

LOAD MANAGEMENT FOR WIND DRIVEN
AC GENERATOR

by

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INTRODUCTION

Presently extensive research is being done to find alternate sources of energy so that the dependence on fossil fuel can be reduced. Wind energy has received much attention in recent years because of the fact that it produces no waste to pollute the environment in comparison with other sources such as nuclear power or coal.

To develop wind energy much research work is being done to make it into a practical source. Here at Kansas State University research is being carried out to make a practical economical unit which can supply an average farm load. Savonius rotors are used to extract power from the wind to drive an AC generator. The problem faced is to keep the frequency constant or nearly constant which is a requirement for most electrical appliances. The variation in the frequency is because of the variation in the wind speed.

One way to maintain nearly constant frequency is to manage the load so as to keep the average power output of the machine proportional to the wind power. For this reason some experiments were performed on an AC generator so as to examine the behavior of the machine by switching various loads on and off.

Chapter 1 has the general information on wind energy, history of wind mills, the power available in the wind and means to extract this power. A brief description of different rotors which are now in use is also given in this chapter.

Chapter 2 contains an analysis of the transient behavior of the system. A wind turbine has a random mechanical output due to the random speed of the wind. It is then desirable to study the output of the AC generator when the load is being managed to match available wind power

to actual generator output. To make the analysis easier, the wind turbine may be viewed as behaving like a DC motor . This is a reasonable approximation because of the fact that by varying the voltage input to the DC motor a similar effect can be experienced as when the wind turbine is subjected to a varying wind.

Experimental work was also performed on the same model (DC motor driving an AC generator) in the laboratory. Several different graphs showing the power vs. frequency characteristics are presented so as to display the behavior of the machine under load. Different resistive loads were connected, including an electrolysis cell which was used to represent a lead acid battery (characteristics of electrolysis cells and lead acid batteries are quite similar). For each load two different graphs are plotted, one for constant excitation of the AC generator and the second for variable excitation from an exciter. The speed of the exciter was proportional to the shaft speed of the generator. Chapter 3 contains the details of this experimental work.

Chapter 1: General Discussion of Wind Power

The fast development of technology has made developed countries highly dependent on oil as a fuel. The Arabs have a monopoly on this fuel and the past four years have shown that it is not wise to depend on Middle Eastern oil reserves. Great effort is being made by the developed countries to find an alternative source of energy. Currently, extensive research is being done on wind energy so as to make it a practical source of energy which may partially replace oil.

Wind energy has a long history. Windmills have been in use for many centuries for irrigation and grinding grain. In the sixteenth century the Dutch took the lead in improving the design of windmills, particularly the rotors.¹ The first advanced 100 KW turbine, built by the Russians, appeared in 1931.¹ The English designed a large wind turbine for generating electricity in 1940, as did the French and Germans.¹ The Americans built their largest wind turbine, capable of delivering 1.25 MW, in Vermont in 1940.² It was in the early 1940's that research on wind energy nearly came to an end because of the availability of cheaper oil and coal to produce electricity. It was not until recently, due to the trouble with oil producing countries, that the research on wind energy started again. In 1975 NASA commissioned its multi-million dollar wind turbine project capable of producing 100 KW on an experimental basis.¹

Different kinds of wind turbines can be used to extract power from the wind. They can be roughly divided into three categories in terms of the orientation of their axes of rotation:

- (1) Vertical Axis Rotors in which the axis of rotation is perpendicular to both the surface of the earth and the wind direction (e.g., Savonius and Darrieus).

- (ii) Cross Wind Horizontal Axis Rotor in which the axis of rotation is both horizontal to the earth's surface and perpendicular to the direction of the wind stream.
- (iii) Horizontal Axis Rotor in which the axis of rotation is parallel to the direction of the wind stream.

Savonius falls in the first category, i.e., a vertical axis machine. This machine has the advantage that it does not have to be turned into the wind as the direction of the wind stream varies, thus eliminating equipment (i.e., Servo motors) which changes the direction of the turbine in response to wind direction. The construction is typically that of a vertical cylinder sliced in half from the top to bottom with the two halves pulled apart. It has a fairly high efficiency (31%), but is inefficient per unit weight since all the area swept by the wind is occupied by the metal. Thus, in order to develop 1000 KW in a wind of 30 MPH, a cylinder about 100 ft. in diameter and 300 ft. in height would require almost 30,000 sq. ft. of metal covering; whereas, the same power can be extracted from other turbines using about 1/30 of the metal area required by Savonius.²

This is just the comparison of the rotors. Savonius rotors may still be competitive with the propeller-type rotors because, although the two propeller rotor takes far less steel than the Savonius, the massive steel tower on which the propeller blade is mounted accounts for a large tonnage of steel. The Savonius can be operated without mounting it on a tower. Keeping this in view, the Savonius has some advantages over the other rotors and it is because of this that the Savonius has been selected for additional experimental work at Kansas State University.

Figure 1 shows the curve C_p (ratio of the output power of the turbine to the power available in the wind) versus the ratio of blade tip speed u to wind velocity v for a Savonius rotor.¹ It is seen that for maximum C_p the ratio of blade tip speed to free flow wind speed is about unity. It is self-starting, has a high starting torque, and can be easily used for driving electrical generators or water pumps.

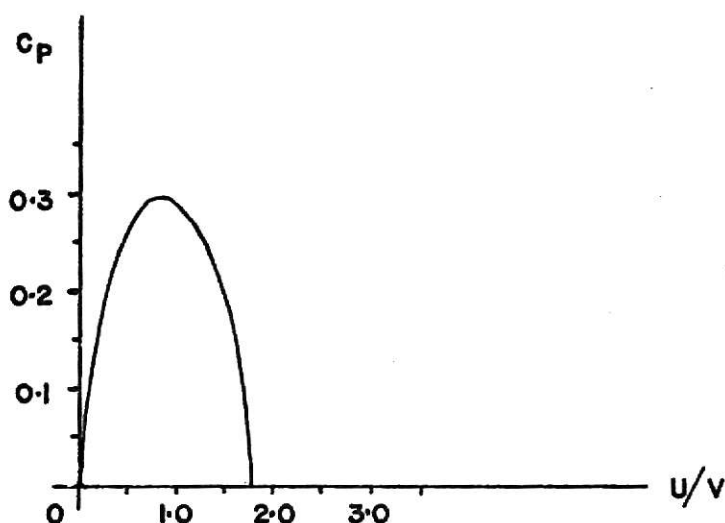


Figure 1
 C_p versus $\frac{u}{v}$ for a
Savonius rotor

The Darrieus rotor is also a vertical axis machine which was invented in 1920 in France. Extensive research has been done on it. These rotors have low starting torques; in fact, they are not self-starting and are usually combined with auxiliary rotors to increase the starting torque. They have a high tip speed to wind speed ratio and, therefore, have relatively high power output for a given rotor weight and cost. These machines have high efficiency and with low capital cost may well be suited for all sizes of applications. There are various types of Darrieus rotors and they can be designed to operate with two, three or more blades.¹ Figure 2 shows the curve of the power coefficient C_p versus the ratio of blade tip speed to wind velocity.

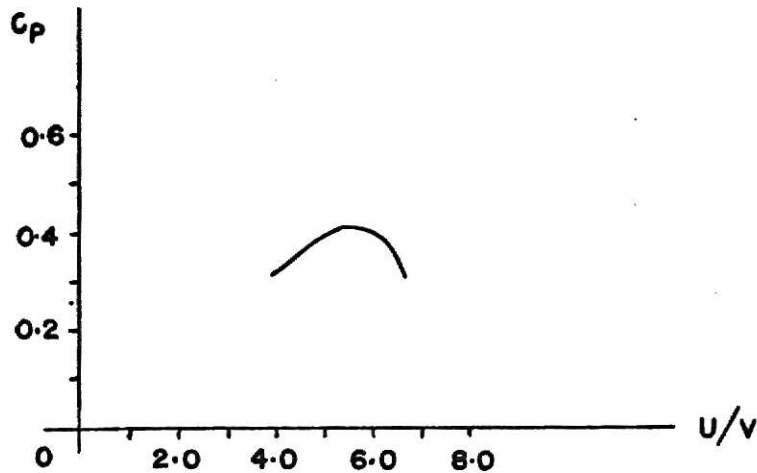


Figure 2
 C_p versus $\frac{u}{v}$ for a
 Darrieus rotor

Horizontal axis rotors have a large variety of different rotors; i.e., propeller-type two bladed, three bladed, and farm windmill multi-bladed. The two bladed propeller-type is very common for obtaining large outputs because of its high C_p as seen from the graph in Figure 3. These propeller-type rotors are efficient, have high RPM and high power-out per given rotor weight, but with relatively low torque.

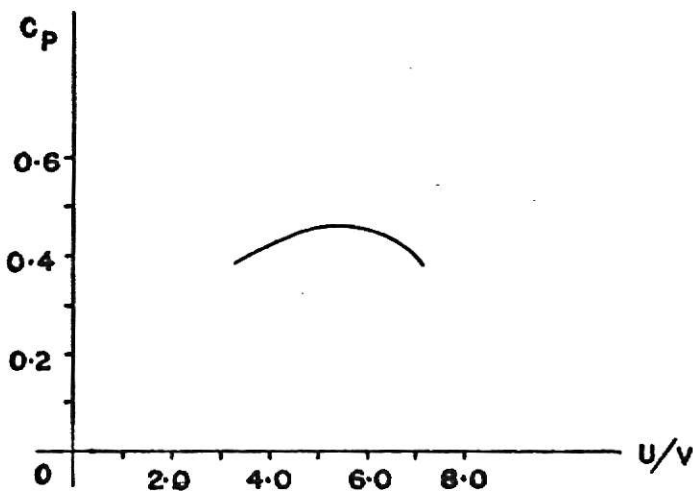


Figure 3
 C_p versus $\frac{u}{v}$ for a
 two bladed propeller-
 type rotor

The disadvantage with all wind turbines is that the power output is not constant, but varies as some function of the wind speed. The power in

the wind is given by the formula³

$$P_w = 5.09 \times 10^{-3} V^3 \text{ Watts/ft.}^2 \quad (V \text{ in Mph}) \text{ ----- 1.1}$$

and, if the coefficient of performance of the wind turbine is C_p , then the power out of the wind turbine is

$$P_o = C_p 5.09 \times 10^{-3} V^3 \text{ Watts/ft.}^2 \text{ ----- 1.2}$$

The efficiency of these turbines does not usually exceed 0.4. From this we see that the power out of the wind turbine varies as the cube of the wind speed and from the C_p curves it is seen that the optimum rotor speed for a given wind speed is one at which C_p is maximum.

Above a certain rated wind speed it is desirable to have a constant power output from the wind. One method of doing this is to change the pitch of the blade by some mechanism to "spoil" the air flow (reduce C_p) so the output of the turbine stays within certain limits.

The research conducted at Kansas State University is mainly focusing on developing wind units capable of handling agricultural loads of 5 kw or more. The electric machine under consideration to be coupled to the wind turbine is an AC generator. The reason for selecting an AC generator is that their initial price is less compared to other machines and they require less maintenance. The disadvantage of such a generator is that the frequency varies with shaft speed. The requirement for many applications is such that the frequency has to be maintained nearly constant irrespective of the random input from the wind turbine. A proposed method to maintain a constant frequency is to use some kind of feedback system connecting the load and the generator, and adjust the load according to the mechanical input of the generator. The feedback system will consist of a microprocessor

circuit which will sense the frequency and adjust the load accordingly. This will be discussed in the next chapter.

To determine the performance of an AC generator under these conditions (i.e., the mechanical input to the generator is not constant) and to determine the behavior of voltage and current in the machine due to switching of load resistors, a 3-phase AC generator was tested in a laboratory under varying conditions such as might be encountered in a farm load.

It is seen from Figure 1 that if C_p has a small value then torque is small also because of the relation

$$\text{Torque (T)} = \frac{\text{Power (P}_o\text{) out from the turbine}}{\text{angular velocity } (\omega)}$$

i.e.,

$$\text{Torque (T)} = \frac{C_p (\text{Power (P}_w\text{) available in the wind})}{\text{angular velocity}}$$

From this it is seen that torque is proportional to C_p . If the torque is small when $\frac{u}{v}$ is small and if the generator is loaded the turbine may not start or it might take a long time in reaching its desired angular velocity. One reason for the analysis of the AC generator is to see how the load presented to the turbine behaves when the generator excitation is proportional to the angular velocity or when the excitation to the field is from a constant source. The power output of the generator should be relatively low when the machine is running at low speeds to prevent stalling. Chapter 2 has a transient analysis of the machine. Chapter 3 discusses the details of the behavior of the generator for different loads.

Chapter 2: Transient Analysis of an AC Generator

In this chapter there is a transient analysis of an AC generator. The power output of the wind turbine is not constant but varies as some function of wind velocity. Therefore, there tends to be a variation in the angular velocity of the generator which results in an undesirable variation in frequency. To study the variation in angular velocity ω and the stability of the system, the wind turbine is modelled as a DC motor as shown in Figure 4. A DC motor can behave almost in the same manner as the wind turbine if the voltage input to the motor is varied similar to the wind power input to the wind turbine. The dynamic equations of a motor driving a generator are well known, which makes the analysis easier. The time constant of the motor should match the time constant of the turbine, so the response time of the model should be similar to that of the wind turbine system. The approximation was made in the motor circuit that the value of L_a was zero. This makes the analysis easier and has minimal effect on the result as will be seen later in this chapter.

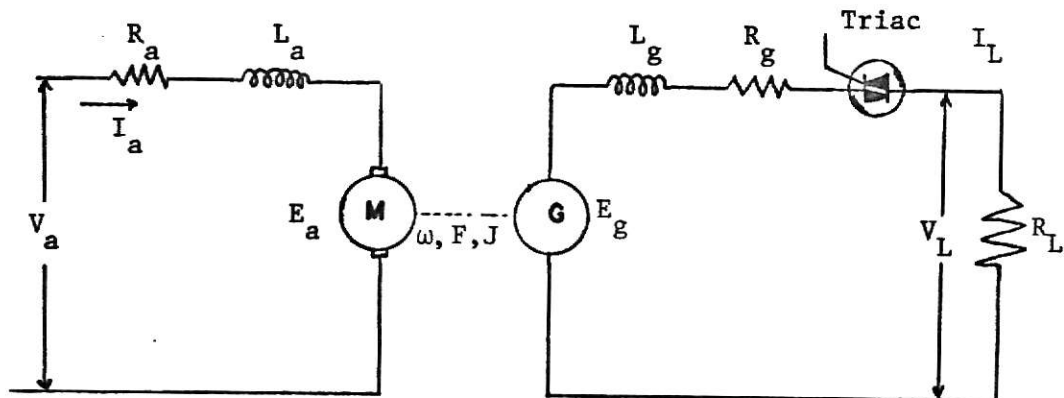


Figure 4

Circuit diagram of the DC motor and AC generator

If E_a and E_g are the induced voltages⁴ in the motor and generator, respectively, and E_g is the line to neutral voltage, then

$$E_a = K\phi N\omega$$

where

ϕ is the flux in wb/m²

N is the number of conductors

ω is the angular velocity rad/sec.

If we assume that the fields of the DC motor and AC generator are constant (not functions of angular velocity), then

$$E_a = K_a \omega \text{ ----- } 2.1$$

$$E_g = K_g \omega \text{ ----- } 2.2$$

The torque developed by the motor is given by the equation

$$T = K\phi I_a$$

and since ϕ is constant

$$\text{Torque } T = K_t I_a \text{ ----- } 2.3$$

This torque can also be expressed as

$$T = J \frac{d\omega}{dt} + F\omega + \frac{P_g}{\omega} \text{ ----- } 2.4$$

where

$$T_L = \frac{P_g}{\omega} = K_R I_L \text{ ----- } 2.5$$

J = combined inertia of motor and generator

F = equivalent viscous friction of motor and generator

T_L = opposing load torque

If $L_a = 0$, then

$$I_a = \frac{V_a - E_a}{R_a} \text{ ----- } 2.6$$

Substituting equation 2.1 in 2.6

$$I_a = \frac{V_a - K_a \omega}{R_a} \text{ ----- } 2.7$$

From Kirchhoffs voltage law in the generator circuit,

$$E_g = I_L (R_g + R_L) + L_g \frac{dI_L}{dt} \text{ ----- } 2.8$$

The Laplace transforms of equations 2.3, 2.4, 2.5, and 2.7 yield

$$T(s) = K_t I_a(s) \text{ ----- } 2.9$$

$$T(s) - T_L(s) = Js \omega(s) + F\omega(s) \text{ ----- } 2.10$$

$$T_L(s) = K_R I_L(s) \text{ ----- } 2.11$$

$$I_a(s) = [V_a(s) - K_a \omega(s)] \frac{1}{R_a} \text{ ----- } 2.12$$

Laplace transform of equation 2.8 is

$$E_g(s) = I_L(s) (R_g + R_L) + L_g s I_L(s)$$

$$= I_L(s) [R_g + R_L + L_g s]$$

$$E_g(s) = I_L(s) [1 + s \tau_g] [R_g + R_L] \text{ ----- } 2.13$$

where

$$\tau_g = \frac{L_g}{[R_g + R_L]}$$

is the generator time constant.

The block diagram for the equation 2.10 is shown in Figure 5.

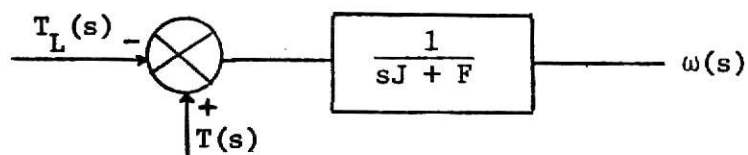


Figure 5

"Block diagram representation of equation 2.10"

The complete diagram for the Laplace transformed equations already derived is shown in Figure 6.

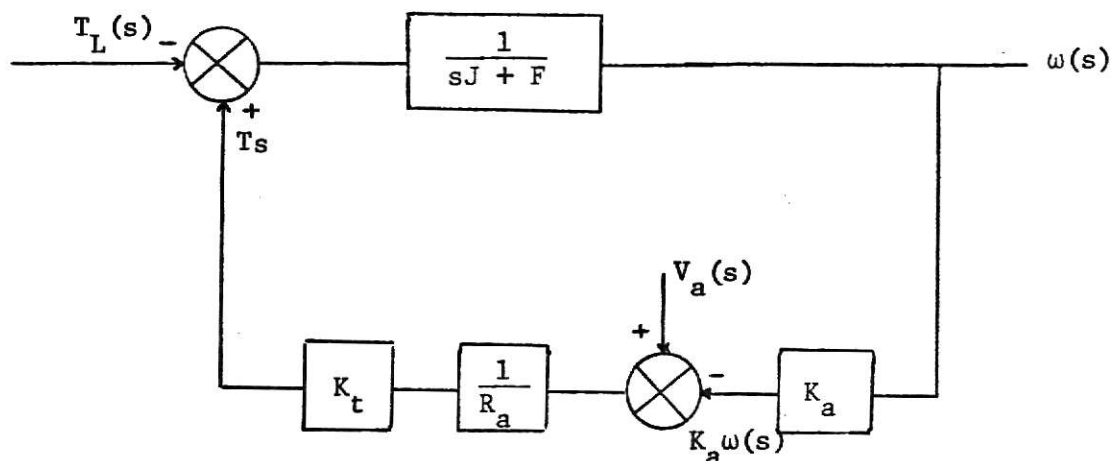


Figure 6

"Block diagram of the motor circuit with torque $T_L(s)$ due to the generator"

Solving equations 2.9, 2.10 and 2.12 for $\omega(s)$ can be partly accomplished with the following steps:

$$\frac{\omega(s)}{T(s) - T_L(s)} = \frac{1}{sJ + F}$$

$$T(s) = \frac{K_t}{R_a} [V_a(s) - K_a \omega(s)]$$

$$\omega(s) = \frac{1}{sJ + F} \left[\frac{K_t}{R_a} \{V_a(s) - K_a \omega(s)\} - T_L(s) \right]$$

grouping the $\omega(s)$ terms

$$\left\{ 1 + \frac{K_t K_a}{R_a (sJ + F)} \right\} \omega(s) = \frac{1}{sJ + F} \left\{ \frac{K_t}{R_a} V_a(s) - T_L(s) \right\} \quad \text{--- 2.14}$$

$T_L(s)$ is the opposing torque and is given by the equation 2.11. Figure 6 is not a complete diagram of the system. To complete the system diagram and to keep the variation in $\omega(s)$ within reasonable limits, there should be a feedback path which will adjust the value of T_L accordingly. Thus, the complete diagram of the system might be as shown in Figure 7.

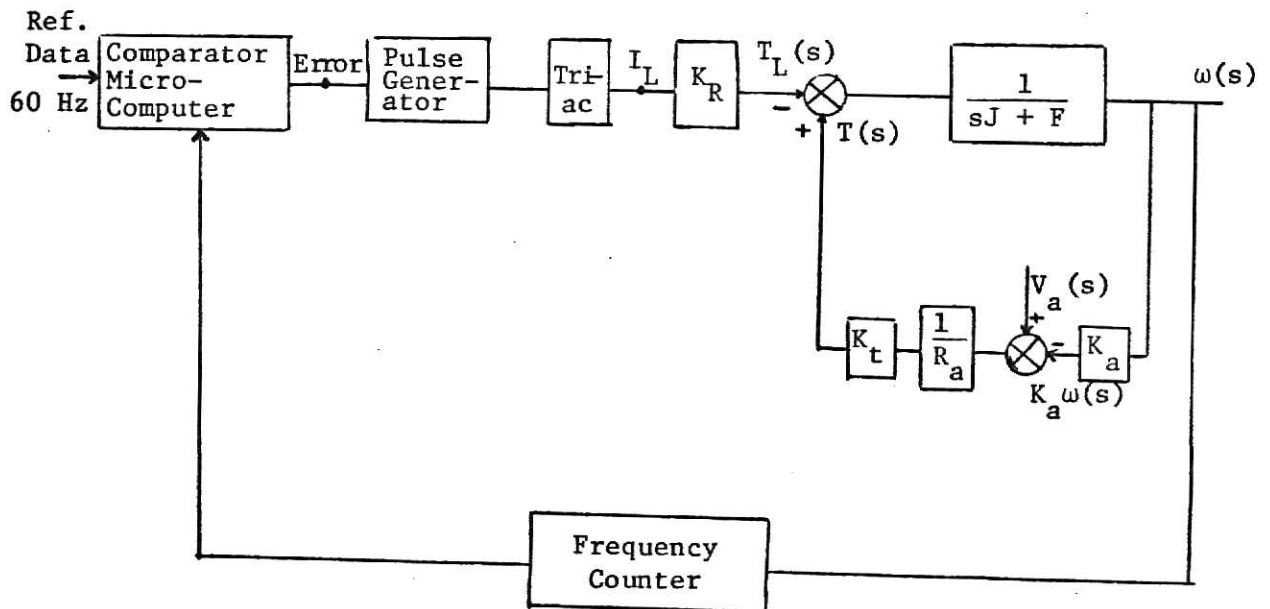


Figure 7

Block diagram of the complete system showing the feedback path which is responsible for maintaining the frequency constant

In Figure 7 the feedback is the actual frequency from the frequency counter which is matched with the reference frequency, i.e., 60 Hz in the microcomputer. A discrete error is sent to the pulse generator. The pulse generator generates a pulse train with the pulse width of γ seconds, i.e., the width is proportional to the error. It is this pulse which triggers the triac for that duration, thus allowing the load current I_L to flow in load R_L which in turn changes the load torque T_L (refer to equation 2.11). That is, if the value of $\omega(t)$ is greater than 60 Hz, turning on the load will cause $\omega(t)$ to decrease. On the other hand, if $\omega(t)$ is low (i.e., generator operating at low frequency), no pulse will be generated. There will be no load current I_L and from equation 2.11 T_L will be zero. With no opposing torque, the value of $\omega(t)$ will tend to increase.

Thus it is seen that the frequency feedback eventually varies the opposing torque T_L .

We assume that pulses of constant width γ and period τ are generated by the pulse generator, as shown in Figure 8.

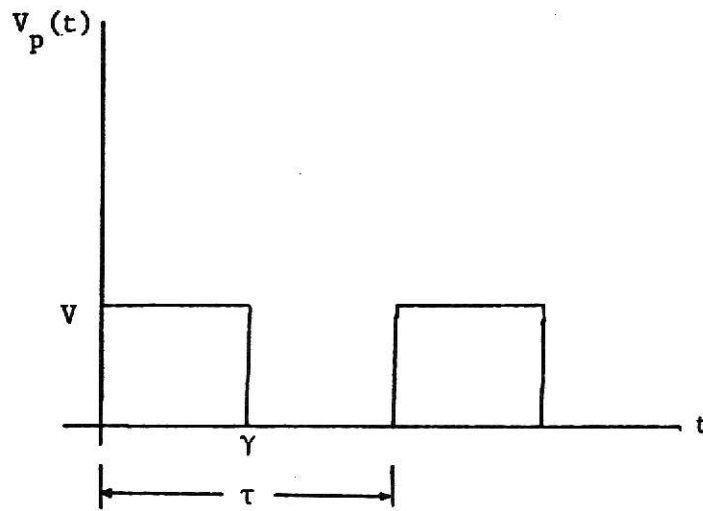


Figure 8

Pulses of width γ seconds and time period τ

The phase voltage from the generator is given by

$$E_g(t) = V_1 \sin \omega t$$

For one phase, Figure 9 shows the generator voltage.

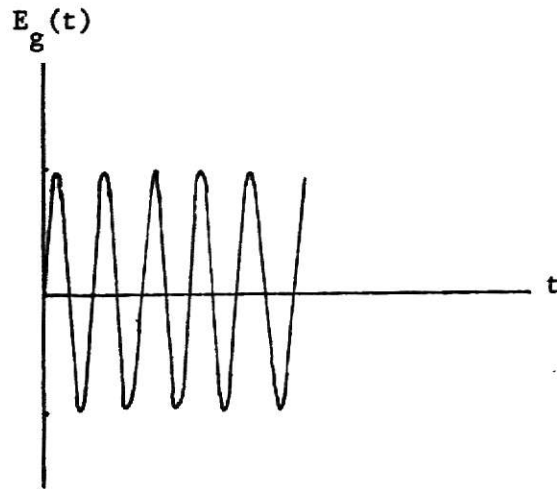


Figure 9

Voltage from the generator

The output voltage at the load will be of the form as seen in Figure 10.

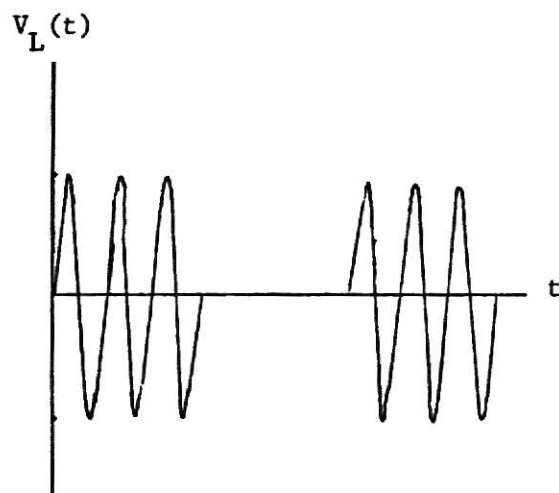


Figure 10

Voltage at the load

Since the load is assumed resistive there will be no phase difference between V_L and I_L and $V_L(t)$ has the same form as the load current I_L .

$$I_L(t) = \frac{V_L(t)}{R_L}$$

Further it can be shown that the opposing torque T_L developed in the generator⁴ due to the current I_L is given by

$$T_L = K V_L(t) I_L(t)$$

and since the current and voltage are both sinusoidal with no phase difference

$$T_L \propto \sin^2 \omega t \quad \text{-----} \quad 2.15$$

Thus because of the load current which has the same wave shape as in Figure 10, it is seen that the load torque T_L per phase will be as shown in Figure 11.

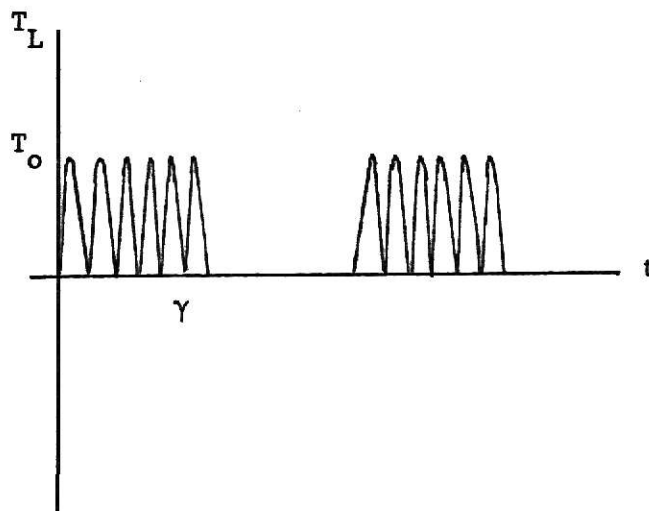


Figure 11

Load torque T_L

The torques due to the other two phases have the same shape and are spaced 120° apart so the total torque from the three phases is essentially a constant. We will approximate the per phase torque by a constant equal to one third of the average three phase torque. This will be a pulse train with amplitude T_o and width γ seconds as shown in Figure 12.

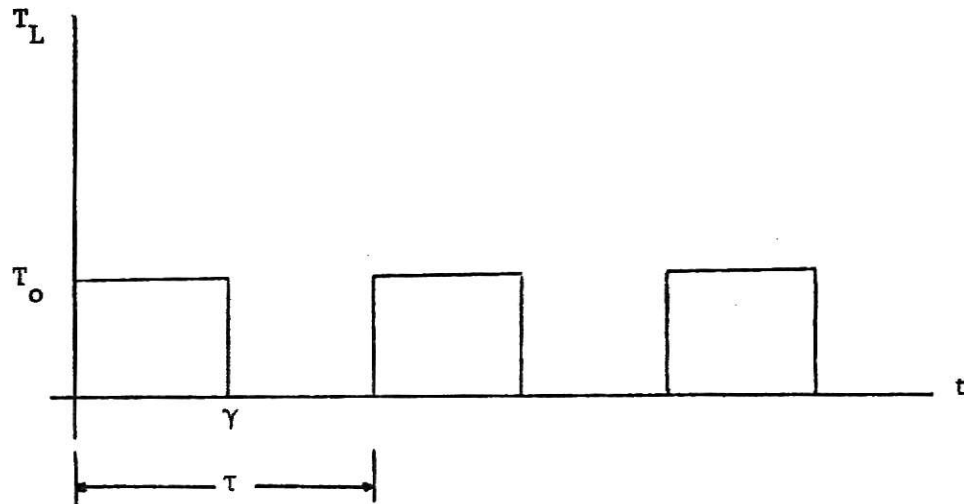


Figure 12

Pulse train representing load torque

Now considering equation 2.14, there are two variables, $V_a(s)$ and $T_L(s)$. Super position applies because the model is linear. We may find a solution by assuming first that $T_L = 0$ and then $V_a = 0$ and then adding the separate results. If $T_L(s) = 0$, then

$$\begin{aligned} \frac{\omega(s)}{V_a(s)} &= \frac{\frac{K_t}{(sJ + F) R_a}}{1 + \frac{K_t K_a}{R_a (sJ + F)}} \\ &= \frac{\frac{K_t}{R_a J}}{s + \frac{R_a F + K_t K_a}{R_a J}} \end{aligned}$$

$$\frac{\omega(s)}{V_a(s)} = \frac{K_1}{s + \alpha} \quad \text{-----} \quad 2.16$$

where

$$K_1 = \frac{K_t}{R_a J}$$

and

$$\alpha = \frac{R_a F + K_t K_a}{R_a J}$$

is the system time constant.

In the second case, $V_a(s) = 0$, and

$$\begin{aligned} \frac{\omega(s)}{T_L(s)} &= \frac{-\frac{1}{(sJ + F)}}{1 + \frac{K_t K_a}{R_a (sJ + F)}} \\ &= \frac{-\frac{1}{J}}{s + \frac{R_a F + K_t K_a}{R_a J}} \end{aligned}$$

$$\frac{\omega(s)}{T_L(s)} = \frac{K_2}{s + \alpha} \quad \text{-----} \quad 2.17$$

where

$$K_2 = -\frac{1}{J}$$

and

$$\alpha = \frac{R_a F + K_t K_a}{R_a J}$$

We can model the two equations 2.16 and 2.17 as shown in Figure 13.

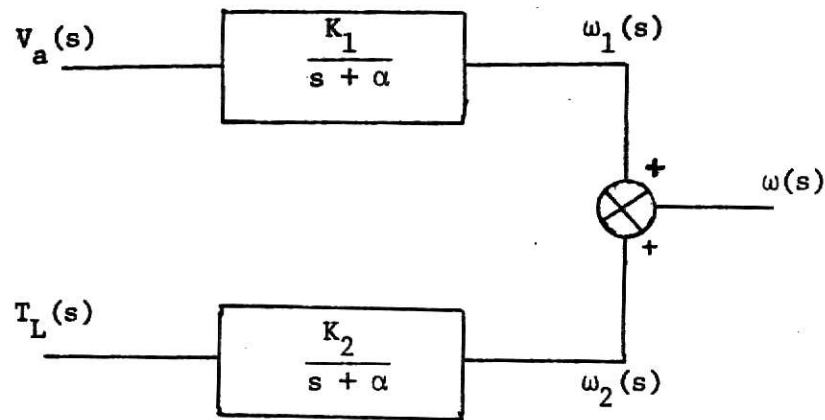


Figure 13

Simplified block diagram showing the two variables $V_a(s)$ and $T_L(s)$ as input to the system

We can solve for $\omega_1(s)$ and $\omega_2(s)$ separately and add them to get the desired output frequency $\omega(s)$

$$\omega(s) = \omega_1(s) + \omega_2(s)$$

Equation 2.16 can be written as

$$\omega_1(s) = \frac{K_1 V_a(s)}{s + \alpha}$$

If $V_a(s) = \frac{V_o}{s}$ (a step input), then

$$\omega_1(s) = \frac{K_1 V_o}{s(s + \alpha)}$$

Taking the inverse Laplace transform gives

$$\omega_1(t) = \frac{V_o K_1}{\alpha} (1 - e^{-\alpha t}) \mu(t) \text{ ----- 2.18}$$

where $\mu(t)$ is a step function.

Likewise, equation 2.17 can be written as

$$\omega_2(s) = \frac{K_2 T_L(s)}{s + \alpha}$$

To see the variation in $\omega_2(s)$ due to the variation in $T_L(s)$ the pulse train shown in Figure 12 is applied to the system. Mathematically the pulse train can be written as

$$T_L(t) = \sum_{n=0}^{\infty} \{ \mu(t - n\tau) - \mu(t - [n\tau + \gamma]) \} T_o$$

where

n is the number of a given pulse

and

τ is the time period of the pulse.

Laplace transform of this pulse train is

$$\mathcal{L}[T_L(t)] = T_L(s) = \sum_{n=0}^{\infty} \left\{ \frac{1}{s} e^{(-n\tau)s} - \frac{1}{s} e^{-(n\tau + \gamma)s} \right\} T_o$$

Equation 2.17 becomes

$$\omega_2(s) = \frac{K_2 T_L(s)}{s + \alpha} \text{-----} 2.19$$

Substituting $T_L(s)$ in equation 2.19 gives

$$\begin{aligned} \omega_2(s) &= \sum_{n=0}^{\infty} \left[\frac{K_2}{(s + \alpha)} \left\{ \frac{1}{s} e^{(-n\tau)s} - \frac{1}{s} e^{-(n\tau + \gamma)s} \right\} T_o \right] \\ &= \sum_{n=0}^{\infty} \left[K_2 T_o \left\{ \frac{e^{(-n\tau)s}}{s(s + \alpha)} - \frac{e^{-(n\tau + \gamma)s}}{s(s + \alpha)} \right\} \right] \end{aligned}$$

Breaking into parts by partial fraction

$$\begin{aligned} \omega_2(s) &= \sum_{n=0}^{\infty} \left[K_2 T_o \left\{ \left[\frac{e^{(-n\tau)s}}{s} - \frac{e^{(-n\tau)s}}{(s + \alpha)} \right] \frac{1}{\alpha} \right. \right. \\ &\quad \left. \left. - \left[\frac{e^{-(n\tau + \gamma)s}}{s} - \frac{e^{-(n\tau + \gamma)s}}{(s + \alpha)} \right] \frac{1}{\alpha} \right\} \right] \end{aligned}$$

The inverse Laplace transform of $\omega_2(s)$ is

$$\begin{aligned}\omega_2(t) &= \sum_{n=0}^{\infty} \left[\frac{K_2 T_o}{\alpha} \left\{ \left[\mu(t - n\tau) - e^{-\alpha(t - n\tau)} \mu(t - n\tau) \right] \right. \right. \\ &\quad \left. \left. - \left[\mu(t - n\tau - \gamma) - e^{-\alpha(t - n\tau - \gamma)} \mu(t - n\tau - \gamma) \right] \right\} \right] \\ \omega_2(t) &= \sum_{n=0}^{\infty} \left[\frac{K_2 T_o}{\alpha} \left\{ \left[1 - e^{-\alpha(t - n\tau)} \right] \mu(t - n\tau) \right. \right. \\ &\quad \left. \left. - \left[1 - e^{-\alpha(t - n\tau - \gamma)} \right] \mu(t - n\tau - \gamma) \right\} \right] \text{----- 2.20}\end{aligned}$$

Equation 2.18 gives the transient in angular velocity when a step voltage is applied and equation 2.20 gives the transient in angular velocity when rated torque is applied to the system as a series of pulses. If we suppose that the system is operating at a steady state, ω rad/sec., then by applying the rated torque the variation in the angular velocity can be seen.

The rated torque cannot actually be applied as a step function because the electrical time constant of the generator limits the rate of increase of load current and hence T_L . The electrical time constant can be evaluated from the circuit in Figure 14 which shows a simple single phase circuit diagram of the AC generator with load connected.

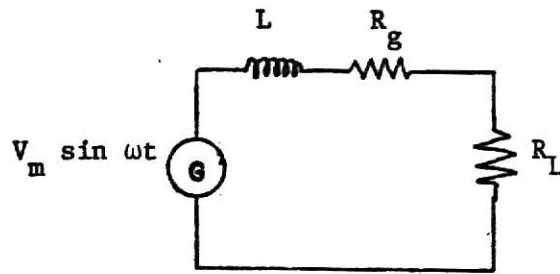


Figure 14

Equivalent circuit of generator

The differential equation for this circuit is

$$\frac{L}{dt} \frac{di}{dt} + Ri = V_m \sin \omega t$$

where

$$R = R_g + R_L$$

Taking the Laplace transform

$$Ls I(s) + RI(s) = \frac{V_m \omega}{s^2 + \omega^2}$$

or

$$I(s) = \frac{V_m \omega}{(s^2 + \omega^2)(s + \frac{R}{L})} \cdot \frac{1}{L}$$

The inverse Laplace transform is

$$I(t) = \frac{V_m \omega}{L (\omega^2 + \frac{R^2}{L^2})} \left[e^{-\frac{R}{L} t} - \cos \omega t + \frac{R}{\omega L} \sin \omega t \right] \text{-----} 2.21$$

A 3 phase permanent magnet generator has been purchased for wind experimentation at Kansas State University, so ratings appropriate to this machine will be used for illustration. The machine is rated at 5 kVA at 1800 rpm, 216 volts line to line or 125 volts line to neutral when connected in a 3 phase wye connection. For machines of this size, R_g is perhaps one percent of R_L and is usually neglected in calculations. The inductive reactance of this machine has not been measured but a typical value is $X_L = 1$ per unit. The actual value would be

$$X_L = \frac{[\text{rated volts per phase}]^2}{\text{rated VA per phase}} = \frac{(125)^2}{5000/3} = 9.38 \, \Omega$$

The corresponding inductance at 60 Hz is

$$L = \frac{X_L}{\omega} = \frac{9.38}{377} = .0249 \, \text{h}$$

where X_L is the synchronous reactance per phase.

The rated load resistance R_L would be 1 per unit or 9.38 ohms. The electrical time constant is then

$$\tau_e = \frac{L}{R} = \frac{.0249}{9.38} = 2.65 \, \text{msec}$$

By comparison, one half cycle of a 60 Hz wave has a period of

$$\tau_{60} = \frac{1}{2(60)} = 8.33 \, \text{msec}$$

Therefore, the electrical transient has essentially disappeared by the end of the first half cycle of the output wave. Zero crossing switches will be used to control the load current, so the load current and torque will be present for an integer number of half cycles. The electrical transient will thus be a relatively small part of the total torque pulse. This allows us to assume the unit step form of torque variation without major error.

Preliminary measurements indicate that the permanent magnet generator is between 87 and 88% efficient. We will assume 86% with the losses equally divided between electrical losses and mechanical losses. If the output electrical power is 5kw, the electrical power generated, P_g , must be greater than 5kw by about 7%. The load torque is then, from equation 2.5,

$$T_L = \frac{P_g}{\omega} = \frac{(1.07)(5000/3)}{377} = 4.73 \text{ Nm}$$

The viscous friction torque per phase of the generator is

$$F_g \omega = \frac{(.07)(5000/3)}{377} = .31 \text{ Nm}$$

so the viscous friction F_g of the generator is

$$F_g = \frac{.31}{377} = 8.21 \times 10^{-4} \frac{\text{Nm sec}}{\text{rad}}$$

For illustration purposes, let us assume that the motor has the same viscous friction, so the total viscous friction is

$$F = 2F_g = 1.642 \times 10^{-3} \frac{\text{Nm sec}}{\text{rad}}$$

The inertia J depends on the construction details of the motor and generator and can easily vary by a factor of two between machines of the same rating. The actual inertia of the generator was not available at the time of this writing, and the motor is only a model for the actual wind turbine, which makes a good inertia value difficult to determine. Textbook examples of motor-generator sets of about this size⁶ suggest values of combined inertia of about $J = 1 \text{ kg-m}^2$. We will use this figure to illustrate the technique.

Suppose our DC motor is rated as follows:

$$P_{in} = 7 \text{ kw}$$

$$P_{out} = 6 \text{ kw}$$

$$I_a = 28 \text{ A}$$

$$E_a = 250 \text{ volts}$$

$$R_a = 0.3 \text{ ohms}$$

$$n = 1800 \text{ rpm}$$

$$\omega = 1800 \left(\frac{2\pi}{60} \right) = 188.5 \text{ rad/sec.}$$

From equation 2.1

$$K_a = \frac{E_a}{\omega} = \frac{250}{188.5} = 1.33 \frac{\text{volt sec.}}{\text{rad.}}$$

From equation 2.4 for steady state conditions

$$T = F\omega + \frac{P_g}{\omega} = (1.64 \times 10^{-3})(188.5) + 4.73 = 5.04 \text{ Nm}$$

So we can solve for K_t from equation 2.3

$$K_t = \frac{T}{I_a} = \frac{5.04}{28} = 0.18 \frac{\text{Nm}}{\text{A}}$$

The necessary quantities in equation 2.20 can now be computed.

$$K_2 = -\frac{1}{J} = -1 \text{ kg}^{-1} \text{ m}^{-2}$$

$$\alpha = \frac{R_a F + K_t K_a}{R_a J} = \frac{(0.3)(.00164) + (0.18)(1.33)}{(0.3)(1)} = 0.80 \text{ s}^{-1}$$

Equation 2.20 now becomes

$$\begin{aligned} \omega_2(t) = & \frac{(-1)(4.73)}{.8} \sum_{n=0}^{\infty} \left\{ \left[1 - e^{-\alpha(t - n\tau)} \right] u(t - n\tau) \right. \\ & \left. - \left[1 - e^{-\alpha(t - n\tau - \gamma)} \right] u(t - n\tau - \gamma) \right\} \text{-----} \quad 2.21 \end{aligned}$$

If a step voltage is applied to the motor, the angular velocity $\omega_1(t)$ will increase until it reaches a steady state value. If load torque is now applied, the system tends to slow down. This is accounted for by the first minus sign in the above equation for $\omega_2(t)$. After a sufficient number of torque pulses of width γ and period τ have been applied, a new "steady state" condition will be reached where the system is slowing down when the torque is present and speeding up when it is absent, effectively oscillating around a steady state value of $\omega(t)$. We are interested in the variation in $\omega(t)$ which is just $\omega_2(t)$. Suppose $\gamma = 0.1$ sec. and $\tau = 0.2$ sec. Evaluating equation 2.21 for several values of t yields the following values.

$$\begin{aligned}
 \omega_2(0) &= 0 \\
 \omega_2(.1) &= -.45 \\
 \omega_2(.2) &= -.42 \\
 \omega_2(.3) &= -.84 \\
 \omega_2(.4) &= -.78 \\
 &\dots \\
 \omega_2(2n)(.1) &= -2.84 \quad \text{for } n \text{ large} \\
 \omega_2(2n + 1)(.1) &= -3.07
 \end{aligned}$$

The difference in angular velocity between the start and finish of a torque pulse is 0.237 rad/sec. This corresponds to 0.474 rad/sec. in electrical measure for a 4 pole machine or 0.075 Hz. This would imply that the frequency does not change significantly during one torque pulse.

Chapter 3: Behavior of an AC Generator Under Various Loadings

In this chapter are some experimental results obtained when a three phase AC generator was tested in the laboratory and operated under the following conditions:

1. AC generator operated with constant field current.
2. AC generator operated with variable field current obtained from a permanent magnet exciter.

Before studying the different types of excitations, the generator voltage build-up was studied. The generator was operated at no load for various shaft speeds which yield a range of output frequencies. The field of the generator was excited from a three phase rectifier source. Figure 15 shows the output voltage variation with respect to field current. Each curve is drawn at a constant frequency. It is seen from the graph that as the frequency decreases, a greater value of field current is required to maintain the same terminal voltage, e.g., to generate 60 volts at 70 Hz, 1.1 amperes of field current is required, whereas for the same voltage at 40 Hz, 2.7 amperes of field current is required. This can be easily shown from the equation

$$E_g = 4.44 \, k\phi f N \text{ ----- } 3.1$$

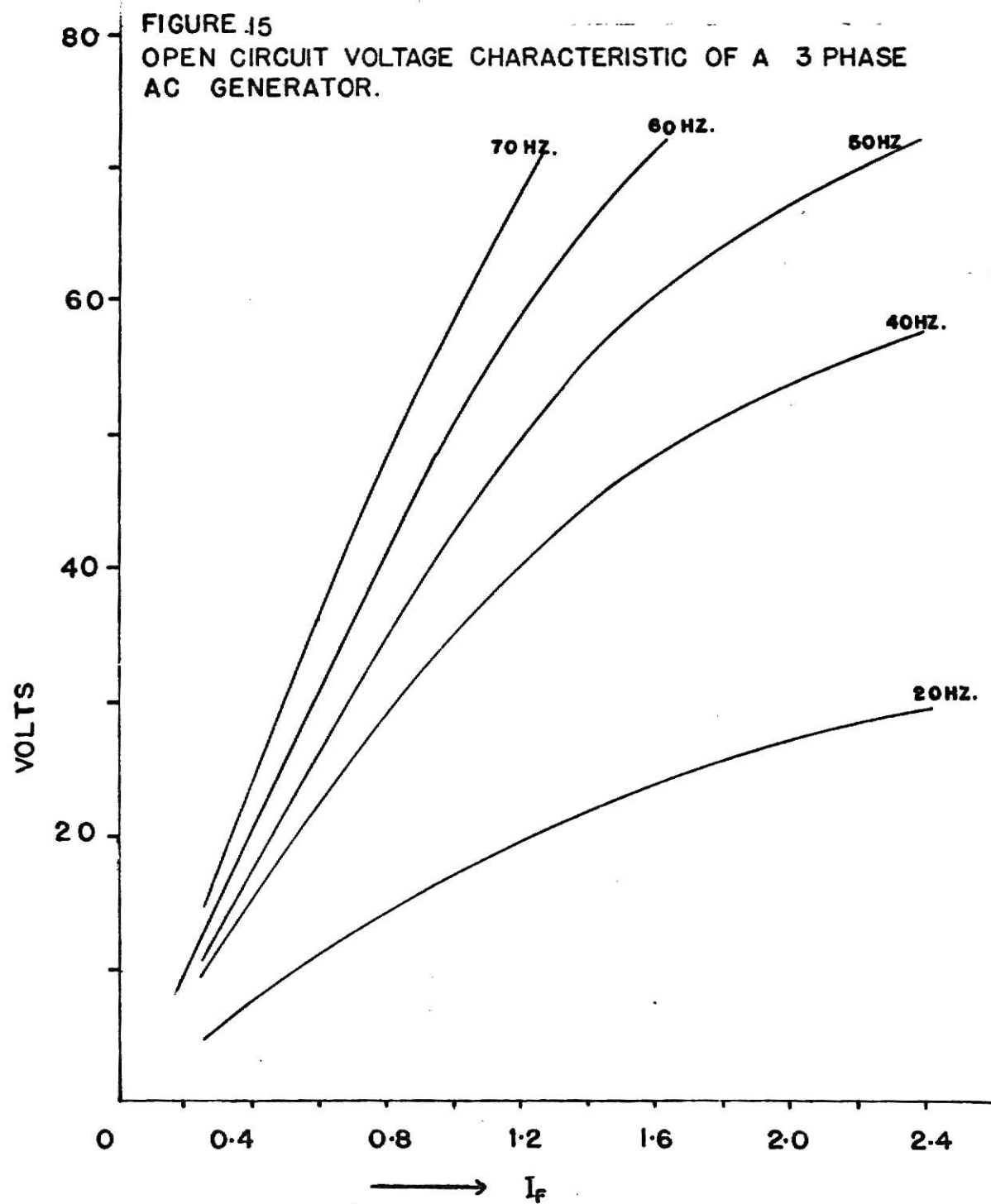
E_g = induced emf in armature

f = frequency in Hz

ϕ = flux in wb/m^2

N = number of turns/phase

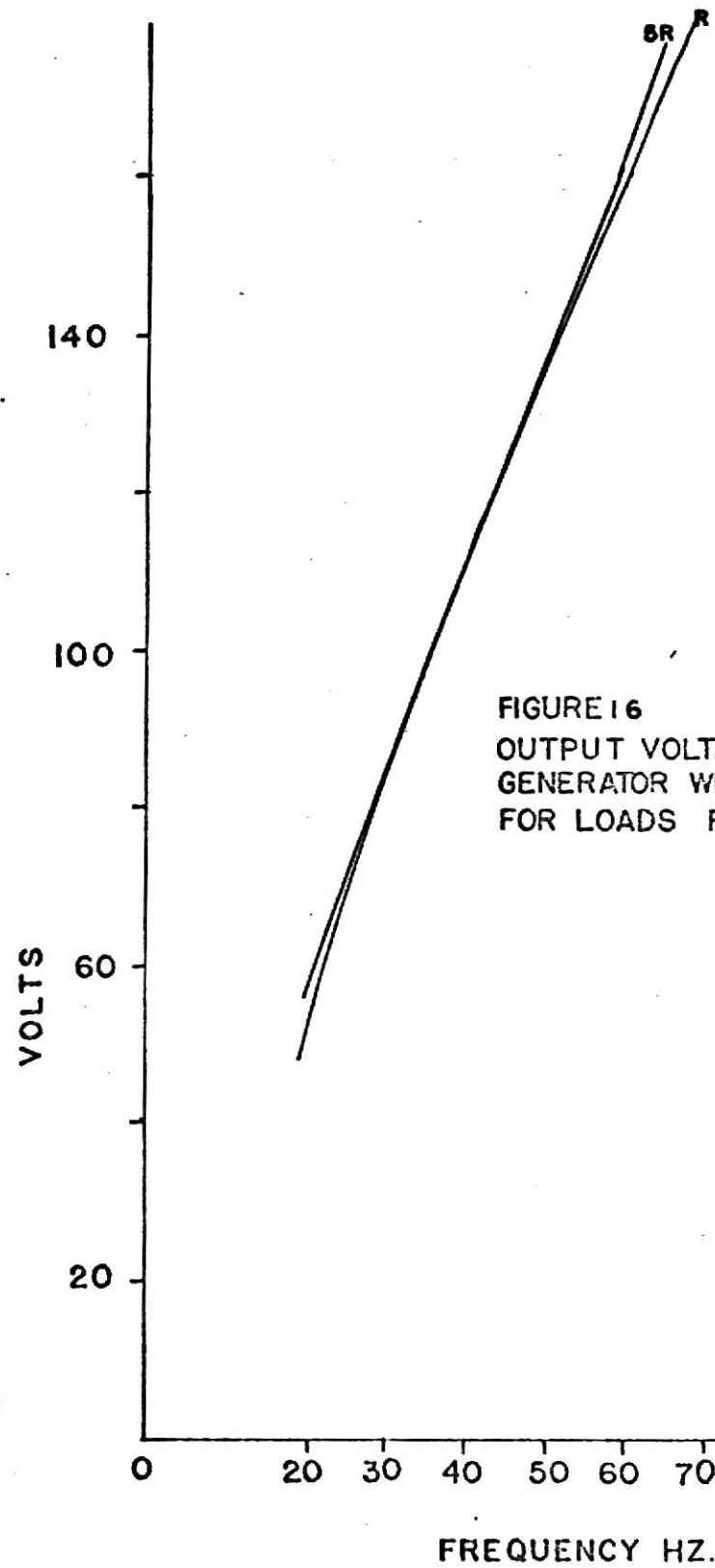
From equation 3.1, as frequency decreases, ϕ has to increase to keep the voltage constant.

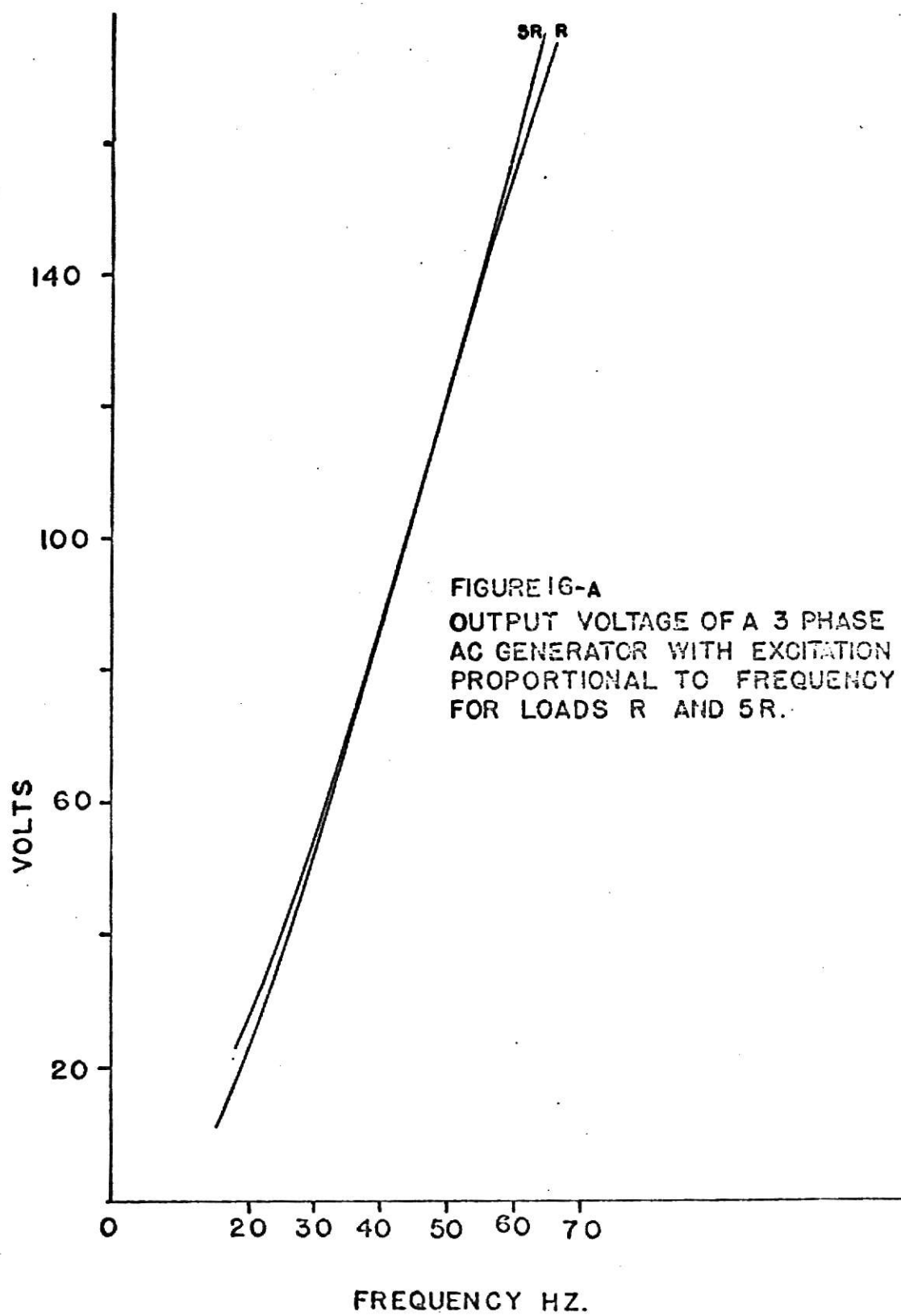


Now consider the case when the generator field is supplied from a constant voltage source. Two resistive loads of value R and $5R$ were used. The value of R was chosen such that it matched the rating of the AC generator, 260 ohms in this case. With this load connected to the generator, the field current I_f was adjusted to yield rated terminal voltage at 60 Hz, and then the load was switched to $5R$ from R . Due to this the voltage will increase. This switching was studied to see if the terminal voltage remains within reasonable limits when output power is changing rapidly. Figure 16 shows the relationship between the output voltage and frequency for the two loads R and $5R$ as labeled. It was seen that the rise in voltage is approximately 1.6 volts when switching takes place at 60 Hz which is about 1% of rated voltage.

Figure 16-A also shows the same behavior of output voltage and frequency for the same two loads R and $5R$, but for this case where the field of the generator is excited by an exciter (DC constant field generator). The adjustments in the field current I_f were again made so as to yield rated voltage at 60 Hz and then load was switched from R to $5R$ and the deviation in voltage examined. It is observed from the graph that the voltage increased by 3 volts which is approximately 2% of the rated voltage. It is thus concluded that the increase in voltage for the two cases is within reasonable limits.

The power output of a wind turbine varies with wind speed and turbine rpm. The load presented to the turbine by the generator should be a function of rpm that will efficiently extract power from the turbine while maintaining frequency within reasonable bounds. It is thus important to see the behavior of output power with frequency for different loads. Three different loads were used: R , $2R$, and $5R$, the value of R being the





same as before. If the field current I_f is supplied by a constant voltage source then it is seen that the power varies as the square of the frequency as shown by the following derivation. The terminal voltage of the generator V_T is

$$V_T = E_g - I_g (r_g + jx_1) \text{ ----- } 3.2$$

E_g is the induced voltage in the armature and is the same as given by equation 3.1

I_g is the armature current

r_g is the armature resistance

and,

x_1 is the synchronous reactance of the armature winding per phase

Substituting equation 3.1 yields

$$V_T = 4.44 K_f \phi N - I_g (r_g + jx_1) \text{ ----- } 3.3$$

If we consider the unsaturated region of the B H curve then current is always proportional to the mmf H. The current is also proportional to the flux ϕ in the linear region. Therefore if I_f is constant then ϕ is also constant and we can write the phase voltage as

$$V_T = K_1 f - I_g (r_g + jx_1) \text{ ----- } 3.4$$

where

$$K_1 = 4.44 K_f N.$$

The theoretical power out P_{th} for all 3 phases is given by

$$P_{th} = 3 \frac{(V_T)^2}{R_L} \text{ ----- } 3.4a$$

$$= \frac{3 \left[K_1 f - I_g r_g \right]^2 + 3 I_g^2 x_1^2}{R_L}$$

$$P_{th} \approx 3 \left[\frac{(K_1 f)^2 + I_g^2 x_1^2}{R_L} \right] = \frac{3f^2 (K_1^2 + I_g^2 [2\pi L_1]^2)}{R_L} \quad \text{---} \quad 3.5$$

This equation neglects the $I_g r_g$ terms since these should be small compared to other terms. Figure 17 shows the actual output power variation with frequency. The curves are drawn for three actual different loads as labeled. The theoretical curve labeled P_{th} is also shown.

The constant K_1 for this curve was estimated from equation 3.4 by assuming $r_g = 0$ and $x_1 = 1$ per unit. Rated current I_g flowing through a 1 per unit reactance produces rated voltage V_T . Equation 3.4 becomes

$$V_T = K_1 f - j V_T$$

so,

$$|K_1| = \frac{|V_T + j V_T|}{f} = \frac{\sqrt{2} V_T}{f} = \frac{\sqrt{2} \left(\frac{120}{\sqrt{3}} \right)}{60} = 1.63$$

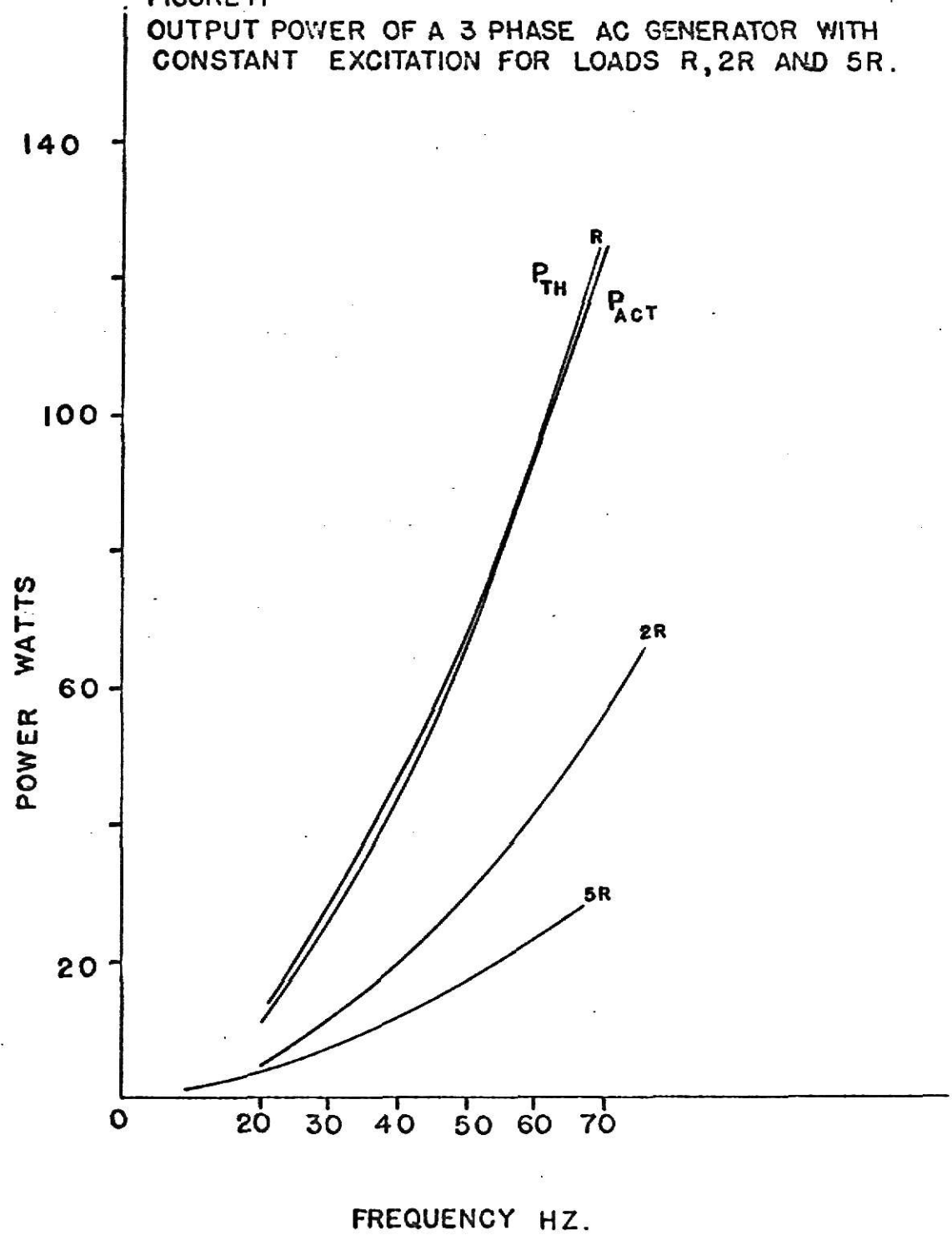
The $\sqrt{3}$ term is due to the line to line voltage of the small machine under test being 120 volts, but phase quantities are desired. This constant was used in equation 3.5 to get the P_{th} curve. Since shape or frequency variation of power is more important than actual values, the P_{th} curve was normalized to the P_{act} curve at 60 Hz. This allows us to easily compare the actual frequency variation of power to the theoretical f^2 variation.

The actual frequency variation of the P_{act} curve was determined from the equation

$$\frac{P_{f_1}}{P_{f_2}} = \left[\frac{f_1}{f_2} \right]^n \quad \text{-----} \quad 3.6$$

FIGURE 17

OUTPUT POWER OF A 3 PHASE AC GENERATOR WITH
CONSTANT EXCITATION FOR LOADS R , $2R$ AND $5R$.



The average value of n for several different sets of frequencies gave the result that

$$P_{act} \propto f^{1.8}$$

As seen from Figure 17 the two curves agree well and it can be said that the actual power is approximately proportional to the square of frequency.

This power behavior was for a fixed resistive load. Another type of load is involved in the proposal to charge batteries by the wind driven generator, which will act as an electrical storage system. The output of these batteries can be used for operating household appliances after being inverted. The advantage with batteries is that a constant frequency output can be achieved irrespective of the input to the generator. In order to match the generator to the wind turbine it is important to know how the power curves behave with this kind of load. Therefore, the battery was simulated by an electrolysis cell which has a voltage-current characteristic similar to a lead acid battery. Figure 18 shows the power curve labeled as P_{act} . The same P_{th} curve as in Figure 17 was also drawn for comparison purposes. This would not be the correct P_{th} curve for an electrolysis cell because the cell resistance does not remain constant and therefore the derivation of equation 3.5 does not hold for this case. Using the same relation as given by equation 3.6 the average value of n was determined and the frequency dependence of P_{act} is

$$P_{act} \propto f^{2.34}$$

Figure 18 also has a curve labeled V . This is the output voltage or voltage of the cell and it is observed that it has a tendency to flatten as the frequency is increased beyond 70 Hz, which is also a characteristic of these cells.

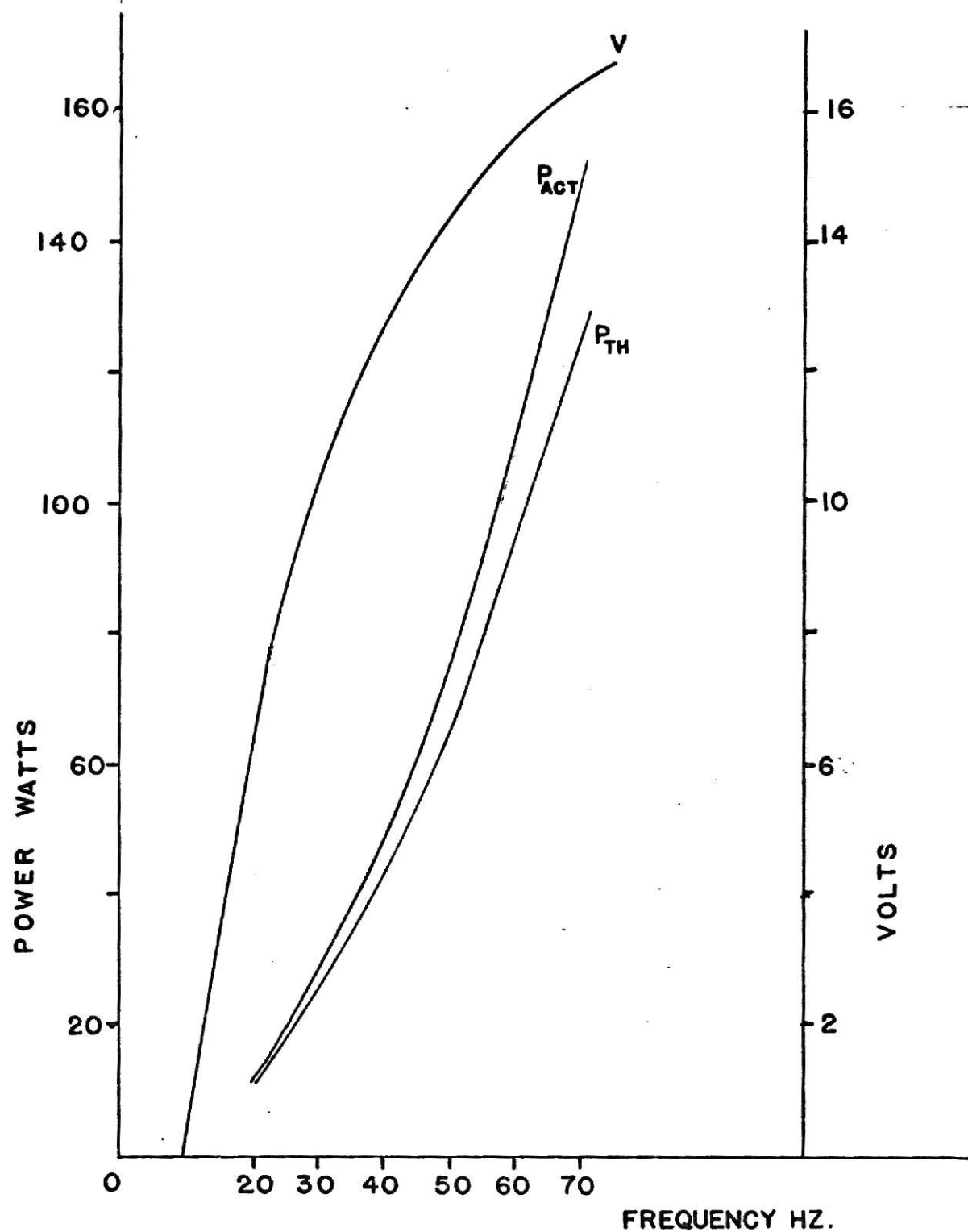


FIGURE 18
OUTPUT POWER AND VOLTAGE OF A 3 PHASE AC GENERATOR
WITH CONSTANT EXCITATION FOR ELECTROLYSIS LOAD .

Power curves take a different shape if the field current I_f is proportional to the frequency (e.g., excitation from permanent magnet exciter). Again the three load resistors R , $2R$ and $5R$ were used as loads. It will be shown that the power out is a function of approximately the fourth power of frequency.

Figure 19 shows the circuit diagram of the exciter with the load being the resistance of the field winding of the generator.

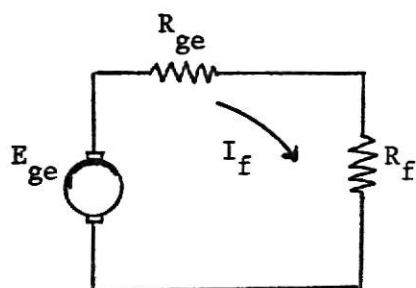


Figure 19

Circuit diagram of exciter with generator field as the load

Let E_{ge} be the induced voltage of the exciter. Then for constant field in the exciter,

$$E_{ge} = K_2 f \text{ ----- } 3.7$$

then,

$$I_f = \frac{E_{ge}}{R_f + R_{ge}}$$

$$= \frac{K_2 f}{R_f + R_{ge}} \text{ ----- } 3.8$$

The same argument holds that if the generator is operated in the unsaturated region or linear region of the B H curve then the generator field flux ϕ is proportional to the field current I_f .

The generator terminal voltage is given by equation 3.3

$$V_T = 4.44 f \phi N - I_g (r_g + jx_1)$$

Substituting in equation 3.3 the value of I_f given by equation 3.8 gives

$$\begin{aligned} V_T &= 4.44 K_f \frac{K_2 f}{(R_f + R_{ge})} fN - I_g (r_g + jx_1) \\ &= K_3 f^2 - I_g (r_g + jx_1) \end{aligned}$$

$$K_3 = \frac{4.44 NK K_2}{(R_f + R_{ge})}$$

The power out (P_{th}) is given by equation 3.4a

$$P_{th} = 3 \left[\frac{V_T^2}{R_L} \right]$$

Substituting the value of V_T in equation 3.4a

$$P_{th} = \frac{3 \left[(K_3 f^2 - I_g r_g)^2 + I_g^2 x_1^2 \right]}{R_L}$$

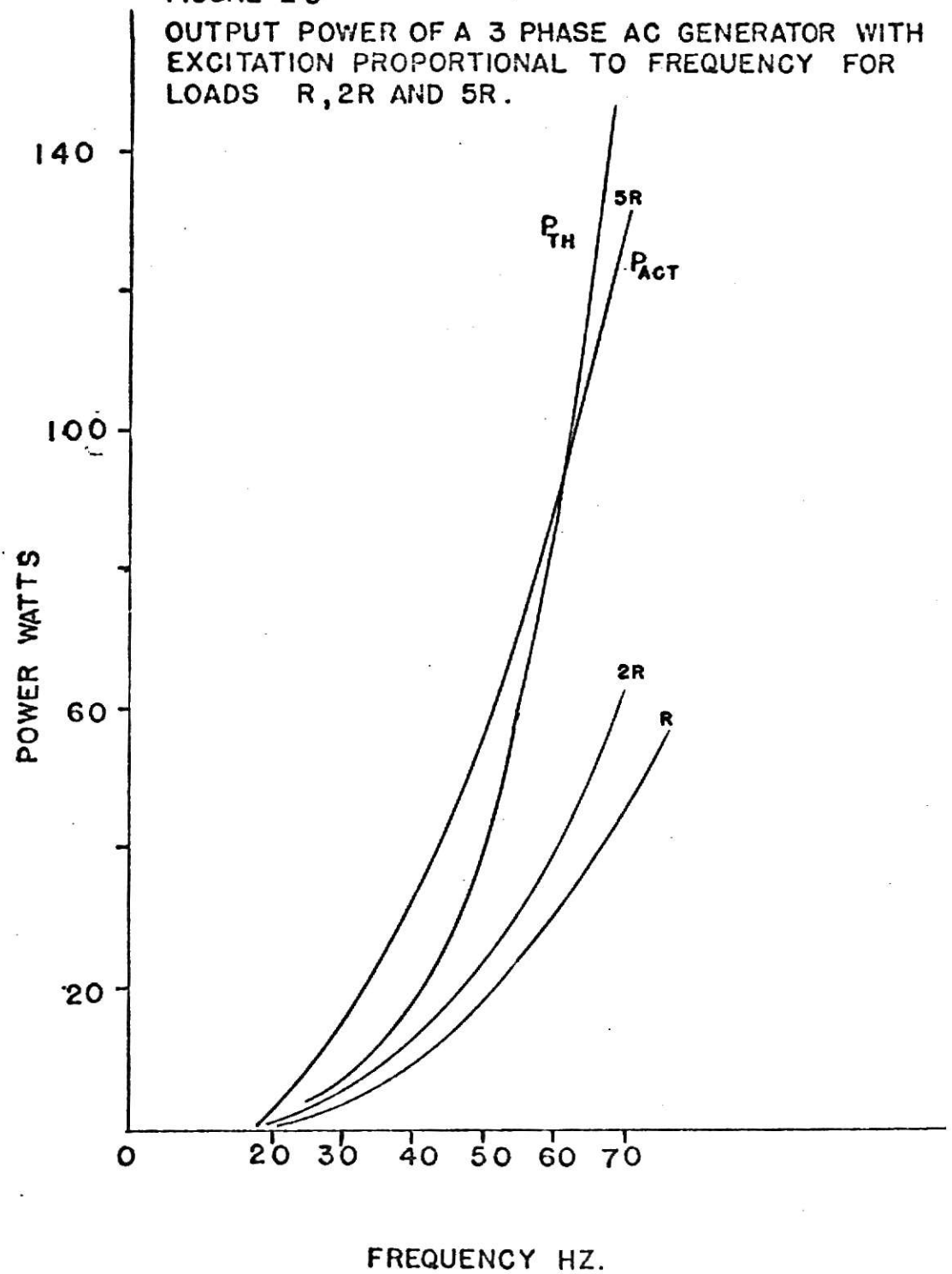
Neglecting the armature resistance term yields

$$P_{th} \approx \frac{3 \left[(K_3 f^2)^2 + I_g^2 x_1^2 \right]}{R_L} \text{-----} 3.9$$

Figure 20 shows the power variation with frequency for this system. The curves are drawn for three actual loads as labeled. The theoretical curve labeled P_{th} is also shown. K_3 was determined at rated specification

FIGURE 20

OUTPUT POWER OF A 3 PHASE AC GENERATOR WITH
EXCITATION PROPORTIONAL TO FREQUENCY FOR
LOADS R , $2R$ AND $5R$.



by using equation 3.9 to be 0.0425 by assuming $x_1 = 1.0$ p.u. for the synchronous machine.

The exponential variation for the P_{act} curve was determined to be

$$P \propto f^{2.59}$$

This is a significant deviation from the predicted f^4 variation. It would appear that some assumed values need closer examination, particularly the per unit value of synchronous reactance.

An electrolysis cell was again connected to the generator but with this frequency dependent excitation. Figure 21 shows the actual power curve labeled as P_{act} . Using the relation given by equation 3.6, the average value of n was found to be approximately 3.87. Thus, the actual relation for P_{act} curve is

$$P_{act} \propto f^{3.87}$$

Figure 21 also shows the output voltage as the cell voltage vs. frequency.

The variation of load power required as a function of frequency is one of the characteristics which are of interest in a wind turbine operating as a prime mover. At low speeds when the output power and torque of the turbine are small the turbine might stall if the generator requires too much power. If the generator presents a very low load at low frequencies the turbine and generator will essentially free wheel at low wind speeds. As the turbine gains speed and develops more torque, the frequency increases and the load comes on.

The curves in Figure 22 explain the concept of speed regulation by load management. It shows the rotor developed power at different wind speeds. The relative power out of a wind rotor varies with rpm and wind speed. This particular curve is typical of a Savonius rotor.

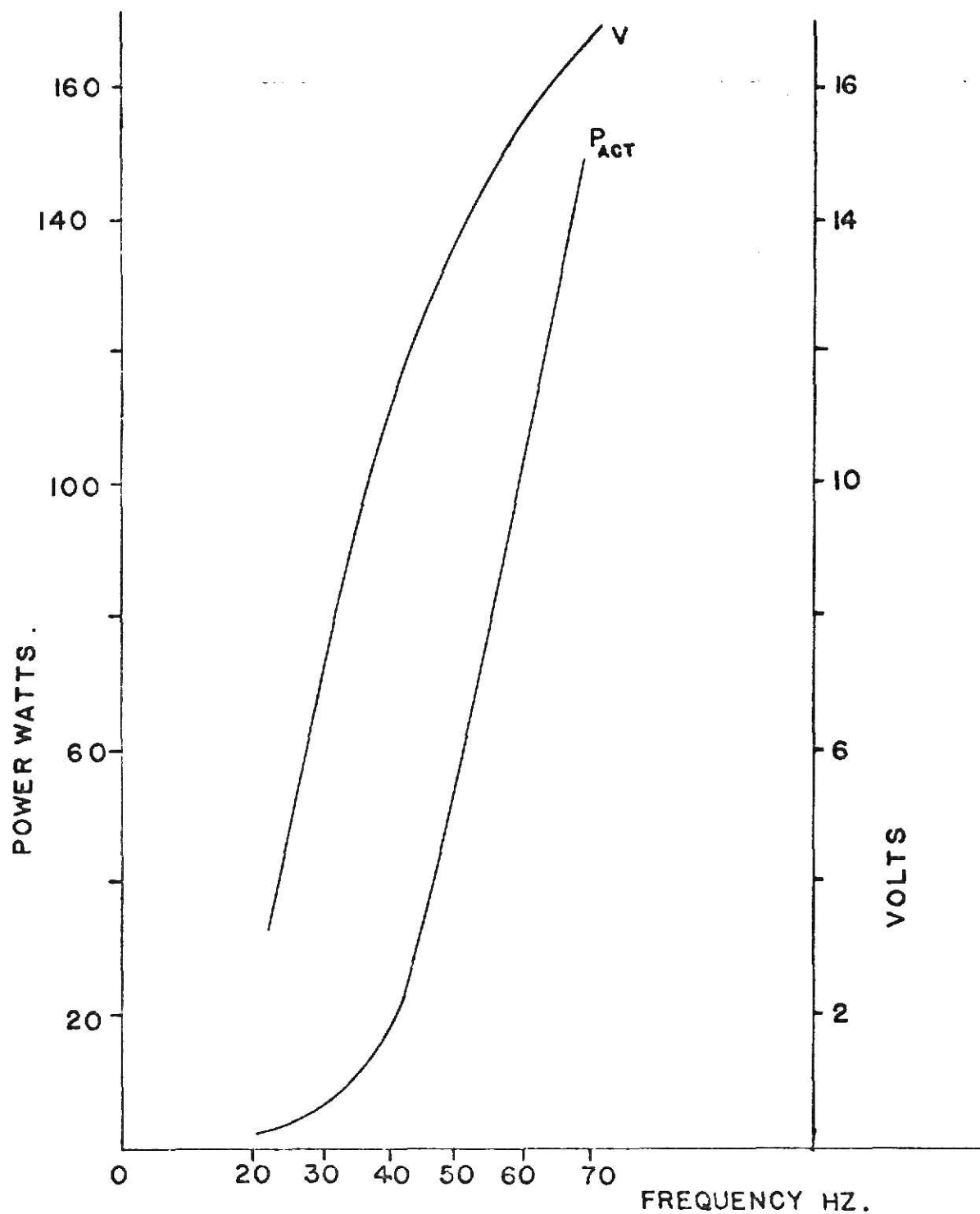


FIGURE 21
OUTPUT POWER AND VOLTAGE OF A 3 PHASE AC GENERATOR
WITH EXCITATION PROPORTIONAL TO FREQUENCY FOR AN
ELECTROLYSIS CELL LOAD.

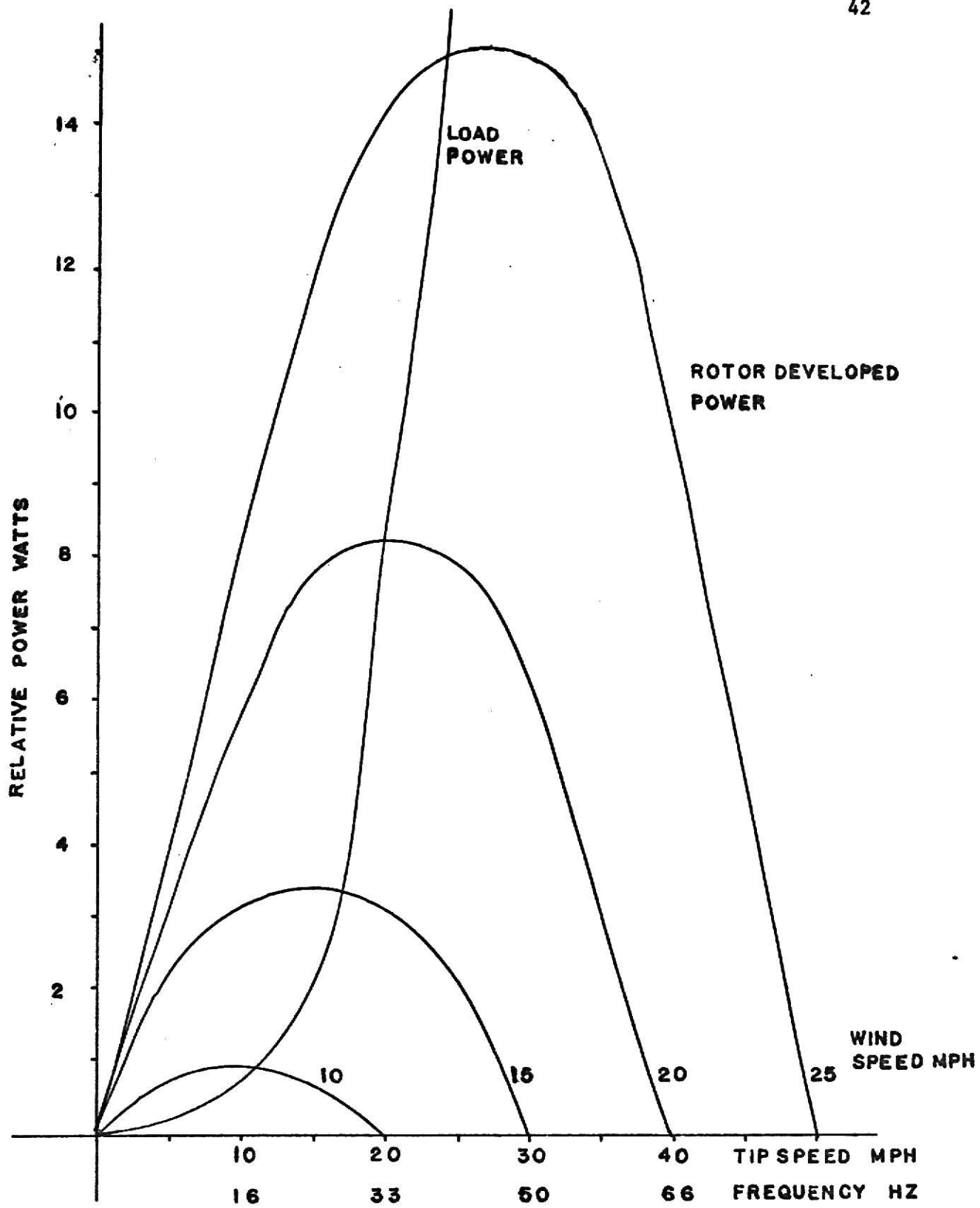


Figure 22

Variation of load power and rotor developed power with wind speed and frequency

Let us suppose a load is attached to the rotor whose power varies as the fourth power of frequency. Such a load curve is shown in Figure 22. If we assume that the system has stopped in low wind and the wind now jumps to 10 mph the shaft power out (rotor developed power) will exceed the load power required and the rotor will accelerate. A stable operating point will be reached where the rotor shaft power out equals the load power required at that rpm. If the wind now jumps to 20 mph the rotor will accelerate until the shaft power out is equal to the load power required. This is a form of rotor speed control by selecting the right load frequency characteristic for the generator.

When loads are changed suddenly there is a transient state in the current and the amplitude of the current may reach a value which is much larger than the steady state value. The transient equation of the current⁵ is

$$i(t) = \frac{V_{\max}}{|Z|} \left\{ \sin(\omega t + \alpha - \theta) - e^{-\frac{R}{L}t} \sin(\alpha - \theta) \right\} \quad \text{--- 3.10}$$

where

$$Z = \sqrt{R^2 + (\omega L)^2}$$

The first term in equation 3.10 varies sinusoidally with time. The second term is non periodic and decays exponentially with time constant $\frac{L}{R}$. This non periodic term is the DC component of the current. It is seen that if the value of the sinusoidal term is not zero at $t = 0$ then the DC component can take an initial value anywhere from 0 to $\frac{V_{\max}}{|Z|}$. To see the transient and its duration a photograph was taken from an oscilloscope. Figure 23 shows the variation in the current wave form, when the generator was switched from $5R$ to R . It is seen that the duration of the transient is approximately one cycle. Figure 24 shows the

circuit diagram of the generator and load. The result in Figure 23 was taken across points c and d.

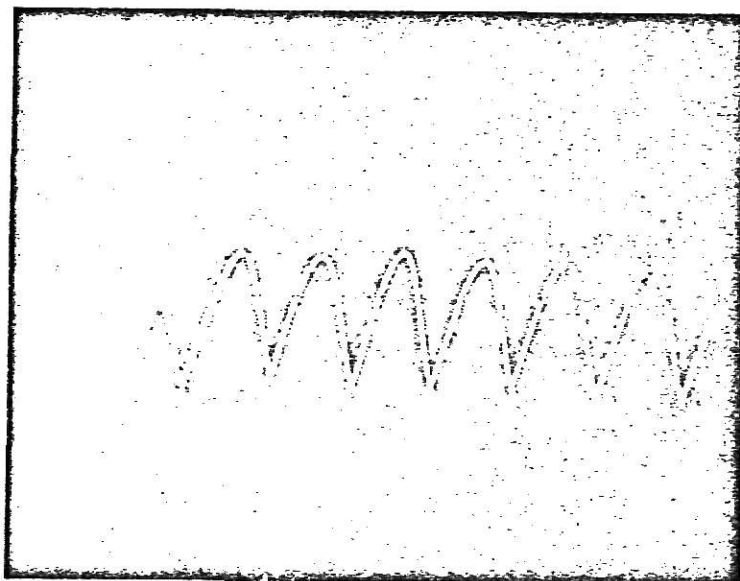


Figure 23

Current wave form switching from 5R to R

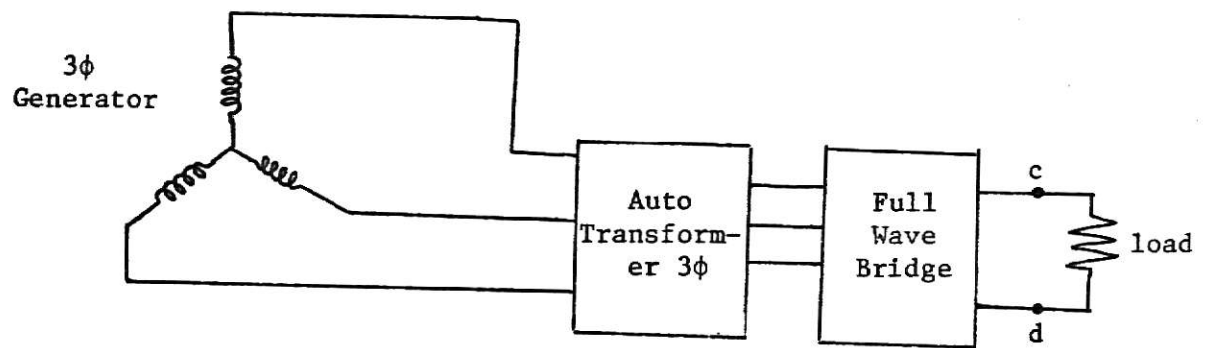


Figure 24

Circuit diagram of 3 ϕ AC generator and load

CONCLUSION

This paper examines the theoretical and experimental basis for frequency regulation by load management. The basic concepts of wind energy were developed in Chapter 1, including various types of rotors, power available from the wind, etc.

The transient behavior of the system was studied in some detail. For the analysis the turbine was modelled as a DC motor. The analysis shows the variation in angular velocity if the electrical load is changed. In the model the input was considered constant, which will not be the case of the wind power input to an actual wind turbine. The analysis shows that if there is some variation in input power, the load can be varied so as to minimize the corresponding change in angular velocity.

The experimental results obtained from the laboratory illustrate two different loads that could be used on this system. Two different excitations were tried to see the variation of power with frequency. The constant excitation case gave experimental results consistent with theory and the power varied approximately as the square of frequency. The excitation which was proportional to frequency showed less consistent results, since for the resistive load P_{act} varied as $f^{2.59}$ instead of as f^4 . The electrolysis cell for this case had power varying approximately as the fourth power of frequency.

From the theory and curve in Figure 22 we conclude that the load can be matched to the prime mover over a narrower range of frequencies by using a generator which has excitation proportional to frequency. Economic reasons may suggest the use of a generator with constant

excitation since the results are similar to those of the generator with variable excitation for resistive load. By doing so the initial costs and maintenance costs can possibly be reduced.

One suggestion for further research in this area is to connect the load to the generator through triac switches and switch the load in and out rapidly to maintain the average load power at that which can be supplied by the given wind speed at a frequency of about 60 Hz.

Table I below shows the comparison of P_{th} and P_{act} for different loads and excitations.

Table I. Comparison of Theoretical and Actual Power Variation With Frequency.

Loads	Constant Excitation		Excitation Proportional to Frequency	
	P_{th}	P_{act}	P_{th}	P_{act}
Resistive Load	$P = Kf^n$ $n = 2$	$P \propto f^{1.8}$	$P = Kf^n$ $n = 4$	$P \propto f^{2.59}$
Electrolysis Cell		$P \propto f^{2.34}$		$P \propto f^{3.87}$

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LOAD MANAGEMENT FOR WIND DRIVEN
AC GENERATOR

by

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ABSTRACT

This report contains the analysis and experimental results of a simulated wind driven AC generator supplying various loads. One of the problems of such a system is to keep the frequency nearly constant. This is difficult because the power in the wind is not constant but varies as the cube of the wind velocity.

The research here is aimed at developing the necessary equations for asynchronous operation where frequency may vary over a range of ± 10 Hz or more. This scheme allows us to use a simple turbine and a simple electrical machine and perhaps hold the cost down on those applications where precise 60 Hz power is not required. Instead of changing the turbine or the generator itself, we can vary the electrical load of the generator in such a manner that the angular velocity of the system remains within a reasonable range.

In order to operate the system in this manner a feedback system would be employed which senses the frequency and then adjusts the load torque in a way to optimally load the turbine.

To develop such a system it was necessary to study the characteristics and behavior of an AC generator under such conditions. Therefore a transient analysis of the system was performed. The transient analysis showed the change in angular velocity of the system as load torque was varied. The load torque was modeled as an infinite pulse train. Such torque pulses might be produced by rapidly switching the load in and out using triac switches. A general equation for angular velocity was developed. Specific values of inertia, friction, rated current, etc., were assumed for a 5 kw generator and the general equation was evaluated. The frequency change during one torque pulse was well under 1 Hz when the

load was being switched at a 5 Hz rate, which implies that even slower switching rates are probably acceptable.

Generator loads experiments on a laboratory model were performed using two different generator excitations. These were:

1. AC generator operated with constant field current.
2. AC generator operated with variable field current, obtained from a permanent magnet exciter.

For the constant excitation case with constant load it was shown that the theoretical power varies as the square of the frequency while the actual variation was $P \propto f^{1.8}$.

For the same excitation an electrolysis cell was used as a load and the power varied as $P \propto f^{2.34}$.

The variable field excitation was accomplished by supplying the field from a permanent magnet exciter whose angular velocity was proportional to the angular velocity of the generator. From theory the power should vary as the fourth power of frequency for a resistive load but experimentally it was observed that the relation was $P \propto f^{2.56}$.