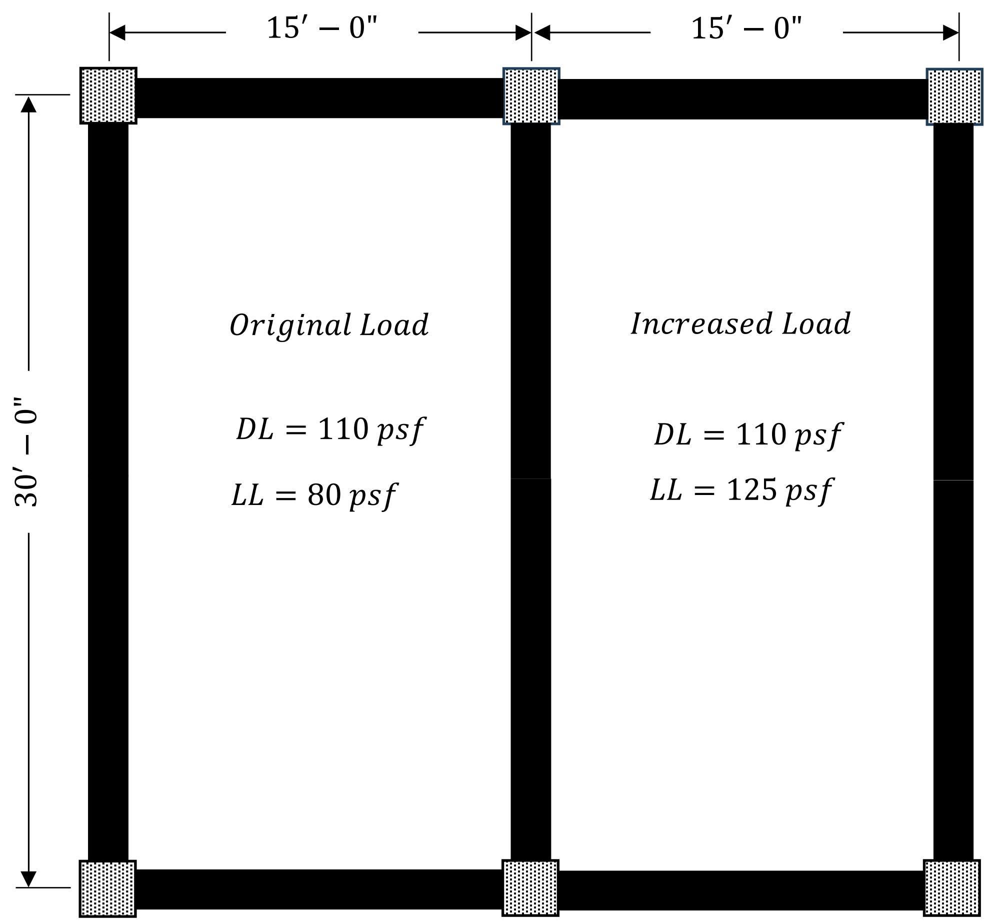

Figure 8. Floor Plan with Original Loading

The dead load acting on the beam is 110 psf, including self-weight, and the live load before the anticipated increase is 80 psf. The increase in live load due to the change in floor usage being light storage will make the new live load 125 psf according to the ASCE/SEI 7-22. The beam is designed as a simply supported beam with uniformly distributed loads, as shown in Figure 9. Table 2 below shows the midspan moments of the existing and anticipated loads for the beam. The existing beam design is shown in Table 3.

Table 2:Distributed Loading and Corresponding Moments

Loading / Moment	Existing Loads	Anticipated Loads
Dead Loads w_{DL}	1.65 kip/ft	1.65 kip/ft
Live Load w_{LL}	1.2 kip/ft	1.875 kip/ft
Unfactored Loads $(w_{DL} + w_{LL})$	2.85 kip/ft	3.525 kip/ft
Unstrengthened Load Limits $(1.1w_{DL} + 0.75w_{LL})$	NA	3.221 kip/ft
Factored Loads $(1.2w_{DL} + 1.6w_{LL})$	3.9 kip/ft	4.98 kip/ft
Dead-Load Moment M_{DL}	185.625 kip-ft	185.625 kip-ft
Live-Load Moment M_{LL}	135 kip-ft	210.938 kip-ft
Service-Load Moment M_s	320.625 kip-ft	396.563 kip-ft
Unstrengthened Moment Limit $(1.1M_{DL} + 0.75M_{LL})$	NA	362.363 kip-ft
Factored Moment M_u	438.75 kip-ft	560.25 kip-ft

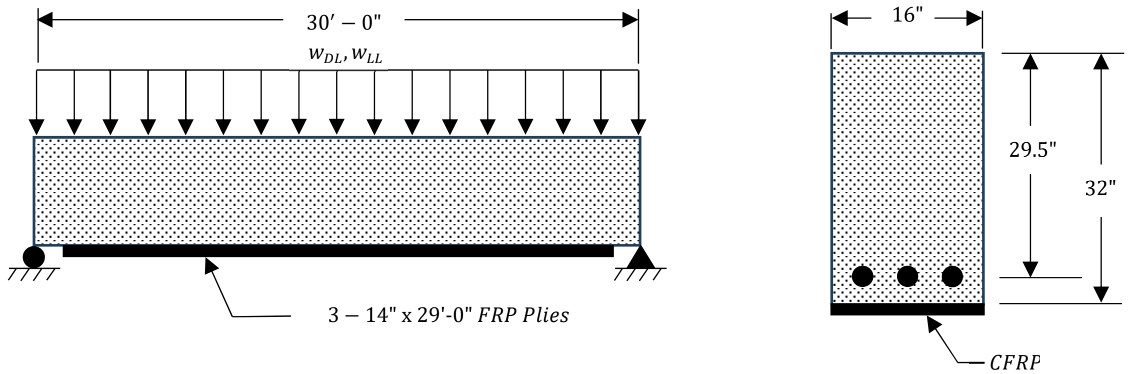

Figure 9. Elevation and Section of Idealized Simply Supported Beam with Externally Bonded CFRP Reinforcement

Table 3:Beam Properties

Beam Properties	**-**
Specified Compressive Stress of Concrete f_c'	4000 psi
Area of Steel A_s	(3) No. 10
Specified Yield Strength of Nonprestressed Steel Reinforcement f_y	60 ksi
Length of Beam l	30 ft
Width of Beam w	16 in.
Depth to the Centroid of the Reinforcing Steel d	29.5 in.
Height of the Beam h	32 in.
Design Moment Capacity	469.75 kip-ft

From the internal force couple, the tension and compression forces are set equal to each other to solve for *a*, which is the depth of the equivalent concrete compression block. With the additional live load that is added to the beam, the factored moment capacity assuming that it is in tension is calculated as follows. It’s sufficient for the original loads.

$$T = A_s f_y \ and \ C = 0.85 f_c' ba$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{3.81 \, in^2 * 60{,}000 \, psi}{0.85 * 4000 \, psi * 16"} = 4.20"$$

$$\phi M_n = A_s f_y \left(d - \frac{a}{2}\right) = 0.9 * 3.81\ in^2 * 60\ ksi * (29.5" - \frac{4.20"}{2}) = 469.75\ kip - ft$$

A strengthening limit must also be checked as part of the design to reduce the risk of structural collapse in the event of bond failure or other deficiencies in the CFRP system. Such failures may occur due to vandalism, accidental damage, or other unforeseen causes. To provide redundancy, the moment capacity of the unstrengthened beam (i.e., without CFRP) should exceed the applicable factored moment demand, with the load factor of dead load being 1.1 and that of live load being 0.75. If the live load is expected to be sustained for extended periods, a factor of 1.0 should be used in place of 0.75 when evaluating this limit. (ACI Committee 440, 2023).

$$(1.1M_{DL} + 0.75M_{LL}) = 362.363\ kip - ft \leq \phi M_n = 469.75\ kip - ft$$

The CFRP properties and design values that have been chosen for the design (shown in Table 4) come from the manufacturer MAPEI. The CFRP is a unidirectional carbon fiber fabric (MapeWrap C Uni-Ax 300) that is used with a two-component epoxy adhesive (MapeWrap 21) which forms an externally bonded CFRP reinforcing system.

Table 4: CFRP Cured Laminate Design Value Properties

Thickness per ply t_f	0.0324 in.
Ultimate tensile strength f_{fu}^*	88 ksi
Modulus of elasticity E_f	7398 ksi
Rupture strain ε_{fu}^*	0.015 in./in.

Following the design steps discussed in Chapter 2 (detailed calculations are included in Appendix A), the addition of the carbon fiber fabric increased the moment capacity of the beam to be 579.2 k-ft, which was greater than the increased required moment of 560.25 k-ft. The following values in Table 5 correspond to the final section analysis results for c, α_1, β_1, and the steel/CFRP strains and stresses after equilibrium and compatibility were satisfied.

Table 5: Force Equilibrium Properties

Depth to neutral axis, c	6.75 in.
Stress block factor, α_1	0.89
Stress block factor, β_1	0.75
Strain in steel, ε_s	0.00635 in./in.
Strain in CFRP, ε_{fe}	0.0062 in./in.
Stress in Steel, f_s	60 ksi
Stress in CFRP, f_{fe}	45.87 ksi

Unlike conventional steel reinforcement, CFRP exhibits linear-elastic behavior up to sudden failure and is sensitive to installation quality, environmental exposure, and sustained loading. Consequently, the design methodology incorporates multiple limit states and corresponding reduction factors to explicitly address less predictable failure modes like delamination and potential failure modes, including improper installation, accidental damage or vandalism, fatigue loading under repeated service conditions, and long-term creep rupture. These reduction factors ensure that none of these brittle failure modes govern the design, thereby maintaining an adequate margin of safety throughout the service life of the strengthened member.

3.2 Steel Plate Design and Comparison to CFRP

Externally bonded steel plates using epoxy adhesives represent an established method for retrofitting and strengthening reinforced concrete beams subjected to increased loading, material deterioration, or updated design requirements. This strengthening technique was first introduced in the 1960s by L'Hermite et al. and Bresson (Czaderski & Meier, 2018). In this section, the steel plate strengthening method is evaluated in comparison with the externally bonded CFRP strengthening technique. Performance criteria including flexural strength, cost, labor requirements, added dead load, and constructability are considered.

A steel plate was installed at the soffit of the reinforced concrete beam to increase its flexural capacity under the same increased loading condition used for the CFRP strengthening scheme. The plate was assumed to be fully bonded or mechanically anchored such that composite action between the steel plate and the concrete section was achieved. The steel plate had a width of 5 in. and a thickness of 0.25 in. and was fabricated from ASTM A36 structural steel. All other parameters, including beam geometry and internal reinforcement, as well as the loads, were maintained identical to those of the CFRP-strengthened section in order to permit direct comparison of strengthening effectiveness. Detailed calculations for design strength using steel plate are included in Appendix A. The strength of the steel plated beam is listed in Table 6, alongside that of the CFRP strengthened beam.

Table 6: CFRP and Steel Plate Strength Comparison

Strengthening Scheme	Design Flexural Strength	M_u	Percent Utilization
CFRP	579.2 kip-ft	560.25 kip-ft	96.7%
Steel Plate	562.6 kip-ft	560.25 kip-ft	99.6%

These results indicate that, for identical beam geometry and internal reinforcement, the externally bonded steel plate system provides greater flexibility in selecting the effective reinforcement width, whereas the CFRP system typically requires application over most of the soffit width. Both strengthening methods were shown to be effective in increasing the flexural capacity of the reinforced concrete beam section.

Concrete and Masonry Restoration Inc. (C & M Restoration Company) was contacted for an estimate of the cost to perform the work for a CFRP beam repair and a steel plate repair. The proposal values are inclusive of all costs associated with material, labor, equipment, standard insurance, and taxes. The proposed cost reflects work completed during normal working hours, Monday through Friday.

The scope of work for both the CFRP and beam repair includes up to 20 square feet of partial depth concrete repairs on the referenced beam. The 20 square feet assumes that 10% of the surface of the beam (sides and soffit) will need to be repaired. Sounding, marking, and measuring of concrete repair areas will be performed by C&M Restoration Company. Once delaminated concrete is identified, the repair boundaries will be saw-cut, and unsound material will be removed to sound concrete to a maximum average depth of 1 inch. Exposed reinforcing steel will be prepared and coated with a zinc-rich primer, and the repair areas will be restored using a fast-curing cementitious repair mortar. New repair surfaces shall be finished to match the plane of the existing concrete.

The CFRP repair will have the beam surface primed and installed per the manufacturer's guidelines. If additional repair quantities are required, a unit price of $939 per square foot shall apply. Alternatively, an equivalent steel plate repair will include a 5 in. wide by 0.25 in. thick plate that runs the length of the beam. For the steel plate approach, a unit price of $759 shall apply if any additional repair quantities are required.

Table 7: CFRP and Steel Plate Cost Comparison

Repair Technique	Total cost	Cost per square foot
CFRP	$18,790	$939
Steel	$15,164	$759

Although CFRP has a higher initial material cost, several additional factors must be considered. CFRP provides superior strength-to-weight performance, excellent corrosion resistance, and a lightweight, low-profile alternative to steel. Its use can reduce installation time and long-term maintenance costs. However, compared to steel, CFRP exhibits brittle behavior and is less effective at correcting significant structural deflection or bowing. Accordingly, CFRP is recommended for applications involving corrosive environments, strengthening without increasing member weight, and situations requiring ease of installation. Steel reinforcement systems are generally preferred where lower initial material cost is the primary consideration.

Strengthening a single beam specimen using CFRP may be less economical due to material supply constraints, as carbon fiber fabric is provided in continuous rolls. The CFRP fabric used in this study (MapeWrap C Uni-Ax 300) is supplied in rolls 164 ft in length. In contrast, steel plate strengthening offers greater flexibility in length control, as plates can be fabricated to the exact dimensions required. CFRP materials must typically be field cut from continuous rolls, making steel plates potentially more suitable for applications involving a limited number of beams.

The additional weight contributed by the CFRP strengthening system is minimal. The dry carbon fiber fabric has a unit weight of 9 oz/yd². For three layers applied over the full beam length of 30 ft and a width of 14 in., the total added weight of the CFRP fabric is calculated as follows:

$$Area\ of\ CFRP = 30' * \frac{14"}{12"} = 35\ ft^2$$

$$Convert\ to\ yd^2 = 35\ ft^2 * \frac{1\ yd^2}{9\ ft^2} = 3.89\ yd^2$$

$$3.89\ yd^2 * \frac{9\ oz.}{yd^2} = 35\ oz.$$

$$35\ oz. * \frac{1\ lb}{16\ oz.} = 2.1875\ lb$$

Thus, the total weight of three CFRP layers is approximately 6.56 lb, excluding the epoxy adhesive, and is negligible compared to weights of other materials.

The additional dead load introduced by the externally bonded steel plate was evaluated using a steel density of 490 lb/ft³. For a steel plate with a width of 5 in. and a thickness of 0.25 in. applied over the full beam length, the corresponding self-weight was calculated to be 127.6 lb, about 20 times the weight of CFRP.

Chapter 4 - Conclusion

This study evaluated the use of externally bonded Carbon Fiber Reinforced Polymer (CFRP) systems for the flexural strengthening of reinforced concrete beams and compared their performance to that of externally bonded steel plate strengthening. The research demonstrated that CFRP provides an effective, lightweight, and corrosion-resistant solution for increasing the flexural capacity of existing reinforced concrete members subjected to increased service demands, changes in occupancy or building codes.

The design methodology presented, based primarily on ACI 440 provisions and strain compatibility principles, incorporates equilibrium of internal forces, nonlinear concrete behavior, and linear-elastic FRP response. Special attention was given to debonding failure mechanisms, particularly intermediate crack-induced debonding, which governs many strengthened members. By applying appropriate environmental reduction factors and strength reduction factors, the design framework ensures adequate safety margins against brittle failure modes such as FRP rupture, delamination, fatigue degradation, and creep rupture.

The design example confirmed that the addition of externally bonded CFRP sheets increased the beam's flexural capacity from 469.75 kip-ft to 579.2 kip-ft, exceeding the increase in required factored moment of 560.25 kip-ft. Service-level stress checks further verified that both reinforcing steel and CFRP remained within allowable limits, ensuring acceptable long-term performance under sustained and cyclic loading conditions.

A comparative evaluation with externally bonded steel plate strengthening showed that both systems are capable of meeting strength requirements. While the steel plate system achieved similar flexural capacity with slightly lower initial material cost, it introduced substantially greater additional dead load (127.6 lb compared to approximately 6.56 lb for CFRP) and remains susceptible to corrosion and long-term durability concerns. CFRP, although more expensive on a per-square-foot basis, offers superior strength-to-weight performance, reduced installation complexity, minimal increase in structural mass, and enhanced durability in aggressive environments.

Overall, CFRP strengthening is particularly advantageous in applications where weight addition must be minimized, corrosion resistance is critical, and constructability constraints exist. Steel plate strengthening may remain appropriate where lower initial cost is the governing factor and environmental exposure is limited.

The findings of this study support the continued adoption of CFRP systems as a reliable and structurally efficient strengthening technique for reinforced concrete beams. When properly designed and installed in accordance with established guidelines, CFRP systems provide a durable, high-performance alternative to traditional strengthening methods, enabling existing infrastructure to safely meet evolving load demands and modern code requirements.

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Appendix A - Calculations

A1.1 CFRP Design Steps

Before the design, determine the ultimate moment acting on the beam.

$$w_{DL} = 110\,psf * 15\,ft = 1.65\frac{k}{ft}$$

$$w_{LL} = 125\,psf * 15\,ft = 1.875\frac{k}{ft}$$

$$w_u = 1.2w_{DL} + 1.6w_{LL} = \left(1.2 * 1.65\frac{k}{ft}\right) + \left(1.6 * 1.875\frac{k}{ft}\right) = 4.98\frac{k}{ft}$$

$$M_u = \frac{w_u l^2}{8} = \frac{4.98\frac{k}{ft} * (30\,ft)^2}{8} = \boldsymbol{560.25\,k - ft}$$

Step 1: Calculate the CFRP system design material properties by applying the appropriate environmental reduction factors

$$f_{fu} = C_E * f_{fu}^* = 0.95 * 88\,ksi = 83.6\,ksi$$

$$\varepsilon_{fu} = C_E * \varepsilon_{fu}^* = 0.95 * 0.015\ \ in./in. = 0.01425\ \ in./in.$$

Step 2: Properties of Concrete, Reinforcing Steel, and CFRP

$$\beta_1 = 0.85 \quad Table\ 22.2.2.4.3$$

$$E_c = 57{,}000\sqrt{f_c'} = 57{,}000 * \sqrt{4000\,psi} = 3{,}604{,}997\,psi$$

$$A_s = 3(1.27\,in^2) = 3.81\,in^2$$

$$A_f = N * t_f * w_f = 3\,plies * 0.0324\frac{in.}{ply} * 14\,in. = 1.3608\,in^2$$

Step 3: Calculate the Existing State of Strain on the Soffit

$$k = \sqrt{\left(\rho_s \frac{E_s}{E_c}\right)^2 + 2\left(\rho_s \frac{E_s}{E_c}\right)} - \left(\rho_s \frac{E_s}{E_c}\right)$$

$$k = \sqrt{\left(\frac{3.81\ in^2}{16\ in. * 29.5\ in.} \frac{29{,}000\ ksi}{3605\ ksi}\right)^2 + 2\left(\frac{3.81\ in^2}{16\ in. * 29.5\ in.} \frac{29{,}000\ ksi}{3605\ ksi}\right)} - \left(\frac{3.81}{16\ in. * 29.5\ in.} \frac{29{,}000\ ksi}{3605\ ksi}\right)$$

$$k = 0.301$$

$$\varepsilon_{bi} = \frac{M_{DL}(d_f - kd)}{I_{cr}E_c} = \frac{2227.5\ kip - in. * [32\ in. - (0.301 * 29.5\ in.)]}{16{,}766\ in^4 * 3605\ ksi} = 0.000852$$

Step 4: Calculate the Design Strain of the CFRP System

$$\varepsilon_{fd} = 0.083 \sqrt{\frac{f_c'}{NE_f t_f}} \leq 0.9\varepsilon_{fu} = 0.083 \sqrt{\frac{4000\ psi}{3 * 7{,}398{,}000\ psi * 0.0324\ in.}} = 0.0062 \leq 0.9 * 0.01425 = 0.0128$$

Since the design strain is less than the rupture strain, debonding controls the design of the CFRP strengthening system.

Step 5: Estimate the Neutral Axis Depth *c*

$$c = 0.20d = 0.20 * 29.5\ in. = 5.9\ in.$$

Step 6: Determine the Effective Level of Strain in the CFRP Reinforcement

$$\varepsilon_{fe} = 0.003\left(\frac{d_f - c}{c}\right) - \varepsilon_{bi} \leq \varepsilon_{fd} = 0.003\left(\frac{29.5\ in. - 5.9\ in.}{5.9\ in.}\right) - 0.000852 = 0.0111$$

$$\varepsilon_{fe} = \varepsilon_{fd} = 0.0062$$

Since ε_{fe} is not less than ε_{fd}, debonding would be in the failure mode for the estimated neutral axis depth.

It is necessary to check the strain in the concrete because the CFRP system controls the failure, so ε_c may not equal 0.003.

$$\varepsilon_c = (\varepsilon_{fe} + \varepsilon_{bi})\left(\frac{c}{d_f - c}\right) = (0.0062 + 0.000852)\left(\frac{5.9\ in.}{32\ in. - 5.9\ in.}\right) = 0.0016$$

Step 7: Calculate the Strain in the Existing Steel Reinforcement

$$\varepsilon_s = (\varepsilon_{fe} + \varepsilon_{bi})\left(\frac{d - c}{d_f - c}\right) = (0.0062 + 0.000852)\left(\frac{29.5\ in. - 5.9\ in.}{32\ in. - 5.9\ in.}\right) = 0.0064$$

Step 8: Calculate the Stress Level in the CFRP and Steel Reinforcement

$$f_{fe} = E_f \varepsilon_{fe} = (7398\ ksi)(0.0062) = 45.87\ ksi$$

$$f_s = E_s \varepsilon_s \leq f_y = (29{,}000\ ksi)(0.0064) \leq 60\ ksi$$

$$f_s = 184.7\ ksi \leq 60\ ksi, so\ f_s = 60\ ksi$$

Step 9: Calculate the Internal Force Resultants and Check Equilibrium

$$\beta_1 = \frac{4\varepsilon_c' - \varepsilon_c}{6\varepsilon_c' - 2\varepsilon_c} = \frac{4(0.00189) - 0.0016}{6(0.00189) - 2(0.0016)} = 0.732$$

$$\alpha_1 = \frac{3\varepsilon_c'\varepsilon_c - \varepsilon_c^2}{3\beta_1 \varepsilon_c'^2} = \frac{3(0.00189)(0.0016) - (0.0016)^2}{3(0.732)(0.00189)^2} = 0.829$$

$$\varepsilon_c' = \frac{1.7 f_c'}{E_c} = \frac{1.7 * 4000\ psi}{3{,}604{,}997\ psi} = 0.00189$$

$$c = \frac{A_s f_s + A_f f_{fe}}{\alpha_1 f_c' \beta_1 b} = \frac{(3.81\ in.^2)(60\ ksi) + (1.3608\ in.^2)(45.87\ ksi)}{(0.829)(4\ ksi)(0.732)(16\ in.)} = 7.49\ in. \neq 5.9\ in. \quad \boldsymbol{NG}$$

Step 10: Adjust *c* Until Force Equilibrium is Satisfied

$c = 6.794\ in.;\quad \varepsilon_s = 0.006353;\quad f_s = f_y = 60\ ksi;$

$\beta_1 = 0.7510;\quad \alpha_1 = 0.8912;\quad and\ f_{fe} = 45.87\ ksi$

$$c = \frac{A_s f_s + A_f f_{fe}}{\alpha_1 f_c' \beta_1 b} = \frac{(3.81\ in.^2)(60\ ksi) + (1.3608\ in^2)(45.87\ ksi)}{(0.8912)(4\ ksi)(0.7510)(16\ in.)} = 6.794\ in.$$

Step 11: Calculate the Flexural Strength Components from Steel and CFRP Contributions

$$M_{ns} = A_s f_s \left(d - \frac{\beta_1 c}{2}\right) = (3.81\ in.^2)(60\ ksi)\left(29.5\ in. - \frac{0.7510(6.794\ in.)}{2}\right) = 513.4\ kip - ft$$

$$M_{nf} = A_f f_{fe} \left(d_f - \frac{\beta_1 c}{2}\right) = (1.3608\ in.^2)(45.87\ ksi)\left(32\ in. - \frac{0.7510(6.794\ in.)}{2}\right) = 153.2\ kip - ft$$

Step 12: Calculate the Design Flexural Strength of the Member

$$\phi M_n = \phi\left[M_{ns} + \Psi_f M_{nf}\right] = 0.9[513.4\ k - ft + 0.85(153.2\ k - ft)] = 579.2\ k - ft$$

$$\phi M_n = 579.2\ k - ft \geq M_u = 560.25\ k - ft, \qquad GOOD$$

Step 13: Check Service Stresses in the CFRP and Reinforcing Steel

$$\rho_s = \frac{A_s}{bd} = \frac{3.81\ in.^2}{(16\ in.)(29.5\ in.)} = 0.0081$$

$$\rho_f = \frac{A_f}{bd_f} = \frac{1.3608\ in.^2}{(16\ in.)(32\ in.)} = 0.00266$$

$$k = \sqrt{\left(0.0081\left(\frac{29{,}000}{3605}\right) + 0.00266\left(\frac{7398}{3605}\right)\right)^2 + 2\left(0.0081\left(\frac{29{,}000}{3605}\right) + 0.00266\left(\frac{7398}{3605}\right)\left(\frac{32\ in.}{29.5\ in.}\right)\right)}$$

$$- \left(0.0081\left(\frac{29{,}000}{3605}\right) + 0.00266\left(\frac{7398}{3605}\right)\right)$$

$k = 0.3128$

$kd = 0.3128(29.5\ in.) = 9.23\ in.$

$$f_{s,s} = \frac{\left[M_s + \varepsilon_{bi} A_f E_f \left(d_f - \frac{kd}{3}\right)\right](d - kd)E_s}{\left[A_s E_s \left(d - \frac{kd}{3}\right)(d - kd) + A_f E_f \left(d_f - \frac{kd}{3}\right)(d_f - kd)\right]}$$

$$f_{s,s} = \frac{\left[4758.756\ k - in + \left[(0.000852)(1.3608\ in.^2)\ x\ (7398\ ksi)\left(32\ in. - \frac{9.23\ in.}{3}\right)\right]\right]\ x\ [(29.5\ in. - 9.23\ in.)(29{,}000\ ksi)]}{\left(\left[(3.81\ in.^2)(29{,}000\ ksi)\ x\ \left(29.5\ in - \frac{9.23\ in.}{3}\right)(29.5\ in. - 9.23\ in.)\right] + \left[(1.3608\ in.^2)(7398\ ksi)\left(32\ in. - \frac{9.23\ in.}{3}\right)(32\ in. - 9.23\ in.)\right]\right)}$$

$$f_{s,s} = 44.72\ ksi \leq 0.80 f_y = 48\ ksi, GOOD$$

Step 14: Check Creep Rupture Limit at Service of the CFRP

$$f_{f,s} = f_{s,s}\left(\frac{E_f}{E_s}\right)\left(\frac{d_f - kd}{d - kd}\right) - \varepsilon_{bi} E_f = 44.72\ ksi\left(\frac{7398\ ksi}{29{,}000\ ksi}\right)\left(\frac{32\ in. - 9.23\ in.}{29.5\ in. - 9.23\ in.}\right) - (0.000852)(7398\ ksi)$$

$$f_{f,s} \leq 0.55 f_{f,u}$$

$$f_{f,s} = 6.51\ ksi\ \leq (0.55)(83.6\ ksi) = 45.98\ ksi$$

A1.2 Steel Plate Flexural Strength Design

Step 1: Ultimate Flexural Strength

$$M_u = \frac{w_u l^2}{8} = \frac{4.98 \frac{k}{ft} * 30^2\ ft^2}{8} = 560.25\ k - ft$$

Step 2: Determine the Depth of the Equivalent Concrete Compression Block

Assumption: Both rebar reinforcement and steel plate yield

$$a = \frac{A_s f_y + A_{sp} f_{yp}}{0.85 f_c' b} = \frac{3.81\ in^2 * 60\ ksi + (0.25'' * 5'') * 36\ ksi}{0.85 * 4\ ksi * 16''} = 5.029''$$

Step 3: Determine the Distance from the Extreme Compression Fiber to the Neutral Axis

$$c = \frac{a}{\beta_1} = \frac{5.029"}{0.85} = 5.917"$$

Step 4: Determine if the Steel Plate Yields

$$\varepsilon_{sp} = \frac{f_{yp}}{E_s} = \frac{36\ ksi}{29000\ ksi} = 0.00124$$

From Strain Compatibility

$$\varepsilon_{sp} = 0.003\left(\frac{\left(d_{sp} + \frac{t_p}{2}\right) - c}{c}\right) = 0.003\left(\frac{(32" + \frac{0.25"}{2}) - 5.917"}{5.917"}\right) = 0.0133 \geq 0.00124$$

Step 5: Determine if the Steel Rebar Reinforcement Yields

$$\varepsilon_t = 0.003\left(\frac{d-c}{c}\right) = 0.003\left(\frac{29.5" - 5.917"}{5.917"}\right) = 0.0120 \geq 0.00207$$

Net Tensile Strain

$$0.0120 \geq 0.005, Tension - Controlled, \phi = 0.9$$

Step 6: Determine Design Flexural Strength

$$\phi M_n = \phi\left(A_s f_y\left(d - \frac{a}{2}\right) + A_{sp} f_{yp}\left(d_{sp} - \frac{a}{2}\right)\right)$$

$$\phi M_n = 0.9\left(3.81\ in^2 * 60\ ksi * \left(29.5" - \frac{5.029"}{2}\right) + 1.25\ in^2 * 36\ ksi * \left(32.125" - \frac{5.029"}{2}\right)\right) = 563\ k - ft$$

$$\phi M_n = 563\ k - ft\ \geq M_u = 560.25\ k - ft, GOOD$$

Appendix B - Figure Permissions

Figure 1:

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Figure 2:

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INTRODUCTION

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