EXAMINING THE EFFECTS OF OPENINGS AT THE BASE OF SLENDER REINFORCED CONCRETE (TILT-UP) WALL PANELS SUBJECTED TO VARYING WIND PRESSURES

by

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Abstract

This report examines the effects of openings located at the base of reinforced concrete slender wall panels (tilt-up panels) designed in accordance with the American Concrete Institute (ACI) Committee 318-11 *Building Code Requirements for Structural Concrete* Section 14.8 *Alternative Design of Slender Walls*. The parametric study calculates the reinforcement (longitudinal) required for specific panels in accordance with ACI 318-11 Section 14.8 and compares the designs to a finite element analysis conducted with SAP 2000 version 14 to determine the appropriateness of the assumptions made in Section 14.8. Furthermore, this report compares the design of a tilt-up panel designed by Section 14.8 *Alternative Design of Slender Walls* and designed by Section 10.10 *Slenderness Effects in Compression Members*.

Table of Contents

List of Figures

List of Tables

List of Terms

a – Depth of the equivalent rectangular stress block

ACI – American Concrete Institute

A_g – Gross area of concrete

A_s – Area of Steel

A_{se} – Equivalent area of steel including effects of an axially applied load

A_{smin} – Minimum area of steel required

b – Width of concrete section under analysis

c – Depth of the actual concrete compression block

C_e –Exposure factor coefficient

CL – Center line of Panel

C_t – Thermal factor coefficient

D – Dead load

d – Distance from the extreme fiber in compression to flexural reinforcing

DOF – Degree of freedom

D_{panel} – Dead load due to panel weight

E – Seismic load

E_c – Modulus of elasticity of concrete

e_{cc} – distance from the centroid of the concrete panel to the applied axial load

E_s – Modulus of elasticity for steel

F – Fluid pressure

f'_c – Ultimate compressive strength of concrete

FEM – Finite element method

f_r – Modulus of rupture for concrete

f_y – Ultimate yield stress for steel

GC_p – External pressure coefficient

GC_{pi} – Internal pressure coefficient

h – Panel thickness

I – Importance Factor

I – Moment of inertia

I_{cr} – Cracked moment of inertia

I_g – Gross moment of inertia of the concrete section

$\boldsymbol{K}$ – Stiffness matrix

K_b – Sectional bending stiffness

K_d – Wind directionality factor

K_z – Velocity pressure exposure coefficient

K_{zt} – Topographic factor

L – Live load

l – Unbraced length of the panel

Leg – Concrete spanning from the roof to the foundation

L_r – Roof live load

l_w –With of the panel leg

M_{cr} – Cracked moment of inertia of a concrete section

M_n – Nominal moment resisting capacity

MPH – Miles per hour

M_s – Service applied moment including P-Δ effects

M_{sa} – Initial service applied moment

M_u – Maximum factored moment including P-Δ effects

M_{ua} – Maximum factored applied moment

n – Modular ratio, modulus of elasticity of steel to modulus of elasticity of concrete

$\boldsymbol{P}$ – Applied force vector

P_f – Flat roof snow load

P_g – Ground Snow load

P_{sa} – Initial service applied axial load

psi – pounds per square inch

P_u – Ultimate factored axially applied load

P_{ua} – Axially load applied on the panel

P_{um} – Axially load including Panel weight

q_z – Velocity pressure

S – Snow Load

SEASC – Structural Engineers Association of Southern California

$\boldsymbol{U}$ – Nodal displacement vector

V – Wind velocity

W – Wind pressure

W_a – Wind pressure based on serviceability requirements

w_u – factored uniform lateral load

W_u – Ultimate factored wind load

y_t – Distance from the centroid to the extreme fiber in tension

β_1 – Factor accounting for the variation in the actual stress curve for different concrete strengths

Δ – Initial deflection exhibited by application of primary moments

$\Delta_{allowable}$ – Allowable deflection

Δ_{cr} – Deflection exhibited at the cracked moment of inertia

Δ_n – Deflection exhibited at the nominal moment capacity

Δ_s – Service deflection

Δ_{sa} – Initial Service deflection due to lateral loads

ε_{cu} – Ultimate strain in the extreme concrete fiber in compression

ε_t – Strain in the tensile reinforcement

λ – Factor to account for the reduced mechanical properties of lightweight concrete

ϕ – Strength reduction factor

Acknowledgements

I would like to thank the members of my committee, Professor Kimberly Kramer, P.E., S.E., Dr. Don Phillippi, P.E., S.E., Architect, and Dr. Kyle A. Riding P.E., for their guidance in helping writing this report. I would like to thank Ted Strahm of Lithko Contracting Inc. for his helping me understand the construction of tilt-up panels. I would also like to thank Joseph J. Steinbicker, P.E., S.E. president of Steinbicker & Associates, LLC for continual support and guidance in helping me understand the engineering of tilt-up panels.

Dedication

This report is dedicated to my wife, Jamie Carter Cook, who has given me unconditional love, support and inspiration to do the best I can throughout my collegiate career.

Chapter 1 - Introduction

In 1999 *Building Code Requirements for Structural Concrete* published by The American Concrete Institute committee 318 (ACI 318-11) introduced Section 14.8 *Alternative Design of Slender Walls*. Section 14.8 was based on the requirements of the 1982 research report composed by the Structural Engineers Association of Southern California and the Southern California Chapter of the American Concrete Institute (ACI-SEASC) entitled “Test Report on Slender Walls” as well as requirements from the 1997 Uniform Building Code (UBC), which was predominately used for high seismic areas in the western United States. Section 14.8 provides a process of designing slender reinforced concrete walls, tilt-up walls panels, for out-of plane loads caused by wind or seismic effects. Section 14.8 was implemented to provide a design standard after the damage in tilt-up buildings from the 1971 San Fernando earthquake, the 1984 Morgan Hill earthquake and the 1994 Northridge earthquake. Section 14.8 makes four key assumptions in its design process which are:

1. One-Way Bending assumption which is presented in Section 3.1.1.
2. Constant Bending Stiffness which is presented in Section 3.1.2.
3. Bending Stiffness Reduction Factor which is presented in Section 3.1.3.
4. Effect of Axial Load on the Stiffness of the Member which is presented in Section 3.1.4.

Brian Bartels published a report in 2010 titled *Analysis of Vertical Reinforcement in Slender Reinforced Concrete (Tilt-Up) Panels with Openings & Subjected to Varying Wind Pressures* (Bartels, 2010) which examined how varying wind pressure affected the design of tilt-up panels with centralized openings of varying sizes using ACI 318-08 Section 14.8. Also in 2010 Brandon Schwabauer published a report titled *Analysis of Assumption Made in Design of Reinforcement in Slender Reinforced Concrete (Tilt-Up) Panels with Openings* which compared Bartels results to a finite element analysis to determine the appropriateness of the assumptions used in ACI 318-08 Section 14.8.

This parametric study builds upon Bartel's and Schwabauer's research by moving the openings to the base of the panels and examining the effect on the design of the vertical reinforcement in the tilt-up panel using ACI 318-11 Section 14.8. The results of using ACI 318-11 Section 14.8 are compared to the results of a finite element analysis for specific panels. Additionally, this report gives a comparison of a panel designed by Section 14.8 *Alternative Design of Slender Walls* and the same panel being designed by Section 10.10 *Slenderness Effects in Compression Members*. This comparison includes a general cost analysis as well as a constructability analysis.

Chapter 2 - Scope of Research

This parametric study examines a solid tilt-up panel which is idealized as simply supported, pinned at the base and pinned at the roof diaphragm, which is 24 foot wide with an unbraced height of 32 feet. The standard panel has four different opening configurations, (A) 8' X 7', (B) 12' X 12', (C) 18' X 18', and (D) 20' X 20' as shown in Figure 2.1. Each panel is designed for four different wind pressures; (1) 115 mph, (2) 130 mph, (3) 150 mph, and (4) 170 mph and eccentrically applied roof loads, P_u, according to ASCE 7-10 *Minimum Design Loads for Buildings and Other Structures.* In addition to axial and shear forces, moments induced by lateral loads and P-delta effects occur.

Panels A through D are designed using ACI 318-11 Section 14.8 *Alternative Design of Slender Walls*; Panel D is also designed by using ACI 318-11 Section 10.10 *Slenderness Effects in Compression Members*. The alternative design of slender walls procedure has four assumptions implemented:

1. One-Way Bending assumption which is presented in Section 3.1.1.
2. Constant Bending Stiffness which is presented in Section 3.1.2.
3. Bending Stiffness Reduction Factor which is presented in Section 3.1.3.
4. Effect of Axial Load on the Stiffness of the Member which is presented in Section 3.1.4.

The constant bending stiffness assumption (2) uses the portion on each side of the opening, often called the leg or pier shown in Figure 2.2 as the elements resisting the forces created in the panel. The stiffness of the portion above the opening is neglected and the stiffness of the wall pier is taken as the stiffness at mid-height where the deflection is the highest. Panels A and C are analyzed using finite element analysis with SAP 2000 version 14 which examines the bending stiffness increase caused by the portion above the opening in the panel and the appropriateness of the four assumptions made by the ACI 318-11.

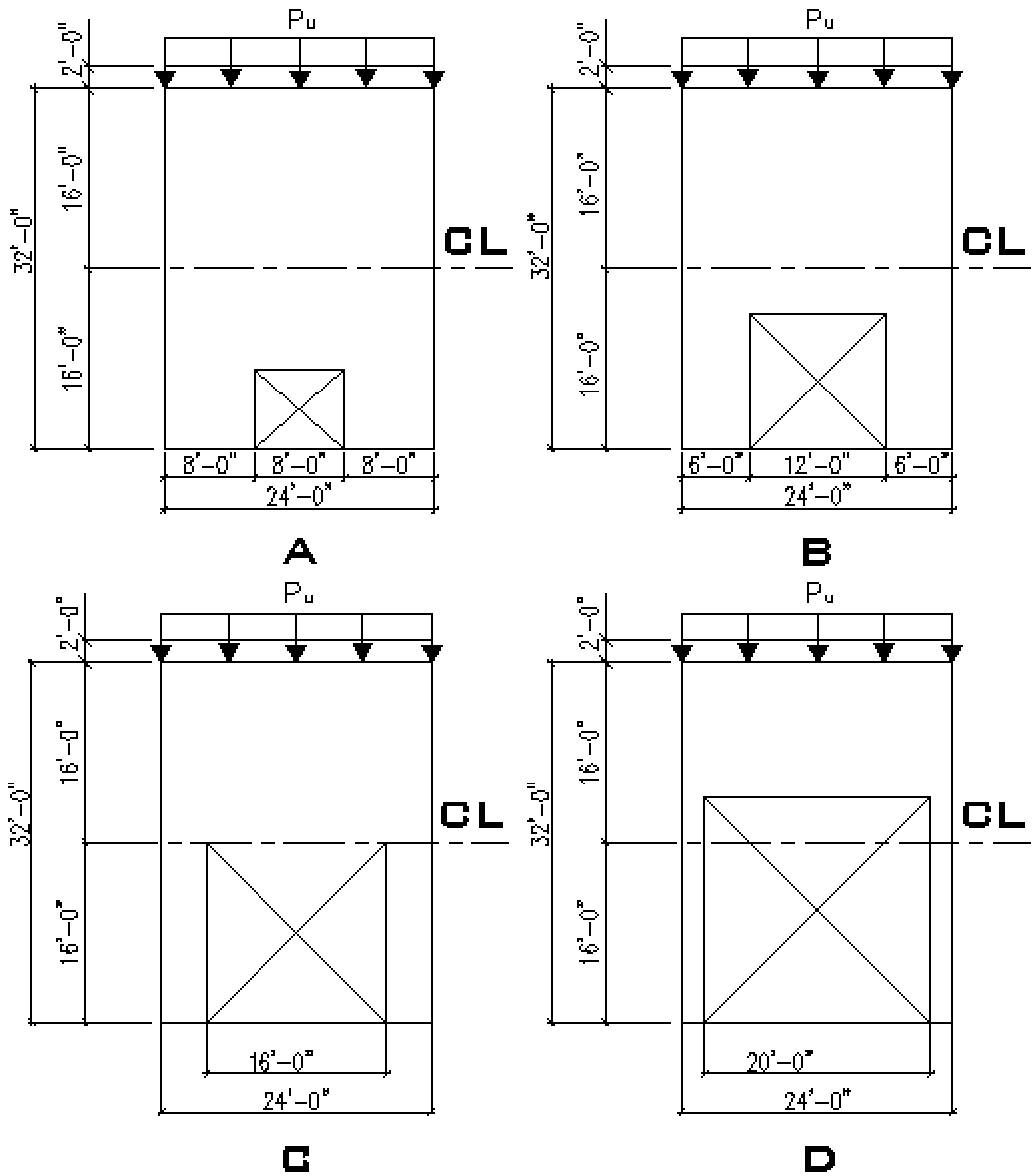

Figure 2.1: Tilt-up Panel Configurations

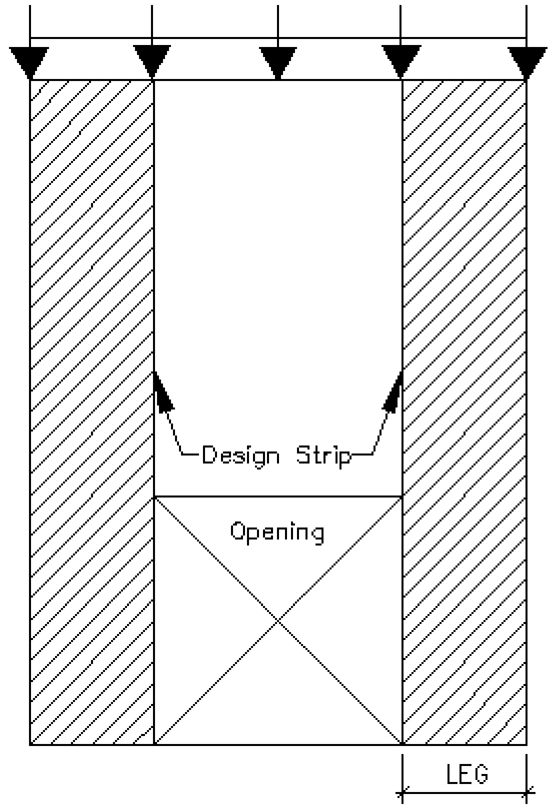

Figure 2.2: Tilt-up Panel Leg

2.1 Tilt-up Panels with Openings at Finished Floor

The panels are idealized as a pin-pin connection, the location of the maximum moment occurs at mid-height of the panel, shown as CL in Figure 2.1. The amount of concrete at the location of the maximum moment is varied based on the opening size. The opening in Panel A was selected to represent a double door; the openings in Panels B, C, and D were chosen to model large door openings such as: garage doors, bay doors, dock doors or other large door openings. According to Tilt-Up Concrete Association (TCA), 32 ft is a common unbraced length for a warehouse structure (Schmitt, 2009). The case study floor plan for the panels being considered is shown in Figure 2.3, similar to Schmidt, Bartels, and Schwabauer's reports. This floor plan represents common warehouse structure consisting of flexible diaphragm – metal deck on joists spanning to load-bearing exterior walls or to interior columns and joist girders.

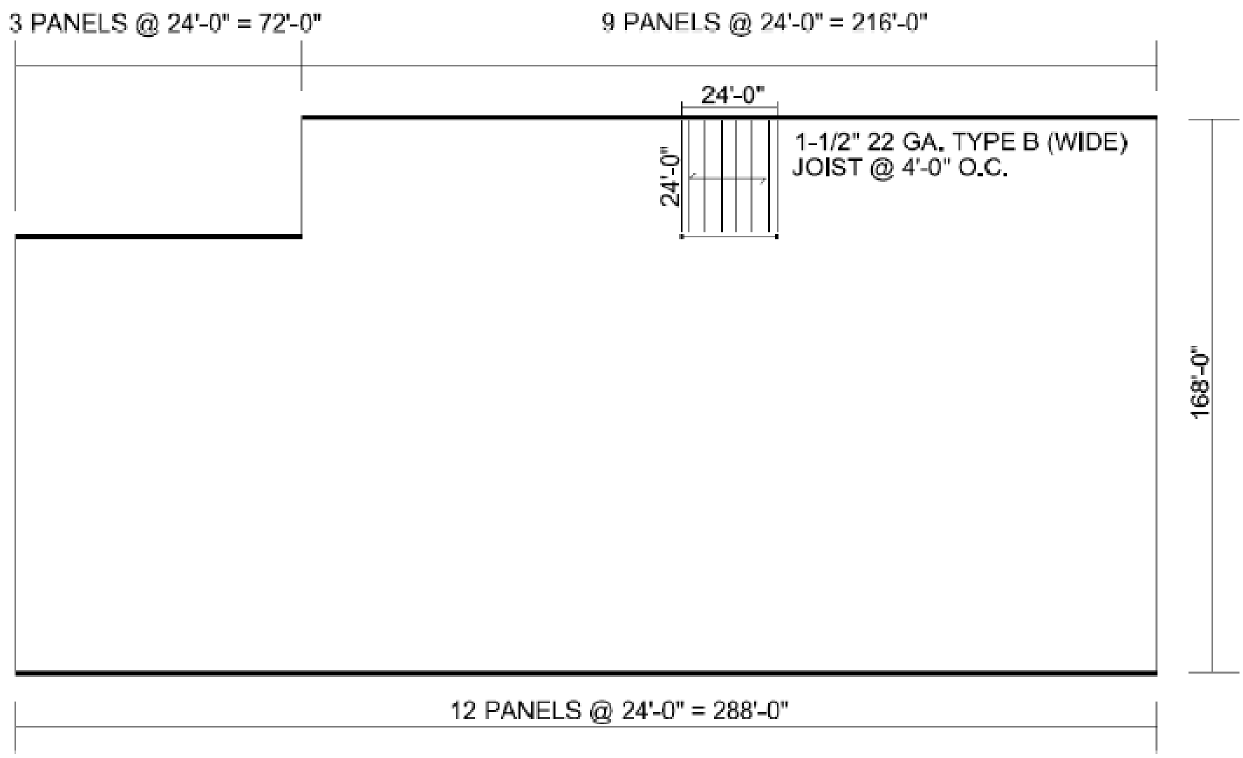

Figure 2.3: Case Study Floor Plan (Bartels 2010)

2.2 Loads

The loads applied to the panels are described here within. All loads are determined in compliance with American Society of Civil Engineers (ASCE) *Minimum Design Loads for Buildings and Other Structures 7-10* (ASCE 7-10). The loads are similar to Bartels and Schwabaurer, except for the wind loads are in strength design according to ASCE 7-10 instead of stress levels according to ASCE 7-05.

2.2.1 Dead Loads

Dead loads applied on the structure are the self weight, weight of fixed equipment, and architectural treatments. The roof structure consists of 1-1/2", 20 gage metal deck spanning to steel joists spaced 4'-0" on center which defines the self-weight of the structure. The superimposed dead loads consists of 6" of rigid insulation, bituminous roofing, mechanical/electrical/plumping equipment supported by the roof structure. Table 2.1 shows the dead loads. Refer to Appendix A for full derivation of loads.

Bituminous Roofing =	1.5	psf
6" Rigid Insulation =	9	psf
1.5 22 Gauge Deck =	2	psf
Joists =	2.5	psf
M/E/P =	4	psf
Total =	19	psf
Use Dead Load =	**20**	**psf**

Table 2.1: Dead Loads (Bartels 2010)

2.2.2 Live Loads

Roof live loads are construction loads. According to Table 4-1 of the ASCE 7-10, roof live load of a flat, ordinary roof is 20 pounds per square foot. The live load reduction permitted by ASCE 7-10 Section 4.9 is not considered to provide a general solution for the tilt-up panels. Refer to Appendix A for full derivation loads.

2.2.3 Snow Loads

Snow loads are climate loads defined by ASCE 7-10 Chapter 7 *Snow Loads* which defines minimum snow loads to be applied to flat roof buildings. The magnitude of the roof snow load depends on the four project specific conditions; ground snow load, exposure of the structure to wind loading, thermal properties of the roof; and importance of the structure for life safety. The ground snow load (p_g) is the average snow load over a 50 year period with a 2% probability of exceedance for a given geographic location and is determined from ASCE 7-10 Table 7-1. The ASCE 7-10 defines the flat roof snow load on a given structure by Equation 2.2-1.

$$p_f = 0.7C_eC_tI_sp_g \qquad \text{Equation 2.2-1}$$

ASCE 7-10 Equation 7.3-1

The exposure factor (C_e) accounts for the effects of the terrain on the wind blowing snow off the roof structure. Exposure C "open terrain with scattered obstructions having heights generally less than 30 ft (ASCE 7-10)" is utilized resulting in an exposure factor of 1.0. Exposure C has been chosen to model the building in an industrial park in an open or suburban location. The thermal factor (C_t) accounts for the effects of heat transfer through the roof and its interaction with the snow on the roof. The thermal factor is determined from ASCE 7-10 Table 7-3. The warehouse is assumed to be heated and insulated corresponding to a thermal factor equal to 1.0. The warehouse building is considered to be Risk Category II (ASCE 7-10 Section 1.5.2) which results in an importance factor (I_s) of 1.0; a warehouse building represents neither a low hazard to human life nor a high hazard of human life. The resultant of Equation 2.2.1 yields a p_f of 14 psf. However, in accordance with ASCE 7-10 Section 7.3, the minimum flat roof snow load is 20 psf and is utilized for calculation purposes. Refer to Appendix A for full derivation of snow loads.

2.2.3 Wind Loads

The wind loads applied to the building are determined according to ASCE 7-10 Chapter 30 *Components and Cladding (C&C).* The C&C procedure is used in lieu of the *Main Wind Force Resisting System* (MWFRS) Chapter 29 procedure - larger lateral pressures result from the smaller effective area when resisting the wind pressure. Components and Cladding gives the largest wind pressures; therefore, govern the design.

ASCE 7-10 Table 30.4-1 is used to determine the design wind pressures. The structure is a warehouse which in categorized as risk category II, from ASCE 7-10 Table 1.5-1. Wind velocities are determined from ASCE 7-10 Figure 26.5-1A *Basic Wind Speeds for Occupancy Category II Buildings and Other Structures*. Wind velocities of 115, 130, 150, and 170 miles per hour, in three-second gusts, are utilized for this parametric study. The velocity wind pressure varies depending on four factors: (1) terrain and height above ground, (2) topographic effect factor, (3) directionality factor; and (4) geographical location. The velocity pressure, $q_{z,}$ is determined:

$$q_z = 0.00256K_zK_{zt}K_dV^2$$

Equation 2.2-2

ASCE 7-10 Equation 30.3-1

The velocity pressure exposure coefficient, $K_{z,}$ depends on the terrain which the structure is located – a rougher terrain slows the wind, reducing the wind pressure acting on the structure. Based on the terrain, the velocity pressure exposure coefficient is determined from ASCE 7-10

Table 30.3-1. For a mean roof height of 34 ft and terrain category C, the exposure coefficient equals 1.0. The topographic factor, K_{zt}, takes into account the increase in the wind velocity when the structure is on the upper half of a hill or escarpment. It is determined by ASCE 7-10 Section 26.8. For this parametric study, the structure is sited on relatively flat terrain; the K_{zt} factor is 1.0. The wind directionality factor, K_d, accounts for the probability of the wind acting perpendicular to the surface of the structure in conjunction with the maximum dead, live, snow loads occurring at the same time. The wind directionality factor is 0.85 from ASCE 7-10 Table 26.6-1 and should only be used in conjunction with other loads.

The design wind pressure, *p*, according to ASCE 7-10 Section 30.4.2 takes into account the equalization of wind pressure when wind is acting on the structure and is adjusted based on the internal pressure coefficient, GC_{pi}, and the external coefficient GC_p. The design wind pressure is calculated by equation 2.2-3.

$$p = q_h[(GC_p) - (GC_{pi})]$$

Equation 2.2-3

ASCE 7-10 Equation 30.4-1

The external pressure coefficient, GC_p, is defined in ASCE 7-10 Figure 30.4-1 for partially enclosed buildings. Warehouse structures tend to have one side (elevation) of the building with large openings (dock doors). This causes the structure to be classified as 'partially enclosed'. Partially enclosed structures have higher internal wind pressures. For this parametric study, partially enclosed is used. For components and cladding, the wind pressures vary depending on the effective wind area of the supporting element. The effective wind area of the panels shown in Figure 2.1 is greater than 500 square feet; the external pressure coefficients are 0.7 for windward walls and -0.8 for leeward walls from ASCE Figure 30.4-1. The internal pressure coefficient, GC_{pi}, is determined by ASCE 7-10 Table 26.11-1. For partially enclosed buildings, the internal pressure coefficient is equal to +/- 0.55. The tabulated wind pressures for the four wind speeds are given in table 2.2. The detail calculations are shown in Appendix A.

Wind Speed (MPH)	Wind Pressure (psf)
115	-38.85
130	-49.65
150	-66.10
170	-84.90

Table: 2.2: Wind Pressures

These wind pressures are applied to the four panels in conjunction with the other loads using the ASCE 7-10 load combinations.

2.3 Previous Reports

This report is a continuation of two previous reports, which research was conducted for reinforced concrete tilt-up wall panels with openings. The reports are *Analysis of Vertical Reinforcement in Slender Reinforced Concrete (Tilt-Up) Panels with Openings Subjected to Varying Wind Pressures* (Bartels, 2010) and *Analysis of Assumptions Made in Design of Reinforcement in Slender Reinforced Concrete (Tilt-Up) Panels with Openings* (Schwabauer, 2010).

2.3.1 Analysis of Vertical Reinforcement in Slender Reinforced Concrete (Tilt-Up) Panels with Openings Subjected to Varying Wind Pressures.

In 2010, Brian Bartels published “Analysis of Vertical Reinforcement in Slender Reinforced Concrete (Tilt-Up) Panels with Openings Subjected to Varying Wind Pressures.” His report investigated how varying wind pressures and varying opening sizes located at the mid-height affected the design of vertical, flexural reinforcement, in a tilt-up panel. The panels analyzed in the report are shown in Figure 2.4. The wind pressures examined were: 90, 110, 130, and 150 MPH, in three second gusts, using the ASCE 7-05 which determines wind pressures at stress levels. This correlates to 115, 130, 150, 170 MPH, in three second gusts, of the ASCE 7-10, which determines wind pressures at strength levels, utilized in this study. The wind design methodology changed from stress levels in the ASCE 7-05 to strength levels in the ASCE 7-10. The resulting wind pressures when used in load combinations are approximately equal when comparing the two standards.

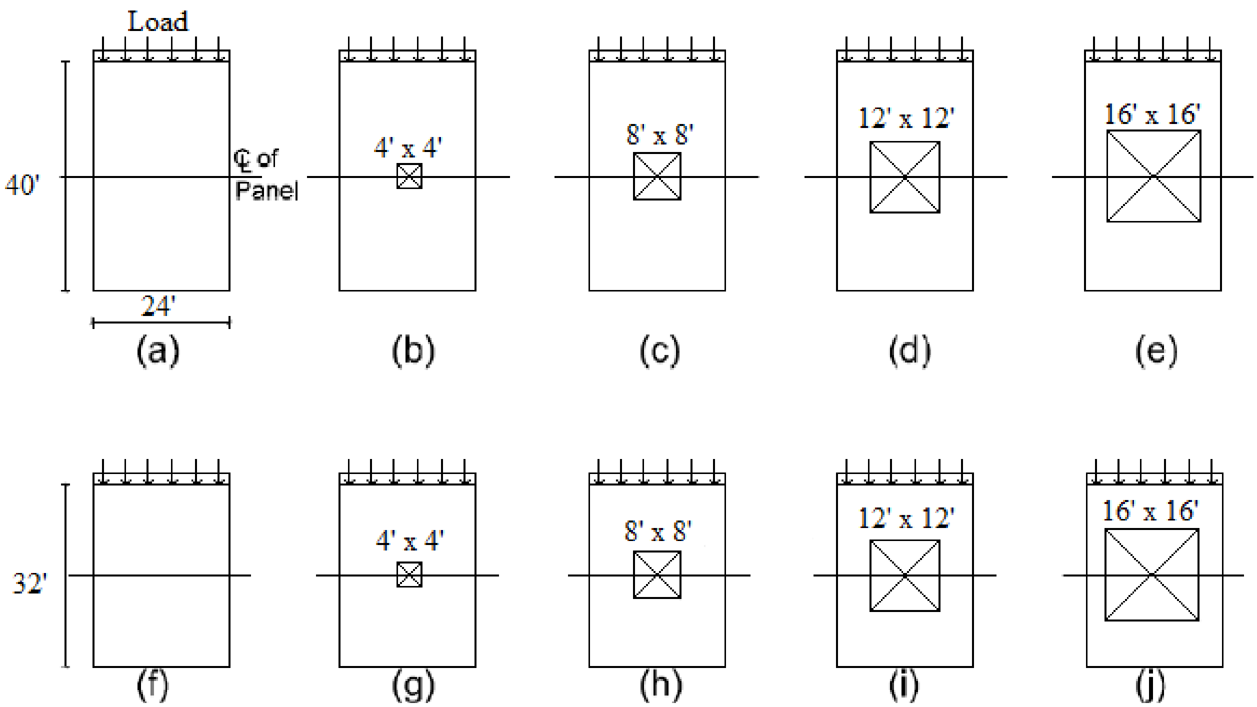

Figure 2.4: Tilt-up Panel Configurations (Bartels 2010)

This report uses the same panel configuration with openings at the base of the panel. Panel A correlates to panel (h); Panel B correlates to panel (i); and Panel C correlates to panel (j) of Bartels' report.

Bartels found that the size of an opening has a large effect on the required vertical reinforcement and thickness of the panel. Examining the results from Bartels' report, for 110 MPH winds, by ACSE 7-05 calculations, panel (h) requires a 7.25" thick panel while Panel (i) requires a 9.25" thick panel. Bartels determined that Panels g and h both require 7.25" panel; however, Panel (g) required 120 #4 vertical reinforcing bars while Panel (h) required 164 #4 bars. This shows openings in tilt-up panels have a dramatic effect on the design of the vertical or longitudinal reinforcement. This parametric study expands on Bartels' research by moving varying sized openings to the base of the panels, which represent door openings. This report examines how the longitudinal reinforcing and panel thickness are affected by openings at the base of the panel for varied wind pressures.

2.3.2 Analysis of Assumptions Made in Design of Reinforcement in Slender Reinforced Concrete (Tilt-Up) Panels with Openings

In 2010, Brandon Schwabauer conducted research to analyze the assumptions made when designing the longitudinal reinforcement for slender reinforced concrete panels using the ACI 318-08 *Building Code Requirements for Structural Concrete* Section 14.8 *Alternate Design of Slender Walls*. In his Master's Report, "Analysis of Assumptions Made in Design of Reinforcement in Slender Reinforced Concrete (Tilt-Up) Panels with Openings," Schwabauer expands on Bartels' research by conducting a finite element analysis on Panel (d) in Figure 2.4. Schwabauer compared the finite element analysis of Panel (d) with the results determined by Bartels for Panel (d). The intent of this comparison was to determine the appropriateness of the four assumptions made by the ACI 318-08 Section 14.8 for one-story, warehouse tilt-up panels:

1. The panel exhibits a one-way bending action.
2. The panel has a constant stiffness equivalent to the panel legs stiffness.
3. The bending stiffness reduction factor.
4. The effect of axial load on the stiffness of the member.

To test the aforementioned assumptions Schwabauer conducted a finite element analysis with SAP 2000 version 14 on panel (d) in Figure 2.4, the resultant analysis is shown in Figure 2.5. Figure 2.5 gives a color gradient of bending stresses in the panel. The blue elements have the largest bending stresses of all the panel elements while the purple elements have the lowest bending stresses. Because of the constraints of SAP 2000 Schwabauer was unable to include P-delta effects in the conducted finite element analysis.

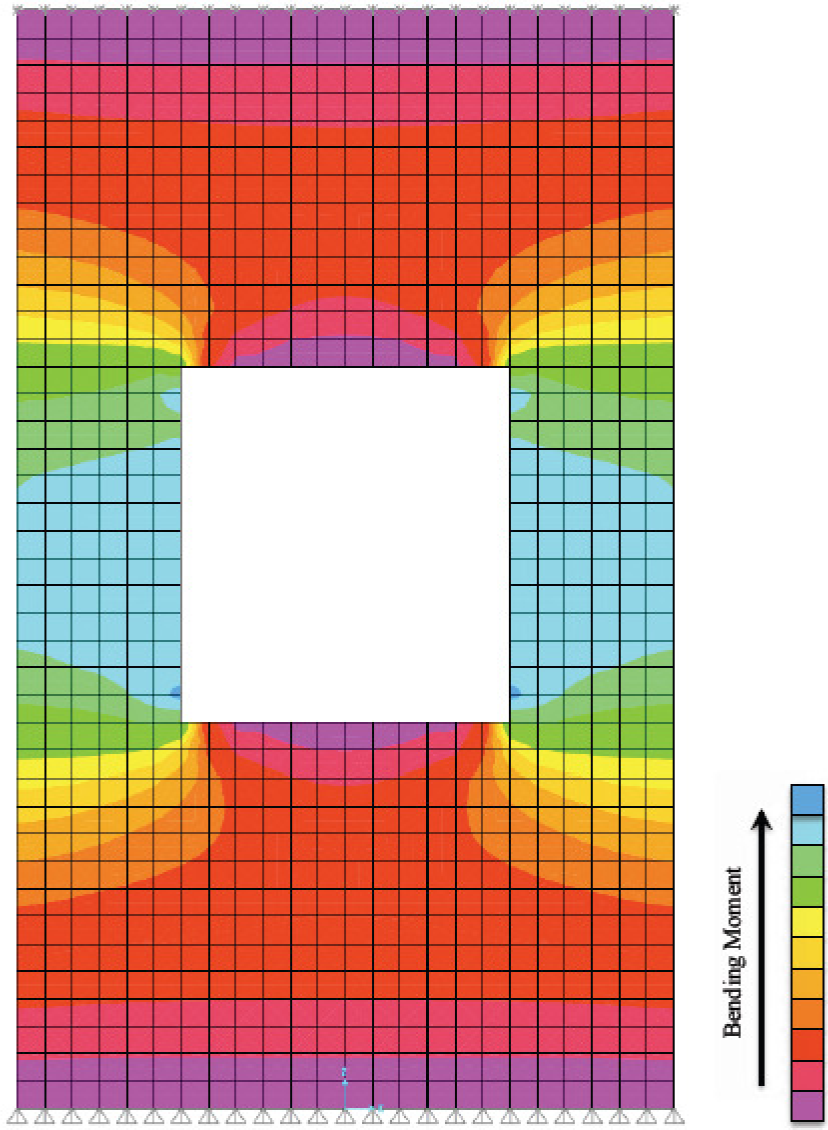

Figure 2.5: Finite Element Analysis of Panel (d) (Schwabauer 2010)

By comparing Bartels results with the finite element analysis he conducted, Schwabauer determined that the bending stresses in panel (d) were within 3% of the bending stresses calculated by Section 14.8 thus verifying that the tilt-up panel under analysis does exhibit assumption (1) one way action, for panel (d). This is verified by the lower bending stresses that occur above and below the window opening. To check assumption (2) constant stiffness, Schwabauer varied the effective moment of inertia above and below the opening to examine the flexural stiffness increase that occurs due to the excess concrete above and below the opening. Even when the flexural stiffness above and below the opening was increased by 20% the maximum bending stress was reduced by 4.5%. This means that the ACI 318 Section 14.8 is at most 4.5% conservative. Assumption (3) bending stiffness reduction factor was tested by reducing the depth of flexural reinforcing by 3/8" (varying the depth of the centroid of reinforcement to the extreme fiber in compression) and reducing the thickness of the panel by ¼" which is the maximum allowed by the ACI 117-90 *Standard Specification for Tolerances for Concrete Construction & Materials*. Implementing these tolerances Schwabauer determined that the bending stiffness for a panel with a constant cross section was reduced by 25%, which is exactly the same as that implemented into assumption (3) the bending stiffness reduction factor. This verified that the bending stiffness reduction factor used by the ACI 318-08 Section 14.8 is appropriate. Assumption (4) the effect of axial load on the stiffness of the member was verified through a moment curvature analysis. Schwabauer determined that at 6% of concrete compressive stress, the maximum allowed by Section 14.8, the moment capacity is approximately 96% of that calculated by Section 14.8 which means it is approximately under conservative by 4%. Further research should be conducted to verify this determination.

The assumptions made by the ACI 318-11 Section 14.8 are appropriate for panel (d) analyzed by Schwabauer. This report further investigates if the four assumptions made by the ACI 318-11 Section 14.8 *Alternative Design of Slender Walls* are appropriate for panels with openings at their base by conducting a finite element analysis of the Panels A and C in Figure 2.1. Panels with an unbraced length of 32' have been selected in lieu of the 40' panel analyzed by Schwabauer because 32' is more common in warehouse structures.

Chapter 3 - ACI 318-11 Section 14.8 Alternative Design of Slender Walls

The four assumptions made by the ACI 318-11 Section 14.8 and the rational of each assumption are examined herein. Additionally, a detailed description of the design procedure using ACI 318-11 Section 14.8, *Alternative Design of Slender Walls* is presented.

3.1 Four Assumptions in ACI 318-11 Section 14.8

The following section gives a detailed description of the assumptions made by ACI 318-11 Section 14.8.

3.1.1 One Way Bending Assumption

ACI 318-11 Section 14.8 idealizes the wall to exhibit one-way bending behavior. This assumption is predicated on the idea that because the panels are analyzed as simply supported members with a constant wind pressure, no two way bending occurs in the panel, the load path is shown in Figure 3.1. However, with an opening located at the base of the panel, by engineering judgment it would appear that the concrete above the opening may display two way bending behavior to transfer the wind load to the legs of the panels and the pin connection at the roof as shown in figure 3.2. The validity of this assumption is tested with a finite element analysis. Refer to Section 6.2.1.

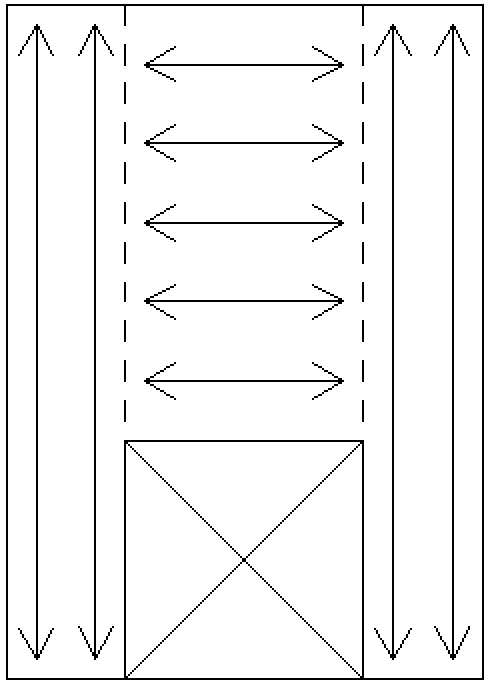

Figure 3.1: Out-of-Plane One-Way-Bending Load Path for Tilt-up Panels

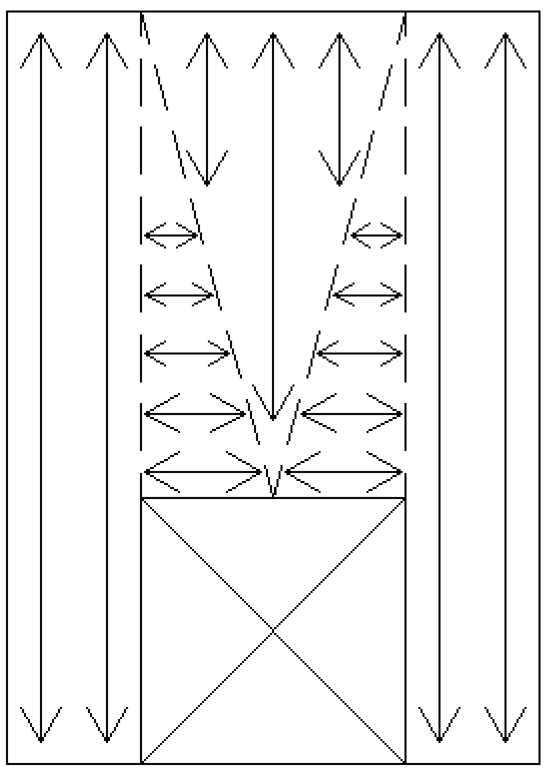

Figure 3.2: Out-of-Plane Two-Way-Bending Load Path for Tilt-up Panels

3.1.2 Constant Bending Stiffness Assumption

The second assumption is a constant bending stiffness along the panel equal to the stiffness of the panel legs at the location of maximum deflection, the panel midheight, where the effective moment of inertia will be the lowest value. The American Concrete Institute Committee 551 (ACI 551) *Design Guide for Tilt-Up Concrete Panels* describes a procedure to evaluate the strength of tilt-up panels according to ACI 318-11. In this design procedure, the constant bending stiffness assumption can be seen. The ACI 551 and the ACI 318-11 use a moment magnifier to determine the ultimate applied load on the panel, see Appendix B for moment magnification derivation. The resulting maximum moment is determined from Equation 3.1-1.

$$M_u = \frac{M_{ua}}{1 - \dfrac{P_u}{(0.75)K_b}}$$ Equation 3.1-1 (ACI 551)

M_{ua} is defined as the maximum applied moment from factored lateral loads and the moment produced by the eccentrically applied roof load. P_u is the factored axial load on the wall panel including the panel weight above midheight. The panel weight above the midheight is used because it will be the location and maximum moment and maximum deflection. The bending stiffness K_b is calculated by Equation 3.1-2.

$$K_b = \frac{48 E_C I_{CR}}{5l^2}$$ Equation 3.1-2 (ACI 551)

E_c is the modulus of elasticity of concrete and l is the unbraced length of the panel. The cracked moment of inertia, I_{cr}, is evaluated from Equation 3.1-3.

$$I_{cr} = \frac{E_s}{E_c}\left(A_s + \frac{P_u h}{f_y 2d}\right)(d-c)^2 + \frac{l_w c^3}{3}$$ Equation 3.1-3

(ACI 318-11 Equation 14-7)

Where c is the depth of the equivalent stress block, d is the depth of flexural reinforcing, h is the thickness of the panel, P_u is the ultimate axially applied roof load and A_s is the area of flexural reinforcing. Equation 3.1-3 is where we can mathematically see the conservatism inherent in the assumptions made by ACI 318-11 Section 14.8. The value for l_w is defined to be the width of the leg. This means that the bending stiffness of the panel is only dependent on the panel legs, the portion of the wall above the opening does not contribute to the stiffness of the panel as shown in Figure 3.3-B. However, if the opening in the panel is located at its base with the panel solid at mid-height where the maximum moment occurs, using Equation 3.1-3 underestimates the cracked moment of inertia of the panel. For example, Panel A is solid at the location of maximum moment. This indicates that the portion of the panel above the opening should contribute to the cracked moment of inertia of the panel as shown in Figure 3.3-C, meaning that the bending stiffness at mid-height of the panel displays a higher moment of inertia than that prescribed by the Equation 3.1-3. Figure 3.3 gives a graphical representation of this assumption.

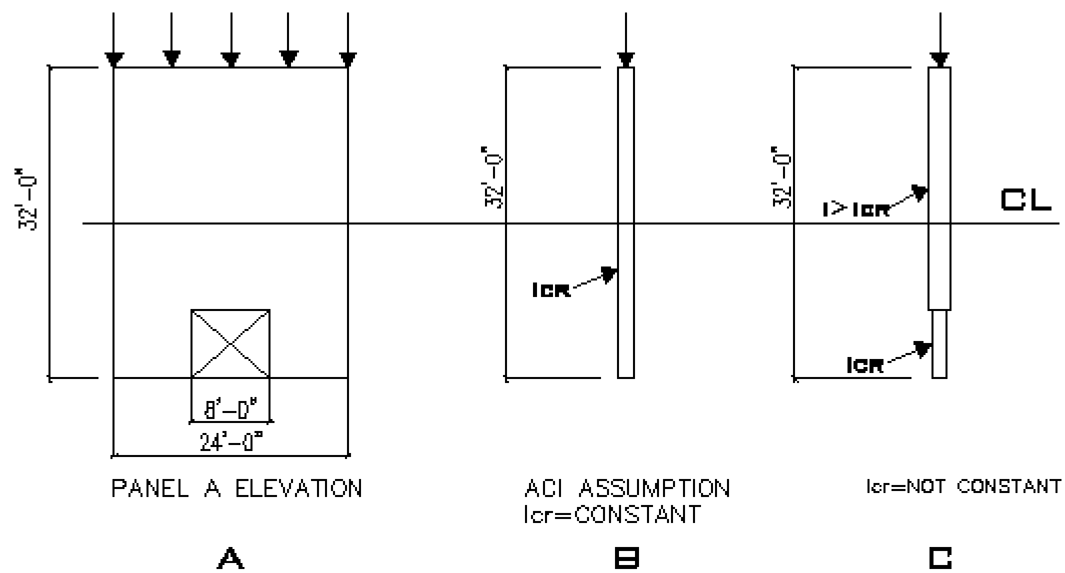

Figure 3.3: Deficiency of Constant Bending Assumption

Figure 3.3-B gives a visual aid of the constant bending stiffness along the length of the panel as prescribed by Section 14.8. Figure 3.3-C shows the actual moment of inertia of the panel varies along the length of the panel. This report does not imply that the value l_w should be

taken as 24 feet, the width of the panel. Rather, this report investigates the increase in stiffness from the additional concrete available to resist the applied bending moment. This assumption is tested by comparing the bending stress in the panel determined by ACI 318-11 Section 14.8 calculations to the finite element performed with computer software, SAP 2000. Refer to Section 6.2.2 for the conclusion of the constant bending stiffness assumption.

3.1.3 Bending Stiffness Reduction Factor Assumption

The third assumption utilized by ACI 318-11 Section 14.8 is the bending stiffness reduction factor. Section 14.8 puts a reduction factor of 0.75 on the bending stiffness of tilt-up panels in equation 3.1-4.

$$M_u = \frac{M_{ua}}{1 - \frac{5P_u l_c^2}{(0.75)48E_c I_{cr}}} \qquad \text{Equation 3.1-4}$$

(ACI 318-11 Equation 14.6)

The 75% bending stiffness reduction factor is taken from ACI 318-11 Section 10.10.6 *Moment Magnification Procedure – Nonsway*, which is taken from a research report by Mirza, Lee and Morgan entitled *ACI Stability Resistance Factor for RC Columns* published in 1987. According to Mirza et. al:

"The actual strength of a reinforced concrete member varies from the calculated nominal strength due to variations in material strengths and dimensions of the member, as well as due to uncertainties inherent in the equations used to compute member strength. Similarly, actual loads that act on a member differ from calculated nominal loads due to variations in constituent material densities, as well as uncertainties inherent in applied loads (Mirza et. al., 1987)."

The same 75% reduction factor is used for the design of slender walls because similar to nonsway slender columns, tilt-up panels are also slender compression elements subjected to flexural loads. The reduction factor is tested by varying the reinforcement placement and the wall panel thickness to the maximum allowed by the *Standard Specification for Tolerances for Concrete Construction & Materials* (ACI 117-90) reported by ACI Committee 117 reapproved in 2002. The panel thickness will be reduced by 1/4” as allowed in ACI 117 Section 4.4.1 and the depth of the reinforcing steel will be reduced by 3/8” according to ACI 117 Section 2.2.2. The moment of inertia is calculated for optimal tilt-up panel construction and at the maximum tolerances allowed by ACI 117 refer to Section 6.2.3 for results.

3.1.4 Effect of Axial Load on the Stiffness of the Member Assumption

The last assumption in ACI 318-11 Section 14.8 pertains to how the increase of the design moment resisting capacity is determined. For small axially applied loading (less than $0.10f'_c$), the axially applied force counteracts a portion of the tensile stress on the steel as the moment is applied to the panel. This phenomenon results in the moment capacity of the section being increased. The axially applied loading also increases the bending stiffness of the section as the P-Δ effects are displayed in the panel. Both of these factors are accounted for by calculating an equivalent are a of steel, A_{se}. In the commentary of ACI 318-11 Section 14.8, the following equation is specified to account for the increase.

$$A_{se} = A_s + \frac{P_u}{f_y}\left(\frac{h}{2d}\right) \qquad \text{Equation 3.1-5}$$

Where A_s is the actual area of flexural reinforcing steel, P_u is the axial load applied at mid-height of the panel, h is the thickness of the panel, d is the distance to the centroid of steel from the extreme fiber in compression, and f_y is the yield strength of the reinforcing steel. Equation 3.1-5 was introduced into the ACI 318-11 in the 2008 code. The previous equation, equation 3.1-6, overestimated the axial load effect on the flexural reinforcing when the member was reinforced with two layers of steel. In order to correct this error, the non-dimensional term $h/2d$ was introduced to reduce the axial load effect for two layers of steel. If a single layer of reinforcement is placed in the center of the panel, the two equations are equivalent. However, if two layers of steel are used, the equivalent area of steel is reduced.

$$A_{se} = A_s + \frac{P_u}{f_y}$$ Equation 3.1-6

Refer to Section 6.2.4 for results of effect of axial load on the stiffness of the member assumption.

3.2 ACI 318-11 Section 14.8 Design Process

A detailed description of the design tilt-up wall panels using ACI 318-11 Section 14.8, *Alternative Design of Slender Walls*, is presented. This design process was initiated in a report entitled *Test Report on Slender Walls* in 1980 through 1982 by the American Concrete Institute – Structural Engineers Association of Southern California (ACI-SEASC). Section 14.8 *Alternative Design of Slender Walls* was first adopted into the ACI 318 code in 1999 based on the results of ACI-SEASC as well as the requirements of the 1997 Uniform Building Code (UBC).

3.2.1 Limitations of ACI 318-11 Section 14.8 Alternative Design of Slender Walls

As with all flexural members, the primary criteria of tilt-up wall panels is that the design moment resisting capacity ϕM_n must be greater than the ultimate applied moment M_u as shown in Equation 3.2-1.

$$\phi M_n \geq M_u$$ Equation 3.2-1

(ACI 318-11 Equation 14-3)

Designing the panels by ACI 318-11 Section 14.8 is allowed by code if the following criteria are met:

1. “The wall panel shall be designed as a simply supported, axial loaded member subjected to an out-of-plane uniform lateral load with maximum moments and deflections occurring at midspan” (ACI 318, 2011).
2. “The cross section shall be constant over the height of the panel” (ACI 318, 2011).
3. “The wall shall be tension-controlled” (ACI 318, 2011).

4. Reinforcing shall provide a design strength such that at all sections the factor moment resisting capacity of the section (ϕM_n) is greater than or equal to the cracking moment of the section (M_{cr}) defined by ACI 318-11 Section 14.8.2.4.
5. "Concentrated gravity loads applied to the wall above the design flexural section shall be assumed to distribute over a width:
   a) Equal to the bearing width, plus a width on each side that increases at a slope of 2 vertical to 1 horizontal down the design section; but
   b) Not greater than the spacing of the concentrated loads; and
   c) Not extending beyond the edges of the wall panel." (ACI 318, 2011).
6. "Vertical stress P_u/Ag at the midheight section shall not exceed $0.06f'_c$." (ACI 318, 2011).

Additionally,

7. "Maximum out-of-plane deflection, $\boldsymbol{\Delta_s}$, due to service loads, including $\boldsymbol{P\Delta}$ effects, shall not exceed $\boldsymbol{l_c}$ (un-braced length)**/150**" (ACI 318, 2011).
8. The minimum steel reinforcing requirements must be met according to Sections 14.3.2 and 14.3.3

According to ACI 318-11 Section 14.8, if one or more of the above criteria are not met, the wall must be designed according to ACI 318-11 Section 14.4, *Walls Designed as Compression Members* in which the tilt-up walls would be designed as slender columns.

3.2.1 Design Moment Strength

Equation 3.2-2 utilized in Section 14.8 is used to determine the design moment strength of tilt-up panels.

$$\phi M_n = \phi A_{se} f_y \left(d - \frac{a}{2} \right) \qquad \text{Equation 3.2-2}$$

(ACI 318-11, 2011)

A_{se} is defined as the equivalent area of steel, ϕ is the strength reduction factor, and f_y is the minimum yield strength of the reinforcing steel. The factor d is defined as the distance from the extreme fiber in compression to the centroid of the reinforcing steel, and *a* is the depth of the equivalent stress block.

The strength reduction factor ϕ is equal to 0.9 for tension-controlled sections according to ACI 318-11 Section 9.3.2.1. The purpose of the strength reduction factor is to account for the following:

1. The statistical probability of under-strength members due to the deviations of actual material strengths to design material strengths.
2. Underestimation of loads applied on the panel.
3. The ductility of the design mode of failure and the reliability of the member under the load effects considered.

From Figure 3.4, the strength reduction factor can be determined. The shaded portion is the tension-controlled region – the region in which slender walls are designed.

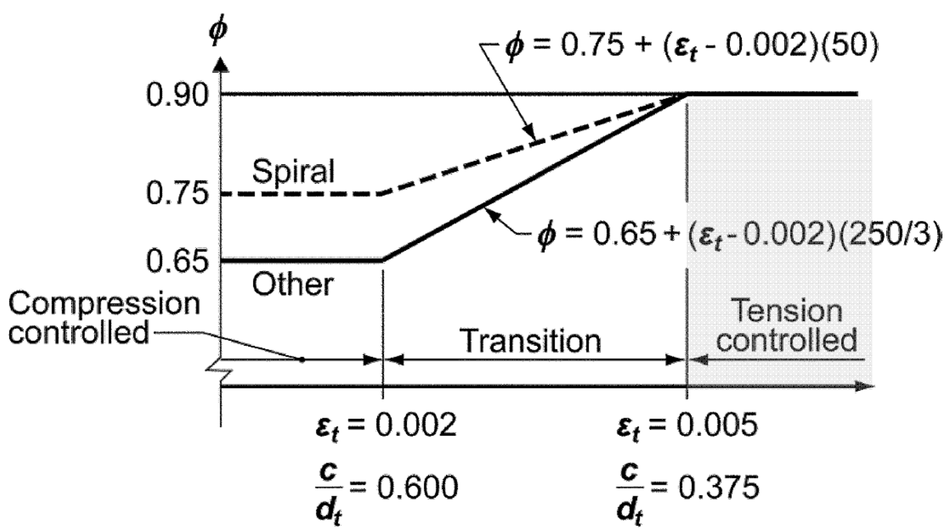

Figure 3.4: Variation of ϕ with Net Tensile Strain in Extreme Tension Steel, ε_t, and c/d_t for Grade 60 Reinforcement and Prestressing Steel (ACI Committee 318, 2011)

A tension controlled section is determined by the strain in the extreme fiber in tension. If this strain is greater than or equal to 0.005 in./in., the section is classified as tension-controlled.

A tension-controlled section ensures that the reinforcing steel will yield before the concrete crushes at the assumed strain limit of 0.003 in./in.. ACI 318-11 Figure R10.3.3, Figure 3.5, represents the strain distribution and net tensile strain in a section.

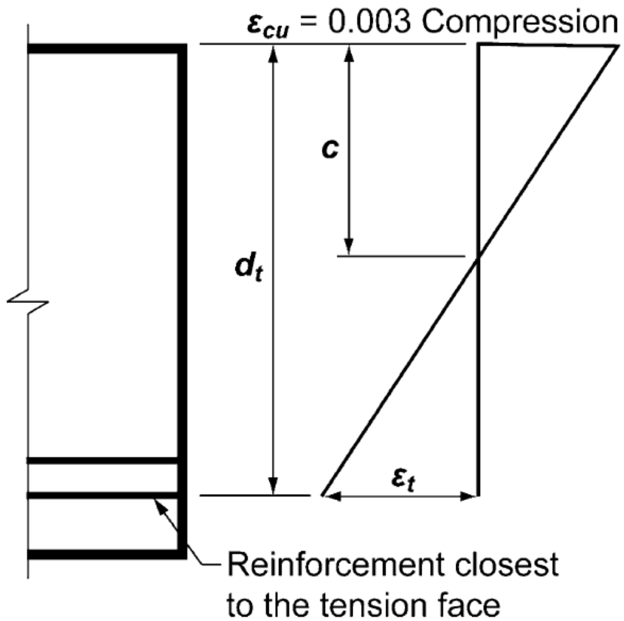

Figure 3.5: Strain Distribution and Net Tensile Strain in Flexural Members (ACI Committee 318, 2011)

The equivalent area of steel is determined by Equation 3.1-5.

$$A_{se} = A_s + \frac{P_u}{f_y}\left(\frac{h}{2d}\right) \qquad \text{Equation 3.1-5}$$

As described in section 3.1.4, the equivalent area of steel equation is one of the assumptions that the ACI 318-11 Section 14.8 makes. Equation 3.1-5 accounts for the increase in the moment resisting capacity due to the axially applied load. When loads are applied to a member that is subjected to bending, the axially applied load offsets a portion of the tensile stress on the reinforcing steel. In addition, utilizing Equation 3.1-5 also accounts for the increase in bending stiffness under P-Δ bending moments in the panel.

The factor *d-a/2* is defined as the moment arm in Equation 3.2-2. This moment arm is derived from the equivalent stress block that is allowed by ACI 318-11. The purpose of the equivalent stress block is to resolve the actual concrete stress distribution of a flexural member, shown in Figure 3.6, to a manageable stress distribution.

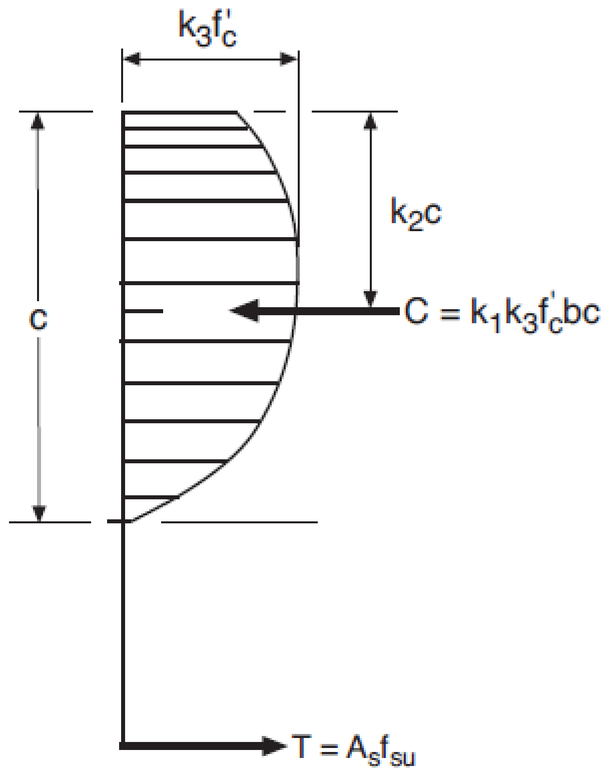

Figure 3.6: Actual Stress Distribution at Nominal Strength in Flexural Members (PCA, 2008)

Figure 3.6 gives a visual representation of the actual stress block of a reinforced concrete member subjected to flexural stresses. In this figure the maximum stress is given by $k_3f'_c$, the average stress is $k_1k_3f'_c$, the centroid of the parabolic curve from the extreme fiber in compression is k_2c and c is the depth of the neutral axis from the extreme compression fiber. If the member fails in a ductile manner f_{su} is equal to f_y. In order to use the equivalent rectangular stress block, a few assumptions should be noted. The first of these is the assumption that the non-linear actual concrete stress can be resolved to an average stress of $0.85f'c$. Additionally, the average stress is assumed to be distributed uniformly across what is known as the equivalent compression zone, *a*. The equivalent compression zone, *a*, is the second assumption that must be recognized. The boundaries of the equivalent compression zone are from the extreme fiber in compression to *a*. The quantity *a* is equivalent to $\beta_1 c$ and *c* is the distance from the extreme fiber in compression to the point of zero strain in the member. The factor β_1 is needed to account for the variation in the actual stress curve for different concrete strengths. For all concrete strengths up to 4000 psi, the factor β_1 is taken to be 0.85. However, as the concrete compressive strength increases, the shape of the stress distribution block becomes more linear. Therefore, in section 10.2.7.3 the ACI 318-11 requires that β_1 decrease at a rate of 0.05 for each 1000 psi increase of concrete compressive strength above 4000 psi. Notably, the upper bound for β_1 is 0.65 for concrete compressive strengths 8000 psi and above. The testing of these assumptions is beyond the scope of this research and is therefore be taken as valid assumptions. Implementing these assumptions yields the equivalent rectangular stress block shown in Figure 3.7.

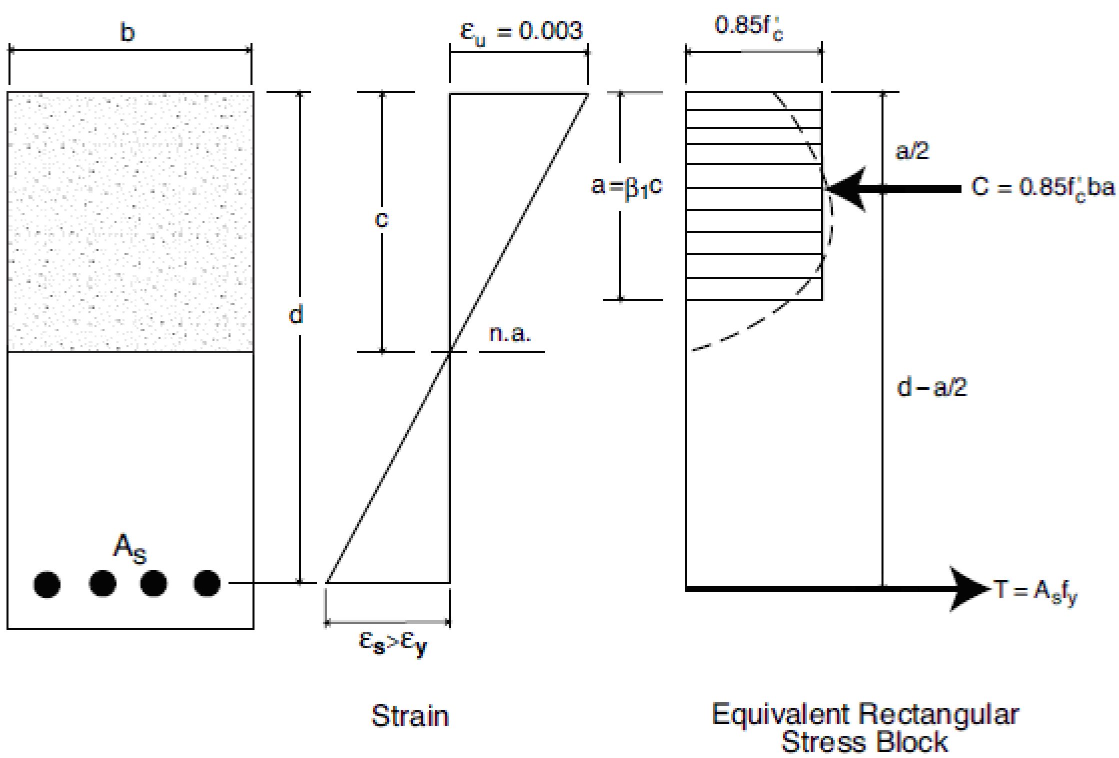

Figure 3.7: Equivalent Rectangular Stress Block (PCA 2008)

The resultant tensile force, T is the total area of tensile reinforcement, A_s multiplied by the yield strength of steel f_y. The resultant compressive force, C is the average concrete compressive strength, $0.85f'c$ multiplied by the area of the equivalent rectangular stress block, ba. The internal moment resisting couple is composed to the resultant tensile force and the resultant compression force combined with a moment arm, $d-a/2$. In order to determine the depth of the equivalent stress block, the internal forces are set equal to one another.

$$T = C \qquad \text{Equation 3.2-3}$$

Therefore,

$$0.85 f'_c\, ba = A_{se} f_y \qquad \text{Equation 3.2-4}$$

Thus,

$$a = \frac{A_{se} f_y}{0.85 f'_c\, b} \qquad \text{Equation 3.2-5}$$

With the information outlined above, the design moment strength of the section can be determined from Equation 3.2-2.

3.2.2 Flexural Cracking Moment

To prevent a sudden brittle failure, ACI 318-11 Section 14.8.2.4 requires that the design flexural strength is greater than the cracking moment at all sections of the panel. The cracking moment can be determined by Equation 3.2-6.

$$M_{cr} = \frac{f_r I_g}{y_t} \qquad \text{Equation 3.2-6}$$

(ACI 318-11, Equation 9-9)

I_g is the gross moment of inertia of the cross section, y_t is the distance from the centroid of the section to the extreme fiber in tension, and f_r is the modulus of rupture defined by Equation 3.2-7.

$$f_r = 7.5\lambda\sqrt{f'_c} \qquad \text{Equation 3.2-7}$$

(ACI 318-11, Equation 9-10)

The factor λ is a function of the weight of the concrete. Its purpose is to account for the reduced mechanical properties of light-weight concrete. For normal-weight concrete, λ is equal to 1.0. Given the above information, the flexural cracking moment can be determined and compared to the design moment strength thus ensuring that a brittle failure does not occur.

3.2.3 Ultimate Applied Moment

Two types of moments are considered when designing tilt-up wall panels: primary and secondary moments. A slender wall panel is initially subjected to primary moments attributed to out-of-plane loading due to wind, seismic or fluid pressures, and the moment from the eccentrically applied roof loading. As a result of the application of the primary moments, the tilt-up panel exhibits an initial deflection, Δ. The secondary moment is the caused by a geometrical nonlinear phenomena known as P-Δ effects. P-Δ effects occur from forces acting on deformed structural members. For tilt-up wall panels the P-Δ are caused by the roof load and the weight of the panel being passed through the deflected shape. When the load is applied at a deflection, additional bending stress is imposed on the panel. At the top and bottom of the panel the secondary stresses will be zero because deflection is prohibited by the connection to the roof and the foundation. The maximum secondary bending stress occur at the mid-height of the panel where the initial deflection is the greatest. This process is illustrated graphically in Figure 3.8.

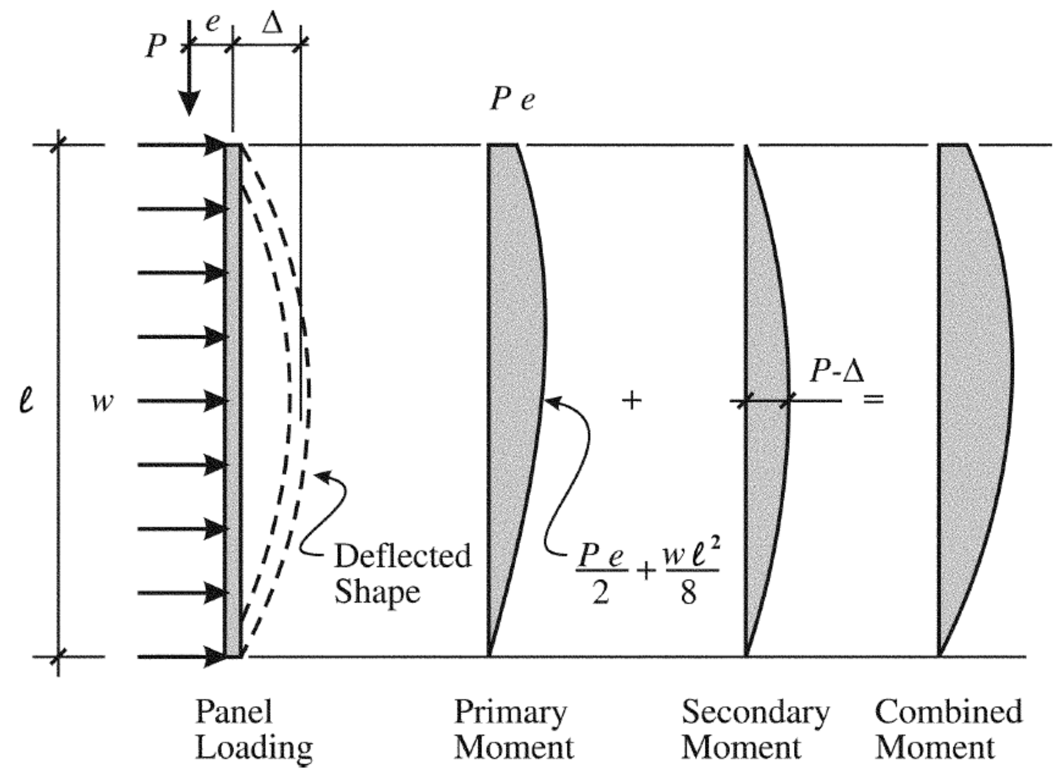

Figure 3.8: Ultimate Applied Moment (ACI Committee 551, 2009)

As prescribed by the ACI 318-11 Section 14.8.2.1 the primary moments are calculated assuming a simply supported flexural member. The primary applied moment due to primary loading is determined from Equation 3.2-8.

$$M_{ua} = w_u \frac{l^2}{8} + P_u \frac{e}{2} \qquad \text{Equation 3.2-8}$$

(ACI 551, 2009)

In the above stated equation, w_u is the factored, uniform lateral load and l is the unbraced length of the panel. The factored applied roof load is P_u and e is the distance from the center of the panel to the location at which the roof load is acting, called the eccentricity. Figure 3.9 shows a practical example of a generic tilt-up panel connection.

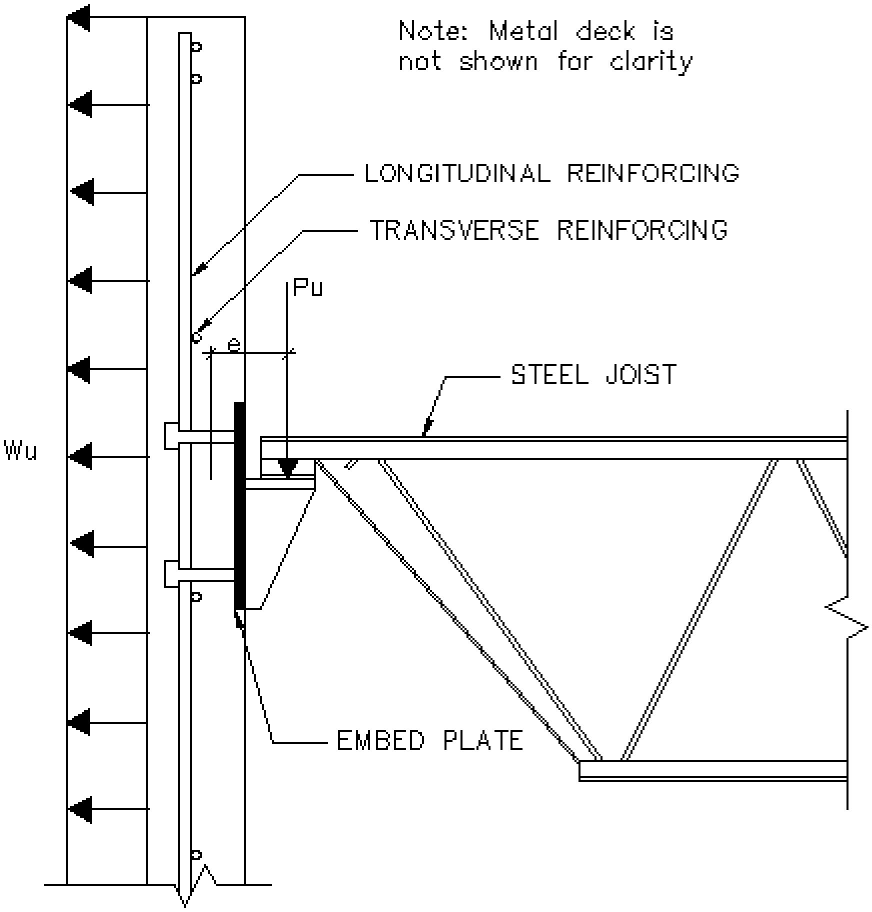

Figure 3.9: Typical Tilt-Up Panel Roof Connection

Equation 3.2-8 yields the applied moment due to primary loading, however it does not account for the secondary P-Δ moment. The moment M_{ua} must be further modified in order to determine the ultimate moment M_u applied to the panel.

The ultimate applied moment must consider the primary applied moment as well as the secondary moment as a result of the P-Δ effects. The moment magnification method and the iterative procedure are the two ways to calculate the ultimate applied moment. The iterative procedure will not be evaluated in detail in this report. Bartels demonstrated the validity of both methods in his 2010 report (Bartels, 2010). For a detailed description of the iterative method refer to (Bartels, 2010). This parametric study utilizes the moment modification method which is prescribed by the ACI 318-11 Section 14.8. The primary equation for the moment magnification is given as Equation 3.1-1 and is derived in Appendix B.

$$M_u = \frac{M_{ua}}{1 - \dfrac{P_u}{(0.75)K_b}} \qquad \text{Equation 3.1-1}$$

(ACI 551, 2009)

The resultant, M_u is the ultimate applied moment considering primary and secondary moments. M_{ua} is the primary applied moment as defined by Equation 3.2-8, P_u is the ultimate applied axial load, and 0.75 is the reduction factor assumptions as described in Section 3.2.3. The factor K_b is the bending stiffness of the section and is calculated by Equation 3.1-2.

$$K_b = \frac{48 E_c I_{cr}}{5 l^2} \qquad \text{Equation 3.1-2}$$

(ACI 551, 2009)

Where E_c is the modulus of elasticity of concrete and l is the unbraced length of the panel. Because the ultimate design strength is assumed to occur when the section is cracked, the bending stiffness K_b is calculated with the cracked moment of inertia, I_{cr}, defined by Equation 3.1-3.

$$I_{cr} = nA_{se}(d - c)^2 + \frac{l_w c^3}{3} \qquad \text{Equation 3.1-3}$$

(ACI 551, 2009)

The width of the panel leg, l_w, is the width of the leg as shown in Figure 2.2. In panels where the leg becomes large the lateral load will not transfer through the entire width of the leg. Therefore, the maximum effective leg width to resist the out of plane loading from the opening is recommended to be 12 times the thickness of the panel by the ACI 551.2R Section 7.2 *Panels with Openings*. Because the member consists of two different materials, steel and concrete, one of the materials must mathematically transform its properties so that the inertia for both components can be added. The modular ratio, n, transforms the material properties of steel to equivalent concrete properties so they can be mathematically combined. The modular ratio is the modulus of elasticity of steel to modulus of elasticity of concrete:

$$n = \frac{E_s}{E_c} \qquad \text{Equation 3.2-9}$$

According to ACI 318-11 Section 14.8.3, the ultimate applied moment must be less than or equal to the design moment strength. Given the information in this section, the ultimate applied moment can be determined. Furthermore, the information outlined in Section 3.2.1, the design moment strength can be determined. The two quantities must be compared to ensure that the tilt-up panel will not fail given the application of the largest moment determined from ASCE 7-10 Chapter 2 *Combinations of Loads*.

3.2.4 Service Deflection

The determination of service load deflections for tilt-up panels are described. As described in Section 3.2.1 and according to ACI 318-11 Section 14.8.4, the maximum wall deflection must be less than or equal to the unbraced length divided by 150. Notably, concrete codes prior to the 1980's wall thickness limitations were based on height to width ratios. However, the tests conducted by ACI-SEASC in 1980 to 1982 conclusively proved that slender wall panels were able to maintain adequate moment resisting capacity even when the panels were subjected to large lateral deflections. However, for the sake of serviceability, the alternative design of slender walls outlined in ACI 318-11 Section 14.8 restricts the lateral deflection.

Prior to the 2008 publication of the ACI 318, it was determined that as the moment due to service loads exceeds 2/3 of the cracking moment, the defection dramatically increases. Therefore, in the 2008 publication, the ACI 318 implemented an equation that utilizes a linear interpolation process to determine service deflections for members where the service moment is greater than 2/3 the cracking moment as shown in Equation 3.2-10. The original equation for service deflection is still utilized for sections where the service moment is less than 2/3 the cracking moment as shown in Equation 3.2-11.

$$\Delta_s = \left(\frac{2}{3}\right)\Delta_{cr} + \frac{\left(M_a - \frac{2}{3}M_{cr}\right)}{\left(M_n - \frac{2}{3}M_{cr}\right)}\left(\Delta_n - \frac{2}{3}\Delta_{cr}\right) \text{ For } M_a \geq \frac{2}{3}M_{cr} \qquad \text{Equation 3.2-10}$$

(ACI 318-11 Equation 14-8)

$$\Delta_s = \frac{M_a}{M_{cr}}\Delta_{cr} \text{ For } M_a < \frac{2}{3}M_{cr} \qquad \text{Equation 3.2-11}$$

(ACI 318-11 Equation 14-9)

and

$$\Delta_{cr} = \frac{5M_{cr}l^2}{48E_cI_g} \qquad \text{Equation 3.2-12}$$

(ACI 318-11 Equation 14-10)

$$\Delta_n = \frac{5M_nl^2}{48E_cI_{cr}} \qquad \text{Equation 3.2-13}$$

(ACI 318-11 Equation 14-11)

Determining the service applied moment is the same as determining the ultimate moment as described in Section 3.2.3 but simply applying service loads rather than factored loads. The primary moment caused by out-of-plane wind loading as well as the moment imposed by the eccentrically applied service roof load must first be determined for service applied loads, then ACI 551 prescribes an iterative process be used to determine the P-Δ effects. Equation 3.2-14 is the iteration equation to be used.

$$M_s = M_{sa} + P_{sa}\Delta_{sa} \qquad \text{Equation 3.2-14}$$

(ACI 551, 2009)

M_{sa} is the primary moment caused by out of plane service loading as well as the eccentrically applied roof load and Δ_{sa} is the initial service deflection. The factor P_{sa} is to include both the service applied roof load as well as the tilt-up panel's weight above the location of maximum moment, the panel midheight. This equation must be evaluated until the service applied moment, M_s, converges at a constant value. Following the determination of the service applied moment, the service deflection can be determined using either Equation 3.2-10 or 3.2-11.

It should be noted that the service applied moment, M_s, is recommended by ACI 318-11 commentary to use the load combination shown in Equation 3.2-15.

$$1.0D + 0.5L + W_a \quad \text{Equation 3.2-15}$$

(ACI 318, 2011)

Where W_a is the winds pressure based on serviceability requirements. Because the ACI 318-11 is used in conjunction with the ASCE 7-05, where the wind loads are in stress levels Equation 3.2-15 must be written in strength levels. Translating Equation 3.2-15 to the strength levels used in the ASCE 7-10 at service levels produces:

$$1.0D + 0.5L + 0.6W \quad \text{Equation 3.2-16}$$

Once the service deflection is determined it must be compared to the allowable deflection to determine if the requirements of ACI 318-11 Section 14.8.4 are met.

3.2.5 Minimum Reinforcement

As with all reinforced concrete structural members, tilt-up wall panels must meet minimum steel requirements to ensure that the section behaves in a ductile manner as well as minimize cracking due to temperature and shrinkage.

The minimum longitudinal reinforcing must comply with ACI 318-11 Section 14.3.2. The minimum longitudinal reinforcing is given by Section 14.3.2 as a ratio of area of reinforcement to gross area of concrete as follows:

(a) "0.0012 for deformed bars not larger than No. 5 with f_y not less than 60,000 psi; or
(b) 0.0015 for other deformed bars; or
(c) 0.0012 for welded wire reinforcement not larger than W31 or D31." (ACI 318, 2011)

The above steel reinforcement area to concrete area ratios are compared to the actual reinforcement ratio. If the above ratios are greater than those determined for strength and serviceability, the minimum must be used.

The minimum horizontal reinforcement must comply with the ACI 318-11 Section 14.3.3. Similar to the longitudinal reinforcing minimum, the transverse minimum is given as the ratio of reinforcing steel area to gross area of concrete as described below:

(d) "0.0020 for deformed bars not larger than No. 5 with f_y not less than 60,000 psi; or
(e) 0.0025 for other deformed bars; or
(f) 0.0020 for welded wire reinforcement not larger than W31 or D31" (ACI 318, 2011)

The code requires that both above minimums for horizontal and longitudinal reinforcing are met. In doing so, the designer will ensure that cracks due to temperature and shrinkage are minimal. In addition, the tilt-up wall panel will behave in a ductile manner in the event of failure. This is a desirable failure mechanism because it is a slow gradual failure and the panel will still have the capacity to carry load after the steel has yielded.

Chapter 4 - Panel with Opening Design Example

This chapter presents a design example of a slender wall with an open following the ACI 318-11 Section 14.8 procedure. Panel D will be utilized as the example panel for this chapter, Panel D can been seen in Figure 2.1. The design parameters are:

Panel Width	=	24'-0"	F'_c	=	4,000	psi
Panel Height	=	34'-0"	F_y	=	60,000	psi
Unbraced Length	=	32'-0"	γ_c	=	150	pcf (normal weight concrete)
Parapet	=	2'	E_s	=	29,000	psi
Opening Size	=	20'X20'	A_s	=	2.64	In^2

An elevation of wall Panel D with a 20 foot by 20 foot opening centered on the wall horizontally and located at the base of the wall, is shown in Figure 4.1.

The process by which tilt-panels are designed is iterative in nature. The area of steel required cannot be directly solved because of the requirement to calculate P-delta effects. Panel D requires two layers of six #6 bars for each leg.

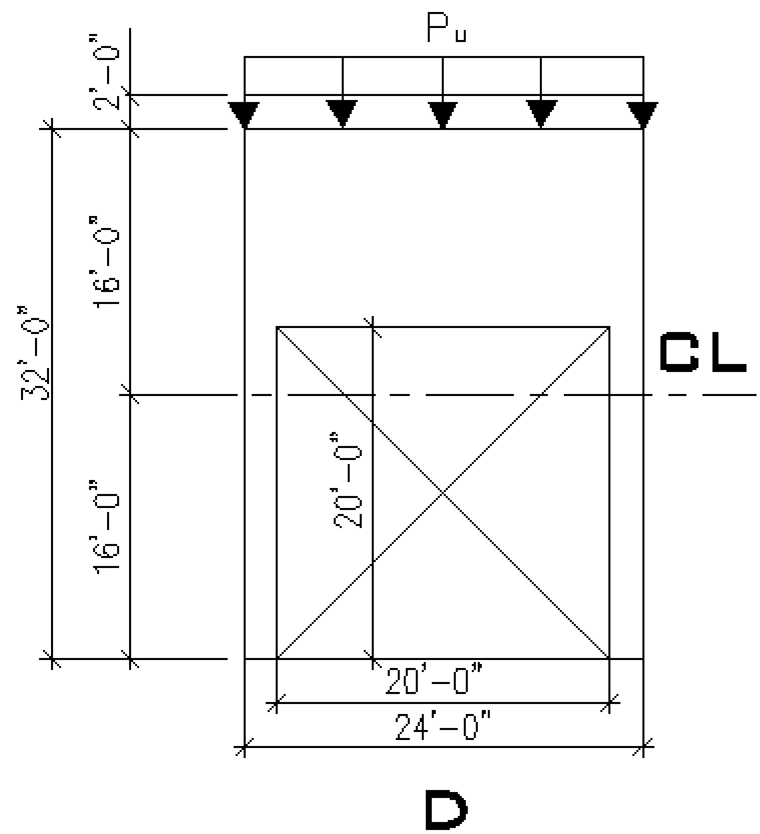

Figure 4.1: Panel 'D' Geometry

4.1 Design Example: Determine Requirements for Strength

The following section gives an example of how to utilize the requirements for strength discussed in Section 3.2. The same process has been used to design all tilt-up wall panels in this report in accordance with ACI 318-11 Section 14.8, *Alternative Design of Slender Walls*.

4.1.1 Determine Applied Loading

In order to determine applied moments, applied loads must be determined. A summarization of the applied loading given in Section 2.2 is summarized in Table 4.1.

Load Summary		
Dead Load	20	psf
Roof Live Load	20	psf
Snow Load	20	psf
Wind Load (115 MPH)	39	psf

Table 4.1: Load Summary

For a wind speed of 115 miles per hour the resultant strength level wind load of 39 pounds per square foot is being utilized in this example. Because of the symmetry of the panel opening in relationship to the overall panel, the roof loads can be resolved as follows:

$$D = L_r = S = \frac{12\,ft(24\,ft)}{2}(20\,psf) = 2.88k \text{ per leg}$$

The panel also supports its self-weight above the panel mid-height. Additionally, each leg supports half of the weight of the concrete above the door opening. For clarification, the panel geometry can be seen in Figure 4.1.

$$D_{panel} = \frac{(14\,ft)(20\,ft)\left(\tfrac{9.25}{12}\,ft\right)(150\,pcf)}{2\left(1000\,\tfrac{lb}{k}\right)} + \left(\tfrac{32}{2}+2\right)(2)\left(\tfrac{9.25}{12}\right)(150\,pcf)\left(\tfrac{1}{1000}\,\tfrac{k}{lb}\right) = 20.35k$$

For design purposes, the wind load is resolved into an equivalent linear load. Each leg supports half of the total wind load on the panel. Note that this report assumes that the opening is for a dock door. The largest wind load will occur when the door is closed. The door is assumed to have capacity to transfer the load to the adjacent panel leg horizontally by one-way bending action.

Figure 4.2 shows that the parapet of the tilt-up panel produces a negative moment thus reducing the applied moment due to the wind load. This study conservatively neglects this negative moment. Omission of this negative moment produces the largest possible ultimate applied moment on the tilt-up panel.

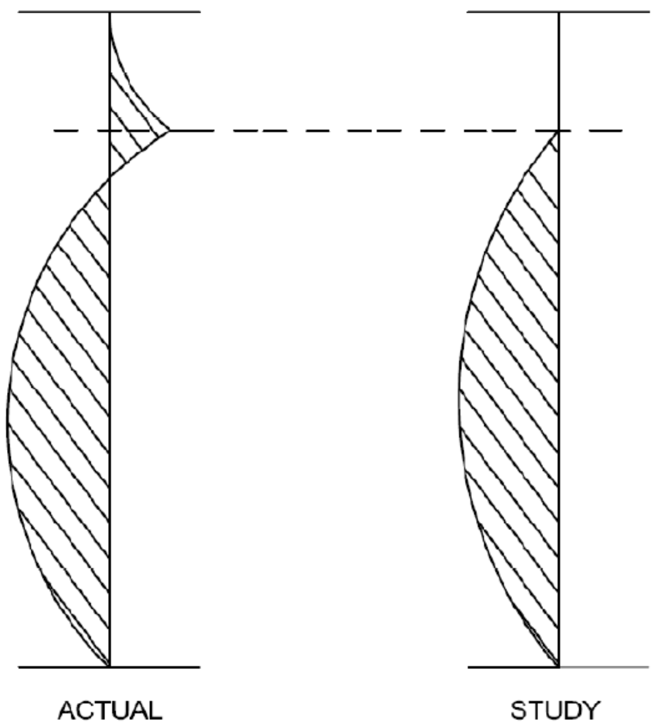

Figure 4.2: Wind Load Moment Diagram

The linear wind load is resolved as follows:

$$W = \frac{24\,ft}{2}(38.85\,psf)\frac{1}{1000\,\frac{lb}{k}} = 0.4662\,plf$$

4.1.2 Combine Loading

According to the ACI 551.R2-10 *Design Guide for Tilt-Up Concrete Panels*, the following load combinations, Equations 4.1-1 through 4.1-6, have the potential to control the design. These load combinations can also be located in the ASCE 7-10 Chapter 2 *Combinations of Loads*.

U = 1.2(D + F + T) + 1.6(L + H) + 0.5(L_r or S or R) Equation 4.1-1
(ACI 551 Equation 9-2)

U = 1.2D + 1.6(Lr or S or R) + (1.0L or 0.5W) Equation 4.1-2

(ACI 551 Equation 9-3)

U = 1.2D + 1.0W + 1.0L + 0.5(L_r or S or R) Equation 4.1-3

(ACI 551 Equation 9-4)

U = 1.2D + 1.0E + 1.0L + 0.2S Equation 4.1-4

(ACI 551 Equation 9-5)

U = 0.9D + 1.0W + 1.6H Equation 4.1-5

(ACI 551 Equation 9-6)

U = 0.9D + 1.0E + 1.6H Equation 4.1-6

(ACI 551 Equation 9-7)

Equation 4.1-1 has the potential to control the design of tilt-up panels supporting large dead loads, live loads, and soil pressure loads or panels spanning multiple stories. Equations 4.1-2 can be the controlling case where the panel supports large dead loads and roof live loads. Equation 4.1-3 must be routinely checked because it typically controls the design of tilt-up panels subjected to the application of gravity load and lateral wind pressures. In seismic areas, Equation 4.1-4 typically governs the design. Equations 4.1-5 and 4.1-6 need to be checked to prevent panel overturning due to in-plane shear loads.

The governing load combinations for the structure in this parametric study are Equations 4.1-2, 4.1-3, and 4.1-5 because the building is situated in areas with high wind pressures. Checking tilt-up panels in seismic regions is beyond the scope of research of this report. For the illustrative purposes, this example analyzes Equation 4.1-3 which governs the design because of the large wind pressure. The results for Equations 4.1-2 and 4.1-5 are in Appendix C. Using Equation 4.1.3, the loads are combined as follows:

$$P_{ua} = 1.2(2.88) + 0.5(2.88) = 4.896k$$
$$P_{um} = 4.896 + 1.2(20.35) = 29.32k$$
$$W_u = 1.0(0.4662) = 0.4662klf$$

4.1.3 Check Stress at Panel Mid-height

Because the modified area of reinforcement, A_{se}, is only accurate for small axial loads as described in section 3.1.4, the panel stress at mid-height must be checked as required by ACI 318-11 Section 14.8.2.6.

$$\text{mid-height stress} = \frac{P_{um}}{A_g} = \frac{(29.32k)(1000\,\tfrac{lb}{k})}{9.25in(2ft)(12\,\tfrac{in}{ft})} = 132\,psi \leq 0.06f'_c = 0.06(4000) = 240\,psi$$

With the above criteria satisfied, the designer can proceed with the design. Equation 4.1-2 typically be the governing equation for this requirement because it is primarily used for combination of gravity loads and will therefore produce the largest stress at the panel mid-height. However, all load combinations must satisfy this requirement. Refer to Appendix C for additional calculations.

4.1.4 Determine Flexural Cracking Moment

The cracking moment for each leg can be determined from Equation 3.2-6 and 3.2-7.

$$f_r = 7.5\lambda\sqrt{f'_c} = 7.5(1.0)\sqrt{4000} = 474.34\text{psi} \qquad \text{Equation 3.2-7}$$

$$M_{cr} = \frac{f_r I_g}{y_t} = \frac{474.34\,psi\left(\tfrac{1}{12}2\,ft(9.25in)^3\right)}{\tfrac{9.25}{2}in\left(1000\,\tfrac{lb}{k}\right)} = 13.38k - ft \qquad \text{Equation 3.2-6}$$

4.1.5 Determine Design Moment Capacity

Next, the design moment strength must be calculated as outlined in Section 3.2.1. Furthermore, it must be verified that the design moment strength is greater than the cracking moment as required by ACI 318-11 Section 14.8.2.4 and discussed in Section 3.2.2. For clarification, it should be noticed that the effects of the compression steel are conservatively neglected as advised by ACI 551.R2-10, *Design Guide for Tilt-Up Concrete Panels*. Testing the conservatism inherit in this procedure is beyond the scope of research for this report. The depth of steel can be seen in Figure 4.3 and is determined as follows:

d = (thickness) – (clear cover) – 0.5 x (bar diameter)

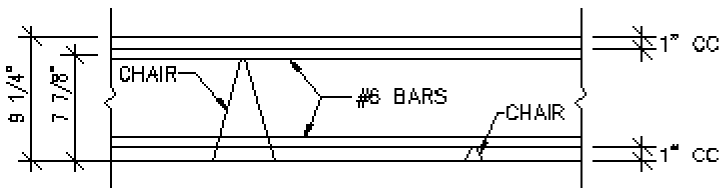

Figure 4.3: Tilt-Up Panel Cross Section

Because tilt-up wall panels are classified as precast-site cast concrete, a ¾ inch clear cover requirement to allow for concrete flow as required by ACI 318-11 Section 7.7.3 is used. This example conservatively uses 1 inch clear cover.

$$d = 9.25 - 1 - 0.5(0.75) = 7.875in$$

The equivalent area of steel is determined,

$$A_{se} = A_s + \frac{P_{um}}{f_y}\left(\frac{h}{2d}\right) = 2.64in^2 + \frac{29.32k}{60ksi}\left(\frac{9.25in}{2(7.875in)}\right) = 2.94in^2 \quad \text{Equation 3.1-5}$$

The depth of the equivalent compression block is calculated from Equation 3.2-5,

$$a = \frac{A_{se} f_y}{0.85 f'_c b} = \frac{2.94in^2(60ksi)}{0.85(4ksi)(2ft)(12\frac{in}{ft})} = 2.16in \quad \text{Equation 3.2-5}$$

The actual depth of the compression block is determined,

$$c = \frac{a}{\beta_1}\frac{2.16in}{0.85} = 2.54in \qquad \frac{c}{d} = \frac{2.54in}{7.875in} = .323 \leq .375$$

Therefore, the section is tension-control as required by Section 14.8.2.3 which makes the strength reduction factor, Φ, equal to 0.9 according to ACI 318-11 Section 9.3.2.1.

$$\phi M_n = \phi A_{se} f_y \left(d - \frac{a}{2} \right) \qquad \text{Equation 3.2-2}$$

$$= 0.9(2.94in^2)(60ksi)\left(7.875in - \frac{2.16in}{2} \right)\left(\frac{1}{12}\frac{ft}{in} \right) = 89.83k - ft$$

$$\phi M_n = 89.83k - ft \geq M_{cr} = 13.38k - ft$$

As a result, the requirements of Section 14.8.2.4 are met thus allowing the design to proceed.

4.1.6 Determine the Ultimate Applied Moment

Next, the ultimate applied moment must be calculated as outlined in Section 3.2.3. This ensures that it is less than the design moment strength as required by ACI 318-11 Section 14.8.3. The primary applied moment is determined from equation 3.2-8.

$$M_{ua} = \frac{w_{um} l_b^{\ 2}}{8} + \frac{P_{ua} e_{cc}}{2} \qquad \text{Equation 3.2-8}$$

$$M_{ua} = \frac{0.4662klf\,(32\,ft)^2}{8} + \frac{4.896k(6.125in)}{2\left(12\,\frac{in}{ft}\right)} = 60.94k - ft$$

The cracked moment of inertia is determined by equation 3.1-3,

$$I_{cr} = nA_{se}(d - c)^2 + \frac{l_w c^3}{3} \qquad \text{Equation 3.1-3}$$

$$I_{cr} = \frac{29000ksi}{3605ksi}\left(2.93in^2\right)\left(7.875in - 2.54in\right)^2 + \frac{2ft\left(2.53in\right)^3}{3}\left(12\tfrac{in}{ft}\right) = 803in^4$$

The flexural bending stiffness is calculated,

$$K_b = \frac{48E_c I_{cr}}{5l_b^2} \qquad \text{Equation 3.1-2}$$

$$K_b = \frac{48\left(3605ksi\right)\left(803in^4\right)}{5\left[32ft\left(12\tfrac{in}{ft}\right)\right]^2} = 188.6k$$

The resultant ultimate applied moment is determined by equation 3.1-1,

$$M_u = \frac{M_{ua}}{1 - \dfrac{P_u}{(0.75)K_b}} \qquad \text{Equation 3.1-1}$$

$$M_u = \frac{60.94k - ft}{1 - \dfrac{29.32k}{0.75(188.6k)}} = 76.9k - ft$$

Determine if the moment resisting capacity is adequate to resist the ultimate applied moment,

$$\phi M_n = 89.83k - ft \geq M_u = 76.9k - ft \qquad \text{Equation 3.2-1}$$

Given the above statement, the panel has adequate strength to resist the ultimate applied moment due to primary and secondary loads. Along with those specifications, all the strength requirements of the ACI 318-11 Section 14.8 are met, as shown in this section. The last step to finalize the design is to ensure that the serviceability requirements are met.

4.2 Design Example: Determine Requirements for Serviceability

As stated in Section 3.2.4, tilt-up wall panels have the capability to support loading even if the panel displays large lateral deflections. For serviceability reasons, the ACI 318-11 Section

14.8 limits the service load deflections. This section gives a detailed example calculation for the service deflection for Panel D in Figure 2.1. Moreover, this section analyzes the minimum steel requirements for the example panel as discussed in section 3.2.5.

4.2.1 Determine the Service Applied Loads

As stated in Section 3.2.4, the ACI 551.R2-10 *Design Guide for Tilt-Up Concrete Panels* recommends utilizing the load combination given in Equation 3.2-16.

$$1.0D + 0.5L + 0.6W \qquad \text{Equation 3.2-16}$$

The service loads are combined as follows,

$$P_{sa} = 1.0(2.88k) + 0.5(2.88k) = 4.32k$$
$$P_{sm} = 4.32 + 1.0(20.35k) = 24.67k$$
$$W_s = 0.6\left(\frac{24\,ft}{2}\right)(38.85\,psf)\frac{1}{1000\,\frac{lb}{k}} = 0.280klf$$

4.2.1 Calculate Service Deflection

As described in Section 3.2.4, an iterative procedure is required to determine the service load deflection. First, the service applied moment and cracking load deflection must be determined from equations 3.2-12 and 3.2-8.

$$\Delta_{cr} = \frac{5M_{cr}l^2}{48E_cI_g} \qquad \text{Equation 3.2-12}$$

$$\Delta_{cr} = \frac{5(13.53k - ft)\left(12\,\frac{in}{ft}\right)\left[32\,ft\left(12\,\frac{in}{ft}\right)\right]^2}{48(3605ksi)\frac{1}{12}(24in)(9.25in)^3} = 0.44in$$

$$M_{sa} = \frac{w_{um}l_b^{\,2}}{8} + \frac{P_{ua}e_{cc}}{2} \qquad \text{Equation 3.2-8}$$

$$M_{sa} = \frac{0.280klf(32ft)^2}{8} + \frac{4.32k(6.125in)}{2(12\frac{in}{ft})} = 36.94k - ft$$

The moment due to service loads is compared to (2/3)M_{cr} to determine which Δ_s equation should be used.

$$\frac{2}{3}M_{cr} = \frac{2}{3}(13.38k - ft) = 8.92k - ft$$

$$M_{sa} = 36.94 \geq \frac{2}{3}M_{cr} = 8.92k - ft \text{ therefore use equation 3.2-10}$$

$$\Delta_s = \left(\frac{2}{3}\right)\Delta_{cr} + \frac{\left(M_a - \frac{2}{3}M_{cr}\right)}{\left(M_n - \frac{2}{3}M_{cr}\right)}\left(\Delta_n - \frac{2}{3}\Delta_{cr}\right) \qquad \text{Equation 3.2-10}$$

Determine the out of plane panel deflection from equation 3.2-13

$$\Delta_n = \frac{5M_n l^2}{48E_c I_{cr}} = \frac{5\left(\frac{89.83k - ft}{0.9}\right)\left(12\frac{in}{ft}\right)\left[32ft\left(12\frac{in}{ft}\right)\right]^2}{48(3605ksi)(803in^4)} = 6.36in \qquad \text{Equation 3.2-13}$$

With the above variables determined, the iterative procedure outlined in Section 3.2.4 can be performed.

$$\Delta_s = \left(\frac{2}{3}\right)(0.44in) + \frac{\left[36.94k - ft - \frac{2}{3}(13.38k - ft)\right]}{\left[\frac{89.83k - ft}{0.9} - \frac{2}{3}(13.38k - ft)\right]}\left(6.36in - \frac{2}{3}(0.44)\right) = 2.15in$$

From equation 3.2-14:

$$M_s = M_{sa} + P_{sa}\Delta_{sa} \qquad \text{Equation 3.2-14}$$

$$M_s = 36.94k - ft + 2.15(24.67k)\left(\tfrac{1}{12}\tfrac{ft}{in}\right) = 41.36k - ft$$

Continue until convergence:

$$\Delta_s = \left(\frac{2}{3}\right)(0.44in) + \frac{\left[41.36k - ft - \frac{2}{3}(13.38k - ft)\right]}{\left[\frac{89.83k - ft}{0.9} - \frac{2}{3}(13.38k - ft)\right]}\left(6.36in - \frac{2}{3}(0.44)\right) = 2.44in$$

$$M_s = 36.94k - ft + 2.44(24.67k)\left(\tfrac{1}{12}\tfrac{ft}{in}\right) = 41.96k - ft$$

$$\Delta_s = \left(\frac{2}{3}\right)(0.44in) + \frac{\left[41.96k - ft - \frac{2}{3}(13.38k - ft)\right]}{\left[\frac{89.83k - ft}{0.9} - \frac{2}{3}(13.38k - ft)\right]}\left(6.36in - \frac{2}{3}(0.44)\right) = 2.48in$$

$$M_s = 36.94k - ft + 2.48(24.67k)\left(\tfrac{1}{12}\tfrac{ft}{in}\right) = 42.04k - ft$$

$$\Delta_s = \left(\frac{2}{3}\right)(0.44in) + \frac{\left[42.04k - ft - \frac{2}{3}(13.38k - ft)\right]}{\left[\frac{89.83k - ft}{0.9} - \frac{2}{3}(13.38k - ft)\right]}\left(6.36in - \frac{2}{3}(0.44)\right) = 2.48in$$

As stated in Section 3.2.4, the service applied deflection must be less than the unbraced length divided by 150. This requirement is verified below.

$$\Delta_{allowable} = \frac{l}{150} = \frac{32ft}{150}\left(12\tfrac{in}{ft}\right) = 2.56in \geq \Delta_s = 2.48in$$

4.2.1 Check Minimum Steel Requirements for Vertical Reinforcing

As stated in Section 3.2.5, the ACI 318-11 Section 14.8 requires that longitudinal reinforcing must be greater than the minimum described in ACI 318-11 Section 14.3.2. Because this example utilizes #6 bars, it is classified as (b) "0.0015 for other deformed bars."

$$A_{s\min} = 0.0015bh = 0.0015(24in)(9.25in) = 0.33in^2 \leq A_{6\#6} = 2.64in^2$$

Therefore, the minimum steel requirements are met. Additionally, the above minimum steel requirement is used as the criteria for the concrete above the opening in the panel. For ease of the design and construction process it is common practice to keep both the panel legs and the panel above the opening the same thickness. However, as the panel legs are required to be thicker, it becomes less economical to keep the thickness of the concrete above the opening the same. Because of this, designers often vary the thickness of this section designing it only with adequate capacity to transfer the wind load to the adjacent legs. Another design techniques is to design the legs as slender confined concrete columns with ACI 318-11 section 10.10 *Slenderness Effects in Compression Members*, this often results in a thinner leg because of the confining action of the tied concrete column. Using both of these design techniques, the designer can reduce the thickness of the entire panel, making the design more economical. All of these design techniques are discussed and examined in detail, see Section 6.1.1. For continuity purposes this design example maintains a constant thickness over the entire panel.

$$A_{s-above} = 0.0015bh = 0.0015(20ft)(9.25in)\left(12\tfrac{in}{ft}\right) = 3.33in^2$$

For ease of construction, it is desirable to utilize the same bar type for the concrete above the opening. As a result, the maximum spacing requirements in ACI 318-11 Section 14.3.5 will control the design, 16 #6 bars (A_s=6.60 in^2) at 1'-4" each face, on center, are utilized.

4.2.1 Design Horizontal Reinforcing

As stated in Section 3.2.5, the horizontal reinforcing is required to comply with ACI 318-11 Section 14.3.3 and is classified as (b) "0.0025 for other deformed bars."

$$A_{s\,\min} = 0.0025bh = 0.0025(34\,ft)(9.25in)\left(12\,\tfrac{in}{ft}\right) = 9.435in^2$$

Therefore, 48 #4 bars (As=9.60in2) at 8.5” on center are used.

4.3 Summary

Figure 4.4 graphically shows the design which this chapter presents. A 9.25” panel with 6 #6 bars at 4 inches on center, each face in the panel legs, has the capacity to satisfy both strength and serviceability requirements. Additionally, 16 #6 longitudinal bars at 1’-4” each face, on center, are required above the door opening while #4 bars at 8.5” on center (not shown for clarification purposes) are required for horizontal reinforcement.

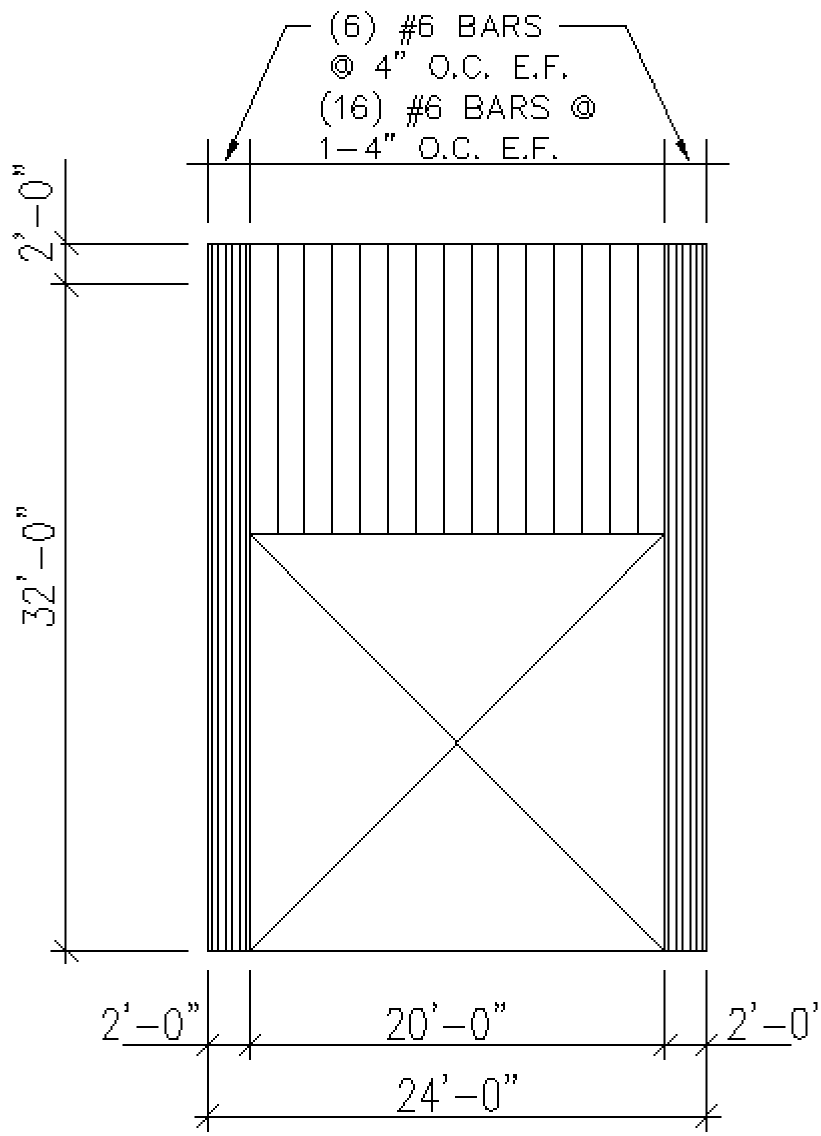

Figure 4.4: Final Design of Panel D

Chapter 5 - Finite Element Analysis Conducted

This chapter gives a detailed description of the Finite Element Analysis conducted in this report.

5.1 Tilt-up Wall panels

For comparison purposes two tilt-up walls from Figure 2.2 are strategically chosen with two different wind speeds as shown in Table 5.1.

FEM Panels	
Tilt-up Panel	Analyzed Wind Speed
A	115 mph
A	170 mph
C	115 mph
C	170 mph

Table 5.1: Tilt-up Panels Analyzed with Finite Element Analysis

Because the maximum moment is at the panel mid-height when assuming a pin-pin connection, Panel A is chosen to quantify the increase in bending stiffness that occurs at this location due to the availability of the concrete above the opening, which correlates to panel (c) in Figure 2.4 analyzed by Bartels and Schwabauer. Consequently, Panel C has been chosen to determine if any bending stiffness increase occurs when the opening extends to the panel mid-height, which correlates to panel (e) analyzed by Bartels and Schwabauer. Panel D is not chosen due to the fact that in a practical application the slender legs would be designed as slender confined columns as described in Section 4.2.1, refer to Section 6.1.1 for results regarding Panel D.

The wind speed has been varied to simulate various possible locations for the case study building shown in Figure 2.3. The 115 mph loading corresponds to the building being located in the central United States while the 170 mph load case represents the building being located in hurricane prone regions predominately on the east coast. Additionally, the wind load was varied to determine how varying the lateral load would affect the design of the tilt-up panel.

5.2 Finite Element Analysis Software

For the purposes of this report, the finite element analysis software program SAP 2000 version 14 is used. SAP 2000 was developed by Computers and Structures Incorporated.

5.3 Discretization of Panels Analyzed

For analysis purposes both Panels A and C are divided into one foot by one foot finite elements as shown in Figure 5.1

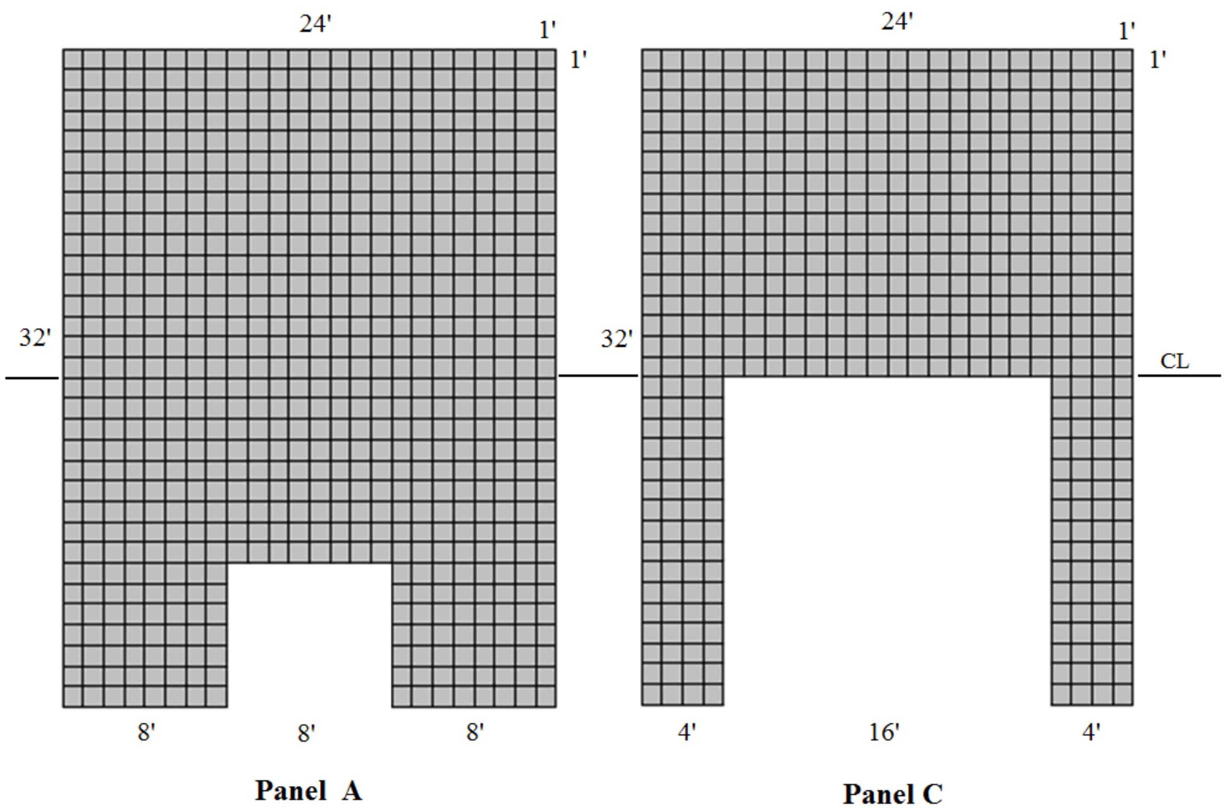

Figure 5.1: Discretization of Tilt-Up Panels A and C

As described in Section E.1 the validity of the results is proportional to the size of the elements utilized. The most accurate results occur if elements with an infinitesimal area could be used. For the purposes of this research one foot by one foot elements have been determined as adequate to achieve convergence of the solution.

5.4 Idealization

The following section gives a detailed description of the isoparametric shell elements being used for the finite element analysis conducted in this report.

5.4.1 Shell element

The shell element, shown in Figure 5.2, is utilized to model the behavioral characteristics of the cracked concrete and steel reinforcing. The shell element has five degrees of freedom per node, three translational and two rotational.

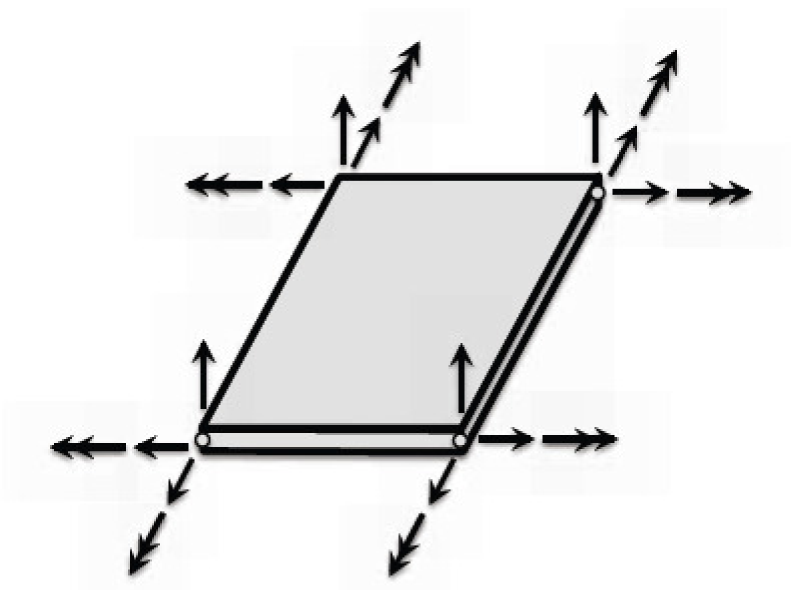

Figure 5.2: Shell Element (Schwabauer, 2010)

5.4.2 Analyzed Panel Section

For analysis purposes and to comply with the analysis procedure used by SAP 2000 a multi-layered shell element is used. A multi-layered shell element is an element in which the shell element is composed of more than one material. The degrees of freedom per node and the isoparametric nature of the element remain the same. The primary difference is that a generic shell element is isotropic while a multi-layered shell element is anisotropic. The multi-layered shell element used in this parametric study is shown in Figure 5.3.

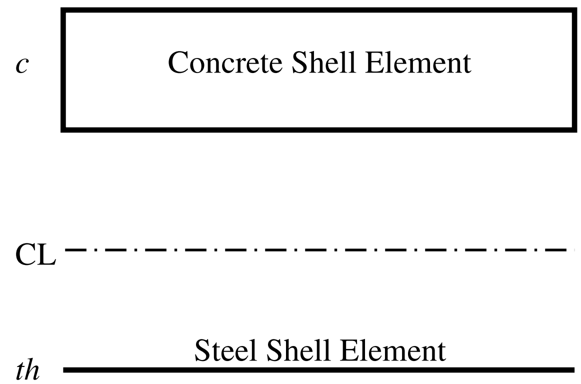

Figure 5.3: Multi-Layered Shell Element

The thicknesses of the steel membrane and the concrete shell (*th* and *c* respectively) as well as their respective locations are predetermined and input into the multi-layered shell element interface of SAP 2000.. This means that the concrete is "pre-cracked" upon inputting it's parameters into SAP 2000. SAP 2000, similar to most finite element programs, does not have the ability to crack the concrete. This is an extremely important realization because simply inputting the thickness of the panel will result in extraneous solutions as can be seen in Figure 5.4.

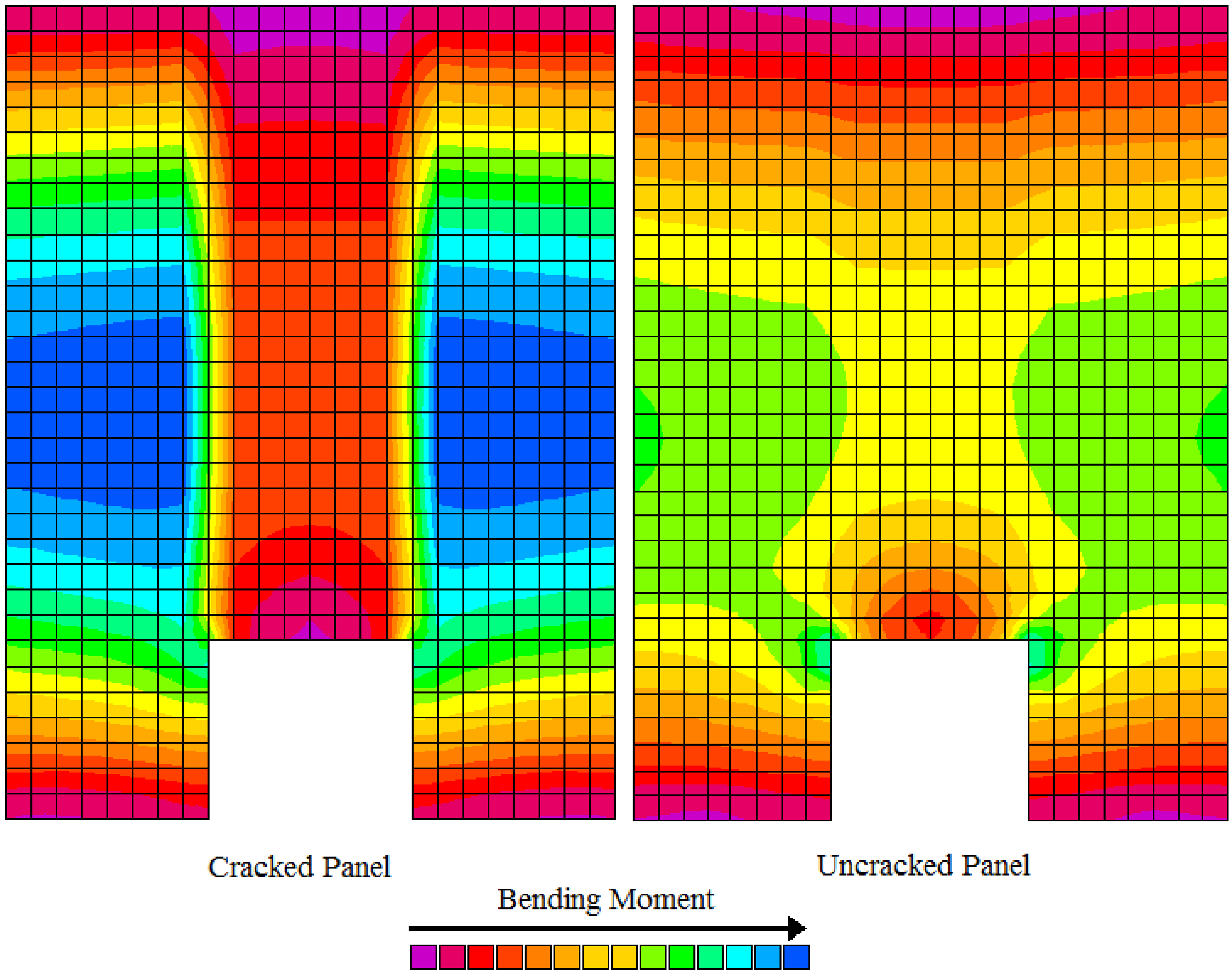

Figure 5.4: Pre-Cracked Panel vs. Uncracked Panel

Notice that the bending moment in the uncracked panel on the right is nearly uniformly distributed across the panel at mid-height. The uncracked concrete section above the door opening drastically increases the bending stiffness of the panel at mid-height.

The compression steel has been left out of the multi-layered shell element composition due to limited effects on the moment resisting capacity of the section. It also this more closely mimics the design equations of ACI 318-11 Section 14.8 *Alternative Design of Slender Walls*.

For each panel two multi-layered shell element have been composed. One models the behavior of the panel leg elements. Each element has been cracked to it's ultimate capacity proportional to the tensile reinforcement in the section. To achieve the most accurate finite element solution the out of plane wind load should be applied to the panel to determine the stresses on a given element. Upon determination of the stresses in each element the magnitude of the panel cracking should be determined and the wind load reapplied. This procedure should continue to be performed until the magnitude of cracking for each individual element converges. This procedure is beyond the scope of research for this report. For the purposes of this report using the ultimate capacity is used which produces comparable stresses in the panel at midheight, the location of maximum moment as predicted by ACI 318-11 Section 14.8. The P-Δ effects and deflections for each panel are not compared as the finite element model used does not produce accurate solutions for secondary stresses and deflections.

The concrete and reinforcing thicknesses as well as their distances from the centerline of the section, for the panel leg elements, are determined by hand calculations, corresponding to the design discussed in Section 3.2.1, before inputting them into SAP 2000.

The second represents the elements located above the door opening. This section is designed with minimum steel requirements as described in Section 3.2.5. Additionally, a yield line analysis is performed and it is determined that this portion of reinforced concrete exhibits one-way bending behavior perpendicular to the door opening. As such, the values for *c* and *th* as well as their distances from the centerline have been extrapolated from the moment strength in a direction perpendicular to the door opening.

5.5 Boundary Conditions

This section describes in detail the boundary condition parameters used the finite element analysis conducted.

5.5.1 Loading

To obtain an accurate comparison base the loading conditions describe in Section 2.2 have been analyzed in the finite element analysis. The eccentrically applied dead and live loads are resolved into a uniformly distributed axial load and an equivalent moment applied at the top of the panel. The wind loading is applied as a surface pressure on each one foot square element. The door openings are intended to model a double door (8’ x 7’) and a loading dock door (16’ x 16’). Both doors are assumed to transfer the wind load equally through one-way bending to the adjacent panel leg. As such the wind load applied to the doors has been resolved do a distributed load applied at the interior adjacent leg.

5.5.1 Fixities

To yield the most comparable solutions both the top and the bottom of the panels have been pinned at every node as required by ACI 318-11 Section 14.8.2.1.

5.6 Assembly and Solving

As described in Chapter E the stiffness matrix for the system becomes exponentially complex proportional to the complexity of the physical system being modeled. The assembled stiffness matrix for Panel C is 2680 by 2680, while the stiffness matrix for Panel A is 3765 by 3765. As such, SAP 2000 is used to both assemble the stiffness matrix and perform the finite element analysis.

Chapter 6 - Results and Conclusion

Using the *Alternative Design of Slender Walls* procedure of ACI 318 Section 14.8, vertical flexural (longitudinal) reinforcement has been determined for tilt-up wall panels subjected to eccentrically applied axial load and out-of-plane wind speed of 115 mph, 130 mph, 150 mph, and 170 mph at an unbraced length of 32 feet. For ease of placement panel thicknesses of 7.25”, 9.25” and 11.25’ are used to match actual wood formwork dimensions generally used in tilt-up construction. Vertical reinforcement for flexure includes #4, #5, and #6 bars.

As with all structural elements, tilt-up wall panels must satisfy both strength and serviceability requirements defined by ACI 318-11 Sections 14.8.3 and 14.8.4. Additionally, all panels have been designed to comply with code limitations for minimum reinforcing requirements. All panels have been designed with the moment Magnifier method discussed in ACI 318-11 Section 14.8. Recall the ultimate applied moment from Equation 3.1-1.

$$M_u = \frac{M_{ua}}{1 - \dfrac{P_u}{(0.75)K_b}} \qquad \text{Equation 3.1-1}$$

Remembering that K_b is the flexural stiffness of the panel which is defined by equation 3.1-2.

$$K_b = \frac{48E_c I_{cr}}{5l_b^2} \qquad \text{Equation 3.1-2}$$

The ultimate applied moment must be less than or equal to the nominal moment strength defined by Equation 3.2-2.

$$\phi M_n = \phi A_{se} f_y \left(d - \frac{a}{2} \right) \qquad \text{Equation 3.2-2}$$

The most economical panels require the least amount of both steel and concrete to compose the panel. This means that the thinnest panel with the smallest area of reinforcement is most desirable. If one layer of steel is used the amount of steel is minimal; however, this often results in a thicker panel. Controversy, if two layers of steel are used the moment arm combining the internal moment couple is maximized thus requiring a thinner panel but a larger area of reinforcing. As such, the superlative designers consider both a single layer of reinforcing and two layers of reinforcing.

6.1 Panels with Openings Designed with ACI 318-11 Section 14.8

The following Tables 6.1 through 6.4 summarize the results obtained for all panels with wind speeds of 115 mph, 130 mph, 150 mph and 170 mph respectively.

Vertical Reinforcement in Panel Legs							
Wind Speed = 115 mph	Opening Size	Panel Thickness	Layers of Steel	Bar Type	Number of Bars	Spacing	Total Area of Steel in^2
	8 ft x 7 ft	7.25 in	2	#6	8	12 in	3.53
	12 ft x 12 ft	7.25 in	2	#6	8	9 in	3.53
	16 ft x 16 ft	7.25 in	2	#5	14	3.5 in	4.30
	20 ft x 20 ft	9.25 in	2	#6	6	4 in	2.65

Table 6.1: Panel Results for 115 mph

Vertical Reinforcement in Panel Legs							
Wind Speed = 130 mph	Opening Size	Panel Thickness	Layers of Steel	Bar Type	Number of Bars	Spacing	Total Area of Steel in^2
	8 ft x 7 ft	7.25 in	2	#6	10	9.5 in	4.42
	12 ft x 12 ft	7.25 in	2	#6	10	6 in	4.42
	16 ft x 16 ft	9.25 in	2	#6	6	8 in	2.65
	20 ft x 20 ft	11.25 in	2	#4	10	2.5 in	1.96

Table 6.2: Panel Results for 130 mph

Vertical Reinforcement in Panel Legs							
	Opening Size	Panel Thickness	Layers of Steel	Bar Type	Number of Bars	Spacing	Total Area of Steel in^2
Wind Speed = 150 mph	8 ft x 7 ft	7.25 in	2	#6	14	6.875 in	6.19
	12 ft x 12 ft	9.25 in	2	#6	8	9 in	3.53
	16 ft x 16 ft	9.25 in	2	#6	8	6 in	3.53
	20 ft x 20 ft	11.25 in	2	#6	6	4 in	2.65

Table 6.3: Panel Results for 150 mph

Vertical Reinforcement in Panel Legs							
	Opening Size	Panel Thickness	Layers of Steel	Bar Type	Number of Bars	Spacing	Total Area of Steel in^2
Wind Speed = 170 mph	8 ft x 7 ft	7.25 in	2	#6	32	3 in	9.82
	12 ft x 12 ft	9.25 in	2	#6	10	7.25 in	4.42
	16 ft x 16 ft	9.25 in	2	#6	12	4 in	5.30
	20 ft x 20 ft	11.25 in	2	#6	8	3 in	3.53

Table 6.4: Panel Results for 170 mph

Tables 6.1 through 6.4 reflect the most economical design for the flexural reinforcing in the tilt-up panels using two layers of reinforcing steel. This is due to the large moments that must be resisted by the panel legs. Solid panels often will not require two layers of reinforcing which can be seen in Bartels report (Bartels, 2010).

The results show that the panel thickness and area of reinforcing are directly proportional to the size of the opening and the applied loading. Primarily the design of tilt-up panels with large loads and large opening is controlled by the tension controlled requirement of the ACI 318-11 Section 14.8.2.3. As the opening becomes larger and the wind load increases the amount of concrete being compressed increases. When the induced moment becomes too large the section becomes compression controlled which is not allowed by the *Alternative Design of Slender Walls*; thus, the panel thickness is required to increase.

6.1.1 Considerations for Panel with 20' x 20' opening

As stated in Section 4.2.1 designing panels with ACI 318-11 Section 14.8 can become uneconomical because it will require a thicker panel. For constructability purposes, in design practice it is common to maintain a constant panel thickness across the width of the panel. However, maintaining a constant panel thickness can become very costly as the panel is required to be thicker. To compensate, designers will often vary the thickness of the panel above the opening. Another technique is to design the panel with ACI 318-11 Section 10.10 *Slenderness Effects in Compression Members*. To analyze these design techniques, Panel D (20' x 20' opening) has been designed in three different configurations as shown in Figure 6.1.

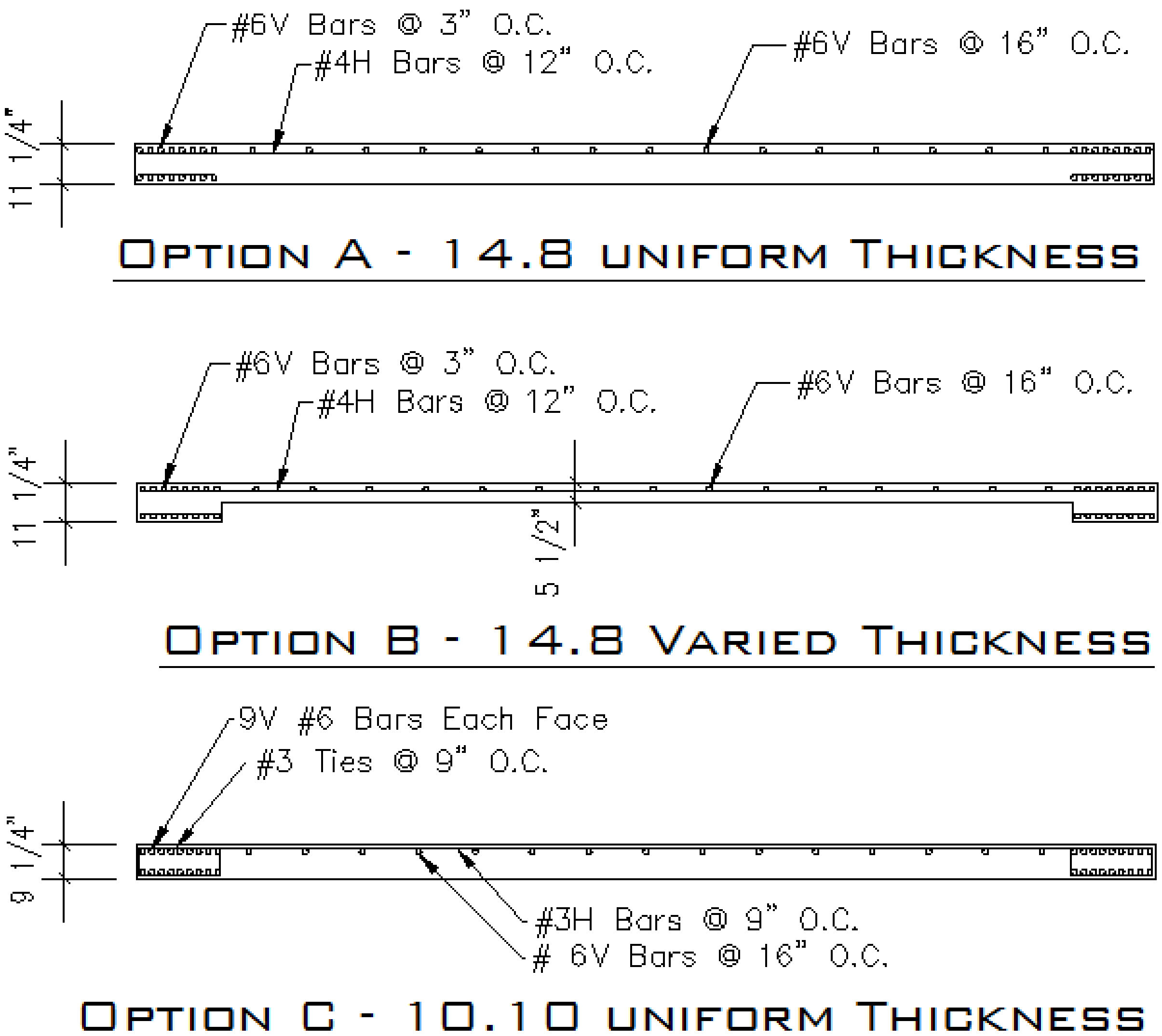

Figure 6.1: Panel D Design Configurations

Option B has been designed to ensure that the 5.5 inch thick section of the panel is sufficient to transfer the applied wind load to the adjacent panel legs. Option C has been designed in accordance with ACI 318-11 Section 10.10 *Slenderness Effects in Compression Members*; meaning the legs behave as a slender column rather than a part of a slender wall.

meaning that the panel legs are designed with horizontal ties which will confine the concrete. The confining action of the columns horizontal ties bars increases the bending stiffness properties of the leg. Testing of this confining action of the steel tie bars is beyond the scope of research for this paper.

Through correspondence with Ted Strahm of Lithko Contracting Incorporated a cost estimate has been determined for an individual panel as well as an overall cost for the tilt-up panels in the warehouse building shown in Figure 2.3. The total building cost includes the cost of the tilt-up panels along longitudinal axis assuming the same opening requirements. The Cost Results are shown in Table 6.5, see Appendix D for calculations.

Cost of Panel with 20' x 20' Opening		
Configuration	Panel Cost	Total Bldg Cost
Option A	$ 3,034.56	$ 72,829.51
Option B	$ 1,666.67	$ 40,000.00
Option C	$ 2,576.20	$ 61,828.78

Table 6.5: Cost of Panel Configurations

As Table 6.5 shows, Option B is the most economical of the three configurations because it requires less concrete. However, the construction of this particular panel may present construction difficulties to the tilt-up contractor. The first difficulty being that the panel will require either two separate concrete pours or the use of hanging forms in order to create the embedded pilasters; both require more labor hours. Additional difficulties include connection to the roof, insulating the interior of the panel, and finishing the interior face of the panel. The last consideration pertains to the lifting the panel. Option B will require 8 lifting inserts while Options A and C only require 4. Variation of lifting inserts requires that the crane be reconfigured to allow for the different lifting configuration prior to lifting of the panel. This can be a time consuming and cumbersome task, resulting in a cost increase on panel lifting day. Controversially, the architect or owner may have a preference on which panel configuration is best for the overall building aesthetics.

From a contractor standpoint Options A and C are the best options for constructability. However, as can be seen from Table 6.5 option B is the most economical when considering the cost of the panel. All of these factors should be considered by the engineering of record when designing tilt-up panels. The engineer's decision may be affected by factors such as, the relationship and confidence with the contractor, economic constrains and architect/owners preference.

6.2 Assumptions of ACI 318-11 Section 14.8 Alternative Design of Slender Walls

The following section compares the results obtained by the finite element analysis with the ACI 318-11 Section 14.8 design process to quantify the appropriateness of the assumptions made by Section 14.8 *Alternative Design of Slender Walls*.

6.2.1 One Way Bending Assumption

As stated in section 3.1.1 the ACI 318-11 assumes that panels with openings behave with one-way bending. To test this assumption two methods are used: a yield line analysis and comparison with the FEM conducted. Each method was performed on Panels A and C in figure 2.2, panels with 8' x 7' and 16' x 16' openings respectively. Because the openings are intended to model doors the wind load applied to the doors are applied as a uniform load on the interior of the panel leg. The validity of the doors ability to transfer the load to the adjacent legs through one way bending is beyond the scope of research for this report.

A yield line analysis is performed by analyzing the portion of the wall above the opening. The connections to the top of the panel as well as the connection to the adjacent legs are modeled with pin connections. This analysis confirms that the portion of the panel above the opening does exhibit one way bending behavior to transferring the applied wind load directly to the legs without two-way bending effects.

The finite element analysis conduct reflects the same results. For example, Panel A (8' x 7' opening) with 170 mph wind pressure applied shown in Figure 6.2.

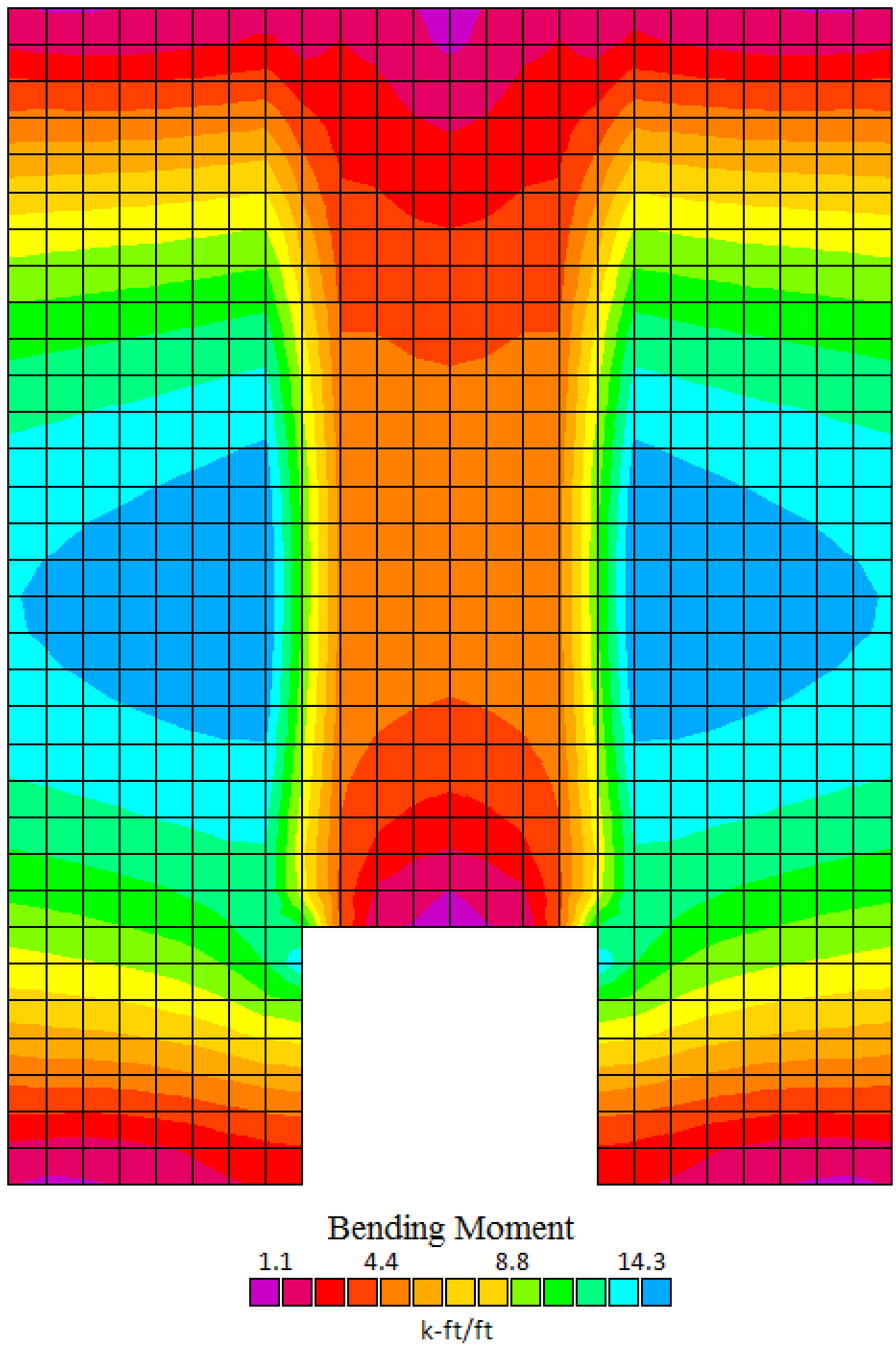

Figure 6.2: Panel A (8’ x 7’ Opening) at 170 mph Wind Speed

The portion of the panel above the opening shows maximum stress in the middle and is distributed through one way bending to the panel legs. It appears that some two-way bending action occurs near the supports at the top of the panel, but as these bending stresses are so small they are considered negligible.

Notice that the maximum applied moment is located in the legs at the panel mid-height where very little bending moment occurs outside the panel legs. The bending moment that does occur above the opening is relatively small in comparison with the maximum moment in the panel. Which means that the overall design of the panel is controlled by the maximum moment in the panel legs.

6.2.2 Constant Bending Assumption

As discussed in section 3.1.2 the second assumption made by the ACI 318-11 Section 14.8 is that the panel has a constant bending stiffness equivalent to the bending stiffness of the panel legs. To test this assumption two panels have been analyzed with finite element analysis. The first panel is Panel A (8’ x 7’ opening) from Figure 2.2 with wind pressures of 115 and 170 mph. The results obtained from both the finite element analysis and the Section 14.8 design procedure are shown in Figure 6.3.

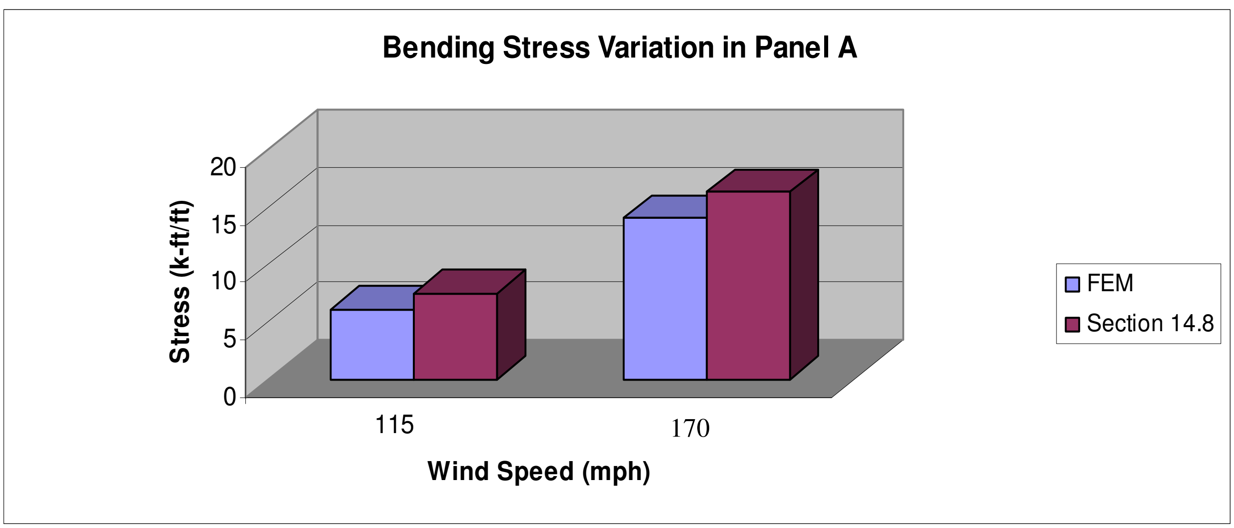

Figure 6.3: Bending Stress Variation in Panel A

The maximum bending stress by Section 14.8 calculations is 7.6 k-ft/ft and16.4 k-ft/ft for 115 mph and 170 mph respectively. The maximum bending stress determined from the finite element analysis is 6.1 k-ft/ft and 14.1 k-ft/ft for 115 mph and 170 mph respectively. The percent differences are 24.1% and 16.6% for 115 mph and 170 mph respectively. This means that the portion of the panel above the opening does provide additional bending stiffness to the panel legs; therefore, making the codes assumption at most 24% conservative. With this knowledge the code could implement a reduction factor proportional to ratio of total wall area to the area of the opening. However, the effects may be negligible for the overall design of the panel, further research should be conduct to check the accuracy of these results and check bending stiffness increases for different panel configurations.

The second panel, Panel D (16' x 16' opening) from figure 2.2 with 115 and 170 mph. The results from both the finite element analysis and the panel design from ACI 318-11 Section 14.8 are shown in Figure 6.4.

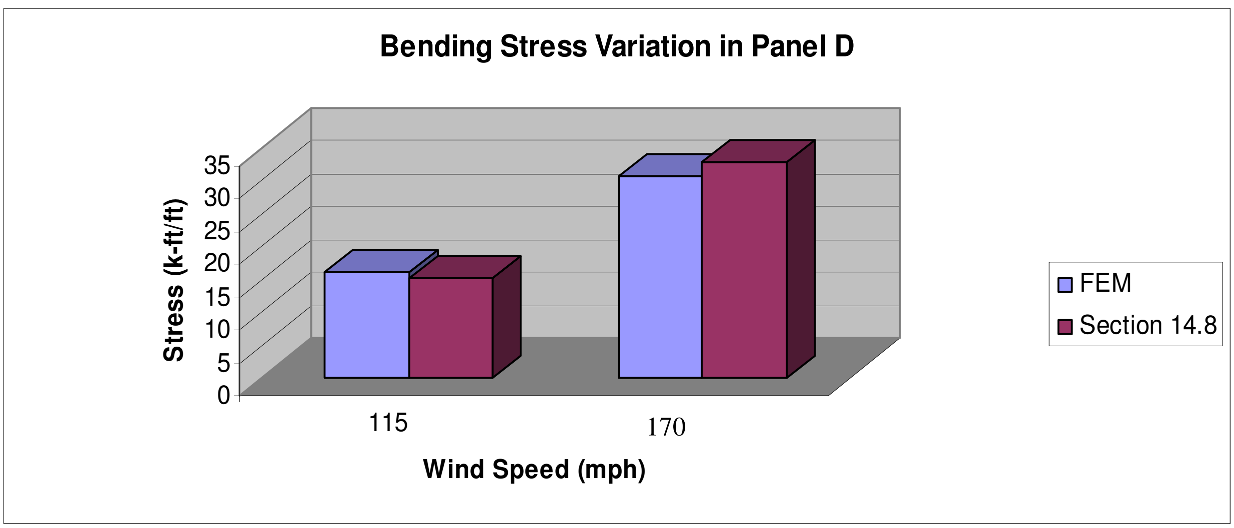

Figure 6.4: Bending Stress Variation for Panel with 16' x 16' Opening

The maximum bending stress by Section 14.8 calculations is 15.2 k-ft/ft and 32.9k-ft/ft for 115 mph and 170 mph respectively. The maximum bending stress determined from the finite element analysis is 16.1 k-ft/ft and 30.7 k-ft/ft for 115 mph and 170 mph respectively. The percent differences are -5.5% and 7.0% for 115 mph and 170 mph respectively. For the 170 mph case the bending stiffness is increased similar to the results seen in Panel A. It should be noticed that the percent difference is significantly less. This is because the opening in the panel extends to the panel mid-height (the location of maximum moment) which does not allow the portion of the panel above the opening to contribute to the bending stiffness of the panel to the same extent as Panel A. For the 115 mph case it should be noted that the finite element analysis gave a higher bending stress than that predicted by the design procedure of the ACI 318-11 Section 14.8. This is caused by the wind load from the adjacent door resulting in a higher bending stress at the portion of the leg closest to the door, as can be seen in Figure 6.5. Section 14.8 predicts that this load will be evenly distributed across the length of the leg however; the finite element analysis shows stress concentrations develop along the edges of the door opening. It is common practice for designers to place two #4 or #5 bars around the perimeter of the opening. Further research may be required to determine if such reinforcing is adequate to resist such stress concentrations.

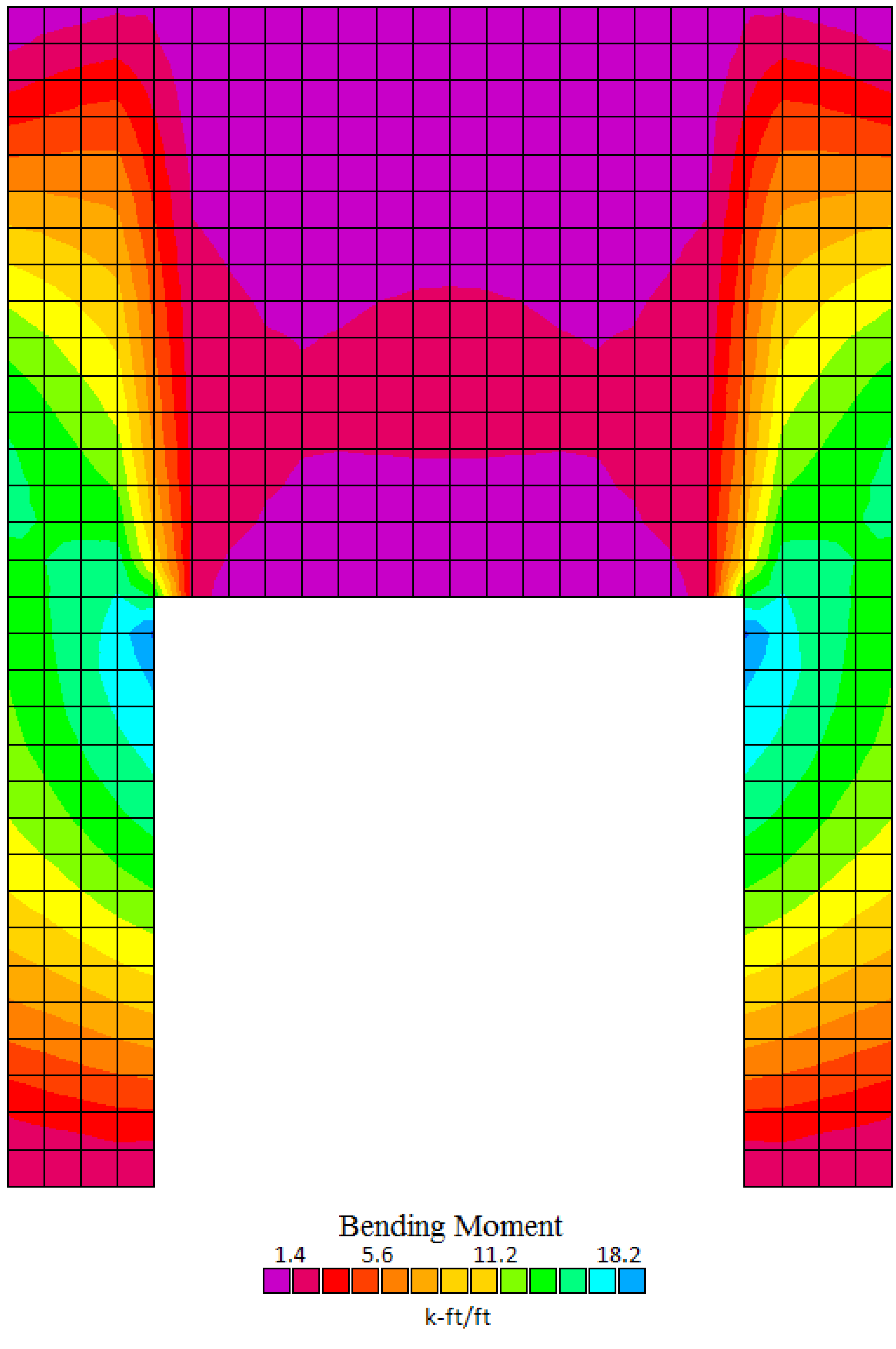

Figure 6.5: Bending Stress for Panel C (16' x16' Opening) and 115 mph Wind

6.2.3 Bending Stiffness Reduction Factor

As stated in Section 3.1.3 the last assumption is the 0.75 reduction factor that the code places on the bending stiffness of the section. This reduction factor is intended to account for variations in workmanship and reinforcing placement. This assumption is analyzed by varying both the thickness of the panel and the location of reinforcement to the worst case as allowed by the *Standard Specification for Tolerances for Concrete Construction & Materials* (ACI 117-90). The tolerance for reinforcing placement is plus or minus 3/8" according to ACI 117-90 Section 2.2.2. The tolerances for wall thicknesses are plus 3/8" and minus 1/4" according to ACI 117-90 Section 4.4.1. For each panel with 170 mph wind, 1/4" was subtracted from the panel thickness and the depth of steel was placed 3/8" closer. This presents the worst case scenario for workmanship and reinforcing placement for all panel configurations. The results obtained from this test are given in Table 6.6.

Varying Reinforcement Depth Results			
Opening Size	I_{cr} Actual (in^4)	I_{cr} with Tolerance (in^4)	Percent Difference
8' x 7'	1510	1293	16.8%
12' x 12'	1681	1501	12.0%
16' x 16'	1573	1395	12.7%
20' x 20'	1726	1574	9.7%

Table 6.6: Evaluation of Bending Stiffness Reduction Factor Results

In Table 6.6 I_{cr} *Actual* is the cracked moment of inertia calculated assuming the panel thickness and reinforcing placement is installed exactly as specified, The I_{cr} *with Tolerance* is the cracked moment of inertia calculated with the worst case allowed by ACI 117-90 as discussed. The data presented in Table 6.6 shows that the code is at most 8.2% conservative. However, it should be noted that in Schwabauer's research the cracked moment of inertia was 25% less than the actual moment of inertia (Schwabauer, 2010). As such, the bending stiffness reduction factor is proven to cover a wide variety of panels. Further research could include performing a similar analysis and a wider variety of panels to determine if the assumption is sufficient.

6.2.4 Effect of Axial Load on the Stiffness of the Member

The effect of axial load on the stiffness of the member was not tested in this research because of the tools available for the conducted research. As stated in Section 5.4.3 when defining the panel section in SAP 2000, the extent to which the concrete is cracked must be input before computing the results. As such, the depth of concrete was input after the effect of axial load on the stiffness of the member assumption had already been used. Other finite element analysis programs that may have the ability to allow the concrete two crack within the constraints of the software include (STRUCTURE, 2010):

1. ADAPT-Floor pro 2010
2. Structural Modeler V8i
3. CADRE Pro
4. ETABS
5. SAFE
6. Perform-3D
7. GT STRUDL
8. IES Visual Analysis 8.0

6.3 Conclusion

Building upon the works of Bartels and Schwabauer this report has given further investigation to the design process described by ACI 551 in accordance with the ACI 318-11 Section 14.8 *Alternative Design of Slender Walls*. The research conducted investigates how openings at the base of tilt-up panels affect the design of tilt-up concrete panels and gives a comparison of designing tilt-up concrete panels with Section 14.8 and Section 10.10 *Slenderness Effects in Compression Members*. In addition, the research has investigated the validity of the four assumptions made by the ACI 318-11 Section 14.8.

Using the design procedures of ACI 551 the panel thickness and area of reinforcing is directly proportional to the size of the opening and the applied loading. Primarily design of tilt-up panels with large loads and large opening is controlled by the tension controlled requirement of Section 14.8.2.3. As the opening becomes larger and the wind load increases the amount of concrete being compressed increases. When the induced moment becomes too large the section becomes compression controlled; thus, requiring the panel thickness to increase.

The four assumptions made by the ACI 318-11 Section 14.8 have been determined to be appropriate. First, the bending stiffness in the panel legs varies only slightly across the length of the leg. Verifying that the two-way bending assumption is relatively accurate in predicting overall behavior of the panel. Secondly, the constant bending stiffness assumption is at most 24% conservative for small openings at the base of the panel. However, as the covers more of the gross area of the panel and extends closer to the panel mid-height the constant bending stiffness assumption closely models the behavior tilt-up panels. The effect of axial loads on the stiffness of the member was not able to be determined because of technological constraints. Lastly this reports shows that a 17% reduction in the bending stiffness of the panel, making the 0.75 bending stiffness reduction factor used by the ACI 318-11 conservative.

As a structural design engineer, safety of the building occupants is the first priority. Tilt-up construction has become increasingly popular due to the speed and economic advantages of the construction process. With the aid of ACI 318-11 Section 14.8 *Alternative Design of Slender Walls* tilt-panels can be designed to ensure the safety of the building occupants while maintaining the positive attributes of tilt-up construction.

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Appendix A - Load Derivation from ASCE 7-10

Building parameters are based off the Tilt-Up building from 2006 IBC Structural/ Seismic Design Manual with modified palan dimensions. All gravity loads are based on a 24'-0" X24'-0" bay, where the joists framing into tilt-up panel are at 4'-0" O.C.

Number of joists framing into panel = 6

Gravity Loads			Reference
Roof			
Dead Load			
Bituminous Roofing =	1.5	psf	
6" Rigid Insulation =	9	psf	
1.5 22 Gauge Deck =	2	psf	
Joists =	2.5	psf	
M/E/P =	4	psf	
Total =	19	psf	
Use Dead Load =	20	psf	
Roof Live Load =	20	psf	ASCE 7-10 Table 4-1
(could be reduced per ASCE 7 Section 4.9)			
Tributary Area of Joists =	96	sf	
Roof Axial Dead Load/Joist =	0.96	k	
Roof Axial Live Load/Joists =	0.96	k	
Total Roof Axial Dead Load =	**5.76**	**k**	
Total Roof Axial Live Load =	**5.76**	**k**	

Snow Loads			Reference
Ground Snow Load, p_g =	20	psf	ASCE 7-10 Figure 7-1
Flat Roof Snow Load, P_f =	14	psf	ASCE 7-10 Section C6.5.6 (Exposure Category "C")
$p_f = 0.7C_eC_tIp_g$			(7-2)
Exposure Factor, C_e =	1.0		Table 7-2
Thermal Factor, C_t =	1.0		Table 7-3
Importance Factor, I =	1.0		Table 7-4
This study assumes the roof has a slope less than or equal to 5 degrees			
Minimum Snow Load			ASCE 7-10 Section 7.3
Pg< 20 psf			
$P_{fmin} = Ip_g$ =	20	psf	
For Locations where Pg is 20 psf or less, but not zero, shall have a 5 psf rain-on-snow surcharge			
Rain-on-Snow			
C_s =	1.0		
p_s =	19	psf	
Snow Load =	20	psf	
Axial Balanced Snow Load/Joist=	0.96	k	
			ASCE 7-10 Section 7.7
Check Snow Drift - Transverse Direction only			
l_u = Length of roof upwind of draft=	168	ft	
Snow Density			
$\gamma = 0.13p_g + 14 < 30$ =	17	pcf	
Height of Balance Snow Load			
$h_b = p_f/\gamma$ =	1.2	ft	
Clear height from h_b to T.O.P.			
h_c = Height of parapet - h_b =	0.8	ft	
Drift Loads Apply			
Height of Snow Drift			ASCE 7-10 Figure 7-9
h_d =	3.04	ft	
Max Intensity of Drift Surcharge			
$p_d = h_c\gamma$ =	13	psf	
Width of Snow Drift			
$w = 4h_d$ =	12.16	ft	

Axial Drift Snow Load / Joist =	0.321	k
Total Axial Snow Load =	7.69	k
(includes balanced Snow load and Drift)		

Wind Load	Reference
Velocity Pressure, q_z	
$q_z=0.00256K_zK_{zt}K_dV^2$	ASCE 7-10 Equation 30.3-1
K_d= 0.85	ASCE 7-10 Table 26.6-1
K_z= 1 For exposure C	ASCE 7-10 Table 30.3-1
K_{zt}= 1	ASCE 7-10 Section 26.8.2
V= 115 MPH	ASCE 7-10 Figure 26.5-1A
130 MPH	ASCE 7-10 Figure 26.5-1A
150 MPH	ASCE 7-10 Figure 26.5-1A
170 MPH	ASCE 7-10 Figure 26.5-1A
q_{z115}= 28.78 psf	ASCE 7-10 Equation 30.3-1
q_{z130}= 36.77 psf	ASCE 7-10 Equation 30.3-1
q_{z150}= 48.96 psf	ASCE 7-10 Equation 30.3-1
q_{z170}= 62.89 psf	ASCE 7-10 Equation 30.3-1
Design Wind Pressure	
For mean roof height < 60ft Section 30.4.2 shall be used	
$p = q_h[(GC_p) - (GC_{pi})]$	ASCE 7-10 Equation 30.4-1
Effective wind area is greater than 500 sf therfore	
GC_p= 0.7 and -0.8	ASCE 7-10 Figure 30.4-1
For partially enclosed buildings	
GC_{pi}= 0.55 and -0.55	ASCE 7-10 Table 26.11-1
Design wind pressure for 115 MPH	
p = 28.49[0.7 - 0.55] = 4.317 psf	
p = 28.49[0.7 + 0.55] = 35.972 psf	
p = 28.49[-0.8 - 0.55] = -38.850 psf	
p = 28.49[-0.8 + 0.55] = -7.194 psf	
Use governing wind pressure, W = -38.850 psf	

Wind Speed (MPH)	Wind Pressure (psf)
115	-38.85
130	-49.65
150	-66.10
170	-84.90

Appendix B - Moment Magnifier Derivation

This appendix gives a detail derivation of the moment magnifier utilized by ACI 318-08 section 14.8. For examples of comparison between the iterative procedure and the moment magnification method see Bartels 2010.

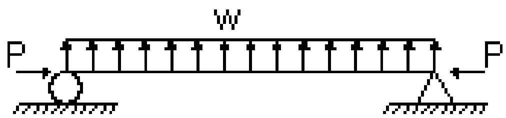

Figure B.1: Section of a Simply Supported Member

$$V^{IV} + K^2 V'' = \frac{q(x)}{EI}$$

$$K^2 = \frac{P}{EI}$$

$$V^C = A\sin(kx) + B\cos(kx) + C_X + D$$

$$V_P = Ex^2$$

$$K^2(2E) = \frac{W}{EI} \rightarrow E = \frac{W}{2EIK^2}$$

$$So, V(x) = A\sin(kx) + B\cos(kx) + C_X + \frac{W}{2EIK^2} X^2$$

<u>Applying Boundary Conditions</u>

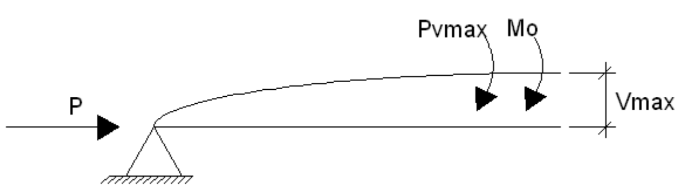

Figure B.2: Moment Magnification Figure

$$V(0)=V''(0)=0$$
$$B+D$$
$$-Bk^2+\frac{W}{EIk^2}=0\Rightarrow B=\frac{W}{EIk^4}\Rightarrow D=-\frac{W}{EIk^4}$$
$$V(L)=V''(C)=0$$
$$A\sin(kL)+\frac{W}{EIk^4}Cos(KL)+CL-\frac{W}{EIk^4}+\frac{W}{EIk^4}L^2=0$$
$$A=\frac{W}{EIk^4}\left\{\frac{1}{\sin(KL)}-\frac{\cos(KL)}{\sin(KL)}\right\}=\frac{W}{EIk^4}\tan\left(\frac{KL}{2}\right)$$
$$C=-\frac{WL}{2EIk^2}$$
$$V(x)=\frac{W}{EIk^4}\left[\tan\left(\frac{KL}{2}\right)\sin(Kx)+\cos(Kx)-1\right]-\frac{WL}{2EIk^2}(L-x)$$
$$With: u=\frac{KL}{2}$$
$$V(x)=\frac{WL^4}{16EIu^4}\left[\tan(u)\sin\left(\frac{2ux}{L}\right)+\cos\left(\frac{2ux}{L}\right)-1\right]-\frac{WL^2x}{8EIu^2}(L-x)$$
$$V''(x)=\frac{W}{EIk^4}\left[-\tan\left(\frac{KL}{2}\right)k^2\sin(Kx)-K^2\cos(Kx)\right]+\frac{W}{EIK^2}$$

$$M(x)=EIV''(x)=\frac{-WL^2}{4EIu^2}\left[\tan u\sin\left(\frac{2ux}{L}\right)+\cos\left(\frac{2ux}{L}\right)-1\right]$$
$$V_{\max}=V\left(\frac{L}{2}\right)=\frac{WL^4}{16EIu^4}\left[\tan(u)\sin(u)+\cos(u)-1\right]-\frac{WL^4}{32EIu^2}$$
$$V_{\max}=\frac{5WL^4}{384EI}\left[\frac{24(\sec(u)-1)-12u^2}{5u^4}\right]$$
$$M_{\max}=M_0+PV_{\max}$$
$$M_{\max}=-\frac{WL^2}{8}-\frac{P(5)WL^4}{384EI}\left[\frac{12(5)(\sec(u)-1)-u^2}{(5)u^4}\right]$$
$$M_{\max}=-\frac{WL^2}{8}\left[\frac{2(\sec(u)-1)}{u^2}\right]$$

Using power series expansion

$$M_{\max} = M_0 \frac{2(1+0.5u^2+0.208u^4+0.0847u^6+0.034u^8+...-1)}{u^2}$$

$$M_{\max} = M_0 \left[1+0.461u^2+0.169u^4+0.0687u^6+...\right]$$

$$M_{\max} = M_0 \left[1+1.028\left(\frac{P}{P_E}\right)+1.031\left(\frac{P}{P_E}\right)^2+1.032\left(\frac{P}{P_E}\right)^3\right]$$

$$M_{\max} = M_0 \left[1+1.028\frac{P}{P_E}\left\{1+\frac{P}{P_E}+\left(\frac{P}{P_E}\right)^2+...\right\}\right]$$

$$M_{\max} = M_0 \left[1+1.028\frac{P}{P_E}\left(\frac{1}{1-\frac{P}{P_E}}\right)\right] = M_0 \left[\frac{1-\frac{P}{P_E}+1.028\frac{P}{P_E}}{1-\frac{P}{P_E}}\right]$$

$$M_{\max} = M_0 \left[\frac{1+0.028\frac{P}{P_E}}{1-\frac{P}{P_E}}\right] \approx M_0 \left[\frac{1}{1-\frac{P}{P_E}}\right]$$

$$M_{\max} = M_0 \left[\frac{1}{1-\frac{P}{P_E}}\right]$$

Appendix C - Panel Results from ACI 318-11 Section 14.8

Alternative Design of Slender Walls

Wind Speed: 115 mph (39 psf)										
Panel A 8' x 7' Opening										
Th(in)	Steel	A_{st}(in^2)	M_{cr}(k-ft)	Δ_s(in)	$\Delta_{sallowable}$(in)					
7.25	2 layers (8) #6 bars	3.53	33.24	1.75	2.56					
Load Case	P_{um}(k)	P_{um}/A_g(psi)	A_{se}(in^2)	a(in)	c/d	I_{cr}(in^4)	K_b(k)	ΦM_n(k-ft)	M_u(k-ft)	Δ_u(in)
1.2D + 1.6Lr + 0.5W	31.6	45.3	3.86	0.709	0.142	807.3	189.5	95.9	40.6	3.43
1.2D +0.5Lr+1.0W	28.4	40.8	3.83	0.703	0.141	802.3	188.3	95.1	76.0	6.46
0.9D + 1.0W	20.21	29.0	3.74	0.688	0.138	789.5	185.3	93.1	70.5	6.09
Panel B 12' x 12' Opening										
Th(in)	Steel	A_{st}(in^2)	M_{cr}(k-ft)	Δ_s(in)	$\Delta_{sallowable}$(in)					
7.25	2 layers (8) #6 bars	3.53	24.93	2.32	2.56					
Load Case	P_{um}(k)	P_{um}/A_g(psi)	A_{se}(in^2)	a(in)	c/d	I_{cr}(in^4)	K_b(k)	ΦM_n(k-ft)	M_u(k-ft)	Δ_u(in)
1.2D + 1.6Lr + 0.5W	31.6	60.4	3.86	0.946	0.189	737.1	173.0	93.8	41.7	3.86
1.2D +0.5Lr+1.0W	28.4	54.4	3.83	0.938	0.188	733.1	172.0	93.1	77.8	7.24
0.9D + 1.0W	20.21	38.7	3.74	0.917	0.184	722.6	169.6	91.2	71.6	6.76
Panel C 16' x 16' Opening										
Th(in)	Steel	A_{st}(in^2)	M_{cr}(k-ft)	Δ_s(in)	$\Delta_{sallowable}$(in)					
7.25	2 layers (14) #5 bars	4.30	16.62	2.49	2.56					
Load Case	P_{um}(k)	P_{um}/A_g(psi)	A_{se}(in^2)	a(in)	c/d	I_{cr}(in^4)	K_b(k)	ΦM_n(k-ft)	M_u(k-ft)	Δ_u(in)
1.2D + 1.6Lr + 0.5W	31.6	90.7	4.60	1.692	0.322	778.4	182.7	110.6	41.0	3.59
1.2D +0.5Lr+1.0W	28.4	81.6	4.57	1.681	0.320	775.6	182.0	110.0	76.7	6.74
0.9D + 1.0W	20.21	58.1	4.49	1.652	0.314	768.4	180.4	108.4	70.8	6.28
Panel D 20' x 20' Opening										
Th(in)	Steel	A_{st}(in^2)	M_{cr}(k-ft)	Δ_s(in)	$\Delta_{sallowable}$(in)					
9.25	2 layers (6) #6 bars	2.65	13.53	2.49	2.56					
Load Case	P_{um}(k)	P_{um}/A_g(psi)	A_{se}(in^2)	a(in)	c/d	I_{cr}(in^4)	K_b(k)	ΦM_n(k-ft)	M_u(k-ft)	Δ_u(in)
1.2D + 1.6Lr + 0.5W	32.5	146.3	2.97	2.183	0.326	808.1	189.7	90.6	41.3	3.49
1.2D +0.5Lr+1.0W	29.3	132.1	2.94	2.160	0.323	803.6	188.6	89.8	76.9	6.52
0.9D + 1.0W	20.91	94.2	2.86	2.100	0.314	791.6	185.8	87.7	71.0	6.11

Wind Speed: 130 mph (50 psf)

Panel A 8' x 7' Opening

Th(in)	Steel	A_{st}(in^2)	M_{cr}(k-ft)	Δ_s(in)	$\Delta_{sallowable}$(in)
7.25	2 layers (10) #6 bars	4.42	33.24	2.20	2.56

Load Case	P_{um}(k)	P_{um}/A_g(psi)	A_{se}(in^2)	a(in)	c/d	I_{cr}(in^4)	K_b(k)	ΦM_n(k-ft)	M_u(k-ft)	Δ_u(in)
1.2D + 1.6Lr + 0.5W	31.6	45.3	4.74	0.872	0.175	931.7	218.7	116.1	49.3	3.61
1.2D +0.5Lr+1.0W	28.4	40.8	4.71	0.866	0.173	927.4	217.7	115.3	93.6	6.88
0.9D + 1.0W	20.21	29.0	4.63	0.850	0.170	916.2	215.0	113.4	87.8	6.53

Panel B 12' x 12' Opening

Th(in)	Steel	A_{st}(in^2)	M_{cr}(k-ft)	Δ_s(in)	$\Delta_{sallowable}$(in)
7.25	2 layers (10) #6 bars	4.42	24.93	2.49	2.56

Load Case	P_{um}(k)	P_{um}/A_g(psi)	A_{se}(in^2)	a(in)	c/d	I_{cr}(in^4)	K_b(k)	ΦM_n(k-ft)	M_u(k-ft)	Δ_u(in)
1.2D + 1.6Lr + 0.5W	31.6	60.4	4.73	1.159	0.223	923.3	216.7	118.0	49.5	3.65
1.2D +0.5Lr+1.0W	28.4	54.4	4.70	1.151	0.221	919.7	215.8	117.3	93.7	6.95
0.9D + 1.0W	20.21	38.7	4.62	1.132	0.217	910.1	213.6	115.5	87.9	6.58

Panel C 16' x 16' Opening

Th(in)	Steel	A_{st}(in^2)	M_{cr}(k-ft)	Δ_s(in)	$\Delta_{sallowable}$(in)
9.25	2 layers (6) #6 bars	2.65	27.06	2.11	2.56

Load Case	P_{um}(k)	P_{um}/A_g(psi)	A_{se}(in^2)	a(in)	c/d	I_{cr}(in^4)	K_b(k)	ΦM_n(k-ft)	M_u(k-ft)	Δ_u(in)
1.2D + 1.6Lr + 0.5W	38.0	85.7	3.02	1.111	0.166	1084.7	254.6	99.6	50.2	3.15
1.2D +0.5Lr+1.0W	34.9	78.5	2.99	1.100	0.164	1077.0	252.8	98.6	95.0	6.01
0.9D + 1.0W	25.07	56.5	2.90	1.065	0.159	1053.2	247.2	95.7	89.0	5.76

Panel D 20' x 20' Opening

Th(in)	Steel	A_{st}(in^2)	M_{cr}(k-ft)	Δ_s(in)	$\Delta_{sallowable}$(in)
11.25	2 layers (10) #4 bars	1.96	20.01	1.94	2.56

Load Case	P_{um}(k)	P_{um}/A_g(psi)	A_{se}(in^2)	a(in)	c/d	I_{cr}(in^4)	K_b(k)	ΦM_n(k-ft)	M_u(k-ft)	Δ_u(in)
1.2D + 1.6Lr + 0.5W	37.8	139.9	2.32	1.704	0.200	1256.2	294.8	95.4	48.9	2.65
1.2D +0.5Lr+1.0W	34.6	128.1	2.29	1.682	0.198	1246.0	292.4	94.3	92.3	5.05
0.9D + 1.0W	24.87	92.1	2.20	1.615	0.190	1214.2	285.0	90.9	87.2	4.89

Wind Speed: 150 mph (66 psf)

Panel A 8' x 7' Opening

Th(in)	Steel	$A_{st}(in^2)$	M_{cr}(k-ft)	Δ_s(in)	$\Delta_{sallowable}$(in)
7.25	2 layers (14) #6 bars	6.19	33.24	2.20	2.56

Load Case	P_{um}(k)	P_{um}/A_g(psi)	$A_{se}(in^2)$	a(in)	c/d	$I_{cr}(in^4)$	K_b(k)	ΦM_n(k-ft)	M_u(k-ft)	Δ_u(in)
1.2D + 1.6Lr + 0.5W	31.6	45.3	6.50	1.194	0.229	1253.0	294.1	161.6	61.2	3.33
1.2D +0.5Lr+1.0W	28.4	40.8	6.47	1.188	0.228	1249.5	293.2	160.9	117.8	6.43
0.9D + 1.0W	20.21	29.0	6.38	1.174	0.225	1240.2	291.1	159.1	112.5	6.18

Panel B 12' x 12' Opening

Th(in)	Steel	$A_{st}(in^2)$	M_{cr}(k-ft)	Δ_s(in)	$\Delta_{sallowable}$(in)
9.25	2 layers (8) #6 bars	3.53	40.59	1.77	2.56

Load Case	P_{um}(k)	P_{um}/A_g(psi)	$A_{se}(in^2)$	a(in)	c/d	$I_{cr}(in^4)$	K_b(k)	ΦM_n(k-ft)	M_u(k-ft)	Δ_u(in)
1.2D + 1.6Lr + 0.5W	38.0	57.1	3.90	0.955	0.138	1570.2	368.5	134.0	61.3	2.66
1.2D +0.5Lr+1.0W	34.9	52.4	3.87	0.947	0.137	1561.3	366.4	133.1	117.7	5.14
0.9D + 1.0W	25.07	37.6	3.77	0.925	0.134	1533.7	359.9	130.1	112.7	5.01

Panel C 16' x 16' Opening

Th(in)	Steel	$A_{st}(in^2)$	M_{cr}(k-ft)	Δ_s(in)	$\Delta_{sallowable}$(in)
9.25	2 layers (8) #6 bars	3.53	27.06	2.48	2.56

Load Case	P_{um}(k)	P_{um}/A_g(psi)	$A_{se}(in^2)$	a(in)	c/d	$I_{cr}(in^4)$	K_b(k)	ΦM_n(k-ft)	M_u(k-ft)	Δ_u(in)
1.2D + 1.6Lr + 0.5W	38.0	85.7	3.91	1.436	0.215	1279.5	300.3	125.8	63.6	3.39
1.2D +0.5Lr+1.0W	34.9	78.5	3.88	1.425	0.213	1273.3	298.8	124.9	121.7	6.52
0.9D + 1.0W	25.07	56.5	3.78	1.390	0.208	1253.9	294.3	122.1	115.3	6.27

Panel D 20' x 20' Opening

Th(in)	Steel	$A_{st}(in^2)$	M_{cr}(k-ft)	Δ_s(in)	$\Delta_{sallowable}$(in)
11.25	2 layers (6) #6 bars	2.65	20.01	2.18	2.56

Load Case	P_{um}(k)	P_{um}/A_g(psi)	$A_{se}(in^2)$	a(in)	c/d	$I_{cr}(in^4)$	K_b(k)	ΦM_n(k-ft)	M_u(k-ft)	Δ_u(in)
1.2D + 1.6Lr + 0.5W	37.8	139.9	3.00	2.206	0.256	1508.2	354.0	121.8	62.0	2.80
1.2D +0.5Lr+1.0W	34.6	128.1	2.97	2.185	0.254	1500.0	352.0	120.8	118.5	5.39
0.9D + 1.0W	24.87	92.1	2.88	2.118	0.246	1474.0	346.0	117.5	113.1	5.23

Wind Speed: 170 mph (85 psf)

Panel A 8' x 7' Opening

Th(in)	Steel	$A_{st}(in^2)$	M_{cr}(k-ft)	Δ_s(in)	$\Delta_{sallowable}$(in)
7.25	2 layers (32) #5 bars	9.82	33.24	2.38	2.56

Load Case	P_{um}(k)	P_{um}/A_g(psi)	$A_{se}(in^2)$	a(in)	c/d	$I_{cr}(in^4)$	K_b(k)	ΦM_n(k-ft)	M_u(k-ft)	Δ_u(in)
1.2D + 1.6Lr + 0.5W	31.6	45.3	10.13	1.861	0.354	1637.8	384.4	239.5	75.2	3.13
1.2D +0.5Lr+1.0W	28.4	40.8	10.09	1.856	0.353	1635.1	383.8	238.9	145.8	6.08
0.9D + 1.0W	20.21	29.0	10.01	1.841	0.350	1628.1	382.1	237.4	140.9	5.90

Panel B 12' x 12' Opening

Th(in)	Steel	$A_{st}(in^2)$	M_{cr}(k-ft)	Δ_s(in)	$\Delta_{sallowable}$(in)
9.25	2 layers (10) #6 bars	4.42	40.59	2.26	2.56

Load Case	P_{um}(k)	P_{um}/A_g(psi)	$A_{se}(in^2)$	a(in)	c/d	$I_{cr}(in^4)$	K_b(k)	ΦM_n(k-ft)	M_u(k-ft)	Δ_u(in)
1.2D + 1.6Lr + 0.5W	38.0	57.1	4.79	1.174	0.175	1688.2	396.2	157.1	77.1	3.11
1.2D +0.5Lr+1.0W	34.9	52.4	4.76	1.166	0.174	1680.9	394.5	156.2	149.2	6.05
0.9D + 1.0W	25.07	37.6	4.66	1.143	0.171	1658.1	389.2	153.3	143.4	5.90

Panel C 16' x 16' Opening

Th(in)	Steel	$A_{st}(in^2)$	M_{cr}(k-ft)	Δ_s(in)	$\Delta_{sallowable}$(in)
9.25	2 layers (12) #6 bars	5.30	27.06	2.41	2.56

Load Case	P_{um}(k)	P_{um}/A_g(psi)	$A_{se}(in^2)$	a(in)	c/d	$I_{cr}(in^4)$	K_b(k)	ΦM_n(k-ft)	M_u(k-ft)	Δ_u(in)
1.2D + 1.6Lr + 0.5W	38.0	85.7	5.66	2.082	0.301	1702.5	399.6	180.5	77.0	3.08
1.2D +0.5Lr+1.0W	34.9	78.5	5.63	2.071	0.300	1697.6	398.4	179.7	149.0	5.99
0.9D + 1.0W	25.07	56.5	5.54	2.036	0.295	1682.6	394.9	177.1	143.2	5.80

Panel D 20' x 20' Opening

Th(in)	Steel	$A_{st}(in^2)$	M_{cr}(k-ft)	Δ_s(in)	$\Delta_{sallowable}$(in)
11.25	2 layers of 8 #6 bars	3.53	20.01	2.45	2.56

Load Case	P_{um}(k)	P_{um}/A_g(psi)	$A_{se}(in^2)$	a(in)	c/d	$I_{cr}(in^4)$	K_b(k)	ΦM_n(k-ft)	M_u(k-ft)	Δ_u(in)
1.2D + 1.6Lr + 0.5W	37.8	139.9	3.88	2.856	0.332	1733.4	406.8	152.0	77.1	3.03
1.2D +0.5Lr+1.0W	34.6	128.1	3.85	2.834	0.329	1726.4	405.2	151.0	148.8	5.88
0.9D + 1.0W	24.87	92.1	3.76	2.768	0.322	1704.9	400.1	148.1	143.0	5.72

Appendix D - Cost Analysis Calculations for Panel D

Option A						
Steel	Bar #	quantity	weight/ft	cost $/ton	linear footage	Total cost
Horizontal	4	64	0.668	350	24	$ 299.26
Vertical F	6	32	1.502	350	34	$ 476.63
Vertical T&S	6	15	1.502	350	14	$ 92.00
					Steel=	$ 867.90

	Cubic feet	cubic yards	$/cy	Cost
Concrete	390	14.44	150	$2,166.67

Option A total cost=	$ 3,034.56

Option B						
Steel	Bar #	quantity	weight/ft	cost $/ton	linear footage	Total cost
Horizontal	4	64	0.668	350	24	$ 299.26
Vertical F	6	32	1.502	350	34	$ 476.63
Vertical T&S	6	15	1.502	350	14	$ 92.00
					Steel=	$ 867.90

	Cubic feet	cubic yards	$/cy	Cost
Concrete	250	9.26	180	$1,666.67

Option A total cost=	$ 1,666.67

Option B						
Steel	Bar #	quantity	weight/ft	cost $/ton	linear footage	Total cost
Horizontal	4	64	0.376	350	20	$ 140.37
Ties	3	43	0.376	350	5.54	$ 26.13
Vertical F	6	36	1.502	350	34	$ 536.21
Vertical T&S	6	15	1.502	350	14	$ 92.00
					Steel=	$ 794.72

	Cubic feet	cubic yards	$/cy	Cost
Concrete	320.6667	11.88	150	$1,781.48

Option A total cost=	$ 2,576.20

Cost of Panel with 20' x 20' Opening		
Configuration	Panel Cost	Total Bldg Cost
Option A	$ 3,034.56	$ 72,829.51
Option B	$ 1,666.67	$ 40,000.00
Option C	$ 2,576.20	$ 61,828.78

*Total building Cost assumes 24 similar panels are used on north and south side of the building from figure 2.3

Appendix E - Introduction to Finite Element Methods

This chapter gives a brief introduction to the finite element method (FEM). The FEM is a structural analysis technique in which a structural system is divided into a finite number elements to examine the structure by components. The FEM can incorporate various different types of structural conditions including: static analysis, dynamic analysis, linear elastic analysis, geometric nonlinearities and materially nonlinearities. This chapter discusses the static analysis process only.

The FEM is based upon a structural analysis technique called matrix structural analysis. Matrix structural analysis is a process in which the stiffness of a structural system is assembled into a mathematical representation of the system known as a stiffness matrix. The stiffness matrix is combined with a force vector and a Degree of Freedom (DOF) vector to compose the entire structural system. Upon assemblage of the stiffness matrix and associated vectors, by way of linear matrix algebra various desirable quantities can be determined including: displacements, rotations, and support reactions. The concept of FEM is a subdivision of the mathematical model into non-overlapping components of simple geometry called finite elements often referred to as elements. The response of each element is express in terms of a finite number of degrees of freedom. The entire response of the mathematical model is considered to be approximated by that of the model obtained by assembling the collection of all individual elements. The larger quantity of elements utilized the more accurate the model becomes. For a detailed description of the FEM as well as matrix structural analysis, refer to Schwabauer 2010.

E.1 Discretization

The discretization process embedded within the FEM is the process by which the engineer divides a model into a finite number of elements. Figure E.1 gives a relatively simple example of the discretization process. The statically indeterminate frame is unsolvable with conventional analysis methods. However, once the frame is divided to three frame members it becomes simple to solve. The point at which the structural elements are divided are called nodes. To analyze the structure the applied loads are transferred across each node. Using this discretization of the frame and nodal load transfers the structural analysis becomes more manageable.

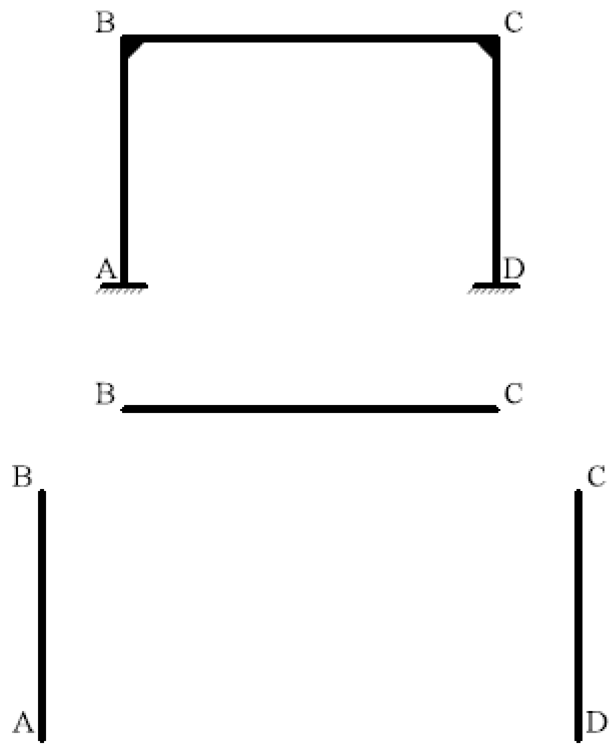

Figure E.1: Discretization of a Frame (Schwabauer, 2010)

Typically finite element methods become necessary as structures become larger and more complex. Using finite element methods a large structure can be divided into smaller pieces allowing for simpler analysis procedures to be utilized.

E.2 Idealization

The Idealization process of the FEM is the physical structure being resolved into a mathematical model. The word model typically has been associated with a scaled representation of another object or structure. However, when discussing a model in terms of the FEM, the word model has a much stricter definition.

"A model is a symbolic device built to simulate and predict aspects of behavior of a system." (Felippa, 2004).

The model is built by assembling elements that have the same aspects of behavior that the physical element has. Each element in a finite element analysis can be expressed in terms of its nodal displacement vector ***U***, applied force vector ***P*** and stiffness matrix ***K***. The mathematical relationship is shown in Equation E.1-1.

$$\boldsymbol{P} = \boldsymbol{KU} \qquad \text{Equation E.1-1}$$

The simplest element utilized in FEM is the truss element shown in Figure E.2.

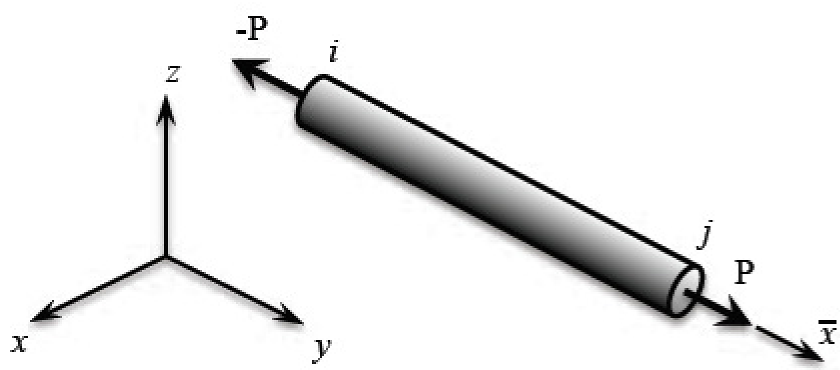

Figure E.1: Truss Element (Schwabauer, 2010)

The truss element has two axial degrees of freedom. This means that the element only has the ability to resist loading the axial direction. Each type of element has a mathematical representation for the degrees of freedom that it can resist. This mathematical representation is referred to as its stiffness matrix. The stiffness matrix for a truss element is derived as follows.

$$\overline{P}_{\bar{x}i} = \frac{AE}{L}\overline{u}_i - \frac{AE}{L}\overline{u}_j$$

$$\overline{P}_{\bar{x}j} = -\frac{AE}{L}\overline{u}_i + \frac{AE}{L}\overline{u}_j$$

Expressing these two equations in matrix terms yields,

$$\begin{Bmatrix} \overline{P}_{xi} \\ \overline{P}_{xj} \end{Bmatrix} = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} \overline{u}_i \\ \overline{u}_j \end{Bmatrix}$$

In the above matrix P is the applied force with the subscript specifying which node the load is applied upon. The vector that is composed of both applied forces, P_{xi} and P_{xj}, is referred to as the force vector. The variable u is defined as the displacement with the subscript identifying which node it applies to. The vector composed of both displacements is known as the displacement vector.

There are numerous element types to model different types of structural members. An Euler Bernoulli plane beam element for example has four degrees of freedom; the ability to resist bending moments as well as shear. The stiffness matrix associated with a beam element is,

$$[k] = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & -\frac{6EI}{L} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$

If the element used has more degrees of freedom the stiffness matrix will have a stiffness term associated that models the resisting capacity of the element for associated degree of freedom.

As the designer it is important to understand the physical system that is being modeled as well as the elements that is being used. For example, if an engineer is attempting to model a flat plate subjected to transverse loading they must choose the correct mathematical model associated with the plate's actual behavior. Four examples of possible mathematical models are listed:

1. A very thin plate modeled by Von Karman's Coupled membrane-bending theory.
2. A thin plate model based on Kirchhoff's Plate theory.

3. A moderately thick plate described by the Mindlin-Reissner plate theory.
4. A very thick plate idealized by three-dimensional elasticity.

It is extremely important that the engineer of record understands the range of applicability of each model type as well as the advantages and disadvantages of each.

E.3 Assembly

Upon the decision of which element types to base the model upon, the engineer must assemble the global stiffness matrix. The global stiffness matrix is defined as the stiffness matrix for the entire structural system. This is obtained by adding stiffness terms of each element at all defined nodes. When the stiffness terms of every element conjoining into every node have been added the global stiffness matrix is complete. A beam with two spans illustrates a simple example of this.

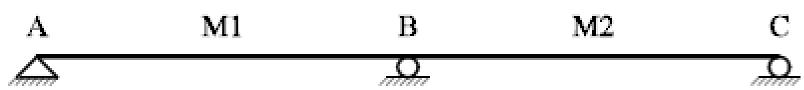

Figure E.3: Two Span Beam (Schwabauer, 2010)

The global stiffness matrix is assembled,

$$[k] = \begin{array}{c} \begin{matrix} v_1 & \theta_1 & v_2 & \theta_2 & v_3 & \theta_3 \end{matrix} \\ \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & 0 \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} & 0 & 0 \\ -\frac{12EI}{L^3}0 & -\frac{6EI}{L^2} & \frac{12EI}{L^3}+\frac{12EI}{L^3} & \frac{6EI}{L^2}-\frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & \frac{6EI}{L^2}-\frac{6EI}{L^2} & \frac{4EI}{L}+\frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ 0 & 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \end{array}$$

Simplifying terms,

$$[k] = \begin{array}{c} \begin{matrix} v_1 & \theta_1 & v_2 & \theta_2 & v_3 & \theta_3 \end{matrix} \\ \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & 0 \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} & 0 & 0 \\ -\frac{12EI}{L^3}0 & -\frac{6EI}{L^2} & \frac{24EI}{L^3} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & \frac{8EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ 0 & 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \end{array}$$

The above stiffness matrix represents the stiffness matrix for the entire structure. As described in section E.1.2, the force vector, displacement vector and stiffness matrix are related by equation E.1-1. The only difference is that both vectors will have six terms in them.

$$\boldsymbol{P = KU} \qquad \text{Equation E.1-1}$$

Upon solving equation E.1-1 numerous desirable quantities can be determined including: displacements, rotations, and support reactions. By extension additional quantities can be determined including: stresses and strains. As the structural system gets more complex, the stiffness matrix and the associated force and displacement vectors will get more complex. However, the mathematics of the system do not change. Even the largest most complex structural systems will utilize the same process described above. However, because the complexity of the matrix increases the statistical probability of human error; computer methods are often utilized to assemble the stiffness matrices of large structural systems.

E.4 Boundary Conditions

In FEM the term boundary conditions refer to the fixities of the physical system as well as the applied loading. The system fixities are implemented in the displacement vector ***U***, the applied loading is seen in the applied force vector ***P***. As with the previous steps, it is extremely important that the designer engineer understands the support conditions of the physical system and how to extrapolate them into a mathematical model. A pin connection for example, will restrain movement of the system in two directions, but does not restrain any rotation. For example the beam shown in Figure E.3 has the following displacement vector ***U***.

$$\{U\} = \begin{Bmatrix} V_1 \\ U_1 \\ \theta_1 \\ V_2 \\ U_2 \\ \theta_2 \\ V_3 \\ U_3 \\ \theta_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \theta_1 \\ 0 \\ U_2 \\ \theta_2 \\ 0 \\ U_3 \\ \theta_3 \end{Bmatrix}$$

Where V is the displacement in the y direction, U is the displacement in the x direction and θ is the rotation about the z-axis. Notice that each node has resistance to movement in the y direction, to model this in the above vector the V term at every node is set equal to zero. The pin connection at node one also restrains movement in the x direction which corresponds to U_1 being set equal to zero. The rollers located at nodes two and three however do not resist loading in the axial direction thus U_1 and U_2 are free variables. Similar to U_1 and U_2, the degrees of freedom that are not explicitly defined as boundary conditions are variable and must be determined when solving. Similarly, if a fixed connection is being utilized the rotational degree of freedom would be set equal to zero. It is extremely important for the design engineer to understand boundary conditions, the displacement vector and understand how to utilize them to model the structural system under analysis.

The applied force vector utilizes the same rational. The applied force at any given node is input in the correct corresponding location in the force vector. An example loading condition is applied to the two span beam shown in Figure E.4.

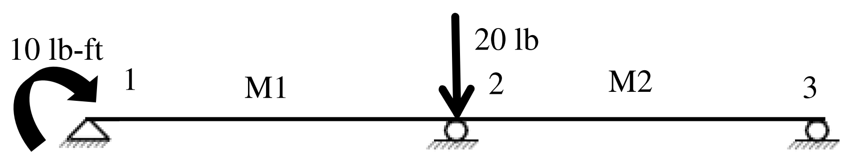

Figure E.4: Two Span Beam with Applied Loading

The applied force vector is resolved as follows,

$$\{P\} = \begin{Bmatrix} P_{y1} \\ P_{x1} \\ M_1 \\ P_{y2} \\ P_{x2} \\ M_2 \\ P_{y3} \\ P_{x3} \\ M_3 \end{Bmatrix} = \begin{Bmatrix} P_{y1} \\ P_{x1} \\ -10lb-ft \\ -20lb \\ P_{x2} \\ M_2 \\ P_{y3} \\ P_{x3} \\ M_3 \end{Bmatrix}$$

At each node, P_y is the applied load in the y direction, P_x is the applied load in the x direction and M is the applied moment about the z axis. The *-10 lb-ft* moment that is applied at node one is input as M_1, while the *-20 lb* load that is applied at node to is input as P_{y2}. The variables that are not defined as boundary conditions will translate to support reactions upon solving. For example the variables P_{y1} and P_{x1} will yield the support reactions at node one in the x and y directions respectively. The engineer must exercise caution to ensure that the correct loading values are input for the correct location.

Establishing the boundary conditions of the physical system is an extremely important process. If the designer does not establish proper boundary conditions, the results will produce extraneous solutions. As with the stiffness matrix described in Section E.1.3 both the force vector and the displacement vector become more complex proportional to the complexity of the physical system being modeled. Because of this computer methods are often utilized to assemble both vectors.

E.5 Solve

With all of the previous steps performed correctly, structural analysis can be performed on the system in order to extrapolate the desired results. Similar to most structural analysis problems finite element problems can be solved with simple algebra hand calculations. The resultant matrix along with the associated force and displacement vectors can be resolved into a system of equations. That system of equation can be solved by Gaussian Elimination. Gaussian Elimination is the process by which equations are manipulated with simple arithmetic until one variable is isolated. Once a variable is isolated it can be back substituted to determine the values of the remaining variables.

While Gaussian Elimination is an excellent mathematical tool, complex structural system can include 1000s upon 1000s of degrees of freedom, making the solving process extremely cumbersome. Because of this complexity, commercial finite element software packages to handle such systems are available to be used. While the technological advances have made the finite element solving process faster, the underlying mathematical principle imprinted within the software are still the same. Some of the different types of solvers utilized are, skyline solvers, sparse solvers and iterative solvers, for a detailed description of these solvers see (Schwabauer, 2010).

Once the finite element matrix has been solved, various quantities can be determined including: support reactions, nodal displacements and element forces and stresses. Though technology has been extremely useful to aid engineers in solving finite element problems, it is extremely important that they understand the process by which finite element analysis problems are solved to ensure that the computer output is not producing extraneous results.

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