ON TESTING THE EQUALITY OF TWO PROPORTIONS

by

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1. INTRODUCTION

Let X_m and Y_n be two independent random variables from binomial populations $B(m,p_x)$ and $B(n, p_y)$, respectively. The statistical problem is to test the following hypotheses:

$$H_o: p_x = p_y$$

against

$$H_1: p_x \neq p_y$$

based on the observations of X_m and Y_n. The following four tests have been suggested in the literature:

a) Z_1-test (See Section 2, (2.1)).

b) Z_2-test (See Section 2, (2.2)).

c) Uniformly most powerful unbiased test (UMPU test) (See Section 2, (2.4)).

d) The unconditional nonrandomized test (UN test) (See Section 2).

In [8], Robbins raised the following question: Between Z_1-test and Z_2-test which one is better with respect to power against various alternatives to H_o? He conjectured that Z_1-test would be better than the Z_2-test in general, but possibly not for all values of m and n. He further pointed out that for the case m = n, it is always true that $|Z_1| \geq |Z_2|$, with equality only when $X_m = Y_n$. In [1], Berengut and Petkau compared Z_1-test with Z_2-test under asymptotic considerations and finite sample cases. They found the Z_1-test tended to produce probabilities of type I error substantially greater than the nominal level, while the Z_2-test tended to produce probabilities of type I error that fluctuate slightly above the nominal level, they concluded that is was difficult to make meaningful comparisons of their power. In [3], Eberhardt and Fligner compared Z_1-test with Z_2-test in terms of asymptotic efficiency. In case of finite sample sizes, they pointed out that the size of the Z_1-test (See [4], p.61 for the definition of the size of a test.) may be much higher than the nominal level. The UMPU test was given in [4]. McDonald, Davis, and Milliken [6] suggested an unconditional nonrandomized test for testing H_0 against H_1. Conover [2] pointed out that the choice between the Z_1-test and the Z_2-test for testing H_0 against H_1 is inconclusive. However, he seemed to prefer the Z_1-test to the Z_2-test because of consistency properties. Other related discussion can be found in [7].

This article gives results from comparing the four tests in the finite-sample-size cases based on their powers and some practical considerations. In Section 2, a brief review of the four tests is given. In Section 3, the powers of the four tests are presented. In Section 4, various comparisons among four tests have been made.

2. THE FOUR TESTS

For testing H_o: $p_x = p_y$ against H_1: $p_x \neq p_y$, the four tests mentioned above are stated below:

a) Nominal level α Z_1-test

$$\begin{aligned} &\text{Reject } H_o \quad \text{if } |Z_1| \geq z_{(1-\frac{\alpha}{2})} \;, \\ &\text{Accept } H_o \quad \text{if } |Z_1| < z_{(1-\frac{\alpha}{2})} \;, \end{aligned} \tag{2.1}$$

where

$$Z_1 = \frac{\hat{p}_x - \hat{p}_y}{\sqrt{\frac{\hat{p}_x(1-\hat{p}_x)}{m} + \frac{\hat{p}_y(1-\hat{p}_y)}{n}}} \;,$$

$$\hat{p}_x = \frac{X_m}{m} \;,$$

$$\hat{p}_y = \frac{Y_n}{n} \;,$$

and $z_{1-\frac{\alpha}{2}}$ is a constant satisfying

$$\Phi(z_{1-\frac{\alpha}{2}}) = 1 - \frac{\alpha}{2}$$

(Φ is the standard normal distribution function).

Remark. In practice, an experimenter first selects a nominal level α ($0 < \alpha < 1$) and hopes that the size of the selected test is close to α. Thus he has some control over the probability of Type I error. But he does not always achieve the goal. In the case of Z_1-test the size of

Z_1-test may be much greater than the selected nominal level α (See Section 4, Table 4.1 and Table 4.2). The term "nominal level" is used here to emphasize that nominal level α is just a number selected by the experimenter to determine what $z_{1-\frac{\alpha}{2}}$ should be used in (2.1). It may not control the size of the selected test.

b) Nominal level α Z_2-test

$$\begin{aligned} &\text{Reject } H_0 \quad \text{if } |Z_2| \geq z_{1-\frac{\alpha}{2}}\ , \\ &\text{Accept } H_0 \quad \text{if } |Z_2| < z_{1-\frac{\alpha}{2}}\ , \end{aligned} \tag{2.2}$$

where

$$Z_2 = \frac{\hat{p}_x - \hat{p}_y}{\sqrt{\hat{p}(1-\hat{p})(\frac{1}{m}+\frac{1}{n})}}\ ,$$

$$\hat{p} = \frac{X_m + Y_n}{m + n}\ .$$

Remark. Both Z_1 and Z_2 are indetermined at $(X_m, Y_n) = (0, 0)$ or (m, n). We shall define that $Z_1 = Z_2 = 0$ at these two points. Thus $(0, 0)$ and (m, n) are always in the acceptance region.

c) Nominal level α UMPU test. (See [4] p. 143).

The test is performed conditionally on the integer points of the line segment $X_m + Y_n = t$ in the positive quadrant of the $X_m\, Y_n$ - plane. Under H_0 the conditional probability of $X_m = x$ given $X_m + Y_n = t$ is the hypergeometric

$$p(x|t) = \frac{\binom{m}{x}\binom{n}{t-x}}{\binom{m+n}{t}} \tag{2.3}$$

for $\max(0, t-n) \le x \le \min(t, m)$. H_0 is rejected if X_m becomes too small or too large. More precisely, a nominal level α UMPU test is defined as:

Given $X_m + Y_n = t$,

$$\text{Accept } H_0 \quad \text{if } c_1(t) < x < c_2(t)$$

$$\text{Reject } H_0 \quad \text{if } x < c_1(t) \text{ or } x > c_2(t) \tag{2.4}$$

Reject H_0 with probability $\gamma_i(t)$, if $x=c_i(t)$ for $i=1,2$; where $c_1(t)$, $c_2(t)$ $\gamma_1(t)$ and $\gamma_2(t)$ are determined by

$$\sum_{c_1(t)<x<c_2(t)} p(x|t)+[1-\gamma_1(t)]p(c_1(t)|t)+[1-\gamma_2(t)]p(c_2(t)|t)=1-\alpha \tag{2.5}$$

and

$$\sum_{x<c_1(t)} xp(x|t) + \sum_{x>c_2(t)} xp(x|t)+c_1(t)\gamma_1(t)p(c_1(t)|t)$$

$$+ c_2(t)\gamma_2(t)p(c_2(t)|t) = \alpha\, t\, \frac{m}{m+n} \tag{2.6}$$

Remark. For the UMPU test one can always choose $c_1(t)$, $c_2(t)$, $\gamma_1(t)$ and $\gamma_2(t)$ in such a way the size of the test is exactly equal to the nominal level. Hence, a nominal level α UMPU test is also a size α UMPU test.

As an example, suppose an investigator wishes to know if the proportions of defective parts made by two production lines differ significantly. Let X_m = the number of defective parts made by the first production line among the m parts checked. Let Y_n = the number of defective parts made by the second production line among n parts checked. He assumes that X_m and Y_n are independent random variables with binomial distributions $B(m, p_x)$ and $B(n, p_y)$, respectively. He wants to test

H_0: $p_x = p_y$ against H_1: $p_x \neq p_y$. Suppost that m = 4 and n = 5 and nominal level α = 0.05. The UMPU test may be described below:

The investigator must determine the critical region for each value of $X_4 + Y_5 = t$ (t = 0, 1, ..., 9). Examine specifically the outcome of t = 5. When t = 5, the range of the random variable X_4 is 0, 1, 2, 3, and 4 with associated probabilities 0.0079, 0.1587, 0.4762, 0.3175 and 0.0397 computed from (2.3). The point X_4 = 0 given t = 5 is inside the rejection region. The points X_4 = 1 given t = 5 and X_4 = 4 given t = 5 are two boundary points. We randomized the two boundary points, reject H_0 with probability 0.1203 when X_4 = 1 given t = 5 is observed, and reject H_0 with probability 0.5794 when X_4 = 4 given t = 5 is observed.

d) Nominal level α unconditional nonrandomized test (See [6]).

The critical region for the UN test is obtained in the following way: First a nominal level α is selected, then for each value of $X_m + Y_n$ = t (t = 0, 1, 2, ..., m+n), the critical region of the UMPU test for testing H_0 against H_1 is determined. Let A be the set of all points "inside" the critical region of the UMPU test for all t = 0, 1, 2, ..., m+n, then A is the critical region of the UN test for testing H_0 against H_1. Finally, the size of the unconditional nonrandomized test is equal to the maximum probability of the set A under H_0. In [5] and [6] the critical regions of the UN test and their sizes for sample sizes up to 20 are given. For example, if m = 4, n = 5, nominal level α = 0.10 then the critical region of the UN test for testing H_0 against H_1 is given in Figure 2.1. Points in the critical region are labeled with a "*".

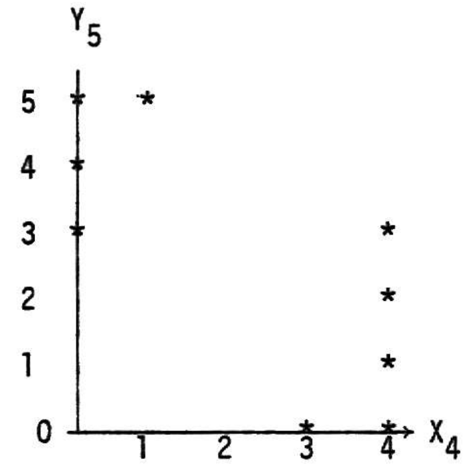

Figure 2.1 - Two-sided critical region of the UN test with nominal level α = 0.10 and with size = 0.078.

3. THE POWERS OF THE FOUR TESTS

For given parameters p_x, p_y; sample sizes (m, n) and a nominal level α, let $P_1(p_x, p_y, \alpha, m, n)$, $P_2(p_x, p_y, \alpha, m, n)$, $P_U(p_x, p_y, \alpha, m, n)$ and $P_{UN}(p_x, p_y, \alpha, m, n)$ be the probabilities of rejection of the null hypothesis, H_0, based on Z_1-test, Z_2-test, UMPU test, and UN test, respectively.

First note that, as has been pointed out in [1],

$$P_i(p_x, p_y, \alpha, m, n) = P_i(1-p_x, 1-p_y, \alpha, m, n) \tag{3.1}$$

for i = 1, 2,.

A computer program was used to calculate $P_1(p_x, p_y, \alpha, m, n)$ and $P_2(p_x, p_y, \alpha, m, n)$ for the case $\alpha = 0.05$; p_x, p_y = 0.1 (0.1) 0.9; m = n = 5 10, 20, 30, 40, 50, 60, 70, 80, 90, 100; m = 10, n = 5; m = 12, n = 8; m = 15, n = 10; m = 20, n = 15; m = 30, n = 20; and m = 30, n = 40. The computed powers of Z_1-test and Z_2-test are presented in Tables 3.1 through 3.16.

The powers of the UMPU test may be computed as follows:

$$P_U(p_x, p_y, \alpha, m, n) = \sum_{t=0}^{m+n} h(t) \tag{3.2}$$

where

$$h(t) = \sum_{\max(0,\, t-m) \le k < c_1(t)} g(k, t) + \sum_{c_2(t) \le k < \min(m,\, t)} g(k, t)$$

$$+ \gamma_1(t)\, g(c_1(t), t) + \gamma_2(t)\, g(c_2(t), t),$$

$$g(k, t) = \begin{cases} \binom{m}{k} p_x^{\,k} (1-p_x)^{m-k} \binom{n}{t-k} p_y^{\,t-k} (1-p_y)^{n-(t-k)}, & \text{for } \max(0,t-n) \\ & \leq k \leq \min(m,t) \\ 0 \quad \text{otherwise;} & \end{cases}$$

$c_i(t)$, $\gamma_i(t)$ are constants determined by (2.5) and (2.6).

Remark. Note that

$$P_U(p_x, p_y, \alpha, m, n) = P_U(1-p_x, 1-p_y, \alpha, m, n) \tag{3.3}$$

for all p_x, p_y. The proof of (3.3) is presented in Appendix A.

A computer program was used to calculate $P_U(p_x, p_y, \alpha, m, n)$ for the case $\alpha = 0.05$; p_x, p_y, $= 0.1$ (0.1) 0.9; $m = n = 5, 10, 20, 30, 40, 50, 60, 70$; $m = 10$, $n = 5$; $m = 15$, $n = 10$; $m = 20$, $n = 15$; $m = 30$, $n = 20$; and $m = 4($ $n = 30$. The computed powers are presented in Table 3.17 through 3.29.

The critical regions of the UN test for sample size up to 20 are given in [5] or [6]. The power of the UN test is equal to

$$P_{UN}(p_x, p_y, \alpha, m, n) = \sum \binom{m}{k} p_x^{\,k} (1-p_x)^{m-k} \binom{n}{\ell} p_y^{\ell} (1-p_y)^{n-\ell} \tag{3.4}$$

where the summation is over all points (k, ℓ), which are in the critical region specified in [5] or [6]. A computer program was used to calculate the power of the UN test for the nominal level $\alpha = 0.05$; p_x, p_y $= 0.1$ (0.1) 0.9; $m = n = 5, 10, 20$; $m = 10$, $n = 5$; $m = 12$, $n = 8$; $m = 15$, $n = 10$; and $m = 20$, $n = 15$. The computed results are presented in Table 3.30 through 3.36.

Remark. Using exactly the same reasoning as in (3.3), we can show:

$$P_{UN}(p_x, p_y, \alpha, n, n) = P_{UN}(1-p_x, 1-p_y, \alpha, m, n) \tag{3.5}$$

for all p_x, p_y.

Table 3.1 - Powers and probabilities of type I error of Z_1-test and Z_2-test for m=n=5, α=0.05, α_1*=0.10937, α_2*=0.06185

p_y	p_x .1	.2	.3	.4	.5
.9	.92981*	.82205	.67976	.51938	.35937
	.82219**	.68767	.56160	.43431	.30805
.8	.82205	.67788	.52539	.37879	.25000
	.68768	.51006	.37770	.27213	.18500
.7	.67976	.52539	.38437	.26467	.17187
	.56160	.37770	.25386	.16914	.11117
.6	.51938	.37879	.26467	.17958	.12500
	.43431	.27213	.16914	.10650	.07250
.5	.35937	.25000	.17187	.12500	.10937
	.30805	.18500	.11117	.07250	.06055
.4	.21753	.14812	.10936	.10157	.12500
	.19222	.11501	.07435	.06175	.07250
.3	.10818	.07880	.07849	.10936	.17187
	.09872	.06488	.05807	.07435	.11117
.2	.03942	.04359	.07880	.14812	.25000
	.03714	.03835	.06488	.11501	.18500
.1	.01041	.03942	.10818	.21753	.35937
	.01012	.03714	.09872	.19222	.30805

* Z_1-test (for Tables 3.1 to 3.16).

** Z_2-test (for Tables 3.1 to 3.16).

α_1* is the size of Z_1-test (for Tables 3.1 to 3.16).

α_2* is the size of Z_2-test (for Tables 3.1 to 3.16).

Table 3.2 - Powers and probabilities of type I error of Z_1-test and Z_2-test for m=n=10, α=0.05, α_1*=0.09488, α_2*=0.04219

p_y	p_x .1	.2	.3	.4	.5
.9	.99433 .98884	.96800 .93666	.90827 .82371	.80160 .65904	.64787 .47136
.8	.96800 .93666	.87294 .80539	.73446 .62163	.57090 .42488	.39964 .25423
.7	.90827 .82371	.73446 .62163	.53750 .41849	.36089 .24767	.22153 .12752
.6	.80160 .65904	.57090 .42488	.36089 .24767	.20901 .12861	.12031 .06221
.5	.64787 .47136	.39964 .25423	.22153 .12752	.12031 .06221	.08831 .04219
.4	.46605 .29121	.24578 .13123	.12896 .05968	.09202 .04031	.12031 .06221
.3	.28492 .14304	.13597 .05827	.09488 .03711	.12896 .05968	.22153 .12752
.2	.13431 .04620	.09069 .02993	.13597 .05827	.24578 .13123	.39964 .25423
.1	.05027 .00904	.13431 .04620	.28492 .14304	.46605 .29121	.64787 .47136

Table 3.3 - Powers and probabilities of type I error of Z_1-test and Z_2-test for m=n=20, α=0.05, α_1*=0.08075, α_2*=0.05337

p_y	.1	.2	p_x .3	.4	.5
.9	.99999 .99998	.99965 .99906	.99423 .99044	.96265 .95143	.86546 .83804
.8	.99965 .99906	.99208 .98169	.94695 .90839	.81533 .75349	.58702 .53085
.7	.99423 .99044	.94695 .90839	.80751 .71324	.57288 .46304	.31670 .24517
.6	.96265 .95143	.81533 .75349	.57288 .46304	.31874 .22183	.13995 .08729
.5	.86546 .83804	.58702 .53085	.31670 .24517	.13995 .08729	.08075 .04253
.4	.67324 .62332	.33125 .29115	.13326 .10401	.07567 .04822	.13995 .08729
.3	.40473 .35741	.14038 .11369	.06857 .05326	.13326 .10401	.31670 .24517
.2	.15384 .13800	.06534 .05022	.14038 .11369	.33125 .29115	.58702 .53085
.1	.03979 .03867	.15384 .13800	.40473 .35741	.67324 .62332	.86546 .83804

Table 3.4 - Powers and probabilities of type I error of Z_1-test and Z_2-test for m=n=30, α=0.05, α_1*=0.06039, α_2*=0.05534

p_y	p_x .1	.2	.3	.4	.5
.9	1.00000 1.00000	.99999 .99999	.99962 .99955	.99316 .99274	.95742 .94899
.8	.99999 .99999	.99919 .99919	.98555 .98528	.90946 .90459	.71345 .69239
.7	.99962 .99955	.98555 .98528	.89619 .89594	.66537 .66312	.36043 .35106
.6	.99316 .99274	.90946 .90459	.66537 .66312	.35023 .34961	.12472 .12333
.5	.95742 .94899	.71345 .69239	.36043 .35106	.12472 .12333	.05219 .05194
.4	.82196 .79609	.42339 .39247	.13558 .12315	.05200 .04882	.12472 .12333
.3	.53874 .49949	.16326 .14227	.05833 .04870	.13558 .12315	.36043 .35106
.2	.21013 .19395	.06037 .04897	.16326 .14227	.42339 .39247	.71345 .69239
.1	.05511 .05437	.21013 .19395	.53874 .49949	.82196 .79609	.95742 .94899

Table 3.5 - Powers and probabilities of type I error of Z_1-test and Z_2-test for m=n=40, α=0.05, α_1*=0.05903, α_2*=0.05903

p_y	p_x .1	.2	.3	.4	.5
.9	1.00000 1.00000	1.00000 1.00000	.99999 .99999	.99930 .99917	.98784 .98633
.8	1.00000 1.00000	.99997 .99997	.99792 .99791	.96870 .96792	.82550 .81658
.7	.99999 .99999	.99792 .99791	.96397 .96396	.79668 .79640	.46124 .45805
.6	.99930 .99917	.96870 .96792	.79668 .79640	.45777 .45773	.15738 .15714
.5	.98784 .98633	.82550 .81658	.46124 .45805	.15738 .15714	.05669 .05667
.4	.90438 .90100	.51685 .49689	.15950 .15216	.05322 .05219	.15738 .15714
.3	.63362 .63225	.18516 .17697	.05415 .04837	.15950 .15216	.46124 .45805
.2	.24694 .24689	.05169 .05108	.18516 .17697	.51685 .49689	.82550 .81658
.1	.05875 .05875	.24694 .24689	.63362 .63225	.90438 .90100	.98784 .98633

Table 3.6 - Powers and probabilities of type I error of Z_1-test and Z_2-test for m=n=50, α=0.05, α_1*=0.06130, α_2*=0.05689

p_y	p_x .1	.2	.3	.4	.5
.9	1.00000 1.00000	1.00000 1.00000	1.00000 1.00000	0.99993 0.99992	0.99722 0.99675
.8	1.00000 1.00000	1.00000 1.00000	0.99971 0.99971	0.98971 0.98945	0.90028 0.89479
.7	1.00000 1.00000	0.99971 0.99971	0.98750 0.98750	0.87644 0.87632	0.54807 0.54554
.6	0.99993 0.99992	0.98971 0.98945	0.87644 0.87632	0.54335 0.54334	0.18375 0.18360
.5	0.99722 0.99675	0.90028 0.89479	0.54807 0.54554	0.18375 0.18360	0.05689 0.05689
.4	0.95888 0.95370	0.61349 0.59531	0.18764 0.18042	0.05319 0.05239	0.18375 0.18360
.3	0.74790 0.73445	0.22771 0.21218	0.05542 0.04979	0.18764 0.18042	0.54807 0.54554
.2	0.30755 0.30271	0.05755 0.05217	0.22771 0.21218	0.61349 0.59531	0.90028 0.89479
.1	0.05941 0.05057	0.30755 0.30271	0.74790 0.73445	0.95888 0.95370	0.99722 0.99675

Table 3.7 - Powers and probabilities of type I error of Z_1-test and Z_2-test for m=n=60, α=0.05, α_1*=0.06288, α_2*=0.05478

			p_x		
p_y	.1	.2	.3	.4	.5
.9	1.00000 1.00000	1.00000 1.00000	1.00000 1.00000	0.99999 0.99999	0.99932 0.99930
.8	1.00000 1.00000	1.00000 1.00000	0.99996 0.99996	0.99673 0.99660	0.94579 0.94237
.7	1.00000 1.00000	0.99996 0.99996	0.99565 0.99565	0.92453 0.92439	0.62137 0.61769
.6	0.99999 0.99999	0.99673 0.99660	0.92453 0.92439	0.61225 0.61223	0.20520 0.20493
.5	0.99932 0.99930	0.94579 0.94237	0.62137 0.61769	0.20520 0.20493	0.05479 0.05478
.4	0.98008 0.97951	0.69123 0.68354	0.21812 0.20981	0.05215 0.05095	0.20520 0.20493
.3	0.81476 0.80215	0.25219 0.24969	0.05523 0.05249	0.21812 0.20981	0.62137 0.61769
.2	0.36010 0.32631	0.05439 0.04993	0.25219 0.24969	0.69123 0.68354	0.94579 0.94237
.1	0.06021 0.05045	0.36010 0.32631	0.81476 0.80215	0.98008 0.97951	0.99932 0.99930

Table 3.8 - Powers and probabilities of type I error of Z_1-test and Z_2-test for m=n=70, α=0.05, α_1*=0.06339, α_2*=0.05221

p_y	p_x .1	.2	.3	.4	.5
.9	1.00000 1.00000	1.00000 1.00000	1.00000 1.00000	1.00000 1.00000	0.99986 0.99984
.8	1.00000 1.00000	1.00000 1.00000	0.99999 0.99999	0.99897 0.99891	0.97038 0.96842
.7	1.00000 1.00000	0.99999 0.99999	0.99848 0.99848	0.95364 0.95350	0.68150 0.67713
.6	1.00000 1.00000	0.99897 0.99891	0.95364 0.95350	0.66854 0.66851	0.22298 0.22258
.5	0.99986 0.99984	0.97038 0.96842	0.68150 0.67713	0.22298 0.22258	0.05154 0.05153
.4	0.99153 0.99098	0.75584 0.74710	0.24420 0.23568	0.04999 0.04854	0.22298 0.22258
.3	0.86352 0.86249	0.29086 0.28025	0.05492 0.05159	0.24420 0.23568	0.68150 0.67713
.2	0.38205 0.38198	0.05183 0.05085	0.29086 0.28025	0.75584 0.74710	0.97038 0.96842
.1	0.05259 0.05109	0.38205 0.38198	0.86352 0.86249	0.99153 0.99098	0.99986 0.99984

Table 3.9 - Powers and probabilities of type I error of Z_1-test and Z_2-test for m=n=80, α=0.05, α_1*=0.05433, α_2*=0.05177

p_y	p_x .1	.2	.3	.4	.5
.9	1.00000 1.00000	1.00000 1.00000	1.00000 1.00000	1.00000 1.00000	0.99997 0.99997
.8	1.00000 1.00000	1.00000 1.00000	1.00000 1.00000	0.99972 0.99967	0.98447 0.98345
.7	1.00000 1.00000	1.00000 1.00000	0.99947 0.99946	0.97199 0.97138	0.74347 0.73103
.6	1.00000 1.00000	0.99972 0.99967	0.97199 0.97138	0.71541 0.71512	0.24093 0.23805
.5	0.99997 0.99997	0.98447 0.98345	0.74347 0.73103	0.24093 0.23805	0.04790 0.04778
.4	0.99620 0.99617	0.80182 0.80095	0.27530 0.26514	0.05166 0.04720	0.24093 0.23805
.3	0.90620 0.90284	0.31489 0.31469	0.05180 0.05132	0.27530 0.26514	0.74347 0.73103
.2	0.43830 0.42287	0.05191 0.05028	0.31489 0.31469	0.80182 0.80095	0.98447 0.98345
.1	0.05168 0.04878	0.43830 0.42287	0.90620 0.90284	0.99620 0.99617	0.99997 0.99997

Table 3.10 - Powers and probabilities of type I error of Z_1-test and Z_2-test for m=n=90, α=0.05, α_1*=0.05788, α_2*=0.05258

p_y	.1	.2	p_x .3	.4	.5
.9	1.00000	1.00000	1.00000	1.00000	0.99999
	1.00000	1.00000	1.00000	1.00000	0.99999
.8	1.00000	1.00000	1.00000	0.99993	0.99156
	1.00000	1.00000	1.00000	0.99992	0.99127
.7	1.00000	1.00000	0.99986	0.98684	0.80364
	1.00000	1.00000	0.99981	0.98358	0.79302
.6	1.00000	0.99993	0.98684	0.77865	0.29021
	1.00000	0.99992	0.98358	0.75558	0.26110
.5	0.99999	0.99156	0.80364	0.29021	0.05384
	0.99999	0.99127	0.79302	0.26110	0.04450
.4	0.99849	0.84777	0.29765	0.05667	0.29021
	0.99830	0.84192	0.29392	0.05205	0.26110
.3	0.93776	0.35069	0.05254	0.29765	0.80364
	0.93174	0.34013	0.04972	0.29392	0.79302
.2	0.49091	0.05404	0.35069	0.84777	0.99156
	0.46913	0.04882	0.34013	0.84192	0.99127
.1	0.05294	0.49091	0.93776	0.99849	0.99999
	0.05071	0.46913	0.93174	0.99830	0.99999

Table 3.11 - Powers and probabilities of type I error of Z_1-test and Z_2-test for m=n=100, α=0.05, α_1*=0.05597, α_2*=0.05597

p_y	p_x .1	.2	.3	.4	.5
.9	1.00000 1.00000	1.00000 1.00000	1.00000 1.00000	1.00000 1.00000	1.00000 1.00000
.8	1.00000 1.00000	1.00000 1.00000	1.00000 1.00000	0.99998 0.99998	0.99569 0.99545
.7	1.00000 1.00000	1.00000 1.00000	0.99996 0.99996	0.99244 0.99244	0.83279 0.83201
.6	1.00000 1.00000	0.99998 0.99998	0.99244 0.99244	0.82607 0.82607	0.30965 0.30963
.5	1.00000 1.00000	0.99569 0.99545	0.83279 0.83201	0.30965 0.30963	0.05597 0.05597
.4	0.99937 0.99933	0.88297 0.87973	0.32232 0.31640	0.05179 0.05143	0.30965 0.30963
.3	0.95585 0.99564	0.37916 0.37073	0.05293 0.05098	0.32232 0.31640	0.83279 0.83201
.2	0.51770 0.51769	0.05190 0.05132	0.37916 0.37073	0.88297 0.87973	0.99569 0.99545
.1	0.05233 0.05195	0.51770 0.51769	0.95585 0.95564	0.99937 0.99933	1.00000 1.00000

Table 3.12 - Powers and probabilities of type I error of Z_1-test and Z_2-test for $m=10$, $n=5$, $\alpha=0.05$, $\alpha_1^*=0.14952$, $\alpha_2^*=0.04999$

p_y	p_x .1	.2	.3	.4	.5
.9	.96800	.90624	.81355	.71206	.61554
	.95103	.87660	.77603	.65337	.51548
.8	.88281	.76482	.62409	.49043	.38431
	.82174	.67703	.53690	.40559	.28734
.7	.75102	.60149	.44967	.32179	.23269
	.63845	.46729	.33068	.22404	.14362
.6	.58876	.43887	.30478	.20503	.14713
	.43218	.28602	.18277	.11308	.06997
.5	.41773	.29626	.20035	.14019	.11951
	.24150	.14939	.09068	.05830	.04797
.4	.26111	.18962	.14530	.12968	.14713
	.10322	.06472	.04718	.04882	.06997
.3	.13960	.13154	.14819	.17956	.23269
	.03194	.03235	.04997	.08641	.14363
.2	.06733	.13112	.21874	.30077	.38431
	.01497	.04800	.10553	.18617	.28734
.1	.04787	.19399	.36946	.51035	.61554
	.03190	.11542	.23383	.37174	.51548

Table 3.13 - Powers and probabilities of type I error of Z_1-test and Z_2-test for m=15, n=10, α=0.05, α_1*=0.08129, α_2*=0.06182

p_y	p_x .1	.2	.3	.4	.5
.9	.99825	.98183	.93156	.83025	.67477
	.99823	.98439	.94115	.85007	.70159
.8	.98440	.90895	.76758	.58674	.39800
	.98048	.90802	.77820	.60721	.42130
.7	.94116	.77862	.56172	.35958	.20487
	.91789	.75818	.55748	.36461	.21046
.6	.85008	.60740	.36617	.19410	.09844
	.78193	.55425	.34505	.18901	.09585
.5	.70163	.42166	.21180	.09886	.06532
	.58417	.34081	.17989	.08996	.06141
.4	.50555	.25215	.11273	.06943	.09844
	.37459	.17400	.08327	.06134	0.9585
.3	.29559	.12902	.07782	.10827	.20487
	.19221	.07972	.06182	.10808	.21046
.2	.12347	.07949	.12497	.23514	.39800
	.06363	.05564	.12205	.25046	.42130
.1	.03879	.13440	.29987	.48740	.67477
	.02375	.11880	.29434	.50528	.70159

Table 3.14 - Powers and probabilities of type I error of Z_1-test and Z_2-test for m=20, n=15, α=0.05, α_1*=0.06816, α_2*=0.06080

p_y	p_x .1	.2	.3	.4	.5
.9	0.99993 0.99992	0.99806 0.99685	0.98600 0.97714	0.93913 0.91632	0.81571 0.78697
.8	0.99791 0.99784	0.97281 0.96814	0.89331 0.86414	0.74606 0.68303	0.53358 0.46680
.7	0.98355 0.98348	0.88786 0.88267	0.69739 0.66714	0.47489 0.41565	0.27346 0.21713
.6	0.93077 0.93092	0.72642 0.72350	0.45801 0.44148	0.24148 0.21082	0.11342 0.08584
.5	0.80422 0.80589	0.50822 0.50739	0.24888 0.24311	0.10460 0.09239	0.06281 0.04790
.4	0.58898 0.59664	0.28604 0.28636	0.11321 0.10985	0.06676 0.05329	0.11342 0.08584
.3	0.32643 0.34667	0.12192 0.12247	0.06708 0.06044	0.12649 0.09853	0.27346 0.21713
.2	0.11341 0.14066	0.06179 0.05647	0.12741 0.11459	0.30023 0.26304	0.53358 0.46680
.1	0.03910 0.03624	0.15568 0.11197	0.35521 0.32433	0.60185 0.57808	0.81571 0.78697

Table 3.15 - Powers and probabilities of type I error of Z_1-test and Z_2-test for m=30, n=20, α=0.05, α_1*=0.07137, α_2*=0.05632

			p_x		
p_y	.1	.2	.3	.4	.5
.9	1.00000 1.00000	0.99991 0.99991	0.99799 0.99751	0.98305 0.97557	0.92069 0.88323
.8	0.99990 0.99987	0.99597 0.99565	0.96334 0.96207	0.85259 0.83892	0.64272 0.59236
.7	0.99749 0.99694	0.96414 0.95939	0.83188 0.82754	0.58753 0.57935	0.32272 0.29834
.6	0.97790 0.97579	0.85238 0.83436	0.59201 0.57757	0.30696 0.30195	0.12072 0.11441
.5	0.89984 0.89670	0.62723 0.60010	0.32310 0.30139	0.12123 0.11565	0.05843 0.05632
.4	0.71360 0.71199	0.34659 0.32823	0.12793 0.11135	0.06126 0.05208	0.12072 0.11441
.3	0.42664 0.42872	0.13251 0.12427	0.06358 0.04560	0.13569 0.10622	0.32272 0.29834
.2	0.15483 0.15885	0.06777 0.04264	0.16168 0.10519	0.38007 0.30546	0.64272 0.59236
.1	0.06492 0.03247	0.23335 0.11371	0.49894 0.36060	0.75907 0.66691	0.92069 0.88323

Table 3.16 - Powers and probabilities of type I error
of Z_1-test and Z_2-test for m=40, n=30, α=0.05, α_1*=0.06098, α_2*=0.05303

	p_x				
p_y	.1	.2	.3	.4	.5
.9	1.00000 1.00000	1.00000 1.00000	0.99993 0.99992	0.99820 0.99755	0.97944 0.97184
.8	1.00000 1.00000	0.99980 0.99977	0.99419 0.99326	0.94811 0.94253	0.78916 0.76461
.7	0.99990 0.99990	0.99355 0.99333	0.93575 0.92996	0.73843 0.72174	0.42591 0.40511
.6	0.99734 0.99734	0.94095 0.94068	0.73307 0.72582	0.40693 0.38849	0.14684 0.13566
.5	0.97292 0.97293	0.76101 0.76091	0.40722 0.40453	0.14422 0.13720	0.05800 0.05178
.4	0.85709 0.85737	0.45229 0.45222	0.14253 0.14131	0.05767 0.05297	0.14684 0.13566
.3	0.56442 0.56753	0.16567 0.16469	0.05919 0.05070	0.16413 0.14362	0.42591 0.40511
.2	0.19656 0.20786	0.06064 0.05070	0.19216 0.15632	0.49330 0.44501	0.78916 0.76461
.1	0.05791 0.04886	0.26452 0.20500	0.61296 0.55854	0.88026 0.84884	0.97944 0.97184

Table 3.17 - Powers of UMPU test for m=n=5, α=0.05

			p_x		
p_y	.1	.2	.3	.4	.5
.9	.78608	.61919	.46102	.32433	.21565
.8	.61919	.45653	.32153	.21576	.13844
.7	.46102	.32153	.21537	.13873	.08780
.6	.32434	.21576	.13873	.08791	.05922
.5	.21566	.13844	.08780	.05922	---
.4	.13664	.08719	.05913	---	.05922
.3	.08544	.05881	---	.05913	.08780
.2	.05812	---	.05881	.08719	.13844
.1	---	.05812	.08544	.13664	.21566

Table 3.18 - Powers of UMPU test for m=n=10, α=0.05

			p_x		
p_y	.1	.2	.3	.4	.5
.9	.98260	.92306	.81244	.65690	.47734
.8	.92306	.78338	.60785	.42747	.26845
.7	.81244	.60785	.41219	.25239	.13890
.6	.65690	.42747	.25239	.13584	.07089
.5	.47734	.26845	.13890	.07089	---
.4	.30357	.14832	.07224	---	.07089
.3	.16440	.07492	---	.07224	.13890
.2	.07827	---	.07492	.14832	.26845
.1	---	.07827	.16440	.30357	.47734

Table 3.19 - Powers of UMPU test for m=n=20, α=0.05

p_y	p_x .1	.2	.3	.4	.5
.9	.99998	.99893	.98824	.93984	.81325
.8	.99893	.98197	.90919	.74546	.50821
.7	.98824	.90919	.72438	.47467	.24540
.6	.93984	.74547	.47467	.23789	.09536
.5	.81325	.50821	.24540	.09536	---
.4	.59596	.27108	.09885	---	.09536
.3	.33763	.00787	---	.09885	.24540
.2	.13045	--	.10787	.27108	.50821
.1	---	.13045	.33762	.59596	.81325

Table 3.20 - Powers of UMPU test for m=n=30, α=0.05

p_y	p_x .1	.2	.3	.4	.5
.9	1.00000	0.99999	0.99926	0.98982	0.93837
.8	0.99999	0.99856	0.97935	0.88660	0.66495
.7	0.99926	0.97935	0.86925	0.62404	0.32725
.6	0.98982	0.88660	0.62404	0.31291	0.10934
.5	0.93837	0.66495	0.32725	0.10934	---
.4	0.78021	0.37083	0.11581	---	0.10934
.3	0.48812	0.13682	---	0.11581	0.32725
.2	0.18555	---	0.13682	0.37083	0.66495
.1	---	0.18555	0.48812	0.78021	0.93837

Table 3.21 - Powers of UMPU test for m=n=40, α=0.05

p_y	p_x .1	.2	.3	.4	.5
.9	1.00000	1.00000	0.99998	0.99906	0.98489
.8	1.00000	0.99995	0.99715	0.96364	0.81195
.7	0.99998	0.99715	0.95542	0.77383	0.44225
.6	0.99906	0.96364	0.77383	0.42778	0.14401
.5	0.98489	0.81195	0.44225	0.14401	---
.4	0.89298	0.49466	0.15231	---	0.14401
.3	0.61571	0.17405	---	0.15231	0.44225
.2	0.23157	---	0.17405	0.49466	0.81195
.1	---	0.23157	0.61571	0.89298	0.98489

Table 3.22 - Powers of UMPU test for m=n=50, α=0.05

	p_x				
p_y	.1	.2	.3	.4	.5
.9	1.00000	1.00000	1.00000	0.99991	0.99626
.8	1.00000	1.00000	0.99959	0.98790	0.89122
.7	1.00000	0.99959	0.98412	0.86046	0.53013
.6	0.99991	0.98790	0.86046	0.51371	0.16901
.5	0.99626	0.89122	0.53013	0.16091	---
.4	0.94933	0.58897	0.17957	---	0.16091
.3	0.71743	0.20736	---	0.17957	0.53013
.2	0.28107	---	0.20736	0.58897	0.89122
.1	---	0.28107	0.71743	0.94933	0.99626

Table 3.23 - Powers of UMPU test for m=n=60, α=0.05

p_y	p_x .1	.2	.3	.4	.5
.9	1.00000	1.00000	1.00000	0.99999	0.99913
.8	1.00000	1.00000	0.99994	0.99618	0.93914
.7	1.00000	0.99994	0.99457	0.91602	0.60818
.6	0.99999	0.99618	0.91602	0.59018	0.19396
.5	0.99913	0.93914	0.60818	0.19396	---
.4	0.97701	0.66991	0.20698	---	0.19396
.3	0.79627	0.24065	---	0.20698	0.60818
.2	0.32982	---	0.24065	0.66991	0.93194
.1	---	0.32982	0.79627	0.97701	0.99913

Table 3.24 - Powers of UMPU test for m=n=70, α=0.05

			p_x		
p_y	.1	.2	.3	.4	.5
.9	1.00000	1.00000	1.00000	1.00000	0.99981
.8	1.00000	1.00000	0.99999	0.99885	0.96685
.7	1.00000	0.99999	0.99822	0.95064	0.67657
.6	1.00000	0.99885	0.95064	0.65749	0.21895
.5	0.99981	0.96685	0.67657	0.21895	---
.4	0.98939	0.73760	0.23441	---	0.21895
.3	0.85012	0.27255	---	0.23441	0.67657
.2	0.38480	--	0.27255	0.73760	0.96685
.1	---	0.38480	0.85012	0.98939	0.99981

Table 3.25 - Powers of UMPU test for m=10, n=5, α=0.05

			p_x		
p_y	.1	.2	.3	.4	.5
.9	.89145	.75541	.59215	.42491	.27868
.8	.74959	.57558	.41292	.27527	.17059
.7	.59219	.41486	.27374	.17007	.10105
.6	.43775	.28068	.17137	.10117	.06238
.5	.30018	.17696	.10220	.06245	---
.4	.18905	.10480	.06270	---	.06238
.3	.10986	.06323	---	.06240	.10105
.2	.06419	---	.06237	.10087	.17059
.1	---	.06226	.09977	.16954	.27868

Table 3.26 - Powers of UMPU test for m=15, n=10, α=0.05

			p_x		
p_y	.1	.2	.3	.4	.5
.9	.99563	.96559	.88549	.74599	.56146
.8	.96847	.86630	.69862	.50125	.31534
.7	.89555	.70850	.49087	.29825	.15876
.6	.76400	.51828	.30472	.15725	.07603
.5	.58259	.33093	.16468	.07714	---
.4	.38237	.17899	.07973	---	.07603
.3	.20564	.08285	---	.07745	.15876
.2	.08907	---	.08118	.17088	.31534
.1	---	.08647	.19520	.36458	.56146

Table 3.27 - Powers of UMPU test for m=20, n=15, α=0.05

p_y	p_x .1	.2	.3	.4	.5
.9	.99984	.99602	.97241	.89710	.74283
.8	.99589	.96044	.85521	.66964	.44131
.7	.97183	.85519	.64857	.41158	.21304
.6	.89685	.67128	.41236	.20718	.08790
.5	.74510	.44486	.21410	.08798	---
.4	.52559	.23736	.09111	---	.08790
.3	.29180	.09878	---	.09078	.21304
.2	.11570	---	.09802	.23441	.44131
.1	---	.11357	.28607	.52002	.74283

Table 3.28 - Powers of UMPU test for m=30, n=20, α=0.05

p_y	p_x .1	.2	.3	.4	.5
.9	1.00000	0.99984	0.99655	0.97240	0.88147
.8	0.99981	0.99385	0.95168	0.81948	0.58321
.7	0.99613	0.95177	0.80145	0.54644	0.28418
.6	0.97074	0.82154	0.54922	0.27585	0.10474
.5	0.88006	0.58813	0.28691	0.10472	---
.4	0.68219	0.31995	0.10958	---	0.10474
.3	0.40190	0.12127	--	0.10873	0.28418
.2	0.15147	--	0.11972	0.31489	0.58321
.1	---	0.14711	0.39248	0.67782	0.88147

Table 3.29 - Powers of UMPU test for m=40, n=30, α=0.05

			p_x		
p_y	.1	.2	.3	.4	.5
.9	1.00000	1.00000	0.99988	0.99685	0.96799
.8	1.00000	0.99969	0.99202	0.93484	0.74712
.7	0.99985	0.99182	0.92222	0.70786	0.38860
.6	0.99644	0.93403	0.70736	0.37584	0.13042
.5	0.96637	0.74736	0.38929	0.12989	---
.4	0.83794	0.43532	0.13708	---	0.13042
.3	0.54695	0.15534	---	0.13667	0.38884
.2	0.20425	---	0.15409	0.43277	0.74712
.1	---	0.20105	0154363	0.83920	0.96799

Table 3.30 - Powers and probabilities of type I error of UN test for m=n=5, α=0.05, α^*=0.02148

p_y	p_x .1	.2	.3	.4	.5
.9	.73610	.54285	.36705	.22449	.12098
.8	.54285	.37581	.24195	.14236	.07450
.7	.36705	.24195	.14945	.08514	.04411
.6	.22449	.14236	.08514	.04804	.02700
.5	.12098	.07450	.04411	.02700	.02148
.4	.05479	.03332	.02134	.01884	.02700
.3	.01905	.01233	.01210	.02134	.04411
.2	.00423	.00467	.01233	.03332	.07450
.1	.00055	.00423	.01905	.05479	.12098

α^* is the size of UN test (for Table 3.30 to 3.36)

Table 3.31 - Powers and probabilities of type I error of UN test for m=n=10, α=0.05, α*=0.04219

			p_x		
p_y	.1	.2	.3	.4	.5
.9	.98884	.93666	.82371	.65904	.47136
.8	.93666	.80539	.62163	.42488	.25423
.7	.82371	.62163	.41849	.24767	.12752
.6	.65904	.42488	.24767	.12861	.06221
.5	.47136	.25423	.12752	.06221	.04219
.4	.29121	.13123	.05968	.04031	.06221
.3	.14304	.05827	.03711	.05968	.12752
.2	.04620	.02993	.05827	.13123	.25423
.1	.00904	.04620	.14304	.29121	.47136

Table 3.32 - Powers and probabilities of type I error of UN test for m=n=20, α=0.05, α^*=0.04643

			p_x		
p_y	.1	.2	.3	.4	.5
.9	.99998	.99888	.98768	.94088	.82166
.8	.99888	.98081	.90140	.72786	.49233
.7	.98768	.90140	.70520	.44668	.22190
.6	.94088	.72786	.44668	.21357	.07999
.5	.82166	.49233	.22190	.07999	.03927
.4	.60854	.26511	.08761	.03996	.07999
.3	.33871	.10502	.04501	.08761	.22190
.2	.11026	.04430	.10502	.26511	.49233
.1	.01684	.11026	.33871	.60854	.82166

Table 3.33 - Powers and probabilities of type I error of UN test for m=10, n=5, α=0.05, α^*=0.04126

p_y	p_x .1	.2	.3	.4	.5
.9	.91997	.80131	.62939	.42917	.24063
.8	.77940	.61655	.44104	.27735	.14664
.7	.61030	.43983	.28955	.16929	.08595
.6	.43914	.28808	.17566	.09735	.05211
.5	.28545	.16990	.09692	.05496	.04126
.4	.16918	.08721	.04870	.03687	.05211
.3	.07494	.03687	.02519	.03948	.08595
.2	.02417	.01226	.02034	.06111	.14664
.1	.00334	.00497	.02889	.10233	.24063

Table 3.34 - Powers and probabilities of type I error of UN test for $m=12$, $n=8$, $\alpha=0.05$, $\alpha^*=0.04337$

p_y	p_x .1	.2	.3	.4	.5
.9	.98725	.92213	.78500	.60976	.43272
.8	.93401	.79791	.59148	.38377	.22490
.7	.82090	.63095	.41298	.22976	.11339
.6	.65802	.44608	.26014	.12802	.05921
.5	.47511	.27606	.14407	.06762	.04306
.4	.30322	.14631	.07096	.04337	.05921
.3	.16300	.06635	.04144	.05656	.11339
.2	.06450	.03174	.05569	.11987	.22490
.1	.01289	.03430	.12348	.26626	.43272

Table 3.35 - Powers and probabilities of type I error of UN test for m=15, n=10, α=0.05, α^*=0.04034

			p_x		
p_y	.1	.2	.3	.4	.5
.9	.99598	.96897	.88629	.73263	.52245
.8	.96355	.86617	.69583	.48302	.28332
.7	.87490	.68948	.47843	.28083	.13736
.6	.72579	.47822	.28221	.14214	.06282
.5	.53909	.28410	.13959	.06410	.04034
.4	.35125	.14198	.05887	.03750	.06282
.3	.19318	.05955	.03128	.05751	.13736
.2	.07969	.02525	.04802	.13403	.28332
.1	.01585	.02568	.11759	.29779	.52245

Table 3.36 - Powers and probabilities of type I error of UN test for m=20, n=15, α=0.05, α^*=0.04656

			p_x		
p_y	.1	.2	.3	.4	.5
.9	.99991	.99672	.97358	.89211	.71580
.8	.99700	.96723	.86096	.66463	.41951
.7	.97356	.87390	.66319	.40823	.19998
.6	.89082	.69084	.43103	.20749	.08155
.5	.72330	.44594	.22445	.08865	.04633
.4	.49632	.22048	.08908	.04612	.08155
.3	.26837	.08052	.03855	.07703	.19998
.2	.09396	.02982	.06914	.20133	.41955
.1	.01275	.05354	.21352	.46314	.71580

4. COMPARISONS OF THE FOUR TESTS

Various comparisons among four tests based on their power and some practical considerations are made in this section. Considering power comparisons of the four tests, it is clear that no one test is uniformly "better" than the others. Thus let us first state, two desired properties of a good test that we hope will help us to make the comparisons easier. This properties are based primarily on practical considerations and are:

(I) The test should not depend on randomization. Since a randomized test has been rarely used by experimenters in practice, dependence on randomization is not a desired property. Among the four tests, the UMPU test is the only one that does not satisfy property I.

(II) The size of the test is close to the predetermined nominal level. An experimenter wants to control the probability of type I error by selecting the nominal level. If the size of a test is higher than the nominal level, then the experimenter has no control of type I error.

The UMPU test certainly satisfies property II. As a matter of fact, for any predetermined nominal level, one can always find a UMPU test such that the size of the test is that of the nominal level. From the way one derives the rejection region of a UN test, it is clear that the size of a UN test is always less than or equal to the nominal level. Hence, the UN test also satisfies property II.

However, the Z_1-test does not satisfy property II. Examine specifically the cases for $\alpha = 0.05$ or 0.01. Let $\alpha_1{}^*$ be the corresponding size of the Z_1-test. Table 4.1 and Table 4.2 below indicate that $\alpha_1{}^*$ may be far higher than the nominal level set by the experimenter. In the case that $\alpha = 0.05$, $m = 10$ and $n = 5$, $\alpha_1{}^*$ is almost three times larger than

Table 4.1 - The sizes of the Z_1-test when the nominal level $\alpha = 0.05$

m	5	10	20	30	40	50	60	70
n	5	10	20	30	40	50	60	70
α_1*	.1094	.0949	.0808	.0604	.0590	.0613	.0629	.0634
m	80	90	100	10	15	20	30	40
n	80	90	100	5	10	15	20	30
α_1*	.0543	.0579	.0560	.1492	.0813	.0682	.0714	.0610

Table 4.2 - The sizes of the Z_1-test when the nominal level $\alpha = 0.01$

m	5	10	20	30	40	50	60	70
n	5	10	20	30	40	50	60	70
α_1*	.0619	.0422	.0167	.0136	.0146	.0132	.0134	.0140
m	80	90	100	10	15	20	30	40
n	80	90	100	5	10	15	20	30
α_1*	.0140	.0137	.0131	.0784	.0307	.0243	.0193	.0149

the nominal level. This result was also pointed out in [1] and [3].

For the Z_2-test, examine the case where the nominal level, α, is 0.05 and 0.01. The sizes of the Z_2-test (denoted by α_2*) for various sample sizes are listed in Table 4.3 and Table 4.4 below. From these two tables, we might say that the sizes of the Z_2-test tend to fluctuate slightly above the nominal level. The studies made by Berengut and Petkau [1] also support the above observation. Hence we conclude that Z_2-test satisfies property II.

Now we shall compare the tests in terms of their powers. Since the size of the Z_1-test could be much higher than the predetermined nominal level for small or moderate sample sizes, it is difficult to make meaningful comparisons between the Z_1-test and the other three tests based on their powers, so the Z_1-test is omitted from the comparisons below.

First let us compare the powers of Z_2-test and UMPU test. Since the sizes of the Z_2-test fluctuate below and above the nominal level while the sizes of the UMPU test are always equal to the nominal level, it is clearly improper just to compare their powers only without considering the sizes of the tests. One reasonable way to compare the two tests, suggested below, takes the sizes of the two tests into account. Let $DEP_{U,2}$ denote the difference of the effective powers between the UMPU test and the Z_2-test. For $p_x \neq p_y$, $DEP_{U,2}$ is defined as

$$DEP_{U,2}(p_x, p_y, \alpha, m, n) = [P_U - \alpha_U^*] - [P_2 - \alpha_2^*] \qquad (4.1)$$

where

Table 4.3 - The sizes of the Z_2-test when the nominal level $\alpha = 0.05$

m	5	10	20	30	40	50	60	70
n	5	10	20	30	40	50	60	70
α_2^*	.0619	.0422	.0534	.0553	.0590	.0569	.0548	.0522
m	80	90	100	10	15	20	30	40
n	80	90	100	5	10	15	20	30
α_2^*	.0517	.0526	.0559	.0500	.0618	.0608	.0563	.0530

Table 4.4 - The sizes of the Z_2-test when the nominal level $\alpha = 0.01$

m	5	10	20	30	40	50	60	70
n	5	10	20	30	40	50	60	70
α_2^*	.0215	.0128	.0111	.0135	.0108	.0120	.0106	.0105
m	80	90	100	10	15	20	30	40
n	80	90	100	5	10	15	20	30
α_2^*	.0102	.0104	.0104	.0093	.0102	.0092	.0093	.0110

$P_U = P_U(p_x, p_y, \alpha, m, n)$,

$\alpha_U{}^*$ = the size of a nominal level α UMPU test with sample sizes (m,n),

$P_2 = P_2(p_x, p_y, \alpha, m, n)$,

$\alpha_2{}^*$ = the size of a nominal level α Z_2-test with sample sizes (m,n).

Remark 1. Equation (4.1) may be rewritten as

$$DEP_{U,2} = (\beta_2 + \alpha_2{}^*) - (\beta_U + \alpha_U{}^*) \tag{4.2}$$

where

$\beta_U = 1 - P_U$ = the probability of Type II error of the UMPU test,

$\beta_2 = 1 - P_2$ = the probability of Type II error of the Z_2-test.

We feel that if neither $\alpha_2{}^*$ nor $\alpha_U{}^*$ differ much from the nominal level α, then $DEP_{U,2}$ is a reasonable measurement to compare two tests. In our case $\alpha_U{}^* \equiv \alpha$ and $\alpha_2{}^*$ is always close to α.

Remark 2. Positive values of $DEP_{U,2}$ indicate that UMPU test is better but negative values of $DEP_{U,2}$ indicate otherwise. $DEP_{U,2}$ has been computed for the case, nominal level $\alpha = 0.05$; $p_x, p_y = 0.1\ (0.1)0.9$; m = n = 5, 10, 20, 30, 40, 50, 60, 70; m = 10, n = 5; m = 15, n = 10; m = 20, n = 15; m = 30, n = 20 and m = 40, n = 30. We present some of the results in Table 4.5 to Table 4.12.

Remark 3. Using equations (3.1) and (3.3), it is easy to show that

$$DEP_{U,2}(p_x, p_y, \alpha, m, n) = DEP_{U,2}(1-p_x, 1-p_y, \alpha, m, n) \quad \ldots \tag{4.3}$$

Hence, in Table 4.5 to Table 4.12, only one part of the values of $DEP_{U,2}$ are presented.

Table 4.5 - $DEP_{U,2}$ values for $m = n = 5$, $\alpha = 0.05$, $\alpha_U^* = 0.05$, $\alpha_2^* = 0.06185$

			p_x		
p_y	.1	.2	.3	.4	.5
.9	-0.02426	-0.05663	-0.08873	-0.09813	-0.08054
.8	-0.05664	-0.04168	-0.04432	-0.04452	-0.03471
.7	-0.08873	-0.04432	-0.02664	-0.01856	-0.01152
.6	-0.09812	-0.04452	-0.01856	-0.00674	-0.00143
.5	-0.08054	-0.03471	-0.01152	-0.00143	---
.4	-0.04373	-0.01597	-0.00337	---	-0.00143
.3	-0.00143	0.00578	---	-0.00337	-0.01152
.2	0.03283	---	0.00578	-0.01597	-0.03471
.1	---	0.03283	-0.00143	-0.04373	-0.08054

Table 4.6 - $DEP_{U,2}$ values for $m = n = 10$, $\alpha = 0.05$, $\alpha_U^* = 0.05$, $\alpha_2^* = 0.04219$

			p_x		
p_y	.1	.2	.3	.4	.5
.9	-0.01397	-0.02141	-0.01908	-0.00995	-0.00183
.8	-0.02141	-0.02982	-0.02159	-0.00522	0.00641
.7	-0.01908	-0.02159	-0.01411	-0.00309	0.00357
.6	-0.00995	-0.00522	-0.00309	-0.00058	0.00087
.5	-0.00183	0.00641	0.00357	0.00087	---
.4	0.00455	0.00928	0.00475	---	0.00087
.3	0.01355	0.03718	---	0.00475	0.00357
.2	0.02426	---	0.03718	0.00928	0.00641
.1	---	0.02426	0.01355	0.00455	-0.00182

Table 4.7 - $DEP_{U,2}$ values for m = n = 20, α = 0.05, α_U^* = 0.05, α_2^* = 0.05337

p_y	.1	.2	p_x .3	.4	.5
.9	0.00337	0.00324	0.00117	-0.00822	-0.02142
.8	0.00324	0.00365	0.00417	-0.00466	-0.01934
.7	0.00117	0.00417	0.01451	0.00826	0.00360
.6	-0.00822	-0.00465	0.01500	0.01943	0.01144
.5	-0.02142	-0.01934	0.00360	0.01144	---
.4	-0.02398	-0.01670	-0.00179	---	0.01144
.3	-0.01641	-0.00245	---	-0.00179	0.00360
.2	-0.00418	---	-0.00245	-0.01670	-0.01934
.1	---	-0.00418	-0.01641	-0.02398	-0.02142

Table 4.8 - $DEP_{U,2}$ values for m = n = 30, α = 0.05, α_U^* = 0.05, α_2^* = 0.05534

p_y	.1	.2	p_x .3	.4	.5
.9	0.00534	0.00534	0.00505	0.00242	-0.00528
.8	0.00534	0.00471	-0.00059	-0.01325	-0.02210
.7	0.00505	-0.00059	-0.02135	-0.03374	-0.01847
.6	-0.00242	-0.01265	-0.03374	-0.03136	-0.00865
.5	-0.00528	-0.02210	-0.01847	-0.00865	---
.4	-0.01054	-0.01630	-0.00153	---	-0.00865
.3	-0.00573	-0.00011	---	-0.00153	-0.01847
.2	-0.00306	---	-0.00011	-0.01630	-0.02210
.1	---	-0.00306	-0.00573	-0.01054	-0.00528

Table 4.9 - $DEP_{U,2}$ values for m = 10, n = 5, α = 0.05, α_U^* = 0.05, α_2^* = 0.04999

p_y	p_x .1	.2	.3	.4	.5
.9	-0.05959	-0.12120	-0.18389	-0.22847	-0.23681
.8	-0.07216	-0.10146	-0.12399	-0.13033	-0.11676
.7	-0.04627	-0.05244	-0.05695	-0.05398	-0.04259
.6	0.00556	-0.00535	-0.01141	-0.01192	-0.00760
.5	0.05867	0.02756	0.01151	0.00414	---
.4	0.08582	0.04007	0.01551	---	-0.00760
.3	0.09488	0.03087	---	-0.02402	-0.04259
.2	0.03228	---	-0.04317	-0.08531	-0.11676
.1	---	-0.05317	-0.13407	-0.20220	-0.23681

Table 4.10 - $DEP_{U,2}$ values for m = 15, n = 10, α = 0.05, α_U^* = 0.05, α_2^* = 0.06182

p_y	p_x .1	.2	.3	.4	.5
.9	0.00922	-0.00698	-0.04384	-0.09226	-0.09831
.8	-0.00019	-0.02990	-0.06776	-0.09414	-0.09414
.7	-0.01052	-0.03786	-0.05479	-0.05454	-0.03988
.6	-0.00611	-0.02415	-0.02851	-0.01994	-0.00800
.5	0.01024	0.00194	-0.00339	-0.00100	---
.4	0.01960	0.01681	0.00831	---	-0.00800
.3	0.02525	0.01495	---	-0.01881	-0.03988
.2	0.03726	---	-0.02905	-0.06776	-0.09414
.1	---	-0.02051	-0.08732	-0.12888	-0.09831

Table 4.11 - $DEP_{U,2}$ values for m = 20, n = 15, α = 0.05, α_U^* = 0.05, α_2^* = 0.0608

			p_x		
p_y	.1	.2	.3	.4	.5
.9	0.01072	0.00997	0.00607	-0.00842	-0.03334
.8	0.00885	0.00310	0.00187	-0.00259	-0.01469
.7	-0.00085	-0.01668	-0.00777	0.00673	0.00671
.6	-0.02327	-0.04142	-0.01832	0.00716	0.01286
.5	-0.04999	-0.05173	-0.01821	0.00639	---
.4	-0.06025	-0.03820	-0.00795	---	0.01286
.3	-0.04407	-0.01289	---	0.00305	0.00670
.2	-0.01416	---	-0.00577	-0.01783	-0.01469
.1	---	0.01240	-0.02746	-0.04726	-0.03334

Table 4.12 - $DEP_{U,2}$ values for m = 40, n = 30, α = 0.05, α_U^* = 0.05, α_2^* = 0.053

			p_x		
p_y	.1	.2	.3	.4	.5
.9	0.00303	0.00303	0.00299	0.00233	-0.00082
.8	0.00303	0.00295	0.00179	-0.00466	-0.01446
.7	0.00298	0.00152	-0.00471	-0.01085	-0.01324
.6	0.00213	-0.00362	-0.01543	-0.00962	-0.00221
.5	-0.00353	-0.01052	-0.01221	-0.00428	---
.4	-0.01640	-0.01387	-0.00120	---	-0.00221
.3	-0.01755	-0.00632	---	-0.00392	-0.01324
.2	-0.00058	---	0.00080	-0.00921	-0.01446
.1	---	-0.00092	-0.01188	-0.00661	-0.00082

For both sample sizes exceeding 30, values of $DEP_{U,2}$ are fairly close to zero. For m = n = 5, 10, and 20 (See Table 4.5 to Table 4.7) values of $DEP_{U,2}$ range from -0.09813 to 0.03282 and are less than 1 percent for all except a few values of p_x and p_y. For the cases m = 10, n = 5; m = 15, n = 10; m = 20, n = 15; and m = 40, n = 30 (See Table 4.8 to Table 4.12), values of $DEP_{U,2}$ range from -0.23681 to 0.09488, and the values of $DEP_{U,2}$ are less than 2 percent for most cases. Hence we conclude that, in terms of $DEP_{U,2}$, Z_2-test is quite comparable with UMPU test.

Next we compare Z_2-test with UN test in terms of their powers and sizes. Since the sizes of UN test for various sample sizes up to 20 are given either in [5] or in [6], an experimenter can always choose nominal level to be equal to the size given in [5] or in [6]. Hence, in the following when we make comparisons between Z_2-test and UN test, we choose the size of UN test given in [5] or in [6] as the nominal level and then compare the two tests the same way we did for UMPU test and Z_2-test. Let $DEP_{UN,2}$ denote the differences of the effective powers between UN test and Z_2-test. For $p_x \neq p_y$, $DEP_{UN,2}$ is defined as:

$$DEP_{UN,2}(p_x, p_y, \alpha, m, n) = (P_{UN} - \alpha_{UN}^*) - (P_2 - \alpha_2^*) \qquad (4.4)$$

where

$P_{UN} = P_{UN}(p_x, p_y, \alpha, m, n)$,

α_{UN}^* = the size of the UN test with sample sizes (m,n).

We have always selected the size of UN test presented in [5] or in [6] as the nominal level α, hence $\alpha_{UN}^* = \alpha$.

Remark 1. Positive values of $DEP_{UN,2}$ indicate that UN test is better but negative values of $DEP_{UN,2}$ indicate otherwise. $DEP_{UN,2}$ has been computed for case p_x, p_y = 0.1 (0.1) 0.9; m = n = 5, 10, 20; m = 10, n = 5; m = 12, n = 18; m = 15, n = 10; m = 20, n = 15 for various nominal levels. When $\alpha = 0.02148$, m = n = 5 or $\alpha = 0.04219$, m = n = 10; $DEP_{UN,2} = 0$ for all p_x, p_y = 0.1 (0.1) 0.9. The computed values of $DEP_{UN,2}$ for the other cases are presented in Tables 4.13 to 4.16

Remark 2. Using equations (3.1) and (3.5), it is easy to see that

$$DEP_{UN,2}(p_x,\ p_y,\ \ ,\ m,\ n) = DEP_{UN,2}(1-p_x,\ 1-p_y,\ \ ,\ m,\ n) \quad ..(4.5)$$

Hence, in Tables 4.13 to Table 4.16, only one part of the values $DEP_{UN,2}$ are presented. From Table 4.13 to Table 4.16, we see that $DEP_{UN,2}$ varies from -0.22130 to 0.00700. Hence, in terms of $DEP_{UN,2}$, Z_2-test is as good as UN test and, in most cases, Z_2-test is better.

Table 4.13 - Values of $DEP_{UN,2}$ for m = 10 and n = 5, $\alpha = \alpha_{UN}^* = 0.04126$, $\alpha_2^* = 0.04999$

p_y	p_x .1	.2	.3	.4	.5
.9	-0.02233	-0.01170	-0.00033	0.00572	0.00786
.8	-0.08847	-0.05175	-0.01752	0.00006	0.00596
.7	-0.15700	-0.08834	-0.03238	-0.00475	0.00400
.6	-0.20550	-0.10878	-0.03965	-0.00700	0.00256
.5	-0.22130	-0.10871	-0.03798	-0.00628	---
.4	-0.19383	-0.09023	-0.02898	---	-0.00628
.3	-0.15016	-0.05992	---	-0.02898	-0.03798
.2	-0.08252	---	-0.05993	-0.09023	-0.10871
.1	---	-0.08252	-0.15016	-0.19383	-0.22130

Table 4.14 - Values of $DEP_{UN,2}$ for m = 12 and n = 18, $\alpha = \alpha_{UN}^* = 0.04337$, $\alpha_2^* = 0.05898$

p_y	p_x .1	.2	.3	.4	.5
.9	0.01243	-0.00588	-0.04556	-0.07155	-0.05860
.8	-0.00314	-0.01188	-0.04104	-0.06159	-0.04975
.7	-0.03660	-0.02317	-0.02565	-0.03200	-0.02384
.6	-0.07238	-0.03726	-0.01556	-0.00984	-0.00479
.5	-0.08768	-0.04273	-0.00993	0.00031	---
.4	-0.07208	-0.03318	-0.00498	---	-0.00479
.3	-0.03587	-0.01337	---	-0.00498	-0.00238
.2	-0.00189	---	-0.01337	-0.03318	-0.00498
.1	---	-0.03587	-0.03587	-0.07208	-0.05860

Table 4.15 - Values of $DEP_{UN,2}$ for m = 20 and n = 15, $\alpha = \alpha_{UN}^* = 0.04656$, $\alpha_2^* = 0.05576$

			p_x		
p_y	.1	.2	.3	.4	.5
.9	0.00919	0.00907	0.00599	-0.01002	-0.03660
.8	0.00836	0.00829	0.00616	-0.00728	-0.02834
.7	-0.00072	0.00043	0.00529	0.00222	-0.00570
.6	-0.03090	-0.02345	-0.00124	0.00595	-0.00526
.5	-0.07339	-0.05223	-0.00938	0.00555	---
.4	-0.09111	-0.05642	-0.01080	---	0.00560
.3	-0.06904	-0.03109	---	-0.01080	-0.00570
.2	-0.03724	---	-0.03109	-0.05642	-0.02834
.1	---	-0.03724	-0.06904	-0.09111	-0.03660

Table 4.16 - Values of $DEP_{UN,2}$ for m = n = 20, $\alpha = \alpha_{UN}^* = 0.04643$, $\alpha_2^* = 0.04976$

			p_x		
p_y	.1	.2	.3	.4	.5
.9	0.00333	0.00333	0.00333	0.00330	0.00279
.8	0.00333	0.00333	0.00333	0.00333	0.00328
.7	0.00333	0.00333	0.00333	0.00333	0.00333
.6	0.00329	0.00333	0.00333	0.00333	0.00333
.5	0.00277	0.00328	0.00333	0.00333	---
.4	-0.00093	0.00292	0.00330	---	0.00333
.3	-0.01260	0.00165	---	0.00330	0.00333
.2	-0.02423	---	0.00165	0.00292	0.00333
.1	---	-0.02423	-0.01260	-0.00093	0.00279

To summarize:

(1) If an experimenter expects to control the probability of Type I error by selecting of the nominal level, one should not use the Z_1-test because the size of Z_1-test could be much higher than the nominal level for small or moderate sample sizes.

(2) The UMPU test depends on randomization and needs to use fairly complete hypergeometric distribution tables which makes it inconvenient to use in practice.

(3) UN test is also inconvenient to use since special tables are needed.

(4) We recommend the Z_2-test for testing H_0 against H_1 for these reasons:

(a) The size of Z_2-test is usually only slightly higher than the nominal level. Thus one can control the probability of Type I error by selecting of a particular nominal level.

(b) Z_2-test is easy to use and the only table needed is the standard normal distribution table.

(c) Z_2-test is a nonrandomized test.

(d) In terms of $DEP_{UN,2}$, Z_2-test is as good as UN test and in most cases it is better.

(e) In terms of $DEP_{U,2}$, Z_2-test is quite comparable with UMPU test.

(f) For sample sizes m, n (both larger or equal to 40), the powers of Z_1 and Z_2 are almost identical.

REFERENCES

[1] Berengut, David and Petkau, Albert John (1979). "On Testing the Equality of Two Proportions," Technical Report No. 267, Department of Statistics, Stanford University.

[2] Conover, W. J. (.978). "A Fundamental Question of Practical Statistics, Letters to the Editor, The American Statistician, 32:114.

[3] Eberhardt, K. R., and Fligner, M. A. (1977). "A Comparison of Two Tests for Equality of Two Proportions," The American Statistician, 31:151-155.

[4] Lehmann, E. L. (1970). Testing Statistical Hypotheses, New York: John Wiley & Sons.

[5] McDonald, L. L., Davis, B. M., and Milliken, G. A. (1975). "A Nonrandomized Unconditional Test for Comparing Two Proportions," Research Paper 94, College of Commerce and Industry, University of Wyoming.

[6] McDonald, L. L., Davis, B. M., and Milliken, G. A. (1977). "A Nonrandomized Unconditional Test for Comparing Two Proportions in 2x2 Contingency Tables," Technometrics, 19, 145-157.

[7] Pape, E. S. (1979). "Tests for Difference in Proportion," Letters to the Editor, The American Statistician, 33:92.

[8] Robbins, Herbert (1977). "A Fundamental Question of Practical Statistics," Letter to the Editor, The American Statistician, 31.

APPENDIX A

In this section we prove equation (3.3). Let (k, l) be a possible value of (X_m, Y_n) and let $t = k + l$. Then define

$$A_t = \{(k', l') \mid k' + l' = t\} \quad ,$$

$$A_t^* = \{k' \mid (k', l') \in A_t\} \quad .$$

Clearly, under H_0

$$p(k|t) = p(m-k|m+n-t) \tag{A.1}$$

for all $k \in A_t^*$, where p is the hypergeometric probability density function defined in (2.3). Since a UMPU test is determined by the hypergeometric probability density function, it is clear that if (k, l) is a point in the sample space with probability $\gamma(t)$, to reject H_0 then the point (m-k, n-l) also has probability $\gamma(t)$ to reject H_0 in a UMPU test, where $0 \leq \gamma(t) \leq 1$.

Let $g(k, l, p_x, p_y)$ be the joint pdf of (X_m, Y_n), then clearly

$$\begin{aligned} g(k, l; p_x, p_y) &= g(m-k, n-l; 1-p_x, 1-p_y) \\ g(m-k, n-l; p_x, p_y) &= g(k, l; 1-p_x, 1-p_y) \ . \end{aligned} \tag{A.2}$$

When $p_x = p_x^\circ$ and $p_y = p_y^\circ$, the contribution of the two points (k, l) and (m-k, n-l) to the power of the UMPU test is

$$\gamma(t)[g(k, l; p_x^\circ, p_y^\circ) + g(m-k, n-l; p_x^\circ, p_y^\circ)] \quad , \tag{A.3}$$

while the contribution of the two points (k, l) and (m-k, n-l) to the power of the UMPU test when $p_x = 1-p_x^\circ$ and $p_y = 1-p_y^\circ$ is

$$\gamma(t)[g(m,l; 1-p_x^\circ, 1-p_y^\circ) + g(m-k, n-l; 1-p_x^\circ, 1-p_y^\circ)] \ . \tag{A.4}$$

The equality of (A.3) and (A.4) follows from equation (A.2). Thus the contributions of the two points (k, l) and (m-k, n-l) to the power of a UMPU test are the same when $(p_x, p_y) = (p_x^\circ, p_y^\circ)$ or $(p_x, p_y) = (1-p_x^\circ, 1-p_y^\circ)$. Furthermore we also know that the two points (k, l) and (m-k, n-l) always have the same probability rejecting H_0 in a UMPU test. Hence equation (3.3) holds.

APPENDIX B

In this section we list the computer programs used in computing the powers of the four tests.

```
LEVEL 21.7 ( JAN 73 )                              OS/360  FORTRAN H

        COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                           SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID,NOXREF
      C
      C     *** THIS PROGRAM COMPUTES POWERS OF THE Z1-TEST FOR NOMINAL
      C     *** LEVEL 0.05 AND P1=0.1(0.1)0.5, P2=0.1(0.1)0.9
      C
            REAL*8 EQS(9),PB1,PB2,PB,D1,SD1,Z1,AZ1,BI1,BI2,EQ,EQ1,RM,RN,P1,P2
            READ(5,100) M,N
        100 FORMAT(2I2)
            WRITE(6,101)
        101 FORMAT('1',3X,'P1')
            P1=0.0
            DO 1 I=1,5
            P1=P1+0.1
            P2=0.0
            DO 2 J=1,9
            P2=P2+0.1
            M1=M+1
            EQ1=0.0
            RM=M
            RN=N
            DO 3 II=1,M1
            II1=II-1
            PB1=II1/RM
            N1=N+1
            DO 4 JJ=1,N1
            JJ1=JJ-1
            PB2=JJ1/RN
            PB=(RM*PB1+RN*PB2)/(RM+RN)
            D1=PB*(1-PB)*(1/RM+1/RN)
            SD1=DSQRT(D1)
            IF (SD1 .GT. 0.0) GO TO 14
            Z1=0.0
            GO TO 15
         14 Z1=(PB1-PB2)/SD1
         15 AZ1=DABS(Z1)
      C
      C     *** DETERMINE THE REJECT REGION
      C     *** COMPUTE THE BINOMIAL PROBABILITIES FOR THE REJECT REGION
      C
            IF (AZ1 .LT. 1.96) GO TO 4
            CALL BIN(M,II1,P1,BI1)
            CALL BIN(N,JJ1,P2,BI2)
            EQ=BI1*BI2
            EQ1=EQ1+EQ
          4 CONTINUE
          3 CONTINUE
            EQS(J)=EQ1
          2 CONTINUE
            WRITE (6,201) P1,(EQS(J),J=1,9)
        201 FORMAT('-',1X,F5.1,9(3X,F8.5))
          1 CONTINUE
            STOP
            END
```

```
LEVEL 21.7 ( JAN 73 )                                   OS/360  FORTRAN H

        COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                           SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID,NOXREF
              C
              C      *** THIS PROGRAM COMPUTES POWERS OF THE Z2-TEST FOR NOMINAL
              C      *** LEVEL 0.05 AND P1=0.1(0.1)0.5, P2=0.1(0.1)0.9
              C
 ISN 0002            REAL*8  EOS(9),PB1,PB2,D2,SD2,Z2,AZ2,BI1,BI2,EO,EO1,RM,RN,P1,P2
 ISN 0003            REAL*8 PBD
 ISN 0004            READ(5,100) M,N
 ISN 0005        100 FORMAT(2I2)
 ISN 0006            WRITE(6,101)
 ISN 0007        101 FORMAT('1',3X,'P1')
 ISN 0008            P1=0.0
 ISN 0009            DO 1 I=1,5
 ISN 0010            P1=P1+0.1
 ISN 0011            P2=0.0
 ISN 0012            DO 2 J=1,9
 ISN 0013            P2=P2+0.1
 ISN 0014            M1=M+1
 ISN 0015            EO1=0.0
 ISN 0016            RM=M
 ISN 0017            RN=N
 ISN 0018            DO 3 II=1,M1
 ISN 0019            II1=II-1
 ISN 0020            PB1=II1/RM
 ISN 0021            N1=N+1
 ISN 0022            DO 4 JJ=1,N1
 ISN 0023            JJ1=JJ-1
 ISN 0024            PB2=JJ1/RN
 ISN 0025            D2=PB1*(1-PB1)/RM+PB2*(1-PB2)/RN
 ISN 0026            SD2=DSQRT(D2)
 ISN 0027            IF (SD2 .GT. 0.0) GO TO 14
 ISN 0029            PBD=PB1-PB2
 ISN 0030            IF (PBD .NE. 0.0) GO TO 16
 ISN 0032            Z2=0.0
 ISN 0033            GO TO 15
 ISN 0034         14 Z2=(PB1-PB2)/SD2
 ISN 0035         15 AZ2=DABS(Z2)
              C
              C      *** DETERMINE THE REJECT REGION
              C      *** COMPUTE THE BINOMIAL PROBABILITIES FOR THE REJECT REGION
              C
 ISN 0036            IF (AZ2 .LT. 1.96) GO TO 4
 ISN 0038         16 CALL BIN(M,II1,P1,BI1)
 ISN 0039            CALL BIN(N,JJ1,P2,BI2)
 ISN 0040            EO=BI1*BI2
 ISN 0041            EO1=EO1+EO
 ISN 0042          4 CONTINUE
 ISN 0043          3 CONTINUE
 ISN 0044            EOS(J)=EO1
 ISN 0045          2 CONTINUE
 ISN 0046            WRITE (6,201) P1,(EOS(J),J=1,9)
 ISN 0047        201 FORMAT('-',1X,F5.1,9(3X,F8.5))
 ISN 0048          1 CONTINUE
 ISN 0049            STOP
 ISN 0050            END
```

```
LEVEL 21.7 ( JAN 73 )                                    OS/360  FORTRAN H

          COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID,NOXREF
                  C
                  C     *** THIS PROGRAM COMPUTES POWERS OF THE UNIFORMLY MOST
                  C     *** POWERFUL UNBIASED TEST FOR NOMINAL LEVEL 0.05 AND
                  C     *** P1=0.1(0.1)0.5, P2=0.1(0.1)0.9
                  C
  ISN 0002              INTEGER C1(11),C2(11)
  ISN 0003              REAL*8 R1(11),R2(11),ECS(9),P1,P2,EQ,EQ1,EQ2,EQ3,EQ4,EQT,BI1
  ISN 0004              REAL*8 BI2,ECST
  ISN 0005              READ(5,100) (C1(I),I=1,11)
  ISN 0006              READ(5,100) (C2(I),I=1,11)
  ISN 0007          100 FORMAT(11I2)
  ISN 0008              READ(5,200) (R1(I),I=1,11)
  ISN 0009              READ(5,200) (R2(I),I=1,11)
  ISN 0010          200 FORMAT(10F8.5/F8.5)
  ISN 0011              READ(5,300) M,N
  ISN 0012          300 FORMAT(2I2)
  ISN 0013              WRITE(6,301)
  ISN 0014          301 FORMAT('1',1X,'P1')
  ISN 0015              P1=0.0
  ISN 0016              DO 1 I=1,5
  ISN 0017              P1=P1+0.1
  ISN 0018              P2=0.0
  ISN 0019              DO 2 J=1,9
  ISN 0020              P2=P2+0.1
  ISN 0021              MN=M+N+1
  ISN 0022              EQT=0.0
  ISN 0023              DO 3 IT1=1,MN
  ISN 0024              IT=IT1-1
  ISN 0025              EQ1=0
  ISN 0026              IF (C1(IT1) .EQ. 0) GO TO 11
  ISN 0028              IC1=C1(IT1)
                  C
                  C     *** COMPUTE THE BINOMIAL PROBABILITIES OF THE REJECT REGION
                  C
  ISN 0029              DO 4 IY1=1,IC1
  ISN 0030              IY=IY1-1
  ISN 0031              ITY=IT-IY
  ISN 0032              CALL BIN(N,IY,P2,BI2)
  ISN 0033              CALL BIN(M,ITY,P1,BI1)
  ISN 0034              EQ=BI1*BI2
  ISN 0035              EQ1=EQ1+EQ
  ISN 0036            4 CONTINUE
  ISN 0037           11 IC2=C2(IT1)+1
  ISN 0038              EQ2=0
  ISN 0039              IF (IC2 .GT. IT) GO TO 12
  ISN 0041              DO 5 IY=IC2,IT
  ISN 0042              ITY=IT-IY
  ISN 0043              CALL BIN(N,IY,P2,BI2)
  ISN 0044              CALL BIN(M,ITY,P1,BI1)
  ISN 0045              EQ=BI1*BI2
  ISN 0046              EQ2=EQ2+EQ
  ISN 0047            5 CONTINUE
  ISN 0048           12 ITC1=IT-C1(IT1)
                  C
                  C     *** COMPUTE THE BINOMIAL PROBABILITIES OF THE BOUNDARY POINTS
                  C
```

```
      CALL BIN(N,C1(IT1),P2,BI2)
      CALL BIN(M,ITC1,P1,BI1)
      EQ3=R1(IT1)*BI1*BI2
      ITC2=IT-C2(IT1)
      CALL BIN(N,C2(IT1),P2,BI2)
      CALL BIN(M,ITC2,P1,BI1)
      EQ4=R2(IT1)*BI1*BI2
      EQST=EQ1+EQ2+EQ3+EQ4
      ECT=ECT+EQST
    3 CONTINUE
      EQS(J)=ECT
    2 CONTINUE
      WRITE(6,101) P1,(EQS(J),J=1,9)
  101 FORMAT('-',1X,F5.1,9(3X,F8.5))
    1 CONTINUE
      STOP
      END
```

```
LEVEL 21.7 ( JAN 73 )                                  OS/360  FORTRAN H

          COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID,NOXREF
                C
                C      *** THIS PROGRAM COMPUTES POWERS OF THE UNCONDITIONAL
                C      *** NONRANDOMIZED TEST FOR NOMINAL LEVEL 0.05 AND P1=0.1(0.1)0.9,
                C      *** P2=0.1(0.1)0.5
                C
  ISN 0002             INTEGER C1(21),C2(21)
  ISN 0003             REAL*8 EQS(5),P1,P2,EQ,EQ1,EQ2,EQT,EQST,BI1,BI2
  ISN 0004             READ(5,100) (C1(I),I=1,11)
  ISN 0005             READ(5,100) (C2(I),I=1,11)
  ISN 0006         100 FORMAT(11I2)
  ISN 0007             READ(5,300) M,N
  ISN 0008         300 FORMAT(2I2)
  ISN 0009             WRITE(6,301)
  ISN 0010         301 FORMAT('1',1X,'P1')
  ISN 0011             P1=0.0
  ISN 0012             DO 1 I=1,9
  ISN 0013             P1=P1+0.1
  ISN 0014             P2=0.0
  ISN 0015             DO 2 J=1,5
  ISN 0016             P2=P2+0.1
  ISN 0017             MN=M+N+1
  ISN 0018             EQT=0.0
  ISN 0019             DO 3 IT1=1,MN
  ISN 0020             IT=IT1-1
  ISN 0021             EQ1=0.0
  ISN 0022             IF (C1(IT1) .EQ. 0) GO TO 11
  ISN 0024             IC1=C1(IT1)
                C      *** COMPUTE THE BINOMIAL PROBABILITES FOR THE REJECT REGION
                C
  ISN 0025             DO 4 IY1=1,IC1
  ISN 0026             IY=IY1-1
  ISN 0027             ITY=IT-IY
  ISN 0028             CALL BIN(N,IY,P2,BI2)
  ISN 0029             CALL BIN(M,ITY,P1,BI1)
  ISN 0030             EQ=BI1*BI2
  ISN 0031             EQ1=EQ1+EQ
  ISN 0032           4 CONTINUE
  ISN 0033          11 IC2=C2(IT1)+1
  ISN 0034             EQ2=0
  ISN 0035             IF (IC2 .GT. IT) GO TO 12
  ISN 0037             DO 5 IY=IC2,IT
  ISN 0038             ITY=IT-IY
  ISN 0039             CALL BIN(N,IY,P2,BI2)
  ISN 0040             CALL BIN(M,ITY,P1,BI1)
  ISN 0041             EQ=BI1*BI2
  ISN 0042             EQ2=EQ2+EQ
  ISN 0043           5 CONTINUE
  ISN 0044          12 EQST=EQ1+EQ2
  ISN 0045             EQT=EQT+EQST
  ISN 0046           3 CONTINUE
  ISN 0047             EQS(J)=EQT
  ISN 0048           2 CONTINUE
  ISN 0049             WRITE(6,101) P1,(EQS(J),J=1,5)
  ISN 0050         101 FORMAT('-',1X,F5.1,5(3X,F8.5))
  ISN 0051           1 CONTINUE
  ISN 0052             STOP
```

```
LEVEL 21.7 ( JAN 73 )                            OS/360  FORTRAN H

          COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID,NOXREF
                 C
                 C      *** THIS SUBROUTINE COMPUTES THE BINOMIAL PROBABILITIES
                 C
  ISN 0002              SUBROUTINE BIN(N,L,P,X)
  ISN 0003              REAL*8 P,Q,X
  ISN 0004              Q=1.-P
  ISN 0005              L1=L+1
  ISN 0006              X=Q**N
  ISN 0007              IF(L1 .GE. 2) GO TO 11
  ISN 0009              GO TO 12
  ISN 0010           11 DO 10 I=2,L1
  ISN 0011           10 X=X*P*(N-I+2)/(Q*(I-1.))
  ISN 0012           12 RETURN
  ISN 0013              END
```

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ON TESTING THE EQUALITY OF TWO PROPORTIONS

by

YOW YEU CHIOU

B. A., National Cheng Kung University, 1975

AN ABSTRACT OF A MASTER'S REPORT

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Manhattan, Kansas

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ABSTRACT

Let X_m and Y_n be two independent random variables from binomial population $B(m, p_x)$ and $B(n, p_y)$, respectively. Four tests have been suggested to test the null hypothesis H_o: $p_x = p_y$ against the alternative hypothesis H_1: $p_x \neq p_y$. This article compares these four tests based on some practical considerations and based on their finite sample size powers. We conclude that the Z_2-test is a very reasonable test.