A STUDY OF DESIGN PARAMETERS FOR VISCOUS DAMPERS APPLIED TO INSTRUMENT SERVOMECHANISMS

by

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Approved by:

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NOMENCIATURE

a - Radius of damper mass, in.

Badius of damper casing, in.

C, - Damping coefficient of motor, oz-in.2

J - Servomotor inertia, oz-in.²

J, - Damper mass inertia, oz-in.2

K₁ - Servomotor constant, oz-in.

Ko - Servomotor constant, oz-in.

T - Servomotor torque, oz-in.

t - Time, seconds

T_f - Frictional torque of bearing, oz-in.

W - Width of damper casing, in.

W, - Width of damper mass, in.

V - Applied voltage to control phase of servomotor, volts

θ - Angular position of servomotor shaft, rad.

θ, - Angular position of damper mass (flywheel), rad.

 ω - Angular velocity of motor shaft, $\frac{\text{rad}}{\text{sec}}$

 η - Viscosity of fluid, $\frac{0z-\sec z}{2}$

 α , β - Torque acting on the damper casing and flywheel respectively per unit relative angular velocity of motor and damper mass, $\frac{Oz-in-sec}{rad}$

ζ - Damping ratio

ω - Natural frequency

INTRODUCTION

In the present age of automation, servomechanisms are frequently used in many industrial fields. They perform a variety of tasks limited only by the imagination of the engineer. The design criteria of these mechanisms is to obtain a stability of the system. For satisfactory operation of servo-system, it is not enough that the system is stable but a sufficient margin of stability should exist so that the transients in the system are well damped.

In practical applications, selection of different elements of servomechanism, such as servomotor and controller elements, depend upon various
considerations. For example, a servomotor may have to be chosen for its
efficiency, size, adaptability, and serviceability rather than for its
dynamic properties. Each component by itself may have satisfactory
operational characteristics, but when they operate as a group to form a
servo-system, it is unlikely that they meet the dynamic requirements of
the system. Hence, it is frequently necessary to provide a compensating
element so that satisfactory margin of stability can be obtained in the
system.

Several stabilizing techniques are commonly used by designers of servo-systems depending upon the requirements of the system. Some of them are as follows:

- 1) Cascade compensation,
- 2) Feedback compensation,
- 3) Load compensation.

A particular method of compensation has significant influence on certain elements of the system. The basis for selection of a particular method is then dependent upon the application. For example, the "cascade compensation" method is preferred when the dynamic characteristics of the servomotor as well as controller is to be altered. The "load compensation" technique has greater influence in changing the characteristics of the servomotor by altering the dynamics of the servo load.

The alteration of the servo load could be brought about by applying to the servomotor shaft, a viscous friction torque proportional to the angular velocity of the motor shaft. However, this method is inefficient, since it involves power loss proportional to the rate of angular rotation of motor shaft. A modification of the above technique, commonly used in practice when "load compensation" method is preferred, is "viscous type inertial damping."

Essentially this device, known as "viscous inertial damper" consists of a mass coupled through viscous fluid and rotating on a frictionless bearing attached to servomotor shaft. The advantage of this type of damping device over the previous one is that it absorbs negligible power and offers no damping torque to the motor shaft when the coupled mass rotates at the same velocity as the servomotor shaft.

An analysis of such a damping device, which directly influences the characteristics of the servomotor is desirable as an aid in the design of servomechanisms.

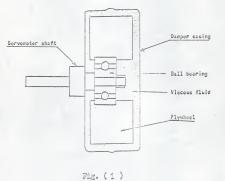
The object of this study is to analyze the various design parameters of the viscous inertial damper to determine the effect upon the dynamic characteristics of the servomotor.

DESCRIPTION AND OPERATION

Figure (1) shows the schematic of the damper. A flywheel rides on a frictionless bearing on servomotor shaft. A thin metal casing rigidly fixed to the motor shaft, encases the flywheel completely. The casing is filled with viscous fluid which serves as a coupling medium to the flywheel and casing. The casing is made from a cylindrical box, the two halves are joined and soldered around the periphery. Two holes in the casing allow the chamber to be filled with fluid. An exploded view of the assembly of these parts is shown in Figure (2). A pictorial view of the servomotor and the various parts of the damper are shown in Figure (3).

When an input signal, proportional to the desired change in the output position of the mechanism is brought into the system through the controlling elements, an amplified error signal is directed from the amplifying unit of the mechanism. This amplified error signal, in the form of voltage or current when applied to the control phase of the servomotor results in the rotation of motor shaft to a new position depending upon the amount of error signal. As the servomotor accelerates to a new position, the feedback element in the servomechanism operates in such a direction that the error signal becomes zero. In these operations there always exists a possibility that the system will oscillate about the desired final position. To bring these oscillations well within the limit of reasonable time, determined by the particular application, a well designed damping device is necessary.

The main criteria in the design of the damping element in the system should be that it offers optimum damping as demanded by the system. If



DESCRIPTION OF FIGURE (2)

- A Upper halve of damper casing (inside radius b = 0.644 inch).
- B Spacer -inertia damper.
- C Retaining ring.
- D Ball bearing
- E Damper mass (Flywheel) (Range of radii 0.5 .64 inch).
- F Lower halve of damper casing with assembly bearing shaft.
- G Set screw.

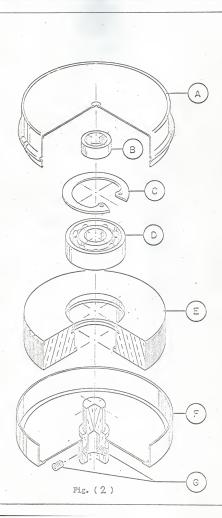






Fig. (3)

the damping offered is insufficient the system oscillates excessively about its final position where as too much damping may result in large time delays for a steady state condition. The over-shoot amplitude ratio and the damping frequency should be so adjusted that the stability requirements of the system are properly met.

In the design of damping element it is essential to study the inherent characteristics of the servomotor, chosen for some other considerations. Depending upon these characteristics and the requirements of the system for stability considerations a suitable damper could be designed. Time constant of the servomotor serves as a good indicator of motor performance under various conditions of damping ratio.

The properties of the viscous inertial damper are such that the flywheel inside the casing accelerates depending upon the drag of the fluid.
This drag is small at lower relative angular rate of rotation of motor shaft
and flywheel. However, sudden acceleration in the servomotor shaft as a
result of application of error signal, causes a large difference in angular
velocities of motor shaft and flywheel. In other words, the relative
angular velocity is high which in turn results in greater damping torque
on servomotor shaft. Analyzing the motion of the flywheel inside the casing,
it is seen that the acceleration of flywheel depends upon the inertia of the
flywheel and the dragging torque of the fluid. Once the flywheel picks up
speed and rotates at the same angular velocity as the motor shaft, there
is no damping torque offered to the motor shaft.

From the above discussions it is apparent that the amount of damping offered by the device is a function of various parameters such as the viscosity of the fluid, the inertia of the flywheel, thickness of the fluid film and the amount of fluid in the damper.

THEORY AND MATHEMATICAL DERIVATION

Since the function of the damper is to alter the dynamic characteristics of the motor shaft, it is convenient to develop the basic equations of motion for the damper including the servomotor. The role of the damper parameters in changing the performance of motor can then be easily studied.

Considering the armature of the servomotor and casing of the damper as a free body, a differential equation for the angular motion of the motor shaft was written by the principle of Newton's second law of motion.

Referring to Figure (4) the differential equation is as follows:

$$T - T_v - T_f - C_1 \frac{d\theta}{dt} = \frac{d^2\theta}{dt^2}$$
 (1)

where

T = motor torque, oz-in.

 $\mathbf{T}_{\mathbf{V}}^{=}$ viscous damping torque on motor shaft, oz-in.

 ${\rm T_{\it f}}{\rm =}$ frictional torque of bearing, oz-in.

The frictional torque of the bearing could be assumed to be independent of angular velocity 4 *. To identify the viscous torque in Equation (1) it is necessary to know the viscous forces that are acting on the sides of the damper casing. From the fundamental concepts of viscosity, viscous force is given as follows:

$$F_{S} = \mu A \frac{dv}{dy}$$
 (2)

^{*}Figure in the square bracket refers to the references at the end.

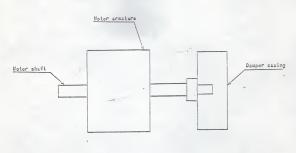
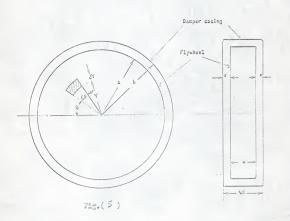


Fig. (4)



where

 $F_{_S}=$ viscous shearing force, oz. $dv=\mbox{linear relative velocity of damper casing and damper mass,} \frac{\mbox{in}}{\mbox{sec}} dy=\mbox{thickness of fluid film, in.}$

For convenience of analysis, laminar flow conditions in the damper are assumed.

For the computation of viscous torque consider an infinitesimal element of area on the side walls of the casing as shown in Figure (5).

Viscous torque on infinitesimal area = δF_S xY.

Applying Equation (2) infinitesimal viscous force,

$$\delta F_{s} = \frac{\gamma \delta \theta x d \gamma x \mu}{e} \frac{\left(\frac{d\theta}{dt} - \frac{d\theta}{dt}\right)}{e} \gamma$$
(4)

where

e = thickness of fluid film, in.

Y=radius of elemental area, in.

 $\delta\theta$ = infinitesimal change in angle.

δΥ = infinitesimal change in radius,

Integrating over the entire range and taking into account both sides of damper walls (assuming that the damper is completely filled with fluid), viscous torque on side walls of damper casing,

$$= 2 \int_0^b \int_0^{2\pi} \frac{\mu(\frac{d\theta}{dt} - \frac{d\theta}{dt}) \gamma^3}{e} d\theta d\gamma$$

$$= \frac{\pi\mu(\frac{d\theta}{dt} - \frac{d\theta}{dt}) b^4}{(3)}$$

In a similar way viscous torque on the circumferential area of the damper casing wall was written and is as follows:

Torque on circumferential area,

$$= \frac{2\pi b w \mu \left(\frac{d\theta}{dt} - \frac{d\theta_1}{dt}\right) b \times b}{b-a}$$
(4)

Hence, adding Equation (3) and (4), a total viscous torque on damper casing is given by Equation (5),

$$T_{\mathbf{v}} = \frac{2\pi b^3 \frac{3}{W_{11}}}{b - a} + \frac{\pi b^4 \mu}{e} \left(\frac{d\theta}{dt} - \frac{d\theta}{dt}\right)$$

$$= \beta \left(\frac{d\theta}{dt} - \frac{d\theta}{dt}\right)$$
(5)

where

$$\beta = \left[\frac{2\pi b^3 w}{b-a} + \frac{\pi b^4}{e} \right] \mu$$

From Equation (5) it is seen that β is the viscous torque acting upon the damper casing per unit relative angular velocity of motor shaft and flywheel. It is also interesting to note that the damping torque per unit relative angular velocity β is directly proportional to ' μ ' viscosity of the fluid. On the basis of Equation (5) and neglecting the area of flywheel occupied by bearing and motor shaft, a similar expression for the dragging torque on the flywheel was written as follows:

$$T_{vm} = \alpha \left(\frac{d\theta}{dt} - \frac{d\theta_1}{dt}\right) \tag{6}$$

where

$$\alpha = \left[\frac{2\pi a^3 w}{b-a} + \frac{\pi a^4}{e} \right] \mu$$

 $T_{vm} = viscous$ torque on the damper mass.

Differential equation for the motion of damper mass is then given as,

$$\alpha \left(\frac{d\theta}{dt} - \frac{d\theta_1}{dt}\right) - T_f = J_1 \frac{d^2\theta_1}{dt^2}$$
 (7)

To solve differential Equations (1) and (7) simultaneously for angular motion of motor shaft, differentiate Equation (1) with respect to time variable.

$$\frac{dT}{dt} - \beta \left(\frac{d^2\theta}{dt^2} - \frac{d^2\theta}{dt^2} \right) - C_1 \frac{d^2\theta}{dt^2} = J \frac{d^3\theta}{dt^3}$$
 (8)

Solving Equation (8) for $\frac{d^2\theta}{dt^2}$ and Equation (1) for $\frac{d^01}{dt}$ and substituting in Equation (7) and rearranging the terms in descending order of derivatives Equation (7) could be written as follows:

$$JJ_{1} \frac{d^{3}\theta}{dt^{3}} + \left[\alpha J + (C_{1} + \beta) J_{1}\right] \frac{d^{2}\theta}{dt^{2}} + \alpha C_{1} \frac{d\theta}{dt} + (\alpha + \beta) T_{f}$$

$$= J_{1} \frac{dT}{dt} + \alpha T \tag{9}$$

For convenience of analysis, frequently torque speed curves of servomotor are approximated by straight lines over the operating range of servomotor. Under such conditions an equation for the torque of the servomotor as a function of applied voltage and the angular velocity of motor shaft could be written as follows:

$$T = K_1 V_c = K_2 \frac{d\theta}{dt} \tag{10}$$

where

 ${\rm K}_1$ and ${\rm K}_2$ are motor constants which could be evaluated from the torque speed characteristics of servomotor.

Differentiation of Equation (10) with respect to time gives the following equation:

$$\frac{dT}{dt} = K_1 \frac{dV_c}{dt} - K_2 \frac{d^2\theta}{dt^2}$$
 (11)

Substitution of Equations (10) and (11) in Equation (9) results in the following equation:

Equation (12) is the basic equation governing the angular motion of motor shaft under damping conditions.

To find the performance of motor for transient conditions it is usual to apply a step input of constant voltage or current. Thus, assuming that a step voltage of constant magnitude is applied to the motor.

$$\frac{dV_{c}}{dt} = 0$$

Since the coefficients of the derivatives are independent of time, simplification of Equation (12) is done as follows:

$$\begin{split} & \omega = \frac{d\theta}{dt}, \ \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}, \ \frac{d^2\omega}{dt^2} = \frac{d^3\theta}{dt^3} \\ & m = \frac{JJ_1}{a} \\ & C = J + (C_1 + \beta + K_2) \frac{J}{a} \\ & K = (C_1 + K_2) \\ & P = K_1 V_c \\ & q = -(1 + \frac{\beta}{a}) T_f \end{split}$$

Equation (12) is then given as follows:

$$m \frac{d^2 \omega}{dt^2} + C \frac{d\omega}{dt} + K\omega = p + q$$
 (13)

Equation (13) is recognized as a second order differential equation in angular velocity of motor shaft with constant coefficients. Complimentary equation of differential Equation (13) is given by,

$$(mD^2 + CD + K) \omega = 0$$
 (14)

where

$$D = \frac{d}{dt}$$
 is a differential operator.

The two roots of Equation (14) are given by,

$$D_{1,2} = -\frac{C}{2m} \pm \sqrt{\frac{C^2}{4m^2} - \frac{K}{m}}$$

making a substitution as follows:

$$\frac{K}{m} = \omega_n^2$$
 and $\frac{C}{m} = 2\zeta\omega_n$

where ω_n = natural frequency,

and $\zeta = damping ratio.$

Two roots of Equation (14) are,

$$\mathrm{D}_{1,2} = -\ \omega_\mathrm{n} \zeta \ \pm \sqrt{(\zeta^2 - 1)} \ \omega_\mathrm{n}.$$

The complimentary solution of the differential Equation (14) is given by,

$$\omega_c = c_1 e^{xt} + c_2 e^{yt}$$

where

$$x = -\omega_n \zeta + \sqrt{(\zeta^2 - 1)} \omega_n$$

$$y = -\omega_n \zeta - (\zeta^2 - 1) \omega_n$$

 $t = time variable.$

Particular integral of the differential Equation (14) can be written by inspection as,

$$\omega_p = \frac{P + q}{K}$$

Hence, the complete solution is the sum of complimentary solution plus the particular integral. Thus, the solution of differential Equation (14) is,

$$\omega = c_1 e^{xt} + c_2 e^{yt} + \frac{P + q}{K}$$
 (15)

Where the constants \mathbf{C}_1 and \mathbf{C}_2 must be evaluated from initial conditions of the system.

For numerical analysis of the problem initial conditions of the system are assumed as follows:

at
$$t = 0$$
 $\omega = 0$

at
$$t = 0$$
 $\frac{du}{dt} = 0$

Substitution of these initial conditions in Equation (15) results in two equations in ${\bf C}_1$ and ${\bf C}_2$, which upon solving simultaneously gives the values of constants as follows:

$$C_{1} = \frac{(P + q) y}{(x - y) K}$$

$$C_{2} = \frac{(P + q) x}{(x - y) K}$$

where P, q, x, y, and K are as defined earlier.

Substitution of above values of constants in Equation (15) results in final solution of differential Equation (12) as given below:

$$\omega = \frac{(P + q) y}{(x - y) K} e^{xt} - \frac{(P + q) x}{(x - y) K} e^{yt} + \frac{P + q}{K}$$
(16)

Equation (16) is the response equation of the servomotor in terms of velocity for a step input of constant voltage applied to the control phase.

The servomotor considered for numerical analysis is two phase A.C. motor. General specifications, mechanical and electrical data and the motor characteristics are given in Appendix A. Values of motor constants \mathbf{K}_1 and \mathbf{K}_2 are calculated from motor characteristic curves and are as shown below:

$$K_1 = .494$$
 $K_2 = .002385$

It is found that values of $\rm K_1$ and $\rm K_2$ are fairly constant between the torque values of 0.5 to 3.0 (oz-in.) which means that torque speed curves for different values of voltages could be approximated as straight lines within the operating range.

Many calculations were carried on IBM 1620 Computer and various graphs plotted from the numerical data. The Fortran listing of the program is given in Appendix B.

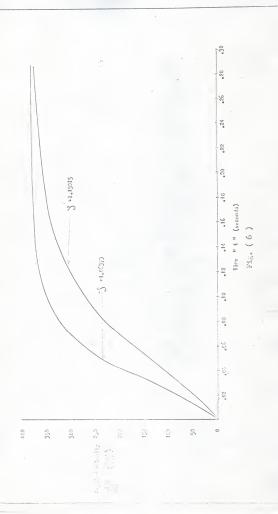
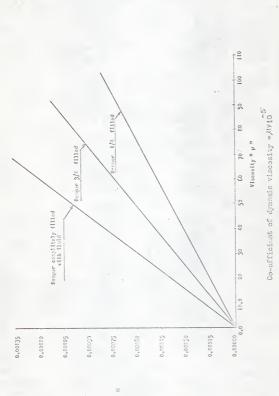
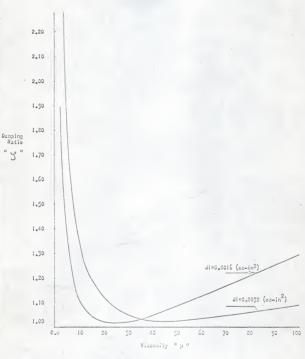
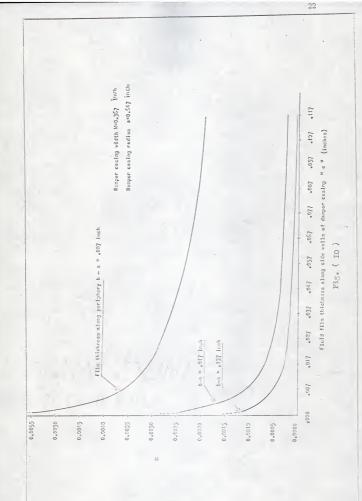


Fig. (7)





Co-efficient of dynamic viscosity = $\mu \times 10^{-5}$ Fig. (8)



DISCUSSION AND CONCLUSIONS

The important variable that influences the change in the response curve is the damping ratio '\z' as defined earlier. From Figure (6) it is seen that the response of the servomotor is delayed as the value of damping ratio increases. This could be interpreted in a way that if the system oscillates excessively about its final position, higher values of damping ratio are desirable. However, greater values of '\z' may also decrease the sensitivity of the system.

The factors governing the values of damping ratio '\z' are 'J_1' damping mass and '\a' the viscous torque on damping mass per unit of relative angular velocity of motor and damper mass. The latter is directly proportional to dynamic viscosity of fluid and shows a straight line relationship in Figure (7). The slope of the line depends upon the amount of fluid inside the damper.

A wide range of viscous fluids are available which can provide high viscosity but the viscosity of these fluids is by no means independent of temperature. However, if the range of operating temperature is small the viscosity of the oil and the dimensions of the damper could be adjusted to give required amount of damping for a reasonable operating temperature.

Figure (8) shows the graph of damping ratio '\z' and the viscosity of the fluid for different values of damper mass. It is seen that for each value of damping mass there is a minimum value of '\z' over a range of viscosity values. Also, the values of '\z' decreases rapidly in the lower range as the viscosity number increases where as '\z' increases slowly once it passes the minimum value. Changing the rotor inertia has similar effect on damping ratio but on a lower scale as shown in Figure (9).

Figure (5) shows the plot of oil film thickness between side walls of casing and flywheel for various values of oil film thickness around periphery of the casing and flywheel. Values of 'a' remain fairly constant over the thick film range but are very high for thin film. When the film thickness approaches zero 'a' tends to infinity. In the limiting case it could be interpreted that the flywheel and the damper casing are rigidly fixed together and the whole damper acting as a inertial wheel.

From the above discussion it is seen that the necessary amount of damping could be obtained from viscous inertial damper by adjusting its various parameters so that the desired response of servomotor could be obtained.

ACKNOWLEDGMENTS

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REFERENCES

- Hopkin, A. M., "Transient Presonse of Small Two Phase Induction Motors." AIEE Transactions Vol. 70, 1951, pp.881-886.
- Hall, A. C., "Damper Stabilized Instrument Servomechanisms." AIEE Transactions, Vol. 68, 1949, pp.229-306.
- 3. Binder, R. C., "Fluid Mechanics," Prentice Hall, Inc., 1949.
- Shigley, J. F., "Mechanical Engineering Design," McGraw-Hill Book Company, 1963.

APPENDIX A

General Specification of A.C. servomotor:

Type A - 10475

115 volts/36 V.C.T.

3.5 watts output

2-phase, 2 poles, 60 cycle

Mechanical characteristics:

Rotor inertia, oz-in.² 0.076

Stall torque, oz-in. 4.50

No load speed, RPM 3500

Maximum power output (watts)3.50

Torque at maximum output

oz-in. 2.4

Speed at maximum power

output 1950

Time constant (seconds) 0.019

Damping coefficient

oz-in-sec/rad 0.0104

Theoretical accel.at stall

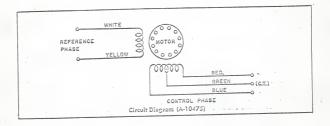
rad 22800

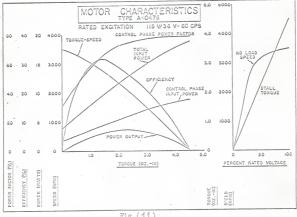
Weight, oz 21

Electrical characteristics:	Reference phase	Phase:
Voltage volts	115	36 C.T.
Frequency cps	60	60
Current (stall) amns	. 12	.310

Power input (stall) watts	9.9	8.8
Power factor (no load)	•25	•31
Power factor (stall)	• 72	• 79
R (stall) ohms	690	92
X (stall) ohms	666	71
Z (stall) ohms	960	116
Effective Resistance (stall)ohms	1340	147
Par. turning cap MFD	1.9	14

 $T_{f} = 0.03$, oz-in.





APPENDIX B

```
C CALCULATION OF DAMPER PERFORMANCE
   V0=702
   AJ1=.00002
   SM1= 000005
   VC=10.
   AK2= .002385
   AK1=.494
   H1=0.25
   H2= .5
   A=0.5935
   B=0.644
   W=0.376
   W1=0.3
   AF=(W-W1)/2.0
   T1=.03
   PI=3.1415927
   C1=.0104
13 VD=VD+1.0
   V=VD/900000.
   AL=((2°*PI*(A**3)*W1*V*H1)/(8-A))+((PI*(A**4)*V*H2)/AE)
   BT=(2.0%PI*(B**3)*(((WI*H1)+.038)*V)/(B-A))+((PI*(B**4)*V*H2)/AE)
   AJM=0.076/386.4
   AJ=AJM+SM1
   AM=(AJ*AJ1)/AL
   Q=-((AL+ST)*T1)/AL
   \Delta K = (C1 + \Delta K2)
   C=(((C1+BT+AK2)/AL)*AJ1)+AJ
   P=AK1*VC
   ZT=C/((AK*AM*4.00)**0.5)
   X1=-(.5*C)/AM
   X2=(((C/AM)**2)**25)-(AK/AM)
   X=X1+(X2**.5)
   Y=X1-(X2**,5)
   PQ = (P+Q)/((X-Y)*AK)
   PUNCH5 . AM . C . Q . X 2
 5 FORMAT(4F18,10)
   T=0.0
 8 T=T+.02
   VL1 = (PO*Y)*EXP(X*T)
   V1.2 = -(PQ \times X) \times EXP(Y \times T)
   VL3=PQ*(X-Y)
   VL=VL1+VL2+VL3
   Pt NCH4 , ZT , VL1 , VL2 , VL3 , VL , VD , T
 4 FORMAT(F8.5.3F13.3.F12.6.F7.3.F6.2)
   IF(T-0,2)8,9,9
 9 T=0.0
15 T=T+0.4
   VL1=(PQ*Y)*EXP(X*T)
   VL2=-(PO*X)*FXP(Y*T)
   VL3=PQ*(X-Y)
   PUNCH4, ZT, VL1, VL2, VL3, VL, VD, T
   IF(T-1.2)15,10,10
10 IF(VD-100.)13,14,14
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requirements of the degree

MASTER OF SCIENCE

Department of Mechanical Engineering

KANSAS STATE UNIVERSITY Manhattan, Kansas

1966

"Load compensation" method of stabilizing servomechanisms usually employs a viscous inertial type damper attached to servomotor shaft.

Inside the damper, a flywheel rides on a frictionless bearing. A thin metal casing rigidly fixed to motor shaft encases the flywheel. The damper is filled with viscous fluid. Such a damping device, when used in servo systems, has the effect of altering the dynamic characteristics of servomotor.

The damping torque offered by the viscous inertial damper is a function of various parameters such as viscosity of the fluid inertia of the damper mass (flywheel) and the dimensions of the damper. Results of variations of these parameters on dynamic properties of the servomotor are studied in this report.

Basic equations of motion for servomotor with damper casing and damper mass are derived from Newton's second law of motion. The equations are as follows:

$${\tt T} \, - \, \beta \, \, (\frac{{\tt d}\theta}{{\tt d}\, t} \, - \, \frac{{\tt d}\theta}{{\tt d}\, t}) \, - \, C_1 \, \, \frac{{\tt d}\theta}{{\tt d}\, t} \, - \, {\tt T}_{\tt f} = \, {\tt J} \, \frac{{\tt d}^2\theta}{{\tt d}\, t^2}$$

and

$$\alpha$$
 ($\frac{d\theta}{dt} - \frac{d\theta}{dt}$) - $T_f = J_1 = \frac{d^2\theta_1}{dt^2}$

These two differential equations are solved simultaneously for angular velocity of motor shaft.

It was found that the flywheel inertia and viscosity of the fluid has greater influence on the damping ratio of motor response. Thick fluid film has little effect on damping torque per unit relative angular velocity of motor shaft and flywheel, but thinner fluid film increases

its value rapidly. Viscosity has the linear effect on the damping torque per unit relative angular velocity but depends upon the amount of fluid in the damper.